**Question 14**

**Formal model**
Firms choose quantities sequential (leader and follower). The payoff of the firm $i$ is given by

$$
\pi_i(q_i, q_j) = q_i[P(Q) - c],
$$

(with $p = \max\{a - Q; 0\}$ with $Q = q_1 + q_2, a > c$) to solve for the backwards-induction outcome of this game, we first compute firm 2's reaction to a $q_1$ by firm 1.

$$
\max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2[a - q_1 - q_2 - c],
$$

$R_2(q_1)$ maximizes

$$
\frac{a - q_1 - c}{2},
$$

with $q_1 < a - c$

Firm 1 can anticipate the quantity choice $R_2(q_1)$ of firm 2, therefore her choice of $q_1$ will be made with the Reaction of the choice:

$$
\max_{q_1 \geq 0} \pi_1(q_1, R_2(q_1)) = \max_{q_1 \geq 0} q_1[a - q_1 - R_2(q_1) - c]
$$

$$
= \max_{q_1 \geq 0} \frac{a - q_1 - c}{2},
$$

which yields

$$
q_1^* = \frac{a - c}{2} \quad \text{and} \quad R_2(q_1^*) = \frac{a - c}{4}.
$$

The aggregate quantity $(3(a - c)/4)$ is higher than the one in Bertrand Model $(2(a - c)/3) \Rightarrow$ prices are lower.

**Economic interpretation**
We have this kind of model if there is a big, already existing firm and a second firm enters the market. The small firm will be the follower because it will first observe what the big firm do and than act. Doing that and (more important) the bigger firm knowing that the new firm will act that way, will put the second firm in a bader position. In that case knowing more and the other one knowing that you know more makes your position worse.

**First mover advantage**
Higher profit, can benefit from the anticipations of the followers firm reaction.

**Second mover advantage**
The second mover can have an advantage because of lower costs. A second mover can learn from the experiences oft he first mover firm and may not face such high research and development costs if they are able to create their own similar product using existing technology. A second mover can also save marketing costs.

---

**Question 15**

Suppose that persons has to choose among three or more outcomes and let $O$ be the set of outcomes that a group is evaluating. Let $R_i$ denote person $i$'s preferences over $O$, and let $R$ be a relation that tells us the group’s preference between pairs of outcomes in $O$, as some institution that operates on individual preferences identifies. The **social preference function** $G(R_1, \ldots, R_n; O) = R$ defines the group’s preference relation, $R$, between every pair of outcomes in $O$.

**A1:** The social preference function is complete and transitive.

**A2:** Every individual preference relation that is complete and transitive is admissible.

**A3:** If $x P y$ for all persons $i$ in a group, then $x P y$ in the social preference order.

**A4:** If $G(R, \{x, y\}) = x P y$ and if $R'$ is another preference profile such that each person’s preference between $x$ and $y$ is the same in $R'$ as in $R$, then $G(R', \{x, y\}) = x P y$. 
A5: No person $i$ is decisive for every pair of outcomes in $O$. (a person $i$ is decisive for $x$ over $y$ if $G(R, \{x, y\}) = x P y$ for every $R$ in which $i$ prefer $x$ to $y$ and the others prefer $y$ to $x$).

**Arrow’s Impossibility Theorem**

- If $O$ consists of three or more outcomes, the only institutions, $G$, that satisfy condition A1-A4 are institutions that violate condition A5, nondictatorship.

- Equivalent restatement: Nondictatorial institutions that satisfy A2 through A4 carry no guarantee that they yield transitive social preference orders.

**Why is it relevant for political economy?**

Although we assume that the individual preferences are transitive, there need not exist a transitive social preference. Terminology such as „the public interest” and „community goals” becomes immediately suspect because we cannot speak of a group „maximizing its utility or welfare” when the preferences of that group are intransitive.

