Einführung in Wissensbasierte Systeme
WS 2020/21, 3.0 VU, 184.737

Exercise Sheet 3 – Answer-Set Programming and Probabilistic Reasoning

Exercise 3.1

Consider the following answer-set programs:

- $\mathcal{P}_1 := \{P(a) \leftarrow \neg B(a). \quad \neg B(a).\}$.
- $\mathcal{P}_2 := \{P(b). \quad B(a).\}$.
- $\mathcal{P}_3 := \{B(a). \quad \neg P(X) \vee Z(X) \leftarrow B(X), \mathit{not}\ P(X).\}$.
- $\mathcal{P}_4 := \{B(a). \quad \neg F(a) \leftarrow B(a). \quad P(a) \leftarrow F(a).\}$.
- $\mathcal{P}_5 := \{B(a). \quad P(X) \vee F(X) \leftarrow B(X).\}$.
- $\mathcal{P}_6 := \{F(a). \quad F(X) \leftarrow B(X), \mathit{not}\ P(X). \quad P(X) \leftarrow B(X), \mathit{not}\ F(X).\}$.

Classify them based on their syntax. That is, decide whether they are a ground, Horn, normal, non-disjunctive, basic, or merely a collection of facts. Moreover, use the translation presented in the lecture to transform the given answer-set programs into their corresponding default theories and determine the cases where there is a one-to-one correspondence between the answer sets of the original program and the extensions of its respective translation.

Exercise 3.1

Solution.

- $\mathcal{P}_1 := \{P(a) \leftarrow \neg B(a). \quad \neg B(a).\}.$

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- $\mathcal{P}_1 := \{P(a) \leftarrow \neg B(a). \quad \neg B(a).\}$.
- $\mathcal{P}_1$ has two rules, $P(a) \leftarrow \neg B(a).$ and $\neg B(a).$.

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- The rule $\neg B(a).$ is a fact (shorthand for a rule with no body).

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- The negation $\neg$ in the literal $\neg B(a)$ is strong negation.

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- The atoms in $\mathcal{P}_1$ are $P(a)$ and $B(a)$. P and B are predicates and a is a ground term.

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- There are no variables in $\mathcal{P}_1$.
- There is no default negation in $\mathcal{P}_1$.

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A rule is:

- ground if: has no variables.

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- non-disjunctive: if no disjunction in head.

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	ground	Horn	normal	non-disjunctive	basic	facts
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A rule is:

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- non-disjunctive: if no disjunction in head.
- normal if: non-disjunctive and no strong negation.

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	ground	Horn	normal	non-disjunctive	basic	facts
$\mathcal{P}_1$	yes		no	yes	yes	

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- ground if: has no variables.
- non-disjunctive: if no disjunction in head.
- normal if: non-disjunctive and no strong negation.
- basic: if no default negation and no non-empty head.

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	ground	Horn	normal	non-disjunctive	basic	facts
$\mathcal{P}_1$	yes	no	no	yes	yes	

A rule is:

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- basic: if no default negation and no non-empty head.
- Horn if: normal and basic.

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	ground	Horn	normal	non-disjunctive	basic	facts
$\mathcal{P}_1$	yes	no	no	yes	yes	no

A rule is:

- ground if: has no variables.
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- $\mathcal{P}_6 := \{F(a). \quad F(X) \leftarrow B(X), \mathit{not}\ P(X). \quad P(X) \leftarrow B(X), \mathit{not}\ F(X).\}$.

	ground	Horn	normal	non-disjunctive	basic	facts
$\mathcal{P}_1$	yes	no	no	yes	yes	no
$\mathcal{P}_2$	yes	yes	yes	yes	yes	yes
$\mathcal{P}_3$	no	no	no	no	no	no
$\mathcal{P}_4$	yes	no	no	yes	yes	no
$\mathcal{P}_5$	no	no	no	no	yes	no
$\mathcal{P}_6$	no	no	yes	yes	no	no

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- $\mathcal{P}_1 := \{P(a) \leftarrow \neg B(a). \quad \neg B(a).\}$.
- Schema for each rule: rule $a \leftarrow b_1, b_2, \text{not } c_1, \text{not } c_2.$ becomes default $\frac{b_1 \wedge b_2 : \neg c_1, \neg c_2}{a}$

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- Schema for each rule: rule $a \leftarrow b_1, b_2, \text{not } c_1, \text{not } c_2.$ becomes default $\frac{b_1 \wedge b_2 \,:\, \neg c_1, \neg c_2}{a}$
- $\delta(\mathcal{P}_1) = \left(\emptyset, \left\{\frac{\neg B(a) \,:\, \emptyset}{P(a)}, \frac{\top \,:\, \emptyset}{\neg B(a)}\right\}\right)$ with its only extension being $Cn(\{P(a), \neg B(a)\})$. which corresponds to the only answer set of $\mathcal{P}_1$, i.e. $\{P(a), \neg B(a)\}$.

