3. Bestimmen Sie den Erwartungswert und die Varianz der Diskreten Gleichverteilung $D(a, b)$. (hier ist die Formel

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

und natürlich auch die Gauss'sche Summenformel

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

nützlich).

$$\underline{E(x)} = \sum_{i=1}^{n} x_i p_i = \frac{1}{b-a+1} \left[a + a+1 + a+2 + a+3 \,.\, + a+n \right] \qquad n = b-a, \quad b = a+n$$

$$\frac{1}{n+1} \qquad \left[\underbrace{a+a+a+a}_{a\cdot\cancel{(n+1)}} + \dots + 1+2+3+\dots n \right] = a\cdot\cancel{(n+1)} + \frac{n\,\cancel{(n+1)}}{2}$$

$$\Rightarrow E(x) = \left(a + \frac{n}{2} \right) = a + \frac{b-a}{2} = a + \frac{b}{2} - \frac{a}{2} = \underline{\frac{a+b}{2}}$$

$$V(x) = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot p_i = \frac{1}{b-a-1} \sum_{i=1}^{n} (x_i - \mu)^2 \qquad \mu = \frac{a+b}{2}$$

$$\sum_{i=1}^{n} (x_i - \mu)^2 p_i = \left[(a-\mu)^2 + (a+1-\mu)^2 + (a+2-\mu)^2 \dots (n+a-\mu)^2 \right] \frac{1}{b-a+1}$$

$$a - \mu = a - \frac{a+b}{2} = \frac{a-b}{2}$$

$$\rightarrow \left[\left(\frac{a-b}{2}\right)^2 + \left(\frac{a-b}{2} + 1\right)^2 + \left(\frac{a-b}{2} + 2\right)^2 \right] \qquad y = \frac{a-b}{2}$$

$0 + 0 + y^2$

$1 + 2y + y^2$

$4 + 4y + y^2$

$9 + 6y + y^2$

$16 + 8y + y^2$

$n^2 + 2ny + y^2 \qquad y = \frac{a-b}{n} \qquad n = b-a$

$$\Sigma = \frac{n(n+1)(2n+1)}{6} + \frac{2y\, n(n+1)}{2} + (n+1)y^2$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \mu)^2 p_i = \frac{1}{\cancel{n+1}} \left(\frac{n\,\cancel{(n+1)}(2n+1)}{6} + y\,n\,\cancel{(n+1)} + \cancel{(n+1)}\,y^2 \right.$$

$$= \frac{2n+1}{6} + yn + y^2 = \frac{(b-a)(2(b-a)+1)}{6} + \frac{a-b}{2}(b-a) + \frac{(a-b)^2}{4}$$

$$= \frac{(b-a)(2(b-a)+1)}{6} + \frac{a-b}{2} \cdot (b-a) + \frac{(b-a)^2}{4}$$

$$b-a \cdot \left(\frac{2b-2a+1}{6} + \frac{a-b}{2} + \frac{b-a}{4} \right) = (b-a) \cdot \left(\frac{4b-4a+1 + 6a-6b + 3b-3a}{12} \right)$$

$$= \frac{b-a}{12} \cdot \left(\frac{b-a+2}{1} \right)$$

1. Beweisen Sie die Behauptung aus dem Skriptum: $X \sim \Gamma(\alpha,\lambda)$ und $Y \sim \Gamma(\beta,\lambda)$ seien unabhängig, $S = X+Y$ und $Q = X/Y$. Dann ist S nach $\Gamma(\alpha+\beta,\lambda)$ verteilt, Q hat eine Betaverteilung zweiter Art, und S und Q sind unabhängig.

$$g(x,y) = \left(x+y, \frac{x}{y}\right), \quad g^{-1}(s,q) = \left(\frac{sq}{1+q}, \frac{s}{q+1}\right)$$

$$\Rightarrow \frac{dg^{-1}(s,q)}{d(s,q)} = \begin{vmatrix} \frac{q}{1+q} & \frac{1}{1+q} \\ \frac{s}{(1+q)^2} & -\frac{s}{(1+q)^2} \end{vmatrix} =$$

$$= \frac{-qs}{(1+q)^3} - \frac{s}{(1+q)^3} = \frac{-s(q+1)}{(1+q)^{\cancel{3}\,2}} = \frac{-s}{(1+q)^2}$$

