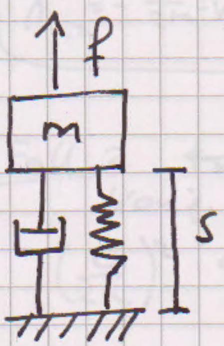
1.2 Dynamikmatrix



$$x = \begin{pmatrix} s \\ \dot{s} \end{pmatrix} \qquad \mathbf{u} = u = f$$

$$m\ddot{s} = f - c\dot{s} - ks$$

$$\left.\begin{aligned} \ddot{s} &= \frac{f}{m} - \frac{c}{m}\dot{s} - \frac{k}{m}s \\ \dot{s} &= \dot{s} \end{aligned}\right\} \text{Zustände}$$

$$\dot{x} = Ax + Bu$$

$$\begin{pmatrix} \dot{s} \\ \ddot{s} \end{pmatrix} = A\begin{pmatrix} s \\ \dot{s} \end{pmatrix} + B \cdot f = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{pmatrix}\begin{pmatrix} s \\ \dot{s} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \cdot f$$

$$\underline{A = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{pmatrix}} \qquad \underline{B = \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}} \qquad y = c^T x = (1 \quad 0) \cdot x$$

Eigenwerte d. Dynamikmatrix

$$\begin{vmatrix} 0-\lambda & 1 \\ -\frac{k}{m} & -\frac{c}{m} - \lambda \end{vmatrix} = 0$$

$$-\lambda\left(-\frac{c}{m} - \lambda\right) + \frac{k}{m} = 0$$

$$\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Stabilität

Fall 1 Reele Ew

$$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} \geq 0$$

$$\left(\frac{c}{2m}\right)^2 \geq \frac{k}{m}$$

asymptot. stabil gdw.

$$\frac{c}{2m} > \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

sonst nicht! da dann ~~...~~ 1 Ew positiv

$$\lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Fall 2 ~~...~~ Konj. kompl. Ew

$$\left(\frac{c}{2m}\right)^2 < \frac{k}{m}$$

↳ ~~...~~

$$\mathrm{Re}\{\lambda_{1,2}\} = -\frac{c}{2m} < 0$$

↳ asymptotisch stabil wenn $c, m > 0$

Satz 3.4

asymptotisch stabil gdw. $\mathrm{Re}\{\lambda\} < 0 \ \forall \lambda$

λ .. Ew

$$\hat{\Phi}(s) = (sE - A)^{-1} =$$

3.65

$$= \begin{bmatrix} s-0 & -1 \\ -\frac{k}{m} & s+\frac{c}{m} \end{bmatrix}^{-1}$$

Nullstellen

$(s-A)(s-B)$

$$\cancel{\det}$$

$$\begin{vmatrix} s & -1 \\ +\frac{k}{m} & s+\frac{c}{m} \end{vmatrix} = s\cdot\left(s+\frac{c}{m}\right) + \frac{k}{m} = s^2 + \frac{c}{m}s + \frac{k}{m}$$

$\boxed{\text{Eigenwerte}}$

$$\det \overset{!}{=} 0 \Rightarrow -\frac{\frac{c}{m}}{2} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} = \frac{-c \pm \sqrt{c^2-4km}}{2m} = \underline{\underline{\lambda_{1,2}}}$$

$$\operatorname{adj}(\ldots) = \begin{pmatrix} s+\frac{c}{m} & 1 \\ -\frac{k}{m} & s \end{pmatrix}$$

$$\frac{\operatorname{adj}}{\det} = \begin{pmatrix} \frac{s+\frac{c}{m}}{s\cdot(s+\frac{c}{m})+\frac{k}{m}} & \frac{1}{s\cdot(s+\frac{c}{m})+\frac{k}{m}} \\ \frac{-\frac{k}{m}}{s(s+\frac{c}{m})+\frac{k}{m}} & \frac{s}{s\cdot(s+\frac{c}{m})+\frac{k}{m}} \end{pmatrix}$$