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- $\mathcal{P}_2 := \{P(b). \quad B(a).\}$.
- Schema for each rule: rule $a \leftarrow b_1, b_2, \text{not } c_1, \text{not } c_2.$ becomes default $\frac{b_1 \wedge b_2 \,:\, \neg c_1, \neg c_2}{a}$
- $\delta(\mathcal{P}_2) = \left(\emptyset, \left\{\frac{\top : \emptyset}{P(b)}, \frac{\top : \emptyset}{B(a)}\right\}\right)$ with its only extension being $Cn(\{P(b), B(a)\})$. which corresponds to the only answer set of $\mathcal{P}_2$, i.e. $\{P(b), B(a)\}$.

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Moreover, use the translation presented in the lecture to transform the given answer-set programs into their corresponding default theories and determine the cases where there is a one-to-one correspondence between the answer sets of the original program and the extensions of its respective translation.

- $\mathcal{P}_3 := \{B(a). \quad \neg P(X) \vee Z(X) \leftarrow B(X), \mathit{not}\, P(X).\}.$

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Moreover, use the translation presented in the lecture to transform the given answer-set programs into their corresponding default theories and determine the cases where there is a one-to-one correspondence between the answer sets of the original program and the extensions of its respective translation.

- $\mathcal{P}_3 := \{B(a). \quad \neg P(X) \vee Z(X) \leftarrow B(X), \mathit{not}\ P(X).\}$.
- Schema for each rule: rule $a \leftarrow b_1, b_2, \text{not } c_1, \text{not } c_2.$ becomes default $\frac{b_1 \wedge b_2 \,:\, \neg c_1, \neg c_2}{a}$
- $\delta(\mathcal{P}_3) = \left(\emptyset, \left\{ \frac{\top \,:\, \emptyset}{B(a)}, \frac{B(x) \,:\, \neg P(x)}{\neg P(x) \vee Z(X)} \right\}\right)$

  with its only extension being $Cn(\{B(a), \neg P(a) \vee Z(a)\})$.

  By contrast the two answer sets of $\mathcal{P}_3$ are $\{B(a), Z(a)\}$ and $\{B(a), \neg P(a)\}$.

Exercise 3.1

- $\delta(\mathcal{P}_4) = \left(\emptyset, \left\{\frac{\top : \emptyset}{B(a)}, \frac{B(a) : \emptyset}{\neg F(a)}, \frac{F(a) : \emptyset}{P(a)}\right\}\right)$ with its only extension being $Cn(\{B(a), \neg F(a)\})$ which corresponds to the only answer set of $\mathcal{P}_4$, i.e. $\{\neg F(a), B(a)\}$.
- $\delta(\mathcal{P}_5) = \left(\emptyset, \left\{\frac{\top : \emptyset}{B(a)}, \frac{B(a) : \emptyset}{P(a) \vee F(a)}\right\}\right)$ with its only extension being $Cn(\{B(a), P(a) \vee F(a)\})$. By contrast the two answer sets of $\mathcal{P}_5$ are $\{B(a), F(a)\}$ and $\{B(a), P(a)\}$.
- $\delta(\mathcal{P}_6) = \left(\emptyset, \left\{\frac{\top : \emptyset}{B(a)}, \frac{B(x) : \neg P(x)}{F(x)}, \frac{B(x) : \neg F(x)}{P(x)}\right\}\right)$ with its extensions being $Cn(\{B(a), F(a)\})$ and $Cn(\{B(a), P(a)\})$ which corresponds to the only answer set of $\mathcal{P}_6$: $\{F(a)\}$.

Exercise 3.3

Consider the following disjunctive logic program $\mathcal{P}$ (a, b, c, and d are propositional atoms):

$$\mathcal{P} = \left\{ \begin{aligned} b \vee c &\leftarrow \mathit{not}\, d. \\ a \vee c &\leftarrow \mathit{not}\, c. \\ a &\leftarrow \mathit{not}\, c. \\ b &\leftarrow \mathit{not}\, c. \end{aligned} \right\}.$$

(i) Determine all answer sets of $\mathcal{P}$. For each proposed answer set S, argue formally that it is indeed an answer set (using the Gelfond-Lifschitz reduct $\mathcal{P}^S$).

(ii) Determine which rule, let it be called r, ought to be removed if one intends to reduce the set of answer sets by at least one. That is, determine some $r \in \mathcal{P}$ such that

$$AS(\mathcal{P} \setminus \{r\}) \subset AS(\mathcal{P})$$

holds, where $AS(\mathcal{Q})$ denotes the set of all answer sets of a given logic program $\mathcal{Q}$.

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$$\mathcal{P} = \left\{ \begin{array}{r} b \vee c \leftarrow \mathit{not}\ d. \\ a \vee c \leftarrow \mathit{not}\ c. \\ a \leftarrow \mathit{not}\ c. \\ b \leftarrow \mathit{not}\ c. \end{array} \right\}.$$

(i) ▶ Propositional atoms $\{a, b, c, d\}$ so any subset of these might be answer set.

