VU Discrete Mathematics

Exercises for 9th December 2025

49) Let K be a field containing $\mathbb{Z}_p$ as a subset and consider an element $a \in K$ that satisfies $a^p = a$. Prove that $a \in \mathbb{Z}_p$.

50) Give a reason why the Chinese remainder theorem (suitably modified) can be applied to the following system of congruence relations over $\mathbb{Q}[x]$ and use it to solve the system.

$$(x+1)P(x) \equiv 2x+1 \ \ (x^2+x+1), \qquad (x+2)P(x) \equiv 3x+3 \ \ (x^2+2x+3).$$

51) Determine all elements of the ring $R = \mathbb{Z}_2[x]/_{(x^2+1)}$ as well as its table for the multiplication. Is this ring a field?

52) Examine whether x^2+1 is a unit of $\mathbb{Q}[x]/_{(x^3+x^2-3x+1)}$. If so, determine its multiplicative inverse!

53) Show that $\sqrt{2}+\sqrt{3}$ is algebraic over $\mathbb{Q}$ and determine its minimal polynomial.

54) Each element of $\mathbb{Q}(\sqrt[3]{2})$ can be uniquely represented in the form $a + b\sqrt[3]{2} + c\sqrt[3]{4}$ where $a, b, c \in \mathbb{Q}$. Why is this true?
Consequently, each element of $\mathbb{Q}(\sqrt[3]{2})$ can be identified with a triple $(a, b, c) \in \mathbb{Q}^3$. Which triple corresponds to $(a + b\sqrt[3]{2} + c\sqrt[3]{4})(a' + b'\sqrt[3]{2} + c'\sqrt[3]{4})$?