

Übungsaufgaben zur VU Computermathematik Serie 6

Test your code with examples.

Exercise 6.1: *Breaking down integers.*

- (a) Write a function `int_break(n)` that takes an integer n and returns a list l where the entry $l[k]$ contains the $(k+1)$ th digit of n .
- (b) Write a second function `counter(n)`, which takes the list generated by `int_break(n)` and reverses its order. You are not allowed to use the list function `reverse()`.

Hint: Use the math function `floor`.

Exercise 6.2: *Counting odd and even numbers.*

- (a) Write two functions `odd` and `even` that get as input an integer n and as output the number of odd and even digits, respectively.
- (b) Extend the functions of item (a) to floats and count the number of odd and even fractional digits (Nachkommastellen). It is sufficient to count e.g. 10 digits.
- (c) Write a function, which takes an integer n and counts the number of digits that are a prime number.

Exercise 6.3: *Sequences. I*

Let N be an arbitrary natural number. Then we define $a_0 := N$, $a_{n+1} := a_n/2$ if a_n is even, $a_{n+1} := 3a_n + 1$ if a_n is odd, for all $n \geq 0$.

- (a) Write a code that implements the sequence (a_n) .
- (b) Verify (numerically) that for arbitrary initial value N , the sequence ends up in the cycle $4 \rightarrow 2 \rightarrow 1 \rightarrow 4$.

Exercise 6.4: *Sequences. II*

- (a) Write a code that numerically tests Fermat's last problem. Fermat's last problem states that for integers $n > 2$, $a, b, c \geq 1$ the equation $a^n + b^n = c^n$ does not hold.
- (b) Write a function that implements the sequences $x_{n+1} := \frac{2x_n^3}{3x_n^2 - 1}$ and $y_{n+1} := \frac{1}{2} \left(y_n + \frac{1}{y_n} \right)$. Choose an initial value > 2 . What is the exact limit of both sequences?

Exercise 6.5: *Print.*

- (a) Write a function which takes two strings $s1$ and $s2$ and joins them together and then prints them.
- (b) Write a function which takes a string flips the first and last word of the sentence. Don't forget the capitalization.

(c) Write a function which produces the following output

1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

Exercise 6.6: *Vector product, tensor product.*

The tensor product between two number $a \in \mathbb{R}^{n_1} = \mathbb{R}^{n_1 \times 1}$ and $b \in \mathbb{R}^{n_2} = \mathbb{R}^{n_2 \times 1}$ is defined by $a \otimes b := ab^\top$.

- (a) Write a function `vec_prod(a, b)` which takes two lists a, b of length 3 and returns the vector product $a \times b$.
- (b) Write a function `tensor_prod(a, b)` which takes two lists a, b (not necessarily the same length) and returns the tensor product.

Exercise 6.7: *Dictionaries.*

- (a)
 - Grab the string 'get me' from the following dictionary.

```
1 d = {'key1': [1, 2, {'key2': ['do not get confused', {'tough': [1, 2, [['get me']]]}]]}]}
```

- Grab the string 'get me' from the dictionary

```
1 d = {'key2': [1, [], {'bug': {'bug': 'get me'}}]}
```

- (b) Write a function which takes a dictionary and reverses all of its *keys*, but not the values. Example $\{\text{'key1'} : \text{val1}, \text{'key2'} : \text{val2}\}$ becomes $\{\text{'1yek'} : \text{val1}, \text{'2yek'} : \text{val2}\}$.

Exercise 6.8: *Debugging*

Go to the webpage <https://docs.python.org/3.6/library/pdb.html> and study the module *pdb* (p stands for python and db for debugger).

- (a) What does the following code do?

```
1 import pdb; pdb.set_trace()
```

- (b) Use (a) to set a debug point at line 6 of

```
1 from math import sin
2
3 print("This is a debugging test.")
4
5 def f(x):
6
7     x = x + 1.0
8     a
9     return sin(x)**2 + x
10
11
12 print("value of f(x)", f(1))
```

Read the section *Debugger Commands* on the webpage and explain what the commands "n", "s", "l" and "c" do. Now save the previous code in a file "my_debug.py" and run **python3 my_debug.py** in the command line. Use the commands n, l, s and c to navigate in debug mode.