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- Propositional atoms $\{a, b, c, d\}$ so any subset of these might be answer set.
- Observe: if c false, then a, b, and $a \vee c$ must hold.

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$$\mathcal{P} = \left\{ \begin{array}{r} b \vee c \leftarrow \mathit{not}\ d. \\ a \vee c \leftarrow \mathit{not}\ c. \\ a \leftarrow \mathit{not}\ c. \\ b \leftarrow \mathit{not}\ c. \end{array} \right\}.$$

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- Propositional atoms $\{a, b, c, d\}$ so any subset of these might be answer set.
- Observe: if c false, then a, b, and $a \vee c$ must hold.

$\Rightarrow$ We try $S_1 = \{a, b\}$ first.

Exercise 3.3

⇒ We try $S_1 = \{a, b\}$ first.

- Gelfond-Lifschitz reduct: take candidate set,
  1) delete all rules where negative body is in conflict with candidate
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- Candidate $S_1 = \{a, b\}$:

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- d not contained in S_1, first rule is kept.

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$$\text{Gelfond-Lifschitz reduct } \mathcal{P}^{S_1} = \left\{ \begin{array}{r} b \vee c \leftarrow . \\ a \vee c \leftarrow . \\ a \leftarrow . \\ b \leftarrow . \end{array} \right\} .$$

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- d not contained in S_1, first rule is kept.
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- Is S_1 a minimal model of $\mathcal{P}^{S_1} = \{b \vee c. \quad a \vee c. \quad a. \quad b.\}$?

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⇒ Yes!

- S_1 is an answer set.

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- Is S_1 a minimal model of $\mathcal{P}^{S_1} = \{b \lor c. \quad a \lor c. \quad a. \quad b.\}$?

⇒ Yes!

- S_1 is an answer set.
- No superset of S_1 is an answer-set.

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$$\mathcal{P} = \left\{ \begin{array}{r} b \vee c \leftarrow \mathit{not}\, d. \\ a \vee c \leftarrow \mathit{not}\, c. \\ a \leftarrow \mathit{not}\, c. \\ b \leftarrow \mathit{not}\, c. \end{array} \right\}.$$

(i)

- Other answer-sets?
- Observe: if c false, then a, b, and $a \vee c$ must hold.

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(i)
- Other answer-sets?
- Observe: if c false, then a, b, and $a \lor c$ must hold.
- Observe: case for c false done, now assume c is true.
  Intuitively, none of the rules with not c in the body can fire.
  So, only the first rule might fire with $b \lor c$ in the head.

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$\Rightarrow$ We try $S_2 = \{c\}$.

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  So, only the first rule might fire with $b \vee c$ in the head.

⇒ We try $S_2 = \{c\}$.

- The GL-reduct is $\mathcal{P}^{S_2} = \{b \vee c.\}$.

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$$\mathcal{P} = \left\{ \begin{array}{r} b \vee c \leftarrow \textit{not}\, d. \\ a \vee c \leftarrow \textit{not}\, c. \\ a \leftarrow \textit{not}\, c. \\ b \leftarrow \textit{not}\, c. \end{array} \right\}.$$

(i)

- Other answer-sets?
- Observe: if c false, then a, b, and $a \vee c$ must hold.
- Observe: case for c false done, now assume c is true.
  Intuitively, none of the rules with not c in the body can fire.
  So, only the first rule might fire with $b \vee c$ in the head.

⇒ We try $S_2 = \{c\}$.

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- No superset of $\{c\}$ can be an answer-set.

⇒ We found all answer-sets.

Exercise 3.3

$$\mathcal{P} = \left\{ \begin{aligned} b \vee c &\leftarrow not\ d. \\ a \vee c &\leftarrow not\ c. \\ a &\leftarrow not\ c. \\ b &\leftarrow not\ c. \end{aligned} \right\}.$$

(ii) Determine which rule, let it be called r, ought to be removed if one intends to reduce the set of answer sets by at least one. That is, determine some $r \in \mathcal{P}$ such that

$$AS(\mathcal{P} \setminus \{r\}) \subset AS(\mathcal{P})$$

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- Answer-sets: $S_1 = \{a, b\}$, $S_2 = \{c\}$.
- Remove $r = b \vee c \leftarrow \text{not}\ d$.
- Then $\mathcal{P} \setminus \{r\}$ has only rules with not c in body, this leads back to S_1.

Exercise 3.2

Find three non-disjunctive, fact-free, non-empty answer-set programs $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ such that

- $\mathcal{P}_1$ has no answer sets,
- $\mathcal{P}_2$ has only the empty set as answer set, and
- $\mathcal{P}_3$ has at least three answer sets, one of size 1, one of size 2, and one of size 3.

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- To confirm:
  - assume $S_1 := \{\}$ is an answer set, then $\mathcal{P}_1^{S_1} = \{P(a).\}$.
    Minimal model of $\mathcal{P}_1^{S_1}$ is $\{P(a)\} \neq S_1$.

