Mid-term Exam
Signal and Image Processing
WS 2020

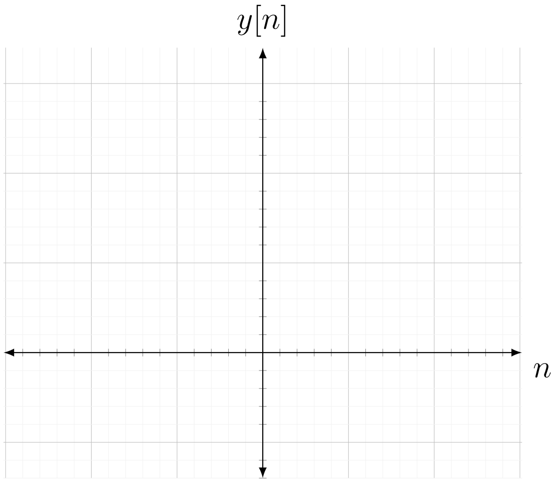
Research Group of Neuroinformatics
University of Vienna

Novermber 25th, 2020

Part	**A**	**B**	**C**	**Total**
Score	**/ 30**	**/ 40**	**/30**	**/ 100**

Part A: Convolution and system [30 P]

a.) Find the output $y[n]$ of the LTI system with impulse response $h[n] = 1\delta[n] + 3\delta[n-1] + 2\delta[n-2]$ given the input $x[n] = 3\delta[n+2] + 1\delta[n] + -3\delta[n-2]$. Write down the mathematical expression for $y[n]$ AND plot $y[n]$ in the below coordinate. **(Carefully label both axes!)**
Hint: $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$

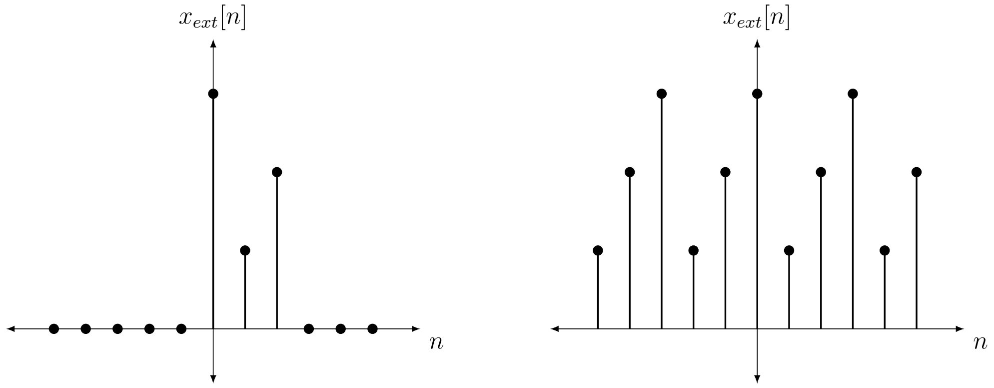


[/ 12]

b.) Let $g[n] = \sum_{i=-\infty}^{n} h[i]$ be the impulse response of a LTI system. Is the system causal? Justify your answer!

[/ 4]

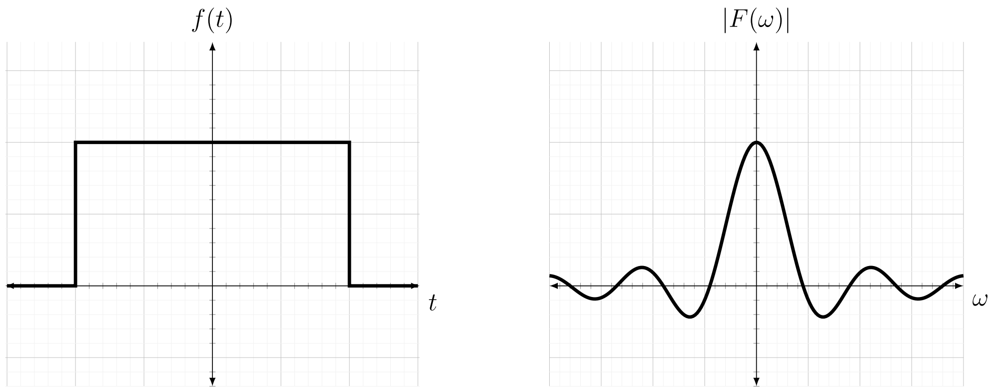
c.) Assuming we have the signal $x[n] = [3, 1, 2]$ and two plots below are the different extensions of $x[n]$ for different types of convolution. Which extension is used for the linear convolution and which one is for the circular convolution? Write down your answer "Linear" or "Circular" within the parentheses.



