

# Software Engineering VU

## [194.020]

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# Summary

# Informal modeling

## Strengths

- Describe particular views of the system
- Omit some information or specify it informally
- Graphical notation facilitates communication

## Weaknesses

- Precise meaning of models is often unclear
- Incomplete and informal models hamper tool support
- Many details are hard to depict visually

# Design

- Informal models
- Formal models
- Design by contract

# Formal modeling

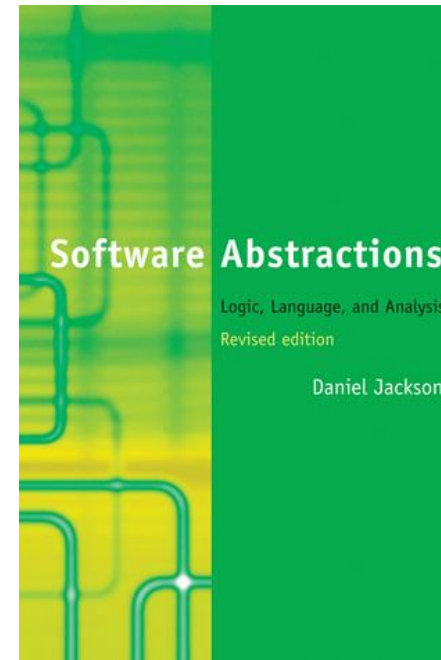
- Notations and tools are based on **math** and are **precise**
- Typically used to describe **some aspect** of a system
- Formal models enable **automatic analysis**
  - Finding ill-formed examples
  - Checking properties

# Alloy

- Alloy is a formal modeling language based on **set theory**
- An Alloy model specifies a **collection of constraints** describing a set of structures
- The **Alloy analyzer** reasons about the constraints of a model
  - Generates sample structures
  - Generates counterexamples for invalid properties
  - Visualizes structures

# Alloy documentation and download

- <https://alloytools.org>
- The documentation includes
  - Tutorials
  - Book by Daniel Jackson



# Design

- Informal models
- Formal models
  - Static models
  - Analyzing models
- Design by contract

# Signatures

- A signature declares a set of atoms
  - Think of signatures as classes
  - Think of atoms as immutable objects
  - Different signatures declare disjoint sets
- Extends-clauses declare subset relations
  - File and Dir are disjoint subsets of FSObject

```
sig FSObject {}
```

```
sig File extends FSObject {}  
sig Dir extends FSObject {}
```

# Operations on sets

- Standard set operators
  - + (union)
  - & (intersection)
  - - (difference)
  - **in** (subset)
  - = (equality)
  - # (cardinality)
  - **none** (empty set)
  - **univ** (universal set)

```
sig File extends FSObject {}  
sig Dir extends FSObject {}
```

```
{f: FSObject | f in File + Dir} >= #Dir
```

```
 #(File + Dir) >= #Dir
```

# More on signatures

- Signatures can be abstract
  - Like abstract classes
  - **Closed-world assumption**: the declared set contains exactly the elements of the declared subsets
- Signatures may constrain the cardinalities of the declared sets
  - **one**: singleton set
  - **lone**: singleton or empty
  - **some**: non-empty set

```
abstract sig FSObject {}  
sig File extends FSObject {}  
sig Dir extends FSObject {}
```

```
FSObject = File + Dir
```

```
one sig Root extends Dir {}
```

# Fields

- A field declares a relation on atoms
  - $f$  is a binary relation with domain  $A$  and range given by expression  $e$
  - Think of fields as associations

```
sig A {  
  f: e  
}
```

- Range expressions may denote multiplicities
  - **one**: singleton set (default)
  - **lone**: singleton or empty set
  - **some**: non-empty set
  - **set**: any set

```
abstract sig FSObject {  
  parent: lone Dir  
}
```

```
sig Dir extends FSObject {  
  contents: set FSObject  
}
```

# Operations on relations

- Standard relation operators
  - $\rightarrow$  (cross product)
  - $.$  (relational join)
  - $\sim$  (transposition)
  - $^{\wedge}$  (transitive closure)
  - $*$  (reflexive transitive closure)
  - $<:$  (domain restriction)
  - $>:$  (range restriction)
  - $++$  (override)
  - **iden** (identity relation)
  - $[ ]$  (box join:  $e1[e2] = e2.e1$ )

```
abstract sig FSOobject {  
    parent: lone Dir  
}
```

