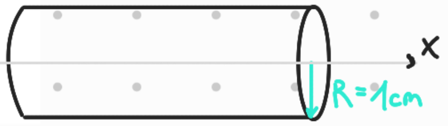
1 Hemodynamics

1.1 Casson fluid – flow rate

Blood flows in a tube of radius 1 cm because of a pressure gradient of 0.4 dynes/cm³. Treating the blood as a Casson fluid with yield stress 0.06 dynes/cm², what percentage of the total volume flowrate is from blood traveling in the central non-flowing "core" of the flow?

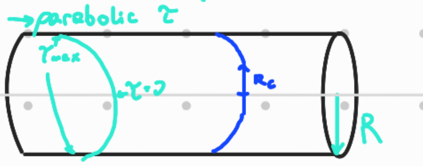
$$\frac{dp}{dx} = 0{,}4\ \frac{dyn}{cm^3}$$

$$\tau_y = 0{,}06\ \frac{dyn}{cm^2}$$



$$\tau(R_c) = \tau_y = \frac{R_c}{2}\cdot\frac{dp}{dx} \Rightarrow R_c = 0{,}3\,cm$$

Newtainoin flow
→ parabolic τ



Casson
→ plug in middle
no change in τ

steady flow: $\tau(r) = \frac{r}{2}\frac{dp}{dx}$

casson relationship: $\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\mu\dot{\gamma}}$

$$\sqrt{\frac{r}{2}\frac{dp}{dx}} = \sqrt{\tau_y} + \sqrt{\mu\dot{\gamma}} \leftarrow \text{rate of strain: } \dot{\gamma} = \frac{du}{dr}$$

↳ dynamic viscosity

$$u(r) = \frac{1}{\mu}\int\left(\sqrt{\frac{r}{2}\frac{dp}{dx}} - \sqrt{\tau_y}\right)^2 dr$$

$$= \frac{1}{\mu}\int\left(\sqrt{\frac{r}{R_c}\tau_y} - \sqrt{\tau_y}\right)^2 dr = \frac{\tau_y}{\mu}\int\left(\sqrt{\frac{r}{R_c}} - 1\right)^2 dr$$

$$= \frac{\tau_y}{\mu}\int \frac{r}{R_c} - 2\sqrt{\frac{r}{R_c}} + 1\, dr = \frac{\tau_y}{\mu}\left(\frac{r^2}{2R_c} - \frac{4\sqrt{R_c}}{3R_c}\cdot r^{3/2} + r + C\right)$$

$$u(R) = 0 = \frac{\tau_y}{\mu}\left(\frac{R^2}{2R_c} - \frac{4\sqrt{R_c}}{3R_c}R^{3/2} + R + C\right) \Rightarrow C = -\frac{R^2}{2R_c} + \frac{4\sqrt{R_c}}{3R_c}R^{3/2} - R$$

Q_f: $r > R_c$ fluid flow
Q_p: $r \leq R_c$ travel as plug (no flow)
$\Big\}\ \oplus\ Q_{ges}$