**Question 16**

**Agenda paradox**

A chairman determines the agenda which is the specific order of voting on alternatives. We have 3 outcomes $(x,y,z)$ and the persons $(1,2,3)$ defining their social preferences:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>x</td>
<td>z</td>
<td>y</td>
</tr>
<tr>
<td>2nd</td>
<td>y</td>
<td>x</td>
<td>z</td>
</tr>
<tr>
<td>3rd</td>
<td>z</td>
<td>y</td>
<td>x</td>
</tr>
</tbody>
</table>

If the chairman set $x$ against $y$ and then the winner against $z$, $x$ would beat $y$ and $z$ would beat $x$, therefore $z>x>y$. But the chairman could also first pair $z$ and $x$ with the winner put to a vote against $y$: $z$ would beat $x$ but lose against $y \Rightarrow y>z>x$. Finally, the chairman could first place $z$ and $y$ against each other and then place that vote’s winner, $y$, against $x$. The social preference would be $x>y>z$.

Thus, a committee chairman can, by selecting an appropriate agenda, control the final outcome. It is therefore always important to consider who sets the agenda and why, or if possible, be the one who sets the agenda.

**Winner-turns-loser paradox**

Instead of voting repeatedly over pairs of alternatives, each person orders his preferences among the $m$ alternatives. The first ranked alternative receives $m$ points, the second $m-1$ and so on. Here is a seven-person preference profile:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 points</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>3 points</td>
<td>b</td>
<td>c</td>
<td>x</td>
<td>b</td>
<td>c</td>
<td>x</td>
<td>b</td>
</tr>
<tr>
<td>2 points</td>
<td>c</td>
<td>x</td>
<td>a</td>
<td>c</td>
<td>x</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>1 point</td>
<td>x</td>
<td>a</td>
<td>b</td>
<td>x</td>
<td>a</td>
<td>b</td>
<td>x</td>
</tr>
</tbody>
</table>

$x$ has 13, $a$ 18, $b$ 19 and $c$ 20 points. Now suppose these were political candidates and after announcing these results, $x$ dies. A recalculation awards alternative $a$ 15, $b$ 14 and $c$ 13 points. The original winner, $c$, is now the loser and $a$ is the winner.

This shows that an seemingly unimportant alternative $(x)$ can have a tremendous effect on the end result and adding such alternatives can make losers winners.
**Question 17**

**Definition 2 (Repeated game)**

A repeated game results from finite or indefinite repetition of a game G, called a ‘constituent’ game or ‘stage’ game. In the repeated game G (T), a strategy mentions player’s decisions at the first stage t = 1 and all the decisions that she (or he) will choose at the subsequent stages (2 ≤ t ≤ T) taking into account the history of the game. At each stage, every player knows the past decisions taken by all her (or his) rivals and all the payoffs of the game for the past stages. Once each player has chosen her (or his) strategy, the game is over.

**Example**

Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Cooperate (C)</th>
<th>Defect (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate (C)</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>Defect (D)</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

The repetition oft he game gives the opportunity to cooperate and reach a higher pay-off for both players ( (2,2) instead of the single game NE (1,1) ). In case of a non-cooperative game, any oft he countries could deviate at any time, but if we assume that the game is going to be played up to infinity, it will be better for both players to cooperate and get long-term gains rather than to deviate and get a short-term gain.

**Question 18**

Axelrod invited people to submit computer programs that specified a strategy for playing a prisoner’s dilemma repeated a finite but large (200) number of times. Winner oft he „league tournament“ was TFT (tit-for-tat, each player plays what the opponent have played in the last round).

Second round of experiment: he gave the people the opportunity to design programs that would beat TFT. Again TFT won.

There was also an „evolution tournament“. First the group using nice strategies got lower and the one using bad strategies grewed. But than, the bad strategies began to fight the bad strategies and again the TFT won also this tournament.

**Implications**

The strategy has to follow the rules

- Don’t be envious
- Don’t be the first to defect
- Reciprocate both cooperation and defection
- Don’t be too clever (the strategy should be understandable to the opponent)
**Question 19**

2 possibilities to operationalize payoffs in infinitely repeated games

1. limit of average payoffs: \( \hat{U}_i = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} u_i(t) \)

   This criterion treats the periods differently since the value of a given payoff decreases in the time. Differences in any finite number of periods do not matter.

2. discounted sum of the payoffs: \( \sum_{t=1}^{\infty} \delta^{t-1} u_i(t) = u_i(1) + \delta u_i(2) + \delta^2 u_i(3) + ... \)

   \( 0 \leq \delta = 1/(1+a) \leq 1, \) where a features the players' preference for the present.

   This criterion treats all periods symmetrically. A change in the payoff in a single period can matter.