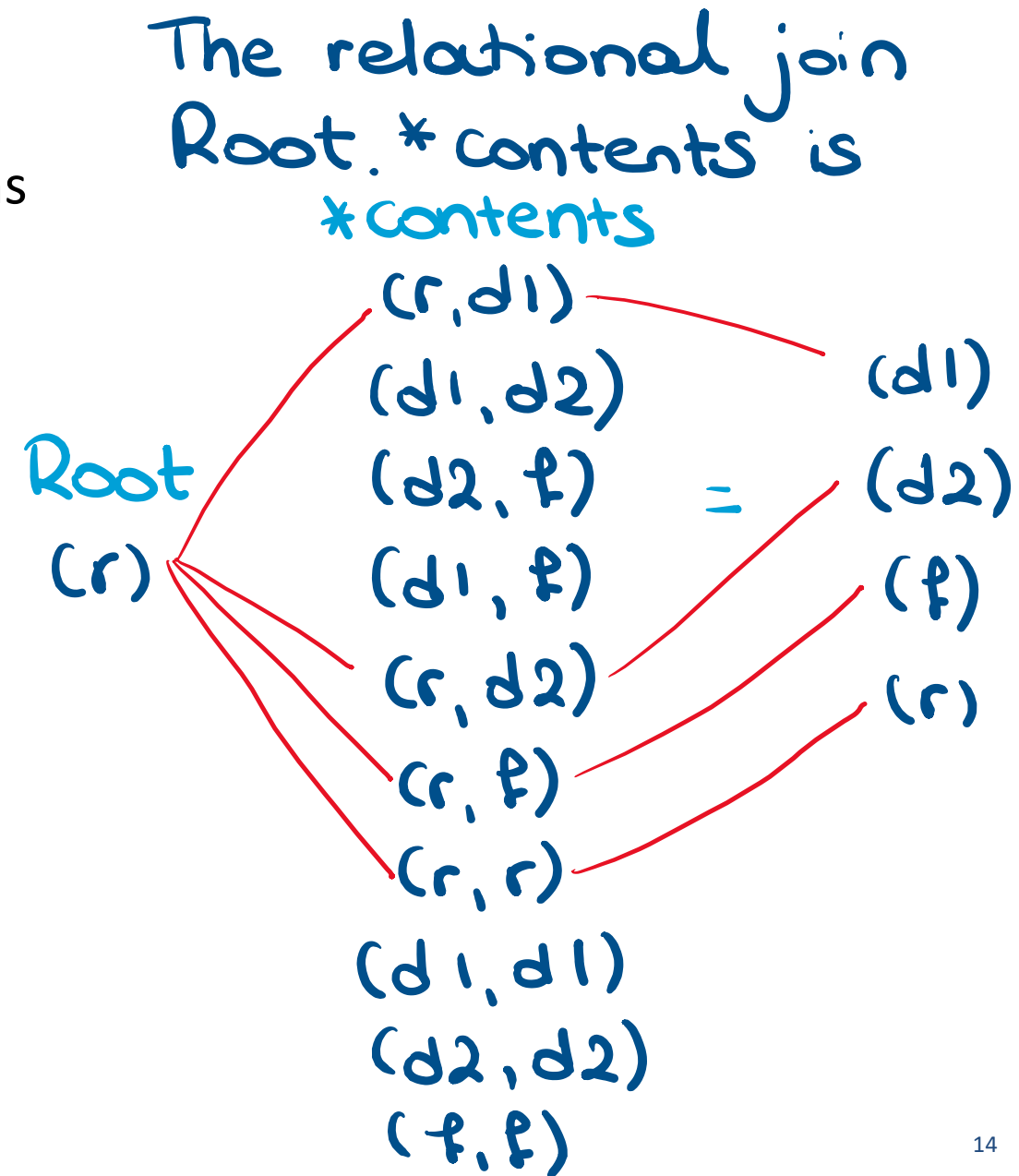
```
sig Dir extends FSOobject {  
    contents: set FSOobject  
}
```

```
one sig Root extends Dir {}
```

```
FSOobject in Root.*contents
```

# Relational join: Example

- Consider a structure with four FSObject atoms
  - r: Root, d1,d2: Dir, f: File
- And contents relation
  - (r,d1) (d1,d2) (d2,f)
- The reflexive transitive closure \*contents is
  - (r,d1) (d1,d2) (d2,f)
  - (d1,f) (r,d2) (r,f)
  - (r,r) (d1,d1) (d2,d2) (f,f)



# More on fields

- Fields may range over relations
- Relation declarations may include multiplicities on both sides
  - **one, lone, some, set** (default)

```
sig University {  
  enrollment: Student set -> one Program  
}
```

- Range expressions may depend on other fields

```
sig University {  
  students: set Student,  
  enrollment: students set -> one Program  
}
```

# Constraints

- Boolean operators
  - **!** or **not** (negation)
  - **&&** or **and** (conjunction)
  - **||** or **or** (disjunction)
  - **=>** or **implies** (implication)
  - **else** (alternative)
  - **<=>** or **iff** (equivalence)
- Four equivalent constraints

```
F => G else H
F implies G else H
(F && G) || ((!F) && H)
(F and G) or ((not F) and H)
```

- Quantified expressions
  - **some e**  
e has at least one tuple
  - **no e**  
e has no tuples
  - **lone e**  
e has at most one tuple
  - **one e**  
e has exactly one tuple

```
no Root.parent
```

# Quantification

- Alloy supports five quantifiers

- **all**  $x: e \mid F$

$F$  holds for every  $x$  in  $e$

- **some**  $x: e \mid F$

$F$  holds for at least one  $x$  in  $e$

- **no**  $x: e \mid F$

$F$  holds for no  $x$  in  $e$

- **lone**  $x: e \mid F$

$F$  holds for at most one  $x$  in  $e$

- **one**  $x: e \mid F$

$F$  holds for exactly one  $x$  in  $e$

- Quantifiers may have the following forms:

- **all**  $x: e \mid F$

- **all**  $x: e1, y: e2 \mid F$

- **all**  $x, y: e \mid F$

- **all disj**  $x, y: e \mid F$

- E.g., the contents relation is acyclic

```
no d: Dir | d in d.^contents
```

# Predicates and functions

- Predicates are named, parameterized formulas

```
pred p[x1:e1,...,xn:en]{F}
```

```
pred isLeaf[f:FSObject] {  
  f in File || no f.contents  
}
```

returns bool

- Functions are named, parameterized expressions

```
fun f[x1:e1,...,xn:en]: e{E}
```

```
fun leaves[f:FSObject]: set FSObject {  
  {x: f.*contents | isLeaf[x]}  
}
```

returns e

# Exploring the model

- The Alloy analyzer can search for structures that satisfy the constraints M in a model
- Find instance of a predicate
  - A solution to M && **some**  $x_1:e_1, \dots, x_n:e_n \mid F$
- Find instance of a function
  - A solution to M && **some**  $x_1:e_1, \dots, x_n:e_n, res:e \mid res = E$

```
pred p[x1:e1,...,xn:en]{F}
```

```
run p
```

```
fun f[x1:e1,...,xn:en]: e {E}
```

```
run f
```

# Exploring the model: Scopes

- The existence of a structure that satisfies the constraints in a model is in general **undecidable**
- The Alloy analyzer searches exhaustively for structures **up to a given size**
  - The problem becomes **finite**, and thus, **decidable**

