Hybrid Systems
Modeling, Analysis and Control

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Aims of the Course

Where do we find such systems?

Your mobile phone, your car, your washer, your home

Your energy supplier, your public transportation, your cells

What are the consequences?

The infrastructure of our society relies on their dependability

However, modeling, analysis and control is very challenging

What are you going to learn?

Mathematical principles underlying such systems

How to model, analyse and control hybrid systems

Course Organization

182.732 VU Hybrid Systems (3 ECTS):

Dedicated to teaching the fundamentals of CPS

No homeworks, but with a final exam. Midterm wanted?

182.733 LU Hybrid Systems (3 ECTS, Optional):

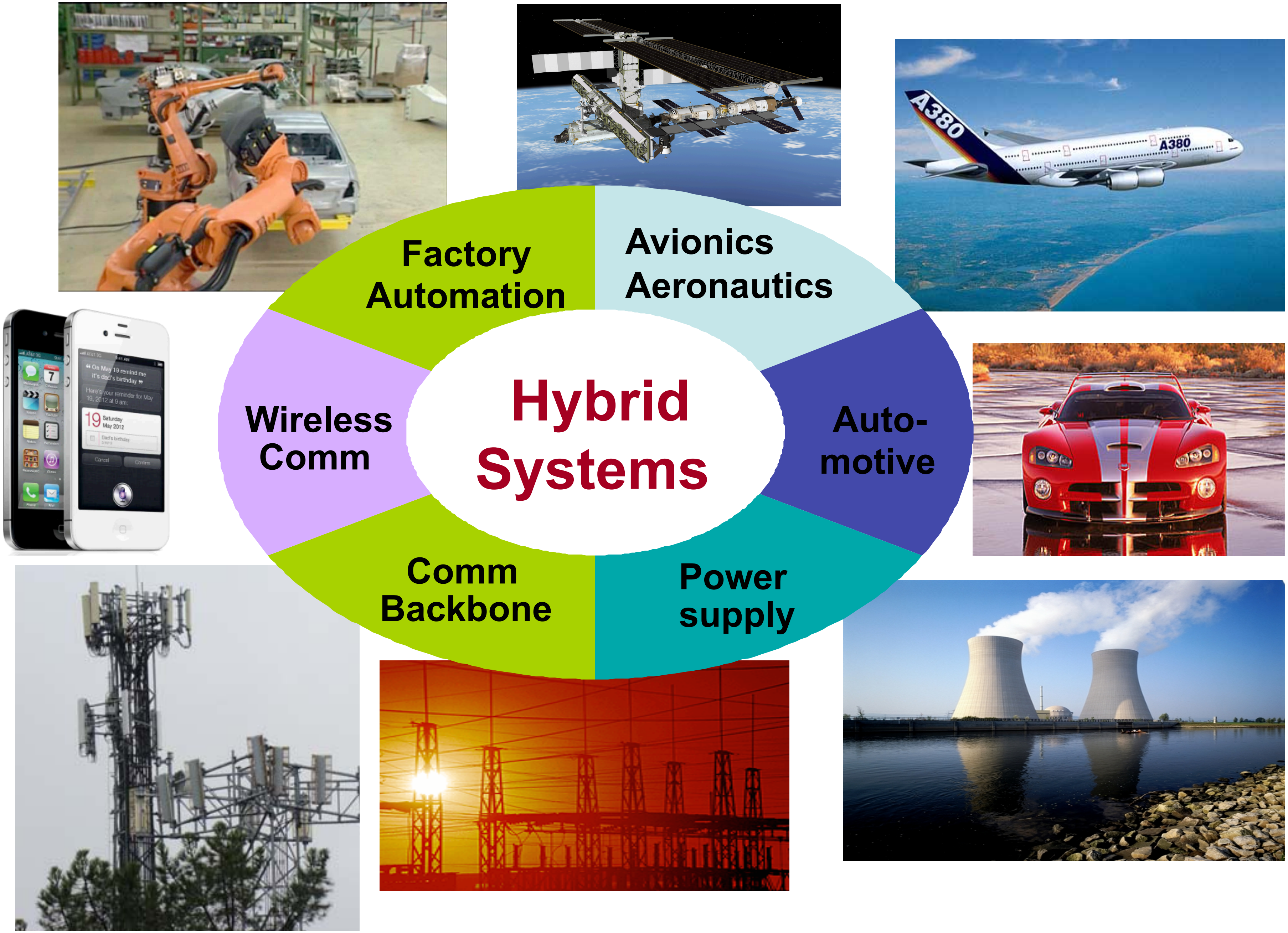
Dedicated to applying the knowledge acquired in the VU

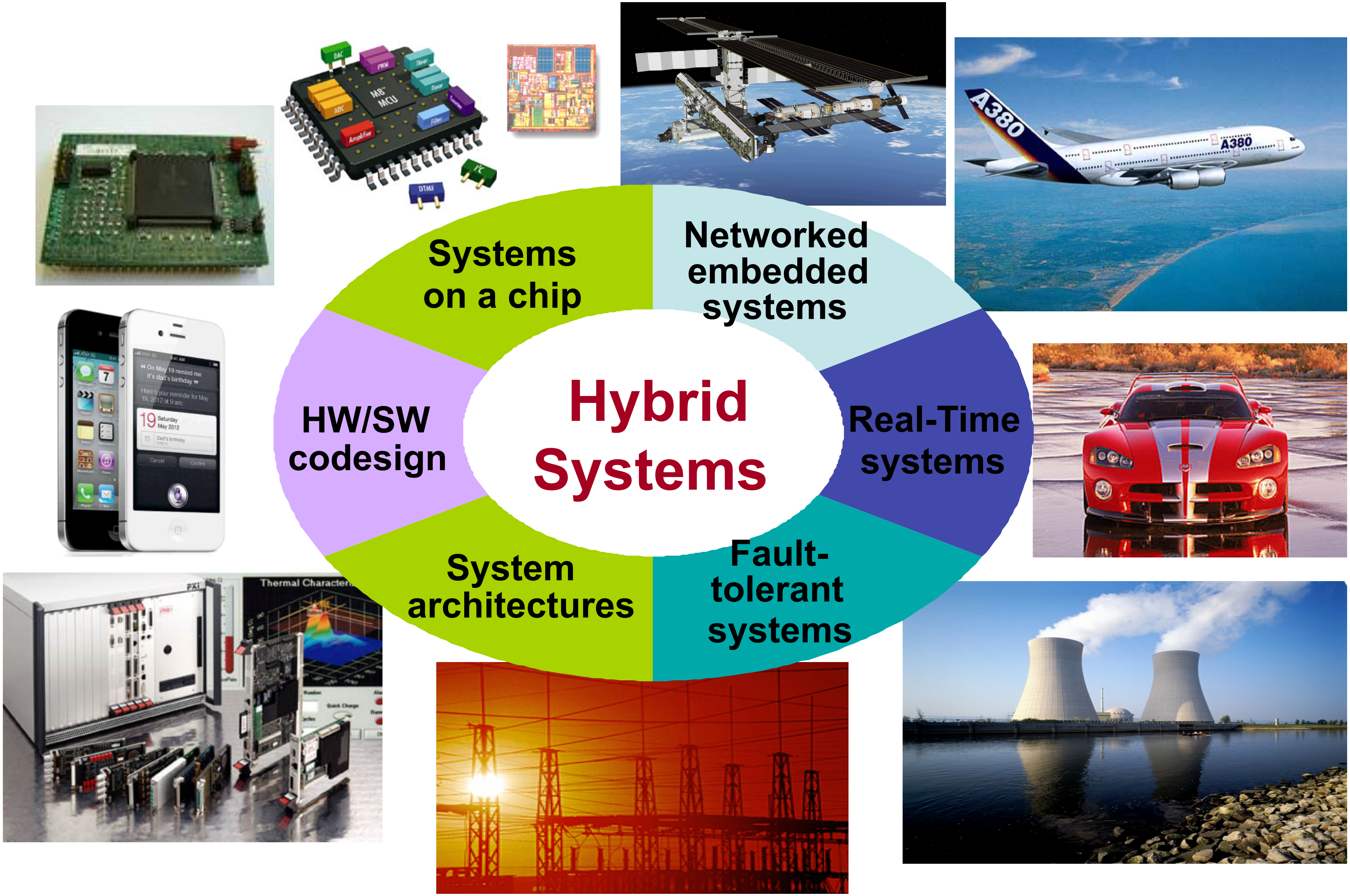
A group project. You may also propose your own project.

