Modelling on the Edge of Chaos: Cellular Automata and Agents Representing Complex Dynamical Systems and Building Structures

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Folien übertragen[1] in Latex / Lyx. - modifiziert/gekürzt und möglicherweise mit Fehlern

Contents

[1] von Georg Regal

1 Introduction

1.1 Attractors

- Phase-Space: Space in which all possible states of a system are represented (Gibbs, 1901)
- Example: Pendulum (damped)

1.2 Tipping Point

- Systems can have different attractors
- Stability within a certain range of parameters (self-stabilisation)
- If range is exceeded, systems can "switch" to other attractor
- Tipping Point
- Example: Climate System of the Earth
  - To a certain degree self-stabilising over certain range of parameters
  - If e.g. global temperature rises over tipping point: "runaway greenhouse effect"
  - Positive feedback loops can be induced
    - Methane release from Siberian thawing
    - Melting of Greenland ice
  - Eventually stabilisation at different attractor (not necessary positive for humanity though)

2 Basic Ideas of Cellular Automata

- Theory of Computation (Alan Turing)
- Automata Theory
  - John von Neumann
  - Stanislav Ulam
  - John Horton Conway
  - "The Game of Life"
- Cellular Automata are Turing Complete!
- Simulation of Complex Systems by Interaction of using "Simple" Rules

2.1 Complex Systems

Complex Dynamical Systems often show Huge Effects on Small Changes in the Starting Condition of the Model

- 3 Body Problem (Henri Poincaré)
- Weather Forecast (Edward Lorenz)
- Uncertainty Relation (Werner Heisenberg)

2.2 Building Cellular Automata

- The Cell
- The Lattice
- Neighbourhoods
- Transition Rules
  - Explicit
  - Totalistic
  - Legal

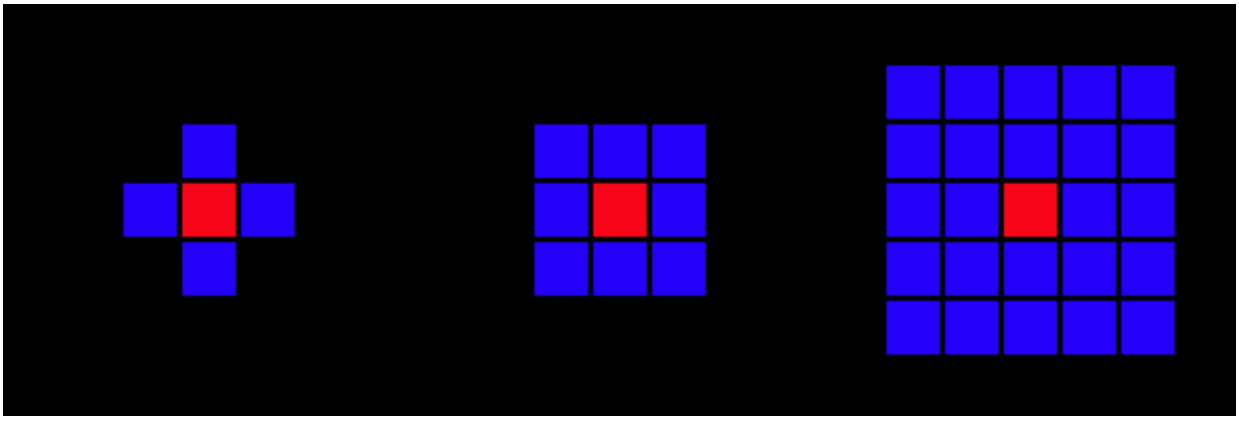

Figure 1: von Neumann / Moore / extended Moore Neigbourhood

Rules

1. Every group of states of the neighbourhood cells is related a state of the core cell. E.g. consider a one-dimensional CA: a rule could be "011 -> x0x", what means that the core cell becomes a 0 in the next time step (generation) if the left cell is 0, the right cell is 1 and the core cell is 1. Every possible state has to be described.
2. "Totalistic" Rules: the state of the next state core cell is only dependent upon the sum of the states of the neighbourhood cells. E.g. if the sum of the adjacent cells is 4 the state of the core cell is 1, in all other cases the state of the core cell is 0.
3. "Legal" Rules: a special kind of totalistic rules are the legal rules. As it is not of advantage in most cases to use rules that produce a pattern from total zero-state lattices (all cells in the automaton are 0), Wolfram defined the so called legal rules . These rules are a subset of all possible

Algorithm Summary

- CAs develop in Space and Time
- Discrete Simulation Method
- Cells Arranged to n-dimensional Lattices

- Finite and Discrete Cell States
- Cells have Identical Properties and Transition Rules
- Future State of Cell only Depending on
  - Neighbourhood of Cell and
  - Defined Transition Rules

3 Behaviour of CAs

- Universal Computation (capable to perform any finite algorithm)
- Classes (L = probability, that cell will be alive at next cycle)
  - 1: Limit Points
  - 2: Limit Cycles (0<< L < 0,3)
  - 3: Chaotic/Strange Attractors (L ~ 0,5)
  - 4: More Complex Behaviour (Univ. Comp.) (L ~ 0,3)
  - "On the Edge of Chaos" the Lambda Parameter

4 Applications

- Game of Life
- Billiard / HPP, FHP - Gas Models
- Ising Model
- Self-Reproduction
- Chemical Waves (Belusov-Zhabotinsky Reaction)
- Reaction-Diffusion Systems

Game of Life

- a cell that is dead at the time step t, becomes alive at time t+1 if exactly three of the eight neighbouring cells at time t were alive.
- a cell that is alive at time t dies at time t+1 if at time t less than two or more than three cells are alive.

5 After the Hype...

- Rich "Theoretical" Results (Automata Theory)
- But, did CAs replace Differential Equations in Modelling... ?
- Disadvantages of CAs – Simple Rules but: How to find the Right Ones? – Scaling Problems – Various Practical Problems (Constant # of Particles...)
- New Ideas?

6 Agent Based Systems?

- (Autonomous) Software Agents
- New Software Engineering Paradigma
- Agent Modelling
- Differences to CAs?
  - Not the Lattice is in the Center but the Individual and the Interaction, which can be more Complex and "Realistic"