329–332) Man bestimme die Funktionalmatrix zu $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2$:

331) $\mathbf{f}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{x-z}{y+1}} \\ z \cdot e^{-\frac{x}{y}} \end{pmatrix}$

Um die zur Funktion $f(x,y,z) = \binom{g(x,y,z)}{h(x,y,z)}$ *zugehörige Jacobi Matrix zu bestimmen, müssen die einzelnen partiellen Ableitungen, also* g_x, g_y, g_z *und* h_x, h_y, h_z *bestimmt und in der Form* $\begin{pmatrix} g_x & g_y & g_z \\ h_x & h_y & h_z \end{pmatrix}$ *angeordnet werden.*

$$g(x,y,z) = \left(\frac{x-z}{y+1}\right)^{1/2}$$

$$g_x = \left(\frac{1}{2}\right) * \left(\frac{x-z}{y+1}\right)^{-\frac{1}{2}} * \frac{1}{y+1} = \frac{1}{2(y+1)\frac{\sqrt{x-z}}{\sqrt{y+1}}} = \frac{\sqrt{y+1}}{2(y+1)\sqrt{x-z}} = \frac{1}{2\sqrt{(y+1)*(x-z)}}$$

$$g_y = (x-z)^{\frac{1}{2}}\left(-\frac{1}{2}\right)(y+1)^{-\frac{3}{2}} * 1 = -\frac{(x-z)^{\frac{1}{2}}}{2(y+1)^{\frac{3}{2}}} = -\frac{\sqrt{x-z}}{2\sqrt{y+1}^3}$$

$$g_z = \left(\frac{1}{2}\right) * \left(\frac{x-z}{y+1}\right)^{-\frac{1}{2}} * \frac{(-1)}{y+1} = \cdots (wie\ g_x) \ldots = -\frac{1}{2\sqrt{(y+1)*(x-z)}}$$

$$h(x,y,z) = z * e^{-\frac{x}{y}}$$

$$h_x = z * \left(-\frac{1}{y}\right) * e^{-\frac{x}{y}} = -\frac{z}{y} * e^{-\frac{x}{y}}$$

$$h_y = z * (-x)(-1)y^{-2} * e^{-\frac{x}{y}} = \frac{zx}{y^2} * e^{-\frac{x}{y}}$$

$$h_z = 1 * e^{-\frac{x}{y}} = e^{-\frac{x}{y}}$$

Jacobi Matrix:

$$\begin{pmatrix} \frac{1}{2\sqrt{(y+1)*(x-z)}} & -\frac{\sqrt{x-z}}{2\sqrt{y+1}^3} & -\frac{1}{2\sqrt{(y+1)*(x-z)}} \\ -\frac{z}{y} * e^{-\frac{x}{y}} & \frac{zx}{y^2} * e^{-\frac{x}{y}} & e^{-\frac{x}{y}} \end{pmatrix}$$