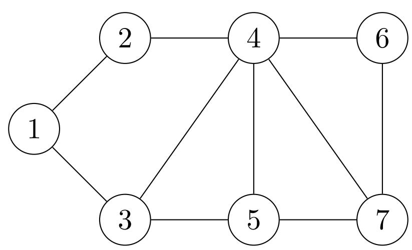
Einführung in Artificial Intelligence SS 2025, 4.0 VU, 192.027

Exercise Sheet 5 – CSP, Planning, and Decision Theory

For the discussion part of this exercise, mark and upload your solved exercises in **TUWEL** until Wednesday, June 11, 23:55 CEST. The registration for a solution discussion ends on Friday, June 13, 23:55 CEST. Be sure that you tick only those exercises that you can solve and explain!

In the discussion, you will be asked questions about your solutions of examples you checked. The discussion will be evaluated with 0–25 points, which are weighted with the fraction of checked examples and rounded to the next integer. There is *no minimum number of points* needed for a positive grade (i.e., you do not need to participate for a positive grade, but you can get at most ≈80% without doing exercises).
Note, however, that *your registration is binding*. Thus, *if* you register for a solution discussion, then it is *mandatory* to show up. Not coming to the discussion after registration will lead to a reduction of examination attempts from 4 to 2.
Please ask questions in the **TUWEL** forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 5.1: Consider the 3-colorability problem for the following graph:

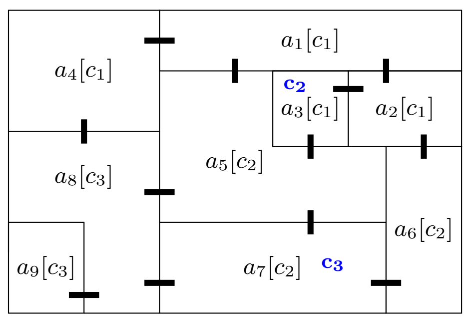


Find a valid 3-coloring for the graph using the colors red, blue, and green. Select the first node by using the *degree heuristic*. After that, select the nodes according to the *minimum-remaining-values heuristic* and the *degree heuristic* if the remaining values of two or more nodes are equal. If you still have multiple options after applying both heuristics, select the node with the smallest number. Furthermore, select the colors according to the *least-constraining-value heuristic*.

Exercise 5.2: You are a rebel spy who has spent the past month gathering intelligence in an obscure Mid-Rim world's ISB field office, but unfortunately your cover was blown while you were busy snooping in the data vault and now you need to get out of the building and to the agreed-upon extraction point as fast as possible. However, you know that the one way out that could conceivably still be open to you is a hidden tunnel accessible only from an area of the building your security clearance is not sufficiently high for. Luckily, you truly must have friends everywhere, as one of the supervisors "forgot" his code cylinder in the break room earlier today, and another one's is being checked over in the engineers' office, so there is still a chance of you obtaining the correct clearance codes. You have created a detailed map of the building showing which areas are accessible from one another via checkpoints and what their respective security levels are. You are able to move into a given area if you are in possession of the correct security code for it and are standing in an area that is connected to it by a checkpoint. Finding your way to the tunnel entrance will require stealing the codes you do not already have along the way. This is possible if you are in the same area as such a code (cylinder), but as speed is of the essence, you need to keep your moves to a minimum.

(a) Design two STRIPS actions, one for moving from one area of the building into another, and one for stealing a code (cylinder). Define the variables to model different aspects of this exercise on your own and describe them in detail.

(b) According to your map, the building is divided into areas $a_1, \ldots, a_9$, each of which has one of three different clearance levels corresponding to codes c_1, c_2, c_3 (indicated in square brackets). You are already in possession of code c_1 and have marked the supervisors' code cylinders in blue. Bold lines connecting two areas represent the checkpoints. Your current location is in area a_1, while the tunnel entrance you need to reach is in a_9. Formulate the initial state of this setting and use progression planning to find a plan to make it out of the building in one piece. What do the goal states look like?



Exercise 5.3: Consider the following planning problem: There are three shipping containers, C_1, C_2, and C_3, in front of you. Container C_1 is empty, C_2 contains a red item, and C_3 contains a blue item. Using a roboter arm, you can access the contents of the containers using the following actions:

$$\begin{aligned}
&\textbf{Action}(\mathit{Grasp}(x, y),\\
&\quad \mathsf{Precond}: \mathit{free} \wedge \mathit{contains}(x, y) \wedge \mathit{pos}(x),\\
&\quad \mathsf{Effect}: \mathit{hold}(y) \wedge \mathit{empty}(x) \wedge \neg \mathit{free} \wedge \neg \mathit{contains}(x, y))\\
&\textbf{Action}(\mathit{Ungrasp}(x, y),\\
&\quad \mathsf{Precond}: \mathit{pos}(x) \wedge \mathit{empty}(x) \wedge \mathit{hold}(y),\\
&\quad \mathsf{Effect}: \mathit{free} \wedge \mathit{contains}(x, y) \wedge \neg \mathit{empty}(x) \wedge \neg \mathit{hold}(y))\\
&\textbf{Action}(\mathit{Move}(x, y),\\
&\quad \mathsf{Precond}: \mathit{pos}(x),\\
&\quad \mathsf{Effect}: \neg \mathit{pos}(x) \wedge \mathit{pos}(y))
\end{aligned}$$

The meaning of the predicates is as follows:

free: the roboter arm is free;

$\mathit{hold}(x)$: the roboter arm holds x;

$\mathit{pos}(y)$: the roboter arm is over container y;

$\mathit{contains}(x, y)$: container x contains item y;

$\mathit{empty}(x)$: container x is empty.

The initial state S is

$$\{free, pos(C_1), contains(C_2, r), contains(C_3, b), empty(C_1)\}$$

and the goal state is

$$\{free, pos(C_3), contains(C_2, b), contains(C_3, r), empty(C_1)\}.$$

Find the shortest possible plan for getting from the initial state to the goal state. Use the STRIPS state-space search algorithm starting in S, i.e., use *progression planning*.

Exercise 5.4: Assume there is a lottery with tickets for €5 and there are three possible prizes: €1000 with a probability of 0.05%, €100 with probability 0.1%, and €1 otherwise.

(a) What is the expected monetary value of a lottery ticket?

(b) When is it rational to buy a ticket? Give an inequality involving utilities with the following utilities: $U(S_k) = 0$, $U(S_{k+5}) = 5 \cdot U(S_{k+1})$, $U(S_{k+100}) = 50 \cdot U(S_{k+5})$, but there is no information about $U(S_{k+1000})$. (S_n denotes the state of possessing n Euros.)

(c) Define $U(S_{k+5})$ and $U(S_{k+1000})$ such that a rational agent whose utility function satisfies the equations in Subtask (b) chooses to buy a lottery ticket.

Exercise 5.5: In 1713, Nicolas Bernoulli investigated a problem, nowadays referred to as the *St. Petersburg paradox*, which works as follows. You have the opportunity to play a game in which a fair coin is tossed repeatedly until it comes up heads. If the first head appears on the n-th toss, you win 2^n Euros.

(a) Show that the expected monetary value of this game is not finite.

(b) Daniel Bernoulli, the cousin of Nicolas, resolved the apparent paradox in 1738 by suggesting that the utility of money is measured on a logarithmic scale, i.e., $U(S_n) = a \log_2 n + b$, where S_n ($n > 0$) is the state of having n Euros and a, b are constants. What is the expected utility of the game under this assumption? Assume, for simplicity, an initial wealth of 0 Euros and that no stake has to be paid in order to play the game.