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  - assume $S_2 = \{P(a)\}$ is an answer set, then
    GL-reduct $\mathcal{P}_2^{S_2} = \{P(a) \leftarrow P(a).\}$.
    Minimal model of $\mathcal{P}_2^{S_2} = \{\} \neq S_2$. No answer set!

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$$
\begin{aligned}
\mathcal{P}_3 := \{ & O(a) \leftarrow \mathit{not}\ P(a), \mathit{not}\ Q(a). \\
& P(a) \leftarrow \mathit{not}\ O(a), \mathit{not}\ Q(a). \\
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- $\mathcal{P}_3$ has the answer sets
  - $S_1 := \{O(a)\}$
  - $S_2 := \{P(a), A(a)\}$,
  - $S_3 := \{Q(a), A(a), B(a)\}$.

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  - $S_2 := \{P(a), A(a)\}$,
  - $\mathcal{P}_3^{S_2} = \{P(a).\ A(a) \leftarrow P(a).\ A(a) \leftarrow Q(a).\ B(a) \leftarrow Q(a).\}$ with minimal model S_2.
  - $S_3 := \{Q(a), A(a), B(a)\}$.
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Exercise 3.4

Let $\mathcal{P}$ be a Horn logic program and $M_1, M_2 \subseteq HB(\mathcal{P})$ classical models of $\mathcal{P}$. Prove that $M_1 \cap M_2$ is also a classical model of $\mathcal{P}$. What about $M_1 \cup M_2$?

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- Horn logic program: rules have no negation (neither strong nor weak), exactly one atom in head (no disjunction).
- Classical model: satisfies every rule of program.
- Rule (classically) satisfied if body false or head true.

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  - Therefore, r is satisfied in $M_1 \cap M_2$ in this case, too.

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  - From $a \in M_1$ and $a \in M_2$ follows $a \in M_1 \cap M_2$.
  - Therefore, r is satisfied in $M_1 \cap M_2$ in this case, too.
- In both cases r is satisfied in $M_1 \cap M_2$ and because r was picked arbitrarily from $\mathcal{P}$, it follows that $M_1 \cap M_2$ satisfies every $r \in \mathcal{P}$, that is, $M_1 \cap M_2$ is a classical model of $\mathcal{P}$.

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- We show: the union of two classical models may be no classical model.
- Consider program $P = \{a \leftarrow b, c\}$.
- Let $M_1 = \{b\}$ and $M_2 = \{c\}$.
- M_1 and M_2 are classical models of P.

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- We show: the union of two classical models may be no classical model.
- Consider program $P = \{a \leftarrow b, c\}$.
- Let $M_1 = \{b\}$ and $M_2 = \{c\}$.
- M_1 and M_2 are classical models of P.
- But $M_1 \cup M_2 \not\models P$ as $a \notin M_1 \cup M_2$.

Exercise 3.5

The *n* queens puzzle is defined as follows:

Given an $n \times n$ chessboard, place n queens on the board such that

(i) no queens share the same row;

(ii) no queens share the same column; and

(iii) no queens are on the same diagonal.

a) Define all required predicates, rules, and constraints to represent the problem as an answer-set program $\mathcal{P}$ such that
   - each row i of the board is denoted by a fact $row(i)$,
   - each column j is denoted by a fact $col(j)$,
   - $queen(i, j)$ denotes that a queen is placed on row i and column j, and
   - each answer set of $\mathcal{P}$ represents a valid solution of the n queens puzzle.

b) Find all solutions for a 4×4 chessboard.

c) How many solutions does a 2×2 chessboard have?

Exercise 3.5

a)

Create board:	$\texttt{row}(1..n).$
	$\texttt{col}(1..n).$
Guess:	$\texttt{queen}(I, J) \vee \neg\texttt{queen}(I, J) \leftarrow \texttt{row}(I), \texttt{col}(J).$
Check:	$\texttt{atLeastOne}(I) \leftarrow \texttt{queen}(I, J).$
At least 1 queen per row:	$\leftarrow \texttt{row}(I), \textit{not}\ \texttt{atLeastOne}(I).$
No two queens per row:	$\leftarrow \texttt{queen}(I, J_1), \texttt{queen}(I, J_2), J_1 \neq J_2.$
No two queens per column:	$\leftarrow \texttt{queen}(I_1, J), \texttt{queen}(I_2, J), I_1 \neq I_2.$
No two queens per diagonal:	$\leftarrow \texttt{queen}(I_1, J_1), \texttt{queen}(I_2, J_2), (I_1, J_1) \neq (I_2, J_2), I_1 + J_1 = I_2 + J_2.$
	$\leftarrow \texttt{queen}(I_1, J_1), \texttt{queen}(I_2, J_2), (I_1, J_1) \neq (I_2, J_2), I_1 - J_1 = I_2 - J_2.$

Exercise 3.5

a)

