Financial Management and Reporting

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Lectures 1 and 2: Forwards and Futures

Content

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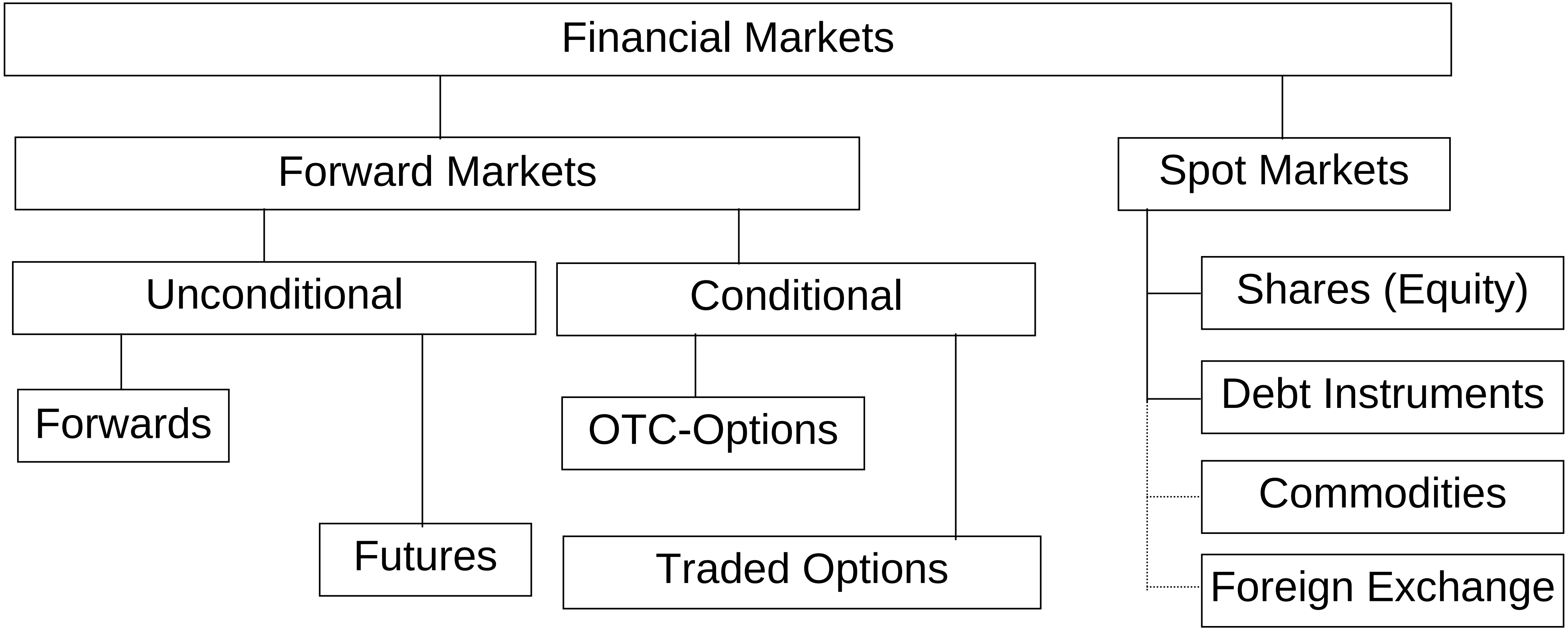
1. Introduction



2. Spot vs Forward Market

Forward contracts are unconditional transactions.

They comprise the **obligation** to **buy or sell** a specific **underlying** instrument, at a **price** agreed upon today, at a specific date **in the future**.

This is an important difference to option contracts, which are conditional forward transactions.

Options comprise the right (not the obligation) to buy or sell a specific amount of an underlying instrument, at a price agreed upon today, on or up to the last trading day.

Agreement and Settlement: Spot transactions versus forward transactions

- **Spot transactions:**

t = 0 (today)

Completion of the agreement (to buy/sell an asset) and **settlement** (delivery of the asset and payment)

- **Forward transactions:**

t = 0 (today) t = T (future)

Settlement
(delivery of the asset and payment)

Completion of the agreement
(to buy/sell an asset)

The conclusion of the agreement (to buy/sell an asset) and the settlement (delivery of the asset and payment) take place at different points in time.

The price is already determined at the conclusion of the agreement (t=0).

Obligation means:

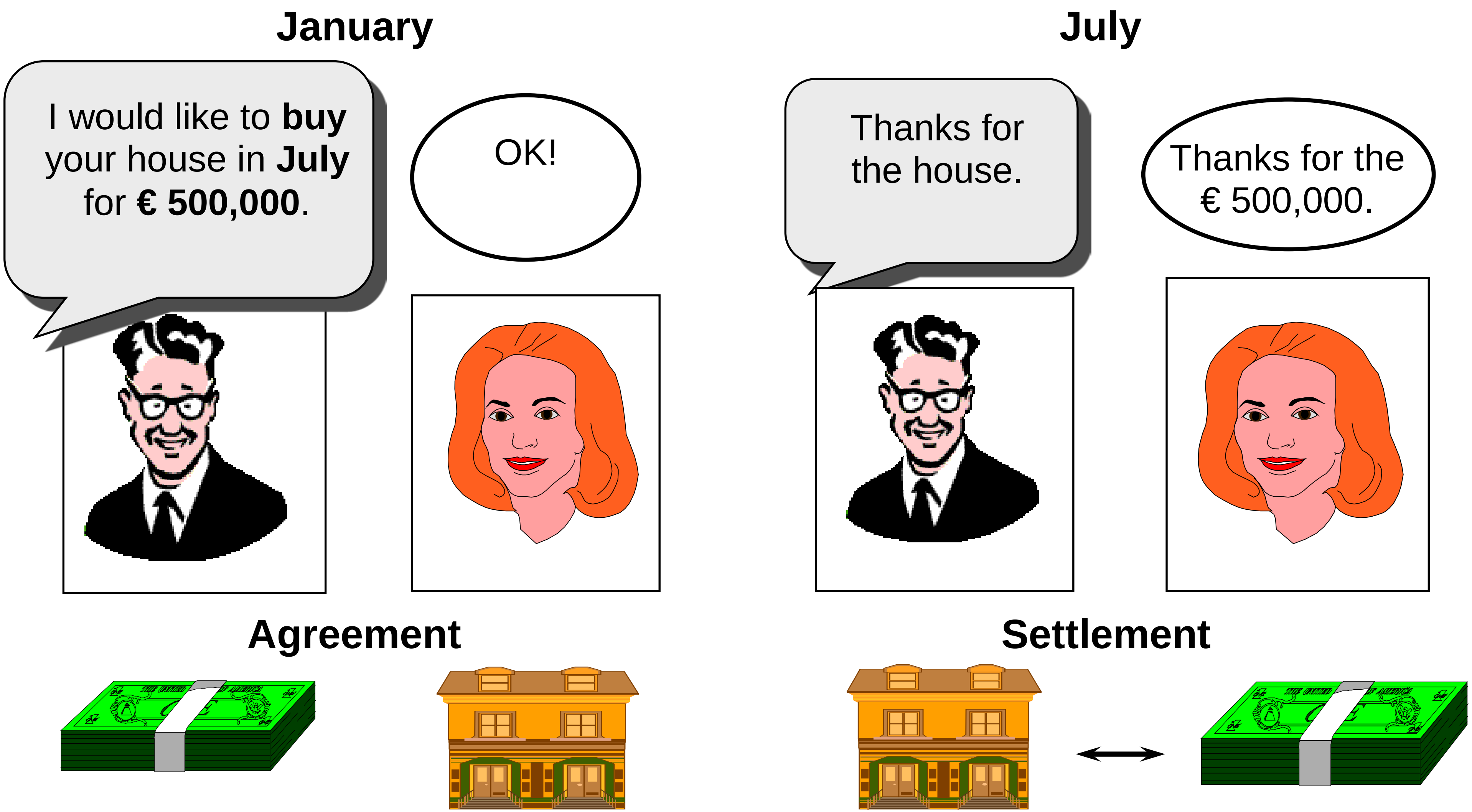
- **Long position**: Buying a forward contract

  At the maturity date, the buyer is obliged to take delivery of the underlying instrument of the futures contract (or settle in cash), and to pay.

- **Short position**: Selling a forward contract

  At the maturity date, the seller is obliged to deliver the underlying instrument of the futures contract (or settle in cash), and receives the payment.

Example of a forward contract:



3. Forward Contracts - Basics

➔ (Forward-) Contract between two parties.

A forward contract is an agreement to buy/sell a predetermined **asset** at a pre-determined **price** at a predetermined point in **time**.

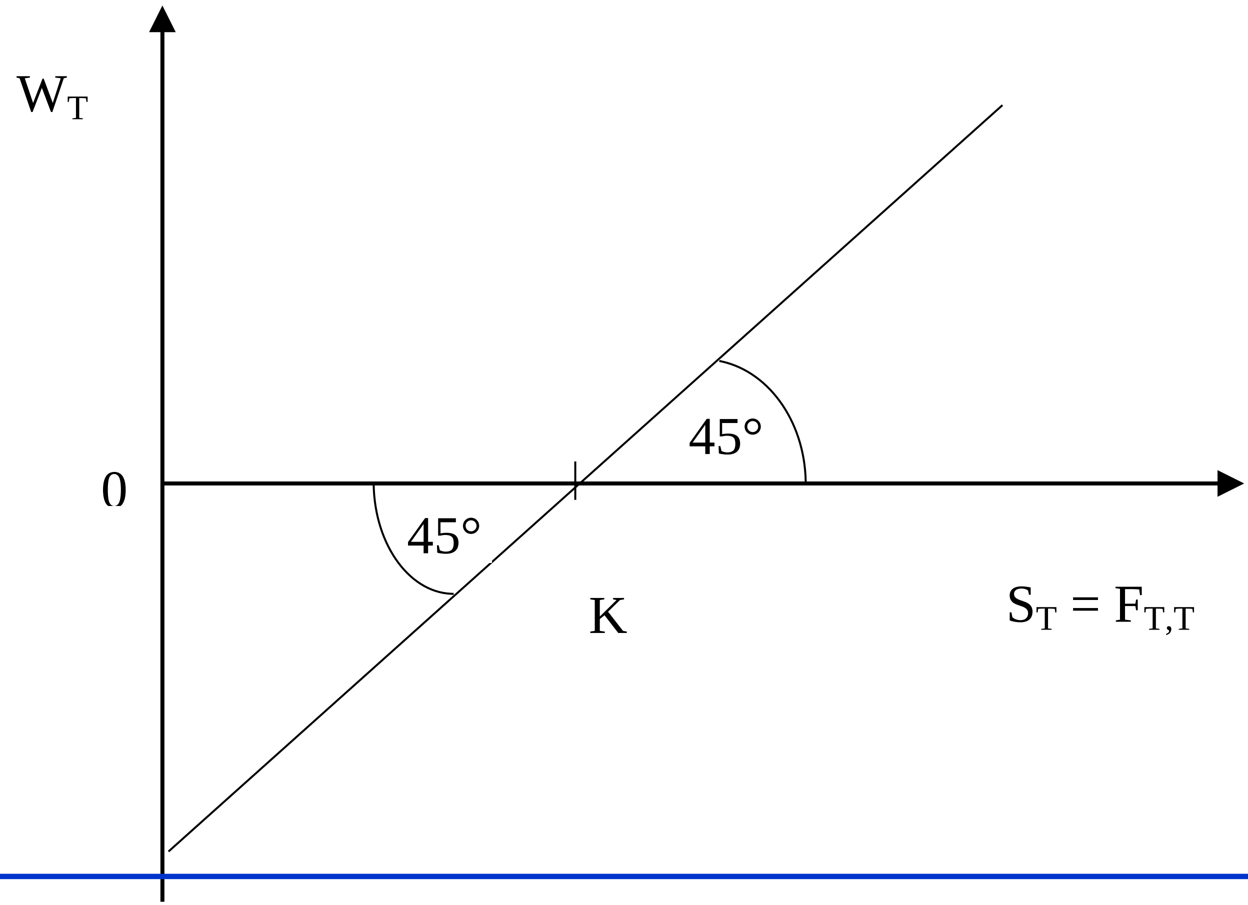
Buyer:	**Forward purchase**	➔	**long Position**
Seller:	**Forward sell**	➔	**short Position**

Delivery price, forward price, contract value

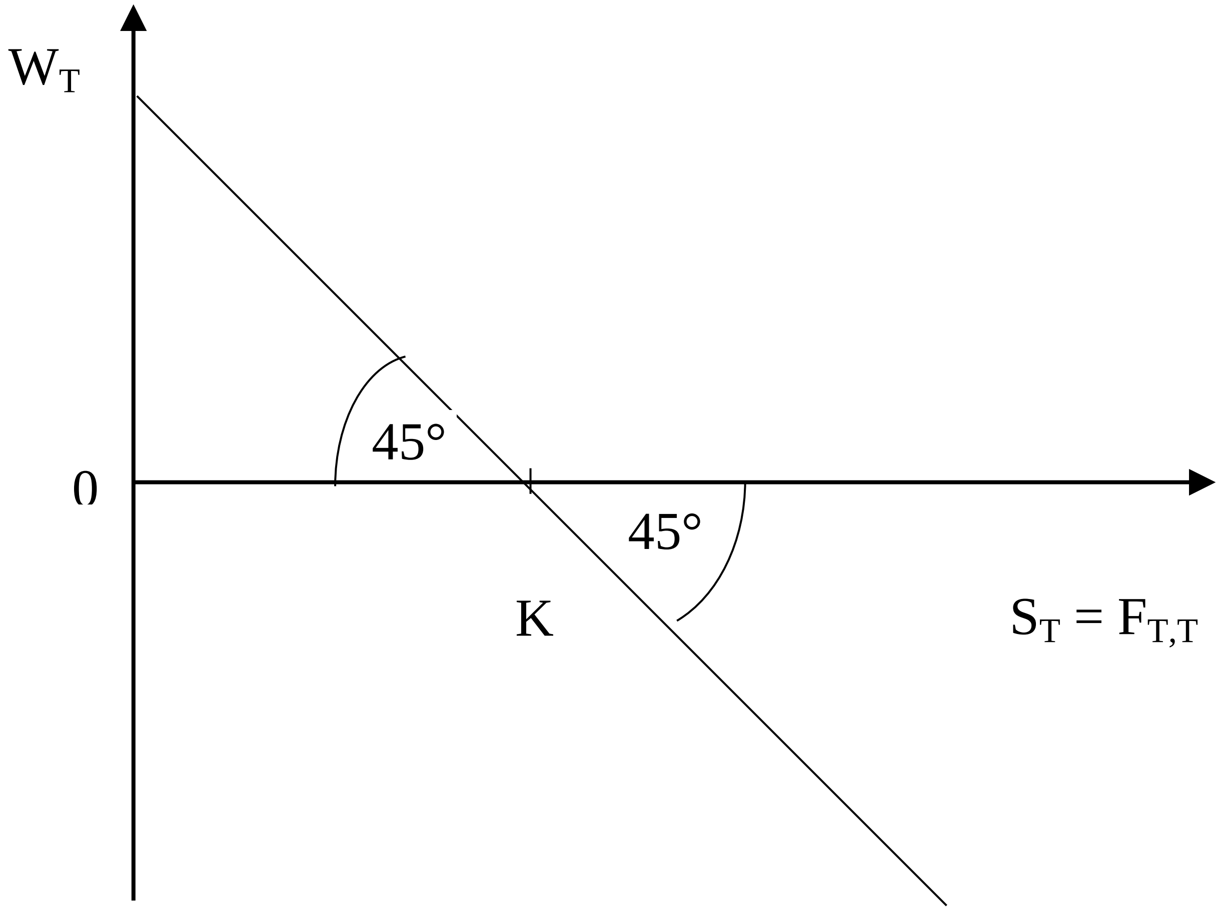
	$t = 0$	$t = T$
Delivery price (K)	K	K
Forward price (F)	$F_{0,T} = K$ (normally: $F_{0,T} \neq S_0$)	$F_{T,T} = S_T$
Contract value (W)	$W_0 = F_{0,T} - K = 0$	$W_T = F_{T,T} - K$ (or $W_T = S_T - K$)