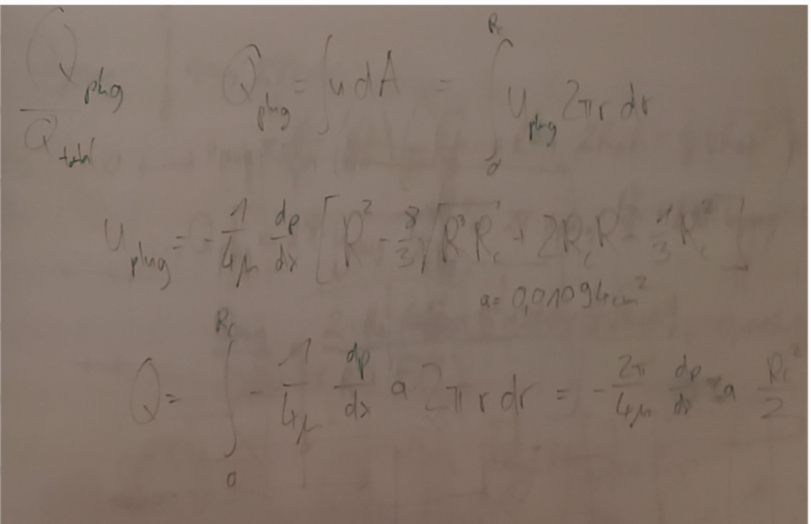
↱ u_{plug} from S.31 in pdF! & task 1.3

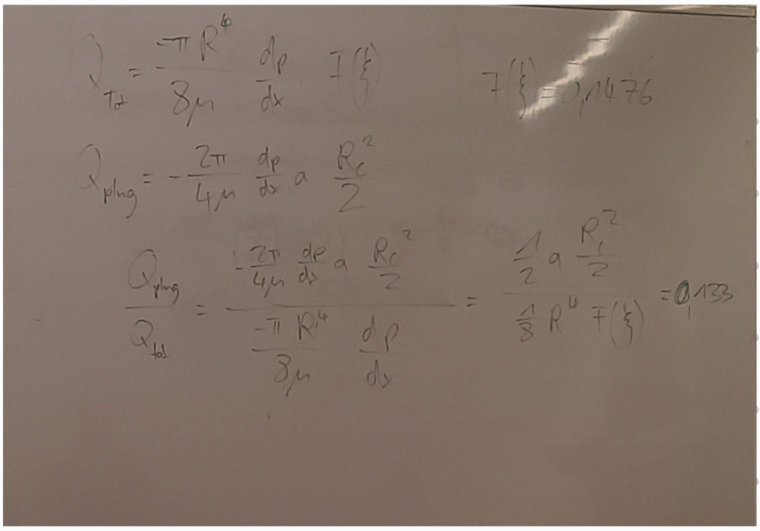
✓ $$Q_p = \int_0^{R_c} u(r)\,2\pi r\, dr = \frac{1}{\mu}\left(-3{,}154\cdot 10^{-3}\ dyn\cdot cm\right)$$

$$\left(Q_f = \int_{R_c}^{R} u(r)\,2\pi r\, dr = \frac{1}{\mu}\left(-20{,}103\cdot 10^{-3}\ dyn\cdot cm\right)\right)$$

$$\Rightarrow \frac{Q_p}{Q_p+Q_f} = 13{,}56\%\ \text{travels as plug}$$

$$Q_{total} = -\frac{\pi R^4}{8\mu}\frac{dp}{dx}\cdot\frac{R_c}{R}$$ S.32 in pdF





1.2 Casson fluid – pressure drop

A 40 cm long tube (internal diameter, 1 mm) is filled with blood, which has been citrated to prevent it from clotting. (Assume that citration does not alter the rheological properties of the blood.) Using the property values for blood: $\mu = 3.5$ cP, $\tau_y = 0.05$ dynes/cm², $\rho = 1.06$ g/cm3.

(a) At what pressure difference between the tube ends (Δp) does the blood begin to flow?

(b) What is the flow rate (Q) when $\Delta p = 20$ Pa? How long would it take for 10ml of blood to flow though the tube? $V = 10\ cm^3$

(c) In comparison, how long would it take for 10ml water (a Newtonian fluid with $\mu = 1$ cP,) to travel through the tube with the same pressure difference between the tube ends.