Computer network with engine and wings

Computer with eyes, ears and voice

Computer network with engine and wheels


Hybrid Systems
Factory Automation
Avionics Aeronautics
Auto-motive
Power supply
Comm Backbone
Wireless Comm
A380


Hybrid Systems
Systems on a chip
Networked embedded systems
Real-Time systems
Fault-tolerant systems
System architectures
HW/SW codesign
A380

Prerequisites

Computer Science:

Finite automata theory, logics and boolean algebra
Abstraction, temporal logics, formal verification

Control Theory:

Differential and difference equations, linear algebra
Approximation, observability, controllability, stability

Literature: Books

- **Lygeros, Tomlin, Sastry.** Hybrid Systems: Modeling analysis and control
- **Tabuada.** Verification and control of hybrid systems: A symbolic approach
- **Lee and Varaiya.** Structure and interpretation of signals and systems

- **Alur.** Principles of Embedded Computation
- **Lee and Seshia.** Introduction to Embedded Systems: A CPS Approach
- **Clarke, Grumberg and Peled.** Model checking

Literature: Articles

R. Alur and D. Dill. A theory of timed automata. Theoretical Computer Science 126:183 – 235, 1994 (prelim. versions app. in Proc. of 17th ICALP, LNCS 443, 1990, and Real Time: Theory in Practice, LNCS 600, 1991

R. Alur, C. Courcoubetis, N. Halbwachs, T.A. Henzinger, P.-H. Ho, X. Nicollin, A. Olivero, J. Sifakis, S. Yovine. The Algorithmic Analysis of Hybrid Systems. Theoretical Computer Science 138:3-34, 1995

T.A. Henzinger. The Theory of Hybrid Automata. Proceedings of LICS'96, the 11th Annual Symposium on Logic in Computer Science, IEEE Computer Society Press, pp. 278-292, 1996.

A. Chutinan and B.H. Krogh. Computing Polyhedral Approximations to Flow Pipes for Dynamic Systems. In CDC'98, the 37th IEEE Conference on Decision and control, pp. 2089 – 2095, IEEE Press, 1998.

Literature: Articles

R. Alur, T.A. Henzinger, G. Lafferriere, and G.J. Pappas. Discrete Abstractions of Hybrid Systems. Proceedings of the IEEE, 2000.

T.A. Henzinger and R. Majumdar. Symbolic Model Checking for Rectangular Hybrid Systems. In TACAS'00, the Proc. of the 6th Int. Conf. on Tools and Algorithms for the Construction and Analysis of Systems, LNCS 1785, pp. 142 – 156, Springer, 2000.

R. Alur, R. Grosu, Y. Hur, V. Kumar, and I. Lee. Modular Specification of Hybrid Systems in CHARON. In Proc. of HSCC'00, the 3rd Int. Conf. on Hybrid Systems: Computation and Control, Pittsburgh, March, 2000, LNCS 179, pp. 6 – 19, Springer, 2000.

Literature: Articles

R. Alur, R. Grosu, I. Lee, O. Sokolsky. Compositional Refinement for Hierarchical Hybrid Systems. In Proc. of HSCC'01, the 4th International Conf. on Hybrid Systems: Computation and Control, Rome, Italy, March, 2001, pp. 33 – 49, Springer, LNCS 2034.

G. Batt, C. Belta and R. Weiss. Model Checking Genetic Regulatory Networks with Parameter Uncertainty. In Proc. of *HSCC'07, the 10th Int. Conf. on Hybrid Systems : Computation and Control*, Pisa, Italy, 2007.

C. Le Guernic and A. Girard. Reachability Analysis of Linear Systems using Support Functions. Nonlinear Analysis: Hybrid Systems, 42(2):250 – 262, Electronic Edition, 2010.

Literature: Articles

C. Le Guernic and A. Girard. Reachability Analysis of Linear Systems using Support Functions. Nonlinear Analysis: Hybrid Systems, 42(2):250 – 262, Electronic Edition, 2010.

G. Frehse, C. Le Guernic, A. Donze, R. Ray, O. Lebeltel, R. Ripado, A. Girard, T. Dang, O. Maler. SpaceEx: Scalable Verication of Hybrid Systems. In Proc. of *CAV'11, The 23rd Int. Conf. on Computer Aided Verification*, Snowbird, USA, LNCS 6806, pp. 379 – 395, 2011.

R. Grosu, G. Batt, F. Fenton, J. Glimm, C. Le Guernic, S.A. Smolka and E. Bartocci. From Cardiac Cells to Genetic Regulatory Networks. In Proc. of CAV'11, the *23rd Int. Conf. on Computer Aided Verification*, Cliff Lodge, Snowbird, Utah, USA, July, 2011, pp. 396 – 411, Springer, LNCS 6806.

