Chapter 1
Systems of Measurement

Conceptual Problems

***1 •**

Determine the Concept The fundamental physical quantities in the SI system include mass, length, and time. Force, being the product of mass and acceleration, is not a fundamental quantity. $\boxed{(c)\text{ is correct.}}$

2 •

Picture the Problem We can express and simplify the ratio of m/s to m/s^2 to determine the final units.

Express and simplify the ratio of m/s to m/s^2:

$$\frac{\frac{\text{m}}{\text{s}}}{\frac{\text{m}}{\text{s}^2}} = \frac{\text{m}\cdot\text{s}^2}{\text{m}\cdot\text{s}} = \text{s and } \boxed{(d)\text{ is correct.}}$$

3 •

Determine the Concept Consulting Table 1-1 we note that the prefix giga means 10^9. $\boxed{(c)\text{ is correct.}}$

4 •

Determine the Concept Consulting Table 1-1 we note that the prefix mega means 10^6. $\boxed{(d)\text{ is correct.}}$

***5 •**

Determine the Concept Consulting Table 1-1 we note that the prefix pico means 10^{-12}. $\boxed{(a)\text{ is correct.}}$

6 •

Determine the Concept Counting from left to right and ignoring zeros to the left of the first nonzero digit, the last significant figure is the first digit that is in doubt. Applying this criterion, the three zeros after the decimal point are not significant figures, but the last zero is significant. Hence, there are four significant figures in this number. $\boxed{(c)\text{ is correct.}}$

7 •

Determine the Concept Counting from left to right, the last significant figure is the first digit that is in doubt. Applying this criterion, there are six significant figures in this number. $\boxed{(e)\text{ is correct.}}$

8 •

Determine the Concept The advantage is that the length measure is always with you. The disadvantage is that arm lengths are not uniform; if you wish to purchase a board of ″two arm lengths″ it may be longer or shorter than you wish, or else you may have to physically go to the lumberyard to use your own arm as a measure of length.

9 •

(*a*) True. You cannot add ″apples to oranges″ or a length (distance traveled) to a volume (liters of milk).

(*b*) False. The distance traveled is the product of speed (length/time) multiplied by the time of travel (time).

(*c*) True. Multiplying by any conversion factor is equivalent to multiplying by 1. Doing so does not change the value of a quantity; it changes its units.

Estimation and Approximation

***10** ••

Picture the Problem Because θ is small, we can approximate it by $\theta \approx D/r_m$ provided that it is in radian measure. We can solve this relationship for the diameter of the moon.

Express the moon's diameter D in terms of the angle it subtends at the earth θ and the earth-moon distance r_m:

$$D = \theta r_m$$

Find θ in radians:

$$\theta = 0.524° \times \frac{2\pi\,\text{rad}}{360°} = 0.00915\,\text{rad}$$

Substitute and evaluate D:

$$D = (0.00915\,\text{rad})(384\,\text{Mm}) = \boxed{3.51 \times 10^6\,\text{m}}$$

***11 ••**

Picture the Problem We'll assume that the sun is made up entirely of hydrogen. Then we can relate the mass of the sun to the number of hydrogen atoms and the mass of each.

Express the mass of the sun M_S as the product of the number of hydrogen atoms N_H and the mass of each atom M_H:

$$M_S = N_H M_H$$

Solve for N_H:

$$N_H = \frac{M_S}{M_H}$$

Substitute numerical values and evaluate N_H:

$$N_H = \frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}} = \boxed{1.19\times10^{57}}$$

12 ••

Picture the Problem Let P represent the population of the United States, r the rate of consumption and N the number of aluminum cans used annually. The population of the United States is roughly 3×10^8 people. Let's assume that, on average, each person drinks one can of soft drink every day. The mass of a soft-drink can is approximately 1.8×10^{-2} kg.

(*a*) Express the number of cans N used annually in terms of the daily rate of consumption of soft drinks r and the population P:

$$N = rP\Delta t$$

Substitute numerical values and approximate N:

$$N = \left(\frac{1\,\text{can}}{\text{person}\cdot\text{d}}\right)\left(3\times10^8\ \text{people}\right)\times\left(1\,\text{y}\right)\left(365.24\,\frac{\text{d}}{\text{y}}\right) \approx \boxed{10^{11}\ \text{cans}}$$

(*b*) Express the total mass of aluminum used per year for soft drink cans M as a function of the number of cans consumed and the mass m per can:

$$M = Nm$$

Substitute numerical values and evaluate M:

$$M = \left(10^{11}\ \text{cans/y}\right)\left(1.8\times10^{-2}\ \text{kg/can}\right) \approx \boxed{2\times10^{9}\ \text{kg/y}}$$

(c) Express the value of the aluminum as the product of M and the value at recycling centers:

$$\begin{aligned}\text{Value} &= (\$1/\text{kg})M \\ &= (\$1/\text{kg})\left(2\times10^{9}\ \text{kg/y}\right) \\ &= \$2\times10^{9}/\text{y} \\ &= \boxed{2\ \text{billion dollars/y}}\end{aligned}$$

13 ••

Picture the Problem We can estimate the number of words in *Encyclopedia Britannica* by counting the number of volumes, estimating the average number of pages per volume, estimating the number of words per page, and finding the product of these measurements and estimates. Doing so in *Encyclopedia Britannica* leads to an estimate of approximately 200 million for the number of words. If we assume an average word length of five letters, then our estimate of the number of letters in *Encyclopedia Britannica* becomes 10^9.

(a) Relate the area available for one letter s^2 and the number of letters N to be written on the pinhead to the area of the pinhead:

$Ns^2 = \frac{\pi}{4}d^2$ where d is the diameter of the pinhead.

Solve for s to obtain:

$$s = \sqrt{\frac{\pi d^2}{4N}}$$

Substitute numerical values and evaluate s:

$$s = \sqrt{\frac{\pi\left[\left(\tfrac{1}{16}\ \text{in}\right)\left(2.54\ \frac{\text{cm}}{\text{in}}\right)\right]^2}{4\left(10^9\right)}} \approx \boxed{10^{-8}\ \text{m}}$$

(b) Express the number of atoms per letter n in terms of s and the atomic spacing in a metal d_{atomic}:

$$n = \frac{s}{d_{\text{atomic}}}$$

Substitute numerical values and evaluate n:

$$n = \frac{10^{-8}\ \text{m}}{5\times10^{-10}\ \text{atoms/m}} \approx \boxed{20\ \text{atoms}}$$

***14** ••

Picture the Problem The population of the United States is roughly 3×10^8 people. Assuming that the average family has four people, with an average of two cars per family, there are about 1.5×10^8 cars in the United States. If we double that number to include trucks, cabs, etc., we have 3×10^8 vehicles. Let's assume that each vehicle uses, on average, about 12 gallons of gasoline per week.

(*a*) Find the daily consumption of gasoline *G*:

$$G = \left(3\times10^8\text{ vehicles}\right)\left(2\text{ gal/d}\right) = 6\times10^8\text{ gal/d}$$

Assuming a price per gallon P = \$1.50, find the daily cost *C* of gasoline:

$$C = GP = \left(6\times10^8\text{ gal/d}\right)\left(\$1.50/\text{gal}\right) = \$9\times10^8/\text{d} \approx \boxed{\$1\text{ billion dollars/d}}$$

(*b*) Relate the number of barrels *N* of crude oil required annually to the yearly consumption of gasoline *Y* and the number of gallons of gasoline *n* that can be made from one barrel of crude oil:

$$N = \frac{Y}{n} = \frac{G\Delta t}{n}$$

Substitute numerical values and estimate *N*:

$$N = \frac{\left(6\times10^8\text{ gal/d}\right)\left(365.24\text{ d/y}\right)}{19.4\text{ gal/barrel}} \approx \boxed{10^{10}\text{ barrels/y}}$$

15 ••

Picture the Problem We'll assume a population of 300 million (fairly accurate as of September, 2002) and a life expectancy of 76 y. We'll also assume that a diaper has a volume of about half a liter. In (*c*) we'll assume the disposal site is a rectangular hole in the ground and use the formula for the volume of such an opening to estimate the surface area required.

(*a*) Express the total number *N* of disposable diapers used in the United States per year in terms of the number of children *n* in diapers and the number of diapers *D* used by each child in 2.5 y:

$$N = nD$$

Use the daily consumption, the number of days in a year, and the estimated length of time a child is in diapers to estimate the number of diapers *D* required per child:

$$D = \frac{3\text{diapers}}{\text{d}}\times\frac{365.24\text{ d}}{\text{y}}\times2.5\text{ y} \approx 3\times10^3\text{ diapers/child}$$

Use the assumed life expectancy to estimate the number of children n in diapers:

$$n = \left(\frac{2.5\,\text{y}}{76\,\text{y}}\right)\left(300 \times 10^6 \text{ children}\right) \approx 10^7 \text{ children}$$

Substitute to obtain:

$$N = \left(10^7 \text{ children}\right) \times \left(3 \times 10^3 \text{ diapers/child}\right) \approx \boxed{3 \times 10^{10} \text{ diapers}}$$

(*b*) Express the required landfill volume V in terms of the volume of diapers to be buried:

$$V = NV_{\text{one diaper}}$$

Substitute numerical values and evaluate V:

$$V = \left(3 \times 10^{10} \text{ diapers}\right)\left(0.5\,\text{L/diaper}\right) \approx \boxed{1.5 \times 10^7 \text{ m}^3}$$

(*c*) Express the required volume in terms of the volume of a rectangular parallelepiped:

$$V = Ah$$

Solve and evaluate h:

$$A = \frac{V}{h} = \frac{1.5 \times 10^7 \text{ m}^3}{10\,\text{m}} = 1.5 \times 10^6 \text{ m}^2$$

Use a conversion factor to express this area in square miles:

$$A = 1.5 \times 10^6 \text{ m}^2 \times \frac{1 \text{ mi}^2}{2.590\,\text{km}^2} \approx \boxed{0.6\,\text{mi}^2}$$

16 •••

Picture the Problem The number of bits that can be stored on the disk can be found from the product of the capacity of the disk and the number of bits per byte. In part (*b*) we'll need to estimate (*i*) the number of bits required for the alphabet, (*ii*) the average number of letters per word, (*iii*) an average number of words per line, (*iv*) an average number of lines per page, and (*v*) a book length in pages.

(*a*) Express the number of bits N_{bits} as a function of the number of bits per byte and the capacity of the hard disk N_{bytes}:

$$N_{\text{bits}} = N_{\text{bytes}}\left(8\,\text{bits/byte}\right) = \left(2 \times 10^9 \text{ bytes}\right)\left(8\,\text{bits/byte}\right) = \boxed{1.60 \times 10^{10} \text{ bits}}$$

(*b*) Assume an average of 8 letters/word and 8 bits/character to estimate the number of bytes required per word:

$$8\,\frac{\text{bits}}{\text{character}}\times 8\,\frac{\text{characters}}{\text{word}} = 64\,\frac{\text{bits}}{\text{word}} = 8\,\frac{\text{bytes}}{\text{word}}$$

Assume 10 words/line and 60 lines/page:

$$600\,\frac{\text{words}}{\text{page}}\times 8\,\frac{\text{bytes}}{\text{word}} = 4800\,\frac{\text{bytes}}{\text{page}}$$

Assume a book length of 300 pages and approximate the number bytes required:

$$300\,\text{pages}\times 4800\,\frac{\text{bytes}}{\text{page}} = 1.44\times 10^{6}\ \text{bytes}$$

Divide the number of bytes per disk by our estimated number of bytes required per book to obtain an estimate of the number of books the 2-gigabyte hard disk can hold:

$$N_{\text{books}} = \frac{2\times 10^{9}\ \text{bytes}}{1.44\times 10^{6}\ \text{bytes/book}} \approx \boxed{1400\,\text{books}}$$

***17** ••

Picture the Problem Assume that, on average, four cars go through each toll station per minute. Let R represent the yearly revenue from the tolls. We can estimate the yearly revenue from the number of lanes N, the number of cars per minute n, and the \$6 toll per car C.

$$R = NnC = 14\,\text{lanes}\times 4\,\frac{\text{cars}}{\text{min}}\times 60\,\frac{\text{min}}{\text{h}}\times 24\,\frac{\text{h}}{\text{d}}\times 365.24\,\frac{\text{d}}{\text{y}}\times\frac{\$6}{\text{car}} = \boxed{\$177\text{M}}$$

Units

18 •

Picture the Problem We can use the metric prefixes listed in Table 1-1 and the abbreviations on page EP-1 to express each of these quantities.

(*a*)

$$1{,}000{,}000\,\text{watts} = 10^{6}\ \text{watts} = \boxed{1\,\text{MW}}$$

(*b*)

$$0.002\text{gram} = 2\times 10^{-3}\ \text{g} = \boxed{2\,\text{mg}}$$

(*c*)

$$3\times 10^{-6}\ \text{meter} = \boxed{3\,\mu\text{m}}$$

(*d*)

$$30{,}000\,\text{seconds} = 30\times 10^{3}\ \text{s} = \boxed{30\,\text{ks}}$$

19 •

Picture the Problem We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without prefixes.

(*a*)

$$40\,\mu\text{W} = 40\times10^{-6}\text{ W} = \boxed{0.000040\,\text{W}}$$

(*b*)

$$4\,\text{ns} = 4\times10^{-9}\text{ s} = \boxed{0.000000004\,\text{s}}$$

(*c*)

$$3\,\text{MW} = 3\times10^{6}\text{ W} = \boxed{3{,}000{,}000\,\text{W}}$$

(*d*)

$$25\,\text{km} = 25\times10^{3}\text{ m} = \boxed{25{,}000\,\text{m}}$$

***20** •

Picture the Problem We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without abbreviations.

(*a*) $10^{-12}\text{ boo} = \boxed{1\,\text{picoboo}}$

(*b*) $10^{9}\text{ low} = \boxed{1\,\text{gigalow}}$

(*c*) $10^{-6}\text{ phone} = \boxed{1\,\text{microphone}}$

(*d*) $10^{-18}\text{ boy} = \boxed{1\,\text{attoboy}}$

(*e*) $10^{6}\text{ phone} = \boxed{1\,\text{megaphone}}$

(*f*) $10^{-9}\text{ goat} = \boxed{1\,\text{nanogoat}}$

(*g*) $10^{12}\text{ bull} = \boxed{1\,\text{terabull}}$

21 ••

Picture the Problem We can determine the SI units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(*a*) Because x is in meters, C_1 and C_2t must be in meters:

$$\boxed{C_1\text{ is in m; }C_2\text{ is in m/s}}$$

(*b*) Because x is in meters, ½C_1t^2 must be in meters:

$$\boxed{C_1\text{ is in m/s}^2}$$

(*c*) Because v^2 is in m^2/s^2, $2C_1x$ must be in m^2/s^2:

$$\boxed{C_1\text{ is in m/s}^2}$$

(*d*) The argument of trigonometric function must be dimensionless; i.e. without units. Therefore, because x

$$\boxed{C_1\text{ is in m; }C_2\text{ is in s}^{-1}}$$

is in meters:

(e) The argument of an exponential function must be dimensionless; i.e. without units. Therefore, because v is in m/s:

$$\boxed{C_1 \text{ is in m/s; } C_2 \text{ is in s}^{-1}}$$

22 ••

Picture the Problem We can determine the US customary units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(a) Because x is in feet, C_1 and C_2t must be in feet:

$$\boxed{C_1 \text{ is in ft; } C_2 \text{ is in ft/s}}$$

(b) Because x is in feet, $\frac{1}{2}C_1t^2$ must be in feet:

$$\boxed{C_1 \text{ is in ft/s}^2}$$

(c) Because v^2 is in ft^2/s^2, $2C_1x$ must be in ft^2/s^2:

$$\boxed{C_1 \text{ is in ft/s}^2}$$

(d) The argument of trigonometric function must be dimensionless; i.e. without units. Therefore, because x is in feet:

$$\boxed{C_1 \text{ is in ft; } C_2 \text{ is in s}^{-1}}$$

(e) The argument of an exponential function must be dimensionless; i.e. without units. Therefore, because v is in ft/s:

$$\boxed{C_1 \text{ is in ft/s; } C_2 \text{ is in s}^{-1}}$$

Conversion of Units

23 •

Picture the Problem We can use the formula for the circumference of a circle to find the radius of the earth and the conversion factor 1 mi = 1.61 km to convert distances in meters into distances in miles.

(a) The Pole-Equator distance is one-fourth of the circumference:

$$c = \boxed{4 \times 10^7 \text{ m}}$$

(*b*) Use the formula for the circumference of a circle to obtain:

$$R = \frac{c}{2\pi} = \frac{4\times10^{-7}\text{ m}}{2\pi} = \boxed{6.37\times10^{6}\text{ m}}$$

(*c*) Use the conversion factors 1 km = 1000 m and 1 mi = 1.61 km:

$$c = 4\times10^{7}\text{ m}\times\frac{1\text{ km}}{10^{3}\text{ m}}\times\frac{1\text{ mi}}{1.61\text{ km}} = \boxed{2.48\times10^{4}\text{ mi}}$$

and

$$R = 6.37\times10^{6}\text{ m}\times\frac{1\text{ km}}{10^{3}\text{ m}}\times\frac{1\text{ mi}}{1.61\text{ km}} = \boxed{3.96\times10^{3}\text{ mi}}$$

24 •

Picture the Problem We can use the conversion factor 1 mi = 1.61 km to convert speeds in km/h into mi/h.

Find the speed of the plane in km/s:

$$v = 2(340\text{ m/s}) = 680\text{ m/s}$$
$$= \left(680\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ km}}{10^{3}\text{ m}}\right)\left(3600\frac{\text{s}}{\text{h}}\right) = \boxed{2450\text{ km/h}}$$

Convert v into mi/h:

$$v = \left(2450\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ mi}}{1.61\text{ km}}\right) = \boxed{1520\text{ mi/h}}$$

***25** •

Picture the Problem We'll first express his height in inches and then use the conversion factor 1 in = 2.54 cm.

Express the player's height into inches:

$$h = 6\text{ ft}\times\frac{12\text{ in}}{\text{ft}} + 10.5\text{ in} = 82.5\text{ in}$$

Convert h into cm:

$$h = 82.5\text{ in}\times\frac{2.54\text{ cm}}{\text{in}} = \boxed{210\text{ cm}}$$

26 •

Picture the Problem We can use the conversion factors 1 mi = 1.61 km, 1 in = 2.54 cm, and 1 m = 1.094 yd to complete these conversions.

(*a*) $$100\,\frac{\text{km}}{\text{h}}=100\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{mi}}{1.61\,\text{km}}=\boxed{62.1\,\frac{\text{mi}}{\text{h}}}$$

(*b*) $$60\,\text{cm}=60\,\text{cm}\times\frac{1\,\text{in}}{2.54\,\text{cm}}=\boxed{23.6\,\text{in}}$$

(*c*) $$100\,\text{yd}=100\,\text{yd}\times\frac{1\,\text{m}}{1.094\,\text{yd}}=\boxed{91.4\,\text{m}}$$

27 •

Picture the Problem We can use the conversion factor 1.609 km = 5280 ft to convert the length of the main span of the Golden Gate Bridge into kilometers.

Convert 4200 ft into km:

$$4200\,\text{ft}=4200\,\text{ft}\times\frac{1.609\,\text{km}}{5280\,\text{ft}}=\boxed{1.28\,\text{km}}$$

***28** •

Picture the Problem Let *v* be the speed of an object in mi/h. We can use the conversion factor 1 mi = 1.61 km to convert this speed to km/h.

Multiply *v* mi/h by 1.61 km/mi to convert *v* to km/h:

$$v\,\frac{\text{mi}}{\text{h}}=v\,\frac{\text{mi}}{\text{h}}\times\frac{1.61\,\text{km}}{\text{mi}}=\boxed{1.61v\,\text{km/h}}$$

29 •

Picture the Problem Use the conversion factors 1 h = 3600 s, 1.609 km = 1 mi, and 1 mi = 5280 ft to make these conversions.

(*a*) $$1.296\times10^5\,\frac{\text{km}}{\text{h}^2}=\left(1.296\times10^5\,\frac{\text{km}}{\text{h}^2}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)=\boxed{36.0\,\frac{\text{km}}{\text{h}\cdot\text{s}}}$$

(*b*) $$1.296\times10^5\,\frac{\text{km}}{\text{h}^2}=\left(1.296\times10^5\,\frac{\text{km}}{\text{h}^2}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)^2\left(\frac{10^3\,\text{m}}{\text{km}}\right)=\boxed{10.0\,\frac{\text{m}}{\text{s}^2}}$$

(*c*) $$60\,\frac{\text{mi}}{\text{h}}=\left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)=\boxed{88.0\,\frac{\text{ft}}{\text{s}}}$$

(*d*) $$60\,\frac{\text{mi}}{\text{h}}=\left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{1.609\,\text{km}}{1\,\text{mi}}\right)\left(\frac{10^3\,\text{m}}{\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)=\boxed{26.8\,\frac{\text{m}}{\text{s}}}$$

30 •

Picture the Problem We can use the conversion factor 1 L = 1.057 qt to convert gallons into liters and then use this gallons-to-liters conversion factor to convert barrels into cubic meters.

$$(a)\ 1\,\text{gal} = (1\,\text{gal})\left(\frac{4\,\text{qt}}{\text{gal}}\right)\left(\frac{1\,\text{L}}{1.057\,\text{qt}}\right) = \boxed{3.784\,\text{L}}$$

$$(b)\ 1\,\text{barrel} = (1\,\text{barrel})\left(\frac{42\,\text{gal}}{\text{barrel}}\right)\left(\frac{3.784\,\text{L}}{\text{gal}}\right)\left(\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{0.1589\,\text{m}^3}$$

31 •

Picture the Problem We can use the conversion factor given in the problem statement and the fact that 1 mi = 1.609 km to express the number of square meters in one acre.

Multiply by 1 twice, properly chosen, to convert one acre into square miles, and then into square meters:

$$1\,\text{acre} = (1\,\text{acre})\left(\frac{1\,\text{mi}^2}{640\,\text{acres}}\right)\left(\frac{1609\,\text{m}}{\text{mi}}\right)^2$$

$$= \boxed{4050\,\text{m}^2}$$

32 ••

Picture the Problem The volume of a right circular cylinder is the area of its base multiplied by its height. Let d represent the diameter and h the height of the right circular cylinder; use conversion factors to express the volume V in the given units.

(*a*) Express the volume of the cylinder:

$$V = \tfrac{1}{4}\pi d^2 h$$

Substitute numerical values and evaluate V:

$$V = \tfrac{1}{4}\pi(6.8\,\text{in})^2(2\,\text{ft})$$

$$= \tfrac{1}{4}\pi(6.8\,\text{in})^2(2\,\text{ft})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^2$$

$$= \boxed{0.504\,\text{ft}^3}$$

(*b*) Use the fact that 1 m = 3.281 ft to convert the volume in cubic feet into cubic meters:

$$V = (0.504\,\text{ft}^3)\left(\frac{1\,\text{m}}{3.281\,\text{ft}}\right)^3$$

$$= \boxed{0.0143\,\text{m}^3}$$

(*c*) Because 1 L = 10^{-3} m^3:

$$V = (0.0143\,\text{m}^3)\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right) = \boxed{14.3\,\text{L}}$$

***33** ••

Picture the Problem We can treat the SI units as though they are algebraic quantities to simplify each of these combinations of physical quantities and constants.

(*a*) Express and simplify the units of v^2/x:

$$\frac{(\text{m/s})^2}{\text{m}} = \frac{\text{m}^2}{\text{m}\cdot\text{s}^2} = \boxed{\frac{\text{m}}{\text{s}^2}}$$

(*b*) Express and simplify the units of $\sqrt{x/a}$:

$$\sqrt{\frac{\text{m}}{\text{m/s}^2}} = \sqrt{\text{s}^2} = \boxed{\text{s}}$$

(*c*) Noting that the constant factor $\frac{1}{2}$ has no units, express and simplify the units of $\frac{1}{2}at^2$:

$$\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s})^2 = \left(\frac{\text{m}}{\text{s}^2}\right)(\text{s}^2) = \boxed{\text{m}}$$

Dimensions of Physical Quantities

34 •

Picture the Problem We can use the facts that each term in an equation must have the same dimensions and that the arguments of a trigonometric or exponential function must be dimensionless to determine the dimensions of the constants.

(*a*)

$$x = C_1 + C_2 \; t$$

$$L \qquad \boxed{L} \qquad \boxed{\frac{L}{T}} \; T$$

(*b*)

$$x = \tfrac{1}{2} \; C_1 \; t^2$$

$$L \qquad \boxed{\frac{L}{T^2}} \; T^2$$

(*c*)

$$v^2 = 2 \; C_1 \; x$$

$$\frac{L^2}{T^2} \qquad \boxed{\frac{L}{T^2}} \; L$$

(*d*)

$$x = C_1 \cos C_2 \; t$$

$$L \qquad \boxed{L} \qquad \boxed{\frac{1}{T}} \; T$$

(*e*)

$$v = C_1 \exp(-C_2 \; t)$$

$$\frac{L}{T} \qquad \boxed{\frac{L}{T}} \qquad \boxed{\frac{1}{T}} \; T$$

35 ••

Picture the Problem Because the exponent of the exponential function must be dimension the dimension of λ must be $\boxed{T^{-1}.}$

***36 ••**

Picture the Problem We can solve Newton's law of gravitation for *G* and substitute the dimensions of the variables. Treating them as algebraic quantities will allow us to express the dimensions in their simplest form. Finally, we can substitute the SI units for the dimensions to find the units of *G*.

Solve Newton's law of gravitation for *G* to obtain:

$$G=\frac{Fr^2}{m_1m_2}$$

Substitute the dimensions of the variables:

$$G=\frac{\frac{ML}{T^2}\times L^2}{M^2}=\boxed{\frac{L^3}{MT^2}}$$

Use the SI units for *L, M,* and *T*:

$$\text{Units of } G \text{ are } \boxed{\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$$

37 ••

Picture the Problem Let *m* represent the mass of the object, *v* its speed, and *r* the radius of the circle in which it moves. We can express the force as the product of *m*, *v*, and *r* (each raised to a power) and then use the dimensions of force *F,* mass *m,* speed *v,* and radius *r* to obtain three equations in the assumed powers. Solving these equations simultaneously will give us the dependence of *F* on *m, v,* and *r.*

Express the force in terms of powers of the variables:

$$F=m^av^br^c$$

Substitute the dimensions of the physical quantities:

$$MLT^{-2}=M^a\left(\frac{L}{T}\right)^bL^c$$

Simplify to obtain:

$$MLT^{-2}=M^aL^{b+c}T^{-b}$$

Equate the exponents to obtain:

$a = 1$,
$b + c = 1$, and
$-b = -2$

Solve this system of equations to obtain:

$a = 1$, $b = 2$, and $c = -1$

Substitute in equation (1):

$$F=mv^2r^{-1}=\boxed{m\frac{v^2}{r}}$$

38 ••

Picture the Problem We note from Table 1-2 that the dimensions of power are ML^2/T^3. The dimensions of mass, acceleration, and speed are M, L/T^2, and L/T respectively.

Express the dimensions of *mav*:

$$[mav] = M \times \frac{L}{T^2} \times \frac{L}{T} = \frac{ML^2}{T^3}$$

From Table 1-2:

$$[P] = \frac{ML^2}{T^3}$$

Comparing these results, we see that the product of mass, acceleration, and speed has the dimensions of power.

39 ••

Picture the Problem The dimensions of mass and velocity are M and L/T, respectively. We note from Table 1-2 that the dimensions of force are ML/T^2.

Express the dimensions of momentum:

$$[mv] = M \times \frac{L}{T} = \frac{ML}{T}$$

From Table 1-2:

$$[F] = \frac{ML}{T^2}$$

Express the dimensions of force multiplied by time:

$$[Ft] = \frac{ML}{T^2} \times T = \frac{ML}{T}$$

Comparing these results, we see that momentum has the dimensions of force multiplied by time.

40 ••

Picture the Problem Let *X* represent the physical quantity of interest. Then we can express the dimensional relationship between *F, X,* and *P* and solve this relationship for the dimensions of *X*.

Express the relationship of *X* to force and power dimensionally:

$$[F][X] = [P]$$

Solve for $[X]$:

$$[X] = \frac{[P]}{[F]}$$

Substitute the dimensions of force and power and simplify to obtain:

$$[X]=\frac{\frac{ML^2}{T^3}}{\frac{ML}{T^2}}=\frac{L}{T}$$

Because the dimensions of velocity are L/T, we can conclude that:

$$\boxed{[P]=[F][v]}$$

Remarks: While it is true that $P = Fv$, dimensional analysis does not reveal the presence of dimensionless constants. For example, if $P = \pi Fv$, the analysis shown above would fail to establish the factor of π.

***41** ••

Picture the Problem We can find the dimensions of C by solving the drag force equation for C and substituting the dimensions of force, area, and velocity.

Solve the drag force equation for the constant C:

$$C=\frac{F_{\text{air}}}{Av^2}$$

Express this equation dimensionally:

$$[C]=\frac{[F_{\text{air}}]}{[A][v]^2}$$

Substitute the dimensions of force, area, and velocity and simplify to obtain:

$$[C]=\frac{\frac{ML}{T^2}}{L^2\left(\frac{L}{T}\right)^2}=\boxed{\frac{M}{L^3}}$$

42 ••

Picture the Problem We can express the period of a planet as the product of these factors (each raised to a power) and then perform dimensional analysis to determine the values of the exponents.

Express the period T of a planet as the product of r^a, G^b, and M_S^c:

$$T = Cr^aG^bM_S^c \qquad (1)$$

where C is a dimensionless constant.

Solve the law of gravitation for the constant G:

$$G=\frac{Fr^2}{m_1m_2}$$

Express this equation dimensionally:

$$[G]=\frac{[F][r]^2}{[m_1][m_2]}$$

Substitute the dimensions of F, r, and m:

$$[G]=\frac{\frac{ML}{T^2}\times(L)^2}{M\times M}=\frac{L^3}{MT^2}$$

Noting that the dimension of time is represented by the same letter as is the period of a planet, substitute the dimensions in equation (1) to obtain:

$$T=(L)^a\left(\frac{L^3}{MT^2}\right)^b(M)^c$$

Introduce the product of M^0 and L^0 in the left hand side of the equation and simplify to obtain:

$$M^0L^0T^1=M^{c-b}L^{a+3b}T^{-2b}$$

Equate the exponents on the two sides of the equation to obtain:

$0 = c - b$,
$0 = a + 3b$, and
$1 = -2b$

Solve these equations simultaneously to obtain:

$$a=\tfrac{3}{2},\ b=-\tfrac{1}{2},\ \text{and}\ c=-\tfrac{1}{2}$$

Substitute in equation (1):

$$T=Cr^{3/2}G^{-1/2}M_S^{-1/2}=\boxed{\frac{C}{\sqrt{GM_S}}r^{3/2}}$$

Scientific Notation and Significant Figures

***43 •**

Picture the Problem We can use the rules governing scientific notation to express each of these numbers as a decimal number.

(*a*) $3\times10^4=\boxed{30{,}000}$

(*b*) $6.2\times10^{-3}=\boxed{0.0062}$

(*c*) $4\times10^{-6}=\boxed{0.000004}$

(*d*) $2.17\times10^5=\boxed{217{,}000}$

44 •

Picture the Problem We can use the rules governing scientific notation to express each of these measurements in scientific notation.

(*a*) $3.1\,\text{GW}=\boxed{3.1\times10^9\ \text{W}}$

(*c*) $2.3\,\text{fs}=\boxed{2.3\times10^{-15}\ \text{s}}$

(*b*) $10\,\text{pm} = 10 \times 10^{-12}\ \text{m} = \boxed{10^{-11}\ \text{m}}$ (*d*) $4\,\mu\text{s} = \boxed{4 \times 10^{-6}\ \text{s}}$

45 •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) The number of significant figures in each factor is three; therefore the result has three significant figures:

$$(1.14)(9.99 \times 10^4) = \boxed{1.14 \times 10^5}$$

(*b*) Express both terms with the same power of 10. Because the first measurement has only two digits after the decimal point, the result can have only two digits after the decimal point:

$$(2.78 \times 10^{-8}) - (5.31 \times 10^{-9}) = (2.78 - 0.531) \times 10^{-8} = \boxed{2.25 \times 10^{-8}}$$

(*c*) We'll assume that 12 is exact. Hence, the answer will have three significant figures:

$$\frac{12\pi}{4.56 \times 10^{-3}} = \boxed{8.27 \times 10^3}$$

(*d*) Proceed as in (*b*):

$$27.6 + (5.99 \times 10^2) = 27.6 + 599 = 627 = \boxed{6.27 \times 10^2}$$

46 •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Note that both factors have four significant figures.

$$(200.9)(569.3) = \boxed{1.144 \times 10^5}$$

(*b*) Express the first factor in scientific notation and note that both factors have three significant figures.

$$(0.000000513)(62.3 \times 10^7) = (5.13 \times 10^{-7})(62.3 \times 10^7) = \boxed{3.20 \times 10^2}$$

(*c*) Express both terms in scientific notation and note that the second has only three significant figures. Hence the result will have only three significant figures.

$$28401+\left(5.78\times10^{4}\right)=\left(2.841\times10^{4}\right)+\left(5.78\times10^{4}\right)=\left(2.841+5.78\right)\times10^{4}=\boxed{8.62\times10^{4}}$$

(*d*) Because the divisor has three significant figures, the result will have three significant figures.

$$\frac{63.25}{4.17\times10^{-3}}=\boxed{1.52\times10^{4}}$$

***47 •**

Picture the Problem Let N represent the required number of membranes and express N in terms of the thickness of each cell membrane.

Express N in terms of the thickness of a single membrane:

$$N=\frac{1\,\text{in}}{7\,\text{nm}}$$

Convert the units into SI units and simplify to obtain:

$$N=\frac{1\,\text{in}}{7\,\text{nm}}\times\frac{2.54\,\text{cm}}{\text{in}}\times\frac{1\,\text{m}}{100\,\text{cm}}\times\frac{1\,\text{nm}}{10^{-9}\,\text{m}}=\boxed{4\times10^{6}}$$

48 •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Both factors and the result have three significant figures:

$$\left(2.00\times10^{4}\right)\left(6.10\times10^{-2}\right)=\boxed{1.22\times10^{3}}$$

(*b*) Because the second factor has three significant figures, the result will have three significant figures:

$$\left(3.141592\right)\left(4.00\times10^{5}\right)=\boxed{1.26\times10^{6}}$$

(*c*) Both factors and the result have three significant figures:

$$\frac{2.32\times10^{3}}{1.16\times10^{8}}=\boxed{2.00\times10^{-5}}$$

(*d*) Write both terms using the same power of 10. Note that the result will have only three significant figures:

$$\left(5.14\times10^{3}\right)+\left(2.78\times10^{2}\right)=\left(5.14\times10^{3}\right)+\left(0.278\times10^{3}\right)=\left(5.14+0.278\right)\times10^{3}=\boxed{5.42\times10^{3}}$$

(*e*) Follow the same procedure used in (*d*):

$$\left(1.99\times10^{2}\right)+\left(9.99\times10^{-5}\right)$$
$$=\left(1.99\times10^{2}\right)+\left(0.000000999\times10^{2}\right)$$
$$=\boxed{1.99\times10^{2}}$$

***49** •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) The second factor and the result have three significant figures:

$$3.141592654\times(23.2)^{2}=\boxed{1.69\times10^{3}}$$

(*b*) We'll assume that 2 is exact. Therefore, the result will have two significant figures:

$$2\times3.141592654\times0.76=\boxed{4.8}$$

(*c*) We'll assume that 4/3 is exact. Therefore the result will have two significant figures:

$$\frac{4}{3}\pi\times(1.1)^{3}=\boxed{5.6}$$

(*d*) Because 2.0 has two significant figures, the result has two significant figures:

$$\frac{(2.0)^{5}}{3.141592654}=\boxed{10}$$

General Problems

50 •

Picture the Problem We can use the conversion factor 1 mi = 1.61 km to convert 100 km/h into mi/h.

Multiply 100 km/h by 1 mi/1.61 km to obtain:

$$100\frac{\text{km}}{\text{h}}=100\frac{\text{km}}{\text{h}}\times\frac{1\text{mi}}{1.61\text{km}}$$
$$=\boxed{62.1\text{mi/h}}$$

***51** •

Picture the Problem We can use a series of conversion factors to convert 1 billion seconds into years.

Multiply 1 billion seconds by the appropriate conversion factors to convert into years:

$$10^9\ \text{s} = 10^9\ \text{s} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1\,\text{day}}{24\,\text{h}} \times \frac{1\,\text{y}}{365.24\,\text{days}} = \boxed{31.7\,\text{y}}$$

52 •

Picture the Problem In both the examples cited we can equate expressions for the physical quantities, expressed in different units, and then divide both sides of the equation by one of the expressions to obtain the desired conversion factor.

(*a*) Divide both sides of the equation expressing the speed of light in the two systems of measurement by 186,000 mi/s to obtain:

$$1 = \frac{3 \times 10^8\ \text{m/s}}{1.86 \times 10^5\ \text{mi/h}} = 1.61 \times 10^3\ \text{m/mi}$$

$$= \left(1.61 \times 10^3\ \frac{\text{m}}{\text{mi}}\right)\left(\frac{1\,\text{km}}{10^3\ \text{m}}\right)$$

$$= \boxed{1.61\,\text{km/mi}}$$

(*b*) Find the volume of 1.00 kg of water:

Volume of 1.00 kg = 10^3 g is 10^3 cm^3

Express 10^3 cm^3 in ft^3:

$$(10\,\text{cm})^3\left(\frac{1\,\text{in}}{2.54\,\text{cm}}\right)^3\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^3$$

$$= 0.0353\,\text{ft}^3$$

Relate the weight of 1 ft^3 of water to the volume occupied by 1 kg of water:

$$\frac{1.00\,\text{kg}}{0.0353\,\text{ft}^3} = 62.4\frac{\text{lb}}{\text{ft}^3}$$

Divide both sides of the equation by the left-hand side to obtain:

$$1 = \frac{62.4\dfrac{\text{lb}}{\text{ft}^3}}{\dfrac{1.00\,\text{kg}}{0.0353\,\text{ft}^3}} = \boxed{2.20\,\text{lb/kg}}$$

53 ••

Picture the Problem We can use the given information to equate the ratios of the number of uranium atoms in 8 g of pure uranium and of 1 atom to its mass.

Express the proportion relating the number of uranium atoms N_U in 8 g of pure uranium to the mass of 1 atom:

$$\frac{N_U}{8\,\text{g}} = \frac{1\,\text{atom}}{4.0 \times 10^{-26}\ \text{kg}}$$

Solve for and evaluate N_U:

$$N_U = (8\,\text{g})\left(\frac{1\,\text{atom}}{4.0\times10^{-26}\,\text{kg}}\right) = \boxed{2.0\times10^{23}}$$

54 ••

Picture the Problem We can relate the weight of the water to its weight per unit volume and the volume it occupies.

Express the weight w of water falling on the acre in terms of the weight of one cubic foot of water, the depth d of the water, and the area A over which the rain falls:

$$w = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)Ad$$

Find the area A in ft^2:

$$A = (1\,\text{acre})\left(\frac{1\,\text{mi}^2}{640\,\text{acre}}\right)\left(\frac{5280\,\text{ft}}{\text{mi}}\right)^2 = 4.356\times10^4\,\text{ft}^2$$

Substitute numerical values and evaluate w:

$$w = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(4.356\times10^4\,\text{ft}^2\right)(1.4\,\text{in})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right) = \boxed{3.17\times10^5\,\text{lb}}$$

55 ••

Picture the Problem We can use the definition of density and the formula for the volume of a sphere to find the density of iron. Once we know the density of iron, we can use these same relationships to find what the radius of the earth would be if it had the same mass per unit volume as iron.

(*a*) Using its definition, express the density of iron:

$$\rho = \frac{m}{V}$$

Assuming it to be spherical, express the volume of an iron nucleus as a function of its radius:

$$V = \tfrac{4}{3}\pi r^3$$

Substitute to obtain:

$$\rho = \frac{3m}{4\pi r^3} \qquad (1)$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{3\left(9.3\times10^{-26}\ \text{kg}\right)}{4\pi\left(5.4\times10^{-15}\ \text{m}\right)^3} = \boxed{1.41\times10^{17}\ \text{kg/m}^3}$$

(*b*) Because equation (1) relates the density of any spherical object to its mass and radius, we can solve for *r* to obtain:

$$r = \sqrt[3]{\frac{3m}{4\pi\rho}}$$

Substitute numerical values and evaluate *r*:

$$r = \sqrt[3]{\frac{3\left(5.98\times10^{24}\ \text{kg}\right)}{4\pi\left(1.41\times10^{17}\ \text{kg/m}^3\right)}} = \boxed{216\,\text{m}}$$

56 ••

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Because all of the factors have two significant figures, the result will have two significant figures:

$$\frac{\left(5.6\times10^{-5}\right)\left(0.0000075\right)}{2.4\times10^{-12}} = \frac{\left(5.6\times10^{-5}\right)\left(7.5\times10^{-6}\right)}{2.4\times10^{-12}} = \boxed{1.8\times10^{2}}$$

(*b*) Because the factor with the fewest significant figures in the first term has two significant figures, the result will have two significant figures. Because its last significant figure is in the tenth's position, the difference between the first and second term will have its last significant figure in the tenth's position:

$$\left(14.2\right)\left(6.4\times10^{7}\right)\left(8.2\times10^{-9}\right) - 4.06 = 7.8 - 4.06 = \boxed{3.4}$$

(*c*) Because all of the factors have two significant figures, the result will have two significant figures:

$$\frac{\left(6.1\times10^{-6}\right)^2\left(3.6\times10^{4}\right)^3}{\left(3.6\times10^{-11}\right)^{1/2}} = \boxed{2.9\times10^{8}}$$

(*d*) Because the factor with the fewest significant figures has two significant figures, the result will have two significant figures.

$$\frac{(0.000064)^{1/3}}{(12.8\times10^{-3})(490\times10^{-1})^{1/2}} = \frac{(6.4\times10^{-5})^{1/3}}{(12.8\times10^{-3})(490\times10^{-1})^{1/2}} = \boxed{0.45}$$

***57 ••**

Picture the Problem We can use the relationship between an angle θ, measured in radians, subtended at the center of a circle, the radius R of the circle, and the length L of the arc to answer these questions concerning the astronomical units of measure.

(*a*) Relate the angle θ subtended by an arc of length S to the distance R:

$$\theta = \frac{S}{R} \qquad (1)$$

Solve for and evaluate S:

$$S = R\theta = (1\,\text{parsec})(1\,\text{s})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\times\left(\frac{1°}{60\,\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{360°}\right) = \boxed{4.85\times10^{-6}\,\text{parsec}}$$

(*b*) Solve equation (1) for and evaluate R:

$$R = \frac{S}{\theta} = \frac{1.496\times10^{11}\,\text{m}}{(1\,\text{s})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1°}{60\,\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{360°}\right)} = \boxed{3.09\times10^{16}\,\text{m}}$$

(*c*) Relate the distance D light travels in a given interval of time Δt to its speed c and evaluate D for Δt = 1 y:

$$D = c\Delta t = \left(3\times10^{8}\,\frac{\text{m}}{\text{s}}\right)(1\,\text{y})\left(3.156\times10^{7}\,\frac{\text{s}}{\text{y}}\right) = \boxed{9.47\times10^{15}\,\text{m}}$$

(*d*) Use the definition of 1 AU and the result from part (*c*) to obtain:

$$1c\cdot\text{y} = \left(9.47\times10^{15}\text{ m}\right)\left(\frac{1\text{AU}}{1.496\times10^{11}\text{ m}}\right)$$
$$= \boxed{6.33\times10^{4}\text{ AU}}$$

(*e*) Combine the results of parts (*b*) and (*c*) to obtain:

$$1\text{parsec} = \left(3.08\times10^{16}\text{ m}\right)\times\left(\frac{1c\cdot\text{y}}{9.47\times10^{15}\text{ m}}\right)$$
$$= \boxed{3.25c\cdot\text{y}}$$

58 ••

Picture the Problem Let N_e and N_p represent the number of electrons and the number of protons, respectively and ρ the critical average density of the universe. We can relate these quantities to the masses of the electron and proton using the definition of density.

(*a*) Using its definition, relate the required density ρ to the electron density N_e/V:

$$\rho = \frac{m}{V} = \frac{N_e m_e}{V}$$

Solve for N_e/V:

$$\frac{N_e}{V} = \frac{\rho}{m_e} \qquad (1)$$

Substitute numerical values and evaluate N_e/V:

$$\frac{N_e}{V} = \frac{6\times10^{-27}\text{ kg/m}^3}{9.11\times10^{-31}\text{ kg/electron}}$$
$$= \boxed{6.59\times10^{3}\text{ electrons/m}^3}$$

(*b*) Express and evaluate the ratio of the masses of an electron and a proton:

$$\frac{m_e}{m_p} = \frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = 5.46\times10^{-4}$$

Rewrite equation (1) in terms of protons:

$$\frac{N_p}{V} = \frac{\rho}{m_p} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{\frac{N_p}{V}}{\frac{N_e}{V}} = \frac{m_e}{m_p} \text{ or } \frac{N_p}{V} = \frac{m_e}{m_p}\left(\frac{N_e}{V}\right)$$

Substitute numerical values and use the result from part (*a*) to evaluate N_p/V:

$$\frac{N_p}{V} = \left(5.46\times10^{-4}\right)\times\left(6.59\times10^{3}\ \text{protons/m}^3\right) = \boxed{3.59\ \text{protons/m}^3}$$

***59** ••

Picture the Problem We can use the definition of density to relate the mass of the water in the cylinder to its volume and the formula for the volume of a cylinder to express the volume of water used in the detector's cylinder. To convert our answer in kg to lb, we can use the fact that 1 kg weighs about 2.205 lb.

Relate the mass of water contained in the cylinder to its density and volume:

$$m = \rho V$$

Express the volume of a cylinder in terms of its diameter d and height h:

$$V = A_{\text{base}}h = \frac{\pi}{4}d^2h$$

Substitute to obtain:

$$m = \rho\frac{\pi}{4}d^2h$$

Substitute numerical values and evaluate m:

$$m = \left(10^3\ \text{kg/m}^3\right)\left(\frac{\pi}{4}\right)(39.3\ \text{m})^2(41.4\ \text{m}) = 5.02\times10^7\ \text{kg}$$

Convert 5.02×10^7 kg to tons:

$$m = 5.02\times10^7\ \text{kg}\times\frac{2.205\ \text{lb}}{\text{kg}}\times\frac{1\ \text{ton}}{2000\ \text{lb}} = 55.4\times10^3\ \text{ton}$$

$$\boxed{\text{The 50,000-ton claim is conservative. The actual weight is closer to 55,000 tons.}}$$

60 •••

Picture the Problem We'll solve this problem two ways. First, we'll substitute two of the ordered pairs in the given equation to obtain two equations in C and n that we can solve simultaneously. Then we'll use a spreadsheet program to create a graph of log T as a function of log m and use its curve-fitting capability to find n and C. Finally, we can identify the data points that deviate the most from a straight-line plot by examination of the graph.

1st Solution for (*a*)

(*a*) To estimate *C* and *n*, we can apply the relation $T = Cm^n$ to two arbitrarily selected data points. We'll use the 1st and 6th ordered pairs. This will produce simultaneous equations that can be solved for *C* and *n*.

$$T_1 = Cm_1^n$$
and
$$T_6 = Cm_6^n$$

Divide the second equation by the first to obtain:

$$\frac{T_6}{T_1} = \frac{Cm_6^n}{Cm_1^2} = \left(\frac{m_6}{m_1}\right)^n$$

Substitute numerical values and solve for *n* to obtain:

$$\frac{1.75\,\text{s}}{0.56\,\text{s}} = \left(\frac{1\,\text{kg}}{0.1\,\text{kg}}\right)^n$$
or
$$3.125 = 10^n \Rightarrow n = \boxed{0.4948}$$
and so a ″judicial″ guess is that $n = 0.5$.

Substituting this value into the second equation gives:

$$T_5 = Cm_5^{0.5}$$
so
$$1.75\,\text{s} = \text{C}(1\,\text{kg})^{0.5}$$

Solving for *C* gives:

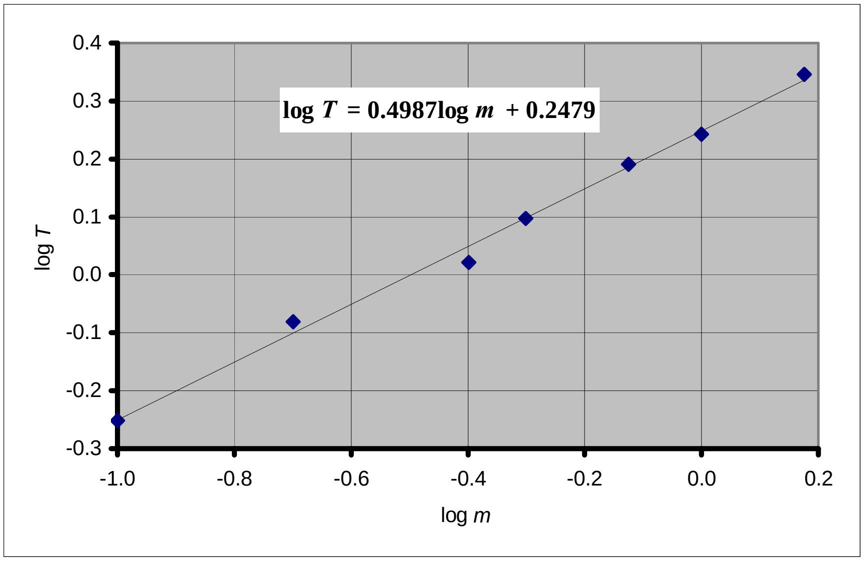
$$C = \boxed{1.75\,\text{s/kg}^{0.5}}$$

2nd Solution for (*a*)

Take the logarithm (we'll arbitrarily use base 10) of both sides of $T = Cm^n$ and simplify to obtain:

$$\log(T) = \log(Cm^n) = \log C + \log m^n$$
$$= n \log m + \log C$$
which, we note, is of the form $y = mx + b$. Hence a graph of log T vs. log m should be linear with a slope of *n* and a log *T*-intercept log *C*.

The graph of log *T* vs. log *m* shown below was created using a spreadsheet program. The equation shown on the graph was obtained using Excel's ″Add Trendline″ function. (Excel's ″Add Trendline″ function uses regression analysis to generate the trendline.)



Comparing the equation on the graph generated by the Add Trendline function to $\log(T) = n\log m + \log C$, we observe:

$$n = \boxed{0.499}$$

and

$$C = 10^{0.2479} = \boxed{1.77\ \text{s/kg}^{1/2}}$$

or

$$T = \boxed{\left(1.77\ \text{s/kg}^{1/2}\right)m^{0.499}}$$

(*b*) From the graph we see that the data points that deviate the most from a straight-line plot are:

$$\boxed{m = 0.02\ \text{kg},\ T = 0.471\ \text{s, and } m = 1.50\ \text{kg},\ T = 2.22\ \text{s}}$$

(*b*) From the graph we see that the points generated using the data pairs (0.02 kg, 0.471 s) and (0.4 kg, 1.05 s) deviate the most from the line representing the best fit to the points plotted on the graph.

Remarks: Still another way to find *n* and *C* is to use your graphing calculator to perform regression analysis on the given set of data for log *T* versus log *m*. The slope yields *n* and the *y*-intercept yields log *C*.

61 •••

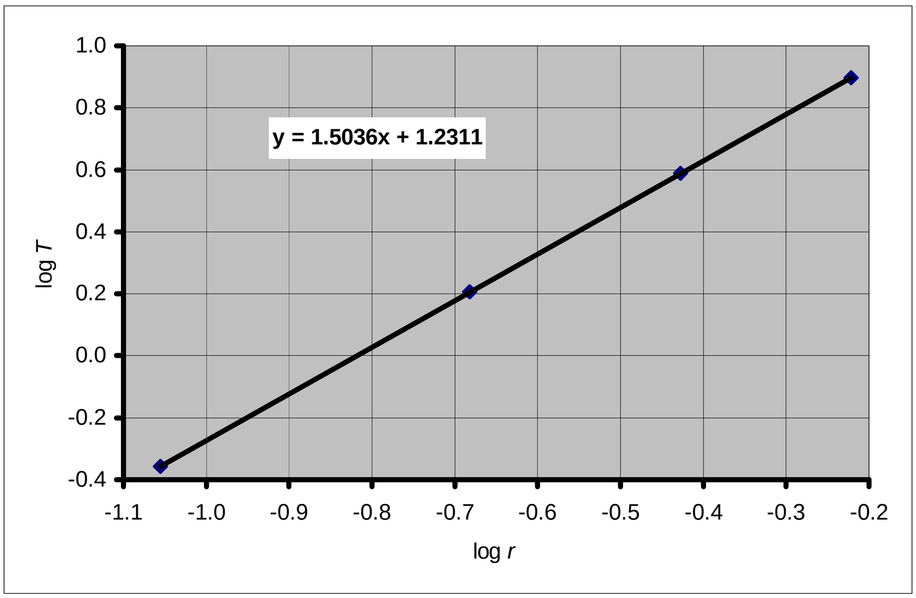
Picture the Problem We can plot log T versus log r and find the slope of the best-fit line to determine the exponent n. We can then use any of the ordered pairs to evaluate C. Once we know n and C, we can solve $T = Cr^n$ for r as a function of T.

(*a*) Take the logarithm (we'll arbitrarily use base 10) of both sides of $T = Cr^n$ and simplify to obtain:

$$\log(T) = \log(Cr^n) = \log C + \log r^n$$
$$= n\log r + \log C$$

Note that this equation is of the form $y = mx + b$. Hence a graph of log T vs. log r should be linear with a slope of n and a log T -intercept log C.

The graph of log T versus log r shown below was created using a spreadsheet program. The equation shown on the graph was obtained using Excel's ″Add Trendline″ function. (Excel's ″Add Trendline″ function uses regression analysis to generate the trendline.)



From the regression analysis we observe that:

$$n = \boxed{1.50}$$

and

$$C = 10^{1.2311} = \boxed{17.0\,\text{y}/(\text{Gm})^{3/2}}$$

or $$T = \boxed{\left(17.0\,\text{y}/(\text{Gm})^{3/2}\right)r^{1.50}} \qquad (1)$$

(*b*) Solve equation (1) for the radius of the planet's orbit:

$$r = \left(\frac{T}{17.0\,\text{y}/(\text{Gm})^{3/2}}\right)^{2/3}$$

Substitute numerical values and evaluate r:

$$r = \left(\frac{6.20\,\text{y}}{17.0\,\text{y}/(\text{Gm})^{3/2}}\right)^{2/3} = \boxed{0.510\ \text{Gm}}$$

***62 •••**

Picture the Problem We can express the relationship between the period T of the pendulum, its length L, and the acceleration of gravity g as $T = CL^a g^b$ and perform dimensional analysis to find the values of a and b and, hence, the function relating these variables. Once we've performed the experiment called for in part (*b*), we can determine an experimental value for C.

(*a*) Express T as the product of L and g raised to powers a and b:

$$T = CL^a g^b \quad (1)$$

where C is a dimensionless constant.

Write this equation in dimensional form:

$$[T] = [L]^a [g]^b$$

Noting that the symbols for the dimension of the period and length of the pendulum are the same as those representing the physical quantities, substitute the dimensions to obtain:

$$T = L^a \left(\frac{L}{T^2}\right)^b$$

Because L does not appear on the left-hand side of the equation, we can write this equation as:

$$L^0 T^1 = L^{a+b} T^{-2b}$$

Equate the exponents to obtain:

$$a + b = 0 \text{ and } -2b = 1$$

Solve these equations simultaneously to find a and b:

$$a = \tfrac{1}{2} \text{ and } b = -\tfrac{1}{2}$$

Substitute in equation (1) to obtain:

$$T = CL^{1/2} g^{-1/2} = \boxed{C\sqrt{\frac{L}{g}}} \quad (2)$$

(*b*) If you use pendulums of lengths 1 m and 0.5 m; the periods should be about:

$$T(1\,\text{m}) = \boxed{2\,\text{s}}$$

and

$$T(0.5\,\text{m}) = \boxed{1.4\,\text{s}}$$

(*c*) Solve equation (2) for C:

$$C = T\sqrt{\frac{g}{L}}$$

Evaluate C with L = 1 m and T = 2 s:

$$C = (2\,\text{s})\sqrt{\frac{9.81\,\text{m/s}^2}{1\,\text{m}}} = 6.26 \approx 2\pi$$

Substitute in equation (2) to obtain:

$$T = \boxed{2\pi\sqrt{\frac{L}{g}}}$$

63 •••

Picture the Problem The weight of the earth's atmosphere per unit area is known as the atmospheric pressure. We can use this definition to express the weight w of the earth's atmosphere as the product of the atmospheric pressure and the surface area of the earth.

Using its definition, relate atmospheric pressure to the weight of the earth's atmosphere:

$$P = \frac{w}{A}$$

Solve for w:

$$w = PA$$

Relate the surface area of the earth to its radius R:

$$A = 4\pi R^2$$

Substitute to obtain:

$$w = 4\pi R^2 P$$

Substitute numerical values and evaluate w:

$$w = 4\pi(6370\,\text{km})^2\left(\frac{10^3\,\text{m}}{\text{km}}\right)^2\left(\frac{39.37\,\text{in}}{\text{m}}\right)^2\left(14.7\frac{\text{lb}}{\text{in}^2}\right) = \boxed{1.16\times10^{19}\,\text{lb}}$$

Chapter 2
Motion in One Dimension

Conceptual Problems

1 •

Determine the Concept The "average velocity" is being requested as opposed to "average speed".

The average velocity is defined as the change in position or displacement divided by the change in time.

$$v_{av} = \frac{\Delta y}{\Delta t}$$

The change in position for any "round trip" is zero by definition. So the ***average velocity*** for any round trip must also be zero.

$$v_{av} = \frac{\Delta y}{\Delta t} = \frac{0}{\Delta t} = \boxed{0}$$

***2** •

Determine the Concept The important concept here is that "average speed" is being requested as opposed to "average velocity".

Under all circumstances, including **constant acceleration**, the definition of the average speed is the ratio of the total distance traveled ($H + H$) to the total time elapsed, in this case $2H/T$. $\boxed{(d)\text{ is correct.}}$

Remarks: **Because this motion involves a round trip, if the question asked for "average velocity," the answer would be zero.**

3 •

Determine the Concept Flying with the wind, the speed of the plane relative to the ground (v_{PG}) is the sum of the speed of the wind relative to the ground (v_{WG}) and the speed of the plane relative to the air ($v_{PG} = v_{WG} + v_{PA}$). Flying into or against the wind the speed relative to the ground is the difference between the wind speed and the true air speed of the plane ($v_g = v_w – v_t$). Because the ground speed landing against the wind is smaller than the ground speed landing with the wind, it is safer to land *against* the wind.

4 •

Determine the Concept The important concept here is that $a = dv/dt$, where a is the acceleration and v is the velocity. Thus, the acceleration is positive if dv is positive; the acceleration is negative if dv is negative.

(*a*) Let's take the direction a car is moving to be the positive direction:

Because the car is moving in the direction we've chosen to be positive, its velocity is positive ($dx > 0$). If the car is braking, then its velocity is decreasing ($dv < 0$) and its acceleration (dv/dt) is negative.

(*b*) Consider a car that is moving to

Because the car is moving in the direction the right but choose the positive direction to be to the left:

opposite to that we've chosen to be positive, its velocity is negative ($dx < 0$). If the car is braking, then its velocity is increasing ($dv > 0$) and its acceleration (dv/dt) is positive.

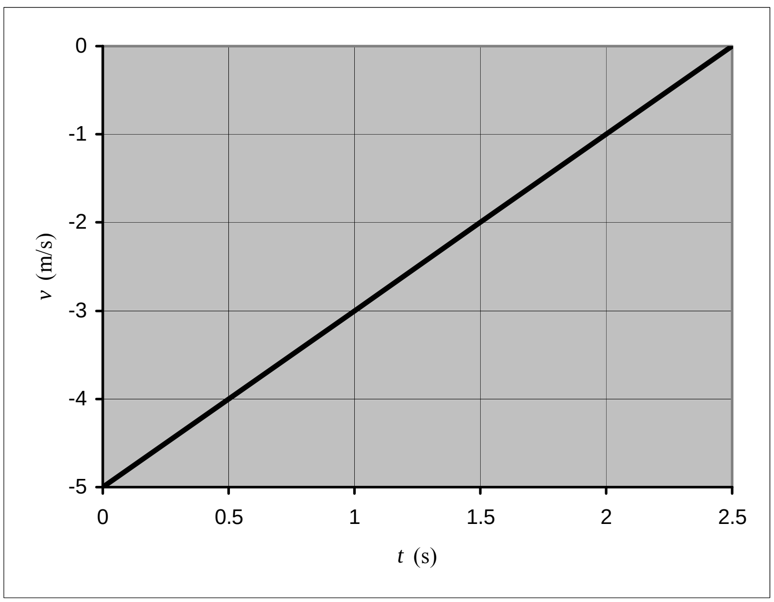
***5 •**

Determine the Concept The important concept is that when both the acceleration and the velocity are in the same direction, the speed increases. On the other hand, when the acceleration and the velocity are in opposite directions, the speed decreases.

(*a*) Because your velocity remains negative, your displacement must be *negative.*

(*b*) Define the direction of your trip as the negative direction. During the last five steps gradually slow the speed of walking, until the wall is reached.

(*c*) A graph of *v* as a function of *t* that is consistent with the conditions stated in the problem is shown below:



6 •

Determine the Concept True. We can use the definition of average velocity to express the displacement Δx as $\Delta x = v_{av}\Delta t$. Note that, if the acceleration is constant, the average velocity is also given by $v_{av} = (v_i + v_f)/2$.

7 •

Determine the Concept Acceleration is the slope of the velocity versus time curve, $a = dv/dt$; while velocity is the slope of the position versus time curve, $v = dx/dt$. The speed of an object is the magnitude of its velocity.

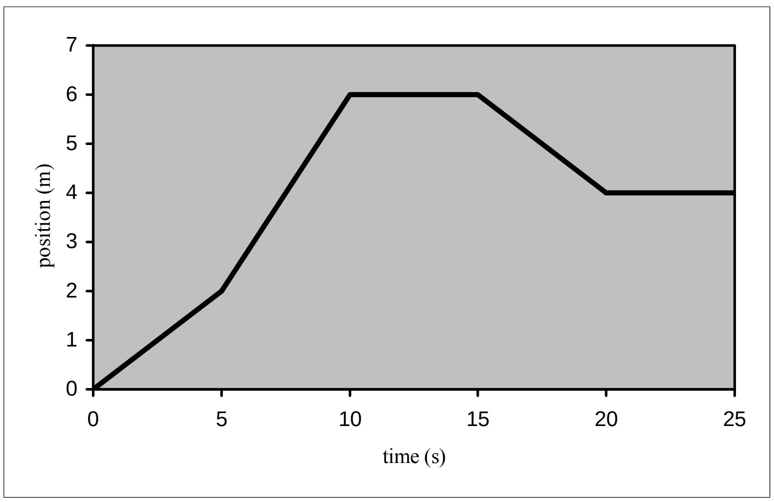
(*a*) True. Zero acceleration implies that the velocity is constant. If the velocity is constant (including zero), the speed must also be constant.

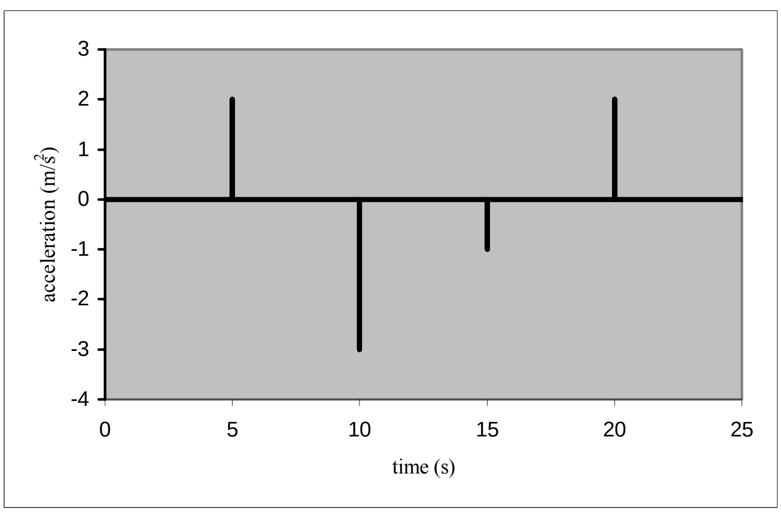
(*b*) True in one dimension.

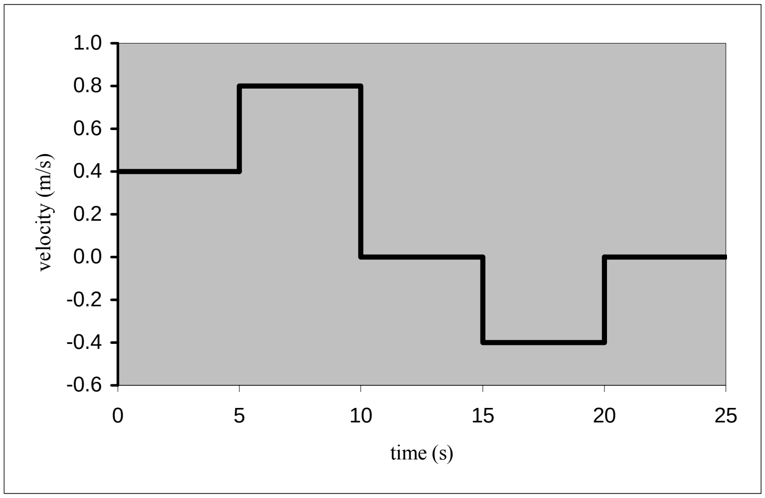
Remarks: The answer to (*b*) would be False in more than one dimension. In one dimension, if the speed remains constant, then the object cannot speed up, slow down, or reverse direction. Thus, if the speed remains constant, the velocity remains constant, which implies that the acceleration remains zero. (In more than one-dimensional motion, an object can change direction while maintaining constant speed. This constitutes a change in the direction of the velocity.) Consider a ball moving in a circle at a constant rotation rate. The speed (magnitude of the velocity) is constant while the velocity is tangent to the circle and always changing. The acceleration is always pointing inward and is certainly NOT zero.

***8** ••

Determine the Concept Velocity is the slope of the position versus time curve and acceleration is the slope of the velocity versus time curve. See the graphs below.







9 •

Determine the Concept False. The average velocity is defined (for any acceleration) as the change in position (the displacement) divided by the change in time $v_{\text{av}} = \Delta x/\Delta t$. It is always valid. If the acceleration remains constant the average velocity is also given by

$$v_{\text{av}} = \frac{v_{\text{i}} + v_{\text{f}}}{2}$$

Consider an engine piston moving up and down as an example of non-constant velocity. For one complete cycle, $v_{\text{f}} = v_{\text{i}}$ and $x_{\text{i}} = x_{\text{f}}$ so $v_{\text{av}} = \Delta x/\Delta t$ is zero. The formula involving the mean of v_{f} and v_{i} cannot be applied because the acceleration is not constant, and yields an incorrect nonzero value of v_{i}.

10 •

Determine the Concept This can occur if the rocks have different initial speeds. Ignoring air resistance, the acceleration is constant. Choose a coordinate system in which the origin is at the point of release and upward is the positive direction. From the constant-acceleration equation

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

we see that the only way two objects can have the same acceleration ($-g$ in this case) and cover the same distance, $\Delta y = y - y_0$, in different times would be if the initial velocities of the two rocks were different. Actually, the answer would be the same whether or not the acceleration is constant. It is just easier to see for the special case of constant acceleration.

***11 ••**

Determine the Concept Neglecting air resistance, the balls are in free fall, each with the same free-fall acceleration, which is a constant.

At the time the second ball is released, the first ball is already moving. Thus, during any time interval their velocities will increase by exactly the same amount. What can be said about the speeds of the two balls? *The first ball will always be moving faster than the second ball.*

This being the case, what happens to the separation of the two balls while they are both falling? *Their separation increases*. $\boxed{(a)\text{ is correct.}}$

12 ••

Determine the Concept The slope of an $x(t)$ curve at any point in time represents the speed at that instant. The way the slope changes as time increases gives the sign of the acceleration. If the slope becomes less negative or more positive as time increases (as you move to the right on the time axis), then the acceleration is positive. If the slope becomes less positive or more negative, then the acceleration is negative. The slope of the slope of an $x(t)$ curve at any point in time represents the acceleration at that instant.

The slope of curve (a) is negative and becomes more negative as time increases.

Therefore, the velocity is negative and the acceleration is negative.

The slope of curve (b) is positive and constant and so the velocity is positive and constant.

Therefore, the acceleration is zero.

The slope of curve (c) is positive and decreasing.

Therefore, the velocity is positive and the acceleration is negative.

The slope of curve (d) is positive and increasing.

Therefore, the velocity and acceleration are positive. We need more information to conclude that a is constant.

The slope of curve (e) is zero.

Therefore, the velocity and acceleration are zero.

$\boxed{(d)\text{ best shows motion with constant positive acceleration.}}$

***13 •**

Determine the Concept The slope of a $v(t)$ curve at any point in time represents the acceleration at that instant. Only one curve has a constant and positive slope.
$\boxed{(b)\text{ is correct.}}$

14 •

Determine the Concept No. The word average implies an interval of time rather than an instant in time; therefore, the statement makes no sense.

***15 •**

Determine the Concept Note that the ″average velocity″ is being requested as opposed to the ″average speed.″

Yes. In any roundtrip, A to B, and back to A, the average velocity is zero.

$$v_{\text{av}(\text{A}\to\text{B}\to\text{A})}=\frac{\Delta x}{\Delta t}=\frac{\Delta x_{\text{AB}}+\Delta x_{\text{BA}}}{\Delta t}=\frac{\Delta x_{\text{AB}}+\left(-\Delta x_{\text{BA}}\right)}{\Delta t}=\frac{0}{\Delta t}=\boxed{0}$$

On the other hand, the average velocity between A and B is not generally zero.

$$v_{\text{av}(\text{A}\to\text{B})}=\frac{\Delta x_{\text{AB}}}{\Delta t}\neq\boxed{0}$$

Remarks: Consider an object launched up in the air. Its average velocity on the way up is NOT zero. Neither is it zero on the way down. However, over the round trip, it is zero.

16 •

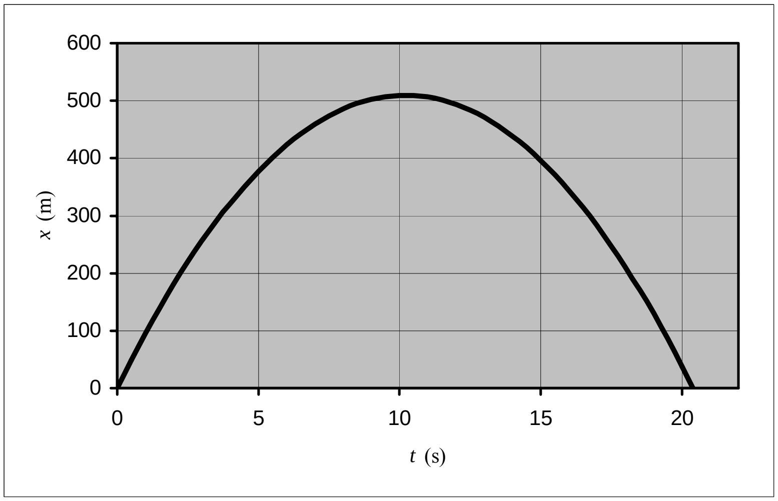
Determine the Concept An object is farthest from the origin when it is farthest from the time axis. In one-dimensional motion starting from the origin, the point located farthest from the time axis in a distance-versus-time plot is the farthest from its starting point. Because the object's initial position is at $x = 0$, point B represents the instant that the object is farthest from $x = 0$. $\boxed{(b)\text{ is correct.}}$

17 •

Determine the Concept No. If the velocity is constant, a graph of position as a function of time is linear with a constant slope equal to the velocity.

18 •

Determine the Concept Yes. The average velocity in a time interval is defined as the displacement divided by the elapsed time $v_{\text{av}}=\Delta x/\Delta t$. The fact that $v_{\text{av}} = 0$ for some time interval, Δt, implies that the displacement Δx over this interval is also zero. Because the instantaneous velocity is defined as $v=\lim_{\Delta t\to 0}\left(\Delta x/\Delta t\right)$, it follows that v must also be zero. As an example, in the following graph of x versus t, over the interval between $t = 0$ and $t \approx 21$ s, $\Delta x = 0$. Consequently, $v_{\text{av}} = 0$ for this interval. Note that the instantaneous velocity is zero only at $t \approx 10$ s.



19 ••

Determine the Concept In the one-dimensional motion shown in the figure, the velocity is a minimum when the slope of a position-versus-time plot goes to zero (i.e., the curve becomes horizontal). At these points, the slope of the position-versus-time curve is zero; therefore, the speed is zero. $\boxed{(b)\text{ is correct.}}$

***20** ••

Determine the Concept In one-dimensional motion, the velocity is the slope of a position-versus-time plot and can be either positive or negative. On the other hand, the speed is the magnitude of the velocity and can only be positive. We'll use v to denote velocity and the word "speed" for how fast the object is moving.

(*a*)
curve a: $v(t_2) < v(t_1)$
curve b: $v(t_2) = v(t_1)$
curve c: $v(t_2) > v(t_1)$
curve d: $v(t_2) < v(t_1)$

(*b*)
curve a: $\text{speed}(t_2) < \text{speed}(t_1)$
curve b: $\text{speed}(t_2) = \text{speed}(t_1)$
curve c: $\text{speed}(t_2) < \text{speed}(t_1)$
curve d: $\text{speed}(t_2) > \text{speed}(t_1)$

21 •

Determine the Concept Acceleration is the slope of the velocity-versus-time curve, $a = dv/dt$, while velocity is the slope of the position-versus-time curve, $v = dx/dt$.

(*a*) False. Zero acceleration implies that the velocity is *not changing*. The velocity could be any constant (including zero). But, if the velocity is constant and nonzero, the particle must be moving.

(*b*) True. Again, zero acceleration implies that the velocity remains constant. This means that the *x*-versus-*t* curve has a constant slope (i.e., a straight line). Note: This does not necessarily mean a zero-slope line.

22 •

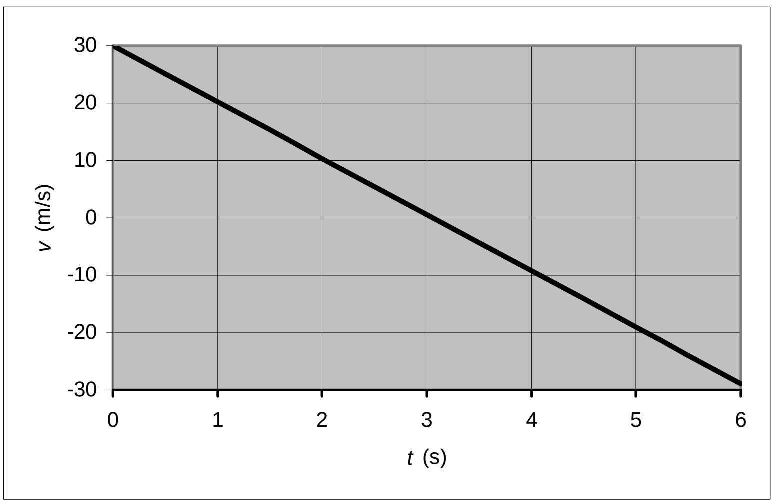
Determine the Concept Yes. If the velocity is changing the acceleration is not zero. The velocity is zero and the acceleration is nonzero any time an object is *momentarily* at rest. If the acceleration were also zero, the velocity would never change; therefore, the object would have to remain at rest.

Remarks: **It is important conceptually to note that when both the acceleration and the velocity have the same sign, the speed increases. On the other hand, when the acceleration and the velocity have opposite signs, the speed decreases.**

23 •

Determine the Concept In the absence of air resistance, the ball will experience a constant acceleration. Choose a coordinate system in which the origin is at the point of release and the upward direction is positive.

The graph shows the velocity of a ball that has been thrown straight upward with an initial speed of 30 m/s as a function of time. Note that the slope of this graph, the acceleration, is the same at every point, including the point at which $v = 0$ (at the top of its flight). Thus, $\boxed{v_{\text{top of flight}} = 0}$ and $\boxed{a_{\text{top of flight}} = -g}$.



The acceleration is the slope (–*g*).

24 •

Determine the Concept The "average speed" is being requested as opposed to "average velocity." We can use the definition of average speed as distance traveled divided by the elapsed time and expression for the average speed of an object when it is experiencing constant acceleration to express v_{av} in terms of v_0.

The average speed is defined as the total distance traveled divided by the change in time:

$$v_{av} = \frac{\text{total distance traveled}}{\text{total time}} = \frac{H + H}{T} = \frac{2H}{T}$$

Find the average speed for the upward flight of the object:

$$v_{\text{av,up}} = \frac{v_0 + 0}{2} = \frac{H}{\frac{1}{2}T}$$

Solve for H to obtain:

$$H = \tfrac{1}{4}v_0 T$$

Find the average speed for the downward flight of the object:

$$v_{\text{av,down}} = \frac{0 + v_0}{2} = \frac{H}{\frac{1}{2}T}$$

Solve for H to obtain:

$$H = \tfrac{1}{4}v_0 T$$

Substitute in our expression for v_{av} to obtain:

$$v_{\text{av}} = \frac{2\left(\frac{1}{4}v_0 T\right)}{T} = \boxed{\frac{v_0}{2}}$$

Because $v_0 \neq 0$, the average speed is not zero.

Remarks: 1) Because this motion involves a roundtrip, if the question asked for ″average velocity″, the answer would be zero. 2) Another easy way to obtain this result is take the absolute value of the velocity of the object to obtain a graph of its speed as a function of time. A simple geometric argument leads to the result we obtained above.

25 •

Determine the Concept In the absence of air resistance, the bowling ball will experience constant acceleration. Choose a coordinate system with the origin at the point of release and upward as the positive direction. Whether the ball is moving upward and slowing down, is momentarily at the top of its trajectory, or is moving downward with ever increasing velocity, its acceleration is constant and equal to the acceleration due to gravity. $\boxed{(b)\text{ is correct.}}$

26 •

Determine the Concept Both objects experience the same constant acceleration. Choose a coordinate system in which downward is the positive direction and use a constant-acceleration equation to express the position of each object as a function of time.

Using constant-acceleration equations, express the positions of both objects as functions of time:

$$x_{\text{A}} = x_{0,\text{A}} + v_0 t + \tfrac{1}{2}gt^2$$

and

$$x_{\text{B}} = x_{0,\text{B}} + v_0 t + \tfrac{1}{2}gt^2$$

where $v_0 = 0$.

Express the separation of the two objects by evaluating $x_{\text{B}} - x_{\text{A}}$:

$$x_{\text{B}} - x_{\text{A}} = x_{0,\text{B}} - x_{0.\text{A}} = 10\,\text{m}$$

and $\boxed{(d)\text{ is correct.}}$

***27** ••

Determine the Concept Because the Porsche accelerates uniformly, we need to look for a graph that represents constant acceleration. We are told that the Porsche has a constant acceleration that is positive (the velocity is increasing); therefore we must look for a velocity-versus-time curve with a positive constant slope and a nonzero intercept.

$\boxed{(c)\text{ is correct.}}$

***28** ••

Determine the Concept In the absence of air resistance, the object experiences constant acceleration. Choose a coordinate system in which the downward direction is positive.

Express the distance D that an object, released from rest, falls in time t:

$$D = \tfrac{1}{2}gt^2$$

Because the distance fallen varies with the square of the time, during the first two seconds it falls four times the distance it falls during the first second.

$$\boxed{(a)\text{ is correct.}}$$

29 ••

Determine the Concept In the absence of air resistance, the acceleration of the ball is constant. Choose a coordinate system in which the point of release is the origin and upward is the positive y direction.

The displacement of the ball halfway to its highest point is:

$$\Delta y = \frac{\Delta y_{\text{max}}}{2}$$

Using a constant-acceleration equation, relate the ball's initial and final velocities to its displacement and solve for the displacement:

$$v^2 = v_0^2 + 2a\Delta y = v_0^2 - 2g\Delta y$$

Substitute $v_0 = 0$ to determine the maximum displacement of the ball:

$$\Delta y_{\text{max}} = -\frac{v_0^2}{2(-g)} = \frac{v_0^2}{2g}$$

Express the velocity of the ball at half its maximum height:

$$v^2 = v_0^2 - 2g\Delta y = v_0^2 - 2g\frac{\Delta y_{\text{max}}}{2}$$

$$= v_0^2 - g\Delta y_{\text{max}} = v_0^2 - g\frac{v_0^2}{2g} = \frac{v_0^2}{2}$$

Solve for v:

$$v = \frac{\sqrt{2}}{2}v_0 \approx 0.707v_0$$

and $\boxed{(c)\text{ is correct.}}$

30 •

Determine the Concept As long as the acceleration remains constant the following constant-acceleration equations hold. If the acceleration is not constant, they do not, in general, give correct results except by coincidence.

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2a\Delta x \qquad v_{\text{av}} = \frac{v_{\text{i}} + v_{\text{f}}}{2}$$

(*a*) False. From the first equation, we see that (*a*) is true if and only if the acceleration is constant.

(*b*) False. Consider a rock thrown straight up into the air. At the "top" of its flight, the velocity is zero but it is changing (otherwise the velocity would remain zero and the rock would hover); therefore the acceleration is not zero.

(*c*) True. The definition of average velocity, $v_{av} = \Delta x/\Delta t$, requires that this always be true.

***31 •**
Determine the Concept Because the acceleration of the object is constant, the constant-acceleration equations can be used to describe its motion. The special expression for average velocity for constant acceleration is $v_{av} = \dfrac{v_i + v_f}{2}$. $\boxed{(c)\text{ is correct.}}$

32 •
Determine the Concept The constant slope of the *x*-versus-*t* graph tells us that the velocity is constant and the acceleration is zero. A linear position versus time curve implies a constant velocity. The negative slope indicates a constant negative velocity. The fact that the velocity is constant implies that the acceleration is also constant and zero. $\boxed{(e)\text{ is correct.}}$

33 ••
Determine the Concept The velocity is the slope of the tangent to the curve, and the acceleration is the rate of change of this slope. Velocity is the slope of the position-versus-time curve. A parabolic $x(t)$ curve opening upward implies an increasing velocity. The acceleration is positive. $\boxed{(a)\text{ is correct.}}$

34 ••
Determine the Concept The acceleration is the slope of the tangent to the velocity as a function of time curve. For constant acceleration, a velocity-versus- time curve must be a straight line whose slope is the acceleration. Zero acceleration means that slope of $v(t)$ must also be zero. $\boxed{(c)\text{ is correct.}}$

35 ••
Determine the Concept The acceleration is the slope of the tangent to the velocity as a function of time curve. For constant acceleration, a velocity-versus- time curve must be a straight line whose slope is the acceleration. The acceleration and therefore the slope can be positive, negative, or zero. $\boxed{(d)\text{ is correct.}}$

36 ••
Determine the Concept The velocity is positive if the curve is above the $v = 0$ line (the *t* axis), and the acceleration is negative if the tangent to the curve has a negative slope. Only graphs (*a*), (*c*), and (*e*) have positive *v*. Of these, only graph (*e*) has a negative slope. $\boxed{(e)\text{ is correct.}}$

37 ••

Determine the Concept The velocity is positive if the curve is above the $v = 0$ line (the t axis), and the acceleration is negative if the tangent to the curve has a negative slope. Only graphs (b) and (d) have negative v. Of these, only graph (d) has a negative slope.

$\boxed{(d)\text{ is correct.}}$

38 ••

Determine the Concept A linear velocity-versus-time curve implies constant acceleration. The displacement from time $t = 0$ can be determined by integrating v-versus-t — that is, by finding the area under the curve. The initial velocity at $t = 0$ can be read directly from the graph of v-versus-t as the v-intercept; i.e., $v(0)$. The acceleration of the object is the slope of $v(t)$. The average velocity of the object is given by drawing a horizontal line that has the same area under it as the area under the curve. Because all of these quantities can be determined $\boxed{(e)\text{ is correct.}}$

***39** ••

Determine the Concept The velocity is the slope of a position versus time curve and the acceleration is the rate at which the velocity, and thus the slope, changes.

Velocity

(a) Negative at t_0 and t_1.
(b) Positive at t_3, t_4, t_6, and t_7.
(c) Zero at t_2 and t_5.

Acceleration

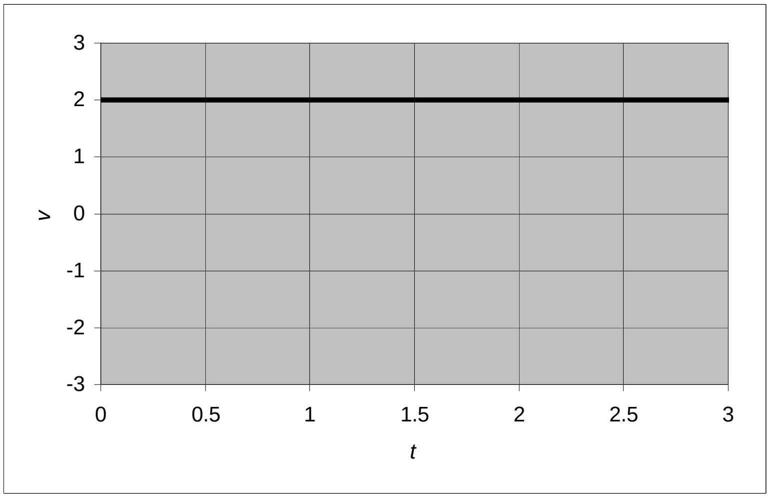
(a) Negative at t_4.
(b) Positive at t_2 and t_6.
(c) Zero at t_0, t_1, t_3, t_5, and t_7.

The acceleration is positive at points where the slope increases as you move toward the right.

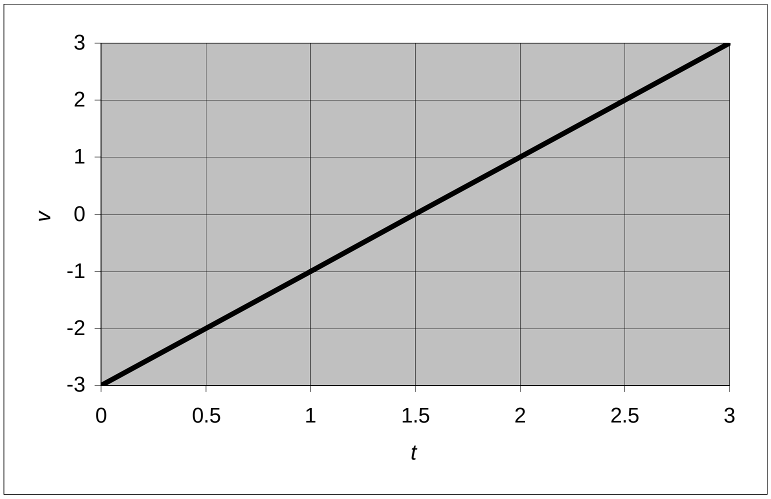
40 ••

Determine the Concept Acceleration is the slope of a velocity-versus-time curve.

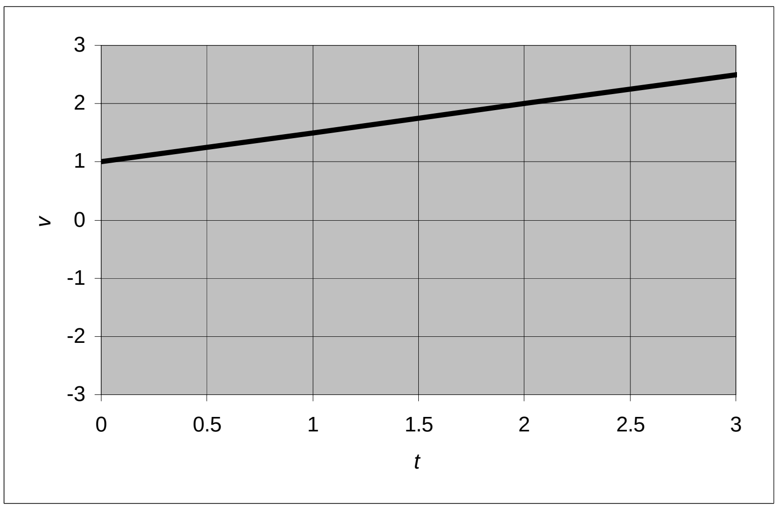
(a) Acceleration is zero and constant while velocity is not zero.



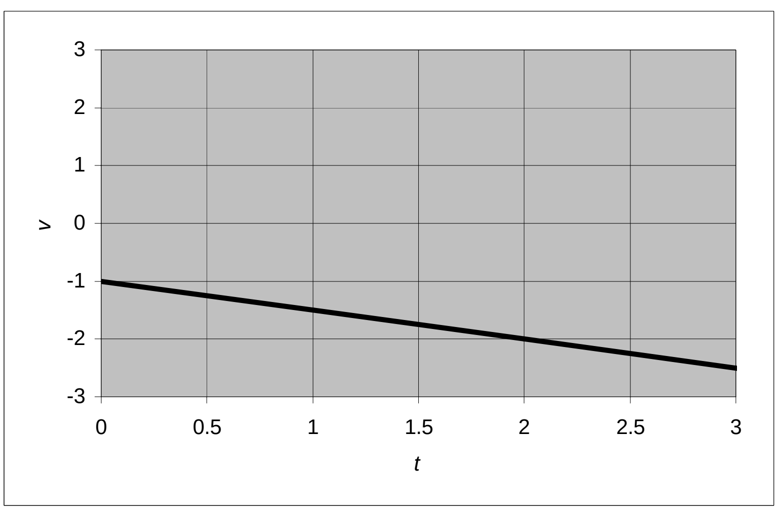
(*b*) Acceleration is constant but not zero.



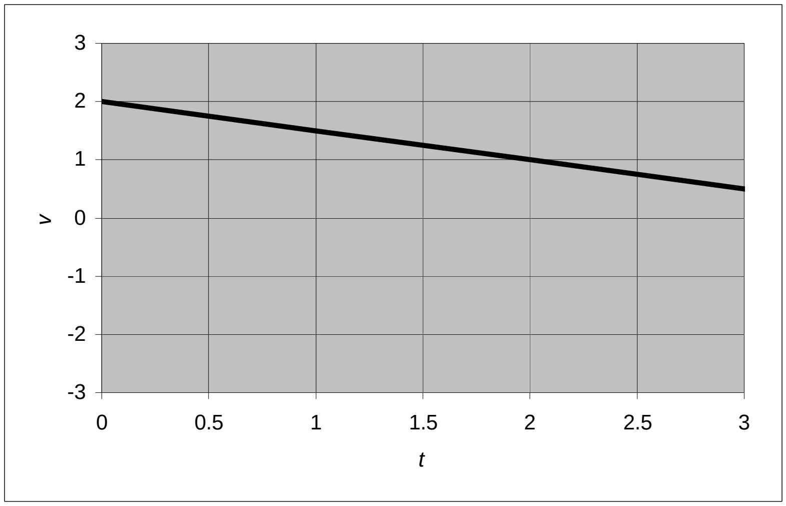
(*c*) Velocity and acceleration are both positive.



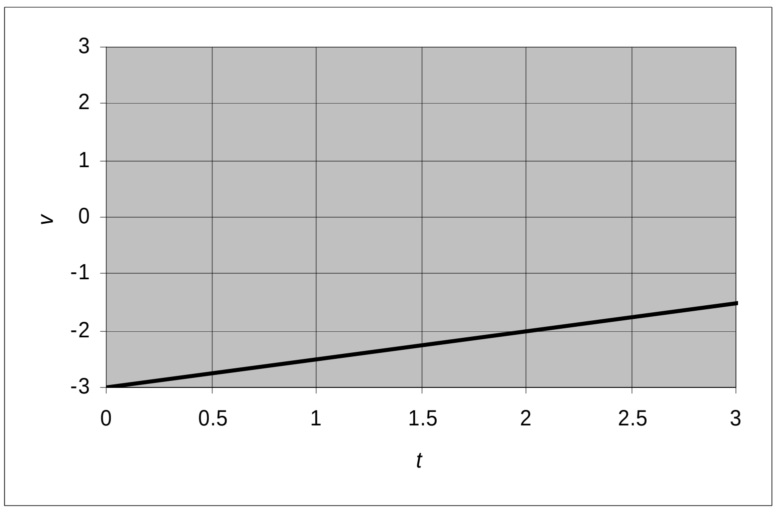
(*d*) Velocity and acceleration are both negative.



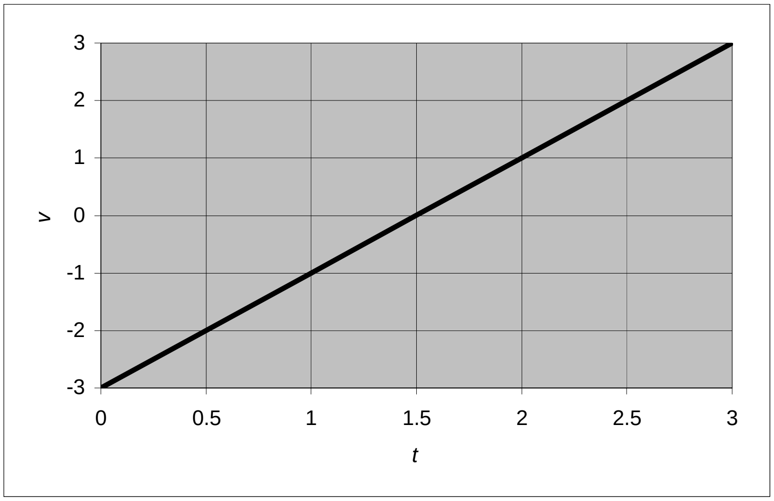
(*e*) Velocity is positive and acceleration is negative.



(*f*) Velocity is negative and acceleration is positive.



(*g*) Velocity is momentarily zero at the intercept with the *t* axis but the acceleration is not zero.



41 ••

Determine the Concept Velocity is the slope and acceleration is the slope of the slope of a position-versus-time curve. Acceleration is the slope of a velocity- versus-time curve.

(*a*) For constant velocity, *x*-versus-*t* must be a straight line; *v*-versus-*t* must be a horizontal straight line; and *a*-versus-*t* must be a straight horizontal line at $a = 0$.

(*a*), (*f*), and (*i*) are the correct answers.

(*b*) For velocity to reverse its direction *x*-versus-*t* must have a slope that changes sign and *v*-versus-*t* must cross the time axis. The acceleration cannot remain zero at all times.

(*c*) and (*d*) are the correct answers.

(*c*) For constant acceleration, *x*-versus-*t* must be a straight line or a parabola, *v*-versus-*t* must be a straight line, and *a*-versus-*t* must be a horizontal straight line.

(*a*), (*d*), (*e*), (*f*), (*h*), and (*i*) are the correct answers.

(*d*) For non-constant acceleration, *x*-versus-*t* must not be a straight line or a parabola; *v*-versus-*t* must not be a straight line, or *a*-versus-*t* must not be a horizontal straight line.

(*b*), (*c*), and (*g*) are the correct answers.

For two graphs to be mutually consistent, the curves must be consistent with the definitions of velocity and acceleration.

Graphs (*a*) and (*i*) are mutually consistent. Graphs (*d*) and (*h*) are mutually consistent. Graphs (*f*) and (*i*) are also mutually consistent.

Estimation and Approximation

42 •

Picture the Problem Assume that your heart beats at a constant rate. It does not, but the average is pretty stable.

(*a*) We will use an average pulse rate of 70 bpm for a seated (resting) adult. One's pulse rate is defined as the number of heartbeats per unit time:

$$\text{Pulse rate} = \frac{\#\text{ of heartbeats}}{\text{Time}}$$

and

$$\#\text{ of heartbeats} = \text{Pulse rate} \times \text{Time}$$

The time required to drive 1 mi at 60 mph is (1/60) h or 1 min:

$$\#\text{ of heartbeats} = (70\text{ beats/min})(1\text{ min}) = \boxed{70\text{ beats}}$$

(*b*) Express the number of heartbeats during a lifetime in terms of the pulse rate and the life span of an individual:

$$\#\text{of heartbeats} = \text{Pulse rate} \times \text{Time}$$

Assuming a 95-y life span, calculate the time in minutes:

$$\text{Time} = (95\text{ y})(365.25\text{ d/y})(24\text{ h/d})(60\text{ min/h}) = 5.00\times10^{7}\text{ min}$$

Substitute numerical values and evaluate the number of heartbeats:

$$\#\text{of heartbeats} = (70\text{ beats/min})(5.00\times10^{7}\text{ min}) = \boxed{3.50\times10^{9}\text{ beats}}$$

***43** ••

Picture the Problem In the absence of air resistance, Carlos' acceleration is constant. Because all the motion is downward, let's use a coordinate system in which downward is positive and the origin is at the point at which the fall began.

(*a***)** Using a constant-acceleration equation, relate Carlos' final velocity to his initial velocity, acceleration, and distance fallen and solve for his final velocity:

$$v^2 = v_0^2 + 2a\Delta y$$

and, because $v_0 = 0$ and $a = g$,

$$v = \sqrt{2g\Delta y}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{2(9.81\text{ m/s}^2)(150\text{ m})} = \boxed{54.2\text{ m/s}}$$

(*b*) While his acceleration by the snow is not constant, solve the same constant- acceleration equation to get an estimate of his average acceleration:

$$a = \frac{v^2 - v_0^2}{2\Delta y}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{-\left(54\,\text{m/s}^2\right)^2}{2(1.22\text{m})} = -1.20\times10^3\text{m/s}^2$$
$$= \boxed{-123g}$$

Remarks: The final velocity we obtained in part (*a*), approximately 121 mph, is about the same as the terminal velocity for an "average" man. This solution is probably only good to about 20% accuracy.

44 ••

Picture the Problem Because we're assuming that the accelerations of the skydiver and the mouse are constant to one-half their terminal velocities, we can use constant-acceleration equations to find the times required for them to reach their ″upper-bound″ velocities and their distances of fall. Let's use a coordinate system in which downward is the positive *y* direction.

(*a*) Using a constant-acceleration equation, relate the upper-bound velocity to the free-fall acceleration and the time required to reach this velocity:

$$v_{\text{upper bound}} = v_0 + g\Delta t$$
or, because $v_0 = 0$,
$$v_{\text{upper bound}} = g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_{\text{upper bound}}}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{25\,\text{m/s}}{9.81\,\text{m/s}^2} = 2.55\,\text{s}$$

Using a constant-acceleration equation, relate the skydiver's distance of fall to the elapsed time Δt:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$
or, because *v*0 = 0 and $a = g$,
$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values and evaluate Δy:

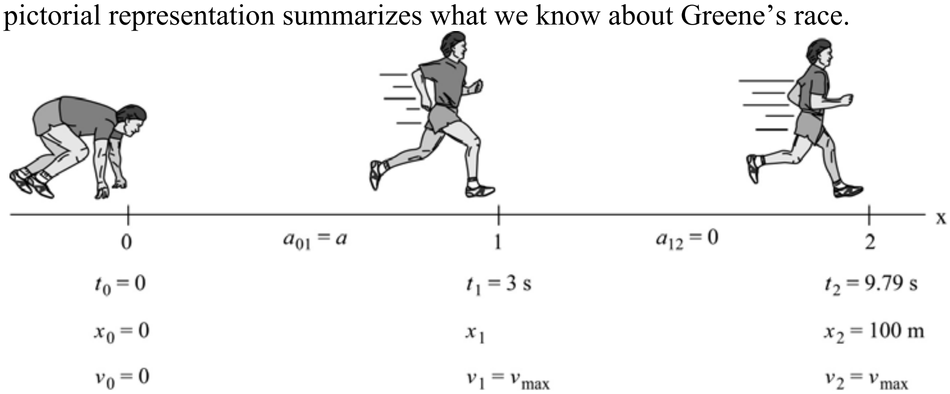
$$\Delta y = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(2.55\,\text{s})^2 = \boxed{31.9\,\text{m}}$$

(*b*) Proceed as in (*a*) with $v_{\text{upper bound}}$ = 0.5 m/s to obtain:

$$\Delta t = \frac{0.5\,\text{m/s}}{9.81\,\text{m/s}^2} = \boxed{0.0510\,\text{s}}$$
and
$$\Delta y = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(0.0510\,\text{s})^2 = \boxed{1.27\,\text{cm}}$$

45 ••

Picture the Problem This is a constant-acceleration problem. Choose a coordinate system in which the direction Greene is running is the positive x direction. During the first 3 s of the race his acceleration is positive and during the rest of the race it is zero. The pictorial representation summarizes what we know about Greene's race.



Express the total distance covered by Greene in terms of the distances covered in the two phases of his race:

$$100\,\text{m} = \Delta x_{01} + \Delta x_{12}$$

Express the distance he runs getting to his maximum velocity:

$$\Delta x_{01} = v_0 \Delta t_{01} + \tfrac{1}{2} a_{01} (\Delta t_{01})^2 = \tfrac{1}{2} a (3\,\text{s})^2$$

Express the distance covered during the rest of the race at the constant maximum velocity:

$$\Delta x_{12} = v_{max} \Delta t_{12} + \tfrac{1}{2} a_{12} (\Delta t_{12})^2 = (a \Delta t_{01}) \Delta t_{12} = a(3\,\text{s})(6.79\,\text{s})$$

Substitute for these displacements and solve for a:

$$100\,\text{m} = \tfrac{1}{2} a (3\,\text{s})^2 + a(3\,\text{s})(6.79\,\text{s})$$

and

$$a = \boxed{4.02\,\text{m/s}^2}$$

***46** ••

Determine the Concept This is a constant-acceleration problem with $a = -g$ if we take upward to be the positive direction.

At the maximum height the ball will reach, its speed will be near zero and when the ball has just been tossed in the air its speed is near its maximum value. What conclusion can you draw from the image of the ball near its maximum height?

Because the ball is moving slowly its blur is relatively short (i.e., there is less blurring).

To estimate the initial speed of the ball:

a) Estimate how far the ball being tossed moves in 1/30 s:

The ball moves about 3 ball diameters in 1/30 s.

b) Estimate the diameter of a tennis ball:

The diameter of a tennis ball is approximately 5 cm.

c) Now one can calculate the approximate distance the ball moved in 1/30 s:

$$\text{Distance traveled} = (3\text{ diameters}) \times (5\text{ cm/diameter}) = 15\text{ cm}$$

d) Calculate the average speed of the tennis ball over this distance:

$$\text{Average speed} = \frac{15\text{ cm}}{\frac{1}{30}\text{ s}} = 450\text{ cm/s} = 4.50\text{ m/s}$$

e) Because the time interval is very short, the average speed of the ball is a good approximation to its initial speed:

$\therefore\ v_0 = 4.5$ m/s

f) Finally, use the constant-acceleration equation $v^2 = v_0^2 + 2a\Delta y$ to solve for and evaluate Δy:

$$\Delta y = \frac{-v_0^2}{2a} = \frac{-(4.5\text{ m/s})^2}{2(-9.81\text{ m/s}^2)} = \boxed{1.03\text{ m}}$$

Remarks: **This maximum height is in good agreement with the height of the higher ball in the photograph.**

***47 ••**

Picture the Problem The average speed of a nerve impulse is approximately 120 m/s. Assume an average height of 1.7 m and use the definition of average speed to estimate the travel time for the nerve impulse.

Using the definition of average speed, express the travel time for the nerve impulse:

$$\Delta t = \frac{\Delta x}{v_{\text{av}}}$$

Substitute numerical values and evaluate Δ*t*:

$$\Delta t = \frac{1.7\text{ m}}{120\text{ m/s}} = \boxed{14.2\text{ ms}}$$

Speed, Displacement, and Velocity

48 •

Picture the Problem Think of the electron as traveling in a straight line at constant speed and use the definition of average speed.

(*a*) Using its definition, express the average speed of the electron:

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time of flight}} = \frac{\Delta s}{\Delta t}$$

Solve for and evaluate the time of flight:

$$\Delta t = \frac{\Delta s}{\text{Average speed}} = \frac{0.16\text{ m}}{4\times 10^{7}\text{ m/s}} = 4\times 10^{-9}\text{s} = \boxed{4.00\text{ ns}}$$

(*b*) Calculate the time of flight for an electron in a 16-cm long current carrying wire similarly.

$$\Delta t = \frac{\Delta s}{\text{Average speed}} = \frac{0.16\text{ m}}{4\times 10^{-5}\text{ m/s}} = 4\times 10^{3}\text{s} = \boxed{66.7\text{ min}}$$

***49** •

Picture the Problem In this problem the runner is traveling in a straight line but not at constant speed - first she runs, then she walks. Let's choose a coordinate system in which her initial direction of motion is taken as the positive *x* direction.

(*a*) Using the definition of average velocity, calculate the average velocity for the first 9 min:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{2.5\text{ km}}{9\text{ min}} = \boxed{0.278\text{ km/min}}$$

(*b*) Using the definition of average velocity, calculate her average speed for the 30 min spent walking:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{-2.5\text{ km}}{30\text{ min}} = \boxed{-0.0833\text{ km/min}}$$

(*c*) Express her average velocity for the whole trip:

$$v_{\text{av}} = \frac{\Delta x_{\text{round trip}}}{\Delta t} = \frac{0}{\Delta t} = \boxed{0}$$

(*d*) Finally, express her average speed for the whole trip:

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{2(2.5\text{ km})}{30\text{ min} + 9\text{ min}} = \boxed{0.128\text{ km/min}}$$

50 •

Picture the Problem The car is traveling in a straight line but not at constant speed. Let the direction of motion be the positive *x* direction.

(*a*) Express the total displacement of the car for the entire trip:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2$$

Find the displacement for each leg of the trip:

$$\Delta x_1 = v_{av,1}\Delta t_1 = (80\text{ km/h})(2.5\text{ h}) = 200\,\text{km}$$

and

$$\Delta x_2 = v_{av,2}\Delta t_2 = (40\text{ km/h})(1.5\text{ h}) = 60.0\,\text{km}$$

Add the individual displacements to get the total displacement:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = 200\,\text{km} + 60.0\,\text{km} = \boxed{260\,\text{km}}$$

(*b*) As long as the car continues to move in the same direction, the average velocity for the total trip is given by:

$$v_{av} \equiv \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{260\,\text{km}}{2.5\text{ h} + 1.5\text{ h}} = \boxed{65.0\,\text{km/h}}$$

51 •

Picture the Problem However unlikely it may seem, imagine that both jets are flying in a straight line at constant speed.

(*a*) The time of flight is the ratio of the distance traveled to the speed of the supersonic jet.

$$t_{\text{supersonic}} = \frac{s_{\text{Atlantic}}}{\text{speed}_{\text{supersonic}}} = \frac{5500\text{ km}}{2(0.340\,\text{km/s})(3600\,\text{s/h})} = \boxed{2.25\text{ h}}$$

(*b*) The time of flight is the ratio of the distance traveled to the speed of the subsonic jet.

$$t_{\text{subsonic}} = \frac{s_{\text{Atlantic}}}{\text{speed}_{\text{subsonic}}} = \frac{5500\text{ km}}{0.9(0.340\,\text{km/s})(3600\,\text{s/h})} = \boxed{4.99\,\text{h}}$$

(*c*) Adding 2 h on both the front and the back of the supersonic trip, we obtain the average speed of the supersonic flight.

$$\text{speed}_{\text{av, supersonic}} = \frac{5500\,\text{km}}{2.25\,\text{h} + 4.00\,\text{h}} = \boxed{880\,\text{km/h}}$$

(*d*) Adding 2 h on both the front and the back of the subsonic trip, we obtain the average speed of the subsonic flight.

$$\text{speed}_{\text{av, subsonic}} = \frac{5500\,\text{km}}{5.00\,\text{h} + 4.00\,\text{h}} = \boxed{611\,\text{km/h}}$$

***52** •

Picture the Problem In free space, light travels in a straight line at constant speed, *c*.

(*a*) Using the definition of average speed, solve for and evaluate the time required for light to travel from the sun to the earth:

$$\text{average speed} = \frac{s}{t}$$

and

$$t = \frac{s}{\text{average speed}} = \frac{1.5\times10^{11}\text{ m}}{3\times10^{8}\text{ m/s}}$$

$$= 500\text{ s} = \boxed{8.33\text{ min}}$$

(*b*) Proceed as in (*a*) this time using the moon-earth distance:

$$t = \frac{3.84\times10^{8}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{1.28\text{ s}}$$

(*c*) One light-year is the distance light travels in a vacuum in one year:

$$1\text{ light - year} = 9.48\times10^{15}\text{ m} = \boxed{9.48\times10^{12}\text{ km}}$$

$$= \left(9.48\times10^{12}\text{ km}\right)\left(1\text{ mi}/1.61\text{ km}\right)$$

$$= \boxed{5.89\times10^{12}\text{ mi}}$$

53 •

Picture the Problem In free space, light travels in a straight line at constant speed, *c*.

(*a*) Using the definition of average speed (equal here to the assumed constant speed of light), solve for the time required to travel the distance to Proxima Centauri:

$$t = \frac{\text{distance traveled}}{\text{speed of light}} = \frac{4.1\times10^{16}\text{ m}}{3\times10^{8}\text{ m/s}}$$

$$= 1.37\times10^{8}\text{ s} = \boxed{4.33\text{ y}}$$

(*b*) Traveling at $10^{-4}c$, the delivery time (t_{total}) will be the sum of the time for the order to reach Hoboken and the time for the pizza to be delivered to Proxima Centauri:

$$t_{\text{total}} = t_{\text{order to be sent to Hoboken}} + t_{\text{order to be delivered}}$$

$$= 4.33\,y + \frac{4.1\times10^{13}\text{ km}}{\left(10^{-4}\right)\left(3\times10^{8}\text{ m/s}\right)}$$

$$= 4.33\text{ y} + 4.33\times10^{6}\text{ y}$$

$$\approx 4.33\times10^{6}\text{ y}$$

$$\boxed{\text{Since } 4.33\times10^{6}\text{ y} >> 1000\text{ y, Gregor does not have to pay.}}$$

54 •

Picture the Problem The time for the second 50 km is equal to the time for the entire journey less the time for the first 50 km. We can use this time to determine the average speed for the second 50 km interval from the definition of average speed.

Using the definition of average speed, find the time required for the total journey:

$$t_{\text{total}} = \frac{\text{total distance}}{\text{average speed}} = \frac{100\text{ km}}{50\text{ km/h}} = 2\text{ h}$$

Find the time required for the first 50 km:

$$t_{\text{1st 50 km}} = \frac{50\,\text{km}}{40\,\text{km/h}} = 1.25\,\text{h}$$

Find the time remaining to travel the last 50 km:

$$t_{\text{2nd 50 km}} = t_{\text{total}} - t_{\text{1st 50 km}} = 2\,\text{h} - 1.25\,\text{h} = 0.75\,\text{h}$$

Finally, use the time remaining to travel the last 50 km to determine the average speed over this distance:

$$\text{Average speed}_{\text{2nd 50 km}} = \frac{\text{distance traveled}_{\text{2nd 50 km}}}{\text{time}_{\text{2nd 50 km}}} = \frac{50\,\text{km}}{0.75\,\text{h}} = \boxed{66.7\,\text{km/h}}$$

***55 ••**

Picture the Problem Note that both the arrow and the sound travel a distance *d*. We can use the relationship between distance traveled, the speed of sound, the speed of the arrow, and the elapsed time to find the distance separating the archer and the target.

Express the elapsed time between the archer firing the arrow and hearing it strike the target:

$$\Delta t = 1\,\text{s} = \Delta t_{\text{arrow}} + \Delta t_{\text{sound}}$$

Express the transit times for the arrow and the sound in terms of the distance, *d*, and their speeds:

$$\Delta t_{\text{arrow}} = \frac{d}{\left|v_{\text{arrow}}\right|} = \frac{d}{40\,\text{m/s}}$$

and

$$\Delta t_{\text{sound}} = \frac{d}{\left|v_{\text{sound}}\right|} = \frac{d}{340\,\text{m/s}}$$

Substitute these two relationships in the expression obtained in step 1 and solve for *d*:

$$\frac{d}{40\,\text{m/s}} + \frac{d}{340\,\text{m/s}} = 1\,\text{s}$$

and $d = \boxed{35.8\,\text{m}}$

56 ••

Picture the Problem Assume both runners travel parallel paths in a straight line along the track.

(*a*) Using the definition of average speed, find the time for Marcia:

$$t_{\text{Marcia}} = \frac{\text{distance run}}{\text{Marcia's speed}} = \frac{\text{distance run}}{1.15(\text{John's speed})} = \frac{100\,\text{m}}{1.15(6\,\text{m/s})} = 14.5\,\text{s}$$

Find the distance covered by John in 14.5 s and the difference between that distance and 100 m:

$$x_{\text{John}} = (6\,\text{m/s})(14.5\,\text{s}) = 87.0\,\text{m}$$

and Marcia wins by

$$100\,\text{m} - 87\,\text{m} = \boxed{13.0\,\text{m}}$$

(*b*) Using the definition of average speed, find the time required by John to complete the 100-m run:

$$t_{\text{John}} = \frac{\text{distance run}}{\text{John's speed}} = \frac{100\,\text{m}}{6\,\text{m/s}} = 16.7\,\text{s}$$

Marsha wins by 16.7 s – 14.5 s = 2.2 s

Alternatively, the time required by John to travel the last 13.0 m is

$(13\text{ m})/(6\text{ m/s}) = \boxed{2.17\,\text{s}}$

57 •

Picture the Problem The average velocity in a time interval is defined as the displacement divided by the time elapsed; that is $v_{\text{av}} = \Delta x/\Delta t$.

(*a*) $\Delta x_a = 0$

$$v_{\text{av}} = \boxed{0}$$

(*b*) $\Delta x_b = 1$ m and $\Delta t_b = 3$ s

$$v_{\text{av}} = \boxed{0.333\,\text{m/s}}$$

(*c*) $\Delta x_c = -6$ m and $\Delta t_c = 3$ s

$$v_{\text{av}} = \boxed{-2.00\,\text{m/s}}$$

(*d*) $\Delta x_d = 3$ m and $\Delta t_d = 3$ s

$$v_{\text{av}} = \boxed{1.00\,\text{m/s}}$$

58 ••

Picture the Problem In free space, light travels in a straight line at constant speed *c*. We can use Hubble's law to find the speed of the two planets.

(*a*) Using Hubble's law, calculate the speed of the first galaxy:

$$v_{\text{a}} = \left(5\times10^{22}\,\text{m}\right)\left(1.58\times10^{-18}\,\text{s}^{-1}\right) = \boxed{7.90\times10^{4}\,\text{m/s}}$$

(*b*) Using Hubble's law, calculate the speed of the second galaxy:

$$v_{\text{b}} = \left(2\times10^{25}\,\text{m}\right)\left(1.58\times10^{-18}\,\text{s}^{-1}\right) = \boxed{3.16\times10^{7}\,\text{m/s}}$$

(*c*) Using the relationship between distance, speed, and time for both galaxies, determine how long ago they were both located at the same place as the earth:

$$t = \frac{r}{v} = \frac{r}{rH} = \frac{1}{H} = 6.33\times10^{17}\,\text{s} = 20.1\times10^{9}\,\text{y} = \boxed{20.1\,\text{billion years}}$$

***59 ••**

Picture the Problem Ignoring the time intervals during which members of this relay time get up to their running speeds, their accelerations are zero and their average speed can be found from its definition.

Using its definition, relate the average speed to the total distance traveled and the elapsed time:

$$|v_{av}| = \frac{\text{distance traveled}}{\text{elapsed time}}$$

Express the time required for each animal to travel a distance L:

$$t_{cheetah} = \frac{L}{v_{cheetah}},$$

$$t_{falcon} = \frac{L}{v_{falcon}},$$

and

$$t_{sailfish} = \frac{L}{v_{sailfish}}$$

Express the total time, Δt:

$$\Delta t = L\left(\frac{1}{v_{cheetah}} + \frac{1}{v_{falcon}} + \frac{1}{v_{sailfish}}\right)$$

Use the total distance traveled by the relay team and the elapsed time to calculate the average speed:

$$|v_{av}| = \frac{3L}{L\left(\frac{1}{113\,\text{km/h}} + \frac{1}{161\,\text{km/h}} + \frac{1}{105\,\text{km/h}}\right)} = \boxed{122\,\text{km/h}}$$

Calculate the average of the three speeds:

$$\text{Average}_{\text{three speeds}} = \frac{113\,\text{km/h} + 161\,\text{km/h} + 105\,\text{km/h}}{3} = \boxed{126\,\text{km/h} = 1.03 v_{av}}$$

60 ••

Picture the Problem Perhaps the easiest way to solve this problem is to think in terms of the relative velocity of one car relative to the other. Solve this problem from the reference frame of car A. In this frame, car A remains at rest.

Find the velocity of car B relative to car A:

$$v_{rel} = v_B - v_A = (110 - 80)\text{ km/h} = 30\text{ km/h}$$

Find the time before car B reaches car A:

$$\Delta t = \frac{\Delta x}{v_{rel}} = \frac{45\,\text{km}}{30\,\text{km/h}} = 1.5\,\text{h}$$

Find the distance traveled, relative to the road, by car A in 1.5 h:

$$d = (1.5\,\text{h})(80\,\text{km/h}) = \boxed{120\,\text{km}}$$

***61 ••**

Picture the Problem One way to solve this problem is by using a graphing calculator to plot the positions of each car as a function of time. Plotting these positions as functions of time allows us to visualize the motion of the two cars relative to the (fixed) ground. More importantly, it allows us to see the motion of the two cars relative to each other. We can, for example, tell how far apart the cars are at any given time by determining the length of a vertical line segment from one curve to the other.

(*a*) Letting the origin of our coordinate system be at the intersection, the position of the slower car, $x_1(t)$, is given by:

$x_1(t) = 20t$
where x_1 is in meters if t is in seconds.

Because the faster car is also moving at a constant speed, we know that the position of this car is given by a function of the form:

$x_2(t) = 30t + b$

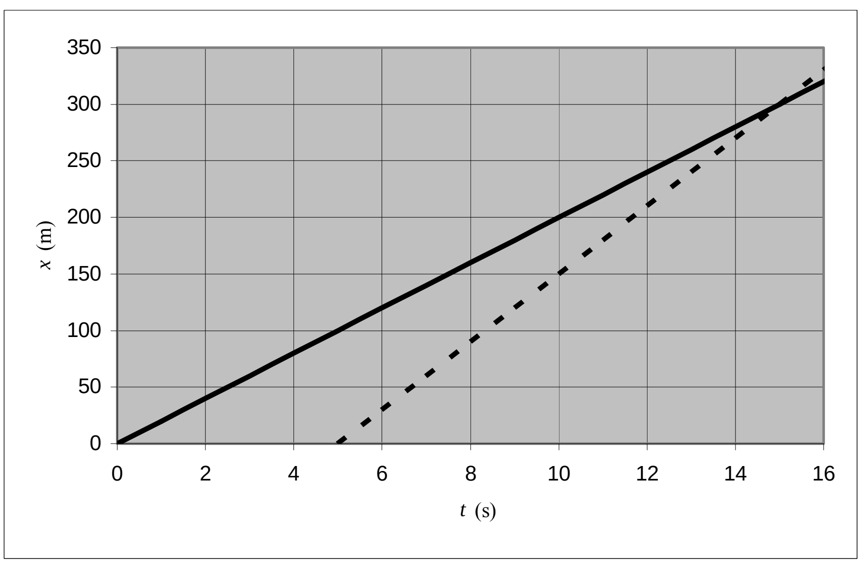
We know that when t = 5 s, this second car is at the intersection (i.e., $x_2(5\text{ s}) = 0$). Using this information, you can convince yourself that:

$b = -150$ m

Thus, the position of the faster car is given by:

$$x_2(t) = 30t - 150$$

One can use a graphing calculator, graphing paper, or a spreadsheet to obtain the graphs of $x_1(t)$ (the solid line) and $x_2(t)$ (the dashed line) shown below:



(*b*) Use the time coordinate of the intersection of the two lines to determine the time at which the second car overtakes the first:

From the intersection of the two lines, one can see that the second car will "overtake" (catch up to) the first car at $\boxed{t = 15\text{ s}}$.

(*c*) Use the position coordinate of the intersection of the two lines to determine the distance from the intersection at which the second car catches up to the first car:

From the intersection of the two lines, one can see that the distance from the intersection is $\boxed{300\text{ m}}$.

(*d*) Draw a vertical line from $t = 5$ s to the red line and then read the position coordinate of the intersection of this line and the red line to determine the position of the first car when the second car went through the intersection:

From the graph, when the second car passes the intersection, the first car was $\boxed{100\text{ m ahead}}$.

62 •

Picture the Problem Sally's velocity relative to the ground (v_{SG}) is the sum of her velocity relative to the moving belt (v_{SB}) and the velocity of the belt relative to the ground (v_{BG}). Joe's velocity relative to the ground is the same as the velocity of the belt relative to the ground. Let D be the length of the moving sidewalk.

Express D in terms of v_{BG} (Joe's speed relative to the ground):

$$D = (2\text{ min})v_{BG}$$

Solve for v_{BG}:

$$v_{BG} = \frac{D}{2\text{ min}}$$

Express D in terms of $v_{BG} + v_{SG}$ (Sally's speed relative to the ground):

$$D = (1\text{ min})(v_{BG} + v_{SG}) = (1\text{ min})\left(\frac{D}{2\text{ min}} + v_{SG}\right)$$

Solve for v_{SG}:

$$v_{SG} = \frac{D}{1\text{ min}} - \frac{D}{2\text{ min}} = \frac{D}{2\text{ min}}$$

Express D in terms of $v_{BG} + 2v_{SB}$ (Sally's speed for a fast walk relative to the ground):

$$D = t_f(v_{BG} + 2v_{SB}) = t_f\left(\frac{D}{2\text{ min}} + \frac{2D}{2\text{ min}}\right) = t_f\left(\frac{3D}{2\text{ min}}\right)$$

Solve for t_f as time for Sally's fast walk:

$$t_f = \frac{2\text{ min}}{3} = \boxed{40.0\text{ s}}$$

63 ••

Picture the Problem The speed of Margaret's boat relative to the riverbank ($\left|v_{\text{BR}}\right|$) is the sum or difference of the speed of her boat relative to the water ($\left|v_{\text{BW}}\right|$) and the speed of the water relative to the riverbank ($\left|v_{WR}\right|$), depending on whether she is heading with or against the current. Let D be the distance to the marina.

Express the total time for the trip:

$$t_{\text{tot}} = t_1 + t_2$$

Express the times of travel with the motor running in terms of D, $\left|v_{WR}\right|$ and $\left|v_{\text{BW}}\right|$:

$$t_1 = \frac{D}{\left|v_{\text{BW}}\right| - \left|v_{\text{WR}}\right|} = 4\,\text{h}$$

and

$$t_2 = \frac{D}{\left|v_{\text{BW}}\right| + \left|v_{\text{WR}}\right|}$$

Express the time required to drift distance D and solve for $\left|v_{\text{WR}}\right|$:

$$t_3 = \frac{D}{\left|v_{\text{WR}}\right|} = 8\,\text{h}$$

and

$$\left|v_{\text{WR}}\right| = \frac{D}{8\,\text{h}}$$

From $t_1 = 4$ h, find $\left|v_{\text{BW}}\right|$:

$$\left|v_{\text{BW}}\right| = \frac{D}{4\ \text{h}} + \left|v_{\text{WR}}\right| = \frac{D}{4\ \text{h}} + \frac{D}{8\ \text{h}} = \frac{3D}{8\ \text{h}}$$

Solve for t_2:

$$t_2 = \frac{D}{\left|v_{\text{BW}}\right| + \left|v_{\text{WR}}\right|} = \frac{D}{\frac{3D}{8\,\text{h}} + \frac{D}{8\,\text{h}}} = 2\,\text{h}$$

Add t_1 and t_2 to find the total time:

$$t_{\text{tot}} = t_1 + t_2 = \boxed{6\,\text{h}}$$

Acceleration

64 •

Picture the Problem In part (*a*), we can apply the definition of average acceleration to find a_{av}. In part (*b*), we can find the change in the car's velocity in one second and add this change to its velocity at the beginning of the interval to find its speed one second later.