( ) ( )

[/ 5]

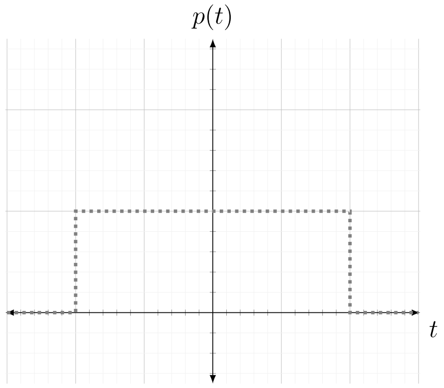
d.) **Continuous convolution**: Given a continuous rectangular signal $f(t)$ with amplitude spectrum $|F(\omega)|$ as shown below, **sketch** the self-convolved signal $p(t) = f(t) * f(t) = \int_{-\infty}^{\infty} f(\tau)f(t-\tau)d\tau$ in the time domain and the amplitude spectrum in the frequency domain AND explain how you arrive at these results. (Note: No need to label both axes.)

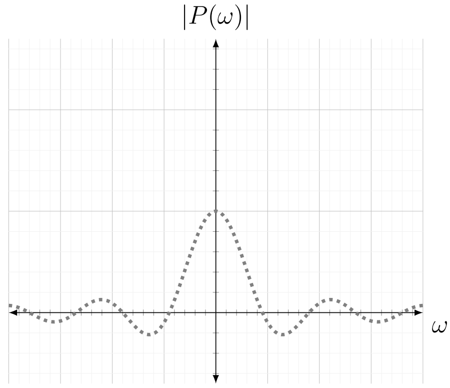


[/ 9]

(Plot your answer in the next page)

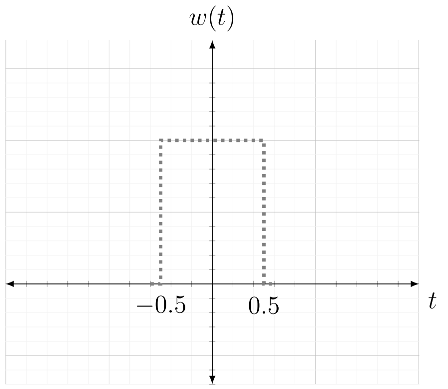
Sketch your answer below, where $f(t)$ and $|F(\omega)|$ are already plotted with gray dotted lines:

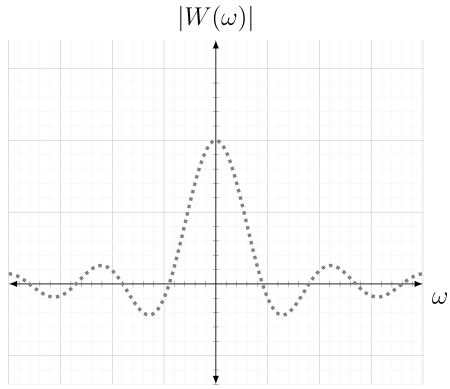




Part B: DFT and Filter Design [40 P]

a.) Assume a one second long windowing function $w(t)$ with amplitude spectrum $W(\omega)$, as plotted below with gray dotted lines. Now, we extend $w(t)$ symmetrically to a length of two seconds while keeping the same amplitude. Please plot the extended $w(t)$ and its corresponding amplitude spectrum! Explain the changes in main lobe and side lobes in the spectrum! **(Carefully label both axes!)**