$1\ pa = 10 \frac{dyn}{cm^2}$

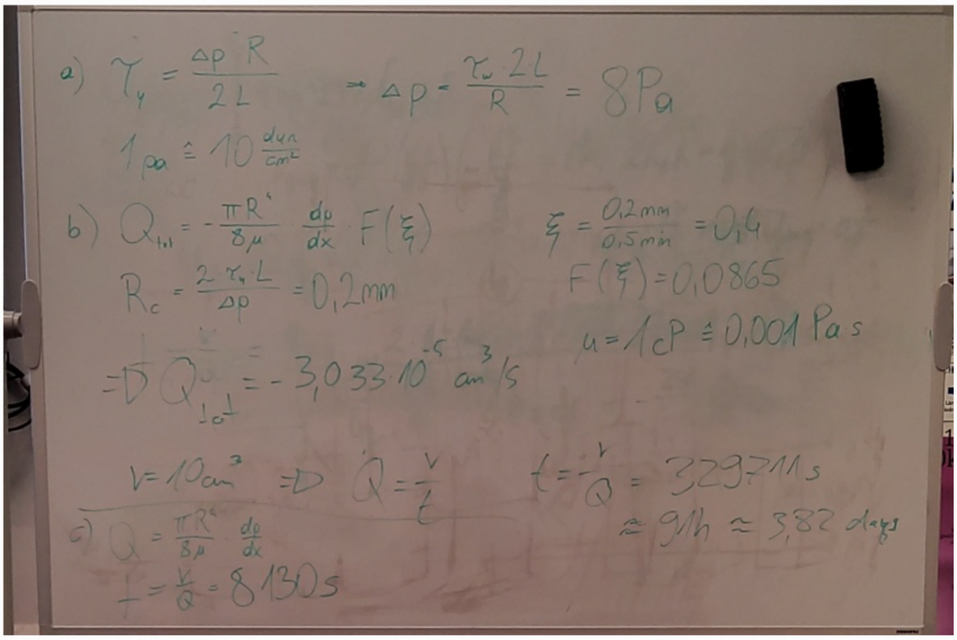
$1cP = 0{,}001\ Pa \cdot s$

a) begins to flow when shear stress equals yield stress

$$\tau_w > \tau_y = \frac{\Delta p R}{2L} \Rightarrow \Delta p = \frac{\tau_w \cdot 2 \cdot L}{d/2} = \frac{5 \cdot 10^{-3} Pa \cdot 2 \cdot 0{,}4m}{0{,}5 \cdot 10^{-3} m} = 8Pa$$

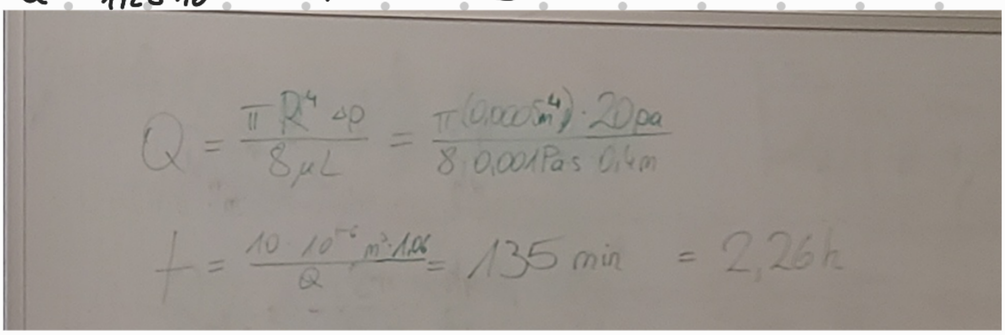
b) $$R_c = \frac{2 \tau_y L}{\Delta p} = \frac{5 \cdot 10^{-3} Pa \cdot 2 \cdot 0{,}4m}{200 \frac{dyn}{cm^2}} = 0{,}2mm$$

$$t = \frac{V}{Q} = \frac{10ml \cdot 1{,}06}{1000} \cdot \frac{1}{3{,}036 \cdot 10^{-4}} = 34{,}46s$$



c) Newtonain fluid: $$Q = \frac{\pi R^4 \Delta p}{8 \mu L} = 1{,}23 \cdot 10^{-9} \frac{L}{s}$$

$$t = \frac{V}{Q} = \frac{0{,}01}{1{,}23 \cdot 10^{-9}} = 8{,}13 \cdot 10^{3} s$$