Value of a forward contract at maturity (t = T):

(a) Long Position (forward buying position)



(b) Short Position (forward selling position)



Example

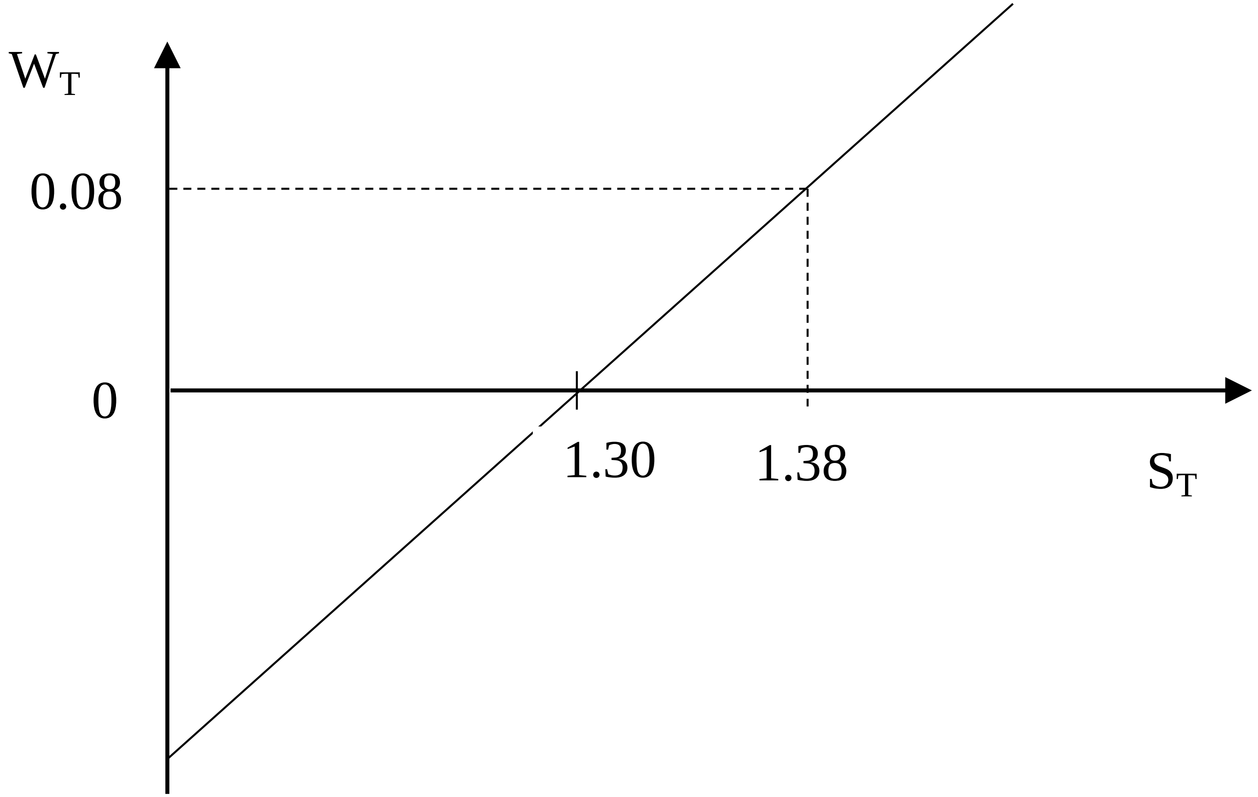
A US-investor negotiates a forward contract to buy in 3 months **€ 1 Mio**. at a price of **$ 1.30 per €** (= **long** position; uncovered long position).

Calculate the **profit** of this forward contract, when

(a) the spot price increases to $ **1.38** per €

(b) the spot price decreases to $ **1.25** per €

Ad (a): S_T = 1.38 $/€



$W_T = S_T - K = 1.38 - 1.30 = 0.08$ $/€

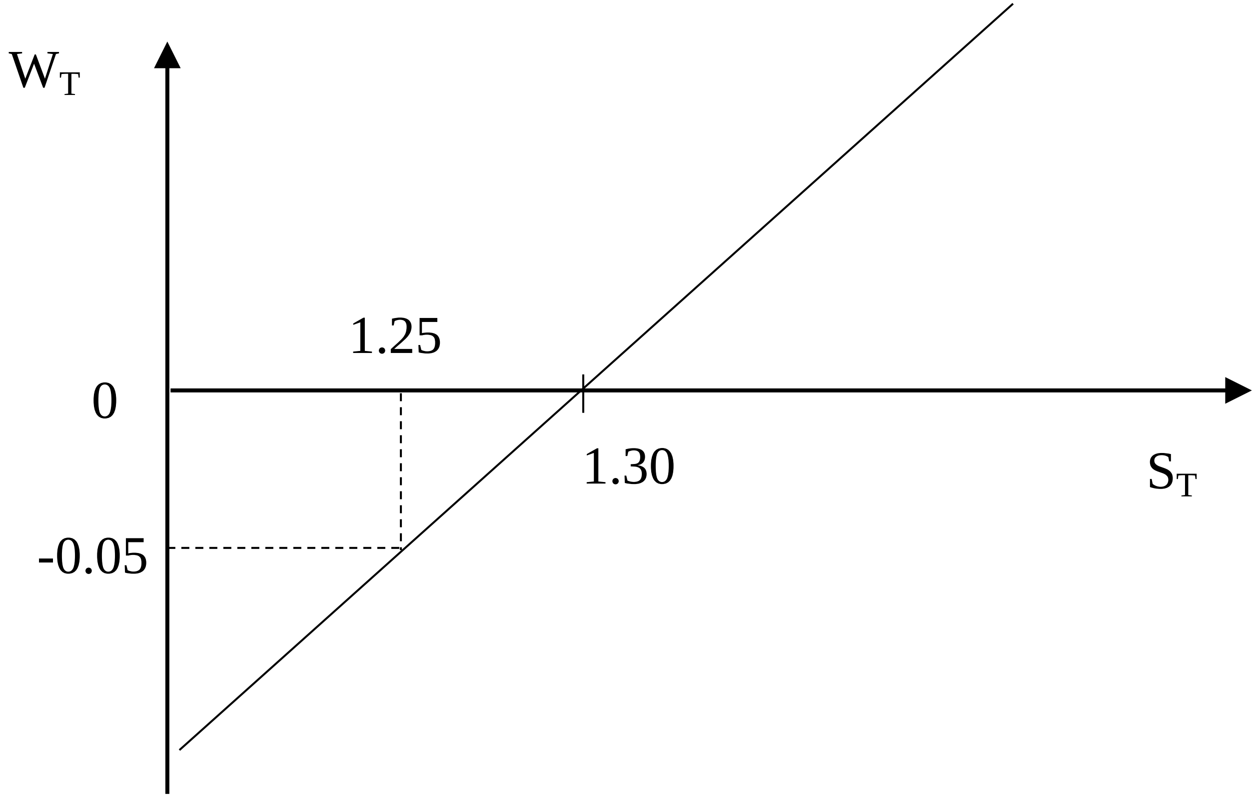
For a contract volume of € 1 Mio:

Profit = $ 80,000

In fulfilling the long position the investor buys € 1 Mio for $ 1,300,000.

But this position already has a value of $ 1,380,000 at maturity.

Ad (b): S_T = 1.25 $/€



$W_T = S_T - K = 1.25 - 1.30 = -0.05$ \$/€

For a contract volume of € 1 Mio:

Profit = \$ -50,000

Thus, a spot market price of $S_T = 1.25$ \$/€ generates for the investor a loss.

At maturity, she has to buy € 1 Mio. at \$ 1,300,000, although the €-position only has a value of \$ 1,250,000.

Three groups of investors can be observed in forward/futures markets:

- **Hedger:** A market participant that wants to hedge (secure) against current or future financial risks. ➔ Selling of current or future risks of (typically existing) spot market positions.

- **Trader:** Deliberately takes open positions (risks). He/she is expecting to generate (based on favourable pricen changes: rising or falling prices) a profit.

- **Arbitrageure:** Market participants trying to generate riskless profits by implement two or more transactions (typically in different markets) at the same time, e.g. attempting to profit from discrepancies between forward and spot prices.

❍ Hedging

➔ Passing on the risk of future price changes to other market participants.

(a) Hedging of an **existing position** **(short hedge)**:

➔ Opposite position in the forward/futures market

Aim**:** Profit of the spot markt transaction (**LONG**)
+ Profit of the forward/futures market (**SHORT**)
= 0

(b) Hedging a **forthcoming transaction** in the spot market **(long hedge)**:

➔ Forward transaction with a similar sign (Lock-in-Effect)

Aim**:** Profit of the forward/futures market (**LONG**)
= Profit based on an immediate transaction on the spot market (**LONG**)

➔ earlier buy on the spot market

Example

15.1.2007 — 15.3.2008

Andritz:

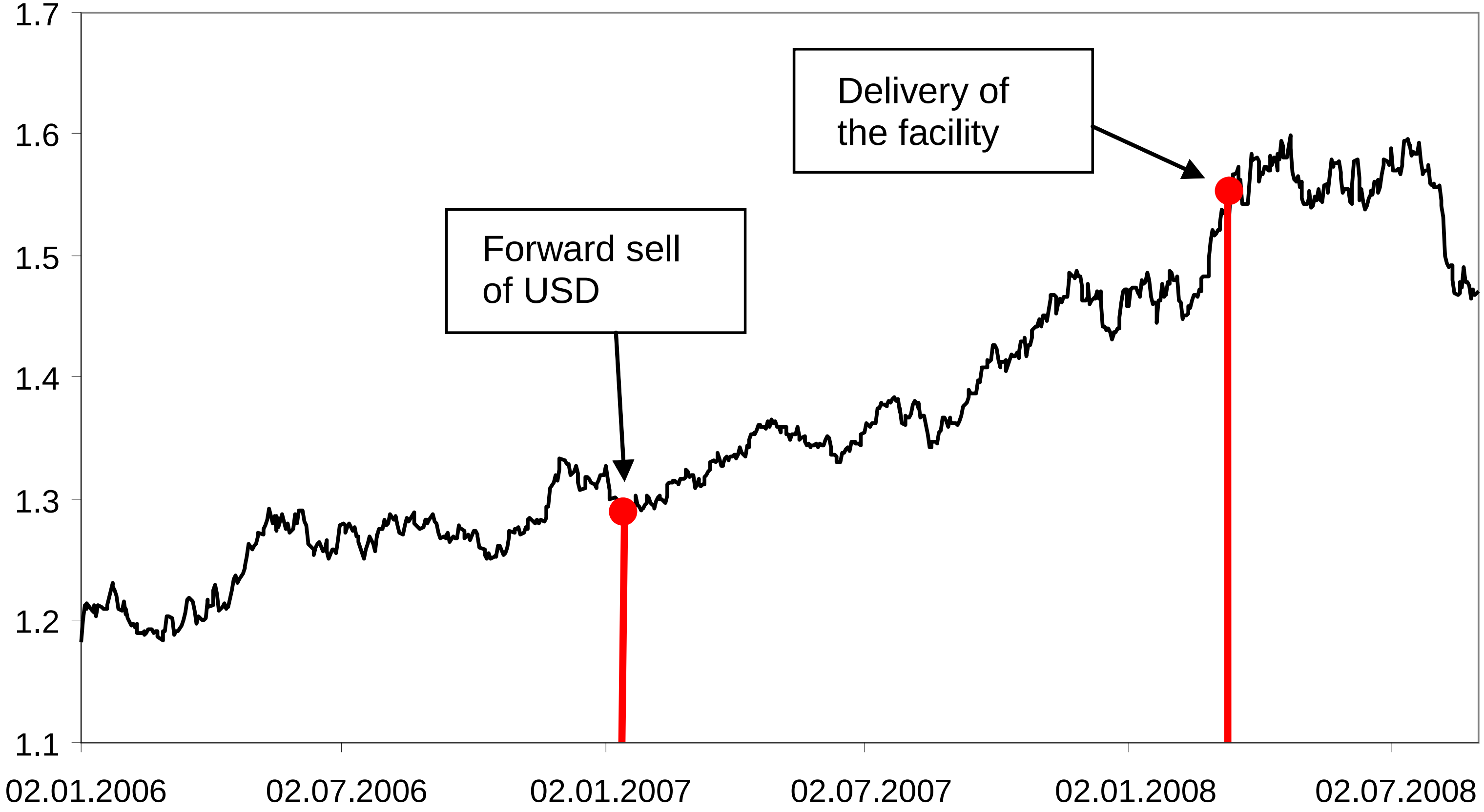
Sale of an industrial facility
(USA in $; price = $200 Mio)

FX
1.2935 $/€ (or 0.7731 €/$)
154.69 Mio EUR

Forward selling position
(short hedge)
Forward price:
1.3137 $/€ (or 0.7612 €/$)
€-Spot Rate (14M) = 4.05% p.a.
$-Spot Rate (14M) = 5.45% p.a.