[/ 8]

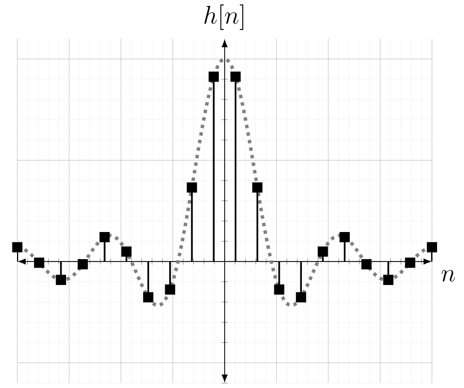
b.) If the windowing function $w(t)$ is further extended to become infinitely long, what will its amplitude spectrum look like? Please explain your answer!

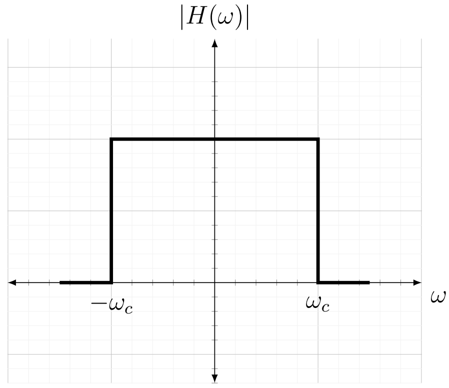
[/ 3]

c.) Assume we would like to apply a window function $w(t)$ to a raw signal $x(t)$, with the windowed signal denoted as $\hat{x}(t) = w(t) \cdot x(t)$. If you want the amplitude spectrum of $\hat{x}(t)$ to approximate the amplitude spectrum of $x(t)$ as closely as possible, will you use a longer window function or a shorter one? Justify your answer!

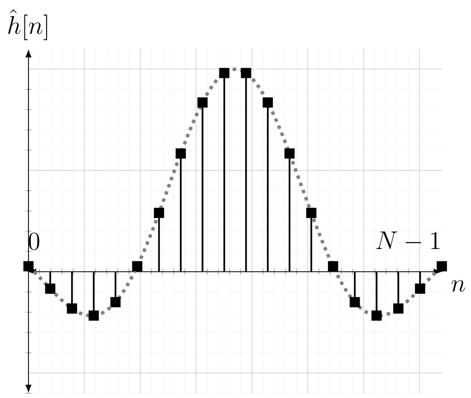
[/ 4]

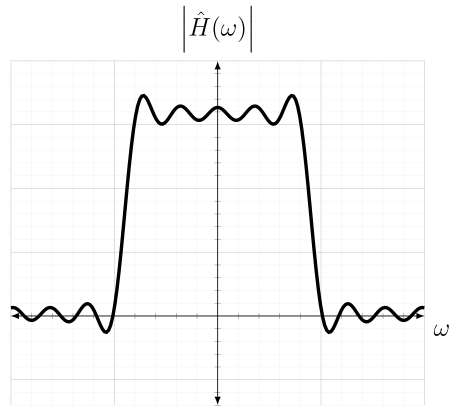
An ideal low-pass filter $h[n]$ and its spectrum $|H(\omega)|$ are sketched below.





Such an idea filter is physically impossible to realize, because $h[n]$ is both non-causal and infinitely long. Hence, we have to apply a window function to $h[n]$ to limit its length and shift $h[n]$ to ensure it is causal such that we obtain a physically possible low pass filter $\hat{h}[n]$, as visualized below.





d.) As you can notice that in the pass-band and stop-band in the spectrum of $\hat{H}(\omega)$, the ripples (vibration) constantly exist, which is known as the truncating effect. Explain why does the truncating effect appear considering that the truncating a signal is equivalent to apply a finite time window to the infinitely long signal?

[/ 6]

e.) If we further increase the number of N, i.e., we extend the window function length, will the extent of the ripples in $\hat{H}(\omega)$ increase or decrease? Justify your answer!

