

192.067 VO Deductive Databases
January 28, 2022

Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)
[REDACTED]	[REDACTED]	[REDACTED]

1.) Consider the following two databases:

$$D_1 = \{PartOf(b, a), PartOf(c, a), PartOf(d, b)\}$$

$$D_2 = \{PartOf(a, b), PartOf(b, a)\}$$

Furthermore, consider the program P consisting of the following two rules:

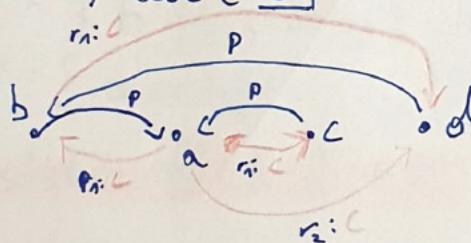
$$\begin{aligned} r_1 &= Contains(X, Y) \leftarrow PartOf(Y, X) & PartOf(A, B) \rightarrow Contains(B, A) \\ r_2 &= Contains(X, Z) \leftarrow Contains(X, Y), PartOf(Z, Y) \end{aligned}$$

Compute the answer to the Datalog query $(P, Contains)$ over the database D_1 .

Compute the answer to the Datalog query $(P, Contains)$ over the database D_2 .

(12 points)

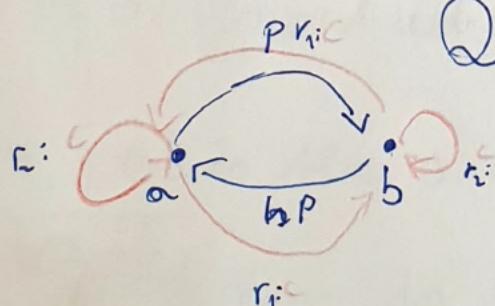
$(P, Contains)$ over D_1



$$\text{Output} = \{(d, b), (a, b), (a, c)\}$$

$$\text{Output} = \{(b, d), (a, c), (a, b), (a, d)\}$$

$(P, Contains)$ over D_2



$$\text{Output} = \{(a, b), (b, a), (a, a), (b, b)\}$$

2.) Consider a program P consisting of the following three rules:

$$\begin{aligned} b &\leftarrow \text{not } a \\ a &\leftarrow b \\ a &\leftarrow \text{not } b \end{aligned}$$

Present at least one stable model of P . Justify your answer (including the computation of the program reduct). (7 points)

Which of the three rules should be deleted from P so that the resulting program P' has exactly two stable models? Explain your answer. (5 points)

(12 points)

I)

Candidates:

$$M^0 = \{\emptyset\}, M^1 = \{a\}, M^2 = \{b\}, M^3 = \{a, b\}$$

Reducts,

$$P^{M^0} = \{b \leftarrow, a \leftarrow b, a \leftarrow\}$$

$$P^{M^1} = \{a \leftarrow b, a \leftarrow\}$$

$$P^{M^2} = \{b \leftarrow, a \leftarrow b,\}$$

$$P^{M^3} = \{a \leftarrow b\}$$

A Model is stable iff $\models I \vdash P^L$ minimal

$$M^0 \models P^{M^0} \models \models \Rightarrow \text{No}$$

$$M^1 \models P^{M^1} \models \models \Rightarrow \text{Yes, and minimal}$$

$$M^2 \models P^{M^2} \models \models \Rightarrow \text{No}$$

$$M^3 \models P^{M^3} \models \models \Rightarrow \text{Yes, but not minimal as } \{\emptyset\} \subset \{a, b\}$$

Therefore $M^1 = \{a\}$ is the only stable model of P

II) Rule

~~a \leftarrow b~~

~~a \leftarrow b~~ has to be removed. Then M_1 and M_2 are stable models.

→ Extra slot

* Task 2.) continued

Program $J = \{ b \leftarrow \text{not } a \\ a \leftarrow \text{not } b \}$

Model candidates: same as before

Reducts

$$J^{u_0} = \{ b \leftarrow ;, a \leftarrow \}$$

$$J^{u_1} = \{ a \}$$

$$J^{u_2} = \{ b \}$$

$$J^{u_3} = \{ \emptyset \}$$

Stable 2:

$$M^0 \models J^{u_0} \models \top \Rightarrow \text{No}$$

$$M^1 \models J^{u_1} \models \top \Rightarrow \text{Yes, and minimal}$$

$$M^2 \models J^{u_2} \models \top \Rightarrow \text{Yes, and minimal}$$

$$M^3 \models J^{u_3} \models \top \Rightarrow \text{Yes, but not minimal}$$

Therefore $M_1 = \{a\}$ and $M_2 = \{b\}$ are the two stable Models

3.) Consider a program P consisting of the following rules:

$$\begin{aligned}r_1 &= a \leftarrow b, \text{not } c \\r_2 &= c \leftarrow \text{not } a \\r_3 &= b \leftarrow a\end{aligned}$$

Present a set U_1 of atoms from P such that U_1 is unfounded w.r.t. $(P, \{\}, \{\})$.

Present a set U_2 of atoms from P such that U_2 is unfounded w.r.t. $(P, \{a\}, \{\})$. It should be the case that $U_1 \neq U_2$.

Justify your answer.

