1 Knee Joint

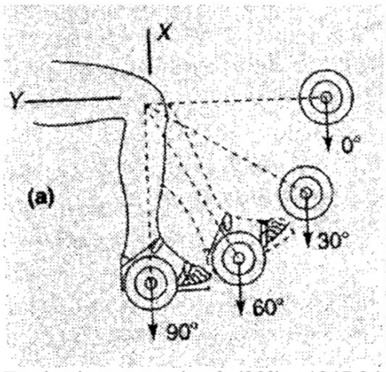
Consider a simple strengthening exercise for the quadriceps, as illustrated in the left figure below. The weight is lifted by the lower leg at different angles. Use the free body diagram in the right figure to answer the following points:

i) Derive an expression for the muscle tension f_Q of the quadriceps required for static equilibrium as a function of the flexion angle Θ

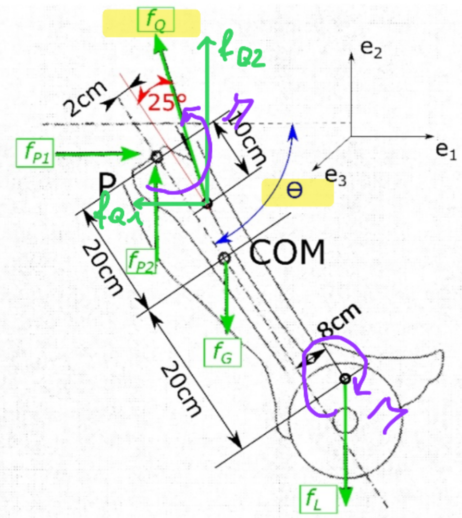
ii) Determine an expressions for the reaction forces in joint P

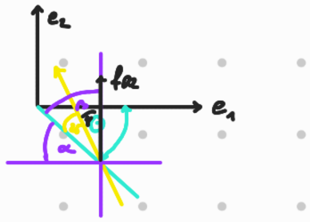
iii) For $f_L = 150$ N, $f_G = 30$ N and $\Theta = 0°, 30°, 60°, 90°$ calculate the required muscle tension.

iv) Sketch $f_Q(\Theta)$ (with the inserted force values) from $\Theta = 0°$ to $90°$



To check your results: f_Q (30°) = 1045.9 N





$\alpha + \beta = 90 \Rightarrow \beta = 90 - \Theta$

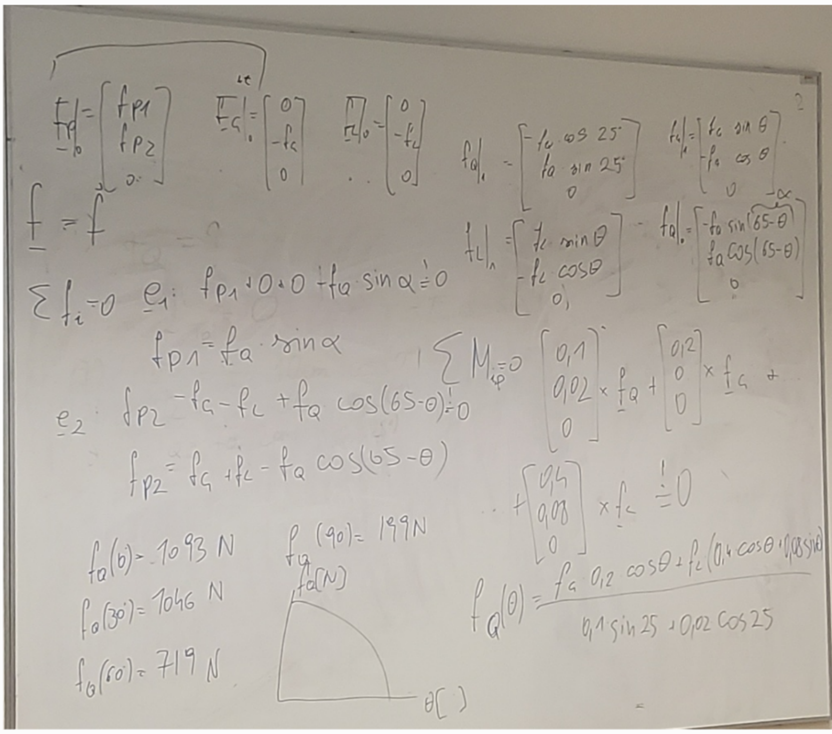
$\alpha = \Theta$

$\gamma = \beta - 25 = 65 - \Theta$

i)
$$f_{Q2} = f_Q \cdot \cos(65-\Theta)$$
$$f_{Q1} = f_Q \cdot \sin(65-\Theta)$$
$$\Sigma f_1 = 0 = f_{P1} - f_{Q1}$$
$$\Sigma f_2 = 0 = f_{P2} + f_{Q2} - f_G - f_L$$
$$\Sigma M = 0 = f_Q(10 \cdot \sin(25) + 2\cos(25)) = f_G \cdot \cos(\Theta) \cdot 20 + f_L(\sin(\Theta) \cdot 8 + \cos(\Theta) \cdot 40)$$
$$\Rightarrow f_Q(\Theta) = \frac{4 f_L \cdot \sin(\Theta) + 10(f_G + 2f_L)\cos(\Theta)}{5\sin(25) + \cos(25)}$$