3.67

$$\det = (s-\lambda_1)\cdot(s-\lambda_2)$$

$$\frac{s+\frac{c}{m}}{(s-\lambda_1)\cdot(s-\lambda_2)} = \frac{A}{s-\lambda_1} + \frac{B}{s-\lambda_2}$$

$$s+\frac{c}{m} = A(s-\lambda_2) + B(s-\lambda_1)$$

$$s := \lambda_1$$

$$\lambda_1 + \frac{c}{m} = A(\lambda_1 - \lambda_2) \Rightarrow \underline{\underline{A = \frac{\lambda_1 + \frac{c}{m}}{\lambda_1 - \lambda_2}}}$$

$$\frac{-c+\sqrt{c^2-4km}}{2m} + \frac{c}{m} = A\left(\frac{-c+\sqrt{}}{2m} + \frac{c+\sqrt{}}{2m}\right)$$

$$\frac{c+\sqrt{}}{2m} = A\left(\frac{2\cdot\sqrt{}}{2m}\right) = A\,\frac{\sqrt{}}{m} \quad |\cdot 2m$$

$$|: 2\cdot\sqrt{}$$

$$\frac{c+\sqrt{c^2-4km}}{2\cdot\sqrt{c^2-4km}} = A = \frac{\lambda_1 + \frac{c}{m}}{\lambda_1 - \lambda_2}$$

$s := \lambda_2$

$\lambda_2 + \frac{c}{m} = B(\lambda_2 - \lambda_1)$

$B = \frac{\lambda_2 + \frac{c}{m}}{\lambda_2 - \lambda_1}$

$-\frac{k}{m} = C(s-\lambda_2) + D(s-\lambda_1)$

$s := \lambda_1$

$C = -\frac{k}{m(\lambda_1 - \lambda_2)}$

$s := \lambda_2$

$D = -\frac{k}{m(\lambda_2 - \lambda_1)}$

$1 = E(s-\lambda_2) + F(s-\lambda_1)$

$E = \frac{1}{\lambda_1 - \lambda_2} \qquad F = \frac{1}{\lambda_2 - \lambda_1}$

$s = G(s-\lambda_2) + H(s-\lambda_1)$

$G = \frac{\lambda_1}{\lambda_1 - \lambda_2} \qquad H = \frac{\lambda_2}{\lambda_2 - \lambda_1}$

$A \dots H = \text{Konstant}$

$$\hat{\Phi}(s) = \begin{pmatrix} \frac{A}{s-\lambda_1} + \frac{B}{s-\lambda_2} & \frac{E}{s-\lambda_1} + \frac{F}{s-\lambda_2} \\ \frac{C}{s-\lambda_1} + \frac{D}{s-\lambda_2} & \frac{G}{s-\lambda_1} + \frac{H}{s-\lambda_2} \end{pmatrix}$$

$\bullet\!\!-\!\!\circ$

Transitionsmatrix

$$\underline{\Phi(t)} = \begin{pmatrix} A\cdot e^{\lambda_1 t} + B\cdot e^{\lambda_2 t} & E\cdot e^{\lambda_1 t} + F e^{\lambda_2 t} \\ C e^{\lambda_1 t} + D e^{\lambda_2 t} & G\cdot e^{\lambda_1 t} + H e^{\lambda_2 t} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{\lambda_1 + \frac{c}{m}}{\lambda_1 - \lambda_2} e^{\lambda_1 t} + \frac{\lambda_2 + \frac{c}{m}}{\lambda_2} e^{\lambda_2 t} & \frac{1}{\lambda_1 - \lambda_2} e^{\lambda_1 t} + \frac{1}{\lambda_2 - \lambda_1} e^{\lambda_2 t} \\ -\frac{k}{m(\lambda_1 - \lambda_2)} e^{\lambda_1 t} - \frac{k}{m(\lambda_2 - \lambda_1)} e^{\lambda_2 t} & \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{\lambda_1 t} + \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{\lambda_2 t} \end{pmatrix}$$