Delivery
Payment
(200 Mio $)

Price development: USD per EUR (January 2006 until August 2008)



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USD dropped until 15.3.2008 from 0.7731 €/$ (=1/1.2935 $/€) to 0.6380 €/$ (=1/1.5674 $/€)

➔ Value of spot market transaction:

127.608 Mio EUR (=0.6380 · 200 Mio)

➔ Value of spot market plus forward market transaction:

152.242 Mio EUR (=0.7612 · 200 Mio)

Total: +24.634 Mio EUR

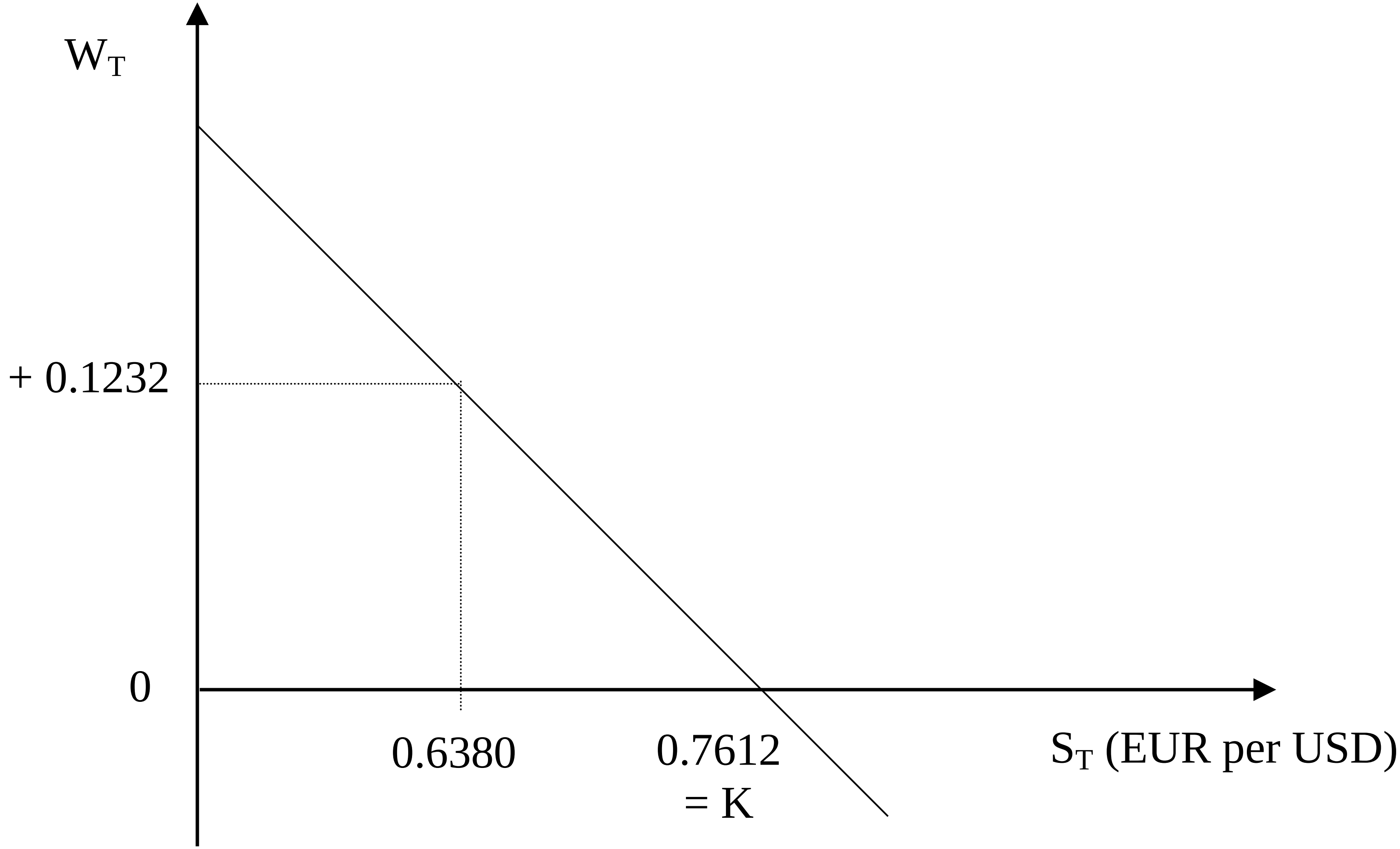
➔ **Hint: Concentrate on your comparative advantages!**

Cash flows at delivery (maturity) date (t = T = 15.3.2008):

	t = 0 (15.1.2007)	t = T (15.3.2008)	
Sold facility		+200 Mio USD ⇓	Revenue
Forward (short)		-200 Mio USD ⇓	Fulfillment of the Forward (Delivery of USD)
		+152.242 Mio €	Revenue from fulfillment of the forward
Total	0	+152.242 Mio €	

➔ The revenue of € 152.242 Mio is **certain** (i.e., without FX risk) and already known at t=0!

Value of the forward contract (EUR per USD !), short position:



Alternative: Immediate sell of USD on 15.1.2007 (t = 0).

➔ Transactions:

	t = 0 (15.1.2007)		**t = T (15.3.2008)**
Sold facility			+ 200 Mio USD
			⇓
Loan in USD	+188,043,563.43 $	⇐ 5.45% p.a. ($)	-200 Mio USD
	⇓		
Sell: USD (1.2935 USD je €)	-188,043,563.43 $ ⇓		
Amount realized: EUR	145,375,773.81 €		
	⇓		
Investment	-145,375,773.81 €	4.05% p.a. (€) ⇒	+152,244,779.13 €
Total	0		+152.244 Mio €

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4. Relationship between Spot and Forward Prices

To understand how forwards and futures markets work, it is necessary to understand how prices of futures contracts relate to the spot prices of the underlying assets.

The market force that most strongly affects the pricing of forwards/futures contracts is **arbitrage**.

Arbitrageurs look for **profits** offered by the **difference between forward /futures and spot prices**.

As arbitrageurs trade between the forward/futures and spot markets, **prices will adjust** until such profit opportunities are eliminated.

Why do we normally observe: $\boxed{F_{0,T} \neq S_0}$

Holding the **asset underlying** a forwards/futures contract is on the one hand **costly** (financing costs, storage costs, insurance costs, etc.) and can on the other hand provide **income** (dividends, coupons, etc.).

The relationship between forward/futures and spot price can be summarized in terms of the **cost of carry**.

For a forward/futures contract with a remaining time to maturity T the **arbitrage free relationship** between forwards/futures price ($F_{0,T}$) and spot price (S_0) is

$$F_{0,T} = S_0 \cdot \left(1 + c_{0,T}\right) \tag{1}$$

where

$c_{0,T}$ = cost of carry between today (t = 0) and the expiration date (t = T) of the contract

Dependent on the forward/futures contract, c can be positve ($c > 0$) or negative ($c < 0$).

Now lets in a first step assume that

- there are no transaction costs,
- the margin system (futures contracts) is cost neutral (i.e. investors earn interest on their margin payments),
- there are no additional costs involved in holding the underlying asset (depart from financing costs),
- the underlying asset has no payouts, such as dividends or coupons, and
- that arbitrageurs can borrow and lend at the same rate between today ($t = 0$) and the expration date of the contract ($t = T$).

Under this assumptions the following relationship must hold:

$$F_{0,T} = S_0 \cdot (1 + r_{0,T}) \qquad (2)$$

where

$r_{0,T}$ = Spot rate for borrowing/lending between today (t = 0) and the expiration date (t = T) of the contract

In (2) the cost of carry only consists of a financing cost component. $r_{0,T}$ represents the interest that can be earned (from t = 0 to t = T) on the amount S_0 when the investor invests in the forwards/futures contract instead buying/selling the underlying asset (at the price S_0).

The arbitrage portfolio behind equation (2) consists of the following components:

	t = 0	t = T
Buy the underlying asset (spot market)	$-S_0$	$+\tilde{S}_T$
Borrow to finance the purchase	$+S_0$	$-S_0 \cdot (1 + r_{0,T})$
Go short forward/futures	0	$F_{0,T} - \tilde{S}_T$
Arbitrageportefeuille	0	$F_{0,T} - S_0 \cdot (1 + r_{0,T})$

where

$\tilde{S}_T$ = Spot price of the underlying asset at maturity of the forward/futures contract (random variable)

This strategy requires no net investment at time t. Therefore, the arbitrage portfolio has to have a value of zero at maturity of the forwards/futures contract, i.e.

$$F_{0,T} - S_0 \cdot (1 + r_{0,T}) \quad = \quad 0$$

This leads to the **fundamental no-arbitrage relationship**:

$$F_{0,T} = S_0 \cdot \left(1 + r_{0,T}\right).$$

Market forces will always drive the relationship between spot and forwards /futures prices towards this **fundamental no-arbitrage equation**. It is therefore the relationship that will prevail in **equilibrium**.

If this relationship is violated **arbitrageurs will react** accordingly. Depending on whether the forward/futures price is too high or too low relative to the spot price a **cash-and-carry** or a **reverse cash-and-carry arbitrage** opportunity will emerge.

- **Cash-and-carry arbitrage**

A cash-and-carry arbitrage opportunity emerges if the forward/futures price is above its no-arbitrage value, i.e.

$$F_{0,T} > S_0 \cdot (1 + r_{0,T}).$$

Example

Let's assume that one troy ounce of gold is currently selling in the spot market for \$420 (S_0), the current forward/futures price for delivery in one year is \$440 ($F_{0,T}$) and the one year spot rate is 2% p.a.

In this case, the fundamental no-arbitrage relationship is violated:

$$440 > 420 \cdot (1 + 0.02)$$

$$440 > 428.4$$

The forward/futures price is greater than the no-arbitrage price, so a **cash-and-carry arbitrage strategy** is appropriate:

	t = 0	t = T
Buy 100 troy ounces of gold in the spot market	-42,000	$+\tilde{S}_T \cdot 100$
Borrow to finance the purchase (2% p.a.)	+42,000	-42,840
Go **short** (**sell**) one forwards/futures contract (**100** troy ounces) (delivery in 1 year, $F_{0,T}$ = 440)	0	$44{,}000 - \tilde{S}_T \cdot 100$
Net	0	+$1,160

* 1 contract (= minimum amount) equals 100 troy ounces

The arbitrageur is **locking in a profit of $1,160** (= $11.60 · **100**) no matter what the price of gold is one year from now.

Since no investment is required to earn this certain profit, arbitrageurs can make unlimited amounts by repeating this arbitrage strategy over and over.

Eventually the **pressure of their trading** will cause the **forward/futures price to fall** and the **spot price to rise** so that the profit drops to zero and the fundamental no-arbitrage relationship is restored.

- **Reverse cash-and-carry arbitrage**

Now suppose the no-arbitrage equation is violated in the opposite direction so that

$$F_{0,T} < S_0 \cdot (1 + r_{0,T}).$$

The forward/futures price is now less than the no-arbitrage price, and arbitrage profits can be earned by following the reverse cash-and-carry strategy.

Example:

Let's assume that the current spot price for one troy ounce of gold is still \$420 (S_0), but that the forward/futures price for delivery in one year from now is only \$415 ($F_{0,T}$). Again, the one year spot rate is 2% p.a.

Also in this case the fundamental no-arbitrage relationship is violated:

$$415 < 420 \cdot (1 + 0.02)$$

$$415 < 428.4$$

This violation of the no-arbitrage relationship provides a reverse cash-and-carry arbitrage opportunity:

	t = 0	t = T
Short (**sell**) of 100 troy ounces of gold (spot market)	+42,000	$-\tilde{S}_T \cdot 100$
Lending of sales proceeds (at 2% p.a.)	-42,000	+42,840
Go **long** (**buy**) 1 gold forward/futures contract (100 troy Ounces, delivery in 1 year, $F_{0,T}$ = 415)	0	$\tilde{S}_T \cdot 100 - 41{,}500$
Net	0	+$1,340

Or: $1,340 = $13.40 · 100

A reverse-cash-and-carry arbitrage requires that the **underlying asset to be sold in the spot market**.

There are two main possibilites to reach this requirement:

(1) Classical short selling

Short selling involves selling assets that one does not own. Therefore, it is necessary to **borrow the underlying asset**, e.g. from a broker or a financial institution.

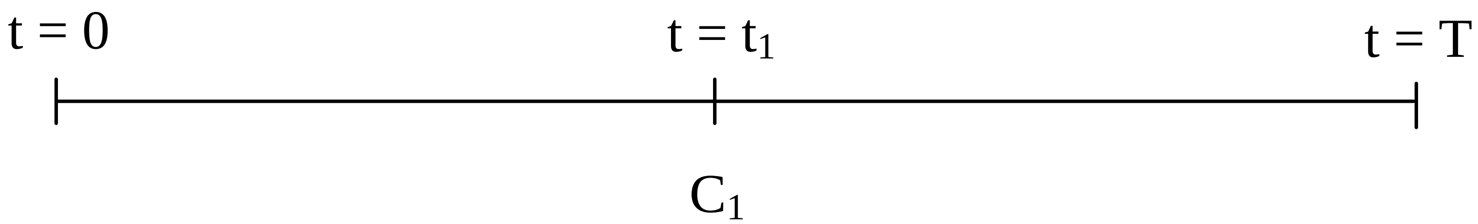
(2) Sell of an existing long position

Another possibility is that a market participant, **owning the underlying asset** with a time **horizon beyond the expiration date** of the forward/futures contract, sells the asset in the spot market and simultaneously buys it back by entering into a long forward/futures position. Therefore, in a reverse cash-and-carry arbitrage such an investor can earn an additional amount of money.

- **Arbitrage with payouts**

Holding the underlying asset can be associated with **payouts**. Some, like dividends for stocks or coupons for bonds, are **positive**. Others, such as the costs of storage or insurance for commodities, like e.g. corn or wheat, are **negative**.

These payouts change the fundamental no-arbitrage equation. Suppose the underlying asset has a payout of C_1 at time t_1:



Then the fundamental no-arbitrage equation (2) becomes

$$F_{0,T} = S_0 \cdot (1 + r_{0,T}) - C_1 \cdot (1 + r_{t_1,T}), \tag{3}$$

where $r_{t1,T}$ is the interest (not annualized) earned between t_1 and T.

$C_1 > 0$: E.g., coupon or dividend **payments**

$C_1 < 0$: E.g., storage or insurance **costs**

Equation (3) is again an equilibrium relationship. If it does **not** hold, **arbitrage** profits are available.

Example: continued

Let's again assume that one troy ounce of gold is currently selling in the spot market for \$420 (S_0) and the one year spot rate is 2% p.a.

In addition, after 6 month the holder of the asset has to pay storage costs of \$2 per ounce.

The forward rate rate for the 6 month period from t = 6 months to t = T = 12 months is 1% (not annualized, i.e. for 6 months).

The current forward/futures price for delivery in one year ($F_{0,T}$) is **(a)** \$440 and **(b)** \$415.

(a) Cash-and-carry arbitrage

The forward/futures price of $440 is above the no-arbitrage price:

$$F_{0,T} > S_0 \cdot (1 + r_{0,T}) - C_1 \cdot (1 + r_{t_1,T})$$

$$440 > 420 \cdot (1 + 0.02) - (-2) \cdot (1.01)$$

$$440 > 430.42$$

Arbitrage strategy: Cash-and-carry arbitrage

	$t = 0$	$t = t_1$	$t = T$
Buy 100 troy ounces of gold (spot market)	-42,000		$+\tilde{S}_T \cdot 100$
Borrow to finance the purchase (2% p.a.)	+42,000		-42,840
Storage costs for 100 troy ounces of gold		-200	
Borrow to finance the storage costs (1%)		+200	-202
Short one forward/futures contract (100 troy ounces) (delivery in 1 year, $F_{0,T} = 440$)	0		$44{,}000 - \tilde{S}_T \cdot 100$
Net	0	0	+$958

Or: $\$958 = \$9.58 \cdot 100$

At time t=0, the arbitrageur goes short the relative overpriced forward/futures contract and buys the relative underpriced gold in the spot market.