Verification Tools for Hybrid Systems

HyTech: LHA

http://embedded.eecs.berkeley.edu/research/hytech/

PHAVer: LHA + affine dynamics

http://www-verimag.imag.fr/~frehse/

d/dt: affine dynamics + controller synthesis

http://www-verimag.imag.fr/~tdang/Tool-ddt/ddt.html

Matisse Toolbox: zonotopes

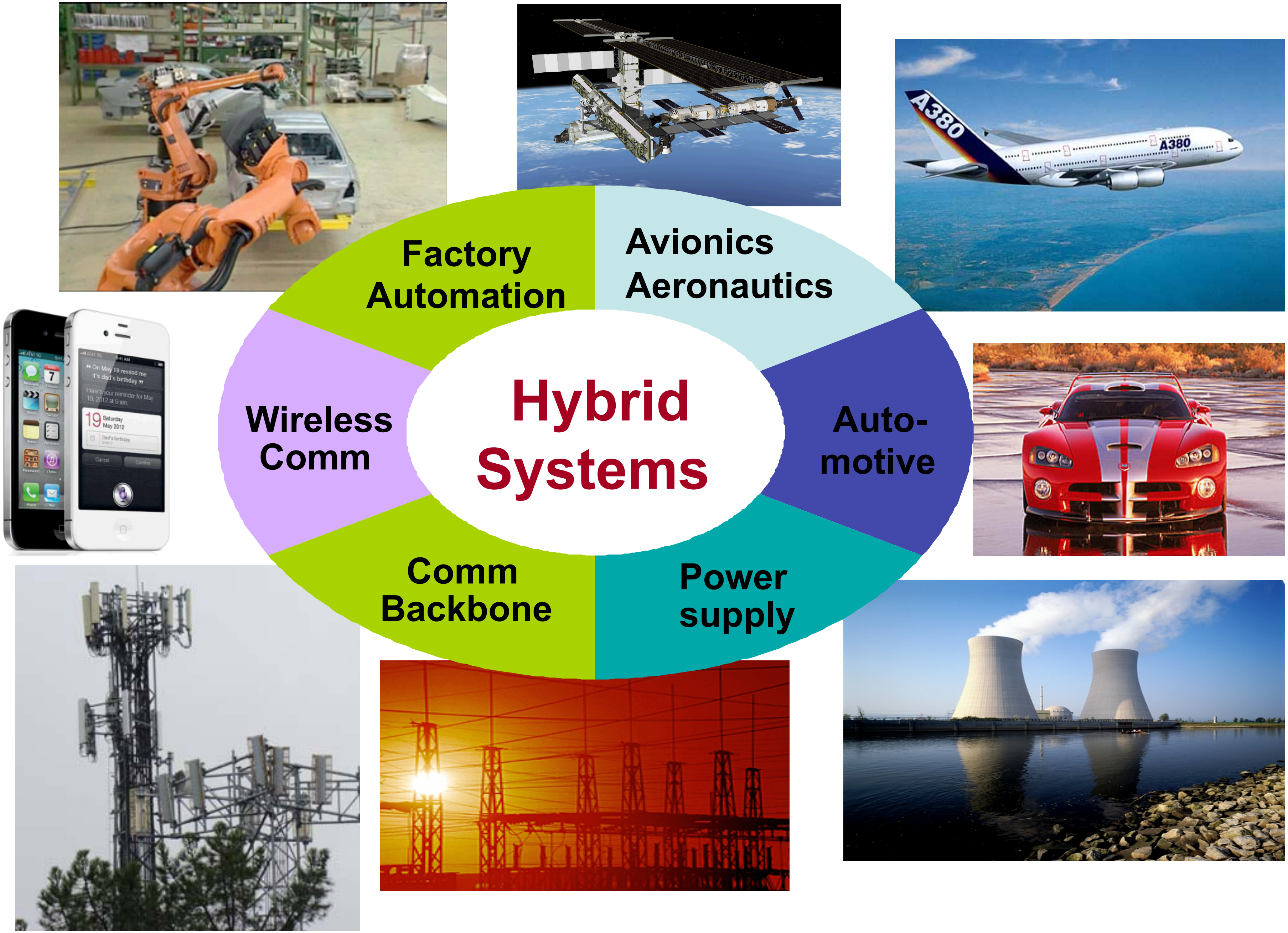
http://www.seas.upenn.edu/~agirard/Software/MATISSE/

HSOLVER: nonlinear systems

http://hsolver.sourceforge.net/

SpaceEx: LHA + affine dynamics

http://spaceex.imag.fr/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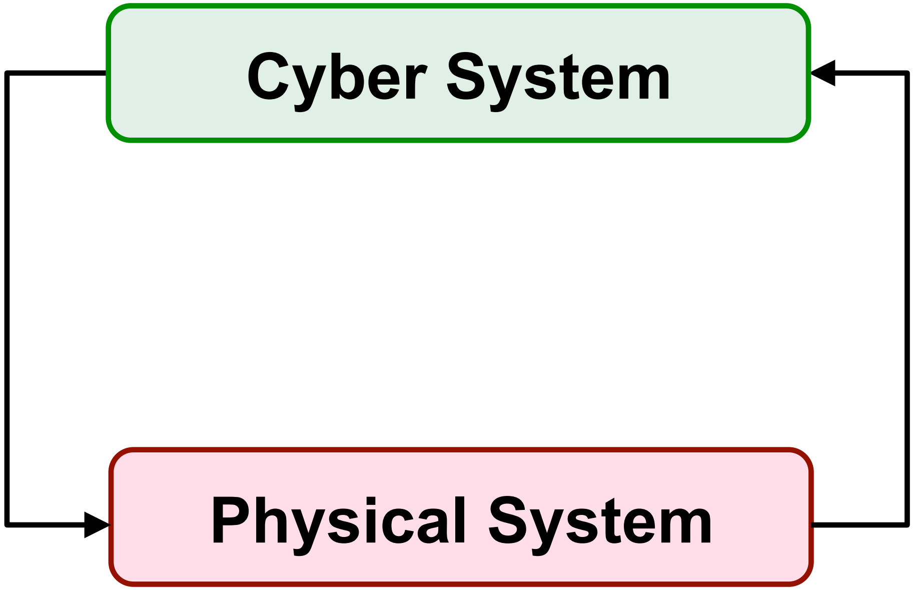
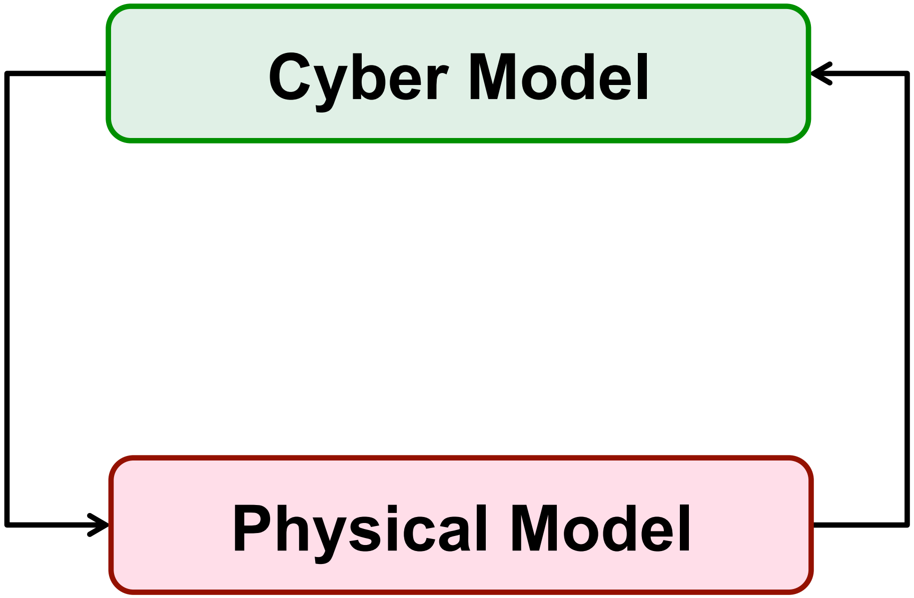
Cyber-Physical System

