

$$56) \quad g(t) = e^t \quad 0 \leq t < T$$

$$\omega = \frac{2\pi}{T^*} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

$$h(t) = \begin{cases} g(t) & 0 \leq t < T \\ g(t-T) & -T < t < 0 \end{cases} \quad h(t+2T) = h(t)$$

$$a_n = \frac{1}{T} \cdot \left( \int_{-T}^0 e^{-t} \cdot \cos(\omega n t) dt + \int_0^T e^t \cdot \cos(\omega n t) dt \right) =$$

$$= \frac{1}{T} \cdot \left[ -\frac{1}{\omega^2 n^2 + 1} - \frac{\omega n}{\omega^2 n^2 + 1} \cdot \sin(-n \cdot \pi) \cdot e^T + \frac{1}{\omega^2 n^2 + 1} \cdot \cos(-n \cdot \pi) \cdot e^T \right. \\ \left. + \frac{\omega n}{\omega^2 n^2 + 1} \cdot \sin(n \pi) \cdot e^T + \frac{1}{\omega^2 n^2 + 1} \cdot \cos(n \pi) \cdot e^T \right. \\ \left. - \frac{\omega n}{\omega^2 n^2 + 1} \cdot \sin(0) - \frac{1}{\omega^2 n^2 + 1} \right]$$

$$= \frac{1}{T} \cdot \left[ -\frac{2}{\omega^2 n^2 + 1} + \frac{2}{\omega^2 n^2 + 1} \cdot \cos(n \pi) \cdot e^T \right]$$

$$\Rightarrow a_n = \begin{cases} \frac{1}{T} \cdot \left[ -\frac{2}{\omega^2 n^2 + 1} - \frac{2}{\omega^2 n^2 + 1} \cdot e^T \right] & \text{wenn } n \text{ ungerade} \\ \frac{1}{T} \cdot \left[ -\frac{2}{\omega^2 n^2 + 1} + \frac{2}{\omega^2 n^2 + 1} \cdot e^T \right] & \text{wenn } n \text{ gerade} \end{cases}$$

$b_n = 0$  weil  $h(t)$  gerade Fkt.

$$\Rightarrow S_f(t) = \frac{1}{T} \cdot [e^T - 1] + \sum_{n=1}^{\infty} a_n \cdot \cos(n \omega t)$$

$$\int \underbrace{e^{-t}}_u \cdot \underbrace{\cos(nwt)}_{dv} dt = \frac{1}{nw} \sin(nwt) \cdot e^{-t} + \frac{1}{nw} \int \underbrace{e^{-t}}_u \cdot \underbrace{\sin(nwt)}_{dv} dt =$$

$$u = e^{-t} \Rightarrow du = -e^{-t} dt$$

$$dv = \sin(nwt) \Rightarrow v = -\frac{1}{nw} \cos(nwt)$$

$$dv = \cos(nwt) dt \Rightarrow v = \frac{1}{nw} \sin(nwt)$$

$$= \frac{1}{nw} \sin(nwt) \cdot e^{-t} + \frac{1}{nw} \cdot \left[ -\frac{1}{nw} \cos(nwt) \cdot e^{-t} - \frac{1}{nw} \int e^{-t} \cdot \cos(nwt) dt \right]$$

$$\left[ \int e^{-t} \cdot \cos(nwt) dt \right] \cdot \left( 1 + \frac{1}{\omega_n^2} \right) = \frac{1}{\omega_n} \sin(nwt) \cdot e^{-t} - \frac{1}{\omega_n} \cos(nwt) \cdot e^{-t} \cdot \frac{1}{1 + \frac{1}{\omega_n^2}}$$

$$\int e^{-t} \cdot \cos(nwt) dt = \frac{1}{\omega_n + \frac{1}{\omega_n}} \sin(nwt) \cdot e^{-t} - \frac{1}{\omega_n^2 + 1} \cos(nwt) \cdot e^{-t}$$

$$= \frac{\omega_n}{\omega_n^2 + 1} \sin(nwt) \cdot e^{-t} - \frac{1}{\omega_n^2 + 1} \cos(nwt) \cdot e^{-t}$$


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$$\int \underbrace{e^t}_u \cdot \underbrace{\cos(nwt)}_{dv} dt = \frac{1}{nw} \sin(nwt) e^t - \frac{1}{nw} \int \underbrace{e^t}_u \cdot \underbrace{\sin(nwt)}_{dv} dt =$$

$$u = e^t \Rightarrow du = e^t dt$$

$$dv = \sin(nwt) dt \Rightarrow v = -\frac{1}{nw} \cos(nwt)$$

$$dv = \cos(nwt) dt \Rightarrow v = \frac{1}{nw} \sin(nwt)$$

$$= \frac{1}{nw} \sin(nwt) \cdot e^t - \frac{1}{nw} \cdot \left[ -\frac{1}{nw} \cos(nwt) e^t + \frac{1}{nw} \int e^t \cdot \cos(nwt) dt \right]$$

$$\left[ \int e^t \cdot \cos(nwt) dt \right] \cdot \left( 1 + \frac{1}{\omega_n^2} \right) = \frac{1}{\omega_n} \sin(nwt) e^t + \frac{1}{\omega_n} \cos(nwt) e^t \cdot \frac{1}{1 + \frac{1}{\omega_n^2}}$$

$$\int e^t \cdot \cos(nwt) dt = \frac{\omega_n}{\omega_n^2 + 1} \sin(nwt) \cdot e^t + \frac{1}{\omega_n^2 + 1} \cos(nwt) \cdot e^t$$


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$$a_0 = \frac{1}{T} \left[ \int_{-T}^0 e^{-t} dt + \int_0^T e^t dt \right] = \frac{1}{T} \left[ -e^{-t} \Big|_{-T}^0 + e^t \Big|_0^T \right] = \frac{1}{T} \left[ -1 + e^T + e^T - 1 \right] =$$

$$= \frac{1}{T} \left[ 2 \cdot e^T - 2 \right] = \frac{2}{T} \left[ e^T - 1 \right]$$