(*a*) Apply the definition of average acceleration:

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{80.5\,\text{km/h} - 48.3\,\text{km/h}}{3.7\,\text{s}}$$

$$= 8.70\frac{\text{km}}{\text{h}\cdot\text{s}}$$

Convert to m/s²:

$$a_{av} = \left(8.70\times10^3\,\frac{\text{m}}{\text{h}\cdot\text{s}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{2.42\,\text{m/s}^2}$$

(*b*) Express the speed of the car at the end of 4.7 s:

$$v(4.7\,\text{s}) = v(3.7\,\text{s}) + \Delta v_{1\,\text{s}} = 80.5\,\text{km/h} + \Delta v_{1\,\text{s}}$$

Find the change in the speed of the car in 1 s:

$$\Delta v = a_{av}\Delta t = \left(8.70\,\frac{\text{km}}{\text{h}\cdot\text{s}}\right)(1\,\text{s}) = 8.70\,\text{km/h}$$

Substitute and evaluate v(4.7 s):

$$v(4.7\,\text{s}) = 80.5\,\text{km/h} + 8.7\,\text{km/h} = \boxed{89.2\,\text{km/h}}$$

65 •

Picture the Problem Average acceleration is defined as $a_{av} = \Delta v/\Delta t$.

The average acceleration is defined as the change in velocity divided by the change in time:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{(-1\text{m/s}) - (5\text{m/s})}{(8\text{s}) - (5\text{s})} = \boxed{-2.00\,\text{m/s}^2}$$

66 ••

Picture the Problem The important concept here is the difference between average acceleration and instantaneous acceleration.

(*a*) The average acceleration is defined as the change in velocity divided by the change in time:

$a_{av} = \Delta v/\Delta t$

Determine v at t = 3 s, t = 4 s, and t = 5 s:

v(3 s) = 17 m/s
v(4 s) = 25 m/s
v(5 s) = 33 m/s

Find a_{av} for the two 1-s intervals:

a_{av}(3 s to 4 s) = (25 m/s – 17 m/s)/(1 s) = 8 m/s²

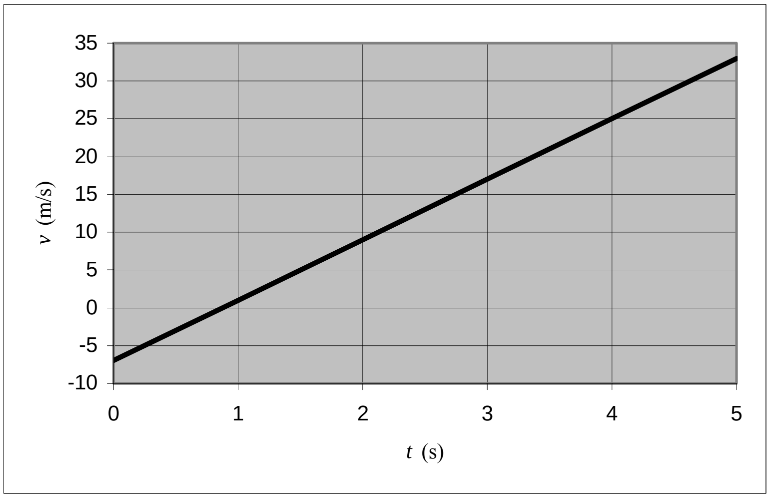
and

a_{av}(4 s to 5 s) = (33 m/s – 25 m/s)/(1 s) = 8 m/s²

The instantaneous acceleration is defined as the time derivative of the velocity or the slope of the velocity-versus-time curve:

$$a = \frac{dv}{dt} = \boxed{8.00\,\text{m/s}^2}$$

(*b*) The given function was used to plot the following spreadsheet-graph of *v*-versus-*t*:



67 ••

Picture the Problem We can closely approximate the instantaneous velocity by the average velocity in the limit as the time interval of the average becomes small. This is important because all we can ever obtain from any measurement is the average velocity, v_{av}, which we use to approximate the instantaneous velocity v.

(*a*) Find $x(4\text{ s})$ and $x(3\text{ s})$:

$x(4\text{ s}) = (4)^2 - 5(4) + 1 = -3\text{ m}$
and
$x(3\text{ s}) = (3)^2 - 5(3) + 1 = -5\text{ m}$

Find Δx:

$$\Delta x = x(4\text{ s}) - x(3\text{ s}) = (-3\text{ m}) - (-5\text{ m}) = \boxed{2\text{ m}}$$

Use the definition of average velocity:

$$v_{av} = \Delta x/\Delta t = (2\text{ m})/(1\text{ s}) = \boxed{2\text{ m/s}}$$

(*b*) Find $x(t + \Delta t)$:

$$x(t+\Delta t) = (t+\Delta t)^2 - 5(t+\Delta t) + 1 = (t^2 + 2t\Delta t + (\Delta t)^2) - 5(t+\Delta t) + 1$$

Express $x(t + \Delta t) - x(t) = \Delta x$:

$$\Delta x = \boxed{(2t-5)\Delta t + (\Delta t)^2}$$

where Δx is in meters if t is in seconds.

(*c*) From (*b*) find $\Delta x/\Delta t$ as $\Delta t \to 0$:

$$\frac{\Delta x}{\Delta t} = \frac{(2t-5)\Delta t + (\Delta t)^2}{\Delta t} = 2t - 5 + \Delta t$$

and

$$v = \lim_{\Delta t \to 0}(\Delta x / \Delta t) = \boxed{2t - 5}$$

where v is in m/s if t is in seconds.

Alternatively, we can take the derivative of $x(t)$ with respect to time to obtain the instantaneous velocity.

$$v(t) = dx(t)/dt = \frac{d}{dt}(at^2 + bt + 1)$$
$$= 2at + b = 2t - 5$$

***68 ••**

Picture the Problem The instantaneous velocity is dx/dt and the acceleration is dv/dt.

Using the definitions of instantaneous velocity and acceleration, determine v and a:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[At^2 - Bt + C\right] = 2At - B$$

and

$$a = \frac{dv}{dr} = \frac{d}{dt}\left[2At - B\right] = 2A$$

Substitute numerical values for A and B and evaluate v and a:

$$v = 2(8\text{m/s}^2)t - 6\text{ m/s}$$
$$= \boxed{(16\text{ m/s}^2)t - 6\text{m/s}}$$

and

$$a = 2(8\text{ m/s}^2) = \boxed{16.0\text{ m/s}^2}$$

69 ••

Picture the Problem We can use the definition of average acceleration ($a_{av} = \Delta v/\Delta t$) to find a_{av} for the three intervals of constant acceleration shown on the graph.

(*a*) Using the definition of average acceleration, find a_{av} for the interval AB:

$$a_{\text{av, AB}} = \frac{15\text{ m/s} - 5\text{ m/s}}{3\text{ s}} = \boxed{3.33\text{ m/s}^2}$$

Find a_{av} for the interval BC:

$$a_{\text{av, BC}} = \frac{15\text{ m/s} - 15\text{ m/s}}{3\text{ s}} = \boxed{0}$$

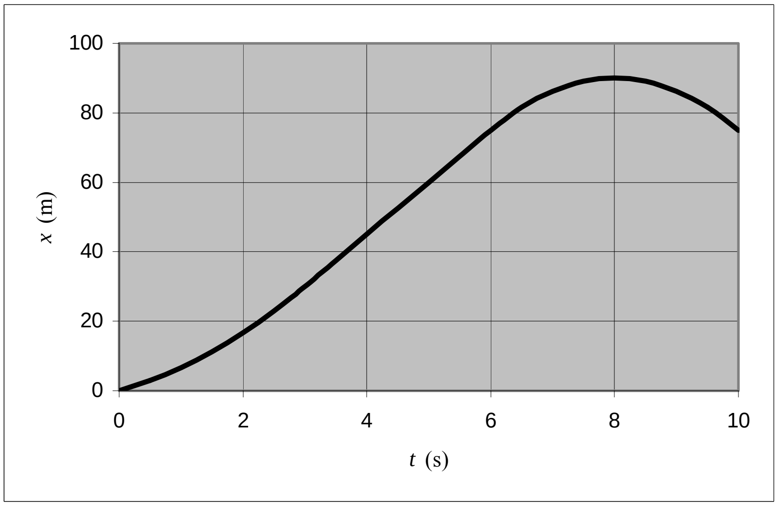
Find a_{av} for the interval CE:

$$a_{\text{av, CE}} = \frac{-15\text{ m/s} - 15\text{m/s}}{4\text{ s}} = \boxed{-7.50\text{m/s}^2}$$

(*b*) Use the formulas for the areas of trapezoids and triangles to find the area under the graph of v as a function of t.

$$\Delta x = (\Delta x)_{A\to B} + (\Delta x)_{B\to C} + (\Delta x)_{C\to D} + (\Delta x)_{D\to E}$$
$$= \tfrac{1}{2}(5\,\text{m/s} + 15\,\text{m/s})(3\,\text{s}) + (15\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}(15\,\text{m/s})(2\,\text{s}) + \tfrac{1}{2}(-15\,\text{m/s})(2\,\text{s})$$
$$= \boxed{75.0\,\text{m}}$$

(*c*) The graph of displacement, *x*, as a function of time, *t*, is shown in the following figure. In the region from B to C the velocity is constant so the *x*- versus-*t* curve is a straight line.



(*d*) Reading directly from the figure, we can find the time when the particle is moving the slowest.

$\boxed{\text{At point D, } t = 8\text{ s, the graph crosses the time axis; therefore, } v = 0.}$

Constant Acceleration and Free-Fall

***70** •

Picture the Problem Because the acceleration is constant (–*g*) we can use a constant-acceleration equation to find the height of the projectile.

Using a constant-acceleration equation, express the height of the object as a function of its initial velocity, the acceleration due to gravity, and its displacement:

$$v^2 = v_0^2 + 2a\Delta y$$

Solve for $\Delta y_{max} = h$:

Because $v(h) = 0$,

$$h = \frac{-v_0^2}{2(-g)} = \frac{v_0^2}{2g}$$

From this expression for *h* we see that the maximum height attained is proportional to the square of the launch speed:

$$h \propto v_0^2$$

Therefore, doubling the initial speed gives four times the height:

$$h_{2v_0} = \frac{(2v_0)^2}{2g} = 4\left(\frac{v_0^2}{2g}\right) = 4h_{v_0}$$

and $\boxed{(a)\,\text{is correct.}}$

71 •

Picture the Problem Because the acceleration of the car is constant we can use constant-acceleration equations to describe its motion.

(*a*) Using a constant-acceleration equation, relate the velocity to the acceleration and the time:

$$v = v_0 + at = 0 + (8\,\text{m/s}^2)(10\,\text{s})$$
$$= \boxed{80.0\,\text{m/s}}$$

(*b*) sing a constant-acceleration equation, relate the displacement to the acceleration and the time:

$$\Delta x = x - x_0 = v_0 t + \frac{a}{2}t^2$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{1}{2}(8\,\text{m/s}^2)(10\,\text{s})^2 = \boxed{400\,\text{m}}$$

(*c*) Use the definition of v_{av}:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{400\,\text{m}}{10\,\text{s}} = \boxed{40.0\,\text{m/s}}$$

Remarks: Because the area under a velocity-versus-time graph is the displacement of the object, we could solve this problem graphically.

72 •

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\,\Delta x$$

Solve for and evaluate the displacement:

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{(15^2 - 5^2)\text{m}^2/\text{s}^2}{2(2\,\text{m/s}^2)}$$
$$= \boxed{50.0\,\text{m}}$$

***73** •

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\,\Delta x$$

Solve for the acceleration:

$$a=\frac{v^2-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate *a*:

$$a=\frac{\left(15^2-10^2\right)\text{m}^2/\text{s}^2}{2(4\,\text{m})}=\boxed{15.6\,\text{m/s}^2}$$

74 •

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2=v_0^2+2\,a\,\Delta x$$

Solve for and evaluate *v*:

$$v=\sqrt{(1\,\text{m/s})^2+2\left(4\,\text{m/s}^2\right)(1\,\text{m})}$$
$$=\boxed{3.00\,\text{m/s}}$$

Using the definition of average acceleration, solve for the time:

$$t=\frac{\Delta v}{a_{\text{av}}}=\frac{3\,\text{m/s}-1\,\text{m/s}}{4\,\text{m/s}^2}=\boxed{0.500\,\text{s}}$$

75 ••

Picture the Problem In the absence of air resistance, the ball experiences constant acceleration. Choose a coordinate system with the origin at the point of release and the positive direction upward.

(*a*) Using a constant-acceleration equation, relate the displacement of the ball to the acceleration and the time:

$$\Delta y=v_0t+\tfrac{1}{2}at^2$$

Setting Δy = 0 (the displacement for a round trip), solve for and evaluate the time required for the ball to return to its starting position:

$$t_{\text{round trip}}=\frac{2v_0}{g}=\frac{2(20\,\text{m/s})}{9.81\,\text{m/s}^2}=\boxed{4.08\,\text{s}}$$

(*b*) Using a constant-acceleration equation, relate the final speed of the ball to its initial speed, the acceleration, and its displacement:

$$v_{\text{top}}^2=v_0^2+2a\Delta y$$

or, because $v_{\text{top}}=0$ and $a=-g$,

$$0=v_0^2+2(-g)H$$

Solve for and evaluate *H*:

$$H=\frac{v_0^2}{2g}=\frac{(20\,\text{m/s})^2}{2\left(9.81\,\text{m/s}^2\right)}=\boxed{20.4\,\text{m}}$$

(*c*) Using the same constant-acceleration equation with which we began part (*a*), express the displacement as a function of time:

$$\Delta y=v_0t+\tfrac{1}{2}at^2$$

Substitute numerical values to obtain:

$$15\,\text{m} = (20\,\text{m/s})t - \left(\frac{9.81\,\text{m/s}^2}{2}\right)t^2$$

Solve the quadratic equation for the times at which the displacement of the ball is 15 m:

The solutions are $t = \boxed{0.991\,\text{s}}$ (this corresponds to passing 15 m on the way up) and $t = \boxed{3.09\,\text{s}}$ (this corresponds to passing 15 m on the way down).

76 ••

Picture the Problem This is a multipart constant-acceleration problem using two different constant accelerations. We'll choose a coordinate system in which downward is the positive direction and apply constant-acceleration equations to find the required times.

(*a*) Using a constant-acceleration equation, relate the time for the slide to the distance of fall and the acceleration:

$$\Delta y = y - y_0 = h - 0 = v_0 t_1 + \tfrac{1}{2}at_1^2$$

or, because $v_0 = 0$,

$$h = \tfrac{1}{2}at_1^2$$

Solve for t_1:

$$t_1 = \sqrt{\frac{2h}{g}}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \sqrt{\frac{2(460\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{9.68\,\text{s}}$$

(*b*) Using a constant-acceleration equation, relate the velocity at the bottom of the mountain to the acceleration and time:

$$v_1 = v_0 + a_1 t_1$$

or, because $v_0 = 0$ and $a_1 = g$,

$$v_1 = g t_1$$

Substitute numerical values and evaluate v_1:

$$v_1 = (9.81\,\text{m/s}^2)(9.68\,\text{s}) = \boxed{95.0\,\text{m/s}}$$

(*c*) Using a constant-acceleration equation, relate the time required to stop the mass of rock and mud to its average speed and the distance it slides:

$$\Delta t = \frac{\Delta x}{v_{\text{av}}}$$

Because the acceleration is constant:

$$v_{\text{av}} = \frac{v_1 + v_{\text{f}}}{2} = \frac{v_1 + 0}{2} = \frac{v_1}{2}$$

Substitute to obtain:

$$\Delta t = \frac{2\Delta x}{v_1}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(8000\,\text{m})}{95.0\,\text{m/s}} = \boxed{168\,\text{s}}$$

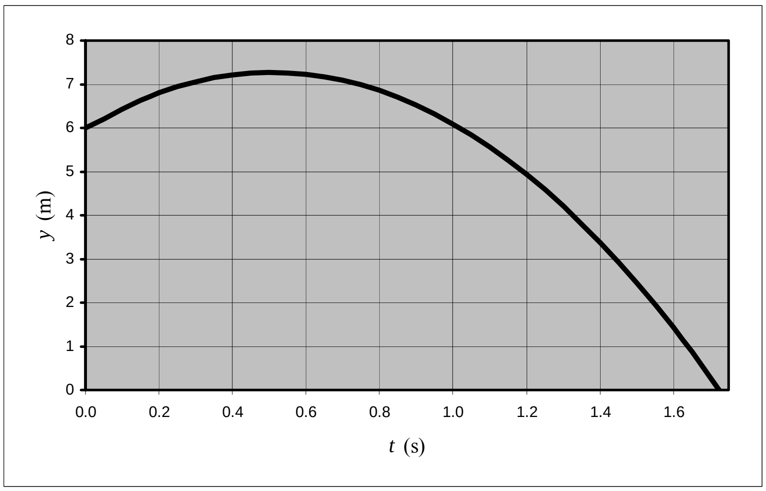
***77** ••

Picture the Problem In the absence of air resistance, the brick experiences constant acceleration and we can use constant-acceleration equations to describe its motion. Constant acceleration implies a parabolic position-versus-time curve.

(*a*) Using a constant-acceleration equation, relate the position of the brick to its initial position, initial velocity, acceleration, and time into its fall:

$$y = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$
$$= 6\,\text{m} + (5\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2$$

The following graph of $y = 6\,\text{m} + (5\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2$ was plotted using a spreadsheet program:



(*b*) Relate the greatest height reached by the brick to its height when it falls off the load and the additional height it rises Δy_{max}:

$$h = y_0 + \Delta y_{max}$$

Using a constant-acceleration equation, relate the height reached by the brick to its acceleration and initial velocity:

$$v_{top}^2 = v_0^2 + 2(-g)\Delta y_{max}$$

or, because $v_{top} = 0$,

$$0 = v_0^2 + 2(-g)\Delta y_{max}$$

Solve for Δy_{max}:

$$\Delta y_{max} = \frac{v_0^2}{2g}$$

Substitute numerical values and evaluate Δy_{max}:

$$\Delta y_{max} = \frac{(5\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 1.27\,\text{m}$$

Substitute to obtain:

$$h = y_0 + \Delta y_{max} = 6\,\text{m} + 1.27\,\text{m} = \boxed{7.27\,\text{m}}$$

Note: The graph shown above confirms this result.

(*c*) Using the quadratic formula, solve for *t* in the equation obtained in part (*a*):

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{-g}{2}\right)(-\Delta y)}}{2\left(\frac{-g}{2}\right)}$$

$$= \left(\frac{v_0}{g}\right)\left(1 \pm \sqrt{1 - \frac{2g(\Delta y)}{v_0^2}}\right)$$

With y_{bottom} = 0 and y_o = 6 m or Δy = –6 m, we obtain:

$t = \boxed{1.73\,\text{s}}$ and t = –0.708 s. Note: The second solution is nonphysical.

(*d*) Using a constant-acceleration equation, relate the speed of the brick on impact to its acceleration and displacement, and solve for its speed:

$$v = \sqrt{2gh}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(7.27\,\text{m})} = \boxed{11.9\,\text{m/s}}$$

78 ••

Picture the Problem In the absence of air resistance, the acceleration of the bolt is constant. Choose a coordinate system in which upward is positive and the origin is at the bottom of the shaft (y = 0).

(*a*) Using a constant-acceleration equation, relate the position of the bolt to its initial position, initial velocity, and fall time:

$$y_{bottom} = 0 = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$

Solve for the position of the bolt when it came loose:

$$y_0 = -v_0 t + \tfrac{1}{2} g t^2$$

Substitute numerical values and evaluate y_0:

$$y_0 = -(6\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}(9.81\,\text{m/s}^2)(3\,\text{s})^2 = \boxed{26.1\,\text{m}}$$

(*b*) Using a constant-acceleration equation, relate the speed of the bolt to its initial speed, acceleration, and fall time:

$$v = v_0 + at$$

Substitute numerical values and evaluate $|v|$:

$$v = 6\,\text{m/s} - (9.81\,\text{m/s}^2)(3\text{s}) = -23.4\,\text{m/s}$$

and

$$|v| = \boxed{23.4\,\text{m/s}}$$

***79** ••

Picture the Problem In the absence of air resistance, the object's acceleration is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release. In this coordinate system, $a = g$ and $y = 120$ m at the bottom of the fall.

Express the distance fallen in the last second in terms of the object's position at impact and its position 1 s before impact:

$$\Delta y_{\text{last second}} = 120\text{ m} - y_{1\,\text{s before impact}} \quad (1)$$

Using a constant-acceleration equation, relate the object's position upon impact to its initial position, initial velocity, and fall time:

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

or, because $y_0 = 0$ and $v_0 = 0$,

$$y = \tfrac{1}{2} g t_{\text{fall}}^2$$

Solve for the fall time:

$$t_{\text{fall}} = \sqrt{\frac{2y}{g}}$$

Substitute numerical values and evaluate t_{fall}:

$$t_{\text{fall}} = \sqrt{\frac{2(120\,\text{m})}{9.81\,\text{m/s}^2}} = 4.95\,\text{s}$$

We know that, one second before impact, the object has fallen for 3.95 s. Using the same constant-acceleration equation, calculate the object's position 3.95 s into its fall:

$$y(3.95\text{ s}) = \tfrac{1}{2}(9.81\text{ m/s}^2)(3.95\,\text{s})^2 = 76.4\,\text{m}$$

Substitute in equation (1) to obtain:

$$\Delta y_{\text{last second}} = 120\,\text{m} - 76.4\,\text{m} = \boxed{43.6\,\text{m}}$$

80 ••

Picture the Problem In the absence of air resistance, the acceleration of the object is constant. Choose a coordinate system with the origin at the point of release and downward as the positive direction.

Using a constant-acceleration equation, relate the height to the initial and final velocities and the acceleration; solve for the height:

$$v_{\text{f}}^2 = v_0^2 + 2a\Delta y$$

or, because $v_0 = 0$,

$$h = \frac{v_{\text{f}}^2}{2g} \quad (1)$$

Using the definition of average velocity, find the average velocity of the object during its final second of fall:

$$v_{av} = \frac{v_{f\text{-}1s} + v_f}{2} = \frac{\Delta y}{\Delta t} = \frac{38\,\text{m}}{1\,\text{s}} = 38\,\text{m/s}$$

Express the sum of the final velocity and the velocity 1 s before impact:

$$v_{f\text{-}1s} + v_f = 2(38\,\text{m/s}) = 76\,\text{m/s}$$

From the definition of acceleration, we know that the change in velocity of the object, during 1 s of fall, is 9.81 m/s:

$$\Delta v = v_f - v_{f\text{-}1s} = 9.81\ \text{m/s}$$

Add the equations that express the sum and difference of $v_{f-1\,s}$ and v_f and solve for v_f:

$$v_f = \frac{76\,\text{m/s} + 9.81\,\text{m/s}}{2} = 42.9\,\text{m/s}$$

Substitute in equation (1) and evaluate h:

$$h = \frac{(42.9\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \boxed{93.8\,\text{m}}$$

***81** •

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant. Choose a coordinate system with the origin at the bottom of the trajectory and the upward direction positive. Let $v_{f\text{-}1/2}$ be the speed one-half second before impact and v_f the speed at impact.

Using a constant-acceleration equation, express the final speed of the stone in terms of its initial speed, acceleration, and displacement:

$$v_f^2 = v_0^2 + 2a\Delta y$$

Solve for the initial speed of the stone:

$$v_0 = \sqrt{v_f^2 + 2g\Delta y} \qquad (1)$$

Find the average speed in the last half second:

$$v_{av} = \frac{v_{f\text{-}1/2} + v_f}{2} = \frac{\Delta x_{\text{last half second}}}{\Delta t} = \frac{45\,\text{m}}{0.5\,\text{s}}$$

$$= 90\,\text{m/s}$$

and

$$v_{f\text{-}1/2} + v_f = 2(90\,\text{m/s}) = 180\,\text{m/s}$$

Using a constant-acceleration equation, express the change in speed of the stone in the last half second in terms of the acceleration and the elapsed time; solve for the change in its speed:

$$\Delta v = v_f - v_{f\text{-}1/2} = g\Delta t$$

$$= (9.81\,\text{m/s}^2)(0.5\,\text{s})$$

$$= 4.91\,\text{m/s}$$

Add the equations that express the sum and difference of $v_{f-½}$ and v_f and solve for v_f:

$$v_f = \frac{180\,\text{m/s} + 4.91\,\text{m/s}}{2} = 92.5\,\text{m/s}$$

Substitute in equation (1) and evaluate v_0:

$$v_0 = \sqrt{(92.5\text{m/s})^2 + 2(9.81\text{m/s}^2)(-200\text{m})} = \boxed{68.1\,\text{m/s}}$$

Remarks: The stone may be thrown either up or down from the cliff and the results after it passes the cliff on the way down are the same.

82 ••

Picture the Problem In the absence of air resistance, the acceleration of the object is constant. Choose a coordinate system in which downward is the positive direction and the object starts from rest. Apply constant-acceleration equations to find the average velocity of the object during its descent.

Express the average velocity of the falling object in terms of its initial and final velocities:

$$v_{av} = \frac{v_0 + v_f}{2}$$

Using a constant-acceleration equation, express the displacement of the object during the 1st second in terms of its acceleration and the elapsed time:

$$\Delta y_{1\text{st second}} = \frac{gt^2}{2} = 4.91\,\text{m} = 0.4\,h$$

Solve for the displacement to obtain:

$h = 12.3$ m

Using a constant-acceleration equation, express the final velocity of the object in terms of its initial velocity, acceleration, and displacement:

$$v_f^2 = v_0^2 + 2g\Delta y$$

or, because $v_0 = 0$,

$$v_f = \sqrt{2g\Delta y}$$

Substitute numerical values and evaluate the final velocity of the object:

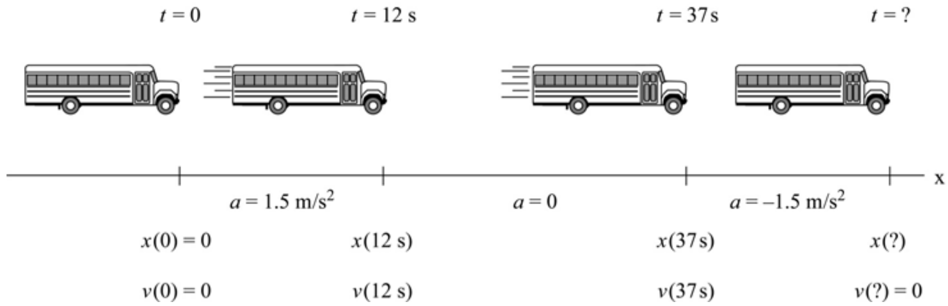
$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(12.3\,\text{m})} = 15.5\,\text{m/s}$$

Substitute in the equation for the average velocity to obtain:

$$v_{av} = \frac{0 + 15.5\,\text{m/s}}{2} = \boxed{7.77\,\text{m/s}}$$

83 ••

Picture the Problem This is a three-part constant-acceleration problem. The bus starts from rest and accelerates for a given period of time, and then it travels at a constant velocity for another period of time, and, finally, decelerates uniformly to a stop. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



(*a*) Express the total displacement of the bus during the three intervals of time.

$$\Delta x_{\text{total}} = \Delta x(0 \to 12\,\text{s}) + \Delta x(12\,\text{s} \to 37\,\text{s}) + \Delta x(37\,\text{s} \to \text{end})$$

Using a constant-acceleration equation, express the displacement of the bus during its first 12 s of motion in terms of its initial velocity, acceleration, and the elapsed time; solve for its displacement:

$$\Delta x(0 \to 12\,\text{s}) = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$,

$$\Delta x(0 \to 12\,\text{s}) = \tfrac{1}{2}at^2 = 108\,\text{m}$$

Using a constant-acceleration equation, express the velocity of the bus after 12 seconds in terms of its initial velocity, acceleration, and the elapsed time; solve for its velocity at the end of 12 s:

$$v_{12\,\text{s}} = v_0 + a_{0\to 12\,\text{s}}\Delta t = (1.5\,\text{m/s}^2)(12\,\text{s}) = 18\,\text{m/s}$$

During the next 25 s, the bus moves with a constant velocity. Using the definition of average velocity, express the displacement of the bus during this interval in terms of its average (constant) velocity and the elapsed time:

$$\Delta x(12\,\text{s} \to 37\,\text{s}) = v_{12\,\text{s}}\Delta t = (18\,\text{m/s})(25\,\text{s}) = 450\,\text{m}$$

Because the bus slows down at the same rate that its velocity increased during the first 12 s of motion, we can conclude that its displacement during this braking period is the same as during its acceleration period and the time to brake to a stop is equal to the time that was required for the bus to accelerate to its cruising speed of 18 m/s. Hence:

$$\Delta x(37\,\text{s} \to 49\text{s}) = 108\,\text{m}$$

Add the displacements to find the distance the bus traveled:

$$\Delta x_{\text{total}} = 108\,\text{m} + 450\,\text{m} + 108\,\text{m} = \boxed{666\,\text{m}}$$

(*b*) Use the definition of average velocity to calculate the average velocity of the bus during this trip:

$$v_{av}=\frac{\Delta x_{total}}{\Delta t}=\frac{666\,\text{m}}{49\,\text{s}}=\boxed{13.6\,\text{m/s}}$$

Remarks: One can also solve this problem graphically. Recall that the area under a velocity as a function-of-time graph equals the displacement of the moving object.

***84** ••

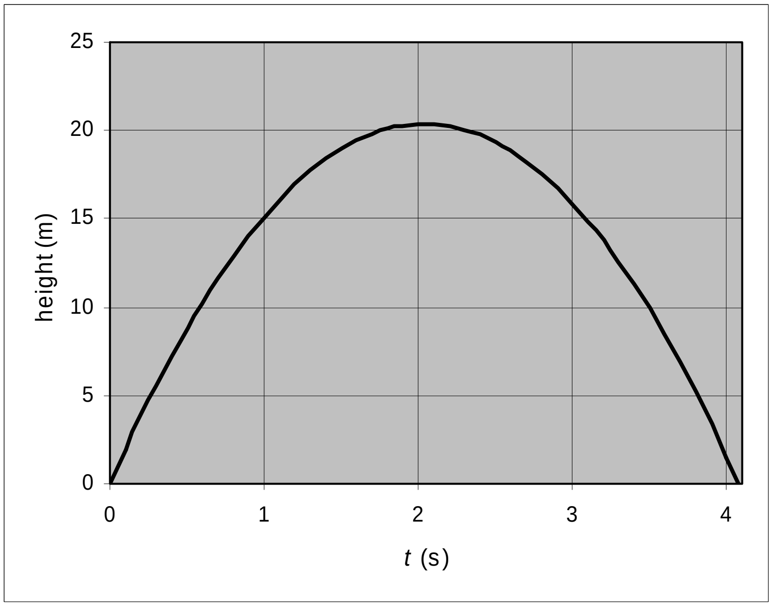
Picture the Problem While we can solve this problem analytically, there are many physical situations in which it is not easy to do so and one has to rely on numerical methods; for example, see the spreadsheet solution shown below. Because we're neglecting the height of the release point, the position of the ball as a function of time is given by $y=v_0t-\frac{1}{2}gt^2$. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	20	v_0
B2	9.81	g
B5	0	t
B6	B5 + 0.1	$t+\Delta t$
C6	$B$1*B6 – 0.5*$B$2*B6^2	$v_0t-\frac{1}{2}gt^2$

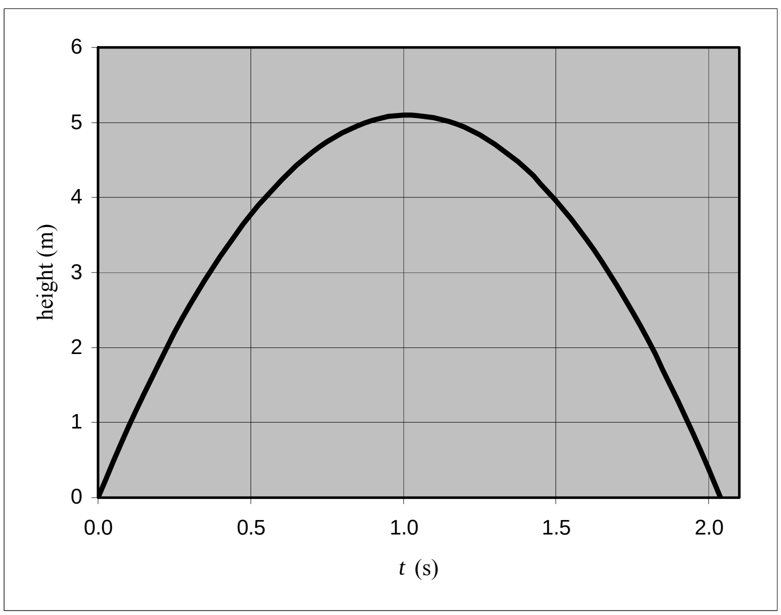
(*a*)

	A	B	C
1	v0 =	20	m/s
2	g =	9.81	m/s^2
3		t	height
4		(s)	(m)
5		0.0	0.00
6		0.1	1.95
7		0.2	3.80
44		3.9	3.39
45		4.0	1.52
46		4.1	–0.45

The graph shown below was generated from the data in the previous table. Note that the maximum height reached is a little more than 20 m and the time of flight is about 4 s.



(b) In the spreadsheet, change the value in cell B1 from 20 to 10. The graph should automatically update. With an initial velocity of 10 m/s, the maximum height achieved is approximately 5 m and the time-of-flight is approximately 2 s.



***85 ••**

Picture the Problem Because the accelerations of both Al and Bert are constant, constant-acceleration equations can be used to describe their motions. Choose the origin of the coordinate system to be where Al decides to begin his sprint.

(a) Using a constant-acceleration equation, relate Al's initial velocity, his acceleration, and the time to reach the end of the trail to his

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

displacement in reaching the end of the trail:

Substitute numerical values to obtain:

$$35\,\text{m} = (0.75\,\text{m/s})t + \tfrac{1}{2}(0.5\,\text{m/s}^2)t^2$$

Solve for the time required for Al to reach the end of the trail:

$$t = \boxed{10.4\,\text{s}}$$

(*b*) Using constant-acceleration equations, express the positions of Bert and Al as functions of time. At the instant Al turns around at the end of the trail, $t = 0$. Also, $x = 0$ at a point 35 m from the end of the trail:

$$x_{\text{Bert}} = x_{\text{Bert,0}} + (0.75\,\text{m/s})t$$

and

$$x_{\text{Al}} = x_{\text{Al,0}} - (0.85\,\text{m/s})t = 35\,\text{m} - (0.85\,\text{m/s})t$$

Calculate Bert's position at $t = 0$. At that time he has been running for 10.4 s:

$$x_{\text{Bert,0}} = (0.75\,\text{m/s})(10.4\,\text{s}) = 7.80\,\text{m}$$

Because Bert and Al will be at the same location when they meet, equate their position functions and solve for *t*:

$$7.80\,\text{m} + (0.75\,\text{m/s})t = 35\,\text{m} - (0.85\,\text{m/s})t$$

and

$$t = 17.0\text{ s}$$

To determine the elapsed time from when Al began his accelerated run, we need to add 10.4 s to this time:

$$t_{\text{start}} = 17.0\,\text{s} + 10.4\,\text{s} = \boxed{27.4\,\text{s}}$$

(*c*) Express Bert's distance from the end of the trail when he and Al meet:

$$d_{\text{end of trail}} = 35\,\text{m} - x_{\text{Bert,0}} - d_{\text{Bert runs until he meets Al}}$$

Substitute numerical values and evaluate $d_{\text{end of trail}}$:

$$d_{\text{end of trail}} = 35\,\text{m} - 7.80\,\text{m} - (17\,\text{s})(0.75\,\text{m/s}) = \boxed{14.5\,\text{m}}$$

86 ••

Picture the Problem Generate two curves on one graph with the first curve representing Al's position as a function of time and the second curve representing Bert's position as a function of time. Al's position, as he runs toward the end of the trail, is given by $x_{\text{Al}} = v_0 t + \frac{1}{2}a_{\text{Al}}t^2$ and Bert's position by $x_{\text{Bert}} = x_{0,\text{Bert}} + v_{\text{Bert}}t$. Al's position, once he's reached the end of the trail and is running back toward Bert, is given by $x_{\text{Al}} = x_{\text{Al,0}} + v_{\text{Al}}(t - 10.5\,\text{s})$. The coordinates of the intersection of the two curves give the time and place where they meet. A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

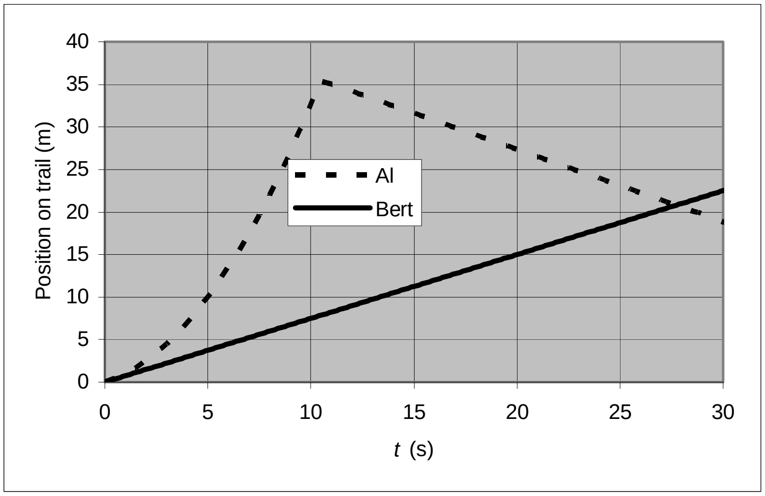
Cell	Content/Formula	Algebraic Form

B1	0.75	v_0
B2	0.50	a_{Al}
B3	–0.85	t
B10	B9 + 0.25	$t + \Delta t$
C10	$B$1*B10 + 0.5*$B$2*B10^2	$v_0 t + \frac{1}{2} a_{Al} t^2$
C52	$C$51 + $B$3*(B52 – $B$51)	$x_{Al,0} + v_{Al}(t - 10.5\,\mathrm{s})$
F10	$F$9 + $B$1*B10	$x_{0,\mathrm{Bert}} + v_{\mathrm{Bert}} t$

(*b*) and (*c*)

	A	B	C	D	E	F
1	v0 =	0.75	m/s			
2	a(Al) =	0.5	m/s^2			
3	v(Al) =	–0.85	m/s			
4						
5		t (s)	x (m)			x (m)
6						
7						
8			Al			Bert
9		0.00	0.00			0.00
10		0.25	0.20			0.19
11		0.50	0.44			0.38
49		10.00	32.50			7.50
50		10.25	33.95			7.69
51		10.50	35.44	*Al reaches		7.88
52		10.75	35.23	end of trail		8.06
53		11.00	35.01	and starts		8.25
54		11.25	34.80	back toward		8.44
55		11.50	34.59	Bert		8.63
56		11.75	34.38			8.81
119		27.50	20.99			20.63
120		27.75	20.78			20.81
121		28.00	20.56			21.00
122		28.25	20.35			21.19
127		29.50	19.29			22.13
128		29.75	19.08			22.31
129		30.00	18.86			22.50

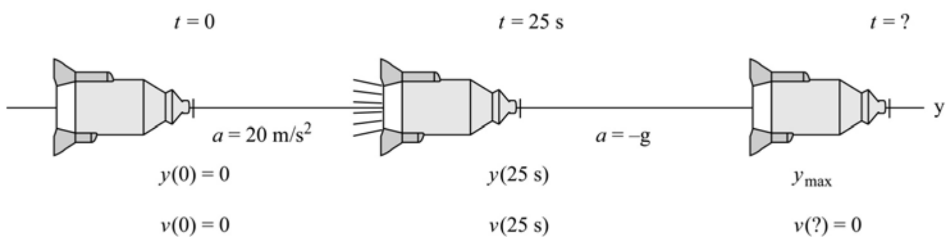
The graph shown below was generated from the spreadsheet; the positions of both Al and Bert were calculated as functions of time. The dashed curve shows Al's position as a function of time for the two parts of his motion. The solid line that is linear from the origin shows Bert's position as a function of time.



Note that the spreadsheet and the graph (constructed from the spreadsheet data) confirm the results in Problem 85 by showing Al and Bert meeting at about 14.5 m from the end of the trail after an elapsed time of approximately 28 s.

87 ••

Picture the Problem This is a two-part constant-acceleration problem. Choose a coordinate system in which the upward direction is positive. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



(*a*) Express the highest point the rocket reaches, *h*, as the sum of its displacements during the first two stages of its flight:

$$h = \Delta x_{\text{1st stage}} + \Delta x_{\text{2nd stage}}$$

Using a constant-acceleration equation, express the altitude reached in the first stage in terms of the rocket's initial velocity, acceleration, and burn time; solve for the first stage altitude:

$$\begin{aligned} x_{\text{1st stage}} &= x_0 + v_0 t + \tfrac{1}{2} a_{\text{1st stage}} t^2 \\ &= \tfrac{1}{2}(20\ \text{m/s}^2)(25\ \text{s})^2 \\ &= 6250\ \text{m} \end{aligned}$$

Using a constant-acceleration equation, express the velocity of the rocket at the end of its first stage in terms of its initial velocity, acceleration, and displacement; calculate its end-of-first-stage velocity:

$$\begin{aligned} v_{\text{1st stage}} &= v_0 + a_{\text{1st stage}} t \\ &= (20\ \text{m/s}^2)(25\ \text{s}) \\ &= 500\ \text{m/s} \end{aligned}$$

Using a constant-acceleration equation, express the final velocity of the rocket during the remainder of its climb in terms of its shut-off velocity, free-fall acceleration, and displacement; solve for its displacement:

$$v_{\text{highest point}}^2 = v_{\text{shutoff}}^2 + 2a_{\text{2nd stage}}\Delta y_{\text{2nd stage}}$$

and, because $v_{\text{highest point}} = 0$,

$$\Delta y_{\text{2nd stage}} = \frac{-v_{\text{shutoff}}^2}{-2g} = \frac{(500\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 1.2742\times10^4\ \text{m}$$

Substitute in the expression for the total height to obtain:

$$h = 6250\,\text{m} + 1.27\times10^4\ \text{m} = \boxed{19.0\,\text{km}}$$

(*b*) Express the total time the rocket is in the air in terms of the three segments of its flight:

$$\Delta t_{\text{total}} = \Delta t_{\text{powered climb}} + \Delta t_{\text{2nd segment}} + \Delta t_{\text{descent}} = 25\,\text{s} + \Delta t_{\text{2nd segment}} + \Delta t_{\text{descent}}$$

Express $\Delta t_{\text{2nd segment}}$ in terms of the rocket's displacement and average velocity:

$$\Delta t_{\text{2nd segment}} = \frac{\text{Displacement}}{\text{Average velocity}}$$

Substitute numerical values and evaluate $\Delta t_{\text{2nd segment}}$:

$$\Delta t_{\text{2nd segment}} = \frac{1.2742\times10^4\ \text{m}}{\left(\dfrac{0+500\,\text{m/s}}{2}\right)} = 50.97\ \text{s}$$

Using a constant-acceleration equation, relate the fall distance to the descent time:

$$\Delta y = v_0 t + \tfrac{1}{2}g(\Delta t_{\text{descent}})^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2}g(\Delta t_{\text{descent}})^2$$

Solve for $\Delta t_{\text{descent}}$:

$$\Delta t_{\text{descent}} = \sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate $\Delta t_{\text{descent}}$:

$$\Delta t_{\text{descent}} = \sqrt{\frac{2(1.90\times10^4\ \text{m})}{9.81\,\text{m/s}^2}} = 62.2\,\text{s}$$

Substitute and calculate the total time the rocket is in the air:

$$\Delta t = 25\,\text{s} + 50.97\,\text{s} + 62.2\,\text{s} = 138\,\text{s} = \boxed{2\,\text{min}\ 18\,\text{s}}$$

(*c*) Using a constant-acceleration equation, express the impact velocity of the rocket in terms of its initial downward velocity, acceleration under free-fall, and time of descent; solve for its impact velocity:

$$v_{\text{impact}} = v_0 + g\Delta t_{\text{descent}}$$

and, because $v_0 = 0$,

$$v_{\text{impact}} = g\Delta t = (9.81\,\text{m/s}^2)(62.2\,\text{s}) = \boxed{610\,\text{m/s}}$$

88 ••

Picture the Problem In the absence of air resistance, the acceleration of the flowerpot is constant. Choose a coordinate system in which downward is positive and the origin is at the point from which the flowerpot fell. Let t = time when the pot is at the top of the window, and $t + \Delta t$ the time when the pot is at the bottom of the window. To find the distance from the ledge to the top of the window, first find the time t_{top} that it takes the pot to fall to the top of the window.

Using a constant-acceleration equation, express the distance y below the ledge from which the pot fell as a function of time:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

Since $a = g$ and $v_0 = y_0 = 0$,

$$y = \tfrac{1}{2}gt^2$$

Express the position of the pot as it reaches the top of the window:

$$y_{top} = \tfrac{1}{2}gt_{top}^2$$

Express the position of the pot as it reaches the bottom of the window:

$$y_{bottom} = \tfrac{1}{2}g\left(t_{top} + \Delta t_{window}\right)^2$$

where $\Delta t_{window} = t_{top} - t_{bottom}$

Subtract y_{bottom} from y_{top} to obtain an expression for the displacement Δy_{window} of the pot as it passes the window:

$$\Delta y_{window} = \tfrac{1}{2}g\left[\left(t_{top} + \Delta t_{window}\right)^2 - t_{top}^2\right] = \tfrac{1}{2}g\left[2t_{top}\Delta t_{window} + \left(\Delta t_{window}\right)^2\right]$$

Solve for t_{top}:

$$t_{top} = \frac{\dfrac{2\Delta y_{window}}{g} - \left(\Delta t_{window}\right)^2}{2\Delta t_{window}}$$

Substitute numerical values and evaluate t_{top}:

$$t_{top} = \frac{\dfrac{2(4\,\text{m})}{9.81\,\text{m/s}^2} - (0.2\,\text{s})^2}{2(0.2\,\text{s})} = 1.839\,\text{s}$$

Substitute this value for t_{top} to obtain the distance from the ledge to the top of the window:

$$y_{top} = \tfrac{1}{2}(9.81\,\text{m/s}^2)(1.939\,\text{s})^2 = \boxed{18.4\,\text{m}}$$

***89 ••**

Picture the Problem The acceleration of the glider on the air track is constant. Its average acceleration is equal to the instantaneous (constant) acceleration. Choose a coordinate system in which the initial direction of the glider's motion is the positive direction.

Using the definition of acceleration, express the average acceleration of the glider in terms of the glider's velocity change and the elapsed time:

$$a = a_{av} = \frac{\Delta v}{\Delta t}$$

Using a constant-acceleration equation, express the average velocity of the glider in terms of the displacement of the glider and the elapsed time:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{v_0 + v}{2}$$

Solve for and evaluate the initial velocity:

$$v_0 = \frac{2\Delta x}{\Delta t} - v = \frac{2(100\,\text{cm})}{8\,\text{s}} - (-15\,\text{cm/s})$$
$$= \boxed{40.0\,\text{cm/s}}$$

Substitute this value of v_0 and evaluate the average acceleration of the glider:

$$a = \frac{-15\,\text{cm/s} - (40\,\text{cm/s})}{8\,\text{s}}$$
$$= \boxed{-6.88\,\text{cm/s}^2}$$

90 ••

Picture the Problem In the absence of air resistance, the acceleration of the rock is constant and its motion can be described using the constant-acceleration equations. Choose a coordinate system in which the downward direction is positive and let the height of the cliff, which equals the displacement of the rock, be represented by *h*.

Using a constant-acceleration equation, express the height *h* of the cliff in terms of the initial velocity of the rock, acceleration, and time of fall:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$, $a = g$, and $\Delta y = h$,

$$h = \tfrac{1}{2}gt^2$$

Using this equation, express the displacement of the rock during the

a) first two-thirds of its fall, and

$$\tfrac{2}{3}h = \tfrac{1}{2}gt^2 \qquad (1)$$

b) its complete fall in terms of the time required for it to fall this distance.

$$h = \tfrac{1}{2}g(t + 1\,\text{s})^2 \qquad (2)$$

Substitute equation (2) in equation (1) to obtain a quadratic equation in *t*:

$$t^2 - (4\text{ s})t - 2\text{ s}^2 = 0$$

Solve for the positive root:

$$t = 4.45\,\text{s}$$

Evaluate $\Delta t = t + 1$ s:

$$\Delta t = 4.45\text{ s} + 1\text{ s} = 5.45\text{ s}$$

Substitute numerical values in equation (2) and evaluate *h*:

$$h = \tfrac{1}{2}(9.81\,\text{m/s}^2)(5.45\,\text{s})^2 = \boxed{146\,\text{m}}$$

91 •••

Picture the Problem Assume that the acceleration of the car is constant. The total distance the car travels while stopping is the sum of the distances it travels during the driver's reaction time and the time it travels while braking. Choose a coordinate system in which the positive direction is the direction of motion of the automobile and apply a constant-acceleration equation to obtain a quadratic equation in the car's initial speed v_0.

(*a*) Using a constant-acceleration equation, relate the velocity of the car to its initial velocity, acceleration, and displacement during braking:

$$v^2 = v_0^2 + 2a\Delta x_{\text{brk}}$$

or, because the final velocity is zero,

$$0 = v_0^2 + 2a\Delta x_{\text{brk}}$$

Solve for the distance traveled during braking:

$$\Delta x_{\text{brk}} = -\frac{v_0^2}{2a}$$

Express the total distance traveled by the car as the sum of the distance traveled during the reaction time and the distance traveled while slowing down:

$$\Delta x_{\text{tot}} = \Delta x_{\text{react}} + \Delta x_{\text{brk}} = v_0\Delta t_{\text{react}} - \frac{v_0^2}{2a}$$

Rearrange this quadratic equation to obtain:

$$v_0^2 - 2a\Delta t_{\text{react}}v_0 + 2a\Delta x_{\text{tot}} = 0$$

Substitute numerical values and simplify to obtain:

$$v_0^2 - 2\left(-7\ \text{m/s}^2\right)\left(0.5\ \text{s}\right)v_0 + 2\left(-7\ \text{m/s}^2\right)\left(4\ \text{m}\right) = 0$$

or

$$v_0^2 + \left(7\ \text{m/s}\right)v_0 - 56\ \text{m}^2/\text{s}^2 = 0$$

Solve the quadratic equation for the positive root to obtain:

$$v_0 = 4.7613558\ \text{m/s}$$

Convert this speed to mi/h::

$$v_0 = \left(4.7613558\ \text{m/s}\right)\left(\frac{1\ \text{mi/h}}{0.477\ \text{m/s}}\right) = \boxed{10.7\ \text{mi/h}}$$

(*b*) Find the reaction-time distance:

$$\Delta x_{\text{react}} = v_0\Delta t_{\text{react}} = (4.76\ \text{m/s})(0.5\ \text{s}) = 2.38\ \text{m}$$

Express and evaluate the ratio of the reaction distance to the total distance:

$$\frac{\Delta x_{\text{react}}}{\Delta x_{\text{tot}}} = \frac{2.38\ \text{m}}{4\ \text{m}} = \boxed{0.595}$$

92 ••

Picture the Problem Assume that the accelerations of the trains are constant. Choose a coordinate system in which the direction of the motion of the train on the left is the positive direction. Take $x_o = 0$ as the position of the train on the left at $t = 0$.

Using a constant-acceleration equation, relate the distance the train on the left will travel before the trains pass to its acceleration and the time-to-passing:

$$x_L = \tfrac{1}{2}a_L t^2 = \tfrac{1}{2}\left(1.4\,\text{m/s}^2\right)t^2 = \left(0.7\,\text{m/s}^2\right)t^2$$

Using a constant-acceleration equation, relate the position of the train on the right to its initial velocity, position, and acceleration:

$$x_R = 40\,\text{m} - \tfrac{1}{2}a_R t^2 = 40\,\text{m} - \tfrac{1}{2}\left(2.2\,\text{m/s}^2\right)t^2$$

Equate x_L and x_R and solve for t:

$$0.7t^2 = 40 - 1.1t^2$$

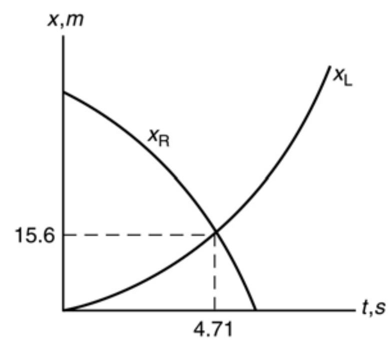
and

$$t = 4.71\,\text{s}$$

Find the position of the train initially on the left, x_L, as they pass:

$$x_L = \tfrac{1}{2}(1.4\,\text{m/s}^2)(4.71\,\text{s})^2 = \boxed{15.6\,\text{m}}$$

Remarks: One can also solve this problem by graphing the functions for x_L and x_R. The coordinates of the intersection of the two curves give one the time-to-passing and the distance traveled by the train on the left.



93 ••

Picture the Problem In the absence of air resistance, the acceleration of the stones is constant. Choose a coordinate system in which the downward direction is positive and the origin is at the point of release of the stones.

Using constant-acceleration equations, relate the positions of the two stones to their initial positions, accelerations, and time-of-fall:

$$x_1 = \tfrac{1}{2}gt^2$$

and

$$x_2 = \tfrac{1}{2}g(t - 1.6\,\text{s})^2$$

Express the difference between x_1 and x_2:

$$x_1 - x_2 = 36\text{ m}$$

Substitute for x_1 and x_2 to obtain:

$$36\,\text{m} = \tfrac{1}{2}gt^2 - \tfrac{1}{2}g\left(t - 1.6\,\text{s}\right)^2$$

Solve this equation for the time t at which the stones will be separated by 36 m:

$t = 3.09$ s

Substitute this result in the expression for x_2 and solve for x_2:

$$x_2 = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(3.09\,\text{s} - 1.6\,\text{s}\right)^2 = \boxed{10.9\,\text{m}}$$

***94 ••**

Picture the Problem The acceleration of the police officer's car is positive and constant and the acceleration of the speeder's car is zero. Choose a coordinate system such that the direction of motion of the two vehicles is the positive direction and the origin is at the stop sign.

Express the velocity of the car in terms of the distance it will travel until the police officer catches up to it and the time that will elapse during this chase:

$$v_{\text{car}} = \frac{d_{\text{caught}}}{t_{\text{car}}}$$

Letting t_1 be the time during which she accelerates and t_2 the time of travel at $v_1 = 110$ km/h, express the time of travel of the police officer:

$$t_{\text{officer}} = t_1 + t_2$$

Convert 110 km/h into m/s:

$$v_1 = \left(110\,\text{km/h}\right)(10^3\ \text{m/km})(1\,\text{h}/3600\,\text{s}) = 30.6\,\text{m/s}$$

Express and evaluate t_1:

$$t_1 = \frac{v_1}{a_{\text{motorcycle}}} = \frac{30.6\ \text{m/s}}{6.2\ \text{m/s}^2} = 4.94\,\text{s}$$

Express and evaluate d_1:

$$d_1 = \tfrac{1}{2}v_1 t_1 = \tfrac{1}{2}(30.6\,\text{m/s})(4.94\,\text{s}) = 75.6\,\text{m}$$

Determine d_2:

$$d_2 = d_{\text{caught}} - d_1 = 1400\,\text{m} - 75.6\,\text{m} = 1324.4\,\text{m}$$

Express and evaluate t_2:

$$t_2 = \frac{d_2}{v_1} = \frac{1324.4\,\text{m}}{30.6\,\text{m/s}} = 43.3\,\text{s}$$

Express the time of travel of the car:

$t_{\text{car}} = 2.0$ s + 4.93 s + 43.3 s = 50.2 s

Finally, find the speed of the car:

$$v_{car} = \frac{d_{caught}}{t_{car}} = \frac{1400\text{ m}}{50.2\text{ s}} = 27.9\text{ m/s}$$
$$= (27.9\text{ m/s})\left(\frac{1\text{ mi/h}}{0.447\text{ m/s}}\right)$$
$$= \boxed{62.4\text{ mi/h}}$$

95 ••

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release of the stone and apply constant-acceleration equations.

Using a constant-acceleration equation, express the height of the cliff in terms of the initial position of the stones, acceleration due to gravity, and time for the first stone to hit the water:

$$h = \tfrac{1}{2}gt_1^2$$

Express the displacement of the second stone when it hits the water in terms of its initial velocity, acceleration, and time required for it to hit the water.

$$d_2 = v_{02}t_2 + \tfrac{1}{2}gt_2^2$$
where $t_2 = t_1 - 1.6$ s.

Because the stones will travel the same distances before hitting the water, equate h and d_2 and solve for t.

$$\tfrac{1}{2}gt_1^2 = v_{02}t_2 + \tfrac{1}{2}gt_2^2$$
or
$$\tfrac{1}{2}(9.81\text{m/s}^2)t_1^2 = (32\text{m/s})(t_1 - 1.6\text{s}) + \tfrac{1}{2}(9.81\text{m/s}^2)(t_1 - 1.6\text{s})^2$$

Solve for t_1 to obtain:

$$t_1 = 2.37\text{ s}$$

Substitute for t_1 and evaluate h:

$$h = \tfrac{1}{2}(9.81\text{ m/s}^2)(2.37\text{ s})^2 = \boxed{27.6\text{ m}}$$

96 •••

Picture the Problem Assume that the acceleration of the passenger train is constant. Let $x_p = 0$ be the location of the passenger train engine at the moment of sighting the freight train's end; let $t = 0$ be the instant the passenger train begins to slow (0.4 s after the passenger train engineer sees the freight train ahead). Choose a coordinate system in which the direction of motion of the trains is the positive direction and use constant-acceleration equations to express the positions of the trains in terms of their initial positions, speeds, accelerations, and elapsed time.

(*a*) Using constant-acceleration equations, write expressions for the positions of the front of the passenger train and the rear of the

$$x_p = (29\text{ m/s})(t + 0.4\text{ s}) - \tfrac{1}{2}at^2$$
$$x_f = (360\text{ m}) + (6\text{ m/s})(t + 0.4\text{ s})$$
where x_p and x_f are in meters if t is in

freight train, x_p and x_f, respectively: seconds.

Equate $x_f = x_p$ to obtain an equation for t:

$$\tfrac{1}{2}at^2 - (23\,\text{m/s})t + 350.8\,\text{m} = 0$$

Find the discriminant $(D = \text{B}^2 - 4\text{AC})$ of this equation:

$$D = (23\,\text{m/s})^2 - 4\left(\frac{a}{2}\right)(350.8\,\text{m})$$

The equation must have real roots if it is to describe a collision. The necessary condition for real roots is that the discriminant be greater than or equal to zero:

If $(23\text{ m/s})^2 - a\,(701.6\text{ m}) \geq 0$, then

$$a \leq \boxed{0.754\,\text{m/s}^2}$$

(b) Express the relative speed of the trains:

$$v_{\text{rel}} = v_{\text{pf}} = v_{\text{p}} - v_{\text{f}} \qquad (1)$$

Repeat the previous steps with $a = 0.754\text{ m/s}^2$ and a 0.8 s reaction time. The quadratic equation that guarantees real roots with the longer reaction time is:

$$\tfrac{1}{2}(0.754\,\text{m/s}^2)t^2 - (23\,\text{m/s})t + 341.6\,\text{m} = 0$$

Solve for t to obtain the collision times:

$t = 25.6$ s and $t = 35.4$ s

Note that at $t = 35.4$ s, the trains have already collided; therefore this root is not a meaningful solution to our problem.

Note: In the graph shown below, you will see why we keep only the smaller of the two solutions.

Now we can substitute our value for t in the constant-acceleration equation for the passenger train and solve for the distance the train has moved prior to the collision:

$$x_p = (29\text{ m/s})(25.6\text{ s} + 0.8\text{ s}) - (0.377\text{ m/s}^2)(25.6\text{ s})^2 = 518\,\text{m}$$

Find the speeds of the two trains:

$$v_p = v_{op} + at = (29\text{ m/s}) + (-0.754\text{ m/s}^2)(25.5\text{ s}) = 9.77\text{ m/s}$$

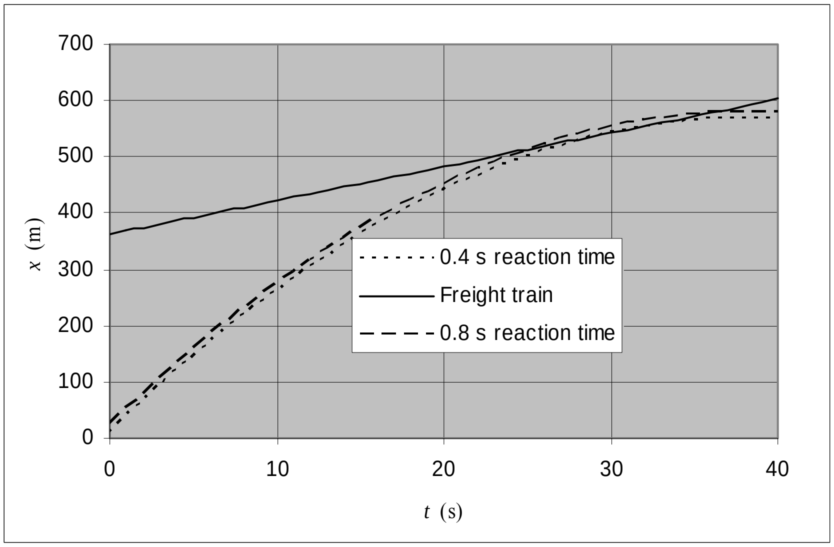
and

$$v_f = v_{of} = 6\text{ m/s}$$

Substitute in equation (1) and evaluate the relative speed of the trains:

$$v_{\text{rel}} = 9.77\,\text{m/s} - 6.00\,\text{m/s} = \boxed{3.77\,\text{m/s}}$$

The graph shows the location of both trains as functions of time. The solid straight line is for the constant velocity freight train; the dashed curves are for the passenger train, with reaction times of 0.4 s for the lower curve and 0.8 s for the upper curve.



Remarks: A collision occurs the first time the curve for the passenger train crosses the curve for the freight train. The smaller of two solutions will always give the time of the collision.

97 •

Picture the Problem In the absence of air resistance, the acceleration of an object near the surface of the earth is constant. Choose a coordinate system in which the upward direction is positive and the origin is at the surface of the earth and apply constant-acceleration equations.

Using a constant-acceleration equation, relate the velocity to the acceleration and displacement:

$$v^2 = v_0^2 + 2a\Delta y$$

or, because $v = 0$ and $a = -g$,

$$0 = v_0^2 - 2g\Delta y$$

Solve for the height to which the projectile will rise:

$$h = \Delta y = \frac{v_0^2}{2g}$$

Substitute numerical values and evaluate *h*:

$$h = \frac{(300\,\text{m/s})^2}{2(9.81\text{ m/s}^2)} = \boxed{4.59\,\text{km}}$$

***98** •

Picture the Problem This is a composite of two constant accelerations with the acceleration equal to one constant prior to the elevator hitting the roof, and equal to a different constant after crashing through it. Choose a coordinate system in which the upward direction is positive and apply constant-acceleration equations.

(*a*) Using a constant-acceleration equation, relate the velocity to the acceleration and displacement:

$$v^2 = v_0^2 + 2\,a\,\Delta y$$

or, because $v = 0$ and $a = -g$,

$$0 = v_0^2 - 2\,g\,\Delta y$$

Solve for v_0:

$$v_0 = \sqrt{2g\Delta y}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{2(9.81\,\text{m/s}^2)(10^4\,\text{m})} = \boxed{443\,\text{m/s}}$$

(*b*) Find the velocity of the elevator just before it crashed through the roof:

$$v_f = 2 \times 443\ \text{m/s} = 886\ \text{m/s}$$

Using the same constant-acceleration equation, this time with $v_0 = 0$, solve for the acceleration:

$$v^2 = 2a\Delta y$$

Substitute numerical values and evaluate a:

$$a = \frac{(886\,\text{m/s})^2}{2(150\,\text{m})} = 2.62\times 10^3\ \text{m/s}^2$$
$$= \boxed{267g}$$

99 ••

Picture the Problem Choose a coordinate system in which the upward direction is positive. We can use a constant-acceleration equation to find the beetle's velocity as its feet lose contact with the ground and then use this velocity to calculate the height of its jump.

Using a constant-acceleration equation, relate the beetle's maximum height to its launch velocity, velocity at the top of its trajectory, and acceleration once it is airborne; solve for its maximum height:

$$v_{\text{highest point}}^2 = v_{\text{launch}}^2 + 2a\Delta y_{\text{free fall}}$$
$$= v_{\text{launch}}^2 + 2(-g)h$$

Because $v_{\text{highest point}} = 0$:

$$h = \frac{v_{\text{launch}}^2}{2g}$$

Now, in order to determine the beetle's launch velocity, relate its time of contact with the ground to its acceleration and push-off distance:

$$v_{\text{launch}}^2 = v_0^2 + 2a\Delta y_{\text{launch}}$$

or, because $v_0 = 0$,

$$v_{\text{launch}}^2 = 2a\Delta y_{\text{launch}}$$

Substitute numerical values and evaluate v_{launch}^2:

$$v_{\text{launch}}^2 = 2(400)(9.81\,\text{m/s}^2)(0.6\times 10^{-2}\,\text{m})$$
$$= 47.1\,\text{m}^2/\text{s}^2$$

Substitute to find the height to which the beetle can jump:

$$h = \frac{v_{\text{launch}}^2}{2g} = \frac{47.1\,\text{m}^2/\text{s}^2}{2(9.81\,\text{m/s}^2)} = \boxed{2.40\,\text{m}}$$

Using a constant-acceleration equation, relate the velocity of the beetle at its maximum height to its launch velocity, free-fall acceleration while in the air, and time-to-maximum height:

$$v = v_0 + at$$

or

$$v_{\text{max. height}} = v_{\text{launch}} - gt_{\text{max. height}}$$

and, because $v_{\text{max height}} = 0$,

$$0 = v_{\text{launch}} - gt_{\text{max. height}}$$

Solve for $t_{\text{max height}}$:

$$t_{\text{max height}} = \frac{v_{\text{launch}}}{g}$$

For zero displacement and constant acceleration, the time-of-flight is twice the time-to-maximum height:

$$t_{\text{flight}} = 2t_{\text{max. height}} = \frac{2v_{\text{launch}}}{g}$$

$$= \frac{2(6.86\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{1.40\,\text{s}}$$

100 •

Picture the Problem Because its acceleration is constant we can use the constant-acceleration equations to describe the motion of the automobile.

Using a constant-acceleration equation, relate the velocity to the acceleration and displacement:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\Delta x$$

Solve for the acceleration *a:*

$$a = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{-\left[(98\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})\right]^2}{2(50\text{m})}$$

$$= \boxed{-7.41\,\text{m/s}^2}$$

Express the ratio of *a* to *g* and then solve for *a*:

$$\frac{a}{g} = \frac{-7.41\,\text{m/s}^2}{9.81\,\text{m/s}^2} = -0.755$$

and $a = \boxed{-0.755g}$

Using the definition of average acceleration, solve for the stopping time:

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \frac{\Delta v}{a_{\text{av}}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{(-98\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{-7.41\,\text{m/s}^2}$$

$$= \boxed{3.67\,\text{s}}$$

***101 ••**

Picture the Problem In the absence of air resistance, the puck experiences constant acceleration and we can use constant-acceleration equations to describe its position as a function of time. Choose a coordinate system in which downward is positive, the particle starts from rest ($v_o = 0$), and the starting height is zero ($y_0 = 0$).

Using a constant-acceleration equation, relate the position of the falling puck to the acceleration and the time. Evaluate the y-position at successive equal time intervals Δt, $2\Delta t$, $3\Delta t$, etc:

$$y_1 = \frac{-g}{2}(\Delta t)^2 = \frac{-g}{2}(\Delta t)^2$$

$$y_2 = \frac{-g}{2}(2\Delta t)^2 = \frac{-g}{2}(4)(\Delta t)^2$$

$$y_3 = \frac{-g}{2}(3\Delta t)^2 = \frac{-g}{2}(9)(\Delta t)^2$$

$$y_4 = \frac{-g}{2}(4\Delta t)^2 = \frac{-g}{2}(16)(\Delta t)^2$$

etc.

Evaluate the changes in those positions in each time interval:

$$\Delta y_{10} = y_1 - 0 = \left(\frac{-g}{2}\right)(\Delta t)^2$$

$$\Delta y_{21} = y_2 - y_1 = 3\left(\frac{-g}{2}\right)(\Delta t)^2 = 3\Delta y_{10}$$

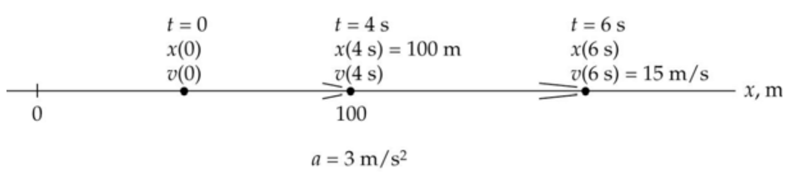
$$\Delta y_{32} = y_3 - y_2 = 5\left(\frac{-g}{2}\right)(\Delta t)^2 = 5\Delta y_{10}$$

$$\Delta y_{43} = y_4 - y_3 = 7\left(\frac{-g}{2}\right)(\Delta t)^2 = 7\Delta y_{10}$$

etc.

102 ••

Picture the Problem Because the particle moves with a constant acceleration we can use the constant-acceleration equations to describe its motion. A pictorial representation will help us organize the information in the problem and develop our solution strategy.



Using a constant-acceleration equation, find the position x at $t = 6$ s. To find x at $t = 6$ s, we first need to find v_0 and x_0:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Using the information that when $t = 4$ s, $x = 100$ m, obtain an equation in x_0 and v_0:

$$x(4\,\text{s}) = 100\,\text{m} = x_0 + v_0(4\,\text{s}) + \tfrac{1}{2}(3\,\text{m/s}^2)(4\,\text{s})^2$$

or

$$x_0 + (4\,\text{s})v_0 = 76\,\text{m}$$

Using the information that when $t = 6$ s, $v = 15$ m/s, obtain a second equation in x_0 and v_0:

$$v(6\,\text{s}) = v_0 + (3\,\text{m/s}^2)(6\,\text{s})$$

Solve for v_0 to obtain:

$v_0 = -3$ m/s

Substitute this value for v_0 in the previous equation and solve for x_0:

$x_0 = 88$ m

Substitute for x_0 and v_0 and evaluate x at $t = 6$ s:

$$x(6\,\text{s}) = 88\,\text{m} + (-3\,\text{m/s})\,(6\,\text{s}) + \tfrac{1}{2}(3\,\text{m/s}^2)\,(6\,\text{s})^2 = \boxed{124\,\text{m}}$$

***103 ••**

Picture the Problem We can use constant-acceleration equations with the final velocity $v = 0$ to find the acceleration and stopping time of the plane.

(*a*) Using a constant-acceleration equation, relate the known velocities to the acceleration and displacement:

$$v^2 = v_0^2 + 2a\Delta x$$

Solve for a:

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{-(60\,\text{m/s})^2}{2(70\,\text{m})} = \boxed{-25.7\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the final and initial speeds of the plane to its acceleration and stopping time:

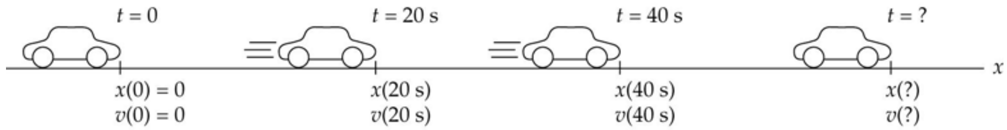
$$v = v_0 + a\Delta t$$

Solve for and evaluate the stopping time:

$$\Delta t = \frac{v - v_0}{a} = \frac{0 - 60\,\text{m/s}}{-25.7\,\text{m/s}^2} = \boxed{2.33\,\text{s}}$$

104 ••

Picture the Problem This is a multipart constant-acceleration problem using three different constant accelerations (+2 m/s^2 for 20 s, then zero for 20 s, and then –3 m/s^2 until the automobile stops). The final velocity is zero. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



Add up all the displacements to get the total:

$$\Delta x_{03} = \Delta x_{01} + \Delta x_{12} + \Delta x_{23}$$

Using constant-acceleration formulas, find the first displacement:

$$\Delta x_{01} = v_0 t_1 + \tfrac{1}{2} a_{01} t_1^2$$
$$= 0 + \tfrac{1}{2}(2\text{ m/s}^2)(20\text{ s})^2 = 400\text{ m}$$

The speed is constant for the second displacement. Find the displacement:

$$\Delta x_{12} = v_1 (t_2 - t_1)$$
where $\mathrm{v}_1 = v_0 + a_{01} t_1 = 0 + a_{01} t_1$ and
$$\Delta x_{12} = a_{01} t_1 (t_2 - t_1)$$
$$= (2\text{ m/s}^2)(20\text{ s})(20\text{ s}) = 800\ \text{ m}$$

Find the displacement during the braking interval:

$$v_3^2 = v_2^2 + 2a_{23}\Delta x_{23}$$
where $v_2 = v_1 = a_{01} t_1$ and $v_3 = 0$ and
$$\Delta x_{23} = \frac{0^2 - (a_{01} t_1)^2}{2a_{23}} = \frac{-[(2\text{ m/s})(20\text{ s})]^2}{2(-3\text{m/s}^2)}$$
$$= 267\text{m}$$

Add the displacements to get the total:

$$\Delta x_{03} = \Delta x_{01} + \Delta x_{12} + \Delta x_{23} = 1467\text{ m}$$
$$= \boxed{1.47\text{ km}}$$

Remarks: Because the area under the curve of a velocity-versus-time graph equals the displacement of the object experiencing the acceleration, we could solve this problem by plotting the velocity as a function of time and finding the area bounded by it and the time axis.

***105** ••

Picture the Problem Note: No material body can travel at speeds faster than light. When one is dealing with problems of this sort, the kinematic formulae for displacement, velocity and acceleration are no longer valid, and one must invoke the special theory of relativity to answer questions such as these. For now, ignore such subtleties. Although the formulas you are using (i.e., the constant- acceleration equations) are not quite correct, your answer to part (*b*) will be wrong by about 1%.

(*a*) This part of the problem is an exercise in the conversion of units. Make use of the fact that 1 c·y = 9.47×10^{15} m and 1 y = 3.16×10^{7} s:

$$g = (9.81\text{m/s}^2)\left(\frac{1c\cdot\text{y}}{9.47\times10^{15}\text{m}}\right)\left(\frac{(3.16\times10^{7}\text{s})^2}{(1\text{ y})^2}\right) = \boxed{1.03c\cdot\text{y}/\text{y}^2}$$

(*b*) Let $t_{1/2}$ represent the time it takes to reach the halfway point. Then the total trip time is:

$$t = 2\,t_{1/2} \qquad (1)$$

Use a constant- acceleration equation to relate the half-distance to Mars Δx to the initial speed, acceleration, and half-trip time $t_{1/2}$:

$$\Delta x = v_0 t + \tfrac{1}{2} a t_{1/2}^2$$

Because $v_0 = 0$ and $a = g$:

$$t_{1/2} = \sqrt{\frac{2\Delta x}{a}}$$

The distance from Earth to Mars at closest approach is 7.8×10^{10} m. Substitute numerical values and evaluate $t_{1/2}$:

$$t_{1/2} = \sqrt{\frac{2\left(3.9\times10^{10}\text{ m}\right)}{9.81\text{m/s}^2}} = 8.92\times10^4\text{ s}$$

Substitute for $t_{1/2}$ in equation (1) to obtain:

$$t = 2\left(8.92\times10^4\text{ s}\right) = 1.78\times10^5\text{ s} \approx \boxed{2\text{d}}$$

Remarks: Our result in part (*b*) seems remarkably short, considering how far Mars is and how low the acceleration is.

106 •

Picture the Problem Because the elevator accelerates uniformly for half the distance and uniformly decelerates for the second half, we can use constant-acceleration equations to describe its motion

Let $t_{1/2} = 40$ s be the time it takes to reach the halfway mark. Use the constant-acceleration equation that relates the acceleration to the known variables to obtain:

$$\Delta y = v_0 t + \tfrac{1}{2} a t^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2} a t^2$$

Solve for a:

$$a = \frac{2\,\Delta y}{t_{1/2}^2}$$

Substitute numerical values and evaluate a:

$$a = \frac{2\left(\tfrac{1}{2}\right)\left(1173\,\text{ft}\right)\left(1\,\text{m}/3.281\,\text{ft}\right)}{\left(40\,\text{s}\right)^2} = 0.223\,\text{m/s}^2$$

$$= \boxed{0.0228g}$$

107 ••

Picture the Problem Because the acceleration is constant, we can describe the motions of the train using constant-acceleration equations. Find expressions for the distances traveled, separately, by the train and the passenger. When are they equal? Note that the train is accelerating and the passenger runs at a constant minimum velocity (zero acceleration) such that she can just catch the train.

1. Using the subscripts "train" and "p" to refer to the train and the passenger and the subscript "c" to identify "critical" conditions, express the position of the train and the passenger:

$$x_{\text{train,c}}(t_{\text{c}}) = \frac{a_{\text{train}}}{2} t_{\text{c}}^2$$

and

$$x_{\text{p,c}}(t_{\text{c}}) = v_{\text{p,c}}(t_{\text{c}} - \Delta t)$$

Express the critical conditions that must be satisfied if the passenger is to catch the train:

$$v_{\text{train,c}} = v_{\text{p,c}}$$

and

$$x_{\text{train,c}} = x_{\text{p,c}}$$

2. Express the train's average velocity.

$$v_{\text{av}}(0 \text{ to } t_{\text{c}}) = \frac{0 + v_{\text{train,c}}}{2} = \frac{v_{\text{train,c}}}{2}$$

3. Using the definition of average velocity, express v_{av} in terms of $x_{\text{p,c}}$ and t_{c}.

$$v_{\text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{0 + x_{\text{p,c}}}{0 + t_{\text{c}}} = \frac{x_{\text{p,c}}}{t_{\text{c}}}$$

4. Combine steps 2 and 3 and solve for $x_{\text{p,c}}$.

$$x_{\text{p,c}} = \frac{v_{\text{train,c}} t_{\text{c}}}{2}$$

5. Combine steps 1 and 4 and solve for t_{c}.

$$v_{\text{p,c}}(t_{\text{c}} - \Delta t) = \frac{v_{\text{train,c}} t_{\text{c}}}{2}$$

or

$$t_{\text{c}} - \Delta t = \frac{t_{\text{c}}}{2}$$

and

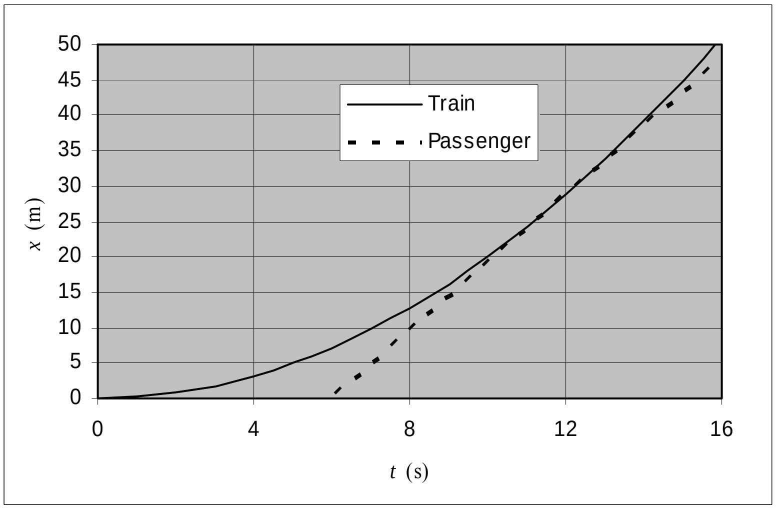
$t_{\text{c}} = 2\,\Delta t = 2\text{ (6 s)} = 12\text{ s}$

6. Finally, combine steps 1 and 5 and solve for $v_{\text{train, c}}$.

$$v_{\text{p,c}} = v_{\text{train,c}} = a_{\text{train}} t_{\text{c}} = (0.4\,\text{m/s}^2)(12\,\text{s})$$

$$= \boxed{4.80\text{ m/s}}$$

The graph shows the location of both the passenger and the train as a function of time. The parabolic solid curve is the graph of $x_{\text{train}}(t)$ for the accelerating train. The straight dashed line is passenger's position $x_{\text{p}}(t)$ if she arrives at $\Delta t = 6.0$ s after the train departs. When the passenger catches the train, our graph shows that her speed and that of the train must be equal ($v_{\text{train,c}} = v_{\text{p,c}}$). Do you see why?



108 •••

Picture the Problem Both balls experience constant acceleration once they are in flight. Choose a coordinate system with the origin at the ground and the upward direction positive. When the balls collide they are at the same height above the ground.

Using constant-acceleration equations, express the positions of both balls as functions of time. At the ground $y = 0$.

$$y_A = h - \tfrac{1}{2}gt^2$$
and
$$y_B = v_0 t - \tfrac{1}{2}gt^2$$

The conditions at collision are that the heights are equal and the velocities are related:

$$y_A = y_B$$
and
$$v_A = -2v_B$$

Express the velocities of both balls as functions of time:

$$v_A = -gt$$
and
$$v_B = v_0 - gt$$

Substituting the position and velocity functions into the conditions at collision gives:

$$h - \tfrac{1}{2}gt_c^2 = v_0 t_c - \tfrac{1}{2}gt_c^2$$
and
$$-gt_c = -2(v_0 - gt_c)$$
where t_c is the time of collision.

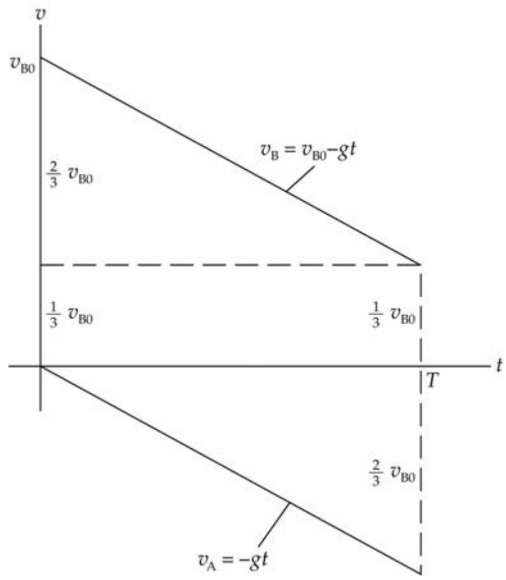
We now have two equations and two unknowns, t_c and v_0. Solving the equations for the unknowns gives:

$$t_c = \sqrt{\frac{2h}{3g}} \text{ and } v_0 = \sqrt{\frac{3gh}{2}}$$

Substitute the expression for t_c into the equation for y_A to obtain the height at collision:

$$y_A = h - \tfrac{1}{2}g\left(\frac{2h}{3g}\right) = \boxed{\frac{2h}{3}}$$

Remarks: We can also solve this problem graphically by plotting velocity- versus-time for both balls. Because ball A starts from rest its velocity is given by $v_A = -gt$. Ball B initially moves with an unknown velocity v_{B0} and its velocity is given by $v_B = v_{B0} - gt$. The graphs of these equations are shown below with T representing the time at which they collide.



The height of the building is the sum of the sum of the distances traveled by the balls. Each of these distances is equal to the magnitude of the area ″under″ the corresponding *v*-versus-*t* curve. Thus, the height of the building equals the area of the parallelogram, which is $v_{B0}T$. The distance that A falls is the area of the lower triangle, which is (1/3) $v_{B0}T$. Therefore, the ratio of the distance fallen by A to the height of the building is 1/3, so the collision takes place at 2/3 the height of the building.

109 •••

Picture the Problem Both balls are moving with constant acceleration. Take the origin of the coordinate system to be at the ground and the upward direction to be positive. When the balls collide they are at the same height above the ground. The velocities at collision are related by $v_A = 4v_B$.

Using constant-acceleration equations, express the positions of both balls as functions of time:

$$y_A = h - \tfrac{1}{2}gt^2$$
and
$$y_B = v_0 t - \tfrac{1}{2}gt^2$$

The conditions at collision are that the heights are equal and the velocities are related:

$$y_A = y_B$$
and
$$v_A = 4v_B$$

Express the velocities of both balls as functions of time:

$$v_A = -gt \text{ and } v_B = v_0 - gt$$

Substitute the position and velocity functions into the conditions at collision to obtain:

$$h - \tfrac{1}{2}gt_c^2 = v_0 t_c - \tfrac{1}{2}gt_c^2$$

and

$$-gt_c = 4(v_0 - gt_c)$$

where t_c is the time of collision.

We now have two equations and two unknowns, t_c and v_0. Solving the equations for the unknowns gives:

$$t_c = \sqrt{\frac{4h}{3g}} \text{ and } v_0 = \sqrt{\frac{3gh}{4}}$$

Substitute the expression for t_c into the equation for y_A to obtain the height at collision:

$$y_A = h - \tfrac{1}{2}g\left(\frac{4h}{3g}\right) = \boxed{\frac{h}{3}}$$

***110 ••**

Determine the Concept The problem describes two intervals of constant acceleration; one when the train's velocity is increasing, and a second when it is decreasing.

(*a*) Using a constant-acceleration equation, relate the half-distance Δx between stations to the initial speed v_0, the acceleration a of the train, and the time-to-midpoint Δt:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

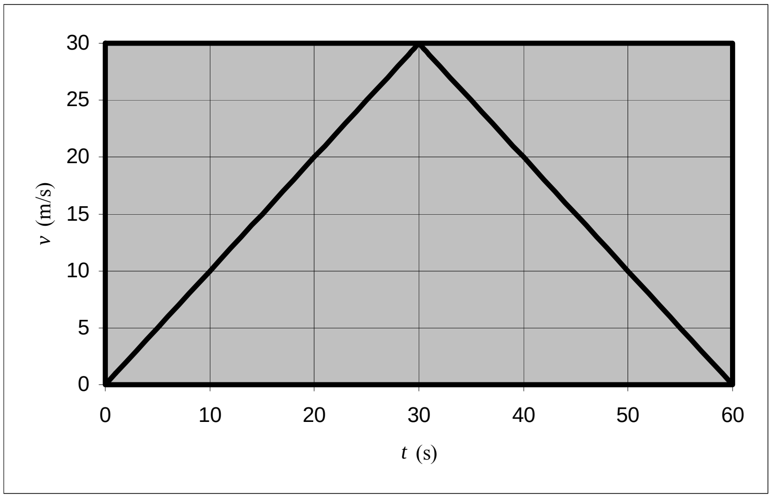
Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a}}$$

Substitute numerical values and evaluate the time-to-midpoint Δt:

$$\Delta t = \sqrt{\frac{2(450\,\text{m})}{1\,\text{m/s}^2}} = 30.0\,\text{s}$$

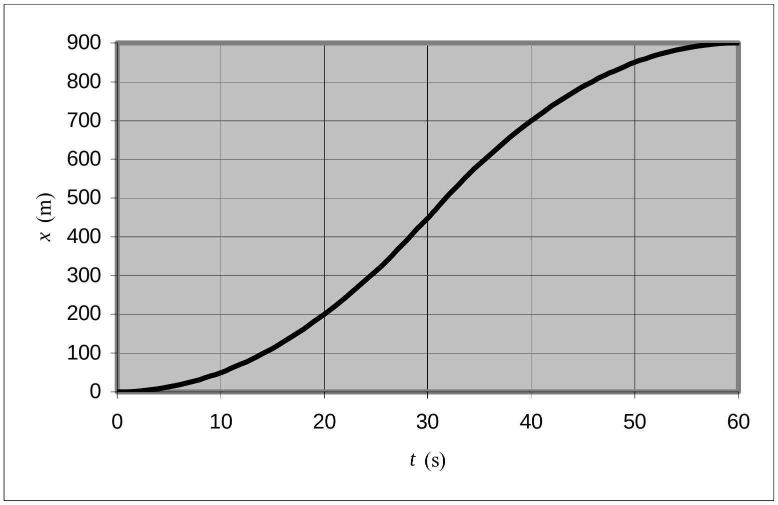
Because the train accelerates uniformly and from rest, the first part of its velocity graph will be linear, pass through the origin, and last for 30 s. Because it slows down uniformly and at the same rate for the second half of its journey, this part of its graph will also be linear but with a negative slope. The graph of v as a function of t is shown below.



(b) The graph of x as a function of t is obtained from the graph of v as a function of t by finding the area under the velocity curve. Looking at the velocity graph, note that when the train has been in motion for 10 s, it will have traveled a distance of

$$\tfrac{1}{2}(10\,\text{s})(10\,\text{m/s}) = 50\,\text{m}$$

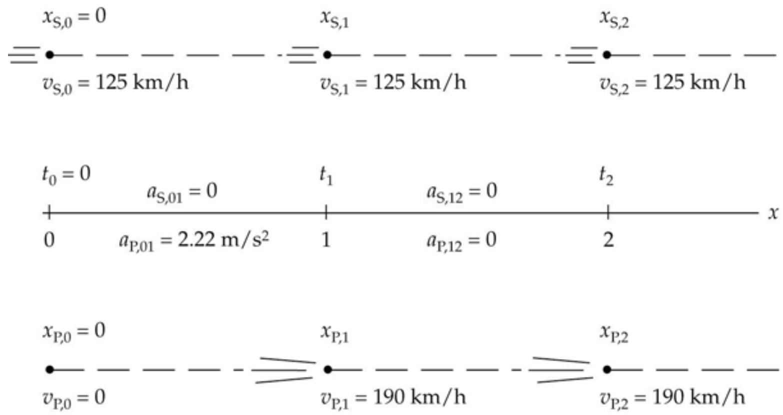
and that this distance is plotted above 10 s on the following graph.



Selecting additional points from the velocity graph and calculating the areas under the curve will confirm the graph of x as a function of t that is shown.

111 ••

Picture the Problem This is a two-stage constant-acceleration problem. Choose a coordinate system in which the direction of the motion of the cars is the positive direction. The pictorial representation summarizes what we know about the motion of the speeder's car and the patrol car.



Convert the speeds of the vehicles and the acceleration of the police car into SI units:

$$8\frac{\text{km}}{\text{h}\cdot\text{s}} = 8\frac{\text{km}}{\text{h}\cdot\text{s}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 2.22\,\text{m/s}^2,$$

$$125\frac{\text{km}}{\text{h}} = 125\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 34.7\,\text{m/s},$$

and

$$190\frac{\text{km}}{\text{h}} = 190\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 52.8\,\text{m/s}$$

(*a*) Express the condition that determines when the police car catches the speeder; i.e., that their displacements will be the same:

$$\Delta x_{P,02} = \Delta x_{S,02}$$

Using a constant-acceleration equation, relate the displacement of the patrol car to its displacement while accelerating and its displacement once it reaches its maximum velocity:

$$\begin{aligned}\Delta x_{P,02} &= \Delta x_{P,01} + \Delta x_{P,12} \\ &= \Delta x_{P,01} + v_{P,1}(t_2 - t_1)\end{aligned}$$

Using a constant-acceleration equation, relate the displacement of the speeder to its constant velocity and the time it takes the patrol car to catch it:

$$\begin{aligned}\Delta x_{S,02} &= v_{S,02}\Delta t_{02} \\ &= (34.7\,\text{m/s})t_2\end{aligned}$$

Calculate the time during which the police car is speeding up:

$$\begin{aligned}\Delta t_{P,01} &= \frac{\Delta v_{P,01}}{a_{P,01}} = \frac{v_{P,1} - v_{P,0}}{a_{P,01}} \\ &= \frac{52.8\,\text{m/s} - 0}{2.22\,\text{m/s}^2} = 23.8\,\text{s}\end{aligned}$$

Express the displacement of the patrol car:

$$\Delta x_{\text{P},01} = v_{\text{P},0}\Delta t_{\text{P},01} + \tfrac{1}{2}a_{\text{P},01}\Delta t_{\text{P},01}^2 = 0 + \tfrac{1}{2}\left(2.22\ \text{m/s}^2\right)\left(23.8\ \text{s}\right)^2 = 629\ \text{m}$$

Equate the displacements of the two vehicles:

$$\Delta x_{\text{P},02} = \Delta x_{\text{P},01} + \Delta x_{\text{P},12} = \Delta x_{\text{P},01} + v_{\text{P},1}\left(t_2 - t_1\right) = 629\ \text{m} + \left(52.8\ \text{m/s}\right)\left(t_2 - 23.8\ \text{s}\right)$$

Solve for the time to catch up to obtain:

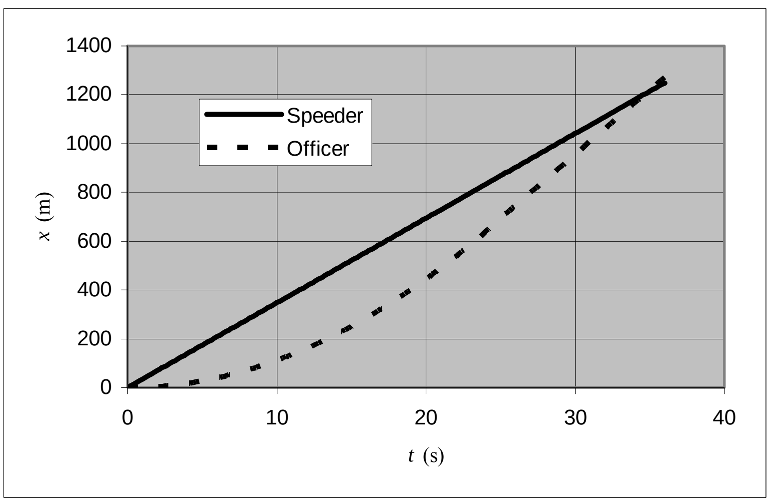
$$(34.7\ \text{m/s})\ t_2 = 629\ \text{m} + (52.8\ \text{m/s})(t_2 - 23.8\ \text{s})$$

$$\therefore\ t_2 = \boxed{34.7\ \text{s}}$$

(*b*) The distance traveled is the displacement, $\Delta x_{02,\text{S}}$, of the speeder during the catch:

$$\Delta x_{\text{S},02} = v_{\text{S},02}\Delta t_{02} = \left(34.7\ \text{m/s}\right)\left(34.7\ \text{s}\right) = \boxed{1.20\ \text{km}}$$

(*c*) The graphs of x_S and x_P are shown below. The straight line (solid) represents $x_\text{S}(t)$ and the parabola (dashed) represents $x_\text{P}(t)$.



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Picture the Problem The accelerations of both cars are constant and we can use constant-acceleration equations to describe their motions. Choose a coordinate system in which the direction of motion of the cars is the positive direction, and the origin is at the initial position of the police car.

(*a*) The collision will ***not*** occur if, during braking, the displacements of the two cars differ by less than 100 m.

$$\Delta x_\text{P} - \Delta x_\text{S} < 100\ \text{m}.$$

Using a constant-acceleration equation, relate the speeder's initial and final speeds to its displacement and acceleration and solve for the displacement:

$$v_s^2 = v_{0,s}^2 + 2a_s\Delta x_s$$

or, because $v_s = 0$,

$$\Delta x_s = \frac{-v_{0,s}^2}{2a_s}$$

Substitute numerical values and evaluate Δx_S:

$$\Delta x_S = \frac{-(34.7\ \text{m/s})^2}{2(-6\ \text{m/s}^2)} = 100\ \text{m}$$

Using a constant-acceleration equation, relate the patrol car's initial and final speeds to its displacement and acceleration and solve for the displacement:

$$v_p^2 = v_{0,p}^2 + 2a_p\Delta x_p$$

or, assuming $v_p = 0$,

$$\Delta x_p = \frac{-v_{0,p}^2}{2a_p}$$

Substitute numerical values and evaluate Δx_p:

$$\Delta x_P = \frac{-(52.8\ \text{m/s})^2}{2(-6\ \text{m/s}^2)} = 232\ \text{m}$$

Finally, substitute these displacements into the inequality that determines whether a collision occurs:

232 m – 100 m = 132 m

Because this difference is greater than 100 m, $\boxed{\text{the cars collide}}$.

(*b*) Using constant-acceleration equations, relate the positions of both vehicles to their initial positions, initial velocities, accelerations, and time in motion:

$$x_S = 100\ \text{m} + (34.7\ \text{m/s})t - (3\ \text{m/s}^2)t^2$$

and

$$x_P = (52.8\ \text{m/s})t - (3\ \text{m/s}^2)t^2$$

Equate these expressions and solve for *t*:

$$100\ \text{m} + (34.7\ \text{m/s})\,t - (3\ \text{m/s}^2)\,t^2 = (52.8\ \text{m/s})\,t - (3\text{m/s}^2)\,t^2$$

and

$$t = \boxed{5.52\ \text{s}}$$

(*c*) $\boxed{\text{If you take the reaction time into account, the collision will occur sooner and be more severe.}}$

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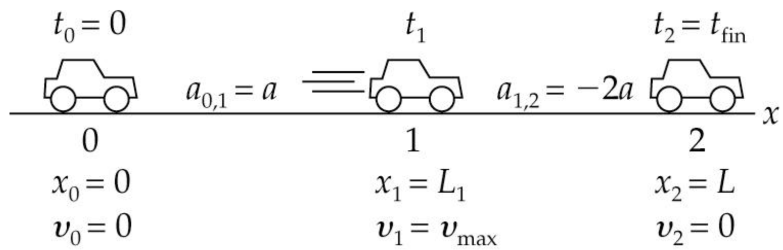
Picture the Problem Lou's acceleration is constant during both parts of his trip. Let t_1 be the time when the brake is applied; L_1 the distance traveled from $t = 0$ to $t = t_1$. Let t_{fin} be the time when Lou's car comes to rest at a distance L from the starting line. A pictorial representation will help organize the given information and plan the solution.



(*a*) Express the total length, *L*, of the course in terms of the distance over which Lou will be accelerating, Δx_{01}, and the distance over which he will be braking, Δx_{12}:

$$L = \Delta x_{01} + \Delta x_{12}$$

Express the final velocity over the first portion of the course in terms of the initial velocity, acceleration, and displacement; solve for the displacement:

$$v_1^2 = v_0^2 + 2a_{01}\Delta x_{01}$$

or, because $v_0 = 0$, $\Delta x_{01} = L_1$, and $a_{01} = a$,

$$\Delta x_{01} = L_1 = \frac{v_1^2}{2a} = \frac{v_{max}^2}{2a}$$

Express the final velocity over the second portion of the course in terms of the initial velocity, acceleration, and displacement; solve for the displacement:

$$v_2^2 = v_1^2 + 2a_{12}\Delta x_{12}$$

or, because $v_2 = 0$ and $a_{12} = -2a$,

$$\Delta x_{12} = \frac{v_1^2}{4a} = \frac{L_1}{2}$$

Substitute for Δx_{01} and Δx_{12} to obtain:

$$L = \Delta x_{01} + \Delta x_{12} = L_1 + \tfrac{1}{2}L_1 = \tfrac{3}{2}L_1$$

and

$$L_1 = \boxed{\tfrac{2}{3}L}$$

(*b*) Using the fact that the acceleration was constant during both legs of the trip, express Lou's average velocity over each leg:

$$v_{av,01} = v_{av,12} = \frac{v_{max}}{2}$$

Express the time for Lou to reach his maximum velocity as a function of L_1 and his maximum velocity:

$$\Delta t_{01} = \frac{\Delta x_{01}}{v_{av,01}} = \frac{2L_1}{v_{max}}$$

and

$$\Delta t_{01} \propto L_1 = \frac{2}{3}L$$

Having just shown that the time required for the first segment of the trip is proportional to the length of the segment, use this result to express Δt_{01} (= t_1) in terms t_{fin}:

$$t = \boxed{\tfrac{2}{3}t_{fin}}$$

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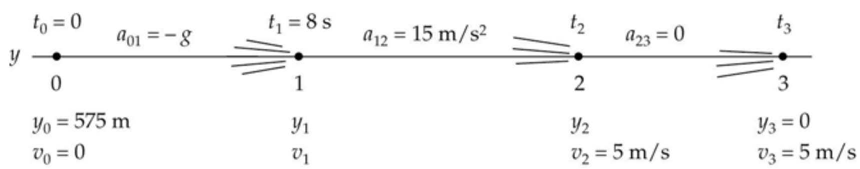
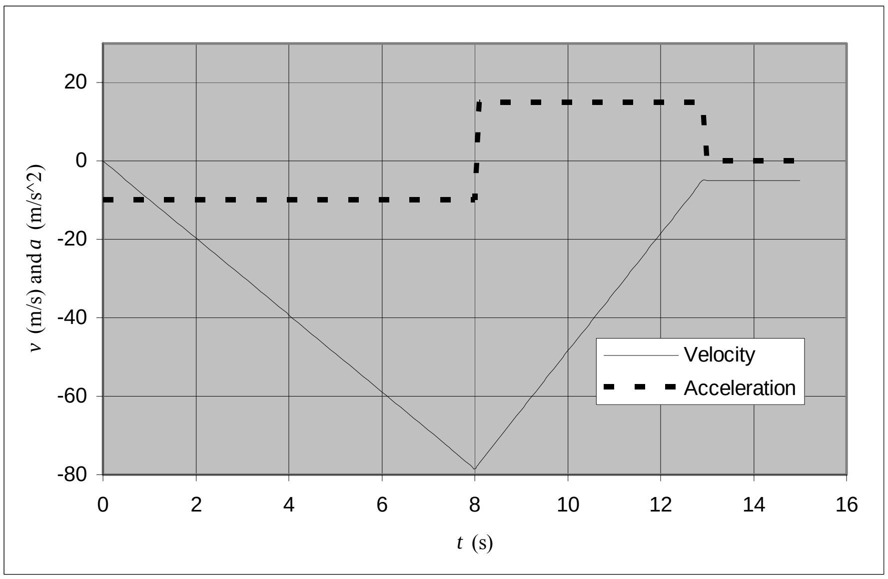
Picture the Problem There are three intervals of constant acceleration described in this problem. Choose a coordinate system in which the upward direction (shown to the left below) is positive. A pictorial representation will help organize the details of the problem and plan the solution.



(*a*) The graphs of *a*(*t*) (dashed lines) and *v*(*t*) (solid lines) are shown below.



(*b*) Using a constant-acceleration equation, express her speed in terms of her acceleration and the elapsed time; solve for her speed after 8 s of fall:

$$v_1 = v_0 + a_{01}t_1 = 0 + (9.81\,\text{m/s}^2)(8\,\text{s}) = \boxed{78.5\,\text{m/s}}$$

(*c*) Using the same constant-acceleration equation that you used in part (*b*), determine the duration of her constant upward acceleration:

$$v_2 = v_1 + a_{12}\Delta t_{12}$$

$$\Delta t_{12} = \frac{v_2 - v_1}{a_{12}} = \frac{-5\,\text{m/s} - (-78.5\,\text{m/s})}{15\,\text{m/s}^2} = \boxed{4.90\,\text{s}}$$

(*d*) Find her average speed as she slows from 78.5 m/s to 5 m/s:

$$v_{av} = \frac{v_1 + v_2}{2} = \frac{78.5\,\text{m/s} + 5\,\text{m/s}}{2} = 41.8\,\text{m/s}$$

Use this value to calculate how far she travels in 4.90 s:

$$\Delta y_{12} = v_{av}\Delta t_{12} = (41.8\text{ m/s})(4.90\text{ s})$$
$$= 204\text{ m}$$

She travels 204 m while slowing down.

(*e*) Express the total time in terms of the times for each segment of her descent:

$$t_{total} = \Delta t_{01} + \Delta t_{12} + \Delta t_{23}$$

We know the times for the intervals from 0 to 1 and 1 to 2 so we only need to determine the time for the interval from 2 to 3. We can calculate Δt_{23} from her displacement and constant velocity during that segment of her descent.

$$\Delta y_{23} = \Delta y_{total} - \Delta y_{01} - \Delta y_{12}$$
$$= 575\text{ m} - \left(\frac{78.5\text{ m/s}}{2}\right)(8\text{ s}) - 204\text{ m}$$
$$= 57.0\text{ m}$$

Add the times to get the total time:

$$t_{total} = t_{01} + t_{12} + t_{23}$$
$$= 8\text{ s} + 4.9\text{ s} + \frac{57.0\text{ m}}{5\text{ m/s}}$$
$$= \boxed{24.3\text{ s}}$$

(*f*) Using its definition, calculate her average velocity:

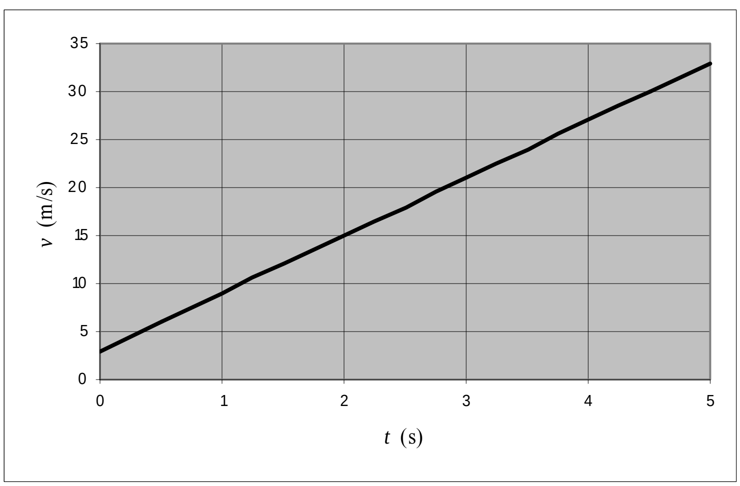
$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{-1500\text{ m}}{209\text{ s}} = \boxed{-7.18\text{ m/s}}$$

Integration of the Equations of Motion

***115** •

Picture the Problem The integral of a function is equal to the "area" between the curve for that function and the independent-variable axis.

(*a*) The graph is shown below:



The distance is found by determining the area under the curve. You can accomplish this easily because the shape of the area under the curve is a trapezoid.

A = (36 blocks)(2.5 m/block) = $\boxed{90\text{ m}}$

or

$$A=\left(\frac{33\,\text{m/s}+3\,\text{m/s}}{2}\right)(5\,\text{s}-0\,\text{s})=90\,\text{m}$$

Alternatively, we could just count the blocks and fractions thereof.

There are approximately 36 blocks each having an area of (5 m/s)(0.5 s) = 2.5 m.

(b) To find the position function $x(t)$, we integrate the velocity function $v(t)$ over the time interval in question:

$$x(t)=\int_0^t v(t')\text{d}t'$$

$$=\int_0^t \left[\left(6\,\text{m/s}^2\right)t'+(3\,\text{m/s})\right]\text{d}t'$$

and

$$\boxed{x(t)=\left(3\,\text{m/s}^2\right)t^2+(3\,\text{m/s})t}$$

Now evaluate $x(t)$ at 0 s and 5 s respectively and subtract to obtain Δx:

$$\Delta x = x(5\,\text{s})-x(0\,\text{s})=90\text{ m}-0\text{ m}$$

$$=\boxed{90.0\,\text{m}}$$

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Picture the Problem The integral of $v(t)$ over a time interval is the displacement (change in position) during that time interval. The integral of a function is equivalent to the "area" between the curve for that function and the independent-variable axis. Count the grid boxes.

(a) Find the area of the shaded gridbox:

$$Area=(1\,\text{m/s})(1\,\text{s})=\boxed{1\,\text{m per box}}$$

(b) Find the approximate area under curve for 1 s $\le t \le$ 2 s:

$$\Delta x_{1\text{ s to }2\text{ s}}=\boxed{1.2\,\text{m}}$$

Find the approximate area under curve for 2 s $\le t \le$ 3 s:

$$\Delta x_{2\text{ s to }3\text{ s}}=\boxed{3.2\,\text{m}}$$

(c) Sum the displacements to obtain the total in the interval 1 s $\le t \le$ 3 s:

$$\Delta x_{1\text{ s to }3\text{ s}}=1.2\text{ m}+3.2\text{ m}$$
$$=4.4\text{ m}$$

Using its definition, express and evaluate v_{av}:

$$v_{\text{av}}=\frac{\Delta x_{1\,\text{s to }3\,\text{s}}}{\Delta t_{1\,\text{s to }3\,\text{s}}}=\frac{4.4\,\text{m}}{2\,\text{s}}=\boxed{2.20\,\text{m/s}}$$

(d) Because the velocity of the particle is dx/dt, separate the

$$dx=\left(0.5\text{m/s}^3\right)dt$$

so

variables and integrate over the interval 1 s ≤ t ≤ 3 s to determine the displacement in this time interval:

$$\Delta x_{1\text{s}\to 3\text{s}} = \int_{x_0}^{x} dx' = \left(0.5\text{m/s}^3\right)\int_{1\,\text{s}}^{3\,\text{s}} t'^2\, dt'$$

$$= \left(0.5\text{m/s}^3\right)\left[\frac{t'^3}{3}\right]_{1\text{s}}^{3\text{s}} = \boxed{4.33\text{m}}$$

This result is a little smaller than the sum of the displacements found in part (*b*).

Calculate the average velocity over the 2-s interval from 1 s to 3 s:

$$v_{\text{av}\,(1\text{s}-3\text{s})} = \frac{\Delta x_{1\text{s}-3\text{s}}}{\Delta t_{1\text{s}-3\text{s}}} = \frac{4.33\text{m}}{2\text{s}} = 2.17\text{m/s}$$

Calculate the initial and final velocities of the particle over the same interval:

$$v(1\,\text{s}) = \left(0.5\,\text{m/s}^3\right)(1\,\text{s})^2 = 0.5\,\text{m/s}$$

$$v(3\,\text{s}) = \left(0.5\,\text{m/s}^3\right)(3\,\text{s})^2 = 4.5\,\text{m/s}$$

Finally, calculate the average value of the velocities at $t = 1$ s and $t = 3$ s:

$$\frac{v(1\,\text{s}) + v(3\,\text{s})}{2} = \frac{0.5\,\text{m/s} + 4.5\,\text{m/s}}{2}$$

$$= 2.50\,\text{m/s}$$

This average is not equal to the average velocity calculated above.

Remarks: The fact that the average velocity was not equal to the average of the velocities at the beginning and the end of the time interval in part (*d*) is a consequence of the acceleration not being constant.

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Picture the Problem Because the velocity of the particle varies with the square of the time, the acceleration is not constant. The displacement of the particle is found by integration.

Express the velocity of a particle as the derivative of its position function:

$$v(t) = \frac{dx(t)}{dt}$$

Separate the variables to obtain:

$$dx(t) = v(t)dt$$

Express the integral of x from $x_o = 0$ to x and t from $t_0 = 0$ to t:

$$x(t) = \int_{t_0=0}^{x(t)} dx' = \int_{t_0=0}^{t} v(t')dt'$$

Substitute for $v(t')$ to obtain:

$$x(t) = \int_{t_0=0}^{t} \left[\left(7\,\text{m/s}^3\right)t'^2 - (5\,\text{m/s})\right]dt'$$

$$= \boxed{\left(\tfrac{7}{3}\,\text{m/s}^3\right)t^3 - (5\,\text{m/s})t}$$

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Picture the Problem The graph is one of constant negative acceleration. Because

$v_x = v(t)$ is a linear function of t, we can make use of the slope-intercept form of the equation of a straight line to find the relationship between these variables. We can then differentiate $v(t)$ to obtain $a(t)$ and integrate $v(t)$ to obtain $x(t)$.

Find the acceleration (the slope of the graph) and the velocity at time 0 (the v-intercept) and use the slope-intercept form of the equation of a straight line to express $v_x(t)$:

$$\boxed{a = -10\,\text{m/s}^2}$$

$$\boxed{v_x(t) = 50\,\text{m/s} + (-10\,\text{m/s}^2)t}$$

Find $x(t)$ by integrating $v(t)$:

$$x(t) = \int \left[\left(-10\,\text{m/s}^2\right)t + 50\,\text{m/s}\right]dt$$
$$= (50\,\text{m/s})t - \left(5\,\text{m/s}^2\right)t^2 + C$$

Using the fact that $x = 0$ when $t = 0$, evaluate C:

$$0 = (50\,\text{m/s})(0) - \left(5\,\text{m/s}^2\right)(0)^2 + C$$

and

$C = 0$

Substitute to obtain:

$$\boxed{x(t) = (50\,\text{m/s})t - \left(5\,\text{m/s}^2\right)t^2}$$

Note that this expression is quadratic in t and that the coefficient of t^2 is negative and equal in magnitude to half the constant acceleration.

Remarks: We can check our result for $x(t)$ by evaluating it over the 10-s interval shown and comparing this result with the area bounded by this curve and the time axis.

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Picture the Problem During any time interval, the integral of $a(t)$ is the change in velocity and the integral of $v(t)$ is the displacement. The integral of a function equals the "area" between the curve for that function and the independent-variable axis.

(*a*) Find the area of the shaded grid box in Figure 2-37:

Area = (0.5 m/s²)(0.5 s)

$$= \boxed{0.250\,\text{m/s per box}}$$

(*b*) We start from rest ($v_0 = 0$) at $t = 0$. For the velocities at the other times, count boxes and multiply by the 0.25 m/s per box that we found in part (*a*):

Examples:

$v(1\text{ s})$ = (3.7 boxes)[(0.25 m/s)/box]

$$= \boxed{0.925\,\text{m/s}}$$

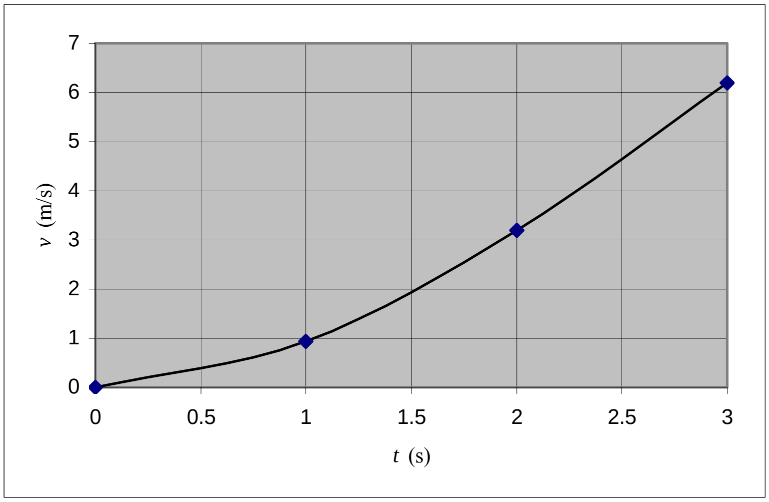
$v(2\text{ s})$ = (12.9 boxes)[(0.25 m/s)/box]

$$= \boxed{3.22\,\text{m/s}}$$

and

$$v(3\text{ s}) = (24.6\text{ boxes})[(0.25\text{ m/s})/\text{box}]$$
$$= \boxed{6.15\text{ m/s}}$$

(*c*) The graph of *v* as a function of *t* is shown below:



$$Area = (1.0\text{ m/s})(1.0\text{ s}) = 1.0\text{ m per box}$$

Count the boxes under the *v*(*t*) curve to find the distance traveled:

$$x(3\text{ s}) = \Delta x(0 \to 3\text{ s})$$
$$= (7\text{ boxes})[(1.0\text{ m})/\text{box}]$$
$$= \boxed{7.00\text{ m}}$$

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Picture the Problem The integral of *v*(*t*) over a time interval is the displacement (change in position) during that time interval. The integral of a function equals the "area" between the curve for that function and the independent-variable axis. Because acceleration is the slope of a velocity versus time curve, this is a non-constant-acceleration problem. The derivative of a function is equal to the "slope" of the function at that value of the independent variable.

(*a*) To obtain the data for *x*(*t*), we must estimate the accumulated area under the *v*(*t*) curve at each time interval:

Find the area of a shaded grid box in Figure 2-38:

$$A = (1\text{ m/s})(0.5\text{ s}) = 0.5\text{ m per box.}$$

We start from rest ($v_o = 0$) at $t_o = 0$. For the position at the other times, count boxes and multiply by the 0.5 m per box that we found above. Remember to add the offset from the origin, $x_o = 5$ m, and that boxes below the $v = 0$ line are counted as negative:

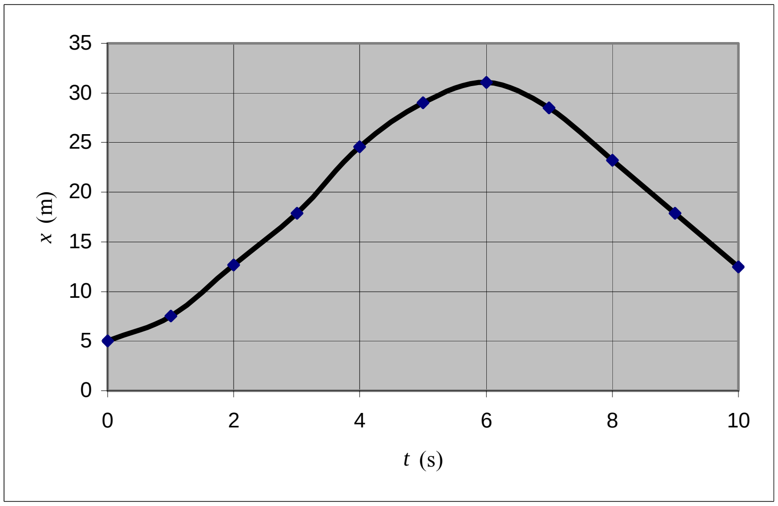
Examples:

$$x(3\text{ s}) = (25.8\text{ boxes})\left(\frac{0.5\text{ m}}{\text{box}}\right) + 5\text{ m}$$
$$= 17.9\text{ m}$$

$$x(5\text{ s}) = (48.0\text{ boxes})\left(\frac{0.5\text{ m}}{\text{box}}\right) + 5\text{ m}$$
$$= 29.0\text{ m}$$

$$x(10\,\text{s}) = (51.0\,\text{boxes})\left(\frac{0.5\,\text{m}}{\text{box}}\right) - (36.0\,\text{boxes})\left(\frac{0.5\,\text{m}}{\text{box}}\right) + 5\,\text{m}$$
$$= 12.5\,\text{m}$$

A graph of *x* as a function of *t* follows:

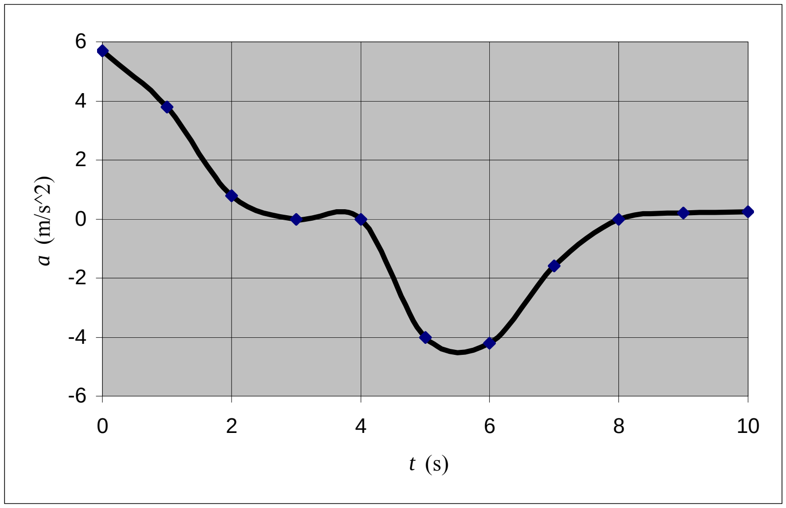


(*b*) To obtain the data for *a*(*t*), we must estimate the slope ($\Delta v/\Delta t$) of the *v*(*t*) curve at each time. A good way to get reasonably reliable readings from the graph is to enlarge several fold:

Examples:

$$a(1\,\text{s}) = \frac{v(1.25\,\text{s}) - v(0.75\,\text{s})}{0.5\,\text{s}}$$
$$= \frac{4.9\,\text{m/s} - 3.0\,\text{m/s}}{0.5\,\text{s}} = 3.8\,\text{m/s}^2$$
$$a(6\,\text{s}) = \frac{v(6.25\,\text{s}) - v(5.75\,\text{s})}{0.5\,\text{s}}$$
$$= \frac{-1.7\,\text{m/s} - 0.4\,\text{m/s}}{0.5\,\text{s}} = -4.2\,\text{m/s}^2$$

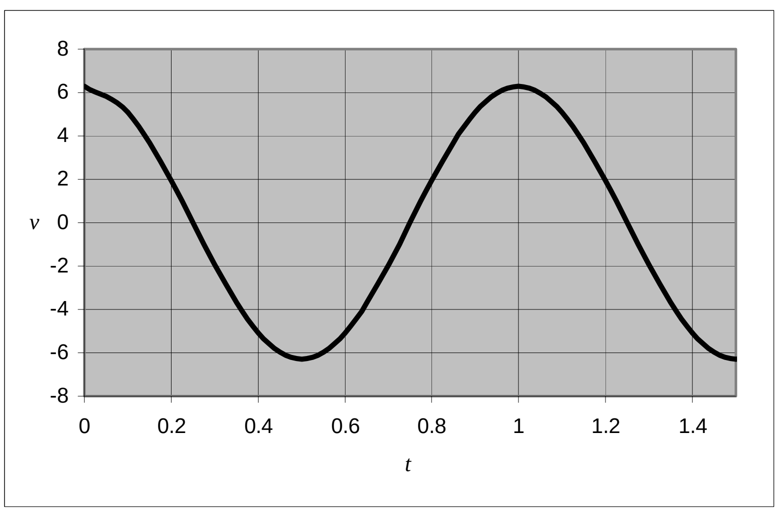
A graph of a as a function of t follows:



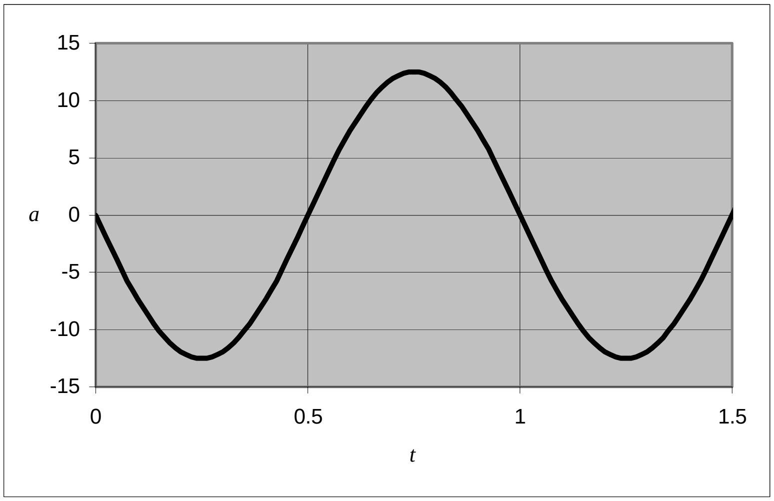
***121** ••

Picture the Problem Because the position of the body is not described by a parabolic function, the acceleration is not constant.

Select a series of points on the graph of $x(t)$ (e.g., at the extreme values and where the graph crosses the t axis), draw tangent lines at those points, and measure their slopes. In doing this, you are evaluating $v = dx/dt$ at these points. Plot these slopes above the times at which you measured the slopes. Your graph should closely resemble the following graph.



Select a series of points on the graph of $v(t)$ (e.g., at the extreme values and where the graph crosses the t axis), draw tangent lines at those points, and measure their slopes. In doing this, you are evaluating $a = dv/dt$ at these points. Plot these slopes above the times at which you measured the slopes. Your graph should closely resemble the graph shown below.



122 ••

Picture the Problem Because the acceleration of the rocket varies with time, it is not constant and integration of this function is required to determine the rocket's velocity and position as functions of time. The conditions on x and v at $t = 0$ are known as ***initial conditions***.

(*a*) Integrate $a(t)$ to find $v(t)$:

$$v(t) = \int a(t)\,dt = b\int t\,dt = \tfrac{1}{2}bt^2 + \mathrm{C}$$

where C, the constant of integration, can be determined from the initial conditions.