At the end of the year (t = T) he or she delivers the bought 100 troy ounces in the forward/futures contract.

In addition, at time t_1 the arbitrageur has to pay $200 of storage costs.

(b) Reverse cash-and-carry arbitrage

In this second case the forward/futures price is below the no-arbitrage price:

$$F_{0,T} < S_0 \cdot (1 + r_{0,T}) - C_1 \cdot (1 + r_{t_1,T})$$

$$415 < 420 \cdot (1 + 0.02) - (-2) \cdot (1.01)$$

$$415 < 430.42$$

The arbitrageur should perform a reverse cash-and-carry arbitrage:

	t = 0	$t = t_1$	t = T
Short 100 troy ounces of gold (spot market)	+42,000		$-\tilde{S}_T \cdot 100$
Lending the sales proceeds (at 2% p.a.)	-42,000		+42,840
Saved storage costs		+200	
Lending of the saved storage costs (1%)		-200	+202
Long 1 gold forward/futures contract (100 ounces) (Delivery in 1 year, $F_{0,T} = 415$)	0		$\tilde{S}_T \cdot 100 - 41{,}500$
Net	0	0	+1,542

Or: $\$1{,}542 = \$15.42 \cdot 100$

- **Transaction costs and arbitrage**

Up to this point, we have assumed that abritrageurs can trade without incurring transaction costs.

Of course, this is not the case in the real world.

The costs of trading in forward/futures and spot markets will affect the fundamental no-arbitrage relationships.

Types of transactions costs:

Bid-Ask spreads in spot as well as forward/futures markets

Different borrowing and leding rates

Short selling costs

Transaction fees (all other costs that traders encount in the marketplace)

To include transaction costs in the pricing relationship we use the following notation (for simplicity, we will ignore payouts for now):

$S_{0,b}$ = **bid price** of the underlying asset (spot market)
$S_{0,a}$ = **ask price** of the underlying asset (spot market)
$r_{0,T,b}$ = **borrowing rate** between today (t=0) and expiration (t=T)
$r_{0,T,l}$ = **lending rate** between today (t=0) and expiration (t=T)
TF = **transaction fees**

Different to equation (2), the forward/futures price now can not be determined exactly.

Thus, the transaction costs of performing arbitrages introduce some "play" into forward/futures prices.

The following relationship shows that the forward/futures price is between a **no-arbitrage lower bound** and a **no-arbitrage upper bound**.

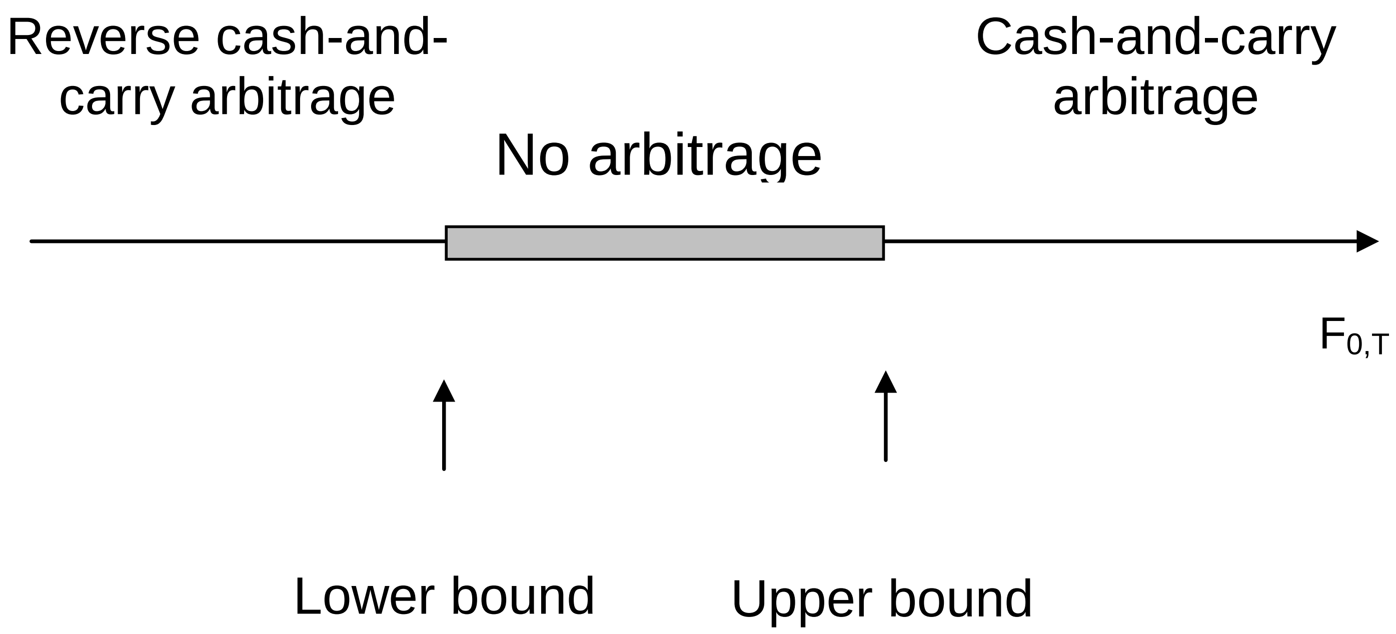
$$S_{0,b} \cdot \left(1 + r_{0,T,l}\right) - TF \;\leq\; F_{0,T} \;\leq\; S_{0,a} \cdot \left(1 + r_{0,T,b}\right) + TF \qquad (4)$$

The size of this no arbitrage bound solely depends on the **size of the transactions costs**.

The lower the transactions costs are, the smaller the bound will be and the closer the forward/futures price will be related to the spot market price of the underlying asset.

A necessary condition for a "good" forward/futures contract is, therefore, a **liquid spot market**, a **liquid forward/futures market** and **low additional transaction costs** (fees).

No arbitrage opportunities exist if the forward/futures price is between the price bounds, as the inflows from arbitrage strategies do not cover transaction costs incurred.



- **Basis**

As the delivery date of a forward/futures contract approaches, the **forward/futures price converges to** the **spot price** of the underlying asset.

Due to the decreasing time to maturity, the cost-of-carry also decrease.

When the delivery date is reached, the futures price equals - or is very close to - the spot price.

This occurs because a forward/futures contract that **expires immediately** is **exactly the same** as a spot transaction.

There is no "future" for such a contract, so the price for immediate delivery through a forward/futures contract must be the same as that for immediate delivery on the spot market.

This movement of the forward/futures price to the spot price is called **delivery date convergence**.

The difference between the spot price and the forward/futures price is called the spot-forward/futures basis:

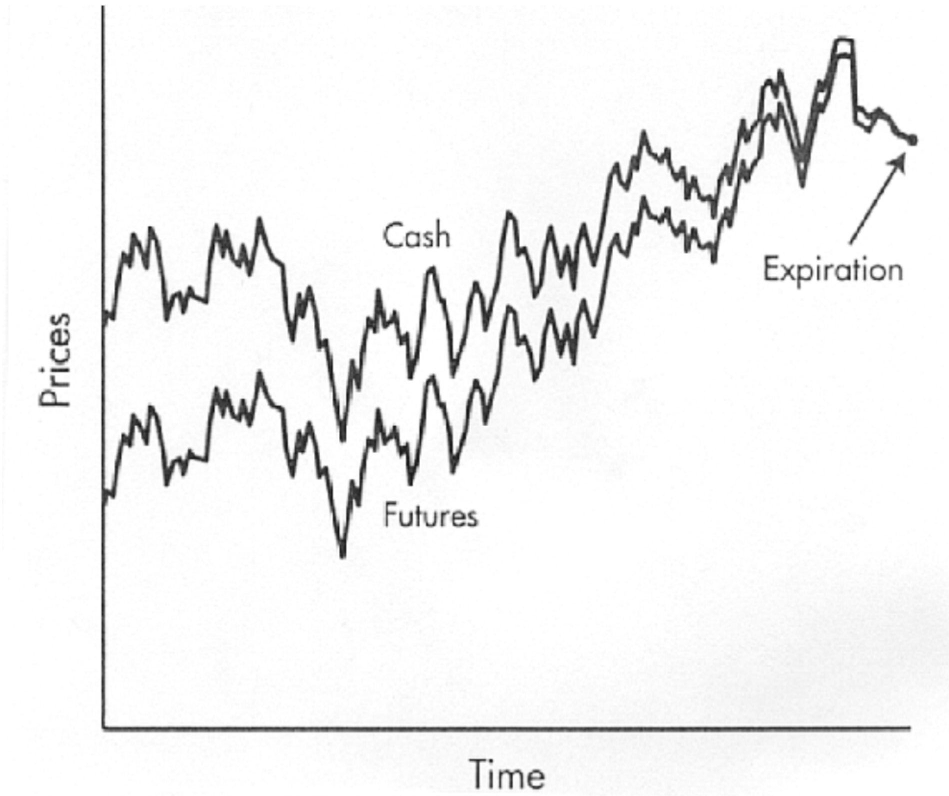
$$Basis_t = S_t - F_{t,T} \tag{5}$$

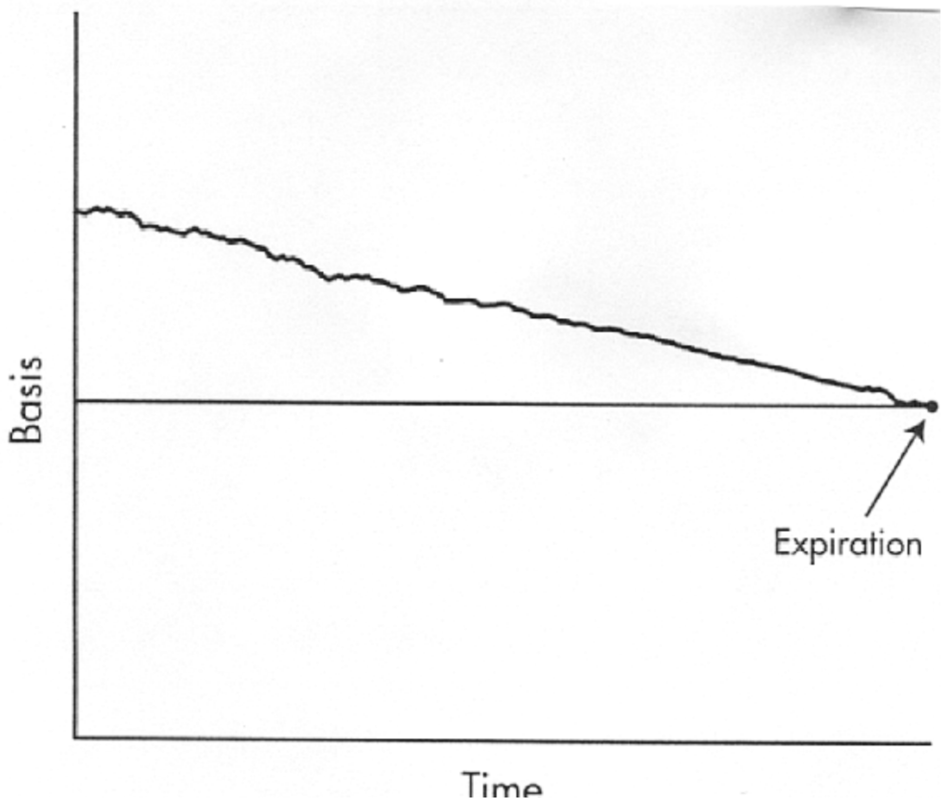
When a contract expires at T, delivery date convergence ensures that the basis is equal to zero:

$$Basis_T = S_T - F_{T,T} = 0$$

The following figure illustrates the movement of the forward/futures price to the spot price.

Spot-forward/futures price convergence (e.g. bond futures with coupon > $r_{0,T}$):





5. FX-Forwards

- An **FX-long (buying) position** (FX-short (selling) position) in an **FX-Forward** (Devisentermingeschäft) entitles to **buy** (sell; see e.g. case of Andritz below) a predetermined **amount** of foreign exchange at a predetermined point in **time** at a predetermined **price**.

- Price = **Forward price (Terminkurs)**

- With existing money market rates the forward price is given via arbitrage relations.

Example

Exchange rate $/€ (quoted price): 1.25 $ per €; 12M-EURIBOR: 1.85% p.a.; 12M-$-LIBOR: 2.03% p.a. Calculate the arbitrage free forward price (FP) for a FX-buying position of 1 Mio. USD with a time to maturity of 12 months.

Action	completion (t=0)	one year later (t=1)
(a) FX-Forward:		
Forward purchase of 1 Mio USD	0	+ 1,000,000 [USD]
(= forward sell of €)		- ($FP \cdot$ 1,000,000) [EUR]
(b) Synthetic replication:		
Investment in USD (2.03%)	- 980,103.89 [USD]	+1,000,000 [USD]
Spot market: Purchase of USD	+ 980,103.89 [USD]	
(1.25 USD per €)	- 784,083.11 [EUR]	
Loan in € (1.85%)	+784,083.11 [EUR]	- 798,588.65 [EUR]

- Arbitrage free forward price: 0.79859 € per $ (or 1.25221 $ per €)

- If forward price < 0.79859 € per $:

Arbitrage portfolio:

- FX-long position (forward market)
- Loan in USD
- Investment in €
- Spot market: Sell of USD

- **Equation for the forward price:**

(***Note****: Due to the quotation convention of $ per € the forward price FP [€ per $] corresponds exactly to the reciprocal of the quoted forward price [$ per €]*):

$$FP_T = S \cdot \frac{1 + T \cdot r_{Home}(T)}{1 + T \cdot r_{Foreign}(T)} = 0.80 \cdot \frac{1 + 0.0185}{1 + 0.0203} = 0.79859 \quad [€ \text{ per } \$]$$

FP_T	Forward price for maturity T (Home currency for <u>one unit of foreign exchange</u>; from a EUR viewpoint)
S	Spot market price (Home currency for one unit of foreign exchange; from a EUR viewpoint)
$r_{Home}(T)$	Interest rate of the home country for maturity T p.a. (MMY: Money Market Yield)
$r_{Foreign}(T)$	Foreign interest rate for maturity T p.a. (MMY)

or based on quoted prices:

$$FP_{T,Q} = S_Q \cdot \frac{1 + T \cdot r_{Foreign}(T)}{1 + T \cdot r_{Home}(T)} = 1.25 \cdot \frac{1 + 0.0203}{1 + 0.0185} = 1.25221 \text{ [\$ per €]}$$

$FP_{T,Q}$	Quoted forward price for maturity *T* (*Foreign exchange for <u>one unit of home currency</u>;* from a EUR viewpoint)
S_Q	Spot market price (*Foreign exchange for one unit of home currency;* from a EUR viewpoint)
$r_{Home}(T)$	Interest rate of the home country for maturity T p.a. (MMY)
$r_{Foreign}(T)$	Foreign interest rate for maturity T p.a. (MMY)

whereby: $FP_{T,Q} = \frac{1}{FP_T} = \frac{1}{0.79859} = 1.25221 \text{ [\$ per €]}$

Under MMY the interest rate calculation takes place on a linear basis (i.e. without compound interest (Zinseszinsen); see calculations above).