Create board:	$\texttt{row}(1..n).$
	$\texttt{col}(1..n).$
Guess:	$\texttt{queen}(I, J) \lor \neg\texttt{queen}(I, J) \leftarrow \texttt{row}(I), \texttt{col}(J).$
Check:	$\texttt{atLeastOne}(I) \leftarrow \texttt{queen}(I, J).$
At least 1 queen per row:	$\leftarrow \texttt{row}(I), \textit{not}\ \texttt{atLeastOne}(I).$
No two queens per row:	$\leftarrow \texttt{queen}(I, J_1), \texttt{queen}(I, J_2), J_1 \neq J_2.$
No two queens per column:	$\leftarrow \texttt{queen}(I_1, J), \texttt{queen}(I_2, J), I_1 \neq I_2.$
No two queens per diagonal:	$\leftarrow \texttt{queen}(I_1, J_1), \texttt{queen}(I_2, J_2), (I_1, J_1) \neq (I_2, J_2), I_1 + J_1 = I_2 + J_2.$
	$\leftarrow \texttt{queen}(I_1, J_1), \texttt{queen}(I_2, J_2), (I_1, J_1) \neq (I_2, J_2), I_1 - J_1 = I_2 - J_2.$

b) Solutions for a 4×4 chessboard:
$\{\texttt{queen}(3, 1)\texttt{queen}(1, 2)\texttt{queen}(4, 3)\texttt{queen}(2, 4)\}$,
$\{\texttt{queen}(2, 1)\texttt{queen}(4, 2)\texttt{queen}(1, 3)\texttt{queen}(3, 4)\}$

Exercise 3.5

a)

Create board:	$\texttt{row}(1..n).$
	$\texttt{col}(1..n).$
Guess:	$\texttt{queen}(I,J) \vee \neg\texttt{queen}(I,J) \leftarrow \texttt{row}(I), \texttt{col}(J).$
Check:	$\texttt{atLeastOne}(I) \leftarrow \texttt{queen}(I,J).$
At least 1 queen per row:	$\leftarrow \texttt{row}(I), \textit{not}\ \texttt{atLeastOne}(I).$
No two queens per row:	$\leftarrow \texttt{queen}(I,J_1), \texttt{queen}(I,J_2), J_1 \neq J_2.$
No two queens per column:	$\leftarrow \texttt{queen}(I_1,J), \texttt{queen}(I_2,J), I_1 \neq I_2.$
No two queens per diagonal:	$\leftarrow \texttt{queen}(I_1,J_1), \texttt{queen}(I_2,J_2), (I_1,J_1) \neq (I_2,J_2), I_1 + J_1 = I_2 + J_2.$
	$\leftarrow \texttt{queen}(I_1,J_1), \texttt{queen}(I_2,J_2), (I_1,J_1) \neq (I_2,J_2), I_1 - J_1 = I_2 - J_2.$

b) Solutions for a 4×4 chessboard:
$\{\texttt{queen}(3,1)\texttt{queen}(1,2)\texttt{queen}(4,3)\texttt{queen}(2,4)\}$,
$\{\texttt{queen}(2,1)\texttt{queen}(4,2)\texttt{queen}(1,3)\texttt{queen}(3,4)\}$

c) For a 2×2 chessboard there are 0 solutions, since both queens are at same row, colum, or diagonal.

Exercise 3.6

A company has three printers. On average, the first printer, P_1, produces 4500 sheets, the second printer, P_2, 400, and the third, P_3, 100 sheets a month. Moreover, usually only 4490 prints from P_1, 395 prints from P_2, and 50 prints from P_3 are printed without paper jam. If you choose a random print, what is the probability that it was printed without producing a paper jam? Moreover, what is the probability that, given a print that produced a paper jam, it comes from P_2?

Use the random variable J to express that a sheet produced a paper jam, and X_{P_i}, for $i \in \{1, 2, 3\}$, expressing that a printed paper is from printer P_i.

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- Overall number of sheets: $4500 + 400 + 100 = 5000$.

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- Overall number of sheets: $4500 + 400 + 100 = 5000$.
- $P(X_{P_1}) := \frac{4500}{5000} = 0.9$;
- $P(X_{P_2}) := \frac{400}{5000} = 0.08$;
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- From marginalisation we know: $P(J) = P(J, X_{P_1}) + P(J, X_{P_2}) + P(J, X_{P_3})$

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$\Rightarrow$ $P(J, X_{P_i}) = P(J|X_{P_i})P(X_{P_i})$ for $i \in \{1, 2, 3\}$.