[/ 6]

The phase spectrum is another vital aspect of filter design. For an ideal low pass filter, the phase spectrum should be either constantly equal to zero or linear.

f.) Explain why do zero-phase or linear-phase filters are desirable!

[/ 4]

g.) If we would like to design a zero-phase FIR filter $h_{FIR}[n]$, what special property should the filter's frequency response function $H_{FIR}[\omega]$ hold? (Hint: Consider the Euler's formula)

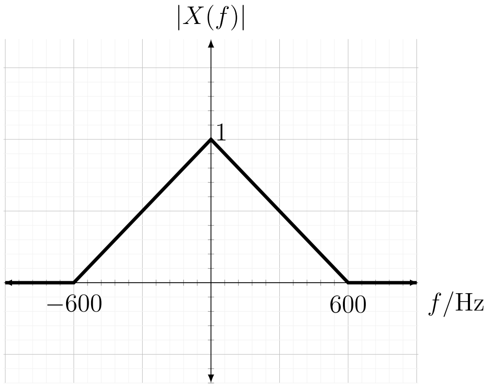
[/ 4]

h.) Usually, IIR filters do not have a linear phase spectrum. However, it is still possible to implement a zero-phase IIR filter. Describe how to implement such a filter! Is this zero-phase IIR filter a causal filter? Justify your answer!

[/ 5]

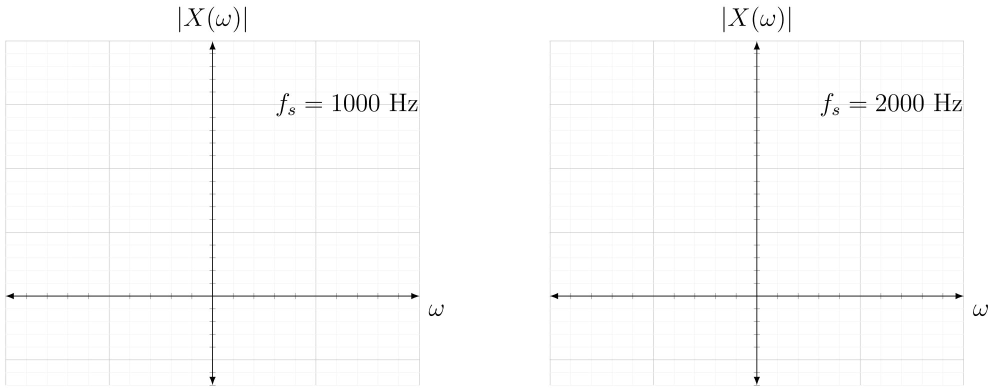
Part C: Resampling [30 P]

Assume we have a signal $x(t)$ with spectrum $X(f)$ as shown below. Note that this spectrum is plotted based on the absolute frequency f and the spectrum we asked to plot later are based on angular frequency ω.



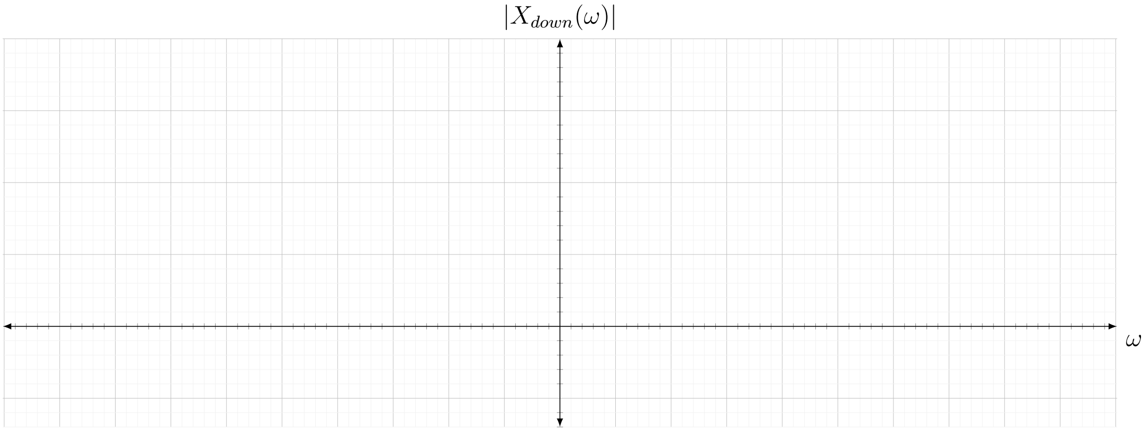
General hint: $\omega = 2\pi \frac{f}{f_s}$