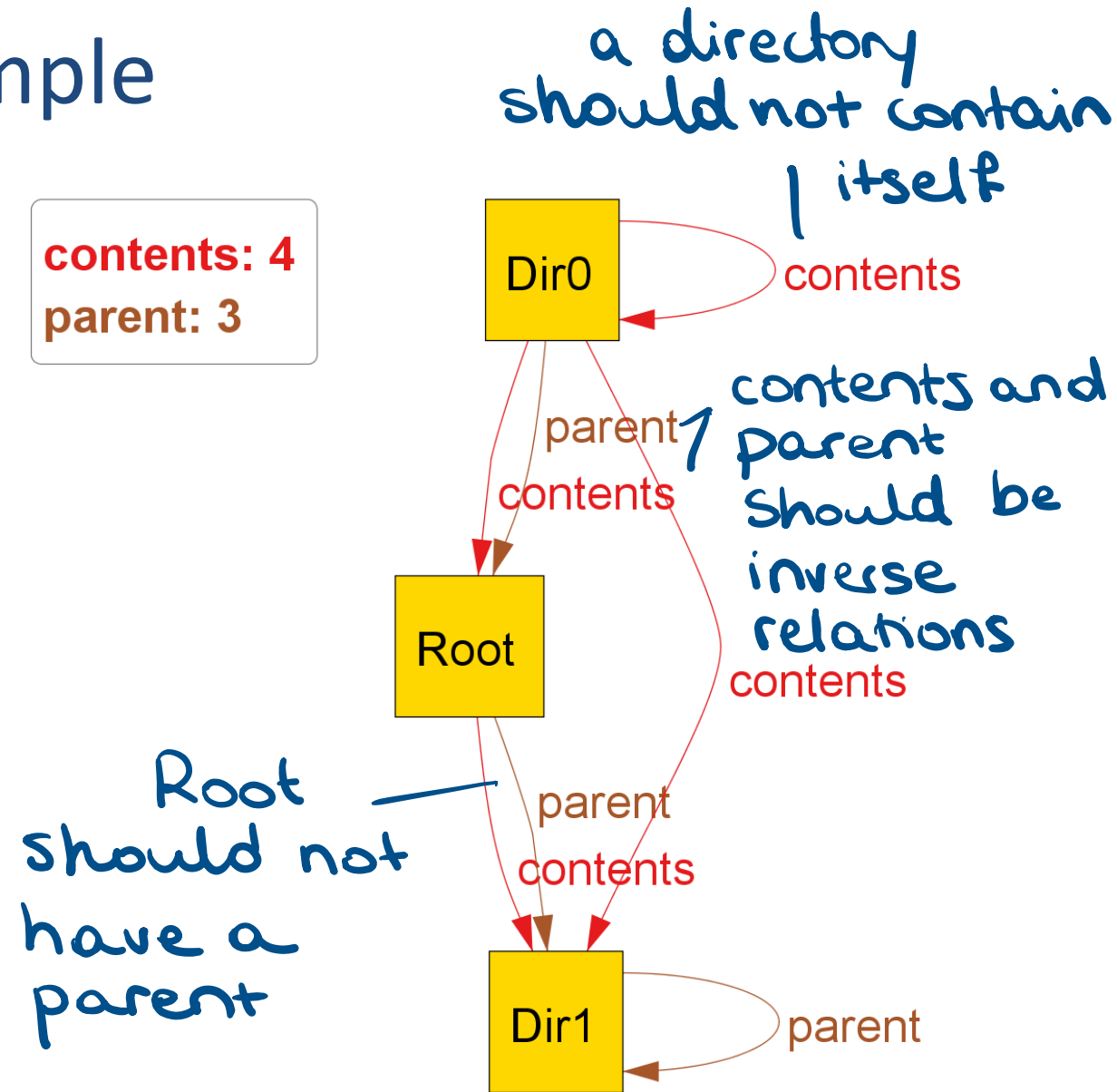
```
run isLeaf
run isLeaf for 5
run isLeaf for 5 Dir, 2 File
run isLeaf for exactly 5 Dir
run isLeaf for 5 but 3 Dir
run isLeaf for 5 but exactly 3 Dir
```

DEMO

# Exploring the model: Example

```
abstract sig FSOBJECT {  
  parent: lone Dir  
}  
  
sig File extends FSOBJECT {}  
  
sig Dir extends FSOBJECT {  
  contents: set FSOBJECT  
}  
  
one sig Root extends Dir {}
```

contents: 4  
parent: 3



# Adding constraints

- Facts add constraints that always hold
  - **run** searches for solutions that satisfy all constraints

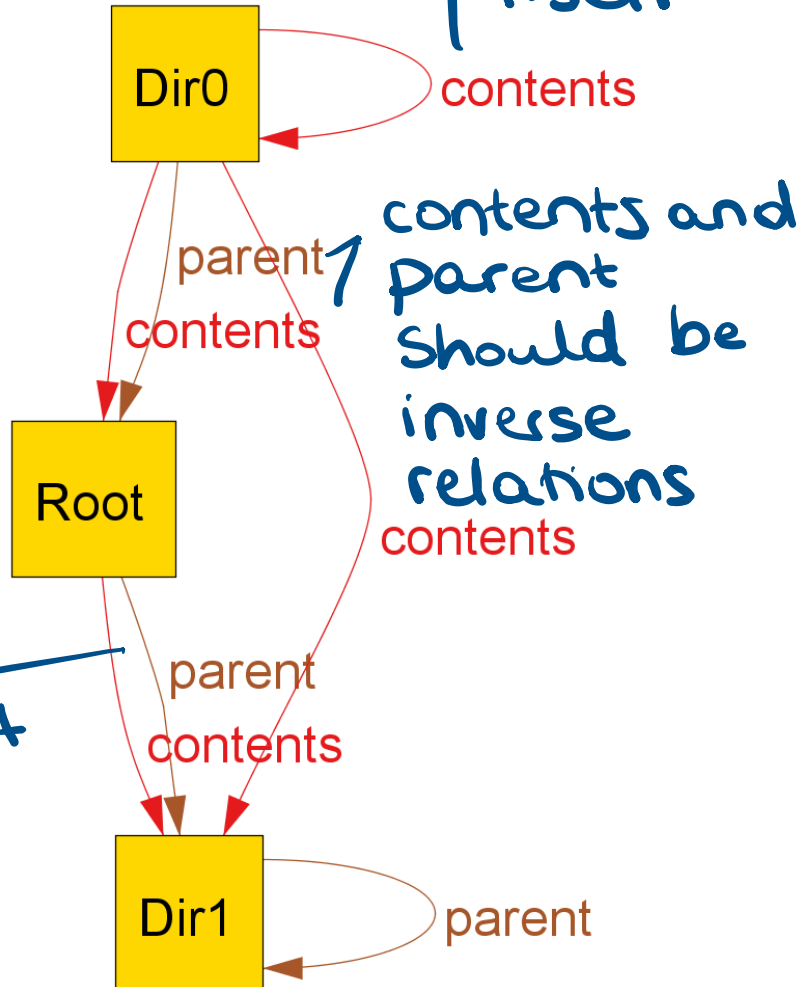
```
fact { F }  
fact f { F }  
sig S {...}{F}
```

