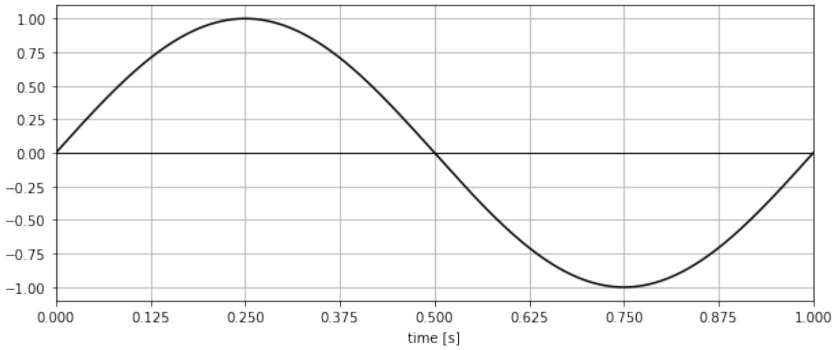
Task 1 (Analytic signal) [19 points]

The plot below shows the signal $x = \sin(2\pi t)$ for $t \in [0, 1]$.

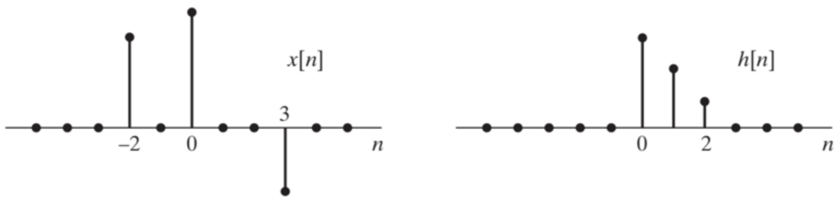


(a) [6 points] Sketch the frequency spectrum and the phase of $x(t)$. Carefully label all axes!

(b) [5 points] Let z be the analytic signal of x. Draw z in the complex plane. Indicate which point corresponds to $t_1 = 0$ and $t_2 = \frac{1}{4}$. Carefully label all axes!

(c) [8 points] Draw the imaginary part, instantaneous amplitude, instantaneous phase and instantaneous frequency of z. Carefully label all axes!

Task 2 (LTI systems) [16 points]



(a) [10 points] Find the output $y[n]$ of the LTI system with impulse response

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$$

given the input

$$x[n] = 4\delta[n+2] + 5\delta[n] + -2\delta[n-3].$$

(b) [3 points] Let $g[n] = \sum_{i=-\infty}^{n} h[i]$ be the impulse response of a LTI system. Is the system causal? Justify your answer!

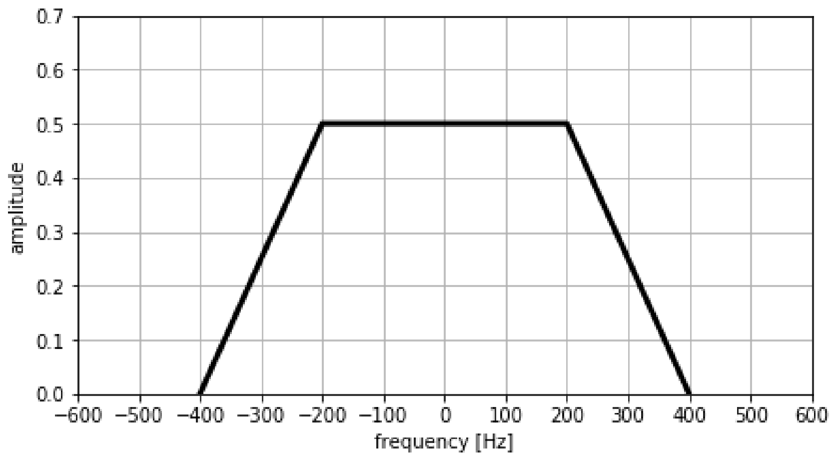
(c) [3 points] Is the system $g[n]$ stable? Justify your answer!

Task 3 (Sampling) [17 points]

(a) [2 points] Suppose we want to sample a continuous signal $x(t)$. Describe how to construct the sampling function $s(t)$.

(b) [4 points] Describe how to obtain the sampled continuous signal $x_s(t)$ and the discrete signal $x[n]$.

(c) [2 points] The plot below shows the frequency spectrum of a continuous signal $x(t)$. What is the minimal sampling rate for sampling this signal without loss of information?



(d) [9 points] Suppose we sample $x(t)$ once with a sampling frequency of 400Hz, once with 800Hz and once with 1000Hz. Draw the spectrum of the sampled signal $x_s(t)$ (without filtering) for each of the sampling rates in the range of [−1500Hz, 1500Hz].