$$\Rightarrow f_{S,Q}(s,q) = \left|\frac{dg^{-1}(s,q)}{d(s,q)}\right| f_X\left(\frac{sq}{1+q}\right) f_Y\left(\frac{s}{1+q}\right)$$

$$= \frac{s}{(1+q)^2} \frac{\lambda^\alpha}{\Gamma(\alpha)} \left(\frac{sq}{1+q}\right)^{\alpha-1} e^{-\lambda\left(\frac{sq}{1+q}\right)} \cdot \frac{\lambda^\beta}{\Gamma(\beta)} \cdot \left(\frac{s}{1+q}\right)^{\beta-1} \cdot e^{-\lambda\left(\frac{s}{1+q}\right)} =$$

$$= \frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)} \frac{s^{\alpha-1} \cdot q^{\alpha-1}}{(1+q)^{\alpha-1}} \cdot \frac{s^{\beta-1}}{(1+q)^{\beta-1}} \cdot e^{-\lambda\left(\frac{\overbrace{sq+s}^{s(q+1)}}{1+q}\right)}$$

$$= \frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)} \cdot \underbrace{s^{\alpha+\beta-1} e^{-\lambda s}}_{\text{nur von } s} \cdot \underbrace{\frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}}}_{\text{nur von } q} \Rightarrow S \text{ und } Q \text{ unabhängig}$$

für x:

$s = x+y \qquad y = s-x$

$Q = \frac{x}{y} \qquad y = \frac{x}{q} \Rightarrow s - x = \frac{x}{q}$

$\Rightarrow sq = x + xq$

$\Rightarrow x = \frac{sq}{1+q}$

analog f. y:

$x = s-y$

$x = qy \qquad \Rightarrow s - y = qy \rightarrow s = y \cdot (q+1)$

$\Rightarrow y = \frac{s}{q+1}$

$$\frac{\partial}{\partial q}\left(\frac{sq}{1+q}\right) = \frac{s}{(1+q)^2}$$

$$= \frac{\lambda^{\alpha+\beta} \cdot s^{\alpha+\beta-1}\, e^{-\lambda s}}{\Gamma(\alpha)\Gamma(\beta)} \cdot \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}}$$

für $s>0$

→ Randverteilungen:

$$F_Q(q) = \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}} \cdot \int_0^\infty \frac{\lambda^{\alpha+\beta} \cdot s^{\alpha+\beta-1} \cdot e^{-\lambda s}}{\Gamma(\alpha)\Gamma(\beta)}\, ds =$$

$$= \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}} \cdot \frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)} \cdot \underbrace{\int_0^\infty s^{\alpha+\beta-1} \cdot e^{-\lambda s}\, ds}_{\Gamma(\alpha+\beta)} = \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}} \underbrace{\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}}_{\frac{1}{B(\alpha,\beta)}} \cdot \lambda^{\alpha+\beta}$$

$$= \lambda^{\alpha+\beta} \cdot \underbrace{\frac{1}{B(\alpha,\beta)} \cdot \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}}}_{B_2(\alpha,\beta)} = \lambda^{\alpha+\beta} \cdot \underline{B_2(\alpha,\beta)} = B_2(\alpha,\beta) \text{ (für } \lambda=1)$$

$$F_S(s) = \frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)} \cdot s^{\alpha+\beta-1} \cdot e^{-\lambda s}\, ds \cdot \int_0^\infty \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}}\, dq$$

$\Rightarrow$ damit $F_S(s) \sim \Gamma(\alpha+\beta, \lambda)$ muss $\int_0^\infty \frac{q^{\alpha-1}}{(1+q)^{\alpha+\beta}}\, dq \overset{!}{=} \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = B(\alpha,\beta) = \int_0^1 q^{\alpha-1} \cdot (1+q)^{\beta-1}$

mies, probieren wirs anders

Mit Faltung?: $f * g(z) = \int_0^z f(u)\, g(s-u)\, dx$

$$f_X(x) = \frac{\lambda^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x}$$

$$f_Y(x) = \frac{\lambda^\beta x^{\beta-1}}{\Gamma(\beta)} e^{-\lambda}$$

$$\Rightarrow f_S(s) = \int_0^\infty \frac{\lambda^\alpha x^{\alpha-1}}{\Gamma(\alpha)} \cdot \frac{\lambda^\beta (s-x)^{\beta-1}}{\Gamma(\beta)} e^{-\lambda x} e^{-\lambda(s-x)} dx$$

$$\underline{f_S(s)} = \frac{\lambda^{\alpha+\beta} e^{-\lambda s}}{\Gamma(\alpha)\Gamma(\beta)} \cdot \int_0^\infty x^{\alpha-1} (s-x)^{\beta-1} dx$$