Integrate $v(t)$ to find $x(t)$:

$$x(t) = \int v(t)dt = \int\left[\tfrac{1}{2}bt^2 + \mathrm{C}\right]dt = \tfrac{1}{6}bt^3 + \mathrm{C}t + \mathrm{D}$$

where D is a second constant of integration.

Using the initial conditions, find the constants C and D:

$$v(0) = 0 \;\Rightarrow\; \mathrm{C} = 0$$

and

$$x(0) = 0 \;\Rightarrow\; \mathrm{D} = 0$$

$$\therefore \boxed{x(t) = \tfrac{1}{6}bt^3}$$

(*b*) Evaluate $v(5\text{ s})$ and $x(5\text{ s})$ with $\mathrm{C} = \mathrm{D} = 0$ and $b = 3\text{ m/s}^2$:

$$v(5\text{ s}) = \frac{1}{2}(3\text{m/s}^2)(5\text{s})^2 = \boxed{37.5\text{ m/s}}$$

and

$$x(5\text{ s}) = \frac{1}{6}(3\text{m/s}^2)(5\text{s})^3 = \boxed{62.5\text{ m}}$$

123 ••

Picture the Problem The acceleration is a function of time; therefore it is not constant. The instantaneous velocity can be determined by integration of the acceleration and the average velocity from the displacement of the particle during the given time interval.

(*a*) Because the acceleration is the derivative of the velocity, integrate the acceleration to find the ***instantaneous velocity*** $v(t)$.

$$a(t)=\frac{dv}{dt}\Rightarrow v(t)=\int_{v_0=0}^{v(t)}dv'=\int_{t_0=0}^{t}a(t')dt'$$

Calculate the instantaneous velocity using the acceleration given.

$$v(t)=\left(0.2\,\text{m/s}^3\right)\int_{t_0=0}^{t}t'dt'$$

and

$$\boxed{v(t)=\left(0.1\,\text{m/s}^3\right)t^2}$$

(*b*) To calculate the ***average velocity***, we need the displacement:

$$v(t)\equiv\frac{dx}{dt}\Rightarrow x(t)=\int_{x_0=0}^{x(t)}dx'=\int_{t_0=0}^{t}v(t')dt'$$

Because the velocity is the derivative of the displacement, integrate the velocity to find Δx.

$$x(t)=\left(0.1\,\text{m/s}^3\right)\int_{t_0=0}^{t}t'^2\,dt'=\left(0.1\,\text{m/s}^3\right)\frac{t^3}{3}$$

and

$$\begin{aligned}\Delta x&=x(7\text{ s})-x(2\text{ s})\\&=\left(0.1\,\text{m/s}^3\right)\left[\frac{(7\text{ s})^3-(2\text{ s})^3}{3}\right]\\&=11.2\text{ m}\end{aligned}$$

Using the definition of the ***average velocity***, calculate v_{av}.

$$v_{\text{av}}=\frac{\Delta x}{\Delta t}=\frac{11.2\text{ m}}{5\text{ s}}=\boxed{2.23\text{ m/s}}$$

124 •

Determine the Concept Because the acceleration is a function of time, it is not constant. Hence we'll need to integrate the acceleration function to find the velocity as a function of time and integrate the velocity function to find the position as a function of time. The important concepts here are the definitions of velocity, acceleration, and average velocity.

(*a*) Starting from $t_o = 0$, integrate the instantaneous acceleration to obtain the instantaneous velocity as a function of time:

From $a=\dfrac{dv}{dt}$

it follows that

$$\int_{v_0}^{v}dv'=\int_{0}^{t}(a_0+bt')dt'$$

and

$$\boxed{v=v_0+a_0t+\tfrac{1}{2}bt^2}$$

(*b*) Now integrate the instantaneous velocity to obtain the position as a function of time:

From $v=\dfrac{dx}{dt}$

it follows that

$$\int_{x_0}^{x} dx' = \int_{t_0=0}^{t} v(t')\,dt'$$
$$= \int_{t_0}^{t}\left(v_9 + a_0 t' + \frac{b}{2}t'^2\right)dt'$$

and

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3}$$

(*c*) The definition of the average velocity is the ratio of the displacement to the total time elapsed:

$$v_{av} \equiv \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{v_0 t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3}{t}$$

and

$$\boxed{v_{av} = v_0 + \tfrac{1}{2}a_0 t + \tfrac{1}{6}bt^2}$$

Note that v_{av} is not the same as that due to constant acceleration:

$$\left(v_{\text{constant acceleration}}\right)_{av} = \frac{v_0 + v}{2}$$
$$= \frac{v_0 + \left(v_0 + a_0 t + \tfrac{1}{2}bt^2\right)}{2}$$
$$= v_0 + \tfrac{1}{2}a_0 t + \tfrac{1}{4}bt^2$$
$$\neq v_{av}$$

General Problems

125 •••

Picture the Problem The acceleration of the marble is constant. Because the motion is downward, choose a coordinate system with downward as the positive direction. The equation $g_{exp} = (1\text{ m})/(\Delta t)^2$ originates in the constant-acceleration equation $\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$. Because the motion starts from rest, the displacement of the marble is 1 m, the acceleration is the experimental value g_{exp}, and the equation simplifies to $g_{exp} = (1\text{ m})/(\Delta t)^2$.

Express the percent difference between the accepted and experimental values for the acceleration due to gravity:

$$\%\text{ difference} = \frac{\left|g_{accepted} - g_{exp}\right|}{g_{accepted}}$$

Using a constant-acceleration equation, express the velocity of the marble in terms of its initial velocity, acceleration, and displacement:

$$v_f^2 = v_0^2 + 2a\Delta y$$

or, because $v_0 = 0$ and $a = g$,

$$v_f^2 = 2g\Delta y$$

Solve for v_f:

$$v_f = \sqrt{2g\Delta y}$$

Let v_1 be the velocity the ball has reached when it has fallen 0.5 cm,

$$v_1 = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(0.005\text{ m}\right)} = 0.313\text{ m/s}$$

and v_2 be the velocity the ball has reached when it has fallen 0.5 m to obtain.

and

$$v_2 = \sqrt{2(9.81\,\text{m/s}^2)(0.5\,\text{m})} = 3.13\,\text{m/s}$$

Using a constant-acceleration equation, express v_2 in terms of v_1, g and Δt:

$$v_2 = v_1 + g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_2 - v_1}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{3.13\,\text{m/s} - 0.313\,\text{m/s}}{9.81\,\text{m/s}^2} = 0.2872\,\text{s}$$

Calculate the experimental value of the acceleration due to gravity from $g_{\text{exp}} = (1\text{ m})/(\Delta t)^2$:

$$g_{\text{exp}} = \frac{1\,\text{m}}{(0.2872\,\text{s})^2} = \boxed{12.13\,\text{m/s}^2}$$

Finally, calculate the percent difference between this experimental result and the value accepted for g at sea level.

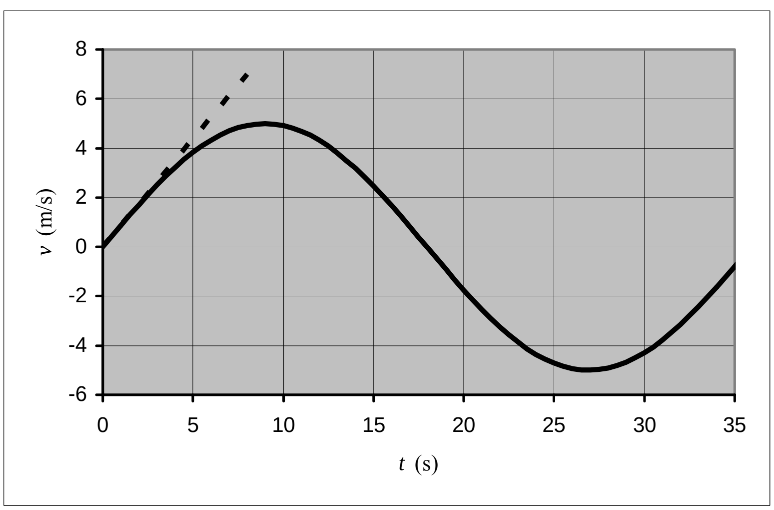
$$\%\text{ difference} = \frac{\left|9.81\,\text{m/s}^2 - 12.13\,\text{m/s}^2\right|}{9.81\,\text{m/s}^2}$$

$$= \boxed{23.6\,\%}$$

***126 •••**

Picture the Problem We can obtain an average velocity, $v_{\text{av}} = \Delta x/\Delta t$, over fixed time intervals. The instantaneous velocity, $v = dx/dt$ can only be obtained by differentiation.

(*a*) The graph of x versus t is shown below:



(*b*) Draw a tangent line at the origin and measure its rise and run. Use this ratio to obtain an approximate value for the slope at the origin:

The tangent line appears to, at least approximately, pass through the point (5, 4). Using the origin as the second point,

$$\Delta x = 4\text{ cm} - 0 = 4\text{ cm}$$

and

$$\Delta t = 5\text{ s} - 0 = 5\text{ s}$$

Therefore, the slope of the tangent line and the velocity of the body as it passes through the origin is approximately:

$$v(0) = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{4\,\text{cm}}{5\,\text{s}} = \boxed{0.800\,\text{cm/s}}$$

(*c*) Calculate the average velocity for the series of time intervals given by completing the table shown below:

t_0	t	Δt	x_0	x	Δx	$v_{av}=\Delta x/\Delta t$
(s)	(s)	(s)	(cm)	(cm)	(cm)	(m/s)
0	6	6	0	4.34	4.34	0.723
0	3	3	0	2.51	2.51	0.835
0	2	2	0	1.71	1.71	0.857
0	1	1	0	0.871	0.871	0.871
0	0.5	0.5	0	0.437	0.437	0.874
0	0.25	0.25	0	0.219	0.219	0.875

(*d*) Express the time derivative of the position:

$$\frac{dx}{dt} = A\omega\cos\omega t$$

Substitute numerical values and evaluate $\frac{dx}{dt}$ at $t = 0$:

$$\frac{dx}{dt} = A\omega\cos 0 = A\omega$$

$$= (0.05\,\text{m})(0.175\,\text{s}^{-1})$$

$$= \boxed{0.875\,\text{cm/s}}$$

(*e*) Compare the average velocities from part (*c*) with the instantaneous velocity from part (*d*):

As Δt, and thus Δx, becomes small, the value for the average velocity approaches that for the instantaneous velocity obtained in part (*d*). For $\Delta t = 0.25$ s, they agree to three significant figures.

127 •••

Determine the Concept Because the velocity varies nonlinearly with time, the acceleration of the object is not constant. We can find the acceleration of the object by differentiating its velocity with respect to time and its position function by integrating the velocity function. The important concepts here are the definitions of acceleration and velocity.

(*a*) The acceleration of the object is the derivative of its velocity with respect to time:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[v_{\max}\sin(\omega t)\right]$$

$$= \boxed{\omega v_{\max}\cos(\omega t)}$$

Because a varies sinusoidally with time it is *not* constant.

(*b*) Integrate the velocity with respect to time from 0 to t to obtain the change in position of the body:

$$\int_{x_0}^{x} dx' = \int_{t_0}^{t} [v_{max} \sin(\omega t')] dt'$$

and

$$x - x_0 = \left[\frac{-v_{max}}{\omega}\cos(\omega t')\right]_0^t$$

$$= \frac{-v_{max}}{\omega}\cos(\omega t) + \frac{v_{max}}{\omega}$$

or

$$\boxed{x = x_0 + \frac{v_{max}}{\omega}[1 - \cos(\omega t)]}$$

Note that, as given in the problem statement, $x(0\text{ s}) = x_0$.

128 •••

Picture the Problem Because the acceleration of the particle is a function of its position, it is not constant. Changing the variable of integration in the definition of acceleration will allow us to determine its velocity and position as functions of position.

(*a*) Because $a = dv/dt$, we must integrate to find $v(t)$. Because a is given as a function of x, we'll need to change variables in order to carry out the integration. Once we've changed variables, we'll separate them with v on the left side of the equation and x on the right:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} = \left(2\,\text{s}^{-2}\right)x$$

or, upon separating variables,

$$vdv = \left(2\,\text{s}^{-2}\right)xdx$$

Integrate from x_o and v_o to x and v:

$$\int_{v_0=0}^{v} v'dv' = \int_{x_0}^{x} \left(2\,\text{s}^{-2}\right)x'dx'$$

and

$$v^2 - v_0^2 = \left(2\,\text{s}^{-2}\right)\left(x^2 - x_0^2\right)$$

Solve for v to obtain:

$$v = \sqrt{v_0^2 + \left(2\,\text{s}^{-2}\right)\left(x^2 - x_0^2\right)}$$

Now set $v_o = 0$, $x_o = 1$ m, $x = 3$ m, $b = 2\ \text{s}^{-2}$ and evaluate the speed:

$$v = \pm\sqrt{\left(2\,\text{s}^{-2}\right)\left[(3\,\text{m})^2 - (1\,\text{m})^2\right]}$$

and

$$|v| = \boxed{4.00\,\text{m/s}}$$

(*b*) Using the definition of *v*, separate the variables, and integrate to get an expression for *t*:

$$v(x)=\frac{dx}{dt}$$

and

$$\int_0^t dt' = \int_{x_0}^{x}\frac{dx'}{v(x')}$$

To evaluate this integral we first must find $v(x)$. Show that the acceleration is always positive and use this to find the sign of $v(x)$.

$a = (2\ \text{s}^{-2})x$ and $x_0 = 1$ m. x_0 is positive, so a_0 is also positive. v_0 is zero and a_0 is positive, so the object moves in the direction of increasing *x*. As *x* increases the acceleration remains positive, so the velocity also remains positive. Thus,

$$v=\sqrt{(2\,\text{s}^{-2})(x^2-x_0^2)}.$$

Substitute $\sqrt{(2\,\text{s}^{-2})(x^2-x_0^2)}$ for *v* and evaluate the integral. (It can be found in standard integral tables.)

$$t=\int_0^t dt' = \int_{x_0}^{x}\frac{dx'}{v(x')}$$

$$=\int_{x_0}^{x}\frac{dx'}{\sqrt{(2\,\text{s}^{-2})(x'^2-x_0^2)}}$$

$$=\frac{1}{\sqrt{(2\,\text{s}^{-2})}}\int_{x_0}^{x}\frac{dx'}{\sqrt{x'^2-x_0^2}}$$

$$=\frac{1}{\sqrt{(2\,\text{s}^{-2})}}\ln\left(\frac{x+\sqrt{x^2-x_0^2}}{x_0}\right)$$

Evaluate this expression with $x_o = 1$ m and $x = 3$ m to obtain:

$$t=\boxed{1.25\,\text{s}}$$

129 •••

Picture the Problem The acceleration of this particle is not constant. Separating variables and integrating will allow us to express the particle's position as a function of time and the differentiation of this expression will give us the acceleration of the particle as a function of time.

(*a*) Write the definition of velocity:

$$v=\frac{dx}{dt}$$

We are given that $x = bv$, where $b = 1$ s. Substitute for *v* and separate variables to obtain:

$$\frac{dx}{dt}=\frac{x}{b}\Rightarrow dt=b\frac{dx}{x}$$

Integrate and solve for $x(t)$:

$$\int_{t_0}^{t}dt'=b\int_{x_0}^{x}\frac{dx'}{x'}\Rightarrow (t-t_0)=b\ln\left(\frac{x}{x_0}\right)$$

and

$$x(t)=\boxed{x_0 e^{(t-t_0)/b}}$$

(*b*) Differentiate twice to obtain *v*(*t*) and *a*(*t*):

$$v=\frac{dx}{dt}=\frac{1}{b}x_0 e^{(t-t_0)/b}$$

and

$$a=\frac{dv}{dt}=\frac{1}{b^2}x_0 e^{(t-t_0)/b}$$

Substitute the result in part (*a*) to obtain the desired results:

$$v(t)=\frac{1}{b}x(t)$$

and

$$a(t)=\frac{1}{b^2}x(t)$$

so

$$a(t)=\frac{1}{b}v(t)=\frac{1}{b^2}x(t)$$

$$\boxed{\text{Because the numerical value of } b\text{, expressed in SI units, is one, the numerical values of } a, v, \text{and } x \text{ are the same at each instant in time.}}$$

130 •••

Picture the Problem Because the acceleration of the rock is a function of time, it is not constant. Choose a coordinate system in which downward is positive and the origin at the point of release of the rock.

Separate variables in $a(t)=dv/dt=ge^{-bt}$ to obtain:

$$dv=ge^{-bt}dt$$

Integrate from $t_o=0$, $v_o=0$ to some later time *t* and velocity *v*:

$$v=\int_0^v dv'=\int_0^t ge^{-bt'}dt'=\frac{g}{-b}\left[e^{-bt'}\right]_0^t$$

$$=\frac{g}{b}\left(1-e^{-bt}\right)=v_{\text{term}}\left(1-e^{-bt}\right)$$

where

$$v_{\text{term}}=\frac{g}{b}$$

Separate variables in $v=dy/dt=v_{\text{term}}\left(1-e^{-bt}\right)$ to obtain:

$$dy=v_{\text{term}}\left(1-e^{-bt}\right)dt$$

Integrate from $t_o=0$, $y_o=0$ to some later time *t* and position *y*:

$$\int_0^y dy'=\int_0^t v_{\text{term}}\left(1-e^{-bt'}\right)dt'$$

$$y = v_{\text{term}}\left[t' + \frac{1}{b}e^{-bt'}\right]_0^t$$

$$= \boxed{v_{\text{term}}t - \frac{v_{\text{term}}}{b}\left(1 - e^{-bt}\right)}$$

This last result is very interesting. It says that throughout its free-fall, the object experiences drag; therefore it has not fallen as far at any given time as it would have if it were falling at the constant velocity, v_{term}.

On the other hand, just as the velocity of the object asymptotically approaches v_{term}, the distance it has covered during its free-fall as a function of time asymptotically approaches the distance it would have fallen if it had fallen with v_{term} throughout its motion.

$$y\left(t_{\text{large}}\right) \to v_{\text{term}}t - \frac{v}{b} \to v_{\text{term}}t$$

This should not be surprising because in the expression above, the first term grows linearly with time while the second term approaches a constant and therefore becomes less important with time.

***131 •••**

Picture the Problem Because the acceleration of the rock is a function of its velocity, it is not constant. Choose a coordinate system in which downward is positive and the origin is at the point of release of the rock.

Rewrite $a = g - bv$ explicitly as a differential equation:

$$\frac{dv}{dt} = g - bv$$

Separate the variables, v on the left, t on the right:

$$\frac{dv}{g - bv} = dt$$

Integrate the left-hand side of this equation from 0 to v and the right-hand side from 0 to t:

$$\int_0^v \frac{dv'}{g - bv'} = \int_0^t dt'$$

and

$$-\frac{1}{b}\ln\left(\frac{g - bv}{g}\right) = t$$

Solve this expression for v.

$$v = \frac{g}{b}\left(1 - e^{-bt}\right)$$

Finally, differentiate this expression with respect to time to obtain an expression for the acceleration and

$$a = \frac{dv}{dt} = \boxed{ge^{-bt}}$$

complete the proof.

132 •••

Picture the Problem The skydiver's acceleration is a function of her velocity; therefore it is not constant. Expressing her acceleration as the derivative of her velocity, separating the variables, and then integrating will give her velocity as a function of time.

(*a*) Rewrite $a = g - cv^2$ explicitly as a differential equation:

$$\frac{dv}{dt} = g - cv^2$$

Separate the variables, with *v* on the left, and *t* on the right:

$$\frac{dv}{g - cv^2} = dt$$

Eliminate *c* by using $c = \frac{g}{v_T^2}$:

$$\frac{dv}{g - \frac{g}{v_T^2}v^2} = \frac{dv}{g\left[1 - \left(\frac{v}{v_T}\right)^2\right]} = dt$$

or

$$\frac{dv}{1 - \left(\frac{v}{v_T}\right)^2} = gdt$$

Integrate the left-hand side of this equation from 0 to *v* and the right-hand side from 0 to *t*:

$$\int_0^v \frac{dv'}{1 - \left(\frac{v'}{v_T}\right)^2} = g\int_0^t dt' = gt$$

The integral can be found in integral tables:

$$v_T \tanh^{-1}(v/v_T) = gt$$

or

$$\tanh^{-1}(v/v_T) = (g/v_T)t$$

Solve this equation for *v* to obtain:

$$v = v_T \tanh\left(\frac{g}{v_T}\right)t$$

Because *c* has units of m^{-1}, and *g* has units of m/s^2, $(cg)^{-1/2}$ will have units of time. Let's represent this expression with the time-scale factor *T*:

i.e., $T = (cg)^{-1/2}$

The skydiver falls with her terminal velocity when $a = 0$. Using this definition, relate her terminal velocity to the acceleration due to gravity and the constant c in the acceleration equation:

$$0 = g - cv_T^2$$

and

$$v_T = \sqrt{\frac{g}{c}}$$

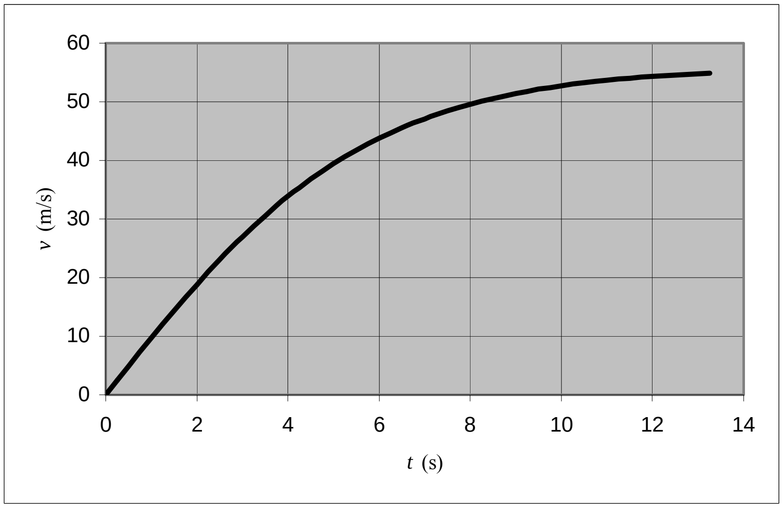
Convince yourself that T is also equal to v_T/g and use this relationship to eliminate g and v_T in the solution to the differential equation:

$$v(t) = \boxed{v_T \tanh\left(\frac{t}{T}\right)}$$

(*b*) The following table was generated using a spreadsheet and the equation we derived in part (*a*) for $v(t)$. The cell formulas and their algebraic forms are:

Cell	Content/Formula	Algebraic Form
D1	56	v_T
D2	5.71	T
B7	B6 + 0.25	$t + 0.25$
C7	$B$1*TANH(B7/$B$2)	$v_T \tanh\left(\frac{t}{T}\right)$

	A	B	C	D	E
1	vT =	56	m/s		
2	T=	5.71	s		
3					
4					
5		time (s)	*v* (m/s)		
6		0.00	0.00		
7		0.25	2.45		
8		0.50	4.89		
9		0.75	7.32		
10		1.00	9.71		
54		12.00	54.35		
55		12.25	54.49		
56		12.50	54.61		
57		12.75	54.73		
58		13.00	54.83		
59		13.25	54.93		



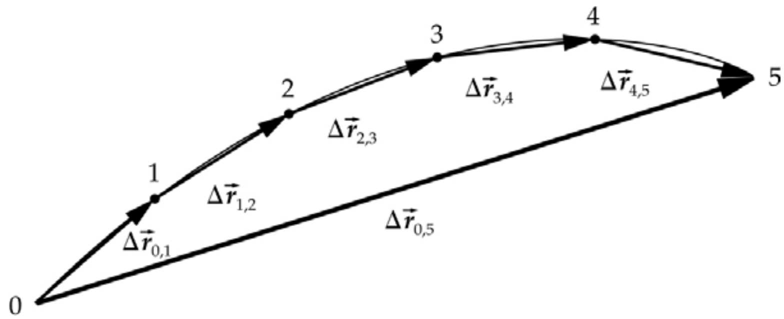
Note that the velocity increases linearly over time (i.e., with constant acceleration) for about time *T*, but then it approaches the terminal velocity as the acceleration decreases.

Chapter 3
Motion in Two and Three Dimensions

Conceptual Problems

***1 •**

Determine the Concept The distance traveled along a path can be represented as a sequence of displacements.



Suppose we take a trip along some path and consider the trip as a sequence of many very small displacements. The net displacement is the vector sum of the very small displacements, and the total distance traveled is the sum of the magnitudes of the very small displacements. That is,

$$\textit{total distance} = \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|$$

where N is the number of very small displacements. (For this to be exactly true we have to take the limit as N goes to infinity and each displacement magnitude goes to zero.) Now, using "the shortest distance between two points is a straight line," we have

$$\left|\Delta\vec{r}_{0,N}\right| \le \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|,$$

where $\left|\Delta\vec{r}_{0,N}\right|$ is the magnitude of the net displacement.

Hence, we have shown that the magnitude of the displacement of a particle is less than or equal to the distance it travels along its path.

2 •

Determine the Concept The displacement of an object is its final position vector minus its initial position vector ($\Delta\vec{r} = \vec{r}_f - \vec{r}_i$). The displacement can be less but never more than the distance traveled. Suppose the path is one complete trip around the earth at the equator. Then, the displacement is 0 but the distance traveled is $2\pi R_e$.

3 •

Determine the Concept The important distinction here is that ***average velocity*** is being requested, as opposed to ***average speed***.

The average velocity is defined as the displacement divided by the elapsed time.

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{0}{\Delta t} = 0$$

The displacement for any trip around the track is zero. Thus we see that no matter how fast the race car travels, the average velocity is always zero at the end of each complete circuit.

What is the correct answer if we were asked for *average speed*?

The average speed is defined as the distance traveled divided by the elapsed time.

$$v_{av} \equiv \frac{\text{total distance}}{\Delta t}$$

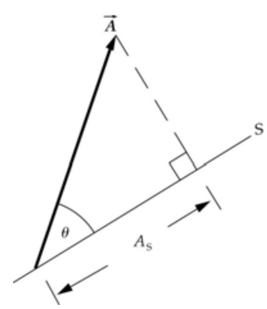
For one complete circuit of any track, the total distance traveled will be greater than zero and the average is not zero.

4 •

False. Vectors are quantities with magnitude and direction that can be added and subtracted like displacements. Consider two vectors that are equal in magnitude and oppositely directed. Their sum is zero, showing by counterexample that *the statement is false.*

5 •

Determine the Concept We can answer this question by expressing the relationship between the magnitude of vector $\vec{A}$ and its component A_S and then using properties of the cosine function.



Express A_S in terms of A and θ:

$$A_S = A\cos\theta$$

Take the absolute value of both sides of this expression:

$$|A_S| = |A\cos\theta| = A|\cos\theta|$$

and

$$|\cos\theta| = \frac{|A_S|}{A}$$

Using the fact that $0 < |\cos\theta| \le 1$, substitute for $|\cos\theta|$ to obtain:

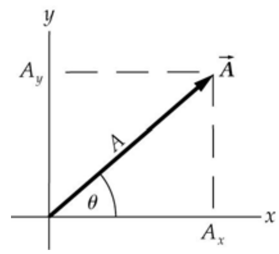
$$0 < \frac{|A_S|}{A} \le 1 \text{ or } 0 < |A_S| \le A$$

No. The magnitude of a component of a vector must be less than or equal to the magnitude of the vector.

If the angle θ shown in the figure is equal to 0° or multiples of 180°, then the magnitude of the vector and its component are equal.

***6** •

Determine the Concept The diagram shows a vector $\vec{A}$ and its components A_x and A_y. We can relate the magnitude of $\vec{A}$ is related to the lengths of its components through the Pythagorean theorem.



Suppose that $\vec{A}$ is equal to zero. Then $A^2 = A_x^2 + A_y^2 = 0$.

But $A_x^2 + A_y^2 = 0 \Rightarrow A_x = A_y = 0$.

No. If a vector is equal to zero, each of its components must be zero too.

7 •

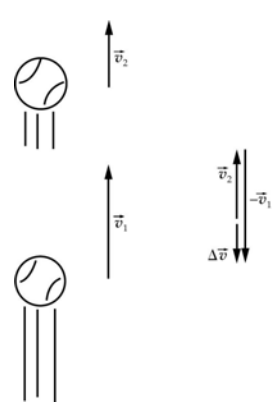
Determine the Concept No. Consider the special case in which $\vec{B} = -\vec{A}$. If $\vec{B} = -\vec{A} \ne 0$, then $\vec{C} = 0$ and the magnitudes of the components of $\vec{A}$ and $\vec{B}$ are larger than the components of $\vec{C}$.

***8** •

Determine the Concept The *instantaneous acceleration* is the limiting value, as Δt approaches zero, of $\Delta\vec{v}/\Delta t$. Thus, the acceleration vector is in the same direction as $\Delta\vec{v}$.

False. Consider a ball that has been thrown upward near the surface of the earth and is slowing down. *The direction of its motion is upward.*

The diagram shows the ball's velocity vectors at two instants of time and the determination of $\Delta\vec{v}$. Note that because $\Delta\vec{v}$ is downward so is the *acceleration* of the ball.



9 •

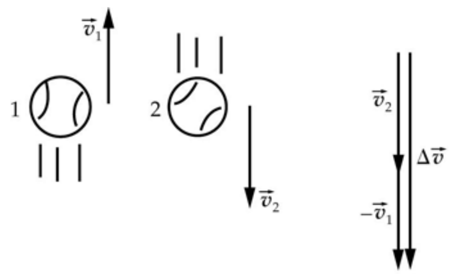
Determine the Concept The *instantaneous acceleration* is the limiting value, as Δt approaches zero, of $\Delta\vec{v}/\Delta t$ and is in the same direction as $\Delta\vec{v}$.

Other than through the definition of $\vec{a}$, the instantaneous velocity and acceleration vectors are unrelated. Knowing the direction of the velocity at one instant tells one nothing about how the velocity is changing at that instant. $\boxed{(e)\text{ is correct.}}$

10 •

Determine the Concept The changing velocity of the golf ball during its flight can be understood by recognizing that it has both horizontal and vertical components. The nature of its acceleration near the highest point of its flight can be understood by analyzing the vertical components of its velocity on either side of this point.

At the highest point of its flight, the ball is still *traveling horizontally* even though its vertical velocity is momentarily zero. The figure to the right shows the vertical components of the ball's velocity just before and just after it has reached its highest point. The change in velocity during this short interval is a non-zero, downward-pointing vector. Because the acceleration is proportional to the change in velocity, it must also be nonzero.



$\boxed{(d)\text{ is correct.}}$

Remarks: Note that v_x is nonzero and v_y is zero, while a_x is zero and a_y is nonzero.

11 •

Determine the Concept The change in the velocity is in the same direction as the acceleration. Choose an *x-y* coordinate system with east being the positive *x* direction and north the positive *y* direction.

Given our choice of coordinate system, the *x* component of $\vec{a}$ is negative and so $\vec{v}$ will decrease. The *y* component of $\vec{a}$ is positive and so $\vec{v}$ will increase toward the north.

$\boxed{(c)\text{ is correct.}}$

***12** •

Determine the Concept The average velocity of a particle, $\vec{v}_{\text{av}}$, is the ratio of the particle's displacement to the time required for the displacement.

(*a*) We can calculate $\Delta\vec{r}$ from the given information and Δt is known. $\boxed{(a)\text{ is correct.}}$

(*b*) We do not have enough information to calculate $\Delta\vec{v}$ and cannot compute the particle's average acceleration.

(*c*) We would need to know how the particle's velocity varies with time in order to compute its instantaneous velocity.

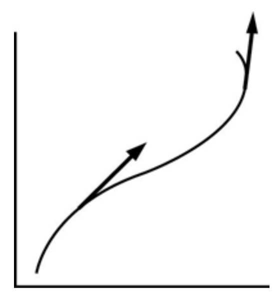
(*d*) We would need to know how the particle's velocity varies with time in order to compute its instantaneous acceleration.

13 ••

Determine the Concept The velocity vector is always in the direction of motion and, thus, tangent to the path.

(*a*) The velocity vector, as a consequence of always being in the direction of motion, is tangent to the path.

(*b*) A sketch showing two velocity vectors for a particle moving along a path is shown to the right.



14 •

Determine the Concept An object experiences acceleration whenever either its speed changes or it changes direction.

The acceleration of a car moving in a straight path at constant speed is zero. In the other examples, either the magnitude or the direction of the velocity vector is changing and, hence, the car is accelerated. (*b*) is correct.

***15** •

Determine the Concept The velocity vector is defined by $\vec{v} = d\vec{r}/dt$, while the acceleration vector is defined by $\vec{a} = d\vec{v}/dt$.

(*a*) A car moving along a straight road while braking.

(*b*) A car moving along a straight road while speeding up.

(*c*) A particle moving around a circular track at constant speed.

16 •

Determine the Concept A particle experiences accelerated motion when either its speed or direction of motion changes.

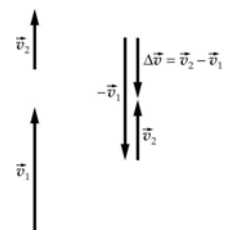
A particle moving at constant speed in a circular path is accelerating because the direction of its velocity vector is changing.

If a particle is moving at constant velocity, it is not accelerating.

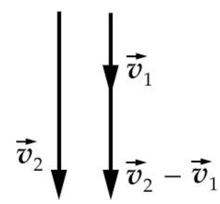
17 ••

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector,* $\Delta\vec{v}$.

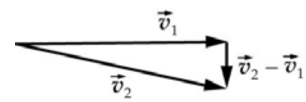
(*a*) The sketch for the dart thrown upward is shown to the right. The acceleration vector is in the direction of the *change* in the velocity vector $\Delta\vec{v}$.



(*b*) The sketch for the falling dart is shown to the right. Again, the acceleration vector is in the direction of the *change* in the velocity vector $\Delta\vec{v}$.



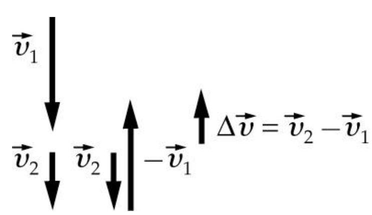
(*c*) The acceleration vector is in the direction of the *change* in the velocity vector … and hence is downward as shown the right:



***18** ••

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector*, $\Delta\vec{v}$.

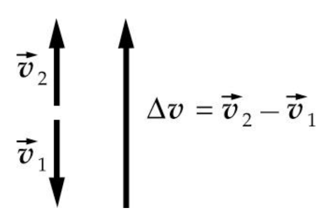
The drawing is shown to the right.



19 ••

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector,* $\Delta\vec{v}$.

The sketch is shown to the right.



20 •

Determine the Concept We can decide what the pilot should do by considering the speeds of the boat and of the current.

Give up. The speed of the stream is equal to the maximum speed of the boat in still water. The best the boat can do is, while facing directly upstream, maintain its position relative to the bank. $\boxed{(d)\text{ is correct.}}$

***21** •

Determine the Concept True. In the absence of air resistance, both projectiles experience the same downward acceleration. Because both projectiles have initial vertical velocities of zero, their vertical motions must be identical.

22 •

Determine the Concept In the absence of air resistance, the horizontal component of the projectile's velocity is constant for the duration of its flight.

At the highest point, the speed is the horizontal component of the initial velocity. The vertical component is zero at the highest point. $\boxed{(e)\text{ is correct.}}$

23 •

Determine the Concept In the absence of air resistance, the acceleration of the ball depends only on the *change in its velocity* and is independent of its velocity.

As the ball moves along its trajectory between points A and C, the vertical component of its velocity decreases and the *change* in its velocity is a downward pointing vector. Between points C and E, the vertical component of its velocity increases and the *change* in its velocity is also a downward pointing vector. There is no change in the horizontal component of the velocity. $\boxed{(d)\text{ is correct.}}$

24 •

Determine the Concept In the absence of air resistance, the horizontal component of the velocity remains constant throughout the flight. The vertical component has its maximum values at launch and impact.

(*a*) The speed is greatest at A and E.

(*b*) The speed is least at point C.

(*c*) The speed is the same at A and E. The horizontal components are equal at these points but the vertical components are oppositely directed.

25 •

Determine the Concept Speed is a scalar quantity, whereas acceleration, equal to the *rate of change* of velocity, is a vector quantity.

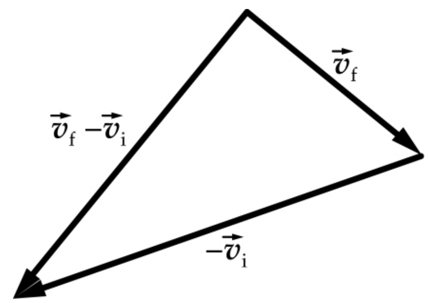
(*a*) False. Consider a ball on the end of a string. The ball can move with constant *speed* (a scalar) even though its *acceleration* (a vector) is always changing direction.

(*b*) True. From its definition, if the acceleration is zero, the velocity must be constant and so, therefore, must be the speed.

26 •

Determine the Concept The average acceleration vector is defined by $\vec{a}_{av} = \Delta\vec{v}/\Delta t$.

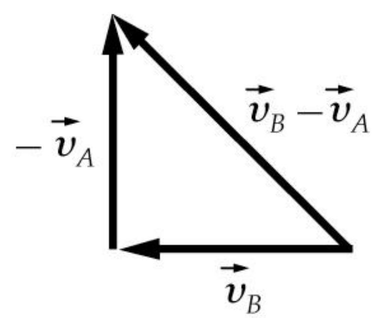
The direction of $\vec{a}_{av}$ is that of $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, as shown to the right.



27 •

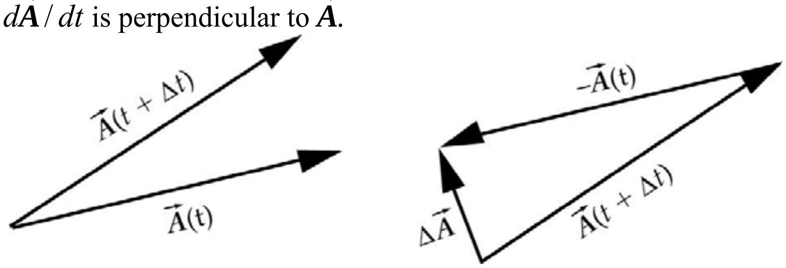
Determine the Concept The velocity of B relative to A is $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$.

The direction of $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ is shown to the right.



***28 ••**

(*a*) The vectors $\vec{A}(t)$ and $\vec{A}(t+\Delta t)$ are of equal length but point in slightly different directions. $\Delta\vec{A}$ is shown in the diagram below. Note that $\Delta\vec{A}$ is nearly perpendicular to $\vec{A}(t)$. For very small time intervals, $\Delta\vec{A}$ and $\vec{A}(t)$ are perpendicular to one another. Therefore, $d\vec{A}/dt$ is perpendicular to $\vec{A}$.



(*b*) If $\vec{A}$ represents the position of a particle, the particle must be undergoing circular motion (i.e., it is at a constant distance from some origin). The velocity vector is tangent to the particle's trajectory; in the case of a circle, it is perpendicular to the circle's radius.

(*c*) Yes, it could in the case of uniform circular motion. The speed of the particle is constant, but its heading is changing constantly. The acceleration vector in this case is always perpendicular to the velocity vector.

29 ••

Determine the Concept The velocity vector is in the same direction as *the change in the position vector* while the acceleration vector is in the same direction as *the change in the velocity vector*. Choose a coordinate system in which the y direction is north and the x direction is east.

(*a*)

Path	Direction of velocity vector
AB	north
BC	northeast
CD	east
DE	southeast
EF	south

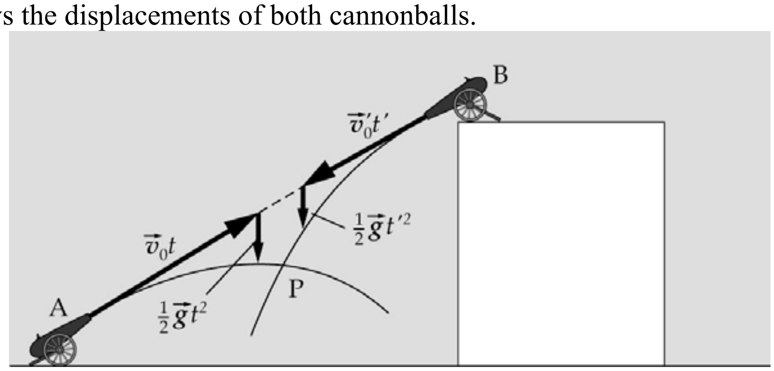
(*b*)

Path	Direction of acceleration vector
AB	north
BC	southeast
CD	0
DE	southwest
EF	north

(*c*) The magnitudes are comparable, but larger for DE since the radius of the path is smaller there.

***30** ••

Determine the Concept We'll assume that the cannons are identical and use a constant-acceleration equation to express the displacement of each cannonball as a function of time. Having done so, we can then establish the condition under which they will have the same vertical position at a given time and, hence, collide. The modified diagram shown below shows the displacements of both cannonballs.



Express the displacement of the cannonball from cannon A at any time t after being fired and before any collision:

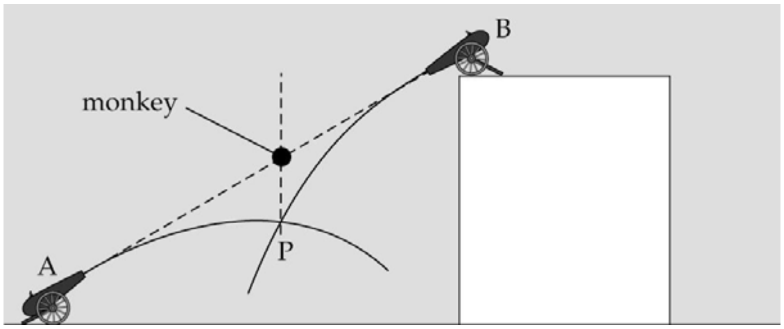
$$\Delta\vec{r} = \vec{v}_0 t + \tfrac{1}{2}\vec{g}t^2$$

Express the displacement of the cannonball from cannon A at any time t' after being fired and before any collision:

$$\Delta\vec{r}' = \vec{v}_0' t' + \tfrac{1}{2}\vec{g}t'^2$$

If the guns are fired simultaneously, $t = t'$ and the balls are the same distance $\frac{1}{2}gt^2$ below the line of sight at all times. Therefore, they should fire the guns simultaneously.

Remarks: This is the "monkey and hunter" problem in disguise. If you imagine a monkey in the position shown below, and the two guns are fired simultaneously, and the monkey begins to fall when the guns are fired, then the monkey and the two cannonballs will all reach point P at the same time.



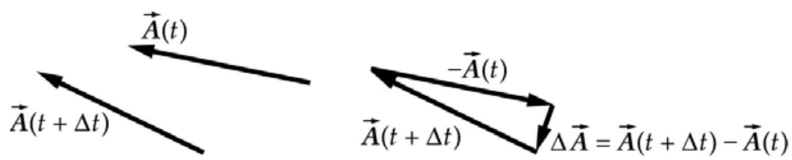
31 ••

Determine the Concept The droplet leaving the bottle has the same horizontal velocity as the ship. During the time the droplet is in the air, it is also moving horizontally with the same velocity as the rest of the ship. Because of this, it falls into the vessel, which has the same horizontal velocity. Because you have the same horizontal velocity as the ship does, you see the same thing as if the ship were standing still.

32 •

Determine the Concept

(*a*) Because $\vec{A}$ and $\vec{D}$ are tangent to the path of the stone, either of them could represent the velocity of the stone.



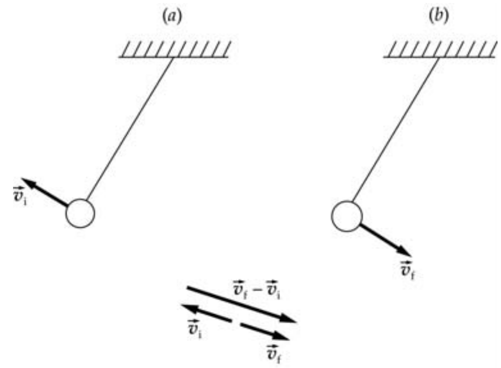
(*b*) Let the vectors $\vec{A}(t)$ and $\vec{B}(t+\Delta t)$ be of equal length but point in slightly different directions as the stone moves around the circle. These two vectors and $\Delta\vec{A}$ are shown in the diagram above. Note that $\Delta\vec{A}$ is nearly perpendicular to $\vec{A}(t)$. For very small time intervals, $\Delta\vec{A}$ and $\vec{A}(t)$ are perpendicular to one another. Therefore, $d\vec{A}/dt$ is perpendicular to $\vec{A}$ and only the vector $\vec{E}$ could represent the acceleration of the stone.

33 •

Determine the Concept True. An object accelerates when its velocity changes; that is, when either its speed or its direction changes. When an object moves in a circle the direction of its motion is continually changing.

34 ••

Picture the Problem In the diagram, (*a*) shows the pendulum just before it reverses direction and (*b*) shows the pendulum just after it has reversed its direction. The acceleration of the bob is in the direction of the *change* in the velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ and is tangent to the pendulum trajectory at the point of reversal of direction. This makes sense because, at an extremum of motion, $v = 0$, so there is no centripetal acceleration. However, because the velocity is reversing direction, the tangential acceleration is nonzero.



35 •

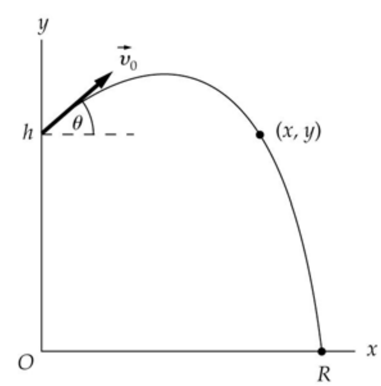
Determine the Concept The principle reason is aerodynamic drag. When moving through a fluid, such as the atmosphere, the ball's acceleration will depend strongly on its velocity.

Estimation and Approximation

***36** ••

Picture the Problem During the flight of the ball the acceleration is constant and equal to 9.81 m/s^2 directed downward. We can find the flight time from the vertical part of the motion, and then use the horizontal part of the motion to find the horizontal distance. We'll assume that the release point of the ball is 2 m above your feet.

Make a sketch of the motion. Include coordinate axes, initial and final positions, and initial velocity components:



Obviously, how far you throw the ball will depend on how fast you can throw it. A major league baseball pitcher can throw a fastball at 90 mi/h or so. Assume that you can throw a ball at two-thirds that speed to obtain:

$$v_0 = 60\,\text{mi/h} \times \frac{0.447\,\text{m/s}}{1\,\text{mi/h}} = 26.8\,\text{m/s}$$

There is no acceleration in the x direction, so the horizontal motion is one of constant velocity. Express the horizontal position of the ball as a function of time:

$$x = v_{0x}t \qquad (1)$$

Assuming that the release point of the ball is a distance h above the ground, express the vertical position of the ball as a function of time:

$$y = h + v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad (2)$$

(a) For $\theta = 0$ we have:

$$v_{0x} = v_0 \cos\theta_0 = (26.8\,\text{m/s})\cos 0° = 26.8\,\text{m/s}$$

and

$$v_{0y} = v_0 \sin\theta_0 = (26.8\,\text{m/s})\sin 0° = 0$$

Substitute in equations (1) and (2) to obtain:

$$x = (26.8\,\text{m/s})t$$

and

$$y = 2\,\text{m} + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 2\,\text{m} - \frac{4.91\,\text{m/s}^2}{(26.8\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = 2\,\text{m} - \frac{4.91\,\text{m/s}^2}{(26.8\,\text{m/s})^2}R^2$$

Solve for R to obtain:

$$R = \boxed{17.1\,\text{m}}$$

(b) Using trigonometry, solve for v_{0x} and v_{0y}:

$$v_{0x} = v_0 \cos\theta_0 = (26.8\,\text{m/s})\cos 45° = 19.0\,\text{m/s}$$

and

$$v_{0y} = v_0 \sin\theta_0 = (26.8\,\text{m/s})\sin 45° = 19.0\,\text{m/s}$$

Substitute in equations (1) and (2) to obtain:

$$x = (19.0\,\text{m/s})t$$

and

$$y = 2\,\text{m} + (19.0\,\text{m/s})t + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 2\,\text{m} + x - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$. Hence:

$$0 = 2\,\text{m} + R - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}R^2$$

or

$$R^2 - (73.60\,\text{m})R - 147.2\,\text{m}^2 = 0$$

Solve for R (you can use the ″solver″ or ″graph″ functions of your calculator) to obtain:

$$R = \boxed{75.6\,\text{m}}$$

(c) Solve for v_{0x} and v_{0y}:

$$v_{0x} = v_0 = 26.8\,\text{m/s}$$

and

$$v_{0y} = 0$$

Substitute in equations (1) and (2) to obtain:

$$x = (26.8\,\text{m/s})t$$

and

$$y = 14\,\text{m} + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 14\,\text{m} - \frac{4.905\,\text{m/s}^2}{(26.8\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = 14\,\text{m} - \frac{4.905\,\text{m/s}^2}{(26.8\,\text{m/s})^2}R^2$$

Solve for R to obtain:

$$R = \boxed{45.3\,\text{m}}$$

(d) Using trigonometry, solve for v_{0x} and v_{0y}:

$$v_{0x} = v_{0y} = 19.0\text{ m / s}$$

Substitute in equations (1) and (2) to obtain:

$$x = (19.0\,\text{m/s})t$$

and

$$y = 14\,\text{m} + (19.0\,\text{m/s})t + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 14\,\text{m} + x - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$:

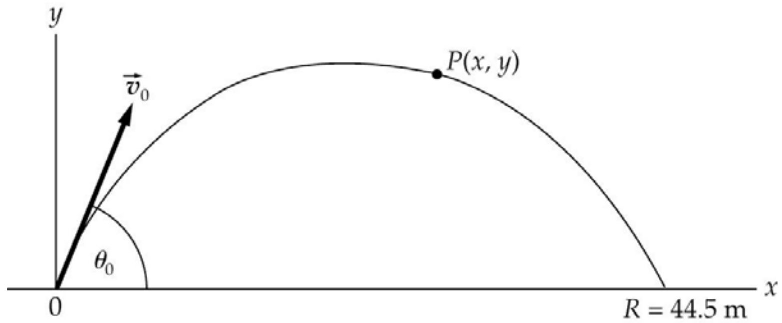
$$0 = 14\,\text{m} + R - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}R^2$$

Solve for R (you can use the ″solver″ or ″graph″ function of your calculator) to obtain:

$$R = \boxed{85.6\,\text{m}}$$

37 ••

Picture the Problem We'll ignore the height of Geoff's release point above the ground and assume that he launched the brick at an angle of 45°. Because the velocity of the brick at the highest point of its flight is equal to the horizontal component of its initial velocity, we can use constant-acceleration equations to relate this velocity to the brick's x and y coordinates at impact. The diagram shows an appropriate coordinate system and the brick when it is at point P with coordinates (x, y).



Using a constant-acceleration equation, express the x coordinate of the brick as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$ and $a_x = 0$,

$$x = v_{0x}t$$

Express the y coordinate of the brick as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$ and $a_y = -g$,

$$y = v_{0y}t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_{0x}^2}x^2$$

Use the brick's coordinates when it strikes the ground to obtain:

$$0 = (\tan\theta_0)R - \frac{g}{2v_{0x}^2}R^2$$

where R is the range of the brick.

Solve for v_{0x} to obtain:

$$v_{0x} = \sqrt{\frac{gR}{2\tan\theta_0}}$$

Substitute numerical values and evaluate v_{0x}:

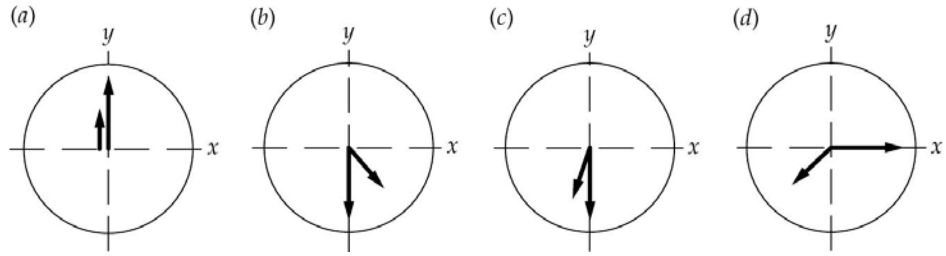
$$v_{0x} = \sqrt{\frac{(9.81\,\text{m/s}^2)(44.5\,\text{m})}{2\tan 45°}} = \boxed{14.8\,\text{m/s}}$$

Note that, at the brick's highest point, $v_y = 0$.

Vectors, Vector Addition, and Coordinate Systems

38 •

Picture the Problem Let the positive y direction be straight up, the positive x direction be to the right, and $\vec{A}$ and $\vec{B}$ be the position vectors for the minute and hour hands. The pictorial representation below shows the orientation of the hands of the clock for parts (*a*) through (*d*).



(*a*) The position vector for the minute hand at12:00 is:

$$\vec{A}_{12:00} = \boxed{(0.5\,\text{m})\hat{\boldsymbol{j}}}$$

The position vector for the hour hand at 12:00 is:

$$\vec{B}_{12:00} = \boxed{(0.25\,\text{m})\hat{\boldsymbol{j}}}$$

(*b*) At 3:30, the minute hand is positioned along the –*y* axis, while the hour hand is at an angle of (3.5 h)/12 h × 360° = 105°, measured clockwise from the top.

The position vector for the minute hand is:

$$\vec{A}_{3:30} = \boxed{-(0.5\,\text{m})\hat{\boldsymbol{j}}}$$

Find the *x*-component of the vector representing the hour hand:

$$B_x = (0.25\,\text{m})\sin 105° = 0.241\,\text{m}$$

Find the *y*-component of the vector representing the hour hand:

$$B_y = (0.25\,\text{m})\cos 105° = -0.0647\,\text{m}$$

The position vector for the hour hand is:

$$\vec{B}_{3:30} = \boxed{(0.241\,\text{m})\hat{\boldsymbol{i}} - (0.0647\,\text{m})\hat{\boldsymbol{j}}}$$

(*c*) At 6:30, the minute hand is positioned along the –*y* axis, while the hour hand is at an angle of (6.5 h)/12 h × 360° = 195°, measured clockwise from the top.

The position vector for the minute hand is:

$$\vec{A}_{6:30} = \boxed{-(0.5\,\text{m})\hat{\boldsymbol{j}}}$$

Find the *x*-component of the vector representing the hour hand:

$$B_x = (0.25\,\text{m})\sin 195° = -0.0647\,\text{m}$$

Find the *y*-component of the vector representing the hour hand:

$$B_y = (0.25\,\text{m})\cos 195° = -0.241\,\text{m}$$

The position vector for the hour hand is:

$$\vec{B}_{6:30} = \boxed{-(0.0647\,\text{m})\hat{\boldsymbol{i}} - (0.241\,\text{m})\hat{\boldsymbol{j}}}$$

(*d*) At 7:15, the minute hand is positioned along the +*x* axis, while the hour hand is at an angle of (7.25 h)/12 h × 360° = 218°, measured clockwise from the top.

The position vector for the minute hand is:

$$\vec{A}_{7:15} = \boxed{(0.5\,\text{m})\hat{i}}$$

Find the x-component of the vector representing the hour hand:

$$B_x = (0.25\,\text{m})\sin 218° = -0.154\,\text{m}$$

Find the y-component of the vector representing the hour hand:

$$B_y = (0.25\,\text{m})\cos 218° = -0.197\,\text{m}$$

The position vector for the hour hand is:

$$\vec{B}_{7:15} = \boxed{-(0.154\,\text{m})\hat{i} - (0.197\,\text{m})\hat{j}}$$

(e) Find $\vec{A} - \vec{B}$ at 12:00:

$$\vec{A} - \vec{B} = (0.5\,\text{m})\hat{j} - (0.25\,\text{m})\hat{j} = \boxed{(0.25\,\text{m})\hat{j}}$$

Find $\vec{A} - \vec{B}$ at 3:30:

$$\vec{A} - \vec{B} = -(0.5\,\text{m})\hat{j} - \left[(0.241\,\text{m})\hat{i} - (0.0647\,\text{m})\hat{j}\right] = \boxed{-(0.241\,\text{m})\hat{i} - (0.435\,\text{m})\hat{j}}$$

Find $\vec{A} - \vec{B}$ at 6:30:

$$\vec{A} - \vec{B} = -(0.5\,\text{m})\hat{j} - \left[(0.0647\,\text{m})\hat{i} - (0.241\,\text{m})\hat{j}\right] = \boxed{-(0.0647\,\text{m})\hat{i} - (0.259\,\text{m})\hat{j}}$$

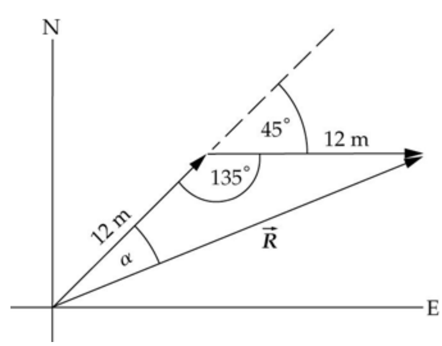
Find $\vec{A} - \vec{B}$ at 7:15:

$$\vec{A} - \vec{B} = (0.5\,\text{m})\hat{j} - \left[-(0.152\,\text{m})\hat{i} - (0.197\,\text{m})\hat{j}\right] = \boxed{(0.152\,\text{m})\hat{i} + (0.697\,\text{m})\hat{j}}$$

***39** •

Picture the Problem The resultant displacement is the vector sum of the individual displacements.

The two displacements of the bear and its resultant displacement are shown to the right:



Using the law of cosines, solve for the resultant displacement:

$$R^2 = (12\,\text{m})^2 + (12\,\text{m})^2 - 2(12\,\text{m})(12\,\text{m})\cos 135°$$

and

$$R = \boxed{22.2\,\text{m}}$$

Using the law of sines, solve for α:

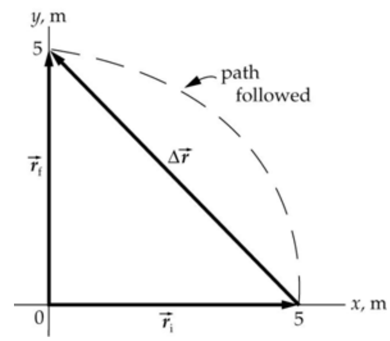
$$\frac{\sin\alpha}{12\,\text{m}} = \frac{\sin 135°}{22.2\,\text{m}}$$

$\therefore$ α = 22.5° and the angle with the horizontal is 45° – 22.5° = $\boxed{22.5°}$

40 •

Picture the Problem The resultant displacement is the vector sum of the individual displacements.

(*a*) Using the endpoint coordinates for her initial and final positions, draw the student's initial and final position vectors and construct her displacement vector.



Find the magnitude of her displacement and the angle this displacement makes with the positive *x*-axis:

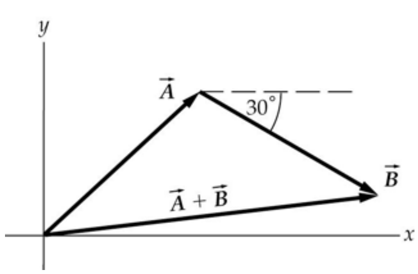
$\boxed{\text{Her displacement is } 5\sqrt{2}\text{ m @ }135°.}$

(*b*) $\boxed{\text{His initial and final positions are the same as in } (a)\text{, so his displacement is also } 5\sqrt{2}\text{ @ }135°.}$

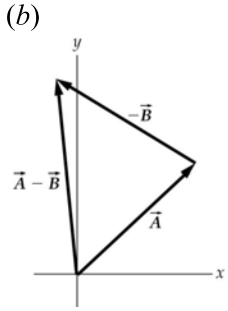
***41** •

Picture the Problem Use the standard rules for vector addition. Remember that changing the sign of a vector reverses its direction.

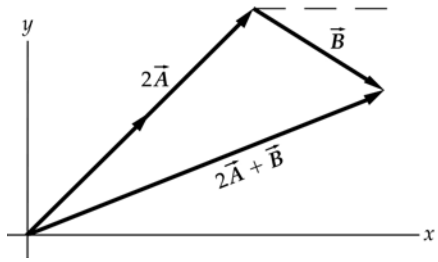
(*a*)



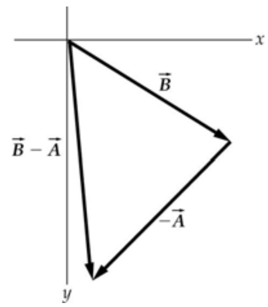
(*b*)



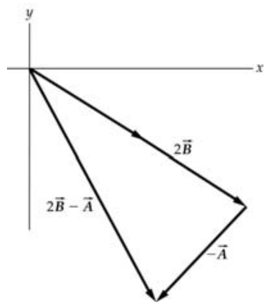
(*c*)



(*d*)

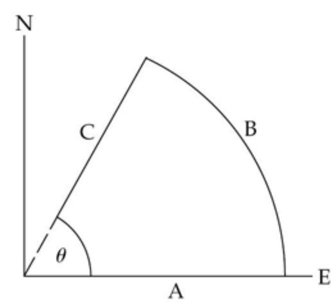


(*e*)



42 •

Picture the Problem The figure shows the paths walked by the Scout. The length of path A is 2.4 km; the length of path B is 2.4 km; and the length of path C is 1.5 km:



(*a*) Express the distance from the campsite to the end of path C:

$$2.4\text{ km} - 1.5\text{ km} = \boxed{0.9\text{ km}}$$

(*b*) Determine the angle θ subtended by the arc at the origin (campsite):

$$\theta_{\text{radians}} = \frac{\text{arc length}}{\text{radius}} = \frac{2.4\text{ km}}{2.4\text{ km}}$$

$$= 1\text{ rad} = 57.3°$$

$$\boxed{\begin{array}{l}\text{His direction from camp}\\ \text{is 1 rad north of east.}\end{array}}$$

(*c*) Express the total distance as the sum of the three parts of his walk:

$$d_{\text{tot}} = d_{\text{east}} + d_{\text{arc}} + d_{\text{toward camp}}$$

Substitute the given distances to find the total:

$$d_{\text{tot}} = 2.4\text{ km} + 2.4\text{ km} + 1.5\text{ km}$$
$$= 6.3\text{ km}$$

Express the ratio of the magnitude of his displacement to the total distance he walked and substitute to obtain a numerical value for this ratio:

$$\frac{\text{Magnitude of his displacement}}{\text{Total distance walked}} = \frac{0.9\,\text{km}}{6.3\,\text{km}} = \boxed{\frac{1}{7}}$$

43 •

Picture the Problem The direction of a vector is determined by its components.

$$\theta = \tan^{-1}\left(\frac{-3.5\,\text{m/s}}{5.5\,\text{m/s}}\right) = -32.5°$$

The vector is in the fourth quadrant and $\boxed{(b)\text{ is correct.}}$

44 •

Picture the Problem The components of the resultant vector can be obtained from the components of the vectors being added. The magnitude of the resultant vector can then be found by using the Pythagorean Theorem.

A table such as the one shown to the right is useful in organizing the information in this problem. Let $\vec{D}$ be the sum of vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.

Vector	x-component	y-component
$\vec{A}$	6	−3
$\vec{B}$	−3	4
$\vec{C}$	2	5
$\vec{D}$		

Determine the components of $\vec{D}$ by adding the components of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

$D_x = 5$ and $D_y = 6$

Use the Pythagorean Theorem to calculate the magnitude of $\vec{D}$:

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(5)^2 + (6)^2} = 7.81$$

and $\boxed{(d)\text{ is correct.}}$

45 •

Picture the Problem The components of the given vector can be determined using right-triangle trigonometry.

Use the trigonometric relationships between the magnitude of a vector and its components to calculate the x- and y-components of each vector.

	A	θ	A_x	A_y
(a)	10 m	30°	8.66 m	5 m
(b)	5 m	45°	3.54 m	3.54 m
(c)	7 km	60°	3.50 km	6.06 km
(d)	5 km	90°	0	5 km

(*e*)	15 km/s	150°	−13.0 km/s	7.50 km/s
(*f*)	10 m/s	240°	−5.00 m/s	−8.66 m/s
(*g*)	8 m/s^2	270°	0	−8.00 m/s^2

***46 •**

Picture the Problem Vectors can be added and subtracted by adding and subtracting their components.

Write $\vec{A}$ in component form:

$A_x = (8\text{ m})\cos 37° = 6.4\text{ m}$

$A_y = (8\text{ m})\sin 37° = 4.8\text{ m}$

$\therefore \vec{A} = (6.4\,\text{m})\hat{i} + (4.8\,\text{m})\hat{j}$

(*a*), (*b*), (*c*) Add (or subtract) *x*- and *y*-components:

$$\vec{D} = \boxed{(0.4\text{m})\hat{i} + (7.8\text{m})\hat{j}}$$

$$\vec{E} = \boxed{(-3.4\text{m})\hat{i} - (9.8\text{m})\hat{j}}$$

$$\vec{F} = \boxed{(-17.6\text{m})\hat{i} + (23.8\text{m})\hat{j}}$$

(*d*) Solve for $\vec{G}$ and add components to obtain:

$$\vec{G} = -\frac{1}{2}\left(\vec{A} + \vec{B} + 2\vec{C}\right)$$

$$= \boxed{(1.3\,\text{m})\hat{i} - (2.9\,\text{m})\hat{j}}$$

47 ••

Picture the Problem The magnitude of each vector can be found from the Pythagorean theorem and their directions found using the inverse tangent function.

(*a*) $\vec{A} = 5\hat{i} + 3\hat{j}$

$$A = \sqrt{A_x^2 + A_y^2} = \boxed{5.83}$$

and, because $\vec{A}$ is in the 1st quadrant,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \boxed{31.0°}$$

(*b*) $\vec{B} = 10\hat{i} - 7\hat{j}$

$$B = \sqrt{B_x^2 + B_y^2} = \boxed{12.2}$$

and, because $\vec{B}$ is in the 4th quadrant,

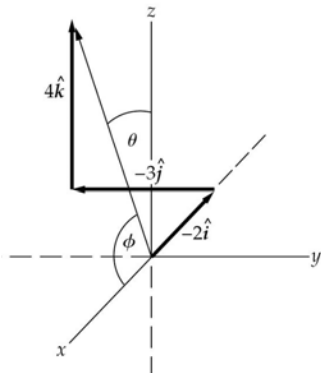
$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \boxed{-35.0°}$$

(*c*) $\vec{C} = -2\hat{i} - 3\hat{j} + 4\hat{k}$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \boxed{5.39}$$

$$\theta = \cos^{-1}\left(\frac{C_z}{C}\right) = \boxed{42.1°}$$

where θ is the polar angle measured from the positive *z*-axis and



$$\phi = \cos^{-1}\left(\frac{C_x}{C}\right) = \cos^{-1}\left(\frac{-2}{\sqrt{29}}\right) = \boxed{112°}$$

48 •

Picture the Problem The magnitude and direction of a two-dimensional vector can be found by using the Pythagorean Theorem and the definition of the tangent function.

(*a*) $\vec{A} = -4\hat{i} - 7\hat{j}$

$$A = \sqrt{A_x^2 + A_y^2} = \boxed{8.06}$$

and, because $\vec{A}$ is in the 3rd quadrant,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \boxed{240°}$$

$\vec{B} = 3\hat{i} - 2\hat{j}$

$$B = \sqrt{B_x^2 + B_y^2} = \boxed{3.61}$$

and, because $\vec{B}$ is in the 4th quadrant,

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \boxed{-33.7°}$$

$\vec{C} = \vec{A} + \vec{B} = -\hat{i} - 9\hat{j}$

$$C = \sqrt{C_x^2 + C_y^2} = \boxed{9.06}$$

and, because $\vec{C}$ is in the 3rd quadrant,

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{264°}$$

(*b*) Follow the same steps as in (*a*).

$A = \boxed{4.12}$; $\theta = \boxed{-76.0°}$

$B = \boxed{6.32}$; $\theta = \boxed{71.6°}$

$C = \boxed{3.61}$; $\theta = \boxed{33.7°}$

49 •

Picture the Problem The components of these vectors are related to the magnitude of each vector through the Pythagorean Theorem and trigonometric functions. In parts (*a*) and (*b*), calculate the rectangular components of each vector and then express the vector in rectangular form.

(a) Express $\vec{v}$ in rectangular form:

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

Evaluate v_x and v_y:

v_x = (10 m/s) cos 60° = 5 m/s
and
v_y = (10 m/s) sin 60° = 8.66 m/s

Substitute to obtain:

$$\vec{v} = \boxed{(5\,\text{m/s})\hat{i} + (8.66\,\text{m/s})\hat{j}}$$

(b) Express $\vec{v}$ in rectangular form:

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Evaluate A_x and A_y:

A_x = (5 m) cos 225° = –3.54 m
and
A_y = (5 m) sin 225° = –3.54 m

Substitute to obtain:

$$\vec{A} = \boxed{(-3.54\,\text{m})\hat{i} + (-3.54\,\text{m})\hat{j}}$$

(c) There is nothing to calculate as we are given the rectangular components:

$$\vec{r} = \boxed{(14\text{m})\hat{i} - (6\text{m})\hat{j}}$$

50 •

Picture the Problem While there are infinitely many vectors $\vec{B}$ that can be constructed such that $A = B$, the simplest are those which lie along the coordinate axes.

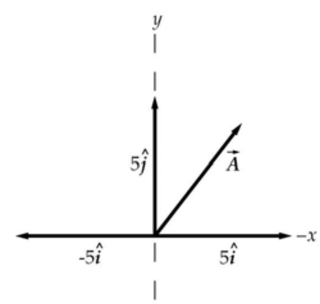
Determine the magnitude of $\vec{A}$:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{3^2 + 4^2} = 5$$

Write three vectors of the same magnitude as $\vec{A}$:

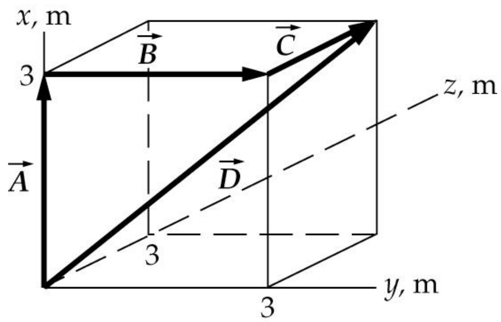
$$\vec{B}_1 = 5\hat{i},\ \vec{B}_2 = -5\hat{i},\ \text{and}\ \vec{B}_3 = 5\hat{j}$$

The vectors are shown to the right:



***51 ••**

Picture the Problem While there are several walking routes the fly could take to get from the origin to point C, its displacement will be the same for all of them. One possible route is shown in the figure.



Express the fly's displacement $\vec{D}$ during its trip from the origin to point C and find its magnitude:

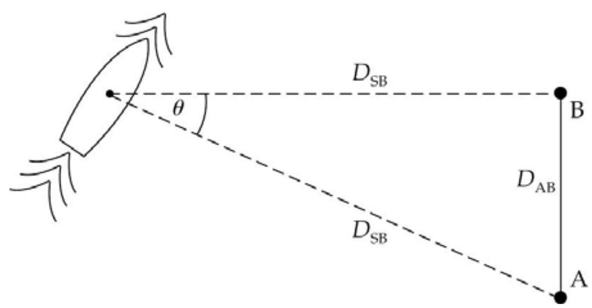
$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = (3\,\text{m})\hat{i} + (3\,\text{m})\hat{j} + (3\,\text{m})\hat{k}$$

and

$$D = \sqrt{(3\,\text{m})^2 + (3\,\text{m})^2 + (3\,\text{m})^2} = \boxed{5.20\,\text{m}}$$

***52 •**

Picture the Problem The diagram shows the locations of the transmitters relative to the ship and defines the distances separating the transmitters from each other and from the ship. We can find the distance between the ship and transmitter B using trigonometry.



Relate the distance between A and B to the distance from the ship to A and the angle θ:

$$\tan\theta = \frac{D_{AB}}{D_{SB}}$$

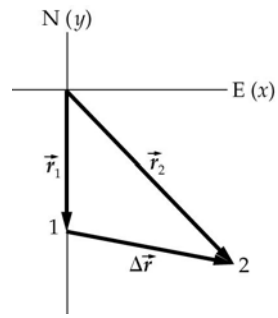
Solve for and evaluate the distance from the ship to transmitter B:

$$D_{SB} = \frac{D_{AB}}{\tan\theta} = \frac{100\,\text{km}}{\tan 30°} = \boxed{173\,\text{km}}$$

Velocity and Acceleration Vectors

53 •

Picture the Problem For constant speed and direction, the instantaneous velocity is identical to the average velocity. Take the origin to be the location of the stationary radar and construct a pictorial representation.



Express the average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

Determine the position vectors:

$$\vec{r}_1 = (-10\,\text{km})\hat{j}$$

and

$$\vec{r}_2 = (14.1\,\text{km})\hat{i} + (-14.1\,\text{km})\hat{j}$$

Find the displacement vector:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (14.1\,\text{km})\hat{i} + (-4.1\,\text{km})\hat{j}$$

Substitute for $\Delta\vec{r}$ and Δt to find the average velocity.

$$\vec{v}_{av} = \frac{(14.1\,\text{km})\hat{i} + (-4.1\,\text{km})\hat{j}}{1\,\text{h}} = \boxed{(14.1\,\text{km/h})\hat{i} + (-4.1\,\text{km/h})\hat{j}}$$

54 •

Picture the Problem The average velocity is the change in position divided by the elapsed time.

(*a*) The average velocity is:

$$v_{av} = \frac{\Delta r}{\Delta t}$$

Find the position vectors and the displacement vector:

$$\vec{r}_0 = (2\text{m})\hat{i} + (3\text{m})\hat{j}$$

$$\vec{r}_2 = (6\text{m})\hat{i} + (7\text{m})\hat{j}$$

and

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (4\,\text{m})\hat{i} + (4\,\text{m})\hat{j}$$

Find the magnitude of the displacement vector for the interval between $t = 0$ and $t = 2$ s:

$$\Delta r_{02} = \sqrt{(4\text{m})^2 + (4\text{m})^2} = 5.66\text{m}$$

Substitute to determine v_{av}:

$$v_{av} = \frac{5.66\text{ m}}{2\text{ s}} = \boxed{2.83\text{ m/s}}$$

and

$$\theta = \tan^{-1}\left(\frac{4\text{ m}}{4\text{ m}}\right) = \boxed{45.0°}\text{ measured}$$

from the positive x axis.

(b) Repeat (a), this time using the displacement between $t = 0$ and $t = 5$ s to obtain:

$$\vec{r}_5 = (13\text{ m})\hat{i} + (14\text{ m})\hat{j},$$

$$\Delta\vec{r}_{05} = \vec{r}_5 - \vec{r}_0 = (11\text{ m})\hat{i} + (11\text{ m})\hat{j},$$

$$\Delta r_{05} = \sqrt{(11\text{ m})^2 + (11\text{ m})^2} = 15.6\text{ m},$$

$$v_{av} = \frac{15.6\text{ m}}{5\text{ s}} = \boxed{3.11\text{ m/s}},$$

and

$$\theta = \tan^{-1}\left(\frac{11\text{ m}}{11\text{ m}}\right) = \boxed{45.0°}\text{ measured}$$

from the positive x axis.

***55 •**

Picture the Problem The magnitude of the velocity vector at the end of the 2 s of acceleration will give us its speed at that instant. This is a constant-acceleration problem.

Find the final velocity vector of the particle:

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = v_{x0}\hat{i} + a_y t\hat{j}$$
$$= (4.0\text{ m/s})\hat{i} + (3.0\text{ m/s}^2)(2.0\text{ s})\hat{j}$$
$$= (4.0\text{ m/s})\hat{i} + (6.0\text{ m/s})\hat{j}$$

Find the magnitude of $\vec{v}$:

$$v = \sqrt{(4.0\text{ m/s})^2 + (6.0\text{ m/s})^2} = 7.21\text{ m/s}$$

and $\boxed{(b)\text{ is correct.}}$

56 •

Picture the Problem Choose a coordinate system in which north coincides with the positive y direction and east with the positive x direction. Expressing the west and north velocity vectors is the first step in determining $\Delta\vec{v}$ and $\vec{a}_{av}$.

(a) The magnitudes of $\vec{v}_W$ and $\vec{v}_N$ are 40 m/s and 30 m/s, respectively. The change in the magnitude of the particle's velocity during this time is:

$$\Delta v = v_N - v_W$$
$$= \boxed{-10\text{ m/s}}$$

(*b*) The change in the direction of the velocity is from west to north.

The change in direction is $\boxed{90°}$

(*c*) The change in velocity is:

$$\Delta\vec{v} = \vec{v}_{\text{N}} - \vec{v}_{\text{W}} = (30\,\text{m/s})\hat{j} - (-40\,\text{m/s})\hat{i}$$
$$= (40\,\text{m/s})\hat{i} + (30\,\text{m/s})\hat{j}$$

Calculate the magnitude and direction of $\Delta\vec{v}$:

$$|\Delta v| = \sqrt{(40\,\text{m/s})^2 + (30\,\text{m/s})^2} = \boxed{50\,\text{m/s}}$$

and

$$\theta_{+x\,\text{axis}} = \tan^{-1}\frac{30\,\text{m/s}}{40\,\text{m/s}} = \boxed{36.9°}$$

(*d*) Find the average acceleration during this interval:

$$\vec{a}_{\text{av}} \equiv \Delta\vec{v}/\Delta t = \frac{(40\,\text{m/s})\hat{i} + (30\,\text{m/s})\hat{j}}{5\,\text{s}}$$
$$= (8\,\text{m/s}^2)\hat{i} + (6\,\text{m/s}^2)\hat{j}$$

The magnitude of this vector is:

$$a_{\text{av}} = \sqrt{(8\,\text{m/s}^2)^2 + (6\,\text{m/s}^2)^2} = \boxed{10\,\text{m/s}^2}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{6\,\text{m/s}^2}{8\,\text{m/s}^2}\right) = \boxed{36.9°}\ \text{measured}$$

from the positive *x* axis.

57 •

Picture the Problem The initial and final positions and velocities of the particle are given. We can find the average velocity and average acceleration using their definitions by first calculating the given displacement and velocities using unit vectors $\hat{i}$ and $\hat{j}$.

(*a*) The average velocity is:

$$\vec{v}_{\text{av}} \equiv \Delta\vec{r}/\Delta t$$

The displacement of the particle during this interval of time is:

$$\Delta\vec{r} = (100\,\text{m})\hat{i} + (80\,\text{m})\hat{j}$$

Substitute to find the average velocity:

$$\vec{v}_{\text{av}} = \frac{(100\,\text{m})\hat{i} + (80\,\text{m})\hat{j}}{3\,\text{s}}$$
$$= \boxed{(33.3\,\text{m/s})\hat{i} + (26.7\,\text{m/s})\hat{j}}$$

(*b*) The average acceleration is:

$$\vec{a}_{\text{av}} = \Delta\vec{v}/\Delta t$$

Find $\vec{v}_1$, $\vec{v}_2$, and $\Delta\vec{v}$:

$$\vec{v}_1 = (28.3\,\text{m/s})\hat{i} + (28.3\,\text{m/s})\hat{j}$$

and

$$\vec{v}_2 = (19.3\,\text{m/s})\hat{i} + (23.0\,\text{m/s})\hat{j}$$

$$\therefore\ \Delta\vec{v} = (-9.00\,\text{m/s})\hat{i} + (-5.30\,\text{m/s})\hat{j}$$

Using $\Delta t = 3$ s, find the average acceleration:

$$\vec{a}_{\text{av}} = \boxed{(-3.00\,\text{m/s}^2)\hat{i} + (-1.77\,\text{m/s}^2)\hat{j}}$$

***58 ••**

Picture the Problem The acceleration is constant so we can use the constant-acceleration equations in vector form to find the velocity at $t = 2$ s and the position vector at $t = 4$ s.

(*a*) The velocity of the particle, as a function of time, is given by:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

Substitute to find the velocity at $t = 2$ s:

$$\vec{v} = (2\text{ m/s})\hat{i} + (-9\text{ m/s})\hat{j} + \left[(4\text{ m/s}^2)\hat{i} + (3\text{ m/s}^2)\hat{j}\right](2\text{s})$$

$$= \boxed{(10\text{ m/s})\hat{i} + (-3\text{ m/s})\hat{j}}$$

(*b*) Express the position vector as a function of time:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Substitute and simplify:

$$\vec{r} = (4\text{ m})\hat{i} + (3\text{ m})\hat{j} + \left[(2\text{ m/s})\hat{i} + (\text{-}9\text{ m/s})\hat{j}\right](4\text{ s}) + \tfrac{1}{2}\left[(4\text{ m/s}^2)\hat{i} + (3\text{ m/s}^2)\hat{j}\right](4\text{ s})^2$$

$$= \boxed{(44\text{ m})\hat{i} + (-9\text{ m})\hat{j}}$$

Find the magnitude and direction of $\vec{r}$ at $t = 4$ s:

$$r(4\text{ s}) = \sqrt{(44\text{ m})^2 + (-9\text{ m})^2} = \boxed{44.9\text{ m}}$$

and, because $\vec{r}$ is in the 4th quadrant,

$$\theta = \tan^{-1}\left(\frac{-9\text{ m}}{44\text{ m}}\right) = \boxed{-11.6°}$$

59 ••

Picture the Problem The velocity vector is the time-derivative of the position vector and the acceleration vector is the time-derivative of the velocity vector.

Differentiate $\vec{r}$ with respect to time:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[(30t)\hat{i} + (40t - 5t^2)\hat{j}\right]$$

$$= \boxed{30\hat{i} + (40 - 10t)\hat{j}}$$

where $\vec{v}$ has units of m/s if t is in seconds.

Differentiate $\vec{v}$ with respect to time:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[30\hat{i} + (40 - 10t)\hat{j}\right] = \boxed{\left(-10\,\text{m/s}^2\right)\hat{j}}$$

60 ••

Picture the Problem We can use the constant-acceleration equations in vector form to solve the first part of the problem. In the second part, we can eliminate the parameter t from the constant-acceleration equations and express y as a function of x.

(a) Use $\vec{v} = \vec{v}_0 + \vec{a}t$ with $\vec{v}_0 = 0$ to find $\vec{v}$:

$$\boxed{\vec{v} = \left[\left(6\text{m/s}^2\right)\hat{i} + \left(4\text{m/s}^2\right)\hat{j}\right]t}$$

Use $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ with $\vec{r}_0 = (10\,\text{m})\hat{i}$ to find $\vec{r}$:

$$\boxed{\vec{r} = \left[(10\text{m}) + \left(3\text{m/s}^2\right)t^2\right]\hat{i} + \left[\left(2\text{m/s}^2\right)t^2\right]\hat{j}}$$

(b) Obtain the x and y components of the path from the vector equation in (a):

$$x = 10\,\text{m} + \left(3\,\text{m/s}^2\right)t^2$$

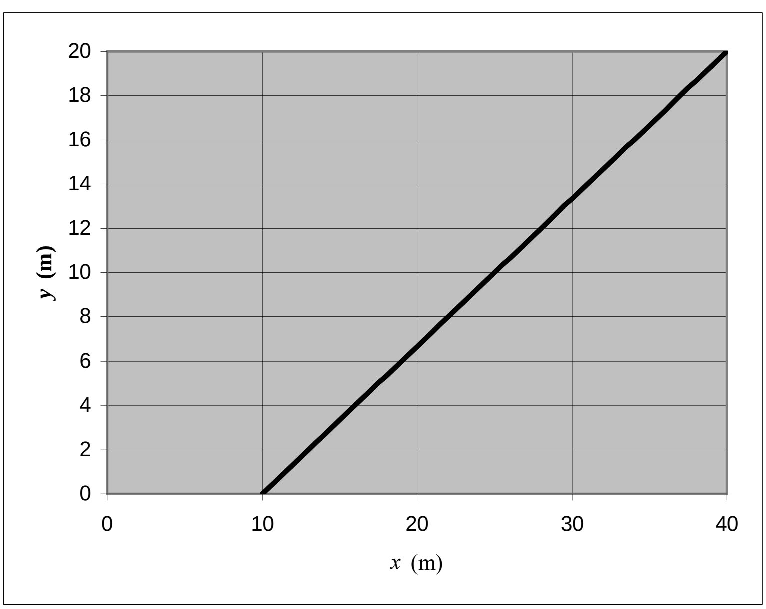
and

$$y = \left(2\,\text{m/s}^2\right)t^2$$

Eliminate the parameter t from these equations and solve for y to obtain:

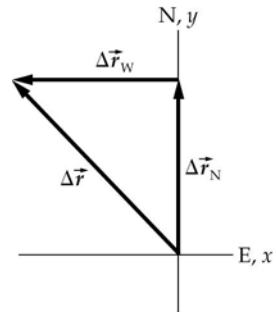
$$\boxed{y = \frac{2}{3}x - \frac{20}{3}\text{m}}$$

Use this equation to plot the graph shown below. Note that the path in the xy plane is a straight line.



61 •••

Picture the Problem The displacements of the boat are shown in the figure. We need to determine each of the displacements in order to calculate the average velocity of the boat during the 30-s trip.



(*a*) Express the average velocity of the boat:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

Express its total displacement:

$$\Delta\vec{r} = \Delta\vec{r}_N + \Delta\vec{r}_W = \tfrac{1}{2}a_N(\Delta t_N)^2\,\hat{j} + v_W\Delta t_W(-\hat{i})$$

To calculate the displacement we first have to find the speed after the first 20 s:

$$v_W = v_{N,\,f} = a_N\Delta t_N = 60\ \text{m/s}$$

so

$$\Delta\vec{r} = \tfrac{1}{2}a_N(\Delta t_N)^2\,\hat{j} - (60\ \text{m/s})\Delta t_W\hat{i} = (600\text{m})\hat{j} - (600\text{m})\hat{i}$$

Substitute to find the average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(600\text{m})(-\hat{i}+\hat{j})}{30\text{s}} = \boxed{(20\text{m/s})(-\hat{i}+\hat{j})}$$

(*b*) The average acceleration is given by:

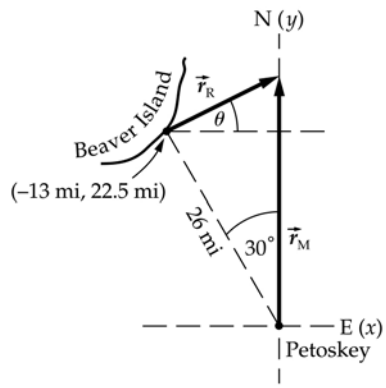
$$\vec{a}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(-60\ \text{m/s})\hat{i} - 0}{30\ \text{s}} = \boxed{(-2\ \text{m/s}^2)\hat{i}}$$

(*c*) The displacement of the boat from the dock at the end of the 30-s trip was one of the intermediate results we obtained in part (*a*).

$$\Delta\vec{r} = (600\text{m})\hat{j} + (-600\text{m})\hat{i} = \boxed{(600\text{m})(-\hat{i}+\hat{j})}$$

***62** •••

Picture the Problem Choose a coordinate system with the origin at Petoskey, the positive x direction to the east, and the positive y direction to the north. Let $t = 0$ at 9:00 a.m. and θ be the angle between Robert's velocity vector and the easterly direction and let "M" and "R" denote Mary and Robert, respectively. You can express the positions of Mary and Robert as functions of time and then equate their north (y) and east (x) coordinates at the time they rendezvous.



Express Mary's position as a function of time:

$$\vec{r}_M = v_M t\,\hat{j} = (8t)\hat{j}$$

where $\vec{r}_M$ is in miles if t is in hours.

Note that Robert's initial position coordinates (x_i, y_i) are:

$(x_i, y_i) = (-13\text{ mi}, 22.5\text{ mi})$

Express Robert's position as a function of time:

$$\begin{aligned}\vec{r}_R &= [x_i + (v_R\cos\theta)(t-1)]\,\hat{i} + [y_i + (v_R\sin\theta)(t-1)]\,\hat{j}\\ &= [-13 + \{6(t-1)\cos\theta\}]\,\hat{i} + [22.5 + \{6(t-1)\sin\theta\}]\,\hat{j}\end{aligned}$$

where $\vec{r}_R$ is in miles if t is in hours.

When Mary and Robert rendezvous, their coordinates will be the same. Equating their north and east coordinates yields:

East: $-13 + 6t\cos\theta - 6\cos\theta = 0$ (1)

North: $22.5 + 6t\sin\theta - 6\sin\theta = 8t$ (2)

Solve equation (1) for $\cos\theta$:

$$\cos\theta = \frac{13}{6(t-1)} \tag{3}$$

Solve equation (2) for $\sin\theta$:

$$\sin\theta = \frac{8t - 22.5}{6(t-1)} \tag{4}$$

Square and add equations (3) and (4) to obtain:

$$\sin^2\theta + \cos^2\theta = 1 = \left[\frac{8t-22.5}{6(t-1)}\right]^2 + \left[\frac{13}{6(t-1)}\right]^2$$

Simplify to obtain a quadratic equation in t:

$$28t^2 - 288t + 639 = 0$$

Solve (you could use your calculator's "solver" function) this

$$t = \boxed{3.24\,\text{h} = 3\,\text{h}\ 15\,\text{min}}$$

equation for the smallest value of t (both roots are positive) to obtain:

Now you can find the distance traveled due north by Mary:

$$r_M = v_M t = (8\,\text{mi/h})(3.24\,\text{h}) = \boxed{25.9\,\text{mi}}$$

Finally, solving equation (3) for θ and substituting 3.24 h for t yields:

$$\theta = \cos^{-1}\left[\frac{13}{6(t-1)}\right] = \cos^{-1}\left[\frac{13}{6(3.24-1)}\right] = 14.7°$$

and so Robert should head $\boxed{14.7° \text{ north of east.}}$

Remarks: Another solution that does not depend on the components of the vectors utilizes the law of cosines to find the time *t* at which Mary and Robert meet and then uses the law of sines to find the direction that Robert must head in order to rendezvous with Mary.

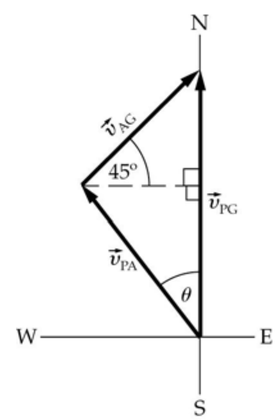
Relative Velocity

63 ••

Picture the Problem Choose a coordinate system in which north is the positive y direction and east is the positive x direction. Let θ be the angle between north and the direction of the plane's heading. The velocity of the plane relative to the ground, $\vec{v}_{PG}$, is the sum of the velocity of the plane relative to the air, $\vec{v}_{PA}$, and the velocity of the air relative to the ground, $\vec{v}_{AG}$. i.e.,

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

The pilot must head in such a direction that the east-west component of $\vec{v}_{PG}$ is zero in order to make the plane fly due north.



(*a*) From the diagram one can see that:

$$v_{AG} \cos 45° = v_{PA} \sin\theta$$

Solve for and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{56.6\,\text{km/h}}{250\,\text{km/h}}\right) = \boxed{13.1° \text{ west of north}}$$

(*b*) Because the plane is headed due north, add the north components of

$$\left|\vec{v}_{PG}\right| = (250\text{ km/h}) \cos 13.1° + (80\text{ km/h}) \sin 45°$$

$\vec{v}_{PA}$ and $\vec{v}_{AG}$ to determine the plane's ground speed:

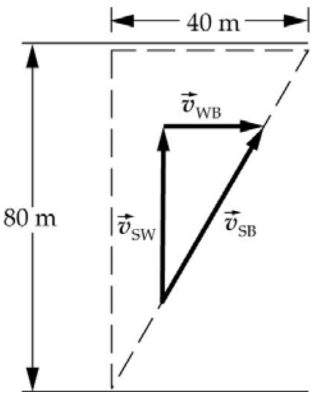
$$= \boxed{300\,\text{km/h}}$$

64 ••

Picture the Problem Let $\vec{v}_{SB}$ represent the velocity of the swimmer relative to the bank; $\vec{v}_{SW}$ the velocity of the swimmer relative to the water; and $\vec{v}_{WB}$ the velocity of the water relative to the shore; i.e.,

$$\vec{v}_{SB} = \vec{v}_{SW} + \vec{v}_{WB}$$

The current of the river causes the swimmer to drift downstream.



(*a*) The triangles shown in the figure are similar right triangles. Set up a proportion between their sides and solve for the speed of the water relative to the bank:

$$\frac{v_{WB}}{v_{SW}} = \frac{40\,\text{m}}{80\,\text{m}}$$

and

$$v_{WB} = \tfrac{1}{2}(1.6\,\text{m/s}) = \boxed{0.800\,\text{m/s}}$$

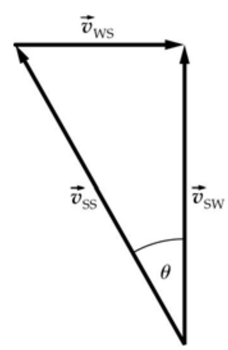
(*b*) Use the Pythagorean Theorem to solve for the swimmer's speed relative to the shore:

$$v_{SB} = \sqrt{v_{SW}^2 + v_{WS}^2}$$

$$= \sqrt{(1.6\,\text{m/s})^2 + (0.8\,\text{m/s})^2}$$

$$= \boxed{1.79\,\text{m/s}}$$

(*c*) The swimmer should head in a direction such that the upstream component of her velocity is equal to the speed of the water relative to the shore:



Use a trigonometric function to evaluate θ:

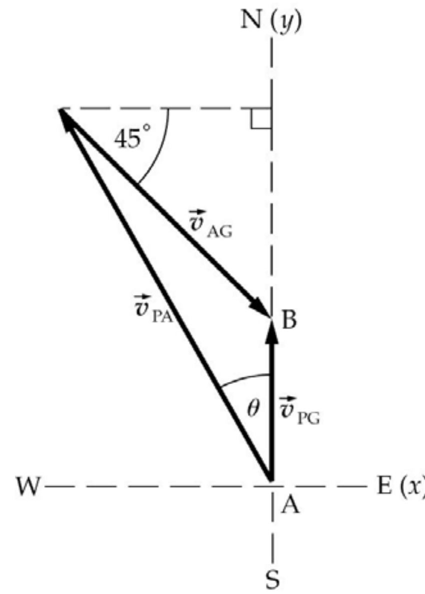
$$\theta = \sin^{-1}\left(\frac{0.8\,\text{m/s}}{1.6\,\text{m/s}}\right) = \boxed{30.0°}$$

***65 ••**

Picture the Problem Let the velocity of the plane relative to the ground be represented by $\vec{v}_{PG}$; the velocity of the plane relative to the air by $\vec{v}_{PA}$, and the velocity of the air relative to the ground by $\vec{v}_{AG}$. Then

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} \quad (1)$$

Choose a coordinate system with the origin at point A, the positive x direction to the east, and the positive y direction to the north. θ is the angle between north and the direction of the plane's heading. The pilot must head so that the east-west component of $\vec{v}_{PG}$ is zero in order to make the plane fly due north.



Use the diagram to express the condition relating the eastward component of $\vec{v}_{AG}$ and the westward component of $\vec{v}_{PA}$. This must be satisfied if the plane is to stay on its northerly course. [Note: this is equivalent to equating the x-components of equation (1).]

$$(50 \text{ km/h}) \cos 45° = (240 \text{ km/h}) \sin\theta$$

Now solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{(50\,\text{km/h})\cos 45°}{240\,\text{km/h}}\right] = \boxed{8.47°}$$

Add the north components of $\vec{v}_{PA}$ and $\vec{v}_{AG}$ to find the velocity of the plane relative to the ground:

$$v_{PG} + v_{AG}\sin 45° = v_{PA}\cos 8.47°$$

and

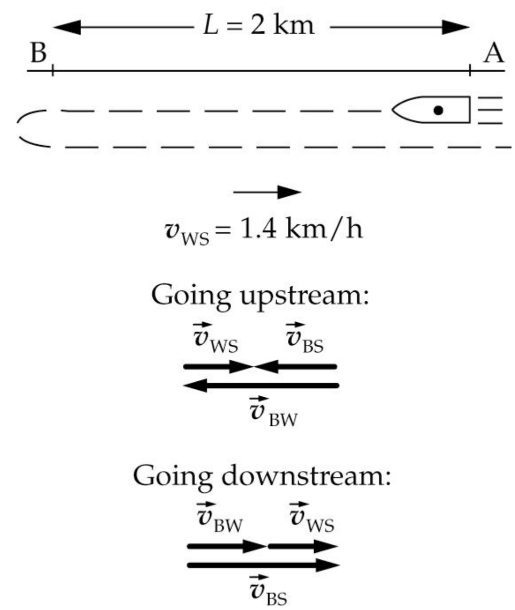
$$\begin{aligned} v_{PG} &= (240 \text{ km/h})\cos 8.47° \\ &\quad - (50 \text{ km/h})\sin 45° \\ &= 202 \text{ km/h} \end{aligned}$$

Finally, find the time of flight:

$$t_{\text{flight}} = \frac{\text{distance travelled}}{v_{PG}} = \frac{520\,\text{km}}{202\,\text{km/h}} = \boxed{2.57\,\text{h}}$$

66 ••

Picture the Problem Let $\vec{v}_{BS}$ be the velocity of the boat relative to the shore; $\vec{v}_{BW}$ be the velocity of the boat relative to the water; and $\vec{v}_{WS}$ represent the velocity of the water relative to the shore. Independently of whether the boat is going upstream or downstream:



$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$

Going upstream, the speed of the boat relative to the shore is reduced by the speed of the water relative to the shore.

Going downstream, the speed of the boat relative to the shore is increased by the same amount.

For the upstream leg of the trip:

$$v_{BS} = v_{BW} - v_{WS}$$

For the downstream leg of the trip:

$$v_{BS} = v_{BW} + v_{WS}$$

Express the total time for the trip in terms of the times for its upstream and downstream legs:

$$t_{total} = t_{upstream} + t_{downstream} = \frac{L}{v_{BW} - v_{WS}} + \frac{L}{v_{BW} + v_{WS}}$$

Multiply both sides of the equation by $(v_{BW} - v_{WS})(v_{BW} + v_{WS})$ (the product of the- denominators) and rearrange the terms to obtain:

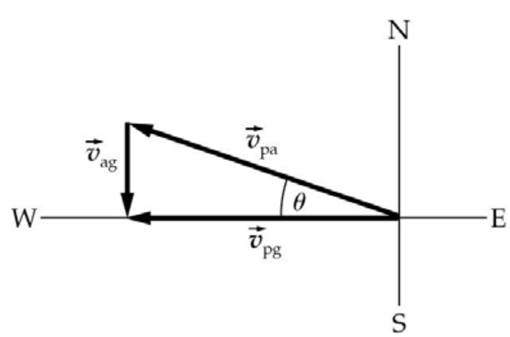
$$v_{BW}^2 - \frac{2L}{t_{total}} v_{BW} - v_{WS}^2 = 0$$

Solve the quadratic equation for v_{BW}. (Only the positive root is physically meaningful.)

$$v_{BW} = \boxed{5.18\,\text{km/h}}$$

67 ••

Picture the Problem Let $\vec{v}_{pg}$ be the velocity of the plane relative to the ground; $\vec{v}_{ag}$ be the velocity of the air relative to the ground; and $\vec{v}_{pa}$ the velocity of the plane relative to the air. Then, $\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$. The wind will affect the flight times differently along these two paths.



The velocity of the plane, relative to the ground, on its eastbound leg is equal to its velocity on its westbound leg. Using the diagram, find the velocity of the plane relative to the ground for both directions:

$$v_{pg} = \sqrt{v_{pa}^2 - v_{ag}^2} = \sqrt{(15\,\text{m/s})^2 - (5\,\text{m/s})^2} = 14.1\,\text{m/s}$$

Express the time for the east-west roundtrip in terms of the distances and velocities for the two legs:

$$t_{\text{roundtrip,EW}} = t_{\text{eastbound}} + t_{\text{westbound}} = \frac{\text{radius of the circle}}{v_{\text{pg,eastbound}}} + \frac{\text{radius of the circle}}{v_{\text{pg,westbound}}} = \frac{2\times10^3\,\text{m}}{14.1\,\text{m/s}} = 141\,\text{s}$$

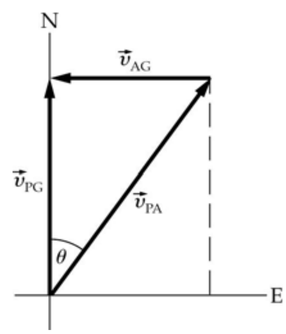
Use the distances and velocities for the two legs to express and evaluate the time for the north-south roundtrip:

$$t_{\text{roundtrip,NS}} = t_{\text{northbound}} + t_{\text{southbound}} = \frac{\text{radius of the circle}}{v_{\text{pg,northbound}}} + \frac{\text{radius of the circle}}{v_{\text{pg,southbound}}} = \frac{10^3\,\text{m}}{(15\,\text{m/s}) - (5\,\text{m/s})} + \frac{10^3\,\text{m}}{(15\,\text{m/s}) + (5\,\text{m/s})} = 150\,\text{s}$$

Because $t_{\text{roundtrip,EW}} < t_{\text{roundtrip,NS}}$, you should fly your plane across the wind.

68 •

Picture the Problem This is a relative velocity problem. The given quantities are the direction of the velocity of the plane relative to the ground and the velocity (magnitude and direction) of the air relative to the ground. Asked for is the direction of the velocity of the air relative to the ground. Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, draw a vector addition diagram and solve for the unknown quantity.



Calculate the heading the pilot must take:

$$\theta = \sin^{-1}\frac{30\text{ kts}}{150\text{ kts}} = \boxed{11.5°}$$

Because this is also the angle of the plane's heading clockwise from north, it is also its azimuth or the required true heading:

Az = (011.5°)

***69 ••**

Picture the Problem The position of B relative to A is the vector from A to B; i.e.,

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

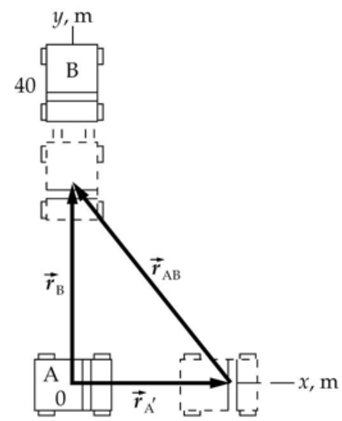
The velocity of B relative to A is

$$\vec{v}_{AB} = d\vec{r}_{AB}/dt$$

and the acceleration of B relative to A is

$$\vec{a}_{AB} = d\vec{v}_{AB}/dt$$

Choose a coordinate system with the origin at the intersection, the positive x direction to the east, and the positive y direction to the north.



(*a*) Find $\vec{r}_B$, $\vec{r}_A$, and $\vec{r}_{AB}$:

$$\vec{r}_B = \left[40\text{m} - \tfrac{1}{2}\left(2\text{m/s}^2\right)t^2\right]\hat{j}$$

$$\vec{r}_A = \left[(20\text{m/s})t\right]\hat{i}$$

and

$$\begin{aligned}\vec{r}_{AB} &= \vec{r}_B - \vec{r}_A \\ &= \left[(-20\text{m/s})t\right]\hat{i} \\ &\quad + \left[40\text{m} - \tfrac{1}{2}\left(2\text{m/s}^2\right)t^2\right]\hat{j}\end{aligned}$$

Evaluate $\vec{r}_{AB}$ at $t = 6$ s:

$$\vec{r}_{AB}(6\text{s}) = \boxed{(120\text{ m})\,\hat{i} + (4\text{ m})\,\hat{j}}$$

(*b*) Find $\vec{v}_{AB} = d\vec{r}_{AB}/dt$:

$$\begin{aligned}\vec{v}_{AB} &= \frac{d\vec{r}_{AB}}{dt} = \frac{d}{dt}\left[\{(-20\text{ m/s})t\}\vec{i}\right. \\ &\quad \left. + \left\{40\text{ m} - \tfrac{1}{2}\left(2\text{ m/s}^2\right)t^2\right\}\hat{j}\right] \\ &= (-20\text{ m/s})\,\hat{i} + (-2\text{ m/s}^2)\,t\,\hat{j}\end{aligned}$$

Evaluate $\vec{v}_{AB}$ at $t = 6$ s:

$$\vec{v}_{AB}(6\text{ s}) = \boxed{(-20\text{ m/s})\hat{i} - (12\text{ m/s})\hat{j}}$$

(*c*) Find $\vec{a}_{AB} = d\vec{v}_{AB}/dt$:

$$\begin{aligned}\vec{a}_{AB} &= \frac{d}{dt}\left[(-20\text{ m/s})\,\hat{i} + (-2\text{ m/s}^2)\,t\,\hat{j}\right] \\ &= \boxed{\left(-2\text{ m/s}^2\right)\hat{j}}\end{aligned}$$

Note that $\vec{a}_{AB}$ is independent of time.

***70 •••**

Picture the Problem Let h and h' represent the heights from which the ball is dropped and to which it rebounds, respectively. Let v and v' represent the speeds with which the ball strikes the racket and rebounds from it. We can use a constant-acceleration equation to relate the pre- and post-collision speeds of the ball to its drop and rebound heights.

(*a*) Using a constant-acceleration equation, relate the impact speed of the ball to the distance it has fallen:

$$v^2 = v_0^2 + 2gh$$

or, because $v_0 = 0$,

$$v = \sqrt{2gh}$$

Relate the rebound speed of the ball to the height to which it rebounds:

$$v^2 = v'^2 - 2gh'$$

or because $v = 0$,

$$v' = \sqrt{2gh'}$$

Divide the second of these equations by the first to obtain:

$$\frac{v'}{v} = \frac{\sqrt{2gh'}}{\sqrt{2gh}} = \sqrt{\frac{h'}{h}}$$

Substitute for h' and evaluate the ratio of the speeds:

$$\frac{v'}{v} = \sqrt{\frac{0.64h}{h}} = 0.8 \Rightarrow v' = \boxed{0.8v}$$

(*b*) Call the speed of the racket V. In a reference frame where the racket is unmoving, the ball initially has speed V, moving *toward* the racket. After it "bounces" from the racket, it will have speed 0.8 V, moving *away* from the racket.

In the reference frame where the racket is moving and the ball initially unmoving, we need to add the speed of the racket to the speed of the ball in the racket's rest frame. Therefore, the ball's speed is:

$$v' = V + 0.8V = 1.8V = 45\,\text{m/s}$$

$$\approx \boxed{100\,\text{mi/h}}$$

This speed is close to that of a tennis pro's serve. Note that this result tells us that the ball is moving significantly faster than the racket.

(*c*) From the result in part (*b*), the ball can never move more than twice as fast as the racket.

Circular Motion and Centripetal Acceleration

71 •

Picture the Problem We can use the definition of centripetal acceleration to express a_c in terms of the speed of the tip of the minute hand. We can find the tangential speed of the tip of the minute hand by using the distance it travels each revolution and the time it takes to complete each revolution.

Express the acceleration of the tip of the minute hand of the clock as a function of the length of the hand and the speed of its tip:

$$a_c = \frac{v^2}{R}$$

Use the distance the minute hand travels every hour to express its speed:

$$v = \frac{2\pi R}{T}$$

Substitute to obtain:

$$a_c = \frac{4\pi^2 R}{T^2}$$

Substitute numerical values and evaluate a_c:

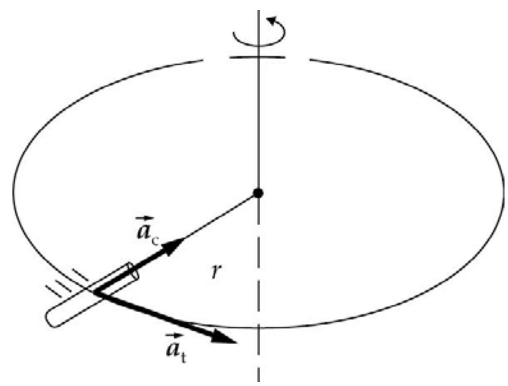
$$a_c = \frac{4\pi^2(0.5\,\text{m})}{(3600\,\text{s})^2} = \boxed{1.52\times10^{-6}\,\text{m/s}^2}$$

Express the ratio of a_c to g:

$$\frac{a_c}{g} = \frac{1.52\times10^{-6}\,\text{m/s}^2}{9.81\,\text{m/s}^2} = \boxed{1.55\times10^{-7}}$$

72 •

Picture the Problem The diagram shows the centripetal and tangential accelerations experienced by the test tube. The tangential acceleration will be zero when the centrifuge reaches its maximum speed. The centripetal acceleration increases as the tangential speed of the centrifuge increases. We can use the definition of centripetal acceleration to express a_c in terms of the speed of the test tube. We can find the tangential speed of the test tube by using the distance it travels each revolution and the time it takes to complete each revolution. The tangential acceleration can be found from the change in the tangential speed as the centrifuge is spinning up.



(*a*) Express the acceleration of the centrifuge arm as a function of the length of its arm and the speed of the test tube:

$$a_c = \frac{v^2}{R}$$

Use the distance the test tube travels every revolution to express its speed:

$$v = \frac{2\pi R}{T}$$

Substitute to obtain:

$$a_c = \frac{4\pi^2 R}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(0.15\,\text{m})}{\left(\frac{1\,\text{min}}{15000\,\text{rev}}\times\frac{60\,\text{s}}{\text{min}}\right)^2} = \boxed{3.70\times10^{5}\,\text{m/s}^2}$$

(*b*) Express the tangential acceleration in terms of the difference between the final and initial tangential speeds:

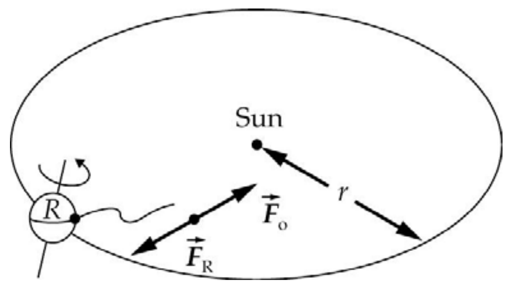
$$a_t = \frac{v_f - v_i}{\Delta t} = \frac{\frac{2\pi R}{T} - 0}{\Delta t} = \frac{2\pi R}{T\Delta t}$$

Substitute numerical values and evaluate a_T:

$$a_t = \frac{2\pi(0.15\,\text{m})}{\left(\frac{1\,\text{min}}{15000\,\text{rev}} \times \frac{60\,\text{s}}{\text{min}}\right)(75\,\text{s})} = \boxed{3.14\,\text{m/s}^2}$$

73 •

Picture the Problem The diagram includes a pictorial representation of the earth in its orbit about the sun and a force diagram showing the force on an object at the equator that is due to the earth's rotation, $\vec{F}_R$, and the force on the object due to the orbital motion of the earth about the sun, $\vec{F}_o$. Because these are centripetal forces, we can calculate the accelerations they require from the speeds and radii associated with the two circular motions.



Express the radial acceleration due to the rotation of the earth:

$$a_R = \frac{v_R^2}{R}$$

Express the speed of the object on the equator in terms of the radius of the earth R and the period of the earth's rotation T_R:

$$v_R = \frac{2\pi R}{T_R}$$

Substitute for v_R in the expression for a_R to obtain:

$$a_R = \frac{4\pi^2 R}{T_R^2}$$

Substitute numerical values and evaluate a_R:

$$a_R = \frac{4\pi^2(6370\times10^3\,\text{m})}{\left[(24\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)\right]^2} = 3.37\times10^{-2}\,\text{m/s}^2 = \boxed{3.44\times10^{-3}\,g}$$

Note that this effect gives rise to the well-known latitude correction for g.

Express the radial acceleration due to the orbital motion of the earth:

$$a_o = \frac{v_o^2}{r}$$

Express the speed of the object on the equator in terms of the earth-sun distance r and the period of the earth's motion about the sun T_o:

$$v_o = \frac{2\pi r}{T_o}$$

Substitute for v_o in the expression for a_o to obtain:

$$a_o = \frac{4\pi^2 r}{T_o^2}$$

Substitute numerical values and evaluate a_o:

$$a_o = \frac{4\pi^2 (1.5\times10^{11}\text{ m})}{\left[(365\text{ d})\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\right]^2}$$

$$= 5.95\times10^{-3}\text{ m/s}^2 = \boxed{6.07\times10^{-4}\,g}$$

74 ••

Picture the Problem We can relate the acceleration of the moon toward the earth to its orbital speed and distance from the earth. Its orbital speed can be expressed in terms of its distance from the earth and its orbital period. From tables of astronomical data, we find that the sidereal period of the moon is 27.3 d and that its mean distance from the earth is 3.84×10^8 m.

Express the centripetal acceleration of the moon:

$$a_c = \frac{v^2}{r}$$

Express the orbital speed of the moon:

$$v = \frac{2\pi r}{T}$$

Substitute to obtain:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2 (3.84\times10^8\text{ m})}{\left(27.3\text{ d}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}\right)^2}$$

$$= 2.72\times10^{-3}\text{ m/s}^2$$

$$= \boxed{2.78\times10^{-4}\,g}$$

Remarks: Note that $\dfrac{a_c}{g} = \dfrac{\textit{radius of earth}}{\textit{distance from earth to moon}}$ **(a_c is just the acceleration due to the earth's gravity evaluated at the moon's position). This is Newton's famous ″falling apple″ observation.**

75 •

Picture the Problem We can find the number of revolutions the ball makes in a given period of time from its speed and the radius of the circle along which it moves. Because the ball's centripetal acceleration is related to its speed, we can use this relationship to express its speed.

Express the number of revolutions per minute made by the ball in terms of the circumference c of the circle and the distance x the ball travels in time t:

$$n = \frac{x}{c} \qquad (1)$$

Relate the centripetal acceleration of the ball to its speed and the radius of its circular path:

$$a_c = g = \frac{v^2}{R}$$

Solve for the speed of the ball:

$$v = \sqrt{Rg}$$

Express the distance x traveled in time t at speed v:

$$x = vt$$

Substitute to obtain:

$$x = \sqrt{Rg}\,t$$

The distance traveled per revolution is the circumference c of the circle:

$$c = 2\pi R$$

Substitute in equation (1) to obtain:

$$n = \frac{\sqrt{Rg}\,t}{2\pi R} = \frac{1}{2\pi}\sqrt{\frac{g}{R}}\,t$$

Substitute numerical values and evaluate n:

$$n = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{0.8\,\text{m}}}(60\,\text{s}) = \boxed{33.4\,\text{min}^{-1}}$$

Remarks: The ball will oscillate at the end of this string as a simple pendulum with a period equal to 1/*n*.

Projectile Motion and Projectile Range

76 •

Picture the Problem Neglecting air resistance, the accelerations of the ball are constant and the horizontal and vertical motions of the ball are independent of each other. We can use the horizontal motion to determine the time-of-flight and then use this information to determine the distance the ball drops. Choose a coordinate system in which the origin is at the point of release of the ball, downward is the positive y direction, and the horizontal direction is the positive x direction.

Express the vertical displacement of the ball:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Find the time of flight from $v_x = \Delta x/\Delta t$:

$$\Delta t = \frac{\Delta x}{v_x} = \frac{(18.4\,\text{m})(3600\,\text{s/h})}{(140\,\text{km/h})(1000\,\text{m/km})} = 0.473\,\text{s}$$

Substitute to find the vertical displacement in 0.473 s:

$$\Delta y = \tfrac{1}{2}(9.81\,\text{m/s}^2)(0.473\,\text{s})^2 = \boxed{1.10\,\text{m}}$$

77 •

Picture the Problem In the absence of air resistance, the maximum height achieved by a projectile depends on the vertical component of its initial velocity.

The vertical component of the projectile's initial velocity is:

$$v_{0y} = v_0 \sin\theta_0$$

Use the constant-acceleration equation:

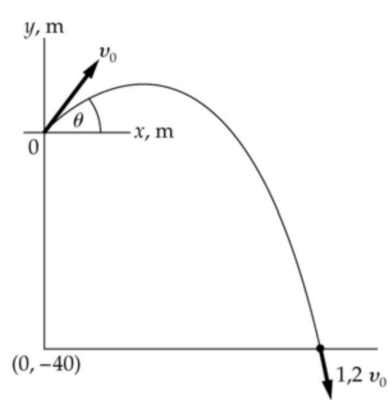
$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

Set $v_y = 0$, $a = -g$, and $\Delta y = h$ to obtain:

$$h = \boxed{\frac{(v_0 \sin\theta_0)^2}{2g}}$$

***78** ••

Picture the Problem Choose the coordinate system shown to the right. Because, in the absence of air resistance, the horizontal and vertical speeds are independent of each other, we can use constant-acceleration equations to relate the impact speed of the projectile to its components.



The horizontal and vertical velocity components are:

$v_{0x} = v_x = v_0\cos\theta$
and
$v_{0y} = v_0\sin\theta$

Using a constant-acceleration equation, relate the vertical

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $a_y = -g$ and $\Delta y = -h$, component of the velocity to the vertical displacement of the projectile:

$$v_y^2 = (v_0 \sin\theta)^2 + 2gh$$

Express the relationship between the magnitude of a velocity vector and its components, substitute for the components, and simplify to obtain:

$$\begin{aligned} v^2 &= v_x^2 + v_y^2 = (v_0 \cos\theta)^2 + v_y^2 \\ &= v_0^2(\sin^2\theta + \cos^2\theta) + 2gh \\ &= v_0^2 + 2gh \end{aligned}$$

Substitute for v:

$$(1.2v_0)^2 = v_0^2 + 2gh$$

Set $v = 1.2\ v_0$, $h = 40$ m and solve for v_0:

$$v_0 = \boxed{42.2\,\text{m/s}}$$

Remarks: Note that *v* is independent of *θ*. This will be more obvious once conservation of energy has been studied.

79 ••

Picture the Problem Example 3-12 shows that the dart will hit the monkey unless the dart hits the ground before reaching the monkey's line of fall. What initial speed does the dart need in order to just reach the monkey's line of fall? First, we will calculate the fall time of the monkey, and then we will calculate the horizontal component of the dart's velocity.

Using a constant-acceleration equation, relate the monkey's fall distance to the fall time:

$$h = \tfrac{1}{2}gt^2$$

Solve for the time for the monkey to fall to the ground:

$$t = \sqrt{\frac{2h}{g}}$$

Substitute numerical values and evaluate t:

$$t = \sqrt{\frac{2(11.2\,\text{m})}{9.81\,\text{m/s}^2}} = 1.51\,\text{s}$$

Let θ be the angle the barrel of the dart gun makes with the horizontal. Then:

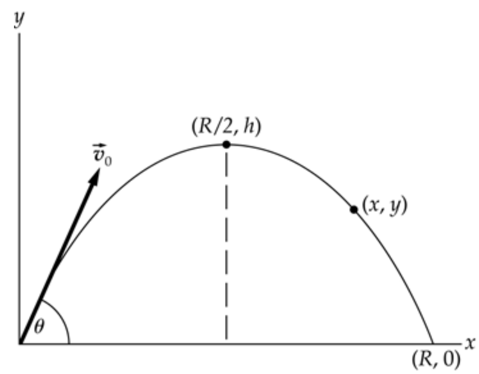
$$\theta = \tan^{-1}\left(\frac{10\,\text{m}}{50\,\text{m}}\right) = 11.3°$$

Use the fact that the horizontal velocity is constant to determine v_0:

$$v_0 = \frac{v_x}{\cos\theta} = \frac{(50\,\text{m}/1.51\,\text{s})}{\cos 11.3°} = \boxed{33.8\,\text{m/s}}$$

80 ••

Picture the Problem Choose the coordinate system shown in the figure to the right. In the absence of air resistance, the projectile experiences constant acceleration in both the x and y directions. We can use the constant-acceleration equations to express the x and y coordinates of the projectile along its trajectory as functions of time. The elimination of the parameter t will yield an expression for y as a function of x that we can evaluate at (R, 0) and (R/2, h). Solving these equations simultaneously will yield an expression for θ.



Express the position coordinates of the projectile along its flight path in terms of the parameter t:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2 \qquad (1)$$

Evaluate equation (1) at (R, 0) to obtain:

$$R = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Evaluate equation (1) at (R/2, h) to obtain:

$$h = \frac{(v_0 \sin\theta)^2}{2g}$$

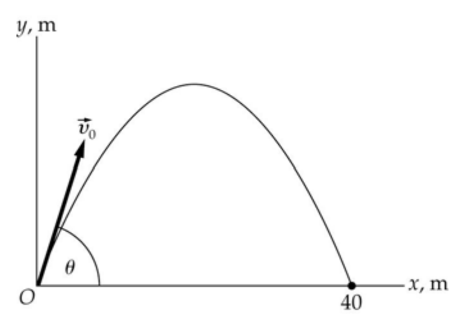
Equate R and h and solve the resulting equation for θ:

$$\theta = \tan^{-1}(4) = \boxed{76.0°}$$

Remarks: Note that this result is independent of v_0.

81 ••

Picture the Problem In the absence of air resistance, the motion of the ball is uniformly accelerated and its horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure to the right and use constant-acceleration equations to relate the x and y components of the ball's initial velocity.



Use the components of v_0 to express θ in terms of v_{0x} and v_{0y}:

$$\theta = \tan^{-1}\frac{v_{0y}}{v_{0x}} \qquad (1)$$

Use the Pythagorean relationship between the velocity and its components to express v_0:

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} \qquad (2)$$

Using a constant-acceleration equation, express the vertical speed of the projectile as a function of its initial upward speed and time into the flight:

$$v_y = v_{0y} + a_y t$$

Because $v_y = 0$ halfway through the flight (at maximum elevation):

$$v_{0y} = (9.81\text{ m/s}^2)(1.22\text{ s}) = 12.0\text{ m/s}$$

Determine v_{0x}:

$$v_{0x} = \frac{\Delta x}{\Delta t} = \frac{40\text{ m}}{2.44\text{ s}} = 16.4\text{ m/s}$$

Substitute in equation (2) and evaluate v_0:

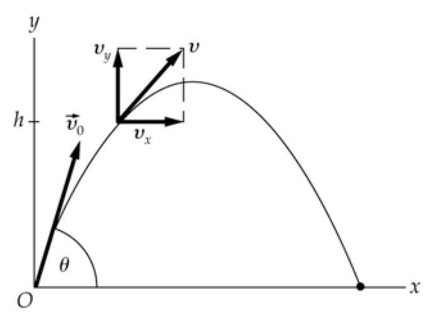
$$v_0 = \sqrt{(16.4\text{ m/s})^2 + (12.0\text{ m/s})^2} = \boxed{20.3\text{ m/s}}$$

Substitute in equation (1) and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{12.0\text{ m/s}}{16.4\text{ m/s}}\right) = \boxed{36.2°}$$

***82** ••

Picture the Problem In the absence of friction, the acceleration of the ball is constant and we can use the constant-acceleration equations to describe its motion. The figure shows the launch conditions and an appropriate coordinate system. The speeds v, v_x, and v_y are related through the Pythagorean Theorem.



The squares of the vertical and horizontal components of the object's velocity are:

$$v_y^2 = v_0^2 \sin^2\theta - 2gh$$

and

$$v_x^2 = v_0^2 \cos^2\theta$$

The relationship between these variables is:

$$v^2 = v_x^2 + v_y^2$$

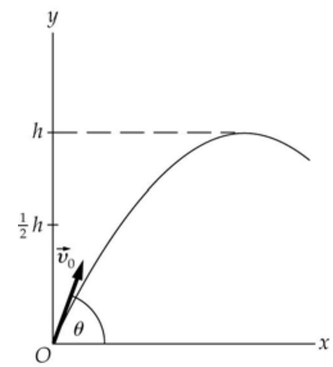
Substitute and simplify to obtain:

$$v^2 = \boxed{v_0^2 - 2gh}$$

Note that v is independent of θ ... as was to be shown.

83 ••

Picture the Problem In the absence of air resistance, the projectile experiences constant acceleration during its flight and we can use constant-acceleration equations to relate the speeds at half the maximum height and at the maximum height to the launch angle θ of the projectile.