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$$P(J) = P(J|X_{P_1})P(X_{P_1}) + P(J|X_{P_2})P(X_{P_2}) + P(J|X_{P_3})P(X_{P_3})$$

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- How to get $P(J|X_{P_i})$ for $i \in \{1, 2, 3\}$?
- For printer P_1 there are 4490 pages printed without jam, and 4500 printed total, so 10 out of 4500 are jammed.
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- $P(J|X_{P_2}) := \frac{400-395}{400} = 0.0125$;

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$$\begin{aligned} P(J) &= P(J|X_{P_1})P(X_{P_1}) + P(J|X_{P_2})P(X_{P_2}) + P(J|X_{P_3})P(X_{P_3}) \\ &= 0.00\dot{2} \cdot 0.9 + 0.0125 \cdot 0.08 + 0.5 \cdot 0.02 = 0.013 = 1.3\,\% \end{aligned}$$

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$$
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P(J) &= P(J|X_{P_1})P(X_{P_1}) + P(J|X_{P_2})P(X_{P_2}) + P(J|X_{P_3})P(X_{P_3}) \\
&= 0.00\dot{2} \cdot 0.9 + 0.0125 \cdot 0.08 + 0.5 \cdot 0.02 = 0.013 = 1.3\,\%
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- Probability of not jamming: $1 - P(J) = 0.987 = 98.7\,\%$

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- Need to compute $P(X_{P_2}|J)$.
- Bayes' rule states:

$$P(X_{P_2}|J) = \frac{P(J|X_{P_2})P(X_{P_2})}{P(J)}$$

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- We have those numbers already.
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-

$$P(X_{P_2}|J) = \frac{P(J|X_{P_2})P(X_{P_2})}{P(J)} = \frac{0.0125 \cdot 0.08}{0.013} = 0.076923 \approx 0.0769 = 7.69\,\%$$

Exercise 3.7

Prove that conditional independence in a Bayesian network is symmetric, i.e., show that, for each node X, Y, and Z of the network,
if $P(X|Y,Z) = P(X|Z)$, then $P(Y|X,Z) = P(Y|Z)$.

Hint: Use a generalised version of the Bayes' rule, $P(Y|X,Z) = P(X,Z|Y)P(Y)/P(X,Z)$, as well as the product rule $P(X,Y) = P(X|Y)P(Y)$.

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- Use Bayes's rule
  $P(Y|X,Z) = \frac{P(X,Z|Y)P(Y)}{P(X,Z)}$

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- Use Bayes's rule
  $P(Y|X,Z) = \frac{P(X,Z|Y)P(Y)}{P(X,Z)}$
- Use Product rule
  $\frac{P(X,Z|Y)P(Y)}{P(X,Z)} = \frac{P(X|Z,Y)P(Z|Y)P(Y)}{P(X|Z)P(Z)}$

Exercise 3.7

Prove that conditional independence in a Bayesian network is symmetric, i.e., show that, for each node X, Y, and Z of the network,
if $P(X|Y,Z) = P(X|Z)$, then $P(Y|X,Z) = P(Y|Z)$.

Hint: Use a generalised version of the Bayes' rule, $P(Y|X,Z) = P(X,Z|Y)P(Y)/P(X,Z)$, as well as the product rule $P(X,Y) = P(X|Y)P(Y)$.

- Use Bayes's rule
  $P(Y|X,Z) = \frac{P(X,Z|Y)P(Y)}{P(X,Z)}$
- Use Product rule
  $\frac{P(X,Z|Y)P(Y)}{P(X,Z)} = \frac{P(X|Z,Y)P(Z|Y)P(Y)}{P(X|Z)P(Z)}$
- Use assumption
  $\frac{P(X|Z,Y)P(Z|Y)P(Y)}{P(X|Z)P(Z)} = \frac{P(X|Z)P(Z|Y)P(Y)}{P(X|Z)P(Z)} = \frac{P(Z|Y)P(Y)}{P(Z)}$

Exercise 3.7

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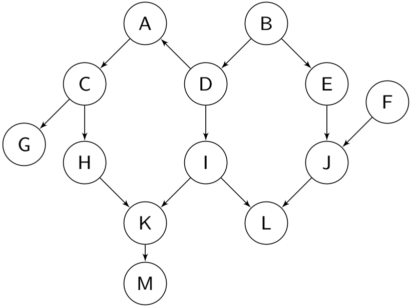
- Use Bayes's rule
  $P(Y|X,Z) = \frac{P(X,Z|Y)P(Y)}{P(X,Z)}$
- Use Product rule
  $\frac{P(X,Z|Y)P(Y)}{P(X,Z)} = \frac{P(X|Z,Y)P(Z|Y)P(Y)}{P(X|Z)P(Z)}$
- Use assumption
  $\frac{P(X|Z,Y)P(Z|Y)P(Y)}{P(X|Z)P(Z)} = \frac{P(X|Z)P(Z|Y)P(Y)}{P(X|Z)P(Z)} = \frac{P(Z|Y)P(Y)}{P(Z)}$
- Use Bayes's rule again
  $\frac{P(Z|Y)P(Y)}{P(Z)} = P(Y|Z)$

Exercise 3.8

Consider the following graph of a Bayesian network:



Answer the following questions and give an explanation of your answer.

(i) Is A conditionally independent of J?

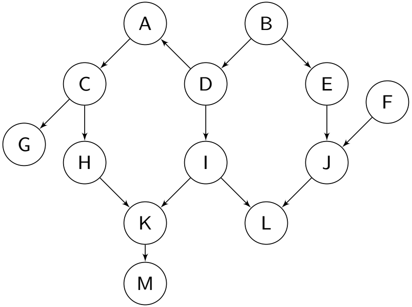
(ii) Is A conditionally independent of J given evidence M?

