Problem Set 3

> **Instructions**
>
> - Write your name and matriculation number on the sheets you hand in and staple all sheets together.
>
> - Hand in your solutions in time and put them in the correct box with course number 389.170, which is located on the ground floor (EG). Note the different floor numbering at the institute!
>
> - Legible writing and a clear layout of your derivations and solutions are absolutely necessary.
>
> - Provide solutions in full detail. No credits will be attributed to solutions without sufficient mathematical foundation.

Problem 3.1 Consider the probability density function

$$f_{\mathsf{x}}(x) = \frac{1}{4}e^{-|x|} + \frac{1}{3}\delta(x) + \frac{1}{6}\delta(x-7),$$

where $\delta(.)$ is the dirac function.

a) Calculate the probability $P\{0 < \mathsf{x} \leq 7\}$.

b) Calculate the probability $P\{\mathsf{x} = 0\}$.

c) Calculate the mean power $E\{\mathsf{x}^2\}$.

Problem 3.2 Consider a joint probability density function

$$f_{\mathsf{x},\mathsf{y}}(x,y) = \begin{cases} b \cdot e^{-(x+y)}, & 0 < x < a \quad \text{and} \quad 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

a) Find a constant b (in terms of a) so that $f_{\mathsf{x},\mathsf{y}}(x,y)$ is a valid joint probability density function.

b) Find an expression for the joint cumulative distribution function $F_{\mathsf{x},\mathsf{y}}(x,y)$.

Problem 3.3 Consider the following joint probability density function of a 2-D random variable (x,y):

$$f_{\mathsf{x},\mathsf{y}}(x,y) = \begin{cases} x+y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Sketch the given pdf.

b) Calculate and sketch the marginal densities $f_x(x)$ and $f_y(y)$.

c) Calculate the mean and the variance of x and y.

d) Calculate the probability $P\{x + y \le 0.3\}$.

Problem 3.4 Consider two random variables x, y whose joint pdf $f_{x,y}(x, y)$ is shown in Figure 1.

a) Provide a mathematical expression for $f_{x,y}(x, y)$.

b) Provide expressions for the marginal pdf's $f_x(x), f_y(y)$ and sketch them.

c) Calculate the conditional cdf $F_{x,y|\mathcal{A}}(x, y|\mathcal{A})$ for $\mathcal{A} = \{x \le 0\}$.

d) Calculate and sketch the conditional pdf $f_{x,y|\mathcal{A}}(x, y|\mathcal{A})$ for $\mathcal{A} = \{x \le 0\}$.

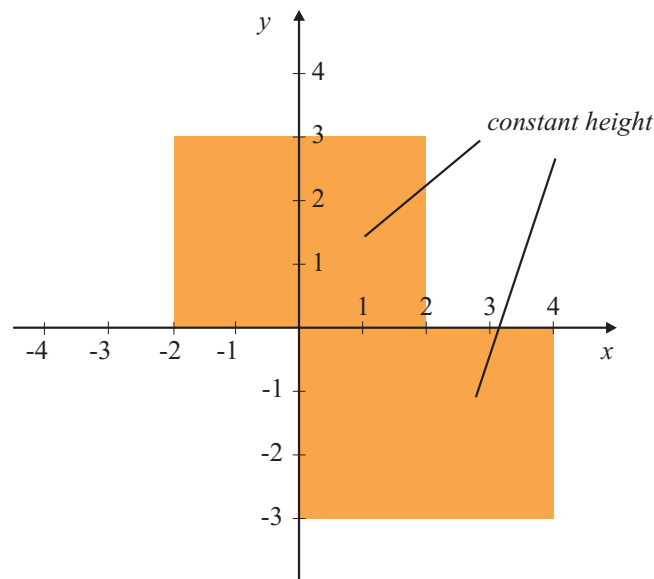e) Calculate the conditional means $\mu_{y|\mathcal{A}}$ and $\mu_{x|\mathcal{A}}$ for $\mathcal{A} = \{x \le 0\}$.

Figure 1: Joint pdf of x, y.