Einführung in Künstliche Intelligenz SS 2015, 2.0 VU, 184.735

Exercise Sheet 2 – Learning and Neural Networks

For the presentation part of this exercise, mark your solved exercises in **TUWEL** until **Sunday, May 17, 23:55 CET**. Be sure that you tick only those exercises that you can solve and explain in detail with the necessary theoretical background! In particular note that ticking exercises which you do not understand can result in the denial of **all exercises** of this sheet!

Please ask questions in the **TISS** Forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 1 (3 pts.): Consider a classification problem with three attributes A, B, C with the domains $V(A) = \{a_1, a_2\}$, $V(B) = \{b_1, b_2\}$, $V(C) = \{c_1, c_2\}$ and two classes $K = \{T, F\}$. For this problem, build a decision tree using the rule from the lecture of choosing the attribute maximising information gain in each step.

	A	B	C	Class
1	a_1	b_2	c_1	F
2	a_2	b_2	c_1	F
3	a_1	b_2	c_2	F
4	a_1	b_1	c_1	T
5	a_1	b_1	c_2	F
6	a_2	b_1	c_2	F

Exercise 2 (2 pts.): Compute the information gain for an attribute A that takes a different value for each example. What problem do you see?

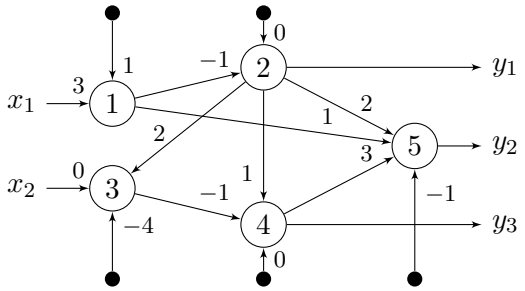
Exercise 3 (2 pts.): Consider again the problem in Exercise 1. Again construct a decision tree, but in every step choose the argument that maximises the *relative information gain*, i.e., the ratio between their gain and their own intrinsic information.

$$GainR(A) = \frac{Gain(A)}{H(A)}$$

Thereby we have $H(A) := \sum_{a \in V(A)} \frac{|E_a|}{V} \log_2 \frac{V}{|E_a|}$, where $V(A)$ denotes the set of possible values for the attribute A and E_a is the set of samples with $A = a$ and $V := \sum_{a \in V(A)} |E_a|$ is the total number of samples.

Exercise 4 (2 pts.): Suppose that an attribute splits the set of examples E into subsets E_k and that each subset has p_k positive and n_k negative examples. Show that the attribute has zero information gain if the ratio $\frac{p_k}{p_k + n_k}$ is equal for all k.

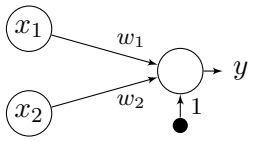
Exercise 5 (1 pt.): Consider a neural network with five nodes of the following form:



The numbers beneath the arrows denote the respective weights. Nodes 1 and 3 have the *identity function* as their activation function. Nodes 2 and 4 use the hard limiter with threshold 0 and node 5 uses a sigmoid function $(1/(1+e^{-x}))$. The arrows from the black dots give the respective bias weights. What is the output produced by the network when the input is $(x_1, x_2) = (1, 1)$?

Exercise 6 (2 pts.): Construct a neural network with 4 input nodes x_1, x_2, x_3, x_4 and one output node y which produces a parity bit of the respective input values, namely $y = x_1$ XOR x_2 XOR x_3 XOR x_4 holds. You are allowed to introduce hidden layers with additional nodes of arbitrary depth, but every node should use the hard limiter with threshold 0 as activation function. Please note that the fixed input a_0 is 1, as presented in the lecture.

Exercise 7 (2 pts.): Consider a 2-layer perceptron with two input neurons and one output neuron of the following form:



Train the perceptron using the *Perceptron learning rule* and the identity transfer function on the following training data: $f(0,0) = 1$, $f(1,0) = 0$ and $f(0,1) = 0$. The weights are initialised to 0 and the learning rate α is 1. The bias weight will not be learned and stays 1.

Exercise 8 (1 pt.): Suppose you have a neural network with linear activation functions. That is, for each unit the output is some constant c times the weighted sum of the inputs. For simplicity, assume that the activation function is the same linear function at each node: $g(x) = c \cdot x + d$.

Assume that the neural network has one hidden layer. For a given assignment to the weights w, write down equations for the value of the units in the output layer as a function of w and the input layer x, without any explicit mention of the output of the hidden layer. Show that there is a network with no hidden units that computes the same function.