$f_s = 400\text{Hz}$

$f_s = 800\text{Hz}$

$f_s = 1000\text{Hz}$

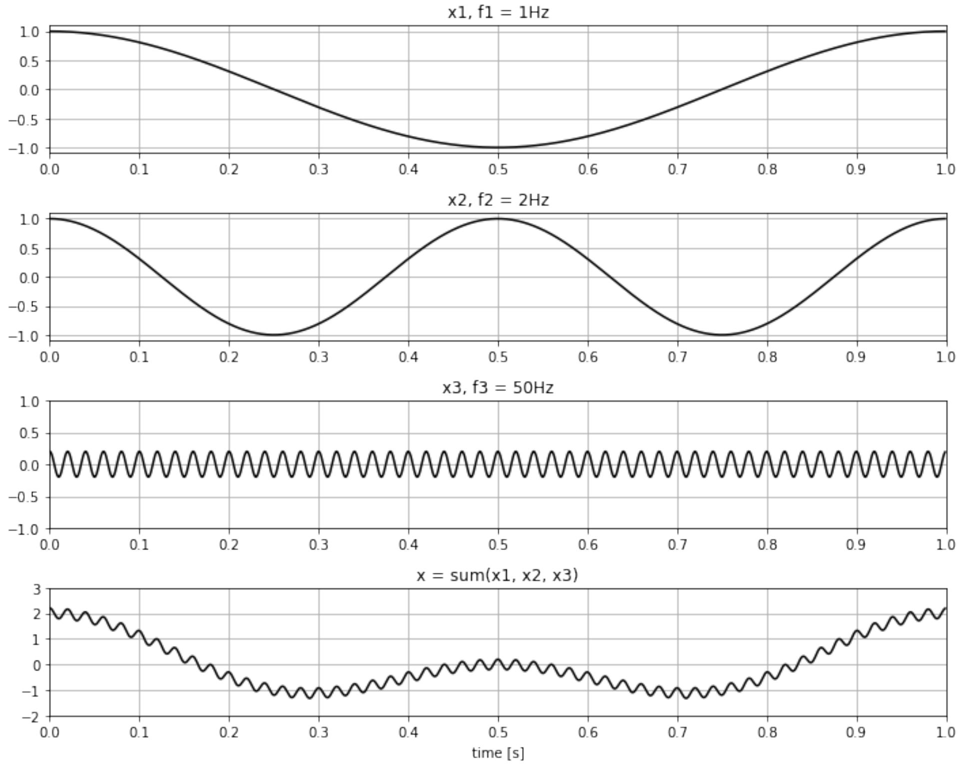
Task 4 (Filters) [33 points]

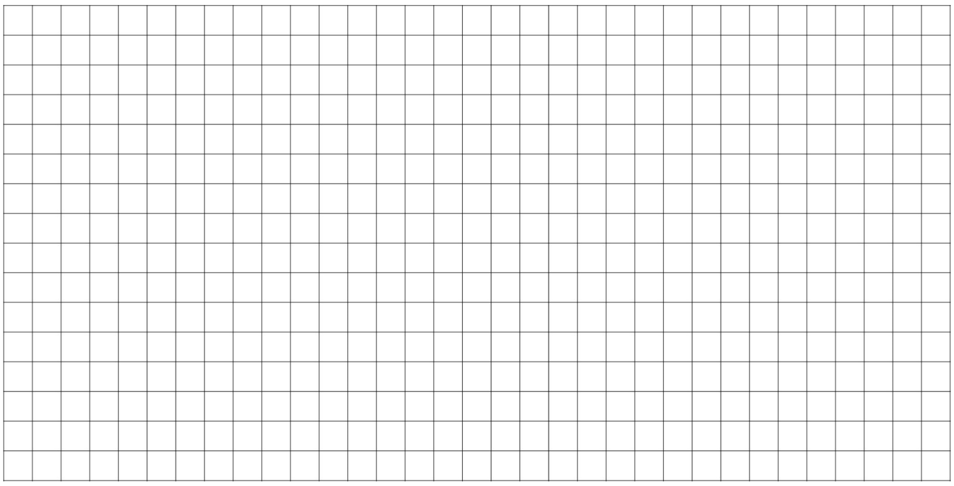
(a) [4 points] Sketch the frequency response of an ideal lowpass filter with cut-off frequency ω_c.

(b) [3 points] Describe how to obtain a $100-200$Hz band-pass filter using two low-pass filters.

(c) [6 points] Name three differences between finite impulse response (FIR) and infinite impulse response (IIR) filters.

(d) [4 points] The plot below shows $1s$ long sections of three infinite signals $x_1(t), x_2(t)$ and $x_3(t)$, as well as their sum $x(t) = \sum_{i=1}^{3} x_i(t)$. Sketch the signal which would result from applying an ideal 30Hz low-pass filter with zero-phase to $x(t)$.





(e) [8 points] Now suppose we instead filter the signals x_1, x_2 and x with a 30Hz FIR low-pass filter with a linear phase $p(f) = f\pi$, where f is the frequency in Hz. Sketch the resulting signals.

(f) [8 points] Describe how the above plots would differ, if we instead used a IIR filter with non-linear phase. Why does this occur and how can the phenomenon be avoided when dealing with offline data?

Task 5 (Amplitude modulation) [15 points]

The term *amplitude modulation* (AM) refers to the process of scaling a signal with a high, constant frequency (carrier signal) by the amplitude of a lower frequency signal (modulator signal).

(a) [6 points] You are tasked to broadcast an AM radio signal and are given a speech signal with a range from 80Hz to 320Hz. Using a carrier frequency of 800kHz describe how you can transfer the speech signal to the carrier frequency.

(b) [3 points] Which frequency components does the resulting modulated signal contain?

(c) [6 points] Describe how the receiver can reconstruct the original signal from the broadcasted signal.