(iii) Which subset-minimal evidence would be sufficient such that F is conditionally independent of M given this evidence?

(iv) Which subset-minimal evidence would be sufficient such that A is conditionally independent of J given this evidence?

Exercise 3.8

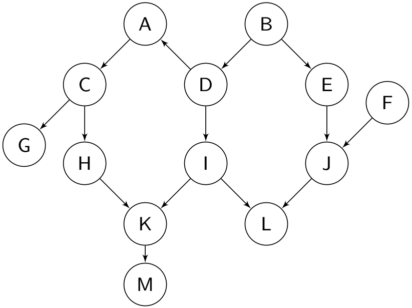
Consider the following graph of a Bayesian network:



X and Y are conditionally independent given evidence $\boldsymbol{E}$
$\Leftrightarrow$ each undirected path between X and Y is blocked by $\boldsymbol{E}$.

Exercise 3.8

Consider the following graph of a Bayesian network:

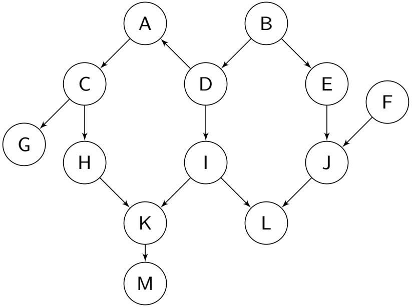


X and Y are conditionally independent given evidence $\boldsymbol{E}$
$\Leftrightarrow$ each undirected path between X and Y is blocked by $\boldsymbol{E}$.

Path P between X and Y is blocked by $\boldsymbol{E}$ if there is $Z \in P$ with

Exercise 3.8

Consider the following graph of a Bayesian network:



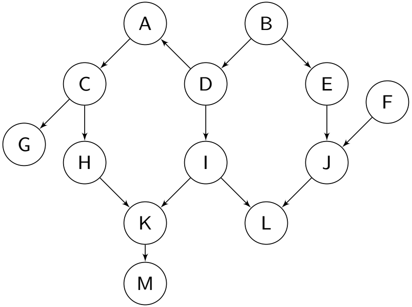
X and Y are conditionally independent given evidence $\boldsymbol{E}$
$\Leftrightarrow$ each undirected path between X and Y is blocked by $\boldsymbol{E}$.

Path P between X and Y is blocked by $\boldsymbol{E}$ if there is $Z \in P$ with

- $Z \in \boldsymbol{E}$ has incoming and outgoing edge,

Exercise 3.8

Consider the following graph of a Bayesian network:



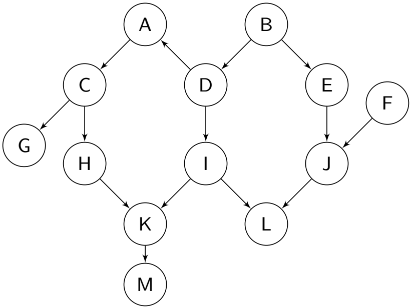
X and Y are conditionally independent given evidence $\boldsymbol{E}$
$\Leftrightarrow$ each undirected path between X and Y is blocked by $\boldsymbol{E}$.

Path P between X and Y is blocked by $\boldsymbol{E}$ if there is $Z \in P$ with

- $Z \in \boldsymbol{E}$ has incoming and outgoing edge,
- $Z \in \boldsymbol{E}$ has two outgoing edges, or

Exercise 3.8

Consider the following graph of a Bayesian network:



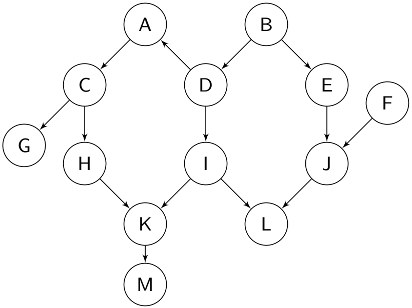
X and Y are conditionally independent given evidence $\boldsymbol{E}$
$\Leftrightarrow$ each undirected path between X and Y is blocked by $\boldsymbol{E}$.

Path P between X and Y is blocked by $\boldsymbol{E}$ if there is $Z \in P$ with

- $Z \in \boldsymbol{E}$ has incoming and outgoing edge,
- $Z \in \boldsymbol{E}$ has two outgoing edges, or
- neither Z nor its descendants in $\boldsymbol{E}$ and both edges are incoming.

Exercise 3.8

Consider the following graph of a Bayesian network:



Answer the following questions and give an explanation of your answer.