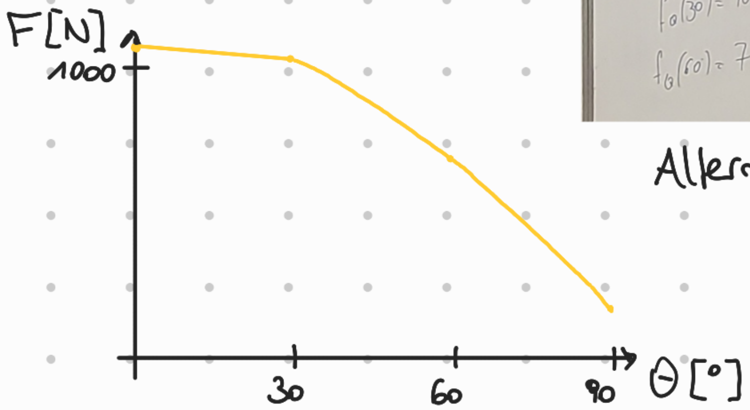
ii)
$$f_{P1} = f_{Q1} = f_Q \cdot \sin(65-\Theta)$$
$$f_{P2} = f_G + f_L - f_Q \cdot \cos(65-\Theta)$$

iii)
$$f_Q(0) = 1092{,}93\text{N}$$
$$f_Q(30°) = 1045{,}87\text{N}$$
$$f_Q(60°) = 718{,}56\text{N}$$
$$f_Q(90°) = 198{,}72\text{N}$$



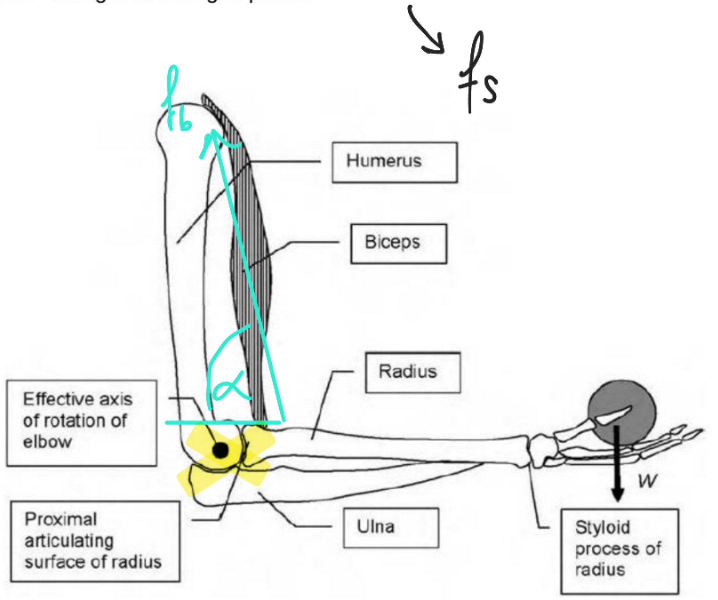
Alternative way: rotate coordinate system

iv)



2 Elbow Joint

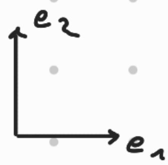
Consider the following simplified diagram of an arm holding a weight.
Calculate the force exerted by the biceps on the radius in order to hold the arm plus a weight of 1.5 kg in place.

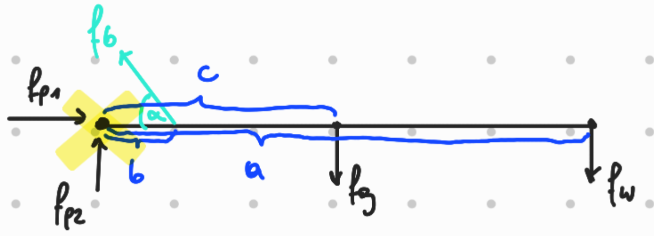


Begin by drawing an appropriate free body diagram. Estimate all the lengths and weights needed for your calculation. For this you can either research literature or measure on your own arm. In any case you need to be able to justify our assumptions.

