Einführung in Künstliche Intelligenz SS 2014, 2.0 VU, 184.735

Exercise Sheet 3 – CSPs, Planning, and Making Simple Decisions

For the presentation part of this exercise, mark your solved exercises in **TUWEL** until
Tuesday, June 10, 13:00 CET.
Be sure that you tick only those exercises that you can solve and explain on the blackboard!

Please ask questions in the **TISS** Forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 1 (4 pts.): Consider the following cryptarithmetic puzzle. Every letter corresponds to exactly one digit. In particular, the digits corresponding to different letters are different and S and M should not be 0.

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   S E N D
 + M O R E
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 M O N E Y
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(a) Describe the corresponding CSP with its variables and constraints and specify the initial domain of each variable.

(b) Find a solution of the puzzle.

Exercise 2 (3 pts.):

(a) Given a single ternary constraint $A + B = C$. Transform this constraint into 3 binary constraints achieving the same functionality using auxiliary variables.

(b) Show how constraints with $n \geq 4$ variables can be transformed in a similar way.

(c) Show how unary constraints can be eliminated by altering the domains of variables.

Exercise 3 (2 pts.): Consider the following planning problem: You are given three containers C_1, C_2, and C_3. Thereby, C_1 contains a red package, C_2 a green package, and C_3 is initially empty. Using a roboter arm, you can access the contents of the containers. You can use the following actions:

$$
\begin{aligned}
&\mathbf{Action}(\mathit{Grasp}(x, y, z),\\
&\quad \mathsf{Precond}: \mathit{empty} \wedge \mathit{full}(y, x) \wedge \mathit{pos}(y)\\
&\quad \mathsf{Effect}: \mathit{hold}(x) \wedge \mathit{free}(y) \wedge \mathit{pos}(z) \wedge \neg \mathit{empty} \wedge \neg \mathit{full}(y, x) \wedge \neg \mathit{pos}(y)\\
&\mathbf{Action}(\mathit{Ungrasp}(x, y, z),\\
&\quad \mathsf{Precond}: \mathit{pos}(x) \wedge \mathit{free}(x) \wedge \mathit{hold}(y)\\
&\quad \mathsf{Effect}: \mathit{empty} \wedge \mathit{full}(x, y) \wedge \mathit{pos}(z) \wedge \neg \mathit{pos}(x) \wedge \neg \mathit{free}(x) \wedge \neg \mathit{hold}(y)
\end{aligned}
$$

The meaning of the predicates is as follows:

- empty: roboter arm is empty;
- $\mathit{hold}(x)$: roboter arm holds x;
- $\mathit{pos}(y)$: roboter arm is over container y;
- $\mathit{full}(x, y)$: container x contains package y;
- $\mathit{free}(x)$: container x is empty.

The initial state S is $\{empty, pos(C_1), full(C_1, r), full(C_2, g), free(C_3)\}$.

The goal state is $\{full(C_1, g), full(C_2, r), pos(C_3)\}$.

Find the shortest possible plan for getting from the initial state to the goal state. Use the STRIPS state-space search algorithm starting in S, i.e., use *progression planning*.

Exercise 4 (2 pts.): Consider a situation where a space traveller is in a far away galaxy which is in a state of war consisting of multiple planets. The goal for our adventurer is to find a safe and peaceful planet. Therefore, he explores the different planets of the galaxy by travelling between them. For getting from one planet to another, the two planets must be connected by an intergalactic space ferry which is still operating. Unfortunately, not all space ferries are operating—some might be shut down temporarily or even destroyed. The traveller is able to reinstall space ferries that have been shut down if he is on one of the planets that it connects. Design two STRIPS actions, one for reinstalling space ferries and one for travelling from one planet to a connected one. Introduce variables for modeling the different aspects of this exercise and describe them in detail.

Exercise 5 (2 pts.): Assume there is a lottery with tickets for 1 Dollar and there are two possible prizes: a 100 Dollar prize with a probability of $\frac{1}{500}$, and a 500.000 Dollar prize with a probability of $\frac{1}{1.000.000}$.

(a) What is the expected monetary value of a lottery ticket?

(b) When is it rational to buy a ticket? Give an equation involving utilities. To this end, you may assume a current wealth of possessing k Dollars with $U(S_k) = 0$. Further, $U(S_{k+100}) = 200 \cdot U(S_{k+1})$, but there is no information about $U(S_{k+500.000})$.

Exercise 6 (2 pts.): In 1738, J. Bernoulli investigated the St. Petersburg paradox, which works as follows. You have the opportunity to play a game in which a fair coin its tossed repeatedly until it comes up heads. If the first head appears on the n-th toss, you win 2^n dollars.

(a) Show that the expected monetary value of this game is not finite.

(b) Bernoulli resolved the apparent paradox by suggesting that the utility of money is measured on a logarithmic scale, i.e., $U(S_n) = a\log_2 n + b$, where S_n ($n > 0$) is the state of having n dollars and a, b are constants. What is the expected utitlity of the game under this assumption? Assume, for simplicity, an initial wealth of 0 dollars and that no stake has to be paid in order to play the game.