Name: _____

Program code, registr. no.: _____

Date: _____

Achieved points: _____

Distributed Algorithms 182.702

Quiz 2 (Ch.3 + Ch.2, SS 2021), Form: A

Question 1. Tick the correct statements:

1. (a) The end-to-end delay τ of a message m is defined as the time between $comp(i)$ where m is sent and $del(i, j, m)$.
   (b) Suppose that some problem P has a worst-case lower bound of $\Omega(n \log n)$ messages for all correct algorithms solving P. An algorithm $\mathcal{A}$ for P that has some execution where only n messages are sent cannot be correct.
2. (a) Consider two schedules σ and σ' and the configurations $C = \sigma(C^0)$ and $C' = \sigma'(C^0)$ reached when starting from C^0 in the distributed computing model of textbook. If $\sigma = \mathrm{del}(i, j, m_1), \mathrm{del}(k, j, m_2), \mathrm{comp}(j)$ and $\sigma' = \mathrm{del}(i, j, m_1), \mathrm{comp}(j), \mathrm{del}(k, j, m_2), \mathrm{comp}(j)$, then $C = C'$.
   (b) If one drops $inbuf_i[*]' = \emptyset$ after a step, then an event $comp(i)$ cannot uniquely determine the corresponding step of p_i.
   (c) A norm function, bound to some assertion A, is defined upon an arbitrary partial order $(W, <)$
   (d) An assertion is a unary predicate on the transition relation.
3. (a) Any deterministic leader election algorithm requires processors with unique identifiers.
   (b) There is no asynchronous leader election algorithm in rings with message complexity at most $O(n^2)$.
   (c) A time-bounded leader election algorithm has a time complexity upper bound that is independent of the id's.
4. (a) A liveness property is a set of event traces such that any finite sequence of events has some extension that is contained in this set.
   (b) Fairness is a safety property.
   (c) An asynchronous execution of a message passing system, where every processor takes an infinite number of steps, is always admissible.

Question 2.

1. What is the asymptotic message complexity of a flooding algorithm like the one in Algorithm 2?
   _____
2. A ring where messages travel in one direction only is called _____
3. If P is a safety property and $\beta \in P$ with $\beta = \phi^1\phi^2\phi^3\ldots$, what can be said about the finite trace $\phi^2\phi^3$?
   _____
4. Give the definition of a deterministic transition relation Φ_i for processor p_i:
   _____

Question 3.

[*More on backside!*]

1. Consider the pseudo-code statements *"on receiving M from parent: send M to all children; terminate"* that appear in our simple spanning-tree broadcast Algorithm 1. State the enabling condition and the actions of the corresponding transition (for processor $p_i \neq p_{root}$), assuming that there are local variables $terminated_i$ $parent_i$, and $children_i$.

2. List three possibilities how the adversary can affect the execution of an asynchronous distributed computation.

3. Consider a directed simple finite graph G that is (i) weakly connected (i.e., the undirected shadow graph is connected) and (ii) where every node $v \in V(G)$ has exactly one outgoing edge $(v, v') \in E(G)$, directed to some node $v' \in V(G)$.

   (a) Show constructively that G contains a cycle.

   (b) Show that there cannot be more than one cycle in G.

4. Consider the following simple asynchronous algorithm, which assumes a fixed spanning tree already in the initial configuration.

   1 *Code for processor* p_i, $0 \leq i \leq n-1$, *with given spanning tree rooted at* p_0

```
2    VAR parent = ... // initially specifies parent in spanning tree, or NULL at p_0
3        start = ... // initially false, becomes true once during the execution
4        next := ⊥

5    // When start becomes true (happens only on p_i ≠ p_0)
6        send message M(i) to parent
7        parent := NULL
8        start := false

9    // On reception of M(k) received from p_j
10       if parent = NULL then
11           next := k
12       else
13           send message M(k) to parent
14       endif
15       parent := j // In any case: adjust parent to p_j
16
```

Prove the invariant $A(C) = A1(C) \wedge A2(C)$ defined by

(A1) For every processor p_i, $parent_i(C)$ is either NULL or else one of the processors p_j or p_ℓ connected by a link (p_j, p_ℓ) in the original spanning tree.

(A2) Messages can be in transit only on a link (p_j, p_ℓ) in the original spanning tree.