**Question 20**

**Folk theorem**

Correctly formulate the theorem. Let \( a_i \) be the lowest payoff attainable by player i facing a coalition of all her (or his) rivals - i.e. her (or his) security strategy. Any payoff vector \( u \) such that: \( u_i > a_i \) for all i, is called an 'individually rational' (or sometimes 'enforceable') payoff vector. The folk theorem can then be stated as follows.

**Theorem 5 (Friedman, 1971)**

In repeated game \( G_0(\infty) \), if \( u \) represents a vector of individually rational payoffs in the constituent game, the indefinite repetition of this payoff vector can be generated by some strategies forming a NE of the repeated game (provided that players are not too impatient, i.e. \( \delta \) is high enough).

**significance**

cooperation (held together by a threat) can make all parties better of in indefinitely repeated games

**sources for cooperation in repeated PD games**

<table>
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<tr>
<th>Player 1</th>
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</tr>
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</tr>
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</table>

the strategy of the players could be:

- play C as long as the rival player plays C
- if the rival plays D, play D forever

= the one who deviates obtaines at his deviation a gain of 1 (3-2) payoff and thereafter in each stage a loss of 1 (2-1) payoff. Hence the threat is good enough to maintain cooperation throughout the repeated game
possible strategy of player 1:

- play C and D alternatively as long as the rival choses C
- if the rival plays D, play D forever

⇒ from a collective point this equilibrium is not as good as the outcome (C, C), but it is also a NE

strategy for both players:

- TFT „tit-for-tat“: begin by playing D, then copy each move of the rival at the previous stage

Question 21

Possible contribution of biology to evolutionary game theory (EGT)

Experiments in biology had lead researchers to find stable constellations of different behaviour within and across species. Biologists brought a lot of new ideas on how to extend the standard mathematical treatment. Many phenomena observed by biologists could not be described by the classical game theory and new tools needed to be developed that are now more and more frequently applied in other fields, e.g. economics. The evolution of animals in biology is might be compared to learning processes of people (e.g. managers) in business environment. The successful application of game theory to evolution has brought further insights to human behavior. Whereas game theory traditionally assumes rational actors, in the real world this does not always describe human behavior. EGT has predicted behaviors in animals where strong assumptions of rationality cannot be made. In economics we do not observe the human gene pool, but the behavior of agents, because an individual agent can change his behavior as he accumulates experience.

Question 22

Replicator dynamics – basic idea

The fraction of the population playing a particular strategy will increase if it performs better in terms of the utility function than the population average. If a type gets less than the average payoff, then its percentage in the population will decrease.

Evolution of the groups: only the fittest will survive

Formal description

$N_t$ is the number of population

$n_t^x$ plays $x$, $n_t^x'$ plays $x'$

$s_t(x) = \frac{n_t^x}{n_t^x + n_t^x'} = \frac{n_t^x}{N_t}$

$u_t(x) = \lambda_t(x) u_t(x, x) + s_t(x') u_t(x, x')$

$ar{u_t} = \bar{u_t}(x) u_t(x) + s_t(x') u_t(x')$

dynamics: $\delta(x) = \delta(x) \cdot (u(x) - \bar{u}) = F(a)$ ← evolution process of populations

Question 23

Hawk-Dove Game

The members of a very large population are fighting for the use of a territory. $V$ is the value of that resource for any member of this population. Individuals meet at random and can play one or the other of the two following strategies: behave like a Hawk (aggressively), or behave like a
Dove (nicely). The Hawk is always ready to fight, the Dove will always avoid fighting. Every fight has a cost, noted C.

When a Hawk meets a Dove, the latter refuses the fight and leaves the place. Of course, the Hawk gets all the benefit. When two Doves meet, the payoff is shared pacifically, $V/2$ for each. Finally, when two Hawks meet each other, they fight until they get half of the value of the resource minus the cost of fighting, that is $(V-C)/2$.

The fraction $s$ of the population playing the Hawk strategy will increase if it performs better than the population average. If one assumes this Malthusian dynamics, one gets the following equation of replication:

$$F(s) = s(1-s)[s(V-C)/2 + (1-s)V/2]$$

The equation has three roots: $s=0$, $s=V/C$ and $s=1$. The phase diagram, where instead of $s=1/4$ we would write $s=V/C$.

