TECHNISCHE UNIVERSITÄT WIEN

Probability Theory

Introduction

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  - All otcomes are equally probable. We have

    $$P(\text{one head in 3 tosses}) = \frac{\text{number of outcomes with 1 head}}{\text{total number of outcomes}} = \frac{3}{8}$$

Counting: Examples

(2) Playing cards: Poker hands

- A full deck of cards consists of 52 cards:
  - 13 values in hierarchical order (ranks): $2, \ldots 9, 10$, J, Q, K, A
  - 4 suits: $\heartsuit$, $\spadesuit$, $\diamondsuit$, $\clubsuit$

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- Poker hands
  - consist of 5 cards
  - One pair refers to two cards of the same rank and three others (pairwise different additional rank)
  - Example: $\{5\spadesuit, 5\heartsuit, 8\diamondsuit, 10\clubsuit, Q\heartsuit\}$
    Note: a full house, e.g. $\{5\spadesuit, 5\heartsuit, 8\diamondsuit, 8\clubsuit, 8\heartsuit\}$, is <u>not</u> a hand with exactly one pair

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  (d) between 20% and 40%
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We will come back to this question!

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HW Question: Can you think of an example where the possible outcomes are not equally probable?

Sets

- A set S is a collection of elements.
- $x \in S$ ... an element x in the set S
- $A \subset S$ ... the set A is a subset of S
- $\emptyset$ ... the empty set
- $A^c = S \backslash A = \{x : x \notin A\}$ ... complement
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$ ... intersection
- $A \cup B = \{x : x \in A \text{ or } x \in B\}$ ... union
- $A \backslash B = \{x : x \in A \text{ and } x \notin B\}$ ... difference
- $A \triangle B = (A \backslash B) \cup (B \backslash A)$ ... symmetric difference
- $|A|$ ... number of elements in A

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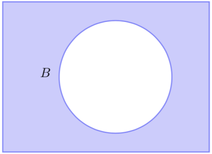
Some properties:

- $A \cup B = B \cup A, \quad A \cap B = B \cap A$
- $A \cap \emptyset = \emptyset, A \cup \emptyset = A, A \cap S = A, \quad A \cup S = S$
- $(A \cup B) \cap C = (A \cap C) \cup (B \cap C), (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
- $(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$
- Complement laws: $A \cup A^c = S,\ A \cap A^c = \emptyset,\ (A^c)^c = A,\ \emptyset^c = S,\ S^c = \emptyset$
- De Morgan's laws: $(A \cup B)^c = A^c \cap B^c,\ (A \cap B)^c = A^c \cup B^c$

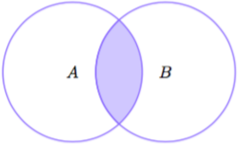
Venn Diagrams

- Venn diagrams giver an easy way to visualize set operations

$B^c = S \backslash B$



$A \cap B$



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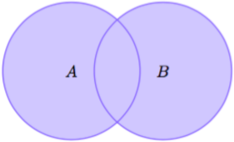

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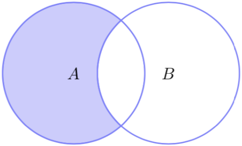

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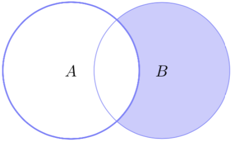
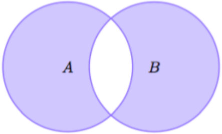


$A \triangle B = (A \backslash B) \cup (B \backslash A)$



Example

- Start with a set of S of all natural numbers less than 20. Consider two subsets

$$A = \{\text{the set of all odd numbers}\}$$
$$B = \{\text{the set of all natural numbers divisible by 3}\}$$

Consider different set operations.

  - Express the set of all numbers divisible by 6 in terms of A and B.
  - What is $A^c \cup B$? State in "words" and as a Venn diagram.

Example: Answer

- Start with a set of S of all natural numbers less than 20. Consider two subsets

$$
\begin{aligned}
S &= \{1, 2, \ldots, 18, 19\} \\
A &= \{\text{all odd numbers}\} = \{1, 3, 5, \ldots, 17, 19\} \\
B &= \{\text{all natural numbers divisible by 3}\} = \{3, 6, 9, 12, 15, 18\}
\end{aligned}
$$

Consider different set operations.

- Write the set of all numbers divisible by 6 in the form of A and B.

$$
\begin{aligned}
&\{\text{all natural numbers divisible by 6}\} \\
&\qquad = \{\text{all even numbers divisible by 3}\} \\
&\qquad = A^c \cap B
\end{aligned}
$$

- What is $A^c \cup B$? State in "words" and as a Venn diagram.

$$
\begin{aligned}
A^c \cup B &= \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\} \\
&= \{\text{all numbers either even or divisible by 3}\}
\end{aligned}
$$

The Cartesian product

- The Cartesian product of the sets A and B is the set of the ordered pairs

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

- Example

$$\begin{array}{cc|cccc} & & & B & & \\ & \times & 1 & 2 & 3 & 4 \\ \hline & 1 & (1,1) & (1,2) & (1,3) & (1,4) \\ A & 2 & (2,1) & (2,2) & (2,3) & (2,4) \\ & 3 & (3,1) & (3,2) & (3,3) & (3,4) \end{array}$$

$$A \times B = \{1, 2, 3\} \times \{1, 2, 3, 4\}$$

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    - 7 people sing
    - 4 play guitar
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    How many people are in the band?

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$$\text{Size of the band} = |S| + |G| - |S \cap G| = 7 + 4 - 2 = 9$$

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$$\text{To choose } \underbrace{\text{one from } \{a_1, a_2, a_3\}}_{\text{Action1}} \text{ AND } \underbrace{\text{one from } \{b_1, b_2\}}_{\text{Action 2}} \text{ is the same}$$

$$\text{as to choose } \underbrace{\text{one from } \{a_1b_1, a_1b_2, a_2b_1, a_2b_2, a_3b_1, a_3b_2\}}_{\text{Action1, Action2}}$$

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- Example:
   - There are 8 participants in the 400m final. In how many ways can gold, silver and bronze medals be awarded? $8 \cdot 7 \cdot 6 = 330$ ways

Questions

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She thinks black and jeans go with everything.
Here is her wardrobe:

- shirts: 4R, 5B, 2G
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How many different outfits can she wear?

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Hint: A tree diagram is an easy way to represent answer.

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    - How many permutations of $\{a, b, c, d, e, f, g, h, i\}$ are there?

      There are $9!$ possibilities

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- Question: How many permutations of 8 out of a set of 15 are there?

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      In total 4 possibilities

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In total $\binom{4}{3} = \frac{4!}{3!\,1!} = 4$ possibilities

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permutations and combinations

abc	acb	bac	bca	cab	cba	$\{a, b, c\}$
abd	adb	bad	bda	dab	dba	$\{a, b, d\}$
acd	adc	cad	cda	dac	dca	$\{a, c, d\}$
bcd	bdc	cbd	cdb	dbc	dcb	$\{b, c, d\}$

permutations: ${}_4P_3 = 3! \cdot \binom{4}{3} = 24$

combinations: ${}_4C_3 = \binom{4}{3}$

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(2) What is is the probability to get exactly 3 heads? $\frac{\binom{10}{3}}{2^{10}} = \frac{120}{1024} = 0.117$

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  - Random experiment
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    - Ω ... sure event
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Questions

Thank you for your attention!