The angle the initial velocity makes with the horizontal is related to the initial velocity components.

$$\tan\theta = \frac{v_{0y}}{v_{0x}}$$

Write the equation $v_y^2 = v_{0y}^2 + 2a\Delta y$, for $\Delta y = h$ and $v_y = 0$:

$$\Delta y = h \Rightarrow 0 = v_{0y}^2 - 2gh \qquad (1)$$

Write the equation $v_y^2 = v_{0y}^2 + 2a\Delta y$, for $\Delta y = h/2$:

$$\Delta y = \frac{h}{2} \Rightarrow v_y^2 = v_{0y}^2 - 2g\frac{h}{2} \qquad (2)$$

We are given $v_y = (3/4)v_0$. Square both sides and express this using the components of the velocity. The x component of the velocity remains constant.

$$v_{0x}^2 + v_y^2 = \left(\frac{3}{4}\right)^2\left(v_{0x}^2 + v_{0y}^2\right) \qquad (3)$$

where we have used $v_x = v_{0x}$.

(Equations 1, 2, and 3 constitute three equations and four unknowns v_{0x}, v_{0y}, v_y, and h. To solve for any of these unknowns, we first need a fourth equation. However, to solve for the ratio (v_{0y}/v_{0x}) of two of the unknowns, the three equations are sufficient. That is because dividing both sides of each equation by v_{0x}^2 gives three equations and three unknowns v_y/v_{0x}, v_{0y}/v_{0x}, and h/v_{0x}^2.

Solve equation 2 for gh and substitute in equation 1:

$$v_{0y}^2 = 2\left(v_{0y}^2 - v_h^2\right) \Rightarrow v_y^2 = \frac{v_{0y}^2}{2}$$

Substitute for v_y^2 in equation 3:

$$v_{0x}^2 + \frac{1}{2}v_{0y}^2 = \left(\frac{3}{4}\right)^2\left(v_{0x}^2 + v_{0y}^2\right)$$

Divide both sides by v_{0x}^2 and solve for v_{0y}/v_{0x} to obtain:

$$1+\frac{1}{2}\frac{v_{0y}^2}{v_{0x}^2}=\frac{9}{16}\left(1+\frac{v_{0y}^2}{v_{0x}^2}\right)$$

and

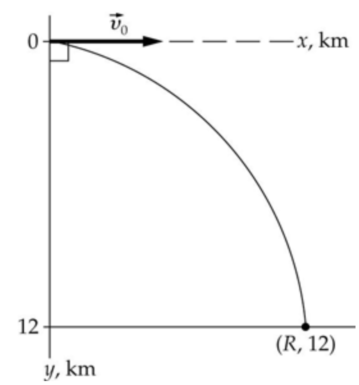
$$\frac{v_{0y}}{v_{0x}}=\sqrt{7}$$

Using $\tan\theta = v_{0y}/v_{0x}$, solve for θ:

$$\theta=\tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)=\tan^{-1}\left(\sqrt{7}\right)=\boxed{69.3°}$$

84 •

Picture the Problem The horizontal speed of the crate, in the absence of air resistance, is constant and equal to the speed of the cargo plane. Choose a coordinate system in which the direction the plane is moving is the positive *x* direction and downward is the positive *y* direction and apply the constant-acceleration equations to describe the crate's displacements at any time during its flight.



(*a*) Using a constant-acceleration equation, relate the vertical displacement of the crate Δy to the time of fall Δt:

$$\Delta y=v_{0y}\Delta t+\tfrac{1}{2}g(\Delta t)^2$$

or, because $v_{0y}=0$,

$$\Delta y=\tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t=\sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t=\sqrt{\frac{2\left(12\times10^3\text{ m}\right)}{9.81\text{m/s}^2}}=\boxed{49.5\text{s}}$$

(*b*) The horizontal distance traveled in 49.5 s is:

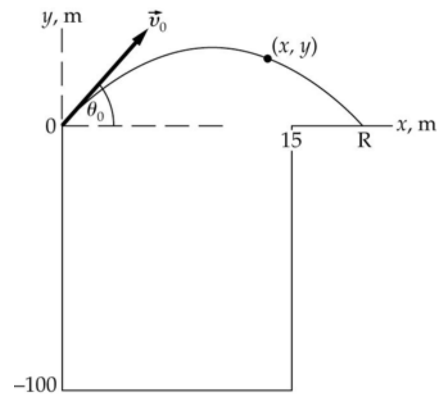
$$R=\Delta x=v_{0x}\Delta t$$
$$=(900\text{km/h})\left(\frac{1\text{h}}{3600\text{s}}\right)(49.5\text{s})$$
$$=\boxed{12.4\text{km}}$$

(*c*) Because the velocity of the plane is constant, it will be directly over the crate when it hits the ground; i.e., the distance to the aircraft will be the elevation of the aircraft.

$$\Delta y=\boxed{12.0\text{km}}$$

***85 ••**

Picture the Problem In the absence of air resistance, the accelerations of both Wiley Coyote and the Roadrunner are constant and we can use constant-acceleration equations to express their coordinates at any time during their leaps across the gorge. By eliminating the parameter t between these equations, we can obtain an expression that relates their y coordinates to their x coordinates and that we can solve for their launch angles.



(*a*) Using constant-acceleration equations, express the x coordinate of the Roadrunner while it is in flight across the gorge:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $a_x = 0$ and $v_{0x} = v_0 \cos\theta_0$,

$$x = (v_0 \cos\theta_0)t$$

Using constant-acceleration equations, express the y coordinate of the Roadrunner while it is in flight across the gorge:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $a_y = -g$ and $v_{0y} = v_0 \sin\theta_0$,

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2 \qquad (1)$$

Letting R represent the Roadrunner's range and using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, solve for and evaluate its launch speed:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(16.5\,\text{m})(9.81\,\text{m/s}^2)}{\sin 30°}}$$

$$= \boxed{18.0\,\text{m/s}}$$

(*b*) Letting R represent Wiley's range, solve equation (1) for his launch angle:

$$\theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

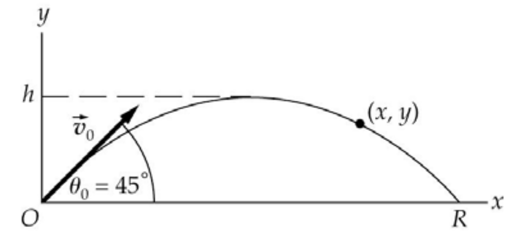
Substitute numerical values and evaluate θ_0:

$$\theta_0 = \frac{1}{2}\sin^{-1}\left[\frac{(14.5\,\text{m})(9.81\,\text{m/s}^2)}{(18.0\,\text{m/s})^2}\right]$$

$$= \boxed{13.0°}$$

86 •

Picture the Problem Because, in the absence of air resistance, the vertical and horizontal accelerations of the cannonball are constant, we can use constant-acceleration equations to express the ball's position and velocity as functions of time and acceleration. The maximum height of the ball and its time-of-flight are related to the components of its launch velocity.



(*a*) Using a constant-acceleration equation, relate *h* to the initial and final speeds of the cannonball:

$$v^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $v = 0$ and $a_y = -g$,

$$0 = v_{0y}^2 - 2g\Delta y$$

Find the vertical component of the firing speed:

$$v_{0y} = v_0 \sin\theta = (300\text{ m/s})\sin 45° = 212\text{ m/s}$$

Solve for and evaluate *h*:

$$h = \frac{v_{0y}^2}{2g} = \frac{(212\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \boxed{2.29\,\text{km}}$$

(*b*) The total flight time is:

$$\Delta t = t_{\text{up}} + t_{\text{dn}} = 2t_{\text{up}} = 2\frac{v_{0y}}{g} = \frac{2(212\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{43.2\,\text{s}}$$

(*c*) Express the *x* coordinate of the ball as a function of time:

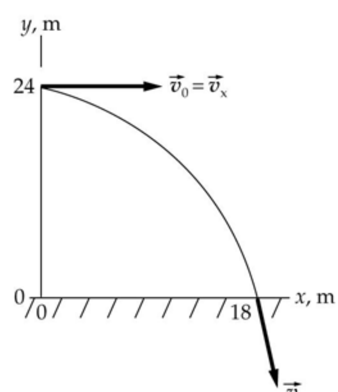
$$x = v_{0x}\Delta t = (v_0\cos\theta)\Delta t$$

Evaluate *x* (= *R*) when $\Delta t = 43.2$ s:

$$x = [(300\,\text{m/s})\cos 45°](43.2\,\text{s}) = \boxed{9.16\,\text{km}}$$

87 ••

Picture the Problem Choose a coordinate system in which the origin is at the base of the tower and the *x*- and *y*-axes are as shown in the figure to the right. In the absence of air resistance, the horizontal speed of the stone will remain constant during its fall and a constant-acceleration equation can be used to determine the time of fall. The final velocity of the stone will be the vector sum of its *x* and *y* components.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the stone (the height of the tower) as a function of the fall time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Solve for and evaluate the time of fall:

$$\Delta t = \sqrt{-\frac{2\Delta y}{g}} = \sqrt{-\frac{2(-24\,\text{m})}{9.81\,\text{m/s}^2}} = 2.21\,\text{s}$$

Use the definition of average velocity to find the velocity with which the stone was thrown from the tower:

$$v_x = v_{0x} \equiv \frac{\Delta x}{\Delta t} = \frac{18\,\text{m}}{2.21\,\text{s}} = \boxed{8.14\,\text{m/s}}$$

(*b*) Find the *y* component of the stone's velocity after 2.21 s:

$$v_y = v_{0y} - gt$$
$$= 0 - (9.81\,\text{m/s2})(2.21\,\text{s})$$
$$= -21.7\,\text{m/s}$$

Express *v* in terms of its components:

$$v = \sqrt{v_x^2 + v_y^2}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(8.14\,\text{m/s})^2 + (-21.7\,\text{m/s})^2}$$
$$= \boxed{23.2\,\text{m/s}}$$

88 ••

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and its horizontal and vertical motions are independent of each other. We can use constant-acceleration equations to express the horizontal and vertical displacements of the projectile in terms of its time-of-flight.

Using a constant-acceleration equation, express the horizontal displacement of the projectile as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = v_0\cos\theta$ and $a_x = 0$,

$$\Delta x = (v_0\cos\theta)\Delta t$$

Using a constant-acceleration equation, express the vertical displacement of the projectile as a function of time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = v_0\sin\theta$ and $a_y = -g$,

$$\Delta y = (v_0\cos\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values to obtain the quadratic equation:

$$-200\,\text{m} = (60\,\text{m/s})(\sin 60°)\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Solve for Δt:

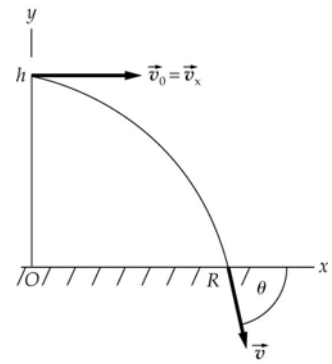
$$\Delta t = 13.6\text{ s}$$

Substitute for Δt and evaluate the horizontal distance traveled by the projectile:

$$\Delta x = (60\text{ m/s})(\cos 60°)(13.6\text{ s}) = \boxed{408\text{ m}}$$

89 ••

Picture the Problem In the absence of air resistance, the acceleration of the cannonball is constant and its horizontal and vertical motions are independent of each other. Choose the origin of the coordinate system to be at the base of the cliff and the axes directed as shown and use constant- acceleration equations to describe both the horizontal and vertical displacements of the cannonball.



Express the direction of the velocity vector when the projectile strikes the ground:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Express the vertical displacement using a constant-acceleration equation:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Set $\Delta x = -\Delta y$ ($R = -h$) to obtain:

$$\Delta x = v_x\Delta t = \tfrac{1}{2}g(\Delta t)^2$$

Solve for v_x:

$$v_x = \frac{\Delta x}{\Delta t} = \tfrac{1}{2}g\Delta t$$

Find the y component of the projectile as it hits the ground:

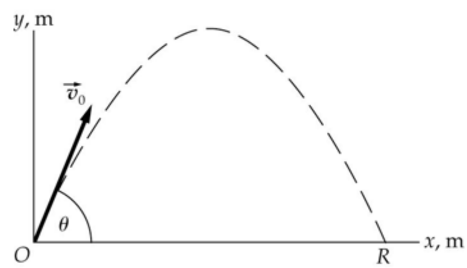
$$v_y = v_{0y} + a\Delta t = -g\Delta t = -2v_x$$

Substitute and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(-2) = \boxed{-63.4°}$$

90 •

Picture the Problem In the absence of air resistance, the vertical and horizontal motions of the projectile experience constant accelerations and are independent of each other. Use a coordinate system in which up is the positive y direction and horizontal is the positive x direction and use constant-acceleration equations to describe the horizontal and vertical displacements of the projectile as functions of the time into the flight.



(*a*) Use a constant-acceleration equation to express the horizontal displacement of the projectile as a function of time:

$$\Delta x = v_{0x}\Delta t = (v_0 \cos\theta)\Delta t$$

Evaluate this expression when $\Delta t = 6$ s:

$$\Delta x = (300\,\text{m/s})(\cos 60°)(6\,\text{s}) = \boxed{900\,\text{m}}$$

(*b*) Use a constant-acceleration equation to express the vertical displacement of the projectile as a function of time:

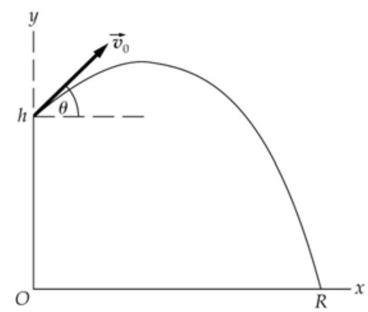
$$\Delta y = (v_0 \sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Evaluate this expression when $\Delta t = 6$ s:

$$\Delta y = (300\,\text{m/s})(\sin 60°)(6\,\text{s}) - \tfrac{1}{2}(9.81\,\text{m/s}^2)(6\,\text{s})^2 = \boxed{1.38\,\text{km}}$$

91 ••

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and the horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure with the origin at the base of the cliff and the axes oriented as shown and use constant-acceleration equations to find the range of the cannonball.



Using a constant-acceleration equation, express the horizontal displacement of the cannonball as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = v_0\cos\theta$ and $a_x = 0$,

$$\Delta x = (v_0 \cos\theta)\Delta t$$

Using a constant-acceleration equation, express the vertical displacement of the cannonball as a function of time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $y = -40$ m, $a = -g$, and $v_{0y} = v_0\sin\theta$,

$$-40\,\text{m} = (42.2\,\text{m/s})(\sin 30°)\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Solve the quadratic equation for Δt:

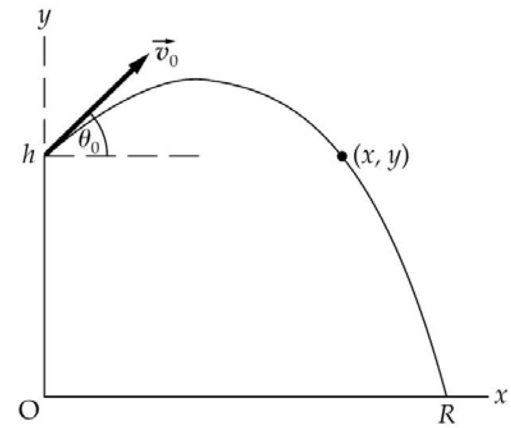
$$\Delta t = 5.73 \text{ s}$$

Calculate the range:

$$R = \Delta x = (42.2\,\text{m/s})(\cos 30°)(5.73\,\text{s}) = \boxed{209\,\text{m}}$$

***92** ••

Picture the Problem Choose a coordinate system in which the origin is at ground level. Let the positive *x* direction be to the right and the positive *y* direction be upward. We can apply constant-acceleration equations to obtain parametric equations in time that relate the range to the initial horizontal speed and the height *h* to the initial upward speed. Eliminating the parameter will leave us with a quadratic equation in *R,* the solution to which will give us the range of the arrow. In (*b*), we'll find the launch speed and angle as viewed by an observer who is at rest on the ground and then use these results to find the arrow's range when the horse is moving at 12 m/s.



(*a*) Use constant-acceleration equations to express the horizontal and vertical coordinates of the arrow's motion:

$$R = \Delta x = x - x_0 = v_{0x}t$$

and

$$y = h + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

where

$$v_{0x} = v_0\cos\theta \text{ and } v_{0y} = v_0\sin\theta$$

Solve the *x*-component equation for time:

$$t = \frac{R}{v_{0x}} = \frac{R}{v_0\cos\theta}$$

Eliminate time from the *y*-component equation:

$$y = h + v_{0y}\frac{R}{v_{0x}} - \frac{1}{2}g\left(\frac{R}{v_{0x}}\right)^2$$

and, at (*R*, 0),

$$0 = h + (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for the range to obtain:

$$R = \frac{v_0^2}{2g}\sin 2\theta\left(1 + \sqrt{1 + \frac{2gh}{v_0^2\sin^2\theta}}\right)$$

Substitute numerical values and evaluate *R*:

$$R = \frac{(45\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin 20°\left(1 + \sqrt{1 + \frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(45\,\text{m/s})^2(\sin^2 10°)}}\right) = \boxed{81.6\,\text{m}}$$

(*b*) Express the speed of the arrow in the horizontal direction:

$$v_x = v_{\text{arrow}} + v_{\text{archer}} = (45\,\text{m/s})\cos 10° + 12\,\text{m/s} = 56.3\,\text{m/s}$$

Express the vertical speed of the arrow:

$$v_y = (45\,\text{m/s})\sin 10° = 7.81\,\text{m/s}$$

Express the angle of elevation from the perspective of someone on the ground:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{7.81\,\text{m/s}}{56.3\,\text{m/s}}\right) = 7.90°$$

Express the arrow's speed relative to the ground:

$$v_0 = \sqrt{v_x^2 + v_y^2} = \sqrt{(56.3\,\text{m/s})^2 + (7.81\,\text{m/s})^2} = 56.8\,\text{m/s}$$

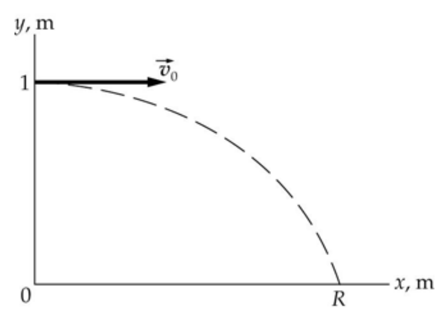
Substitute numerical values and evaluate *R*:

$$R = \frac{(56.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin 15.8°\left(1 + \sqrt{1 + \frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(56.8\,\text{m/s})^2(\sin^2 7.9°)}}\right) = \boxed{104\,\text{m}}$$

Remarks: An alternative solution for part (*b*) is to solve for the range in the reference frame of the archer and then add to it the distance the frame travels, relative to the earth, during the time of flight.

93 •

Picture the Problem In the absence of air resistance, the horizontal and vertical motions are independent of each other. Choose a coordinate system oriented as shown in the figure to the right and apply constant-acceleration equations to find the time-of-flight and the range of the spud-plug.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the plug:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Solve for and evaluate the flight time Δt:

$$\Delta t = \sqrt{-\frac{2\Delta y}{g}} = \sqrt{-\frac{2(-1.00\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{0.452\,\text{s}}$$

(*b*) Using a constant-acceleration equation, express the horizontal displacement of the plug:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $a_x = 0$ and $v_{0x} = v_0$,

$$\Delta x = v_0\Delta t$$

Substitute numerical values and evaluate R:

$$\Delta x = R = (50\,\text{m/s})(0.452\,\text{s}) = \boxed{22.6\,\text{m}}$$

94 ••

Picture the Problem An extreme value (i.e., a maximum or a minimum) of a function is determined by setting the appropriate derivative equal to zero. Whether the extremum is a maximum or a minimum can be determined by evaluating the second derivative at the point determined by the first derivative.

Evaluate $dR/d\theta_0$:

$$\frac{dR}{d\theta_0} = \frac{v_0^2}{g}\frac{d}{d\theta_0}[\sin(2\theta_0)] = \frac{2v_0^2}{g}\cos(2\theta_0)$$

Set $dR/d\theta_0 = 0$ for extrema and solve for θ_0:

$$\frac{2v_0^2}{g}\cos(2\theta_0) = 0$$

and

$$\theta_0 = \tfrac{1}{2}\cos^{-1}0 = 45°$$

Determine whether 45° is a maximum or a minimum:

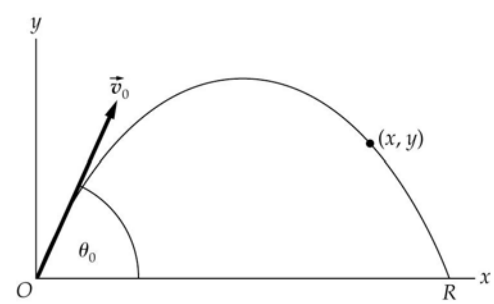
$$\left.\frac{d^2R}{d\theta_0^{\,2}}\right|_{\theta_0=45°} = \left[-4\left(v_0^2/g\right)\sin 2\theta_0\right]_{\theta_0=45°}$$

$$< 0$$

$\therefore$ R is a maximum at $\theta_0 = 45°$

95 •

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of a bullet in flight on the moon as a function of t. Eliminating this parameter will yield an expression for y as a function of x that we can use to find the range of the bullet. The necessity that the centripetal acceleration of an object in orbit at the surface of a body equal the acceleration due to gravity at the surface will allow us to determine the required muzzle velocity for orbital motion.



(*a*) Using a constant-acceleration equation, express the x coordinate of a bullet in flight on the moon:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_xt^2$$

or, because $x_0 = 0$, $a_x = 0$ and $v_{0x} = v_0\cos\theta_0$,

$$x = (v_0\cos\theta_0)t$$

Using a constant-acceleration equation, express the y coordinate of a bullet in flight on the moon:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $a_y = -g_{\text{moon}}$ and $v_{0y} = v_0\sin\theta_0$,

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}g_{\text{moon}}t^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta_0)x - \frac{g_{\text{moon}}}{2v_0^2\cos^2\theta_0}x^2$$

When $y = 0$ and $x = R$:

$$0 = (\tan\theta_0)R - \frac{g_{\text{moon}}}{2v_0^2\cos^2\theta_0}R^2$$

and

$$R = \frac{v_0^2}{g_{\text{moon}}}\sin 2\theta_0$$

Substitute numerical values and evaluate R:

$$R = \frac{(900\,\text{m/s})^2}{1.67\,\text{m/s}^2}\sin 90° = 4.85\times10^5\,\text{m}$$
$$= \boxed{485\,\text{km}}$$

This result is probably not very accurate because it is about 28% of the moon's radius (1740 km). This being the case, we can no longer assume that the ground is "flat" because of the curvature of the moon.

(*b*) Express the condition that the centripetal acceleration must satisfy for an object in orbit at the surface of the moon:

$$a_c = g_{\text{moon}} = \frac{v^2}{r}$$

Solve for and evaluate v:

$$v = \sqrt{g_{\text{moon}}r} = \sqrt{(1.67\,\text{m/s}^2)(1.74\times10^6\,\text{m})}$$
$$= \boxed{1.70\,\text{km/s}}$$

96 •••

Picture the Problem We can show that $\Delta R/R = -\Delta g/g$ by differentiating R with respect to g and then using a differential approximation.

Differentiate the range equation with respect to g:

$$\frac{dR}{dg} = \frac{d}{dg}\left(\frac{v_0^2}{g}\sin 2\theta_0\right) = -\frac{v_0^2}{g^2}\sin 2\theta_0$$
$$= -\frac{R}{g}$$

Approximate dR/dg by $\Delta R/\Delta g$:

$$\frac{\Delta R}{\Delta g}=-\frac{R}{g}$$

Separate the variables to obtain:

$$\frac{\Delta R}{R}=-\frac{\Delta g}{g}$$

i.e., for small changes in gravity ($g\approx g\pm\Delta g$), the fractional change in R is linearly opposite to the fractional change in g.

Remarks: This tells us that as gravity increases, the range will decrease, and vice versa. This is as it must be because *R* is inversely proportional to *g*.

97 •••

Picture the Problem We can show that $\Delta R/R = 2\Delta v_0/ v_0$ by differentiating R with respect to v_0 and then using a differential approximation.

Differentiate the range equation with respect to v_0:

$$\frac{dR}{dv_0}=\frac{d}{dv_0}\left(\frac{v_0^2}{g}\sin 2\theta_0\right)=\frac{2v_0}{g}\sin 2\theta_0$$

$$=2\frac{R}{v_0}$$

Approximate dR/dv_0 by $\Delta R/\Delta v_0$:

$$\frac{\Delta R}{\Delta v_0}=2\frac{R}{v_0}$$

Separate the variables to obtain:

$$\frac{\Delta R}{R}=2\frac{\Delta v_0}{v_0}$$

i.e., for small changes in the launch velocity ($v_0\approx v_0\pm\Delta v_0$), the fractional change in R is twice the fractional change in v_0.

Remarks: This tells us that as launch velocity increases, the range will increase twice as fast, and vice versa.

98 •••

Picture the Problem Choose a coordinate system in which the origin is at the base of the surface from which the projectile is launched. Let the positive x direction be to the right and the positive y direction be upward. We can apply constant-acceleration equations to obtain parametric equations in time that relate the range to the initial horizontal speed and the height h to the initial upward speed. Eliminating the parameter will leave us with a quadratic equation in R, the solution to which is the result we are required to establish.

Write the constant-acceleration equations for the horizontal and vertical parts of the projectile's

$$x=v_{0x}t$$

and

motion:

$$y = h + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

where

$v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$

Solve the x-component equation for time:

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta}$$

Using the x-component equation, eliminate time from the y-component equation to obtain:

$$y = h + (\tan\theta)x - \frac{g}{2v_0^2 \cos^2\theta}x^2$$

When the projectile strikes the ground its coordinates are (R, 0) and our equation becomes:

$$0 = h + (\tan\theta)R - \frac{g}{2v_0^2 \cos^2\theta}R^2$$

Using the plus sign in the quadratic formula to ensure a physically meaningful root (one that is positive), solve for the range to obtain:

$$R = \boxed{\left(1 + \sqrt{1 + \frac{2gh}{v_0^2 \sin^2\theta_0}}\right)\frac{v_0^2}{2g}\sin 2\theta_0}$$

***99 ••**

Picture the Problem We can use trigonometry to relate the maximum height of the projectile to its range and the sighting angle at maximum elevation and the range equation to express the range as a function of the launch speed and angle. We can use a constant-acceleration equation to express the maximum height reached by the projectile in terms of its launch angle and speed. Combining these relationships will allow us to conclude that $\tan\phi = \tfrac{1}{2}\tan\theta$.

Referring to the figure, relate the maximum height of the projectile to its range and the sighting angle ϕ:

$$\tan\phi = \frac{h}{R/2}$$

Express the range of the rocket and use the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$ to rewrite the expression as:

$$R = \frac{v^2}{g}\sin(2\theta) = 2\frac{v^2}{g}\sin\theta\cos\theta$$

Using a constant-acceleration equation, relate the maximum height of a projectile to the vertical component of its launch speed:

$$v_y^2 = v_{0y}^2 - 2gh$$

or, because $v_y = 0$ and $v_{0y} = v_0\sin\theta$,

$$v_0^2 \sin^2\theta = 2gh$$

Solve for the maximum height h:

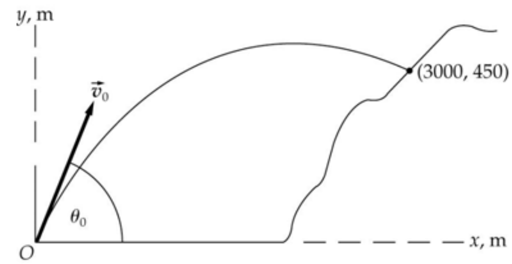
$$h = \frac{v^2}{2g}\sin^2\theta$$

Substitute for R and h and simplify to obtain:

$$\tan\phi = \frac{2\frac{v^2}{2g}\sin^2\theta}{2\frac{v^2}{g}\sin\theta\cos\theta} = \boxed{\tfrac{1}{2}\tan\theta}$$

100 •

Picture the Problem In the absence of air resistance, the horizontal and vertical displacements of the projectile are independent of each other and describable by constant-acceleration equations. Choose the origin at the firing location and with the coordinate axes as shown in the figure and use constant-acceleration equations to relate the vertical displacement to vertical component of the initial velocity and the horizontal velocity to the horizontal displacement and the time of flight.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the projectile as a function of its time of flight:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $a_y = -g$,

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Solve for v_{0y}:

$$v_{0y} = \frac{\Delta y + \frac{1}{2}g(\Delta t)^2}{\Delta t}$$

Substitute numerical values and evaluate v_{0y}:

$$v_{0y} = \frac{450\,\text{m} + \frac{1}{2}(9.81\,\text{m/s}^2)(20\,\text{s})^2}{20\,\text{s}}$$

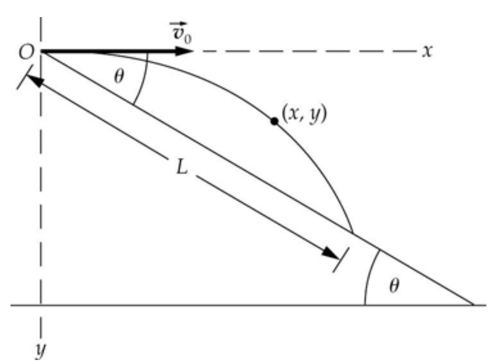
$$= \boxed{121\,\text{m/s}}$$

(*b*) The horizontal velocity remains constant, so:

$$v_{0x} = v_x = \frac{\Delta x}{\Delta t} = \frac{3000\,\text{m}}{20\,\text{s}} = \boxed{150\,\text{m/s}}$$

***101** ••

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant and the horizontal and vertical motions are independent of each other. Choose a coordinate system with the origin at the throwing location and the axes oriented as shown in the figure and use constant- acceleration equations to express the x and y coordinates of the stone while it is in flight.



Using a constant-acceleration equation, express the x coordinate of the stone in flight:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0$ and $a_x = 0$,

$$x = v_0 t$$

Using a constant-acceleration equation, express the y coordinate of the stone in flight:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = 0$ and $a_y = g$,

$$y = \tfrac{1}{2}gt^2$$

Referring to the diagram, express the relationship between θ, y and x at impact:

$$\tan\theta = \frac{y}{x}$$

Substitute for x and y and solve for the time to impact:

$$\tan\theta = \frac{gt^2}{2v_0 t} = \frac{g}{2v_0}t$$

Solve for t to obtain:

$$t = \frac{2v_0}{g}\tan\theta$$

Referring to the diagram, express the relationship between θ, L, y and x at impact:

$$x = L\cos\theta = \frac{y}{\tan\theta}$$

Substitute for y to obtain:

$$\frac{gt^2}{2g} = L\cos\theta$$

Substitute for t and solve for L to obtain:

$$L = \boxed{\frac{2v_0^2\tan\theta}{g\cos\theta}}$$

102 •••

Picture the Problem The equation of a particle's trajectory is derived in the text so we'll use it as our starting point in this derivation. We can relate the coordinates of the point of impact (x, y) to the angle ϕ and use this relationship to eliminate y from the equation for the cannonball's trajectory. We can then solve the resulting equation for x and relate the horizontal component of the point of impact to the cannonball's range.

The equation of the cannonball's trajectory is given in the text:

$$y(x) = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Relate the x and y components of a point on the ground to the angle ϕ:

$$y(x) = (\tan\phi)x$$

Express the condition that the cannonball hits the ground:

$$(\tan\phi)x = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Solve for x to obtain:

$$x = \frac{2v_0^2 \cos^2\theta_0 (\tan\theta_0 - \tan\phi)}{g}$$

Relate the range of the cannonball's flight R to the horizontal distance x:

$$x = R\cos\phi$$

Substitute to obtain:

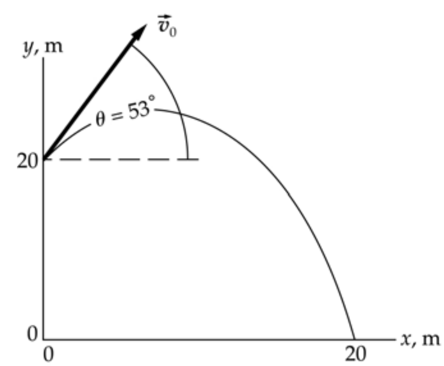
$$R\cos\phi = \frac{2v_0^2 \cos^2\theta_0 (\tan\theta_0 - \tan\phi)}{g}$$

Solve for R:

$$R = \boxed{\frac{2v_0^2 \cos^2\theta_0 (\tan\theta_0 - \tan\phi)}{g\cos\phi}}$$

103 ••

Picture the Problem In the absence of air resistance, the acceleration of the rock is constant and the horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure with the origin at the base of the building and the axes oriented as shown and apply constant-acceleration equations to relate the horizontal and vertical displacements of the rock to its time of flight.



Find the horizontal and vertical components of v_0:

$$v_{0x} = v_0 \cos 53° = 0.602 v_0$$
$$v_{0y} = v_0 \sin 53° = 0.799 v_0$$

Using a constant-acceleration equation, express the horizontal displacement of the projectile:

$$\Delta x = 20\,\text{m} = v_{0x}\Delta t = (0.602 v_0)\Delta t$$

Using a constant-acceleration equation, express the vertical displacement of the projectile:

$$\Delta y = -20\,\text{m} = v_{0y}\Delta t - \tfrac{1}{2} g(\Delta t)^2$$
$$= (0.799 v_0)\Delta t - \tfrac{1}{2} g(\Delta t)^2$$

Solve the x-displacement equation for Δt:

$$\Delta t = \frac{20\,\text{m}}{0.602 v_0}$$

Substitute Δt into the expression for Δy:

$$-20\text{ m} = (0.799 v_0)\Delta t - (4.91\text{ m/s}^2)(\Delta t)^2$$

Solve for v_0 to obtain:

$$v_0 = \boxed{10.8\,\text{m/s}}$$

Find Δt at impact:

$$\Delta t = \frac{20\,\text{m}}{(10.8\,\text{m/s})\cos 53°} = 3.08\,\text{s}$$

Using constant-acceleration equations, find v_y and v_x at impact:

$$v_x = v_{0x} = 6.50\,\text{m/s}$$

and

$$v_y = v_{0y} - g\Delta t = -21\,\text{m/s}$$

Express the velocity at impact in vector form:

$$\vec{v} = \boxed{(6.50\,\text{m/s})\hat{\boldsymbol{i}} + (-21.6\,\text{m/s})\hat{\boldsymbol{j}}}$$

104 ••

Picture the Problem The ball experiences constant acceleration, except during its collision with the wall, so we can use the constant-acceleration equations in the analysis of its motion. Choose a coordinate system with the origin at the point of release, the positive x axis to the right, and the positive y axis upward.

Using a constant-acceleration equation, express the vertical displacement of the ball as a function of Δt:

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

When the ball hits the ground, $\Delta y = -2$ m:

$$-2\,\text{m} = (10\,\text{m/s})\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Solve for the time of flight:

$$t_{\text{flight}} = \Delta t = 2.22\text{ s}$$

Find the horizontal distance traveled in this time:

$$\Delta x = (10\text{ m/s})\,(2.22\text{ s}) = 22.2\text{ m}$$

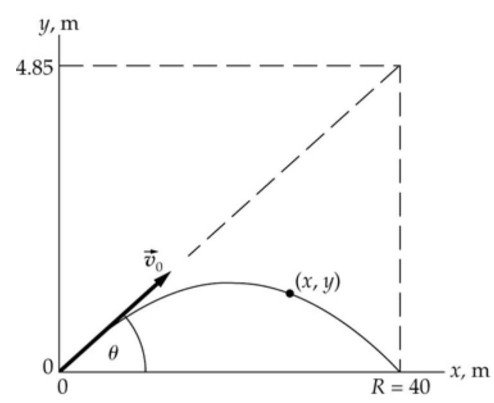
The distance from the wall is:

$$\Delta x - 4\text{ m} = \boxed{18.2\,\text{m}}$$

Hitting Targets and Related Problems

105 •

Picture the Problem In the absence of air resistance, the acceleration of the pebble is constant. Choose the coordinate system shown in the diagram and use constant-acceleration equations to express the coordinates of the pebble in terms of the time into its flight. We can eliminate the parameter t between these equations and solve for the launch velocity of the pebble. We can determine the launch angle from the sighting information and, once the range is known, the time of flight can be found using the horizontal component of the initial velocity.



Referring to the diagram, express θ in terms of the given distances:

$$\theta = \tan^{-1}\left(\frac{4.85\,\text{m}}{40\,\text{m}}\right) = 6.91°$$

Use a constant-acceleration equation to express the horizontal position of the pebble as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0\cos\theta)t \qquad (1)$$

Use a constant-acceleration equation to express the vertical position of the
pebble as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for v_0 to obtain:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta}}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{\frac{(40\,\text{m})(9.81\,\text{m/s}^2)}{\sin 13.8°}} = \boxed{40.6\,\text{m/s}}$$

Substitute in equation (1) to relate R to t_{flight}:

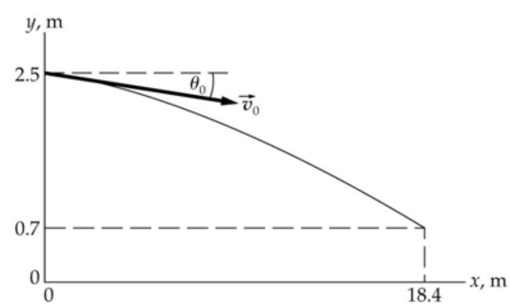
$$R = (v_0\cos\theta)t_{\text{flight}}$$

Solve for and evaluate the time of flight:

$$t_{\text{flight}} = \frac{40\,\text{m}}{(40.6\,\text{m/s})\cos 6.91°} = \boxed{0.992\,\text{s}}$$

***106** ••

Picture the Problem The acceleration of the ball is constant (zero horizontally and –g vertically) and the vertical and horizontal components are independent of each other. Choose the coordinate system shown in the figure and assume that v and t are unchanged by throwing the ball slightly downward.



Express the horizontal displacement of the ball as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $a_x = 0$,
$\Delta x = v_{0x}\Delta t$

Solve for the time of flight if the ball were thrown horizontally:

$$\Delta t = \frac{\Delta x}{v_{0x}} = \frac{18.4\text{ m}}{37.5\text{ m/s}} = 0.491\text{ s}$$

Using a constant-acceleration equation, express the distance the ball would drop (vertical displacement) if it were thrown horizontally:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values and evaluate Δy:

$$\Delta y = -\tfrac{1}{2}\left(9.81\text{ m/s}^2\right)(0.491\text{ s})^2 = -1.18\text{ m}$$

The ball must drop an additional 0.62 m before it gets to home plate.

$y = (2.5 - 1.18)$ m
= 1.32 m above ground

Calculate the initial downward speed the ball must have to drop 0.62 m in 0.491 s:

$$v_y = \frac{-0.62\text{ m}}{0.491\text{s}} = -1.26\text{ m}$$

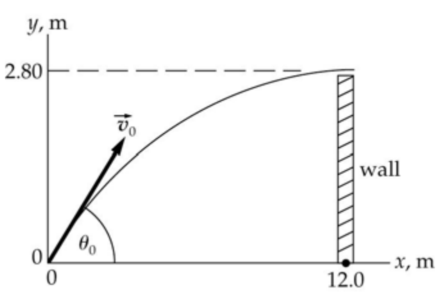
Find the angle with horizontal:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-1.26\text{ m/s}}{37.5\text{ m/s}}\right) = \boxed{-1.92°}$$

Remarks: One can readily show that $\sqrt{v_x^2 + v_y^2}$ = 37.5 m/s to within 1%; so the assumption that *v* and *t* are unchanged by throwing the ball downward at an angle of 1.93° is justified.

107 ••

Picture the Problem The acceleration of the puck is constant (zero horizontally and –*g* vertically) and the vertical and horizontal components are independent of each other. Choose a coordinate system with the origin at the point of contact with the puck and the coordinate axes as shown in the figure and use constant-acceleration equations to relate the variables v_{0y}, the time *t* to reach the wall, v_{0x}, v_0, and θ_0.



Using a constant-acceleration equation for the motion in the *y* direction, express v_{0y} as a function of the puck's displacement Δy:

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $v_y = 0$ and $a_y = -g$,

$$0 = v_{0y}^2 - 2g\Delta y$$

Solve for and evaluate v_{0y}:

$$v_{0y}=\sqrt{2g\Delta y}=\sqrt{2(2.80\,\text{m})(9.81\,\text{m/s}^2)}=\boxed{7.41\,\text{m/s}}$$

Find t from the initial velocity in the y direction:

$$t=\frac{v_{0y}}{g}=\frac{7.41\,\text{m/s}}{9.81\,\text{m/s}^2}=\boxed{0.756\,\text{s}}$$

Use the definition of average velocity to find v_{0x}:

$$v_{0x}=v_x=\frac{\Delta x}{t}=\frac{12.0\,\text{m}}{0.756\,\text{s}}=\boxed{15.9\,\text{m/s}}$$

Substitute numerical values and evaluate v_0:

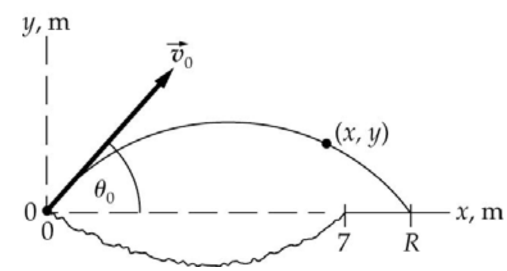
$$v_0=\sqrt{v_{0x}^2+v_{0y}^2}=\sqrt{(15.9\,\text{m/s})^2+(7.41\,\text{m/s})^2}=\boxed{17.5\,\text{m/s}}$$

Substitute numerical values and evaluate θ:

$$\theta=\tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)=\tan^{-1}\left(\frac{7.41\,\text{m/s}}{15.9\,\text{m/s}}\right)=\boxed{25.0°}$$

108 ••

Picture the Problem In the absence of air resistance, the acceleration of Carlos and his bike is constant and we can use constant-acceleration equations to express his x and y coordinates as functions of time. Eliminating the parameter t between these equations will yield y as a function of x … an equation we can use to decide whether he can jump the creek bed as well as to find the minimum speed required to make the jump.



(*a*) Use a constant-acceleration equation to express Carlos' horizontal position as a function of time:

$$x=x_0+v_{0x}t+\tfrac{1}{2}a_xt^2$$

or, because $x_0=0$, $v_{0x}=v_0\cos\theta$, and $a_x=0$,

$$x=(v_0\cos\theta)t$$

Use a constant-acceleration equation to express Carlos' vertical position as a function of time:

$$y=y_0+v_{0y}t+\tfrac{1}{2}a_yt^2$$

or, because $y_0=0$, $v_{0y}=v_0\sin\theta$, and $a_y=-g$,

$$y=(v_0\sin\theta)t-\tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Substitute $y = 0$ and $x = R$ to obtain:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for and evaluate R:

$$R = \frac{v_0^2}{g}\sin(2\theta_0) = \frac{(11.1\,\text{m/s})^2}{9.81\,\text{m/s}^2}\sin 20°$$
$$= 4.30\,\text{m}$$

$$\boxed{\text{He should apply the brakes!}}$$

(b) Solve the equation we used in the previous step for $v_{0,\text{min}}$:

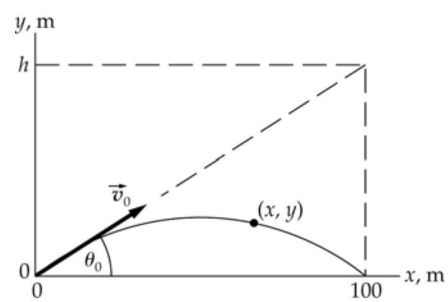
$$v_{0,\text{min}} = \sqrt{\frac{Rg}{\sin(2\theta_0)}}$$

Letting $R = 7$ m, evaluate $v_{0,\text{min}}$:

$$v_{0,\text{min}} = \sqrt{\frac{(7\,\text{m})(9.81\,\text{m/s}^2)}{\sin 20°}}$$
$$= \boxed{14.2\,\text{m/s} = 51.0\,\text{km/h}}$$

109 •••

Picture the Problem In the absence of air resistance, the bullet experiences constant acceleration along its parabolic trajectory. Choose a coordinate system with the origin at the end of the barrel and the coordinate axes oriented as shown in the figure and use constant-acceleration equations to express the x and y coordinates of the bullet as functions of time along its flight path.



Use a constant-acceleration equation to express the bullet's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0\cos\theta)t$$

Use a constant-acceleration equation to express the bullet's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Let $y = 0$ when $x = R$ to obtain:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for the angle above the horizontal that the rifle must be fired to hit the target:

$$\theta_0 = \tfrac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \tfrac{1}{2}\sin^{-1}\left[\frac{(100\,\text{m})(9.81\,\text{m/s}^2)}{(250\,\text{m/s})^2}\right] = 0.450°$$

Note: A second value for θ_0, 89.6° is physically unreasonable.

Referring to the diagram, relate h to θ_0 and solve for and evaluate h:

$$\tan\theta_0 = \frac{h}{100\,\text{m}}$$

and

$$h = (100\,\text{m})\tan(0.450°) = \boxed{0.785\,\text{m}}$$

General Problems

110 •

Picture the Problem The sum and difference of two vectors can be found from the components of the two vectors. The magnitude and direction of a vector can be found from its components.

(*a*) The table to the right summarizes the components of $\vec{A}$ and $\vec{B}$.

Vector	x component (m)	y component (m)
$\vec{A}$	0.707	0.707
$\vec{B}$	0.866	–0.500

(*b*) The table to the right shows the components of $\vec{S}$.

Vector	x component (m)	y component (m)
$\vec{A}$	0.707	0.707
$\vec{B}$	0.866	–0.500
$\vec{S}$	1.57	0.207

Determine the magnitude and direction of $\vec{S}$ from its components:

$$S = \sqrt{S_x^2 + S_y^2} = \boxed{1.59\,\text{m}}$$

and, because $\vec{S}$ is in the 1st

$$\theta_S = \tan^{-1}\left(\frac{S_y}{S_x}\right) = \boxed{7.50°}$$

(*c*) The table to the right shows the components of $\vec{D}$:

Vector	x component (m)	y component (m)
$\vec{A}$	0.707	0.707
$\vec{B}$	0.866	−0.500
$\vec{D}$	−0.159	1.21

Determine the magnitude and direction of $\vec{D}$ from its components:

$$D = \sqrt{D_x^2 + D_y^2} = \boxed{1.22\,\text{m}}$$

and, because $\vec{D}$ is in the 2nd quadrant,

$$\theta_D = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \boxed{97.5°}$$

***111 •**

Picture the Problem A vector quantity can be resolved into its components relative to any coordinate system. In this example, the axes are orthogonal and the components of the vector can be found using trigonometric functions.

The x and y components of $\vec{g}$ are related to g through the sine and cosine functions:

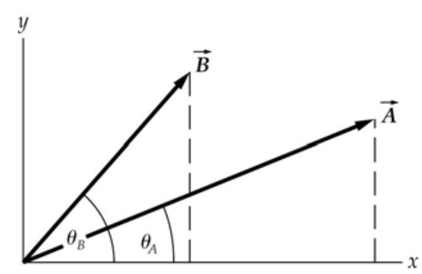
$$g_x = g\sin 30° = \boxed{4.91\,\text{m/s}^2}$$

and

$$g_y = g\cos 30° = \boxed{8.50\,\text{m/s}^2}$$

112 •

Picture the Problem The figure shows two arbitrary, co-planar vectors that (as drawn) do not satisfy the condition that $A/B = A_x/B_x$.



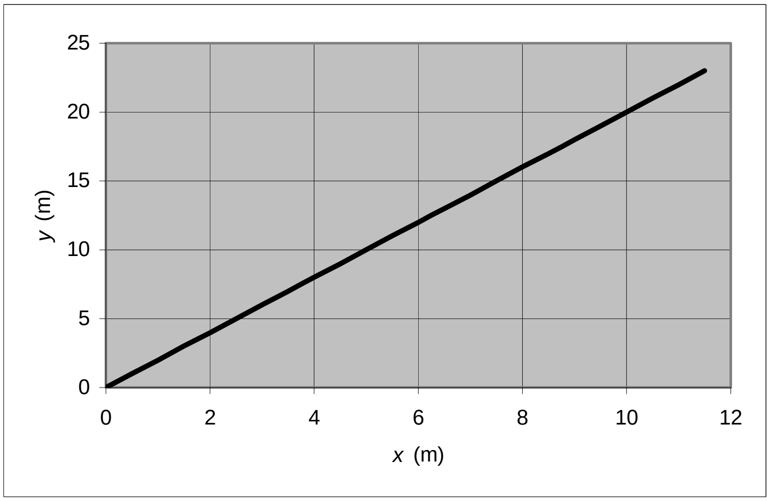
Because $A_x = A\cos\theta_A$ and $B_x = B\cos\theta_B$, $\dfrac{\cos\theta_A}{\cos\theta_B} = 1$ for the condition to be satisfied.

$\therefore$ $A/B = A_x/B_x$ if and only if $\vec{A}$ and $\vec{B}$ are parallel ($\theta_A = \theta_B$) or on opposite sides of the x-axis ($\theta_A = -\theta_B$).

113 •

Picture the Problem We can plot the path of the particle by substituting values for t and evaluating r_x and r_y coordinates of $\vec{r}$. The velocity vector is the time derivative of the position vector.

(*a*) We can assign values to t in the parametric equations $x = (5\text{ m/s})t$ and $y = (10\text{ m/s})t$ to obtain ordered pairs (x, y) that lie on the path of the particle. The path is shown in the following graph:



(*b*) Evaluate $d\vec{\mathbf{r}}/dt$:

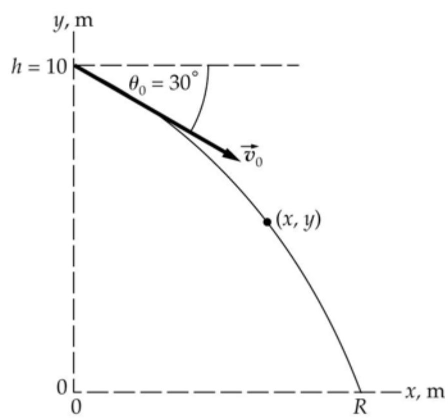
$$\vec{v} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[(5\,\text{m/s})t\,\hat{\boldsymbol{i}} + (10\text{m/s})t\,\hat{\boldsymbol{j}}\right]$$
$$= \boxed{(5\,\text{m/s})\hat{\boldsymbol{i}} + (10\text{m/s})\hat{\boldsymbol{j}}}$$

Use its components to find the magnitude of $\vec{v}$:

$$v = \sqrt{v_x^2 + v_y^2} = \boxed{11.2\,\text{m/s}}$$

114 ••

Picture the Problem In the absence of air resistance, the hammer experiences constant acceleration as it falls. Choose a coordinate system with the origin and coordinate axes as shown in the figure and use constant-acceleration equations to describe the *x* and *y* coordinates of the hammer along its trajectory. We'll use the equation describing the vertical motion to find the time of flight of the hammer and the equation describing the horizontal motion to determine its range.



Using a constant-acceleration equation, express the *x* coordinate of the hammer as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$x = (v_0\cos\theta_0)t$$

Using a constant-acceleration equation, express the *y* coordinate of the hammer as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = h$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = h + (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Substitute numerical values to obtain:

$$y = 10\,\text{m} + (4\,\text{m/s})(\sin 30°)t - \tfrac{1}{2}(9.81\,\text{m/s}^2)t^2$$

Substitute the conditions that exist when the hammer hits the ground:

$$0 = 10\,\text{m} - (4\,\text{m/s})\sin 30°\, t - \tfrac{1}{2}(9.81\,\text{m/s}^2)\, t^2$$

Solve for the time of fall to obtain:

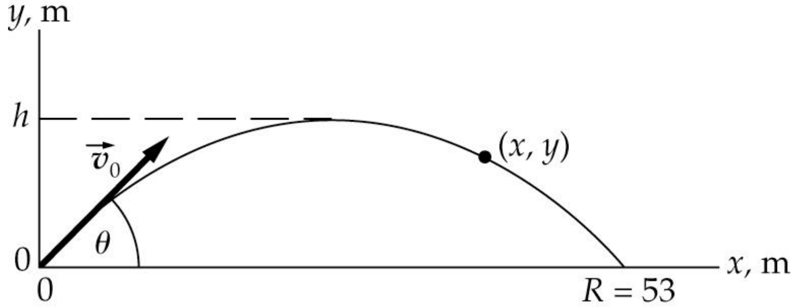
$t = 1.24$ s

Use the x-coordinate equation to find the horizontal distance traveled by the hammer in 1.24 s:

$$R = (4\,\text{m/s})(\cos 30°)(1.24\,\text{s}) = \boxed{4.29\,\text{m}}$$

115 ••

Picture the Problem We'll model Zacchini's flight as though there is no air resistance and, hence, the acceleration is constant. Then we can use constant- acceleration equations to express the x and y coordinates of Zacchini's motion as functions of time. Eliminating the parameter t between these equations will leave us with an equation we can solve for θ. Because the maximum height along a parabolic trajectory occurs (assuming equal launch and landing elevations) occurs at half range, we can use this same expression for y as a function of x to find h.



Use a constant-acceleration equation to express Zacchini's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0 \cos\theta)t$$

Use a constant-acceleration equation to express Zacchini's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Use Zacchini's coordinates when he lands in a safety net to obtain:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for his launch angle θ:

$$\theta=\tfrac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

Substitute numerical values and evaluate θ:

$$\theta=\tfrac{1}{2}\sin^{-1}\left[\frac{(53\,\text{m})(9.81\,\text{m/s}^2)}{(24.2\,\text{m/s})^2}\right]=\boxed{31.3°}$$

Use the fact that his maximum height was attained when he was halfway through his flight to obtain:

$$h=(\tan\theta)\frac{R}{2}-\frac{g}{2v_0^2\cos^2\theta}\left(\frac{R}{2}\right)^2$$

Substitute numerical values and evaluate h:

$$h=(\tan 31.3°)\frac{53\,\text{m}}{2}-\frac{9.81\,\text{m/s}^2}{2(24.2\,\text{m/s})^2\cos^2 31.3°}\left(\frac{53\,\text{m}}{2}\right)^2=\boxed{8.06\,\text{m}}$$

116 ••

Picture the Problem Because the acceleration is constant; we can use the constant-acceleration equations in vector form and the definitions of average velocity and average (instantaneous) acceleration to solve this problem.

(*a*) The average velocity is given by:

$$\vec{\boldsymbol{v}}_{\text{av}}=\frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}=\frac{\vec{\boldsymbol{r}}_2-\vec{\boldsymbol{r}}_1}{\Delta t}$$
$$=(3\ \text{m/s})\hat{\boldsymbol{i}}+(-2.5\ \text{m/s})\hat{\boldsymbol{j}}$$

The average velocity can also be expressed as:

$$\vec{\boldsymbol{v}}_{\text{av}}=\frac{\vec{\boldsymbol{v}}_1+\vec{\boldsymbol{v}}_2}{2}$$

and

$$\vec{v}_1=2\vec{v}_{\text{av}}-\vec{v}_2$$

Substitute numerical values to obtain:

$$\vec{\boldsymbol{v}}_1=\boxed{(1\,\text{m/s})\hat{\boldsymbol{i}}+(1\,\text{m/s})\hat{\boldsymbol{j}}}$$

(*b*) The acceleration of the particle is given by:

$$\vec{\boldsymbol{a}}=\frac{\Delta\vec{\boldsymbol{v}}}{\Delta t}=\frac{\vec{\boldsymbol{v}}_2-\vec{\boldsymbol{v}}_1}{\Delta t}$$
$$=\boxed{(2\ \text{m/s}^2)\hat{\boldsymbol{i}}+(-3.5\ \text{m/s}^2)\hat{\boldsymbol{j}}}$$

(*c*) The velocity of the particle as a function of time is:

$$\vec{\boldsymbol{v}}(t)=\vec{\boldsymbol{v}}_1+\vec{\boldsymbol{a}}t=\boxed{[(1\,\text{m/s})+(2\ \text{m/s}^2)t]\hat{\boldsymbol{i}}+[(1\,\text{m/s})+(-3.5\ \text{m/s}^2)t]\hat{\boldsymbol{j}}}$$

(*d*) Express the position vector as a function of time:

$$\vec{\boldsymbol{r}}(t)=\vec{\boldsymbol{r}}_1+\vec{\boldsymbol{v}}_1t+\tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

Substitute numerical values and evaluate $\vec{r}(t)$:

$$\vec{r}(t) = \boxed{[(4\text{ m}) + (1\text{ m/s})t + (1\text{ m/s}^2)t^2]\hat{i} + [(3\text{ m}) + (1\text{ m/s})t + (-1.75\text{ m/s}^2)t^2]\hat{j}}$$

***117 ••**

Picture the Problem In the absence of air resistance, the steel ball will experience constant acceleration. Choose a coordinate system with its origin at the initial position of the ball, the x direction to the right, and the y direction downward. In this coordinate system $y_0 = 0$ and $a = g$. Letting (x, y) be a point on the path of the ball, we can use constant-acceleration equations to express both x and y as functions of time and, using the geometry of the staircase, find an expression for the time of flight of the ball. Knowing its time of flight, we can find its range and identify the step it strikes first.

The angle of the steps, with respect to the horizontal, is:

$$\theta = \tan^{-1}\left(\frac{0.18\text{ m}}{0.3\text{ m}}\right) = 31.0°$$

Using a constant-acceleration equation, express the x coordinate of the steel ball in its flight:

$$x = x_0 + v_0 t + \tfrac{1}{2}a_y t^2$$

or, because $x_0 = 0$ and $a_y = 0$,

$$x = v_0 t$$

Using a constant-acceleration equation, express the y coordinate of the steel ball in its flight:

$$y = y_0 + v_{0y} t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = 0$, and $a_y = g$,

$$y = \tfrac{1}{2}gt^2$$

The equation of the dashed line in the figure is:

$$\frac{y}{x} = \tan\theta = \frac{gt}{2v_0}$$

Solve for the flight time:

$$t = \frac{2v_0}{g}\tan\theta$$

Find the x coordinate of the landing position:

$$x = \frac{y}{\tan\theta} = \frac{2v_0^2}{g}\tan\theta$$

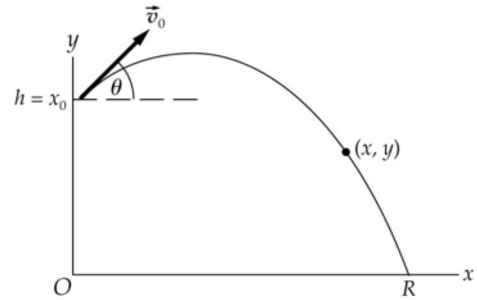
Substitute the angle determined in the first step:

$$x = \frac{2(3\text{ m/s})^2}{9.81\text{ m/s}^2}\tan 31° = 1.10\text{ m}$$

$$\boxed{\text{The first step with } x > 1.10\text{ m is the 4th step.}}$$

118 ••

Picture the Problem Ignoring the influence of air resistance, the acceleration of the ball is constant once it has left your hand and we can use constant-acceleration equations to express the x and y coordinates of the ball. Elimination of the parameter t will yield an equation from which we can determine v_0. We can then use the y equation to express the time of flight of the ball and the x equation to express its range in terms of x_0, v_0, θ and the time of flight.



Use a constant-acceleration equation to express the ball's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0 \cos\theta)t \qquad (1)$$

Use a constant-acceleration equation to express the ball's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = x_0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = x_0 + (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \qquad (2)$$

Eliminate the parameter t to obtain:

$$y = x_0 + (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

For the throw while standing on level ground we have:

$$0 = (\tan\theta)x_0 - \frac{g}{2v_0^2\cos^2\theta}x_0^2$$

and

$$x_0 = \frac{v_0^2}{g}\sin 2\theta = \frac{v_0^2}{g}\sin 2(45°) = \frac{v_0^2}{g}$$

Solve for v_0:

$$v_0 = \sqrt{gx_0}$$

At impact equation (2) becomes:

$$0 = x_0 + \left(\sqrt{gx_0}\sin\theta\right)t_{\text{flight}} - \tfrac{1}{2}gt_{\text{flight}}^2$$

Solve for the time of flight:

$$t_{\text{flight}} = \sqrt{\frac{x_0}{g}}\left(\sin\theta + \sqrt{\sin^2\theta + 2}\right)$$

Substitute in equation (1) to express the range of the ball when thrown from an elevation x_0 at an angle θ with the horizontal:

$$R = \left(\sqrt{gx_0}\cos\theta\right)t_{\text{flight}} = \left(\sqrt{gx_0}\cos\theta\right)\sqrt{\frac{x_0}{g}}\left(\sin\theta + \sqrt{\sin^2\theta + 2}\right) = x_0\cos\theta\left(\sin\theta + \sqrt{\sin^2\theta + 2}\right)$$

Substitute θ = 0°, 30°, and 45°:

$$x(0°) = \boxed{1.41x_0}$$

$$x(30°) = \boxed{1.73x_0}$$

and

$$x(45°) = \boxed{1.62x_0}$$

119 •••

Picture the Problem Choose a coordinate system with its origin at the point where the motorcycle becomes airborne and with the positive x direction to the right and the positive y direction upward. With this choice of coordinate system we can relate the x and y coordinates of the motorcycle (which we're treating as a particle) using Equation 3-21.

(*a*) The path of the motorcycle is given by:

$$y(x) = (\tan\theta)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

For the jump to be successful, $h < y(x)$. Solving for v_0, we find:

$$v_{\min} > \boxed{\frac{x}{\cos\theta}\sqrt{\frac{g}{2(x\tan\theta - h)}}}$$

(*b*) Use the values given to obtain:

$$v_{\min} > \boxed{26.0\,\text{m/s or}\,58.0\,\text{mph}}$$

(*c*) In order for our expression for $v_{\min}$ to be real valued; i.e., to predict values for $v_{\min}$ that are physically meaningful, $x\tan\theta - h > 0$.

$$\therefore\ h_{\max} < x\tan\theta$$

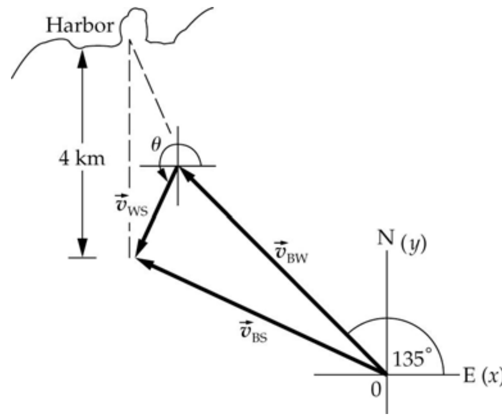
The interpretation is that the bike "falls away" from traveling on a straight-line path due to the free-fall acceleration downwards. No matter what the initial speed of the bike, it must fall a little bit before reaching the other side of the pit.

120 •••

Picture the Problem Let the origin be at the position of the boat when it was engulfed by the fog. Take the x and y directions to be east and north, respectively. Let $\vec{v}_{BW}$ be the velocity of the boat relative to the water, $\vec{v}_{BS}$ be the velocity of the boat relative to the shore, and $\vec{v}_{WS}$ be the velocity of the water with respect to the shore. Then

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

θ is the angle of $\vec{v}_{WS}$ with respect to the x (east) direction.



(*a*) Find the position vector for the boat at $t = 3$ h:

$$\vec{r}_{boat} = \{(32\text{ km})(\cos 135°)t\}\hat{i} + \{(32\text{ km})(\sin 135°)t - 4\text{ km}\}\hat{j}$$
$$= \{(-22.6\text{ km})t\}\hat{i} + \{(22.6\text{ km})t - 4\text{ km}\}\hat{j}$$

Find the coordinates of the boat at $t = 3$ h:

$$r_x = [(10\text{ km/h})\cos 135° + v_{WS}\cos\theta](3\text{ h})$$
and
$$r_y = [(10\text{ km/h})\sin 135° + v_{WS}\sin\theta](3\text{ h})$$

Simplify the expressions involving r_x and r_y and equate these simplified expressions to the x and y components of the position vector of the boat:

$$3v_{WS}\cos\theta = -1.41\text{ km/h}$$
and
$$3v_{WS}\sin\theta = -2.586\text{ km/h}$$

Divide the second of these equations by the first to obtain:

$$\tan\theta = \frac{-2.586\text{ km}}{-1.41\text{ km}}$$
or
$$\theta = \tan^{-1}\left(\frac{-2.586\text{ km}}{-1.41\text{ km}}\right) = 61.4° \text{ or } 241.4°$$

Because the boat has drifted south, use $\theta = 241.4°$ to obtain:

$$v_{WS} = \frac{v_x}{\cos\theta} = \frac{-\frac{1.41\text{ km/h}}{3}}{\cos(241.4°)}$$
$$= \boxed{0.982\text{ km/h at } \theta = 241.4°}$$

(*b*) Letting ϕ be the angle between east and the proper heading for the boat, express the components of the velocity of the boat with respect to the shore:

$$v_{BS,x} = (10\text{ km/h})\cos\phi + (0.982\text{ km/h})\cos(241.3°)$$

$$v_{BS,y} = (10\text{ km/h})\sin\phi + (0.982\text{ km/h})\sin(241.3°)$$

For the boat to travel northwest:

$$v_{BS,x} = -v_{BS,y}$$

Substitute the velocity components, square both sides of the equation, and simplify the expression to obtain the equations:

$$\sin\phi + \cos\phi = 0.133,$$
$$\sin^2\phi + \cos^2\phi + 2\sin\phi\cos\phi = 0.0177,$$
and
$$1 + \sin(2\phi) = 0.0177$$

Solve for ϕ:

$$\phi = 129.6° \text{ or } 140.4°$$

Because the current pushes south, the boat must head more northerly than 135°:

Using 129.6°, the correct heading is $\boxed{39.6° \text{ west of north}}$.

(*c*) Find v_{BS}:

$$v_{BS,x} = -6.84\text{ km/h}$$
and
$$v_{BS} = v_{Bx}/\cos 135° = 9.68\text{ km/h}$$

To find the time to travel 32 km, divide the distance by the boat's actual speed:

$$t = (32\text{ km})/(9.68\text{ km/h}) = \boxed{3.31\text{ h} = 3\text{ h } 18\text{ min}}$$

***121 ••**

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and the equation of a projectile for equal initial and final elevations, which was derived from the constant-acceleration equations, is applicable. We can use the equation giving the range of a projectile for equal initial and final elevations to evaluate the ranges of launches that exceed or fall short of 45° by the same amount.

Express the range of the projectile as a function of its initial speed and angle of launch:

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

Let $\theta_0 = 45° \pm \theta$:

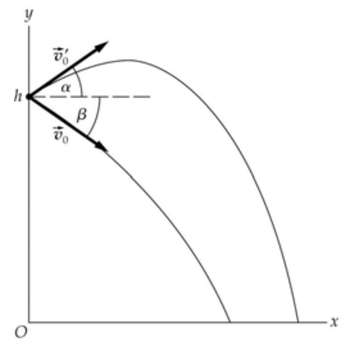
$$R = \frac{v_0^2}{g}\sin(90° \pm 2\theta) = \frac{v_0^2}{g}\cos(\pm 2\theta)$$

Because $\cos(-\theta) = \cos(+\theta)$ (the cosine function is an *even* function):

$$\boxed{R(45° + \theta) = R(45° - \theta)}$$

122 ••

Picture the Problem In the absence of air resistance, the acceleration of both balls is that due to gravity and the horizontal and vertical motions are independent of each other. Choose a coordinate system with the origin at the base of the cliff and the coordinate axes oriented as shown and use constant-acceleration equations to relate the *x* and *y* components of the ball's speed.



Independently of whether a ball is thrown upward at the angle α or downward at β, the vertical motion is described by:

$$v_y^2 = v_{0y}^2 + 2a\Delta y$$
$$= v_{0y}^2 - 2gh$$

The horizontal component of the motion is given by:

$v_x = v_{0x}$

Find v at impact from its components:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + v_{0y}^2 - 2gh}$$
$$= \boxed{\sqrt{v_0^2 - 2gh}}$$

Chapter 4
Newton's Laws

Conceptual Problems

***1** ••

Determine the Concept A reference frame in which the law of inertia holds is called an inertial reference frame.

If an object with no net force acting on it is at rest or is moving with a constant speed in a straight line (i.e., with constant velocity) relative to the reference frame, then the reference frame is an inertial reference frame. Consider sitting at rest in an accelerating train or plane. The train or plane is not an inertial reference frame even though you are at rest relative to it. In an inertial frame, a dropped ball lands at your feet. You are in a noninertial frame when the driver of the car in which you are riding steps on the gas and you are pushed back into your seat.

2 ••

Determine the Concept A reference frame in which the law of inertia holds is called an inertial reference frame. A reference frame with acceleration *a* relative to the initial frame, and with *any* velocity relative to the initial frame, is inertial.

3 •

Determine the Concept No. If the net force acting on an object is zero, its acceleration is zero. The only conclusion one can draw is that the *net* force acting on the object is zero.

***4** •

Determine the Concept An object accelerates when a *net* force acts on it. The fact that an object is accelerating tells us nothing about its velocity other than that it is always changing.

Yes, the object must have an acceleration relative to the inertial frame of reference. According to Newton's 1st and 2nd laws, an object must accelerate, relative to any inertial reference frame, in the direction of the net force. If there is ″only a single nonzero force,″ then this force is the net force.

Yes, the object's velocity may be momentarily zero. During the period in which the force is acting, the object may be momentarily at rest, but its velocity cannot remain zero because it must continue to accelerate. Thus, its velocity is always changing.

5 •

Determine the Concept No. Predicting the direction of the subsequent motion correctly requires knowledge of the initial velocity as well as the acceleration. While the acceleration can be obtained from the net force through Newton's 2nd law, the velocity can only be obtained by integrating the acceleration.

6 •

Determine the Concept An object in an inertial reference frame accelerates if there is a *net* force acting on it. Because the object is moving at constant velocity, the net force acting on it is zero. $\boxed{(c)\text{ is correct.}}$

7 •

Determine the Concept The mass of an object is an intrinsic property of the object whereas the weight of an object depends directly on the local gravitational field. Therefore, the mass of the object would not change and $w_{\text{grav}} = mg_{\text{local}}$. Note that if the gravitational field is zero then the gravitational force is also zero.

***8** •

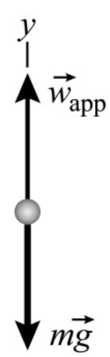
Determine the Concept If there is a force on her in addition to the gravitational force, she will experience an additional acceleration relative to her space vehicle that is proportional to the net force required producing that acceleration and inversely proportional to her mass.

She could do an experiment in which she uses her legs to push off from the wall of her space vehicle and measures her acceleration and the force exerted by the wall. She could calculate her mass from the ratio of the force exerted by the wall to the acceleration it produced.

***9** •

Determine the Concept One's apparent weight is the reading of a scale in one's reference frame.

Imagine yourself standing on a scale that, in turn, is on a platform accelerating upward with an acceleration *a*. The free-body diagram shows the force the gravitational field exerts on you, $m\vec{g}$, and the force the scale exerts on you, $\vec{w}_{\text{app}}$. The scale reading (the force the scale exerts on you) is your apparent weight.



Choose the coordinate system shown in the free-body diagram and apply $\sum \vec{F} = m\vec{a}$ to the scale:

$$\sum F_y = w_{\text{app}} - mg = ma_y$$

or

$$w_{\text{app}} = mg + ma_y$$

So, your apparent weight would be greater than your true weight when observed from a reference frame that is accelerating upward. That is, when the surface on which you are standing has an acceleration *a* such that a_y is positive: $\boxed{a_y > 0}$.

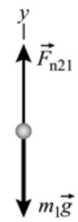
10 ••

Determine the Concept Newton's 2nd law tells us that forces produce *changes* in the velocity of a body. If two observers pass each other, each traveling at a constant velocity, each will experience no net force acting on them, and so each will feel as if he or she is standing still.

11 •

Determine the Concept Neither block is accelerating so the net force on each block is zero. Newton's 3rd law states that objects exert equal and opposite forces on each other.

(a) and (b) Draw the free-body diagram for the forces acting on the block of mass m_1:



Apply $\sum \vec{F} = m\vec{a}$ to the block 1:

$$\sum F_y = F_{n21} - m_1 g = m_1 a_1$$

or, because $a_1 = 0$,

$$F_{n21} - m_1 g = 0$$

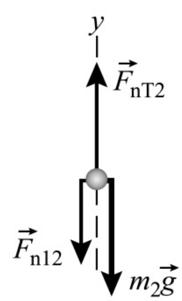
Therefore, the magnitude of the force that block 2 exerts on block 1 is given by:

$$F_{n21} = \boxed{m_1 g}$$

From Newton's 3rd law of motion we know that the force that block 1 exerts on block 2 is equal to, but opposite in direction, the force that block 2 exerts on block 1.

$$\vec{F}_{n21} = -\vec{F}_{n12} \Rightarrow F_{n12} = \boxed{m_1 g}$$

(c) and (d) Draw the free-body diagram for the forces acting on block 2:



Apply $\sum \vec{F} = m\vec{a}$ to block 2:

$$\sum F_{2y} = F_{nT2} - F_{n12} - m_2 g = m_2 a_2$$

or, because $a_2 = 0$,

$$F_{nT2} = F_{nT2} + m_2 g = m_1 g + m_2 g$$
$$= (m_1 + m_2)g$$

and the normal force that the table exerts on body 2 is

$$F_{nT2} = \boxed{(m_1 + m_2)g}$$

From Newton's 3rd law of motion we know that the force that block 2 exerts on the table is equal to, but opposite in direction, the force that the table exerts on block 2.

$$\vec{F}_{nT2} = -\vec{F}_{n2T} \Rightarrow F_{n2T} = \boxed{(m_1 + m_2)g}$$

***12** •

(*a*) True. By definition, action-reaction force pairs cannot act on the same object.

(*b*) False. Action equals reaction independent of any motion of the two objects.

13 •

Determine the Concept Newton's 3rd law of motion describes the interaction between the man and his less massive son. According to the 3rd law description of the interaction of two objects, these are action-reaction forces and therefore must be equal in magnitude. $\boxed{(b)\text{ is correct.}}$

14 •

Determine the Concept According to Newton's 3rd law the reaction force to a force exerted by object A on object B is the force exerted by object B on object A. The bird's weight is a gravitational field force exerted by the earth on the bird. Its reaction force is the gravitational force the bird exerts on the earth. $\boxed{(b)\text{ is correct.}}$

15 •

Determine the Concept We know from Newton's 3rd law of motion that the reaction to the force that the bat exerts on the ball is the force the ball exerts on the bat and is equal in magnitude but oppositely directed. The action-reaction pair consists of the force with which the bat hits the ball and the force the ball exerts on the bat. These forces are equal in magnitude, act in opposite directions. $\boxed{(c)\text{ is correct.}}$

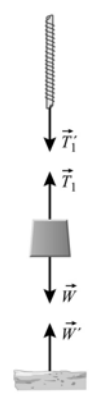
16 •

Determine the Concept The statement of Newton's 3rd law given in the problem is not complete. It is important to remember that the action and reaction forces act on different bodies. The reaction force does not cancel out because it does not act on the same body as the external force.

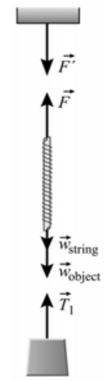
***17** •

Determine the Concept The force diagrams will need to include the ceiling, string, object, and earth if we are to show all of the reaction forces as well as the forces acting on the object.

(*a*) The forces acting on the 2.5-kg object are its weight $\vec{W}$, and the tension $\vec{T}_1$, in the string. The reaction forces are $\vec{W}\,'$ acting on the earth and $\vec{T}_1\,'$ acting on the string.



(b) The forces acting on the string are its weight, the weight of the object, and $\vec{F}$, the force exerted by the ceiling. The reaction forces are $\vec{T}_1$ acting on the string and $\vec{F}\,'$ acting on the ceiling.



18 •

Determine the Concept Identify the objects in the block's environment that are exerting forces on the block and then decide in what directions those forces must be acting if the block is sliding *down* the inclined plane.

Because the incline is frictionless, the force the incline exerts on the block must be normal to the surface. The second object capable of exerting a force on the block is the earth and its force; the weight of the block acts directly downward. The magnitude of the normal force is less than that of the weight because it supports only a portion of the weight. $\boxed{\text{The forces shown in FBD } (c) \text{ satisfy these conditions.}}$

19 •

Determine the Concept In considering these statements, one needs to decide whether they are consistent with Newton's laws of motion. A good strategy is to try to think of a counterexample that would render the statement false.

(a) True. If there are no forces acting on an object, the *net* force acting on it must be zero and, hence, the acceleration must be zero.

(b) False. Consider an object moving with constant velocity on a frictionless horizontal surface. While the *net* force acting on it is zero (it is not accelerating), gravitational and normal forces are acting on it.

(c) False. Consider an object that has been thrown vertically upward. While it is still rising, the direction of the gravitational force acting on it is downward.

(d) False. The mass of an object is an intrinsic property that is independent of its location (the gravitational field in which it happens to be situated).

20 •

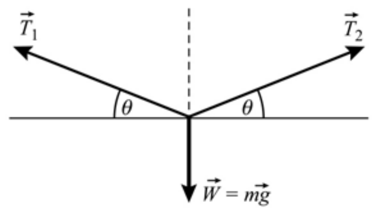
Determine the Concept In considering these alternatives, one needs to decide which alternatives are consistent with Newton's 3rd law of motion. According to Newton's 3rd law, the magnitude of the gravitational force exerted by her body on the earth is equal and opposite to the force exerted by the earth on her. $\boxed{(a) \text{ is correct.}}$

***21 •**

Determine the Concept In considering these statements, one needs to decide whether they are consistent with Newton's laws of motion. In the absence of a *net* force, an object moves with constant velocity. $\boxed{(d) \text{ is correct.}}$

22 •

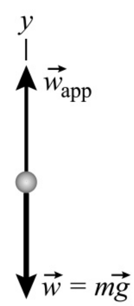
Determine the Concept Draw the free-body diagram for the towel. Because the towel is hung at the center of the line, the magnitudes of $\vec{T}_1$ and $\vec{T}_2$ are the same.



No. To support the towel, the tension in the line must have a vertical component equal to the towel's weight. Thus $\theta > 0$.

23 •

Determine the Concept The free-body diagram shows the forces acting on a person in a descending elevator. The upward force exerted by the scale on the person, $\vec{w}_{app}$, is the person's apparent weight.



Apply $\sum F_y = ma_y$ to the person and solve for w_{app}:

$$w_{app} - mg = ma_y$$

or

$$w_{app} = mg + ma_y = m(g + a_y)$$

$$\boxed{\begin{array}{l}\text{Because } w_{app} \text{ is independent of } v, \\ \text{the velocity of the elevator has} \\ \text{no effect on the person's apparent} \\ \text{weight.}\end{array}}$$

Remarks: Note that a nonconstant velocity will alter the apparent weight.

Estimation and Approximation

24 ••

Picture the Problem Assuming a stopping distance of 25 m and a mass of 80 kg, use Newton's 2nd law to determine the force exerted by the seat belt.

The force the seat belt exerts on the driver is given by:

$F_{net} = ma$, where m is the mass of the driver.

Using a constant-acceleration equation, relate the velocity of the car to its stopping distance and acceleration:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$-v_0^2 = 2a\Delta x$$

Solve for a:

$$a = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = -\frac{\left(90\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{10^3\,\text{m}}{\text{km}}\right)^2}{2(25\,\text{m})} = -12.5\,\text{m/s}^2$$

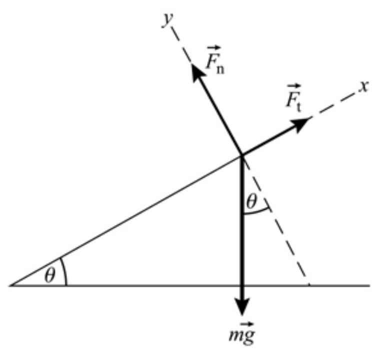
Substitute for a and evaluate F_{net}:

$$F_{\text{net}} = (80\,\text{kg})(-12.5\,\text{m/s}^2) = \boxed{-1.00\,\text{kN}}$$

F_{net} is negative because it is opposite the direction of motion.

***25** •••

Picture the Problem The free-body diagram shows the forces acting on you and your bicycle as you are either ascending or descending the grade. The magnitude of the normal force acting on you and your bicycle is equal to the component of your weight in the *y* direction and the magnitude of the tangential force is the *x* component of your weight. Assume a combined mass (you plus your bicycle) of 80 kg.



(*a*) Apply $\sum F_y = ma_y$ to you and your bicycle and solve for F_n:

$F_n - mg\cos\theta = 0$, because there is no acceleration in the *y* direction.

$\therefore F_n = mg\cos\theta$

Determine θ from the information concerning the grade:

$\tan\theta = 0.08$

and

$\theta = \tan^{-1}(0.08) = 4.57°$

Substitute to determine F_n:

$$F_n = (80\text{ kg})(9.81\text{ m/s}^2)\cos 4.57° = \boxed{782\,\text{N}}$$

Apply $\sum F_x = ma_x$ to you and your bicycle and solve for F_t, the tangential force exerted by the road on the wheels:

$F_t - mg\sin\theta = 0$, because there is no acceleration in the *x* direction.

Evaluate F_t:

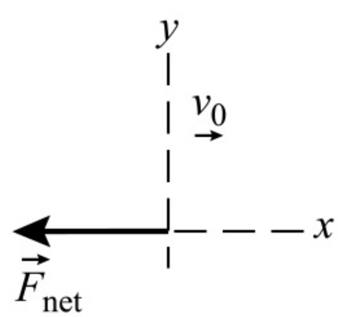
$$F_t = (80\text{ kg})(9.81\text{ m/s}^2)\sin 4.57° = \boxed{62.6\text{ N}}$$

(*b*) Because there is no acceleration, the forces are the same going up and going down the incline.

Newton's First and Second Laws: Mass, Inertia, and Force

26 •

Picture the Problem The acceleration of the particle can be found from the stopping distance by using a constant-acceleration equation. The mass of the particle and its acceleration are related to the net force through Newton's second law of motion. Choose a coordinate system in which the direction the particle is moving is the positive x direction and apply $\vec{F}_{net} = m\vec{a}$.



Use Newton's 2nd law to relate the mass of the particle to the net force acting on it and its acceleration:

$$m = \frac{F_{net}}{a_x}$$

Because the force is constant, use a constant-acceleration equation with $v_x = 0$ to determine a:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

and

$$a_x = \frac{-v_{0x}^2}{2\Delta x}$$

Substitute to obtain:

$$m = \frac{2\Delta x F_{net}}{v_{0x}^2}$$

Substitute numerical values and evaluate m:

$$m = \frac{2(62.5\text{ m})(15.0\text{ N})}{(25.0\text{ m/s})^2} = 3.00\text{ kg}$$

and $\boxed{(b)\text{ is correct.}}$

27 •

Picture the Problem The acceleration of the object is related to its mass and the *net* force acting on it by $F_{net} = F_0 = ma$.

(*a*) Use Newton's 2nd law of motion to calculate the acceleration of the object:

$$a = \frac{F_{net}}{m} = \frac{2F_0}{m}$$

$$= 2(3\text{ m/s}^2) = \boxed{6.00\text{ m/s}^2}$$

(*b*) Let the subscripts 1 and 2 distinguish the two objects. The ratio of the two masses is found from Newton's 2nd law:

$$\frac{m_2}{m_1}=\frac{F_0/a_2}{F_0/a_1}=\frac{a_1}{a_2}=\frac{3\,\text{m/s}^2}{9\,\text{m/s}^2}=\boxed{\frac{1}{3}}$$

(*c*) The acceleration of the two-mass system is the net force divided by the total mass $m = m_1 + m_2$:

$$a=\frac{F_{\text{net}}}{m}=\frac{F_0}{m_1+m_2}$$
$$=\frac{F_0/m_1}{1+m_2/m_1}=\frac{a_1}{1+1/3}$$
$$=\tfrac{3}{4}a_1=\boxed{2.25\,\text{m/s}^2}$$

28 •

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it by $F_{\text{net}}=ma$. Let m be the mass of the ship, a_1 be the acceleration of the ship when the net force acting on it is F_1, and a_2 be its acceleration when the net force is $F_1 + F_2$.

Using Newton's 2nd law, express the net force acting on the ship when its acceleration is a_1:

$$F_1 = ma_1$$

Express the net force acting on the ship when its acceleration is a_2:

$$F_1 + F_2 = ma_2$$

Divide the second of these equations by the first and solve for the ratio F_2/F_1:

$$\frac{F_1+F_2}{F_1}=\frac{ma_1}{ma_2}$$

and

$$\frac{F_2}{F_1}=\frac{a_2}{a_1}-1$$

Substitute for the accelerations to determine the ratio of the accelerating forces and solve for F_2:

$$\frac{F_2}{F_1}=\frac{(16\,\text{km/h})/(10\,\text{s})}{(4\,\text{km/h})/(10\,\text{s})}-1=3$$

or

$$F_2=\boxed{3F_1}$$

***29** ••

Picture the Problem Because the deceleration of the bullet is constant, we can use a constant-acceleration equation to determine its acceleration and Newton's 2nd law of motion to find the average resistive force that brings it to a stop.

Apply $\sum\vec{F}=m\vec{a}$ to express the force exerted on the bullet by the wood:

$$F_{\text{wood}} = ma$$

Using a constant-acceleration

$$v^2=v_0^2+2a\Delta x$$

equation, express the final velocity of the bullet in terms of its acceleration and solve for the acceleration:

and

$$a=\frac{v^2-v_0^2}{2\Delta x}=\frac{-v_0^2}{2\Delta x}$$

Substitute to obtain:

$$F_{\text{wood}}=-\frac{mv_0^2}{2\Delta x}$$

Substitute numerical values and evaluate F_{wood}:

$$F_{\text{wood}}=-\frac{(1.8\times10^{-3}\text{ kg})(500\text{ m/s})^2}{2(0.06\text{ m})}$$

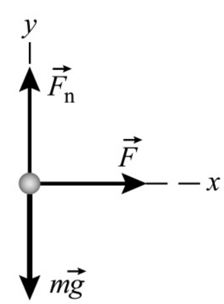
$$=\boxed{-3.75\text{ kN}}$$

where the negative sign means that the direction of the force is opposite the velocity.

***30 ••**

Picture the Problem The pictorial representation summarizes what we know about the motion. We can find the acceleration of the cart by using a constant-acceleration equation.

The free-body diagram shows the forces acting on the cart as it accelerates along the air track. We can determine the net force acting on the cart using Newton's 2nd law and our knowledge of its acceleration.



(*a*) Apply $\sum F_x = ma_x$ to the cart to obtain an expression for the net force F:

$$F = ma$$

Using a constant-acceleration equation, relate the displacement of the cart to its acceleration, initial speed, and travel time:

$$\Delta x = v_0\Delta t+\tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

Solve for *a*:

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute for *a* in the force equation to obtain:

$$F = m\frac{2\Delta x}{(\Delta t)^2} = \frac{2m\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{2(0.355\,\text{kg})(1.5\,\text{m})}{(4.55\,\text{s})^2} = \boxed{0.0514\,\text{N}}$$

(*b*) Using a constant-acceleration equation, relate the displacement of the cart to its acceleration, initial speed, and travel time:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a'(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a'(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a'}}$$

If we assume that air resistance is negligible, the net force on the cart is still 0.0514 N and its acceleration is:

$$a' = \frac{0.0514\,\text{N}}{0.722\,\text{kg}} = 0.0713\,\text{m/s}^2$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(1.5\,\text{m})}{0.0713\,\text{m/s}^2}} = \boxed{6.49\,\text{s}}$$

31 •

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it according to $\vec{F}_{\text{net}} = m\vec{a}$. Let *m* be the mass of the object and choose a coordinate system in which the direction of $2F_0$ in (*b*) is the positive *x* and the direction of the left-most F_0 in (a) is the positive *y* direction. Because both force and acceleration are vector quantities, find the resultant force in each case and then find the resultant acceleration.

(*a*) Calculate the acceleration of the object from Newton’s 2nd law of motion:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Express the net force acting on the object:

$$\vec{F}_{\text{net}} = F_x\hat{i} + F_y\hat{j} = F_0\hat{i} + F_0\hat{j}$$

and

Find the magnitude and direction of this net force:

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{2}F_0$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{F_0}{F_0}\right) = 45°$$

Use this result to calculate the magnitude and direction of the acceleration:

$$a = \frac{F_{\text{net}}}{m} = \frac{\sqrt{2}F_0}{m} = \sqrt{2}a_0$$
$$= \sqrt{2}\left(3\,\text{m/s}^2\right)$$
$$= \boxed{4.24\,\text{m/s}^2 \text{ @ } 45.0° \text{ from each force.}}$$

(*b*) Calculate the acceleration of the object from Newton's 2nd law of motion:

$$\vec{a} = \vec{F}_{\text{net}}/m$$

Express the net force acting on the object:

$$\vec{F}_{\text{net}} = F_x\hat{i} + F_y\hat{j}$$
$$= \left(-F_0 \sin 45°\right)\hat{i} + \left(2F_0 + F_0 \cos 45°\right)\hat{j}$$

Find the magnitude and direction of this net force:

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(-F_0 \sin 45°\right)^2 + \left(2F_0 + F_0 \cos 45°\right)^2} = 2.80F_0$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{2F_0 + F_0 \cos 45°}{-F_0 \sin 45°}\right) = -75.4°$$
$$= 14.6° \text{ from } 2\vec{F}_0$$

Use this result to calculate the magnitude and direction of the acceleration:

$$a = \frac{F_{\text{net}}}{m} = 2.80\frac{F_0}{m} = 2.80a_0$$
$$= 2.80\left(3\,\text{m/s}^2\right)$$
$$= \boxed{8.40\,\text{m/s}^2 \text{ @ } 14.6° \text{ from } 2\vec{F}_0}$$

32 •

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it according to $\vec{a} = \vec{F}_{\text{net}}/m$.

Apply $\vec{a} = \vec{F}_{\text{net}}/m$ to the object to obtain:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{(6\,\text{N})\hat{i} - (3\,\text{N})\hat{j}}{1.5\,\text{kg}}$$
$$= \boxed{\left(4.00\,\text{m/s}^2\right)\hat{i} - \left(2.00\,\text{m/s}^2\right)\hat{j}}$$

Find the magnitude of $\vec{a}$:

$$a = \sqrt{a_x^2 + a_y^2}$$
$$= \sqrt{\left(4.00\,\text{m/s}^2\right)^2 + \left(2.00\,\text{m/s}^2\right)^2}$$
$$= \boxed{4.47\,\text{m/s}^2}$$

33 •

Picture the Problem The mass of the particle is related to its acceleration and the *net* force acting on it by Newton’s 2nd law of motion. Because the force is constant, we can use constant-acceleration formulas to calculate the acceleration. Choose a coordinate system in which the positive *x* direction is the direction of motion of the particle.

The mass is related to the net force and the acceleration by Newton’s 2nd law:

$$m = \frac{\sum \vec{F}}{\vec{a}} = \frac{F_x}{a_x}$$

Because the force is constant, the acceleration is constant. Use a constant-acceleration equation to find the acceleration:

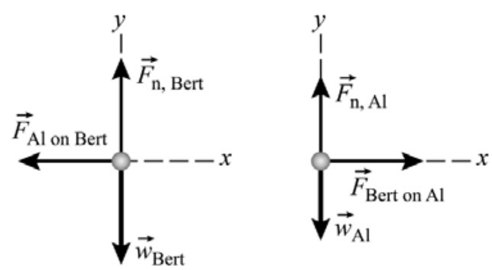
$$\Delta x = v_{0x}t + \tfrac{1}{2}a_x(\Delta t)^2\text{, where } v_{0x} = 0,$$
so
$$a_x = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute this result into the first equation and solve for and evaluate the mass *m* of the particle:

$$m = \frac{F_x}{a_x} = \frac{F_x(\Delta t)^2}{2\Delta x} = \frac{(12\,\text{N})(6\,\text{s})^2}{2(18\,\text{m})}$$
$$= \boxed{12.0\,\text{kg}}$$

***34** •

Picture the Problem The speed of either Al or Bert can be obtained from their accelerations; in turn, they can be obtained from Newtons 2nd law applied to each person. The free-body diagrams to the right show the forces acting on Al and Bert. The forces that Al and Bert exert on each other are action-and-reaction forces.



(*a*) Apply $\sum F_x = ma_x$ to Bert and solve for his acceleration:

$$-F_{\text{Al on Bert}} = m_{\text{Bert}}a_{\text{Bert}}$$
$$a_{\text{Bert}} = \frac{-F_{\text{Al on Bert}}}{m_{\text{Bert}}} = \frac{-20\,\text{N}}{100\,\text{kg}}$$
$$= -0.200\,\text{m/s}^2$$

Using a constant-acceleration equation, relate Bert’s speed to his initial speed, speed after 1.5 s, and acceleration and solve for his speed at the end of 1.5 s:

$$v = v_0 + a\Delta t$$
$$= 0 + (-0.200\,\text{m/s}^2)(1.5\,\text{s})$$
$$= \boxed{-0.300\,\text{m/s}}$$

(*b*) From Newton's 3rd law, an equal but oppositely directed force acts on Al while he pushes Bert. Because the ice is frictionless, Al speeds off in the opposite direction. Apply Newton's 2nd law to the forces acting on Al and solve for his acceleration:

$$\sum F_{x,\mathrm{Al}} = F_{\mathrm{Bert\,on\,Al}} = m_{\mathrm{Al}} a_{\mathrm{Al}}$$

and

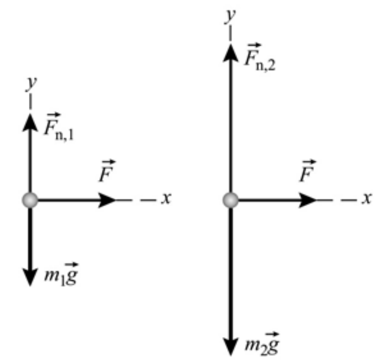
$$a_{\mathrm{Al}} = \frac{F_{\mathrm{Bert\,on\,Al}}}{m_{\mathrm{Al}}} = \frac{20\,\mathrm{N}}{80\,\mathrm{kg}} = 0.250\,\mathrm{m/s^2}$$

Using a constant-acceleration equation, relate Al's speed to his initial speed, speed after 1.5 s, and acceleration; solve for his speed at the end of 1.5 s:

$$v = v_0 + a\Delta t = 0 + (0.250\,\mathrm{m/s^2})(1.5\,\mathrm{s}) = \boxed{0.375\,\mathrm{m/s}}$$

35 •

Picture the Problem The free-body diagrams show the forces acting on the two blocks. We can apply Newton's second law to the forces acting on the blocks and eliminate F to obtain a relationship between the masses. Additional applications of Newton's 2nd law to the sum and difference of the masses will lead us to values for the accelerations of these combinations of mass.



(*a*) Apply $\sum F_x = ma_x$ to the two blocks:

$$\sum F_{x,1} = F = m_1 a_1$$

and

$$\sum F_{x,2} = F = m_2 a_2$$

Eliminate F between the two equations and solve for m_2:

$$m_2 = \frac{a_1}{a_2} m_1 = \frac{12\,\mathrm{m/s^2}}{3\,\mathrm{m/s^2}} m_1 = 4m_1$$

Express and evaluate the acceleration of an object whose mass is $m_2 - m_1$ when the net force acting on it is F:

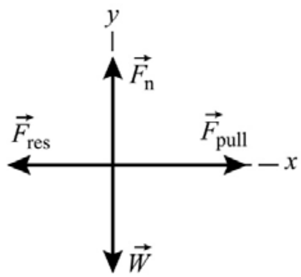
$$a = \frac{F}{m_2 - m_1} = \frac{F}{4m_1 - m_1} = \frac{F}{3m_1} = \tfrac{1}{3} a_1 = \tfrac{1}{3}\left(12\,\mathrm{m/s^2}\right) = \boxed{4.00\,\mathrm{m/s^2}}$$

(*b*) Express and evaluate the acceleration of an object whose mass is $m_2 + m_1$ when the net force acting on it is F:

$$a = \frac{F}{m_2 + m_1} = \frac{F}{4m_1 + m_1} = \frac{F}{5m_1} = \tfrac{1}{5} a_1 = \tfrac{1}{5}\left(12\,\mathrm{m/s^2}\right) = \boxed{2.40\,\mathrm{m/s^2}}$$

36 •

Picture the Problem Because the velocity is constant, the net force acting on the log must be zero. Choose a coordinate system in which the positive x direction is the direction of motion of the log. The free-body diagram shows the forces acting on the log when it is accelerating in the positive x direction.



(*a*) Apply $\sum F_x = ma_x$ to the log when it is moving at constant speed:

$$F_{pull} - F_{res} = ma_x = 0$$

Solve for and evaluate F_{res}:

$$F_{res} = F_{pull} = \boxed{250\,\text{N}}$$

(*b*) Apply $\sum F_x = ma_x$ to the log when it is accelerating to the right:

$$F_{pull} - F_{res} = ma_x$$

Solve for and evaluate F_{pull}:

$$\begin{aligned} F_{pull} &= F_{res} + ma_x \\ &= 250\text{ N} + (75\text{ kg})\,(2\text{ m/s}^2) \\ &= \boxed{400\,\text{N}} \end{aligned}$$

37 •

Picture the Problem The acceleration can be found from Newton's 2nd law. Because both forces are constant, the net force and the acceleration are constant; hence, we can use the constant-acceleration equations to answer questions concerning the motion of the object at various times.

(*a*) Apply Newton's 2nd law to the object to obtain:

$$\begin{aligned} \vec{a} &= \frac{\vec{F}_{net}}{m} = \frac{\vec{F}_1 + \vec{F}_2}{m} \\ &= \frac{(6\,\text{N})\hat{i} + (-14\,\text{N})\hat{j}}{4\,\text{kg}} \\ &= \boxed{(1.50\,\text{m/s}^2)\hat{i} + (-3.50\,\text{m/s}^2)\hat{j}} \end{aligned}$$

(*b*) Using a constant-acceleration equation, express the velocity of the object as a function of time and solve for its velocity when $t = 3$ s:

$$\begin{aligned} \vec{v} &= \vec{v}_0 + \vec{a}t \\ &= 0 + \left[(1.50\,\text{m/s}^2)\hat{i} + (-3.50\,\text{m/s}^2)\hat{j}\right](3\,\text{s}) \\ &= \boxed{(4.50\,\text{m/s})\hat{i} + (-10.5\,\text{m/s})\hat{j}} \end{aligned}$$

(*c*) Express the position of the object in terms of its average velocity and evaluate this expression at $t = 3$ s:

$$\begin{aligned} \vec{r} &= \vec{v}_{av}t \\ &= \tfrac{1}{2}\vec{v}t \\ &= \boxed{(6.75\,\text{m})\hat{i} + (-15.8\,\text{m})\hat{j}} \end{aligned}$$

Mass and Weight

***38 •**

Picture the Problem The mass of the astronaut is independent of gravitational fields and will be the same on the moon or, for that matter, out in deep space.

Express the mass of the astronaut in terms of his weight on earth and the gravitational field at the surface of the earth:

$$m=\frac{w_{\text{earth}}}{g_{\text{earth}}}=\frac{600\ \text{N}}{9.81\ \text{N/kg}}=61.2\ \text{kg}$$

and $\boxed{(c)\ \text{is correct.}}$

39 •

Picture the Problem The weight of an object is related to its mass and the gravitational field through $w = mg$.

(*a*) The weight of the girl is:

$$w=mg=(54\ \text{kg})(9.81\ \text{N/kg})=\boxed{530\ \text{N}}$$

(*b*) Convert newtons to pounds:

$$w=\frac{530\ \text{N}}{4.45\ \text{N/lb}}=\boxed{119\ \text{lb}}$$

40 •

Picture the Problem The mass of an object is related to its weight and the gravitational field.

Find the weight of the man in newtons:

$$165\ \text{lb}=(165\ \text{lb})(4.45\ \text{N/lb})=734\ \text{N}$$

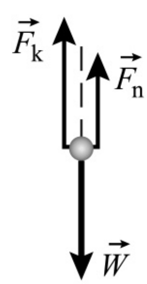
Calculate the mass of the man from his weight and the gravitational field:

$$m=\frac{w}{g}=\frac{734\ \text{N}}{9.81\ \text{N/kg}}=\boxed{74.8\ \text{kg}}$$

Contact Forces

***41 •**

Picture the Problem Draw a free-body diagram showing the forces acting on the block. $\vec{F}_k$ is the force exerted by the spring, $\vec{W}=m\vec{g}$ is the weight of the block, and $\vec{F}_\text{n}$ is the normal force exerted by the horizontal surface. Because the block is resting on a surface, $F_\text{k}+F_\text{n}=W$.



(*a*) Calculate the force exerted by the spring on the block:

$$F_x=kx=(600\ \text{N/m})(0.1\ \text{m})=\boxed{60.0\ \text{N}}$$

(*b*) Choosing the upward direction to be positive, sum the forces acting on the block and solve for F_n:

$$\sum \vec{F} = 0 \Rightarrow F_k + F_n - W = 0$$

and

$$F_n = W - F_k$$

Substitute numerical values and evaluate F_n:

$$F_n = (12\ \text{kg})(9.81\ \text{N/kg}) - 60\ \text{N}$$
$$= \boxed{57.7\ \text{N}}$$

42 •

Picture the Problem Let the positive *x* direction be the direction in which the spring is stretched. We can use Newton's 2nd law and the expression for the force exerted by a stretched (or compressed) spring to express the acceleration of the box in terms of its mass *m*, the stiffness constant of the spring *k*, and the distance the spring is stretched *x*.

Apply Newton's 2nd law to the box to obtain:

$$a = \frac{\sum F}{m}$$

Express the force exerted on the box by the spring:

$$F = -kx$$

Substitute to obtain:

$$a = \frac{-kx}{m}$$

Substitute numerical values and evaluate *a*:

$$a = -\frac{(800\ \text{N/m})(0.04\ \text{m})}{6\ \text{kg}}$$
$$= \boxed{-5.33\ \text{m/s}^2}$$

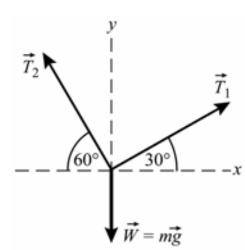
where the minus sign tells us that the box's acceleration is toward its equilibrium position.

Free-Body Diagrams: Static Equilibrium

43 •

Picture the Problem Because the traffic light is not accelerating, the *net* force acting on it must be zero; i.e., $\vec{T}_1 + \vec{T}_2 + m\vec{g} = 0$.

Construct a free-body diagram showing the forces acting on the knot and choose the coordinate system shown:



Apply $\sum F_x = ma_x$ to the knot: $T_1\cos30° - T_2\cos60° = ma_x = 0$

Solve for T_2 in terms of T_1:

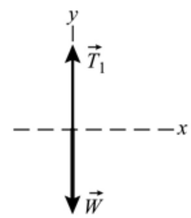
$$T_2 = \frac{\cos 30°}{\cos 60°}T_1 = 1.73T_1$$

$\therefore$ $\boxed{T_2 \text{ is greater than } T_1}$

44 •

Picture the Problem Draw a free-body diagram showing the forces acting on the lamp and apply $\sum F_y = 0$.

From the FBD, it is clear that T_1 supports the full weight mg = 418 N.



Apply $\sum F_y = 0$ to the lamp to obtain: $T_1 - w = 0$

Solve for T_1: $T_1 = w = mg$

Substitute numerical values and evaluate T_1:

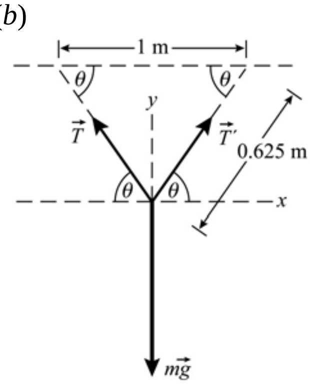
$$T_1 = (42.6\,\text{kg})(9.81\,\text{m/s}^2) = 418\,\text{N}$$

and $\boxed{(b) \text{ is correct.}}$

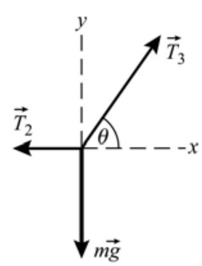
***45** ••

Picture the Problem The free-body diagrams for parts (*a*), (*b*), and (*c*) are shown below. In both cases, the block is in equilibrium under the influence of the forces and we can use Newton's 2nd law of motion and geometry and trigonometry to obtain relationships between θ and the tensions.

(*a*) and (*b*)



(*c*)



(*a*) Referring to the FBD for part (*a*), use trigonometry to determine θ:

$$\theta = \cos^{-1}\frac{0.5\,\text{m}}{0.625\,\text{m}} = \boxed{36.9°}$$

(*b*) Noting that $T = T'$, apply $\sum F_y = ma_y$ to the 0.500-kg block and solve for the tension T:

$$2T\sin\theta - mg = 0 \text{ since } a = 0$$

and

$$T = \frac{mg}{2\sin\theta}$$

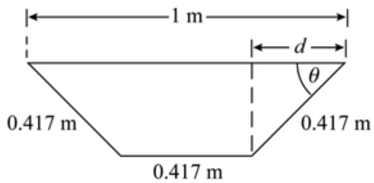
Substitute numerical values and evaluate T:

$$T = \frac{(0.5\,\text{kg})(9.81\,\text{m/s}^2)}{2\sin 36.9°} = \boxed{4.08\,\text{N}}$$

(*c*) The length of each segment is:

$$\frac{1.25\,\text{m}}{3} = 0.417\,\text{m}$$

Find the distance d:



$$d = \frac{1\,\text{m} - 0.417\,\text{m}}{2} = 0.2915\,\text{m}$$

Express θ in terms of d and solve for its value:

$$\theta = \cos^{-1}\left(\frac{d}{0.417\,\text{m}}\right) = \cos^{-1}\left(\frac{0.2915\,\text{m}}{0.417\,\text{m}}\right) = 45.7°$$

Apply $\sum F_y = ma_y$ to the 0.250-kg block and solve for the tension T_3:

$$T_3\sin\theta - mg = 0 \text{ since } a = 0.$$

and

$$T_3 = \frac{mg}{\sin\theta}$$

Substitute numerical values and evaluate T_3:

$$T_3 = \frac{(0.25\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 45.7°} = \boxed{3.43\,\text{N}}$$

Apply $\sum F_x = ma_x$ to the 0.250-kg block and solve for the tension T_2:

$$T_3\cos\theta - T_2 = 0 \text{ since } a = 0.$$

and

$$T_2 = T_3\cos\theta$$

Substitute numerical values and evaluate T_2:

$$T_2 = (3.43\,\text{N})\cos 45.7° = \boxed{2.40\,\text{N}}$$

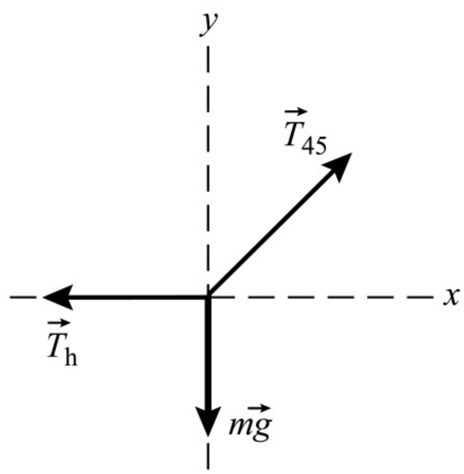
By symmetry:

$$T_1 = T_3 = \boxed{3.43\,\text{N}}$$

46 •

Picture the Problem The suspended body is in equilibrium under the influence of the forces $\vec{T}_h$, $\vec{T}_{45}$, and $m\vec{g}$; i.e., $\vec{T}_h + \vec{T}_{45} + m\vec{g} = 0$

Draw the free-body diagram of the forces acting on the knot just above the 100-N body. Choose a coordinate system with the positive x direction to the right and the positive y direction upward. Apply the conditions for translational equilibrium to determine the tension in the horizontal cord.



If the system is to remain in static equilibrium, the vertical component of T_{45} must be exactly balanced by, and therefore equal to, the tension in the string suspending the 100-N body:

$$T_v = T_{45}\sin 45° = mg$$

Express the horizontal component of T_{45}:

$$T_h = T_{45}\cos 45°$$

Because $T_{45}\sin 45° = T_{45}\cos 45°$:

$$T_h = mg = \boxed{100\ \text{N}}$$

47 •

Picture the Problem The acceleration of *any* object is directly proportional to the *net* force acting on it. Choose a coordinate system in which the positive x direction is the same as that of $\vec{F}_1$ and the positive y direction is to the right. Add the two forces to determine the net force and then use Newton's 2nd law to find the acceleration of the object. If $\vec{F}_3$ brings the system into equilibrium, it must be true that $\vec{F}_3 + \vec{F}_1 + \vec{F}_2 = 0$.

(*a*) Find the components of $\vec{F}_1$ and $\vec{F}_2$:

$$\vec{F}_1 = (20\ \text{N})\hat{\boldsymbol{i}}$$

$$\vec{F}_2 = \{(-30\ \text{N})\sin 30°\}\hat{\boldsymbol{i}} + \{(30\ \text{N})\cos 30°\}\hat{\boldsymbol{j}} = (-15\ \text{N})\hat{\boldsymbol{i}} + (26\ \text{N})\hat{\boldsymbol{j}}$$

Add $\vec{F}_1$ and $\vec{F}_2$ to find $\vec{F}_{tot}$:

$$\vec{F}_{tot} = (5\ \text{N})\hat{\boldsymbol{i}} + (26\ \text{N})\hat{\boldsymbol{j}}$$

Apply $\sum \vec{F} = m\vec{a}$ to find the acceleration of the object:

$$\vec{a} = \frac{\vec{F}_{tot}}{m}$$
$$= \boxed{(0.500\,\text{m/s}^2)\hat{i} + (2.60\,\text{m/s}^2)\hat{j}}$$

(*b*) Because the object is in equilibrium under the influence of the three forces, it must be true that:

$$\vec{F}_3 + \vec{F}_1 + \vec{F}_2 = 0$$
and
$$\vec{F}_3 = -\left(\vec{F}_1 + \vec{F}_2\right)$$
$$= \boxed{(-5.00\,\text{N})\hat{i} + (-26.0\,\text{N})\hat{j}}$$

***48** •

Picture the Problem The acceleration of the object equals the net force, $\vec{T} - m\vec{g}$, divided by the mass. Choose a coordinate system in which upward is the positive y direction. Apply Newton's 2nd law to the forces acting on this body to find the acceleration of the object as a function of T.

(*a*) Apply $\sum F_y = ma_y$ to the object:

$$T - w = T - mg = ma_y$$

Solve this equation for a as a function of T:

$$a_y = \frac{T}{m} - g$$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{5\,\text{N}}{5\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{-8.81\,\text{m/s}^2}$$

(*b*) Proceed as in (*a*) with T = 10 N:

$$a = \boxed{-7.81\,\text{m/s}^2}$$

(*c*) Proceed as in (*a*) with T = 100 N:

$$a = \boxed{10.2\,\text{m/s}^2}$$

49 ••

Picture the Problem The picture is in equilibrium under the influence of the three forces shown in the figure. Due to the symmetry of the support system, the vectors $\vec{T}$ and $\vec{T}'$ have the same magnitude T. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Apply the condition for translational equilibrium to obtain an expression for T as a function of θ and w.

(*a*) Referring to Figure 4-37, apply the condition for translational equilibrium in the vertical direction and solve for T:

$$\sum F_y = 2T\sin\theta - w = 0$$
and
$$T = \boxed{\frac{w}{2\sin\theta}}$$

$T_{\min}$ occurs when $\sin\theta$ is a maximum:

$$\theta = \sin^{-1} 1 = \boxed{90°}$$

$T_{\max}$ occurs when $\sin\theta$ is a minimum. Because the function is undefined when $\sin\theta$ = 0, we can conclude that:

$$\boxed{T \to T_{\max} \text{ as } \theta \to 0°}$$

(*b*) Substitute numerical values in the result in (*a*) and evaluate *T*:

$$T = \frac{(2\,\text{kg})(9.81\,\text{m/s}^2)}{2\sin 30°} = \boxed{19.6\,\text{N}}$$

Remarks: θ = 90° requires wires of infinite length; therefore it is not possible. As θ gets small, *T* gets large without limit.

***50** •••

Picture the Problem In part (*a*) we can apply Newton's 2nd law to obtain the given expression for *F*. In (*b*) we can use a symmetry argument to find an expression for tan θ_0. In (*c*) we can use our results obtained in (*a*) and (*b*) to express x_i and y_i.

(*a*) Apply $\sum F_y = 0$ to the balloon:

$$F + T_\text{i} \sin\theta_\text{i} - T_{\text{i}-1} \sin\theta_{\text{i}-1} = 0$$

Solve for *F* to obtain:

$$F = \boxed{T_{\text{i}-1} \sin\theta_{\text{i}-1} - T_\text{i} \sin\theta_\text{i}}$$

(*b*) By symmetry, each support must balance half of the force acting on the entire arch. Therefore, the vertical component of the force on the support must be *NF*/2. The horizontal component of the tension must be T_H. Express tanθ_0 in terms of *NF*/2 and T_H:

$$\tan\theta_0 = \frac{NF/2}{T_\text{H}} = \frac{NF}{2T_\text{H}}$$

By symmetry, $\theta_{\text{N}+1} = -\theta_0$. Therefore, because the tangent function is odd:

$$\tan\theta_0 = \boxed{-\tan\theta_{\text{N}+1} = \frac{NF}{2T_\text{H}}}$$

(*c*) Using $T_\text{H} = T_\text{i}\cos\theta_\text{i} = T_{\text{i}-1}\cos\theta_{\text{i}-1}$, divide both sides of our result in (*a*) by T_H and simplify to obtain:

$$\frac{F}{T_\text{H}} = \frac{T_{\text{i}-1}\sin\theta_{\text{i}-1}}{T_{\text{i}-1}\cos\theta_{\text{i}-1}} - \frac{T_\text{i}\sin\theta_\text{i}}{T_\text{i}\cos\theta_\text{i}}$$

$$= \tan\theta_{\text{i}-1} - \tan\theta_\text{i}$$

Using this result, express tan θ_1:

$$\tan\theta_1 = \tan\theta_0 - \frac{F}{T_\text{H}}$$

Substitute for tan θ_0 from (*a*):

$$\tan\theta_1 = \frac{NF}{2T_\text{H}} - \frac{F}{T_\text{H}} = (N-2)\frac{F}{2T_\text{H}}$$

Generalize this result to obtain:

$$\boxed{\tan\theta_\mathrm{i} = (N-2i)\frac{F}{2T_\mathrm{H}}}$$

Express the length of rope between two balloons:

$$\ell_{\text{between balloons}} = \frac{L}{N+1}$$

Express the horizontal coordinate of the point on the rope where the ith balloon is attached, x_i, in terms of x_{i-1} and the length of rope between two balloons:

$$x_\mathrm{i} = x_{\mathrm{i}-1} + \frac{L}{N+1}\cos\theta_{\mathrm{i}-1}$$

Sum over all the coordinates to obtain:

$$\boxed{x_\mathrm{i} = \frac{L}{N+1}\sum_{j=0}^{\mathrm{i}-1}\cos\theta_j}$$

Proceed similarly for the vertical coordinates to obtain:

$$\boxed{y_\mathrm{i} = \frac{L}{N+1}\sum_{j=0}^{\mathrm{i}-1}\sin\theta_j}$$

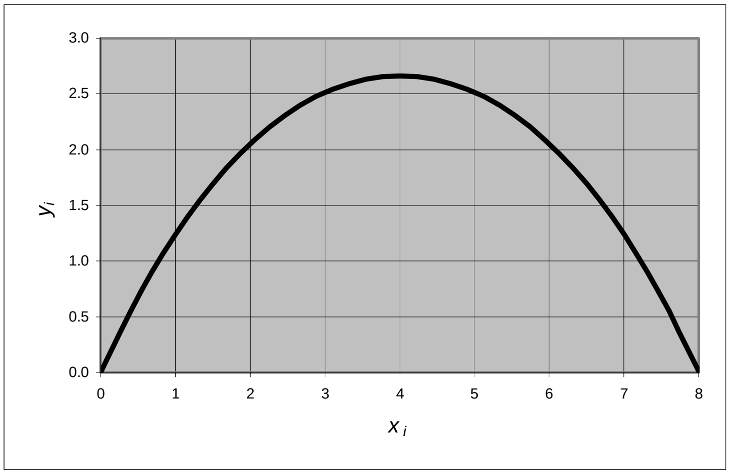
(*d*) A spreadsheet program is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
C9	($B$2-2*B9)/(2*$B$4)	$(N-2\mathrm{i})\frac{F}{2T_\mathrm{H}}$
D9	SIN(ATAN(C9))	$\sin(\tan^{-1}\theta_\mathrm{i})$
E9	COS(ATAN(C9))	$\cos(\tan^{-1}\theta_\mathrm{i})$
F10	F9+$B$1/($B$2+1)*E9	$x_{\mathrm{i}-1} + \frac{L}{N+1}\cos\theta_{\mathrm{i}-1}$
G10	G9+$B$1/($B$2+1)*D9	$y_{\mathrm{i}-1} + \frac{L}{N+1}\cos\theta_{\mathrm{i}-1}$

	A	B	C	D	E	F	G
1	L =	10	m				
2	N =	10					
3	F =	1	N				
4	TH=	3.72	N				
5							
6							
7							
8		I	tan(thetai)	sin(thetai)	cos(thetai)	xi	yi
9		0	1.344	0.802	0.597	0.000	0.000
10		1	1.075	0.732	0.681	0.543	0.729
11		2	0.806	0.628	0.778	1.162	1.395
12		3	0.538	0.474	0.881	1.869	1.966
13		4	0.269	0.260	0.966	2.670	2.396

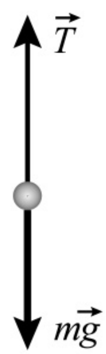
14		5	0.000	0.000	1.000	3.548	2.632
15		6	–0.269	–0.260	0.966	4.457	2.632
16		7	–0.538	–0.474	0.881	5.335	2.396
17		8	–0.806	–0.628	0.778	6.136	1.966
18		9	–1.075	–0.732	0.681	6.843	1.395
19		10	–1.344	–0.802	0.597	7.462	0.729
20		11				8.005	0.000

(*e*) A horizontal component of tension 3.72 N gives a spacing of 8 m. At this spacing, the arch is 2.63 m high, tall enough for someone to walk through.



51 ••

Picture the Problem We know, because the speed of the load is changing in parts (*a*) and (*c*), that it is accelerating. We also know that, if the load is accelerating in a particular direction, there must be a *net* force in that direction. A free-body diagram for part (*a*) is shown to the right. We can apply Newton's 2nd law of motion to each part of the problem to relate the tension in the cable to the acceleration of the load. Choose the upward direction to be the positive *y* direction.



(*a*) Apply $\sum F_y = ma_y$ to the load and solve for *T*:

$T - mg = ma$

and

$$T = ma_y + mg = m(a_y + g) \qquad (1)$$

Substitute numerical values and evaluate *T*:

$$T = (1000\,\text{kg})(2\,\text{m/s}^2 + 9.81\,\text{m/s}^2) = \boxed{11.8\,\text{kN}}$$

(*b*) Because the crane is lifting the

$$T = mg = \boxed{9.81\,\text{kN}}$$

load at constant speed, $a = 0$:

(*c*) Because the acceleration of the load is downward, a is negative. Apply $\sum F_y = ma_y$ to the load:

$$T - mg = ma_y$$

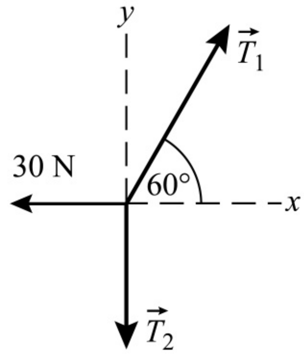
Substitute numerical values in equation (1) and evaluate T:

$$T = (1000\text{ kg})(9.81\text{ m/s}^2 - 2\text{ m/s}^2) = \boxed{7.81\text{ kN}}$$

52 ••

Picture the Problem Draw a free-body diagram for each of the depicted situations and use the conditions for translational equilibrium to find the unknown tensions.

(*a*)



$$\Sigma F_x = T_1\cos 60° - 30\text{ N} = 0$$

and

$$T_1 = (30\text{ N})/\cos 60° = \boxed{60.0\text{ N}}$$

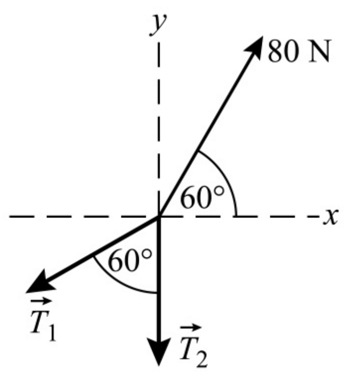
$$\Sigma F_y = T_1\sin 60° - T_2 = 0$$

and

$$T_2 = T_1\sin 60° = \boxed{52.0\text{ N}}$$

$$\therefore\ m = T_2/g = \boxed{5.30\text{ kg}}$$

(*b*)



$$\Sigma F_x = (80\text{ N})\cos 60° - T_1\sin 60° = 0$$

and

$$T_1 = (80\text{ N})\cos 60°/\sin 60° = \boxed{46.2\text{ N}}$$

$$\Sigma F_y = (80\text{ N})\sin 60° - T_2 - T_1\cos 60° = 0$$

$$T_2 = (80\text{ N})\sin 60° - (46.2\text{ N})\cos 60° = \boxed{46.2\text{ N}}$$

$$m = T_2/g = \boxed{4.71\text{ kg}}$$

(*c*)

$$\Sigma F_x = -T_1\cos 60° + T_3\cos 60° = 0$$

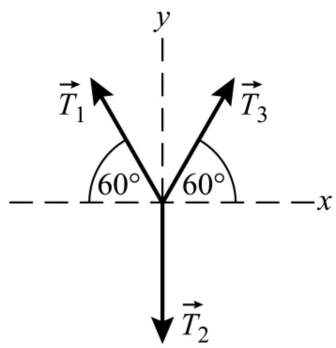
and

$$T_1 = T_3$$

$$\Sigma F_y = 2T_1\sin 60° - mg = 0$$

and

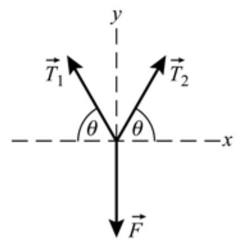
$$T_1 = T_3 = (58.9\text{ N})/(2\sin 60°) = \boxed{34.0\text{ N}}$$



$\therefore\ m = T_1/g = \boxed{3.46\ \text{kg}}$

53 ••

Picture the Problem Construct the free-body diagram for that point in the rope at which you exert the force $\vec{F}$ and choose the coordinate system shown on the FBD. We can apply Newton's 2nd law to the rope to relate the tension to F.



(*a*) Noting that $T_1 = T_2 = T$, apply $\sum F_y = ma_y$ to the car:

$2T\sin\theta - F = ma_y = 0$ because the car's acceleration is zero.

Solve for and evaluate T:

$$T = \frac{F}{2\sin\theta} = \frac{400\ \text{N}}{2\sin 3°} = \boxed{3.82\ \text{kN}}$$

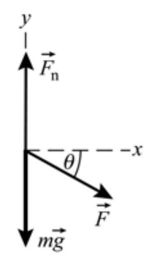
(*b*) Proceed as in part (*a*):

$$T = \frac{600\ \text{N}}{2\sin 4°} = \boxed{4.30\ \text{kN}}$$

Free-Body Diagrams: Inclined Planes and the Normal Force

***54** •

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it across the frictionless floor. We can apply Newton's 2nd law of motion to the box to find its acceleration.



