Zuname:

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DISCRETE MATHEAMTICS

DRMOTA, 100 MIN.

1) a) Let $A(x)$ be the generating fuction of the sequence (a_n) and $B(x)$ the generating function of the sequence (b_n). What is the generating function of the sequence

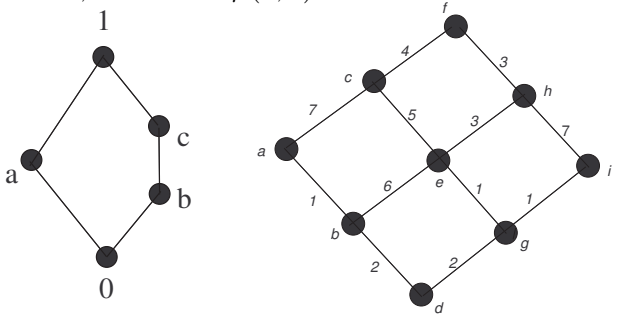
$$c_n = \sum_{k=0}^{n} a_k b_{n-k} \quad ?$$

b) Furthermore let (d_n) be the sequence that is determined by the relation

$$d_n = \sum_{k=0}^{n} d_k - 2n \quad (n \geq 0).$$

Determine the corresponding generating function $D(x)$ and then d_n.

2) Determine the value of the Moebius function $\mu(0,1)$ of the following partial order (on the left hand side).
Furthermore, determine $\mu(0,1)$ if there an additional relation $a \leq c$.



3) Determine with the help of the Dijkstra-Algorithmu a shortest path from f to d (in the right hand side network).

4) a) Which of the following polynomials is irreducible over $\mathbb{Z}_3$:

$$f(x) = x^3 + x^2 + 1, \qquad g(x) = x^3 + 2x + 1.$$

b) Furthermore solve the following system of congruence relations:

$$x^3 \equiv 1 \bmod 3,$$
$$12x \equiv 15 \bmod 21.$$

Remark: Redude the system first to a system of the form $x \equiv a_i \bmod m_i$.

Vienna, March 12, 2021