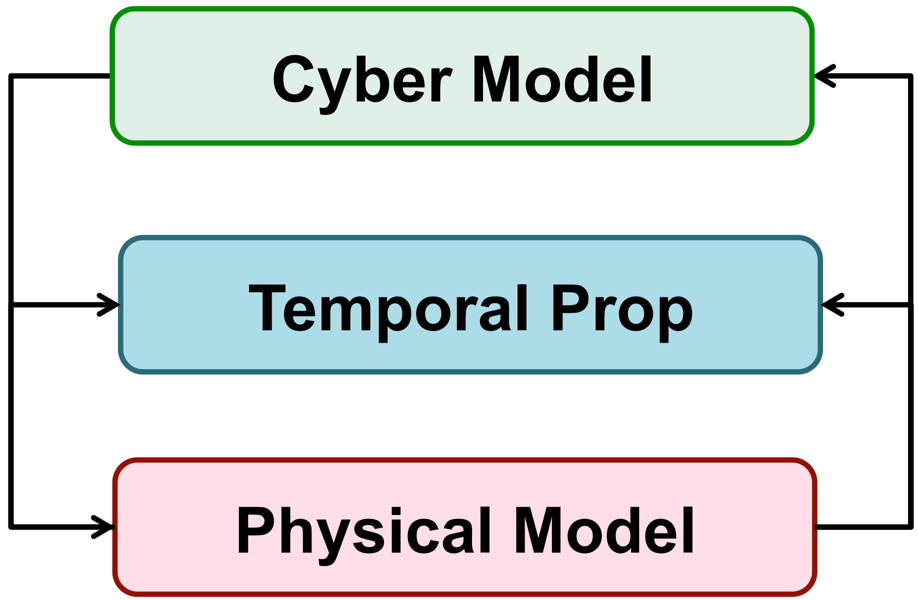


Cyber-Physical Systems

Cyber-Physical Models

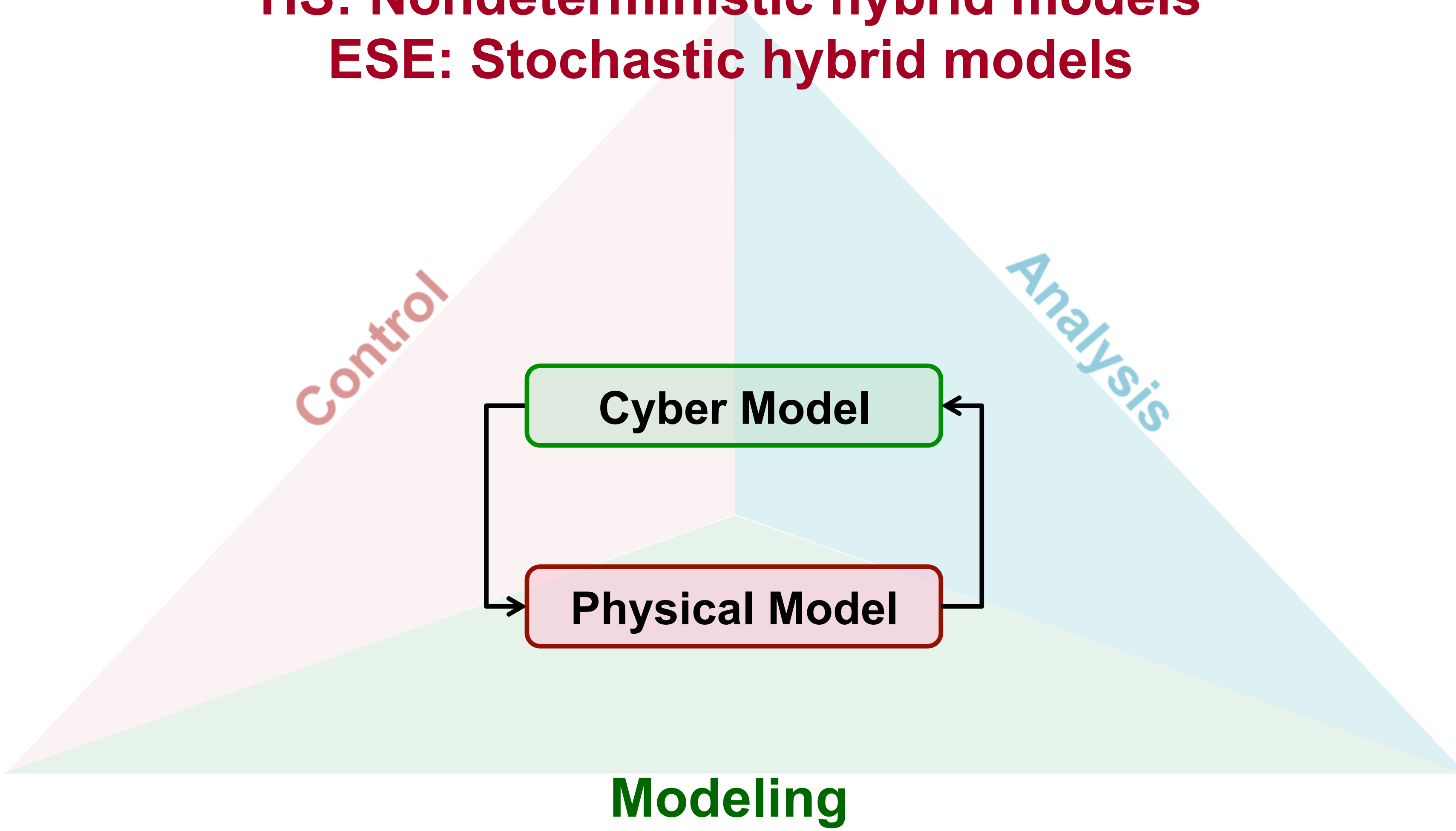


Analysis and Synthesis



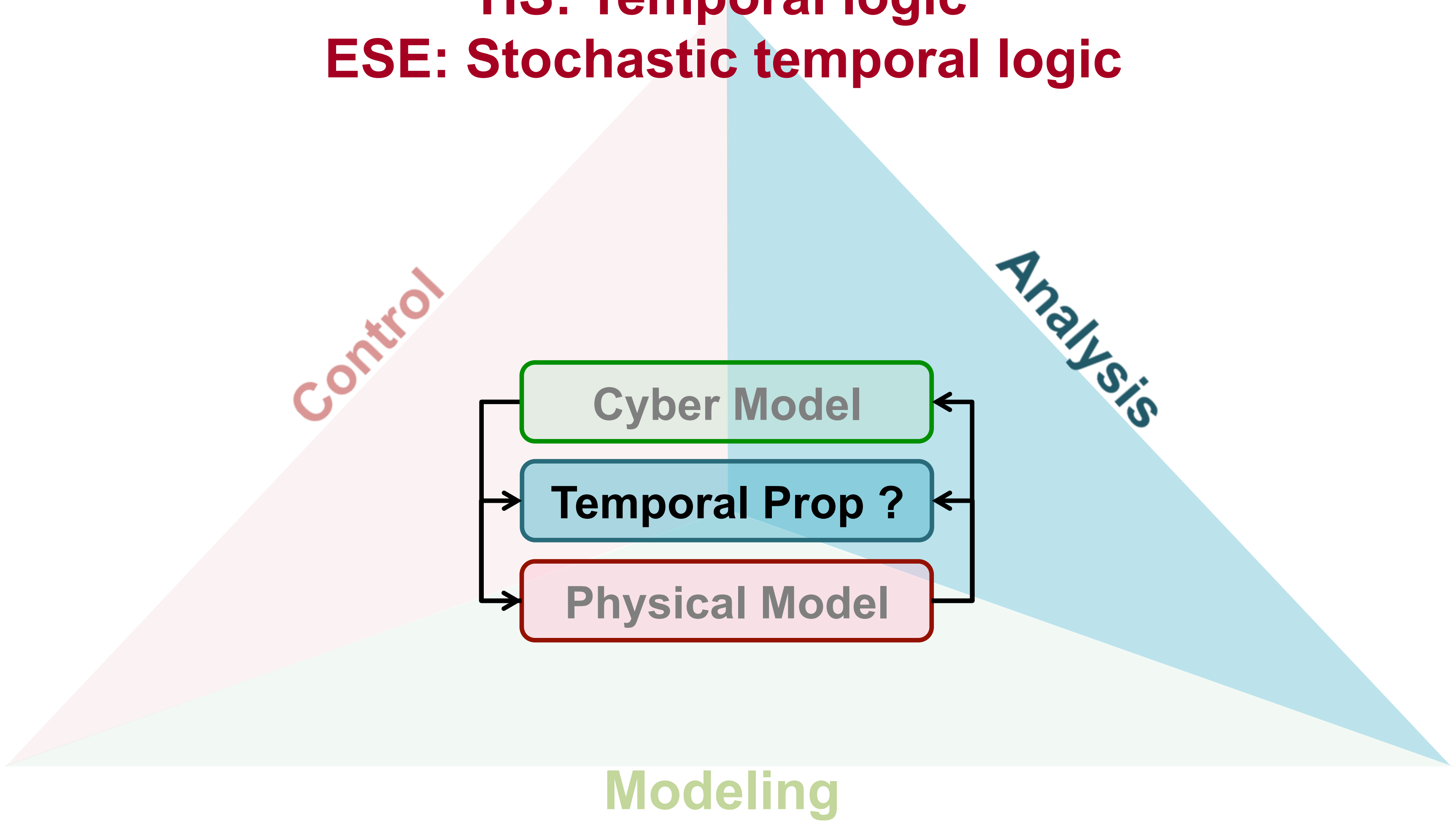
Modeling (Abstraction)

HS: Nondeterministic hybrid models
ESE: Stochastic hybrid models