Apply $\sum F_x = ma_x$ to the box:

$$F\cos\theta = ma_x$$

Solve for a_x:

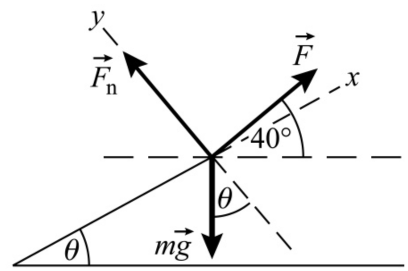
$$a_x = \frac{F\cos\theta}{m}$$

Substitute numerical values and evaluate a_x:

$$a_x = \frac{(250\ \text{N})\cos 35°}{20\ \text{kg}} = \boxed{10.2\ \text{m/s}^2}$$

55 •

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it up the frictionless incline. We can apply Newton's 2nd law of motion to the box to determine the smallest force that will move it up the incline at constant speed.



Apply $\sum F_x = ma_x$ to the box as it moves up the incline with constant speed:

$$F_{min}\cos(40° - \theta) - mg\sin\theta = 0$$

Solve for F_{min}:

$$F_{min} = \frac{mg\sin\theta}{\cos(40° - \theta)}$$

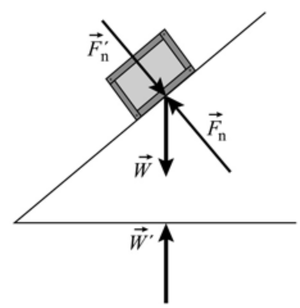
Substitute numerical values and evaluate F_{min}:

$$F_{min} = \frac{(20\,kg)(9.81\,m/s^2)}{\cos 25°} = \boxed{56.0\,N}$$

56 •

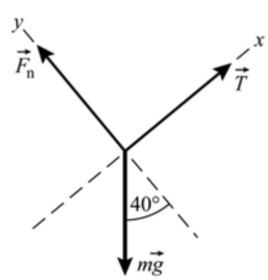
Picture the Problem Forces always occur in equal and opposite pairs. If object *A* exerts a force, $\vec{F}_{B,A}$ on object *B*, an equal but opposite force, $\vec{F}_{A,B} = -\vec{F}_{B,A}$ is exerted by object *B* on object *A*.

The forces acting on the box are its weight, $\vec{W}$, and the normal reaction force of the inclined plane on the box, $\vec{F}_n$. The reaction forces are the forces the box exerts on the inclined plane and the gravitational force the box exerts on the earth. The reaction forces are indicated with primes.



57 •

Picture the Problem Because the block whose mass is *m* is in equilibrium, the sum of the forces $\vec{F}_n$, $\vec{T}$, and $m\vec{g}$ must be zero. Construct the free-body diagram for this object, use the coordinate system shown on the free-body diagram, and apply Newton's 2nd law of motion.



Apply $\sum F_x = ma_x$ to the block on the incline:

$T - mg\sin 40° = ma_x = 0$ because the system is in equilibrium.

Solve for m:

$$m = \frac{T}{g \sin 40°}$$

The tension must equal the weight of the 3.5-kg block because that block is also in equilibrium:

$T = (3.5 \text{ kg})g$

and

$$m = \frac{(3.5\,\text{kg})g}{g \sin 40°} = \frac{3.5\,\text{kg}}{\sin 40°}$$

Because this expression is not included in the list of solution candidates,

$\boxed{(d) \text{ is correct.}}$

Remarks: Because the object whose mass is *m* does not hang vertically, its mass must be greater than 3.5 kg.

***58** •

Picture the Problem The balance(s) indicate the tension in the string(s). Draw free-body diagrams for each of these systems and apply the condition(s) for equilibrium.

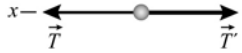
(*a*)

$$\sum F_y = T - mg = 0$$

and

$$T = mg = (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98.1\,\text{N}}$$

(*b*)

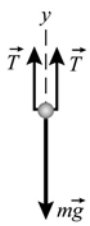


$$\sum F_x = T - T' = 0$$

or, because $T' = mg$,

$$T = T' = mg$$
$$= (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98.1\,\text{N}}$$

(*c*)



$$\sum F_y = 2T - mg = 0$$

and

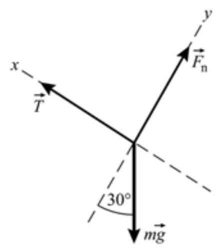
$$T = \tfrac{1}{2} mg$$
$$= \tfrac{1}{2}(10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{49.1\,\text{N}}$$

(*d*)

$$\sum F_x = T - mg \sin 30° = 0$$

and

$$T = mg \sin 30°$$
$$= (10\,\text{kg})(9.81\,\text{m/s}^2)\sin 30° = \boxed{49.1\,\text{N}}$$



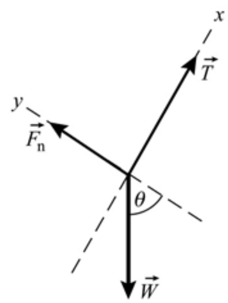
Remarks: Note that (*a*) and (*b*) give the same answers … a rather surprising result until one has learned to draw FBDs and apply the conditions for translational equilibrium.

59 ••

Picture the Problem Because the box is held in place (is in equilibrium) by the forces acting on it, we know that

$$\vec{T} + \vec{F}_n + \vec{W} = 0$$

Choose a coordinate system in which the positive *x* direction is in the direction of $\vec{T}$ and the positive *y* direction is in the direction of $\vec{F}_n$. Apply Newton's 2nd law to the block to obtain expressions for $\vec{T}$ and $\vec{F}_n$.



(*a*) Apply $\sum F_x = ma_x$ to the box: $T - mg\sin\theta = 0$

Solve for *T*: $T = mg\sin\theta$

Substitute numerical values and evaluate *T*: $T = (50\,\text{kg})(9.81\,\text{m/s}^2)\sin 60° = \boxed{425\,\text{N}}$

Apply $\sum F_y = ma_y$ to the box: $F_n - mg\cos\theta = 0$

Solve for F_n: $F_n = mg\cos\theta$

Substitute numerical values and evaluate F_n: $F_n = (50\,\text{kg})(9.81\,\text{m/s}^2)\cos 60° = \boxed{245\,\text{N}}$

(*b*) Using the result for the tension from part (*a*) to obtain:

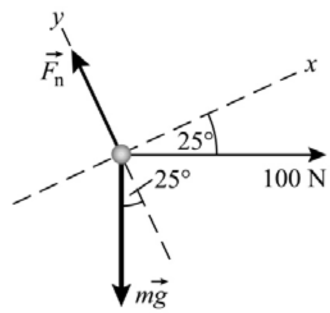
$T_{90°} = mg\sin 90° = \boxed{mg}$

and

$T_{0°} = mg\sin 0° = \boxed{0}$

60 ••

Picture the Problem Draw a free-body diagram for the box. Choose a coordinate system in which the positive x-axis is parallel to the inclined plane and the positive y-axis is in the direction of the normal force the incline exerts on the block. Apply Newton's 2nd law of motion to both the x and y directions.



(a) Apply $\sum F_y = ma_y$ to the block:

$$F_n - mg\cos 25° - (100\,\text{N})\sin 25° = 0$$

Solve for F_n:

$$F_n = mg\cos 25° + (100\,\text{N})\sin 25°$$

Substitute numerical values and evaluate F_n:

$$F_n = (12\,\text{kg})(9.81\,\text{m/s}^2)\cos 25° + (100\,\text{N})\sin 25° = \boxed{149\,\text{N}}$$

(b) Apply $\sum F_x = ma_x$ to the block:

$$(100\,\text{N})\cos 25° - mg\sin 25° = ma$$

Solve for a:

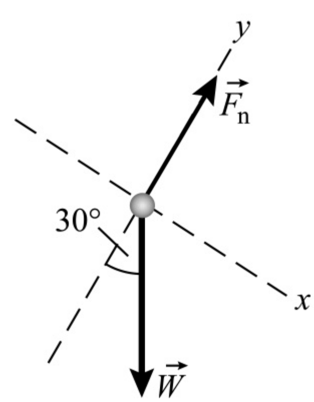
$$a = \frac{(100\,\text{N})\cos 25°}{m} - g\sin 25°$$

Substitute numerical values and evaluate a:

$$a = \frac{(100\,\text{N})\cos 25°}{12\,\text{kg}} - (9.81\,\text{m/s}^2)\sin 25° = \boxed{3.41\,\text{m/s}^2}$$

***61** ••

Picture the Problem The scale reading (the boy's apparent weight) is the force the scale exerts on the boy. Draw a free-body diagram for the boy, choosing a coordinate system in which the positive x-axis is parallel to and down the inclined plane and the positive y-axis is in the direction of the normal force the incline exerts on the boy. Apply Newton's 2nd law of motion in the y direction.



Apply $\sum F_y = ma_y$ to the boy to find F_n. Remember that there is no acceleration in the y direction:

$$F_n - W\cos 30° = 0$$

Substitute for W to obtain:

$$F_n - mg\cos 30° = 0$$

Solve for F_n:

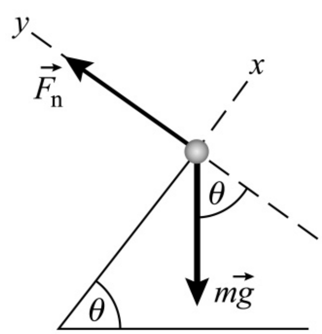
$$F_n = mg\cos 30°$$

Substitute numerical values and evaluate F_n:

$$F_n = (65\,\text{kg})(9.81\,\text{m/s}^2)\cos 30° = \boxed{552\,\text{N}}$$

62 ••

Picture the Problem The free-body diagram for the block sliding up the incline is shown to the right. Applying Newton's 2nd law to the forces acting in the x direction will lead us to an expression for a_x. Using this expression in a constant-acceleration equation will allow us to express h as a function of v_0 and g.



The height h is related to the distance Δx traveled up the incline:

$$h = \Delta x \sin\theta$$

Using a constant-acceleration equation, relate the final speed of the block to its initial speed, acceleration, and distance traveled:

$$v^2 = v_0^2 + 2a_x\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a_x\Delta x$$

Solve for Δx to obtain:

$$\Delta x = \frac{-v_0^2}{2a_x}$$

Apply $\sum F_x = ma_x$ to the block and solve for its acceleration:

$$-mg\sin\theta = ma_x$$

and

$$a_x = -g\sin\theta$$

Substitute these results in the equation for h and simplify:

$$h = \Delta x\sin\theta = \left(\frac{v_0^2}{2g\sin\theta}\right)\sin\theta = \boxed{\frac{v_0^2}{2g}}$$

which is independent of the ramp's angle θ.

Free-Body Diagrams: Elevators

63 •

Picture the Problem Because the elevator is descending at constant speed, the object is in equilibrium and $\vec{T} + m\vec{g} = 0$. Draw a free-body diagram of the object and let the upward direction be the positive y direction. Apply Newton's 2nd law with $a = 0$.

Because the downward speed is constant, the acceleration is zero. Apply $\sum F_y = ma_y$ and solve for T:

$T - mg = 0 \Rightarrow T = mg$
and
$\boxed{(a)\text{ is correct.}}$

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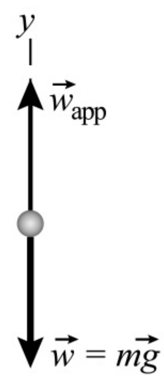
Picture the Problem The sketch to the right shows a person standing on a scale in a descending elevator. To its right is a free-body diagram showing the forces acting on the person. The force exerted by the scale on the person, $\vec{w}_{app}$, is the person's apparent weight. Because the elevator is slowing down while descending, the acceleration is directed upward.



Apply $\sum F_y = ma_y$ to the person:

$w_{app} - mg = ma_y$

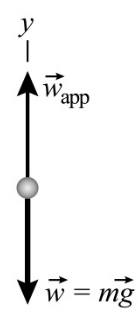
Solve for w_{app}:

$w_{app} = mg + ma_y > mg$

The apparent weight will be higher. Because an upward acceleration is required to "slow" a downward velocity, the normal force exerted on you by the scale (your *apparent weight*) must be greater than your weight.

***65** •

Picture the Problem The sketch to the right shows a person standing on a scale in the elevator immediately after the cable breaks. To its right is the free-body diagram showing the forces acting on the person. The force exerted by the scale on the person, $\vec{w}_{app}$, is the person's apparent weight.



From the free-body diagram we can see that $\vec{w}_{\text{app}} + m\vec{g} = m\vec{a}$ where $\vec{g}$ is the local gravitational field and $\vec{a}$ is the acceleration of the reference frame (elevator). When the elevator goes into free fall ($\vec{a} = \vec{g}$), our equation becomes $\vec{w}_{\text{app}} + m\vec{g} = m\vec{a} = m\vec{g}$. This tells us that $\vec{w}_{\text{app}} = 0$. $\boxed{(e)\text{ is correct.}}$

66 •

Picture the Problem The free-body diagram shows the forces acting on the 10-kg block as the elevator accelerates upward. Apply Newton's 2nd law of motion to the block to find the minimum acceleration of the elevator required to break the cord.

Apply $\sum F_y = ma_y$ to the block: $T - mg = ma_y$

Solve for a_y to determine the minimum breaking acceleration:

$$a_y = \frac{T - mg}{m} = \frac{T}{m} - g$$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{150\,\text{N}}{10\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{5.19\,\text{m/s}^2}$$

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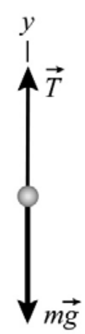
Picture the Problem The free-body diagram shows the forces acting on the 2-kg block as the elevator ascends at a constant velocity. Because the acceleration of the elevator is zero, the block is in equilibrium under the influence of $\vec{T}$ and $m\vec{g}$. Apply Newton's 2nd law of motion to the block to determine the scale reading.



(*a*) Apply $\sum F_y = ma_y$ to the block to obtain:

$$T - mg = ma_y \quad (1)$$

For motion with constant velocity, $a_y = 0$:

$$T - mg = 0 \text{ and } T = mg$$

Substitute numerical values and evaluate T:

$$T = (2\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{19.6\,\text{N}}$$

(*b*) As in part (*a*), for constant velocity, $a = 0$:

$$T - mg = ma_y$$

and

$$T = (2\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{19.6\,\text{N}}$$

(*c*) Solve equation (1) for *T* and simplify to obtain:

$$T = mg + ma_y = m(g + a_y) \qquad (2)$$

Because the elevator is ascending and its speed is increasing, we have $a_y = 3\ \text{m/s}^2$. Substitute numerical values and evaluate *T*:

$$T = (2\,\text{kg})(9.81\,\text{m/s}^2 + 3\,\text{m/s}^2) = \boxed{25.6\,\text{N}}$$

(*d*) For $0 < t < 5$ s: $a_y = 0$ and

$$T_{0\to 5\,\text{s}} = \boxed{19.6\ \text{N}}$$

Using its definition, calculate *a* for 5 s < t < 9 s:

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 10\,\text{m/s}}{4\,\text{s}} = -2.5\,\text{m/s}^2$$

Substitute in equation (2) and evaluate *T*:

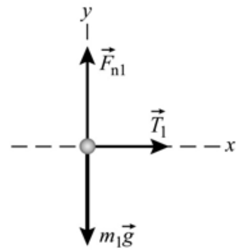
$$T_{5\,\text{s}\to 9\,\text{s}} = (2\,\text{kg})(9.81\,\text{m/s}^2 - 2.5\,\text{m/s}^2) = \boxed{14.6\,\text{N}}$$

Free-Body Diagrams: Ropes, Tension, and Newton's Third Law

68 •

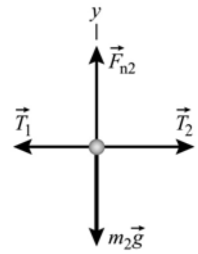
Picture the Problem Draw a free-body diagram for each object and apply Newton's 2nd law of motion. Solve the resulting simultaneous equations for the ratio of T_1 to T_2.

Draw the FBD for the box to the left and apply $\sum F_x = ma_x$:



$$T_1 = m_1 a_1$$

Draw the FBD for the box to the right and apply $\sum F_x = ma_x$:



$$T_2 - T_1 = m_2 a_2$$

The two boxes have the same acceleration:

$$a_1 = a_2$$

Divide the second equation by the first:

$$\frac{T_1}{T_2 - T_1} = \frac{m_1}{m_2}$$

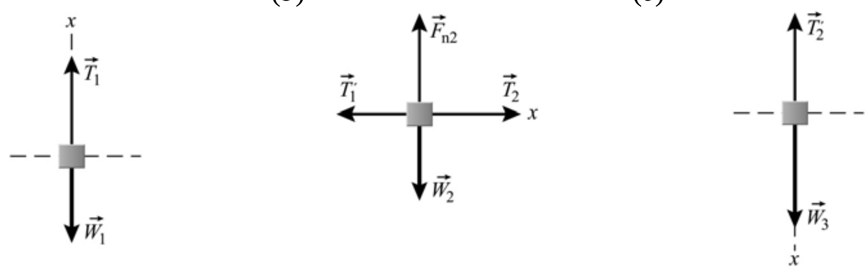
Solve for the ratio T_1/T_2 :

$$\frac{T_1}{T_2} = \frac{m_1}{m_1 + m_2} \text{ and } \boxed{(d) \text{ is correct.}}$$

69 ••

Picture the Problem Call the common acceleration of the boxes a. Assume that box 1 moves upward, box 2 to the right, and box 3 downward and take this direction to be the positive x direction. Draw free-body diagrams for each of the boxes, apply Newton's 2nd law of motion, and solve the resulting equations simultaneously.

(*a*) (*b*) (*c*)



(*a*) Apply $\sum F_x = ma_x$ to the box whose mass is m_1:

$$T_1 - w_1 = m_1 a$$

Apply $\sum F_x = ma_x$ to the box whose mass is m_2:

$$T_2 - T_1 = m_2 a$$

Noting that $T_2 = T_2'$, apply $\sum F_x = ma_x$ to the box whose mass is m_3:

$$w_3 - T_2 = m_3 a$$

Add the three equations to obtain:

$$w_3 - w_1 = (m_1 + m_2 + m_3)a$$

Solve for a:

$$a = \frac{(m_3 - m_1)g}{m_1 + m_2 + m_3}$$

Substitute numerical values and evaluate a:

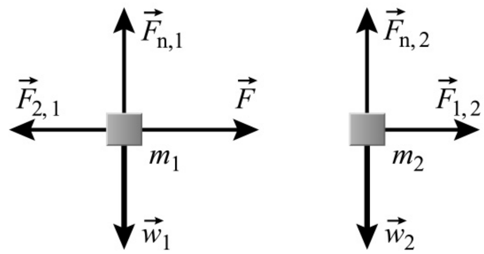
$$a = \frac{(2.5\,\text{kg} - 1.5\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\,\text{kg} + 3.5\,\text{kg} + 2.5\,\text{kg}} = \boxed{1.31\,\text{m/s}^2}$$

(*b*) Substitute for the acceleration in the equations obtained above to find the tensions:

$$T_1 = \boxed{16.7\,\text{N}} \text{ and } T_2 = \boxed{21.3\,\text{N}}$$

***70** ••

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Let $\vec{F}_{2,1}$ be the contact force exerted by m_2 on m_1 and $\vec{F}_{1,2}$ be the force exerted by m_1 on m_2. These forces are equal and opposite so $\vec{F}_{2,1} = -\vec{F}_{1,2}$. The free-body diagrams for the blocks are shown to the right. Apply Newton's 2nd law to each block separately and use the fact that their accelerations are equal.



(*a*) Apply $\sum F_x = ma_x$ to the first block:

$$F - F_{2,1} = m_1 a_1 = m_1 a$$

Apply $\sum F_x = ma_x$ to the second block:

$$F_{1,2} = m_2 a_2 = m_2 a \quad (1)$$

Add these equations to eliminate $F_{2,1}$ and $F_{1,2}$ and solve for $a = a_1 = a_2$:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

Substitute your value for a into equation (1) and solve for $F_{1,2}$:

$$F_{1,2} = \boxed{\frac{Fm_2}{m_1 + m_2}}$$

(*b*) Substitute numerical values in the equations derived in part (*a*) and evaluate a and $F_{1,2}$:

$$a = \frac{3.2\,\text{N}}{2\,\text{kg} + 6\,\text{kg}} = \boxed{0.400\,\text{m/s}^2}$$

and

$$F_{1,2} = \frac{(3.2\,\text{N})(6\,\text{kg})}{2\,\text{kg} + 6\,\text{kg}} = \boxed{2.40\,\text{N}}$$

Remarks: Note that our results for the acceleration are the same as if the force *F* had acted on a single object whose mass is equal to the sum of the masses of the two blocks. In fact, because the two blocks have the same acceleration, we can consider them to be a single system with mass $m_1 + m_2$.

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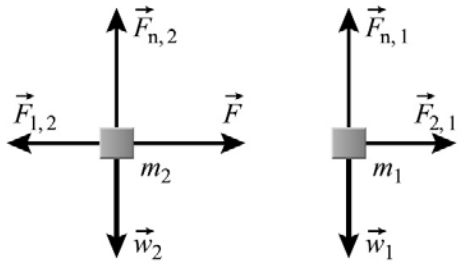
Picture the Problem Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Let $\vec{F}_{2,1}$ be the contact force exerted by m_2 on m_1 and $\vec{F}_{1,2}$ be the force exerted by m_1 on m_2. These forces are equal and opposite so $\vec{F}_{2,1} = -\vec{F}_{1,2}$. The free-body diagrams for the blocks are shown. We can apply Newton's 2nd law to each block separately and use the fact that their accelerations are equal.



(*a*) Apply $\sum F_x = ma_x$ to the first block:

$$F - F_{1,2} = m_2 a_2 = m_2 a$$

Apply $\sum F_x = ma_x$ to the second block:

$$F_{2,1} = m_1 a_1 = m_1 a \qquad (1)$$

Add these equations to eliminate $F_{2,1}$ and $F_{1,2}$ and solve for $a = a_1 = a_2$:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

Substitute your value for a into equation (1) and solve for $F_{2,1}$:

$$F_{2,1} = \boxed{\frac{Fm_1}{m_1 + m_2}}$$

(*b*) Substitute numerical values in the equations derived in part (*a*) and evaluate a and $F_{2,1}$:

$$a = \frac{3.2\,\text{N}}{2\,\text{kg} + 6\,\text{kg}} = \boxed{0.400\,\text{m/s}^2}$$

and

$$F_{2,1} = \frac{(3.2\,\text{N})(2\,\text{kg})}{2\,\text{kg} + 6\,\text{kg}} = \boxed{0.800\,\text{N}}$$

Remarks: Note that our results for the acceleration are the same as if the force *F* had acted on a single object whose mass is equal to the sum of the masses of the two blocks. In fact, because the two blocks have the same acceleration, we can consider them to be a single system with mass $m_1 + m_2$.

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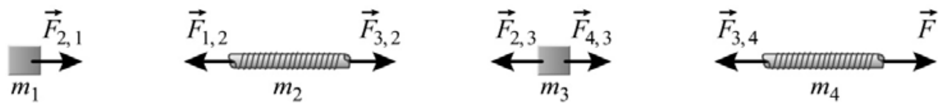
Picture the Problem The free-body diagrams for the boxes and the ropes are below. Because the vertical forces have no bearing on the problem they have not been included. Let the numeral 1 denote the 100-kg box to the left, the numeral 2 the rope connecting the boxes, the numeral 3 the box to the right and the numeral 4 the rope to which the force $\vec{F}$ is applied. $\vec{F}_{3,4}$ is the tension force exerted by m_3 on m_4, $\vec{F}_{4,3}$ is the tension force exerted by m_4 on m_3, $\vec{F}_{2,3}$ is the tension force exerted by m_2 on m_3, $\vec{F}_{3,2}$ is the tension force exerted by m_3 on m_2, $\vec{\boldsymbol{F}}_{1,2}$ is the tension force exerted by m_1 on m_2, and $\vec{\boldsymbol{F}}_{2,1}$ is the tension force exerted by m_2 on m_1. The equal and opposite pairs of forces are $\vec{\boldsymbol{F}}_{2,1} = -\vec{\boldsymbol{F}}_{1,2}$, $\vec{\boldsymbol{F}}_{3,2} = -\vec{\boldsymbol{F}}_{2,3}$, and $\vec{\boldsymbol{F}}_{4,3} = -\vec{\boldsymbol{F}}_{3,4}$. We can apply Newton's 2nd law to each box and rope separately and use the fact that their accelerations are equal.



Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the box whose mass is m_1:

$$F_{2,1} = m_1 a_1 = m_1 a \quad (1)$$

Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the rope whose mass is m_2:

$$F_{3,2} - F_{1,2} = m_2 a_2 = m_2 a \quad (2)$$

Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the box whose mass is m_3:

$$F_{4,3} - F_{2,3} = m_3 a_3 = m_3 a \quad (3)$$

Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the rope whose mass is m_4:

$$F - F_{3,4} = m_4 a_4 = m_4 a$$

Add these equations to eliminate $F_{2,1}$, $F_{1,2}$, $F_{3,2}$, $F_{2,3}$, $F_{4,3}$, and $F_{3,4}$ and solve for F:

$$F = (m_1 + m_2 + m_3 + m_4)a = (202\,\text{kg})(1.0\,\text{m/s}^2) = \boxed{202\,\text{N}}$$

Use equation (1) to find the tension at point A:

$$F_{2,1} = (100\,\text{kg})(1.0\,\text{m/s}^2) = \boxed{100\,\text{N}}$$

Use equation (2) to find the tension at point B:

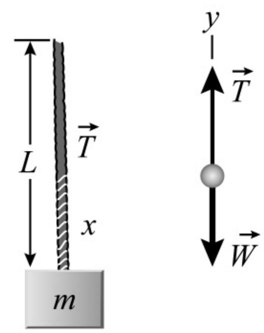
$$F_{3,2} = F_{1,2} + m_2 a = 100\,\text{N} + (1\,\text{kg})(1.0\,\text{m/s}^2) = \boxed{101\,\text{N}}$$

Use equation (3) to find the tension at point C:

$$F_{4,3} = F_{2,3} + m_3 a = 101\,\text{N} + (100\,\text{kg})(1.0\,\text{m/s}^2) = \boxed{201\,\text{N}}$$

73 ••

Picture the Problem Because the distribution of mass in the rope is uniform, we can express the mass m' of a length x of the rope in terms of the total mass of the rope M and its length L. We can then express the total mass that the rope must support at a distance x above the block and use Newton's 2nd law to find the tension as a function of x.



Set up a proportion expressing the mass m' of a length x of the rope as a function of M and L and solve for m':

$$\frac{m'}{x}=\frac{M}{L}\Rightarrow m'=\frac{M}{L}x$$

Express the total mass that the rope must support at a distance x above the block:

$$m+m'=m+\frac{M}{L}x$$

Apply $\sum F_y = ma_y$ to the block and a length x of the rope:

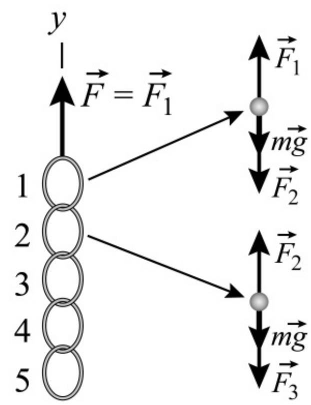
$$T-w=T-\left(m+\frac{M}{L}x\right)g$$
$$=\left(m+\frac{M}{L}x\right)a$$

Solve for T to obtain:

$$T=\boxed{(a+g)\left(m+\frac{M}{L}x\right)}$$

***74** ••

Picture the Problem Choose a coordinate system with the positive y direction upward and denote the top link with the numeral 1, the second with the numeral 2, etc.. The free-body diagrams show the forces acting on links 1 and 2. We can apply Newton's 2nd law to each link to obtain a system of simultaneous equations that we can solve for the force each link exerts on the link below it. Note that the net force on each link is the product of its mass and acceleration.



(*a*) Apply $\sum F_y = ma_y$ to the top link and solve for F:

$$F-5mg=5ma$$

and

$$F=5m(g+a)$$

Substitute numerical values and evaluate F:

$$F = 5(0.1\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2) = \boxed{6.16\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ to a single link:

$$F_{1\,\text{link}} = m_{1\,\text{link}}a = (0.1\,\text{kg})(2.5\,\text{m/s}^2) = \boxed{0.250\,\text{N}}$$

(*c*) Apply $\sum F_y = ma_y$ to the 1st through 5th links to obtain:

$$F - F_2 - mg = ma\,, \quad (1)$$
$$F_2 - F_3 - mg = ma\,, \quad (2)$$
$$F_3 - F_4 - mg = ma\,, \quad (3)$$
$$F_4 - F_5 - mg = ma\,, \text{ and} \quad (4)$$
$$F_5 - mg = ma \quad (5)$$

Add equations (2) through (5) to obtain:

$$F_2 - 4mg = 4ma$$

Solve for F_2 to obtain:

$$F_2 = 4mg + 4ma = 4m(a + g)$$

Substitute numerical values and evaluate F_2:

$$F_2 = 4(0.1\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2) = \boxed{4.92\,\text{N}}$$

Substitute for F_2 to find F_3, and then substitute for F_3 to find F_4:

$$F_3 = \boxed{3.69\,\text{N}} \text{ and } F_4 = \boxed{2.46\,\text{N}}$$

Solve equation (5) for F_5:

$$F_5 = m(g + a)$$

Substitute numerical values and evaluate F_5:

$$F_5 = (0.1\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2) = \boxed{1.23\,\text{N}}$$

75 •

Picture the Problem A *net* force is required to accelerate the object. In this problem the net force is the difference between $\vec{T}$ and $\vec{W}(= m\vec{g})$. The free-body diagram of the object is shown to the right. Choose a coordinate system in which the upward direction is positive.

Apply $\sum \vec{F} = m\vec{a}$ to the object to obtain:

$$F_{\text{net}} = T - W = T - mg$$

Solve for the tension in the lower portion of the rope:

$$T = F_{\text{net}} + mg = ma + mg = m(a + g)$$

Using its definition, find the acceleration of the object:

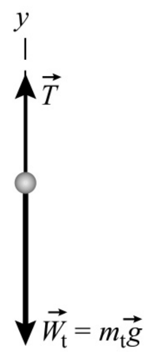
$a \equiv \Delta v/\Delta t = (3.5\text{ m/s})/(0.7\text{ s}) = 5.00\text{ m/s}^2$

Substitute numerical values and evaluate T:

$T = (40\text{ kg})(5.00\text{ m/s}^2 + 9.81\text{ m/s}^2) = 592\text{ N}$ and $\boxed{(a)\text{ is correct.}}$

76 •

Picture the Problem A net force in the downward direction is required to accelerate the truck downward. The net force is the difference between $\vec{W}_\text{t}$ and $\vec{T}$. A free-body diagram showing these forces acting on the truck is shown to the right. Choose a coordinate system in which the downward direction is positive.



Apply $\sum F_y = ma_y$ to the truck to obtain:

$T - m_\text{t}g = m_\text{t}a_y$

Solve for the tension in the lower portion of the cable:

$T = m_\text{t}g + m_\text{t}a_y = m_\text{t}(g + a_y)$

Substitute to find the tension in the rope:

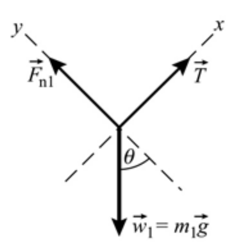
$T = m_\text{t}(g - 0.1g) = 0.9m_\text{t}g$

and $\boxed{(c)\text{ is correct.}}$

77 ••

Picture the Problem Because the string does not stretch or become slack, the two objects must have the same speed and therefore the magnitude of the acceleration is the same for each object. Choose a coordinate system in which up the incline is the positive x direction for the object of mass m_1 and downward is the positive x direction for the object of mass m_2. This idealized pulley acts like a piece of polished pipe; i.e., its only function is to change the direction the tension in the massless string acts. Draw a free-body diagram for each of the two objects, apply Newton's 2nd law of motion to both objects, and solve the resulting equations simultaneously.

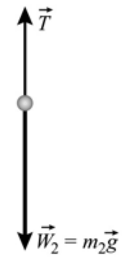
(*a*) Draw the FBD for the object of mass m_1:



Apply $\sum F_x = ma_x$ to the object whose mass is m_1:

$T - m_1g\sin\theta = m_1a$

Draw the FBD for the object of mass m_2:



Apply $\sum F_x = ma_x$ to the object whose mass is m_2:

$$m_2 g - T = m_2 a$$

Add the two equations and solve for a:

$$a = \boxed{\frac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2}}$$

Substitute for a in either of the equations containing the tension and solve for T:

$$T = \boxed{\frac{g m_1 m_2 (1 + \sin\theta)}{m_1 + m_2}}$$

(*b*) Substitute the given values into the expression for a:

$$a = \boxed{2.45\,\text{m/s}^2}$$

Substitute the given data into the expression for T:

$$T = \boxed{36.8\,\text{N}}$$

78 •

Picture the Problem The magnitude of the accelerations of Peter and the counterweight are the same. Choose a coordinate system in which up the incline is the positive x direction for the counterweight and downward is the positive x direction for Peter. The pulley changes the direction the tension in the rope acts. Let Peter's mass be m_P. Ignoring the mass of the rope, draw free-body diagrams for the counterweight and Peter, apply Newton's 2nd law to each of them, and solve the resulting equations simultaneously.

(*a*) Using a constant-acceleration equation, relate Peter's displacement to her acceleration and descent time:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2} a (\Delta t)^2$$

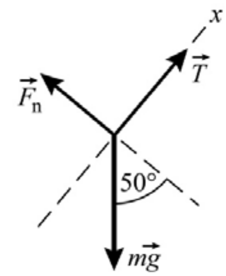
Solve for the common acceleration of Peter and the counterweight:

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

$$a = \frac{2(3.2\,\text{m})}{(2.2\,\text{s})^2} = 1.32\,\text{m/s}^2$$

Draw the FBD for the counterweight:



Apply $\sum F_x = ma_x$ to the counterweight:

$$T - mg\sin 50° = ma$$

Draw the FBD for Peter:

Apply $\sum F_x = ma_x$ to Peter:

$$m_P g - T = m_P a$$

Add the two equations and solve for m:

$$m = \frac{m_P(g-a)}{a + g\sin 50°}$$

Substitute numerical values and evaluate m:

$$m = \frac{(50\,\text{kg})(9.81\,\text{m/s}^2 - 1.32\,\text{m/s}^2)}{1.32\,\text{m/s}^2 + (9.81\,\text{m/s}^2)\sin 50°} = \boxed{48.0\,\text{kg}}$$

(*b*) Substitute for m in the force equation for the counterweight and solve for T:

$$T = m(a + g\sin 50°)$$

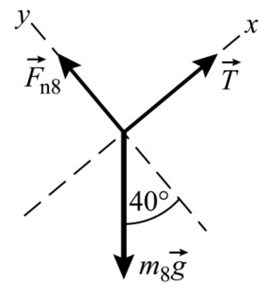
(*b*) Substitute numerical values and evaluate T:

$$T = (48.0\,\text{kg})\left[1.32\,\text{m/s}^2 + (9.81\,\text{m/s}^2)\sin 50°\right] = \boxed{424\,\text{N}}$$

79 ••

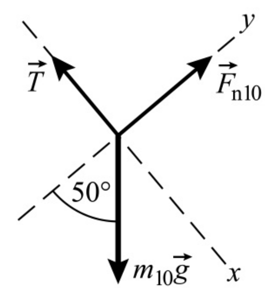
Picture the Problem The magnitude of the accelerations of the two blocks are the same. Choose a coordinate system in which up the incline is the positive *x* direction for the 8-kg object and downward is the positive *x* direction for the 10-kg object. The peg changes the direction the tension in the rope acts. Draw free-body diagrams for each object, apply Newton's 2nd law of motion to both of them, and solve the resulting equations simultaneously.

(*a*) Draw the FBD for the 3-kg object:



Apply $\sum F_x = ma_x$ to the 3-kg block: $T - m_8 g \sin 40° = m_3 a$

Draw the FBD for the 10-kg object:



Apply $\sum F_x = ma_x$ to the 10-kg block: $m_{10} g \sin 50° - T = m_{10} a$

Add the two equations and solve for and evaluate a:

$$a = \frac{g\left(m_{10}\sin 50° - m_8 \sin 40°\right)}{m_8 + m_{10}}$$

$$= \boxed{1.37\,\text{m/s}^2}$$

Substitute for a in the first of the two force equations and solve for T:

$$T = m_8 g \sin 40° + m_8 a$$

Substitute numerical values and evaluate T:

$$T = (8\,\text{kg})\left[\left(9.81\,\text{m/s}^2\right)\sin 40° + 1.37\,\text{m/s}^2\right]$$

$$= \boxed{61.4\,\text{N}}$$

(*b*) Because the system is in equilibrium, set $a = 0$, express the force equations in terms of m_1 and m_2, add the two force equations, and solve for and evaluate the ratio m_1/m_2:

$T - m_1 g \sin 40° = 0$

$m_2 g \sin 50° - T = 0$

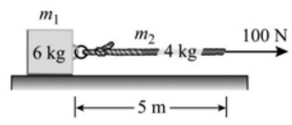
$\therefore\ m_2 g \sin 50° - m_1 g \sin 40° = 0$

and

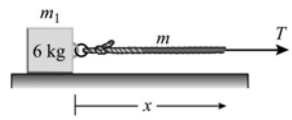
$$\frac{m_1}{m_2} = \frac{\sin 50°}{\sin 40°} = \boxed{1.19}$$

80 ••

Picture the Problem The pictorial representations shown to the right summarize the information given in this problem. While the mass of the rope is distributed over its length, the rope and the 6-kg block have a common acceleration. Choose a coordinate system in which the direction of the 100-N force is the positive *x* direction. Because the surface is horizontal and frictionless, the only force that influences our solution is the 100-N force.



Part (a)



Part (b)

(*a*) Apply $\sum F_x = ma_x$ to the objects shown for part (*a*):

$$100\text{ N} = (m_1 + m_2)a$$

Solve for *a* to obtain:

$$a = \frac{100\text{ N}}{m_1 + m_2}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{100\text{ N}}{10\text{ kg}} = \boxed{10.0\text{ m/s}^2}$$

(*b*) Let *m* represent the mass of a length *x* of the rope. Assuming that the mass of the rope is uniformly distributed along its length:

$$\frac{m}{x} = \frac{m_2}{L_{\text{rope}}} = \frac{4\text{ kg}}{5\text{ m}}$$

and

$$m = \left(\frac{4\text{ kg}}{5\text{ m}}\right)x$$

Let *T* represent the tension in the rope at a distance *x* from the point at which it is attached to the 6-kg block. Apply $\sum F_x = ma_x$ to the system shown for part (*b*) and solve for *T*:

$$T = (m_1 + m)a$$

$$= \left[6\text{ kg} + \left(\frac{4\text{ kg}}{5\text{ m}}\right)x\right]\left(10\text{ m/s}^2\right)$$

$$= \boxed{60\text{ N} + (8\text{ N/m})x}$$

***81** ••

Picture the Problem Choose a coordinate system in which upward is the positive *y* direction and draw the free-body diagram for the frame-plus- painter. Noting that $\vec{F} = -\vec{T}$, apply Newton's 2[nd] law of motion.

(*a*) Letting $m_{\text{tot}} = m_{\text{frame}} + m_{\text{painter}}$,

$$2T - m_{\text{tot}}g = m_{\text{tot}}a$$

and

apply $\sum F_y = ma_y$ to the frame-plus-painter and solve T:

$$T = \frac{m_{tot}(a+g)}{2}$$

Substitute numerical values and evaluate T:

$$T = \frac{(75\,\text{kg})(0.8\,\text{m/s}^2 + 9.81\,\text{m/s}^2)}{2} = 398\,\text{N}$$

Because $F = T$:

$$F = \boxed{398\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ with $a = 0$ to obtain:

$$2T - m_{tot}g = 0$$

Solve for T:

$$T = \tfrac{1}{2}m_{tot}g$$

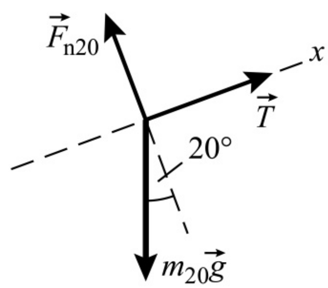
Substitute numerical values and evaluate T:

$$T = \tfrac{1}{2}(75\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{368\,\text{N}}$$

82 •••

Picture the Problem Choose a coordinate system in which up the incline is the positive x direction and draw free-body diagrams for each block. Noting that $\vec{a}_{20} = -\vec{a}_{10}$, apply Newton's 2nd law of motion to each block and solve the resulting equations simultaneously.

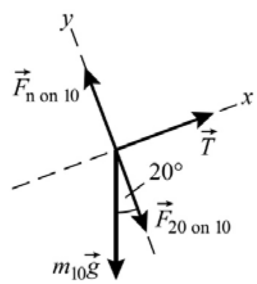
Draw a FBD for the 20-kg block:



Apply $\sum F_x = ma_x$ to the block to obtain:

$$T - m_{20}g\sin 20° = m_{20}a_{20}$$

Draw a FBD for the 10-kg block. Because all the surfaces, including the surfaces between the blocks, are frictionless, the force the 20-kg block exerts on the 10-kg block must be normal to their surfaces as shown to the right.



Apply $\sum F_x = ma_x$ to the block to obtain:

$$T - m_{10}g\sin 20° = m_{10}a_{10}$$

Because the blocks are connected by a taut string: $a_{20} = -a_{10}$

Substitute for a_{20} and eliminate T between the two equations to obtain: $a_{10} = \boxed{1.12\,\text{m/s}^2}$

and

$a_{20} = \boxed{-1.12\,\text{m/s}^2}$

Substitute for either of the accelerations in the force equations and solve for T: $T = \boxed{44.8\,\text{N}}$

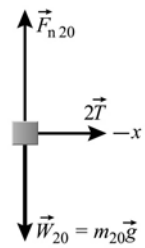
83 •••

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and draw free-body diagrams for each block. Because of the pulley, the string exerts a force of $2T$. Apply Newton's 2nd law of motion to both blocks and solve the resulting equations simultaneously.

(a) Noting the effect of the pulley, express the distance the 20-kg block moves in a time Δt:

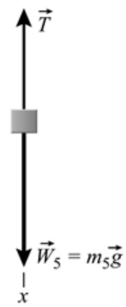
$$\Delta x_{20} = \tfrac{1}{2}\Delta x_5 = \tfrac{1}{2}(10\,\text{cm}) = \boxed{5.00\,\text{cm}}$$

(b) Draw a FBD for the 20-kg block:



Apply $\sum F_x = ma_x$ to the block to obtain: $2T = m_{20}a_{20}$

Draw a FBD for the 5-kg block:



Apply $\sum F_x = ma_x$ to the block to obtain: $m_5g - T = m_5a_5$

Using a constant-acceleration equation, relate the displacement of the 5-kg block to its acceleration

$$\Delta x_5 = \tfrac{1}{2}a_5(\Delta t)^2$$

and the time during which it is accelerated:

Using a constant-acceleration equation, relate the displacement of the 20-kg block to its acceleration and the time during which it is accelerated:

$$\Delta x_{20} = \tfrac{1}{2}a_{20}(\Delta t)^2$$

Divide the first of these equations by the second to obtain:

$$\frac{\Delta x_5}{\Delta x_{20}} = \frac{\tfrac{1}{2}a_5(\Delta t)^2}{\tfrac{1}{2}a_{20}(\Delta t)^2} = \frac{a_5}{a_{20}}$$

Use the result of part (*a*) to obtain:

$$a_5 = 2a_{20}$$

Let $a_{20} = a$. Then $a_5 = 2a$ and the force equations become:

$$2T = m_{20}a$$

and

$$m_5 g - T = m_5(2a)$$

Eliminate T between the two equations to obtain:

$$a = a_{20} = \frac{m_5 g}{2m_5 + \tfrac{1}{2}m_{20}}$$

Substitute numerical values and evaluate a_{20} and a_5:

$$a_{20} = \frac{(5\,\text{kg})(9.81\,\text{m/s}^2)}{2(5\,\text{kg}) + \tfrac{1}{2}(20\,\text{kg})} = \boxed{2.45\,\text{m/s}^2}$$

and

$$a_5 = 2(2.45\,\text{m/s}^2) = \boxed{4.91\,\text{m/s}^2}$$

Substitute for either of the accelerations in either of the force equations and solve for T:

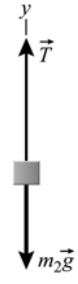
$$T = \boxed{24.5\,\text{N}}$$

Free-Body Diagrams: The Atwood's Machine

***84** ••

Picture the Problem Assume that $m_1 > m_2$. Choose a coordinate system in which the positive y direction is downward for the block whose mass is m_1 and upward for the block whose mass is m_2 and draw free-body diagrams for each block. Apply Newton's 2nd law of motion to both blocks and solve the resulting equations simultaneously.

Draw a FBD for the block whose mass is m_2:



Apply $\sum F_y = ma_y$ to this block:

$T - m_2g = m_2a_2$

Draw a FBD for the block whose mass is m_1:

Apply $\sum F_y = ma_y$ to this block:

$m_1g - T = m_1a_1$

Because the blocks are connected by a taut string, let a represent their common acceleration:

$a = a_1 = a_2$

Add the two force equations to eliminate T and solve for a:

$m_1g - m_2g = m_1a + m_2a$
and

$$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2}g}$$

Substitute for a in either of the force equations and solve for T:

$$\boxed{T = \frac{2m_1m_2g}{m_1 + m_2}}$$

85 ••

Picture the Problem The acceleration can be found from the given displacement during the first second. The ratio of the two masses can then be found from the acceleration using the first of the two equations derived in Problem 89 relating the acceleration of the Atwood’s machine to its masses.

Using a constant-acceleration equation, relate the displacement of the masses to their acceleration and solve for the acceleration:

$\Delta y = v_0t + \frac{1}{2}a(\Delta t)^2$
or, because $v_0 = 0$,
$\Delta y = \frac{1}{2}a(\Delta t)^2$

Solve for and evaluate a:

$$a = \frac{2\Delta y}{(\Delta t)^2} = \frac{2(0.3\,\text{m})}{(1\,\text{s})^2} = 0.600\,\text{m/s}^2$$

Solve for m_1 in terms of m_2 using the first of the two equations given in Problem 84:

$$m_1 = m_2\frac{g + a}{g - a} = \frac{10.41\,\text{m/s}^2}{9.21\,\text{m/s}^2}m_2 = 1.13m_2$$

Find the second mass for m_2 or $m_1 = 1.2$ kg:

$$m_{\text{2nd mass}} = \boxed{1.36\,\text{kg or } 1.06\,\text{kg}}$$

86 ••

Picture the Problem Let F_{nm} be the force the block of mass m_2 exerts on the pebble of mass m. Because $m_2 < m_1$, the block of mass m_2 accelerates upward. Draw a free-body diagram for the pebble and apply Newton's 2nd law and the acceleration equation given in Problem 84.

Apply $\sum F_y = ma_y$ to the pebble: $F_{nm} - mg = ma$

Solve for F_{nm}: $F_{nm} = m(a + g)$

From Problem 84: $$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

Substitute for a and simplify to obtain: $$F_{nm} = m\left(\frac{m_1 - m_2}{m_1 + m_2} g + g\right) = \boxed{\frac{2m_1 m}{m_1 + m_2} g}$$

87 ••

Picture the Problem Note from the free-body diagrams for Problem 89 that the net force exerted by the accelerating blocks is $2T$. Use this information, together with the expression for T given in Problem 84, to derive an expression for $F = 2T$.

From Problem 84 we have: $$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

The net force, F, exerted by the Atwood's machine on the hanger is: $$F = 2T = \boxed{\frac{4m_1 m_2 g}{m_1 + m_2}}$$

If $m_1 = m_2 = m$, then: $$F = \frac{4m^2 g}{2m} = 2mg$$ … as expected.

If either m_1 or $m_2 = 0$, then: $F = 0$ … also as expected.

88 •••

Picture the Problem Use a constant-acceleration equation to relate the displacement of the descending (or rising) mass as a function of its acceleration and then use one of the results from Problem 84 to relate a to g. Differentiation of our expression for g will allow us to relate uncertainty in the time measurement to uncertainty in the measured value for g … and to the values of m_2 that would yield an experimental value for g that is good to within 5%.

(*a*) Use the result given in Problem 84 to express g in terms of a:

$$g = a\frac{m_1 + m_2}{m_1 - m_2} \qquad (1)$$

Using a constant-acceleration equation, express the displacement, L, as a function of t and solve for the acceleration:

$$\Delta y = L = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$ and $\Delta t = t$,

$$a = \frac{2L}{t^2} \qquad (2)$$

Substitute this expression for a:

$$g = \boxed{\frac{2L}{t^2}\left(\frac{m_1 + m_2}{m_1 - m_2}\right)}$$

(*b*) Evaluate dg/dt to obtain:

$$\frac{dg}{dt} = -4Lt^{-3}\left(\frac{m_1 + m_2}{m_1 - m_2}\right)$$

$$= \frac{-2}{t}\left[\frac{2L}{t^2}\right]\left(\frac{m_1 + m_2}{m_1 - m_2}\right) = \frac{-2g}{t}$$

Divide both sides of this expression by g and multiply both sides by dt:

$$\boxed{\frac{dg}{g} = -2\frac{dt}{t}}$$

(*c*) We have:

$$\frac{dg}{g} = \pm 0.05 \text{ and } \frac{dt}{t} = \pm 0.025$$

Solve the second of these equations for t to obtain:

$$t = \frac{dt}{0.025} = \frac{1\,\text{s}}{0.025} = 4\,\text{s}$$

Substitute in equation (2) to obtain:

$$a = \frac{2(3\,\text{m})}{(4\,\text{s})^2} = 0.375\,\text{m/s}^2$$

Solve equation (1) for m_2 to obtain:

$$m_2 = \frac{g - a}{g + a}m_1$$

Evaluate m_2 with m_1 = 1 kg:

$$m_2 = \frac{9.81\,\text{m/s}^2 - 0.375\,\text{m/s}^2}{9.81\,\text{m/s}^2 + 0.375\,\text{m/s}^2}(1\,\text{kg})$$

$$= 0.926\,\text{kg}$$

Solve equation (1) for m_1 to obtain:

$$m_1 = m_2\frac{g + a}{g - a}$$

Substitute numerical values to obtain:

$$m_1 = (0.926\,\text{kg})\frac{9.81\,\text{m/s}^2 + 0.375\,\text{m/s}^2}{9.81\,\text{m/s}^2 - 0.375\,\text{m/s}^2}$$

$$= 1.08\,\text{kg}$$

Because the masses are interchangeable: $m_2 = \boxed{0.926\,\text{kg or }1.08\,\text{kg}}$

***89** ••

Picture the Problem We can reason to this conclusion as follows: In the two extreme cases when the mass on one side or the other is zero, the tension is zero as well, because the mass is in free-fall. By symmetry, the maximum tension must occur when the masses on each side are equal. An alternative approach that is shown below is to treat the problem as an extreme-value problem.

Express m_2 in terms of M and m_1:

$$m_2 = M - m_1$$

Substitute in the equation from Problem 84 and simplify to obtain:

$$T = \frac{2gm_1(M - m_1)}{m_1 + M - m_1} = 2g\left(m_1 - \frac{m_1^2}{M}\right)$$

Differentiate this expression with respect to m_1 and set the derivative equal to zero for extreme values:

$$\frac{dT}{dm_1} = 2g\left(1 - \frac{2m_1}{M}\right) = 0 \text{ for extreme values}$$

Solve for m_1 to obtain:

$$m_1 = \tfrac{1}{2}M$$

Show that $m_1 = M/2$ is a maximum value by evaluating the second derivative of T with respect to m_1 at $m_1 = M/2$:

$$\frac{d^2T}{dm_1^2} = -\frac{4g}{M} < 0,\text{ independently of } m_1$$

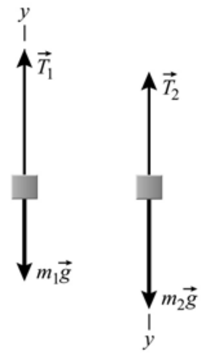
and we have shown that

$$\boxed{T \text{ is a maximum when } m_1 = m_2 = \tfrac{1}{2}M.}$$

Remarks: An alternative solution is to use a graphing calculator to show that *T* as a function of m_1 is concave downward and has its maximum value when $m_1 = m_2 = M/2$.

90 •••

Picture the Problem The free-body diagrams show the forces acting on the objects whose masses are m_1 and m_2. The application of Newton's 2nd law to these forces and the accelerations the net forces are responsible for will lead us to an expression for the tension in the string as a function of m_1 and m_2. Examination of this expression as for $m_2 >> m_1$ will yield the predicted result.



(*a*) Apply $\sum F_y = ma_y$ to the objects whose masses are m_1 and m_2 to obtain:

$$T_1 - m_1g = m_1a_1$$

and

$$m_2g - T_2 = m_2a_2$$

Assume that the role of the pulley is simply to change the direction the tension acts. Then $T_1 = T_2 = T$. Because the two objects have a common acceleration, let $a = a_1 = a_2$. Eliminate a between the two equations and solve for T to obtain:

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

Divide the numerator and denominator of this fraction by m_2:

$$T = \frac{2m_1 g}{1 + \dfrac{m_1}{m_2}}$$

Take the limit of this fraction as $m_2 \to \infty$ to obtain:

$$T = \boxed{2m_1 g}$$

(*b*) Imagine the situation when $m_2 >> m_1$:

Under these conditions, the object whose mass is m_2 is essentially in free-fall, so the object whose mass is m_1 is accelerating *upward* with an acceleration of magnitude g.

Under these conditions, the net force acting on the object whose mass is m_1 is $m_1 g$ and:

$T - m_1 g = m_1 g \Rightarrow T = 2m_1 g$.
Note that this result agrees with that obtained using more analytical methods.

General Problems

91 •

Picture the Problem Choose a coordinate system in which the force the tree exerts on the woodpecker's head is in the negative-x direction and determine the acceleration of the woodpecker's head from Newton's 2nd law of motion. The depth of penetration, under the assumption of constant acceleration, can be determined using a constant- acceleration equation. Knowing the acceleration of the woodpecker's head and the depth of penetration of the tree, we can calculate the time required to bring the head to rest.

(*a*) Apply $\sum F_x = ma_x$ to the woodpecker's head to obtain:

$$a_x = \frac{\sum F_x}{m} = \frac{-6\,\text{N}}{0.060\,\text{kg}} = \boxed{-100\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the depth-of-penetration into the bark to the acceleration of the woodpecker's head:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\Delta x$$

Solve for and evaluate Δx:

$$\Delta x = \frac{-v_0^2}{2a} = \frac{-(3.5\,\text{m/s})^2}{2(-100\,\text{m/s}^2)} = \boxed{6.13\,\text{cm}}$$

(*c*) Use the definition of acceleration to express the time required for the woodpecker's head to come to rest:

$$\Delta t = \frac{v - v_0}{a}$$

or, because $v = 0$,

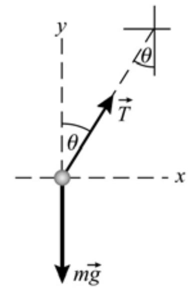
$$\Delta t = \frac{v - v_0}{a}$$

Substitute numerical values and evaluate Δt:

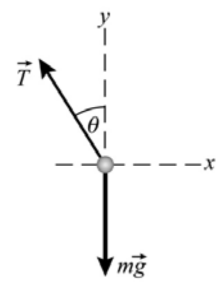
$$\Delta t = \frac{-v_0}{a} = \frac{-3.5\,\text{m/s}}{-100\,\text{m/s}^2} = \boxed{35.0\,\text{ms}}$$

***92 ••**

Picture the Problem The free-body diagram shown to the right shows the forces acting on an object suspended from the ceiling of a car that is accelerating to the right. Choose the coordinate system shown and use Newton's laws of motion and constant- acceleration equations in the determination of the influence of the forces on the behavior of the suspended object.



The second free-body diagram shows the forces acting on an object suspended from the ceiling of a car that is braking while it moves to the right.



(*a*) $\boxed{\text{In accordance with Newton's law of inertia, the object's displacement will be in the direction opposite that of the acceleration.}}$

(*b*) Resolve the tension, T, into its components and apply $\sum \vec{F} = m\vec{a}$ to the object:

$$\Sigma F_x = T\sin\theta = ma$$

and

$$\Sigma F_y = T\cos\theta - mg = 0$$

Take the ratio of these two equations to eliminate T and m:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{ma}{mg}$$

or

$$\tan\theta = \frac{a}{g} \Rightarrow \boxed{a = g\tan\theta}$$

(*c*) $\boxed{\text{Because the acceleration is opposite the direction the car is moving, the accelerometer will swing forward.}}$

Using a constant-acceleration equation, express the velocity of the car in terms of its acceleration and solve for the acceleration:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\Delta x$$

Solve for a:

$$a = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{-(50\,\text{km/h})^2}{2(60\,\text{m})} = \boxed{-1.61\,\text{m/s}^2}$$

Solve the equation derived in (*b*) for θ:

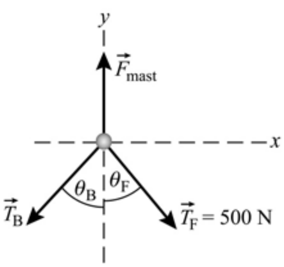
$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{1.61\,\text{m/s}^2}{9.81\,\text{m/s}^2}\right) = \boxed{9.32°}$$

93 ••

Picture the Problem The free-body diagram shows the forces acting at the top of the mast. Choose the coordinate system shown and use Newton's 2nd and 3rd laws of motion to analyze the forces acting on the deck of the sailboat.



Apply $\sum F_x = ma_x$ to the top of the mast:

$$T_F \sin\theta_F - T_B \sin\theta_B = 0$$

Find the angles that the forestay and backstay make with the vertical:

$$\theta_F = \tan^{-1}\left(\frac{3.6\,\text{m}}{12\,\text{m}}\right) = 16.7°$$

and

$$\theta_B = \tan^{-1}\left(\frac{6.4\,\text{m}}{12\,\text{m}}\right) = 28.1°$$

Solve the x-direction equation for T_B:

$$T_B = T_F \frac{\sin\theta_F}{\sin\theta_B} = (500\,\text{N})\frac{\sin 16.7°}{\sin 28.1°} = \boxed{305\,\text{N}}$$

Find the downward forces that T_B and T_F exert on the mast:

$$\sum F_y = F_{mast} - T_F \cos\theta_F - T_B \cos\theta_B = 0$$

Solve for F_{mast} to obtain:

$$F_{mast} = T_F \cos\theta_F + T_B \cos\theta_B$$

Substitute numerical values and evaluate F_{mast}:

$$F_{\text{mast}} = (500\text{ N})\cos 16.7° + (305\text{ N})\cos 28.1° = 748\text{ N}$$

The force that the mast exerts on the deck is the sum of its weight and the downward forces exerted on it by the forestay and backstay:

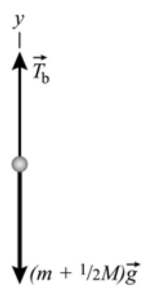
$$F_{\text{mast on the deck}} = 748\text{ N} + 800\text{ N}$$
$$= \boxed{1.55\text{ kN}}$$

94 ••

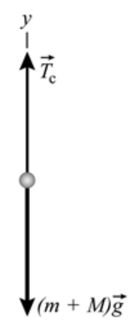
Picture the Problem Let m be the mass of the block and M be the mass of the chain. The free-body diagrams shown below display the forces acting at the locations identified in the problem. We can apply Newton's 2nd law with $a_y = 0$ to each of the segments of the chain to determine the tensions.

(*a*)

(*b*)



(*c*)



(*a*) Apply $\sum F_y = ma_y$ to the block and solve for T_a:

$$T_a - mg = ma_y$$
or, because $a_y = 0$,
$$T_a = mg$$

Substitute numerical values and evaluate T_a:

$$T_a = (50\text{ kg})(9.81\text{ m/s}^2) = \boxed{491\text{ N}}$$

(*b*) Apply $\sum F_y = ma_y$ to the block and half the chain and solve for T_b:

$$T_b - \left(m + \frac{M}{2}\right)g = ma_y$$
or, because $a_y = 0$,
$$T_b = \left(m + \frac{M}{2}\right)g$$

Substitute numerical values and evaluate T_b:

$$T_b = (50\text{ kg} + 10\text{ kg})(9.81\text{ m/s}^2) = \boxed{589\text{ N}}$$

(*c*) Apply $\sum F_y = ma_y$ to the block and chain and solve for T_c:

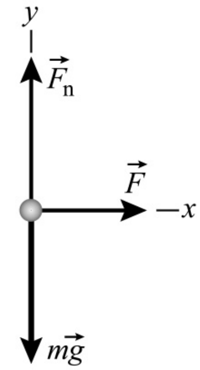
$$T_c - (m + M)g = ma_y$$
or, because $a_y = 0$,
$$T_c = (m + M)g$$

Substitute numerical values and evaluate T_c:

$$T_c = (50\,\text{kg} + 20\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{687\,\text{N}}$$

***95 •••**

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it across a frictionless floor. Because the force is time-dependent, the acceleration will be, too. We can obtain the acceleration as a function of time from the application of Newton's 2nd law and then find the velocity of the box as a function of time by integration. Finally, we can derive an expression for the displacement of the box as a function of time by integration of the velocity function.



(*a*) The velocity is related to the acceleration according to:

$$\frac{dv}{dt} = a(t) \qquad (1)$$

Apply $\sum F_x = ma_x$ to the box and solve for its acceleration:

$$F = ma$$

and

$$a = \frac{F}{m} = \frac{(8\,\text{N/s})t}{24\,\text{kg}} = \left(\tfrac{1}{3}\,\text{m/s}^3\right)t$$

Because the box's acceleration is a function of time, separate variables in equation (1) and integrate to find v as a function of time:

$$v(t) = \int_0^t a(t')dt' = \left(\tfrac{1}{3}\,\text{m/s}^3\right)\int_0^t t'\,dt' = \left(\tfrac{1}{3}\,\text{m/s}^3\right)\frac{t^2}{2} = \left(\tfrac{1}{6}\,\text{m/s}^3\right)t^2$$

Evaluate v at $t = 3$ s:

$$v(3\,\text{s}) = \left(\tfrac{1}{6}\,\text{m/s}^3\right)(3\,\text{s})^2 = \boxed{1.50\,\text{m/s}}$$

(*b*) Integrate $v = dx/dt$ between 0 and 3 s to find the displacement of the box during this time:

$$\Delta x = \int_0^{3\,\text{s}} v(t')dt' = \left(\tfrac{1}{6}\,\text{m/s}^3\right)\int_0^{3\,\text{s}} t'^2\,dt' = \left[\left(\tfrac{1}{6}\,\text{m/s}^3\right)\frac{t'^3}{3}\right]_0^{3\,\text{s}} = \boxed{1.50\,\text{m}}$$

(*c*) The average velocity is given by:

$$v_\text{ave} = \frac{\Delta x}{\Delta t} = \frac{1.5\,\text{m}}{3\,\text{s}} = \boxed{0.500\,\text{m/s}}$$

(*d*) Use Newton's 2nd law to express the average force exerted on the box by the man:

$$F_\text{av} = ma_\text{av} = m\frac{\Delta v}{\Delta t}$$

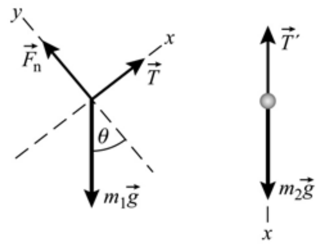
Substitute numerical values and evaluate F_{av}:

$$F_{av} = (24\,\text{kg})\frac{1.5\,\text{m/s} - 0\,\text{m/s}}{3\text{s}} = \boxed{12.0\,\text{N}}$$

96 ••

Picture the Problem The application of Newton's 2nd law to the glider and the hanging weight will lead to simultaneous equations in their common acceleration *a* and the tension *T* in the cord that connects them. Once we know the acceleration of this system, we can use a constant-acceleration equation to predict how long it takes the cart to travel 1 m from rest. Note that the magnitudes of $\vec{T}$ and $\vec{T}'$ are equal.

(*a*) The free-body diagrams are shown to the right. m_1 represents the mass of the cart and m_2 the mass of the hanging weight.



(*b*) Apply $\sum F_x = ma_x$ to the cart and the suspended mass:

$$T - m_1 g\sin\theta = m_1 a_1$$
and
$$m_2 g - T = m_2 a_2$$

Letting *a* represent the common accelerations of the two objects, eliminate *T* between the two equations and solve *a*:

$$a = \frac{m_2 - m_1\sin\theta}{m_1 + m_2}g$$

Substitute numerical values and evaluate *a*:

$$a = \frac{0.075\,\text{kg} - (0.270\,\text{kg})\sin 30°}{0.075\,\text{kg} + 0.270\,\text{kg}} \times (9.81\,\text{m/s}^2) = \boxed{-1.71\,\text{m/s}^2}$$

i.e., the acceleration is down the incline.

Substitute for *a* in either of the force equations to obtain:

$$T = \boxed{0.863\,\text{N}}$$

(*c*) Using a constant-acceleration equation, relate the displacement of the cart down the incline to its initial speed and acceleration:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$
or, because $v_0 = 0$,
$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(1\,\text{m})}{1.71\,\text{m/s}^2}} = \boxed{1.08\,\text{s}}$$

97 ••

Picture the Problem Note that, while the mass of the rope is distributed over its length, the rope and the block have a common acceleration. Because the surface is horizontal and smooth, the only force that influences our solution is $\vec{F}$. The figure misrepresents the situation in that each segment of the rope experiences a gravitational force; the combined effect of which is that the rope must sag.

(*a*) Apply $\vec{a} = \vec{F}_{\text{net}}/m_{\text{tot}}$ to the rope-block system to obtain:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

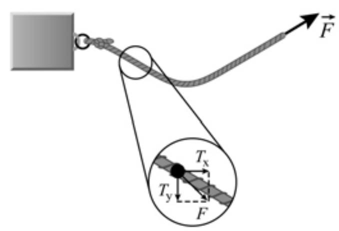
(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the rope, substitute the acceleration of the system obtained in (*a*), and simplify to obtain:

$$F_{\text{net}} = m_2 a = m_2\left(\frac{F}{m_1 + m_2}\right) = \boxed{\frac{m_2}{m_1 + m_2}F}$$

(*c*) Apply $\sum \vec{F} = m\vec{a}$ to the block, substitute the acceleration of the system obtained in (*a*), and simplify to obtain:

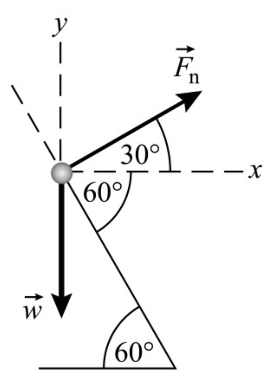
$$T = m_1 a = m_1\left(\frac{F}{m_1 + m_2}\right) = \boxed{\frac{m_1}{m_1 + m_2}F}$$

(*d*) The rope sags and so $\vec{F}$ has both vertical and horizontal components; with its horizontal component being less than $\vec{F}$. Consequently, *a* will be somewhat smaller.



***98 ••**

Picture the Problem The free-body diagram shows the forces acting on the block. Choose the coordinate system shown on the diagram. Because the surface of the wedge is frictionless, the force it exerts on the block must be normal to its surface.



(*a*) Apply $\sum F_y = ma_y$ to the block to obtain:

$$F_n \sin 30° - w = ma_y$$

or, because $a_y = 0$ and $w = mg$,

$$F_n \sin 30° - mg = 0$$

or

$$F_n \sin 30° = mg \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$F_n \cos 30° = ma_x \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{a_x}{g} = \cot 30°$$

Solve for and evaluate a_x:

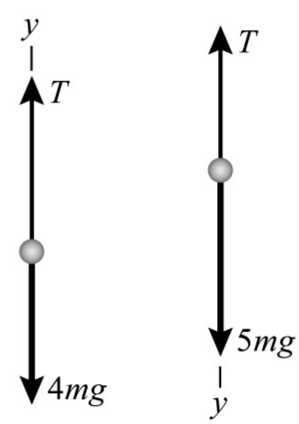
$$a_x = g \cot 30° = (9.81\,\text{m/s}^2)\cot 30° = \boxed{17.0\,\text{m/s}^2}$$

(*b*) An acceleration of the wedge greater than $g \cot 30°$ would require that the normal force exerted on the body by the wedge be greater than that given in part (*a*); i.e., $F_n > mg/\sin 30°$.

Under this condition, there would be a net force in the y direction and the block would accelerate up the wedge.

99 ••

Picture the Problem Because the system is initially in equilibrium, it follows that $T_0 = 5mg$. When one washer is removed on the left side, the remaining washers will accelerate upward (and those on the right side downward) in response to the net force that results. The free-body diagrams show the forces under this unbalanced condition. Applying Newton's 2nd law to each collection of washers will allow us to determine both the acceleration of the system and the mass of a single washer.



(*a*) Apply $\sum F_y = ma_y$ to the rising masses:

$$T - 4mg = (4m)a \qquad (1)$$

Apply $\sum F_y = ma_y$ to the descending masses:

$$5mg - T = (5m)a \qquad (2)$$

Eliminate T between these equations to obtain:

$$a = \tfrac{1}{9}g$$

Use this acceleration in equation (1) or equation (2) to obtain:

$$T = \frac{40}{9}mg$$

Express the difference between T_0 and T and solve for m:

$$T_0 - T = 5mg - \frac{40}{9}mg = 0.3\,\text{N}$$

and

$m = \boxed{0.0550\,\text{kg} = 55.0\,\text{g}}$

(*b*) Proceed as in (*a*) to obtain:

$T - 3mg = 3ma$
and
$5mg - T = 5ma$

Eliminate T and solve for a:

$a = \frac{1}{4}g = \frac{1}{4}(9.81\,\text{m/s}^2) = \boxed{2.45\,\text{m/s}^2}$

Eliminate a in either of the motion equations and solve for T to obtain:

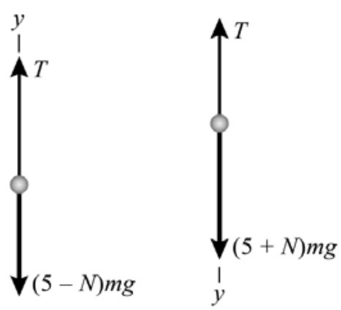
$$T = \frac{15}{4}mg$$

Substitute numerical values and evaluate T:

$$T = \frac{15}{4}(0.0550\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{2.03\,\text{N}}$$

100 ••

Picture the Problem The free-body diagram represents the Atwood's machine with N washers moved from the left side to the right side. Application of Newton's 2nd law to each collection of washers will result in two equations that can be solved simultaneously to relate N, a, and g. The acceleration can then be found from the given data.



Apply $\sum F_y = ma_y$ to the rising washers:

$T - (5 - N)mg = (5 - N)ma$

Apply $\sum F_y = ma_y$ to the descending washers:

$(5 + N)mg - T = (5 + N)ma$

Add these equations to eliminate T:

$$(5 + N)mg - (5 - N)mg = (5 - N)ma + (5 + N)ma$$

Simplify to obtain:

$2Nmg = 10ma$

Solve for N:

$N = 5a/g$

Using a constant-acceleration equation, relate the distance the washers fell to their time of fall:

$\Delta y = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$
or, because $v_0 = 0$,
$\Delta y = \frac{1}{2}a(\Delta t)^2$

Solve for the acceleration:

$$a = \frac{2\Delta y}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

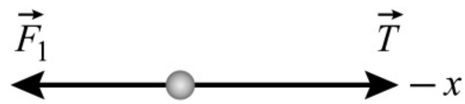
$$a = \frac{2(0.471\,\text{m})}{(0.40\,\text{s})^2} = 5.89\,\text{m/s}^2$$

Substitute in the expression for N:

$$N = 5\left(\frac{5.89\,\text{m/s}^2}{9.81\,\text{m/s}^2}\right) = \boxed{3}$$

101 ••

Picture the Problem Draw the free-body diagram for the block of mass m and apply Newton's 2nd law to obtain the acceleration of the system and then the tension in the rope connecting the two blocks.



(a) Letting T be the tension in the connecting string, apply $\sum F_x = ma_x$ to the block of mass m:

$$T - F_1 = ma$$

Apply $\sum F_x = ma_x$ to both blocks to determine the acceleration of the system:

$$F_2 - F_1 = (m + 2m)a = (3m)a$$

Substitute and solve for a:

$$a = (F_2 - F_1)/3m$$

Substitute for a in the first equation and solve for T:

$$T = \boxed{\tfrac{1}{3}(F_2 + 2F_1)}$$

(b) Substitute for F_1 and F_2 in the equation derived in part (a):

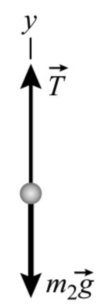
$$T = (2Ct + 2Ct)/3 = 4Ct/3$$

Evaluate this expression for $T = T_0$ and $t = t_0$ and solve for t_0:

$$t_0 = \boxed{\frac{3T_0}{4C}}$$

***102** •••

Picture the Problem Because a constant-upward acceleration has the same effect as an increase in the acceleration due to gravity, we can use the result of Problem 89 (for the tension) with a replaced by $a + g$. The application of Newton's 2nd law to the object whose mass is m_2 will connect the acceleration of this body to tension from Problem 84.



In Problem 84 it is given that, when the support pulley is not accelerating, the tension in the rope and the acceleration of the masses are related according to:

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

Replace a with $a + g$:

$$T = \frac{2m_1m_2}{m_1 + m_2}(a + g)$$

Apply $\sum F_y = ma_y$ to the object whose mass is m_2 and solve for a_2:

$$T - m_2g = m_2a_2$$

and

$$a_2 = \frac{T - m_2g}{m_2}$$

Substitute for T and simplify to obtain:

$$\boxed{a_2 = \frac{(m_1 - m_2)g + 2m_1a}{m_1 + m_2}}$$

The expression for a_1 is the same as for a_2 with all subscripts interchanged (note that a positive value for a_1 represents acceleration upward):

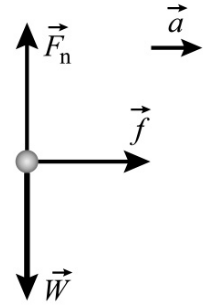
$$\boxed{a_1 = \frac{(m_2 - m_1)g + 2m_2a}{m_1 + m_2}}$$

Chapter 5
Applications of Newton's Laws

Conceptual Problems

1 •

Determine the Concept Because the objects are speeding up (accelerating), there must be a net force acting on them. The forces acting on an object are the normal force exerted by the floor of the truck, the weight of the object, and the friction force; also exerted by the floor of the truck.

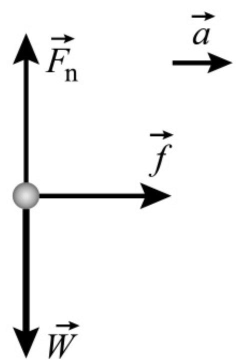


Of these forces, the only one that acts in the direction of the acceleration (chosen to be to the right in the free-body diagram) is the friction force.

The force of friction between the object and the floor of the truck must be the force that causes the object to accelerate.

***2** •

Determine the Concept The forces acting on an object are the normal force exerted by the floor of the truck, the weight of the object, and the friction force; also exerted by the floor of the truck. Of these forces, the only one that acts in the direction of the acceleration (chosen to be to the right in the free-body diagram) is the friction force. Apply Newton's 2nd law to the object to determine how the critical acceleration depends on its weight.



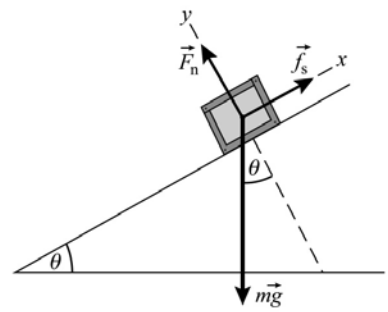
Taking the positive x direction to be to the right, apply $\Sigma F_x = ma_x$ and solve for a_x:

$$f = \mu_s w = \mu_s mg = ma_x$$
and
$$a_x = \mu_s g$$

Because a_x is independent of m and w, the critical accelerations are the same.

3 •

Determine the Concept The forces acting on the block are the normal force $\vec{F}_n$ exerted by the incline, the weight of the block $m\vec{g}$ exerted by the earth, and the static friction force $\vec{f}_s$ exerted by an external agent. We can use the definition of μ_s and the conditions for equilibrium to determine the relationship between μ_s and θ.



Apply $\sum F_x = ma_x$ to the block: $f_s - mg\sin\theta = 0$ (1)

Apply $\sum F_y = ma_y$ in the y direction: $F_n - mg\cos\theta = 0$ (2)

Divide equation (1) by equation (2) to obtain: $\tan\theta = \dfrac{f_s}{F_n}$

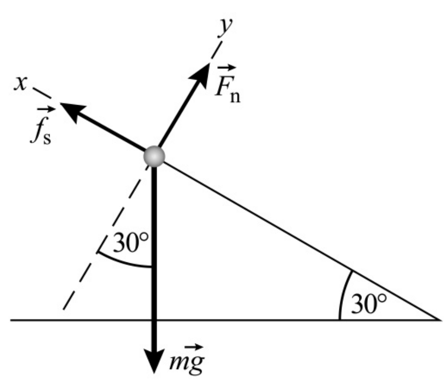
Substitute for f_s ($\le \mu_s F_n$): $\tan\theta \le \dfrac{\mu_s F_n}{F_n} = \mu_s$

and $\boxed{(d) \text{ is correct.}}$

***4** •

Determine the Concept The block is in equilibrium under the influence of $\vec{F}_n$, $m\vec{g}$, and $\vec{f}_s$; i.e.,

$$\vec{F}_n + m\vec{g} + \vec{f}_s = 0$$



We can apply Newton's 2nd law in the x direction to determine the relationship between f_s and mg.

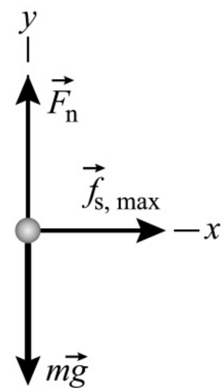
Apply $\sum F_x = 0$ to the block: $f_s - mg\sin\theta = 0$

Solve for f_s: $f_s = mg\sin\theta$

and $\boxed{(d) \text{ is correct.}}$

5 ••

Picture the Problem The forces acting on the car as it rounds a curve of radius R at maximum speed are shown on the free-body diagram to the right. The centripetal force is the static friction force exerted by the roadway on the tires. We can apply Newton's 2nd law to the car to derive an expression for its maximum speed and then compare the speeds under the two friction conditions described.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = f_{s,max} = m\frac{v_{max}^2}{R}$$

and

$$\sum F_y = F_n - mg = 0$$

From the y equation we have: $F_n = mg$

Express $f_{s,max}$ in terms of F_n in the x equation and solve for v_{max}:

$$v_{max} = \sqrt{\mu_s gR}$$

or

$$v_{max} = \text{constant}\sqrt{\mu_s}$$

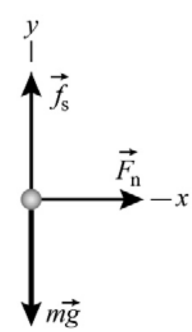
Express v'_{max} for $\mu'_s = \frac{1}{2}\mu_s$:

$$v'_{max} = \text{constant}\sqrt{\frac{\mu_s}{2}} = .707v_{max} \approx 71\%v_{max}$$

and $\boxed{(b) \text{ is correct.}}$

***6 ••**

Picture the Problem The normal reaction force F_n provides the centripetal force and the force of static friction, $\mu_s F_n$, keeps the cycle from sliding down the wall. We can apply Newton's 2nd law and the definition of $f_{s,max}$ to derive an expression for v_{min}.



Apply $\sum \vec{F} = m\vec{a}$ to the motorcycle:

$$\sum F_x = F_n = m\frac{v^2}{R}$$

and

$$\sum F_y = f_s - mg = 0$$

For the minimum speed: $f_s = f_{s,max} = \mu_s F_n$

Substitute for f_s, eliminate F_n between the force equations, and solve for v_{min}:

$$v_{min} = \sqrt{\frac{Rg}{\mu_s}}$$

Assume that $R = 6$ m and $\mu_s = 0.8$ and solve for v_{min}:

$$v_{min} = \sqrt{\frac{(6\,\text{m})(9.81\,\text{m/s}^2)}{0.8}} = \boxed{8.58\,\text{m/s} = 30.9\,\text{km/h}}$$

7 ••

Determine the Concept As the spring is extended, the force exerted by the spring on the block increases. Once that force is greater than the maximum value of the force of static friction on the block, the block will begin to move. However, as it accelerates, it will shorten the length of the spring, decreasing the force that the spring exerts on the block. As this happens, the force of kinetic friction can then slow the block to a stop, which starts the cycle over again. One interesting application of this to the real world is the bowing of a violin string: The string under tension acts like the spring, while the bow acts as the block, so as the bow is dragged across the string, the string periodically sticks and frees itself from the bow.

8 •

True. The velocity of an object moving in a circle is continually changing independently of whether the object's speed is changing. The change in the velocity vector and the acceleration vector and the net force acting on the object all point toward the center of circle. This center-pointing force is called a centripetal force.

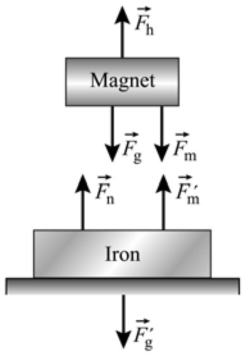
9 •

Determine the Concept A particle traveling in a vertical circle experiences a downward gravitational force plus an additional force that constrains it to move along a circular path. Because the net force acting on the particle will vary with location along its trajectory, neither (*b*), (*c*), nor (*d*) can be correct. Because the velocity of a particle moving along a circular path is continually changing, (*a*) cannot be correct. $\boxed{(e)\text{ is correct.}}$

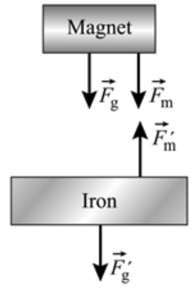
***10** •

Determine the Concept We can analyze these demonstrations by drawing force diagrams for each situation. In both diagrams, h denotes ″hand″, g denotes ″gravitational″, m denotes ″magnetic″, and n denotes ″normal″.

(*a*) Demonstration 1:



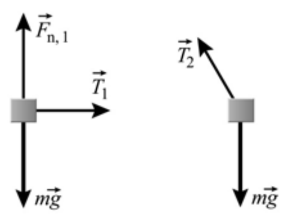
Demonstration 2:



(*b*) Because the magnet doesn't lift the iron in the first demonstration, the force exerted on the iron must be less than its (the iron's) weight. This is still true when the two are falling, but the motion of the iron is not restrained by the table, and the motion of the magnet is not restrained by the hand. Looking at the second diagram, the net force pulling the magnet down is greater than its weight, implying that its acceleration is greater than g. The opposite is true for the iron: the magnetic force acts upwards, slowing it down, so its acceleration will be less than g. Because of this, the magnet will catch up to the iron piece as they fall.

***11 •••**

Picture the Problem The free-body diagrams show the forces acting on the two objects some time after block 2 is dropped. Note that, while $\vec{T}_1 \neq \vec{T}_2$, $T_1 = T_2$.



The only force pulling block 2 to the left is the horizontal component of the tension. Because this force is smaller than the magnitude of the tension, the acceleration of block 1, which is identical to block 2, to the right ($T_1 = T_2$) will always be greater than the acceleration of block 2 to the left.

Because the initial distance from block 1 to the pulley is the same as the initial distance of block 2 to the wall, block 1 will hit the pulley before block 2 hits the wall.

12 •

True. The terminal speed of an object is given by $v_t = (mg/b)^{1/n}$, where b depends on the shape and area of the falling object as well as upon the properties of the medium in which the object is falling.

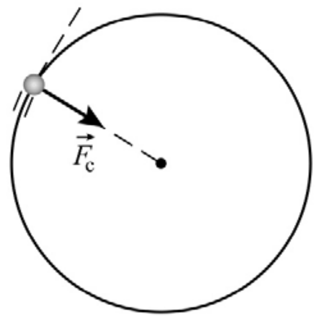
13 •

Determine the Concept The terminal speed of a sky diver is given by $v_t = (mg/b)^{1/n}$, where b depends on the shape and area of the falling object as well as upon the properties of the medium in which the object is falling. The sky diver's orientation as she falls determines the surface area she presents to the air molecules that must be pushed aside.

$\boxed{(d) \text{ is correct.}}$

14 ••

Determine the Concept In your frame of reference (the accelerating reference frame of the car), the direction of the force must point toward the center of the circular path along which you are traveling; that is, in the direction of the centripetal force that keeps you moving in a circle. The friction between you and the seat you are sitting on supplies this force. The reason you seem to be "pushed" to the outside of the curve is that your body's inertia "wants" , in accordance with Newton's law of inertia, to keep it moving in a straight line–that is, tangent to the curve.



***15 •**

Determine the Concept The centripetal force that keeps the moon in its orbit around the earth is provided by the gravitational force the earth exerts on the moon. As described by Newton's 3rd law, this force is equal in magnitude to the force the moon exerts on the earth. $\boxed{(d) \text{ is correct.}}$

16 •

Determine the Concept The only forces acting on the block are its weight and the force the surface exerts on it. Because the loop-the-loop surface is frictionless, the force it exerts on the block must be perpendicular to its surface.

Point A: the weight is downward and the normal force is to the right.	Free-body diagram 3
Point B: the weight is downward, the normal force is upward, and the normal force is greater than the weight so that their difference is the centripetal force.	Free-body diagram 4
Point C: the weight is downward and the normal force is to the left.	Free-body diagram 5
Point D: both the weight and the normal forces are downward.	Free-body diagram 2

17 ••

Picture the Problem Assume that the drag force on an object is given by the Newtonian formula $F_D = \frac{1}{2}CA\rho v^2$, where A is the projected surface area, v is the object's speed, ρ is the density of air, and C a dimensionless coefficient.

Express the net force acting on the falling object:

$$F_{net} = mg - F_D = ma$$

Substitute for F_D under terminal speed conditions and solve for the terminal speed:

$$mg - \frac{1}{2}CA\rho v_T^2 = 0$$

or

$$v_T = \sqrt{\frac{2mg}{CA\rho}}$$

Thus, the terminal velocity depends on the ratio of the mass of the object to its surface area.

For a rock, which has a relatively small surface area compared to its mass, the terminal speed will be relatively high; for a lightweight, spread-out object like a feather, the opposite is true.

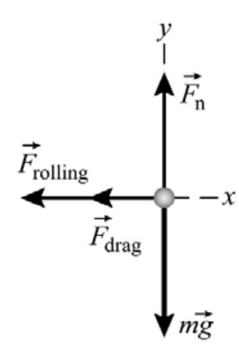
Another issue is that the higher the terminal velocity is, the longer it takes for a falling object to reach terminal velocity. From this, the feather will reach its terminal velocity quickly, and fall at an almost constant speed very soon after being dropped; a rock, if not dropped from a great height, will have almost the same acceleration as if it were in free-fall for the duration of its fall, and thus be continually speeding up as it falls.

An interesting point is that the average drag force acting on the rock will be larger than that acting on the feather precisely *because* the rock's average speed is larger than the feather's, as the drag force increases as v^2. This is another reminder that force is not the same thing as acceleration.

Estimation and Approximation

***18 •**

Picture the Problem The free-body diagram shows the forces on the Tercel as it slows from 60 to 55 mph. We can use Newton's 2nd law to calculate the average force from the rate at which the car's speed decreases and the rolling force from its definition. The drag force can be inferred from the average and rolling friction forces and the drag coefficient from the defining equation for the drag force.



(*a*) Apply $\sum F_x = ma_x$ to the car to relate the average force acting on it to its average velocity:

$$F_{av} = ma_{av} = m\frac{\Delta v}{\Delta t}$$

Substitute numerical values and evaluate F_{av}:

$$F_{\text{av}} = (1020\,\text{kg})\frac{5\dfrac{\text{mi}}{\text{h}}\times 1.609\dfrac{\text{km}}{\text{mi}}\times\dfrac{1\,\text{h}}{3600\,\text{s}}\times\dfrac{1000\,\text{m}}{\text{km}}}{3.92\,\text{s}} = \boxed{581\,\text{N}}$$

(*b*) Using its definition, express and evaluate the force of rolling friction:

$$f_{\text{rolling}} = \mu_{\text{rolling}}F_{\text{n}} = \mu_{\text{rolling}}mg$$
$$= (0.02)(1020\,\text{kg})(9.81\,\text{m/s}^2)$$
$$= \boxed{200\,\text{N}}$$

Assuming that only two forces are acting on the car in the direction of its motion, express their relationship and solve for and evaluate the drag force:

$$F_{\text{av}} = F_{\text{drag}} + F_{\text{rolling}}$$
and
$$F_{\text{drag}} = F_{\text{av}} - F_{\text{rolling}}$$
$$= 581\,\text{N} - 200\,\text{N} = \boxed{381\,\text{N}}$$

(*c*) Convert 57.5 mi/h to m/s:

$$57.5\frac{\text{mi}}{\text{h}} = 57.5\frac{\text{mi}}{\text{h}}\times\frac{1.609\,\text{km}}{\text{mi}}$$
$$\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{10^3\,\text{m}}{\text{km}}$$
$$= 25.7\,\text{m/s}$$

Using the definition of the drag force and its calculated value from (*b*) and the average speed of the car during this 5 mph interval, solve for *C*:

$$F_{\text{drag}} = \tfrac{1}{2}C\rho Av^2 \Rightarrow C = \frac{2F_{\text{drag}}}{\rho Av^2}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{2(381\,\text{N})}{(1.21\,\text{kg/m}^3)(1.91\,\text{m}^2)(25.7\,\text{m/s})^2}$$
$$= \boxed{0.499}$$

19 •

Picture the Problem We can use the dimensions of force and velocity to determine the dimensions of the constant *b* and the dimensions of ρ, r, and v to show that, for $n = 2$, Newton's expression is consistent dimensionally with our result from part (*b*). In parts (*d*) and (*e*), we can apply Newton's 2nd law under terminal velocity conditions to find the terminal velocity of the sky diver near the surface of the earth and at a height of 8 km.

(*a*) Solve the drag force equation for *b* with $n = 1$:

$$b = \frac{F_{\text{d}}}{v}$$

Substitute the dimensions of F_d and v and simplify to obtain:

$$[b] = \frac{\frac{ML}{T^2}}{\frac{L}{T}} = \boxed{\frac{M}{T}}$$

and the units of b are $\boxed{\text{kg/s}}$

(*b*) Solve the drag force equation for b with $n = 2$:

$$b = \frac{F_d}{v^2}$$

Substitute the dimensions of F_d and v and simplify to obtain:

$$[b] = \frac{\frac{ML}{T^2}}{\left(\frac{L}{T}\right)^2} = \boxed{\frac{M}{L}}$$

and the units of b are $\boxed{\text{kg/m}}$

(*c*) Express the dimensions of Newton's expression:

$$[F_d] = \left[\tfrac{1}{2}\rho\pi r^2 v^2\right] = \left(\frac{M}{L^3}\right)(L)^2\left(\frac{L}{T}\right)^2$$

$$= \boxed{\frac{ML}{T^2}}$$

From part (*b*) we have:

$$[F_d] = \left[bv^2\right] = \left(\frac{M}{L}\right)\left(\frac{L}{T}\right)^2$$

$$= \boxed{\frac{ML}{T^2}}$$

(*d*) Letting the downward direction be the positive y direction, apply $\sum F_y = ma_y$ to the sky diver:

$$mg - \tfrac{1}{2}\rho\pi r^2 v_t^2 = 0$$

Solve for and evaluate v_t:

$$v_t = \sqrt{\frac{2mg}{\rho\pi r^2}} = \sqrt{\frac{2(56\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(1.2\,\text{kg/m}^3)(0.3\,\text{m})^2}}$$

$$= \boxed{56.9\,\text{m/s}}$$

(*e*) Evaluate v_t at a height of 8 km:

$$v_t = \sqrt{\frac{2(56\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(0.514\,\text{kg/m}^3)(0.3\,\text{m})^2}}$$

$$= \boxed{86.9\,\text{m/s}}$$

20 ••

Picture the Problem From Newton's 2nd law, the equation describing the motion of falling raindrops and large hailstones is $mg - F_d = ma$ where $F_d = \frac{1}{2}\rho\pi r^2 v^2 = bv^2$ is the drag force. Under terminal speed conditions ($a = 0$), the drag force is equal to the weight of the falling object. Take the radius of a raindrop r_r to be 0.5 mm and the radius of a golf-ball sized hailstone r_h to be 2 cm.

Using $b = \frac{1}{2}\pi\rho r^2$, evaluate b_r and b_h:

$$b_r = \tfrac{1}{2}\pi(1.2\,\text{kg/m}^3)(0.5\times10^{-3}\,\text{m})^2 = 4.71\times10^{-7}\,\text{kg/m}$$

and

$$b_h = \tfrac{1}{2}\pi(1.2\,\text{kg/m}^3)(2\times10^{-2}\,\text{m})^2 = 7.54\times10^{-4}\,\text{kg/m}$$

Express the mass of a sphere in terms of its volume and density:

$$m = \rho V = \frac{4\pi r^3\rho}{3}$$

Using $\rho_r = 10^3$ kg/m^3 and $\rho_h = 920$ kg/m^3, evaluate m_r and m_h:

$$m_r = \frac{4\pi(0.5\times10^{-3}\,\text{m})^3(10^3\,\text{kg/m}^3)}{3} = 5.24\times10^{-7}\,\text{kg}$$

and

$$m_h = \frac{4\pi(2\times10^{-2}\,\text{m})^3(920\,\text{kg/m}^3)}{3} = 3.08\times10^{-2}\,\text{kg}$$

Express the relationship between v_t and the weight of a falling object under terminal speed conditions and solve for v_t:

$$bv_t^2 = mg \Rightarrow v_t = \sqrt{\frac{mg}{b}}$$

Use numerical values to evaluate $v_{t,r}$ and $v_{t,h}$:

$$v_{t,r} = \sqrt{\frac{(5.24\times10^{-7}\,\text{kg})(9.81\,\text{m/s}^2)}{4.71\times10^{-7}\,\text{kg/m}}} = \boxed{3.30\,\text{m/s}}$$

and

$$v_{t,h} = \sqrt{\frac{(3.08\times10^{-2}\,\text{kg})(9.81\,\text{m/s}^2)}{7.54\times10^{-4}\,\text{kg/m}}} = \boxed{20.0\,\text{m/s}}$$

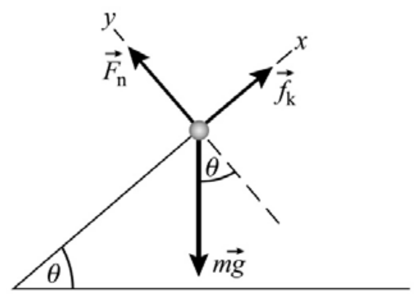
Friction

***21** •

Picture the Problem The block is in equilibrium under the influence of $\vec{F}_n$, $m\vec{g}$, and $\vec{f}_k$; i.e.,

$$\vec{F}_n + m\vec{g} + \vec{f}_k = 0$$

We can apply Newton's 2nd law to determine the relationship between f_k, θ, and mg.



Using its definition, express the coefficient of kinetic friction:

$$\mu_k = \frac{f_k}{F_n} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block: $f_k - mg\sin\theta = ma_x = 0$ because $a_x = 0$

Solve for f_k: $f_k = mg\sin\theta$

Apply $\sum F_y = ma_y$ to the block: $F_n - mg\cos\theta = ma_y = 0$ because $a_y = 0$

Solve for F_n: $F_n = mg\cos\theta$

Substitute in equation (1) to obtain:

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

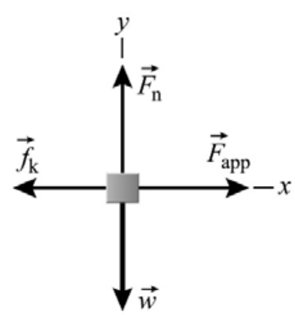
and **(b) is correct.**

22 •

Picture the Problem The block is in equilibrium under the influence of $\vec{F}_n$, $m\vec{g}$, $\vec{F}_{app}$, and $\vec{f}_k$; i.e.,

$$\vec{F}_n + m\vec{g} + \vec{F}_{app} + \vec{f}_k = 0$$

We can apply Newton's 2nd law to determine f_k.



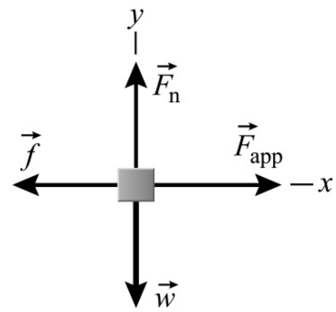
Apply $\sum F_x = ma_x$ to the block: $F_{app} - f_k = ma_x = 0$ because $a_x = 0$

Solve for f_k: $f_k = F_{app} = 20\ \text{N}$

and **(e) is correct.**

***23 •**

Picture the Problem Whether the friction force is that due to static friction or kinetic friction depends on whether the applied tension is greater than the maximum static friction force. We can apply the definition of the maximum static friction to decide whether $f_{s,max}$ or T is greater.



Calculate the maximum static friction force:

$$f_{s,max} = \mu_s F_n = \mu_s w = (0.8)(20\text{ N}) = 16\text{ N}$$

(*a*) Because $f_{s,max} > T$:

$$f = f_s = T = \boxed{15.0\text{ N}}$$

(*b*) Because $T > f_{s,max}$:

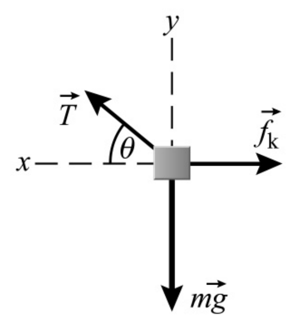
$$f = f_k = \mu_k w = (0.6)(20\text{ N}) = \boxed{12.0\text{ N}}$$

24 •

Picture the Problem The block is in equilibrium under the influence of the forces $\vec{T}$, $\vec{f}_k$, and $m\vec{g}$; i.e.,

$$\vec{T} + \vec{f}_k + m\vec{g} = 0$$

We can apply Newton's 2nd law to determine the relationship between T and f_k.



Apply $\sum F_x = ma_x$ to the block:

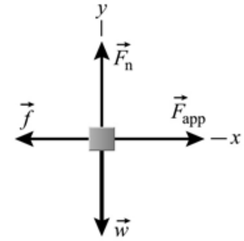
$$T\cos\theta - f_k = ma_x = 0 \text{ because } a_x = 0$$

Solve for f_k:

$$f_k = T\cos\theta \text{ and } \boxed{(b)\text{ is correct.}}$$

25 •

Picture the Problem Whether the friction force is that due to static friction or kinetic friction depends on whether the applied tension is greater than the maximum static friction force.



Calculate the maximum static

$$f_{s,max} = \mu_s F_n = \mu_s w$$

friction force:

$= (0.6)(100\ \text{kg})(9.81\ \text{m/s}^2)$
$= 589\ \text{N}$

Because $f_{s,\max} > F_{app}$, the box does not move and :

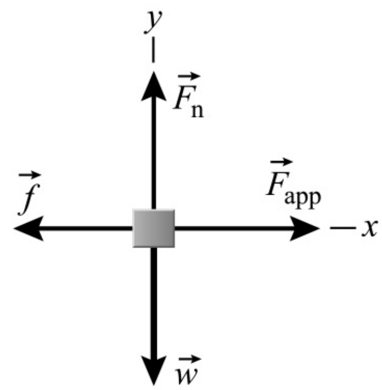
$F_{app} = f_s = \boxed{500\ \text{N}}$

26 •

Picture the Problem Because the box is moving with constant velocity, its acceleration is zero and it is in equilibrium under the influence of $\vec{F}_{app}$, $\vec{F}_n$, $\vec{w}$, and $\vec{f}$; i.e.,

$$\vec{F}_{app} + \vec{F}_n + \vec{w} + \vec{f} = 0$$

We can apply Newton's 2nd law to determine the relationship between f and mg.



The definition of μ_k is:

$$\mu_k = \frac{f_k}{F_n}$$

Apply $\sum F_y = ma_y$ to the box:

$F_n - w = ma_y = 0$ because $a_y = 0$

Solve for F_n:

$F_n = w = 600\ \text{N}$

Apply $\sum F_x = ma_x$ to the box:

$\Sigma F_x = F_{app} - f = ma_x = 0$ because $a_x = 0$

Solve for f_k:

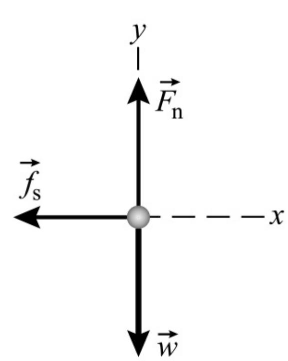
$F_{app} = f_k = 250\ \text{N}$

Substitute to obtain μ_k:

$\mu_k = (250\ \text{N})/(600\ \text{N}) = \boxed{0.417}$

27 •

Picture the Problem Assume that the car is traveling to the right and let the positive x direction also be to the right. We can use Newton's 2nd law of motion and the definition of μ_s to determine the maximum acceleration of the car. Once we know the car's maximum acceleration, we can use a constant-acceleration equation to determine the least stopping distance.



(a) Apply $\sum F_x = ma_x$ to the car: $-f_{s,max} = -\mu_s F_n = ma_x$ (1)

Apply $\sum F_y = ma_y$ to the car and solve for F_n:

$F_n - w = ma_y = 0$

or, because $a_y = 0$,

$F_n = mg$ (2)

Substitute (2) in (1) and solve for $a_{x,max}$:

$$a_{x,max} = \mu_s g = (0.6)(9.81\,\text{m/s}^2) = \boxed{-5.89\,\text{m/s}^2}$$

(b) Using a constant-acceleration equation, relate the stopping distance of the car to its initial velocity and its acceleration and solve for its displacement:

$v^2 = v_0^2 + 2a\Delta x$

or, because $v = 0$,

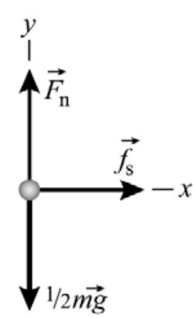
$$\Delta x = \frac{-v_0^2}{2a}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{-(30\,\text{m/s})^2}{2(-5.89\,\text{m/s}^2)} = \boxed{76.4\,\text{m}}$$

***28** •

Picture the Problem The free-body diagram shows the forces acting on the drive wheels, the ones we're assuming support half the weight of the car. We can use the definition of acceleration and apply Newton's 2nd law to the horizontal and vertical components of the forces to determine the minimum coefficient of friction between the road and the tires.



(a) $\boxed{\text{Because } \mu_s > \mu_k,\ f \text{ will be greater if the wheels do not slip.}}$

(b) Apply $\sum F_x = ma_x$ to the car: $f_s = \mu_s F_n = ma_x$ (1)

Apply $\sum F_y = ma_y$ to the car and solve for F_n:

$F_n - \frac{1}{2}mg = ma_y$

Because $a_y = 0$,

$F_n - \frac{1}{2}mg = 0 \Rightarrow F_n = \frac{1}{2}mg$

Find the acceleration of the car:

$$a_x = \frac{\Delta v}{\Delta t} = \frac{(90\,\text{km/h})(1000\,\text{m/km})}{12\,\text{s}}$$

$$= 2.08\,\text{m/s}^2$$

Solve equation (1) for μ_s:

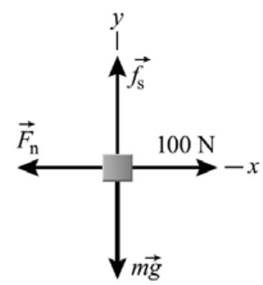
$$\mu_s = \frac{ma_x}{\frac{1}{2}mg} = \frac{2a_x}{g}$$

Substitute numerical values and evaluate a_x:

$$\mu_s = \frac{2(2.08\,\text{m/s}^2)}{9.81\,\text{m/s}^2} = \boxed{0.424}$$

29 •

Picture the Problem The block is in equilibrium under the influence of the forces shown on the free-body diagram. We can use Newton's 2nd law and the definition of μ_s to solve for f_s and F_n.



(*a*) Apply $\sum F_y = ma_y$ to the block and solve for f_s:

$$f_s - mg = ma_y$$

or, because $a_y = 0$,

$$f_s - mg = 0$$

Solve for and evaluate f_s:

$$f_s = mg = (5\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{49.1\,\text{N}}$$

(*b*) Use the definition of μ_s to express F_n:

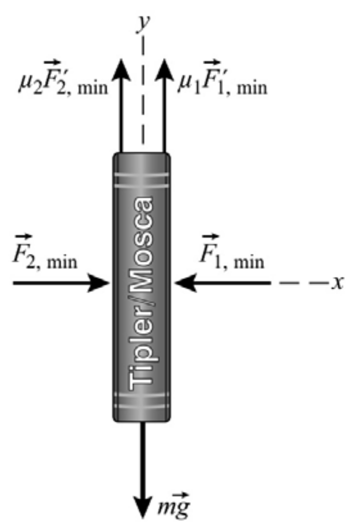
$$F_n = \frac{f_{s,max}}{\mu_s}$$

Substitute numerical values and evaluate F_n:

$$F_n = \frac{49.1\,\text{N}}{0.4} = \boxed{123\,\text{N}}$$

30 •

Picture the Problem The free-body diagram shows the forces acting on the book. The normal force is the net force the student exerts in squeezing the book. Let the horizontal direction be the *x* direction and upward the *y* direction. Note that the normal force is the same on either side of the book because it is not accelerating in the horizontal direction. The book could be accelerating downward. We can apply Newton's 2nd law to relate the minimum force required to hold the book in place to its mass and to the coefficients of static friction. In part (*b*), we can proceed similarly to relate the acceleration of the



book to the coefficients of kinetic friction.

(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the book:

$$\sum F_x = F_{2,\text{min}} - F_{1,\text{min}} = 0$$

and

$$\sum F_y = \mu_{s,1}F'_{1,\text{min}} + \mu_{s,2}F'_{2,\text{min}} - mg = 0$$

Noting that $F'_{1,\text{min}} = F'_{2,\text{min}}$, solve the y equation for F_{min}:

$$F_{\text{min}} = \frac{mg}{\mu_{,1} + \mu_{s,2}}$$

Substitute numerical values and evaluate F_{min}:

$$F_{\text{min}} = \frac{(10.2\,\text{kg})(9.81\,\text{m/s}^2)}{0.32 + 0.16} = \boxed{208\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ with the book accelerating downward, to obtain:

$$\sum F_y = \mu_{k,1}F + \mu_{k,2}F - mg = ma$$

Solve for a to obtain:

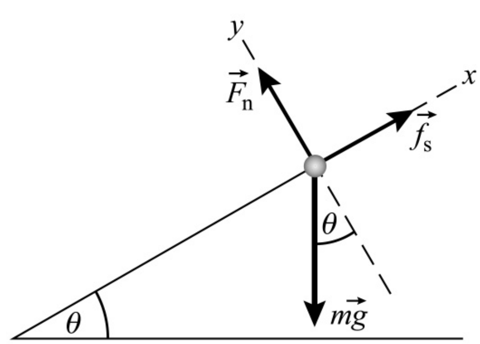
$$a = \frac{\mu_{k,} + \mu_{k,2}}{m}F - g$$

Substitute numerical values and evaluate a:

$$a = \frac{0.2 + 0.09}{10.2\,\text{kg}}(195\,\text{N}) - 9.81\,\text{m/s}^2 = \boxed{-4.27\,\text{m/s}^2}$$

31 •

Picture the Problem A free-body diagram showing the forces acting on the car is shown to the right. The friction force that the ground exerts on the tires is the force f_s shown acting up the incline. We can use the definition of the coefficient of static friction and Newton's 2nd law to relate the angle of the incline to the forces acting on the car.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = f_s - mg\sin\theta = 0 \qquad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \qquad (2)$$

Solve equation (1) for f_s and equation (2) for F_n:

$$f_s = mg\sin\theta$$

and

$$F_n = mg\cos\theta$$

Use the definition of μ_s to relate f_s and F_n:

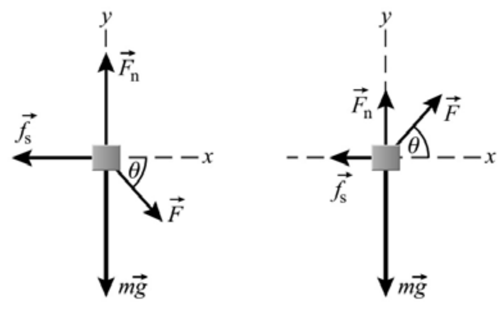
$$\mu_s = \frac{f_s}{F_n} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

Solve for and evaluate θ:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.08) = \boxed{4.57°}$$

***32** •

Picture the Problem The free-body diagrams for the two methods are shown to the right. Method 1 results in the box being pushed into the floor, increasing the normal force and the static friction force. Method 2 partially lifts the box,, reducing the normal force and the static friction force. We can apply Newton's 2nd law to obtain expressions that relate the maximum static friction force to the applied force $\vec{F}$.



(*a*) $\boxed{\text{Method 2 is preferable as it reduces } F_n \text{ and, therefore, } f_s.}$

(*b*) Apply $\sum F_x = ma_x$ to the box:

$$F\cos\theta - f_s = F\cos\theta - \mu_s F_n = 0$$

Method 1: Apply $\sum F_y = ma_y$ to the block and solve for F_n:

$$F_n - mg - F\sin\theta = 0$$
$$\therefore F_n = mg + F\sin\theta$$

Relate $f_{s,max}$ to F_n:

$$f_{s,max} = \mu_s F_n = \mu_s(mg + F\sin\theta) \qquad (1)$$

Method 2: Apply $\sum F_y = ma_y$ to the forces in the y direction and solve for F_n:

$$F_n - mg + F\sin\theta = 0$$
and
$$F_n = mg - F\sin\theta$$

Relate $f_{s,max}$ to F_n:

$$f_{s,max} = \mu_s F_n = \mu_s(mg - F\sin\theta) \qquad (2)$$

Express the condition that must be satisfied to move the box by either method:

$$f_{s,max} = F\cos\theta \qquad (3)$$

Method 1: Substitute (1) in (3) and solve for F:

$$F_1 = \frac{\mu_s mg}{\cos\theta - \mu_s\sin\theta} \qquad (4)$$

Method 2: Substitute (2) in (3) and solve for F:

$$F_2 = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \qquad (5)$$

Evaluate equations (4) and (5) with $\theta = 30°$:

$$F_1(30°) = \boxed{520\text{ N}}$$

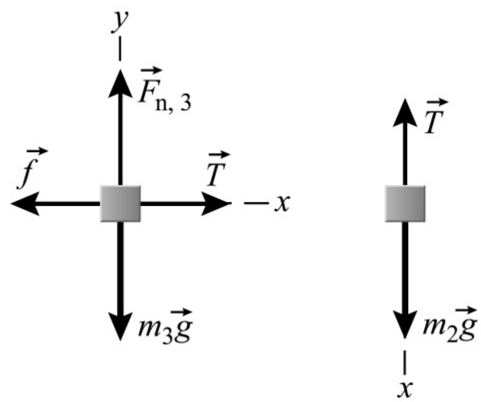
$$F_2(30°) = \boxed{252\text{ N}}$$

Evaluate (4) and (5) with $\theta = 0°$:

$$F_1(0°) = F_2(0°) = \mu_s mg = \boxed{294\text{ N}}$$

33 •

Picture the Problem Draw a free-body diagram for each object. In the absence of friction, the 3-kg box will move to the right, and the 2-kg box will move down. The friction force is indicated by $\vec{f}$ without subscript; it is $\vec{f}_s$ for (*a*) and $\vec{f}_k$ for (*b*). For values of μ_s less than the value found in part (a) required for equilibrium, the system will accelerate and the fall time for a given distance can be found using a constant-acceleration equation.



(*a*) Apply $\sum F_x = ma_x$ to the 3-kg box:

$$T - f_s = 0 \text{ because } a_x = 0 \qquad (1)$$

Apply $\sum F_y = ma_y$ to the 3-kg box, solve for $F_{n,3}$, and substitute in (1):

$$F_{n,3} - m_3 g = 0 \text{ because } a_y = 0$$

and

$$T - \mu_s m_3 g = 0 \qquad (2)$$

Apply $\sum F_x = ma_x$ to the 2-kg box:

$$m_2 g - T = 0 \text{ because } a_x = 0 \qquad (3)$$

Solve (2) and (3) simultaneously and solve for μ_s:

$$\mu_s = \frac{m_2}{m_3} = \boxed{0.667}$$

(*b*) The time of fall is related to the acceleration, which is constant:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2} a (\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a}} \qquad (4)$$

Apply $\sum F_x = ma_x$ to each box:

$$T - \mu_k m_3 g = m_3 a \quad (5)$$

and

$$m_2 g - T = m_2 a \quad (6)$$

Add equations (5) and (6) and solve for a:

$$a = \frac{(m_2 - \mu_k m_3)g}{m_2 + m_3}$$

Substitute numerical values and evaluate a:

$$a = \frac{[2\,\text{kg} - 0.3(3\,\text{kg})](9.81\,\text{m/s}^2)}{2\,\text{kg} + 3\,\text{kg}} = \boxed{2.16\,\text{m/s}^2}$$

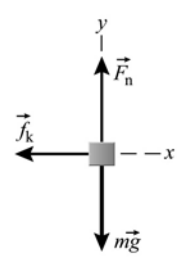
Substitute numerical values in equation (4) and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(2\,\text{m})}{2.16\,\text{m/s}^2}} = \boxed{1.36\,\text{s}}$$

34 ••

Picture the Problem The application of Newton's 2nd law to the block will allow us to express the coefficient of kinetic friction in terms of the acceleration of the block. We can then use a constant-acceleration equation to determine the block's acceleration. The pictorial representation summarizes what we know about the motion.

A free-body diagram showing the forces acting on the block is shown to the right.



Apply $\sum F_x = ma_x$ to the block:

$$-f_k = -\mu_k F_n = ma \quad (1)$$

Apply $\sum F_y = ma_y$ to the block and solve for F_n:

$F_n - mg = 0$ because $a_y = 0$

and

$$F_n = mg \quad (2)$$

Substitute (2) in (1) and solve for μ_k: $\mu_k = -a/g$ (3)

Using a constant-acceleration equation, relate the initial and final velocities of the block to its displacement and acceleration:

$$v_1^2 = v_0^2 + 2a\Delta x$$

or, because $v_1 = 0$, $v_0 = v$, and $\Delta x = d$,

$$0 = v^2 + 2ad$$

Solve for a to obtain:

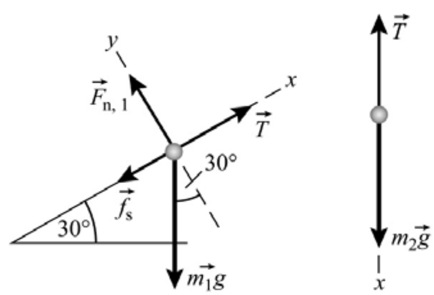
$$a = \frac{-v^2}{2d}$$

Substitute for a in equation (3) to obtain:

$$\mu_k = \boxed{\frac{v^2}{2gd}}$$

***35** ••

Picture the Problem We can find the speed of the system when it has moved a given distance by using a constant-acceleration equation. Under the influence of the forces shown in the free-body diagrams, the blocks will have a common acceleration a. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_k, will allow us to determine the acceleration of the system.



Using a constant-acceleration equation, relate the speed of the system to its acceleration and displacement; solve for its speed:

$$v^2 = v_0^2 + 2a\Delta x$$

and, because $v_0 = 0$,

$$v = \sqrt{2a\Delta x} \qquad (1)$$

Apply $\vec{F}_{net} = m\vec{a}$ to the block whose mass is m_1:

$$\Sigma F_x = T - f_k - m_1 g\sin 30° = m_1 a \qquad (2)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 30° = 0 \qquad (3)$$

Using $f_k = \mu_k F_n$, substitute (3) in (2) to obtain:

$$T - \mu_k m_1 g\cos 30° - m_1 g\sin 30° = m_1 a$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = m_2 a$$

Add the last two equations to eliminate T and solve for a to

$$a = \frac{(m_2 - \mu_k m_1\cos 30° - m_1\sin 30°)g}{m_1 + m_2}$$

obtain:

Substitute numerical values and evaluate a:

$$a = 1.16\,\text{m/s}^2$$

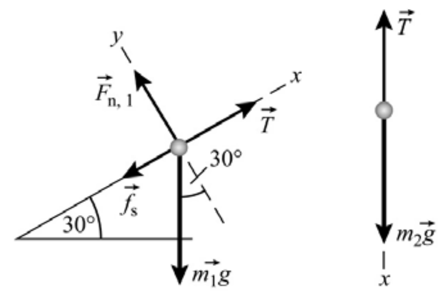
Substitute numerical values in equation (1) and evaluate v:

$$v = \sqrt{2(1.16\,\text{m/s}^2)(0.3\,\text{m})} = 0.835\,\text{m/s}$$

and $\boxed{(a)\text{ is correct.}}$

36 ••

Picture the Problem Under the influence of the forces shown in the free-body diagrams, the blocks are in static equilibrium. While f_s can be either up or down the incline, the free-body diagram shows the situation in which motion is impending up the incline. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_s, will allow us to determine the range of values for m_2.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\Sigma F_x = T \pm f_{s,max} - m_1 g\sin 30° = 0 \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 30° = 0 \quad (2)$$

Using $f_{s,max} = \mu_s F_n$, substitute (2) in (1) to obtain:

$$T \pm \mu_s m_1 g\cos 30° - m_1 g\sin 30° = m_1 a \quad (3)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = 0 \quad (4)$$

Add equations (3) and (4) to eliminate T and solve for m_2:

$$m_2 = m_1(\pm \mu_s \cos 30° + \sin 30°) = (4\,\text{kg})[\pm(0.4)\cos 30° + \sin 30°] \quad (5)$$

Evaluate (5) denoting the value of m_2 with the plus sign as $m_{2,+}$ and the value of m_2 with the minus sign as $m_{2,-}$ to determine the range of values of m_2 for which the system is in static equilibrium:

$$m_{2,+} = 3.39\,\text{kg and m}_{2,-} = 0.614\,\text{kg}$$

$$\therefore \boxed{0.614\,\text{kg} \le m_2 \le 3.39\,\text{kg}}$$

(b) With $m_2 = 1$ kg, the impending motion is down the incline and the static friction force is up the incline. Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T + f_s - m_1 g\sin 30° = 0 \quad (6)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

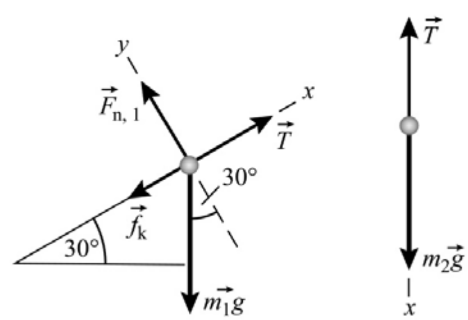
$$m_2 g - T = 0 \quad (7)$$

Add equations (6) and (7) and solve for and evaluate f_s:

$$f_s = (m_1 \sin 30° - m_2)g$$
$$= [(4\text{ kg})\sin 30° - 1\text{ kg}](9.81\text{ m/s}^2)$$
$$= \boxed{9.81\text{N}}$$

37 ••

Picture the Problem Under the influence of the forces shown in the free-body diagrams, the blocks will have a common acceleration a. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_k, will allow us to determine the acceleration of the system. Finally, we can substitute for the tension in either of the motion equations to determine the acceleration of the masses.



Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\Sigma F_x = T - f_k - m_1 g\sin 30° = m_1 a \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 30° = 0 \quad (2)$$

Using $f_k = \mu_k F_n$, substitute (2) in (1) to obtain:

$$T - \mu_k m_1 g\cos 30° - m_1 g\sin 30° = m_1 a \quad (3)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = m_2 a \quad (4)$$

Add equations (3) and (4) to eliminate T and solve for a to obtain:

$$a = \frac{(m_2 - \mu_k m_1 \cos 30° - m_1 \sin 30°)g}{m_1 + m_2}$$

Substituting numerical values and evaluating a yields:

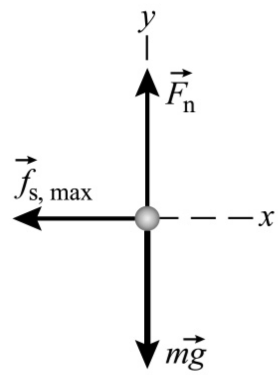
$$a = \boxed{2.36\text{ m/s}^2}$$

Substitute for a in equation (3) to obtain:

$$T = \boxed{37.3\,\text{N}}$$

***38 ••**

Picture the Problem The truck will stop in the shortest possible distance when its acceleration is a maximum. The maximum acceleration is, in turn, determined by the maximum value of the static friction force. The free-body diagram shows the forces acting on the box as the truck brakes to a stop. Assume that the truck is moving in the positive x direction and apply Newton's 2nd law and the definition of $f_{s,max}$ to find the shortest stopping distance.



Using a constant-acceleration equation, relate the truck's stopping distance to its acceleration and initial velocity; solve for the stopping distance:

$$v^2 = v_0^2 + 2a\Delta x$$

or, since $v = 0$,

$$\Delta x_{min} = \sqrt{\frac{-v_0^2}{2a_{max}}}$$

Apply $\vec{F}_{net} = m\vec{a}$ to the block:

$$\Sigma F_x = -f_{s,max} = ma_{max} \qquad (1)$$

and

$$\Sigma F_y = F_n - mg = 0 \qquad (2)$$

Using the definition of $f_{s,max}$, solve equations (1) and (2) simultaneously for a:

$$f_{s,max} \equiv \mu_s F_n$$

and

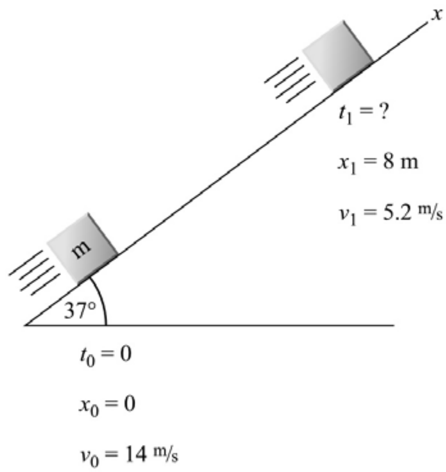
$$a_{max} = -\mu_s g = -(0.3)(9.81\text{ m/s}^2) = -2.943\text{ m/s}^2$$

Substitute numerical values and evaluate Δx_{min}:

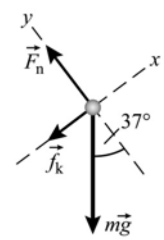
$$\Delta x_{min} = \sqrt{\frac{-(80\,\text{km/h})^2(1000\,\text{km/m})^2(1\,\text{h}/3600\,\text{s})^2}{2(-2.943\,\text{m/s}^2)}} = \boxed{9.16\,\text{m}}$$

39 ••

Picture the Problem We can find the coefficient of friction by applying Newton's 2nd law and determining the acceleration from the given values of displacement and initial velocity. We can find the displacement and speed of the block by using constant-acceleration equations. During its motion up the incline, the sum of the kinetic friction force and a component of the object's weight will combine to bring the object to rest. When it is moving down the incline, the difference between the weight component and the friction force will be the net force.