$s \in [0,\infty]$, $x \in [0,\infty]$ $\to x = st \Rightarrow t \in [0,1]$ darf man das?

Subst. $x = st$ (Stack-Exchange)

$$e^{-\lambda x - \lambda s + \lambda x} = e^{-\lambda s}$$

$$= \frac{\lambda^{\alpha+\beta} e^{-\lambda s} s^{\alpha-1}}{\Gamma(\alpha)\Gamma(\beta)} \underbrace{\int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt}_{B(\alpha,\beta)} = \frac{e^{-\lambda s} s^{\alpha-1}}{\cancel{\Gamma(\alpha)\Gamma(\beta)}} \lambda^{\alpha+\beta} \cdot \frac{\cancel{\Gamma(\alpha)\Gamma(\beta)}}{\Gamma(\alpha+\beta)}$$

$$= \frac{e^{-\lambda s} s^{\alpha-1}}{\Gamma(\alpha+\beta)} \cdot \lambda^{\alpha+\beta} \sim \underline{\Gamma(\alpha+\beta, \lambda)}$$

4. Bestimmen Sie den Erwartungswert und die Varianz der Gammaverteilung $\Gamma(\alpha, \lambda)$.

$$\underline{\mathbb{E}(X)} = \int_0^\infty x \cdot \frac{\lambda^\alpha \cdot x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x} \cdot \frac{\lambda}{\lambda} \, dx = \frac{1}{\Gamma(\alpha) \cdot \lambda} \cdot \underbrace{\int_0^\infty \frac{\lambda^{\alpha+1} \cdot x^\alpha}{\Gamma(\alpha)} \cdot e^{-\lambda x} \, dx}_{\Gamma(\alpha+1)}$$

$$= \frac{\Gamma(\alpha+1)}{\Gamma(\alpha) \cdot \lambda} = \frac{\cancel{\Gamma(\alpha)} \cdot \alpha}{\cancel{\Gamma(\alpha)} \cdot \lambda} = \underline{\frac{\alpha}{\lambda}}$$

$$\mathbb{V}(X) = \mathbb{E}(X^2) - E(X)^2$$

$$\Rightarrow E(X^2) = \int_0^\infty x^2 \cdot \frac{\lambda^\alpha \cdot x^{\alpha-1}}{\Gamma(\alpha)} \cdot e^{-\lambda x} \cdot \frac{\lambda^2}{\lambda^2} = \frac{1}{\Gamma(\alpha) \cdot \lambda^2} \cdot \underbrace{\int_0^\infty \frac{x^{\alpha+1} \cdot \lambda^{\alpha+2}}{\Gamma(\alpha)} \cdot e^{-\lambda x} \, dx}_{\Gamma(\alpha+2)}$$

$$= \frac{\Gamma(\alpha+2)}{\Gamma(\alpha) \cdot \lambda^2} = \frac{\Gamma(\alpha+1) \cdot (\alpha+1)}{\Gamma(\alpha) \cdot \lambda^2} = \frac{\cancel{\Gamma(\alpha)} \cdot (\alpha^2+\alpha)}{\cancel{\Gamma(\alpha)} \cdot \lambda^2} = \frac{\alpha^2+\alpha}{\lambda^2}$$

$$\Rightarrow \underline{\mathbb{V}(X)} = E(X^2) - E(X)^2 = \frac{\alpha^2+\alpha}{\lambda^2} - \frac{\alpha^2}{\lambda^2} = \underline{\frac{\alpha}{\lambda^2}}$$

