Einführung in Artificial Intelligence SS 2025, 4.0 VU, 192.027

Exercise Sheet 4 – CSP and Knowledge Representation

For the discussion part of this exercise, mark and upload your solved exercises in **TUWEL** until Wednesday, June 11, 23:55 CEST. The registration for a solution discussion ends on Friday, June 13, 23:55 CEST. Be sure that you tick only those exercises that you can solve and explain!

In the discussion, you will be asked questions about your solutions of examples you checked. The discussion will be evaluated with 0–25 points, which are weighted with the fraction of checked examples and rounded to the next integer. There is *no minimum number of points* needed for a positive grade (i.e., you do not need to participate for a positive grade, but you can get at most ≈80% without doing exercises).
Note, however, that *your registration is binding*. Thus, *if* you register for a solution discussion, then it is *mandatory* to show up. Not coming to the discussion after registration will lead to a reduction of examination attempts from 4 to 2.
Please ask questions in the **TUWEL** forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 4.1: Consider the following cryptarithmetic puzzle. Every letter corresponds to exactly one digit. In particular, the digits corresponding to different letters need to be different, and I, T, and F should not be 0.

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    I C E
  + T E A
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  F I N E
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(a) Describe this puzzle as a CSP, providing all its variables, constraints, and domains of variables.

(b) Draw the constraint graph.

(c) Find a solution of the puzzle.

Exercise 4.2:

(a) Given a single ternary constraint $A + B = C$. Transform this constraint into 3 binary constraints achieving the same functionality using auxiliary variables.

(b) Show how constraints with $n \geq 4$ variables can be transformed in a similar way.

Exercise 4.3: Assume the following statements:

- Tom only goes swimming when it does not rain.
- If Tom goes swimming, so does Lisa.
- It is raining and Lisa goes swimming.

Given this information, who goes swimming?

To answer this question, represent the statements in propositional logic and use truth tables to determine a model of the conjunction of the three statements. Use the atomic formulas T, L, and R for representing the statements that

- Tom goes swimming (T),
- Lisa goes swimming (L), and
- it is raining (R),

respectively.

Exercise 4.4: Prove or refute whether the following statements hold, for any formula A, B, C and D (i.e., show for each statement that it either holds for any A, B, C, and D or give a counterexample refuting the statement; note that "$\models A$" denotes that A is valid):

- $\models ((A \Rightarrow B) \Rightarrow A) \Rightarrow A$;
- $A, A \Rightarrow (C \vee B) \models B$;
- $A \wedge \neg B, A \Rightarrow C, B \Rightarrow D \models C \vee D$.

Exercise 4.5: Consider the following sentences:

1. Every guitarist looks up to a guitarist who played at Woodstock.
2. Every guitarist who rehearsed a lot also played at Woodstock.
3. Some guitarist did not rehearse a lot, but played at Woodstock.
4. Some guitarist who played at Woodstock does not look up to any guitarist who rehearsed a lot.

(a) Formalise these sentences in first-order logic by using the following predicates:

- $G(x)$: "x is a guitarist";
- $L(x, y)$: "Guitarist x looks up to guitarist y";
- $W(x)$: "Guitarist x played at Woodstock";
- $R(x)$: "Guitarist x rehearsed a lot".

(b) Is the set of formulas resulting from the formalisation in (a) satisfiable? If yes, provide a model; if no, give a justification.

Exercise 4.6: Prove the unsatisfiability of the following propositional formula using resolution:

$$(\neg x \vee \neg y \vee \neg z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg x \vee y \vee z) \wedge \\ (\neg x \vee \neg y \vee z) \wedge (x \vee \neg y \vee \neg z) \wedge (x \vee y \vee \neg z) \wedge z.$$