$\hat{y}(s) = c^T \hat{x}(s)$ 3.59a $(d=0)$

Allgemeine Lsg. mit $u = \sigma(t)$

$\hat{x}(s) = \hat{\Phi}(s)\, x_0 + \hat{\Phi}(s) \cdot B \cdot \hat{u}(s)$ 3.64

$\hat{U}(s) = \frac{1}{s} \quad \bullet\!\!-\!\!\circ \quad u(t) = \sigma(t)$

$$\hat{x}(s) = \hat{\Phi}(s) \begin{pmatrix} s_0 \\ v_0 \end{pmatrix} + \hat{\Phi}(s) \cdot \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \cdot \frac{1}{s} =$$

$$= \begin{pmatrix} \left(\frac{A}{s-\lambda_1} + \frac{B}{s-\lambda_2}\right) s_0 + \left(\frac{E}{s-\lambda_1} + \frac{F}{s-\lambda_2}\right) v_0 \\ \left(\frac{C}{s-\lambda_1} + \frac{D}{s-\lambda_2}\right) s_0 + \left(\frac{G}{s-\lambda_1} + \frac{H}{s-\lambda_2}\right) v_0 \end{pmatrix} + \begin{pmatrix} \left(\frac{E}{s-\lambda_1} + \frac{F}{s-\lambda_2}\right) \frac{1}{s \cdot m} \\ \left(\frac{G}{s-\lambda_1} + \frac{H}{s-\lambda_2}\right) \frac{1}{s \cdot m} \end{pmatrix} =$$

$$= \begin{pmatrix} \left(\frac{A}{s-\lambda_1} + \frac{B}{s-\lambda_2}\right) s_0 + \left(\frac{E}{s-\lambda_1} + \frac{F}{s-\lambda_2}\right) \left(v_0 + \frac{1}{s \cdot m}\right) \\ \left(\frac{C}{s-\lambda_1} + \frac{D}{s-\lambda_2}\right) s_0 + \left(\frac{G}{s-\lambda_1} + \frac{H}{s-\lambda_2}\right) \left(v_0 + \frac{1}{s \cdot m}\right) \end{pmatrix}$$

$$\hat{y}(s) = \begin{pmatrix} 1 & 0 \end{pmatrix} \cdot \hat{x}(s) = \left(\frac{A}{s-\lambda_1} + \frac{B}{s-\lambda_2}\right) s_0 + \left(\frac{E}{s-\lambda_1} + \frac{F}{s-\lambda_2}\right) \left(v_0 + \frac{1}{s \cdot m}\right) =$$

$$y(t) = \left(A \cdot e^{\lambda_1 t} + B e^{\lambda_2 t}\right) s_0 + \left(E \cdot e^{\lambda_1 t} + F e^{\lambda_2 t}\right) v_0 +$$

$$\frac{E}{m} \cdot \left(\frac{1}{\lambda_1} e^{\lambda_1 t} - \frac{1}{\lambda_1} \sigma(t)\right) + \frac{F}{m} \left(\frac{1}{\lambda_2} e^{\lambda_2 t} - \frac{1}{\lambda_2} \sigma(t)\right) =$$

$$= \left(\frac{\lambda_1 + \frac{c}{m}}{\lambda_1 - \lambda_2} e^{\lambda_1 t} + \frac{\lambda_2 + \frac{c}{m}}{\lambda_2 - \lambda_1} e^{\lambda_2 t}\right) \cdot s_0 +$$

$$\left(\frac{1}{\lambda_1 - \lambda_2} e^{\lambda_1 t} + \frac{1}{\lambda_2 - \lambda_1} e^{\lambda_2 t}\right) \cdot v_0 +$$

$$\frac{\frac{1}{\lambda_1}}{m(\lambda_1 - \lambda_2)} \cdot \left(-\sigma(t) + e^{\lambda_1 t}\right) + \frac{\frac{1}{\lambda_2}}{m(\lambda_2 - \lambda_1)} \left(-\sigma(t) + e^{\lambda_2 t}\right)$$

$$\boxed{\begin{array}{ll} \multicolumn{2}{l}{\frac{1}{s-\lambda} \cdot \frac{1}{s} = \frac{A}{s-\lambda} + \frac{B}{s} = + \frac{\frac{1}{\lambda}}{s-\lambda} - \frac{\frac{1}{\lambda}}{s}} \\ \multicolumn{2}{l}{1 = As + B(s-\lambda)} \\ s := 0 & s := \lambda \\ 1 = B \cdot (-\lambda) & 1 = A \cdot \lambda \\ B = -\frac{1}{\lambda} & A = \frac{1}{\lambda} \end{array}}$$

