Examination for "Logic and Computability"

January 20, 2020 — first Exam for WS20/21

Matrikelnummer	FAMILY NAME	First Name

Task 1:
Formalize the following sentence in classical logic

"If not everybody gets a covid vaccine, there is at least someone who does not get a covid vaccine."

Exhibit either a sequent calculus or a natural deduction derivation for the resulting formula (in case it is valid) or an interpretation that falsifies it (in case it is not valid).

Task 2:
Consider the following formulas of classical logic:

(a) $\forall x \forall y (P(x,y) \to P(y,x))$

(b) $\forall x \forall y \forall z (P(x,y) \wedge P(y,z) \to P(x,z))$

(c) $\forall x P(x,x)$

Does $(a), (b) \models (c)$ hold? Motivate your answer.

Task 3:
Let A be the set

$$\{x \mid \ \Phi_x(x) \downarrow \text{ and } \Phi_x(x) > 2\}$$

Is A recursive, r.e. but not recursive, or none of them? (provide a formal proof)

Task 4:
Let I be an infinite set. Prove the following statement
"I is recursively enumerable if and only if $I = R(f)$ for some f that is total, computable, and 1-1."
(Recall that $R(f)$ denotes the range of the function f.)

Task 5:
Let G be the modal formula $\Diamond\Box A \vee \Diamond\neg A$. Prove or refute:
(1) G is valid in every reflexive frame.
(2) If $\mathcal{F} \models G$ for a frame $\mathcal{F}$, then $\mathcal{F}$ is reflexive.

Task 6:
Show by Robinson-resolution that the clause set $\{C, D, E\}$ is unsatisfiable, where
$C = \neg q(f(x), y) \vee \neg q(y, f(z))$
$D = q(x,y) \vee q(y,x) \vee p(x, g(z))$,
$E = \neg p(x, g(x))$.
Specify all used factors, MGUs, and unified literals.

Task 7:
Use the proof of the ADRF theorem to prove that the function $f(g(x), 2) + 3$ is arithmetically definable, if f and g are arithmetically definable. *Hint:* Specify the witnessing formula, following slide 23 of the last set of slides (on incompleteness).