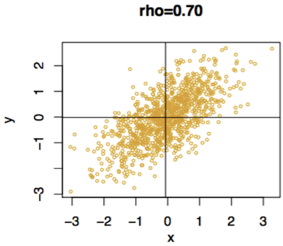
# Covariance and correlation




Please send your comments to: T. Levajković, tijana.levajkovic@tuwien.ac.at

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      * Especially if $X$ and $Y$ are independent, then $Var(X + Y) = Var(X) + Var(Y)$
    - If $X$ and $Y$ are independent, then $Cov(X,Y) = 0$.
      * The converse is not true! If covariance $Cov(X,Y)$ is zero, random variables $X$ and $Y$ might not be independent.

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HW Confirm this result in R.

For computing the covariance use the function `cov`.

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$$\begin{aligned}
\rho(X,Y) &= \phantom{-}1 \qquad \Longleftrightarrow \qquad Y = aX + b \text{ with } a > 0 \\
\rho(X,Y) &= -1 \qquad \Longleftrightarrow \qquad Y = aX + b \text{ with } a < 0
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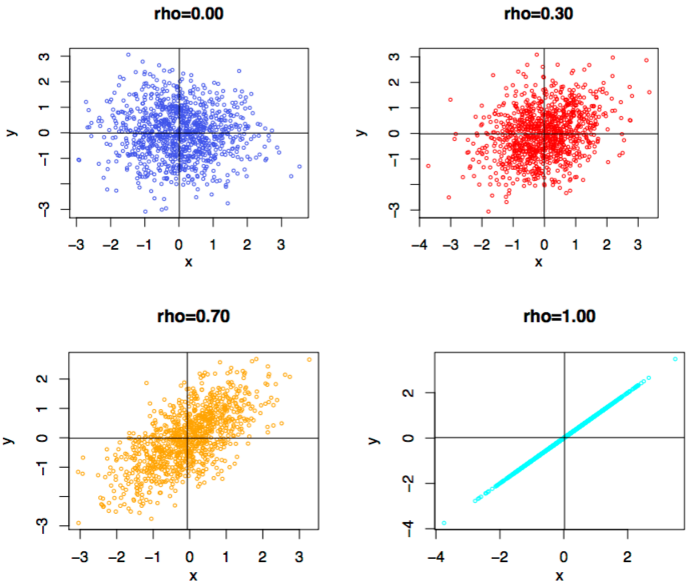

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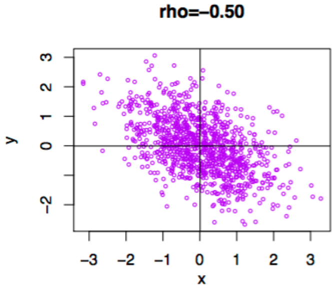


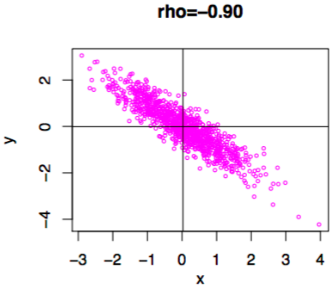

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  - We obtain

$$\begin{aligned} Cov(X, Y) &= Cov(X_1 + X_2, X_2 + X_3) \\ &= Cov(X_1, X_2) + Cov(X_1, X_3) + Cov(X_2, X_2) + Cov(X_2, X_3) \\ &= Cov(X_2, X_2) = Var(X_2) = \frac{1}{4}. \end{aligned}$$

# Examples

HW Let $X$ be a random variable that takes values $-2, -1, 0, 1, 2$, each with probability $\frac{1}{5}$. Let $Y = X^2$. Show that $Cov(X, Y) = 0$, but $X$ and $Y$ are not independent.

HW Let $X_1, \ldots, X_{10}$ be an independent and identically distributed random variables with $\mathcal{N}(3, 12)$. Let

$$Y_1 = \frac{1}{6}\sum_{i=1}^{6} X_i \qquad \text{and} \qquad Y_2 = \frac{1}{4}\sum_{i=7}^{10} X_i$$

(i) What is the distribution of $Y_1 + Y_2$?

(ii) Compute $Cov(2Y_1 - 5, Y_2 + 4)$ and $\rho(Y_1, Y_2)$.

# Examples

HW Let $X \sim bern(p)$.
Let $Y_1$ be the indicator that $X = 1$ and $Y_0$ be the indicator that $X = 0$.
Compute $Cov(Y_0, Y_1) = 0$.

HW Let $Z \sim \mathcal{N}(0, 1)$.
Compute $Cov(Z^2, Z^3)$.

HW Let $T \sim \exp(\lambda)$ and $S = T + 10$.

(i) Does $S$ have an exponential distribution?
Use the pdf of $S$ to answer this question.

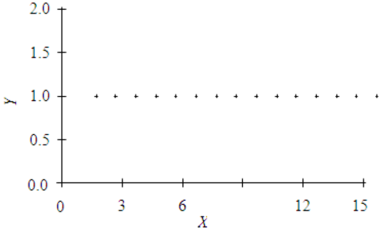
(ii) Compute $\rho(T, S)$.

HW Consider $G \sim U(-3, -1)$, $V \sim U(-1, 1)$ and $W \sim U(1, 3)$ and assume that all are independent. Compute $Cov(W - V, V - G)$.

# Questions

# Multiple-choice questions

(1) A scatterplot of a variable $Y$ versus a variable $X$ produced the scatterplot below. The value of $Y$ for all values of $X$ is exactly 1.0. The correlation coefficient between $Y$ and $X$ is:




a. 1
b. $-1$
c. 0
d. either 1 or $-1$

# Multiple-choice questions

(2) Let $X$ and $Z$ be independent random variables and both $N(0,1)$-distributed. Let

$$Y = \frac{1}{2}(X + Z).$$

Then the correlation between $X$ and $Y$ is

a. smaller than the correlation between $X$ and $Z$
b. equal to the correlation between $X$ and $Z$
c. larger than the correlation between $X$ and $Z$
d. not defined

Next week we start with Statistics

Thank you for your attention!