An evolutionary equilibrium (EE) is any asymptotically stable fixed point of the dynamical process of evolution.

Hence we see immediately, that the mixed strategy $(V/C, (V-C)/C)$ is an EE and the Hawk-Dove game is evolutionary stable.

**Question 24**

Suppose that the population is originally playing a strategy $x$ and that a small percentage of „mutants“, say $\epsilon$, play another strategy $x'$. The basic idea of ESS ist to require that the equilibrium can resist mutant invasion. Once a strategy is fixed in a population, natural selection alone is sufficient to prevent „mutant“ strategies from invading successfully. Hence, the strategy is resistant to „mutations“.
**Question 25**

Usually economists start with an assumption of reality, which is then put to the test. Elinor started with an actual reality instead. She gathered information through field studies and then analyzed this material. Her work was about *governance of common resources*.

Problem: Some shepherds share a common pasture. There are no restrictions on how many sheep each shepherd may place there. However, with too many sheeps there is a great risk that the meadow will be exercised too hard and destroyed. At the same time, each shepherd had to think of their livelihood. So, reducing the number of sheeps may be good for the meadow itself, but will provide less income. So what will happen to the pasture?

The dominant view among economists had long been that pooling natural resources will result in them being overused. This is the simple reason that each individual user will do what is best for himself, rather than what is best for everyone.

Elinor’s research shows a different view. She studied how people in small communities around the world managed common resources. Her research showed that when natural resources are pooled and have shared ownership, the rules for managing those resources evolve over time in a way both economically and ecologically sustainable. Elinor therefore believed that common pooled ownership of natural resources is better than privatization and government involvement. Required however that decision-making is transparent and democratic.

**Main challenge of collective action**

Although a certain result might be beneficial for the group as a whole, as the group does not have a *common mind*, each member of the group might be seeking to satisfy their own interest. Therefore, *some form of supervision* might need to be established that will make decisions on behalf of the group.

**Question 26 – not answered!**

**Question 27**

*Axelrod’s norm game*

A *norm* exists in a given social setting to the extent that individuals usually act in a certain way and are often punished when seen not to be acting in this way.

**AXELROD’S MODELS: The Norms model**
Agents have to decide whether to cooperate or defect. A defecting agent gets a Temptation payoff (T=3) and inflicts each of the other agents a Hurt payoff (H=-1). If, on the other hand, the agent cooperates, no one’s payoff is altered.

The opportunity to defect given to an agent comes with a known chance of being seen by each of the other agents, called S. This probability of being observed is drawn from a uniform distribution between 0 and 1 every time a certain agent is given the opportunity to defect. For each observed defection, agents have to decide whether to punish the defector or not. Punishers incur an Enforcement cost (E = -2) every time they punish (P = -9) a defector.

The strategy of an agent is defined by its propensity to defect (Boldness), and its propensity to punish agents they have observed defecting (Vengefulness). Agents defect when given the opportunity if their Boldness is higher than the probability of being observed (S); and they punish observed defectors with probability Vengefulness.

**Question 28**

Metanorms are a mechanism for the enforcement of norms and cooperation by introducing another incentive to retaliate. They give the opportunity to punish those who did not enforce the norm.

In the metanorms game that Axelrod analysed, players still have the option to punish a defector. However, players also have the opportunity to punish others who have failed to punish defectors. This way, nonpunishment is treated as if it were another form of defection; that is, a player will be vengeful against someone who observed a defection but did not punish it.

Simulation of the evolution of strategies in this metanorms game demonstrated that players had a strong incentive to increase their vengefulness so that they are not punished by others, and this in turn led to a decline of boldness in the attempting of defections. Thus, metanorms can promote and sustain cooperation in a population.

**Question 29**

Problem of big demonstrations like in Brazil. GT should solve for the “game” between the demonstrators and the country. Which strategy should the police and the government follow and what will be the equilibrium?

**Question 30**

Police and bank robber: bank robber flees from the police. He could flee by car where his velocity would be 5 streets per move or he could flee walking and only move 1 street per move. The velocity of the police is the same, but they don’t know whether the bank robber is using the car or not. Every 5th move a someone spots the bank robber and the police knows who he is and they have to try to catch them within 30 moves.