- Facts express value and structural invariants of the model

# Adding constraints: Example

a directory  
should not contain  
| itself

contents: 4  
parent: 3



```
// The contents path is acyclic  
fact { no d: Dir | d in d.^contents }
```

```
// A directory is the parent of its contents  
fact { all d: Dir, o: d.contents | o.parent = d }
```

```
// Root is the root  
fact { no Root.parent }
```

# Checking the model

- Exploring models by manually inspecting instances is cumbersome
- The Alloy analyzer can search for structures that violate a given property
  - It can find counterexamples to an assertion
  - The search is complete for a given scope
- For a model with constraints  $M$ , find a solution to  $M \ \&\& \ !F$

```
assert a { F }
```

```
check a scope
```

# Checking the model: Example

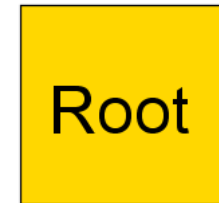
- Finding a counterexample

```
pred isLeaf[f: FSObject] {  
  f in File || no f.contents  
}
```

```
assert nonEmptyRoot { !isLeaf[Root] }  
check nonEmptyRoot for 3
```

- Proving a property

```
assert acyclic { no d: Dir | d in d.^contents }  
check acyclic for 5
```



Validity is checked  
only within the given  
scope

## Executing "Check acyclic for 5"

Solver=sat4j Bitwidth=0 MaxSeq=0 SkolemDepth=1 Symmetry=20  
1212 vars. 63 primary vars. 2423 clauses. 62ms


No counterexample found. Assertion may be valid 0ms.

# Under and over-constrained models

- Missing or weak facts **under-constrain** the model
  - They permit undesired structures
  - Under-constrained models are typically easy to detect during model exploration (using **run**) and assertion checking (using **check**)
- Unnecessary facts **over-constrain** the model
  - They exclude desired structures
- **Inconsistencies** are an extreme case of over-constraining
  - They preclude the existence of any structure
  - All assertion checks will succeed!

```
// The contents path is acyclic
fact acyclic { no d: Dir | d in d.*contents }

assert nonsense { 0 = 1 }
check nonsense
```



# Guidelines to avoid over-constraining

- Simulate model to check consistency
  - Use **run** to ensure that structures exist

```
// The contents path is acyclic
fact acyclic { no d: Dir | d in d.*contents }

pred show {}
run show
```

## Executing "Run show"

Solver=sat4j Bitwidth=0 MaxSeq=0 SkolemDepth=4 Symmetry=20  
0 vars. 0 primary vars. 0 clauses. 16ms.  
No instance found. Predicate may be inconsistent. 0ms.

- Prefer assertions over facts
  - When in doubt, check whether the current model already ensures a desired property before adding it as a fact

DEMO

# Design

- Informal models
- Formal models
  - Static models
  - Analyzing models
- Design by contract

# Consistency and validity

- An Alloy model specifies a collection of constraints  $C$  that describe a set of structures

- Consistency:

A formula  $F$  is consistent (satisfiable) if it evaluates to true in at least one of these structures

$$\exists s \cdot C(s) \wedge F(s)$$

- Validity:

A formula  $F$  is valid if it evaluates to true in all these structures

$$\forall s \cdot C(s) \Rightarrow F(s)$$

# Analyzing models within a scope

- Validity and consistency checking for Alloy is undecidable
- The Alloy analyzer sidesteps this problem by checking validity and consistency **within a given scope**
  - A scope gives a **finite bound on the sizes of the sets** in the model (which makes everything else in the model also finite)
  - Naïve algorithm: enumerate all structures of a model within the bounds and check the formula for each of them

# Consistency checking

Translate constraints and formula into formula over Boolean variables



Check whether this formula has a satisfying assignment

NO



Formula is inconsistent within the given scope

YES

Formula is consistent:  
Translate satisfying assignment back to model

# Translation into Boolean formula

- Internally, Alloy represents all data types as relations
  - A relation is a set of tuples

```
sig Node {  
  next: lone Node  
}
```

next is a binary relation  
in  $\text{Node} \times \text{Node}$

- Constraints and formulas in the model are represented as formulas over relations

```
fact {  
  all n: Node | n != n.next  
}
```

$\forall n. (n, n) \notin \text{next}$

# Translation into Boolean formula

- A relation is translated into Boolean variables
  - Introduce one Boolean variable for each tuple that is potentially contained in the relation

```
sig Node {  
  next: lone Node  
}  
pred show {}  
run show for 3
```

next is a binary relation  
in Node x Node

$n_{00}, n_{01}, n_{02}, n_{10}, n_{11}, n_{12}, n_{20}, n_{21}, n_{22}$

For the given scope, next may contain  
9 different tuples

- Constraints and formulas are translated into Boolean formulas over these variables

```
fact {  
  all n: Node | n != n.next  
}
```

$\neg(n_{00} \wedge n_{01}) \wedge \neg(n_{00} \wedge n_{02}) \wedge \neg(n_{01} \wedge n_{02}) \wedge$   
 $\neg(n_{10} \wedge n_{11}) \wedge \neg(n_{10} \wedge n_{12}) \wedge \neg(n_{11} \wedge n_{12}) \wedge$   
 $\neg(n_{20} \wedge n_{21}) \wedge \neg(n_{20} \wedge n_{22}) \wedge \neg(n_{21} \wedge n_{22}) \wedge$   
 $\neg n_{00} \wedge \neg n_{11} \wedge \neg n_{22}$