$\boxed{1.3.1}$

$$\dddot{x} + \cos(x)^2 \ddot{x} + \dot{x} + e^{-x} u = 0$$

$$\dot{x}_3 = -\cos(x_1)^2 \cdot x_3 - x_2 - e^{-x_1} \cdot u$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_1 = x_2$$

$$\dot{x} = A \cdot x + B \cdot u$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -\cos(x_1)^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -e^{-x_1} \end{pmatrix} \cdot u$$

$$y = C \cdot x = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \cdot x$$

$\boxed{1.3.2}$

$$f(x,u) = \dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -\cos(x_1)^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -e^{-x_1} \end{pmatrix} \cdot u =$$

$$= \begin{pmatrix} x_2 \\ x_3 \\ -x_2 - \cos(x_1)^2 x_3 - e^{-x_1} u \end{pmatrix}$$

$$h(x,u) = y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1$$

Satz 2.6

linearisiertes System:

$x_R = 0$

$u_R = 0$

$$\Delta\dot{x} = A\,\Delta x + B\,\Delta u$$

$$\Delta y = C\,\Delta y$$

$$A_\Delta = \frac{\partial}{\partial x} f(x_R, u_R) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sin(x_1)\cos(x_1)x_3 + e^{-x_1} \cdot u ; & -1 ; & -\cos(x_1)^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

$$B_\Delta = \frac{\partial}{\partial u} f(x_R, u_R) = \begin{pmatrix} 0 \\ 0 \\ -e^{-x_1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$C_\Delta = \frac{\partial}{\partial x} h(x_R, u_R) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & -1 & -1-\lambda \end{vmatrix} = 0$$

$$-\lambda^2 - \lambda^3 - \lambda = 0$$

$$-\lambda(1 + \lambda + \lambda^2) = 0 \qquad \lambda_{2,3} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1} = -\frac{1}{2} \pm \sqrt{-\frac{3}{4}}$$

$$\lambda_1 = 0$$

Nicht $\underbrace{\text{asyptotisch stabil}}_{\lambda_i < 0,\ \forall i}$, da $\lambda_1 = 0$

$\boxed{1.4.1}$

$K_1: \; I_{R_1} = I_{C_1} + I_{R_2}$ $\qquad U_d = 0 \quad i_{ein,p} = i_{ein,n} = 0$

$K_2: \; I_{R_2} + I_{R_{22}} = I_{C_1}$

$\boxed{R} \; U = R \cdot I, \quad I = \frac{U}{R}$

$\boxed{C} \; C \cdot \frac{dU}{dt} = I$

$S_1: \; U_e = U_{R_1} + U_{R_2} + U_{C_2}$

$S_2: \; U_{C_2} + U_{R_2} = U_{C_1} + U_a$

$S_3: \; U_{C_2} + U_{R_{22}} = U_s$

$S_4: \; U_{C_2} = U_{R_3}$

$S_5: \; U_{R_3} + U_{R_{33}} = U_a$

$S_2: \; U_{R_2} = U_{C_1} + U_a - U_{C_2}$

S_2 in S_1: $\; U_e = U_{R_1} + U_{C_1} + U_a \;\rightarrow\; U_{R_1} = U_e - U_{C_1} - U_a$

$K_1: \; \frac{U_{R_1}}{R_1} = \frac{U_{R_2}}{R_2} + C_1 \cdot \dot{U}_{C_1}$

$K_2: \; \frac{U_{R_2}}{R_2} + \frac{U_{R_{22}}}{R_2} = C_2 \cdot \dot{U}_{C_2}$

S_2 in K_1: $\; \frac{U_{R_1}}{R_1} = \frac{U_{C_1} + U_a - U_{C_2}}{R_2} + C_1 \cdot \dot{U}_{C_1}$