(*a*) Draw a free-body diagram for the block as it travels up the incline:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = -f_k - mg\sin 37° = ma \qquad (1)$$

and

$$\Sigma F_y = F_n - mg\cos 37° = 0 \qquad (2)$$

Substitute $f_k = \mu_k F_n$ and F_n from (2) in (1) and solve for μ_k:

$$\mu_k = \frac{-g\sin 37° - a}{g\cos 37°} = -\tan 37° - \frac{a}{g\cos 37°} \qquad (3)$$

Using a constant-acceleration equation, relate the final velocity of the block to its initial velocity, acceleration, and displacement:

$$v_1^2 = v_0^2 + 2a\Delta x$$

Solving for a yields:

$$a = \frac{v_1^2 - v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{(5.2\,\text{m/s})^2 - (14\,\text{m/s})^2}{2(8\,\text{m})} = -10.6\,\text{m/s}^2$$

Substitute for a in (3) to obtain:

$$\mu_k = -\tan 37° - \frac{-10.6\,\text{m/s}^2}{(9.81\,\text{m/s}^2)\cos 37°} = \boxed{0.599}$$

(b) Use the same constant-acceleration equation used above but with $v_1 = 0$ to obtain:

$$0 = v_0^2 + 2a\Delta x$$

Solve for Δx to obtain:

$$\Delta x = \frac{-v_0^2}{2a}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{-(14\,\text{m/s})^2}{2(-10.6\,\text{m/s}^2)} = \boxed{9.25\,\text{m}}$$

(c) When the block slides down the incline, f_k is in the positive x direction:

$$\Sigma F_x = f_k - mg\sin 37° = ma$$

and

$$\Sigma F_y = F_n - mg\cos 37° = 0$$

Solve for a as in part (a):

$$a = g(\mu_k \cos 37° - \sin 37°) = -1.21\,\text{m/s}^2$$

Use the same constant-acceleration equation used in part (b) to obtain:

$$v^2 = v_0^2 + 2a\Delta x$$

Set $v_0 = 0$ and solve for v:

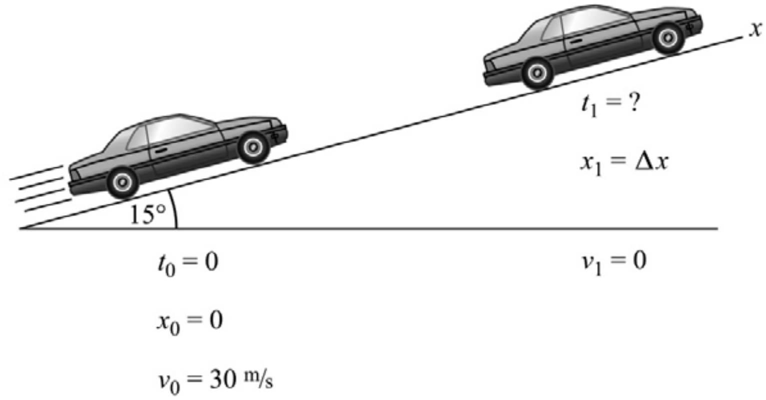
$$v = \sqrt{2a\Delta x}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(-1.21\,\text{m/s}^2)(-9.25\,\text{m})} = \boxed{4.73\,\text{m/s}}$$

40 ••

Picture the Problem We can find the stopping distances by applying Newton's 2nd law to the automobile and then using a constant-acceleration equation. The friction force the road exerts on the tires and the component of the car's weight along the incline combine to provide the net force that stops the car. The pictorial representation summarizes what we know about the motion of the car. We can use Newton's 2nd law to determine the acceleration of the car and a constant-acceleration equation to obtain its stopping distance.



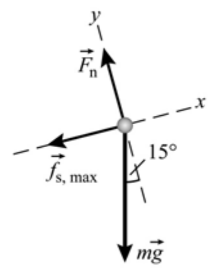
(*a*) Using a constant-acceleration equation, relate the final speed of the car to its initial speed, acceleration, and displacement; solve for its displacement:

$$v_1^2 = v_0^2 + 2a_{\max}\Delta x_{\min}$$

or, because $v_1 = 0$,

$$\Delta x_{\min} = \frac{-v_0^2}{2a_{\max}}$$

Draw the free-body diagram for the car going up the incline:



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\Sigma F_x = -f_{s,\max} - mg\sin 15° = ma \quad (1)$$

and

$$\Sigma F_y = F_n - mg\cos 15° = 0 \quad (2)$$

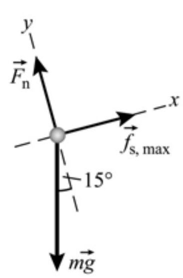
Substitute $f_{s,\max} = \mu_s F_n$ and F_n from (2) in (1) and solve for a:

$$a_{\max} = -g\left(\mu_s \cos 15° + \sin 15°\right) = -9.17\ \text{m/s}^2$$

Substitute numerical values in the expression for $\Delta x_{\min}$ to obtain:

$$\Delta x_{\min} = \frac{-(30\ \text{m/s})^2}{2\left(-9.17\ \text{m/s}^2\right)} = \boxed{49.1\ \text{m}}$$

(*b*) Draw the free-body diagram for the car going down the incline:



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\Sigma F_x = f_{s,max} - mg\sin 15° = ma$$
and
$$\Sigma F_y = F_n - mg\cos 15° = 0$$

Proceed as in (*a*) to obtain a_{max}:

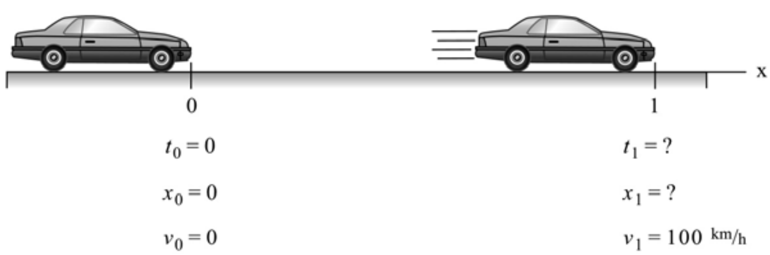
$$a_{max} = g(\mu_s \cos 15° - \sin 15°) = 4.09\,\text{m/s}^2$$

Again, proceed as in (*a*) to obtain the displacement of the car:

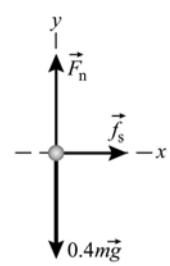
$$\Delta x_{min} = \left|\frac{-v_0^2}{2a_{max}}\right| = \frac{(30\,\text{m/s})^2}{2(4.09\,\text{m/s}^2)} = \boxed{110\,\text{m}}$$

41 ••

Picture the Problem The friction force the road exerts on the tires provides the net force that accelerates the car. The pictorial representation summarizes what we know about the motion of the car. We can use Newton's 2nd law to determine the acceleration of the car and a constant-acceleration equation to calculate how long it takes it to reach 100 km/h.



(*a*) Because 40% of the car's weight is on its two drive wheels and the accelerating friction forces act just on these wheels, the free-body diagram shows just the forces acting on the drive wheels.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\Sigma F_x = f_{s,max} = ma \qquad (1)$$
and
$$\Sigma F_y = F_n - 0.4mg = 0 \qquad (2)$$

Use the definition of $f_{s,max}$ in equation (1) and eliminate F_n between the two equations to obtain:

$$a = 0.4\mu_s g = 0.4(0.7)(9.81\,\text{m/s}^2) = \boxed{2.75\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the initial and final

$$v_1 = v_0 + a\Delta t$$

velocities of the car to its acceleration and the elapsed time; solve for the time:

or, because $v_0 = 0$ and $\Delta t = t_1$,

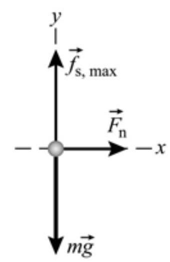
$$t_1 = \frac{v_1}{a}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \frac{(100\,\text{km/h})(1\,\text{h}/3600\,\text{s})(1000\,\text{m/km})}{2.75\,\text{m/s}^2} = \boxed{10.1\,\text{s}}$$

***42 ••**

Picture the Problem To hold the box in place, the acceleration of the cart and box must be great enough so that the static friction force acting on the box will equal the weight of the box. We can use Newton's 2nd law to determine the minimum acceleration required.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the box:

$$\Sigma F_x = F_n = ma_{min} \quad (1)$$

and

$$\Sigma F_y = f_{s,max} - mg = 0 \quad (2)$$

Substitute μF_n for $f_{s,max}$ in equation (2), eliminate F_n between the two equations and solve for and evaluate a_{min}:

$$\mu F_n - mg = 0, \quad \mu(ma_{min}) - mg = 0$$

and

$$a_{min} = \frac{g}{\mu_s} = \frac{9.81\,\text{m/s}^2}{0.6} = \boxed{16.4\,\text{m/s}^2}$$

(*b*) Solve equation (2) for $f_{s,max}$, and substitute numerical values and evaluate $f_{s,max}$:

$$f_{s,max} = mg = (2\text{ kg})(9.81\text{ m/s}^2) = \boxed{19.6\,\text{N}}$$

(*c*) If a is twice that required to hold the box in place, f_s will still have its maximum value given by:

$$f_{s,max} = \boxed{19.6\,\text{N}}$$

(*d*) $\boxed{\text{Because } g/\mu_s \text{ is } a_{min}\text{, the box will not fall if } a \geq g/\mu_s.}$

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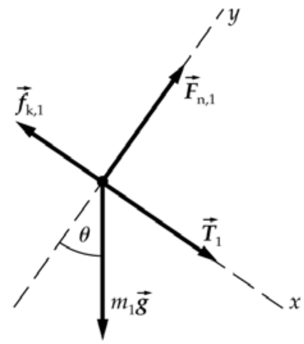
Picture the Problem Note that the blocks have a common acceleration and that the tension in the string acts on both blocks in accordance with Newton's third law of motion. Let down the incline be the positive *x* direction. Draw the free-body diagrams for each block and apply Newton's second law of motion and the definition of the kinetic friction force to each block to obtain simultaneous equations in a_x and T.

Draw the free-body diagram for the block whose mass is m_1:



Apply $\sum \vec{F} = m\vec{a}$ to the upper block:

$$\Sigma F_x = -f_{k,1} + T_1 + m_1 g\sin\theta = m_1 a_x \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos\theta = 0 \quad (2)$$

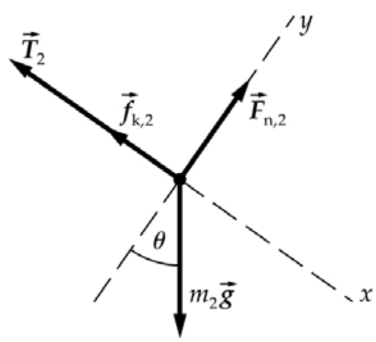
The relationship between $f_{k,1}$ and $F_{n,1}$ is:

$$f_{k,1} = \mu_{k,1} F_{n,1} \quad (3)$$

Eliminate $f_{k,1}$ and $F_{n,1}$ between (1), (2), and (3) to obtain:

$$-\mu_{k,1} m_1 g\cos\theta + T_1 + m_1 g\sin\theta = m_1 a_x \quad (4)$$

Draw the free-body diagram for the block whose mass is m_2:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = -f_{k,2} - T_2 + m_2 g\sin\theta = m_2 a_x \quad (5)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g\cos\theta = 0 \quad (6)$$

The relationship between $f_{k,2}$ and $F_{n,2}$ is:

$$f_{k,2} = \mu_{k,2} F_{n,2} \quad (7)$$

Eliminate $f_{k,2}$ and $F_{n,2}$ between (5), (6), and (7) to obtain:

$$-\mu_{k,2} m_2 g\cos\theta - T_2 + m_2 g\sin\theta = m_2 a_x \quad (8)$$

Noting that $T_2 = T_1 = T$, add equations (4) and (8) to eliminate T and solve for a_x:

$$a_x = \left[\sin\theta - \frac{\mu_{k,1}m_1 + \mu_{k,2}m_2}{m_1 + m_2}\cos\theta\right]g$$

Substitute numerical values and evaluate a_x to obtain:

$$a_x = \boxed{-0.965\,\text{m/s}^2}$$ where the minus sign tells us that the acceleration is directed up the incline.

(*b*) Eliminate a_x between equations (4) and (8) and solve for $T = T_1 = T_2$ to obtain:

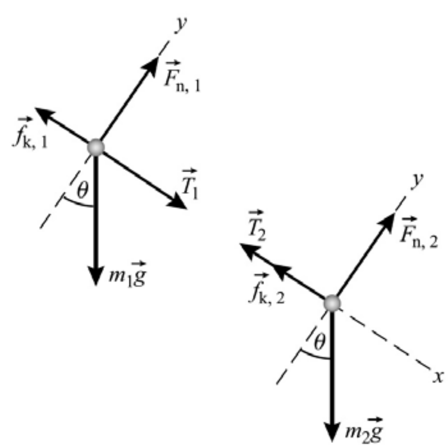
$$T = \frac{m_1 m_2(\mu_{k,2} - \mu_{k,1})g\cos\theta}{m_1 + m_2}$$

Substitute numerical values and evaluate T:

$$T = \boxed{0.184\,\text{N}}$$

***44** ••

Picture the Problem The free-body diagram shows the forces acting on the two blocks as they slide down the incline. Down the incline has been chosen as the positive x direction. T is the force transmitted by the stick; it can be either tensile ($T > 0$) or compressive ($T < 0$). By applying Newton's 2nd law to these blocks, we can obtain equations in T and a from which we can eliminate either by solving them simultaneously. Once we have expressed T, the role of the stick will become apparent.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to block 1:

$$\sum F_x = T_1 + m_1 g\sin\theta - f_{k,1} = m_1 a$$

and

$$\sum F_y = F_{n,1} - m_1 g\cos\theta = 0$$

Apply $\sum \vec{F} = m\vec{a}$ to block 2:

$$\sum F_x = m_2 g\sin\theta - T_2 - f_{k,2} = m_2 a$$

and

$$\sum F_y = F_{n,2} - m_2 g\cos\theta = 0$$

Letting $T_1 = T_2 = T$, use the definition of the kinetic friction force to eliminate $f_{k,1}$ and $F_{n,1}$ between the equations for block 1 and $f_{k,2}$ and $F_{n,1}$ between the equations for block 2 to obtain:

$$m_1 a = m_1 g\sin\theta + T - \mu_1 m_1 g\cos\theta \qquad (1)$$

and

$$m_2 a = m_2 g\sin\theta - T - \mu_2 m_2 g\cos\theta \qquad (2)$$

Add equations (1) and (2) to eliminate T and solve for a:

$$\boxed{a = g\left(\sin\theta - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}\cos\theta\right)}$$

(*b*) Rewrite equations (1) and (2) by dividing both sides of (1) by m_1 and both sides of (2) by m_2 to obtain.

$$a = g\sin\theta + \frac{T}{m_1} - \mu_1 g\cos\theta \qquad (3)$$

and

$$a = g\sin\theta - \frac{T}{m_2} - \mu_2 g\cos\theta \qquad (4)$$

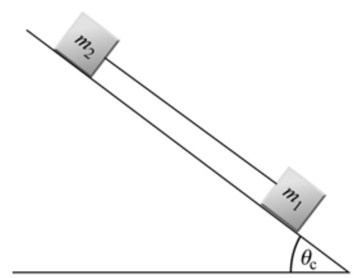
Subtracting (4) from (3) and rearranging yields:

$$\boxed{T = \left(\frac{m_1 m_2}{m_1 - m_2}\right)(\mu_1 - \mu_2)g\cos\theta}$$

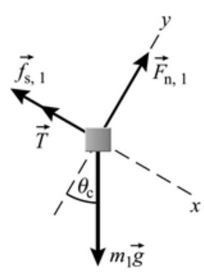
If $\mu_1 = \mu_2, T = 0$ and the blocks move down the incline with the same acceleration of $g(\sin\theta - \mu\cos\theta)$. Inserting a stick between them can't change this; therefore, the stick must exert no force on either block.

45 ••

Picture the Problem The pictorial representation shows the orientation of the two blocks on the inclined surface. Draw the free-body diagrams for each block and apply Newton's 2nd law of motion and the definition of the static friction force to each block to obtain simultaneous equations in θ_c and T.



(*a*) Draw the free-body diagram for the lower block:



Apply $\sum\vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = m_1 g\sin\theta_c - f_{s,1} - T = 0 \qquad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos\theta_c = 0 \qquad (2)$$

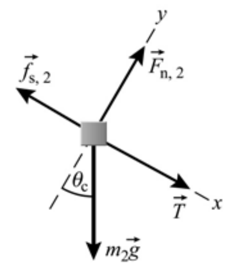
The relationship between $f_{s,1}$ and $F_{n,1}$ is:

$$f_{s,1} = \mu_{s,1}F_{n,1} \qquad (3)$$

Eliminate $f_{s,1}$ and $F_{n,1}$ between (1), (2), and (3) to obtain:

$$m_1 g\sin\theta_c - \mu_{s,1} m_1 g\cos\theta_c - T = 0 \quad (4)$$

Draw the free-body diagram for the upper block:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = T + m_2 g\sin\theta_c - f_{s,2} = 0 \quad (5)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g\cos\theta_c = 0 \quad (6)$$

The relationship between $f_{s,2}$ and $F_{n,2}$ is:

$$f_{s,2} = \mu_{s,2} F_{n,2} \quad (7)$$

Eliminate $f_{s,2}$ and $F_{n,2}$ between (5), (6), and (7) to obtain:

$$T + m_2 g\sin\theta_c - \mu_{s,2} m_2 g\cos\theta_c = 0 \quad (8)$$

Add equations (4) and (8) to eliminate T and solve for θ_c:

$$\theta_c = \tan^{-1}\left[\frac{\mu_{s,1}m_1 + \mu_{s,2}m_2}{m_1 + m_2}\right] = \tan^{-1}\left[\frac{(0.4)(0.2\,\text{kg}) + (0.6)(0.1\,\text{kg})}{0.1\,\text{kg} + 0.2\,\text{kg}}\right] = \boxed{25.0°}$$

(*b*) Because θ_c is greater than the angle of repose ($\tan^{-1}(\mu_{s,1}) = \tan^{-1}(0.4) = 21.8°$) for the lower block, it would slide if $T = 0$. Solve equation (4) for T:

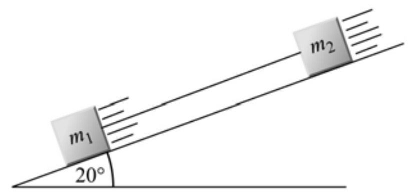
$$T = m_1 g(\sin\theta_C - \mu_{s,1}\cos\theta_C)$$

Substitute numerical values and evaluate T:

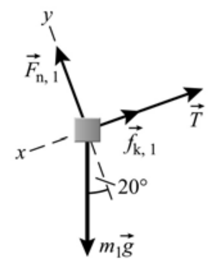
$$T = (0.2\,\text{kg})(9.81\,\text{m/s}^2)[\sin 25° - (0.4)\cos 25°] = \boxed{0.118\,\text{N}}$$

46 ••

Picture the Problem The pictorial representation shows the orientation of the two blocks with a common acceleration on the inclined surface. Draw the free-body diagrams for each block and apply Newton's 2nd law and the definition of the kinetic friction force to each block to obtain simultaneous equations in a and T.



(a) Draw the free-body diagram for the lower block:



Apply $\sum \vec{F} = m\vec{a}$ to the lower block:

$$\Sigma F_x = m_1 g\sin 20° - f_{k,1} - T = m_1 a \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 20° = 0 \quad (2)$$

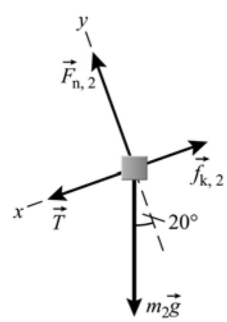
Express the relationship between $f_{k,1}$ and $F_{n,1}$:

$$f_{k,1} = \mu_{k,1} F_{n,1} \quad (3)$$

Eliminate $f_{k,1}$ and $F_{n,1}$ between (1), (2), and (3) to obtain:

$$m_1 g\sin 20° - \mu_{k,1} m_1 g\cos 20° - T = m_1 a \quad (4)$$

Draw the free-body diagram for the upper block:



Apply $\sum \vec{F} = m\vec{a}$ to the upper block:

$$\Sigma F_x = T + m_2 g\sin 20° - f_{k,2} = m_2 a \quad (5)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g\cos 20° = 0 \quad (6)$$

Express the relationship between $f_{k,2}$ and $F_{n,2}$:

$$f_{k,2} = \mu_{k,2} F_{n,2} \quad (7)$$

Eliminate $f_{k,2}$ and $F_{n,2}$ between (5), (2), and (7) to obtain:

$$T + m_2 g \sin 20° - \mu_{k,2} m_2 g \cos 20° = m_2 a \quad (8)$$

Add equations (4) and (8) to eliminate T and solve for a:

$$a = \boxed{g\left(\sin 20° - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}\cos 20°\right)}$$

Substitute the given values and evaluate a:

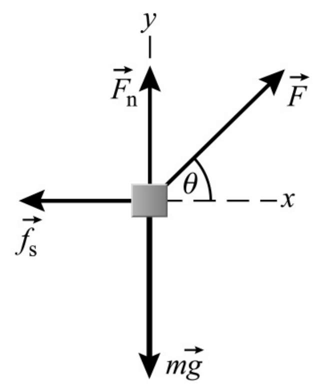
$$a = \boxed{0.944\,\text{m/s}^2}$$

(*b*) Substitute for a in either equation (4) or equation (8) to obtain:

$T = \boxed{-0.426\,\text{N}}$; i.e., the rod is under compression.

***47 ••**

Picture the Problem The vertical component of $\vec{F}$ reduces the normal force; hence, the static friction force between the surface and the block. The horizontal component is responsible for any tendency to move and equals the static friction force until it exceeds its maximum value. We can apply Newton's 2nd law to the box, under equilibrium conditions, to relate F to θ.



(*a*) The static-frictional force opposes the motion of the object, and the maximum value of the static-frictional force is proportional to the normal force F_N. The normal force is equal to the weight minus the vertical component F_V of the force F. Keeping the magnitude F constant while increasing θ from zero results in a decrease in F_V and thus a corresponding decrease in the maximum static-frictional force f_{max}. The object will begin to move if the horizontal component F_H of the force F exceeds f_{max}. An increase in θ results in a decrease in F_H. As θ increases from 0, the decrease in F_N is larger than the decrease in F_H, so the object is more and more likely to slip. However, as θ approaches 90°, F_H approaches zero and no movement will be initiated. If F is large enough and if θ increases from 0, then at some value of θ the block will start to move.

(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\Sigma F_y = F_n + F\sin\theta - mg = 0 \quad (2)$$

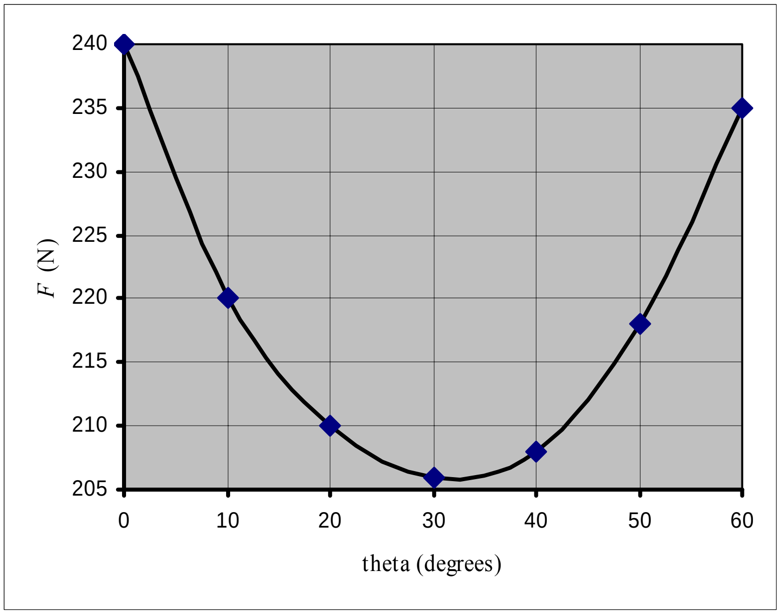
Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Use this function with $mg = 240$ N to generate the table shown below:

θ	(deg)	0	10	20	30	40	50	60
F	(N)	240	220	210	206	208	218	235

The following graph of $F(\theta)$ was plotted using a spreadsheet program.

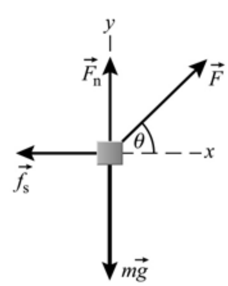


From the graph, we can see that the minimum value for F occurs when $\theta \approx 32°$.

Remarks: An alternative to manually plotting F as a function of θ or using a spreadsheet program is to use a graphing calculator to enter and graph the function.

48 •••

Picture the Problem The free-body diagram shows the forces acting on the block. We can apply Newton's 2nd law, under equilibrium conditions, to relate F to θ and then set its derivative with respect to θ equal to zero to find the value of θ that minimizes F.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\Sigma F_y = F_n + F\sin\theta - mg = 0 \quad (2)$$

Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \quad (3)$$

To find θ_{min}, differentiate F with respect to θ and set the derivative equal to zero for extrema of the function:

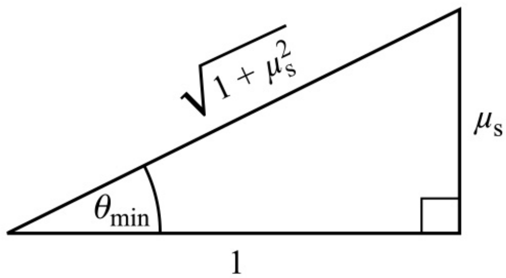
$$\frac{dF}{d\theta} = \frac{(\cos\theta + \mu_s \sin\theta)\frac{d}{d\theta}(\mu_s mg)}{(\cos\theta + \mu_s \sin\theta)^2} - \frac{\mu_s mg \frac{d}{d\theta}(\cos\theta + \mu_s \sin\theta)}{(\cos\theta + \mu_s \sin\theta)^2}$$

$$= \frac{\mu_s mg(-\sin\theta + \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0 \text{ for extrema}$$

Solve for θ_{min} to obtain:

$$\theta_{min} = \boxed{\tan^{-1}\mu_s}$$

(*b*) Use the reference triangle shown below to substitute for $\cos\theta$ and $\sin\theta$ in equation (3):



$$F_{min} = \frac{\mu_s mg}{\frac{1}{\sqrt{1+\mu_s^2}} + \mu_s \frac{\mu_s}{\sqrt{1+\mu_s^2}}} = \frac{\mu_s mg}{\frac{1+\mu_s^2}{\sqrt{1+\mu_s^2}}} = \boxed{\frac{\mu_s}{\sqrt{1+\mu_s^2}} mg}$$

(*c*)

The coefficient of kinetic friction is less than the coefficient of static friction. An analysis identical to the one above shows that the minimum force one should apply to keep the block moving should be applied at an angle given by $\theta_{min} = \tan^{-1}\mu_k$. Therefore, once the block is moving the coefficient of friction will decrease, so the angle can be decreased.

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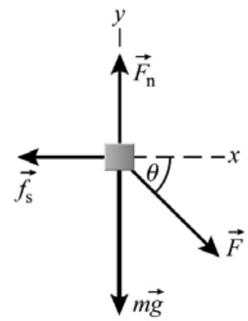
Picture the Problem The vertical component of $\vec{F}$ increases the normal force and the static friction force between the surface and the block. The horizontal component is responsible for any tendency to move and equals the static friction force until it exceeds its maximum value. We can apply Newton's 2nd law to the box, under equilibrium conditions, to relate F to θ.

(*a*) As θ increases from zero, F increases the normal force exerted by the surface and the static friction force. As the horizontal component of F decreases with increasing θ, one would expect F to continue to increase.



(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\Sigma F_y = F_n - F\sin\theta - mg = 0 \quad (2)$$

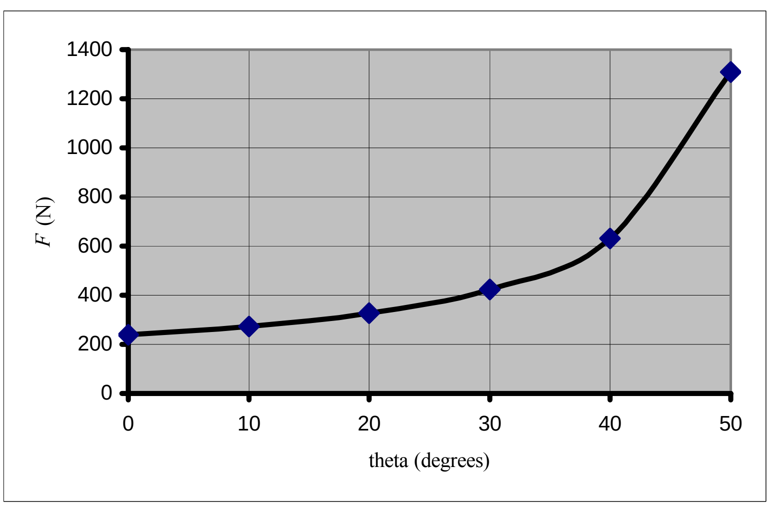
Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta} \quad (3)$$

Use this function with $mg = 240$ N to generate the table shown below.

θ	(deg)	0	10	20	30	40	50	60
F	(N)	240	273	327	424	631	1310	very large

The graph of F as a function of θ, plotted using a spreadsheet program, confirms our prediction that F continues to increase with θ.



(*a*) From the graph we see that: $\theta_{min} = \boxed{0°}$

(*b*) Evaluate equation (3) for $\theta = 0°$ to obtain:

$$F = \frac{\mu_s mg}{\cos 0° - \mu_s \sin 0°} = \boxed{\mu_s mg}$$

(*c*)

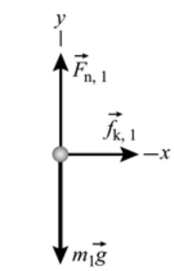
$$\boxed{\text{You should keep the angle at } 0°.}$$

Remarks: An alternative to the use of a spreadsheet program is to use a graphing calculator to enter and graph the function.

50 ••

Picture the Problem The forces acting on each of these masses are shown in the free-body diagrams below. m_1 represents the mass of the 20-kg mass and m_2 that of the 100-kg mass. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{k,1}$ $(= f_{k,2})$ act on both masses but in opposite directions. Newton's 2nd law and the definition of kinetic friction forces can be used to determine the various forces and the acceleration called for in this problem.

(*a*) Draw a free-body diagram showing the forces acting on the 20-kg mass:



Apply $\sum \vec{F} = m\vec{a}$ to this mass:

$$\Sigma F_x = f_{k,1} = m_1 a_1 \quad (1)$$

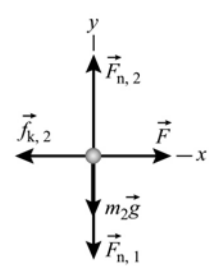
and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Solve equation (1) for $f_{k,1}$:

$$f_{k,1} = m_1 a_1 = (20\text{ kg})(4\text{ m/s}^2) = \boxed{80.0\text{ N}}$$

(*b*) Draw a free-body diagram showing the forces acting on the 100-kg mass:



Apply $\sum F_x = ma_x$ to the 100-kg object and evaluate F_{net}:

$$F_{net} = m_2 a_2$$
$$= (100\text{ kg})(6\text{ m/s}^2) = \boxed{600\text{ N}}$$

Express F in terms of F_{net} and $f_{k,2}$:

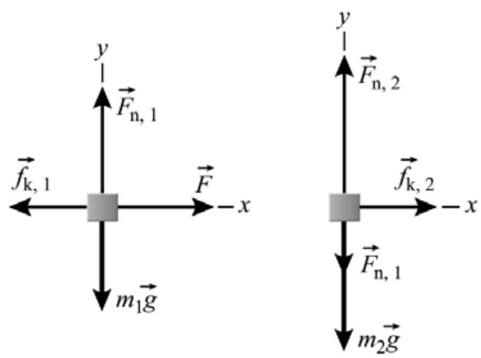
$$F = F_{net} + f_{k,2} = 600\text{ N} + 80\text{ N} = \boxed{680\text{ N}}$$

(c) When the 20-kg mass falls off, the 680-N force acts just on the 100-kg mass and its acceleration is given by Newton's 2nd law:

$$a = \frac{F_{net}}{m} = \frac{680\text{ N}}{100\text{ kg}} = \boxed{6.80\text{ m/s}^2}$$

51 ••

Picture the Problem The forces acting on each of these blocks are shown in the free-body diagrams to the right. m_1 represents the mass of the 60-kg block and m_2 that of the 100-kg block. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{k,1}$ ($= f_{k,2}$) act on both objects but in opposite directions. Newton's 2nd law and the definition of kinetic friction forces can be used to determine the coefficient of kinetic friction and acceleration of the 100-kg block.



(a) Apply $\sum \vec{F} = m\vec{a}$ to the 60-kg block:

$$\Sigma F_x = F - f_{k,1} = m_1 a_1 \qquad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \qquad (2)$$

Apply $\sum F_x = ma_x$ to the 100-kg block:

$$f_{k,2} = m_2 a_2 \qquad (3)$$

Using equation (2), express the relationship between the kinetic friction forces $\vec{f}_{k,1}$ and $\vec{f}_{k,2}$:

$$f_{k,1} = f_{k,2} = f_k = \mu_k F_{n,1} = \mu_k m_1 g \quad (4)$$

Substitute equation (4) into equation (1) and solve for μ_k:

$$\mu_k = \frac{F - m_1 a_1}{m_1 g}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{320\text{ N} - (60\text{ kg})(3\text{ m/s}^2)}{(60\text{ kg})(9.81\text{ m/s}^2)} = \boxed{0.238}$$

(b) Substitute equation (4) into equation (3) and solve for a_2:

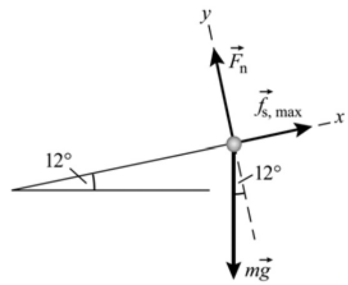
$$a_2 = \frac{\mu_k m_1 g}{m_2}$$

Substitute numerical values and evaluate a_2:

$$a_2 = \frac{(0.238)(60\,\text{kg})(9.81\,\text{m/s}^2)}{100\,\text{kg}} = \boxed{1.40\,\text{m/s}^2}$$

***52 ••**

Picture the Problem The accelerations of the truck can be found by applying Newton's 2nd law of motion. The free-body diagram for the truck climbing the incline with maximum acceleration is shown to the right.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the truck when it is climbing the incline:

$$\Sigma F_x = f_{\text{s,max}} - mg\sin 12° = ma \quad (1)$$

and

$$\Sigma F_y = F_\text{n} - mg\cos 12° = 0 \quad (2)$$

Solve equation (2) for F_n and use the definition of $f_{\text{s,max}}$ to obtain:

$$f_{\text{s,max}} = \mu_\text{s} mg\cos 12° \quad (3)$$

Substitute equation (3) into equation (1) and solve for a:

$$a = g(\mu_\text{s}\cos 12° - \sin 12°)$$

Substitute numerical values and evaluate a:

$$a = (9.81\,\text{m/s}^2)[(0.85)\cos 12° - \sin 12°] = \boxed{6.12\,\text{m/s}^2}$$

(*b*) When the truck is descending the incline with maximum acceleration, the static friction force points down the incline; i.e., its direction is reversed on the FBD. Apply $\sum F_x = ma_x$ to the truck under these conditions:

$$-f_{\text{s,max}} - mg\sin 12° = ma \quad (4)$$

Substitute equation (3) into equation (4) and solve for a:

$$a = -g(\mu_\text{s}\cos 12° + \sin 12°)$$

Substitute numerical values and evaluate a:

$$a = (-9.81\,\text{m/s}^2)[(0.85)\cos 12° + \sin 12°] = \boxed{-10.2\,\text{m/s}^2}$$

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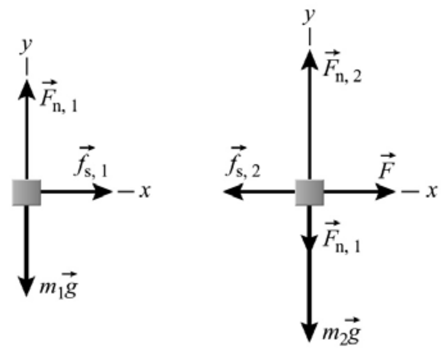
Picture the Problem The forces acting on each of the blocks are shown in the free-body diagrams to the right. m_1 represents the mass of the 2-kg block and m_2 that of the 4-kg block. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{s,1}$ $(= f_{s,2})$ act on both objects but in opposite directions. Newton's 2nd law and the definition of the maximum static friction force can be used to determine the maximum force acting on the 4-kg block for which the 2-kg block does not slide.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 2-kg block:

$\Sigma F_x = f_{s,1,max} = m_1 a_{max}$ (1)

and

$\Sigma F_y = F_{n,1} - m_1 g = 0$ (2)

Apply $\sum \vec{F} = m\vec{a}$ to the 4-kg block:

$\Sigma F_x = F - f_{s,2,max} = m_2 a_{max}$ (3)

and

$\Sigma F_y = F_{n,2} - F_{n,1} - m_2 g = 0$ (4)

Using equation (2), express the relationship between the static friction forces $\vec{f}_{s,1,max}$ and $\vec{f}_{s,2,max}$:

$f_{s,1,max} = f_{s,2,max} = \mu_s m_1 g$ (5)

Substitute (5) in (1) and solve for a_{max}:

$a_{max} = \mu_s g = (0.3)g = 2.94\text{ m/s}^2$

Solve equation (3) for $F = F_{max}$:

$$F_{max} = m_2 a_{max} + \mu_s m_1 g$$

Substitute numerical values and evaluate F_{max}:

$$F_{max} = (4\text{ kg})(2.94\text{ m/s}^2) + (0.3)(2\text{ kg}) \times (9.81\text{ m/s}^2) = \boxed{17.7\text{ N}}$$

(*b*) Use Newton's 2nd law to express the acceleration of the blocks moving as a unit:

$$a = \frac{F}{m_1 + m_2}$$

Substitute numerical values and evaluate a:

$$a = \frac{\frac{1}{2}(17.7\text{ N})}{2\text{ kg} + 4\text{ kg}} = \boxed{1.47\text{ m/s}^2}$$

Because the friction forces are an action-reaction pair, the friction force acting on each block is given by:

$f_s = m_1 a = (2\text{ kg})(1.47\text{ m/s}^2)$
$= \boxed{2.94\text{ N}}$

(*c*) If $F = 2F_{max}$, then m_1 slips on m_2 and the friction force (now kinetic) is given by:

$f = f_k = \mu_k m_1 g$

Use $\sum F_x = ma_x$ to relate the acceleration of the 2-kg block to the net force acting on it and solve for a_1:

$f_k = \mu_k m_1 g = m_1 a_1$
and
$a_1 = \mu_k g = (0.2)g = \boxed{1.96\text{ m/s}^2}$

Use $\sum F_x = ma_x$ to relate the acceleration of the 4-kg block to the net force acting on it:

$F - \mu_k m_1 g = m_2 a_2$

Solve for a_2:

$$a_2 = \frac{F - \mu_k m_1 g}{m_2}$$

Substitute numerical values and evaluate a_2:

$$a_2 = \frac{2(17.7\text{ N}) - (0.2)(2\text{ kg})(9.81\text{ m/s}^2)}{4\text{ kg}}$$
$$= \boxed{7.87\text{ m/s}^2}$$

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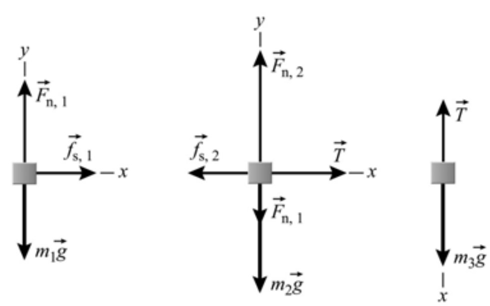
Picture the Problem Let the positive *x* direction be the direction of motion of these blocks. The forces acting on each of the blocks are shown, for the static friction case, on the free-body diagrams to the right. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{s,1}$ ($= f_{s,2}$) act on both objects but in opposite directions. Newton's 2nd law and the definition of the maximum static friction force can be used to determine the maximum acceleration of the block whose mass is m_1.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 2-kg block:

$\Sigma F_x = f_{s,1,max} = m_1 a_{max}$ (1)
and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the 4-kg block:

$$\Sigma F_x = T - f_{s,2,max} = m_2 a_{max} \quad (3)$$

and

$$\Sigma F_y = F_{n,2} - F_{n,1} - m_2 g = 0 \quad (4)$$

Using equation (2), express the relationship between the static friction forces $\vec{f}_{s,1,max}$ and $\vec{f}_{s,2,max}$:

$$f_{s,1,max} = f_{s,2,max} = \mu_s m_1 g \quad (5)$$

Substitute (5) in (1) and solve for a_{max}:

$$a_{max} = \mu_s g = (0.6)g = \boxed{5.89\,\text{m/s}^2}$$

(*b*) Use $\sum F_x = ma_x$ to express the acceleration of the blocks moving as a unit:

$$T = (m_1 + m_2)\, a_{max} \quad (6)$$

Apply $\sum F_x = ma_x$ to the object whose mass is m_3:

$$m_3 g - T = m_3 a_{max} \quad (7)$$

Add equations (6) and (7) to eliminate T and then solve for and evaluate m_3:

$$m_3 = \frac{\mu_s(m_1 + m_2)}{1 - \mu_s} = \frac{(0.6)(10\,\text{kg} + 5\,\text{kg})}{1 - 0.6} = \boxed{22.5\,\text{kg}}$$

(*c*) If m_3 = 30 kg, then m_1 will slide on m_2 and the friction force (now kinetic) is given by:

$$f = f_k = \mu_k m_1 g$$

Use $\sum F_x = ma_x$ to relate the acceleration of the 30-kg block to the net force acting on it:

$$m_3 g - T = m_3 a_3 \quad (8)$$

Noting that $a_2 = a_3$ and that the friction force on the body whose mass is m_2 is due to kinetic friction, add equations (3) and (8) and solve for and evaluate the common acceleration:

$$a_2 = a_3 = \frac{g(m_3 - \mu_k m_1)}{m_2 + m_3} = \frac{(9.81\,\text{m/s}^2)[30\,\text{kg} - (0.4)(5\,\text{kg})]}{10\,\text{kg} + 30\,\text{kg}} = \boxed{6.87\,\text{m/s}^2}$$

With block 1 sliding on block 2, the

$$f_k = \mu_k m_1 g = m_1 a_1 \quad (1')$$

friction force acting on each is kinetic and equations (1) and (3) become:

$$T - f_k = T - \mu_k m_1 g = m_2 a_2 \quad (3')$$

Solve equation (1′) for and evaluate a_1:

$$a_1 = \mu_k g = (0.4)(9.81\,\text{m/s}^2) = \boxed{3.92\,\text{m/s}^2}$$

Solve equation (3′) for T:

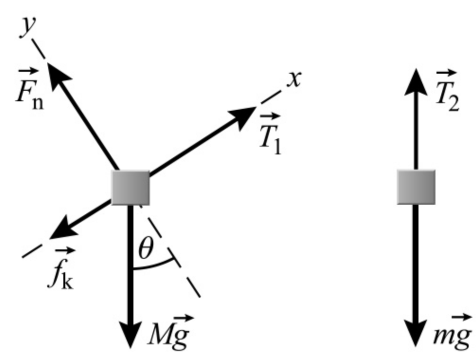
$$T = m_2 a_2 + \mu_k m_1 g$$

Substitute numerical values and evaluate T:

$$T = (10\,\text{kg})(6.87\,\text{m/s}^2) + (0.4)(5\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{88.3\,\text{N}}$$

55 •

Picture the Problem Let the direction of motion be the positive x direction. The free-body diagrams show the forces acting on both the block (M) and the counterweight (m). While $\vec{T}_1 \neq \vec{T}_2$, $T_1 = T_2$. By applying Newton's 2nd law to these blocks, we can obtain equations in T and a from which we can eliminate the tension. Once we know the acceleration of the block, we can use constant-acceleration equations to determine how far it moves in coming to a momentary stop.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block on the incline:

$$\sum F_x = T_1 - Mg\sin\theta - f_k = Ma$$

and

$$\sum F_y = F_n - Mg\cos\theta = 0$$

Apply $\sum \vec{F} = m\vec{a}$ to the counterweight:

$$\sum F_x = mg - T_2 = ma \quad (1)$$

Letting $T_1 = T_2 = T$ and using the definition of the kinetic friction force, eliminate f_k and F_n between the equations for the block on the incline to obtain:

$$T - Mg\sin\theta - \mu_k Mg\cos\theta = Ma \quad (2)$$

Eliminate T from equations (1) and (2) by adding them and solve for a:

$$a = \frac{m - M(\sin\theta + \mu_k\cos\theta)}{m + M}g$$

Substitute numerical values and evaluate a:

$$a = \frac{550\,\text{kg} - (1600\,\text{kg})(\sin 10° + 0.15\cos 10°)}{550\,\text{kg} + 1600\,\text{kg}}(9.81\,\text{m/s}^2) = \boxed{0.163\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the speed of the block at the instant the rope breaks to its acceleration and displacement as it slides to a stop. Solve for its displacement:

$$v_f^2 = v_i^2 + 2a\Delta x$$

or, because $v_f = 0$,

$$\Delta x = \frac{-v_i^2}{2a} \qquad (3)$$

The block had been accelerating up the incline for 3 s before the rope broke, so it has an initial speed of :

$$(0.163\text{ m/s}^2)(3\text{ s}) = 0.489\text{ m/s}$$

From equation (2) we can see that, when the rope breaks ($T = 0$) and:

$$\begin{aligned} a &= -g(\sin\theta + \mu_k \cos\theta) \\ &= -(9.81\,\text{m/s}^2)[\sin 10° + (0.15)\cos 10°] \\ &= -3.15\,\text{m/s}^2 \end{aligned}$$

where the minus sign indicates that the block is being accelerated down the incline, although it is still sliding up the incline.

Substitute in equation (3) and evaluate Δx:

$$\Delta x = \frac{-(0.489\,\text{m/s})^2}{2(-3.15\,\text{m/s}^2)} = \boxed{0.0380\,\text{m}}$$

(*c*) When the block is sliding down the incline, the kinetic friction force will be up the incline. Express the block's acceleration:

$$\begin{aligned} a &= -g(\sin\theta - \mu_k \cos\theta) \\ &= -(9.81\,\text{m/s}^2)[\sin 10° - (0.15)\cos 10°] \\ &= \boxed{-0.254\,\text{m/s}^2} \end{aligned}$$

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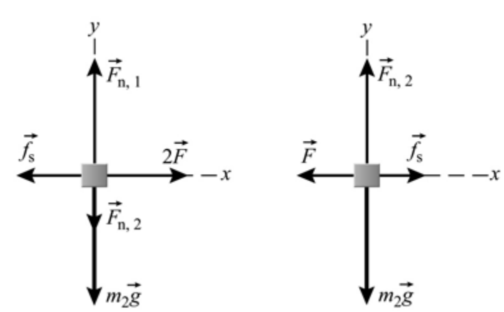
Picture the Problem If the 10-kg block is not to slide on the bracket, the maximum value for $\vec{F}$ must be equal to the maximum value of f_s and will produce the maximum acceleration of this block and the bracket. We can apply Newton's 2nd law and the definition of $f_{s,max}$ to first calculate the maximum acceleration and then the maximum value of F.



(*a*) and (*b*) Apply $\sum \vec{F} = m\vec{a}$ to the 10-kg block when it is experiencing its maximum acceleration:

$$\Sigma F_x = f_{s,max} - F = m_2 a_{2,max} \qquad (1)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g = 0 \qquad (2)$$

Express the static friction force acting on the 10-kg block:

$$f_{s,max} = \mu_s F_{n,2} \quad (3)$$

Eliminate $f_{s,max}$ and $F_{n,2}$ from equations (1), (2) and (3) to obtain:

$$\mu_s m_2 g - F = m_2 a_{2,max} \quad (4)$$

Apply $\sum F_x = ma_x$ to the bracket to obtain:

$$2F - \mu_s m_2 g = m_1 a_{1,max} \quad (5)$$

Because $a_{1,max} = a_{2,max}$, denote this acceleration by a_{max}. Eliminate F from equations (4) and (5) and solve for a_{max}:

$$a_{max} = \frac{\mu_s m_2 g}{m_1 + 2m_2}$$

Substitute numerical values and evaluate a_{max}:

$$a_{max} = \frac{(0.4)(10\,\text{kg})(9.81\,\text{m/s}^2)}{5\,\text{kg} + 2(10\,\text{kg})} = \boxed{1.57\,\text{m/s}^2}$$

Solve equation (4) for $F = F_{max}$:

$$F = \mu_s m_2 g - m_2 a_{max} = m_2(\mu_s g - a_{max})$$

Substitute numerical values and evaluate F:

$$F = (10\,\text{kg})\left[(0.4)(9.81\,\text{m/s}^2) - 1.57\,\text{m/s}^2\right] = \boxed{23.5\,\text{N}}$$

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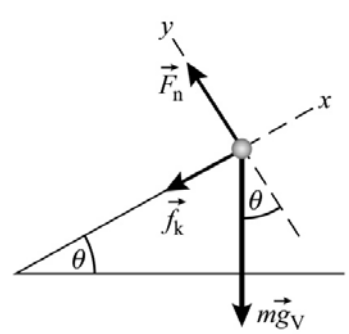
Picture the Problem The free-body diagram shows the forces acting on the block as it is moving up the incline. By applying Newton's 2nd law, we can obtain expressions for the accelerations of the block up and down the incline. Adding and subtracting these equations, together with the data found in the notebook, will lead to values for g_V and μ_k.



Apply $\sum_i \vec{F}_i = m\vec{a}$ to the block when it is moving up the incline:

$$\sum F_x = -f_k - mg_V \sin\theta = ma_{up}$$

and

$$\sum F_y = F_n - mg_V \cos\theta = 0$$

Using the definition of f_k, eliminate F_n between the two equations to obtain:

$$a_{up} = -\mu_k g_V \cos\theta - g_V \sin\theta \quad (1)$$

When the block is moving down the incline, f_k is in the positive x direction, and its acceleration is:

$$a_{down} = \mu_k g_V \cos\theta - g_V \sin\theta \qquad (2)$$

Add equations (1) and (2) to obtain:

$$a_{up} + a_{down} = -2g_V \sin\theta \qquad (3)$$

Solve equation (3) for g_V:

$$g_V = \frac{a_{up} + a_{down}}{-2\sin\theta} \qquad (4)$$

Determine θ from the figure:

$$\theta = \tan^{-1}\left[\frac{0.73\,\text{glapp}}{3.82\,\text{glapp}}\right] = 10.8°$$

Substitute the data from the notebook in equation (4) to obtain:

$$g_V = \frac{1.73\,\text{glapp/plipp}^2 + 1.42\,\text{glapp/plipp}^2}{-2\sin 10.8°} = \boxed{-8.41\,\text{glapp/plipp}^2}$$

Subtract equation (1) from equation (2) to obtain:

$$a_{down} - a_{up} = 2\mu_k g_V \cos\theta$$

Solve for μ_k:

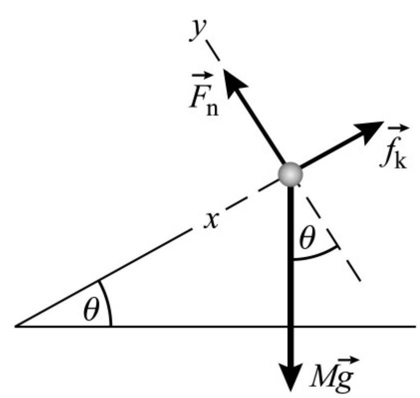
$$\mu_k = \frac{a_{down} - a_{up}}{2g_V \cos\theta}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{-1.42\,\text{glapp/plipp}^2 - 1.73\,\text{glapp/plipp}^2}{2\left(-8.41\,\text{glapp/plipp}^2\right)\cos 10.8°} = \boxed{0.191}$$

***58 ••**

Picture the Problem The free-body diagram shows the block sliding down the incline under the influence of a friction force, its weight, and the normal force exerted on it by the inclined surface. We can find the range of values for m for the two situations described in the problem statement by applying Newton's 2nd law of motion to, first, the conditions under which the block will not move or slide if pushed, and secondly, if pushed, the block will move up the incline.



(*a*) Assume that the block is sliding down the incline with a constant velocity and with no hanging weight (m = 0) and apply $\sum \vec{F} = m\vec{a}$ to

$$\sum F_x = -f_k + Mg\sin\theta = 0$$

and

$$\sum F_y = F_n - Mg\cos\theta = 0$$

the block:

Using $f_k = \mu_k F_n$, eliminate F_n between the two equations and solve for the net force acting on the block:

$$F_{net} = -\mu_k Mg\cos\theta + Mg\sin\theta$$

If the block is moving, this net force must be nonnegative and:

$$(-\mu_k \cos\theta + \sin\theta)Mg \geq 0$$

This condition requires that:

$$\mu_k \leq \tan\theta = \tan 18° = 0.325$$

Because $\mu_k = 0.2$, this condition is satisfied and:

$$m_{min} = 0$$

To find the maximum value, note that the maximum possible value for the tension in the rope is mg. For the block to move down the incline, the component of the block's weight parallel to the incline minus the frictional force must be greater than or equal to the tension in the rope:

$$Mg\sin\theta - \mu_k Mg\cos\theta \geq mg$$

Solve for m_{max}:

$$m_{max} \leq M(\sin\theta - \mu_k \cos\theta)$$

Substitute numerical values and evaluate m_{max}:

$$m_{max} \leq (100\,\text{kg})[\sin 18° - (0.2)\cos 18°] = 11.9\,\text{kg}$$

The range of values for m is:

$$\boxed{0 \leq m \leq 11.9\,\text{kg}}$$

(*b*) If the block is being dragged up the incline, the frictional force will point down the incline, and:

$$Mg\sin\theta + \mu_k Mg\cos\theta < mg$$

Solve for and evaluate m_{min}:

$$m_{min} > M(\sin\theta + \mu_k \cos\theta) = (100\text{ kg})[\sin 18° + (0.2)\cos 18°] = 49.9\,\text{kg}$$

If the block is not to move unless pushed:

$$Mg\sin\theta + \mu_s Mg\cos\theta > mg$$

Solve for and evaluate m_{max}:

$$m_{max} < M(\sin\theta + \mu_s \cos\theta) = (100\text{ kg})[\sin 18° + (0.4)\cos 18°] = 68.9\,\text{kg}$$

The range of values for m is:

$$\boxed{49.9\,\text{kg} \leq m \leq 68.9\,\text{kg}}$$

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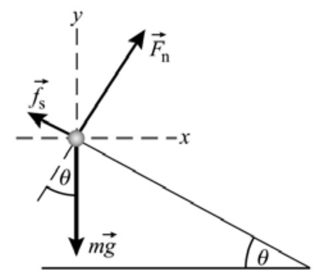
Picture the Problem The free-body diagram shows the forces acting on the 0.5 kg block when the acceleration is a minimum. Note the choice of coordinate system is consistent with the direction of $\vec{F}$. Apply Newton's 2nd law to the block and solve the resulting equations for a_{min} and a_{max}.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 0.5-kg block:

$$\Sigma F_x = F_n \sin\theta - f_s \cos\theta = ma \qquad (1)$$

and

$$\Sigma F_y = F_n \cos\theta + f_s \sin\theta - mg = 0 \qquad (2)$$

Under minimum acceleration, $f_s = f_{s,max}$. Express the relationship between $f_{s,max}$ and F_n:

$$f_{s,max} = \mu_s F_n \qquad (3)$$

Substitute $f_{s,max}$ for f_s in equation (2) and solve for F_n:

$$F_n = \frac{mg}{\cos\theta + \mu_s \sin\theta}$$

Substitute for F_n in equation (1) and solve for $a = a_{min}$:

$$a_{min} = g\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}$$

Substitute numerical values and evaluate a_{min}:

$$a_{min} = \left(9.81\,\text{m/s}^2\right)\frac{\sin 35° - (0.8)\cos 35°}{\cos 35° + (0.8)\sin 35°} = -0.627\,\text{m/s}^2$$

Treat the block and incline as a single object to determine F_{min}:

$$F_{min} = m_{tot} a_{min} = (2.5\text{ kg})(-0.627\text{ m/s}^2) = \boxed{-1.57\,\text{N}}$$

To find the maximum acceleration, reverse the direction of $\vec{f}_s$ and apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F_n \sin\theta + f_s \cos\theta = ma \qquad (4)$$

and

$$\Sigma F_y = F_n \cos\theta - f_s \sin\theta - mg = 0 \qquad (5)$$

Proceed as above to obtain:

$$a_{max} = g\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}$$

Substitute numerical values and evaluate a_{max}:

$$a_{max} = \left(9.81\,\text{m/s}^2\right)\frac{\sin 35° + (0.8)\cos 35°}{\cos 35° - (0.8)\sin 35°} = 33.5\,\text{m/s}^2$$

Treat the block and incline as a single object to determine F_{max}:

$$F_{max} = m_{tot}a_{max} = (2.5\text{ kg})(33.5\text{ m/s}^2) = \boxed{83.8\text{ N}}$$

(*b*) Repeat (*a*) with $\mu_s = 0.4$ to obtain:

$$F_{min} = \boxed{5.75\text{ N}} \text{ and } F_{max} = \boxed{37.5\text{ N}}$$

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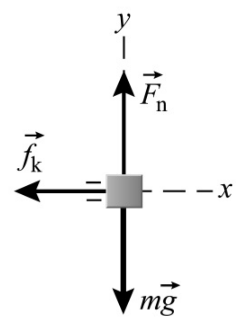
Picture the Problem The kinetic friction force f_k is the product of the coefficient of sliding friction μ_k and the normal force F_n the surface exerts on the sliding object. By applying Newton's 2nd law in the vertical direction, we can see that, on a horizontal surface, the normal force is the weight of the sliding object. Note that the acceleration of the block is opposite its direction of motion.



(*a*) Relate the force of kinetic friction to μ_k and the normal force acting on the sliding wooden object:

$$f_k = \mu_k F_n = \frac{0.11}{\left(1 + 2.3\times10^{-4}v^2\right)^2}mg$$

Substitute v = 10 m/s and evaluate f_k:

$$f_k = \frac{0.11(100\text{ kg})(9.81\text{ m/s}^2)}{\left(1 + 2.3\times10^{-4}(10\text{ m/s})^2\right)^2} = \boxed{103\text{ N}}$$

(*b*) Substitute v = 20 m/s and evaluate f_k:

$$f_k = \frac{0.11(100\text{ kg})(9.81\text{ m/s}^2)}{\left(1 + 2.3\times10^{-4}(20\text{ m/s})^2\right)^2} = \boxed{90.5\text{ N}}$$

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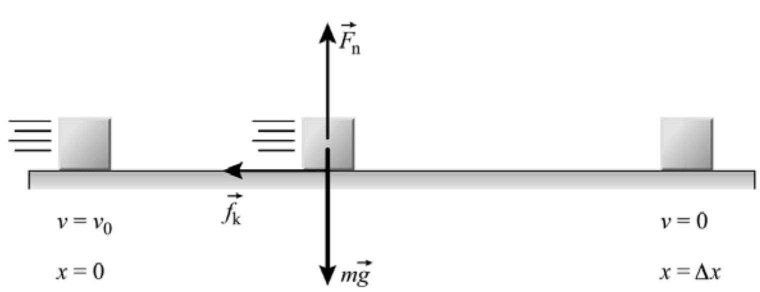
Picture the Problem The pictorial representation shows the block sliding from left to right and coming to rest when it has traveled a distance Δx. Note that the direction of the motion is opposite that of the block's acceleration. The acceleration and stopping distance of the blocks can be found from constant-acceleration equations. Let the direction of motion of the sliding blocks be the positive x direction. Because the surface is horizontal, the normal force acting on the sliding block is the block's weight.



(*a*) Using a constant-acceleration equation, relate the block's stopping distance to its initial speed and acceleration; solve for the stopping distance:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$\Delta x = \frac{-v_0^2}{2a} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the sliding block, introduce Konecny's empirical expression, and solve for the block's acceleration:

$$a = \frac{F_{\text{net},x}}{m} = \frac{-f_\text{k}}{m} = -\frac{0.4F_\text{n}^{0.91}}{m} = -\frac{0.4(mg)^{0.91}}{m}$$

Evaluate a with $m = 10$ kg:

$$a = -\frac{(0.4)[(10\,\text{kg})(9.81\,\text{m/s}^2)]^{0.91}}{10\,\text{kg}} = \boxed{-2.60\,\text{m/s}^2}$$

Substitute in equation (1) and evaluate the stopping distance when $v_0 = 10$ m/s:

$$\Delta x = \frac{-(10\,\text{m/s})^2}{2(-2.60\,\text{m/s}^2)} = \boxed{19.2\,\text{m}}$$

(*b*) Proceed as in (*a*) with $m = 100$ kg to obtain:

$$a = -\frac{(0.4)[(100\,\text{kg})(9.81\,\text{m/s}^2)]^{0.91}}{100\,\text{kg}} = \boxed{-2.11\,\text{m/s}^2}$$

Find the stopping distance as in (*a*):

$$\Delta x = \frac{-(10\,\text{m/s})^2}{2(-2.11\,\text{m/s}^2)} = \boxed{23.7\,\text{m}}$$

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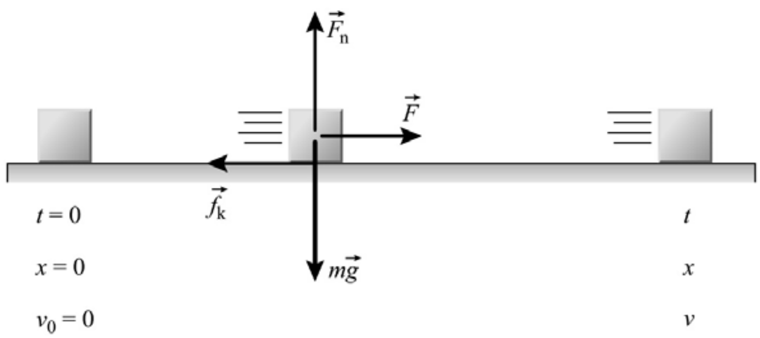
Picture the Problem The kinetic friction force f_k is the product of the coefficient of sliding friction μ_k and the normal force F_n the surface exerts on the sliding object. By applying Newton's 2nd law in the vertical direction, we can see that, on a horizontal surface, the normal force is the weight of the sliding object. We can apply Newton's 2nd law in the horizontal (x) direction to relate the block's acceleration to the net force acting on it. In the spreadsheet program, we'll find the acceleration of the block from this net force (which is velocity dependent), calculate the increase in the block's speed from its acceleration and the elapsed time and add this increase to its speed at end of the previous time interval, determine how far it has moved in this time interval, and add this distance to its previous position to find its current position. We'll also calculate the position of the block x_2, under the assumption that $\mu_\text{k} = 0.11$, using a constant-acceleration equation.



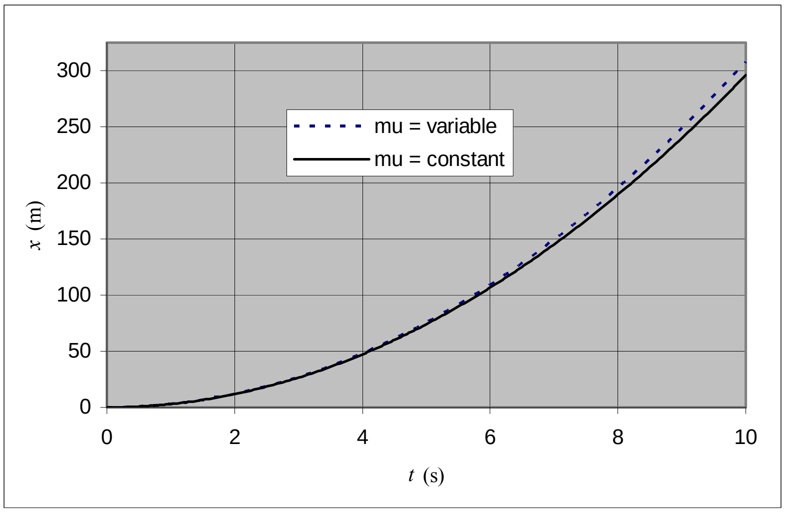
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
C9	C8+$B$6	$t + \Delta t$
D9	D8+F9*$B$6	$v + a\Delta t$
E9	$B$5–($B$3)*($B$2)*$B$5/ (1+$B$4*D9^2)^2	$F - \dfrac{\mu_k mg}{(1 + 2.34\times 10^{-4} v^2)^2}$
F9	E10/$B$5	F_{net} / m
G9	G9+D10*$B$6	$x + v\Delta t$
K9	0.5*5.922*I10^2	$\frac{1}{2} at^2$
L9	J10-K10	$x - x_2$

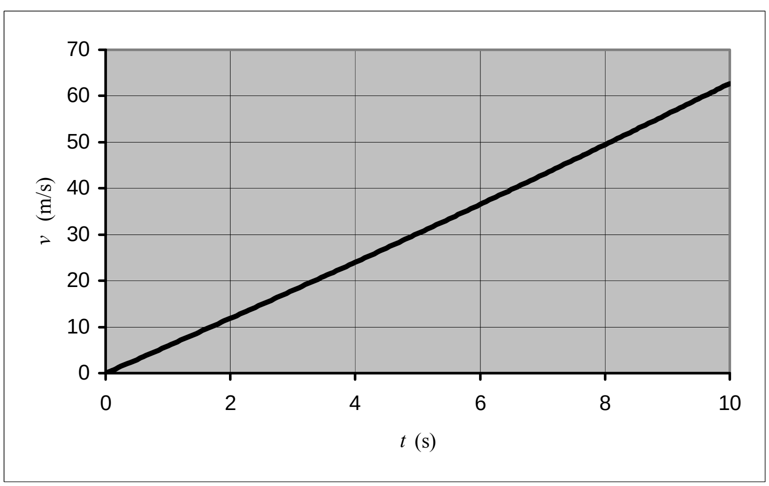
	A	B	C	D	E	F	G	H	I	J
1	g=	9.81	m/s^2							
2	Coeff1=	0.11								
3	Coeff2=	2.30E-04								
4	Mass=	10	kg							
5	Applied Force=	70	N							
6	Time step=	0.05	s				t	x	x2	x–x2
7										
8										
9	t	v	Net force	a	x			mu=variable	mu=constant	
10	0.00	0.00			0.00		0.00	0.00	0.00	0.00
11	0.05	0.30	59.22	5.92	0.01		0.05	0.01	0.01	0.01
12	0.10	0.59	59.22	5.92	0.04		0.10	0.04	0.03	0.01
13	0.15	0.89	59.22	5.92	0.09		0.15	0.09	0.07	0.02
14	0.20	1.18	59.22	5.92	0.15		0.20	0.15	0.12	0.03
15	0.25	1.48	59.23	5.92	0.22		0.25	0.22	0.19	0.04
205	9.75	61.06	66.84	6.68	292.37		9.75	292.37	281.48	10.89
206	9.80	61.40	66.88	6.69	295.44		9.80	295.44	284.37	11.07

207	9.85	61.73	66.91	6.69	298.53		9.85	298.53	287.28	11.25
208	9.90	62.07	66.94	6.69	301.63		9.90	301.63	290.21	11.42
209	9.95	62.40	66.97	6.70	304.75		9.95	304.75	293.15	11.61
210	10.00	62.74	67.00	6.70	307.89		10.00	307.89	296.10	11.79

The displacement of the block as a function of time, for a constant coefficient of friction ($\mu_k = 0.11$) is shown as a solid line on the graph and for a variable coefficient of friction, is shown as a dotted line. Because the coefficient of friction decreases with increasing particle speed, the particle travels slightly farther when the coefficient of friction is variable.

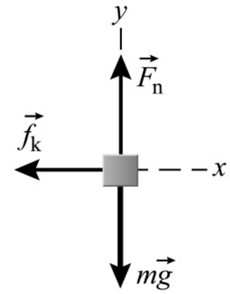


The velocity of the block, with variable coefficient of kinetic friction, is shown below.



63 ••

Picture the Problem The free-body diagram shows the forces acting on the block as it moves to the right. The kinetic friction force will slow the block and, eventually, bring it to rest. We can relate the coefficient of kinetic friction to the stopping time and distance by applying Newton's 2nd law and then using constant-acceleration equations.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block of wood:

$$\sum F_x = -f_k = ma$$

and

$$\sum F_y = F_n - mg = 0$$

Using the definition of f_k, eliminate F_n between the two equations to obtain:

$$a = -\mu_k g \qquad (1)$$

Use a constant-acceleration equation to relate the acceleration of the block to its displacement and its stopping time:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2 \qquad (2)$$

Relate the initial speed of the block, v_0, to its displacement and stopping distance:

$$\Delta x = v_{av} \Delta t = \frac{v_0 + v}{2} \Delta t = \tfrac{1}{2} v_0 \Delta t \text{ since } v = 0. \qquad (3)$$

Use this result to eliminate v_0 in equation (2):

$$\Delta x = -\tfrac{1}{2} a (\Delta t)^2 \qquad (4)$$

Substitute equation (1) in equation (4) and solve for μ_k:

$$\mu_k = \frac{2\Delta x}{g(\Delta t)^2}$$

Substitute for Δx = 1.37 m and Δt = 0.97 s to obtain:

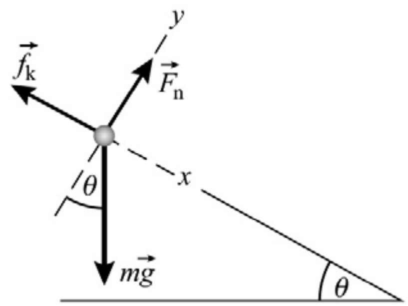
$$\mu_k = \frac{2(1.37\,\text{m})}{(9.81\,\text{m/s}^2)(0.97\,\text{s})^2} = \boxed{0.297}$$

(*b*) Use equation (3) to find v_0:

$$v_0 = \frac{2\Delta x}{\Delta t} = \frac{2(1.37\,\text{m})}{0.97\,\text{s}} = \boxed{2.82\,\text{m/s}}$$

***64** ••

Picture the Problem The free-body diagram shows the forces acting on the block as it slides down an incline. We can apply Newton's 2nd law to these forces to obtain the acceleration of the block and then manipulate this expression algebraically to show that a graph of $a/\cos\theta$ versus $\tan\theta$ will be linear with a slope equal to the acceleration due to gravity and an intercept whose absolute value is the coefficient of kinetic friction.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the block as it slides down the incline:

$$\sum F_x = mg\sin\theta - f_k = ma$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Substitute $\mu_k F_n$ for f_k and eliminate F_n between the two equations to obtain:

$$a = g(\sin\theta - \mu_k\cos\theta)$$

Divide both sides of this equation by $\cos\theta$ to obtain:

$$\frac{a}{\cos\theta} = g\tan\theta - g\mu_k$$

Note that this equation is of the form $y = mx + b$:

Thus, if we graph $a/\cos\theta$ versus $\tan\theta$, we should get a straight line with slope g and y-intercept $-g\mu_k$.

(*b*) A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
C7	θ	
D7	a	
E7	TAN(C7*PI()/180)	$\tan\left(\theta\times\frac{\pi}{180}\right)$
F7	D7/COS(C7*PI()/180)	$\frac{a}{\cos\left(\theta\times\frac{\pi}{180}\right)}$

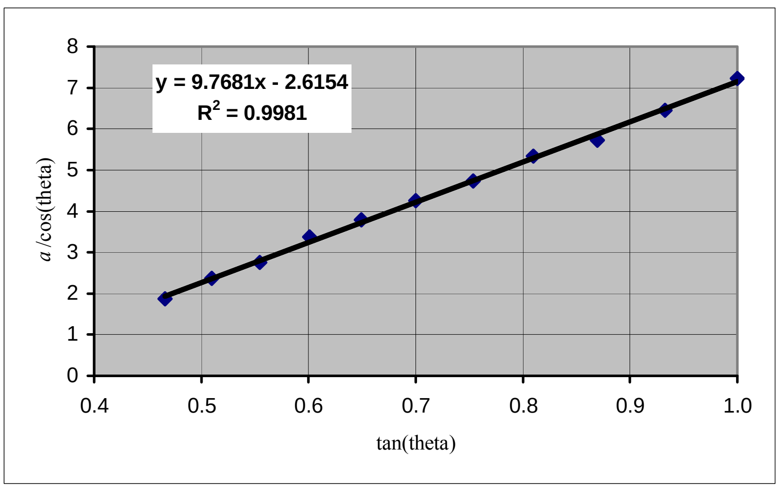
	C	D	E	F
6	theta	a	tan(theta)	a/cos(theta)
7	25	1.691	0.466	1.866
8	27	2.104	0.510	2.362
9	29	2.406	0.554	2.751

10	31	2.888	0.601	3.370
11	33	3.175	0.649	3.786
12	35	3.489	0.700	4.259
13	37	3.781	0.754	4.735
14	39	4.149	0.810	5.338
15	41	4.326	0.869	5.732
16	43	4.718	0.933	6.451
17	45	5.106	1.000	7.220

A graph of $a/\cos\theta$ versus $\tan\theta$ is shown below. From the curve fit (Excel's Trendline was used), $g = 9.77$ m/s^2 and $\mu_k = \dfrac{2.62\text{ m/s}^2}{9.77\text{ m/s}^2} = 0.268.$

The percentage error in g from the commonly accepted value of 9.81 m/s^2 is

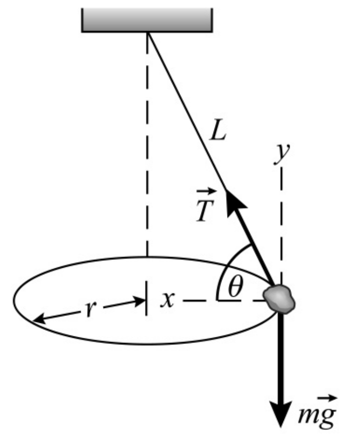
$$100\left(\frac{9.81\text{ m/s}^2 - 9.77\text{ m/s}^2}{9.81\text{ m/s}^2}\right) = \boxed{0.408\%}$$



Motion Along a Curved Path

65 •

Picture the Problem The free-body diagram showing the forces acting on the stone is superimposed on a sketch of the stone rotating in a horizontal circle. The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the horizontal is θ by applying Newton's 2nd law of motion to the forces acting on the stone.



Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$\Sigma F_x = T\cos\theta = ma_c = mv^2/r$ (1)

and

$\Sigma F_y = T\sin\theta - mg = 0$ (2)

Use the right triangle in the diagram to relate r, L, and θ:

$r = L\cos\theta$ (3)

Eliminate T and r between equations (1), (2) and (3) and solve for v_2:

$v^2 = gL\cot\theta\cos\theta$ (4)

Express the velocity of the stone in terms of its period:

$$v = \frac{2\pi r}{t_{1\,\text{rev}}} \quad (5)$$

Eliminate v between equations (4) and (5) and solve for θ:

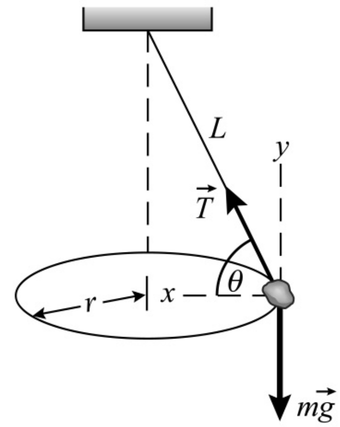
$$\theta = \sin^{-1}\left(\frac{gt_{1\,\text{rev}}^2}{4\pi^2 L}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(9.81\,\text{m/s}^2)(1.22\,\text{s})^2}{4\pi^2(0.85\,\text{m})}\right] = 25.8°$$

and $\boxed{(c)\text{ is correct.}}$

66 •



Picture the Problem The free-body diagram showing the forces acting on the stone is superimposed on a sketch of the stone rotating in a horizontal circle. The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the horizontal is θ by applying Newton's 2nd law of motion to the forces acting on the stone.

Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$$\Sigma F_x = T\cos\theta = ma_c = mv^2/r \quad (1)$$

and

$$\Sigma F_y = T\sin\theta - mg = 0 \quad (2)$$

Use the right triangle in the diagram to relate r, L, and θ:

$$r = L\cos\theta \quad (3)$$

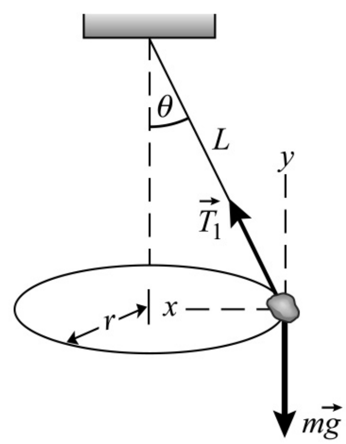
Eliminate T and r between equations (1), (2), and (3) and solve for v:

$$v = \sqrt{gL\cot\theta\cos\theta}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(9.81\,\text{m/s}^2)(0.8\,\text{m})\cot 20^\circ\cos 20^\circ} = \boxed{4.50\,\text{m/s}}$$

67 •



Picture the Problem The free-body diagram showing the forces acting on the stone is superimposed on a sketch of the stone rotating in a horizontal circle. The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the vertical is θ by applying Newton's 2nd law of motion to the forces acting on the stone.

(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$$\Sigma F_x = T\sin\theta = ma_c = mv^2/r \quad (1)$$

and

$$\Sigma F_y = T\cos\theta - mg = 0 \qquad (2)$$

Eliminate T between equations (1) and (2) and solve for v:

$$v = \sqrt{rg\tan\theta}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(0.35\,\text{m})(9.81\,\text{m/s}^2)\tan 30°} = \boxed{1.41\,\text{m/s}}$$

(*b*) Solve equation (2) for T:

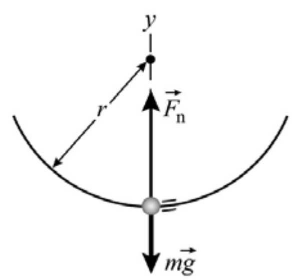
$$T = \frac{mg}{\cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(0.75\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 30°} = \boxed{8.50\,\text{N}}$$

***68** ••

Picture the Problem The sketch shows the forces acting on the pilot when her plane is at the lowest point of its dive. $\vec{F}_n$ is the force the airplane seat exerts on her. We'll apply Newton's 2nd law for circular motion to determine F_n and the radius of the circular path followed by the airplane.



(*a*) Apply $\sum F_y = ma_y$ to the pilot:

$$F_n - mg = ma_c$$

Solve for and evaluate F_n:

$$F_n = mg + ma_c = m(g + a_c) = m(g + 8.5g) = 9.5mg = (9.5)(50\text{ kg})(9.81\text{ m/s}^2) = \boxed{4.66\,\text{kN}}$$

(*b*) Relate her acceleration to her velocity and the radius of the circular arc and solve for the radius:

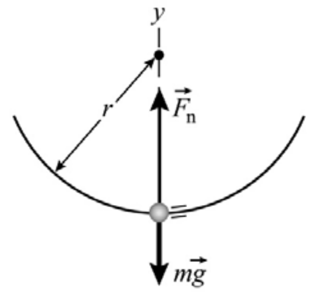
$$a_c = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_c}$$

Substitute numerical values and evaluate r :

$$r = \frac{[(345\,\text{km/h})(1\,\text{h}/3600\,\text{s})(1000\,\text{m/km})]^2}{8.5(9.81\,\text{m/s}^2)} = \boxed{110\,\text{m}}$$

69 ••

Picture the Problem The diagram shows the forces acting on the pilot when her plane is at the lowest point of its dive. $\vec{F}_n$ is the force the airplane seat exerts on her. We'll use the definitions of centripetal acceleration and centripetal force and apply Newton's 2nd law to calculate these quantities and the normal force acting on her.



(*a*) Her acceleration is centripetal and given by:

$$a_c = \frac{v^2}{r}\text{, upward}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{[(180\,\text{km/h})(1\,\text{h}/3600\,\text{s})(10^3\,/\text{km})]^2}{300\,\text{m}}$$

$$= \boxed{8.33\,\text{m/s}^2\text{, upward}}$$

(*b*) The net force acting on her at the bottom of the circle is the force responsible for her centripetal acceleration:

$$F_{net} = ma_c = (65\,\text{kg})(8.33\,\text{m/s}^2)$$

$$= \boxed{541\,\text{N, upward}}$$

(*c*) Apply $\sum F_y = ma_y$ to the pilot:

$$F_n - mg = ma_c$$

Solve for F_n:

$$F_n = mg + ma_c = m(g + a_c)$$

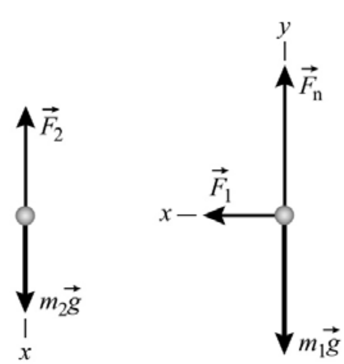
Substitute numerical values and evaluate F_n:

$$F_n = (65\text{ kg})(9.81\text{ m/s}^2 + 8.33\text{ m/s}^2)$$

$$= \boxed{1.18\,\text{kN, upward}}$$

70 ••

Picture the Problem The free-body diagrams for the two objects are shown to the right. The hole in the table changes the direction the tension in the string (which provides the centripetal force required to keep the object moving in a circular path) acts. The application of Newton's 2nd law and the definition of centripetal force will lead us to an expression for r as a function of m_1, m_2, and the time T for one revolution.



Apply $\sum F_x = ma_x$ to both objects and use the definition of centripetal acceleration to obtain:

$m_2g - F_2 = 0$
and
$F_1 = m_1a_c = m_1v^2/r$

Because $F_1 = F_2$ we can eliminate both of them between these equations to obtain:

$$m_2g - m_1\frac{v^2}{r} = 0$$

Express the speed v of the object in terms of the distance it travels each revolution and the time T for one revolution:

$$v = \frac{2\pi r}{T}$$

Substitute to obtain:

$$m_2g - m_1\frac{4\pi^2r^2}{rT^2} = 0$$

or

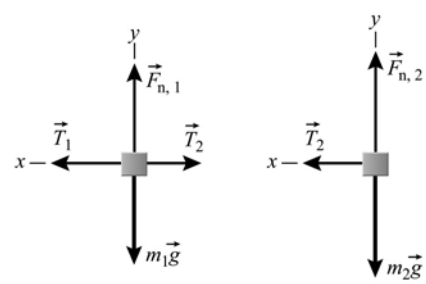
$$m_2g - m_1\frac{4\pi^2r}{T^2} = 0$$

Solve for r:

$$r = \boxed{\frac{m_2gT^2}{4\pi^2m_1}}$$

***71** ••

Picture the Problem The free-body diagrams show the forces acting on each block. We can use Newton's 2nd law to relate these forces to each other and to the masses and accelerations of the blocks.



Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T_1 - T_2 = m_1\frac{v_1^2}{L_1}$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$T_2 = m_2\frac{v_2^2}{L_1 + L_2}$$

Relate the speeds of each block to their common period and their distance from the center of the

$$v_1 = \frac{2\pi L_1}{T} \text{ and } v_2 = \frac{2\pi(L_1 + L_2)}{T}$$

circle:

Solve the first force equation for T_2, substitute for v_2, and simplify to obtain:

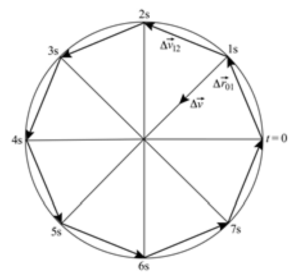
$$\boxed{T_2 = [m_2(L_1 + L_2)]\left(\frac{2\pi}{T}\right)^2}$$

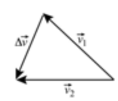
Substitute for T_2 and v_1 in the first force equation to obtain:

$$\boxed{T_1 = [m_2(L_1 + L_2) + m_1L_1]\left(\frac{2\pi}{T}\right)^2}$$

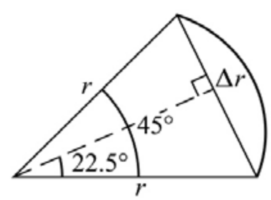
***72 ••**

Picture the Problem The path of the particle and its position at 1-s intervals are shown. The displacement vectors are also shown. The velocity vectors for the average velocities in the first and second intervals are along $\vec{r}_{01}$ and $\vec{r}_{12}$, respectively, and are shown in the lower diagram. $\Delta\vec{v}$ points toward the center of the circle.





Use the diagram to the right to find Δr:



$\Delta r = 2r\sin 22.5° = 2(4\text{ cm})\sin 22.5°$
$= 3.06\text{ cm}$

Find the average velocity of the particle along the chords:

$v_{av} = \Delta r/\Delta t = (3.06\text{ cm})/(1\text{ s})$
$= 3.06\text{ cm/s}$

Using the lower diagram and the fact that the angle between $\vec{v}_1$ and $\vec{v}_2$ is 45°, express Δv in terms of v_1 ($= v_2$):

$\Delta v = 2v_1\sin 22.5°$

Evaluate Δv using v_{av} as v_1:

$\Delta v = 2(3.06\text{ cm/s})\sin 22.5° = 2.34\text{ cm/s}$

Now we can determine $a = \Delta v/\Delta t$:

$$a = \frac{2.34\text{ cm/s}}{1\text{ s}} = \boxed{2.34\text{ cm/s}^2}$$

Find the speed v (= $v_1 = v_2$ …) of the particle along its circular path:

$$v = \frac{2\pi r}{T} = \frac{2\pi(4\,\text{cm})}{8\,\text{s}} = 3.14\,\text{cm/s}$$

Calculate the radial acceleration of the particle:

$$a_c = \frac{v^2}{r} = \frac{(3.14\,\text{cm/s})^2}{4\,\text{cm}} = \boxed{2.46\,\text{cm/s}^2}$$

Compare a_c and a by taking their ratio:

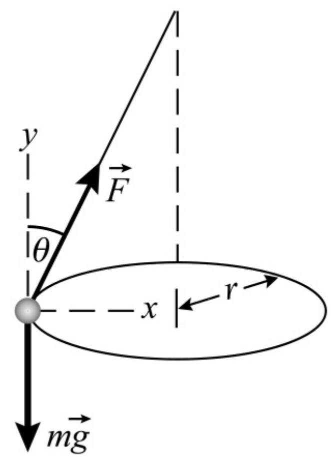
$$\frac{a_c}{a} = \frac{2.46\,\text{cm/s}^2}{2.34\,\text{cm/s}^2} = 1.05$$

or

$$\boxed{a_c = 1.05a}$$

73 ••

Picture the Problem The diagram to the right has the free-body diagram for the child superimposed on a pictorial representation of her motion. The force her father exerts is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive y direction is θ. We can infer her speed from the given information concerning the radius of her path and the period of her motion. Applying Newton's 2nd law as it describes circular motion will allow us to find both the direction and magnitude of $\vec{F}$.



Apply $\sum\vec{F} = m\vec{a}$ to the child:

$$\Sigma F_x = F\sin\theta = mv^2/r$$

and

$$\Sigma F_y = F\cos\theta - mg = 0$$

Eliminate F between these equations and solve for θ:

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Express v in terms of the radius and period of the child's motion:

$$v = \frac{2\pi r}{T}$$

Substitute for v in the expression for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{4\pi^2 r}{gT^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{4\pi^2(0.75\,\text{m})}{(9.81\,\text{m/s}^2)(1.5\,\text{s})^2}\right] = \boxed{53.3°}$$

Solve the y equation for F:

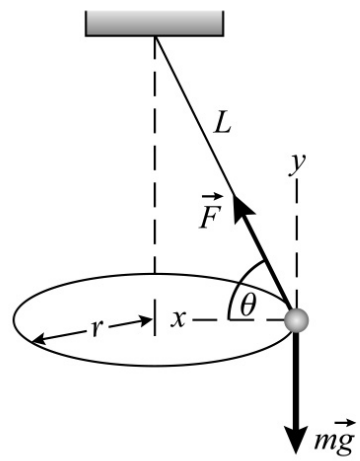
$$F = \frac{mg}{\cos\theta}$$

Substitute numerical values and evaluate F:

$$F = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 53.3°} = \boxed{410\,\text{N}}$$

74 ••

Picture the Problem The diagram to the right has the free-body diagram for the bob of the conical pendulum superimposed on a pictorial representation of its motion. The tension in the string is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive x direction is θ. We can find θ from the y equation and the information provided about the tension. Then, by using the definition of the speed of the bob in its orbit and applying Newton's 2nd law as it describes circular motion, we can find the period T of the motion.



Apply $\sum\vec{F} = m\vec{a}$ to the pendulum bob:

$$\Sigma F_x = F\cos\theta = mv^2/r$$
and
$$\Sigma F_y = F\sin\theta - mg = 0$$

Using the given information that $F = 6mg$, solve the y equation for θ:

$$\theta = \sin^{-1}\left(\frac{mg}{F}\right) = \sin^{-1}\left(\frac{mg}{6mg}\right) = 9.59°$$

With $F = 6mg$, solve the x equation for v:

$$v = \sqrt{6rg\cos\theta}$$

Relate the period T of the motion to the speed of the bob and the radius of the circle in which it moves:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{6rg\cos\theta}}$$

From the diagram, one can see that:

$$r = L\cos\theta$$

Substitute for r in the expression for the period to obtain:

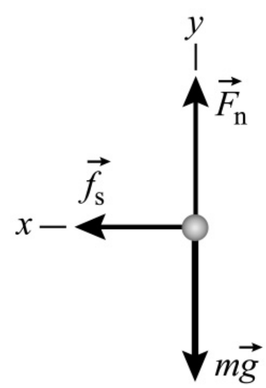
$$T = 2\pi\sqrt{\frac{L}{6g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{0.5\,\text{m}}{6(9.81\,\text{m/s}^2)}} = \boxed{0.579\,\text{s}}$$

75 ••

Picture the Problem The static friction force f_s is responsible for keeping the coin from sliding on the turntable. Using Newton's 2nd law of motion, the definition of the period of the coin's motion, and the definition of the maximum static friction force, we can find the magnitude of the friction force and the value of the coefficient of static friction for the two surfaces.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the coin:

$$\sum F_x = f_s = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n - mg = 0$$

If T is the period of the coin's motion, its speed is given by:

$$v = \frac{2\pi r}{T}$$

Substitute for v in the force equation and simplify to obtain:

$$f_s = \frac{4\pi^2 mr}{T^2}$$

Substitute numerical values and evaluate f_s:

$$f_s = \frac{4\pi^2((0.1\,\text{kg})(0.1\,\text{m})}{(1\,\text{s})^2} = \boxed{0.395\,\text{N}}$$

(*b*) Determine F_n from the y equation:

$$F_n = mg$$

If the coin is about to slide at $r = 16$ cm, $f_s = f_{s,\text{max}}$. Solve for μ_s in terms of $f_{s,\text{max}}$ and F_n:

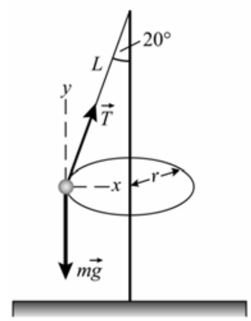
$$\mu_s = \frac{f_{s,\text{max}}}{F_n} = \frac{\frac{4\pi^2 mr}{T^2}}{mg} = \frac{4\pi^2 r}{gT^2}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{4\pi^2(0.16\,\text{m})}{(9.81\,\text{m/s}^2)(1\,\text{s})^2} = \boxed{0.644}$$

76 ••

Picture the Problem The forces acting on the tetherball are shown superimposed on a pictorial representation of the motion. The horizontal component of $\vec{T}$ is the centripetal force. Applying Newton's 2nd law of motion and solving the resulting equations will yield both the tension in the cord and the speed of the ball.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the tetherball:

$$\sum F_x = T\sin 20° = m\frac{v^2}{r}$$

and

$$\sum F_y = T\cos 20° - mg = 0$$

Solve the *y* equation for *T*:

$$T = \frac{mg}{\cos 20°}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{(0.25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 20°} = \boxed{2.61\,\text{N}}$$

(*b*) Eliminate *T* between the force equations and solve for *v*:

$$v = \sqrt{rg\tan 20°}$$

Note from the diagram that:

$$r = L\sin 20°$$

Substitute for *r* in the expression for *v* to obtain:

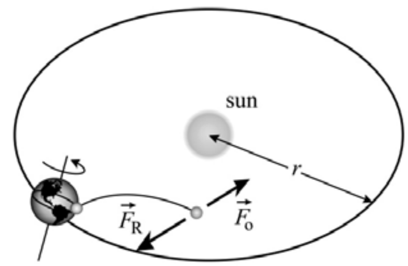
$$v = \sqrt{gL\sin 20°\tan 20°}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(9.81\,\text{m/s}^2)(1.2\,\text{m})\sin 20°\tan 20°} = \boxed{1.21\,\text{m/s}}$$

***77 ••**

Picture the Problem The diagram includes a pictorial representation of the earth in its orbit about the sun and a force diagram showing the force on an object at the equator that is due to the earth's rotation, $\vec{F}_R$, and the force on the object due to the orbital motion of the earth about the sun, $\vec{F}_o$. Because these are centripetal forces, we can calculate the accelerations they require from the speeds and radii associated with the two circular motions.



Express the radial acceleration due to the rotation of the earth:

$$a_R = \frac{v_R^2}{R}$$

Express the speed of the object on the equator in terms of the radius of the earth R and the period of the earth's rotation T_R:

$$v_R = \frac{2\pi R}{T_R}$$

Substitute for v_R in the expression for a_R to obtain:

$$a_R = \frac{4\pi^2 R}{T_R^2}$$

Substitute numerical values and evaluate a_R:

$$a_R = \frac{4\pi^2(6370\,\text{km})(1000\,\text{m/km})}{\left[(24\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)\right]^2}$$

$$= 3.37\times10^{-2}\,\text{m/s}^2$$

$$= \boxed{3.44\times10^{-3}\,g}$$

Express the radial acceleration due to the orbital motion of the earth:

$$a_o = \frac{v_o^2}{r}$$

Express the speed of the object on the equator in terms of the earth-sun distance r and the period of the earth's motion about the sun T_o:

$$v_o = \frac{2\pi r}{T_o}$$

Substitute for v_o in the expression for a_o to obtain:

$$a_o = \frac{4\pi^2 r}{T_o^2}$$

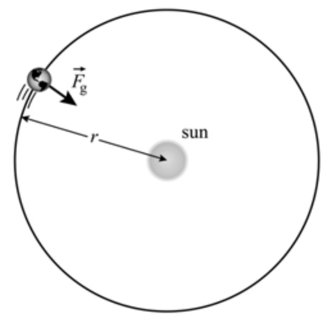
Substitute numerical values and evaluate a_c:

$$a_o = \frac{4\pi^2(1.5\times10^{11}\text{ m})}{\left[(365\text{ d})\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\right]^2}$$

$$= 5.95\times10^{-3}\text{ m/s}^2 = \boxed{6.07\times10^{-4}\,g}$$

78 •

Picture the Problem The most significant force acting on the earth is the gravitational force exerted by the sun. More distant or less massive objects exert forces on the earth as well, but we can calculate the net force by considering the radial acceleration of the earth in its orbit. Similarly, we can calculate the net force acting on the moon by considering its radial acceleration in its orbit about the earth.



(*a*) Apply $\sum F_r = ma_r$ to the earth:

$$F_{\text{on earth}} = m\frac{v^2}{r}$$

Express the orbital speed of the earth in terms of the time it takes to make one trip around the sun (i.e., its period) and its average distance from the sun:

$$v = \frac{2\pi r}{T}$$

Substitute for v to obtain:

$$F_{\text{on earth}} = \frac{4\pi^2 mr}{T^2}$$

Substitute numerical values and evaluate $F_{\text{on earth}}$:

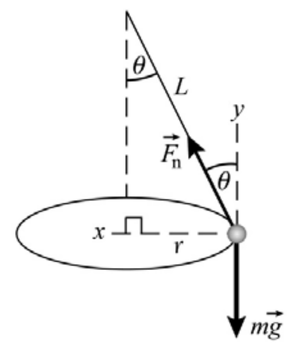
$$F_{\text{on earth}} = \frac{4\pi^2(5.98\times10^{24}\text{ kg})(1.496\times10^{11}\text{ m})}{\left(365.24\text{ d}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}\right)^2} = \boxed{3.55\times10^{22}\text{ N}}$$

(*b*) Proceed as in (*a*) to obtain:

$$F_{\text{on moon}} = \frac{4\pi^2(7.35\times10^{22}\text{ kg})(3.844\times10^{8}\text{ m})}{\left(27.32\text{ d}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}\right)^2} = \boxed{2.00\times10^{20}\text{ N}}$$

79 ••

Picture the Problem The semicircular wire of radius 10 cm limits the motion of the bead in the same manner as would a 10-cm string attached to the bead and fixed at the center of the semicircle. The horizontal component of the normal force the wire exerts on the bead is the centripetal force. The application of Newton's 2nd law, the definition of the speed of the bead in its orbit, and the relationship of the frequency of a circular motion to its period will yield the angle at which the bead will remain stationary relative to the rotating wire.



Apply $\sum \vec{F} = m\vec{a}$ to the bead:

$$\sum F_x = F_n \sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

$$\tan\theta = \frac{v^2}{rg}$$

The frequency of the motion is the reciprocal of its period T. Express the speed of the bead as a function of the radius of its path and its period:

$$v = \frac{2\pi r}{T}$$

Using the diagram, relate r to L and θ:

$$r = L\sin\theta$$

Substitute for r and v in the expression for $\tan\theta$ and solve for θ:

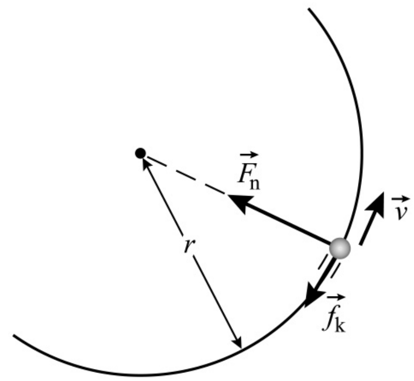
$$\theta = \cos^{-1}\left[\frac{gT^2}{4\pi^2 L}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(9.81\,\text{m/s}^2)(0.5\,\text{s})^2}{4\pi^2(0.1\,\text{m})}\right] = \boxed{51.6^\circ}$$

80 •••

Picture the Problem Note that the acceleration of the bead has two components, the radial component perpendicular to $\vec{v}$, and a tangential component due to friction that is opposite to $\vec{v}$. The application of Newton's 2nd law will result in a differential equation with separable variables. Its integration will lead to an expression for the speed of the bead as a function of time.



Apply $\sum \vec{F} = m\vec{a}$ to the bead in the radial and tangential directions:

$$\sum F_r = F_n = m\frac{v^2}{r}$$

and

$$\sum F_t = -f_k = ma_t = m\frac{dv}{dt}$$

Express f_k in terms of μ_k and F_n:

$$f_k = \mu_k F_n$$

Substitute for F_n and f_k in the tangential equation to obtain the differential equation:

$$\frac{dv}{dt} = -\frac{\mu_k}{r}v^2$$

Separate the variables to obtain:

$$\frac{dv}{v^2} = -\frac{\mu_k}{r}dt$$

Express the integral of this equation with the limits of integration being from v_0 to v on the left-hand side and from 0 to t on the right-hand side:

$$\int_{v_0}^{v} \frac{1}{v'^2}dv' = -\frac{\mu_k}{r}\int_0^t dt'$$

Evaluate these integrals to obtain:

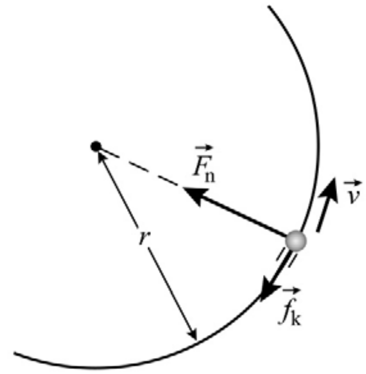
$$-\left(\frac{1}{v} - \frac{1}{v_0}\right) = -\left(\frac{\mu_k}{r}\right)t$$

Solve this equation for v:

$$\boxed{v = v_0\left(\frac{1}{1+\left(\frac{\mu_k v_0}{r}\right)t}\right)}$$

81 •••

Picture the Problem Note that the acceleration of the bead has two components–the radial component perpendicular to $\vec{v}$, and a tangential component due to friction that is opposite to $\vec{v}$. The application of Newton's 2nd law will result in a differential equation with separable variables. Its integration will lead to an expression for the speed of the bead as a function of time.



(*a*) In Problem 81 it was shown that:

$$v = v_0\left(\frac{1}{1+\left(\frac{\mu_k v_0}{r}\right)t}\right)$$

Express the centripetal acceleration of the bead:

$$a_c = \frac{v^2}{r} = \boxed{\frac{v_0^2}{r}\left(\frac{1}{1+\left(\frac{\mu_k v_0}{r}\right)t}\right)^2}$$

(*b*) Apply Newton's 2nd law to the bead:

$$\sum F_r = F_n = m\frac{v^2}{r}$$

and

$$\sum F_t = -f_k = ma_t = m\frac{dv}{dt}$$

Eliminate F_n and f_k to rewrite the radial force equation and solve for a_t:

$$a_t = -\mu_k\frac{v^2}{r} = \boxed{-\mu_k a_c}$$

(*c*) Express the resultant acceleration in terms of its radial and tangential components:

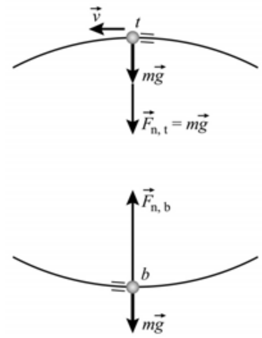
$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(-\mu_k a_c)^2 + a_c^2}$$

$$= \boxed{a_c\sqrt{1+\mu_k^2}}$$

Concepts of Centripetal Force

***82 •**

Picture the Problem The diagram depicts a seat at its highest and lowest points. Let "t" denote the top of the loop and "b" the bottom of the loop. Applying Newton's 2nd law to the seat at the top of the loop will establish the value of mv^2/r; this can then be used at the bottom of the loop to determine $F_{\text{n,b}}$.



Apply $\sum F_r = ma_r$ to the seat at the top of the loop:

$$mg + F_{\text{n,t}} = 2mg = ma_{\text{r}} = mv^2/r$$

Apply $\sum F_r = ma_r$ to the seat at the bottom of the loop:

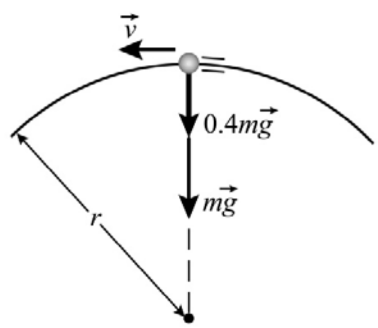
$$F_{\text{n,b}} - mg = mv^2/r$$

Solve for $F_{\text{n,b}}$ and substitute for mv^2/r to obtain:

$$F_{\text{n,b}} = 3mg \text{ and } \boxed{(d) \text{ is correct.}}$$

83 •

Picture the Problem The speed of the roller coaster is imbedded in the expression for its radial acceleration. The radial acceleration is determined by the net radial force acting on the passenger. We can use Newton's 2nd law to relate the net force on the passenger to the speed of the roller coaster.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the passenger:

$$mg + 0.4mg = mv^2/r$$

Solve for v:

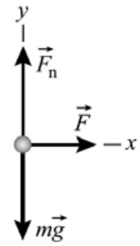
$$v = \sqrt{1.4gr}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{1.4(9.81\,\text{m/s}^2)(12.0\,\text{m})} = \boxed{12.8\,\text{m/s}}$$

84 •

Picture the Problem The force F the passenger exerts on the armrest of the car door is the radial force required to maintain the passenger's speed around the curve and is related to that speed through Newton's 2nd law of motion.



Apply $\sum F_x = ma_x$ to the forces acting on the passenger:

$$F = m\frac{v^2}{r}$$

Solve this equation for v:

$$v = \sqrt{\frac{rF}{m}}$$

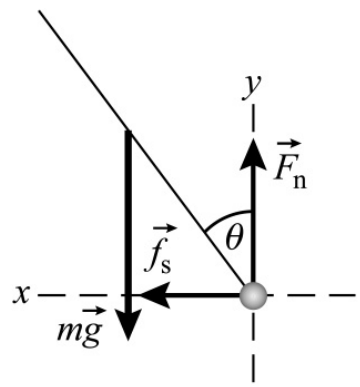
Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{(80\,\text{m})(220\,\text{N})}{70\,\text{kg}}} = 15.9\,\text{m/s}$$

and $\boxed{(a)\text{ is correct.}}$

***85** •••

Picture the Problem The forces acting on the bicycle are shown in the force diagram. The static friction force is the centripetal force exerted by the surface on the bicycle that allows it to move in a circular path. $\vec{F}_n + \vec{f}_s$ makes an angle θ with the vertical direction. The application of Newton's 2nd law will allow us to relate this angle to the speed of the bicycle and the coefficient of static friction.



(a) Apply $\sum \vec{F} = m\vec{a}$ to the bicycle:

$$\sum F_x = f_s = \frac{mv^2}{r}$$

and

$$\sum F_y = F_n - mg = 0$$

Relate F_n and f_s to θ:

$$\tan\theta = \frac{f_s}{F_n} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

Solve for v:

$$v = \sqrt{rg\tan\theta}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(20\,\text{m})(9.81\,\text{m/s}^2)\tan 15°} = \boxed{7.25\,\text{m/s}}$$

(b) Relate f_s to μ_s and F_n:

$$f_s = \tfrac{1}{2}f_{s,max} = \tfrac{1}{2}\mu_s mg$$

Solve for μ_s and substitute for f_s to obtain:

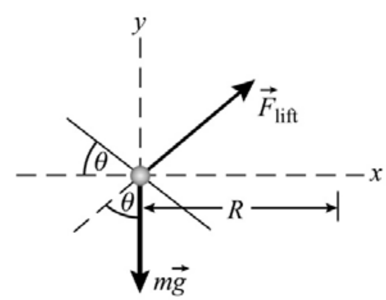
$$\mu_s = \frac{2f_s}{mg} = \frac{2v^2}{rg}$$

Substitute numerical values and evaluate μ_s

$$\mu_s = \frac{2(7.25\,\text{m/s})^2}{(20\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{0.536}$$

86 ••

Picture the Problem The diagram shows the forces acting on the plane as it flies in a horizontal circle of radius R. We can apply Newton's 2nd law to the plane and eliminate the lift force in order to obtain an expression for R as a function of v and θ.



Apply $\sum \vec{F} = m\vec{a}$ to the plane:

$$\sum F_x = F_{lift}\sin\theta = m\frac{v^2}{R}$$

and

$$\sum F_y = F_{lift}\cos\theta - mg = 0$$

Eliminate F_{lift} between these equations to obtain:

$$\tan\theta = \frac{v^2}{Rg}$$

Solve for R:

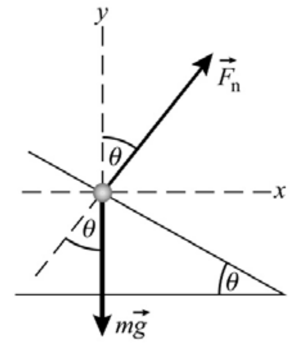
$$R = \frac{v^2}{g\tan\theta}$$

Substitute numerical values and evaluate R:

$$R = \frac{\left(480\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{(9.81\,\text{m/s}^2)\tan 40°} = \boxed{2.16\,\text{km}}$$

87 •

Picture the Problem Under the conditions described in the problem statement, the only forces acting on the car are the normal force exerted by the road and the gravitational force exerted by the earth. The horizontal component of the normal force is the centripetal force. The application of Newton's 2nd law will allow us to express θ in terms of v, r, and g.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

$$\tan\theta = \frac{v^2}{rg}$$

Solve for θ:

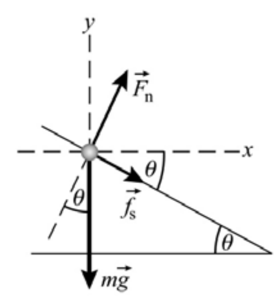
$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left\{\frac{[(90\,\text{km/h})(1\,\text{h}/3600\,\text{s})(1000\,\text{m/km})]^2}{(160\,\text{m})(9.81\,\text{m/s}^2)}\right\} = \boxed{21.7°}$$

***88** ••

Picture the Problem Both the normal force and the static friction force contribute to the centripetal force in the situation described in this problem. We can apply Newton's 2nd law to relate f_s and F_n and then solve these equations simultaneously to determine each of these quantities.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Multiply the *x* equation by $\sin\theta$ and the *y* equation by $\cos\theta$ to obtain:

$$f_s \sin\theta\cos\theta + F_n \sin^2\theta = m\frac{v^2}{r}\sin\theta$$

and

$$F_n \cos^2\theta - f_s \sin\theta\cos\theta - mg\cos\theta = 0$$

Add these equations to eliminate f_s:

$$F_n - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

Solve for F_n:

$$F_n = mg\cos\theta + m\frac{v^2}{r}\sin\theta = m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)$$

Substitute numerical values and evaluate F_n:

$$F_n = (800\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos10° + \frac{(85\,\text{km/h})^2(1000\,\text{m/km})^2(1\,\text{h}/3600\,\text{s})^2}{150\,\text{m}}\sin10°\right] = \boxed{8.25\,\text{kN}}$$

(*b*) Solve the *y* equation for f_s:

$$f_s = \frac{F_n\cos\theta - mg}{\sin\theta}$$

Substitute numerical values and evaluate f_s:

$$f_s = \frac{(8.25\,\text{kN})\cos10° - (800\,\text{kg})(9.81\,\text{m/s}^2)}{\sin10°} = \boxed{1.59\,\text{kN}}$$

(*c*) Express $\mu_{s,\min}$ in terms of f_s and F_n:

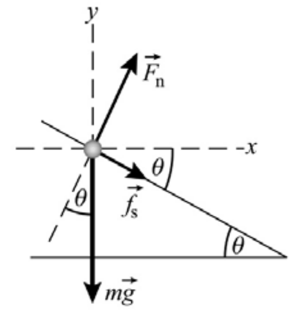
$$\mu_{s,\min} = \frac{f_s}{F_n}$$

Substitute numerical values and evaluate $\mu_{s,\min}$:

$$\mu_{s,\min} = \frac{1.59\ \text{kN}}{8.25\,\text{kN}} = \boxed{0.193}$$

89 ••

Picture the Problem Both the normal force and the static friction force contribute to the centripetal force in the situation described in this problem. We can apply Newton's 2nd law to relate f_s and F_n and then solve these equations simultaneously to determine each of these quantities.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r}$$

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Multiply the x equation by $\sin\theta$ and the y equation by $\cos\theta$:

$$f_s \sin\theta\cos\theta + F_n \sin^2\theta = m\frac{v^2}{r}\sin\theta$$

$$F_n \cos^2\theta - f_s \sin\theta\cos\theta - mg\cos\theta = 0$$

Add these equations to eliminate f_s:

$$F_n - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

Solve for F_n:

$$F_n = mg\cos\theta + m\frac{v^2}{r}\sin\theta = m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)$$

Substitute numerical values and evaluate F_n:

$$F_n = (800\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos10° + \frac{(38\,\text{km/h})^2(1000\,\text{m/km})^2(1\,\text{h}/3600\,\text{s})^2}{150\,\text{m}}\sin10°\right] = \boxed{7.832\,\text{kN}}$$

(*b*) Solve the y equation for f_s:

$$f_s = \frac{F_n\cos\theta - mg}{\sin\theta} = F_n\cot\theta - \frac{mg}{\sin\theta}$$

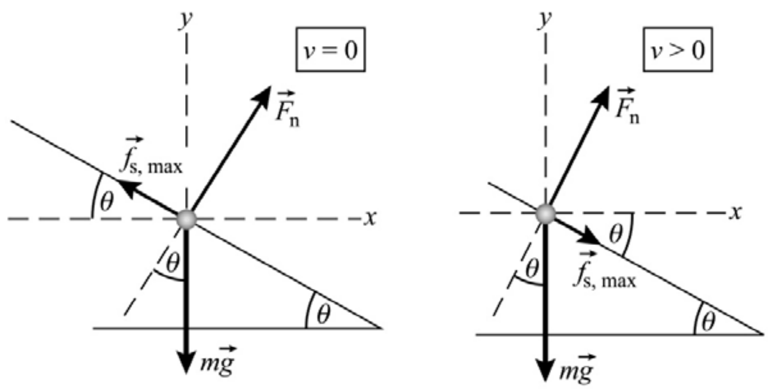
Substitute numerical values and evaluate f_s:

$$f_s = (7.832\,\text{kN})\cot 10° - \frac{(800\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 10°} = \boxed{-777\,\text{N}}$$

The negative sign tells us that f_s points upward along the inclined plane rather than as shown in the force diagram.

***90 •••**

Picture the Problem The free-body diagram to the left is for the car at rest. The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding. In the free-body diagram to the right, the static friction force points in the opposite direction as the tendency of the moving car is to slide toward the outside of the curve.



Apply $\sum \vec{F} = m\vec{a}$ to the car that is at rest:

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0 \quad (1)$$

and

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = 0 \quad (2)$$

Substitute $f_s = f_{s,max} = \mu_s F_n$ in equation (2) and solve for and evaluate the maximum allowable value of θ:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.08) = \boxed{4.57°}$$

Apply $\sum \vec{F} = m\vec{a}$ to the car that is moving with speed v:

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0 \quad (3)$$

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r} \quad (4)$$

Substitute $f_s = \mu_s F_n$ in equations (3) and (4) and simplify to obtain:

$$F_n(\cos\theta - \mu_s \sin\theta) = mg \quad (5)$$

$$F_n(\mu_s \cos\theta + \sin\theta) = m\frac{v^2}{r} \quad (6)$$

Substitute numerical values into (5)

$$0.9904 F_n = mg$$

and (6) to obtain:

and

$$0.1595F_n = m\frac{v^2}{r}$$

Eliminate F_n and solve for r:

$$r = \frac{v^2}{0.1610g}$$

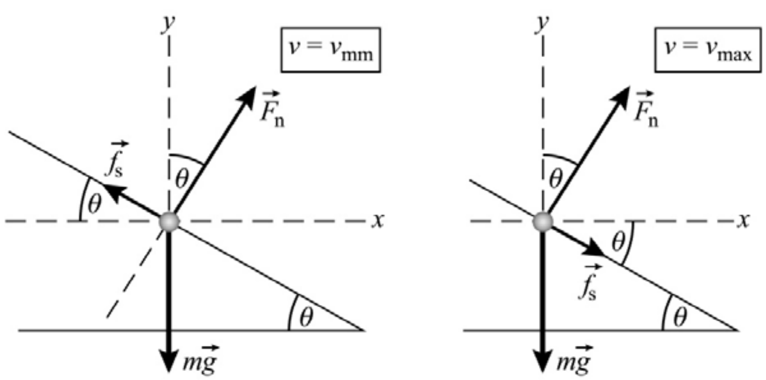
Substitute numerical values and evaluate r:

$$r = \frac{(60\,\text{km/h}\times 1\,\text{h}/3600\,\text{s}\times 1000\,\text{m/km})^2}{0.1610(9.81\,\text{m/s}^2)}$$

$$= \boxed{176\,\text{m}}$$

91 •••

Picture the Problem The free-body diagram to the left is for the car rounding the curve at the minimum (not sliding down the incline) speed. The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding. In the free-body diagram to the right, the static friction force points in the opposite direction as the tendency of the car moving with the maximum safe speed is to slide toward the outside of the curve. Application of Newton's 2nd law and the simultaneous solution of the force equations will yield v_{min} and v_{max}.



Apply $\sum \vec{F} = m\vec{a}$ to a car traveling around the curve when the coefficient of static friction is zero:

$$\sum F_x = F_n \sin\theta = m\frac{v_{min}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Divide the first of these equations by the second to obtain:

$$\tan\theta = \frac{v^2}{rg} \text{ or } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Substitute numerical values and evaluate the banking angle:

$$\theta = \tan^{-1}\left[\frac{(40\,\text{km/h})^2(1000\,\text{m/km})^2(1\text{h}/3600\,\text{s}^2)}{(30\,\text{m})(9.81\,\text{m/s}^2)}\right] = 22.8°$$

Apply $\sum \vec{F} = m\vec{a}$ to the car traveling around the curve at minimum speed:

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = m\frac{v_{\text{min}}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0$$

Substitute $f_s = f_{s,\text{max}} = \mu_s F_n$ in the force equations and simplify to obtain:

$$F_n(\mu_s \cos\theta - \sin\theta) = m\frac{v_{\text{min}}^2}{r}$$

and

$$F_n(\cos\theta + \mu_s \sin\theta) = mg$$

Evaluate these equations for $\theta = 22.8°$ and $\mu_s = 0.3$:

$$0.1102 F_n = m\frac{v_{\text{min}}^2}{r}$$

and

$$1.038 F_n = mg$$

Eliminate F_n between these two equations and solve for v_{min}:

$$v_{\text{min}} = \sqrt{0.106 rg}$$

Substitute numerical values and evaluate v_{min}:

$$v_{\text{min}} = \sqrt{0.106(30\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{5.59\,\text{m/s} = 20.1\,\text{km/h}}$$

Apply $\sum \vec{F} = m\vec{a}$ to the car traveling around the curve at maximum speed:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v_{\text{max}}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Substitute $f_s = f_{s,\text{max}} = \mu_s F_n$ in the force equations and simplify to obtain:

$$F_n(\mu_s \cos\theta + \sin\theta) = m\frac{v_{\text{max}}^2}{r}$$

and

$$F_n(\cos\theta - \mu_s \sin\theta) = mg$$

Evaluate these equations for $\theta = 22.8°$ and $\mu_s = 0.3$:

$$0.6641 F_n = m\frac{v_{\text{max}}^2}{r}$$

and

$$0.8056 F_n = mg$$

Eliminate F_n between these two equations and solve for v_{max}:

$$v_{max} = \sqrt{0.8243rg}$$

Substitute numerical values and evaluate v_{max}:

$$v_{max} = \sqrt{(0.8243)(30\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{15.6\,\text{m/s} = 56.1\,\text{km/h}}$$

Drag Forces

92 •

Picture the Problem We can apply Newton's 2nd law to the particle to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values.

Apply $\sum F_y = ma_y$ to the particle:

$$mg - bv = ma_y$$

When the particle reaches its terminal speed $v = v_t$ and $a_y = 0$:

$$mg - bv_t = 0$$

Solve for b to obtain:

$$b = \frac{mg}{v_t}$$

Substitute numerical values and evaluate b:

$$b = \frac{(10^{-13}\,\text{kg})(9.81\,\text{m/s}^2)}{3\times 10^{-4}\,\text{m/s}} = \boxed{3.27\times 10^{-9}\,\text{kg/s}}$$

93 •

Picture the Problem We can apply Newton's 2nd law to the Ping-Pong ball to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values.

Apply $\sum F_y = ma_y$ to the Ping-Pong ball:

$$mg - bv^2 = ma_y$$

When the Ping-Pong ball reaches its terminal speed $v = v_t$ and $a_y = 0$:

$$mg - bv_t^2 = 0$$

Solve for b to obtain:

$$b = \frac{mg}{v_t^2}$$

Substitute numerical values and evaluate b:

$$b = \frac{(2.3\times10^{-3}\,\text{kg})(9.81\,\text{m/s}^2)}{(9\,\text{m/s})^2} = \boxed{2.79\times10^{-4}\,\text{kg/m}}$$

***94 •**

Picture the Problem Let the upward direction be the positive y direction and apply Newton's 2nd law to the sky diver.

(*a*) Apply $\sum F_y = ma_y$ to the sky diver:

$$F_\text{d} - mg = ma_y$$

or, because $a_y = 0$,

$$F_\text{d} = mg \qquad (1)$$

Substitute numerical values and evaluate F_d:

$$F_\text{d} = (60\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{589\,\text{N}}$$

(*b*) Substitute $F_\text{d} = bv_\text{t}^2$ in equation (1) to obtain:

$$bv_\text{t}^2 = mg$$

Solve for b:

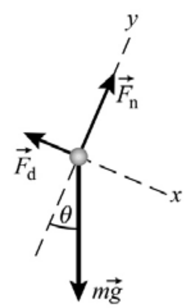
$$b = \frac{mg}{v_\text{t}^2} = \frac{F_\text{d}}{v_\text{t}^2}$$

Substitute numerical values and evaluate b:

$$b = \frac{589\,\text{N}}{(25\,\text{m/s})^2} = \boxed{0.942\,\text{kg/m}}$$

95 ••

Picture the Problem The free-body diagram shows the forces acting on the car as it descends the grade with its terminal velocity. The application of Newton's 2nd law with $a = 0$ and F_d equal to the given function will allow us to solve for the terminal velocity of the car.



Apply $\sum F_x = ma_x$ to the car:

$$mg\sin\theta - F_\text{d} = ma_x$$

or, because $v = v_\text{t}$ and $a_x = 0$,

$$mg\sin\theta - F_\text{d} = 0$$

Substitute for F_d to obtain:

$$mg\sin\theta - 100\,\text{N} - (1.2\,\text{N}\cdot\text{s}^2/\text{m}^2)v_\text{t}^2 = 0$$

Solve for v_t:

$$v_t = \sqrt{\frac{mg\sin\theta - 100\,\text{N}}{1.2\,\text{N}\cdot\text{s}^2/\text{m}^2}}$$

Substitute numerical values and evaluate v_t:

$$v_t = \sqrt{\frac{(800\,\text{kg})(9.81\,\text{m/s}^2)\sin 6° - 100\,\text{N}}{1.2\,\text{N}\cdot\text{s}^2/\text{m}^2}}$$

$$= 24.5\,\text{m/s} = \boxed{88.2\,\text{km/h}}$$

96 •••

Picture the Problem Let the upward direction be the positive *y* direction and apply Newton's 2nd law to the particle to obtain an equation from which we can find the particle's terminal speed.

(*a*) Apply $\sum F_y = ma_y$ to a pollution particle:

$$mg - 6\pi\eta r v = ma_y$$

or, because $a_y = 0$,

$$mg - 6\pi\eta r v_t = 0$$

Solve for v_t to obtain:

$$v_t = \frac{mg}{6\pi\eta r}$$

Express the mass of a sphere in terms of its volume:

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substitute for *m* to obtain:

$$v_t = \frac{2r^2\rho g}{9\eta}$$

Substitute numerical values and evaluate v_t:

$$v_t = \frac{2(10^{-5}\,\text{m})^2(2000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}{9(1.8\times10^{-5}\,\text{N}\cdot\text{s/m}^2)}$$

$$= \boxed{2.42\,\text{cm/s}}$$

(*b*) Use distance equals average speed times the fall time to find the time to fall 100 m at 2.42 cm/s:

$$t = \frac{10^4\,\text{cm}}{2.42\,\text{cm/s}} = 4.13\times10^3\,\text{s} = \boxed{1.15\,\text{h}}$$

***97 •••**

Picture the Problem The motion of the centrifuge will cause the pollution particles to migrate to the end of the test tube. We can apply Newton's 2nd law and Stokes' law to derive an expression for the terminal speed of the sedimentation particles. We can then use this terminal speed to calculate the sedimentation time. We'll use the 12 cm distance from the center of the centrifuge as the average radius of the pollution particles as they settle in the test tube. Let R represent the radius of a particle and r the radius of the particle's circular path in the centrifuge.