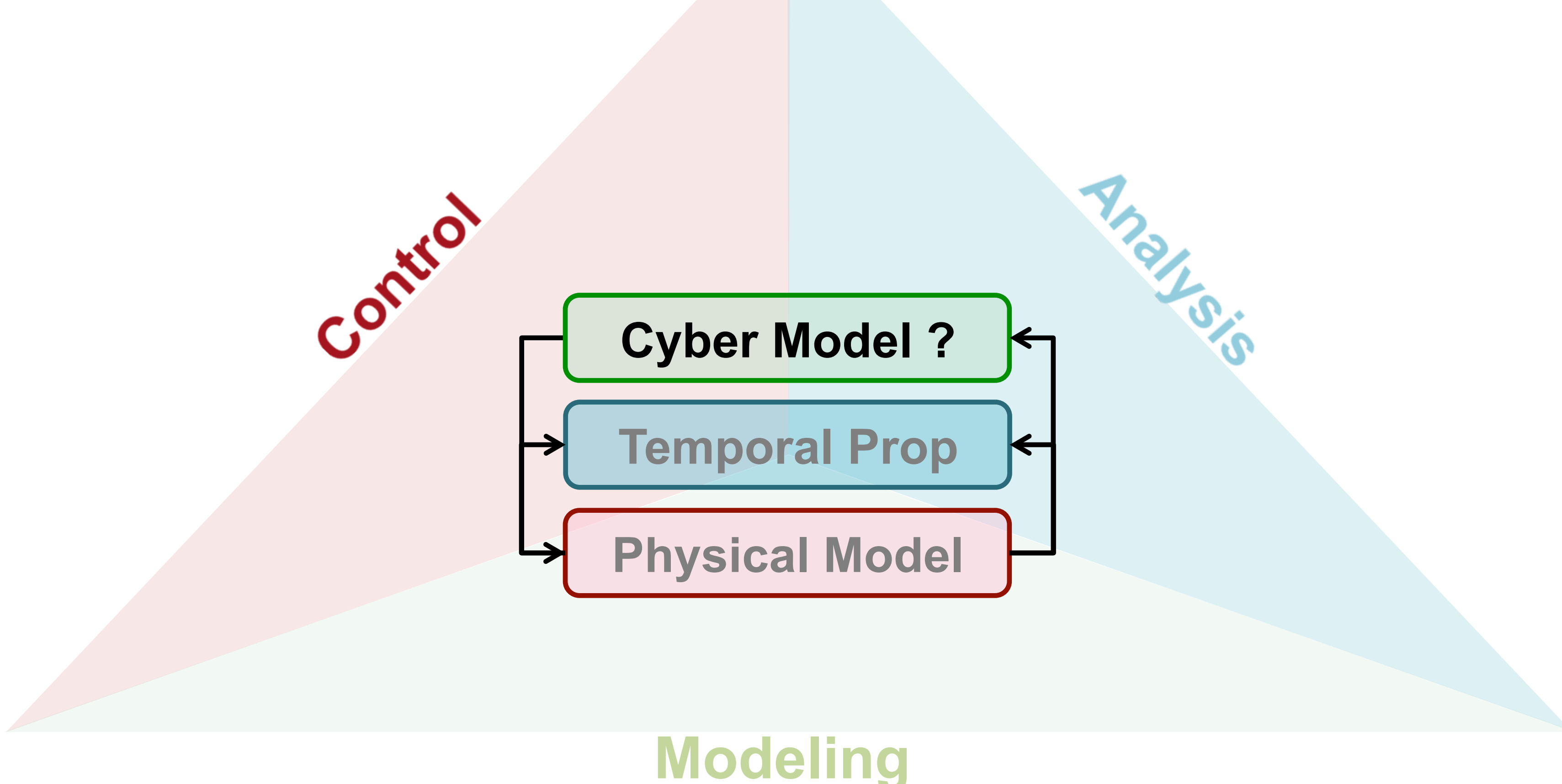
Analysis (Testing, Verification)

HS: Temporal logic
ESE: Stochastic temporal logic



Control (Synthesis)

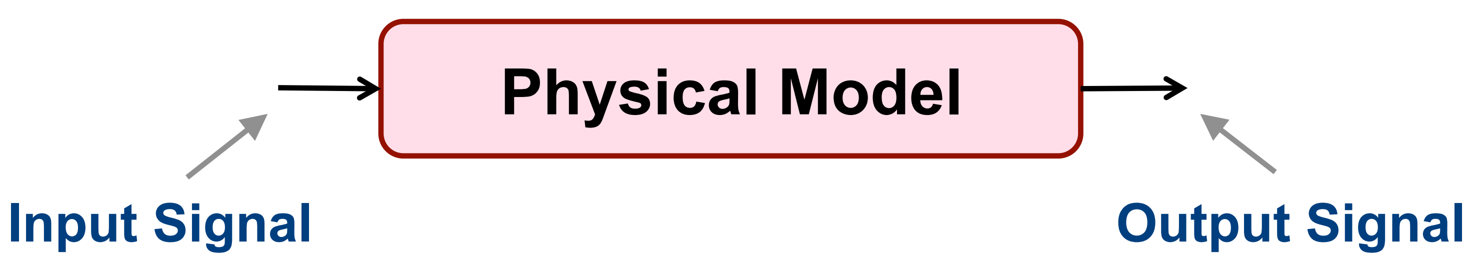
HS: Synthesis of a hybrid system
ESE: Synthesis of a stochastic hybrid system