Whrsch.
Blatt 4
S. 2

7) $f_{X,Y}(x,y) = (x+y)\ [0 \leq x, y \leq 1]$

$$F(x,y) = \int_0^1\int_0^1 f(x,y)\,dx\,dy = \int_0^1\int_0^1 x+y\,dx\,dy = \int_0^1 \left(\frac{x^2}{2}+xy\right)\Big|_0^1 dy =$$

$$= \int_0^1 \frac{1^2}{2} + 1\cdot y - \frac{0^2}{2} - 0\cdot y\,dy = \left(\frac{1}{2}\cdot y + \frac{y^2}{2}\right)\Big|_0^1 =$$

$$= \frac{1}{2} + \frac{1^2}{2} - \frac{0}{2} - \frac{0^2}{2} = \frac{1}{2} + \frac{1}{2} = \underline{\underline{1}} \Rightarrow \text{Dichte}$$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 (x+y)\,dy = \left(xy + \frac{y^2}{2}\right)\Big|_0^1 =$$

$$= x\cdot 1 + \frac{1^2}{2} - x\cdot 0 - \frac{0^2}{2} = x + \frac{1}{2}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 (x+y)\,dx = \frac{1}{2} + y$$

$$E(X) = \int_{-\infty}^{\infty} x\cdot f_X(x)\,dx = \int_0^1 x\cdot\left(x+\frac{1}{2}\right)dx = \int_0^1 x^2 + \frac{1}{2}x\,dx =$$

$$= \left(\frac{x^3}{3} + \frac{x^2}{4}\right)\Big|_0^1 = \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

$$E(Y) = \int_{-\infty}^{\infty} y\cdot f_Y(y)\,dy = \int_0^1 y\cdot\left(y+\frac{1}{2}\right)dy = \frac{7}{12}$$

$$E(XY) = \int_0^1\int_0^1 xy\, f_{X,Y}(x,y)\,dx\,dy = \int_0^1\int_0^1 xy(x+y)\,dx\,dy =$$

$$= \int_0^1\int_0^1 x^2y + xy^2\,dx\,dy = \int_0^1 \left(\frac{x^3y}{3} + \frac{x^2y^2}{2}\right)\Big|_0^1 dy = \int_0^1 \frac{y}{3} + \frac{y^2}{2}\,dy =$$

$$= \left(\frac{y^2}{6} + \frac{y^3}{6}\right)\Big|_0^1 = \frac{1}{6} + \frac{1}{6} = \underline{\frac{1}{3}}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{1}{3} - \frac{7}{12}\cdot\frac{7}{12} = \frac{1}{3} - \frac{49}{144} = -\frac{1}{144}$$

$$V(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \int_0^1 x^2 f_X(x^2)\,dx = \int_0^1 x^2\left(x^2+\frac{1}{2}\right)dx = \int_0^1 x^4 + \frac{x^2}{2}\,dx =$$

$$= \left(\frac{x^5}{5} + \frac{x^3}{6}\right)\Big|_0^1 = \frac{1}{5} + \frac{1}{6} = \frac{11}{30} = E(Y^2)$$

$$\mathbb{V}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \frac{7}{12} - \frac{11}{30} = \frac{13}{60}$$

$$\mathbb{V}(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 = \frac{7}{12} - \frac{11}{30} = \frac{13}{60}$$

$$\varrho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathbb{V}(X)\,\mathbb{V}(Y)}} = \frac{-\frac{1}{144}}{\sqrt{\left(\frac{13}{60}\right)^2}} = -\frac{60}{13 \cdot 144} = -\frac{5}{156}$$

2)

$$P_f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$\rightarrow E(X) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!} =$$

$$= \sum_{x=1}^{\infty} \frac{x \cdot \lambda^x \cdot e^{-\lambda}}{x!} = \sum_{x=1}^{\infty} \frac{\lambda^x e^{-\lambda}}{(x-1)!}$$

$$\Updownarrow$$

$$\lambda \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^x \cdot e^{-\lambda}}{x!}}_{\stackrel{!}{=} \sum_{0}^{\infty} Pf(x) = 1} \leftrightarrow \lambda = E(X)$$

$$\rightarrow V(X) = E(X^2) - E(X)^2 =$$

$$= \sum_{x=0}^{\infty} x^2 \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!} - \lambda^2$$

