This is the first homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by **20:00** on Wednesday **October 13, 2021**.

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(1) **Car licence plates**
A certain state's car licence plates have four letters of the alphabet followed by a four-digit number.

(a) How many different licence plates are possible if all four-letter sequences are permitted and any number from 0000 to 9999 is allowed?

(b) Mary withnessed a hit-and-run accident. She knows that the first letter on the licence plate of the offender's car was a T, that the second letter was an A or H or M, and that the last number was a 9. How many state's licence plates fit this description?

(2) **Symphony orchestra program**
A symphony orchestra has in its repertoire 30 Haydn symphonies, 15 modern works, and 9 Beethoven symphonies. Its program always consists of a Haydn symphony followed by a modern work, and then a Beethoven symphony.

(a) Assume that each piece can be played more than once. How many different programs can it play? How many different programs are there if the three pieces can be played in any order?

(c) Assume that each piece cannot be played more than once. How many different three-piece programs are there if more than one piece from the same category can be played and they can be played in any order?

(3) **Poker game**
A deck of 52 cards has 13 ranks ($2, 3, 4, 5, 6, 7, 8, 9, 10$, J, Q, K, A) and 4 suits ($\heartsuit$, $\spadesuit$, $\diamondsuit$, $\clubsuit$). A poker hand is a set of 5 cards randomly chosen from a deck of 52 cards.

(a) A full house in poker is a hand where three cards share one rank and two cards share another rank. How many ways are there to get a full-house? What is the probability of getting a full-house?

(b) A royal flush in poker is a hand with ten, jack, queen, king, ace in a single suit.
What is the probability of getting a royal flush?

(4) **Coin game**
Two players, A and B, alternately and independently flip a coin and the first player to obtain a head wins. Assume player A flips first. Suppose that $P(\text{head}) = p$, not necessarily $\frac{1}{2}$. What is the probability that the player B wins?

(5) **Student athletes**

A random sample of 400 college students was asked if college athletes should be payed. The following table gives a two-way classification of the responses.

	Should be paid	Should not be paid
Student athlete	80	20
Student nonathlete	220	80

(a) If one student is randomly selected from these 400 students, find the probability that this student

i. is in favor of paying college athletes

ii. is an athlete and favors paying student athletes

iii. is a nonathlete or is against paying students athletes

(b) Are the events *student athlete* and *should be paid* mutually exclusive? Justify your answer.

(6) **Binomial coefficients**

(a) Prove that $\binom{n}{j} = \binom{n}{n-j}$ holds for $n \in \mathbb{N}$ and $0 \leq j \leq n$.

(b) Prove that $\binom{n}{j} = \binom{n-1}{j} + \binom{n-1}{j-1}$ holds for $n \geq 2$ and $0 < j < n$.

(c) Find integers n and r such that the equation $\binom{13}{5} + 2\binom{13}{6} + \binom{13}{7} = \binom{n}{r}$ is true.