Physical Model: Signals

Continuous Signal: Function $f : \mathbb{R} \to \mathbb{R}^n$

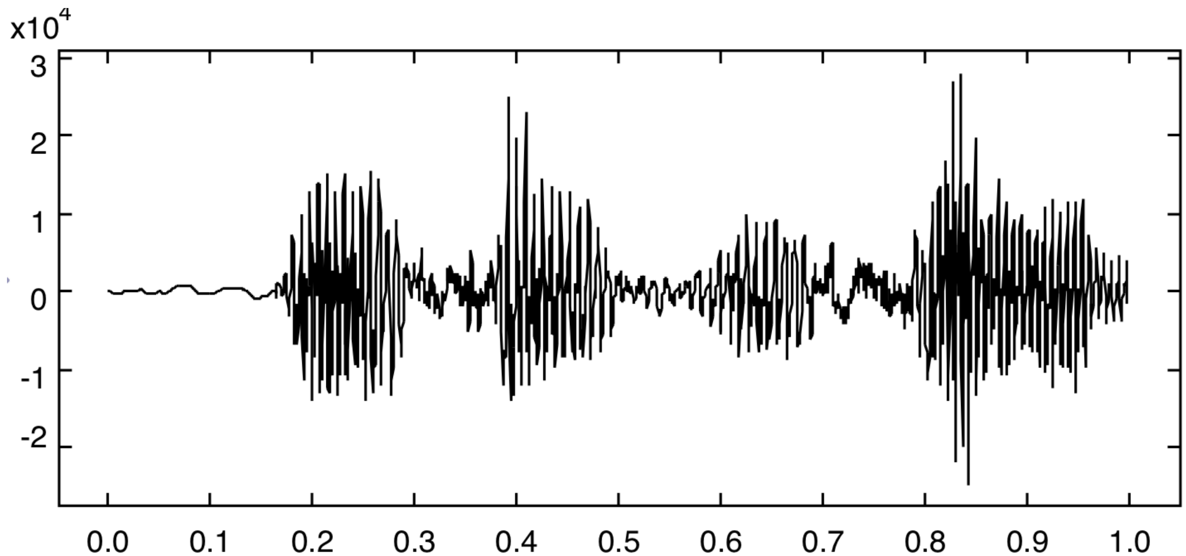
Time — Value domain

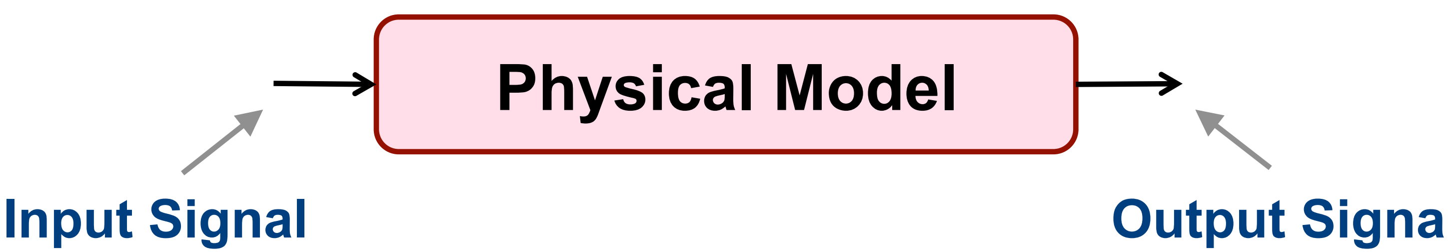


Physical Model: Signals

Continuous Signal (SignalCT): Function $f : \mathbb{R} \rightarrow \mathbb{R}^n$

Audio Signals: *Sound :* Time $\rightarrow$ Pressure

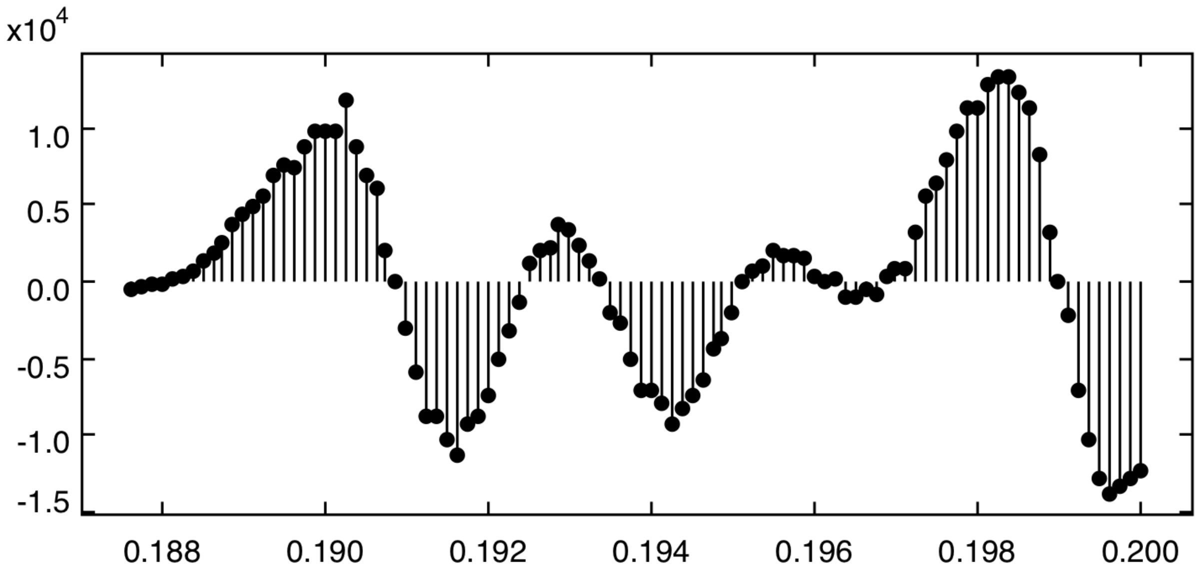


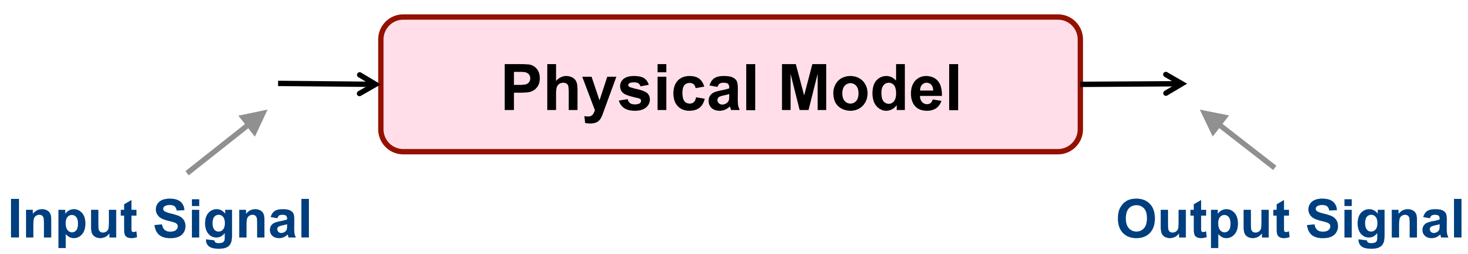


Physical Model: Signals

Discrete-time Signal (SignalDT): **Function** $f : \mathbb{N} \rightarrow \mathbb{R}^n$

