2.1

a) Theory question: For $n+1$ knots and values, what is the highest degree for which an interpolation polynomial can be guaranteed to have a unique existence? also for lower/higher degrees?

b) Interpolate through $\left(\frac{1}{2}, 1\right)\left(1, \frac{3}{2}\right)(2,5)$ through $f'(0)$.

c) How are the Chebychev knots defined? Explain why these are a good choice. (Because they are chosen to minimize the error because they affect the weights. There are formulas for the $\omega_{i,n}^{\text{cheb}}$ and for the errors with $\omega_{i,n}^{\text{cheb}}$)

2.2

a) $\left[0, \frac{3}{2}\pi\right]$, decompose into equidistant mesh with 3 intervals, draw mesh. what is h? Use composite trapezoidal rule, compute $\int_0^{\frac{3}{2}\pi} x\sin(x)dx$.

b) For general $f \in C^{(2)}\left(\left\{0, \frac{3\pi}{2}\right]\right)$ write formula for error $\int_0^{\frac{3}{2}\pi} f(x)dx - T_n(f)$, where $T_n(f)$ is trapezoidal.

c) General estimate find mesh-width h to $\left|\int x\sin(x)dx - T_n(x\sin(x))\right| \leq 0,01$.

hint: use $x\sin x^{(2)} = 2\cos x - x\sin x$

2.3

a) Give example of a matrix where the LU decomposition is impossible but possible with pivot.
b) Compare inverse shift with Rayleigh. (List pros and cons of both and explain what they do)

2.4

a) Define contraction and why/how is it needed/related to fixed point/newton method.
b) When and how does newton method converge quadratically?

2.5

a) Convengence/complexity of neville $p(x)$
b) Is quadrature with $\sum \omega_i = 1$ for constant functions always exact?
c) For Newton Cotes if n->infinity is the quadrature exact? False
d) The midpoint rule is more efficient than gaussian quadrature? False
e) Convergence behaviour of power iteration method? I think $C|\frac{\lambda_2}{\lambda_1}|^l$ where l is iteration step
f) What order is simpson rule? h^4 i.e. 4
g) SVD is used for overdetermined systems? False
etc. more theory.