Express the sedimentation time in terms of the sedimentation speed v_t:

$$\Delta t_{\text{sediment}} = \frac{\Delta x}{v_t}$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to a pollution particle:

$$6\pi\eta R v_t = ma_c$$

Express the mass of the particle in terms of its radius R and density ρ:

$$m = \rho V = \tfrac{4}{3}\pi R^3 \rho$$

Express the acceleration of the pollution particles due to the motion of the centrifuge in terms of their orbital radius r and period T:

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Substitute for m and a_c and simplify to obtain:

$$6\pi\eta R v_t = \tfrac{4}{3}\pi R^3 \rho\left(\frac{4\pi^2 r}{T^2}\right) = \frac{16\pi^3 \rho r R^3}{3T^2}$$

Solve for v_t:

$$v_t = \frac{8\pi^2 \rho r R^2}{9\eta T^2}$$

Find the period T of the motion from the number of revolutions the centrifuge makes in 1 second:

$$T = \frac{1}{800\,\text{rev/min}} = 1.25\times10^{-3}\ \text{min/rev}$$
$$= 1.25\times10^{-3}\ \text{min/rev}\times 60\,\text{s/min}$$
$$= 75.0\times10^{-3}\ \text{s/rev}$$

Substitute numerical values and evaluate v_t:

$$v_t = \frac{8\pi^2\left(2000\,\text{kg/m}^3\right)(0.12\,\text{m})\left(10^{-5}\,\text{m}\right)^2}{9\left(1.8\times10^{-5}\,\text{N}\cdot\text{s/m}^2\right)\left(75\times10^{-3}\,\text{s}\right)^2}$$
$$= 2.08\,\text{m/s}$$

Find the time it takes the particles to move 8 cm as they settle in the test tube:

$$\Delta t_{\text{sediment}} = \frac{\Delta x}{v} = \frac{8\,\text{cm}}{208\,\text{cm/s}}$$
$$= \boxed{38.5\,\text{ms}}$$

In Problem 96 it was shown that the rate of fall of the particles in air is 2.42 cm/s. Find the time required to fall 8 cm in air under the influence of gravity:

$$\Delta t_{air} = \frac{\Delta x}{v} = \frac{8\,\text{cm}}{2.42\,\text{cm/s}} = \boxed{3.31\,\text{s}}$$

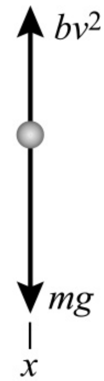
Find the ratio of the two times:

$$\Delta t_{air}/\Delta t_{sediment} \approx \boxed{100}$$

Euler's Method

98 ••

Picture the Problem The free-body diagram shows the forces acting on the baseball sometime after it has been thrown downward but before it has reached its terminal speed. In order to use Euler's method, we'll need to determine how the acceleration of the ball varies with its speed. We can do this by applying Newton's 2nd law to the ball and using its terminal speed to express the constant in the acceleration equation in terms of the ball's terminal speed. We can then use $v_{n+1} = v_n + a_n \Delta t$ to find the speed of the ball at any given time.



Apply Newton's 2nd law to the ball to obtain:

$$mg - bv^2 = m\frac{dv}{dt}$$

Solve for dv/dt to obtain:

$$\frac{dv}{dt} = g - \frac{b}{m}v^2$$

When the ball reaches its terminal speed:

$$0 = g - \frac{b}{m}v_t^2 \Rightarrow \frac{b}{m} = \frac{g}{v_t^2}$$

Substitute to obtain:

$$\frac{dv}{dt} = g\left(1 - \frac{v^2}{v_t^2}\right)$$

Express the position of the ball to obtain:

$$x_{n+1} = x_n + \frac{v_{n+1} + v_n}{2}\Delta t$$

Letting a_n be the acceleration of the ball at time t_n, express its speed when $t = t_n + 1$:

$$v_{n+1} = v_n + a_n \Delta t$$

where

$$a_n = g\left(1 - \frac{v_n^2}{v_t^2}\right)$$

and Δt is an arbitrarily small interval of time.

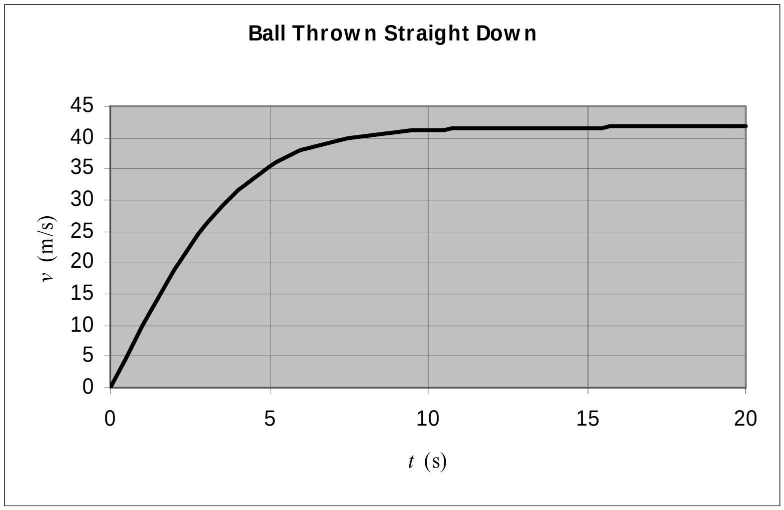
A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A10	B9+$B$1	$t + \Delta t$
B10	B9+0.5*(C9+C10)*$B$1	$x_{n+1} = x_n + \frac{v_{n+1} + v_n}{2}\Delta t$
C10	C9+D9*$B$1	$v_{n+1} = v_n + a_n\Delta t$
D10	$B$4*(1–C10^2/$B$5^2)	$a_n = g\left(1 - \frac{v_n^2}{v_t^2}\right)$

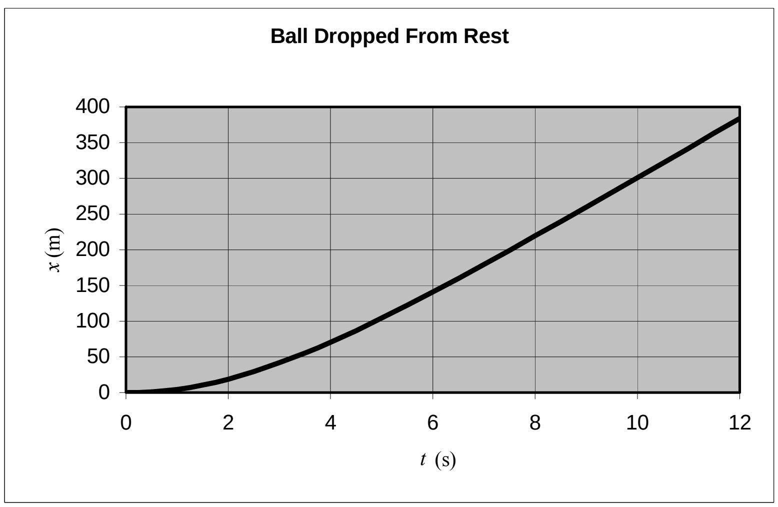
	A	B	C	D
1	Δt=	0.5	s	
2	x0=	0	m	
3	v0=	9.722	m/s	
4	a0=	9.81	m/s^2	
5	vt=	41.67	m/s	
6				
7	t	x	v	a
8	(s)	(m)	(m/s)	(m/s^2)
9	0.0	0	9.7	9.28
10	0.5	6	14.4	8.64
11	1.0	14	18.7	7.84
12	1.5	25	22.6	6.92
28	9.5	317	41.3	0.17
29	10.0	337	41.4	0.13
30	10.5	358	41.5	0.10
38	14.5	524	41.6	0.01
39	15.0	545	41.7	0.01
40	15.5	566	41.7	0.01
41	16.0	587	41.7	0.01
42	16.5	608	41.7	0.00

From the table we can see that the speed of the ball after 10 s is approximately $\boxed{41.4\,\text{m/s.}}$ We can estimate the uncertainty in this result by halving Δt and recalculating the speed of the ball at $t = 10$ s. Doing so yields $v(10\text{ s}) \approx 41.3$ m/s, a difference of about $\boxed{0.02\%.}$

The graph shows the velocity of the ball thrown straight down as a function of time.

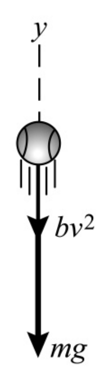


Reset Δt to 0.5 s and set $v_0 = 0$. Ninety-nine percent of 41.67 m/s is approximately 41.3 m/s. Note that the ball will reach this speed in about $\boxed{10.5\,\text{s}}$ and that the distance it travels in this time is about $\boxed{322\,\text{m.}}$ The following graph shows the distance traveled by the ball dropped from rest as a function of time.



***99** ••

Picture the Problem The free-body diagram shows the forces acting on the baseball after it has left your hand. In order to use Euler's method, we'll need to determine how the acceleration of the ball varies with its speed. We can do this by applying Newton's 2nd law to the baseball. We can then use $v_{n+1} = v_n + a_n\Delta t$ and $x_{n+1} = x_n + v_n\Delta t$ to find the speed and



position of the ball.

Apply $\sum F_y = ma_y$ to the baseball:

$$-bv|v| - mg = m\frac{dv}{dt}$$

where $|v| = v$ for the upward part of the flight of the ball and $|v| = -v$ for the downward part of the flight.

Solve for dv/dt:

$$\frac{dv}{dt} = -g - \frac{b}{m}v|v|$$

Under terminal speed conditions ($|v| = -v_t$):

$$0 = -g + \frac{b}{m}v_t^2$$

and

$$\frac{b}{m} = \frac{g}{v_t^2}$$

Substitute to obtain:

$$\frac{dv}{dt} = -g - \frac{g}{v_t^2}v|v| = -g\left(1 + \frac{v|v|}{v_t^2}\right)$$

Letting a_n be the acceleration of the ball at time t_n, express its position and speed when $t = t_n + 1$:

$$y_{n+1} = y_n + \tfrac{1}{2}(v_n + v_{n-1})\Delta t$$

and

$$v_{n+1} = v_n + a_n\Delta t$$

where

$$a_n = -g\left(1 + \frac{v_n|v_n|}{v_t^2}\right)$$

and Δt is an arbitrarily small interval of time.

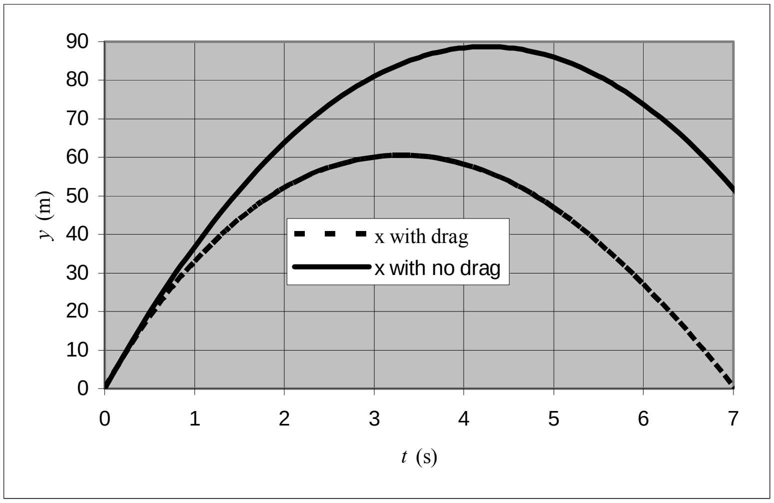
A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
D11	D10+$B$6	$t + \Delta t$
E10	41.7	v_0
E11	E10−$B$4*(1+E10*ABS(E10)/($B$5^2))*$B$6	$v_{n+1} = v_n + a_n\Delta t$
F10	0	y_0
F11	F10+0.5*(E10+E11)*$B$6	$y_{n+1} = y_n + \frac{1}{2}(v_n + v_{n-1})\Delta t$
G10	0	y_0
G11	$E$10*D11−0.5*$B$4*D11^2	$v_0 t - \frac{1}{2}gt^2$

	A	B	C	D	E	F	G
4	g=	9.81	m/s^2				
5	vt=	41.7	m/s				
6	Δt=	0.1	s				
7							
8							
9				t	v	y	y no drag
10				0.0	41.70	0.00	0.00
11				0.1	39.74	4.07	4.12
12				0.2	37.87	7.95	8.14
40				3.0	3.01	60.13	81.00
41				3.1	2.03	60.39	82.18
42				3.2	1.05	60.54	83.26
43				3.3	0.07	60.60	84.25
44				3.4	–0.91	60.55	85.14
45				3.5	–1.89	60.41	85.93
46				3.6	–2.87	60.17	86.62
78				6.8	–28.34	6.26	56.98
79				6.9	–28.86	3.41	54.44
80				7.0	–29.37	0.49	51.80
81				7.1	–29.87	–2.47	49.06

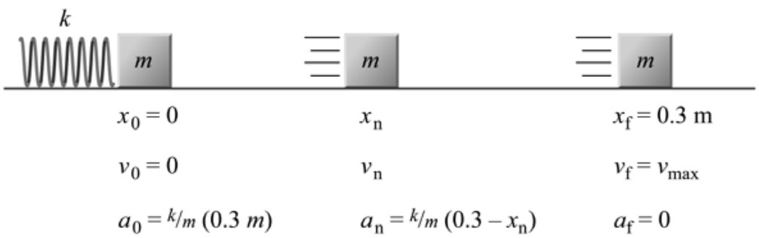
From the table we can see that, after 3.5 s, the ball reaches a height of about $\boxed{60.4\,\text{m.}}$ It reaches its peak a little earlier–at about $\boxed{3.3\,\text{s,}}$ and its height at $t = 3.3$ s is $\boxed{60.6\,\text{m.}}$ The ball hits the ground at about $t = \boxed{7\,\text{s}}$ –so it spends a little longer coming down than going up.

The solid curve on the following graph shows $y(t)$ when there is no drag on the baseball and the dotted curve shows $y(t)$ under the conditions modeled in this problem.



100 ••

Picture the Problem The pictorial representation shows the block in its initial position against the compressed spring, later as the spring accelerates it to the right, and finally when it has reached its maximum speed at $x_f = 0$. In order to use Euler's method, we'll need to determine how the acceleration of the block varies with its position. We can do this by applying Newton's 2nd law to the box. We can then use $v_{n+1} = v_n + a_n\Delta t$ and $x_{n+1} = x_n + v_n\Delta t$ to find the speed and position of the block.



Apply $\sum F_x = ma_x$ to the block:

$$k(0.3\,\text{m} - x_n) = ma_n$$

Solve for a_n:

$$a_n = \frac{k}{m}(0.3\,\text{m} - x_n)$$

Express the position and speed of the block when $t = t_n + 1$:

$$x_{n+1} = x_n + v_n\Delta t$$

and

$$v_{n+1} = v_n + a_n\Delta t$$

where

$$a_n = \frac{k}{m}(0.3\,\text{m} - x_n)$$

and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A10	A9+$B$1	$t + \Delta t$
B10	B9+C10*$B$1	$x_n + v_n \Delta t$
C10	C9+D9*$B$1	$v_n + a_n \Delta t$
D10	($B$4/$B$5)*(0.3–B10)	$\frac{k}{m}(0.3 - x_n)$

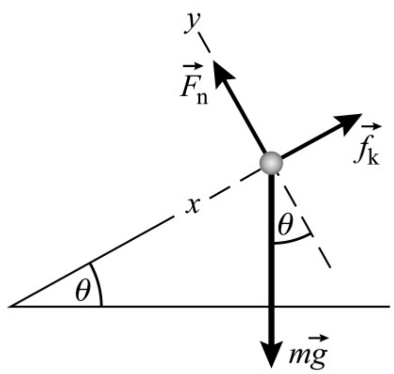
	A	B	C	D
1	Δt=	0.005	s	
2	x0=	0	m	
3	v0=	0	m/s	
4	k =	50	N/m	
5	m =	0.8	kg	
6				
7	t	x	v	a
8	(s)	(m)	(m/s)	(m/s^2)
9	0.000	0.00	0.00	18.75
10	0.005	0.00	0.09	18.72
11	0.010	0.00	0.19	18.69
12	0.015	0.00	0.28	18.63
45	0.180	0.25	2.41	2.85
46	0.185	0.27	2.42	2.10
47	0.190	0.28	2.43	1.34
48	0.195	0.29	2.44	0.58
49	0.200	0.30	2.44	–0.19

From the table we can see that it took about $\boxed{0.200\,\text{s}}$ for the spring to push the block 30 cm and that it was traveling about $\boxed{2.44\,\text{m/s}}$ at that time. We can estimate the uncertainty in this result by halving Δt and recalculating the speed of the ball at t = 10 s. Doing so yields v(0.200 s) ≈ 2.41 m/s, a difference of about $\boxed{1.2\%.}$

General Problems

101 •

Picture the Problem The forces that act on the block as it slides down the incline are shown on the free-body diagram to the right. The acceleration of the block can be determined from the distance-and-time information given in the problem. The application of Newton's 2nd law to the block will lead to an expression for the coefficient of kinetic friction as a function of the block's acceleration and the angle of the incline.



Apply $\sum\vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = mg\sin\theta - f_k = ma$$
and
$$\Sigma F_y = F_n - mg = 0$$

Set $f_k = \mu_k F_n$, F_n between the two equations, and solve for μ_k:

$$\mu_k = \frac{g\sin\theta - a}{g\cos\theta}$$

Using a constant-acceleration equation, relate the distance the block slides to its sliding time:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2 \text{ where } v_0 = 0$$

Solve for a:

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

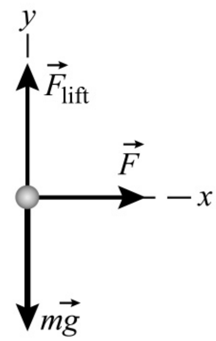
$$a = \frac{2(2.4\,\text{m})}{(5.2\,\text{s})^2} = 0.1775\,\text{m/s}^2$$

Find μ_k for $a = 0.1775$ m/s^2 and $\theta = 28°$:

$$\mu_k = \frac{(9.81\,\text{m/s}^2)\sin 28° - 0.1775\,\text{m/s}^2}{(9.81\,\text{m/s}^2)\cos 28°} = \boxed{0.511}$$

102 •

Picture the Problem The free-body diagram shows the forces acting on the model airplane. The speed of the plane can be calculated from the data concerning the radius of its path and the time it takes to make one revolution. The application of Newton's 2nd law will give us the tension F in the string.



(*a*) Express the speed of the airplane in terms of the circumference of the circle in which it is flying and its period:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(5.7\,\text{m})}{\frac{4}{1.2}\,\text{s}} = \boxed{10.7\,\text{m/s}}$$

(*b*) Apply $\sum F_x = ma_x$ to the model airplane:

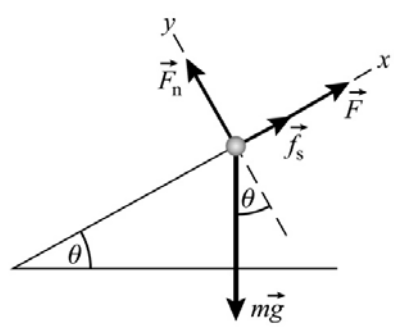
$$F = m\frac{v^2}{r}$$

Substitute numerical values and evaluate F:

$$F = (0.4\,\text{kg})\frac{(10.7\,\text{m/s})^2}{5.7\,\text{m}} = \boxed{8.03\,\text{N}}$$

***103** ••

Picture the Problem The free-body diagram shows the forces acting on the box. If the student is pushing with a force of 200 N and the box is on the verge of moving, the static friction force must be at its maximum value. In part (*b*), the motion is impending up the incline; therefore the direction of $f_{s,max}$ is down the incline.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the box:

$$\sum F_x = f_s + F - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Substitute $f_s = f_{s,max} = \mu_s F_n$, eliminate F_n between the two equations, and solve for μ_s:

$$\mu_s = \tan\theta - \frac{F}{mg\cos\theta}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \tan 30° - \frac{200\,\text{N}}{(800\,\text{N})\cos 30°} = \boxed{0.289}$$

(*b*) Find $f_{s,max}$ from the *x*-direction force equation:

$$f_{s,max} = mg\sin\theta - F$$

Substitute numerical values and evaluate $f_{s,max}$:

$$f_{s,max} = (800\,\text{N})\sin 30° - 200\,\text{N} = 200\,\text{N}$$

If the block is on the verge of sliding up the incline, $f_{s,max}$ must act down the incline. The *x*-direction force equation becomes:

$$-f_{s,max} + F - mg\sin\theta = 0$$

Solve the *x*-direction force equation for *F*:

$$F = mg\sin\theta + f_{s,max}$$

Substitute numerical values and evaluate *F*:

$$F = (800\,\text{N})\sin 30° + 200\,\text{N} = \boxed{600\,\text{N}}$$

104 •

Picture the Problem The path of the particle is a circle if *r* is a constant. Once we have shown that it is, we can calculate its value from its components. The direction of the particle's motion can be determined by examining two positions of the particle at times that are close to each other.

(*a*) and (*b*) Express the magnitude of $\vec{r}$ in terms of its components:

$$r = \sqrt{r_x^2 + r_y^2}$$

Evaluate *r* with $r_x = -10$ m cos ωt and $r_y = 10$ m sinωt:

$$r = \sqrt{[(-10\,\text{m})\cos\omega t]^2 + [(10\,\text{m})\sin\omega t]^2} = \sqrt{100(\cos^2\omega t + \sin^2\omega t)}\,\text{m} = \boxed{10.0\,\text{m}}$$

(*c*) Evaluate r_x and r_y at $t = 0$ s:

$$r_x = -(10\,\text{m})\cos 0° = -10\,\text{m}$$
$$r_y = (10\,\text{m})\sin 0° = 0$$

Evaluate r_x and r_y at $t = \Delta t$, where Δt is small:

$$r_x = -(10\,\text{m})\cos\omega\Delta t \approx -(10\,\text{m})\cos 0° = -10\,\text{m}$$
$$r_y = (10\,\text{m})\sin\omega\Delta t = \Delta y \text{ where } \Delta y \text{ is positive}$$

and $\boxed{\text{the motion is clockwise}}$

(*d*) Differentiate $\vec{r}$ with respect to time to obtain $\vec{v}$:

$$\vec{v} = d\vec{r}/dt = [(10\omega\sin\omega t)m]\,\hat{i} + [(10\omega\cos\omega t)m]\,\hat{j}$$

Use the components of $\vec{v}$ to find its speed:

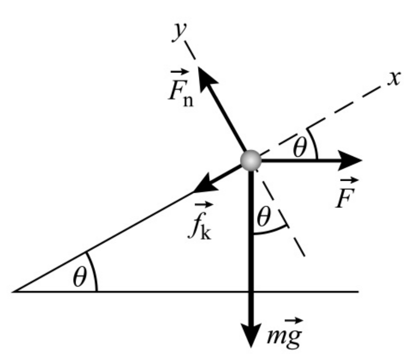
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{[(10\omega\sin\omega t)\text{m}]^2 + [(10\omega\cos\omega t)\text{m}]^2} = (10\,\text{m})\omega = (10\,\text{m})(2\,\text{s}^{-1}) = \boxed{20.0\,\text{m/s}}$$

(*e*) Relate the period of the particle's motion to the radius of its path and its speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi(10\,\text{m})}{20\,\text{m/s}} = \boxed{\pi\,\text{s}}$$

105 ••

Picture the Problem The free-body diagram shows the forces acting on the crate of books. The kinetic friction force opposes the motion of the crate up the incline. Because the crate is moving at constant speed in a straight line, its acceleration is zero. We can determine F by applying Newton's 2nd law to the crate, substituting for f_k, eliminating the normal force, and solving for the required force.



Apply $\sum\vec{F} = m\vec{a}$ to the crate, with both a_x and a_y equal to zero, to the crate:

$$\sum F_x = F\cos\theta - f_k - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - F\sin\theta - mg\cos\theta = 0$$

Substitute $\mu_s F_n$ for f_k and eliminate F_n to obtain:

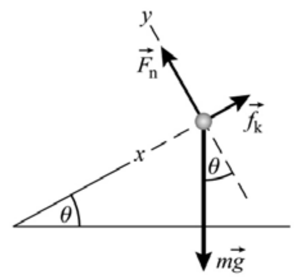
$$F = \frac{mg(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta}$$

Substitute numerical values and evaluate F:

$$F = \frac{(100\,\text{kg})(9.81\,\text{m/s}^2)(\sin 30° + (0.5)\cos 30°)}{\cos 30° - (0.5)\sin 30°} = \boxed{1.49\,\text{kN}}$$

106 ••

Picture the Problem The free-body diagram shows the forces acting on the object as it slides down the inclined plane. We can calculate its speed at the bottom of the incline from its acceleration and displacement and find its acceleration from Newton's 2nd law.



Using a constant-acceleration equation, relate the initial and final velocities of the object to its acceleration and displacement: solve for the final velocity:

$$v^2 = v_0^2 + 2a\Delta x$$

Because $v_0 = 0$, $v = \sqrt{2a\Delta x}$ (1)

Apply $\sum \vec{F} = m\vec{a}$ to the sliding object:

$$\sum F_x = -f_k + mg\sin\theta = ma$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Solve the y equation for F_n and using $f_k = \mu_k F_n$, eliminate both F_n and f_k from the x equation and solve for a:

$$a = g(\sin\theta - \mu_k \cos\theta) \quad (2)$$

Substitute equation (2) in equation (1) and solve for v:

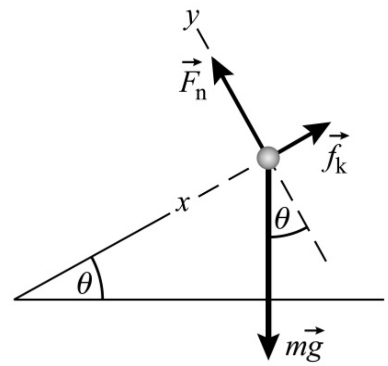
$$v = \sqrt{2g(\sin\theta - \mu_k\cos\theta)\Delta x}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(\sin 30° - (0.35)\cos 30°)(72\,\text{m})} = 16.7\,\text{m/s} \text{ and } \boxed{(d)\text{ is correct.}}$$

***107** ••

Picture the Problem The free-body diagram shows the forces acting on the brick as it slides down the inclined plane. We'll apply Newton's 2nd law to the brick when it is sliding down the incline with constant speed to derive an expression for μ_k in terms of θ_0. We'll apply Newton's 2nd law a second time for $\theta = \theta_1$ and solve the equations simultaneously to obtain an expression for a as a function of θ_0 and θ_1.



Apply $\sum \vec{F} = m\vec{a}$ to the brick when it is sliding with constant speed:

$$\sum F_x = -f_k + mg\sin\theta_0 = 0$$

and

$$\sum F_y = F_n - mg\cos\theta_0 = 0$$

Solve the y equation for F_n and using $f_k = \mu_k F_n$, eliminate both F_n and f_k from the x equation and solve for μ_k:

$$\mu_k = \tan\theta_0$$

Apply $\sum \vec{F} = m\vec{a}$ to the brick when $\theta = \theta_1$:

$$\sum F_x = -f_k + mg\sin\theta_1 = ma$$

and

$$\sum F_y = F_n - mg\cos\theta_1 = 0$$

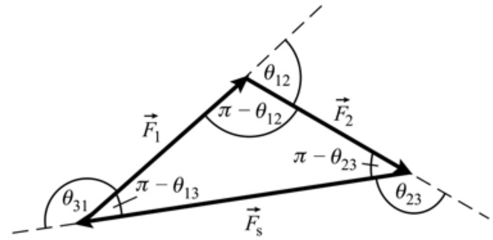
Solve the y equation for F_n, use $f_k = \mu_k F_n$ to eliminate both F_n and f_k from the x equation, and use the expression for μ_k obtained above to obtain:

$$\boxed{a = g(\sin\theta_1 - \tan\theta_0\cos\theta_1)}$$

108 ••

Picture the Problem The fact that the object is in static equilibrium under the influence of the three forces means that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Drawing the corresponding force triangle will allow us to relate the forces to the angles between them through the law of sines and the law of cosines.

(*a*) Using the fact that the object is in static equilibrium, redraw the force diagram connecting the forces head-to-tail:



Apply the law of sines to the triangle:

$$\frac{F_1}{\sin(\pi-\theta_{23})}=\frac{F_2}{\sin(\pi-\theta_{13})}=\frac{F_3}{\sin(\pi-\theta_{12})}$$

Use the trigonometric identity $\sin(\pi-\alpha)=\sin\alpha$ to obtain:

$$\boxed{\frac{F_1}{\sin\theta_{23}}=\frac{F_2}{\sin\theta_{13}}=\frac{F_3}{\sin\theta_{12}}}$$

(*b*) Apply the law of cosines to the triangle:

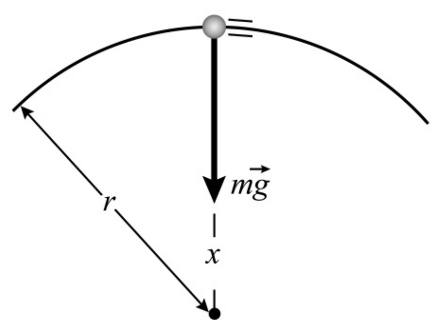
$$F_1^2=F_2^2+F_3^2-2F_2F_3\cos(\pi-\theta_{23})$$

Use the trigonometric identity $\cos(\pi-\alpha)=-\cos\alpha$ to obtain:

$$\boxed{F_1^2=F_2^2+F_3^2+2F_2F_3\cos\theta_{23}}$$

109 ••

Picture the Problem We can calculate the acceleration of the passenger from his/her speed that, in turn, is a function of the period of the motion. To determine the longest period of the motion, we focus our attention on the situation at the very top of the ride when the seat belt is exerting no force on the rider. We can use Newton's 2nd law to relate the period of the motion to the acceleration and speed of the rider.



(*a*) Because the motion is at constant speed, the acceleration is entirely radial and is given by:

$$a_c=\frac{v^2}{r}$$

Express the speed of the motion of the ride as a function of the radius of the circle and the period of its motion:

$$v=\frac{2\pi r}{T}$$

Substitute in the expression for a_c to obtain:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(5\,\text{m})}{(2\,\text{s})^2} = \boxed{49.3\,\text{m/s}^2}$$

(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the passenger when he/she is at the top of the circular path and solve for a_c:

$$\sum F_r = mg = ma_c$$

and

$a_c = g$

Relate the acceleration of the motion to its radius and speed and solve for v:

$$g = \frac{v^2}{r} \Rightarrow v = \sqrt{gr}$$

Express the period of the motion as a function of the radius of the circle and the speed of the passenger and solve for T_m:

$$T_m = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r}{g}}$$

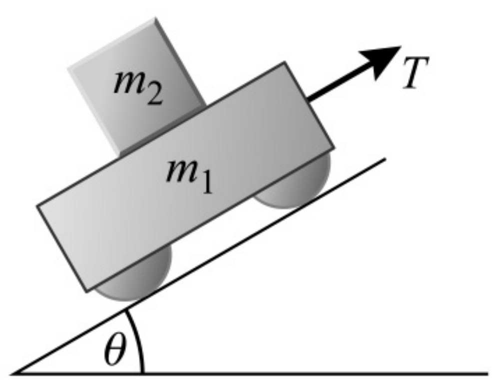
Substitute numerical values and evaluate T_m:

$$T_m = 2\pi\sqrt{\frac{5\,\text{m}}{9.81\,\text{m/s}^2}} = \boxed{4.49\,\text{s}}$$

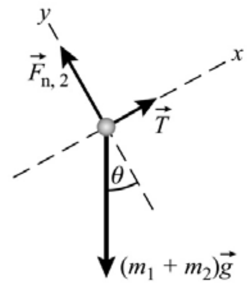
Remarks: The rider is "weightless" under the conditions described in part (*b*).

***110** ••

Picture the Problem The pictorial representation to the right shows the cart and its load on the inclined plane. The load will not slip provided its maximum acceleration is not exceeded. We can find that maximum acceleration by applying Newton's 2nd law to the load. We can then apply Newton's 2nd law to the cart-plus-load system to determine the tension in the rope when the system is experiencing its maximum acceleration.



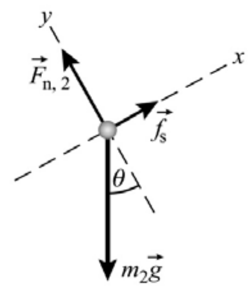
Draw the free-body diagram for the cart and its load:



Apply $\sum F_x = ma_x$ to the cart plus its load:

$$T - (m_1 + m_2)g\sin\theta = (m_1 + m_2)a_{\text{max}} \quad (1)$$

Draw the free-body diagram for the load of mass m_2 on top of the cart:



Apply $\sum \vec{F} = m\vec{a}$ to the load on top of the cart:

$$\sum F_x = f_{\text{s,max}} - m_2 g\sin\theta = m_2 a_{\text{max}}$$

and

$$\sum F_y = F_{\text{n,2}} - m_2 g\cos\theta = 0$$

Using $f_{\text{s,max}} = \mu_s F_{\text{n,2}}$, eliminate $F_{\text{n,2}}$ between the two equations and solve for the maximum acceleration of the load:

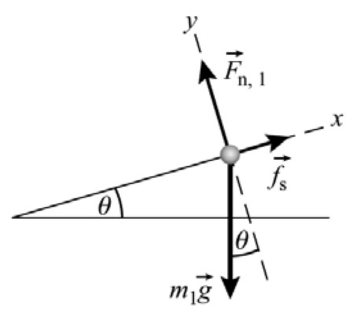
$$a_{\text{max}} = g(\mu_s\cos\theta - \sin\theta) \quad (2)$$

Substitute equation (2) in equation (1) and solve for T:

$$T = \boxed{(m_1 + m_2)g\mu_s\cos\theta}$$

111 ••

Picture the Problem The free-body diagram for the sled while it is held stationary by the static friction force is shown to the right. We can solve this problem by repeatedly applying Newton's 2nd law under the conditions specified in each part of the problem.



(*a*) Apply $\sum F_y = ma_y$ to the sled:

$$F_{n,1} - m_1 g\cos\theta = 0$$

Solve for $F_{n,1}$:

$$F_{n,1} = m_1 g\cos\theta$$

Substitute numerical values and evaluate $F_{n,1}$:

$$F_{n,1} = (200\,\text{N})\cos 15° = \boxed{193\,\text{N}}$$

(*b*) Apply $\sum F_x = ma_x$ to the sled:

$$f_s - m_1 g\sin\theta = 0$$

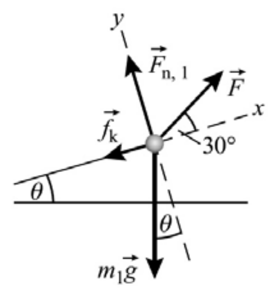
Solve for f_s:

$$f_s = m_1 g\sin\theta$$

Substitute numerical values and evaluate f_s:

$$f_s = (200\,\text{N})\sin 15° = \boxed{51.8\,\text{N}}$$

(*c*) Draw the free-body diagram for the sled when the child is pulling on the rope:



Apply $\sum \vec{F} = m\vec{a}$ to the sled to determine whether it moves:

$$\sum F_x = F_{net} = F\cos 30° - m_1 g\sin\theta - f_{s,max}$$

and

$$\sum F_y = F_{n,1} + F\sin 30° - m_1 g\cos\theta = 0$$

Solve the y-direction equation for $F_{n,1}$:

$$F_{n,1} = -F\sin 30° + m_1 g\cos\theta$$

Substitute numerical values and evaluate $F_{n,1}$:

$$F_{n,1} = -(100\,\text{N})\sin 30° + (200\,\text{N})\cos 15° = 143\,\text{N}$$

Express $f_{s,max}$:

$$f_{s,max} = \mu_s F_{n,1} = (0.5)(143\text{ N}) = 71.5\text{ N}$$

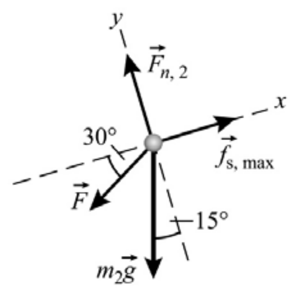
Use the x-direction force equation to evaluate F_{net}:

$$F_{net} = (100\text{ N})\cos 30° - (200\text{ N})\sin 15° - 71.5\text{ N} = -36.7\text{ N}$$

Because the net force is negative, the sled does not move: $\boxed{f_\text{k} \text{ is undetermined}}$

(*d*) Because the sled does not move: $\boxed{\mu_\text{k} \text{ is undetermined}}$

(*e*) Draw the FBD for the child:



Express the net force F_c exerted on the child by the incline:

$$F_\text{c} = \sqrt{F_\text{n2}^2 + f_\text{s,max}^2} \quad (1)$$

Noting that the child is stationary, apply $\sum \vec{F} = m\vec{a}$ to the child:

$$\sum F_x = f_\text{s,max} - F\cos 30° - m_2 g \sin 15° = 0$$

and

$$\sum F_y = F_\text{n2} - m_2 g \sin 15° - F \sin 30° = 0$$

Solve the x equation for $f_{s,max}$ and the y equation for F_n2:

$$f_\text{s,max} = F\cos 30° + m_2 g \sin 15°$$

and

$$F_\text{n2} = m_2 g \sin 15° + F \sin 30°$$

Substitute numerical values and evaluate F_x and F_n2:

$$f_\text{s,max} = (500\,\text{N})\cos 30° + (100\,\text{N})\sin 15° = 459\,\text{N}$$

and

$$F_\text{n2} = (100\,\text{N})\sin 15° + (500\,\text{N})\sin 30° = 276\,\text{N}$$

Substitute numerical values in equation (1) and evaluate F:

$$F_\text{c} = \sqrt{(276\,\text{N})^2 + (459\,\text{N})^2} = \boxed{536\,\text{N}}$$

112 •

Picture the Problem Let *v* represent the speed of rotation of the station, and *r* the distance from the center of the station. Because the O'Neill colony is, presumably, in deep space, the only acceleration one would experience in it would be that due to its rotation.

(*a*) Express the acceleration of anyone who is standing inside the station:

$a = v^2/r$

This acceleration is directed toward the axis of rotation. If someone inside the station drops an apple, the apple will not have any forces acting on it once released, but will move along a straight line at constant speed. However, from the point of view of our observer inside the station, if he views himself as unmoving, the apple is perceived to have an acceleration of mv^2/r directed away from the axis of rotation (a "centrifugal" force).

(*b*) Each deck must rotate the central axis with the same period *T*. Relate the speed of a person on a particular deck to his/her distance *r* from the center:

$$v = \frac{2\pi r}{T}$$

Express the "acceleration of gravity" perceived by someone a distance *r* from the center:

$$\frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$\boxed{\text{i.e., the "acceleration due to gravity" decreases as } r \text{ decreases.}}$$

(*c*) Relate the desired acceleration to the radius of Babylon 5 and its period:

$$a = \frac{4\pi^2 r}{T^2}$$

Solve for *T*:

$$T = \sqrt{\frac{4\pi^2 r}{a}}$$

Substitute numerical values and evaluate *T*:

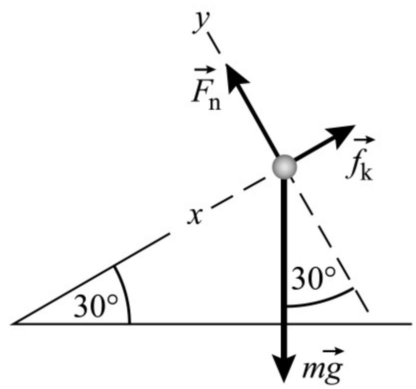
$$T = \sqrt{\frac{4\pi^2\left(0.3\,\text{mi}\times\frac{1.609\,\text{km}}{\text{mi}}\right)}{9.8\,\text{m/s}^2}} = 44.1\,\text{s} = 0.735\,\text{min}$$

Take the reciprocal of this time to find the number of revolutions per minute Babylon 5 has to make in order to provide this "earth-like" acceleration:

$$T^{-1} = \boxed{1.36\,\text{rev/min}}$$

113 ••

Picture the Problem The free-body diagram shows the forces acting on the child as she slides down the incline. We'll first use Newton's 2nd law to derive an expression for μ_k in terms of her acceleration and then use Newton's 2nd law to find her acceleration when riding the frictionless cart. Using a constant-acceleration equation, we'll relate these two accelerations to her descent times and solve for her acceleration when sliding. Finally, we can use this acceleration in the expression for μ_k.



Apply $\sum \vec{F} = m\vec{a}$ to the child as she slides down the incline:

$$\sum F_x = mg\sin 30° - f_k = ma_1$$

and

$$\sum F_y = F_n - mg\cos 30° = 0$$

Using $f_k = \mu_k F_n$, eliminate f_k and F_n between the two equations and solve for μ_k:

$$\mu_k = \tan 30° - \frac{a_1}{g\cos 30°} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the child as she rides the frictionless cart down the incline and solve for her acceleration a_2:

$$mg\sin 30° = ma_2$$

and

$$a_2 = g\sin 30° = 4.91\,\text{m/s}^2$$

Letting s represent the distance she slides down the incline, use a constant-acceleration equation to relate her sliding times to her accelerations and distance traveled down the slide :

$$s = v_0 t_1 + \tfrac{1}{2}a_1 t_1^2 \text{ where } v_0 = 0$$

and

$$s = v_0 t_2 + \tfrac{1}{2}a_2 t_2^2 \text{ where } v_0 = 0$$

Equate these expressions, substitute $t_2 = \frac{1}{2}t_1$ and solve for a_1:

$$a_1 = \tfrac{1}{4}a_2 = \tfrac{1}{4}g\sin 30° = 1.23\,\text{m/s}^2$$

Evaluate equation (1) with $a_1 = 1.23\ \text{m/s}^2$:

$$\mu_k = \tan 30° - \frac{1.23\ \text{m/s}^2}{(9.81\ \text{m/s}^2)\cos 30°} = \boxed{0.433}$$

***114 ••**

Picture the Problem The path of the particle is a circle if *r* is a constant. Once we have shown that it is, we can calculate its value from its components and determine the particle's velocity and acceleration by differentiation. The direction of the net force acting on the particle can be determined from the direction of its acceleration.

(*a*) Express the magnitude of $\vec{r}$ in terms of its components:

$$r = \sqrt{r_x^2 + r_y^2}$$

Evaluate *r* with $r_x = R\sin\omega t$ and $r_y = R\cos\omega t$:

$$r = \sqrt{[R\sin\omega t]^2 + [R\cos\omega t]^2} = \sqrt{R^2(\sin^2\omega t + \cos^2\omega t)} = R = 4.0\ \text{m}$$

$$\boxed{\therefore \text{the path of the particle is a circle centered at the origin.}}$$

(*b*) Differentiate $\vec{r}$ with respect to time to obtain $\vec{v}$:

$$\vec{v} = d\vec{r}/dt = [R\omega\cos\omega t]\,\hat{i} + [-R\omega\sin\omega t]\,\hat{j} = \boxed{[(8\pi\cos 2\pi t)\text{m/s}]\,\hat{i} - [(8\pi\sin 2\pi t)\text{m/s}]\,\hat{j}}$$

Express the ratio $\dfrac{v_x}{v_y}$:

$$\frac{v_x}{v_y} = \frac{8\pi\cos\omega t}{-8\pi\sin\omega t} = -\cot\omega t$$

Express the ratio $-\dfrac{y}{x}$:

$$-\frac{y}{x} = -\frac{R\cos\omega t}{R\sin\omega t} = -\cot\omega t$$

$$\therefore \boxed{\frac{v_x}{v_y} = -\frac{y}{x}}$$

(*c*) Differentiate $\vec{v}$ with respect to time to obtain $\vec{a}$:

$$\vec{a} = d\vec{v}/dt = \boxed{[(-16\pi^2\text{m/s}^2)\sin\omega t]\,\hat{i} + [(-16\pi^2\text{m/s}^2)\cos\omega t]\,\hat{j}}$$

Factor $-4\pi^2/\text{s}^2$ from $\vec{a}$ to obtain:

$$\vec{a} = (-4\pi^2/\text{s}^2)\left[(4\sin\omega t)\hat{i} + (4\cos\omega t)\hat{j}\right]$$
$$= \boxed{(-4\pi^2/\text{s}^2)\vec{r}}$$

Because $\vec{a}$ is in the opposite direction from $\vec{r}$, it is directed toward the center of the circle in which the particle is traveling.

Find the ratio $\dfrac{v^2}{r}$:

$$\frac{v^2}{r} = \frac{(8\pi\ \text{m/s})^2}{4\ \text{m}} = \boxed{16\pi^2\ \text{m/s}^2 = a}$$

(*d*) Apply $\sum\vec{F} = m\vec{a}$ to the particle:

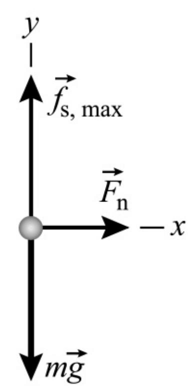
$$F_{\text{net}} = ma = (0.8\ \text{kg})(16\pi^2\ \text{m/s}^2)$$
$$= \boxed{12.8\pi^2\ \text{N}}$$

Because the direction of $\vec{F}_{\text{net}}$ is the same as that of $\vec{a}$:

$\boxed{\vec{F}_{\text{net}}\ \text{is toward the center of the circle.}}$

115 ••

Picture the Problem The free-body diagram showing the forces acting on a rider being held in place by the maximum static friction force is shown to the right. The application of Newton's 2nd law and the definition of the maximum static friction force will be used to determine the period T of the motion. The reciprocal of the period will give us the minimum number of revolutions required per unit time to hold the riders in place.



Apply $\sum\vec{F} = m\vec{a}$ to the riders while they are held in place by friction:

$$\sum F_x = F_{\text{n}} = m\frac{v^2}{r}$$

and

$$\sum F_y = f_{\text{s,max}} - mg = 0$$

Using $f_{\text{s,max}} = \mu_{\text{s}}F_{\text{n}}$ and $v = \dfrac{2\pi r}{T}$, eliminate F_{n} between the force equations and solve for the period of the motion:

$$T = 2\pi\sqrt{\frac{\mu_{\text{s}}r}{g}}$$

Substitute numerical values and evaluate T:

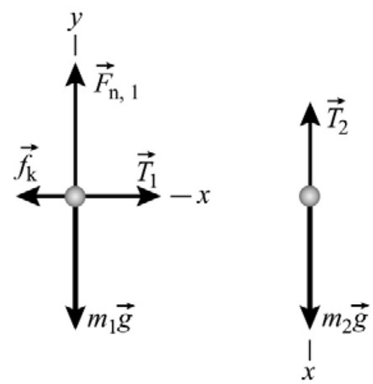
$$T = 2\pi\sqrt{\frac{(0.4)(4\,\text{m})}{9.81\,\text{m/s}^2}} = 2.54\,\text{s} = 0.00423\,\text{min}$$

The number of revolutions per minute is the reciprocal of the period in minutes:

$$\boxed{23.6\,\text{rev/min}}$$

116 ••

Picture the Problem The free-body diagrams to the right show the forces acting on the blocks whose masses are m_1 and m_2. The application of Newton's 2nd law and the use of a constant-acceleration equation will allow us to find a relationship between the coefficient of kinetic friction and m_1. The repetition of this procedure with the additional object on top of the object whose mass is m_1 will lead us to a second equation that, when solved simultaneously with the former equation, leads to a quadratic equation in m_1. Finally, its solution will allow us to substitute in an expression for μ_k and determine its value.



Using a constant-acceleration equation, relate the displacement of the system in its first configuration as a function of its acceleration and fall time:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a_1(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a_1(\Delta t)^2$$

Solve for a_1:

$$a_1 = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate a_1:

$$a_1 = \frac{2(1.5\,\text{m})}{(0.82\,\text{s})^2} = 4.46\,\text{m/s}^2$$

Apply $\sum F_x = ma_x$ to the object whose mass is m_2 and solve for T_1:

$$m_2g - T_1 = m_2a_1$$

and

$$T_1 = m_2(g - a)$$

Substitute numerical values and evaluate T_1:

$$T_1 = (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 4.46\,\text{m/s}^2) = 13.375\,\text{N}$$

Apply $\sum \vec{F} = m\vec{a}$ to the object whose mass is m_1:

$$\sum F_x = T_1 - f_k = m_1 a_1$$

and

$$\sum F_y = F_{n,1} - m_1 g = 0$$

Using $f_k = \mu_k F_n$, eliminate F_n between the two equations to obtain:

$$T_1 - \mu_k m_1 g = m_1 a_1 \qquad (1)$$

Find the acceleration a_2 for the second run:

$$a_2 = \frac{2\Delta x}{(\Delta t)^2} = \frac{2(1.5\,\text{m})}{(1.3\,\text{s})^2} = 1.775\,\text{m/s}^2$$

Evaluate T_2:

$$T_2 = m_2(g - a) = (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 1.775\,\text{m/s}^2) = 20.1\,\text{N}$$

Apply $\sum F_x = ma_x$ to the 1.2-kg object in place:

$$T_2 - \mu_k(m_1 + 1.2\,\text{kg})g = (m_1 + 1.2\,\text{kg})a_2 \qquad (2)$$

Solve equation (1) for μ_k:

$$\mu_k = \frac{T_1 - m_1 a_1}{m_1 g} \qquad (3)$$

Substitute for μ_k in equation (2) and simplify to obtain the quadratic equation in m_1:

$$2.685 m_1^2 + 9.947 m_1 - 16.05 = 0$$

Solve the quadratic equation to obtain:

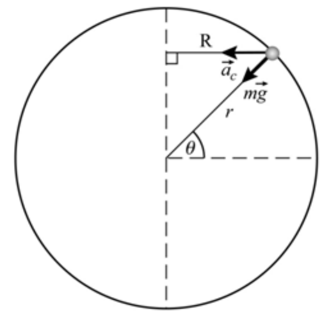
$$m_1 = (-1.85 \pm 3.07)\text{kg} \Rightarrow m_1 = \boxed{1.22\,\text{kg}}$$

Substitute numerical values in equation (3) and evaluate μ_k:

$$\mu_k = \frac{13.375\,\text{N} - (1.22\,\text{kg})(4.66\,\text{m/s}^2)}{(1.22\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.643}$$

***117** •••

Picture the Problem The diagram shows a point on the surface of the earth at latitude θ. The distance R to the axis of rotation is given by $R = r\cos\theta$. We can use the definition of centripetal acceleration to express the centripetal acceleration of a point on the surface of the earth due to the rotation of the earth.



(*a*) Referring to the figure, express a_c for a point on the surface of the earth at latitude θ:

$$a_c = \frac{v^2}{R} \text{ where } R = r\cos\theta$$

Express the speed of the point due to the rotation of the earth:

$$v = \frac{2\pi R}{T}$$

where T is the time for one revolution.

Substitute for v in the expression for a_c and simplify to obtain:

$$a_c = \frac{4\pi^2 r\cos\theta}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(6370\,\text{km})\cos\theta}{[(24\,\text{h})(3600\,\text{s/h})]^2}$$

$$= \boxed{(3.37\,\text{cm/s}^2)\cos\theta,\text{ toward the earth's axis.}}$$

(*b*)

A stone dropped from a hand at a location on earth. The effective weight of the stone is equal to $m\vec{a}_{\text{st, surf}}$, where $\vec{a}_{\text{st, surf}}$ is the acceleration of the falling stone (neglecting air resistance) relative to the local surface of the earth. The gravitational force on the stone is equal to $m\vec{a}_{\text{st,iner}}$, where $\vec{a}_{\text{st, iner}}$ is the acceleration of the local surface of the earth relative to the inertial frame (the acceleration of the surface due to the rotation of the earth). Multiplying through this equation by m and rearranging gives $m\vec{a}_{\text{st, surf}} = m\vec{a}_{\text{st, iner}} - m\vec{a}_{\text{surf, iner}}$, which relates the apparent weight to the acceleration due to gravity and the acceleration due to the earth's rotation. A vector addition diagram can be used to show that the magnitude of $m\vec{a}_{\text{st, surf}}$ is slightly less than that of $m\vec{a}_{\text{st, iner}}$.

(*c*) At the equator, the gravitational acceleration and the radial acceleration are both directed toward the center of the earth. Therefore:

$$g = g_{\text{eff}} + a_c$$
$$= 978\,\text{cm/s}^2 + (3.37\,\text{cm/s}^2)\cos 0°$$
$$= \boxed{981.4\,\text{cm/s}^2}$$

At latitude θ the gravitational acceleration points toward the center of the earth whereas the centripetal acceleration points toward the axis of rotation. Use the law of cosines to relate g_{eff}, g, and a_c:

$$g_{\text{eff}}^2 = g^2 + a_c^2 - 2ga_c\cos\theta$$

Substitute for θ, g_{eff}, and a_c and simplify to obtain the quadratic equation:

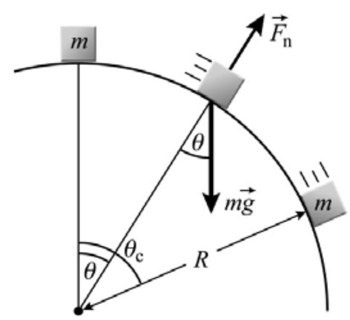
$$g^2 - (4.75\,\text{cm/s}^2)g - 962350\,\text{cm}^2/\text{s}^4 = 0$$

Solve for the physically meaningful (i.e., positive) root to obtain:

$$g = \boxed{983\,\text{cm/s}^2}$$

***118 •••**

Picture the Problem The diagram shows the block in its initial position, an intermediate position, and as it is separating from the sphere. Because the sphere is frictionless, the only forces acting on the block are the normal and gravitational forces. We'll apply Newton's 2nd law and set F_n equal to zero to determine the angle θ_c at which the block leaves the surface.



Taking the inward direction to be positive, apply $\sum F_r = ma_r$ to the block:

$$mg\cos\theta - F_n = m\frac{v^2}{R}$$

Apply the separation condition to obtain:

$$mg\cos\theta_c = m\frac{v^2}{R}$$

Solve for $\cos\theta_c$:

$$\cos\theta_c = \frac{v^2}{gR} \qquad (1)$$

Apply $\sum F_t = ma_t$ to the block:

$$mg\sin\theta = ma_t$$

or

$$a_t = \frac{dv}{dt} = g\sin\theta$$

Note that a is not constant and, hence, we cannot use constant-acceleration equations.

Multiply the left-hand side of the equation by one in the form of $d\theta/d\theta$ and rearrange to obtain:

$$\frac{dv}{dt}\frac{d\theta}{d\theta} = g\sin\theta$$

and

$$\frac{d\theta}{dt}\frac{dv}{d\theta} = g\sin\theta$$

Relate the arc distance s the block travels to the angle θ and the radius R of the sphere:

$$\theta = \frac{s}{R} \text{ and } \frac{d\theta}{dt} = \frac{1}{R}\frac{ds}{dt} = \frac{v}{R}$$

where v is the block's instantaneous speed.

Substitute to obtain:

$$\frac{v}{R}\frac{dv}{d\theta} = g\sin\theta$$

Separate the variables and integrate from $v' = 0$ to v and $\theta = 0$ to θ_c:

$$\int_0^v v'dv' = gR\int_0^{\theta_c} \sin\theta d\theta$$

or

$$v^2 = 2gR(1 - \cos\theta_c)$$

Substitute in equation (1) to obtain:

$$\cos\theta_c = \frac{2gR(1-\cos\theta_c)}{gR} = 2(1-\cos\theta_c)$$

Solve for and evaluate θ_c:

$$\theta_c = \cos^{-1}\left(\frac{2}{3}\right) = \boxed{48.2°}$$

Chapter 6
Work and Energy

Conceptual Problems

***1** •

Determine the Concept A force does work on an object when its point of application moves through some distance and there is a component of the force along the line of motion.

(*a*) False. The *net* force acting on an object is the vector sum of all the forces acting on the object and is responsible for displacing the object. Any or all of the forces contributing to the net force may do work.

(*b*) True. The object could be at rest in one reference frame and moving in another. If we consider only the frame in which the object is at rest, then, because it must undergo a displacement in order for work to be done on it, we would conclude that the statement is true.

(*c*) True. A force that is always perpendicular to the velocity of a particle changes neither it's kinetic nor potential energy and, hence, does no work on the particle.

2 •

Determine the Concept If we ignore the work that you do in initiating the horizontal motion of the box and the work that you do in bringing it to rest when you reach the second table, then neither the kinetic nor the potential energy of the system changed as you moved the box across the room. Neither did any forces acting on the box produce displacements. Hence, we must conclude that the minimum work you did on the box is zero.

3 •

False. While it is true that the person's kinetic energy is not changing due to the fact that she is moving at a constant speed, her gravitational potential energy is continuously changing and so we must conclude that the force exerted by the seat on which she is sitting is doing work on her.

***4** •

Determine the Concept The kinetic energy of any object is proportional to the square of its speed. Because $K = \frac{1}{2}mv^2$, replacing v by $2v$ yields

$K' = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4K.$ Thus doubling the speed of a car quadruples its kinetic energy.

5 •

Determine the Concept No. The work done on any object by any force $\vec{F}$ is defined as $dW = \vec{F} \cdot d\vec{r}$. The direction of $\vec{F}_{\text{net}}$ is toward the center of the circle in which the object is traveling and $d\vec{r}$ is tangent to the circle. No work is done by the net force because $\vec{F}_{\text{net}}$ and $d\vec{r}$ are perpendicular so the dot product is zero.

6 •

Determine the Concept The kinetic energy of any object is proportional to the square of its speed and is always positive. Because $K = \frac{1}{2}mv^2,$ replacing v by $3v$ yields $K' = \frac{1}{2}m(3v)^2 = 9\left(\frac{1}{2}mv^2\right) = 9K.$ Hence tripling the speed of an object increases its kinetic energy by a factor of 9 and $\boxed{(d) \text{ is correct.}}$

***7** •

Determine the Concept The work required to stretch or compress a spring a distance x is given by $W = \frac{1}{2}kx^2$ where k is the spring's stiffness constant. Because $W \propto x^2$, doubling the distance the spring is stretched will require four times as much work.

8 •

Determine the Concept No. We know that if a *net* force is acting on a particle, the particle must be accelerated. If the *net* force does no work on the particle, then we must conclude that the kinetic energy of the particle is constant and that the *net* force is acting perpendicular to the direction of the motion and will cause a departure from straight-line motion.

9 •

Determine the Concept We can use the definition of power as the scalar product of force and velocity to express the dimension of power.

Power is defined as:	$P \equiv \vec{F} \cdot \vec{v}$
Express the dimension of force:	$[M][L/T^2]$
Express the dimension of velocity:	$[L/T]$
Express the dimension of power in terms of those of force and velocity:	$[M][L/T^2][L/T] = [M][L]^2/[T]^3$ and $\boxed{(d) \text{ is correct.}}$

10 •

Determine the Concept The change in gravitational potential energy, over elevation changes that are small enough so that the gravitational field can be considered constant, is $mg\Delta h$, where Δh is the elevation change. Because Δh is the same for both Sal and Joe, their gains in gravitational potential energy are the same. $\boxed{(c) \text{ is correct.}}$

11 •

(*a*) False. The definition of work is not limited to displacements caused by conservative forces.

(*b*) False. Consider the work done by the gravitational force on an object in freefall.

(*c*) True. This is the definition of work done by a conservative force.

***12** ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*; i.e., $F_x = -dU/dx$.

(*a*) Examine the slopes of the curve at each of the lettered points, remembering that F_x is the negative of the slope of the potential energy graph, to complete the table:

Point	dU/dx	F_x
A	+	–
B	0	0
C	–	+
D	0	0
E	+	–
F	0	0

(*b*) Find the point where the slope is steepest:

$\boxed{\text{At point } C \left|F_x\right| \text{ is greatest.}}$

(*c*) If $d^2U/dx^2 < 0$, then the curve is concave downward and the equilibrium is *unstable*.

$\boxed{\text{At point } B \text{ the equilibrium is unstable.}}$

If $d^2U/dx^2 > 0$, then the curve is concave upward and the equilibrium is *stable*.

$\boxed{\text{At point } D \text{ the equilibrium is stable.}}$

Remarks: At point *F*, $d^2U/dx^2 = 0$ and the equilibrium is neither *stable* nor *unstable*; it is said to be *neutral*.

13 •

(*a*) False. Any force acting on an object may do work depending on whether the force produces a displacement … or is displaced as a consequence of the object's motion.

(*b*) False. Consider an element of area under a force-versus-time graph. Its units are N·s whereas the units of work are N·m.

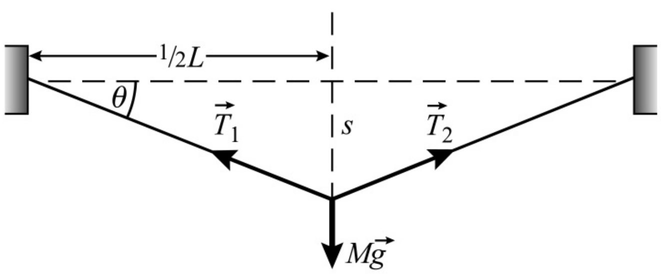
14 •

Determine the Concept Work $dW\left(=\vec{F}\cdot d\vec{s}\right)$ is done when a force $\vec{F}$ produces a displacement $d\vec{s}$. Because $\vec{F}\cdot d\vec{s} \equiv Fds\cos\theta = \left(F\cos\theta\right)ds$, W will be negative if the value of θ is such that $F\cos\theta$ is negative. $\boxed{(d)\text{ is correct.}}$

Estimation and Approximation

***15** ••

Picture the Problem The diagram depicts the situation when the tightrope walker is at the center of rope. M represents her mass and the vertical components of tensions $\vec{T}_1$ and $\vec{T}_2$, equal in magnitude, support her weight. We can apply a condition for static equilibrium in the vertical direction to relate the tension in the rope to the angle θ and use trigonometry to find s as a function of θ.



(*a*) Use trigonometry to relate the sag s in the rope to its length L and θ:

$$\tan\theta = \frac{s}{\frac{1}{2}L} \text{ and } s = \frac{L}{2}\tan\theta$$

Apply $\sum F_y = 0$ to the tightrope walker when she is at the center of the rope to obtain:

$2T\sin\theta - Mg = 0$ where T is the magnitude of $\vec{T}_1$ and $\vec{T}_2$.

Solve for θ to obtain:

$$\theta = \sin^{-1}\left(\frac{Mg}{2T}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(50\,\text{kg})(9.81\,\text{m/s}^2)}{2(5000\,\text{N})}\right] = 2.81°$$

Substitute to obtain:

$$s=\frac{10\,\text{m}}{2}\tan 2.81^\circ=\boxed{0.245\,\text{m}}$$

(*b*) Express the change in the tightrope walker's gravitational potential energy as the rope sags:

$$\Delta U=U_{\text{at center}}-U_{\text{end}}=Mg\Delta y$$

Substitute numerical values and evaluate ΔU:

$$\Delta U=(50\,\text{kg})(9.81\,\text{m/s}^2)(-0.245\,\text{m})=\boxed{-120\,\text{J}}$$

16 •

Picture the Problem You can estimate your change in potential energy due to this change in elevation from the definition of ΔU. You'll also need to estimate the height of one story of the Empire State building. We'll assume your mass is 70 kg and the height of one story to be 3.5 m. This approximation gives us a height of 1170 ft (357 m), a height that agrees to within 7% with the actual height of 1250 ft from the ground floor to the observation deck. We'll also assume that it takes 3 min to ride non-stop to the top floor in one of the high-speed elevators.

(*a*) Express the change in your gravitational potential energy as you ride the elevator to the 102nd floor:

$$\Delta U=mg\Delta h$$

Substitute numerical values and evaluate ΔU:

$$\Delta U=(70\,\text{kg})(9.81\,\text{m/s}^2)(357\,\text{m})=\boxed{245\,\text{kJ}}$$

(*b*) Ignoring the acceleration intervals at the beginning and the end of your ride, express the work done on you by the elevator in terms of the change in your gravitational potential energy:

$$W=Fh=\Delta U$$

Solve for and evaluate F:

$$F=\frac{\Delta U}{h}=\frac{245\,\text{kJ}}{357\,\text{m}}=\boxed{686\,\text{N}}$$

(*c*) Assuming a 3 minute ride to the top, express and evaluate the average power delivered to the elevator:

$$P=\frac{\Delta U}{\Delta t}=\frac{245\,\text{kJ}}{(3\,\text{min})(60\,\text{s/min})}=\boxed{1.36\,\text{kW}}$$

17 •

Picture the Problem We can find the kinetic energy K of the spacecraft from its definition and compare its energy to the annual consumption in the U.S. W by examining the ratio K/W.

Using its definition, express and evaluate the kinetic energy of the spacecraft:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(10000\,\text{kg})\left(3\times10^7\text{ m/s}\right)^2$$
$$= 4.50\times10^{18}\text{ J}$$

Express this amount of energy as a percentage of the annual consumption in the United States:

$$\frac{K}{E} \approx \frac{4.50\times10^{18}\text{ J}}{5\times10^{20}\text{ J}} \approx \boxed{1\%}$$

***18** ••

Picture the Problem We can find the orbital speed of the Shuttle from the radius of its orbit and its period and its kinetic energy from $K = \tfrac{1}{2}mv^2$. We'll ignore the variation in the acceleration due to gravity to estimate the change in the potential energy of the orbiter between its value at the surface of the earth and its orbital value.

(*a*) Express the kinetic energy of the orbiter:

$$K = \tfrac{1}{2}mv^2$$

Relate the orbital speed of the orbiter to its radius r and period T:

$$v = \frac{2\pi r}{T}$$

Substitute and simplify to obtain:

$$K = \tfrac{1}{2}m\left(\frac{2\pi r}{T}\right)^2 = \frac{2\pi^2 mr^2}{T^2}$$

Substitute numerical values and evaluate K:

$$K = \frac{2\pi^2\left(8\times10^4\text{ kg}\right)\left[(200\text{ mi}+3960\text{ mi})(1.609\text{ km/mi})\right]^2}{\left[(90\text{ min})(60\text{ s/min})\right]^2} = \boxed{2.43\text{ TJ}}$$

(*b*) Assuming the acceleration due to gravity to be constant over the 200 mi and equal to its value at the surface of the earth (actually, it is closer to 9 m/s^2 at an elevation of 200 mi), express the change in gravitational potential energy of the orbiter, relative to the surface of the earth, as the Shuttle goes into orbit:

$$\Delta U = mgh$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \left(8\times10^4\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right)\times\left(200\,\text{mi}\right)\left(1.609\,\text{km/mi}\right) = \boxed{0.253\,\text{TJ}}$$

(*c*) No, they shouldn't be equal because there is more than just the force of gravity to consider here. When the shuttle is resting on the surface of the earth, it is supported against the force of gravity by the normal force the earth exerts upward on it. We would need to take into consideration the change in potential energy of the surface of earth in its deformation under the weight of the shuttle to find the actual change in potential energy.

19 •

Picture the Problem Let's assume that the width of the driveway is 18 ft. We'll also assume that you lift each shovel full of snow to a height of 1 m, carry it to the edge of the driveway, and drop it. We'll ignore the fact that you must slightly accelerate each shovel full as you pick it up and as you carry it to the edge of the driveway. While the density of snow depends on the extent to which it has been compacted, one liter of freshly fallen snow is approximately equivalent to 100 mL of water.

Express the work you do in lifting the snow a distance *h*:

$$W = \Delta U = mgh = \rho_{\text{snow}} V_{\text{snow}} gh$$

where ρ is the density of the snow.

Using its definition, express the densities of water and snow:

$$\rho_{\text{snow}} = \frac{m_{\text{snow}}}{V_{\text{snow}}} \text{ and } \rho_{\text{water}} = \frac{m_{\text{water}}}{V_{\text{water}}}$$

Divide the first of these equations by the second to obtain:

$$\frac{\rho_{\text{snow}}}{\rho_{\text{water}}} = \frac{V_{\text{water}}}{V_{\text{snow}}} \text{ or } \rho_{\text{snow}} = \rho_{\text{water}} \frac{V_{\text{water}}}{V_{\text{snow}}}$$

Substitute and evaluate the ρ_{snow}:

$$\rho_{\text{snow}} = \left(10^3\ \text{kg/m}^3\right)\frac{100\,\text{mL}}{\text{L}} = 100\,\text{kg/m}^3$$

Calculate the volume of snow covering the driveway:

$$V_{\text{snow}} = \left(50\,\text{ft}\right)\left(18\,\text{ft}\right)\left(\frac{10}{12}\,\text{ft}\right) = 750\,\text{ft}^3 \times \frac{28.32\,\text{L}}{\text{ft}^3} \times \frac{10^{-3}\,\text{m}^3}{\text{L}} = 21.2\,\text{m}^3$$

Substitute numerical values in the expression for *W* to obtain an estimate (a lower bound) for the work you would do on the snow in removing it:

$$W = \left(100\,\text{kg/m}^3\right)\left(21.2\,\text{m}^3\right)\left(9.81\,\text{m/s}^2\right)\left(1\,\text{m}\right) = \boxed{20.8\,\text{kJ}}$$

Work and Kinetic Energy

***20 •**

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the bullet.

(*a*) Use the definition of *K*:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.015\,\text{kg})(1.2\times10^3\,\text{m/s})^2 = \boxed{10.8\,\text{kJ}}$$

(*b*) Because $K \propto v^2$:

$$K' = \tfrac{1}{4}K = \boxed{2.70\,\text{kJ}}$$

(*c*) Because $K \propto v^2$:

$$K' = 4K = \boxed{43.2\,\text{kJ}}$$

21 •

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the baseball and the jogger.

(*a*) Use the definition of *K*:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.145\,\text{kg})(45\,\text{m/s})^2 = \boxed{147\,\text{J}}$$

(*b*) Convert the jogger's pace of 9 min/mi into a speed:

$$v = \left(\frac{1\,\text{mi}}{9\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1609\,\text{m}}{1\,\text{mi}}\right) = 2.98\,\text{m/s}$$

Use the definition of *K*:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(60\,\text{kg})(2.98\,\text{m/s})^2 = \boxed{266\,\text{J}}$$

22 •

Picture the Problem The work done in raising an object a given distance is the product of the force producing the displacement and the displacement of the object. Because the weight of an object is the gravitational force acting on it and this force acts downward, the work done by gravity is the negative of the weight of the object multiplied by its displacement. The change in kinetic energy of an object is equal to the work done by the *net* force acting on it.

(*a*) Use the definition of *W*:

$$W = \vec{F}\cdot\Delta\vec{y} = F\Delta y$$

$= (80\text{ N})(3\text{ m}) = \boxed{240\text{ J}}$

(*b*) Use the definition of *W*:

$W = \vec{F}\cdot\Delta\vec{y} = -mg\Delta y$, because $\vec{F}$ and $\Delta\vec{y}$ are in opposite directions.

$\therefore\ W = -(6\text{ kg})(9.81\text{ m/s}^2)(3\text{ m})$
$= \boxed{-177\text{ J}}$

(*c*) According to the work-kinetic energy theorem:

$K = W + W_g = 240\text{ J} + (-177\text{ J})$
$= \boxed{63.0\text{ J}}$

23 •

Picture the Problem The constant force of 80 N is the net force acting on the box and the work it does is equal to the *change* in the kinetic energy of the box.

Using the work-kinetic energy theorem, relate the work done by the constant force to the *change* in the kinetic energy of the box:

$$W = K_f - K_i = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)$$

Substitute numerical values and evaluate *W*:

$$W = \tfrac{1}{2}(5\text{ kg})\left[(68\text{ m/s})^2 - (20\text{ m/s})^2\right] = \boxed{10.6\text{ kJ}}$$

***24** ••

Picture the Problem We can use the definition of kinetic energy to find the mass of your friend.

Using the definition of kinetic energy and letting ″1″ denote your mass and speed and ″2″ your girlfriend's, express the equality of your kinetic energies and solve for your girlfriend's mass as a function of both your masses and speeds:

$$\tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}m_2v_2^2$$

and

$$m_2 = m_1\left(\frac{v_1}{v_2}\right)^2 \qquad (1)$$

Express the condition on your speed that enables you to run at the same speed as your girlfriend:

$$v_2 = 1.25v_1 \qquad (2)$$

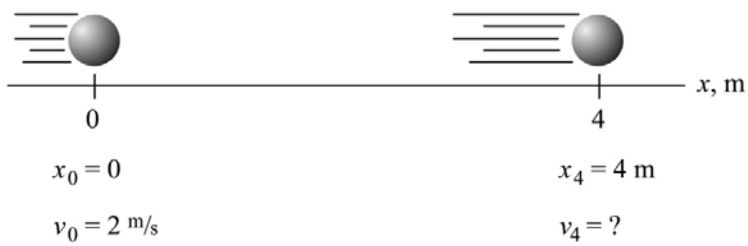
Substitute equation (2) in equation (1) to obtain:

$$m_2 = m_1\left(\frac{v_1}{v_2}\right)^2 = (85\,\text{kg})\left(\frac{1}{1.25}\right)^2 = \boxed{54.4\,\text{kg}}$$

Work Done by a Variable Force

25 ••

Picture the Problem The pictorial representation shows the particle as it moves along the positive x axis. The particle's kinetic energy increases because work is done on it. We can calculate the work done on it from the graph of F_x vs. x and relate its kinetic energy when it is at x = 4 m to its kinetic energy when it was at the origin and the work done on it by using the work-kinetic energy theorem.



(*a*) Calculate the kinetic energy of the particle when it is at x = 0:

$$K_0 = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(3\,\text{kg})(2\,\text{m/s})^2 = \boxed{6.00\,\text{J}}$$

(*b*) Because the force and displacement are parallel, the work done is the area under the curve. Use the formula for the area of a triangle to calculate the area under the F as a function of x graph:

$$W_{0\to4} = \tfrac{1}{2}(\text{base})(\text{altitude}) = \tfrac{1}{2}(4\,\text{m})(6\,\text{N}) = \boxed{12.0\,\text{J}}$$

(*c*) Express the kinetic energy of the particle at x = 4 m in terms of its speed and mass and solve for its speed:

$$v_4 = \sqrt{\frac{2K_4}{m}} \qquad (1)$$

Using the work-kinetic energy theorem, relate the work done on the particle to its *change* in kinetic energy and solve for the particle's kinetic energy at x = 4 m:

$$W_{0\to4} = K_4 - K_0$$

$$K_4 = K_0 + W_{0\to4} = 6.00\,\text{J} + 12.0\,\text{J} = 18.0\,\text{J}$$

Substitute numerical values in equation (1) and evaluate v_4:

$$v_4=\sqrt{\frac{2(18.0\,\text{J})}{3\,\text{kg}}}=\boxed{3.46\,\text{m/s}}$$

***26 ••**

Picture the Problem The work done by this force as it displaces the particle is the area under the curve of *F* as a function of *x*. Note that the constant *C* has units of N/m^3.

Because *F* varies with position non-linearly, express the work it does as an integral and evaluate the integral between the limits *x* = 1.5 m and *x* = 3 m:

$$\begin{aligned}W&=\left(C\,\text{N/m}^3\right)\int_{1.5\,\text{m}}^{3\,\text{m}}x'^3\,dx'\\&=\left(C\,\text{N/m}^3\right)\left[\tfrac{1}{4}x'^4\right]_{1.5\,\text{m}}^{3\,\text{m}}\\&=\frac{\left(C\,\text{N/m}^3\right)}{4}\left[(3\,\text{m})^4-(1.5\,\text{m})^4\right]\\&=\boxed{19C\,\text{J}}\end{aligned}$$

27 ••

Picture the Problem The work done on the dog by the leash as it stretches is the area under the curve of *F* as a function of *x*. We can find this area (the work Lou does holding the leash) by integrating the force function.

Because *F* varies with position non-linearly, express the work it does as an integral and evaluate the integral between the limits *x* = 0 and $x = x_1$:

$$\begin{aligned}W&=\int_0^{x_1}\left(-kx'-ax'^2\right)dx'\\&=\left[-\tfrac{1}{2}kx'^2-\tfrac{1}{3}ax'^3\right]_0^{x_1}\\&=\boxed{-\tfrac{1}{2}kx_1^2-\tfrac{1}{3}ax_1^3}\end{aligned}$$

28 ••

Picture the Problem The work done on an object can be determined by finding the area bounded by its graph of F_x as a function of *x* and the *x* axis. We can find the kinetic energy and the speed of the particle at any point by using the work-kinetic energy theorem.

(*a*) Express *W*, the area under the curve, in terms of the area of one square, A_{square}, and the number of squares *n*:

$$W = n\,A_{\text{square}}$$

Determine the work equivalent of one square:

$$W = (0.5\text{ N})(0.25\text{ m}) = 0.125\text{ J}$$

Estimate the number of squares under the curve between $x = 0$ and $x = 2$ m:

$n \approx 22$

Substitute to determine W:

$$W = 22(0.125\text{ J}) = \boxed{2.75\text{ J}}$$

(b) Relate the kinetic energy of the object at $x = 2$ m, K_2, to its initial kinetic energy, K_0, and the work that was done on it between $x = 0$ and $x = 2$ m:

$$K_2 = K_0 + W_{0\to2}$$
$$= \tfrac{1}{2}(3\text{ kg})(2.40\text{ m/s})^2 + 2.75\text{ J}$$
$$= \boxed{11.4\text{ J}}$$

(c) Calculate the speed of the object at $x = 2$ m from its kinetic energy at the same location:

$$v = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(11.4\text{ J})}{3\text{ kg}}} = \boxed{2.76\text{ m/s}}$$

(d) Estimate the number of squares under the curve between $x = 0$ and $x = 4$ m:

$n \approx 26$

Substitute to determine W:

$$W = 26(0.125\text{ J}) = \boxed{3.25\text{ J}}$$

(e) Relate the kinetic energy of the object at $x = 4$ m, K_4, to its initial kinetic energy, K_0, and the work that was done on it between $x = 0$ and $x = 4$ m:

$$K_4 = K_0 + W_{0\to4}$$
$$= \tfrac{1}{2}(3\text{ kg})(2.40\text{ m/s})^2 + 3.25\text{ J}$$
$$= 11.9\text{ J}$$

Calculate the speed of the object at $x = 4$ m from its kinetic energy at the same location:

$$v = \sqrt{\frac{2K_4}{m}} = \sqrt{\frac{2(11.9\text{ J})}{3\text{ kg}}} = \boxed{2.82\text{ m/s}}$$

***29** ••

Picture the Problem We can express the mass of the water in Margaret's bucket as the difference between its initial mass and the product of the rate at which it loses water and her position during her climb. Because Margaret must do work against gravity in lifting and carrying the bucket, the work she does is the integral of the product of the gravitational field and the mass of the bucket as a function of its position.

(a) Express the mass of the bucket and the water in it as a function of

$$m(y) = 40\text{ kg} - ry$$

its initial mass, the rate at which it is losing water, and Margaret's position, y, during her climb:

Find the rate, $r=\frac{\Delta m}{\Delta y}$, at which Margaret's bucket loses water:

$$r=\frac{\Delta m}{\Delta y}=\frac{20\,\text{kg}}{20\,\text{m}}=1\,\text{kg/m}$$

Substitute to obtain:

$$m(y)=40\,\text{kg}-ry=\boxed{40\,\text{kg}-\frac{1\,\text{kg}}{\text{m}}y}$$

(b) Integrate the force Margaret exerts on the bucket, $m(y)g$, between the limits of $y = 0$ and $y = 20$ m:

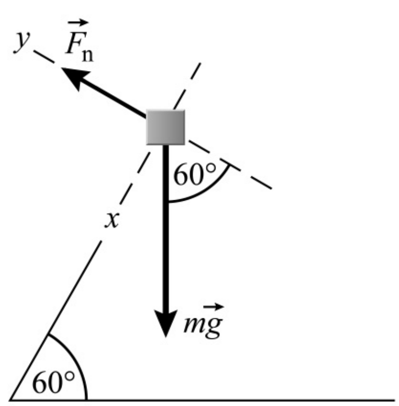
$$W=g\int_0^{20\,\text{m}}\left(40\,\text{kg}-\frac{1\,\text{kg}}{\text{m}}y'\right)dy'=\left(9.81\,\text{m/s}^2\right)\left[(40\,\text{kg})y'-\tfrac{1}{2}(1\,\text{kg/m})y'^2\right]_0^{20\,\text{m}}=\boxed{5.89\,\text{kJ}}$$

Remarks: We could also find the work Margaret did on the bucket, at least approximately, by plotting a graph of *m*(*y*)*g* and finding the area under this curve between *y* = 0 and *y* = 20 m.

Work, Energy, and Simple Machines

30 •

Picture the Problem The free-body diagram shows the forces that act on the block as it slides down the frictionless incline. We can find the work done by these forces as the block slides 2 m by finding their components in the direction of, or opposite to, the motion. When we have determined the work done on the block, we can use the work-kinetic energy theorem or a constant-acceleration equation to calculate its kinetic energy and its speed at any given location.



(*a*) From the free-body diagram, we see that the forces acting on the block are a gravitational force that acts downward and the normal force that the incline exerts perpendicularly to the incline.

Identify the component of *mg* that acts down the incline and calculate the work done by it:

$F_x = mg \sin 60°$

Express the work done by this force:

$W = F_x \Delta x = mg\Delta x \sin 60°$

Substitute numerical values and evaluate *W*:

$$W = (6\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m})\sin 60° = \boxed{102\ \text{J}}$$

Remarks: F_n and mgcos60°, being perpendicular to the motion, do no work on the block

(*b*) The total work done on the block is the work done by the net force:

$$W = F_{\text{net}}\Delta x = mg\Delta x \sin 60° = (6\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m})\sin 60° = \boxed{102\ \text{J}}$$

(*c*) Express the change in the kinetic energy of the block in terms of the distance, Δx, it has moved down the incline:

$\Delta K = K_f - K_i = W = (mg\sin 60°)\Delta x$

or, because $K_i = 0$,

$K_f = W = (mg\sin 60°)\Delta x$

Relate the speed of the block when it has moved a distance Δx down the incline to its kinetic energy at that location:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2mg\Delta x \sin 60°}{m}} = \sqrt{2g\Delta x \sin 60°}$$

Determine this speed when $\Delta x = 1.5$ m:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(1.5\,\text{m})\sin 60°} = \boxed{5.05\,\text{m/s}}$$

(*d*) As in part (*c*), express the change in the kinetic energy of the block in terms of the distance, Δx, it has moved down the incline and

$\Delta K = K_f - K_i$

$= W$

$= (mg \sin 60°)\Delta x$

and

solve for K_f: $K_f = (mg\sin 60°)\Delta x + K_i$

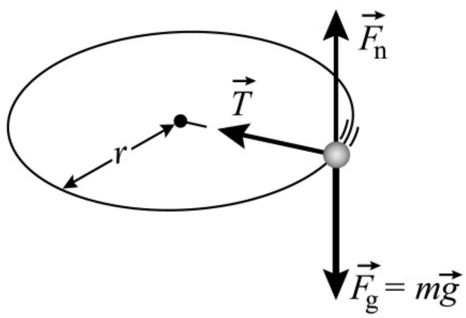
Substitute for the kinetic energy terms and solve for v_f to obtain: $v_f = \sqrt{2g\sin 60°\Delta x + v_i^2}$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(1.5\,\text{m})\sin 60° + (2\,\text{m/s})^2} = \boxed{5.43\,\text{m/s}}$$

31 •

Picture the Problem The free-body diagram shows the forces acting on the object as in moves along its circular path on a frictionless horizontal surface. We can use Newton's 2nd law to obtain an expression for the tension in the string and the definition of work to determine the amount of work done by each force during one revolution.



(*a*) Apply $\sum F_r = ma_r$ to the 2-kg object and solve for the tension:

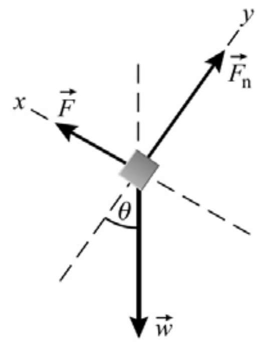
$$T = m\frac{v^2}{r} = (2\,\text{kg})\frac{(2.5\,\text{m/s})^2}{3\,\text{m}} = \boxed{4.17\,\text{N}}$$

(*b*) From the FBD we can see that the forces acting on the object are: $\boxed{\vec{T}, \vec{F}_g, \text{ and } \vec{F}_n}$

$\boxed{\text{Because all of these forces act perpendicularly to the direction of motion of the object, none of them do any work.}}$

***32** •

Picture the Problem The free-body diagram, with $\vec{F}$ representing the force required to move the block at constant speed, shows the forces acting on the block. We can apply Newton's 2nd law to the block to relate F to its weight w and then use the definition of the mechanical advantage of an inclined plane. In the second part of the problem we'll use the definition of work.



(*a*) Express the mechanical advantage M of the inclined plane:

$$M = \frac{w}{F}$$

Apply $\sum F_x = ma_x$ to the block:

$F - w\sin\theta = 0$ because $a_x = 0$.

Solve for F and substitute to obtain:

$$M = \frac{w}{w\sin\theta} = \frac{1}{\sin\theta}$$

Refer to the figure to obtain:

$$\sin\theta = \frac{H}{L}$$

Substitute to obtain:

$$M = \boxed{\frac{1}{\sin\theta} = \frac{L}{H}}$$

(*b*) Express the work done pushing the block up the ramp:

$$W_{\text{ramp}} = FL = mgL\sin\theta$$

Express the work done lifting the block into the truck:

$$W_{\text{lifting}} = mgH = mgL\sin\theta$$

and

$$\boxed{W_{\text{ramp}} = W_{\text{lifting}}}$$

33 •

Picture the Problem We can find the work done per revolution in lifting the weight and the work done in each revolution of the handle and then use the definition of mechanical advantage.

Express the mechanical advantage of the jack:

$$M = \frac{W}{F}$$

Express the work done by the jack in one complete revolution (the weight W is raised a distance p):

$$W_{\text{lifting}} = Wp$$

Express the work done by the force F in one complete revolution:

$$W_{\text{turning}} = 2\pi RF$$

Equate these expressions to obtain: $Wp = 2\pi RF$

Solve for the ratio of W to F: $M = \frac{W}{F} = \boxed{\frac{2\pi R}{p}}$

Remarks: One does the same amount of work turning as lifting; exerting a smaller force over a greater distance.

34 •

Picture the Problem The object whose weight is $\vec{w}$ is supported by two portions of the rope resulting in what is known as a *mechanical advantage* of 2. The work that is done in each instance is the product of the force doing the work and the displacement of the object on which it does the work.

(*a*) If w moves through a distance h: F moves a distance $\boxed{2h}$

(*b*) Assuming that the kinetic energy of the weight does not change, relate the work done on the object to the change in its potential energy to obtain: $W = \Delta U = wh\cos\theta = \boxed{wh}$

(*c*) Because the force you exert on the rope and its displacement are in the same direction: $W = F(2h)\cos\theta = F(2h)$

Determine the tension in the ropes supporting the object:

$$\sum F_{\text{vertical}} = 2F - w = 0$$

and

$$F = \tfrac{1}{2}w$$

Substitute for F: $W = F(2h) = \tfrac{1}{2}w(2h) = \boxed{wh}$

(*d*) The mechanical advantage of the inclined plane is the ratio of the weight that is lifted to the force required to lift it, i.e.: $M = \frac{w}{F} = \frac{w}{\frac{1}{2}w} = \boxed{2}$

Remarks: Note that the mechanical advantage is also equal to the number of ropes supporting the load.

Dot Products

***35** •

Picture the Problem Because $\vec{A}\cdot\vec{B} \equiv AB\cos\theta$ we can solve for $\cos\theta$ and use the fact that $\vec{A}\cdot\vec{B} = -AB$ to find θ.

Solve for θ:

$$\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{AB}$$

Substitute for $\vec{A}\cdot\vec{B}$ and evaluate θ:

$$\theta = \cos^{-1}(-1) = \boxed{180°}$$

36 •

Picture the Problem We can use its definition to evaluate $\vec{A}\cdot\vec{B}$.

Express the definition of $\vec{A}\cdot\vec{B}$:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Substitute numerical values and evaluate $\vec{A}\cdot\vec{B}$:

$$\vec{A}\cdot\vec{B} = (6\,\text{m})(6\,\text{m})\cos 60° = \boxed{18.0\,\text{m}^2}$$

37 •

Picture the Problem The scalar product of two-dimensional vectors $\vec{A}$ and $\vec{B}$ is $A_xB_x + A_yB_y$.

(*a*) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$:

$$\vec{A}\cdot\vec{B} = (3)(-4) + (-6)(2) = \boxed{-24}$$

(*b*) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$:

$$\vec{A}\cdot\vec{B} = (5)(2) + (5)(-4) = \boxed{-10}$$

(*c*) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$:

$$\vec{A}\cdot\vec{B} = (6)(4) + (4)(-6) = \boxed{0}$$

38 •

Picture the Problem The scalar product of two-dimensional vectors $\vec{A}$ and $\vec{B}$ is $AB\cos\theta = A_xB_x + A_yB_y$. Hence the angle between vectors $\vec{A}$ and $\vec{B}$ is given by

$$\theta = \cos^{-1}\frac{A_xB_x + A_yB_y}{AB}.$$

(*a*) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$:

$$\vec{A}\cdot\vec{B} = (3)(-4) + (-6)(2) = -24$$

$$A = \sqrt{(3)^2 + (-6)^2} = \sqrt{45}$$

$$B = \sqrt{(-4)^2 + (2)^2} = \sqrt{20}$$

and

$$\theta = \cos^{-1}\frac{-24}{\sqrt{45}\sqrt{20}} = \boxed{143°}$$

(*b*) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$:

$$\vec{A}\cdot\vec{B} = (5)(2) + (5)(-4) = -10$$

$$A = \sqrt{(5)^2 + (5)^2} = \sqrt{50}$$

$$B = \sqrt{(2)^2 + (-4)^2} = \sqrt{20}$$

and

$$\theta = \cos^{-1}\frac{-10}{\sqrt{50}\sqrt{20}} = \boxed{108°}$$

(*c*) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$:

$$\vec{A}\cdot\vec{B} = (6)(4) + (4)(-6) = \boxed{0}$$

$$A = \sqrt{(6)^2 + (4)^2} = \sqrt{52}$$

$$B = \sqrt{(4)^2 + (-6)^2} = \sqrt{52}$$

and

$$\theta = \cos^{-1}\frac{0}{\sqrt{52}\sqrt{52}} = \boxed{90.0°}$$

39 •

Picture the Problem The work W done by a force $\vec{F}$ during a displacement $\Delta\vec{s}$ for which it is responsible is given by $\vec{F}\cdot\Delta\vec{s}$.

(*a*) Using the definitions of work and the scalar product, calculate the work done by the given force during the specified displacement:

$$W = \vec{F}\cdot\Delta\vec{s}$$
$$= \left(2\,\text{N}\,\hat{i} - 1\,\text{N}\,\hat{j} + 1\,\text{N}\,\hat{k}\right)\cdot\left(3\,\text{m}\,\hat{i} + 3\,\text{m}\,\hat{j} - 2\,\text{m}\,\hat{k}\right)$$
$$= \left[(2)(3) + (-1)(3) + (1)\,(-2)\right]\text{N}\cdot\text{m}$$
$$= \boxed{1.00\,\text{J}}$$

(*b*) Using the definition of work that includes the angle between the force and displacement vectors, solve for the component of $\vec{F}$ in the direction of $\Delta\vec{s}$:

$$W = F\Delta s\cos\theta = (F\cos\theta)\Delta s$$

and

$$F\cos\theta = \frac{W}{\Delta s}$$

Substitute numerical values and evaluate $F\cos\theta$:

$$F\cos\theta = \frac{1\,\text{J}}{\sqrt{(3\,\text{m})^2 + (3\,\text{m})^2 + (-2\,\text{m})^2}} = \boxed{0.213\,\text{N}}$$

40 ••

Picture the Problem The component of a vector that is along another vector is the scalar product of the former vector and a unit vector that is parallel to the latter vector.

(*a*) By definition, the unit vector that is parallel to the vector $\vec{A}$ is:

$$\hat{u}_A = \boxed{\frac{\vec{A}}{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}}$$

(*b*) Find the unit vector parallel to $\vec{B}$:

$$\hat{u}_B = \frac{\vec{B}}{B} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^2}} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

The component of $\vec{A}$ along $\vec{B}$ is:

$$\vec{A}\cdot\hat{u}_B = \left(2\hat{i} - \hat{j} - \hat{k}\right)\cdot\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$$
$$= (2)\left(\frac{3}{5}\right) + (-1)\left(\frac{4}{5}\right) + (-1)(0)$$
$$= \boxed{0.400}$$

***41** ••

Picture the Problem We can use the definitions of the magnitude of a vector and the dot product to show that if $\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$, then $\vec{A}\perp\vec{B}$.

Express $\left|\vec{A}+\vec{B}\right|^2$: $\left|\vec{A}+\vec{B}\right|^2=\left(\vec{A}+\vec{B}\right)^2$

Express $\left|\vec{A}-\vec{B}\right|$: $\left|\vec{A}-\vec{B}\right|^2=\left(\vec{A}-\vec{B}\right)^2$

Equate these expressions to obtain: $\left(\vec{A}+\vec{B}\right)^2=\left(\vec{A}-\vec{B}\right)^2$

Expand both sides of the equation to obtain: $A^2+2\vec{A}\cdot\vec{B}+B^2=A^2-2\vec{A}\cdot\vec{B}+B^2$

Simplify to obtain:

$$4\vec{A}\cdot\vec{B}=0$$

or

$$\vec{A}\cdot\vec{B}=0$$

From the definition of the dot product we have:

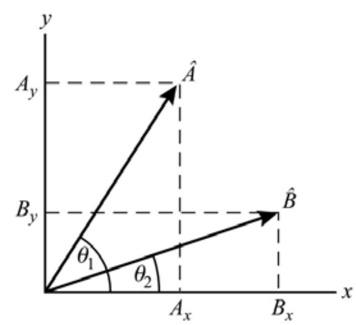
$$\vec{A}\cdot\vec{B}=AB\cos\theta$$

where θ is the angle between $\vec{A}$ and $\vec{B}$.

Because neither $\vec{A}$ nor $\vec{B}$ is the zero vector: $\cos\theta=0 \Rightarrow \theta=90°$ and $\vec{A}\perp\vec{B}$.

42 ••

Picture the Problem The diagram shows the unit vectors $\hat{A}$ and $\hat{B}$ arbitrarily located in the 1st quadrant. We can express these vectors in terms of the unit vectors $\hat{i}$ and $\hat{j}$ and their *x* and *y* components. We can then form the dot product of $\hat{A}$ and $\hat{B}$ to show that $\cos(\theta_1-\theta_2)=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2$.



(*a*) Express $\hat{A}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$:

$$\hat{A}=A_x\hat{i}+A_y\hat{j}$$

where

$$A_x=\boxed{\cos\theta_1} \text{ and } A_y=\boxed{\sin\theta_1}$$

Proceed as above to obtain:

$$\hat{B}=B_x\hat{i}+B_y\hat{j}$$

where

$$B_x=\boxed{\cos\theta_2} \text{ and } B_y=\boxed{\sin\theta_2}$$

(*b*) Evaluate $\hat{A}\cdot\hat{B}$:

$$\hat{A}\cdot\hat{B}=\left(\cos\theta_1\hat{i}+\sin\theta_1\hat{j}\right)\cdot\left(\cos\theta_2\hat{i}+\sin\theta_2\hat{j}\right)=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2$$

From the diagram we note that:

$$\hat{A}\cdot\hat{B}=\cos\left(\theta_1-\theta_2\right)$$

Substitute to obtain:

$$\boxed{\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2}$$

43 •

Picture the Problem In (*a*) we'll show that it does not follow that $\vec{B} = \vec{C}$ by giving a counterexample.

Let $\vec{A} = \hat{i}$, $\vec{B} = 3\hat{i} + 4\hat{j}$ and $\vec{C} = 3\hat{i} - 4\hat{j}$. Form $\vec{A}\cdot\vec{B}$ and $\vec{A}\cdot\vec{C}$:

$$\vec{A}\cdot\vec{B} = \hat{i}\cdot\left(3\hat{i} + 4\hat{j}\right) = 3$$

and

$$\vec{A}\cdot\vec{C} = \hat{i}\cdot\left(3\hat{i} - 4\hat{j}\right) = 3$$

$$\boxed{\text{No. We've shown by a counter-example that } \vec{B} \text{ is not necessarily equal to } \vec{C}.}$$

44 ••

Picture the Problem We can form the dot product of $\vec{A}$ and $\vec{r}$ and require that $\vec{A}\cdot\vec{r} = 1$ to show that the points at the head of all such vectors $\vec{r}$ lie on a straight line. We can use the equation of this line and the components of $\vec{A}$ to find the slope and intercept of the line.

(*a*) Let $\vec{A} = a_x\hat{i} + a_y\hat{j}$. Then:

$$\vec{A}\cdot\vec{r} = \left(a_x\hat{i} + a_y\hat{j}\right)\cdot\left(x\hat{i} + y\hat{j}\right) = a_x x + a_y y = 1$$

Solve for *y* to obtain:

$$y = \boxed{-\frac{a_x}{a_y}x + \frac{1}{a_y}}$$

which is of the form $y = mx + b$ and hence is the equation of a line.

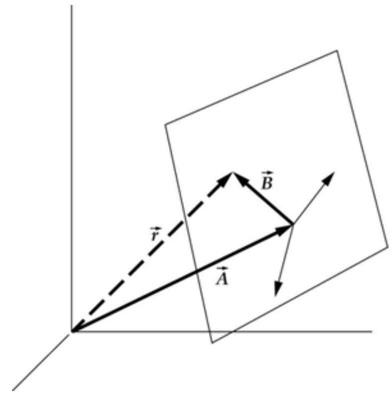
(*b*) Given that $\vec{A} = 2\hat{i} - 3\hat{j}$:

$$m = -\frac{a_x}{a_y} = -\frac{2}{-3} = \boxed{\frac{2}{3}}$$

and

$$b = \frac{1}{a_y} = \frac{1}{-3} = \boxed{-\frac{1}{3}}$$

(*c*) The equation we obtained in (*a*) specifies all vectors whose component parallel to $\vec{A}$ has constant magnitude; therefore, we can write such a vector as $\vec{r}=\frac{\vec{A}}{\left|\vec{A}\right|^2}+\vec{B}$, where $\vec{B}$ is *any* vector perpendicular to $\vec{A}$. This is shown graphically to the right.



Because all possible vectors $\vec{B}$ lie in a plane, the resultant $\vec{r}$ must lie in a plane as well, as is shown above.

***45 ••**

Picture the Problem The rules for the differentiation of vectors are the same as those for the differentiation of scalars and scalar multiplication is commutative.

(*a*) Differentiate $\vec{r}\cdot\vec{r}=r^2=$ constant:

$$\frac{d}{dt}\left(\vec{r}\cdot\vec{r}\right)=\vec{r}\cdot\frac{d\vec{r}}{dt}+\frac{d\vec{r}}{dt}\cdot\vec{r}=2\vec{v}\cdot\vec{r}$$
$$=\frac{d}{dt}\left(\text{constant}\right)=0$$

Because $\vec{v}\cdot\vec{r}=0$:

$$\boxed{\vec{v}\perp\vec{r}}$$

(*b*) Differentiate $\vec{v}\cdot\vec{v}=v^2=$ constant with respect to time:

$$\frac{d}{dt}\left(\vec{v}\cdot\vec{v}\right)=\vec{v}\cdot\frac{d\vec{v}}{dt}+\frac{d\vec{v}}{dt}\cdot\vec{v}=2\vec{a}\cdot\vec{v}$$
$$=\frac{d}{dt}\left(\text{constant}\right)=0$$

Because $\vec{a}\cdot\vec{v}=0$:

$$\boxed{\vec{a}\perp\vec{v}}$$

The results of (*a*) and (*b*) tell us that $\vec{a}$ is perpendicular to $\vec{r}$ and and parallel (or antiparallel) to $\vec{r}$.

(*c*) Differentiate $\vec{v}\cdot\vec{r}=0$ with respect to time:

$$\frac{d}{dt}\left(\vec{v}\cdot\vec{r}\right)=\vec{v}\cdot\frac{d\vec{r}}{dt}+\vec{r}\cdot\frac{d\vec{v}}{dt}$$
$$=v^2+\vec{r}\cdot\vec{a}=\frac{d}{dt}(0)=0$$

Because $v^2 + \vec{r} \cdot \vec{a} = 0$: $\boxed{\vec{r} \cdot \vec{a} = -v^2}$ (1)

Express a_r in terms of θ, where θ is the angle between $\vec{r}$ and $\vec{a}$: $a_r = a\cos\theta$

Express $\vec{r} \cdot \vec{a}$: $\vec{r} \cdot \vec{a} = ra\cos\theta = ra_r$

Substitute in equation (1) to obtain: $ra_r = -v^2$

Solve for a_r: $a_r = \boxed{-\frac{v^2}{r}}$

Power

46 ••

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt}$.

Calculate the rate at which force A does work: $P_A = \frac{5\,\text{J}}{10\,\text{s}} = 0.5\,\text{W}$

Calculate the rate at which force B does work: $P_B = \frac{3\,\text{J}}{5\,\text{s}} = 0.6\,\text{W}$ and $\boxed{P_B > P_A}$

47 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$.

(*a*) If the box moves upward with a constant velocity, the net force acting it must be zero and the force that is doing work on the box is: $F = mg$

The power input of the force is: $P = Fv = mgv$

Substitute numerical values and evaluate P: $P = (5\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m/s}) = \boxed{98.1\,\text{W}}$

(*b*) Express the work done by the force in terms of the rate at which energy is delivered:

$W = Pt = (98.1\text{ W})\,(4\text{ s}) = \boxed{392\text{ J}}$

48 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \dfrac{dW}{dt} = \vec{F}\cdot\vec{v}$.

(*a*) Using the definition of power, express Fluffy's speed in terms of the rate at which he does work and the force he exerts in doing the work:

$$v = \frac{P}{F} = \frac{6\text{ W}}{3\text{ N}} = \boxed{2\text{ m/s}}$$

(*b*) Express the work done by the force in terms of the rate at which energy is delivered:

$W = Pt = (6\text{ W})\,(4\text{ s}) = \boxed{24.0\text{ J}}$

49 •

Picture the Problem We can use Newton's 2nd law and the definition of acceleration to express the velocity of this object as a function of time. The power input of the force accelerating the object is defined to be the rate at which it does work; i.e., $P = dW/dt = \vec{F}\cdot\vec{v}$.

(*a*) Express the velocity of the object as a function of its acceleration and time:

$v = at$

Apply $\sum\vec{F} = m\vec{a}$ to the object:

$a = F/m$

Substitute for *a* in the expression for *v*:

$$v = \frac{F}{m}t = \frac{5\text{ N}}{8\text{ kg}}t = \boxed{\left(\tfrac{5}{8}\text{ m/s}^2\right)t}$$

(*b*) Express the power input as a function of *F* and *v* and evaluate *P*:

$$P = Fv = (5\text{ N})\left(\tfrac{5}{8}\text{ m/s}^2\right)t = \boxed{3.13t\text{ W/s}}$$

(*c*) Substitute *t* = 3 s:

$$P = (3.13\text{ W/s})(3\text{ s}) = \boxed{9.38\text{ W}}$$

50 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$.

(*a*) For $\vec{F}$ = 4 N $\hat{i}$ + 3 N $\hat{k}$ and $\vec{v}$ = 6 m/s $\hat{i}$:

$$P = \vec{F} \cdot \vec{v} = (4\,\text{N}\,\hat{i} + 3\,\text{N}\,\hat{k}) \cdot (6\,\text{m/s}\,\hat{i}) = \boxed{24.0\,\text{W}}$$

(*b*) For $\vec{F}$ = 6 N $\hat{i}$ – 5 N $\hat{j}$ and $\vec{v}$ = – 5 m/s $\hat{i}$ + 4 m/s $\hat{j}$:

$$P = \vec{F} \cdot \vec{v} = (6\,\text{N}\,\hat{i} - 5\,\text{N}\,\hat{j}) \cdot (-5\,\text{m/s}\,\hat{i} + 4\,\text{m/s}\,\hat{j}) = \boxed{-50.0\,\text{W}}$$

(*c*) For $\vec{F}$ = 3 N $\hat{i}$ + 6 N $\hat{j}$ and $\vec{v}$ = 2 m/s $\hat{i}$ + 3 m/s $\hat{j}$:

$$P = \vec{F} \cdot \vec{v} = (3\,\text{N}\,\hat{i} + 6\,\text{N}\,\hat{j}) \cdot (2\,\text{m/s}\,\hat{i} + 3\,\text{m/s}\,\hat{j}) = \boxed{24.0\,\text{W}}$$

***51** •

Picture the Problem Choose a coordinate system in which upward is the positive *y* direction. We can find P_{in} from the given information that $P_{\text{out}} = 0.27P_{\text{in}}$. We can express P_{out} as the product of the tension in the cable *T* and the constant speed *v* of the dumbwaiter. We can apply Newton's 2nd law to the dumbwaiter to express *T* in terms of its mass *m* and the gravitational field *g*.

Express the relationship between the motor's input and output power:

$$P_{\text{out}} = 0.27P_{\text{in}}$$

or

$$P_{\text{in}} = 3.7P_{\text{out}}$$

Express the power required to move the dumbwaiter at a constant speed *v*:

$$P_{\text{out}} = Tv$$

Apply $\sum F_y = ma_y$ to the dumbwaiter:

$$T - mg = ma_y$$

or, because $a_y = 0$,

$$T = mg$$

Substitute to obtain:

$$P_{\text{in}} = 3.7Tv = 3.7mgv$$

Substitute numerical values and evaluate P_{in}:

$$P_{\text{in}} = 3.7(35\,\text{kg})(9.81\,\text{m/s}^2)(0.35\,\text{m/s}) = \boxed{445\,\text{W}}$$

52 ••

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can express P_{drag} as the product of the drag force F_{drag} acting on the skydiver and her terminal velocity v_t. We can apply Newton's 2nd law to the skydiver to express F_{drag} in terms of her mass m and the gravitational field g.

(a) Express the power due to drag force acting on the skydiver as she falls at her terminal velocity v_t:

$$\vec{P}_{drag} = \vec{F}_{drag} \cdot \vec{v}_t$$

or, because $\vec{F}_{drag}$ and $\vec{v}_t$ are antiparallel,

$$P_{drag} = -F_{drag} v_t$$

Apply $\sum F_y = ma_y$ to the skydiver:

$$F_{drag} - mg = ma_y$$

or, because $a_y = 0$,

$$F_{drag} = mg$$

Substitute to obtain, for the magnitude of P_{drag}:

$$P_{drag} = \left| -mgv_t \right| \qquad (1)$$

Substitute numerical values and evaluate P:

$$P_{drag} = \left| -(55\,\text{kg})(9.81\,\text{m/s}^2)(120\,\frac{\text{mi}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1.609\,\text{km}}{\text{mi}}) \right| = \boxed{2.89 \times 10^4\,\text{W}}$$

(b) Evaluate equation (1) with v = 15 mi/h:

$$P_{drag} = \left| -(55\,\text{kg})(9.81\,\text{m/s}^2)\left(15\frac{\text{mi}}{\text{h}}\right) \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1.609\,\text{km}}{\text{mi}}) \right| = \boxed{3.62\,\text{kW}}$$

***53** ••

Picture the Problem Because, in the absence of air resistance, the acceleration of the cannonball is constant, we can use a constant-acceleration equation to relate its velocity to the time it has been in flight. We can apply Newton's 2nd law to the cannonball to find the net force acting on it and then form the dot product of $\vec{F}$ and $\vec{v}$ to express the rate at which the gravitational field does work on the cannonball. Integrating this expression over the time-of-flight T of the ball will yield the desired result.

Express the velocity of the cannonball as a function of time while it is in the air:

$$\vec{v}(t) = 0\hat{i} + (v_0 - gt)\hat{j}$$

Apply $\sum \vec{F} = m\vec{a}$ to the cannonball to express the force acting on it while it is in the air:

$$\vec{F} = -mg\,\hat{j}$$

Evaluate $\vec{F} \cdot \vec{v}$:

$$\vec{F} \cdot \vec{v} = -mg\,\hat{j} \cdot (v_0 - gt)\hat{j} = -mgv_0 + mg^2 t$$

Relate $\vec{F}\cdot\vec{v}$ to the rate at which work is being done on the cannonball:

$$\frac{dW}{dt}=\vec{F}\cdot\vec{v}=-mgv_0+mg^2t$$

Separate the variables and integrate over the time T that the cannonball is in the air:

$$W=\int_0^T\left(-mgv_0+mg^2t\right)dt=\tfrac{1}{2}mg^2T^2-mgv_0T \qquad (1)$$

Using a constant-acceleration equation, relate the speed v of the cannonball when it lands at the bottom of the cliff to its initial speed v_0 and the height of the cliff H:

$$v^2=v_0^2+2a\Delta y$$

or, because $a = g$ and $\Delta y = H$,

$$v^2=v_0^2+2gH$$

Solve for v to obtain:

$$v=\boxed{\sqrt{{v_0}^2+2gH}}$$

Using a constant-acceleration equation, relate the time-of-flight T to the initial and impact speeds of the cannonball:

$$v=v_0-gT$$

Solve for T to obtain:

$$T=\frac{v_0-v}{g}$$

Substitute for T in equation (1) and simplify to evaluate W:

$$W=\tfrac{1}{2}mg^2\frac{v_0^2-2vv_0+v^2}{g^2}-mgv_0\left(\frac{v_0-v}{g}\right)=\tfrac{1}{2}mv^2-\tfrac{1}{2}mv_0^2=\boxed{\Delta K}$$

54 ••

Picture the Problem If the particle is acted on by a *single* force, that force is the *net* force acting on the particle and is responsible for its acceleration. The rate at which energy is delivered by the force is $P=\vec{F}\cdot\vec{v}$.

Express the rate at which this force does work in terms of $\vec{F}$ and $\vec{v}$:

$$P=\vec{F}\cdot\vec{v}$$

The velocity of the particle, in terms of its acceleration and the time that the force has acted is:

$$\vec{v}=\vec{a}t$$

Using Newton's 2nd law, substitute for $\vec{a}$:

$$\vec{v} = \frac{\vec{F}}{m}t$$

Substitute for $\vec{v}$ in the expression for *P* and simplify to obtain:

$$P = \vec{F} \cdot \frac{\vec{F}}{m}t = \frac{\vec{F} \cdot \vec{F}}{m}t = \boxed{\frac{F^2}{m}t}$$

Potential Energy

55 •

Picture the Problem The change in the gravitational potential energy of the earth-man system, near the surface of the earth, is given by $\Delta U = mg\Delta h$, where Δh is measured relative to an arbitrarily chosen reference position.

Express the change in the man's gravitational potential energy in terms of his change in elevation:

$$\Delta U = mg\Delta h$$

Substitute for *m*, *g* and Δh and evaluate ΔU:

$$\Delta U = (80\,\text{kg})(9.81\,\text{m/s}^2)(6\,\text{m}) = \boxed{4.71\,\text{kJ}}$$

56 •

Picture the Problem The water going over the falls has gravitational potential energy relative to the base of the falls. As the water falls, the falling water acquires kinetic energy until, at the base of the falls; its energy is entirely kinetic. The rate at which energy is delivered to the base of the falls is given by $P = dW/dt = -dU/dt$.

Express the rate at which energy is being delivered to the base of the falls; remembering that half the potential energy of the water is converted to electric energy:

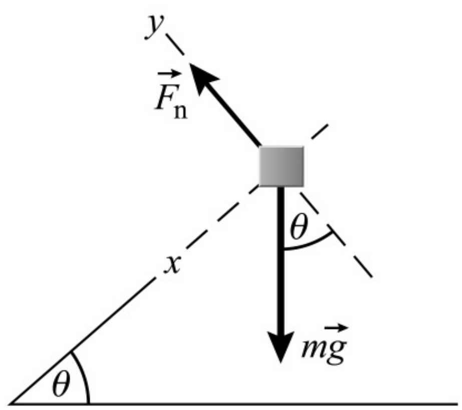
$$P = \frac{dW}{dt} = -\frac{dU}{dt} = -\tfrac{1}{2}\frac{d}{dt}(mgh) = -\tfrac{1}{2}gh\frac{dm}{dt}$$

Substitute numerical values and evaluate *P*:

$$P = -\tfrac{1}{2}(9.81\,\text{m/s}^2)(-128\,\text{m}) \times (1.4\times10^6\ \text{kg/s}) = \boxed{879\,\text{MW}}$$

57 •

Picture the Problem In the absence of friction, the sum of the potential and kinetic energies of the box remains constant as it slides down the incline. We can use the conservation of the mechanical energy of the system to calculate where the box will be and how fast it will be moving at any given time. We can also use Newton's 2nd law to show that the acceleration of the box is constant and constant-acceleration equations to calculate where the box will be and how fast it will be moving at any given time.



(*a*) Express and evaluate the gravitational potential energy of the box, relative to the ground, at the top of the incline:

$U_i = mgh = (2\text{ kg})(9.81\text{ m/s}^2)(20\text{ m}) = \boxed{392\text{ J}}$

(*b*) Using a constant-acceleration equation, relate the displacement of the box to its initial speed, acceleration and time-of-travel:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

Apply $\sum F_x = ma_x$ to the box as it slides down the incline and solve for its acceleration:

$$mg\sin\theta = ma \Rightarrow a = g\sin\theta$$

Substitute for a and evaluate $\Delta x(t = 1\text{ s})$:

$$\Delta x(1\text{s}) = \tfrac{1}{2}(g\sin\theta)(\Delta t)^2 = \tfrac{1}{2}(9.81\text{m/s}^2)(\sin 30°)(1\text{s})^2 = \boxed{2.45\text{m}}$$

Using a constant-acceleration equation, relate the speed of the box at any time to its initial speed and acceleration and solve for its speed when $t = 1$ s:

$v = v_0 + at$ where $v_0 = 0$

and

$$v(1\text{s}) = a\Delta t = (g\sin\theta)\Delta t = (9.81\text{m/s}^2)(\sin 30°)(1\text{s}) = \boxed{4.91\text{m/s}}$$

(*c*) Calculate the kinetic energy of the box when it has traveled for 1 s:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\,\text{kg})(4.91\,\text{m/s})^2 = \boxed{24.1\,\text{J}}$$

Express the potential energy of the box after it has traveled for 1 s in terms of its initial potential energy and its kinetic energy:

$$U = U_\text{i} - K = 392\,\text{J} - 24.1\,\text{J} = \boxed{368\,\text{J}}$$

(*d*) Express the kinetic energy of the box at the bottom of the incline in terms of its initial potential energy and solve for its speed at the bottom of the incline:

$$K = U_i = \tfrac{1}{2}mv^2 = \boxed{392\,\text{J}}$$

and

$$v = \sqrt{\frac{2U_i}{m}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2(392\,\text{J})}{2\,\text{kg}}} = \boxed{19.8\,\text{m/s}}$$

58 •

Picture the Problem The potential energy function $U(x)$ is defined by the equation $U(x) - U(x_0) = -\int_{x_0}^{x} F dx.$ We can use the given force function to determine $U(x)$ and then the conditions on U to determine the potential functions that satisfy the given conditions.

(*a*) Use the definition of the potential energy function to find the potential energy function associated with F_x:

$$U(x) = U(x_0) - \int_{x_0}^{x} F_x dx = U(x_0) - \int_{x_0}^{x} (6\,\text{N}) dx' = \boxed{-(6\,\text{N})(x - x_0)}$$

because $U(x_0) = 0$.

(*b*) Use the result obtained in (*a*) to find $U(x)$ that satisfies the condition that $U(4\text{ m}) = 0$:

$$U(4\,\text{m}) = -(6\,\text{N})(4\,\text{m} - x_0) = 0 \Rightarrow x_0 = 4\,\text{m}$$

and

$$U(x) = -(6\,\text{N})(x - 4\,\text{m}) = \boxed{24\,\text{J} - (6\,\text{N})x}$$

(*c*) Use the result obtained in (*a*) to find *U* that satisfies the condition that *U*(6 m) = 14 J:

$$U(6\,\text{m}) = -(6\,\text{N})(6\,\text{m} - x_0) = 14\,\text{J} \Rightarrow x_0 = 50\,\text{m}$$

and

$$U(x) = -(6\,\text{N})\left(x - \frac{25}{3}\,\text{m}\right) = \boxed{50\,\text{J} - (6\,\text{N})x}$$

59 •

Picture the Problem The potential energy of a stretched or compressed ideal spring U_s is related to its force (stiffness) constant *k* and stretch or compression Δx by $U_s = \frac{1}{2}kx^2$.

(*a*) Relate the potential energy stored in the spring to the distance it has been stretched:

$$U_s = \tfrac{1}{2}kx^2$$

Solve for *x*:

$$x = \sqrt{\frac{2U_s}{k}}$$

Substitute numerical values and evaluate *x*:

$$x = \sqrt{\frac{2(50\,\text{J})}{10^4\ \text{N/m}}} = \boxed{0.100\,\text{m}}$$

(*b*) Proceed as in (*a*) with U_s = 100 J:

$$x = \sqrt{\frac{2(100\,\text{J})}{10^4\ \text{N/m}}} = \boxed{0.141\,\text{m}}$$

***60** ••

Picture the Problem In a simple Atwood's machine, the only effect of the pulley is to connect the motions of the two objects on either side of it; i.e., it could be replaced by a piece of polished pipe. We can relate the kinetic energy of the rising and falling objects to the mass of the system and to their common speed and relate their accelerations to the sum and difference of their masses … leading to simultaneous equations in m_1 and m_2.

Use the definition of the kinetic energy of the system to determine the total mass being accelerated:

$$K = \tfrac{1}{2}(m_1 + m_2)v^2$$

and

$$m_1 + m_2 = \frac{2K}{v^2} = \frac{2(80\,\text{J})}{(4\,\text{m/s})^2} = 10.0\,\text{kg} \quad (1)$$

In Chapter 4, the acceleration of the masses was shown to be:

$$a = \frac{m_1 - m_2}{m_1 + m_2}g$$

Because $v(t) = at$, we can eliminate a in the previous equation to obtain:

$$v(t) = \frac{m_1 - m_2}{m_1 + m_2} gt$$

Solve for $m_1 - m_2$:

$$m_1 - m_2 = \frac{(m_1 + m_2)v(t)}{gt}$$

Substitute numerical values and evaluate $m_1 - m_2$:

$$m_1 - m_2 = \frac{(10\,\text{kg})(4\,\text{m/s})}{(9.81\,\text{m/s}^2)(3\,\text{s})} = 1.36\,\text{kg} \quad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$m_1 = \boxed{5.68\,\text{kg}} \text{ and } m_2 = \boxed{4.32\,\text{kg}}$$

61 ••

Picture the Problem The gravitational potential energy of this system of two objects is the sum of their individual potential energies and is dependent on an arbitrary choice of where, or under what condition(s), the gravitational potential energy is zero. The best choice is one that simplifies the mathematical details of the expression of U. In this problem let's choose $U = 0$ where $\theta = 0$.

(a) Express U for the 2-object system as the sum of their gravitational potential energies; noting that because the object whose mass is m_2 is above the position we have chosen for $U = 0$, its potential energy is positive while that of the object whose mass is m_1 is negative:

$$\begin{aligned} U(\theta) &= U_1 + U_2 \\ &= m_2 g\ell_2 \sin\theta - m_1 g\ell_1 \sin\theta \\ &= \boxed{(m_2\ell_2 - m_1\ell_1) g \sin\theta} \end{aligned}$$

(b) Differentiate U with respect to θ and set this derivative equal to zero to identify extreme values:

$$\frac{dU}{d\theta} = (m_2\ell_2 - m_1\ell_1) g \cos\theta = 0$$

from which we can conclude that $\cos\theta = 0$ and $\theta = \cos^{-1}0$.

To be physically meaningful, $-\pi/2 \le \theta \le \pi/2$:

$$\therefore \theta = \pm\pi/2$$

Express the 2nd derivative of U with respect to θ and evaluate this derivative at $\theta = \pm\pi/2$:

$$\frac{d^2U}{d\theta^2} = -(m_2\ell_2 - m_1\ell_1) g \sin\theta$$

If we assume, in the expression for U that we derived in (*a*), that $m_2\ell_2 - m_1\ell_1 > 0$, then $U(\theta)$ is a sine function and, in the interval of interest, $-\pi/2 \le \theta \le \pi/2$, takes on its minimum value when $\theta = -\pi/2$:

$$\left.\frac{d^2U}{d\theta^2}\right|_{-\pi/2} > 0$$

and $\boxed{U \text{ is a minimum at } \theta = -\pi/2}$

$$\left.\frac{d^2U}{d\theta^2}\right|_{\pi/2} < 0$$

and $\boxed{U \text{ is a maximum at } \theta = \pi/2}$

(*c*) If $m_1\ell_1 = m_2\ell_2$, then

$$(m_2\ell_2 - m_1\ell_1) = 0$$

and $\boxed{U = 0 \text{ independently of } \theta.}$

Remarks: An alternative approach to establishing the *U* is a maximum at $\theta = \pi/2$ is to plot its graph and note that, in the interval of interest, *U* is concave downward with its maximum value at $\theta = \pi/2$.

Force, Potential Energy, and Equilibrium

62 •

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(*a*) Evaluate $F_x = -\dfrac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(Ax^4\right) = \boxed{-4Ax^3}$$

(*b*) Set $F_x = 0$ and solve for x:

$$F_x = 0 \Rightarrow \boxed{x = 0}$$

63 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(*a*) Evaluate $F_x = -\dfrac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(\frac{C}{x}\right) = \boxed{\frac{C}{x^2}}$$

(*b*) Because $C > 0$:

$\boxed{F_x \text{ is positive for } x \ne 0 \text{ and therefore } \vec{F} \text{ is directed away from the origin.}}$

(*c*) Because U is inversely proportional to x and $C > 0$:

$\boxed{U(x) \text{ decreases with increasing } x.}$

(*d*) With $C < 0$:

$\boxed{F_x \text{ is negative for } x \neq 0 \text{ and therefore } \vec{F} \text{ is directed toward from the origin.}}$

Because U is inversely proportional to x and $C < 0$, $U(x)$ becomes less negative as x increases:

$\boxed{U(x) \text{ increases with increasing } x.}$

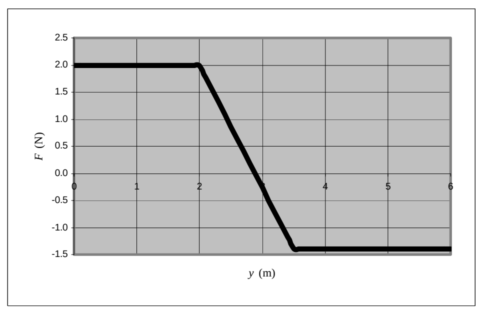
***64** ••

Picture the Problem F_y is defined to be the negative of the derivative of the potential function with respect to y, i.e. $F_y = -dU/dy$. Consequently, we can obtain F_y by examining the slopes of the graph of U as a function of y.

The table to the right summarizes the information we can obtain from Figure 6-40:

	Slope	F_y
Interval	(N)	(N)
$A \to B$	−2	2
$B \to C$	transitional	$-2 \to 1.4$
$C \to D$	1.4	−1.4

The graph of F as a function of y is shown to the right:



65 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, i.e. $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

Evaluate the integral of F_x with respect to x:

$$U(x) = -\int F(x)dx = -\int \frac{a}{x^2}dx$$

$$= \boxed{\frac{a}{x} + U_0}$$

where U_0 is a constant determined by whatever conditions apply to U.

66 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*, that is, $F_x = -dU/dx$. Consequently, given *U* as a function of *x*, we can find F_x by differentiating *U* with respect to *x*. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(3x^2 - 2x^3\right) = \boxed{6x(x-1)}$$

(*b*) We know that, at equilibrium, $F_x = 0$:

When $F_x = 0$, $6x(x - 1) = 0$. Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 1\,\text{m.}}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{dU}{dx} = \frac{d}{dx}\left(3x^2 - 2x^3\right) = 6x - 6x^2$$

and

$$\frac{d^2U}{dx^2} = 6 - 12x$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 6 > 0$$

$$\Rightarrow \boxed{\text{stable equilibrium at } x = 0}$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 1$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=1\,\text{m}} = 6 - 12 < 0$$

$$\Rightarrow \boxed{\text{unstable equilibrium at } x = 1\,\text{m}}$$

67 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*, i.e. $F_x = -dU/dx$. Consequently, given *U* as a function of *x*, we can find F_x by differentiating *U* with respect to *x*. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate the negative of the derivative of *U* with respect to *x*:

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(8x^2 - x^4\right) = 4x^3 - 16x = \boxed{4x(x+2)(x-2)}$$

(*b*) The object is in equilibrium wherever $F_{net} = F_x = 0$:

$4x(x+2)(x-2) = 0 \Rightarrow$ the equilibrium points are $\boxed{x = -2\,\text{m}, 0, \text{and}\ 2\,\text{m}.}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{d^2U}{dx^2} = \frac{d}{dx}\left(16x - 4x^3\right) = 16 - 12x^2$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = -2$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=-2\,\text{m}} = -32 < 0$$

$$\Rightarrow \boxed{\text{unstable equilibrium at } x = -2\,\text{m}}$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 16 > 0$$

$$\Rightarrow \boxed{\text{stable equilibrium at } x = 0}$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 2$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=2\,\text{m}} = -32 < 0$$

$$\Rightarrow \boxed{\text{unstable equilibrium at } x = 2\,\text{m}}$$

Remarks: You could also decide whether the equilibrium positions are stable or unstable by plotting *F*(*x*) and examining the curve at the equilibrium positions.

68 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*, i.e. $F_x = -dU/dx$. Consequently, given *F* as a function of *x*, we can find *U* by integrating F_x with respect to *x*. Examination of d^2U/dx^2 at extreme points will determine the nature of the stability at these locations.