1.3 Casson fluid – flow between two plates

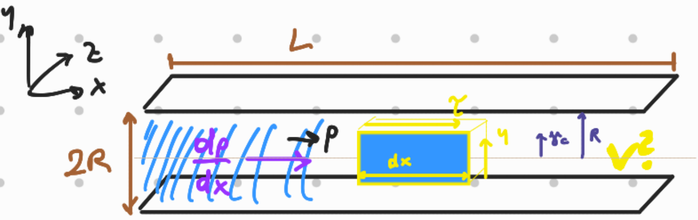
A Casson fluid fills the space between two horizontally positioned, parallel plates. The plates are very large, in comparison to the distance between them. A pressure gradient $\frac{dp}{dx}$ forces the fluid from left to right. Derive an expression for the velocity of the plug flow region in this flow. Specifically show that for plates at a distance *2R* from each other:

kommt aber zur Prüfung erst nicht!

$$u_{plug} = -\frac{1}{2\mu}\frac{\mathrm{dp}}{\mathrm{dx}}\left(R^2 - \frac{y_c^2}{3} - \frac{8}{3}\sqrt{y_c R^3} + 2y_c R\right)$$

Where μ is the parameter from the Casson relationship and R-y_c is the distance between the plate and the edge of the plug flow region (or, in other words, $\boldsymbol{yc}$ is the distance between the center-line between the plates and the edge of the plug flow region)

Hint: The derivation is very similar to the derivation for a cylindrical tube.



no shear stress along y (compared to cylinder)

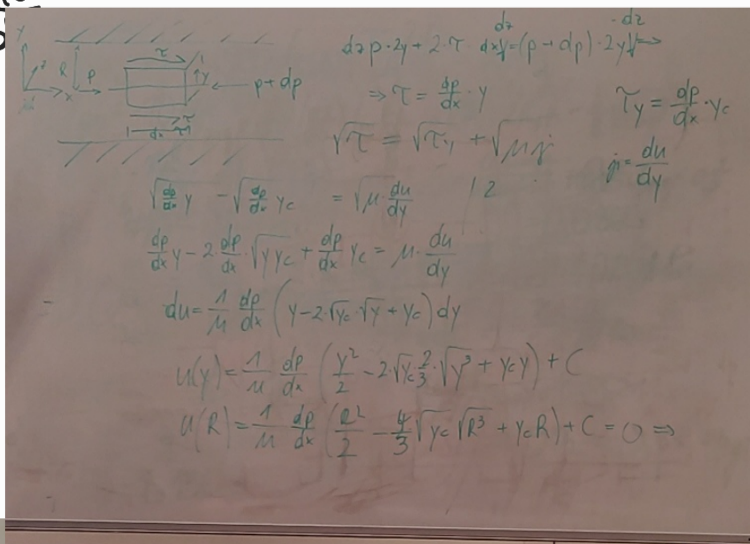
$$p \cdot 2y + 2\tau\, dx\, dz = (p + dp)\, 2y \cdot dz$$