For longer periods, rates available are typically based on **BEY** (Bond Equivalent Yield: with compound interest). Adjustment:

$$FP_T = S \cdot \frac{\left[1 + r_{Home}(T)\right]^T}{\left[1 + r_{Foreign}(T)\right]^T}$$

- If discount rates (*P(T)*) are given:

$$FP_T = S \cdot \frac{P_{Foreign}(T)}{P_{Home}(T)}$$

- The FX-forward price is driven by three risk factors:
  - FX-spot market price
  - Discount factor in FX (same maturity as forward)
  - Discount factor in home currency (same maturity as forward)

Valuation principle: FX-Forward

A **purchase (long) position** in an **FX-Forward** can be decomposed into **two zero bonds**:

A purchase position in a zero bond in FX, with a nominal value equal to the nominal value of the FX-purchase position (i.e. long $, Nom = Nom FX), and

a selling position in a zero bond in home currency, with a nominal value equal to the forward price multiplied by the nominal value (Nom) of the FX-purchase position (i.e. short €, Nom = FP · Nom FX).

A **sale (short) position** in an FX-Forward can be decomposed into **two zero bonds**:

A selling position in a zero bond in FX, with a nominal value equal to the nominal value of the FX-selling position (i.e. short $, Nom = Nom FX), and

a buying position in a zero bond in home currency, with a nominal value equal to the forward price multiplied by the nominal value of the FX-selling position (i.e. long €, Nom = FP · Nom FX).

The **market value** of the **forward** equals the **sum** of the market values of the two zero bonds (in home currency).

The **FX-purchase (long) position** in the **example above** can be decomposed into the following components:

- **Zero bond in USD:**

  Nominal = Nominal of the FX-buying position = **1 Mio USD**

  Risk factors:
  - **1Y-Discount factor in USD**
  - **EUR/USD price (=1/S_Q)**

- **Zero bond in Euro:**

  Nominal = (Nominal of the FX-buying position)·forward price =

  1 Mio USD·0.79858865 (EUR/USD) = **798,588.65 EUR**

  Risk factor: **1Y-Discount factor in Euro**

6. Futures – Mechanics

Forwards, futures: **unconditional forward transactions**

↳ **Obligation** to buy or sell a specific underlying instrument, at a price agreed upon today, at a specific date in the future.

- **Futures are traded on an exchange, forwards are traded OTC**
- Futures contracts are **standardized (contract specifications)**
  - ↳ Maturity, contract size, delivery procedure, underlying (characteristics), etc.

Example: COMEX Gold Futures (26.4.2013, USD per troy unce)

	Current Session (26.4.2013)								Prior Session (25.4.2013)	
	Open	High	Low	**Last**	Time	Settlement	Chg	Vol	Settlement	Op Int
May'13	1466.0	1483.9	1448.0	**1462.0**	17:25	1453.60	-8.2	976	1461.8	1,448
Jun'13	1467.3	1484.8	1447.3	**1462.2**	17:25	1453.60	-8.4	230,240	1462.0	250,714
Aug'13	1467.1	1485.6	1449.6	**1463.7**	17:25	1455.10	-8.6	14,300	1463.7	49,563
Oct'13	1469.0	1485.8	1452.6	**1465.0**	17:25	1456.30	-8.8	3,272	1465.1	8,680
Dec'13	1470.7	1488.3	1452.4	**1462.3**	17:25	1457.50	-8.9	4,837	1466.4	44,076
Feb'14	1481.5	1482.0	1455.0	**1467.2**	17:25	1458.70	-9.0	1,109	1467.7	15,510
Apr'14	1482.8	1484.0	1468.3	**1468.6**	17:25	1459.90	-9.0	235	1468.9	5,593
Jun'14	1481.0	1490.4	1457.0	**1457.0**	17:25	1461.20	-9.0	268	1470.2	8,900
Aug'14	1483.0	1483.0	1483.0	**1483.0**	17:25	1462.60	-9.0	24	1471.6	1,125
Oct'14	1484.0	1484.0	1484.0	**1484.0**	17:25	1464.00	-9.0	5	1473.0	749
Dec'14	1487.6	1490.6	1469.8	**1469.8**	17:25	1465.40	-9.0	9	1474.4	8,645
Feb'15	-	-	-	**1428.2**	17:25	1466.90	-9.0	-	1475.9	12
Jun'15	-	-	-	**1385.0**	17:25	1470.10	-9.1	5	1479.2	8,258
Dec'15	-	-	-	**1450.4**	17:25	1475.70	-9.1	-	1484.8	11,737

- **Daily settlement, marking to market, clearinghouse**

Forward: gains or losses are realized **when the contract expires**

Futures: gains or losses are realized on a **daily basis**

The system of **daily settlement** in the futures market is called **marking to market**.

It is carried out through an intermediary body called the futures **clearinghouse**.

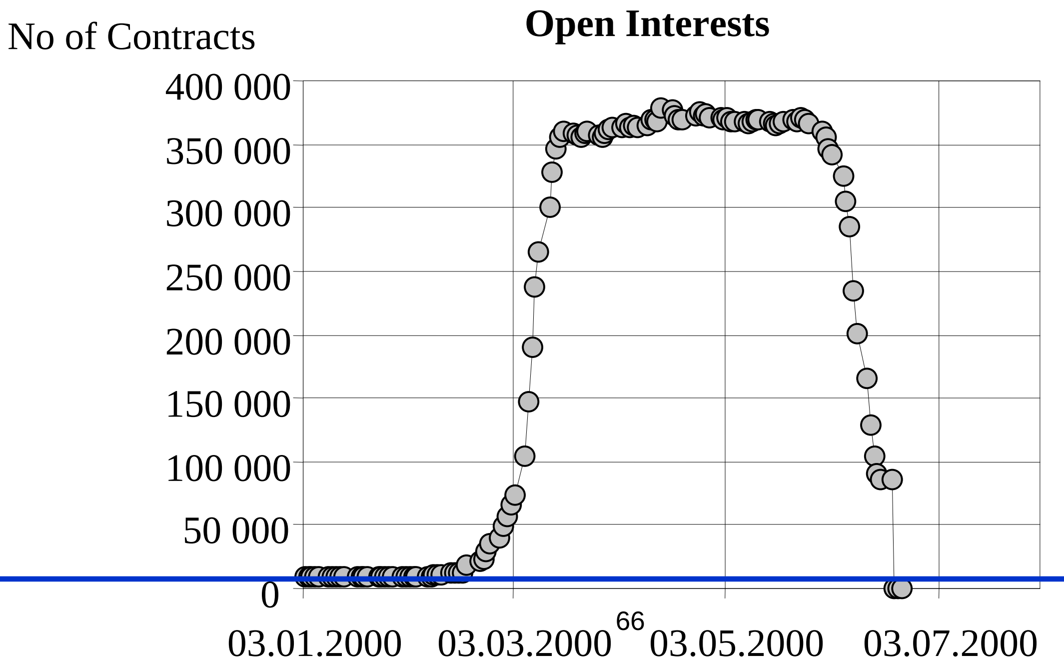
↳ The clearinghouse carries the counterparty risk.

- **Settlement or close-out**

Futures contracts are settled through **physical delivery or cash settlement**.

Very few futures positions are held until maturity: the **majority of contracts are closed out** before.

Open interests for the Standard & Poor's stock index June 2000 futures contract:



- **Summary:** Most important differences between forward and futures contracts

Forward	Futures
Private contract between two parties	Traded on an **exchange**
Not standardized	**Standardized** contract
Settled at the end of the contract	**Settled daily**
Delivery or final cash settlement usually takes place	Contract is **usually closed out** prior to maturity
Counterparty risk	**No counterparty risk**

- Large Derivative Exchanges
  (Source: FIA Annual Volume Survey (2014), Futures Industry, pp. 19ff)

Exchange Rank

Ranked by number of contracts traded and/or cleared. Futures and options volume broken down by subsidiary exchanges.

Rank	Exchange	Jan-Dec 2013 Volume	Jan-Dec 2014 Volume	Annual % Change
1	**CME Group**	**3,161,476,638**	**3,442,766,942**	**8.9%**
	Chicago Mercantile Exchange	1,551,802,526	1,775,988,677	14.4%
	Chicago Board of Trade [1]	1,090,449,431	1,171,499,384	7.4%
	New York Mercantile Exchange	519,224,681	495,278,881	-4.6%
2	**Intercontinental Exchange**	**2,558,489,589**	**2,276,171,019**	**-11.0%**
	ICE Futures Europe [2]	1,118,381,584	993,647,768	-11.2%
	NYSE Amex	549,050,523	473,742,797	-13.7%
	NYSE Arca	451,427,061	438,869,148	-2.8%
	ICE Futures U.S.	433,500,235	364,250,670	-16.0%
	ICE Futures Canada	5,688,295	5,659,335	-0.5%
	Singapore Mercantile Exchange	441,891	1,301	-99.7%
3	**Eurex**	**2,190,727,275**	**2,097,974,756**	**-4.2%**
	Eurex	1,551,889,344	1,490,541,110	-4.0%
	International Securities Exchange	606,765,206	481,279,337	-20.7%
	International Securities Exchange Gemini	32,072,725	126,154,309	293.3%

4	**National Stock Exchange of India**	**2,127,151,585**	**1,880,362,513**	**-11.6%**
5	**BM&FBovespa**	**1,603,706,918**	**1,417,925,815**	**-11.6%**
	Bolsa de Valores de Sao Paulo	910,894,138	790,094,482	-13.3%
	Bolsa de Mercadorias & Futuros	692,812,780	627,831,333	-9.4%
6	**Moscow Exchange**	**1,134,477,258**	**1,413,222,196**	**24.6%**
7	**CBOE Holdings**	**1,187,642,669**	**1,325,391,523**	**11.6%**
	Chicago Board Options Exchange	1,070,865,472	1,193,388,385	11.4%
	C2 Exchange	76,583,750	81,387,833	6.3%
	CBOE Futures Exchange	40,193,447	50,615,305	25.9%
8	**Nasdaq OMX**	**1,142,955,206**	**1,127,130,071**	**-1.4%**
	Nasdaq OMX PHLX	681,995,742	617,770,938	-9.4%
	Nasdaq Options Market	326,388,360	386,177,089	18.3%
	Nasdaq OMX Nordic	98,387,962	90,070,658	-8.5%
	Nasdaq OMX Boston	35,334,377	31,590,712	-10.6%
	Nasdaq OMX Commodities	848,765	1,520,674	79.2%

- Futures and Options Volume
  (Source: FIA Annual Volume Survey (2014), Futures Industry, pp. 19ff)

Global Futures and Options Volume by Category

Based on the number of contracts traded and/or cleared at 75 exchanges worldwide

Category	Jan-Dec 2013	Jan-Dec 2014	% Change
Individual Equity	6,390,404,778	6,493,177,097	1.6%
Equity Index	5,381,657,190	5,827,913,937	8.3%
Interest	3,330,904,991	3,268,154,625	-1.9%
Currency	2,496,423,691	2,119,023,131	-15.1%
Agriculture	1,209,776,849	1,400,153,550	15.7%
Energy	1,315,276,356	1,160,317,682	-11.8%
Non-Precious Metals	646,349,077	872,601,162	35.0%
Precious Metals	433,546,140	370,872,772	-14.5%
Other	347,412,764	355,224,591	2.2%
Total	**21,551,751,836**	**21,867,438,547**	**1.5%**

Note: Other includes contracts based on commodity indices, credit, fertilizer, housing, inflation, lumber, plastics and weather.

A **margin account** is used to **settle the day-to-day changes** in a futures position.

All gains and losses are calculated by looking at the difference between the **previous and current** days' **settlement price**.

(1) Initial margin (**original or additional margin**)

- Original amount that must be put into an account to establish the futures position.
- To determine the initial margin: past volatility

(2) Maintenance margin

- To ensure that the balance in the margin account never becomes negative.
- If the balance in the margin account falls **below the maintenance margin,** the investor receives a **margin call** and is expected to **top up the margin account** to the initial margin level the next day.
- The extra funds deposited: **variation margin**. If the investor does **not provide** the variation margin, the **broker closes out** the position by selling the contract.

