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| **Status** | Beendet |
| **Begonnen** | Mittwoch, 30. April 2025, 13:11 |
| **Abgeschlossen** | Mittwoch, 30. April 2025, 13:34 |
| **Dauer** | 23 Minuten 20 Sekunden |
| **Bewertung** | **10,00** von 10,00 (**100**%) |

**Frage 1**

Richtig

Erreichte Punkte 1,00 von 1,00

Assume that $h(j(d,e)) \lor \forall n(m(e,j(l,n)) \rightarrow h(n))$ is a well-formed formula. Classify each symbol as either a function or a predicate by drag-and-dropping it to the correct location.

**Important:** The symbols should be specified in alphabetical order. For instance, in $a(b) \land c(d)$, the predicate symbols should be specified as $a, c$ and the function symbols as $b, d$. Furthermore, note that constants are 0-ary functions.

- The predicate symbols are h ✓ , m ✓ .
- The function symbols are d ✓ , e ✓ , j ✓ , l ✓ .

**Frage 2**

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{a\ (arity\ 0), b\ (arity\ 0), f\ (arity\ 2), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{x, y\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. Which formulas are **not** well-formed formulas in First-Order Logic?

**Important:** Note that there can be more than one correct answer.

- [x] a. $\exists x Q(f(x,b), g(x)) \land \neg(f(b,a) \rightarrow Q(g(b)))$ ✓
- [x] b. $(P(b,a) \lor Q(g(a))) \rightarrow \forall(x \land y)Q(f(x,b), g(f(x,b)))$ ✓
- [x] c. $(P(f(a),a) \rightarrow Q(g(b))) \rightarrow \forall x Q(f(x,b), g(x))$ ✓
- [ ] d. $\forall x(Q(a,b) \rightarrow Q(a,x)) \land Q(a, f(a,a))$

**Frage 3**

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{f\ (arity\ 2), a\ (arity\ 0), b\ (arity\ 0), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{x\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. How many **different terms** occur in the formula $P(f(a,b), g(a)) \lor \forall x(P(a,x) \to Q(a,x))$? Note that if a term occurs twice, it only counts as one.

*Example:* The formula $P(f(x), g(x), f(a))$ contains 4 unique terms - $x$, $f(x)$, $g(x)$, $f(a)$.

Antwort: 5

**Frage 4**

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{f\ (arity\ 2)\} \subseteq \mathcal{F}$ and $\{x, y\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. Which variables have free occurrences in the formula $\forall x(P(f(x,x), f(x,y)) \land \exists x P(x, f(x,x)))$? Note that variables that have both free and bound occurrences should also be listed.

**Important:** In order for your answer to be properly validated, input your variables comma-separated and with no white-spaces in between. For instance, for the formula $\forall y P(x,y) \land P(x,y)$ your answer should be: *x,y*

Antwort: y

**Frage 5**

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{g\ (arity\ 1), f\ (arity\ 2), a\ (arity\ 0), b\ (arity\ 0)\} \subseteq \mathcal{F}$ and $\{x\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. How many **subformulas** does the formula $(\exists x P(g(x), x) \to Q(f(a,b), g(a))) \to Q(f(a,b), f(a,a))$ have? Note that if a subformula occurs twice, it only counts as one.

Antwort: 6

**Frage 6**

Richtig

Erreichte Punkte 1,00 von 1,00

How is the correct formalization of the natural language sentence "All roads lead to Rome."?

- (●) a. $\forall x(Road(x) \to LeadToRome(x))$ ✓
- ( ) b. $Road(x) \to \forall x\ LeadToRome(x)$
- ( ) c. $\forall x\ \neg Road(x) \lor \forall x\ LeadToRome(x)$
- ( ) d. $\forall x\ Road(x) \to \forall x\ LeadToRome(x)$

**Frage 7**

Richtig

Erreichte Punkte 1,00 von 1,00

Which formula is equivalent to the **negation** of $\exists x\ P(x)$?

- a. $\forall x\ P(x)$
- b. $\forall x\ \neg P(x)$
- c. $\neg\forall x\ P(x)$
- d. $\neg\forall x\ \neg P(x)$

**Frage 8**

Richtig

Erreichte Punkte 1,00 von 1,00

Which one of the following entailments holds?

- a. $\forall x\ P(x) \models \exists y\ P(y)$
- b. $\forall x(P(x) \rightarrow Q(x)) \models \exists y Q(y)$
- c. $\exists x(P(x) \rightarrow Q(x)), P(a) \models \exists y\ Q(y)$
- d. $\exists x(P(x) \wedge Q(x)), P(a) \models \forall x\ Q(x)$

**Frage 9**

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the formula $\varphi := \exists x \forall y \exists z (P(x,x) \wedge P(x,y) \wedge \neg P(z,x))$. Given the structure $\mathcal{M} = (D_\mathcal{M}, I_\mathcal{M})$ with domain $D_\mathcal{M} = \{a,b\}$, your task is to specify the relation $P^\mathcal{M}$ associated to the predicate symbol $P$ by $I_\mathcal{M}$, such that the formula $\varphi$ is satisfied by $\mathcal{M}$.

**Note:** List the tuples in $P^\mathcal{M}$ comma-separated, with no spaces in between. In case $P^\mathcal{M}$ should not contain any elements, input "empty" (without the quotes) in the field below. Furthermore, note that multiple correct answers may be possible.

*Example:* For $\exists x P(x,x)$ and $\mathcal{M} = (\{a,b\}, I_\mathcal{M})$, a possible input could be: *(a,a),(a,b)*

Antwort: (a,a),(a,b)

Frage **10**

Richtig

Erreichte Punkte 1,00 von 1,00

Which of the following interpretations is a model of the formula $\forall x \exists y P(x, y)$? Note that more than one answer may be correct.

**Note:** We consider 0 as a natural number, i.e. $\mathbb{N} = \{0, 1, 2, \ldots\}$

- [ ] a. $\mathcal{M} = (D_\mathcal{M}, I_\mathcal{M})$ with $D_\mathcal{M} = \mathbb{N}$ and $P^\mathcal{M} = \{(n, n') \mid n > n'\}$
- [ ] b. $\mathcal{M} = (D_\mathcal{M}, I_\mathcal{M})$ with $D_\mathcal{M} = \mathbb{N}$ and $P^\mathcal{M} = \{(n, n') \mid n + n' = 10\}$
- [x] c. $\mathcal{M} = (D_\mathcal{M}, I_\mathcal{M})$ with $D_\mathcal{M} = \mathbb{N}$ and $P^\mathcal{M} = \{(n, n') \mid n' - n = 1\}$ ✓
- [x] d. $\mathcal{M} = (D_\mathcal{M}, I_\mathcal{M})$ with $D_\mathcal{M} = \mathbb{N}$ and $P^\mathcal{M} = \{(n, n') \mid n \leq n'\}$ ✓