192.067 VO Deductive Databases
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1.) Consider a program P consisting of the following rules:

$$
\begin{aligned}
a &\leftarrow \\
b &\leftarrow a, c \\
w &\leftarrow c, b \\
c &\leftarrow e \\
c &\leftarrow e, g \\
e &\leftarrow \\
g &\leftarrow c, f
\end{aligned}
$$

List all minimal models of P. Explain your answer. **(10 points)**

Facts

1.)
$a \leftarrow$ $\quad \{a, e\}$
$e \leftarrow$

2)
$c \leftarrow e$ $\quad \{a, e, c\}$

3)
$b \leftarrow a, c$ $\quad \{a, e, b, c\}$

4)
$w \leftarrow c, b$ $\quad \{a, e, b, c, w\}$

$$
\begin{aligned}
&\cancel{a \leftarrow} \\
&\cancel{b \leftarrow a, c} \\
&\cancel{w \leftarrow c, b} \\
&\cancel{c \leftarrow e} \\
&c \leftarrow e, g \\
&\cancel{e \leftarrow} \\
&g \leftarrow c, f
\end{aligned}
$$

→ $\{a, e, b, c, w\}$ is a minimal model

$\{a, e\}$ has to be included and $\{b, c, w\}$ can be inferred from $\{a, e\}$

2.) Consider interpretations $I_1 = \{c, d\}$ and $I_2 = \{e, f\}$, and a program P consisting of the following rules:

$$e \leftarrow not\ c, not\ d$$
$$f \leftarrow not\ c, e$$
$$c \leftarrow not\ e$$

Compute the programs P^{I_1} and P^{I_2}, i.e. the reducts of P with respect to I_1, and with respect to I_2. Is I_1 a stable model of the program P? Is I_2 a stable model of the program P? Justify your answer. **(10 points)**

reduct is defined as $P^I = \{ H(r) \leftarrow B^+(r) \mid r \in P;\ I \cap B^-(r) = \emptyset \}$

P^{I_1}

$c \leftarrow$

P^{I_2}

$e \leftarrow$

$f \leftarrow e$

I^1 is model of P^I

-> not minimal

I^2 is model of P^{I_2}

-> also minimal thus a stable model of P

3.) Consider a program P consisting of the following rules:

$$d \leftarrow c$$
$$c \leftarrow not\ d$$
$$d \leftarrow not\ c$$

List all stable models of P. Justify your answer. **(10 points)**

candidates

$M_0 = \emptyset$ $M_1 = \{c\}$ $M_2 = \{d\}$ $M_3 = \{c, d\}$

1) P^{M_0}

$d \leftarrow c$

$c \leftarrow$

$d \leftarrow$

M_0 nal a model

2) P^{M_1}

$d \leftarrow c$

$c \leftarrow$

M_1 nal a model

3) P^{M_2}

$d \leftarrow c$

$d \leftarrow$

M_2 model of P^{M_2} $\wedge$ minimal $\rightarrow$ stable model

4) P^{M_3}

$d \leftarrow c$

M_3 model of P^{M_3} however nal minimal

4.) Consider an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ satisfying the following:

- $\Delta^{\mathcal{I}} = \{a, b, c\}$,
- $A^{\mathcal{I}} = \{b, c\}$ for the concept name A,
- $B^{\mathcal{I}} = \{a\}$ for the concept name B,
- $P^{\mathcal{I}} = \{(b, b), (a, b)\}$ for the role name P, and
- $R^{\mathcal{I}} = \{(a, a), (b, b)\}$ for the role name R.

Compute the extension of $\cdot^{\mathcal{I}}$ for the following complex concepts (i.e. compute $C^{\mathcal{I}}$ for all complex concepts C listed below):

(1) $B \sqcup \neg A$

(2) $(B \sqcup A) \sqcap \neg B$

(3) $\forall P.A$

(4) $\exists P.A$

(5) $\forall P.(B \sqcap \neg B)$

(6) $\exists R.(B \sqcup \neg B)$

(15 points)

1)
$(B \sqcup \neg A)' = B' \cup \neg A' = \{a\} \cup \{a\} = \{a\}$

2)
$((B \sqcup A) \sqcap \neg B)' = \{a, b, c\} \cap \{b, c\} = \{b, c\}$

3)
$(\forall P.A)' = \{a, b, c\}$

a a ✓   a b $b \in A'$ ✓   a c ✓   b a ✓   b b $b \in A'$ ✓   b c ✓   c a ✓   c b ✓   c c ✓

$\forall R.C \quad \{d_1 \mid \forall d_2 \in \Delta' \; R'(d_1, d_2) \rightarrow d_2 \in C'\}$

4) $(\exists P.A)' = \{a, b\}$

a $\rightarrow b \in A'$ b $\rightarrow b \in A'$ c X

$\forall R.C \quad \{d_1 \mid \exists d_2 \in \Delta'. R'(d_1, d_2) \wedge d_2 \in C'\}$

5)
$(\forall P.\underbrace{(B \sqcap \neg B)}_{=\{\}})' = \{c\}$

$ab \rightarrow b \notin \{\}$ bb $b \notin \{\}$

6)
$(\exists R.\underbrace{(B \sqcup \neg B)}_{\{a,b,c\}})' = \{a, b\}$

a (a,a) ✓   b (b,b) ✓   c X

5.) By defining a suitable interpretation, show that the concept $A \sqcap \neg(\forall R.A)$ is satisfiable. Here A is a concept name and R is a role name. **(15 points)**

$A^I = \{a\}$ $\quad\quad$ $\Delta^I = \{a, b\}$

$R^I = \{(a,b)\}$

$A^I = \{a\}$

$(\forall R.A)^I = \{b\}$ $\quad \Rightarrow \quad$ $\Delta^I \setminus \{b\} = \{a\}$

a $\quad\quad\quad\quad$ b

(a,b) but $b \notin A^I$ $\quad$ ✓

$\rightarrow$ satisfiable

Grading scheme: 0–31 nicht genügend, 32–38 genügend, 39–45 befriedigend, 46–52 gut, 53–60 sehr gut