54) $f(t) = \cos(t) + |\cos(t)|$

$$\Rightarrow f(t) = \begin{cases} 2\cdot\cos(t) & 0 \le t < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le t < \frac{3\pi}{2} \\ 2\cdot\cos(t) & \frac{3\pi}{2} \le t < 2\pi \end{cases}$$

$$a_n = \frac{1}{\pi}\cdot\left[2\cdot\int_0^{\frac{\pi}{2}}\cos(t)\cdot\cos(nt)\,dt + \cancel{\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0\,dt} + 2\cdot\int_{\frac{3\pi}{2}}^{2\pi}\cos(t)\cdot\cos(nt)\,dt\right]$$

$$\int \underbrace{\cos(t)}_{u}\cdot\underbrace{\cos(nt)}_{dv}\,dt = \frac{1}{n}\cdot\sin(nt)\cdot\cos(t) + \frac{1}{n}\cdot\int\underbrace{\sin(t)}_{u}\cdot\underbrace{\sin(nt)}_{dv}\,dt =$$

$u = \cos(t) \Rightarrow du = -\sin(t)\,dt$ $\qquad$ $u = \sin(t) \Rightarrow du = \cos(t)\,dt$

$dv = \cos(nt)\,dt \Rightarrow v = \frac{1}{n}\cdot\sin(nt)$ $\qquad$ $dv = \sin(nt)\,dt \Rightarrow v = -\frac{1}{n}\cdot\cos(nt)$

$$= \frac{1}{n}\cdot\sin(nt)\cdot\cos(t) + \frac{1}{n}\cdot\left[-\frac{1}{n}\cdot\sin(t)\cdot\cos(nt) + \frac{1}{n}\cdot\int\cos(t)\cdot\cos(nt)\,dt\right]$$

$$\left[\int\cos(t)\cdot\cos(nt)\,dt\right]\cdot\left(1-\frac{1}{n^2}\right) = \frac{1}{n}\cdot\sin(nt)\cdot\cos(t) - \frac{1}{n^2}\cdot\sin(t)\cdot\cos(nt) \quad /\cdot\frac{1}{1-\frac{1}{n^2}}$$

$$\int\cos(t)\cdot\cos(nt)\,dt = \frac{n}{n^2-1}\cdot\sin(nt)\cdot\cos(t) - \frac{1}{n^2-1}\cdot\sin(t)\cdot\cos(nt)$$

$$\Rightarrow a_n = \frac{2}{\pi}\cdot\left[-\frac{1}{n^2-1}\cdot\cos\left(n\cdot\frac{\pi}{2}\right) - \frac{1}{n^2-1}\cdot\cos\left(n\cdot\frac{3\pi}{2}\right)\right]$$

$$\boxed{\Rightarrow a_n = \begin{cases} 0 & \text{wenn } n \text{ ungerade} \\ \frac{2}{\pi}\cdot\frac{-2}{n^2-1} = -\frac{4}{\pi\cdot(n^2-1)} & \text{wenn } n \equiv 0 \mod 4 \\ \frac{4}{\pi\cdot(n^2-1)} & \text{wenn } n \equiv 2 \mod 4 \end{cases} \qquad n \ge 2}$$

$$a_0 = \frac{1}{\pi}\cdot\left[2\cdot\int_0^{\frac{\pi}{2}}\cos(t)\,dt + 2\cdot\int_{\frac{3\pi}{2}}^{2\pi}\cos(t)\,dt\right] = \frac{1}{\pi}\cdot[2+2] = \underline{\underline{\frac{4}{\pi}}}$$

$$a_1 = \frac{1}{\pi}\cdot\left[2\cdot\int_0^{\frac{\pi}{2}}\cos^2(t)\,dt + 2\cdot\int_{\frac{3\pi}{2}}^{2\pi}\cos^2(t)\,dt\right]$$

$$\int\underbrace{\cos(t)}_{u}\cdot\underbrace{\cos(t)}_{dv}\,dt = \sin(t)\cdot\cos(t) + \int\underbrace{\sin^2(t)}_{1-\cos^2(t)}\,dt = \sin(t)\cdot\cos(t) + \int dt - \int\cos^2(t)\,dt$$

$u = \cos(t) \Rightarrow du = -\sin(t)\,dt$

$dv = \cos(t)\,dt \Rightarrow v = \sin(t)$

$$\Rightarrow 2 \cdot \int \cos^2(t)\,dt = \sin(t) \cdot \cos(t) + t \quad /:2$$

$$\int \cos^2(t)\,dt = \frac{\sin(t) \cdot \cos(t) + t}{2}$$

$$\Rightarrow a_1 = \frac{1}{\pi} \cdot \left[\frac{\pi \cdot 2}{4} + 2\pi - \frac{3\pi}{2} \right] = \frac{1}{\cancel{\pi}} \cdot \frac{\cancel{4\pi}}{\cancel{4}} = \underline{\underline{1}}$$

$$S_f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos(nt)$$

$$\Rightarrow \boxed{S_f(t) = \frac{2}{\pi} + \cos(t) + \sum_{n=2}^{\infty} a_n \cdot \cos(nt)}$$