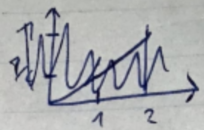
$$= \sum_{x=1}^{\infty} x \cdot \frac{\lambda^x \cdot e^{-\lambda}}{(x-1)!} - \lambda^2 = \lambda \cdot \sum_{x=0}^{\infty} (x+1) \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!} - \lambda^2$$

$$= \lambda \cdot \underbrace{\sum_{x=0}^{\infty} x \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!}}_{\lambda} + \lambda \cdot \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^x \cdot e^{-\lambda}}{x!}}_{1} - \lambda^2 = \lambda^2 + \lambda - \lambda^2$$

$$= \underline{\lambda = V(X)}$$

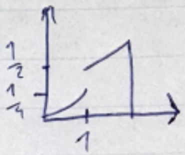
5/ $X := F_X(x) = \begin{cases} 0 & x<0 \\ x^2/4 & 0\le x<1 \\ x/2 & 1\le x<2 \\ 1 & x\ge 2 \end{cases}$

$F_X(x) =$

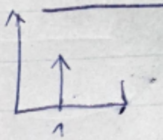


$\Rightarrow$ Dichte $\Leftrightarrow$ $\dfrac{d\,F_X(x)}{dx} = f_X(x) = \begin{cases} 0 & x<0 \\ x/2 & 0\le x<1 \\ 1/2 & 1\le x<2 \\ 0 & x\ge 2 \end{cases}$

$F_X :=$



$p_X(x) =$



$\rightarrow p_X(x) = \begin{cases} \frac{1}{4} & x=1 \\ 0 & x\neq 1 \end{cases}$

$E(X) = \sum_{x=0}^{1} x\cdot p_X(x) + \int_0^1 x\cdot\frac{x}{2}\,dx + \int_1^2 \frac{x}{2}\,dx$

$= \frac{1}{4} + \left[\frac{1}{6}x^3 \Big|_0^1\right] + \left[\frac{x^2}{4}\Big|_1^2\right] = \frac{1}{4} + \frac{1}{6} + 1 - \frac{1}{4}$

$= \frac{7}{6}$

~~$V(X) = E(X^2) - E(X)^2$~~

$= \int$ [illegible]

$$V(X) = E(X^2) - E(X)^2$$

$$\rightarrow \quad \frac{1}{4} + \int_0^1 \frac{x^3}{2}\,dx + \int_1^2 \frac{x^2}{2}\,dx - \left(\frac{7}{6}\right)^2$$

$$\frac{1}{4} + \frac{1}{8} + \left[\frac{1}{6}x^3 \Big|_1^2 \right] + \left(\frac{7}{6}\right)^2 = \frac{1}{4} + \frac{1}{8} + \frac{8}{6} - \frac{1}{6} + \left(\frac{7}{6}\right)^2$$

$$= \frac{9}{24} + \frac{28}{24} - \left(\frac{7}{6}\right)^2 = \frac{37}{24} - \frac{49}{36} = \underline{\frac{13}{72}}$$

8) $$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{\mathbb{V}(X)\,\mathbb{V}(Y)}} = \frac{\mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)}{\sqrt{(\mathbb{E}(X^2) - \mathbb{E}(X)^2)(\mathbb{E}(Y^2) - \mathbb{E}(Y)^2)}}$$

$$\mathbb{E}(X) = \sum_x x p_X(x)$$

$$p_X(X=1) = \left(\frac{3}{9}\cdot\frac{6}{8}\cdot\frac{5}{7}\right)\cdot 3 = \frac{15}{28}$$

$$p_X(X=2) = \left(\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{6}{7}\right)\cdot 3 = \frac{3}{14}$$

$$p_X(X=3) = \frac{3}{9}\cdot\frac{2}{8}\cdot\frac{1}{7} = \frac{1}{84}$$

$$p_X(X=0) = \frac{6}{9}\cdot\frac{5}{8}\cdot\frac{4}{7} = \frac{5}{21}$$

$$p(y) = p(x)$$

$$E(x) = \sum_{x=0}^{3} x\cdot p_X(x) = 0\cdot p_X(0) + 1\cdot p_X(1) + 2\cdot p_X(2) + 3\cdot p_X(3) =$$

$$= 0 + \frac{15}{28} + \frac{2\cdot 3}{14} + \frac{3\cdot 1}{84} = \frac{15}{28} + \frac{3}{7} + \frac{1}{28} = \frac{16}{28} + \frac{12}{28} = \frac{28}{28} = 1$$

