The TUWEL course will be available as soon as the group registration will be closed. You have to tick the prepared exercises in TUWEL at the latest before Friday, 20th April 2012, 13:00 (AUFGABE Ankreuzen 1. Übungsblatt).

**Exercise 1 (2 pts):**
Let \( b > 1 \) be the maximal branching degree in the search tree and let \( d \) be its depth. Estimate the number of nodes, \( n_{bfs}(d) \), generated during a bfs\(^1\) with depth \( d \). Show that \( n_{bfs}(d) = O(b^d) \) and estimate the constant \( c_{bfs} \).

**Exercise 2 (2 pts):**
Let \( b > 1 \) be the maximal branching degree in the search tree and let \( d \) be its depth. Estimate the number of nodes, \( n_{dfid}(d) \), generated during a dfid with depth \( d \). Show that \( n_{dfid}(d) = O(b^d) \) and estimate the constant \( c_{dfid} \). What can you say about the overhead induced by dfid?

**Exercise 3 (1 pt):**
Consider dfid again. Analyze the behavior of
\[
\frac{1}{(1 - \frac{1}{b})^2}
\]
if the branching factor \( b \) increases (you may draw a curve!). For which kind of problems is the overhead induced by dfid low? Compare it to the behaviour of
\[
\frac{1}{(1 - \frac{1}{b})}.
\]

**Exercise 4 (1 pt):**
Give an example that \( A^* \) on graphs (with admissible heuristics) is not optimal. A detailed description of \( A^* \) can be found at [http://en.wikipedia.org/wiki/A*_search_algorithm](http://en.wikipedia.org/wiki/A*_search_algorithm)

**Exercise 5 (2 pts):**
Show the following statement: Consistent heuristics, with \( h(G) = 0 \) for each goal \( G \), are admissible.

**Exercise 6 (2 pts):**
Prove that, if any given heuristics \( h_1, h_2 \) are both admissible, then
\[
h(n) = \max(h_1(n), h_2(n)).
\]
is also admissible.

**Exercise 7 (2 pts):**
Prove that, if any given heuristics \( h_1, h_2 \) are both monotone (consistent), then
\[
h(n) = \max(h_1(n), h_2(n)).
\]
is also monotone (consistent).

\(^1\)bfs: breadth-first search; dfs: depth-first search; dfid: depth-first iterative deepening; ucs: uniform cost search
Exercise 8 (2 pts):
Consider the 8-puzzle discussed in the lecture and the introduced heuristics

- $h_1(n)$: number of misplaced tiles
- $h_2(n)$: total Manhattan distance

Show in a formal way whether the two suggested heuristics $h_1$ and $h_2$ are admissible and/or consistent (monotonic).

Exercise 9 (1 pt):
Consider the following search tree: Integer values at the edges represent the operator costs for an action, $A$ is the start node and the grey nodes are goal nodes.

Determine for the following search strategies the order in which the nodes are expanded and the corresponding goal node. In case you can expand several nodes and the search strategy does not specify the order, choose the nodes in alphabetic sequence. In addition, calculate for each search strategy the set of nodes that is actually kept in memory when the goal node is found.

a) Breadth-First Search
b) Uniform-Cost Search
c) Depth-First Search
d) Depth-Limited Search (Limit $d=2$)
e) Iterative Deepening Search
Exercise 10 (2 pts):
Let the following search problem problem with start node A and grey colored goal node F be given.

Solve this search problem using $A^*$ search by creating an $A^*$ search tree. The heuristic function $h$ is defined as follows:

$$h(A) = 8, h(B) = 3, h(C) = 7, h(D) = 2, h(E) = 1, h(F) = 0$$

Perform the $A^*$ algorithm on the created search tree in order to find the shortest path from $A$ to $F$. In which order are the nodes expanded, and when are which nodes reached? Show the content of the OPEN list (according to the pseudocode proposed by the link below) at each iteration and calculate the value of the evaluation function for the solution and for each expanded node.

The pseudo code of $A^*$ can be found at

http://www.heyes-jones.com/pseudocode.html

More detailed information about the $A^*$ algorithm can be found at

http://en.wikipedia.org/wiki/A*_search_algorithm