Problem Set 1

Problem 1.1 The number of airplanes arriving per minute at the Metropolis International Airport (MMA) is Poisson distributed with parameter $\lambda = 5/4$. Presently, MMA can service up to 3 arriving airplanes per minute. In case more than 3 airplanes arrive per minute, the airplanes in excess are rerouted to a different airport.

a) Find the average number of airplanes arriving per minute.

b) Calculate the most probable number of airplanes arriving per minute.

c) Find the probability that at least one airplane has to be rerouted in a given minute.

d) Calculate the expected number of arriving airplanes being serviced per minute.

e) Calculate the expected number of arriving airplanes being rerouted per minute.

Problem 1.2 The Institute of Unreliable Studies conducts a survey in order to get insight about the income of Michisota's citizens. A random sample of $N = 100$ persons from Michisota is asked the question "is your yearly income over 150 000 credits?". Suppose that 20 % of Michisota's citizens have a yearly income of over 150 000 credits. These persons give a truthful answer with probability 0.6. All others answer truthfully with a probability of 0.8.

a) Let x denote the number of persons in the sample with an income of over 150 000 credits. Calculate the pmf of x. *Note: you don't have to compute all probabilities* $P\{x = n\}$; *just give a formula which is valid for all* $n \in \{0, 1, \ldots, N\}$.

b) Let y denote the number of persons in the sample who answer "yes". Calculate the pmf of y.

c) Let z denote the number of persons in the sample who give a truthful answer. Calculate the pmf of z.

d) Provide a lower bound on the probability that between 10 and 30 persons from the sample have an income of more than 150 000 credits. *Hint: Chebyshev inequality.*

e) Given that a person answered "yes", find the probability that the person's income is over 150 000 credits.

Problem 1.3 Show that if x is Cauchy distributed with parameter α, then $y = 1/x$ is Cauchy distributed with parameter $1/\alpha$.

Problem 1.4 Let $x \sim \mathcal{N}(\mu_x, \sigma_x^2)$. Show that $y = e^x$ has lognormal pdf with parameters $a = \mu_x$ and $b = \sigma_x$.