# Check for satisfying assignments

- Satisfiability of formulas over Boolean variables is a well understood problem
  - Find a satisfying assignment if one exists and return UNSAT otherwise
  - The problem is NP-complete
- In practice, SAT solvers are extremely efficient

A satisfying assignment

$$\neg(n_{00} \wedge n_{01}) \wedge \neg(n_{00} \wedge n_{02}) \wedge \neg(n_{01} \wedge n_{02}) \wedge \\ \neg(n_{10} \wedge n_{11}) \wedge \neg(n_{10} \wedge n_{12}) \wedge \neg(n_{11} \wedge n_{12}) \wedge \\ \neg(n_{20} \wedge n_{21}) \wedge \neg(n_{20} \wedge n_{22}) \wedge \neg(n_{21} \wedge n_{22}) \wedge \\ \neg n_{00} \wedge \neg n_{11} \wedge \neg n_{22}$$

n	0	1	2
0	F	F	F
1	F	F	T
2	F	T	F

# Translation back to model

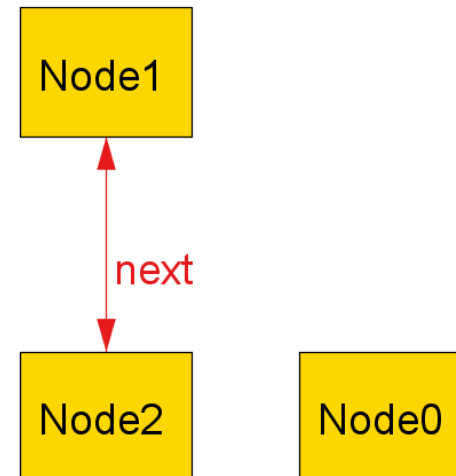
- A satisfying assignment can be translated back to relations

n	0	1	2
0	F	F	F
1	F	F	T
2	F	T	F

$$\text{next} = \{(1, 2), (2, 1)\}$$



next: 2



and then visualized

# Interpretation of UNSAT

- If a Boolean formula has no satisfying assignment, the SAT solver returns UNSAT
- The Boolean formula encodes an Alloy model **within a given scope**
  - There are no structures within this scope, but **larger structures may exist**
  - The model may be, but is **not necessarily inconsistent**

```
sig Node {  
  next: lone Node  
}  
fact { #Node = 4 }  
pred show {}  
run show for 3
```

## Executing "Run show for 3"

Solver=sat4j Bitwidth=0 MaxSeq=0 SkolemDepth=1 Symmetry=20  
0 vars. 0 primary vars. 0 clauses. 0ms.

No instance found. Predicate may be inconsistent. 0ms.

# Validity and invalidity checking

- A formula  $F$  is valid if it evaluates to true in **all** structures that satisfy the constraints  $C$  of the model

$$\forall s. C(s) \Rightarrow F(s)$$

- Enumerating all structures within a given scope is possible but too slow
- Instead of checking validity, the Alloy analyzer **checks for invalidity**, that is, it looks for counterexamples

$$\neg (\forall s. C(s) \Rightarrow F(s)) \equiv \exists s. C(s) \wedge \neg F(s)$$

this is a consistency check

# Validity checking

Translate constraints and negated formula into formula over Boolean vars



Check whether this formula has a satisfying assignment

NO



Formula is valid within the given scope



Formula is invalid:

Translate satisfying assignment back to model

# Interpretation of UNSAT

- Validity checking searches for a counterexample **within a given scope**
  - There are no structures within this scope, but **larger structures may exist**
  - The model may be, but is **not necessarily valid**

```
sig Node { next: Node }
```

```
assert demo { all n: Node | some m: Node | m.next = n }
```

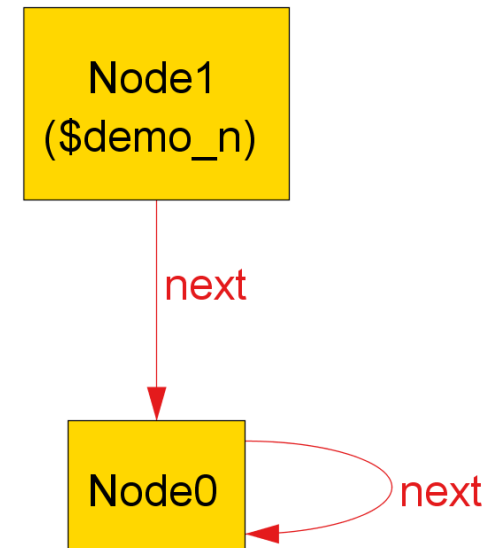
check demo for 1

check demo for 2

next: 2

## Executing "Check demo for 1"

Solver=sat4j Bitwidth=0 MaxSeq=0 SkolemDepth=1 Symmetry=20  
14 vars. 3 primary vars. 18 clauses. 9ms.  
No counterexample found. Assertion may be valid. 0ms.



# Analyzing models: Summary

- Consistency checking
  - Performed by **run** command within a scope
  - Positive answers are definite (structures)
- Validity checking
  - Performed by **check** command within a scope
  - Negative answers are definite (counterexamples)
- Small model hypothesis

Most interesting errors are found by looking at small instances

# Design

- Informal models
- Formal models
  - Static models
  - Analyzing models
- Design by contract

# Design by contract

- Pioneered by Bertrand Meyer in the Eiffel language
- Defining formal, precise, and verifiable interface specifications for software components, with **preconditions, postconditions, and invariants**
- Three key questions that the designer must repeatedly ask:
  - What does this code expect?
  - What does it guarantee?
  - What does it maintain?