Discrete-time audio: $\textit{Sound} : \text{DiscreteTime} \rightarrow \text{Pressure}$

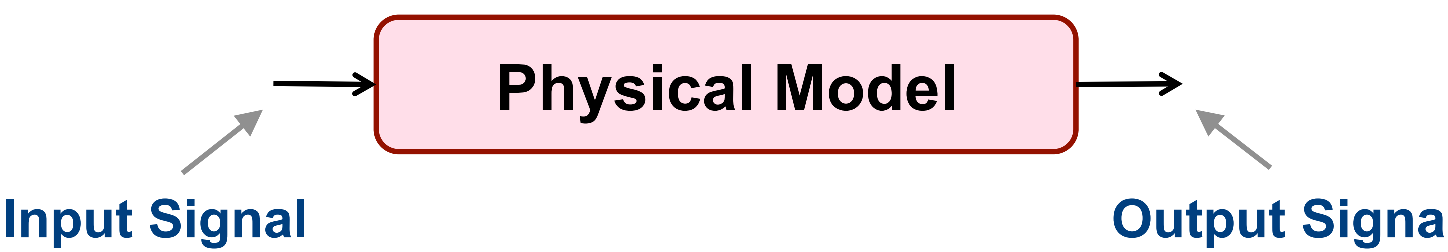




Physical Model: Signals

Discrete-space Signal (SignalDS): Function $f : \mathbb{N}^n \to \mathbb{R}$

Images: $\textit{Image} : \text{VSpace} \times \text{HSpace} \to \text{Intensity}$



Physical Model: Signals

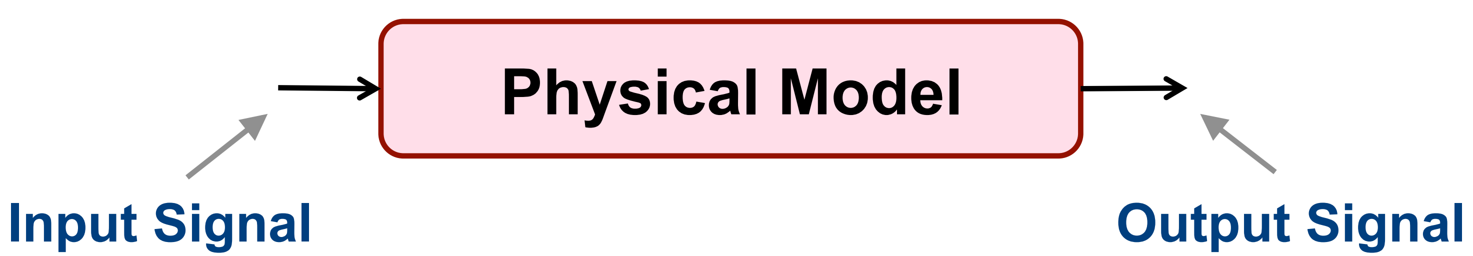
Video Signals (SignalVS): Function $f : \mathbb{N} \rightarrow$ SignalDS

Position, Velocity, Acceleration: $f : \mathbb{R} \rightarrow \mathbb{R}^3$

Temperature: $f : \mathbb{R} \rightarrow (\mathbb{R}^3 \rightarrow \mathbb{R})$

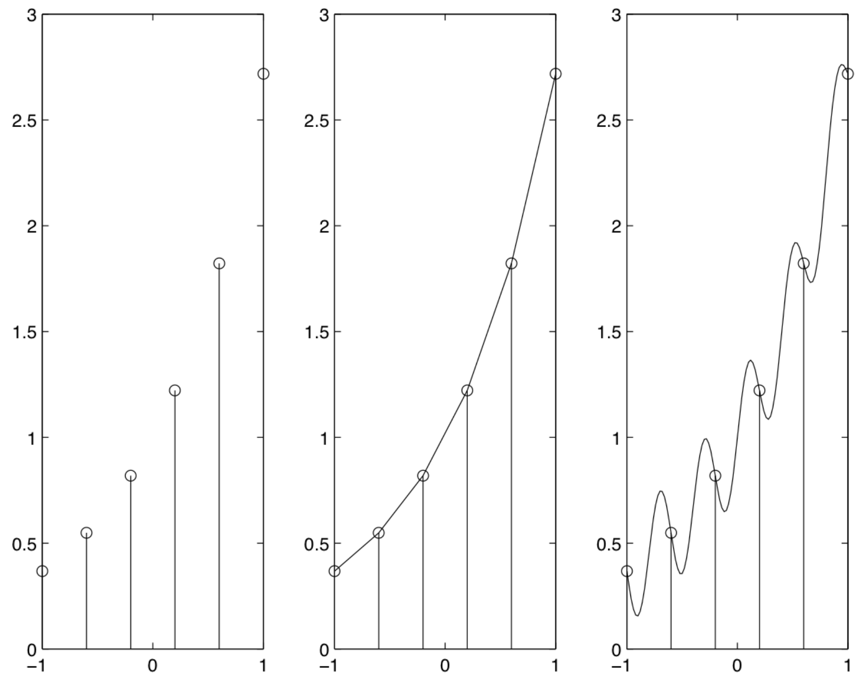
Boolean Sequences: $f : \mathbb{N} \rightarrow \mathbb{B}$