$(S_2 \text{ in } S_1)$ in $(S_2 \text{ in } K_1)$: $\; \frac{U_e - U_{C_1} - U_a}{R_1} = \frac{U_{C_1} + U_a - U_{C_2}}{R_2} + C_1 \dot{U}_{C_1}$

S_4 in S_5: $\; U_{C_2} + U_{R_{33}} = U_a$

$\quad \hookrightarrow U_{R_{33}} = U_a - U_{C_2}$

U_a: $\quad \hookrightarrow U_a = K \cdot U_{C_2}$

S_3 in K_2: $\; U_{R_{22}} = R_2 \cdot \left(-\frac{U_{R_2}}{R_2} + C_2 \cdot \dot{U}_{C_2} \right) = R_2 C_1 \dot{U}_{C_2} - U_{R_2}$

$\quad -U_{C_2} + U_s = R_2 C_2 \dot{U}_{C_2} - (U_{C_1} + U_a - U_{C_2})$

$S_2 \hookrightarrow \; U_s = R_2 C_2 \dot{U}_{C_2} - U_{C_1} - U_a + 2U_{C_2}$

$(S_2 \text{ in } S_1) \text{ in } (S_2 \text{ in } K_1):$

$$\frac{U_e - U_{C1} - (K \cdot U_{C2})}{R_1} = \frac{U_{C1} + K U_{C2} - U_{C2}}{R_2} + C_1 \cdot \dot{U}_{C1} \quad | \cdot R_1 R_2$$

$$R_2 U_e - R_2 U_{C1} - R_2 (K \cdot U_{C2}) = R_1 U_{C1} + R_1 K U_{C2} - U_{C2} \cdot R_1 + R_1 R_2 C_1 \dot{U}_{C1}$$

S_2: $U_S = R_2 C_2 \dot{U}_{C2} - U_{C1} + (2-K) U_{C2}$

•) $\dot{U}_{C2} = \frac{-(2-K) U_{C2} + U_{C1} + U_S}{R_2 C_2} = \frac{K-2}{R_2 C_2} U_{C2} + \frac{1}{R_2 C_2} U_{C1} + \frac{1}{R_2 C_2} U_S$

$$\dot{U}_{C1} = \frac{R_2 U_e - R_2 U_{C1} - K R_2 U_{C2} - R_1 U_{C1} - R_1 K U_{C2} + R_1 U_{C2}}{R_1 R_2 C_1}$$

•) $\dot{U}_{C1} = \left(-\frac{1}{R_1 C_1} - \frac{1}{R_2 C_1}\right) U_{C1} + \left(-\frac{K}{R_1 C_1} + \frac{1-K}{R_2 C_1}\right) U_{C2} + \frac{1}{R_1 C_1} U_e$

$$x = \begin{pmatrix} U_{C1} \\ U_{C2} \end{pmatrix}$$

$$\dot{x} = \underbrace{\begin{pmatrix} -\frac{1}{R_1 C_1} - \frac{1}{R_2 C_1} & \frac{1-K}{R_2 C_1} - \frac{K}{R_1 C_1} \\ \frac{1}{R_2 C_2} & \frac{K-2}{R_2 C_2} \end{pmatrix}}_{A} \cdot x + \underbrace{\begin{pmatrix} \frac{1}{R_1 C_1} & 0 \\ 0 & \frac{1}{R_2 C_2} \end{pmatrix}}_{B} \cdot \underbrace{\begin{pmatrix} U_e \\ U_S \end{pmatrix}}_{U}$$

$$\dot{x} = A \cdot x + B \cdot U$$

$$y = U_a = K \cdot U_{C2}$$

$$y = C \cdot x = \begin{pmatrix} 0 \\ K \end{pmatrix}^T \cdot x$$

$D = 0$

1.4.2 Linearisieren des Systems um eine Ruhelage

$B_\Delta = (b_{u_\Delta}, b_{d_\Delta})$

$$\Delta\dot{x} = A_\Delta \Delta x + b_{u_\Delta}\Delta u + b_{d_\Delta}\Delta d$$