(12 points)

Definition

Set U is unfounded w.r.t. $(P, P_{\text{as}}, \text{Weg})$ if $\forall r \in P$ s.t. one of the following holds

- I. $H(r) \notin U$
- II. ~~exists~~ $\exists p \in B^+(r)$ s.t. $p \in U_{\text{Weg}}$ or $p \in U$
- III. $\exists p \in B^-(r)$ s.t. $p \in P_{\text{as}}$

$$U_1 = \cancel{\{a, b, c\}} \quad \underline{\{b, a\}} \quad \begin{array}{c} \xrightarrow{r_1 \rightarrow \text{II holds: } b \in B^+(r_1) \wedge b \in U} \\ \xrightarrow{r_2 \rightarrow \text{I holds: } c \notin U} \\ \xrightarrow{r_3 \rightarrow \text{II holds: } a \in B^+(r_3) \wedge a \in U} \end{array}$$

$$U_2 = \underline{\{c\}} \quad \begin{array}{c} \xrightarrow{r_1 \rightarrow \text{I holds: } a \notin U} \\ \xrightarrow{r_2 \rightarrow \text{III holds: } a \in B^-(r_2) \wedge a \in P_{\text{as}}} \\ \xrightarrow{r_3 \rightarrow \text{I holds: } b \notin U} \end{array}$$

Both U_1 and U_2 fulfill at least one of the conditions w.r.t. their sets $(P, P_{\text{as}}, \text{Weg})$ Run each rule!

4.) Consider an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ satisfying the following:

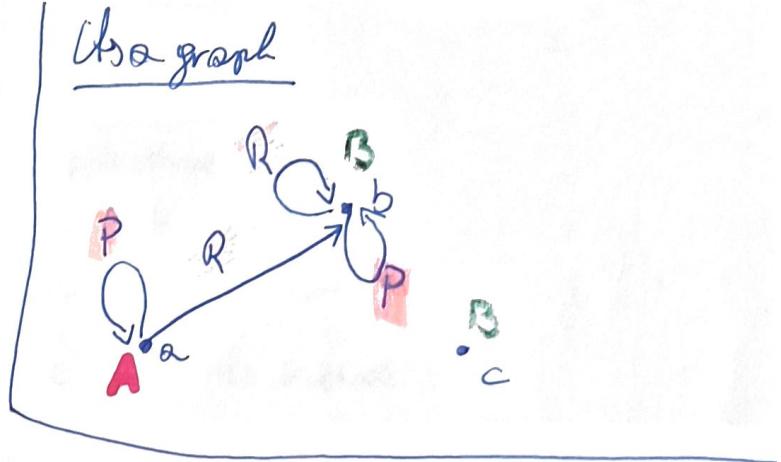
- $\Delta^{\mathcal{I}} = \{a, b, c\}$,
- $A^{\mathcal{I}} = \{a\}$ for the concept name A ,
- $B^{\mathcal{I}} = \{b, c\}$ for the concept name B ,
- $R^{\mathcal{I}} = \{(b, b), (a, b)\}$ for the role name R , and
- $P^{\mathcal{I}} = \{(a, a), (b, b)\}$ for the role name P .

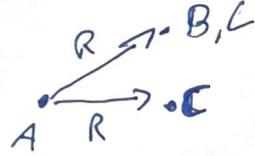
Compute the extension of $\cdot^{\mathcal{I}}$ for the following complex concepts (i.e. compute $C^{\mathcal{I}}$ for all complex concepts C listed below):

- (1) $A \sqcup \neg B$
- (2) $\neg A \sqcap (A \sqcup B)$
- (3) $\exists R.B$
- (4) $\forall R.B$
- (5) $\exists P.(A \sqcup \neg A)$
- (6) $\forall R.(A \sqcap \neg A)$

(12 points)

- (1) $(A \sqcup \neg B)^{\mathcal{I}} = \underline{\underline{\{a\}}}$
- (2) $(\neg A \sqcap (A \sqcup B))^{\mathcal{I}} = \underline{\underline{\{b, c\}}}$
- (3) $(\exists R.B)^{\mathcal{I}} = \underline{\underline{\{a, b\}}}$
- (4) $(\forall R.B)^{\mathcal{I}} = \underline{\underline{\{a, b, c\}}} = \underline{\underline{\{a, b, c\}}}$
- (5) $(\exists P.(A \sqcup \neg A))^{\mathcal{I}} = \underline{\underline{\{a, b\}}}$
- (6) $(\forall R.(A \sqcap \neg A))^{\mathcal{I}} = \underline{\underline{\{c\}}}$





- 5.) By defining a suitable interpretation, show that the concept $A \sqcap (\exists R.B) \sqcap (\forall R.C)$ is satisfiable. (12 points)
Here A, B, C are concept names and R is a role name.

Inclusive Measuring:

Elements that:

- are in A
- all R -Neighbours are in C
- There exists an R -Neighbour in B

Suitable interpretation

$$\mathcal{I} =$$

$$\Delta^{\mathcal{I}} = \{a, b, c\}$$

$$A^{\mathcal{I}} = \{a\} \text{ for the concept name } A$$

$$B^{\mathcal{I}} = \{b\} \text{ for the concept name } B$$

$$C^{\mathcal{I}} = \{b, c\} \text{ for the concept name } C$$

$$R^{\mathcal{I}} = \{(a, b), (a, c)\} \text{ for the role name } R$$

$$(A \sqcap (\exists R.B) \sqcap (\forall R.C))^{\mathcal{I}}$$

$$\{a\} \cap \{a\} \cap \{a, b, c\}$$

$$\{a\} \Rightarrow \text{concept is satisfiable}$$