# Preconditions

- Preconditions express **requirements on the input state** (parameters, heap) of a method
- Semantics
  - Condition must be true at the **entry** of the method

```
// requires 0 <= index < size  
public int getElemAtIndex(int index) {  
    return elems[index];  
}
```

# Checking preconditions

- Approach 1: Return error value

```
// requires 0 <= index < size  
public int getElemAtIndex(int index) {  
    if (index < 0 || index >= size) {  
        return -1;  
    } else {  
        return elems[index];  
    }  
}
```

# Checking preconditions

- Approach 2: Throw exception

```
// requires 0 <= index < size
public int getElemAtIndex(int index) {
    if (index < 0 || index >= size) {
        throw new IndexOutOfBoundsException(index);
    } else {
        return elems[index];
    }
}
```

# Checking preconditions

- Approach 3: Use assertions
  - An assertion is a logical statement that can be made at a particular program point and is expected to be true
  - In Java, assertions can be enabled (they are off by default)

```
// requires 0 <= index < size  
public int getElemAtIndex(int index) {  
    assert (0 <= index && index < size) : "index must be within bounds";  
    return elems[index];  
}
```

# Postconditions

- Postconditions express **guarantees about the result and output state** (out-parameters, heap) of a method
- Semantics
  - Condition must be true at the **normal exit** of the method

```
// requires 0 <= capacity  
// ensures capacity <= elems.length  
public int ensureCapacity(int capacity) {  
    while (capacity > elems.length) {  
        elems = Arrays.copyOf(elems, 2*elems.length);  
    }  
}
```

# Checking postconditions

- Use assertions

```
// requires 0 <= capacity  
// ensures capacity <= elems.length  
public int ensureCapacity(int capacity) {  
    while (capacity > elems.length) {  
        elems = Arrays.copyOf(elems, 2*elems.length);  
    }  
    assert (capacity <= elems.length) : "elems does not have enough capacity";  
}
```

# Pre- and postconditions: Methods and callers

	Obligation	Benefit
Method	Establish post-condition for all calls that satisfy precondition	May assume precondition upon method entry
Caller	Establish precondition of called method	May assume postcondition upon return from call

# Dafny

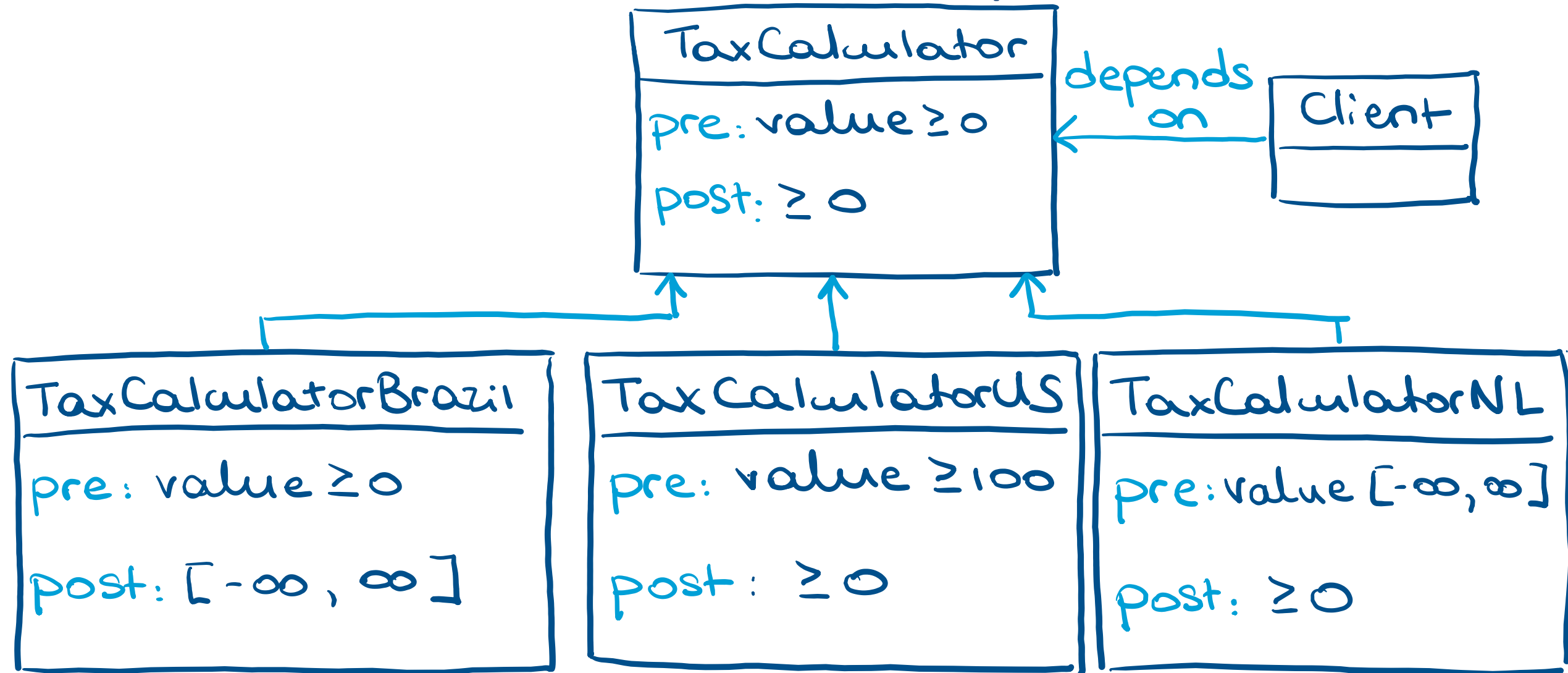
- There are many languages that natively support design by contract
- Dafny is a such a programming language that comes with a **program verifier** for automatically checking the specifications
- Dafny **compiles to mainstream languages**, such as Java, C#, Go, etc.
- Check it out: <https://dafny.org/>
  - Tutorials, examples, documentation, ...