Example

Balance development of a margin account

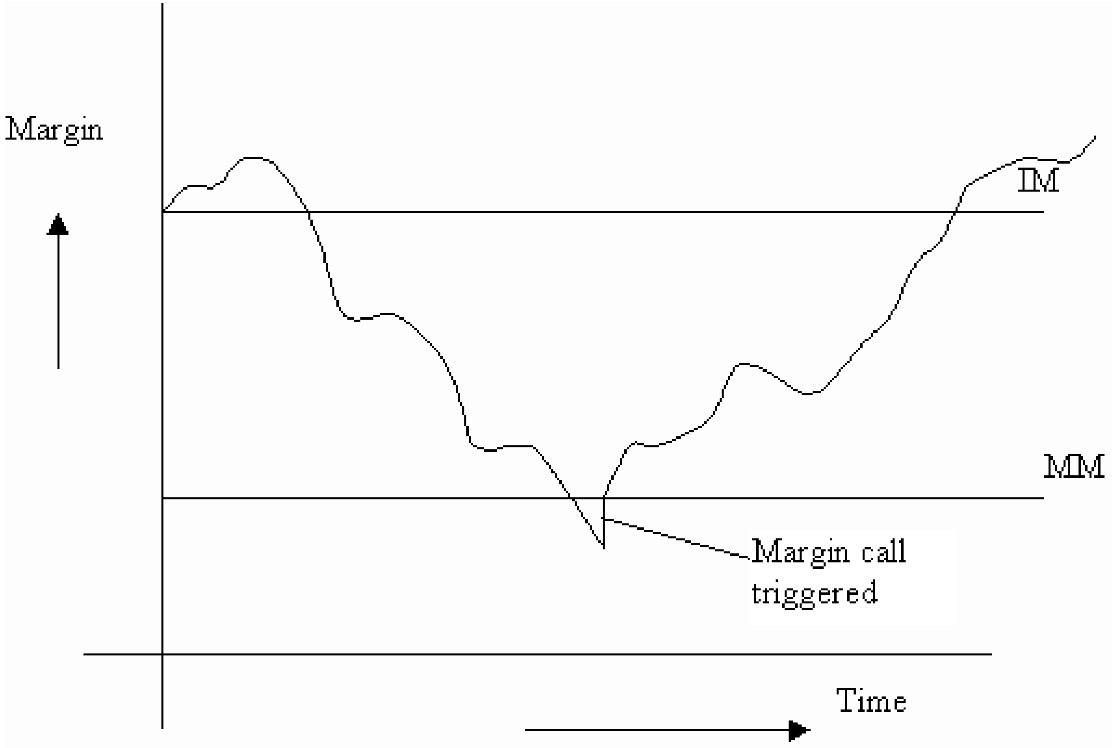
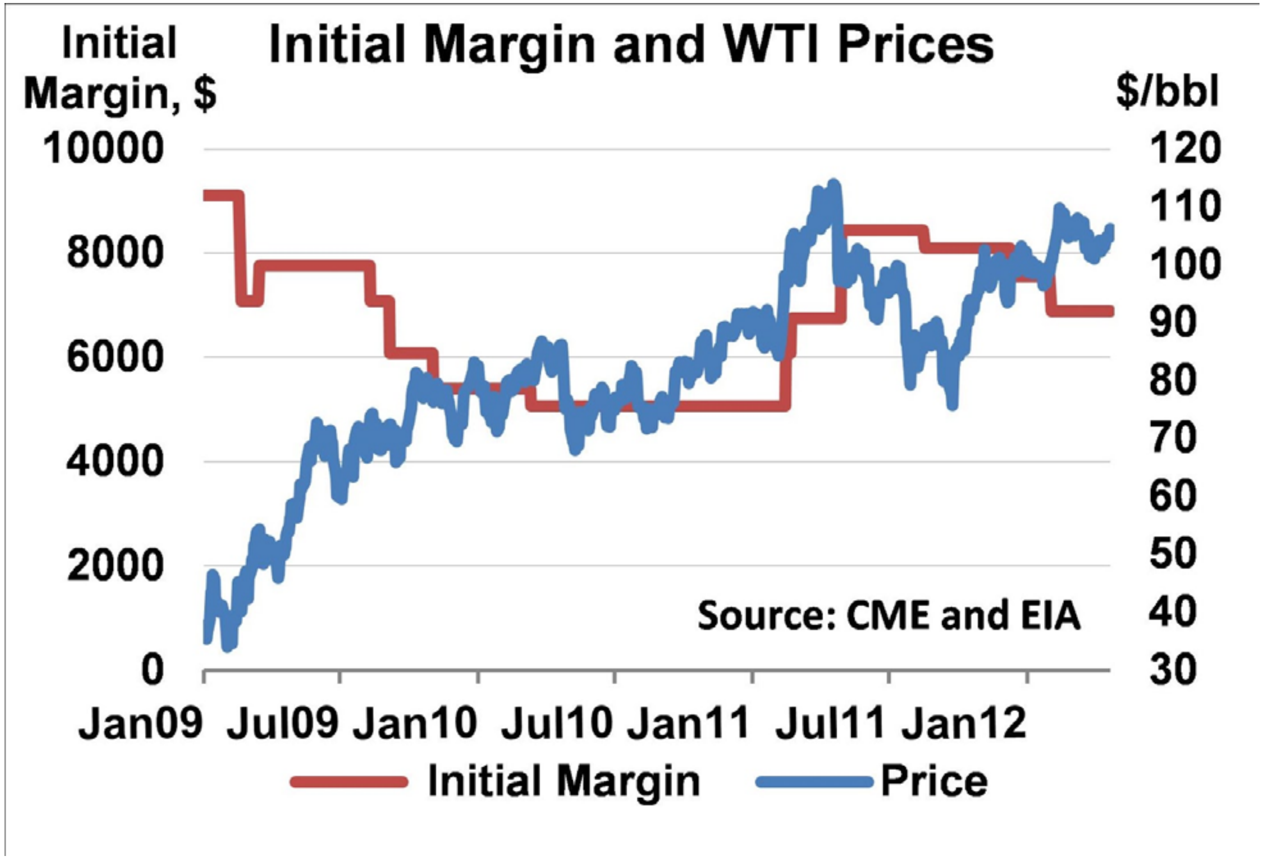


Figure: Typical behavior of Actual Margin over time

IM: Initial Margin, MM: Maintenance margin

Example

West Texas Intermediate (WIT) Futures **Initial Margin** development



Example

On October 1, 2003 investor A establishes a long position in 5 contracts of the COMEX February 2004 Gold futures. One contract consists of 100 troy ounces.

Initial margin: $2,025 per contract (or **$20.25** per troy **ounce**)

➔ ~ 5% of underlying value

Maintenance margin: $1,500 per contract (or **$15.00** per troy **ounce**)

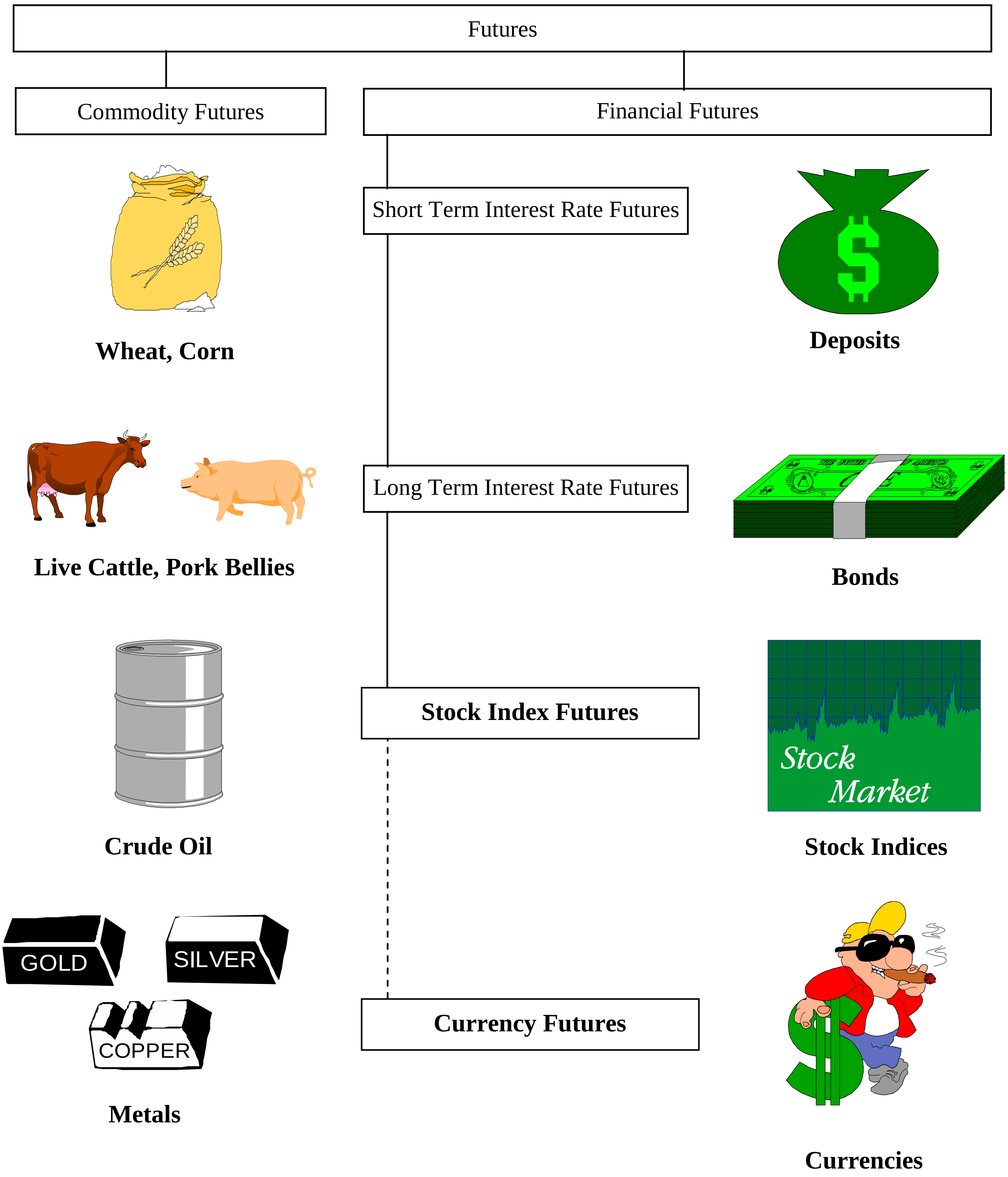
➔ ~ 4% of underlying value

The contract is **entered into** on October 1, 2003 at **$385.8** (=close on that trading day).

The long position is **closed out** on October 24, 2003 at the closing price of **$390.1**.

Date	Gold Feb 2004 futures price $	daily gain/loss $	cumulative gain/loss $	Beginning margin account balance $	**margin call** $	cash **withdrawal** $	Ending margin account balance $
01.10.2003	385.8			10,125			**10,125**
02.10.2003	384.5	-650	-650	9,475			9,475
03.10.2003	370.8	-6,850	-7,500	**2,625**	+7,500		**10,125**
06.10.2003	374.1	1,650	-5,850	11,775		-1,650	**10,125**
07.10.2003	378.6	2,250	-3,600	12,375		-2,250	**10,125**
08.10.2003	376.8	-900	-4,500	9,225			9,225
09.10.2003	370.6	-3,100	-7,600	**6,125**	+4,000		**10,125**
10.10.2003	374.9	2,150	-5,450	12,275		-2,150	**10,125**
13.10.2003	376.5	800	-4,650	10,925		-800	**10,125**
14.10.2003	377	250	-4,400	10,375		-250	**10,125**
15.10.2003	374	-1,500	-5,900	8,625			8,625
16.10.2003	374.1	50	-5,850	8,675			8,675
17.10.2003	373.1	-500	-6,350	8,175			8,175
20.10.2003	375.3	1,100	-5,250	9,275			9,275
21.10.2003	382.9	3,800	-1,450	13,075		-2,950	**10,125**
22.10.2003	387.7	2,400	950	12,525		-2,400	**10,125**
24.10.2003	390.1	1,200	**2,150**	11,325		-1,200	**10,125**
Total					**11,500**	**13,650**	

Types of futures contracts:



7. Stock Index Futures

A stock index futures is a contract to **buy or sell a stock market index** (=portfolio of individual stocks) underlying the futures contract.

Stock index futures are generally **settled in cash**.

Not all stock indices can be used for futures trading.

- ↳ The **arbitrage bound** should **not be too wide**.
- ↳ The market in **individual stocks** that comprise the index should be **liquid**.

❍ Contract specifications

- **Example 1: Standard & Poor's 500 Stock Price Index Futures**

Underlying:	Standard & Poor's 500 Stock Price Index (S&P 500)
Contrakt size:	$250 times the S&P 500 Index *E-mini contract: $50 times the S&P 500 Index*
Contract month:	8 months in the March, June, September, December cycle plus 3 additional December contracts *E-mini contract: 5 months in the quarterly cycle (Mar, Jun, Sep, Dec)*
Last trade:	3:15 p.m. on Thursday prior to the 3rd Friday of the contract month. *E-mini contract: Trading can occur up to 8:30 a.m. on the 3rd Friday of the contract month*
Final settlement price:	Opening price of the index on the 3rd Friday of the contract month.

- **Quotes: S&P 500 E-mini Stock Index Futures, CME, 12.3.2018:**

Month	Open	High	Low	Last	Settle	Volume	Open Interest
MAR 18	2784.00	2800.50	2779.00	2783.00	2784.00	1,210,960	2,313,022
JUN 18	2788.75	2805.25	2783.75	2788.25	2789.00	1,644,022	1,166,354
SEP 18	2800.00	2812.25	2791.75	2795.50	2796.25	2,062	15,875
DEC 18	2805.00	2813.00	2799.50	2800.50	2802.00	53	24,922
MAR 19	-	2808.75	-	2808.75	2809.00	3	41

Maintenance Margin: 5,800 USD

Cash Market (S&P 500 Index): 2,783.02

- **Example 2: DAX Stock Price Index Futures**

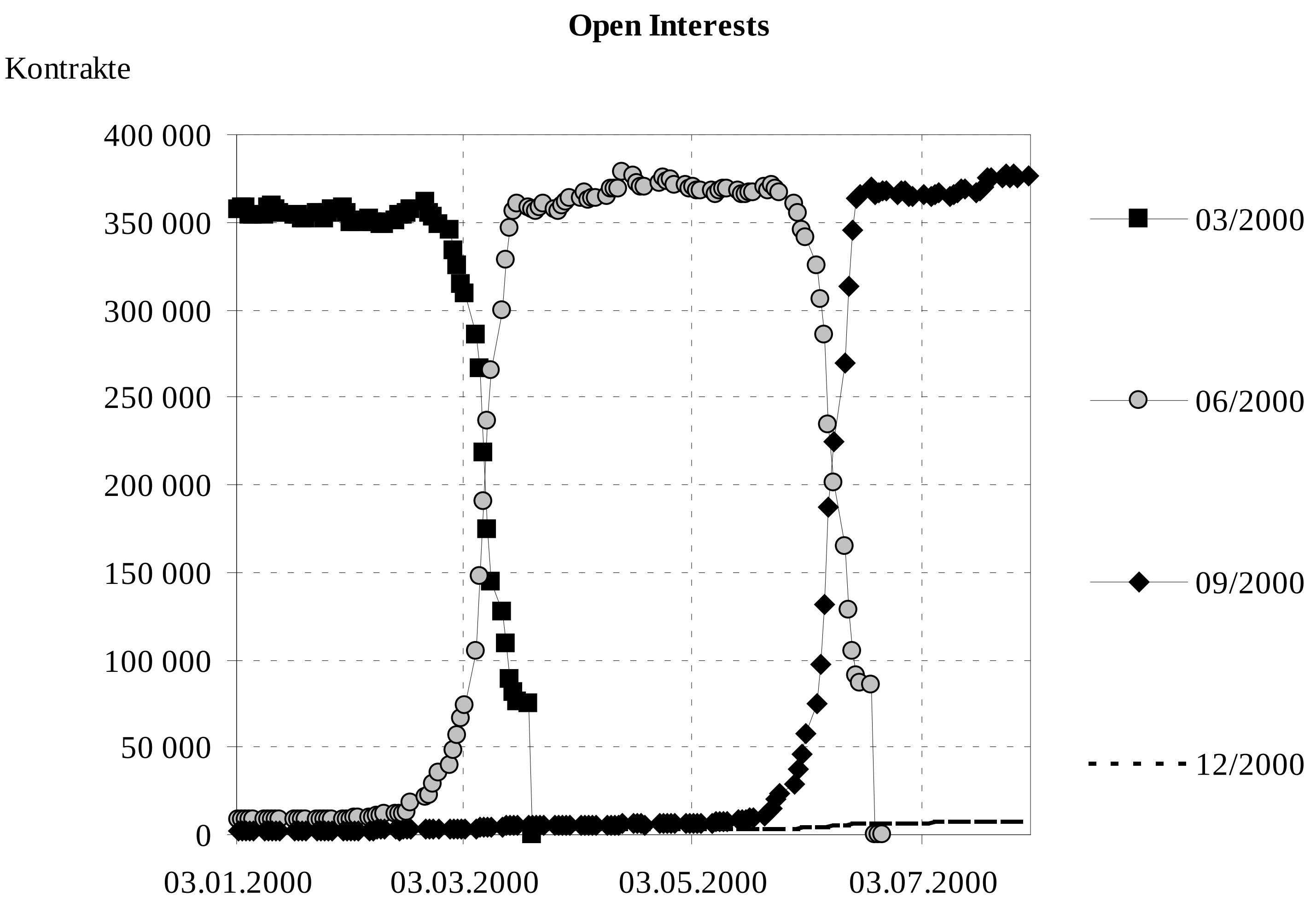
Underlying:	DAX 30 Stock Price Index
Contrakt size:	€25 times the DAX 30 Index
Contract month:	3 nearest months in the cycle March, June, Sep, Dec
Last trading day:	3rd Friday of the expiration month, if that is an exchange day; otherwise the exchange day immediately prior to that Friday.
Daily settlement:	The Daily Settlement Prices for the current maturity month are derived from the volume-weighted av. of the prices of all trans-actions during the minute before 17:30 CET, provided that more than five trades took place within this period.
Final settlement:	Cash settlement based on the final settlement price.
Final settlement price:	Is established by Eurex on the Final Settlement Day of the con-tract (=last trading day) and is determined by the value of the respective index, based on Xetra® auction prices of the respec-tive index component shares. The intraday auction starts at 13:00 CET (for MDAX® component shares at 13:05 CET). For the remaining maturity months, the Daily Settlement Price for a contract is determined based on the average bid/ask spread of the combination order book.

