Knowledge Based Systems, 4.0 VU, 184.730

Exercise Sheet 1

This exercise sheet serves as a **preparation for the mandatory exercise test**, which covers the exercises and related background knowledge. You do *not* need to submit solutions.

For questions regarding exercises please visit the tutoring sessions (times are announced in **TUWEL**). You can find them in Conference room Hahn (room HG 03 06), Favoritenstr. 9-11, Stiege 3, 3rd floor, Institute of Logic and Computation. For questions of general interest, please use the **TISS** Forum or contact us on `kbsci-2019s@kr.tuwien.ac.at`.

Exercise 1.1: To cut personnel costs, the ministry of finance hires you to write rules in Clips-related pseudocode (like the examples on the lecture slides) such that the RBS outputs the correct decision whether a person has to pay income tax or not. You can assume that the input facts (i.e., the facts about a person asking for tax exempt) contain all relevant information. Additionally, draw the RETE-network of your solution.

> Rich people don't have to pay taxes. People with more then 250.000 Euro employed earnings or more then 200.000 Euro self-employed a year are rich and not poor. A person is also tax exempt if they are poor, except if they are married and their spouse earns more then 30.000 Euro as an employee per year (assume the spouse is uniquely identified by their name). A person earning less then 15.000 Euro as an employee and less then 5.000 Euro from self-employment per year is considered poor and not rich. People with less then 30.000 Euro employed earnings who have difficulties to pay for their accommodations are also considered poor and not rich. Moreover, if a person owns more than 1.000.000 Euro and is not in debt, they are considered rich and not poor.

Exercise 1.2: Consider the following template.

```
(deftemplate Person
       (slot name (type SYMBOL) (default NN))
       (slot employedEarningsPerYear (type INTEGER) (default 0))
       (slot selfEmployedEarningsPerYear (type INTEGER) (default 0))
       (slot hasDifficulty (type SYMBOL) (default false))
       (slot wealth (type INTEGER) (default 0))
       (slot inDebt (type SYMBOL) (default false))
       (slot poor (type SYMBOL) (default false))
       (slot rich (type SYMBOL) (default false))
       (slot taxExempt (type SYMBOL) (default false)
       (slot spouseName (type SYMBOL) (default false)
)
```

Given this template and the rules in Exercise 1.1 with its corresponding RETE-Network, use `assert` and `modify` operations to modify the working memory in reaction to the following events:

1. Bob works for a bank and *earns 10.000 Euro per year*. Moreover, his *income from self-employment is 0 Euro per year*.

2. Bob loves embroidering. He starts selling the fruits of his labour, making about *6.000 Euro per year*, which is considered *income from self-employment* by the ministry of finance.
3. Bob's landlord increases his rent, making it difficult for him to retain a roof over his head. That is, he has *difficulties to pay for his accommodation.*
4. In order to pay rent, Bob had to take on a loan. The ministry of finance, thus considers Bob to be *in debt.*
5. Government regulation curtails rent prices. Thereby allowing Bob to *pay his rent* once again *without struggles.*
6. Bob wins *2.000.000 Euro* in the lottery, thus *increasing Bobs wealth.*
7. Bob uses this money to pay back his loan. Bob is now *free from debt* and still has a *wealth of 1.950.000 Euro.*

After each statement, reset the attributes `rich`, `poor` and `taxExempt` by applying

```
(modify 1 (rich false) (poor false) (taxExempt false))
```

then apply RAC until no more rules are applicable, and show the working memory.
(Note: Assume the *Working Memory* to be empty. That is, after the initial assertion "Bob" can be addressed with fact index 1, e.g. `(modify 1 (slot_name value))`

Exercise 1.3: In order to formalise the scenario from Exercise 1.1 into first-order logic,

1. define a suitable signature Σ and
2. specify a theory Γ that represents the rules from Example 1.1.

Exercise 1.4: Classify the following statements as *unsatisfiable*, *refutable*, *satisfiable* and *valid*. Moreover, if φ is both satisfiable and refutable give $\mathcal{I}$ and $\mathcal{J}$ where $\mathcal{I} \not\models \varphi$ and $\mathcal{J} \models \varphi$. For the other two cases, give a syntactic proof.

$$\varphi_1 := \forall x \forall y \exists z (R(x,y) \to (R(x,z) \land R(z,y)))$$
$$\varphi_2 := \forall x \exists y R(x,y) \to \exists y \forall x R(x,y)$$
$$\varphi_3 := \forall x \exists y (P(x) \to \neg P(y))$$
$$\varphi_4 := \forall x \exists y R(x,y) \to \neg \exists y \forall x \neg R(x,y)$$
$$\varphi_5 := \neg \exists x \forall y (P(x) \land \neg R(x,y)) \lor \neg \forall x \exists y (P(x) \to R(x,y))$$