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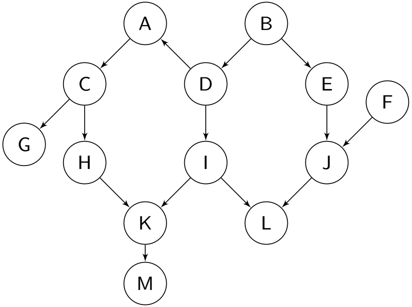
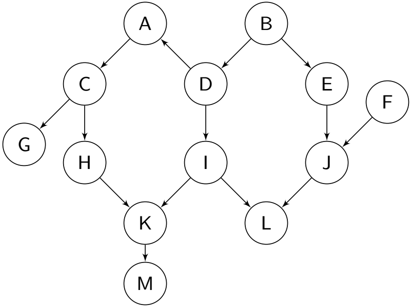


Answer the following questions and give an explanation of your answer.

(i) Is A conditionally independent of J?

Exercise 3.8

Consider the following graph of a Bayesian network:



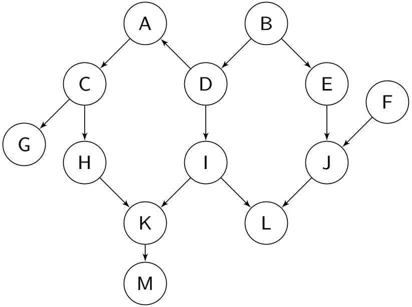
Answer the following questions and give an explanation of your answer.

(i) Is A conditionally independent of J?

$\Rightarrow$ No. Since Path $A \rightarrow D \rightarrow B \rightarrow E \rightarrow J$ in the undirected path of the network is not blocked, A is not conditionally independend of J.

Exercise 3.8

Consider the following graph of a Bayesian network:

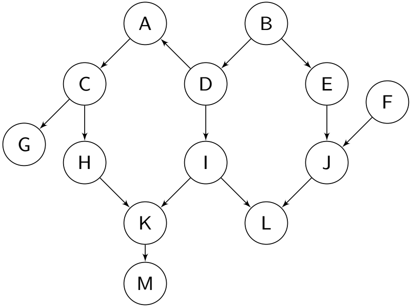


Answer the following questions and give an explanation of your answer.

(ii) Is A conditionally independent of J given evidence M?

Exercise 3.8

Consider the following graph of a Bayesian network:



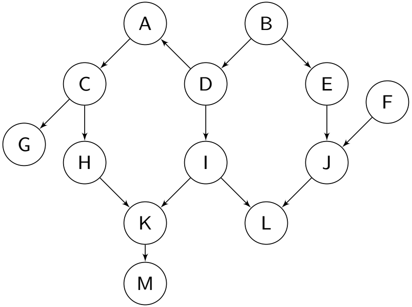
Answer the following questions and give an explanation of your answer.

(ii) Is A conditionally independent of J given evidence M?

$\Rightarrow$ Still No. Since Path A, D, B, E, J in the undirected path of the network is not blocked, A and J are not blocked.

Exercise 3.8

Consider the following graph of a Bayesian network:

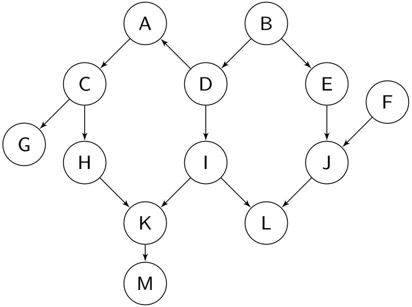


Answer the following questions and give an explanation of your answer.

(iii) Which subset-minimal evidence would be sufficient such that F is conditionally independent of M given this evidence?

Exercise 3.8

Consider the following graph of a Bayesian network:



Answer the following questions and give an explanation of your answer.

(iii) Which subset-minimal evidence would be sufficient such that F is conditionally independent of M given this evidence?

$\Rightarrow$ The empty set, because all paths are blocked.

Exercise 3.8

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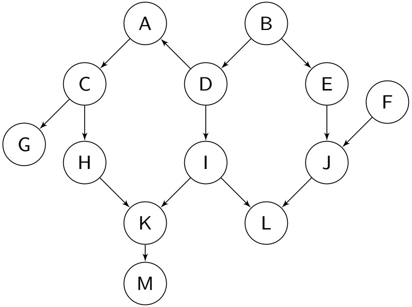
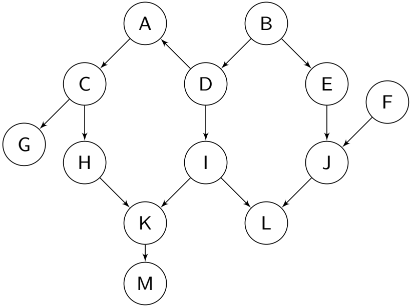


Answer the following questions and give an explanation of your answer.

(iv) Which subset-minimal evidence would be sufficient such that A is conditionally independent of J given this evidence?

Exercise 3.8

Consider the following graph of a Bayesian network:



Answer the following questions and give an explanation of your answer.

(iv) Which subset-minimal evidence would be sufficient such that A is conditionally independent of J given this evidence?

$\Rightarrow$ Since all non-blocked paths from A to J use D, B and E, it is sufficient to give evidence either to B, D or E.