Dafny

DEMO

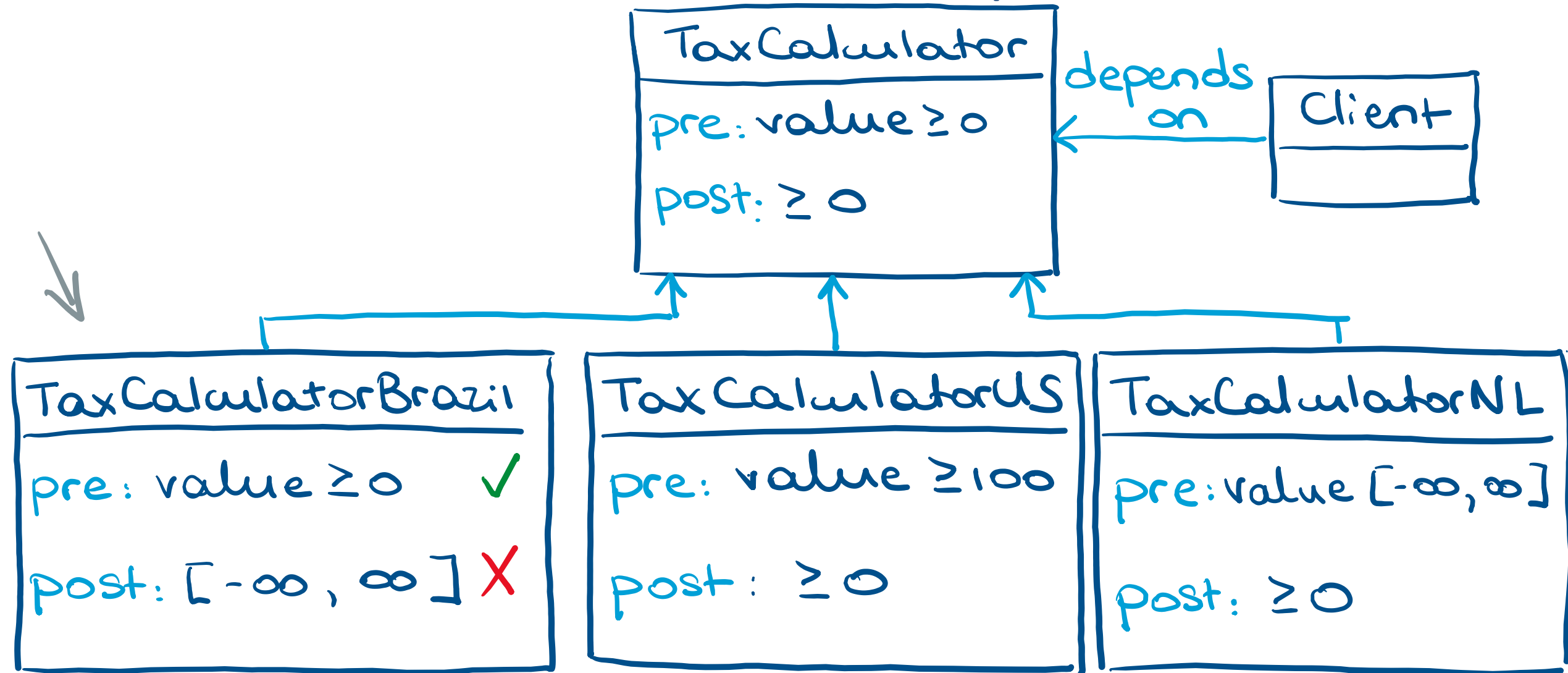
# Inheritance and contracts: Example



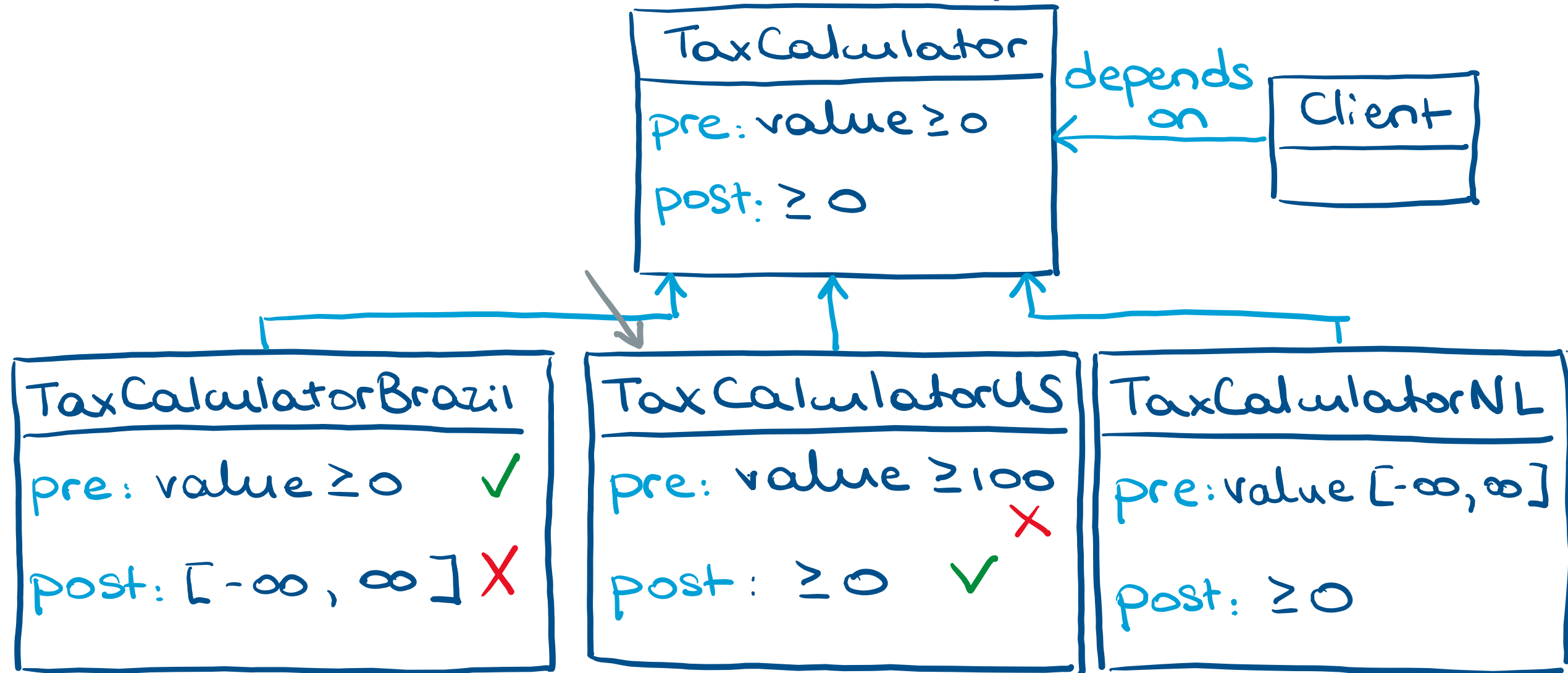
# Inheritance and contracts: Example

- Suppose the client class receives a TaxCalculator in its constructor and uses it in its methods
- Due to polymorphism, any of the child classes can also be passed to the client
- Since the client does not know which tax calculator was given to it, it can only assume that whatever class it received will respect the pre- and post-conditions of the base class (the only class the client knows)

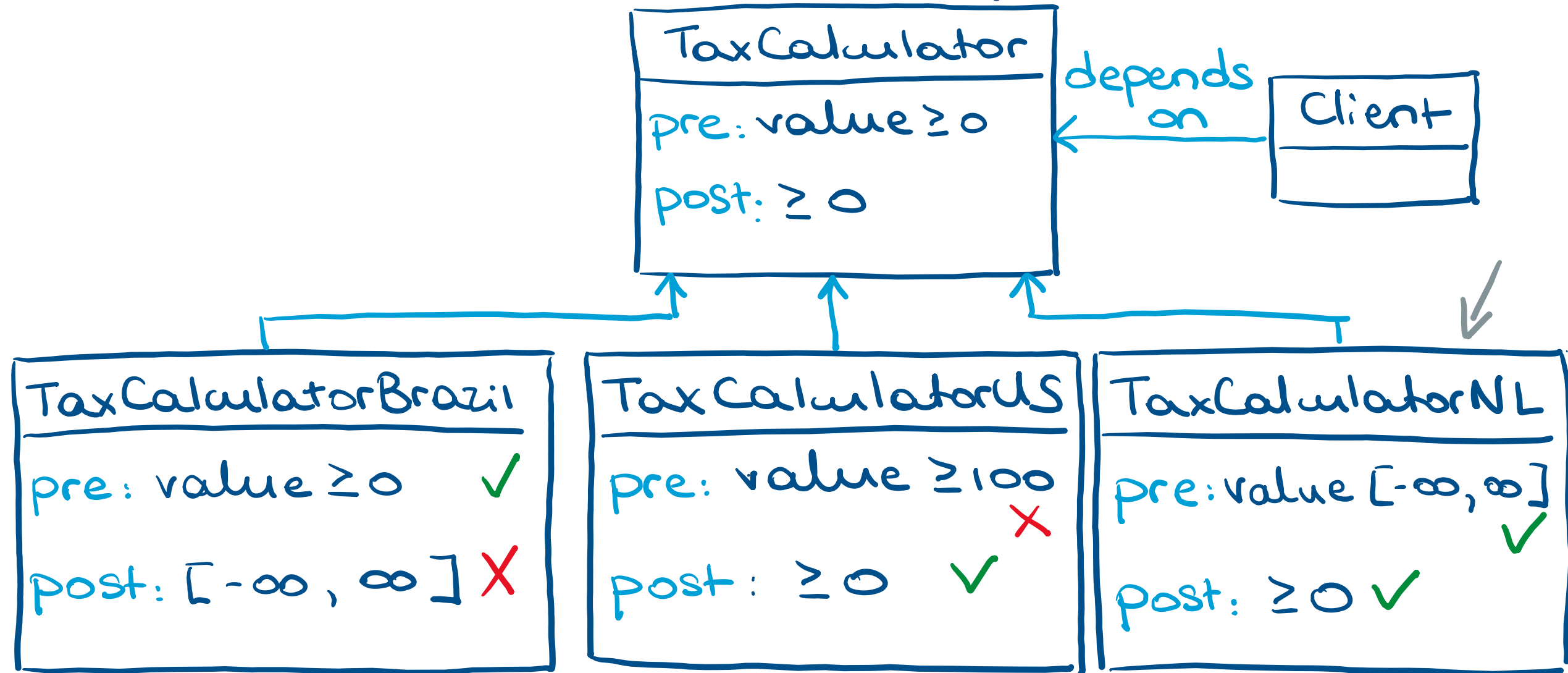
# Inheritance and contracts: Example



# Inheritance and contracts: Example



# Inheritance and contracts: Example



# Inheritance and contracts: Rules

Whenever a subclass S (e.g., TaxCalculatorBrazil) inherits from a base class B (e.g., TaxCalculator):

1. The pre-conditions of subclass S should be the same as or weaker (accept more values) than the pre-conditions of base class B
2. The post-conditions of subclass S should be the same as or stronger (return fewer values) than the post-conditions of base class B

**Liskov substitution principle (LSP):** The idea that a subclass may be used as a substitution for a base class without breaking the system's expected behavior

# Weak or strong pre-conditions?

- An important design decision when modeling contracts is whether to use strong or weak contracts: it's a trade-off!
- Weak pre-condition
  - E.g., the method accepts any input value, including null
  - It's **easy to use for clients**: any call to it will work and the method will never throw an exception related to a pre-condition being violated
  - This puts an **extra burden on the method**, as it must handle any invalid inputs

# Weak or strong pre-conditions?

- An important design decision when modeling contracts is whether to use strong or weak contracts: it's a trade-off!
- Strong pre-condition
  - E.g., the method only accepts positive numbers and does not accept null
  - This puts an **extra burden on the client**, as it must make sure it does not violate the pre-conditions of the method
  - The **method implementation is easier**, as it may assume that inputs are always valid

## Suggested reading

From Book: "Effective Software Testing A Developer's Guide" by M. Aniche

- Section 4.3.1
- Section 4.5.1