$$\Delta y = c_\Delta^T \Delta x + d_{u_\Delta}\Delta u + d_{d_\Delta}\Delta d = C_\Delta \Delta x$$

$$f(x_\Delta, u) = \begin{pmatrix} \left(-\frac{1}{R_1C_1} - \frac{1}{R_2C_1}\right)U_{C_1} + \left(-\frac{K}{R_1C_1} + \frac{1-K}{R_2C_1}\right)U_{C_2} + \frac{1}{R_1C_1}U_e \\ \frac{1}{R_2C_2}U_{C_1} + \frac{K-2}{R_2C_2}U_{C_2} + \frac{1}{R_2C_2}U_s \end{pmatrix}$$

$$\boxed{\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'g - fg'}{g^2}}$$

$$A_\Delta = \frac{\partial}{\partial x} f(x_R, u_R)$$

$$A_{\Delta 1,1} = \frac{\partial}{\partial U_{C_1}} f_1(x,u) = \frac{\partial}{\partial U_{C_1}} \left[\frac{(-R_1 - R_2)U_{C_1}}{R_1R_2C_1(U_{C_1})} + \frac{(R_1(1-K) - R_2K)U_{C_2} + R_2U_e}{R_1R_2C_1(U_{C_1})}\right] =$$

$$= \frac{(-R_1-R_2)\cdot R_1R_2C_1(U_{C_1}) - R_1R_2\dot{C}_1(U_{C_1})\cdot(-R_1-R_2)U_{C_1}}{\left[R_1R_2C_1(U_{C_1})\right]^2} +$$

$$+ \frac{0 - R_1R_2\dot{C}_1(U_{C_1})\cdot\left[(R_1(1-K)-R_2K)U_{C_2} + R_2U_e\right]}{\left[R_1R_2C_1(U_{C_1})\right]^2} =$$

$$= \frac{(-R_1-R_2)\left[C_1(U_{C_1}) - \dot{C}_1(U_{C_1})\cdot U_{C_1}\right] - \dot{C}_1(U_{C_1})\cdot\left[(R_1(1-K)-R_2K)U_{C_2} + R_2U_e\right]}{R_1R_2\left[C_1(U_{C_1})\right]^2}$$

$$A_{\Delta 1,2} = \frac{\partial}{\partial U_{C_2}} f_1(x,u) = -\frac{K}{R_1C_1(U_{C_1})} + \frac{1-K}{R_2C_1(U_{C_1})}$$

$$A_{\Delta 2,1} = \frac{\partial}{\partial U_{C_1}} f_2(x,u) = \frac{1}{R_2C_2}$$

$$A_{\Delta 2,2} = \frac{\partial}{\partial U_{C_2}} f_2(x,u) = \frac{K-2}{R_2C_2} \qquad C_\Delta = \begin{pmatrix} 0 \\ K \end{pmatrix}$$

$$B_{\Delta 1,1} = \frac{\partial}{\partial U_e} f_1(x,u) = \frac{1}{R_1C_1(U_{C_1})} \qquad B_{\Delta 1,2} = B_{\Delta 2,1} = 0$$

$$B_{\Delta 2,2} = \frac{\partial}{\partial U_S} f_2(x,u) = \frac{1}{R_2C_2}$$

$\boxed{1.4.3}$

$$G(s) = \frac{V}{1+2\zeta(sT)+(sT)^2} = \frac{1{,}371\cdot 10^6}{9{,}959\cdot 10^5 + 1600s + s^2} =$$

$$\frac{\frac{1{,}371\cdot 10^6}{9{,}959\cdot 10^5}}{1 + 2\cdot\frac{800}{9{,}959\cdot 10^5}s + \frac{1}{9{,}959\cdot 10^5}\cdot s^2}$$