$$\Rightarrow \tau = \frac{dp}{dx} \cdot y$$

$$\tau_y = \frac{dp}{dx}\, y_c$$

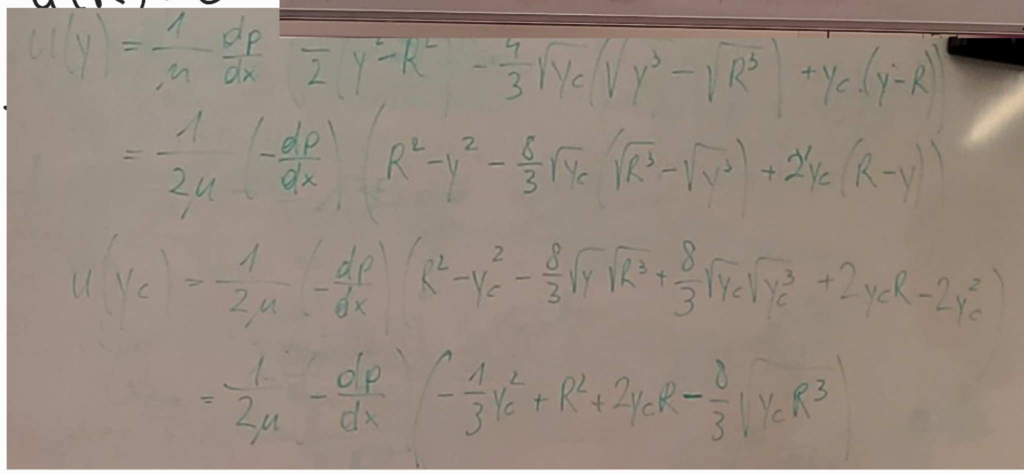
$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\mu \cdot \dot{\gamma}} \qquad \dot{\gamma} = \frac{du}{d}$$

$$\sqrt{\frac{dp}{dx}\, y} - \sqrt{\frac{dp}{dx}\, y_c} = \sqrt{\mu}\, \frac{du}{dy} \quad /^2$$



boundary conditions: $u(R) = 0$

= mein 1. Bsp...



2 The heart

Based on the averaged left-ventricular pressure-volume loop (Figure 1), calculate the pumping power of the female and the male heart (left ventricle only). Assume a resting heart rate of 70 beats per minute and approximate the area under the curve with a rectangle.

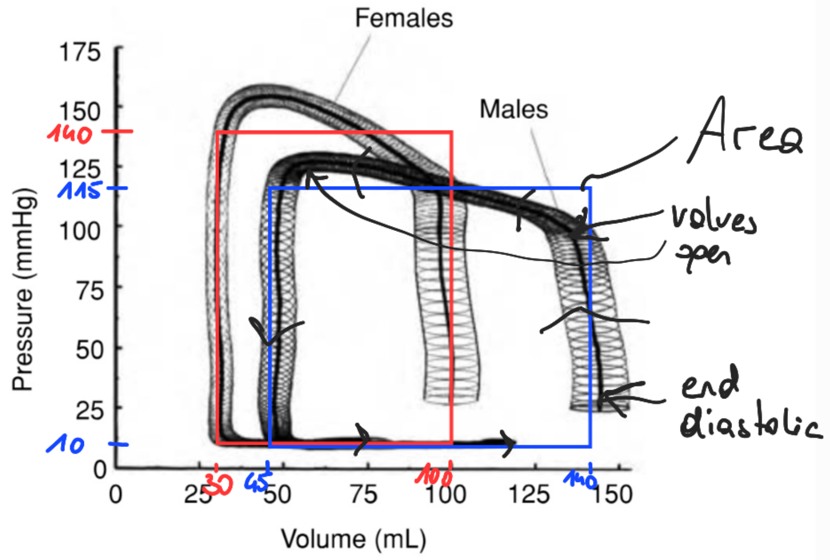


Figure 1: Left ventricular pressure-volume loop

Area = Work done at each heartbeat = Stroke power

$f = 70\,\text{bpm} = \frac{70}{60}\,\frac{1}{s} = 1{,}16\,\text{Hz}$

$1\,\text{mmHg} \mathrel{\hat=} 133{,}32\,\text{Pa} = \frac{N}{m^2} = \frac{kg}{m s^2}$.

$(1\,\text{ml blood} \mathrel{\hat=} 1{,}06\,g)$

$[W] = \frac{m^2 kg}{s^3}$

Female: $A_F = \frac{100-30}{10^6}(140-10) \cdot 133{,}32 = 1{,}213\,\frac{kg m^2}{s^2}$

$P_F = A_F \cdot f = 1{,}41\,W$

Male: $A_m = \frac{140-45}{10^6} \cdot (115-10)\,133{,}32 = 1{,}329\,\frac{kg m^2}{s^2}$

$P_m = A_m \cdot f = 1{,}55\,W$