- **Quotes: Dax Stock Index Futures, Eurex, 12.3.2018:**

Delivery month	Open	High	Low	Last P.	Settlem. Price	Traded contracts	Open interest
Mar 18	12,435.0	12,476.0	12,357.5	12,398.0	**12,425.0**	**112,482**	**139,012**
Jun 18	12,443.5	12,487.0	12,374.5	12,380.0	**12,439.5**	**45,867**	**42,308**
Sep 18	12,430.5	12,433.0	12,360.0	12,420.0	**12,425.0**	72	1,053

Cash Market (Dax Index): 12,418.39

Near by contracts of the Standard & Poor's 500 Stock Index Futures:



❍ Contract size (Examples)

- **Standard & Poor's 500 E-mini Stock Index Futures, June 2018, 12.3.2018**

  Contract size =

- **DAX Stock Index Futures, June 2018, 12.3.2018**

  Contract size =

Important contracts

Index	Country	Exchange
DJIA	US	CME
S&P 500	US	CME
NASDAQ 100	US	CME
S&P/TSX 60	Canada	Montreal Exchange
MEX BOLSA	Mexico	MexDer
BOVESPA	Brazil	BM&F Bovespa
EURO STOXX 50	Europe	Eurex
FTSE 100	UK	NYSE Euronext
CAC40	France	NYSE Euronext
DAX	Germany	Eurex
IBEX 35	Spain	MEFF
FTSE/MIB	Italy	Borsa Italiana
AMX Index	Netherlands	NYSE Euronext
OMXS30	Sweden	Nasdaq OMX
SMI Index	Swiss	Eurex
NIKKEI 225	Japan	OSE
HANG SENG	Hong Kong	Hong Kong Exchange
SPI 200	Australia	ASX
CSI 300	China	China Financial Futures Ex.
Kospi 200	South Korea	KRX
S&P CNX Nifty	India	CME

An investor wishing to enter into a long position to take the **market risk** of a particular stock market index (like the S&P 500) has **two possibilities**:

- **Buying all individual shares** comprised in the index (in line with the weighting of the index) on the spot market, or

- **buying stock index futures** contracts.

Buying futures contracts has the advantage of no **funding costs** for the cash purchase (depart from margin account requirements) but the disadvantage of no **dividend income**.

This leads to the no-arbitrage equation for stock index futures:[1]

$$F_{0,T} = S_0 \cdot (1 + r_{0,T}) - \sum_{j=1}^{N} Div_j \cdot (1 + r_{t_j,T})$$

$\Sigma Div_j \cdot (1+r_{tj,T})$ **Value of all dividends** paid between t=0 (today) and t=T (expiration of the contract) on a portfolio that mimics the index **at time t = T**.

N Number of dividend payments between t = 0 und t = T

$r_{0,T}$ Spot rate between t = 0 and the expiration of the contract (t = T)

1 For simplicity we again assume that there are no transactions costs and we neglect interest effects of the daily settlement.

Example

Suppose the S&P500 spot index is 1,122. We wish to calculate the no-arbitrage price for a futures contract with a remaining time to maturity of 0.2 years. The future value of expected dividends on the S&P500 till the last trading day of this contract amounts to 3.3 index points and the corresponding spot rate is 1.5% p.a.

No-arbitrage futures price: $F_{0,T} = 1{,}122 \cdot (1 + 0.015 \cdot 0.2) - 3.3 = 1{,}122.07$

Sometimes it is assumed that stocks underlying the index provide a dividend yield. If $d_{0,T}$ is the dividend yield rate, the no-arbitrage equation changes to

$$F_{0,T} = S_0 \cdot (1 + r_{0,T} - d_{0,T}).$$

Example

Consider again the above contract of the S&P500 stock index futures.

Now suppose that the stocks underlying the index provide a dividend yield of 0.3% in the remaining time to maturity of 0.2 years.

Then the arbitrage free futures price is given by

$$F_{0,T} = 1{,}122 \cdot (1 + 0.015 \cdot 0.2 - 0.003) = 1{,}122.$$

The **basis** (S_0 - $F_{0,T}$) of a stock index futures contract can be positive or negative.

Negative basis (**$S_0 < F_{0,T}$**), negative cost of carry:

Funding costs > **income from the spot market position** (i.e. dividends).

The futures price is higher, the higher interest rates and the lower dividends are.

Positive basis (**$S_0 > F_{0,T}$**), positive cost of carry:

Funding costs < **income from the spot market position** (i.e. dividends).

The futures price is lower, the lower interest rate and the higher dividends are.

If the index is a **performance index** (like e.g. the DAX index) dividends are already included in the spot market price of the underlying instrument (the spot index value) and the no-arbitrage relationship can be reduced to

$$F_{0,T} = S_0 \cdot \left(1 + r_{0,T}\right).$$

If the arbitrage equations are violated arbitrage opportunities exist.

(a) Hedging

The risks of an equity portfolio comprise on the one hand company specific risks (**unsystematic**) and on the other hand overall market (**systematic**) risk.

- **Unsystematic risks** can be reduced mainly by holding a broadly **diversified portfolio**.

- **Market risk can be hedged: index instruments**

The **relationship** between the return on a portfolio of **stocks** and the return on the **market** (i.e. a particular index) is described by the parameter β.

β measures the **sensitivity** of an equity position and the corresponding stock (market) index, and characterizes the **amount of systematic risk** of the equity position.

Important: Only the systematic (and therefore **not the total risk**) of an equity position can be managed via stock index futures contracts. The **unsystematic risk** of an equity position remains **unaffected** by futures transactions!

Thus, the higher the unsystematic risk component is, the lower is the hedge efficiency.

Reasons for a **high unsystematic risk component** are especially:

- A badly diversified equity portfolio.
- The wrong stock index futures contract.

Therefore:

➔ The stock market index futures contract which is **closest** to the characteristics of the equity portfolio should be used in hedging transactions. This is typically the **"local" stock market index futures** contract of a particular stock market.

The number of contracts (N) can then be calculated by:

$$N = \frac{MV_P}{MV_{UL}} \cdot \beta_P$$

MV_P = Market value of the portfolio

MV_{UL} = Contract size (=market value of the underlying for one contract)

β_P = Beta of the portfolio relative to the index

❍ Short hedge

To hedge an **existing equity portfolio** (long spot market) against falling equity prices, stock index futures can be sold. This procedure is known as short hedge.

Example

In April 2002 a portfolio manager **fears** (on the basis of a market analysis) a **significant price decline** in the US equity market within the coming months.

She manages a broadly diversified portfolio of US equities valued per April 2002 at $5,450,000.

The **beta factor** of this portfolio, measured relative to the S&P500 index, is **1.20**. The **S&P500** is trading at **1,105 points** (S_0).

The portfolio manager decides to hedge the equity position against the impending loss in value with the **S&P 500 Mini Futures**.

The number of contracts is calculated according to:

$$N = \frac{MV_P}{MV_I} \cdot \beta_P = \frac{\$5{,}450{,}000}{1{,}105 \cdot \$50} \cdot 1.2 = \frac{\$5{,}450{,}000}{\$55{,}250} \cdot 1.2 = 118.37$$

The portfolio manager **sells 118 September 2002 contracts** of the S&P 500 Mini Futures at 1,108 ($F_{0,T}$) to hedge the equity portfolio against price fluctuations.

Now suppose **share prices have fallen** (as they did), and the S&P500 is trading at only 890 points (S_T) in September 2002.

The value of the equity position has fallen to $ 4,177,000.

The investor **closes out** the S&P500 Mini Futures position shortly before maturity, by buying back the September contract at a price of 891 (F_T).

Outcome

Equity position	Value in April		$	5,450,000
	Value in Sep		$	4,177,000
	Loss		$	-1,273,000
S&P500 Mini Futures position	Sale in April	(118 · 1,108 · $50 =)	$	6,537,200
	Closing out in Sep	(118 · 891 · $50 =)	$	5,256,900
	Profit		$	+1,280,300

The portfolio manager achieves the following overall outcome:

Portfolio	Profit on the S&P500 Mini Futures position	$	+ 1,280,300
	Loss on the equity position	$	- 1,273,000
	Total change	$	+ 7,300

The overall outcome of the investor's hedging strategy is a profit of $ 7,300.

❍ Long hedge

If equity prices are **expected to rise**, a long hedge can be used **to lock in the current price level**.

Example:

An investor **expects prices** on the German equity market **to rise**. The DAX Index is trading at 2,605 points in March 2003.

The investor plans to build up a diversified equity position amounting to € 2,700,000. The beta factor of the planned portfolio will be 1.3.

The **funds** required for the investment are tied up **in a time deposit** that does not mature for three months.

The investor therefore decides to **buy** June DAX Index Futures to hedge against rising prices. The contract trades at a price of 2,623 points.

The number of contracts is calculated according to:

$$N = \frac{MV_P}{MV_{UL}} \cdot \beta_P = \frac{\text{EUR } 2{,}700{,}000}{2{,}605 \cdot \text{EUR } 25} \cdot 1.3 = \frac{\text{EUR } 2{,}700{,}000}{\text{EUR } 65{,}125} \cdot 1.3 = 53.90$$

The investor therefore buys 54 June 2003 contracts of the DAX Futures at 2,623 to lock in the current price level.

Suppose share **prices rise** untill June 2003 and the DAX reaches 3,370 points in June 2003.

The June DAX Futures position is sold shortly before maturity at a price of 3,375 points.

Outcome

Equity position	Value in March		€	+2,700,000
	Value in June		€	-3,731,000
	Additional investment		€	-1,031,000
DAX Futures position	Purchase in March	(54 ·2,623 ·€ 25 =)	€	-3,541,050
	Sale in June	(54 ·3,375 ·€ 25 =)	€	+4,556,250
	Profit		€	+1,015,200

The investor realizes the following overall outcome:

Portfolio	Profit on the DAX Futures position	€	+ 1,015,200
	Additional investment required for the portfolio	€	- 1,031,000
	Difference	€	- 15,800

(b) Changing Beta

Investors can change the beta factor of their portfolio depending on individual market expectations.

If the beta is not (or is no longer) in line with the desired value, they can manage the resulting market risk by buying or selling index futures.

When the market trend is **bullish** investors can **increase the beta factor** by **buying** index **futures**, to reap greater benefits from the anticipated rally.

When the market trend is **bearish** investors can **reduce the beta factor** by **selling** index **futures**, to reduce their (expected) losses.

Example

In the above short hedge example the portfolio manager needs 118 S&P500 Mini Futures contracts to reduce the **beta** of the (existing) portfolio to **zero**.

To **reduce the portfolio beta** from 1.2 to, e.g. 0.6, she only needs to go **short** 59 contracts rather than 118.

On the other hand to **increase the portfolio beta** from 1.2 to, e.g. 1.8, a **long** position of 59 contracts should be taken.

The later is appropriate, if equity prices are expected to rise.

In general, to reduce the beta (i.e. decrease in risk) of the portfolio from β_{old} to β_{new} (**$\boldsymbol{\beta_{old} > \beta_{new}}$**) a **short position** in

$$N = \frac{MV_P}{MV_{UL}} \cdot (\beta_{old} - \beta_{new})$$

contracts is required. For **$\boldsymbol{\beta_{old} < \beta_{new}}$** (i.e. increase in risk), a **long position** in

$$N = \frac{MV_P}{MV_{UL}} \cdot (\beta_{new} - \beta_{old})$$

contracts is required.

8. Bond Futures

- **Most important contracts**

		Traded contracts (Mio)	
Contract	**Exchange**	**2013**	**2014**
Euro Bund Futures	EUREX	190.3	179.1
Euro Bobl Futures	EUREX	129.5	113.6
Euro Schatz Futures	EUREX	95.5	71.4
30Y US-Treasury Bond Futures	CBOT	98.0	93.2
10Y US-Treasury Note Futures	CBOT	325.9	340.5
5Y US-Treasury Note Futures	CBOT	175.3	196.4
2Y US-Treasury Note Futures	CBOT	57.8	71.8

- **Contract specifications: Eurex**

	Buxl Futures	**Bund Futures**	**Bobl Futures**	**Schatz Futures**
Deliverable grades	German Federal Government Bonds (Bundesanleihen)		German Federal Government Bonds (Bundesanleihen); German Federal Debt Obligations (Bundesobligationen); German Federal Treasury Notes (Bundesschatzanweisungen, Schatz futures only)	
Maturity (years)	24 - 35	8.5 - 10.5	4.5 - 5.5	1.75 - 2.25
Contract size	€100,000			
Delivery months	Up to 3 successive months within the cycle Mar, Jun, Sep and Dec			
Delivery day	10^{th} calendar day of the respective month, if this day is an exchange trading day. Otherwise the immediately following trading day is used.			
Last trading day	2 exchange trading days prior to the delivery day of the relevant month (trading ends at 12:30 CET).			

Min. issue amount: €5 Billion

Example

Quotes of the Bund Futures, 12.3.2018

Delivery Month	Opening Price	Daily High	Daily Low	Closing Price	Settlement Price	Traded Contracts	Open Interests
Jun 18	157.00	157.36	156.91	157.32	157.27	440,187	1,824,378
Sep 18	156.80	156.80	156.80	156.80	156.84	1	141
Dec 18					156.84		

- **Conversion Factor**

LTIR futures: **Physical delivery**

The holder of the **short position** has the **obligation to deliver** the underlying security and the holder of the corresponding **long position** must **accept delivery** against payment of the delivery price.

E.g., the Bund Futures contract provides for the holder of the **short position** to **choose** to deliver any German Federal Government bond that has a maturity of 8.5-10.5 years.

When a particular bond is delivered, a parameter known as its **conversion factor** defines the **price received** by the party with the **short position**. This **delivery price** of the bond is calculated as follows:

$$\text{Delivery price} = F_T \cdot CF + AI$$

CF = conversion factor of the bond delivered
AI = accrued interest since the last coupon date on the bond delivered
F_T = futures price at maturity (final settlement price)

The conversion factor generates a **price** at which a **bond would trade** if its **yield** were **6% p.a.** (Buxl: 4% p.a.) on delivery.

The conversion factor is **necessary**, as the bonds eligible for delivery are **non-homogeneous**. Although they have the same issuer, they **vary** by **coupon level**, **maturity** and therefore price.