estimated

$c = 9$ cm
$a = 20$ cm
$b = 2$ cm





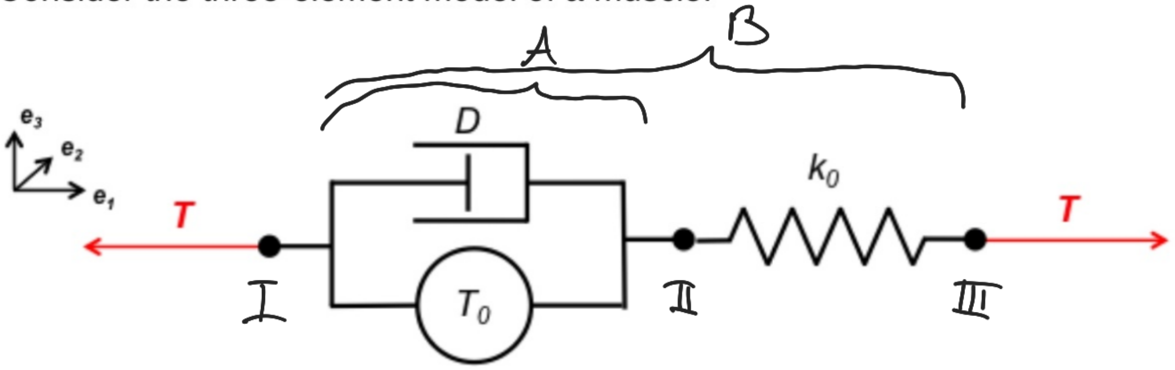
$\alpha \approx 80°$

Arm 6% des Körpergewicht
Oberarm: 3% = 1,68kg
Unterarm: 2% = 1,12kg
Hand: 1% = 0,56kg
$\}$ m_g

$$\sum f_1 = 0 = f_{s2} - f_{e2} - f_a - f_w$$
$$\sum f_2 = 0 = f_{s1} + f_{e1}$$
$$\sum M = 0 = f_b \cdot \sin(\alpha) \cdot b - f_a \cdot c - f_w \cdot a \Rightarrow f_b = \frac{f_w \cdot a + f_a \cdot c}{\sin(\alpha) \cdot b}$$
$$f_b =$$

3 Muscle lumped parameter model

Consider the three-element model of a muscle:



The contractile element produces a constant tension T_0 during activation – otherwise it produces no tension.

→ overall distance stays constant between I & II & III

- For an isometric contraction derive an expression for $T(t)$, assuming that the contractile element is activated from $t = 0$ until $t = C$.

- Sketch $T(t)$

$$F_D = D \cdot \frac{dx}{dt}$$
$$F_T = T_0$$
$$F_k = k \cdot x$$

$$A: \quad x_D = x_T = x$$
$$F_A = D \cdot \frac{dx}{dt} + T_0$$
$$B: \quad x_k + x = x_0 \rightarrow \dot{x}_k + \dot{x} = 0$$
$$T = F_A = F_k \Rightarrow D \cdot \frac{dx}{dt} + T_0 = k \cdot (x_k - x_0)$$

$$\frac{d}{dt} T = \frac{d}{dt}\left(k_0 (x_k - x_0)\right) = k_0 \frac{dx_k}{dt}$$
$$\frac{dx}{dt} = \frac{T - T_0}{D}$$
$$\frac{T - T_0}{D} + \frac{1}{k_0}\frac{dT}{dt} = 0$$
$$-\frac{k_0}{D} \cdot dt = \frac{1}{T - T_0} dT \quad /\int$$
$$-\frac{k_0}{D} \cdot t = \ln(T - T_0) + C \quad /e^{\dots}$$
$$T(t) = A e^{-\frac{k_0}{D} t} + T_0$$
$$T(0) = 0 = A + T_0 \Rightarrow A = -T_0$$
$$0 \leq t \leq C \Rightarrow T(t) = T_0 \left(1 - e^{-\frac{k_0}{D} t}\right)$$

$$t > C \begin{cases} T_0 = 0 \Rightarrow T_2(t) = B e^{-\frac{k_0}{D} t} \\ T_2(C) = T(C) = T_0\left(1 - e^{-\frac{k_0}{D} \cdot C}\right) = B e^{-\frac{k_0}{D} \cdot C} \\ \left(T_0 - T_0 e^{-\frac{k_0}{D} \cdot C}\right) e^{\frac{k_0}{D} C} = B \\ \Rightarrow B = T_0 \left(e^{\frac{k_0}{D} C} - 1\right) \\ T_2(t) = T_0 \left(e^{\frac{k_0}{D} C} - 1\right) e^{-\frac{k_0}{D} t} \end{cases}$$