$$T^2 = \frac{1}{9{,}959\cdot 10^5}$$

$$T = \sqrt{\frac{1}{9{,}959\cdot 10^5}} = 0{,}001$$

$$\zeta = \frac{\frac{800}{9{,}959\cdot 10^5}}{T} = 0{,}8016$$

$$V = \frac{1{,}371\cdot 10^6}{9{,}959\cdot 10^5} = 1{,}3766$$

$\boxed{1.6.1}$ $G(s) = \frac{c}{sT_1+1}$

6.45

$$G(z) = \frac{z-1}{z} Z\left(\frac{G(s)}{s}\right) =$$

lt. Transformations-tabelle

$$= \frac{z-1}{z} Z\left(\frac{\frac{c}{sT_1+1}}{s}\right) = \frac{z-1}{z} Z\left(\frac{c}{s(sT_1+1)}\right) = \frac{\cancel{z-1}}{\cancel{z}} c \cdot \frac{\left(1-e^{-\frac{T_a}{T_1}}\right)\cdot \cancel{z}}{\cancel{(z-1)}\left(z-e^{-\frac{T_a}{T_1}}\right)} =$$

6.68

$$= \frac{1-e^{-\frac{T_a}{T_1}}}{z-e^{-\frac{T_a}{T_1}}} \cdot c = \frac{b_0}{a_0+a_1 \cdot z}$$

$b_0 = c \cdot \left(1-e^{-\frac{T_a}{T_1}}\right)$

$a_0 = -e^{-\frac{T_a}{T_1}}$

$\boxed{n=1}$

$$x_{k+1} = \Phi \cdot x_k + \Gamma u_k$$

6.69a

$$x_{k+1} = \begin{bmatrix} \cancel{0} & -a_0 \\ \cancel{1} & \cancel{-a_1} \end{bmatrix} x_k + \begin{bmatrix} \tilde{b}_0 \\ \cancel{\tilde{b}_1} \end{bmatrix} u_k = \underbrace{\begin{bmatrix} \cancel{0} & e^{-\frac{T_a}{T_1}} \\ \cancel{1} & \cancel{-a_1} \end{bmatrix}}_{\Phi} x_k + \underbrace{\begin{bmatrix} c\cdot\left(1-e^{-\frac{T_a}{T_1}}\right) \\ \cancel{0} \end{bmatrix}}_{\Gamma} u_k$$

$\tilde{b}_i = b_i - a_i b_n$ 6.70

$\tilde{b}_0 = b_0 - a_0 b_1 = b_0$

$\cancel{\tilde{b}_1 = b_1 - a_1 b_1}$

$$\Phi = e^{-\frac{T_a}{T_1}}$$

$$\Gamma = c \cdot \left(1-e^{-\frac{T_a}{T_1}}\right)$$

$$y_k = c^T x_k + b_n u_k = c^T x_k$$ 6.69b

$\boxed{1.6.2}$

$$S_{k+1} = \overbrace{\underbrace{(d_{in} - d_{out})}_{u_k} \cdot T_a}^{\text{In/Output}} \overbrace{- \underbrace{a \cdot S_k}_{d_p} \cdot T_a + d_c \cdot T_a}^{\text{Prozessor}} + \underbrace{S_k}_{\text{letzter Speicherstand}}$$

nächster Speicherstand

$$S_{k+1} = u_k \cdot T_a + S_k (1 - a \cdot T_a) + d_c \cdot T_a$$

$$x_k = \begin{bmatrix} p_k \\ S_k \end{bmatrix} \begin{matrix} \rightarrow \text{Prozessor} \\ \rightarrow \text{Speiche} \end{matrix}$$

Prozessor Verarbeitung

Eingang Prozessor (von Speicher)

$$x_{k+1} = \begin{bmatrix} e^{-\frac{T_a}{T_1}} & \overbrace{a \cdot \left(c \cdot \left(1 - e^{-\frac{T_a}{T_1}}\right)\right)} \\ \cdot T_a & \underbrace{1 - a \cdot T_a} \end{bmatrix} x_k + \begin{bmatrix} 0 \\ T_a \end{bmatrix} u_k$$

din-dout ↳ Speicher

Ausgang Prozessor
Eingang Speicher

Eingang ~~Prozessor~~ vom Speicher
+ letzter Speicherstand

$\boxed{1.6.4}$

$s_{k+1} = s_k$ $\qquad u = 1$

$s_k = A \cdot s_k + B \cdot u = A \cdot s_k + B$

$s_k(1-A) = B$

$s_k = (1-A)/B$