Example

Deliverable bonds for particular Bund Futures contracts

Expiry month Jun 2018			
Deliverable Bond ISIN	**Coupon Rate (%)**	**Maturity Date**	**Conversion Factor**
DE0001102416	0.25	15.02.2027	0.619489
DE0001102424	0.5	15.08.2027	0.62028
DE0001102440	0.5	15.02.2028	0.604713
Expiry month Sep 2018			
Deliverable Bond ISIN	**Coupon Rate (%)**	**Maturity Date**	**Conversion Factor**
DE0001102424	0.5	15.08.2027	0.628154
DE0001102440	0.5	15.02.2028	0.612345

Assumptions made in the conversion factor formula:

- **yield curve is flat at delivery**
- **level: 6% p.a.** (Buxl: 4% p.a.)

$$CF_{Bond} = \frac{PV_{Bond}(\text{flat yield curve at } 6\%)}{100}$$

➔ This is typically not the case in reality.

- **Cheapest to Deliver (CTD)**

The **conversion factor** therefore **creates a bias** which promotes certain bonds for delivery above all others.

The **futures price** will **track the price of the deliverable bond** that presents the **short position** with the **greatest advantage** upon maturity. This bond is called the <u>cheapest to deliver</u> (<u>CTD</u>).

Since the party with the <u>short position receives</u>

$$F_T \cdot CF \; + \; AI$$

and the <u>cost of purchasing a bond</u> is

$$S_T \; + \; AI,$$

we can calculate the **zero basis futures price** F_T:

$$Basis_T = S_T - F_T \cdot CF = 0$$

$$F_T = \frac{S_T}{CF}$$

Hedging of interest rate positions largely comprises (i) choosing a **suitable futures contract** and (ii) determining the **number of contracts** required to hedge the cash position.

There are various procedures to **determine the hedge ratio**. A very common procedure is **the Modified Duration method**. Under this method the necessary number of contracts (N) is calculated as:

$$N = \frac{MV_P}{MV_{CTD}} \cdot \frac{MD_P}{MD_{CTD}} \cdot CF_{CTD}$$

MD_P	=	Modified duration of the bond portfolio
MD_{CTD}	=	Modified duration of the CTD bond
MV_P	=	Market value of the bond portfolio
MV_{CTD}	=	Market value of the CTD bond with a face value equal to the size of one contract (e.g. €100,000 for Bund futures contracts)
CF_{CTD}	=	Conversion factor of the CTD bond

Example

Suppose a bond portfolio consists of german government bonds with a market value of €20 Mio and a modified duration of 8.7% (August 2000). For the next 3 months the portfolio manager **expects an interest rate increase** and therefore plans to hedge the bond portfolio. How many futures contracts are necessary?

➔ Bund futures contract

➔ December 2000 contract

➔ short position

The **CTD** bond of the **December 2000 contract** has the following characteristics:

Price = 94.88
Conversion factor = 0.899414
Modified duration = 7.2%

Number of contracts the portfolio manager has to **sell**:

$$N = \frac{20{,}000{,}000}{94{,}880} \cdot \frac{8.7}{7.2} \cdot 0.899414 = 229.1$$

where €94,880 is the price of one futures contract with a face value of €100,000 (i.e. 0.9488·€100,000).

Opposite to the expectations, **interest rates dropped** till November 2000 by **50 BP**. The price of the CTD bond increased to 98.55, the Bund futures price increased from 105.40 to 109.50 and the market value of the bond portfolio increased to €20,900,000.

Overall performance of the hedged portfolio:

Date	Bond portfolio	Bund futures	
August 2000	€20,000,000	229 contracts sold at 105.40	+€24,136,600
November 2000	€20,900,000	229 contracts bought at 109.50	-€25,075,500
Change	+€900,000		-€938,900

The net change in the value of the portfolio managers' position was therefore only

€900,000 - €938,900 = -€38,900

- **Perfect Hedge versus Cross Hedge**

Perfect hedge: **Losses** in the value of the **cash position** are almost **totally compensated** by changes in the value of the **futures position**

Cross hedge: The hedge position does **not precisely offset** the performance of the hedged portfolio

➔ When the hedged portfolio has **different characteristics** than the unerlying of the futures contract **(no perfect correlation)**

➔ Basis risk

9. Money Market Futures

- **Important contracts**

		No of contracts (Mio)	
Contract	**Exchange**	**2013**	**2014**
3-Month Eurodollar	CME	517.3	664.4
3-Month Euribor	LIFFE		178.8*
3-Month Euribor	EUREX	0.3	0.2

* 2012

The **underlying** of the **Eurodollar** (**Euribor**) contract is a **time deposit in $ (€)**. The interest rate is the **3-month LIBOR** (**3-month Euribor**).

Futures on time deposits **lock in** a rate **for borrowing or lending** when the contract expires just as a futures contract on gold locks in a price for gold.

But short-term interest rate futures differ from futures on metals or agricultural products in that they are **not actually written on an asset that can be bought and sold**. Instead, the asset is a time deposit.

- **Contract specifications**

	3M Eurodollar Futures	3M Euribor Futures
Exchange	CME	LIFFE
Contract size	$1 Mio	€1 Mio
Contract months	**40 months** in the quarterly cycle **March, June, Sep, Dec**, and the 4 nearest serial contract months	**March, June, Sep, Dec**, and four serial months, such that **28 delivery months** are available for trading, with the nearest six delivery months being consecutive calendar months

- **Last trading day:**

2 (London) bank **business days prior** to the **3rd Wednesday** of the contract month.

- **Final settlement:**

Cash Settlement at the Exchange Delivery Settlement Price **(EDSP)**

When a Eurodollar (or Euribor) futures contract expires, the **final settlement price** (expiration futures price or EDSP), is calculated by the following formula:

$$\mathbf{EDSP_{\$} = 100 - LIBOR_{3M}}$$

$$\mathbf{EDSP_{€} = 100 - Euribor_{3M}}$$

$LIBOR_{3M}$: 3M LIBOR Offered Rate on the last trading day, 10.00 London time

$Euribor_{3M}$: 3M Euribor Offered Rate on the last trading day, 10.00 London time

- **Quotation:**

Before expiration, the quoted futures price ($F_{0,T}$) is

$$\mathbf{F_{0,T} = 100 - 3M\ Futures\ Rate}$$

$F_{0,T}$ = Futures price at t = 0 with maturity (last trading day) at t = T

The quoted futures price implies therefore an interest rate of

$$\mathbf{Interest\ rate = 100 - F_{0,T} = 3M\ Futures\ Rate}$$

This is the interest rate that **can be locked in** under the futures contract.

- **3M Eurodollar Futures (CME): Trading statistics 12.3.2018**

Month	Last	Change	Settle	Estimated Volume	Prior Day Open Interest
MAR 18	97.8225	-.0175	97.8225	290,633	1,299,914
APR 18	97.7700	-.0150	97.7700	59,118	176,905
MAY 18	97.7450	-.0200	97.7400	19,311	85,065
JUN 18	97.7000	-.0200	97.6950	343,081	1,687,988
JLY 18	-	-.0050	97.6750	0	1,994
AUG 18	-	-.0100	97.6350	0	2,986
SEP 18	97.5950	-.0150	97.5900	293,374	1,459,172
DEC 18	97.4650	-.0050	97.4600	328,493	1,937,437
MAR 19	97.3700	-.0050	97.3600	288,575	1,397,324
JUN 19	97.2600	-.0050	97.2550	265,774	1,401,339
SEP 19	97.1950	UNCH	97.1900	223,467	918,886
DEC 19	97.1250	+.0050	97.1200	334,009	2,206,516

Month	Last	Change	Settle	Estimated Volume	Prior Day Open Interest
MAR 20	97.1050	+.0050	97.1000	204,001	1,028,176
JUN 20	97.0950	+.0050	97.0900	157,936	892,628
SEP 20	97.0900	+.0050	97.0850	130,036	616,918
DEC 20	97.0650	+.0050	97.0550	160,445	723,242
MAR 21	97.0550	+.0100	97.0500	74,919	526,836
JUN 21	97.0500	+.0100	97.0450	60,499	252,006
SEP 21	97.0450	+.0100	97.0350	58,565	185,685
DEC 21	97.0250	+.0050	97.0150	65,197	291,406
MAR 22	97.0200	+.0100	97.0100	36,033	133,794
JUN 22	97.0100	+.0100	97.0050	46,534	86,339
SEP 22	97.0050	+.0100	96.9950	32,809	67,315
DEC 22	96.9900	+.0150	96.9800	34,371	97,118
MAR 23	96.9750	+.0150	96.9700	1,322	22,121
JUN 23	96.9600	+.0150	96.9550	765	12,917
SEP 23	96.9450	+.0150	96.9450	738	12,212
DEC 23	96.9250	+.0200	96.9350	632	7,828

Month	Last	Change	Settle	Estimated Volume	Prior Day Open Interest
MAR 24	96.9200	+.0200	96.9250	45	2,836
JUN 24	-	+.0200	96.9100	184	2,884
SEP 24	96.9000	+.0200	96.9000	129	1,379
DEC 24	96.8800	+.0200	96.8850	26	1,507
MAR 25	-	+.0200	96.8800	13	336
JUN 25	-	+.0200	96.8700	76	344
SEP 25	96.8550	+.0200	96.8600	37	662
DEC 25	96.8350	+.0200	96.8450	0	517
MAR 26	96.8300	+.0200	96.8400	0	381
JUN 26	96.8200	+.0250	96.8350	14	339
SEP 26	96.8100B	+.0250	96.8250	0	99
DEC 26	96.7900	+.0250	96.8100	0	52
MAR 27	96.7950	+.0250	96.8050	0	81
JUN 27	96.7900	+.0250	96.8050	0	10
SEP 27	-	+.0300	96.7950	12	81
DEC 27	-	+.0300	96.7850	0	13

Example:

September 2000 Eurodollar, last transactions price = 93.30

➔ 3M Futures Rate = 100 - $F_{0,T}$ = 100 - 93.30 = 6.70% p.a.

CME Eurodollar futures transactions prices for August 10, 2000:

Contract	Open	High	Low	Last	Settlement	Volumen	Open Interests
Aug 2000	93.3175	93.3225	93.3175	93.3200	93.3175	415	22,771
Sep 2000	93.3000	93.3000	93.2900	**93.3000**	93.2950	44,722	602,978
Oct 2000	93.1700	93.1700	93.1600	93.1600	93.1650	1,309	9,084
Nov 2000	93.1700	93.1700	93.1550	93.1650	93.1600	701	2,030
Dec 2000	93.1600	93.1800	93.1550	93.1700	93.1600	62,318	546,319
Mar 2001	93.2350	93.2600	93.2250	93.2550	93.2300	71,156	470,923
⋮							

Long position: profit, when the quoted futures price ↑ (the implied interest rate ↓)

Short position:profit, when the quoted futures price ↓ (the implied interest rate ↑)

Profits and losses on a Eurodollar/Euribor futures position **equal** the **changes in interest payments on a 3-month**, $1Mio/€1Mio **time deposit** implied by the changes in the quoted futures price.

The **gain and loss** on a Eurodollar/Euribor futures contract is **$25/€25** for **each basis point move in the rate implied** by the quoted futures price.

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❍ **Example:** Calculating the profit on a Eurodollar position

Suppose that on August 8, 2000, we take a **long position** in a **September Eurodollar** contract that has a quoted price of **93.30**. Then suppose that on August 15, the quoted futures price has fallen 100 basis points to **92.30**.

Loss in the long position: $25·(-100 BP) = -$2,500

This loss is exactly **equal** to the **change in interest payments on a 3-month, $1 Mio Eurodollar time deposit** (implied by the change in the quoted futures price).

The initial futures price of 93.30 represents a yearly interest rate of 6.70% or a **quarterly rate of 1.675%**. This rate implies a **quarterly interest payment** on a $1Mio time deposit of

0.01675·$1,000,000 = $16,750.

The August 15 futures price of 92.30 represents a **yearly interest rate of 7.7%** or 1.925% p.q. Therefore, the **quarterly interest payment** implied for a $1 Mio time deposit has increased to

$$0.01925 \cdot \$1{,}000{,}000 = \$19{,}250.$$

The **$2,500 difference** between the two implied interest payments ($16,750 and $19,250) is exactly **equal** to the **loss** we incur on the long position in the **futures contract**.

Thus, the Eurodollar futures contract represents the dollar amount of interest on a 3-month, $1 Mio Eurodollar time deposit.

Example

Suppose that on August 8, 2000, a firm knows it **must borrow $1 Mio** on September 18 for a period of 90 days. It will **pay the 90-day LIBOR**, which prevails on September 18, and will receive its funds on that day.

The firm is **concernd that interest rates will rise** before September 18 (= last day of trading for the Eurodollar futures contract).

Eurodollar futures price for September expiration:	93.30
3-month LIBOR:	6.60% p.a.

By going **short Eurodollar futures**, the firm will make money if rates rise and thus offset the higher interest it must pay.

To see how the firm hedges by going short one Eurodollar futures contract, **2 interest rate scenarios** are analyzed.

Scenario 1: 3-month LIBOR falls to 6% p.a.

$$\text{Interest paid} = 0.06 \cdot \$1{,}000{,}000 \cdot \frac{1}{4} = \$15{,}000$$

$$\begin{aligned}
\text{Futures profit} &= [(\text{initial futures price}) - \text{EDSP}] \cdot 100 \cdot \$25 \\
&= [(\text{initial futures price}) - (100 - \text{LIBOR})] \cdot 100 \cdot \$25 \\
&= [93.30 - 94.00] \cdot 100 \cdot \$25 \\
&= (-70 \text{ basis points}) \cdot \$25 \\
&= -\$1{,}750
\end{aligned}$$

$$\begin{aligned}
\text{Net interest} &= \text{interest paid} - \text{futures profit} \\
&= \$15{,}000 - (-\$1{,}750) \\
&= \underline{\$16{,}750}
\end{aligned}$$

Scenario 2: 3-month LIBOR rises to 9% p.a.

$$\text{Interest paid} = 0.09 \cdot \$1{,}000{,}000 \cdot \frac{1}{4} = \$22{,}500$$

Futures profit = [(initial futures price) - EDSP]·100·$25

= [(initial futures price) - (100 - LIBOR)]·100·$25

= [93.30 - 91.00]·100·$25 = (+230 basis points)·$25 = +$5,750

Net interest = interest paid - futures profit

= $22,500 - $5,750 = <u>$16,750</u>

The firm pays **$16,750** in interest **under both scenarios**. On an annual basis, this represents interest of

$$\text{Interest rate} = \left(\frac{\$16{,}750}{\$1{,}000{,}000}\right)\cdot\left(\frac{360}{90}\right) = 6.70\%\ \text{p.a.}$$

The firms **net rate** is the **6.70% p.a.** interest rate **implied** in the Eurodollar **futures price**, and **NOT the spot LIBOR** of 6.60% p.a.

Hedging with Eurodollar futures, therefore, allows the firm to **lock in a borrowing rate** equal to the **rate implied** in the **current futures price**.