$$\mathbb{E}(Y) = \mathbb{E}(X) = 1$$

$$\mathbb{E}(XY) = \sum_{x=0}^{3}\sum_{y=0}^{3} x\cdot y\cdot p(x,y)$$

$$p(X=0, Y=0) = \frac{3}{9}\cdot\frac{2}{8}\cdot\frac{1}{7} = \frac{1}{84}$$

$$p(X=0, Y=1) = \left(\frac{3}{9}\cdot\frac{3}{8}\cdot\frac{2}{7}\right)\cdot 3 = \frac{3}{28}$$

$$p(X=0, Y=2) = \left(\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{3}{7}\right)\cdot 3 = \frac{3}{28}$$

$$p(X=0, Y=3) = \left(\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{1}{7}\right) = \frac{1}{84}$$

$p(X=1, Y=0) = \left(\frac{3}{9}\cdot\frac{3}{8}\cdot\frac{2}{7}\right)\cdot 3 = \frac{3}{28}$

$p(X=1, Y=1) = \left(\frac{3}{9}\cdot\frac{3}{8}\cdot\frac{3}{7}\right)\cdot 3 = \frac{9}{56}$

$p(X=1, Y=2) = \left(\frac{3}{9}\cdot\frac{3}{8}\cdot\frac{2}{7}\right)\cdot 3 = \frac{3}{28}$

$p(X=1, Y=3) = 0$

$p(X=2, Y=0) = \left(\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{3}{7}\right)\cdot 3 = \frac{3}{28}$

$p(X=2, Y=1) = \left(\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{3}{7}\right)\cdot 3 = \frac{3}{28}$

$p(X=2, Y=2) = 0$

$p(X=2, Y=3) = 0$

$p(X=3, Y=0) = \frac{3}{9}\cdot\frac{2}{8}\cdot\frac{1}{7} = \frac{1}{84}$

$p(X=3, Y=1) = 0$

$p(X=3, Y=2) = 0$

$p(X=3, Y=3) = 0$

$E(XY) = \sum_{x=0}^{3}\sum_{y=0}^{3} x\,y\,p(x,y) = \cancel{0\cdot 0\cdot\frac{1}{84}} + \cancel{0\cdot 1\cdot\frac{3}{28}} + \cancel{0\cdot 2\cdot\frac{3}{28}} +$
$+ \cancel{0\cdot 3\cdot\frac{1}{84}} + \cancel{1\cdot 0\cdot\frac{3}{28}} + 1\cdot 1\cdot\frac{9}{56} + 1\cdot 2\cdot\frac{3}{28} + \cancel{1\cdot 3\cdot 0} + \cancel{2\cdot 0\cdot\frac{3}{28}} +$
$+ 2\cdot 1\cdot\frac{3}{28} + \cancel{2\cdot 2\cdot 0} + \cancel{2\cdot 3\cdot 0} + \cancel{3\cdot 0\cdot\frac{1}{84}} + \cancel{3\cdot 1\cdot 0} + \cancel{3\cdot 2\cdot 0} + \cancel{3\cdot 3\cdot 0} =$
$= \frac{9}{56} + \frac{6}{28} + \frac{6}{28} = \frac{33}{56}$

$E(X^2) = \sum_{x=0}^{3} x^2 p(x) = 0^2\cdot p_X(0) + 1^2\cdot p_X(1) + 2^2\cdot p_X(2) + 3^2 p_X(3) =$
$= 0 + \frac{1\cdot 15}{28} + \frac{4\cdot 3}{14} + \frac{9\cdot 1}{84} = \frac{3}{2}$

$E(Y^2) = E(X^2)$

$E(X)^2 = E(Y)^2 = 1^2 = 1$

$\rho = \frac{E(XY) - E(X)E(Y)}{\sqrt{(E(X^2)-E(X)^2)(E(Y^2)-E(Y)^2)}} = \frac{\frac{33}{56} - 1\cdot 1}{\sqrt{(\frac{3}{2}-1)(\frac{3}{2}-1)}} =$

$= -\frac{\frac{23}{56}}{(\frac{3}{2}-1)} = -\frac{\frac{23}{56}}{\frac{1}{2}} = -\frac{23\cdot 2}{56} = -\frac{23}{28}$