a.) If $x(t)$ is sampled with sampling frequency $f_s = 1000$ Hz and $f_s = 2000$ Hz, what will the amplitude spectrum of the sampled signal $x[n]$ be? Plot your answer in the below coordinates AND justify your answer! (**Carefully label both axes and plot the spectrum within the angular frequency range $\omega \in [-\pi, \pi]$.**)



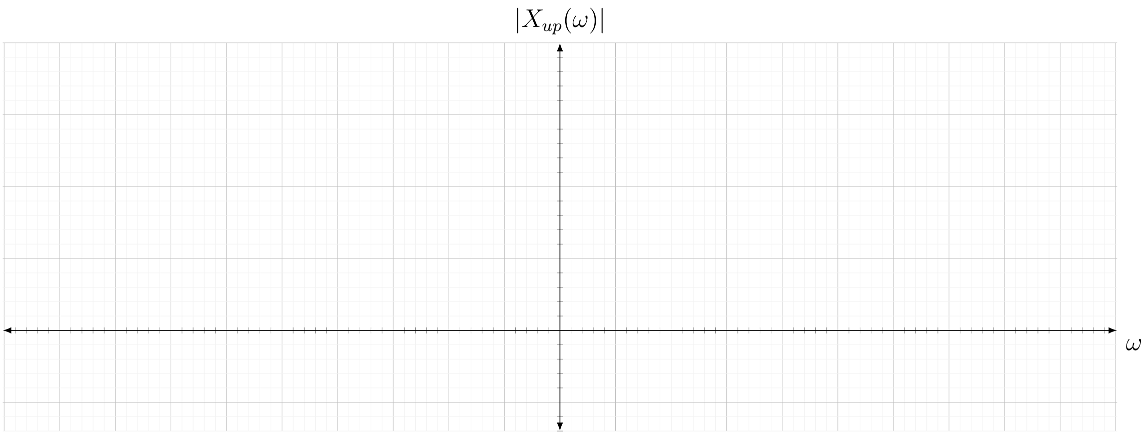
[/ 12]

b.) Let $x[n]$ denote the signal $x(t)$ sampled at $f_s = 2000$ Hz. We would like to decimate $x[n]$ by a factor of two, i.e., $x_{down}[n] = x[2n], \forall n > 0$. Plot the spectrum of $x_{down}[n]$ within the angular frequency range $\omega \in [-3\pi, 3\pi]$ AND explain how you arrive at this result. (**Carefully label both axes!**)



[/ 6]

c.) Let $x[n]$ denote the signal $x(t)$ sampled at $f_s = 2000$ Hz. We would like to up-sample $x[n]$ by a factor of two, i.e., $x_{up}[2n] = x[n], \forall n > 0$. Plot the spectrum of $x_{up}[n]$ within the angular frequency range $\omega \in [-3\pi, 3\pi]$ AND explain how you arrive at this result. (**Carefully label both axes and only consider the case of optimal interpolation!**)



[/ 6]

d.) When decimating the signal in the time domain, we mostly start the decimation from the first sample such that $x_{down}[n] = x[Mn], \forall n > 0$, where M indicates the factor of decimation. What if the decimation did not start from the first but the second sample, i.e., $x_{down}^{offset}[n] = x[Mn + 1], \forall n > 0$? Will the spectrum of $x_{down}^{offset}[n]$ be equal to the spectrum of $x_{down}[n]$? Justify your answer!

[/ 6]