Exercise 1.5: 1. Let $\Sigma := (\{\}, \{R/2, S/2, T/2\})$ be a signature and let Γ be the following set of first-order formulas.

$$\begin{aligned}\Gamma := \{&\forall x R(x,x), \forall x \forall y (R(x,y) \leftrightarrow R(y,x)), \forall x \forall y \forall z (R(x,y) \land R(y,z) \to R(x,z)),\\ &\forall x S(x,x), \forall x \forall y (S(y,x) \lor S(x,y)), \forall x \forall y \forall z (S(x,y) \land S(y,z) \to S(x,z)),\\ &\forall x \forall y ((S(x,y) \land S(y,x)) \to R(x,y)), \forall x \forall y (T(x,y) \leftrightarrow \neg R(x,y) \land S(x,y))\}\end{aligned}$$

Explain informally what the theory (and, in particular, the defined relations) represents. Construct a model of Γ.

2. Consider the signature $\Sigma := (\{a/0, b/0\}, \{E/2, S/2, B/3\})$ and the following first-order theories

$$\begin{aligned}
\Gamma_1 :=& \{\forall x(E(x,a) \vee E(x,b))\} \\
\Gamma_2 :=& \{\forall x \forall y \exists z(\neg E(x,y) \rightarrow (B(x,y,z) \vee B(y,x,z)))\} \\
\Gamma_3 :=& \{\forall x \exists y(S(x,y) \wedge \neg \exists z B(x,y,z))\} \\
\Gamma_4 :=& \{\forall x \exists y S(x,y), \forall x \exists y S(y,x)\}
\end{aligned}$$

Consider the Σ-structures $\mathcal{I}_X := \langle X, I_X, \alpha_X \rangle$ for all $X \in \{\mathbb{N}, \mathbb{Z}, \mathbb{R}\}$ where $\alpha_X := \{\}$ and

$$\begin{aligned}
&I_X(E) := \{(x,x) \mid \forall x \in X\} \\
&I_X(S) := \{(x,y) \mid \forall x, y \in X \ y = x+1\} \\
&I_X(B) := \{(x,y,z) \mid \forall x,y,z \in X \ x < z < y\} \\
&I_X(a) := 0, \ I_X(b) := 1
\end{aligned}$$

Determine which set of formulas is satisfied by which Σ-structure. Support your assertions with informal arguments.

(Note: In the above definition of I_X, addition (+) and $<$ have the standard semantics of addition and smaller-than over the respective domains.)

Exercise 1.6: You are given the following description of the world

- Every Organism reproduces sexually or asexually;
- If something reproduces, it must be an organism;
- If something reproduces sexually or asexually, it reproduces;
- Humans and hammerhead sharks are organisms;
- There exists no organism that is a hammerhead shark and a human;
- Humans reproduce only sexually;
- Having exactly one biological parent is synonymous with asexual reproduction;
- Having exactly two biological parents is synonymous with sexual reproduction;
- Every organism has either exactly one or exactly two biological parents;
- If there exists a hammerhead shark with exactly one biological parent and one with exactly two biological parents, then all hammerhead shark reproduce sexually and asexually.

1. Formalise the assertions above and construct a first-order theory with equality, called Γ. Moreover, characterise the notion of having exactly one (two) biological parent(s), by relying solely on a binary parent predicate and equality "=".

2. Having internalised the theory of the previous point, you encounter an article in the newspaper that reports that the first human was successfully cloned (i.e. it has only one biological parent). Adding this information to your theory you wonder, are all unicorns pink? If so, give a syntactic proof.

Exercise 1.7: Consider an interpretation $\mathcal{I} := \langle \mathcal{U}, I, \alpha \rangle$ over the signature $\Sigma := \{\{\}, \{R/2\}\}$. We say that $a \in \mathcal{U}$ is an *ancestor* of $b \in \mathcal{U}$, if there exists a sequence $x_1, x_2, \ldots, x_n$ ($x_k \in \mathcal{U}$ for $k = 1, \ldots, n$) such that

(i) $I(R)(x_k, x_{k+1})$ for all $k = 1, \ldots, n-1$, and

(ii) $a = x_1$, $b = x_n$.

Use the compactness theorem[1] to show that there is no formula $\varphi(x, y)$ such that for any interpretation $\mathcal{I}$ with domain $\mathcal{U}$ and any $a, b \in \mathcal{U}$ we have

$$\mathcal{I}_{\{x \leftarrow a, y \leftarrow b\}} \models \varphi(x, y) \iff a \text{ is an ancestor of } b.$$

Hint: You may consult the literature on how to prove that *graph reachability* is not first-order expressible and use the same proof idea here!

[1] Recall that the compactness theorem states that a set of formulas is satisfiable iff every finite subset of it is.