Event Stream: $f : \mathbb{N} \rightarrow$ EventSet



Physical Model: Signals

Sampling: **Depends on the nature of the function**



De-Aliasing

Sampling 10x faster

Physical Model

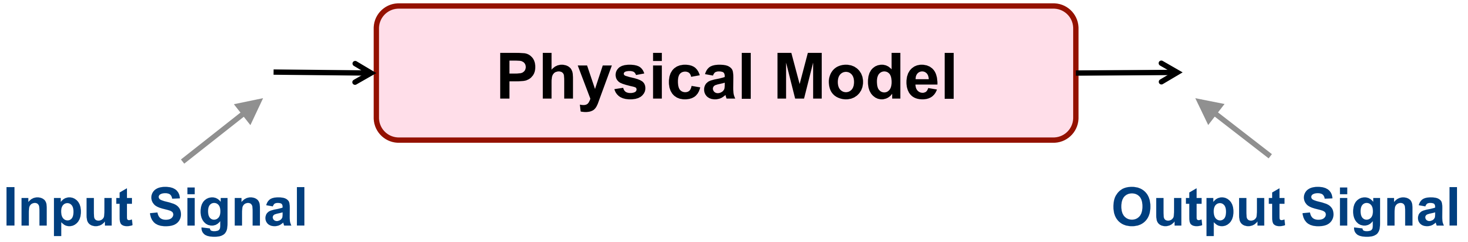
Input Signal

Output Signal

Physical Model: Systems

System: **Function f : Signal $\rightarrow$ Signal**

Input Output



Physical Model: Systems

System: Function f : Signal $\rightarrow$ Signal

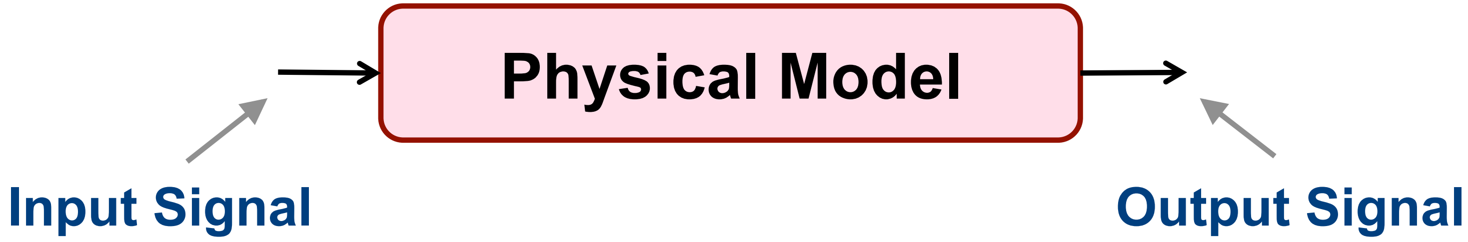
Transmission: Encoding and Decoding

Security: Encryption and decryption

Storage: Compression and decompression

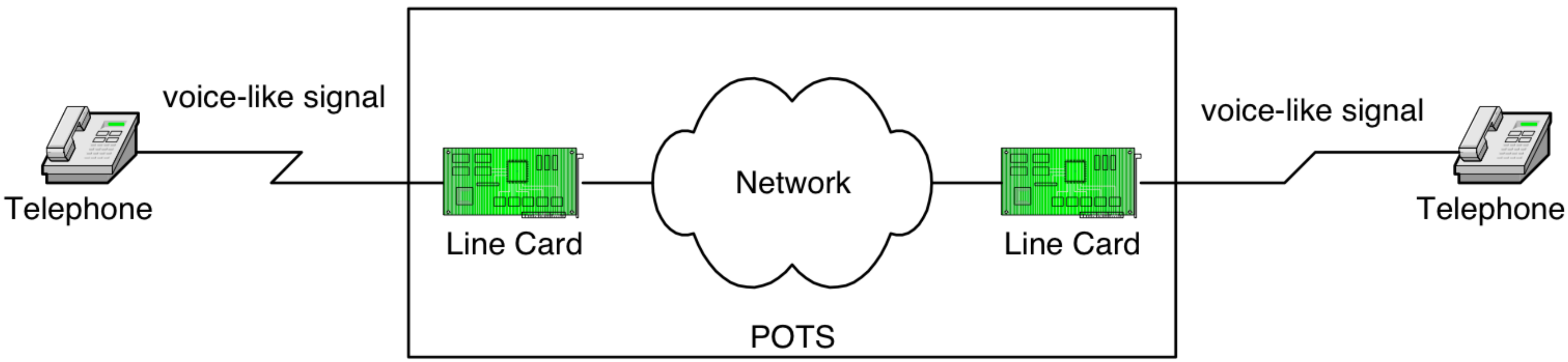
Quality: Denoising, equalizing, filtering

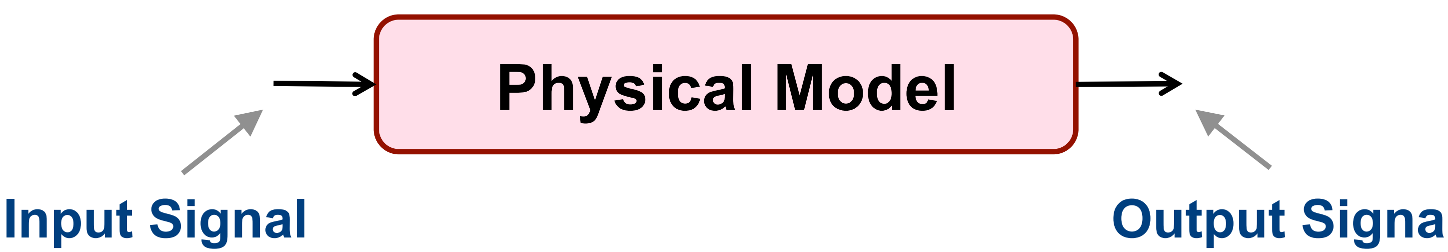
Control: Transform output to control input



Physical Model: Systems

System: **Function *f* : Signal → Signal**

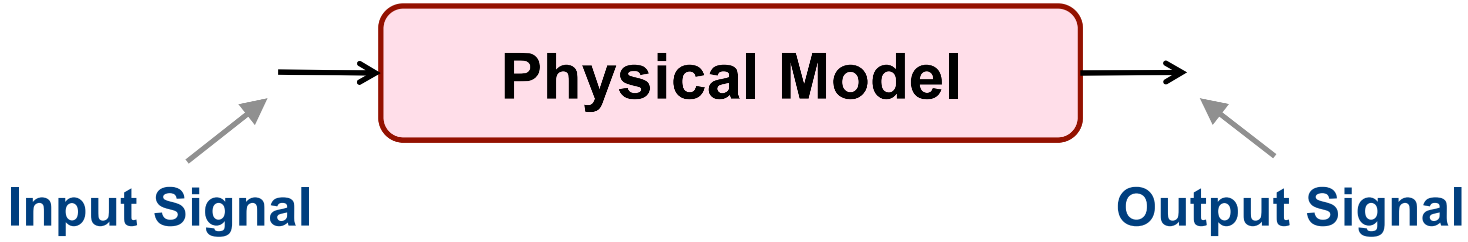




Physical Model: Description

Differential Equations: $\dot{x} = f(x, u, t),\ y = g(x, u, t),\ x(0) = x_0$

Next state equation — Current output equation — initial state



Physical Model: Description

Differential Equations: $\dot{x} = f(x,u,t),\ y = g(x,u,t),\ x(0) = x_0$

- **State vector:** $x \in \mathbb{R}^n$, **input vector:** $u \in \mathbb{R}^k$, **output vector:** $y \in \mathbb{R}^m$
- **Next (infinitesimal) state function:** $f : \mathbb{R}^n \times \mathbb{R}^k \times \mathbb{R} \to \mathbb{R}^n$
  - **Time invariant:** $\dot{x} = f(x,u),\ y = g(x,u)$, no explicit dependence on t
  - **Linear:** $f(a_1x_1 + a_2x_2, u, t) = a_1 f(x_1,u,t) + a_2 f(x_2,u,t)$, similar for u
- **Output (observation) function:** $g : \mathbb{R}^n \times \mathbb{R}^k \times \mathbb{R} \to \mathbb{R}^m$
  - **Moore:** if g depends only on x