Determine the equilibrium locations by setting $F_{net} = F(x) = 0$:

$F(x) = x^3 - 4x = x(x^2 - 4) = 0$

$\therefore$ the positions of stable and unstable equilibrium are at $\boxed{x = -2, 0 \text{ and } 2}$.

Evaluate the negative of the integral of $F(x)$ with respect to x:

$$U(x) = -\int F(x)$$
$$= -\int (x^3 - 4x)dx$$
$$= -\frac{x^4}{4} + 2x^2 + U_0$$

where U_0 is a constant whose value is determined by conditions on $U(x)$.

Differentiate $U(x)$ twice:

$$\frac{dU}{dx} = -F_x = -x^3 + 4x$$

and

$$\frac{d^2U}{dx^2} = -3x^2 + 4$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = -2$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=-2} = -8 < 0$$

$\therefore$ $\boxed{\text{the equilibrium is unstable at } x = -2}$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 4 > 0$$

$\therefore$ $\boxed{\text{the equilibrium is stable at } x = 0}$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 2$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=2} = -8 < 0$$

$\therefore$ $\boxed{\text{the equilibrium is unstable at } x = 2}$

Thus $U(x)$ has a local minimum at $x = 0$ and local maxima at $x = \pm 2$.

69 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, i.e. $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we can examine the graph of U.

(a) Evaluate $F_x = -\frac{dU}{dx}$ for $x \le 3$ m:

$$F_x = -\frac{d}{dx}\left(3x^2 - x^3\right) = 3x\left(2 - x\right)$$

Set $F_x = 0$ to identify those values of x for which the 4-kg object is in equilibrium:

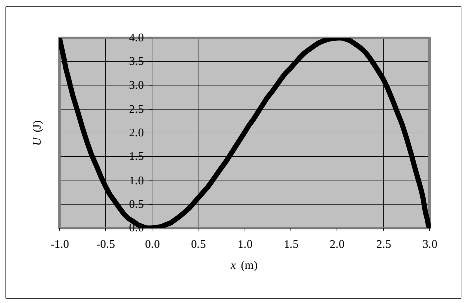
When $F_x = 0$, $3x(2 - x) = 0$.
Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 2\,\text{m.}}$

Evaluate $F_x = -\frac{dU}{dx}$ for $x > 3$ m:

$F_x = 0$
because $U = 0$.

$\boxed{\text{Therefore, the object is in neutral equilibrium for } x > 3\,\text{m.}}$

(b) A graph of $U(x)$ in the interval $-1\text{ m} \le x \le 3$ m is shown to the right:



(c) From the graph, $U(x)$ is a minimum at $x = 0$:

$\therefore$ $\boxed{\text{stable equilibrium at } x = 0}$

From the graph, $U(x)$ is a maximum at $x = 2$ m:

$\therefore$ $\boxed{\text{unstable equilibrium at } x = 2\,\text{m}}$

(d) Relate the kinetic energy of the object to its total energy and its potential energy:

$$K = \tfrac{1}{2}mv^2 = E - U$$

Solve for v:

$$v = \sqrt{\frac{2\left(E - U\right)}{m}}$$

Evaluate $U(x = 2\text{ m})$:

$$U\left(x = 2\,\text{m}\right) = 3\left(2\right)^2 - \left(2\right)^3 = 4\,\text{J}$$

Substitute in the equation for v to obtain:

$$v = \sqrt{\frac{2\left(12\,\text{J} - 4\,\text{J}\right)}{4\,\text{kg}}} = \boxed{2.00\,\text{m/s}}$$

70 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

(*a*) Evaluate the negative of the integral of $F(x)$ with respect to x:

$$U(x) = -\int F(x) = -\int Ax^{-3}dx$$

$$= \frac{1}{2}\frac{A}{x^2} + U_0$$

where U_0 is a constant whose value is determined by conditions on $U(x)$.

For $x > 0$:

$$\boxed{U \text{ decreases as } x \text{ increases}}$$

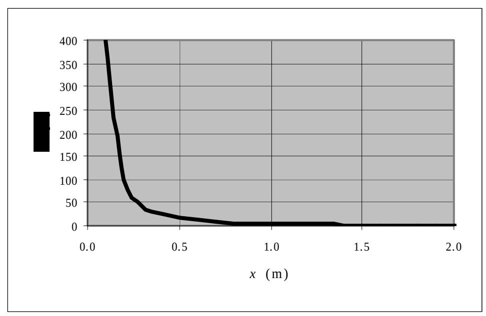
(*b*) As $x \to \infty$, $\frac{1}{2}\frac{A}{x^2} \to 0$:

$\therefore\ U_0 = 0$

and

$$U(x) = \frac{1}{2}\frac{A}{x^2} = \frac{1}{2}\frac{8\,\text{N}\cdot\text{m}^3}{x^2} = \boxed{\frac{4}{x^2}\,\text{N}\cdot\text{m}^3}$$

(*c*) The graph of $U(x)$ is shown to the right:



***71** •••

Picture the Problem Let L be the total length of one cable and the zero of gravitational potential energy be at the top of the pulleys. We can find the value of y for which the potential energy of the system is an extremum by differentiating $U(y)$ with respect to y and setting this derivative equal to zero. We can establish that this value corresponds to a minimum by evaluating the second derivative of $U(y)$ at the point identified by the first derivative. We can apply Newton's 2nd law to the clock to confirm the result we obtain by examining the derivatives of $U(y)$.

(*a*) Express the potential energy of the system as the sum of the potential energies of the clock and counterweights:

$$U(y) = U_{\text{clock}}(y) + U_{\text{weights}}(y)$$

Substitute to obtain:

$$U(y) = \boxed{-mgy - 2Mg\left(L - \sqrt{y^2 + d^2}\right)}$$

(*b*) Differentiate *U*(*y*) with respect to *y*:

$$\frac{dU(y)}{dy}=-\frac{d}{dy}\left[mgy+2Mg\left(L-\sqrt{y^2+d^2}\right)\right]$$

$$=-\left[mg-2Mg\frac{y}{\sqrt{y^2+d^2}}\right]$$

or

$$mg-2Mg\frac{y'}{\sqrt{y'^2+d^2}}=0 \text{ for extrema}$$

Solve for *y*′ to obtain:

$$y'=d\sqrt{\frac{m^2}{4M^2-m^2}}$$

Find $\frac{d^2U(y)}{dy^2}$:

$$\frac{d^2U(y)}{dy^2}=-\frac{d}{dy}\left[mg-2Mg\frac{y}{\sqrt{y^2+d^2}}\right]$$

$$=\frac{2Mgd^2}{\left(y^2+d^2\right)^{3/2}}$$

Evaluate $\frac{d^2U(y)}{dy^2}$ at $y=y'$:

$$\left.\frac{d^2U(y)}{dy^2}\right|_{y'}=\left.\frac{2Mgd^2}{\left(y^2+d^2\right)^{3/2}}\right|_{y'}$$

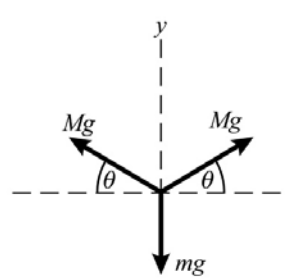
$$=\frac{2Mgd}{\left(\frac{m^2}{4M^2-m^2}+1\right)^{3/2}}$$

$$>0$$

and the potential energy is a minimum at

$$y=\boxed{d\sqrt{\frac{m^2}{4M^2-m^2}}}$$

(*c*) The FBD for the clock is shown to the right:



Apply $\sum F_y=0$ to the clock:

$$2Mg\sin\theta-mg=0$$

and

$$\sin\theta=\frac{m}{2M}$$

Express $\sin\theta$ in terms of y and d:

$$\sin\theta = \frac{y}{\sqrt{y^2 + d^2}}$$

Substitute to obtain:

$$\boxed{\frac{m}{2M} = \frac{y}{\sqrt{y^2 + d^2}}}$$

which is equivalent to the first equation in part (*b*).

This is a point of stable equilibrium. If the clock is displaced downward, θ increases, leading to a larger upward force on the clock. Similarly, if the clock is displaced upward, the net force from the cables decreases. Because of this, the clock will be pulled back toward the equilibrium point if it is displaced away from it.

Remarks: Because we've shown that the potential energy of the system is a minimum at $y = y'$ (i.e., $U(y)$ is concave upward at that point), we can conclude that this point is one of stable equilibrium.

General Problems

***72** •

Picture the Problem 25 percent of the electrical energy generated is to be diverted to do the work required to change the potential energy of the American people. We can calculate the height to which they can be lifted by equating the change in potential energy to the available energy.

Express the change in potential energy of the population of the United States in this process:

$$\Delta U = Nmgh$$

Letting E represent the total energy generated in February 2002, relate the change in potential to the energy available to operate the elevator:

$$Nmgh = 0.25E$$

Solve for h:

$$h = \frac{0.25E}{Nmg}$$

Substitute numerical values and evaluate h:

$$h = \frac{(0.25)(60.7\times10^{9}\ \text{kW}\cdot\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)}{(287\times10^{6})(60\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{323\,\text{km}}$$

73 •

Picture the Problem We can use the definition of the work done in changing the potential energy of a system and the definition of power to solve this problem.

(*a*) Find the work done by the crane in changing the potential energy of its load:

$$W = mgh = (6\times10^{6}\ \text{kg})\,(9.81\ \text{m/s}^2)\,(12\ \text{m}) = \boxed{706\,\text{MJ}}$$

(*b*) Use the definition of power to find the power developed by the crane:

$$P \equiv \frac{dW}{dt} = \frac{706\,\text{MJ}}{60\,\text{s}} = \boxed{11.8\,\text{MW}}$$

74 •

Picture the Problem The power P of the engine needed to operate this ski lift is related to the rate at which it changes the potential energy U of the cargo of the gondolas according to $P = \Delta U/\Delta t$. Because as many empty gondolas are descending as are ascending, we do not need to know their mass.

Express the rate at which work is done as the cars are lifted:

$$P = \frac{\Delta U}{\Delta t}$$

Letting N represent the number of gondola cars and M the mass of each, express the change in U as they are lifted a vertical displacement Δh:

$$\Delta U = NMg\Delta h$$

Substitute to obtain:

$$P \equiv \frac{\Delta U}{\Delta t} = \frac{NMg\Delta h}{\Delta t}$$

Relate Δh to the angle of ascent θ and the length L of the ski lift:

$$\Delta h = L\sin\theta$$

Substitute for Δh in the expression for P:

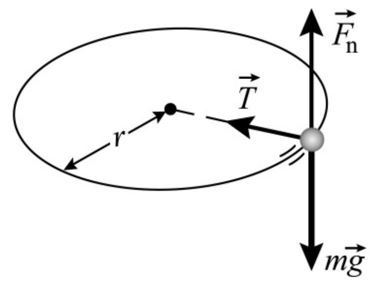
$$P = \frac{NMgL\sin\theta}{\Delta t}$$

Substitute numerical values and evaluate P:

$$P = \frac{12(550\,\text{kg})(9.81\,\text{m/s}^2)(5.6\,\text{km})\sin 30°}{(60\,\text{min})(60\,\text{s/min})} = \boxed{50.4\,\text{kW}}$$

75 •

Picture the Problem The application of Newton's 2nd law to the forces shown in the free-body diagram will allow us to relate R to T. The unknown mass and speed of the object can be eliminated by introducing its kinetic energy.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ the object and solve for R:

$$T = \frac{mv^2}{R} \text{ and } R = \frac{mv^2}{T}$$

Express the kinetic energy of the object:

$$K = \tfrac{1}{2}mv^2$$

Eliminate mv^2 between the two equations to obtain:

$$R = \frac{2K}{T}$$

Substitute numerical values and evaluate R:

$$R = \frac{2(90\,\text{J})}{360\,\text{N}} = \boxed{0.500\,\text{m}}$$

***76** •

Picture the Problem We can solve this problem by equating the expression for the gravitational potential energy of the elevated car and its kinetic energy when it hits the ground.

Express the gravitational potential energy of the car when it is at a distance h above the ground:

$$U = mgh$$

Express the kinetic energy of the car when it is about to hit the ground:

$$K = \tfrac{1}{2}mv^2$$

Equate these two expressions (because at impact, all the potential energy has been converted to kinetic energy) and solve for h:

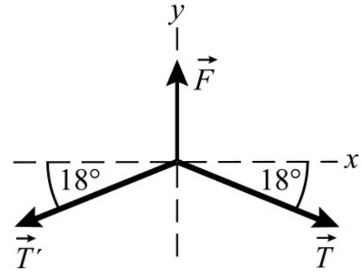
$$h = \frac{v^2}{2g}$$

Substitute numerical values and evaluate h:

$$h = \frac{\left[(100\,\text{km/h})(1\,\text{h}/3600\,\text{s})\right]^2}{2(9.81\,\text{m/s}^2)} = \boxed{39.3\,\text{m}}$$

77 •••

Picture the Problem The free-body diagram shows the forces acting on one of the strings at the bridge. The force whose magnitude is F is one-fourth of the force (103 N) the bridge exerts on the strings. We can apply the condition for equilibrium in the y direction to find the tension in each string. Repeating this procedure at the site of the plucking will yield the restoring force acting on the string. We can find the work done on the string as it returns to equilibrium from the product of the average force acting on it and its displacement.



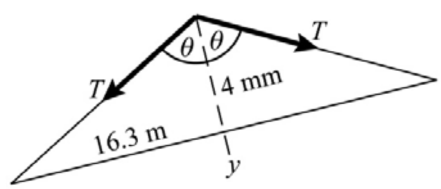
(a) Noting that, due to symmetry, $T' = T$, apply $\sum F_y = 0$ to the string at the point of contact with the bridge:

$$F - 2T\sin 18° = 0$$

Solve for and evaluate T:

$$T = \frac{F}{2\sin 18°} = \frac{\frac{1}{4}(103\,\text{N})}{2\sin 18°} = \boxed{41.7\,\text{N}}$$

(b) A free-body diagram showing the forces restoring the string to its equilibrium position just after it has been plucked is shown to the right:



Express the net force acting on the string immediately after it is released:

$$F_{\text{net}} = 2T\cos\theta$$

Use trigonometry to find θ:

$$\theta = \tan^{-1}\left(\frac{16.3\,\text{cm}}{4\,\text{mm}} \times \frac{10\,\text{mm}}{\text{cm}}\right) = 88.6°$$

Substitute and evaluate F_{net}:

$$F_{\text{net}} = 2(34.4\,\text{N})\cos 88.6° = \boxed{1.68\,\text{N}}$$

(c) Express the work done on the string in displacing it a distance dx':

$$dW = F dx'$$

If we pull the string out a distance x', the magnitude of the force pulling it down is approximately:

$$F = (2T)\frac{x'}{L/2} = \frac{4T}{L}x'$$

Substitute to obtain:

$$dW = \frac{4T}{L}x'\,dx'$$

Integrate to obtain:

$$W = \frac{4T}{L}\int_0^x x'dx' = \frac{2T}{L}x^2$$

where x is the final displacement of the string.

Substitute numerical values to obtain:

$$W = \frac{2(41.7\ \text{N})}{32.6\times10^{-2}\ \text{m}}\left(4\times10^{-3}\ \text{m}\right)^2 = \boxed{4.09\ \text{mJ}}$$

78 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

Evaluate the integral of F_x with respect to x:

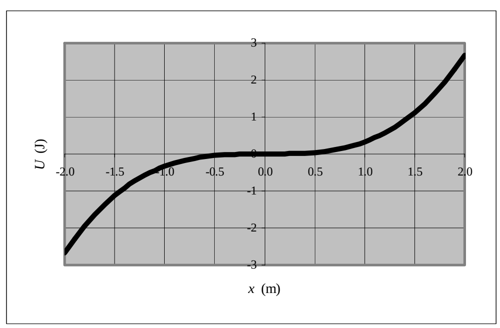
$$U(x) = -\int F(x)dx = -\int\left(-ax^2\right)dx = \tfrac{1}{3}ax^3 + U_0$$

Apply the condition that $U(0) = 0$ to determine U_0:

$U(0) = 0 + U_0 = 0 \Rightarrow U_0 = 0$

$$\therefore U(x) = \boxed{\tfrac{1}{3}ax^3}$$

The graph of $U(x)$ is shown to the right:



***79 ••**

Picture the Problem We can use the definition of work to obtain an expression for the position-dependent force acting on the cart. The work done on the cart can be calculated from its change in kinetic energy.

(*a*) Express the force acting on the cart in terms of the work done on it:

$$F(x)=\frac{dW}{dx}$$

Because *U* is constant:

$$F(x)=\frac{d}{dx}\left(\tfrac{1}{2}mv^2\right)=\frac{d}{dx}\left[\tfrac{1}{2}m(Cx)^2\right]=\boxed{mC^2x}$$

(*b*) The work done by this force changes the kinetic energy of the cart:

$$W=\Delta K=\tfrac{1}{2}mv_1^2-\tfrac{1}{2}mv_0^2=\tfrac{1}{2}mv_1^2-0=\tfrac{1}{2}m(Cx_1)^2=\boxed{\tfrac{1}{2}mC^2x_1^2}$$

80 ••

Picture the Problem The work done by $\vec{F}$ depends on whether it causes a displacement in the direction it acts.

(*a*) Because $\vec{F}$ is along *x*-axis and the displacement is along *y*-axis:

$$W=\int\vec{F}\cdot d\vec{s}=\boxed{0}$$

(*b*) Calculate the work done by $\vec{F}$ during the displacement from x = 2 m to 5 m:

$$W=\int\vec{F}\cdot d\vec{s}=\int_{2\,\text{m}}^{5\,\text{m}}\left(2\,\text{N/m}^2\right)x^2dx=\left(2\,\text{N/m}^2\right)\left[\frac{x^3}{3}\right]_{2\,\text{m}}^{5\,\text{m}}=\boxed{78.0\,\text{J}}$$

81 ••

Picture the Problem The velocity and acceleration of the particle can be found by differentiation. The power delivered to the particle can be expressed as the product of its velocity and the net force acting on it, and the work done by the force and can be found from the change in kinetic energy this work causes.

In the following, if *t* is in seconds and *m* is in kilograms, then *v* is in m/s, *a* is in m/s^2, *P* is in W, and *W* is in J.

(*a*) The velocity of the particle is given by:

$$v=\frac{dx}{dt}=\frac{d}{dt}\left(2t^3-4t^2\right)=\boxed{\left(6t^2-8t\right)}$$

The acceleration of the particle is given by:

$$a=\frac{dv}{dt}=\frac{d}{dt}\left(6t^2-8t\right)=\boxed{\left(12t-8\right)}$$

(*b*) Express and evaluate the rate at which energy is delivered to this particle as it accelerates:

$$P = Fv = mav = m(12t-8)\left(6t^2-8t\right)=\boxed{8mt\left(9t^2-18t+8\right)}$$

(*c*) Because the particle is moving in such a way that its potential energy is not changing, the work done by the force acting on the particle equals the change in its kinetic energy:

$$W=\Delta K=K_1-K_0=\tfrac{1}{2}m\left[\left(v(t_1)\right)^2-\left(v(0)\right)^2\right]=\tfrac{1}{2}m\left[\left(6t_1^2-8t_1\right)\right]^2-0=\boxed{2mt_1^2\left(3t_1-4\right)^2}$$

Remarks: We could also find *W* by integrating *P*(*t*) with respect to time.

82 ••

Picture the Problem We can calculate the work done by the given force from its definition. The power can be determined from $P=\vec{F}\cdot\vec{v}$ and v from the change in kinetic energy of the particle produced by the work done on it.

(*a*) Calculate the work done from its definition:

$$W=\int\vec{F}\cdot d\vec{s}=\int_0^{3\,\text{m}}\left(6+4x-3x^2\right)dx=\left[6x+\frac{4x^2}{2}-\frac{3x^3}{3}\right]_0^{3\,\text{m}}=\boxed{9.00\ \text{J}}$$

(*b*) Express the power delivered to the particle in terms of $F_{x=3\,\text{m}}$ and its velocity:

$$P=\vec{F}\cdot\vec{v}=F_{x=3\,\text{m}}v$$

Relate the work done on the particle to its kinetic energy and solve for its velocity:

$$W=\Delta K=K_{\text{final}}=\tfrac{1}{2}mv^2\ \text{since}\ v_0=0$$

Solve for and evaluate v:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(9\,\text{J})}{3\,\text{kg}}} = 2.45\,\text{m/s}$$

Evaluate $F_{x=3\text{ m}}$:

$$F_{x=3\text{m}} = 6 + 4(3) - 3(3)^2 = -9\,\text{N}$$

Substitute for $F_{x=3\text{ m}}$ and v:

$$P = (-9\,\text{N})(2.45\,\text{m/s}) = \boxed{-22.1\,\text{W}}$$

***83 ••**

Picture the Problem We'll assume that the firing height is negligible and that the bullet lands at the same elevation from which it was fired. We can use the equation $R = \left(v_0^2/g\right)\sin 2\theta$ to find the range of the bullet and constant-acceleration equations to find its maximum height. The bullet's initial speed can be determined from its initial kinetic energy.

Express the range of the bullet as a function of its firing speed and angle of firing:

$$R = \frac{v_0^2}{g}\sin 2\theta$$

Rewrite the range equation using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$:

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Express the position coordinates of the projectile along its flight path in terms of the parameter t:

$$x = (v_0\cos\theta)t$$

and

$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t and make use of the fact that the maximum height occurs when the projectile is at half the range to obtain:

$$h = \frac{(v_0\sin\theta)^2}{2g}$$

Equate R and h and solve the resulting equation for θ:

$$\tan\theta = 4 \Rightarrow \theta = \tan^{-1}4 = 76.0°$$

Relate the bullet's kinetic energy to its mass and speed and solve for the square of its speed:

$$K = \tfrac{1}{2}mv_0^2 \text{ and } v_0^2 = \frac{2K}{m}$$

Substitute for v_0^2 and θ and evaluate R:

$$R = \frac{2(1200\,\text{J})}{(0.02\,\text{kg})(9.81\,\text{m/s}^2)}\sin 2(76°) = \boxed{5.74\,\text{km}}$$

84 ••

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is positive, so W is negative for $x < 0$.

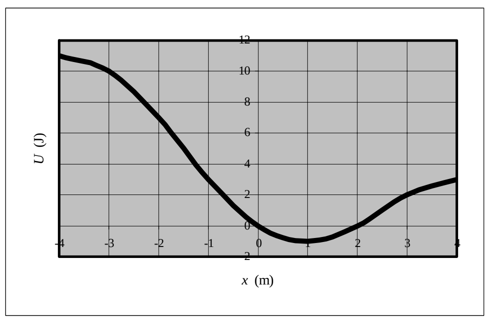
(*a*) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square to complete the table to the right:

x	W
(m)	(J)
–4	–11
–3	–10
–2	–7
–1	–3
0	0
1	1
2	0
3	–2
4	–3

(*b*) Choosing $U(0) = 0$, and using the definition of $\Delta U = -W$, complete the third column of the table to the right:

x	W	ΔU
(m)	(J)	(J)
–4	–11	11
–3	–10	10
–2	–7	7
–1	–3	3
0	0	0
1	1	–1
2	0	0
3	–2	2
4	–3	3

The graph of U as a function of x is shown to the right:



85 ••

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is negative, so W is positive for $x < 0$.

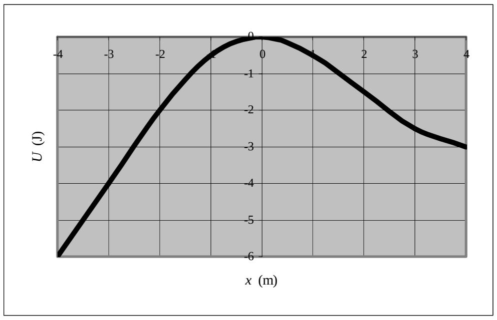
(*a*) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square to complete the table to the right:

x (m)	W (J)
–4	6
–3	4
–2	2
–1	0.5
0	0
1	0.5
2	1.5
3	2.5
4	3

(*b*) Choosing $U(0) = 0$, and using the definition of $\Delta U = -W$, complete the third column of the table to the right:

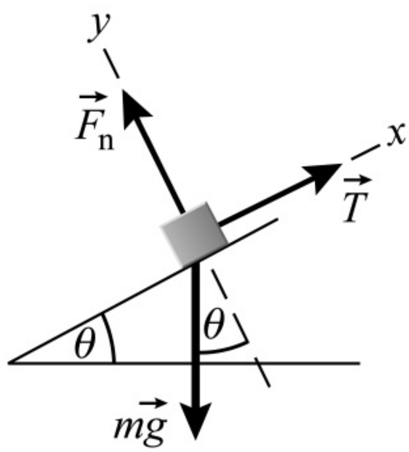
x (m)	W (J)	ΔU (J)
–4	6	–6
–3	4	–4
–2	2	–2
–1	0.5	–0.5
0	0	0
1	0.5	–0.5
2	1.5	–1.5
3	2.5	–2.5
4	3	–3

The graph of U as a function of x is shown to the right:



86 ••

Picture the Problem The pictorial representation shows the box at its initial position 0 at the bottom of the inclined plane and later at position 1. We'll assume that the block is at position 0. Because the surface is frictionless, the work done by the tension will change both the potential and kinetic energy of the block. We'll use Newton's 2nd law to find the acceleration of the block up the incline and a constant-acceleration equation to express v in terms of T, x, M, and θ. Finally, we can express the power produced by the tension in terms of the tension and the speed of the box.



(*a*) Use the definition of work to express the work the tension T does moving the box a distance x up the incline:

$$W = \boxed{Tx}$$

(*b*) Apply $\sum F_x = Ma_x$ to the box:

$$T - Mg\sin\theta = Ma_x$$

Solve for a_x:

$$a_x = \frac{T - Mg\sin\theta}{M} = \frac{T}{M} - g\sin\theta$$

Using a constant-acceleration equation, express the speed of the box in terms of its acceleration and the distance x it has moved up the incline:

$$v^2 = v_0^2 + 2a_x x$$

or, because $v_0 = 0$,

$$v = \sqrt{2a_x x}$$

Substitute for a_x to obtain:

$$v = \boxed{\sqrt{2\left(\frac{T}{M} - g\sin\theta\right)x}}$$

(*c*) The power produced by the tension in the string is given by:

$$P = Tv = \boxed{T\sqrt{2\left(\frac{T}{M} - g\sin\theta\right)x}}$$

87 •••

Picture the Problem We can use the definition of the magnitude of vector to show that the magnitude of $\vec{F}$ is F_0 and the definition of the scalar product to show that its direction is perpendicular to $\vec{r}$. The work done as the particle moves in a circular path can be found from its definition.

(*a*) Express the magnitude of $\vec{F}$:

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(\frac{F_0}{r}y\right)^2 + \left(-\frac{F_0}{r}x\right)^2} = \frac{F_0}{r}\sqrt{x^2 + y^2}$$

Because $r = \sqrt{x^2 + y^2}$:

$$\left|\vec{F}\right| = \frac{F_0}{r}\sqrt{x^2 + y^2} = \frac{F_0}{r}r = \boxed{F_0}$$

Form the scalar product of $\vec{F}$ and $\vec{r}$:

$$\vec{F}\cdot\vec{r} = \left(\frac{F_0}{r}\right)\left(y\,\hat{i} - x\,\hat{j}\right)\cdot\left(x\,\hat{i} + y\,\hat{j}\right) = \left(\frac{F_0}{r}\right)\left(xy - xy\right) = 0$$

Because $\vec{F}\cdot\vec{r} = 0$, $\boxed{\vec{F}\perp\vec{r}}$

(*b*) Because $\vec{F}\perp\vec{r}$, $\vec{F}$ is tangential to the circle and constant. At (5 m, 0), $\vec{F}$ points in the $-\hat{j}$ direction. If $d\vec{s}$ is in the $-\hat{j}$ direction, $dW > 0$. The work it does in one revolution is:

$$W = F_0(2\pi r) = 2\pi(5\,\mathrm{m})F_0 = (10\pi\,\mathrm{m})F_0 \text{ if the rotation is clockwise}$$

and

$$W = (-10\pi\,\mathrm{m})F_0 \text{ if the rotation is counterclockwise.}$$

$$\boxed{W = (10\pi\,\mathrm{m})F_0 \text{ if the rotation is clockwise}, -(10\pi\,\mathrm{m})F_0 \text{ if the rotation is counterclockwise. Because } W \neq 0 \text{ for a complete circuit, } \vec{F} \text{ is not conservative.}}$$

***88** •••

Picture the Problem We can substitute for r and $x\hat{i} + y\hat{j}$ in $\vec{F}$ to show that the magnitude of the force varies as the inverse of the square of the distance to the origin, and that its direction is opposite to the radius vector. We can find the work done by this force by evaluating the integral of F with respect to x from an initial position $x = 2$ m, $y = 0$ m to a final position $x = 5$ m, $y = 0$ m. Finally, we can apply Newton's 2nd law to the particle to relate its speed to its radius, mass, and the constant b.

(*a*) Substitute for *r* and $x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$ in $\vec{F}$ to obtain:

$$\vec{F} = -\left(\frac{b}{\left(x^2 + y^2\right)^{3/2}}\right)\sqrt{x^2 + y^2}\,\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the origin toward the point of application of $\vec{F}$.

Simplify to obtain:

$$\vec{F} = -b\left(\frac{1}{x^2 + y^2}\right)\hat{\boldsymbol{r}} = \boxed{-\frac{b}{r^2}\hat{\boldsymbol{r}}}$$

i.e., the magnitude of the force varies as the inverse of the square of the distance to the origin, and its direction is antiparallel (opposite) to the radius vector $\vec{r} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$.

(*b*) Find the work done by this force by evaluating the integral of *F* with respect to *x* from an initial position *x* = 2 m, *y* = 0 m to a final position *x* = 5 m, *y* = 0 m:

$$W = -\int_{2\,\text{m}}^{5\,\text{m}} \frac{b}{x'^2}dx' = b\left[\frac{1}{x'}\right]_{2\,\text{m}}^{5\,\text{m}}$$

$$= 3\,\text{N}\cdot\text{m}^2\left(\frac{1}{5\,\text{m}} - \frac{1}{2\,\text{m}}\right) = \boxed{-0.900\,\text{J}}$$

(*c*) $\boxed{\text{No work is done as the force is perpendicular to the velocity.}}$

(*d*) Because the particle is moving in a circle, the force on the particle must be supplying the centripetal acceleration keeping it moving in the circle. Apply $\sum F_r = ma_c$ to the particle:

$$\frac{b}{r^2} = m\frac{v^2}{r}$$

Solve for *v*:

$$v = \sqrt{\frac{b}{mr}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{3\,\text{N}\cdot\text{m}^2}{(2\,\text{kg})(7\,\text{m})}} = \boxed{0.463\,\text{m/s}}$$

89 •••

Picture the Problem A spreadsheet program to calculate the potential is shown below. The constants used in the potential function and the formula used to calculate the ″6-12″ potential are as follows:

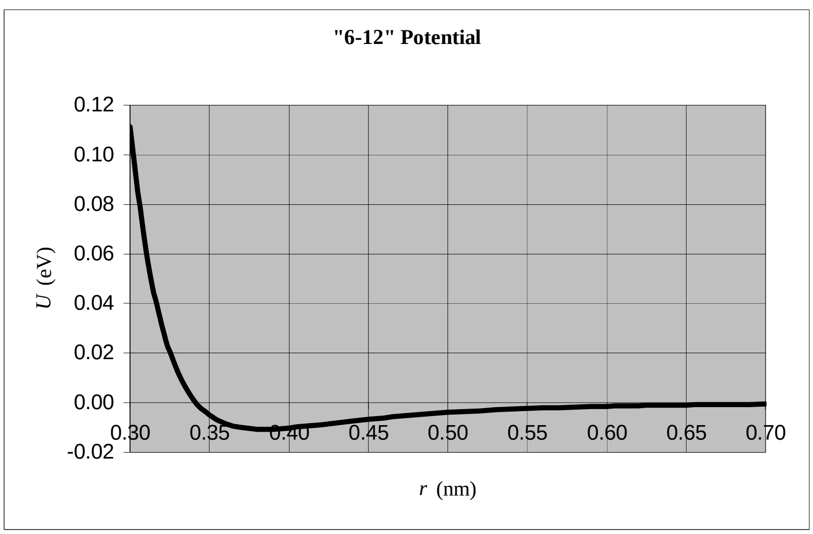
Cell	Content/Formula	Algebraic Form
B2	1.09×10^{-7}	*a*
B3	6.84×10^{-5}	*b*

D8	\$B\$2/C8^12–\$B\$3/C8^6	$\frac{a}{r^{12}} - \frac{b}{r^6}$
C9	C8+0.1	$r + \Delta r$

(*a*)

	A	B	C	D
1				
2	a =	1.09E-07		
3	b =	6.84E-05		
4				
5				
6				
7			*r*	*U*
8			3.00E-01	1.11E-01
9			3.10E-01	6.13E-02
10			3.20E-01	3.08E-02
11			3.30E-01	1.24E-02
12			3.40E-01	1.40E-03
13			3.50E-01	–4.95E-03
45			6.70E-01	–7.43E-04
46			6.80E-01	–6.81E-04
47			6.90E-01	–6.24E-04
48			7.00E-01	–5.74E-04

The graph shown below was generated from the data in the table shown above. Because the force between the atomic nuclei is given by $F = -(dU/dr)$, we can conclude that the shape of the potential energy function supports Feynman's claim.



(*b*) The minimum value is about –0.0107 eV, occurring at a separation of approximately 0.380 nm. Because the function is concave upward (a potential ″well″) at this separation, this separation is one of stable equilibrium, although very shallow.

(*c*) Relate the force of attraction between two argon atoms to the slope of the potential energy function:

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left[\frac{a}{r^{12}} - \frac{b}{r^6}\right] = \frac{12a}{r^{13}} - \frac{6b}{r^7}$$

Substitute numerical values and evaluate *F*(5 Å):

$$F = \frac{12\left(1.09\times10^{-7}\right)}{(0.5\,\text{nm})^{13}} - \frac{6\left(6.84\times10^{-5}\right)}{(0.5\,\text{nm})^{7}} = -4.18\times10^{-2}\,\frac{\text{eV}}{\text{nm}}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}\times\frac{1\,\text{nm}}{10^{-9}\,\text{m}}$$
$$= \boxed{-6.69\times10^{-12}\,\text{N}}$$

where the minus sign means that the force is attractive.

Substitute numerical values and evaluate *F*(3.5 Å):

$$F = \frac{12\left(1.09\times10^{-7}\right)}{(0.35\,\text{nm})^{13}} - \frac{6\left(6.84\times10^{-5}\right)}{(0.35\,\text{nm})^{7}} = 4.68\times10^{-1}\,\frac{\text{eV}}{\text{nm}}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}\times\frac{1\,\text{nm}}{10^{-9}\,\text{m}}$$
$$= \boxed{7.49\times10^{-11}\,\text{N}}$$

where the plus sign means that the force is repulsive.

***90** •••

Picture the Problem A spreadsheet program to plot the Yukawa potential is shown below. The constants used in the potential function and the formula used to calculate the Yukawa potential are as follows:

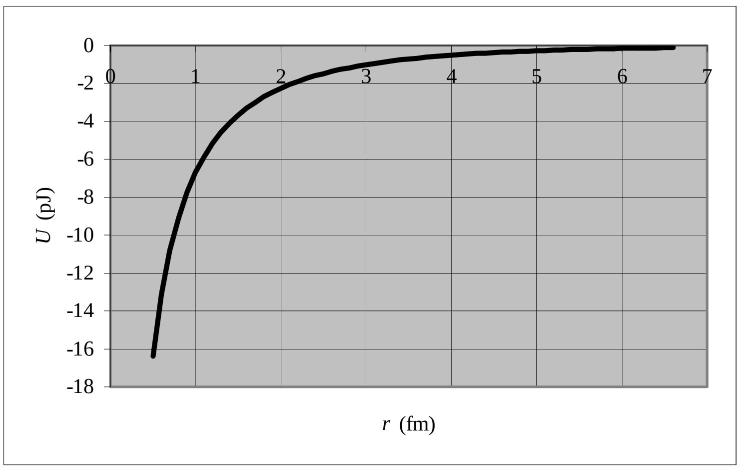
Cell	Content/Formula	Algebraic Form
B1	4	U_0
B2	2.5	a
D8	–$B$1*($B$2/C9)*EXP(–C9/$B$2)	$-U_0\left(\frac{a}{r}\right)e^{-r/a}$
C10	C9+0.1	$r+\Delta r$

(*a*)

	A	B	C	D
1	U0=	4	pJ	
2	a=	2.5	fm	
3				
7				
8			r	U
9			0.5	–16.37
10			0.6	–13.11

11			0.7	–10.80
12			0.8	–9.08
13			0.9	–7.75
14			1	–6.70
64			6	–0.15
65			6.1	–0.14
66			6.2	–0.14
67			6.3	–0.13
68			6.4	–0.12
69			6.5	–0.11
70			6.6	–0.11

U as a function of *r* is shown below.



(*b*) Relate the force between the nucleons to the slope of the potential energy function:

$$F(r) = -\frac{dU(r)}{dr}$$

$$= -\frac{d}{dr}\left[-U_0\left(\frac{a}{r}\right)e^{-r/a}\right]$$

$$= \boxed{-U_0 e^{-r/a}\left(\frac{a}{r^2} + \frac{1}{r}\right)}$$

(*c*) Evaluate $F(2a)$:

$$F(2a) = -U_0 e^{-2a/a}\left(\frac{a}{(2a)^2} + \frac{1}{2a}\right)$$

$$= -U_0 e^{-2}\left(\frac{3}{4a}\right)$$

Evaluate $F(a)$:

$$F(a) = -U_0 e^{-a/a}\left(\frac{a}{(a)^2} + \frac{1}{a}\right)$$

$$= -U_0 e^{-1}\left(\frac{1}{a} + \frac{1}{a}\right) = -U_0 e^{-1}\left(\frac{2}{a}\right)$$

Express the ratio $F(2a)/F(a)$:

$$\frac{F(2a)}{F(a)} = \frac{-U_0 e^{-2}\left(\frac{3}{4a}\right)}{-U_0 e^{-1}\left(\frac{2}{a}\right)} = \frac{3}{8}e^{-1}$$

$$= \boxed{0.138}$$

(d) Evaluate $F(5a)$:

$$F(5a) = -U_0 e^{-5a/a}\left(\frac{a}{(5a)^2} + \frac{1}{5a}\right)$$

$$= -U_0 e^{-5}\left(\frac{6}{25a}\right)$$

Express the ratio $F(5a)/F(a)$:

$$\frac{F(5a)}{F(a)} = \frac{-U_0 e^{-5}\left(\frac{6}{25a}\right)}{-U_0 e^{-1}\left(\frac{2}{a}\right)} = \frac{3}{25}e^{-4}$$

$$= \boxed{2.20 \times 10^{-3}}$$

Chapter 7
Conservation of Energy

Conceptual Problems

***1** •

Determine the Concept Because the peg is frictionless, mechanical energy is conserved as this system evolves from one state to another. The system moves and so we know that $\Delta K > 0$. Because $\Delta K + \Delta U = constant$, $\Delta U < 0$. $\boxed{(a) \text{ is correct.}}$

2 •

Determine the Concept Choose the zero of gravitational potential energy to be at ground level. The two stones have the same initial energy because they are thrown from the same height with the same initial speeds. Therefore, they will have the same total energy at all times during their fall. When they strike the ground, their gravitational potential energies will be zero and their kinetic energies will be equal. Thus, their speeds at impact will be equal. The stone that is thrown at an angle of 30° above the horizontal has a longer flight time due to its initial upward velocity and so they do not strike the ground at the same time. $\boxed{(c) \text{ is correct.}}$

3 •

(*a*) False. Forces that are external to a system can do work on the system to change its energy.

(*b*) False. In order for some object to do work, it must exert a force *over some distance*. The chemical energy stored in the muscles of your legs allows your muscles to do the work that launches you into the air.

4 •

Determine the Concept Your kinetic energy increases at the expense of chemical energy.

***5** •

Determine the Concept As she starts pedaling, chemical energy inside her body is converted into kinetic energy as the bike picks up speed. As she rides it up the hill, chemical energy is converted into gravitational potential and thermal energy. While freewheeling down the hill, potential energy is converted to kinetic energy, and while braking to a stop, kinetic energy is converted into thermal energy (a more random form of kinetic energy) by the frictional forces acting on the bike.

***6** •

Determine the Concept If we define the system to include the falling body and the earth, then no work is done by an external agent and $\Delta K + \Delta U_g + \Delta E_{therm} = 0$. Solving for the change in the gravitational potential energy we find $\Delta U_g = -(\Delta K + \text{friction energy})$.

$\boxed{(b) \text{ is correct.}}$

7 ••

Picture the Problem Because the constant friction force is responsible for a constant acceleration, we can apply the constant-acceleration equations to the analysis of these statements. We can also apply the work-energy theorem with friction to obtain expressions for the kinetic energy of the car and the rate at which it is changing. Choose the system to include the earth and car and assume that the car is moving on a horizontal surface so that $\Delta U = 0$.

(*a*) A constant frictional force causes a constant acceleration. The stopping distance of the car is related to its speed before the brakes were applied through a constant-acceleration equation.

$$v^2 = v_0^2 + 2a\Delta s \text{ where } v = 0.$$

$$\therefore \Delta s = \frac{-v_0^2}{2a} \text{ where } a < 0.$$

Thus, $\Delta s \propto v_0^2$ and statement (*a*) is *false*.

(*b*) Apply the work-energy theorem with friction to obtain:

$$\Delta K = -W_{\text{f}} = -\mu_{\text{k}} mg\Delta s$$

Express the rate at which K is dissipated:

$$\frac{\Delta K}{\Delta t} = -\mu_{\text{k}} mg \frac{\Delta s}{\Delta t}$$

Thus, $\dfrac{\Delta K}{\Delta t} \propto v$ and therefore not constant.

Statement (*b*) is *false*.

(*c*) In part (*b*) we saw that: $K \propto \Delta s$

Because $\Delta s \propto \Delta t$: $K \propto \Delta t$ and statement (*c*) is *false*.

Because none of the above are correct: $\boxed{(d) \text{ is correct.}}$

8 •

Picture the Problem We'll let the zero of potential energy be at the bottom of each ramp and the mass of the block be m. We can use conservation of energy to predict the speed of the block at the foot of each ramp. We'll consider the distance the block travels on each ramp, as well as its speed at the foot of the ramp, in deciding its descent times.

Use conservation of energy to find the speed of the blocks at the bottom of each ramp:

$$\Delta K + \Delta U = 0$$

or

$$K_{\text{bot}} - K_{\text{top}} + U_{\text{bot}} - U_{\text{top}} = 0$$

Because $K_{top} = U_{bot} = 0$: $K_{bot} - U_{top} = 0$

Substitute to obtain: $\frac{1}{2}mv_{bot}^2 - mgH = 0$

Solve for v_{bot}: $v_{bot} = \sqrt{2gH}$ independently of the shape of the ramp.

Because the block sliding down the circular arc travels a greater distance (an arc length is greater than the length of the chord it defines) but arrives at the bottom of the ramp with the same speed that it had at the bottom of the inclined plane, it will require more time to arrive at the bottom of the arc. $\boxed{(b)\text{ is correct.}}$

9 ••

Determine the Concept No. From the work-kinetic energy theorem, no total work is being done on the rock, as its kinetic energy is constant. However, the rod must exert a tangential force on the rock to keep the speed constant. The effect of this force is to cancel the component of the force of gravity that is tangential to the trajectory of the rock.

Estimation and Approximation

***10** ••

Picture the Problem We'll use the data for the "typical male" described above and assume that he spends 8 hours per day sleeping, 2 hours walking, 8 hours sitting, 1 hour in aerobic exercise, and 5 hours doing moderate physical activity. We can approximate his energy utilization using $E_{activity} = AP_{activity}\Delta t_{activity}$, where A is the surface area of his body, $P_{activity}$ is the rate of energy consumption in a given activity, and $\Delta t_{activity}$ is the time spent in the given activity. His total energy consumption will be the sum of the five terms corresponding to his daily activities.

(*a*) Express the energy consumption of the hypothetical male:

$$E = E_{sleeping} + E_{walking} + E_{sitting} + E_{mod.\,act.} + E_{aerobic\,act.}$$

Evaluate $E_{sleeping}$:

$$\begin{aligned} E_{sleeping} &= AP_{sleeping}\Delta t_{sleeping} \\ &= (2\,m^2)(40\,W/m^2)(8\,h)(3600\,s/h) \\ &= 2.30\times10^6\,J \end{aligned}$$

Evaluate $E_{walking}$:

$$\begin{aligned} E_{walking} &= AP_{walking}\Delta t_{walking} \\ &= (2\,m^2)(160\,W/m^2)(2\,h)(3600\,s/h) \\ &= 2.30\times10^6\,J \end{aligned}$$

Evaluate $E_{sitting}$:

$$\begin{aligned} E_{sitting} &= AP_{sitting}\Delta t_{sitting} \\ &= (2\,m^2)(60\,W/m^2)(8\,h)(3600\,s/h) \\ &= 3.46\times10^6\,J \end{aligned}$$

Evaluate $E_{\text{mod. act.}}$:

$$\begin{aligned} E_{\text{mod. act.}} &= AP_{\text{mod. act.}}\Delta t_{\text{mod. act.}} \\ &= \left(2\,\text{m}^2\right)\left(175\,\text{W/m}^2\right)\left(5\,\text{h}\right)\left(3600\,\text{s/h}\right) \\ &= 6.30\times10^6\ \text{J} \end{aligned}$$

Evaluate $E_{\text{aerobic act.}}$:

$$\begin{aligned} E_{\text{aerobic act.}} &= AP_{\text{aerobic act.}}\Delta t_{\text{aerobic act.}} \\ &= \left(2\,\text{m}^2\right)\left(300\,\text{W/m}^2\right)\left(1\,\text{h}\right)\left(3600\,\text{s/h}\right) \\ &= 2.16\times10^6\ \text{J} \end{aligned}$$

Substitute to obtain:

$$\begin{aligned} E &= 2.30\times10^6\ \text{J} + 2.30\times10^6\ \text{J} + 3.46\times10^6\ \text{J} \\ &\quad + 6.30\times10^6\ \text{J} + 2.16\times10^6\ \text{J} \\ &= \boxed{16.5\times10^6\ \text{J}} \end{aligned}$$

Express the average metabolic rate represented by this energy consumption:

$$P_{\text{av}} = \frac{E}{\Delta t} = \frac{16.5\times10^6\ \text{J}}{\left(24\,\text{h}\right)\left(3600\,\text{s/h}\right)} = \boxed{191\,\text{W}}$$

or about twice that of a 100 W light bulb.

(*b*) Express his average energy consumption in terms of kcal/day:

$$E = \frac{16.5\times10^6\ \text{J/day}}{4190\,\text{J/kcal}} = \boxed{3940\,\text{kcal/day}}$$

(*c*) $\dfrac{3940\,\text{kcal}}{175\,\text{lb}} = 22.5\,\text{kcal/lb}$ is higher than the estimate given in the statement of the problem. However, by adjusting the day's activities, the metabolic rate can vary by more than a factor of 2.

11 •

Picture the Problem The rate at which you expend energy, i.e., do work, is defined as *power* and is the ratio of the work done to the time required to do the work.

Relate the rate at which you can expend energy to the work done in running up the four flights of stairs and solve for your running time:

$$P = \frac{\Delta W}{\Delta t} \Rightarrow \Delta t = \frac{\Delta W}{P}$$

Express the work done in climbing the stairs:

$$\Delta W = mgh$$

Substitute for ΔW to obtain:

$$\Delta t = \frac{mgh}{P}$$

Assuming that your weight is 600 N, evaluate Δt:

$$\Delta t = \frac{(600\,\text{N})(4\times 3.5\,\text{m})}{250\,\text{W}} = \boxed{33.6\,\text{s}}$$

12 •

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(*a*) Relate the rest mass consumed to the energy produced and solve for and evaluate *m*:

$$E_0 = mc^2 \Rightarrow m = \frac{E_0}{c^2} \qquad (1)$$

$$m = \frac{1\,\text{J}}{(2.998\times 10^8\,\text{m/s})^2} = \boxed{1.11\times 10^{-17}\,\text{kg}}$$

(*b*) Express the energy required as a function of the power of the light bulb and evaluate *E*:

$$\begin{aligned} E &= 3Pt \\ &= 3(100\,\text{W})(10\,\text{y}) \\ &\quad\times\left(\frac{365.24\,\text{d}}{\text{y}}\right)\left(\frac{24\,\text{h}}{\text{d}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right) \\ &= 9.47\times 10^{10}\,\text{J} \end{aligned}$$

Substitute in equation (1) to obtain:

$$m = \frac{9.47\times 10^{10}\,\text{J}}{(2.998\times 10^8\,\text{m/s})^2} = \boxed{1.05\,\mu\text{g}}$$

***13** •

Picture the Problem There are about 3×10^8 people in the United States. On the assumption that the average family has 4 people in it and that they own two cars, we have a total of 1.5×10^8 automobiles on the road (excluding those used for industry). We'll assume that each car uses about 15 gal of fuel per week.

Calculate, based on the assumptions identified above, the total annual consumption of energy derived from gasoline:

$$(1.5\times 10^8\,\text{auto})\left(15\frac{\text{gal}}{\text{auto}\cdot\text{week}}\right)\left(52\frac{\text{weeks}}{y}\right)\left(2.6\times 10^8\frac{\text{J}}{\text{gal}}\right) = \boxed{3.04\times 10^{19}\,\text{J/y}}$$

Express this rate of energy use as a fraction of the total annual energy use by the US:

$$\frac{3.04\times 10^{19}\,\text{J/y}}{5\times 10^{20}\,\text{J/y}} \approx \boxed{6\%}$$

Remarks: This is an average power expenditure of roughly $9\text{x}10^{11}$ watt, and a total cost (assuming $1.15 per gallon) of about 140 billion dollars per year.

14 •

Picture the Problem The energy consumption of the U.S. works out to an average power consumption of about 1.6×10^{13} watt. The solar constant is roughly 10^3 W/m^2 (reaching the ground), or about 120 W/m^2 of useful power with a 12% conversion efficiency. Letting P represent the daily rate of energy consumption, we can relate the power available at the surface of the earth to the required area of the solar panels using $P = IA$.

Relate the required area to the electrical energy to be generated by the solar panels:

$$P = IA$$

where I is the solar intensity that reaches the surface of the Earth.

Solve for and evaluate A:

$$A = \frac{P}{I} = \frac{2\left(1.6\times10^{13}\ \text{W}\right)}{120\,\text{W/m}^2}$$
$$= 2.67\times10^{11}\ \text{m}^2$$

where the factor of 2 comes from the fact that the sun is only up for roughly half the day.

Find the side of a square with this area:

$$s = \sqrt{2.67\times10^{11}\ \text{m}^2} = \boxed{516\,\text{km}}$$

Remarks: A more realistic estimate that would include the variation of sunlight over the day and account for latitude and weather variations might very well increase the area required by an order of magnitude.

15 •

Picture the Problem We can relate the energy available from the water in terms of its mass, the vertical distance it has fallen, and the efficiency of the process. Differentiation of this expression with respect to time will yield the rate at which water must pass through its turbines to generate Hoover Dam's annual energy output.

Assuming a total efficiency η, use the expression for the gravitational potential energy near the earth's surface to express the energy available from the water when it has fallen a distance h:

$$E = \eta mgh$$

Differentiate this expression with respect to time to obtain:

$$P = \frac{d}{dt}\left[\eta mgh\right] = \eta gh\frac{dm}{dt} = \eta\rho gh\frac{dV}{dt}$$

Solve for dV/dt:

$$\frac{dV}{dt} = \frac{P}{\eta\rho gh} \qquad (1)$$

Using its definition, relate the dam's annual power output to the energy produced:

$$P = \frac{\Delta E}{\Delta t}$$

Substitute numerical values to obtain:

$$P = \frac{4\times10^9\ \text{kW}\cdot\text{h}}{\left(365.24\,\text{d}\right)\left(24\,\text{h/d}\right)} = 4.57\times10^8\ \text{W}$$

Substitute in equation (1) and evaluate dV/dt:

$$\frac{dV}{dt} = \frac{4.57\times10^{8}\ \text{W}}{0.2(1\,\text{kg/L})(9.81\,\text{m/s}^2)(211\,\text{m})} = \boxed{1.10\times10^{6}\ \text{L/s}}$$

The Conservation of Mechanical Energy

16 •

Picture the Problem The work done in compressing the spring is stored in the spring as potential energy. When the block is released, the energy stored in the spring is transformed into the kinetic energy of the block. Equating these energies will give us a relationship between the compressions of the spring and the speeds of the blocks.

Let the numeral 1 refer to the first case and the numeral 2 to the second case. Relate the compression of the spring in the second case to its potential energy, which equals its initial kinetic energy when released:

$$\tfrac{1}{2}kx_2^2 = \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(4m_1)(3v_1)^2 = 18m_1v_1^2$$

Relate the compression of the spring in the first case to its potential energy, which equals its initial kinetic energy when released:

$$\tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m_1v_1^2$$

or

$$m_1v_1^2 = kx_1^2$$

Substitute to obtain:

$$\tfrac{1}{2}kx_2^2 = 18kx_1^2$$

Solve for x_2:

$$x_2 = \boxed{6x_1}$$

17 •

Picture the Problem Choose the zero of gravitational potential energy to be at the foot of the hill. Then the kinetic energy of the woman on her bicycle at the foot of the hill is equal to her gravitational potential energy when she has reached her highest point on the hill.

Equate the kinetic energy of the rider at the foot of the incline and her gravitational potential energy when she has reached her highest point on the hill and solve for h:

$$\tfrac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v^2}{2g}$$

Relate her displacement along the

$$d = h/\sin\theta$$

incline d to h and the angle of the incline:

Substitute for h to obtain:

$$d\sin\theta = \frac{v^2}{2g}$$

Solve for d:

$$d = \frac{v^2}{2g\sin\theta}$$

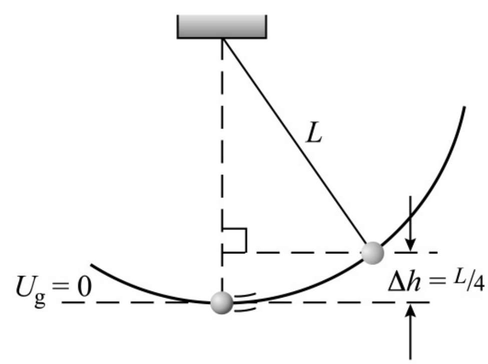
Substitute numerical values and evaluate d:

$$d = \frac{(10\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)\sin 3°} = 97.4\,\text{m}$$

and $\boxed{(c)\text{ is correct.}}$

***18** •

Picture the Problem The diagram shows the pendulum bob in its initial position. Let the zero of gravitational potential energy be at the low point of the pendulum's swing, the equilibrium position. We can find the speed of the bob at it passes through the equilibrium position by equating its initial potential energy to its kinetic energy as it passes through its lowest point.



Equate the initial gravitational potential energy and the kinetic energy of the bob as it passes through its lowest point and solve for v:

$$mg\Delta h = \tfrac{1}{2}mv^2$$

and

$$v = \sqrt{2g\Delta h}$$

Express Δh in terms of the length L of the pendulum:

$$\Delta h = \frac{L}{4}$$

Substitute and simplify:

$$v = \boxed{\sqrt{\frac{gL}{2}}}$$

19 •

Picture the Problem Choose the zero of gravitational potential energy to be at the foot of the ramp. Let the system consist of the block, the earth, and the ramp. Then there are no external forces acting on the system to change its energy and the kinetic energy of the block at the foot of the ramp is equal to its gravitational potential energy when it has reached its highest point.

Relate the gravitational potential energy of the block when it has reached *h*, its highest point on the ramp, to its kinetic energy at the foot of the ramp:

$$mgh = \tfrac{1}{2}mv^2$$

Solve for *h*:

$$h = \frac{v^2}{2g}$$

Relate the displacement *d* of the block along the ramp to *h* and the angle the ramp makes with the horizontal:

$$d = h/\sin\theta$$

Substitute for *h*:

$$d\sin\theta = \frac{v^2}{2g}$$

Solve for *d*:

$$d = \frac{v^2}{2g\sin\theta}$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(7\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)\sin 40°} = \boxed{3.89\ \text{m}}$$

20 •

Picture the Problem Let the system consist of the earth, the block, and the spring. With this choice there are no external forces doing work to change the energy of the system. Let $U_g = 0$ at the elevation of the spring. Then the initial gravitational potential energy of the 3-kg object is transformed into kinetic energy as it slides down the ramp and then, as it compresses the spring, into potential energy stored in the spring.

(*a*) Apply conservation of energy to relate the distance the spring is compressed to the initial potential energy of the block:

$$W_{ext} = \Delta K + \Delta U = 0$$

and, because $\Delta K = 0$,

$$-mgh + \tfrac{1}{2}kx^2 = 0$$

Solve for *x*:

$$x = \sqrt{\frac{2mgh}{k}}$$

Substitute numerical values and evaluate x:

$$x=\sqrt{\frac{2(3\,\text{kg})(9.81\,\text{m/s}^2)(5\,\text{m})}{400\,\text{N/m}}}=\boxed{0.858\,\text{m}}$$

(b) The energy stored in the compressed spring will accelerate the block, launching it back up the incline:

The block will retrace its path, rising to a height of 5 m.

21 •

Picture the Problem With U_g chosen to be zero at the uncompressed level of the spring, the ball's initial gravitational potential energy is negative. The difference between the initial potential energy of the spring and the gravitational potential energy of the ball is first converted into the kinetic energy of the ball and then into gravitational potential energy as the ball rises and slows … eventually coming momentarily to rest.

Apply the conservation of energy to the system as it evolves from its initial to its final state:

$$-mgx+\tfrac{1}{2}kx^2=mgh$$

Solve for h:

$$h=\frac{kx^2}{2mg}-x$$

Substitute numerical values and evaluate h:

$$h=\frac{(600\,\text{N/m})(0.05\,\text{m})^2}{2(0.015\,\text{kg})(9.81\,\text{m/s}^2)}-0.05\,\text{m}=\boxed{5.05\,\text{m}}$$

22 •

Picture the Problem Let the system include the earth and the container. Then the work done by the crane is done by an external force and this work changes the energy of the system. Because the initial and final speeds of the container are zero, the initial and final kinetic energies are zero and the work done by the crane equals the change in the gravitational potential energy of the container. Choose $U_g = 0$ to be at the level of the deck of the freighter.

Apply conservation of energy to the system:

$$W_{ext}=\Delta E_{sys}=\Delta K+\Delta U$$

Because $\Delta K = 0$:

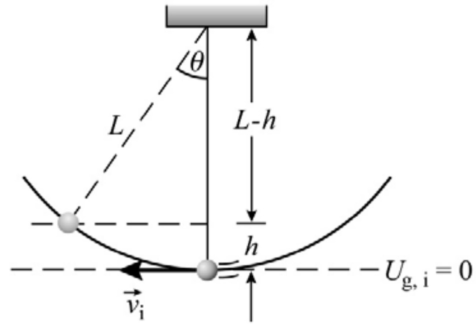
$$W_{ext}=\Delta U=mg\Delta h$$

Evaluate the work done by the crane:

$$W_{ext} = mg\Delta h = (4000\,\text{kg})(9.81\,\text{m/s}^2)(-8\,\text{m}) = \boxed{-314\,\text{kJ}}$$

23 •

Picture the Problem Let the system consist of the earth and the child. Then $W_{ext} = 0$. Choose $U_{g,i} = 0$ at the child's lowest point as shown in the diagram to the right. Then the child's initial energy is entirely kinetic and its energy when it is at its highest point is entirely gravitational potential. We can determine h from energy conservation and then use trigonometry to determine θ.



Using the diagram, relate θ to h and L:

$$\theta = \cos^{-1}\left(\frac{L-h}{L}\right) = \cos^{-1}\left(1-\frac{h}{L}\right)$$

Apply conservation of energy to the system to obtain:

$$\tfrac{1}{2}mv_i^2 - mgh = 0$$

Solve for h:

$$h = \frac{v_i^2}{2g}$$

Substitute to obtain:

$$\theta = \cos^{-1}\left(1-\frac{v_i^2}{2gL}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left(1-\frac{(3.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(6\,\text{m})}\right) = \boxed{25.6°}$$

***24** ••

Picture the Problem Let the system include the two objects and the earth. Then $W_{ext} = 0$. Choose $U_g = 0$ at the elevation at which the two objects meet. With this choice, the initial potential energy of the 3-kg object is positive and that of the 2-kg object is negative. Their sum, however, is positive. Given our choice for $U_g = 0$, this initial potential energy is transformed entirely into kinetic energy.

Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U_g = 0$$

or, because $W_{ext} = 0$,

$$\Delta K = -\Delta U_g$$

Substitute for ΔK and solve for v_f; noting that m represents the sum of the masses of the objects as they are both moving in the final state:

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -\Delta U_g$$

or, because $v_i = 0$,

$$v_f = \sqrt{\frac{-2\Delta U_g}{m}}$$

Express and evaluate ΔU_g:

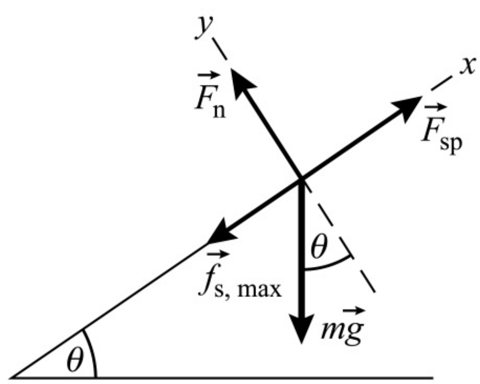
$$\begin{aligned}\Delta U_g &= U_{g,f} - U_{g,i} \\ &= 0 - (3\,\text{kg} - 2\,\text{kg})(0.5\,\text{m}) \\ &\quad \times (9.81\,\text{m/s}^2) \\ &= -4.91\,\text{J}\end{aligned}$$

Substitute and evaluate v_f:

$$v_f = \sqrt{\frac{-2(-4.91\,\text{J})}{5\,\text{kg}}} = \boxed{1.40\,\text{m/s}}$$

25 ••

Picture the Problem The free-body diagram shows the forces acting on the block when it is about to move. F_{sp} is the force exerted by the spring and, because the block is on the verge of sliding, $f_s = f_{s,max}$. We can use Newton's 2nd law, under equilibrium conditions, to express the elongation of the spring as a function of m, k and θ and then substitute in the expression for the potential energy stored in a stretched or compressed spring.



Express the potential energy of the spring when the block is about to move:

$$U = \tfrac{1}{2}kx^2$$

Apply $\sum \vec{F} = m\vec{a}$, under equilibrium conditions, to the block:

$$\sum F_x = F_{sp} - f_{s,max} - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Using $f_{s,max} = \mu_s F_n$ and $F_{sp} = kx$, eliminate $f_{s,max}$ and F_{sp} from the x equation and solve for x:

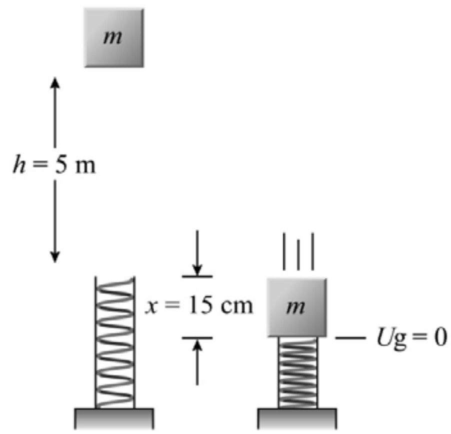
$$x = \frac{mg(\sin\theta + \mu_s\cos\theta)}{k}$$

Substitute for x in the expression for U:

$$U = \tfrac{1}{2}k\left[\frac{mg(\sin\theta + \mu_s\cos\theta)}{k}\right]^2 = \boxed{\frac{[mg(\sin\theta + \mu_s\cos\theta)]^2}{2k}}$$

26 ••

Picture the Problem The mechanical energy of the system, consisting of the block, the spring, and the earth, is initially entirely gravitational potential energy. Let $U_g = 0$ where the spring is compressed 15 cm. Then the mechanical energy when the compression of the spring is 15 cm will be partially kinetic and partially stored in the spring. We can use conservation of energy to relate the initial potential energy of the system to the energy stored in the spring and the kinetic energy of block when it has compressed the spring 15 cm.



Apply conservation of energy to the system:

$$\Delta U + \Delta K = 0$$

or

$$U_{g,f} - U_{g,i} + U_{s,f} - U_{s,i} + K_f - K_i = 0$$

Because $U_{g,f} = U_{s,I} = K_i = 0$:

$$-U_{g,i} + U_{s,f} + K_f = 0$$

Substitute to obtain:

$$-mg(h + x) + \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = 0$$

Solve for v:

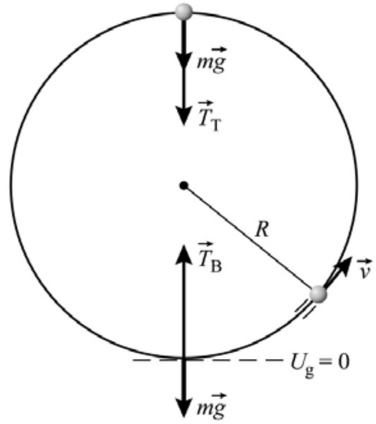
$$v = \sqrt{2g(h + x) - \frac{kx^2}{m}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(5\,\text{m} + 0.15\,\text{m}) - \frac{(3955\,\text{N/m})(0.15\,\text{m})^2}{2.4\,\text{kg}}} = \boxed{8.00\,\text{m/s}}$$

***27 ••**

Picture the Problem The diagram represents the ball traveling in a circular path with constant energy. U_g has been chosen to be zero at the lowest point on the circle and the superimposed free-body diagrams show the forces acting on the ball at the top and bottom of the circular path. We'll apply Newton's 2nd law to the ball at the top and bottom of its path to obtain a relationship between T_T and T_B and the conservation of mechanical energy to relate the speeds of the ball at these two locations.



Apply $\sum F_{radial} = ma_{radial}$ to the ball at the bottom of the circle and solve for T_B:

$$T_B - mg = m\frac{v_B^2}{R}$$

and

$$T_B = mg + m\frac{v_B^2}{R} \quad (1)$$

Apply $\sum F_{radial} = ma_{radial}$ to the ball at the top of the circle and solve for T_T:

$$T_T + mg = m\frac{v_T^2}{R}$$

and

$$T_T = -mg + m\frac{v_T^2}{R} \quad (2)$$

Subtract equation (2) from equation (1) to obtain:

$$T_B - T_T = mg + m\frac{v_B^2}{R} - \left(-mg + m\frac{v_T^2}{R}\right) = m\frac{v_B^2}{R} - m\frac{v_T^2}{R} + 2mg \quad (3)$$

Using conservation of energy, relate the mechanical energy of the ball at the bottom of its path to its mechanical energy at the top of the circle and solve for $m\frac{v_B^2}{R} - m\frac{v_T^2}{R}$:

$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_T^2 + mg(2R)$$

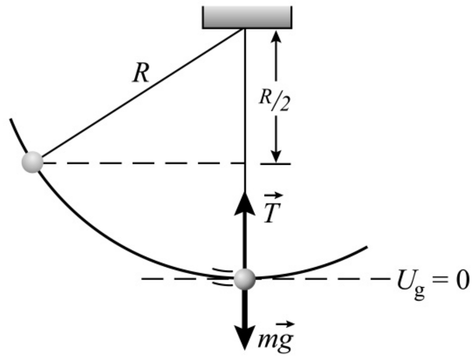
$$m\frac{v_B^2}{R} - m\frac{v_T^2}{R} = 4mg$$

Substitute in equation (3) to obtain:

$$T_B - T_T = \boxed{6mg}$$

28 ••

Picture the Problem Let $U_g = 0$ at the lowest point in the girl's swing. Then we can equate her initial potential energy to her kinetic energy as she passes through the low point on her swing to relate her speed v to R. The FBD show the forces acting on the girl at the low point of her swing. Applying Newton's 2nd law to her will allow us to establish the relationship between the tension T and her speed.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the girl at her lowest point and solve for T:

$$T - mg = m\frac{v^2}{R}$$

and

$$T = mg + m\frac{v^2}{R}$$

Equate the girl's initial potential energy to her final kinetic energy and solve for $\frac{v^2}{R}$:

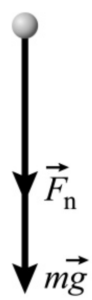
$$mg\frac{R}{2} = \tfrac{1}{2}mv^2 \Rightarrow \frac{v^2}{R} = g$$

Substitute for v^2/R^2 and simplify to obtain:

$$T = mg + mg = \boxed{2mg}$$

29 ••

Picture the Problem The free-body diagram shows the forces acting on the car when it is upside down at the top of the loop. Choose $U_g = 0$ at the bottom of the loop. We can express F_n in terms of v and R by apply Newton's 2nd law to the car and then obtain a second expression in these same variables by applying the conservation of mechanical energy. The simultaneous solution of these equations will yield an expression for F_n in terms of known quantities.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the car at the top of the circle and solve for F_n:

$$F_n + mg = m\frac{v^2}{R}$$

and

$$F_n = m\frac{v^2}{R} - mg \qquad (1)$$

Using conservation of energy, relate the energy of the car at the beginning of its motion to its energy when it is at the top of the loop:

$$mgH = \tfrac{1}{2}mv^2 + mg(2R)$$

Solve for $m\dfrac{v^2}{R}$:

$$m\frac{v^2}{R} = 2mg\left(\frac{H}{R} - 2\right) \qquad (2)$$

Substitute equation (2) in equation (1) to obtain:

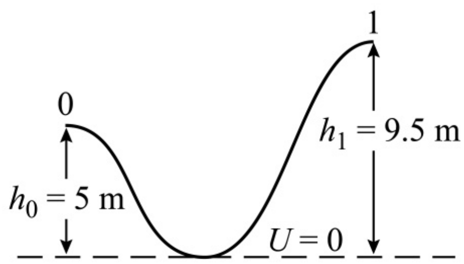
$$F_n = 2mg\left(\frac{H}{R} - 2\right) - mg = mg\left(\frac{2H}{R} - 5\right)$$

Substitute numerical values and evaluate F_n:

$$F_n = (1500\,\text{kg})(9.81\,\text{m/s}^2)\left[\frac{2(23\,\text{m})}{7.5\,\text{m}} - 5\right] = 1.67\times10^4\ \text{N} \Rightarrow \boxed{(c)\ \text{is correct.}}$$

30 •

Picture the Problem Let the system include the roller coaster, the track, and the earth and denote the starting position with the numeral 0 and the top of the second hill with the numeral 1. We can use the work-energy theorem to relate the energies of the coaster at its initial and final positions.



(*a*) Use conservation of energy to relate the work done by external forces to the change in the energy of the system:

$$W_{ext} = \Delta E_{sys} = \Delta K + \Delta U$$

Because the track is frictionless, $W_{ext} = 0$:

$$\Delta K + \Delta U = 0$$

and

$$K_1 - K_0 + U_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh_1 - mgh_0 = 0$$

Solve for v_0:

$$v_0 = \sqrt{v_1^2 + 2g(h_1 - h_0)}$$

If the coaster just makes it to the top of the second hill, $v_1 = 0$ and:

$$v_0 = \sqrt{2g(h_1 - h_0)}$$

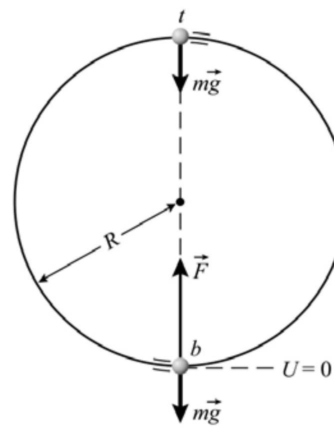
Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{2(9.81\,\text{m/s}^2)(9.5\,\text{m} - 5\,\text{m})} = \boxed{9.40\,\text{m/s}}$$

(*b*) No. Note that the required speed depends only on the difference in the heights of the two hills.

31 ••

Picture the Problem Let the radius of the loop be R and the mass of one of the riders be m. At the top of the loop, the centripetal force on her is her weight (the force of gravity). The two forces acting on her at the bottom of the loop are the normal force exerted by the seat of the car, pushing up, and the force of gravity, pulling down. We can apply Newton's 2nd law to her at both the top and bottom of the loop to relate the speeds at those locations to m and R and, at b, to F, and then use conservation of energy to relate v_t and v_b.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the rider at the bottom of the circular arc:

$$F - mg = m\frac{v_{\text{b}}^2}{R}$$

Solve for F to obtain:

$$F = mg + m\frac{v_{\text{b}}^2}{R} \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the rider at the top of the circular arc:

$$mg = m\frac{v_{\text{t}}^2}{R}$$

Solve for v_{t}^2:

$$v_{\text{t}}^2 = gR$$

Use conservation of energy to relate the energies of the rider at the top and bottom of the arc:

$$K_{\text{b}} - K_{\text{t}} + U_{\text{b}} - U_{\text{t}} = 0$$

or, because $U_{\text{b}} = 0$,

$$K_{\text{b}} - K_{\text{t}} - U_{\text{t}} = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_{\text{b}}^2 - \tfrac{1}{2}mv_{\text{t}}^2 - 2mgR = 0$$

Solve for v_{b}^2:

$$v_b^2 = 5gR$$

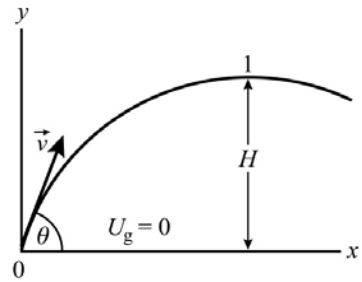
Substitute in equation (1) to obtain:

$$F = mg + m\frac{5gR}{R} = \boxed{6mg}$$

i.e., the rider will feel six times heavier than her normal weight.

***32 ••**

Picture the Problem Let the system consist of the stone and the earth and ignore the influence of air resistance. Then $W_{ext} = 0$. Choose $U_g = 0$ as shown in the figure. Apply the law of the conservation of mechanical energy to describe the energy transformations as the stone rises to the highest point of its trajectory.



Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

and

$$K_1 - K_0 + U_1 - U_0 = 0$$

Because $U_0 = 0$:

$$K_1 - K_0 + U_1 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_x^2 - \tfrac{1}{2}mv^2 + mgH = 0$$

In the absence of air resistance, the horizontal component of $\vec{v}$ is constant and equal to $v_x = v\cos\theta$. Hence:

$$\tfrac{1}{2}m(v\cos\theta)^2 - \tfrac{1}{2}mv^2 + mgH = 0$$

Solve for v:

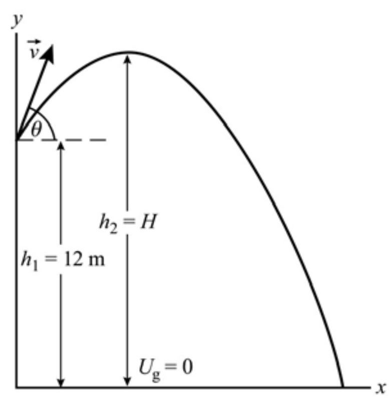
$$v = \sqrt{\frac{2gH}{1-\cos^2\theta}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9.81\,\text{m/s}^2)(24\,\text{m})}{1-\cos^2 53°}} = \boxed{27.2\,\text{m/s}}$$

33 ••

Picture the Problem Let the system consist of the ball and the earth. Then $W_{ext} = 0$. The figure shows the ball being thrown from the roof of a building. Choose $U_g = 0$ at ground level. We can use the conservation of mechanical energy to determine the maximum height of the ball and its speed at impact with the ground. We can use the definition of the work done by gravity to calculate how much work was done by gravity as the ball rose to its maximum height.



(*a*) Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_2 - K_1 + U_2 - U_1 = 0$$

Substitute for the energies to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgh_2 - mgh_1 = 0$$

Note that, at point 2, the ball is moving horizontally and:

$$v_2 = v_1 \cos\theta$$

Substitute for v_2 and h_2:

$$\tfrac{1}{2}m(v_1 \cos\theta)^2 - \tfrac{1}{2}mv_1^2 + mgH - mgh_1 = 0$$

Solve for H:

$$H = h_1 - \frac{v^2}{2g}\left(\cos^2\theta - 1\right)$$

Substitute numerical values and evaluate H:

$$H = 12\,\text{m} - \frac{(30\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\left(\cos^2 40° - 1\right) = \boxed{31.0\,\text{m}}$$

(*b*) Using its definition, express the work done by gravity:

$$W_g = -\Delta U = -\left(U_H - U_{h_i}\right) = -\left(mgH - mgh_i\right) = -mg\left(H - h_i\right)$$

Substitute numerical values and evaluate W_g:

$$W_g = -(0.17\,\text{kg})(9.81\,\text{m/s}^2)(31\,\text{m} - 12\,\text{m}) = \boxed{-31.7\,\text{J}}$$

(*c*) Relate the initial mechanical energy of the ball to its just-before-impact energy:

$$\tfrac{1}{2}mv_i^2 + mgh_i = \tfrac{1}{2}mv_f^2$$

Solve for v_f:

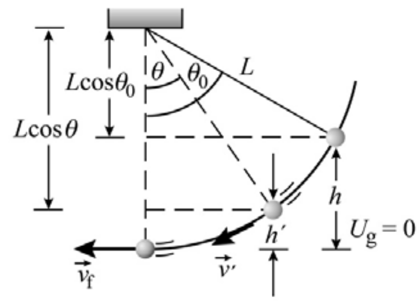
$$v_f = \sqrt{v_i^2 + 2gh_i}$$

Substitute numerical values and evaluate v_f

$$v_f = \sqrt{(30\,\text{m/s})^2 + 2(9.81\,\text{m/s}^2)(12\,\text{m})} = \boxed{33.7\,\text{m/s}}$$

34 ••

Picture the Problem The figure shows the pendulum bob in its release position and in the two positions in which it is in motion with the given speeds. Choose $U_g = 0$ at the low point of the swing. We can apply the conservation of mechanical energy to relate the two angles of interest to the speeds of the bob at the intermediate and low points of its trajectory.



(*a*) Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

where U_f and K_i equal zero.

$$\therefore K_f - U_i = 0$$

Express U_i:

$$U_i = mgh = mgL(1-\cos\theta_0)$$

Substitute for K_f and U_i:

$$\tfrac{1}{2}mv_f^2 - mgL(1-\cos\theta_0) = 0$$

Solve for θ_0:

$$\theta_0 = \cos^{-1}\left(1-\frac{v^2}{2gL}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \cos^{-1}\left[1-\frac{(2.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.8\,\text{m})}\right]$$

$$= \boxed{60.0°}$$

(*b*) Letting primed quantities describe the indicated location, use the law of the conservation of mechanical energy to relate the speed of the bob at this point to θ:

$$K_f' - K_i + U_f' - U_i = 0$$

where $K_i = 0$.

$$\therefore K_f' + U_f' - U_i = 0$$

Express U_f':

$$U_f' = mgh' = mgL(1-\cos\theta)$$

Substitute for K_f', U_f' and U_i:

$$\tfrac{1}{2}m(v_f')^2 + mgL(1-\cos\theta) - mgL(1-\cos\theta_0) = 0$$

Solve for θ:

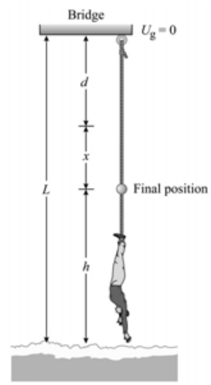
$$\theta = \cos^{-1}\left[\frac{(v_f')^2}{2gL} + \cos\theta_0\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(1.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.8\,\text{m})} + \cos 60°\right] = \boxed{51.3°}$$

***35** ••

Picture the Problem Choose $U_g = 0$ at the bridge, and let the system be the earth, the jumper and the bungee cord. Then $W_{ext} = 0$. Use the conservation of mechanical energy to relate to relate her initial and final gravitational potential energies to the energy stored in the stretched bungee, U_s cord. In part (*b*), we'll use a similar strategy but include a kinetic energy term because we are interested in finding her maximum speed.



(*a*) Express her final height *h* above the water in terms of *L*, *d* and the distance *x* the bungee cord has stretched:

$$h = L - d - x \qquad (1)$$

Use the conservation of mechanical energy to relate her gravitational potential energy as she just touches the water to the energy stored in the stretched bungee cord:

$$W_{ext} = \Delta K + \Delta U = 0$$

Because $\Delta K = 0$ and $\Delta U = \Delta U_g + \Delta U_s$,

$$-mgL + \tfrac{1}{2}kx^2 = 0,$$

where x is the maximum distance the bungee cord has stretched.

Solve for *k*:

$$k = \frac{2mgL}{x^2}$$

Find the maximum distance the bungee cord stretches:

$x = 310\text{ m} - 50\text{ m} = 260\text{ m}.$

Evaluate *k*:

$$k = \frac{2(60\,\text{kg})(9.81\,\text{m/s}^2)(310\,\text{m})}{(260\,\text{m})^2} = 5.40\,\text{N/m}$$

Express the relationship between the forces acting on her when she has finally come to rest and solve for x:

$$F_{\text{net}} = kx - mg = 0$$

and

$$x = \frac{mg}{k}$$

Evaluate x:

$$x = \frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}} = 109\,\text{m}$$

Substitute in equation (1) and evaluate h:

$$h = 310\,\text{m} - 50\,\text{m} - 109\,\text{m} = \boxed{151\,\text{m}}$$

(b) Using conservation of energy, express her total energy E:

$$E = K + U_{\text{g}} + U_{\text{s}} = E_{\text{i}} = 0$$

Because v is a maximum when K is a maximum, solve for K::

$$K = -U_{\text{g}} - U_{\text{s}} = mg(d + x) - \tfrac{1}{2}kx^2 \qquad (1)$$

Use the condition for an extreme value to obtain:

$$\frac{dK}{dx} = mg - kx = 0 \text{ for extreme values}$$

Solve for and evaluate x:

$$x = \frac{mg}{k} = \frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}} = 109\,\text{m}$$

From equation (1) we have:

$$\tfrac{1}{2}mv^2 = mg(d + x) - \tfrac{1}{2}kx^2$$

Solve for v to obtain:

$$v = \sqrt{2g(d + x) - \frac{kx^2}{m}}$$

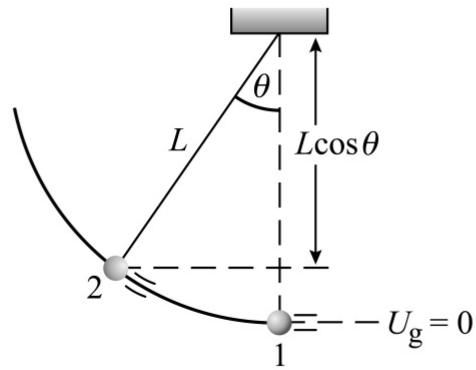
Substitute numerical values and evaluate v for x = 109 m:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m} + 109\,\text{m}) - \frac{(5.4\,\text{N/m})(109\,\text{m})^2}{60\,\text{kg}}} = \boxed{45.3\,\text{m/s}}$$

Because $\dfrac{d^2K}{dx^2} = -k < 0$, x = 109 m corresponds to $K_{\max}$ and so v is a maximum.

36 ••

Picture the Problem Let the system be the earth and pendulum bob. Then $W_{ext} = 0$. Choose $U_g = 0$ at the low point of the bob's swing and apply the law of the conservation of mechanical energy to its motion. When the bob reaches the 30° position its energy will be partially kinetic and partially potential. When it reaches its maximum height, its energy will be entirely potential. Applying Newton's 2nd law will allow us to express the tension in the string as a function of the bob's speed and its angular position.



(*a*) Apply conservation of energy to relate the energies of the bob at points 1 and 2:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_2 - K_1 + U_2 - U_1 = 0$$

Because $U_1 = 0$:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + U_2 = 0$$

Express U_2:

$$U_2 = mgL(1 - \cos\theta)$$

Substitute for U_2 to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgL(1 - \cos\theta) = 0$$

Solve for v_2:

$$v_2 = \sqrt{v_1^2 - 2gL(1 - \cos\theta)}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{(4.5\,\text{m/s})^2 - 2(9.81\,\text{m/s}^2)(3\,\text{m})(1 - \cos 30°)} = \boxed{3.52\,\text{m/s}}$$

(*b*) From (*a*) we have:

$$U_2 = mgL(1 - \cos\theta)$$

Substitute numerical values and evaluate U_2:

$$U_2 = (2\,\text{kg})(9.81\,\text{m/s}^2)(3\,\text{m})(1 - \cos 30°) = \boxed{7.89\,\text{J}}$$

(*c*) Apply $\sum F_{radial} = ma_{radial}$ to the bob to obtain:

$$T - mg\cos\theta = m\frac{v_2^2}{L}$$

Solve for T:

$$T = m\left(g\cos\theta + \frac{v_2^2}{L}\right)$$

Substitute numerical values and evaluate T:

$$T = (2\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos 30° + \frac{(3.52\,\text{m/s})^2}{3\,\text{m}}\right] = \boxed{25.3\,\text{N}}$$

(*d*) When the bob reaches its greatest height:

$$U = U_{max} = mgL(1 - \cos\theta_{max})$$

and

$$K_1 + U_{max} = 0$$

Substitute for K_1 and U_{max}

$$-\tfrac{1}{2}mv_1^2 + mgL(1 - \cos\theta_{max}) = 0$$

Solve for θ_{max}:

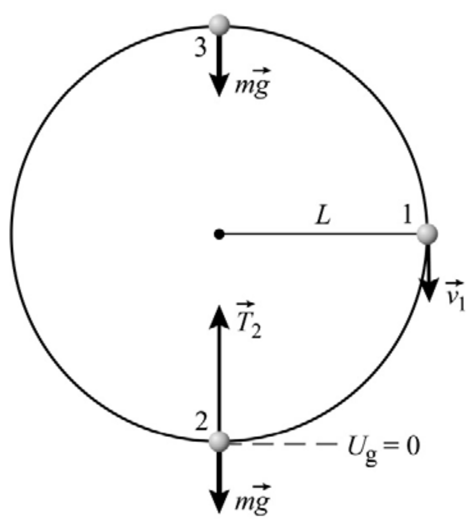
$$\theta_{max} = \cos^{-1}\left(1 - \frac{v_1^2}{2gL}\right)$$

Substitute numerical values and evaluate θ_{max}:

$$\theta_{max} = \cos^{-1}\left[1 - \frac{(4.5\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(3\,\text{m})}\right] = \boxed{49.0°}$$

37 ••

Picture the Problem Let the system consist of the earth and pendulum bob. Then $W_{ext} = 0$. Choose $U_g = 0$ at the bottom of the circle and let points 1, 2 and 3 represent the bob's initial point, lowest point and highest point, respectively. The bob will gain speed and kinetic energy until it reaches point 2 and slow down until it reaches point 3; so it has its maximum kinetic energy when it is at point 2. We can use Newton's 2nd law at points 2 and 3 in conjunction with the law of the conservation of mechanical energy to find the maximum kinetic energy of the bob and the tension in the string when the bob has its maximum kinetic energy.



(*a*) Apply $\sum F_{radial} = ma_{radial}$ to the bob at the top of the circle and solve for v_3^2:

$$mg = m\frac{v_3^2}{L}$$

and

$$v_3^2 = gL$$

Use conservation of energy to express the relationship between K_2, K_3 and U_3 and solve for K_2:

$K_3 - K_2 + U_3 - U_2 = 0$ where $U_2 = 0$

Therefore,

$$K_2 = K_{max} = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mg(2L)$$

Substitute for v_3^2 and simplify to obtain:

$$K_{max} = \tfrac{1}{2}m(gL) + 2mgL = \boxed{\tfrac{5}{2}mgL}$$

(*b*) Apply $\sum F_{radial} = ma_{radial}$ to the bob at the bottom of the circle and solve for T_2:

$$F_{net} = T_2 - mg = m\frac{v_2^2}{L}$$

and

$$T_2 = mg + m\frac{v_2^2}{L} \qquad (1)$$

Use conservation of energy to relate the energies of the bob at points 2 and 3 and solve for K_2:

$K_3 - K_2 + U_3 - U_2 = 0$ where $U_2 = 0$

$$K_2 = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mg(2L)$$

Substitute for v_3^2 and K_2 and solve for v_2^2:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}m(gL) + mg(2L)$$

and

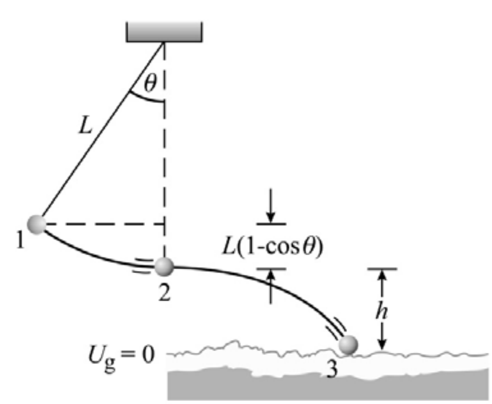
$$v_2^2 = 5gL$$

Substitute in equation (1) to obtain:

$$T_2 = \boxed{6mg}$$

38 ••

Picture the Problem Let the system consist of the earth and child. Then $W_{ext} = 0$. In the figure, the child's initial position is designated with the numeral 1; the point at which the child releases the rope and begins to fall with a 2, and its point of impact with the water is identified with a 3. Choose $U_g = 0$ at the water level. While one could use the law of the conservation of energy between points 1 and 2 and then between points 2 and 3, it is more direct to consider the energy transformations between points 1 and 3. Given our choice of the zero of gravitational potential energy, the initial potential energy at point 1 is transformed into kinetic energy at point 3.



Apply conservation of energy to the energy transformations between points 1 and 3:

$$W_{ext} = \Delta K + \Delta U = 0$$
$$K_3 - K_1 + U_3 - U_1 = 0$$
where U_3 and K_1 are zero.

Substitute for K_3 and U_1;

$$\tfrac{1}{2}mv_3^2 - mg[h + L(1-\cos\theta)] = 0$$

Solve for v_3:

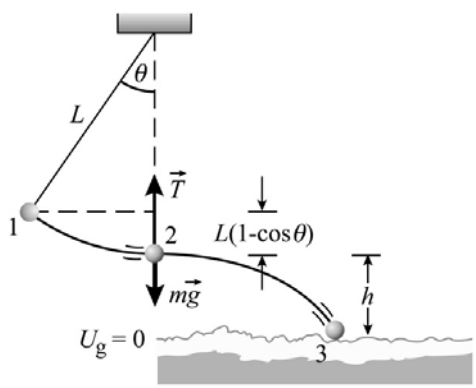
$$v_3 = \sqrt{2g[h + L(1-\cos\theta)]}$$

Substitute numerical values and evaluate v_3:

$$v_3 = \sqrt{2(9.81\,\text{m/s}^2)[3.2\,\text{m} + (10.6\,\text{m})(1-\cos 23°)]} = \boxed{8.91\,\text{m/s}}$$

***39 ••**

Picture the Problem Let the system consist of you and the earth. Then there are no external forces to do work on the system and W_{ext} = 0. In the figure, your initial position is designated with the numeral 1, the point at which you release the rope and begin to fall with a 2, and your point of impact with the water is identified with a 3. Choose U_g = 0 at the water level. We can apply Newton's 2nd law to the forces acting on you at point 2 and apply conservation of energy between points 1 and 2 to determine the maximum angle at which you can begin your swing and then between points 1 and 3 to determine the speed with which you will hit the water.



(*a*) Use conservation of energy to relate your speed at point 2 to your potential energy there and at point 1:

$$W_{ext} = \Delta K + \Delta U = 0$$
or
$$K_2 - K_1 + U_2 - U_1 = 0$$

Because K_1 = 0:

$$\tfrac{1}{2}mv_2^2 + mgh - [mgL(1-\cos\theta) + mgh] = 0$$

Solve this equation for θ:

$$\theta = \cos^{-1}\left[1 - \frac{v_2^2}{2gL}\right] \qquad (1)$$

Apply $\sum F_{radial} = ma_{radial}$ to

$$T - mg = m\frac{v_2^2}{L}$$

yourself at point 2 and solve for T:

and

$$T = mg + m\frac{v_2^2}{L}$$

Because you've estimated that the rope might break if the tension in it exceeds your weight by 80 N, it must be that:

$$m\frac{v_2^2}{L} = 80\,\text{N}$$

or

$$v_2^2 = \frac{(80\text{ N})L}{m}$$

Let's assume your weight is 650 N. Then your mass is 66.3 kg and:

$$v_2^2 = \frac{(80\text{ N})(4.6\,\text{m})}{66.3\text{kg}} = 5.55\,\text{m}^2/\text{s}^2$$

Substitute numerical values in equation (1) to obtain:

$$\theta = \cos^{-1}\left[1 - \frac{5.55\,\text{m}^2/\text{s}^2}{2(9.81\,\text{m/s}^2)(4.6\,\text{m})}\right] = \boxed{20.2°}$$

(*b*) Apply conservation of energy to the energy transformations between points 1 and 3:

$$W_{\text{ext}} = \Delta K + \Delta U = 0$$

$K_3 - K_1 + U_3 - U_1 = 0$ where U_3 and K_1 are zero

Substitute for K_3 and U_1 to obtain:

$$\tfrac{1}{2}mv_3^2 - mg[h + L(1 - \cos\theta)] = 0$$

Solve for v_3:

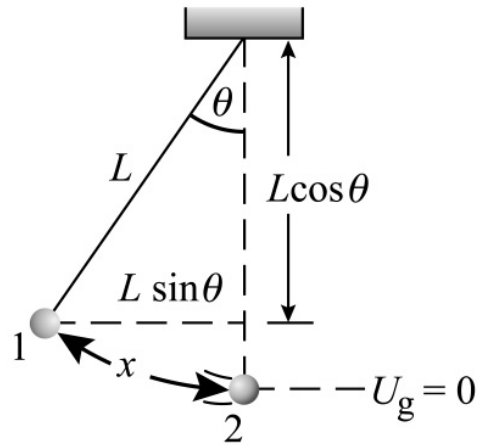
$$v_3 = \sqrt{2g[h + L(1 - \cos\theta)]}$$

Substitute numerical values and evaluate v_3:

$$v_3 = \sqrt{2(9.81\,\text{m/s}^2)[1.8\,\text{m} + (4.6\,\text{m})(1 - \cos 20.2°)]} = \boxed{6.39\,\text{m/s}}$$

40 ••

Picture the Problem Choose $U_g = 0$ at point 2, the lowest point of the bob's trajectory and let the system consist of the bob and the earth. Given this choice, there are no external forces doing work on the system. Because $\theta << 1$, we can use the trigonometric series for the sine and cosine functions to approximate these functions. The bob's initial energy is partially gravitational potential and partially potential energy stored in the stretched spring. As the bob swings down to point 2 this energy is transformed into kinetic energy. By equating these energies, we can derive an expression for the speed of the bob at point 2.



Apply conservation of energy to the system as the pendulum bob swings from point 1 to point 2:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kx^2 + mgL(1-\cos\theta)$$

Note, from the figure, that $x \approx L\sin\theta$ when $\theta << 1$:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}k(L\sin\theta)^2 + mgL(1-\cos\theta)$$

Also, when $\theta << 1$:

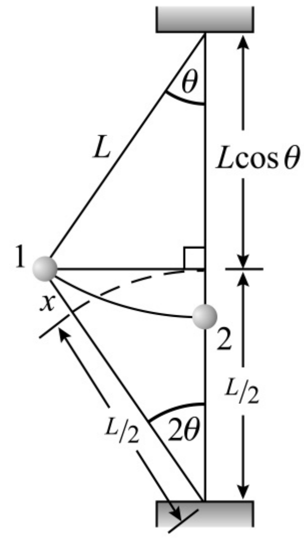
$$\sin\theta \approx \theta \text{ and } \cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

Substitute, simplify and solve for v_2:

$$v_2 = \boxed{L\theta\sqrt{\frac{k}{m} + \frac{g}{L}}}$$

41 •••

Picture the Problem Choose $U_g = 0$ at point 2, the lowest point of the bob's trajectory and let the system consist of the earth, ceiling, spring, and pendulum bob. Given this choice, there are no external forces doing work to change the energy of the system. The bob's initial energy is partially gravitational potential and partially potential energy stored in the stretched spring. As the bob swings down to point 2 this energy is transformed into kinetic energy. By equating these energies, we can derive an expression for the speed of the bob at point 2.



Apply conservation of energy to the system as the pendulum bob swings from point 1 to point 2:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kx^2 + mgL(1-\cos\theta) \quad (1)$$

Apply the Pythagorean theorem to the lower triangle in the diagram to obtain:

$$\left(x+\tfrac{1}{2}L\right)^2 = L^2\left[\sin^2\theta + \left(\tfrac{3}{2}\cos\theta\right)^2\right] = L^2\left[\sin^2\theta + \tfrac{9}{4} - 3\cos\theta + \cos^2\theta\right] = L^2\left(\tfrac{13}{4} - 3\cos\theta\right)$$

Take the square root of both sides of the equation to obtain:

$$x + \tfrac{1}{2}L = L\sqrt{\left(\tfrac{13}{4} - 3\cos\theta\right)}$$

Solve for x:

$$x = L\left[\sqrt{\left(\tfrac{13}{4} - 3\cos\theta\right)} - \tfrac{1}{2}\right]$$

Substitute for x in equation (1):

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kL^2\left[\sqrt{\left(\tfrac{13}{4} - 3\cos\theta\right)} - \tfrac{1}{2}\right]^2 + mgL(1-\cos\theta)$$

Solve for v_2^2 to obtain:

$$v_2^2 = 2gL(1-\cos\theta) + \frac{k}{m}L^2\left[\sqrt{\tfrac{13}{4} - 3\cos\theta} - \tfrac{1}{2}\right]^2$$

$$= L^2\left[2\frac{g}{L}(1-\cos\theta) + \frac{k}{m}\left(\sqrt{\tfrac{13}{4} - 3\cos\theta} - \tfrac{1}{2}\right)^2\right]$$

Finally, solve for v_2:

$$v_2 = \boxed{L\sqrt{2\frac{g}{L}(1-\cos\theta)+\frac{k}{m}\left(\sqrt{\tfrac{13}{4}-3\cos\theta}-\tfrac{1}{2}\right)^2}}$$

The Conservation of Energy

42 •

Picture the Problem The energy of the eruption is initially in the form of the kinetic energy of the material it thrusts into the air. This energy is then transformed into gravitational potential energy as the material rises.

(*a*) Express the energy of the eruption in terms of the height Δh to which the debris rises:

$$E = mg\Delta h$$

Relate the density of the material to its mass and volume:

$$\rho = \frac{m}{V}$$

Substitute for m to obtain:

$$E = \rho Vg\Delta h$$

Substitute numerical values and evaluate E:

$$E = (1600\,\text{kg/m}^3)(4\,\text{km}^3)(9.81\,\text{m/s}^2)(500\,\text{m}) = \boxed{3.14\times10^{16}\,\text{J}}$$

(*b*) Convert 3.13×10^{16} J to megatons of TNT:

$$3.14\times10^{16}\,\text{J} = 3.14\times10^{16}\,\text{J}\times\frac{1\,\text{Mton TNT}}{4.2\times10^{15}\,\text{J}} = \boxed{7.48\,\text{Mton TNT}}$$

43 ••

Picture the Problem The work done by the student equals the change in his/her gravitational potential energy and is done as a result of the transformation of metabolic energy in the climber's muscles.

(*a*) The increase in gravitational potential energy is:

$$\Delta U = mg\Delta h = (80\,\text{kg})(9.81\,\text{m/s}^2)(120\,\text{m}) = \boxed{94.2\,\text{kJ}}$$

(*b*) The energy required to do this work comes from chemical energy stored in the body.

(*c*) Relate the chemical energy expended by the student to the change in his/her potential energy and solve for *E*:

$$0.2E = \Delta U$$

and

$$E = 5\Delta U = 5(94.2\text{ kJ}) = \boxed{471\text{ kJ}}$$

Kinetic Friction

44 •

Picture the Problem Let the car and the earth be the system. As the car skids to a stop on a horizontal road, its kinetic energy is transformed into internal (i.e., thermal) energy. Knowing that energy is transformed into heat by friction, we can use the definition of the coefficient of kinetic friction to calculate its value.

(*a*) The energy dissipated by friction is given by:

$$f\Delta s = \Delta E_{\text{therm}}$$

Apply the work-energy theorem for problems with kinetic friction:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} = \Delta E_{\text{mech}} + f\Delta s$$

or, because $\Delta E_{\text{mech}} = \Delta K = -K_{\text{i}}$ and $W_{\text{ext}} = 0$,

$$0 = -\tfrac{1}{2}mv_{\text{i}}^2 + f\Delta s$$

Solve for $f\Delta s$ to obtain:

$$f\Delta s = \tfrac{1}{2}mv_{\text{i}}^2$$

Substitute numerical values and evaluate $f\Delta s$:

$$f\Delta s = \tfrac{1}{2}(2000\text{ kg})(25\text{ m/s})^2 = \boxed{625\text{ kJ}}$$

(*b*) Relate the kinetic friction force to the coefficient of kinetic friction and the weight of the car and solve for the coefficient of kinetic friction:

$$f_{\text{k}} = \mu_{\text{k}}mg \Rightarrow \mu_{\text{k}} = \frac{f_{\text{k}}}{mg}$$

Express the relationship between the energy dissipated by friction and the kinetic friction force and solve f_{k}:

$$\Delta E_{\text{therm}} = f_{\text{k}}\Delta s \Rightarrow f_{\text{k}} = \frac{\Delta E_{\text{therm}}}{\Delta s}$$

Substitute to obtain:

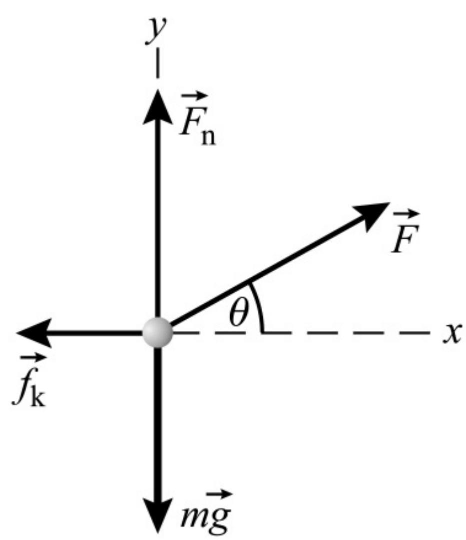
$$\mu_{\text{k}} = \frac{\Delta E_{\text{therm}}}{mg\Delta s}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{625\,\text{kJ}}{(2000\,\text{kg})(9.81\,\text{m/s}^2)(60\,\text{m})} = \boxed{0.531}$$

45 •

Picture the Problem Let the system be the sled and the earth. Then the 40-N force is external to the system. The free-body diagram shows the forces acting on the sled as it is pulled along a horizontal road. The work done by the applied force can be found using the definition of work. To find the energy dissipated by friction, we'll use Newton's 2nd law to determine f_k and then use it in the definition of work. The change in the kinetic energy of the sled is equal to the net work done on it. Finally, knowing the kinetic energy of the sled after it has traveled 3 m will allow us to solve for its speed at that location.



(*a*) Use the definition of work to calculate the work done by the applied force:

$$W_{ext} \equiv \vec{F}\cdot\vec{s} = Fs\cos\theta = (40\,\text{N})(3\,\text{m})\cos 30° = \boxed{104\,\text{J}}$$

(*b*) Express the energy dissipated by friction as the sled is dragged along the surface:

$$\Delta E_{therm} = f\Delta x = \mu_k F_n \Delta x$$

Apply $\sum F_y = ma_y$ to the sled and solve for F_n:

$$F_n + F\sin\theta - mg = 0$$

and

$$F_n = mg - F\sin\theta$$

Substitute to obtain:

$$\Delta E_{therm} = \mu_k \Delta x(mg - F\sin\theta)$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{therm} = (0.4)(3\,\text{m})[(8\,\text{kg})(9.81\,\text{m/s}^2) - (40\,\text{N})\sin 30°] = \boxed{70.2\,\text{J}}$$

(*c*) Apply the work-energy theorem

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta E_{mech} + f\Delta s$$

for problems with kinetic friction:

or, because $\Delta E_{\text{mech}} = \Delta K + \Delta U$ and $\Delta U = 0$,
$W_{\text{ext}} = \Delta K + \Delta E_{\text{therm}}$

Solve for and evaluate ΔK to obtain:

$$\Delta K = W_{\text{ext}} - \Delta E_{\text{therm}} = 104\,\text{J} - 70.2\,\text{J} = \boxed{33.8\,\text{J}}$$

(*d*) Because $K_i = 0$:

$$K_f = \Delta K = \tfrac{1}{2}mv_f^2$$

Solve for v_f:

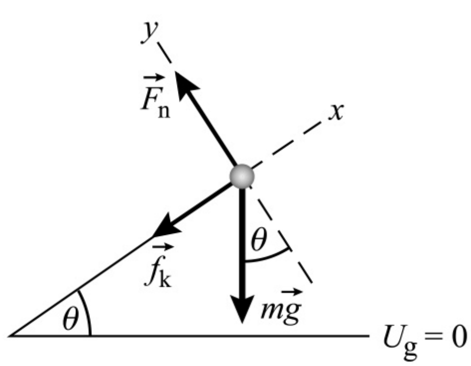
$$v_f = \sqrt{\frac{2\Delta K}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2(33.8\,\text{J})}{8\,\text{kg}}} = \boxed{2.91\,\text{m/s}}$$

***46 •**

Picture the Problem Choose $U_g = 0$ at the foot of the ramp and let the system consist of the block, ramp, and the earth. Then the kinetic energy of the block at the foot of the ramp is equal to its initial kinetic energy less the energy dissipated by friction. The block's kinetic energy at the foot of the incline is partially converted to gravitational potential energy and partially dissipated by friction as the block slides up the incline. The free-body diagram shows the forces acting on the block as it slides up the incline. Applying Newton's 2nd law to the block will allow us to determine f_k and express the energy dissipated by friction.



(*a*) Apply conservation of energy to the system while the block is moving horizontally:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} = \Delta K + \Delta U + f\Delta s$$

or, because $\Delta U = W_{\text{ext}} = 0$,

$$0 = \Delta K + f\Delta s = K_f - K_i + f\Delta s$$

Solve for K_f:

$$K_f = K_i - f\Delta s$$

Substitute for K_f, K_i, and $f\Delta s$ to obtain:

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - \mu_k mg\Delta x$$

Solving for v_f yields:

$$v_f = \sqrt{v_i^2 - 2\mu_k g\Delta x}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{(7\,\text{m/s})^2 - 2(0.3)(9.81\,\text{m/s}^2)(2\,\text{m})} = \boxed{6.10\,\text{m/s}}$$

(*b*) Apply conservation of energy to the system while the block is on the incline:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta K + \Delta U + f\Delta s$$

or, because $K_f = W_{ext} = 0$,

$$0 = -K_i + \Delta U + f\Delta s$$

Apply $\sum F_y = ma_y$ to the block when it is on the incline:

$$F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$$

Express $f\Delta s$:

$$f\Delta s = f_k L = \mu_k F_n L = \mu_k mgL\cos\theta$$

The final potential energy of the block is:

$$U_f = mgL\sin\theta$$

Substitute for U_f, U_i, and $f\Delta s$ to obtain:

$$0 = -K_i + mgL\sin\theta + \mu_k mgL\cos\theta$$

Solving for L yields:

$$L = \frac{\tfrac{1}{2}v_i^2}{g(\sin\theta + \mu_k\cos\theta)}$$

Substitute numerical values and evaluate L:

$$L = \frac{\tfrac{1}{2}(6.10\,\text{m/s})^2}{(9.81\,\text{m/s}^2)(\sin 40° + (0.3)\cos 40°)} = \boxed{2.17\,\text{m}}$$

47 •

Picture the Problem Let the system include the block, the ramp and horizontal surface, and the earth. Given this choice, there are no external forces acting that will change the energy of the system. Because the curved ramp is frictionless, mechanical energy is conserved as the block slides down it. We can calculate its speed at the bottom of the ramp by using the law of the conservation of energy. The potential energy of the block at the top of the ramp or, equivalently, its kinetic energy at the bottom of the ramp is converted into thermal energy during its slide along the horizontal surface.

(*a*) Choosing $U_g = 0$ at point 2 and letting the numeral 1 designate the initial position of the block and the numeral 2 its position at the foot of the ramp, use conservation of energy to relate the block's potential energy at the top of the ramp to its kinetic energy at the bottom:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm}$$

or, because $W_{ext} = K_i = U_f = \Delta E_{therm} = 0$,

$$0 = \tfrac{1}{2} m v_2^2 - mg\Delta h = 0$$

Solve for v_2 to obtain:

$$v_2 = \sqrt{2g\Delta h}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{2(9.81\,\text{m/s}^2)(3\,\text{m})} = \boxed{7.67\,\text{m/s}}$$

(*b*) The energy dissipated by friction is responsible for changing the thermal energy of the system:

$$W_f + \Delta K + \Delta U = \Delta E_{therm} + \Delta K + \Delta U = 0$$

Because $\Delta K = 0$ for the slide:

$$W_f = -\Delta U = -(U_2 - U_1) = U_1$$

Substitute numerical values and evaluate W_f:

$$W_f = mg\Delta h = (2\,\text{kg})(9.81\,\text{m/s}^2)(3\,\text{m}) = \boxed{58.9\,\text{J}}$$

(*c*) The energy dissipated by friction is given by:

$$\Delta E_{therm} = f\Delta s = \mu_k mg\Delta x$$

Solve for μ_k:

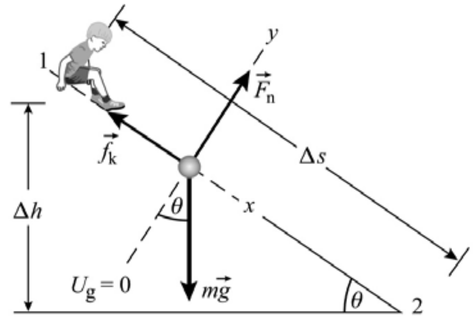
$$\mu_k = \frac{\Delta E_{therm}}{mg\Delta x}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{58.9\,\text{J}}{(2\,\text{kg})(9.81\,\text{m/s}^2)(9\,\text{m})} = \boxed{0.333}$$

48 ••

Picture the Problem Let the system consist of the earth, the girl, and the slide. Given this choice, there are no external forces doing work to change the energy of the system**.** By the time she reaches the bottom of the slide, her potential energy at the top of the slide has been converted into kinetic and thermal energy. Choose $U_g = 0$ at the bottom of the slide and denote the top and bottom of the slide as shown in the figure. We'll use the work-energy theorem with friction to relate these quantities and the forces acting on her during her slide to determine the friction force that transforms some of her initial potential energy into thermal energy.



(*a*) Express the work-energy theorem:

$$W_{ext} = \Delta K + \Delta U + \Delta E_{therm} = 0$$

Because $U_2 = K_1 = W_{ext} = 0$:

$$0 = K_2 - U_1 + \Delta E_{therm} = 0$$

or

$$\Delta E_{therm} = U_1 - K_2 = mg\Delta h - \tfrac{1}{2}mv_2^2$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{therm} = (20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m}) - \tfrac{1}{2}(20\,\text{kg})(1.3\,\text{m/s})^2 = \boxed{611\,\text{J}}$$

(*b*) Relate the energy dissipated by friction to the kinetic friction force and the distance over which this force acts and solve for μ_k:

$$\Delta E_{therm} = f\Delta s = \mu_k F_n \Delta s$$

and

$$\mu_k = \frac{\Delta E_{therm}}{F_n \Delta s}$$

Apply $\sum F_y = ma_y$ to the girl and solve for F_n:

$$F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$$

Referring to the figure, relate Δh to Δs and θ:

$$\Delta s = \frac{\Delta h}{\sin\theta}$$

Substitute for Δs and F_n to obtain:

$$\mu_k = \frac{\Delta E_{therm}}{mg\dfrac{\Delta h}{\sin\theta}\cos\theta} = \frac{\Delta E_{therm}\tan\theta}{mg\Delta h}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{(611\,\text{J})\tan 20°}{(20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m})} = \boxed{0.354}$$

49 ••

Picture the Problem Let the system consist of the two blocks, the shelf, and the earth. Given this choice, there are no external forces doing work to change the energy of the system. Due to the friction between the 4-kg block and the surface on which it slides, not all of the energy transformed during the fall of the 2-kg block is realized in the form of kinetic energy. We can find the energy dissipated by friction and then use the work-energy theorem with kinetic friction to find the speed of either block when they have moved the given distance.

(*a*) The energy dissipated by friction when the 2-kg block falls a distance *y* is given by:

$$\Delta E_{\text{therm}} = f\Delta s = \mu_k m_1 g y$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (0.35)(4\,\text{kg})(9.81\,\text{m/s}^2)y = \boxed{(13.7\,\text{N})y}$$

(*b*) From the work-energy theorem with kinetic friction we have:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}}$$

or, because $W_{\text{ext}} = 0$,

$$\Delta E_{\text{mech}} = -\Delta E_{\text{therm}} = \boxed{-(13.7\,\text{N})y}$$

(*c*) Express the total mechanical energy of the system:

$$\tfrac{1}{2}(m_1 + m_2)v^2 - m_2 g y = -\Delta E_{\text{therm}}$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2(m_2 g y - \Delta E_{\text{therm}})}{m_1 + m_2}} \qquad (1)$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2[(2\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m}) - (13.73\,\text{N})(2\,\text{m})]}{4\,\text{kg} + 2\,\text{kg}}} = \boxed{1.98\,\text{m/s}}$$

***50** ••

Picture the Problem Let the system consist of the particle, the table, and the earth. Then $W_{\text{ext}} = 0$ and the energy dissipated by friction during one revolution is the change in the thermal energy of the system.

(*a*) Apply the work-energy theorem

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}}$$

with kinetic friction to obtain:

or, because $\Delta U = W_{ext} = 0$,

$$0 = \Delta K + \Delta E_{therm}$$

Substitute for ΔK_f and simplify to obtain:

$$\Delta E_{therm} = -\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) = -\left[\tfrac{1}{2}m\left(\tfrac{1}{2}v_0\right)^2 - \tfrac{1}{2}m(v_0)^2\right] = \boxed{\tfrac{3}{8}mv_0^2}$$

(*b*) Relate the energy dissipated by friction to the distance traveled and the coefficient of kinetic friction:

$$\Delta E_{therm} = f\Delta s = \mu_k mg\Delta s = \mu_k mg(2\pi r)$$

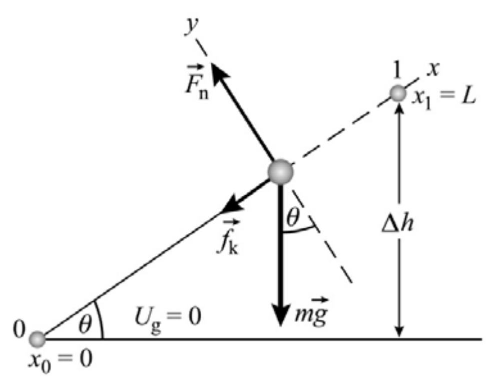
Substitute for ΔE and solve for μ_k to obtain:

$$\mu_k = \frac{\Delta E_{therm}}{2\pi mgr} = \frac{\tfrac{3}{8}mv_0^2}{2\pi mgr} = \boxed{\frac{3v_0^2}{16\pi gr}}$$

(*c*) Because it lost $\tfrac{3}{4}K_i$ in one revolution, it will only require another 1/3 revolution to lose the remaining $\tfrac{1}{4}K_i$.

51 ••

Picture the Problem The box will slow down and stop due to the dissipation of thermal energy. Let the system be the earth, the box, and the inclined plane and apply the work-energy theorem with friction. With this choice of the system, there are no external forces doing work to change the energy of the system. The free-body diagram shows the forces acting on the box when it is moving up the incline.



Apply the work-energy theorem with friction to the system:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta K + \Delta U + \Delta E_{therm}$$

Substitute for ΔK, ΔU, and ΔE_{therm} to obtain:

$$0 = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mg\Delta h + \mu_k F_n L \quad (1)$$

Referring to the FBD, relate the normal force to the weight of the box and the angle of the incline:

$$F_n = mg\cos\theta$$

Relate Δh to the distance L along the

$$\Delta h = L\sin\theta$$

incline:

Substitute in equation (1) to obtain:

$$\mu_k mgL\cos\theta + \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgL\sin\theta = 0 \quad (2)$$

Solving equation (2) for L yields:

$$L = \frac{v_0^2}{2g(\mu_k\cos\theta + \sin\theta)}$$

Substitute numerical values and evaluate L:

$$L = \frac{(3.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)[(0.3)\cos37° + \sin37°]} = \boxed{0.875\,\text{m}}$$

Let v_f represent the box's speed as it passes its starting point on the way down the incline. For the block's descent, equation (2) becomes:

$$\mu_k mgL\cos\theta + \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_1^2 - mgL\sin\theta = 0$$

Set $v_1 = 0$ (the block starts from rest at the top of the incline) and solve for v_f :

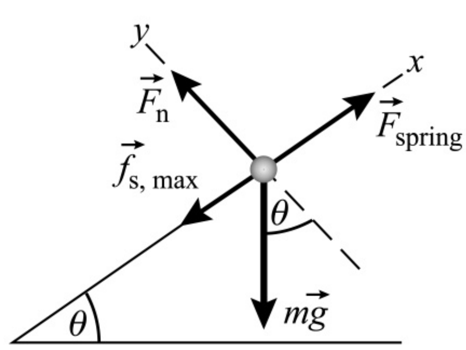
$$v_f = \sqrt{2gL(\sin\theta - \mu_k\cos\theta)}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(0.875\,\text{m})[\sin37° - (0.3)\cos37°]} = \boxed{2.49\,\text{m/s}}$$

52 •••

Picture the Problem Let the system consist of the earth, the block, the incline, and the spring. With this choice of the system, there are no external forces doing work to change the energy of the system. The free-body diagram shows the forces acting on the block just before it begins to move. We can apply Newton's 2nd law to the block to obtain an expression for the extension of the spring at this instant. We'll apply the work-energy theorem with friction to the second part of the problem.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the block

$$\sum F_x = F_{spring} - f_{s,max} - mg\sin\theta = 0$$

when it is on the verge of sliding: and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Eliminate F_n, $f_{s,max}$, and F_{spring} between the two equations to obtain:

$$kd - \mu_s mg\cos\theta - mg\sin\theta = 0$$

Solve for and evaluate d:

$$d = \boxed{\frac{mg}{k}\left(\sin\theta + \mu_s\cos\theta\right)}$$

(b) Begin with the work-energy theorem with friction and no work being done by an external force:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm}$$
$$= \Delta K + \Delta U_g + \Delta U_s + \Delta E_{therm}$$

Because the block is at rest in both its initial and final states, $\Delta K = 0$ and:

$$\Delta U_g + \Delta U_s + \Delta E_{therm} = 0 \qquad (1)$$

Let $U_g = 0$ at the initial position of the block. Then:

$$\Delta U_g = U_{g,final} - U_{g,initial} = mgh - 0$$
$$= mgd\sin\theta$$

Express the change in the energy stored in the spring as it relaxes to its unstretched length:

$$\Delta U_s = U_{s,final} - U_{s,initial} = 0 - \tfrac{1}{2}kd^2$$
$$= -\tfrac{1}{2}kd^2$$

The energy dissipated by friction is:

$$\Delta E_{therm} = f\Delta s = -f_k d = -\mu_k F_n d$$
$$= -\mu_k mgd\cos\theta$$

Substitute in equation (1) to obtain:

$$mgd\sin\theta - \tfrac{1}{2}kd^2 - \mu_k mgd\cos\theta = 0$$

Finally, solve for μ_k:

$$\mu_k = \boxed{\tfrac{1}{2}\left(\tan\theta - \mu_s\right)}$$

Mass and Energy

53 •

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(*a*) Relate the rest mass consumed to the energy produced and solve for and evaluate *m*:

$$E_0 = mc^2$$
$$= \left(1\times10^{-3}\text{ kg}\right)\left(3\times10^{8}\text{ m/s}\right)^2$$
$$= \boxed{9.00\times10^{13}\text{ J}}$$

(*b*) Express kW·h in joules:

$$1\text{kW}\cdot\text{h} = \left(1\times10^{3}\text{ J/s}\right)\left(1\text{h}\right)\left(3600\text{ s/h}\right)$$
$$= 3.60\times10^{6}\text{ J}$$

Convert 9×10^{13} J to kW·h:

$$9\times10^{13}\text{ J} = \left(9\times10^{13}\text{ J}\right)\left(\frac{1\text{kW}\cdot\text{h}}{3.60\times10^{6}\text{ J}}\right)$$
$$= 2.50\times10^{7}\text{ kW}\cdot\text{h}$$

Determine the price of the electrical energy:

$$\text{Price} = \left(2.50\times10^{7}\text{ kW}\cdot\text{h}\right)\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right)$$
$$= \boxed{\$2.5\times10^{6}}$$

(*c*) Relate the energy consumed to its rate of consumption and the time and solve for the latter:

$$E = Pt$$

and

$$t = \frac{E}{P} = \frac{9\times10^{13}\text{ J}}{100\text{ W}}$$
$$= \boxed{9\times10^{11}\text{ s} = 28{,}500\text{ y}}$$

54 •

Picture the Problem We can use the equation expressing the equivalence of energy and matter, $E = mc^2$, to find the mass equivalent of the energy from the explosion.

Solve $E = mc^2$ for *m*:

$$m = \frac{E}{c^2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{5\times10^{12}\text{ J}}{\left(2.998\times10^{8}\text{ m/s}\right)^2}$$
$$= \boxed{5.56\times10^{-5}\text{ kg}}$$

55 •

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

Relate the rest mass of a muon to its rest energy:

$$m_0 = \frac{E}{c^2}$$

Express 1 MeV in joules:

$1\text{ MeV} = 1.6\times10^{-13}\text{ J}$

Substitute numerical values and evaluate m_0:

$$m_0 = \frac{(105.7\,\text{MeV})(1.6\times10^{-13}\text{ J/MeV})}{(3\times10^{8}\text{ m/s})^2} = \boxed{1.88\times10^{-28}\text{ kg}}$$

***56** •

Picture the Problem We can differentiate the mass-energy equation to obtain an expression for the rate at which the black hole gains energy.

Using the mass-energy relationship, express the energy radiated by the black hole:

$$E = 0.01mc^2$$

Differentiate this expression to obtain an expression for the rate at which the black hole is radiating energy:

$$\frac{dE}{dt} = \frac{d}{dt}\left[0.01mc^2\right] = 0.01c^2\frac{dm}{dt}$$

Solve for dm/dt:

$$\frac{dm}{dt} = \frac{dE/dt}{0.01c^2}$$

Substitute numerical values and evaluate dm/dt:

$$\frac{dm}{dt} = \frac{4\times10^{31}\text{ watt}}{(0.01)(2.998\times10^{8}\text{ m/s})^2} = \boxed{4.45\times10^{16}\text{ kg/s}}$$

57 •

Picture the Problem The number of reactions per second is given by the ratio of the power generated to the energy released per reaction. The number of reactions that must take place to produce a given amount of energy is the ratio of the energy per second (power) to the energy released per second.

In Example 7-15 it is shown that the energy per reaction is 17.59 MeV. Convert this energy to joules:

$$17.59\,\text{MeV} = (17.59\,\text{MeV})\times(1.6\times10^{-19}\text{ J/eV}) = 28.1\times10^{-13}\text{ J}$$

The number of reactions per second is:

$$\frac{1000\,\text{J/s}}{28.1\times10^{-13}\text{ J/reaction}} = \boxed{3.56\times10^{14}\text{ reactions/s}}$$

58 •

Picture the Problem The energy required for this reaction is the difference between the rest energy of ^{4}He and the sum of the rest energies of ^{3}He and a neutron.

Express the reaction: $^{4}\text{He} \rightarrow {}^{3}\text{He} + \text{n}$

The rest energy of a neutron (Table 7-1) is: 939.573 MeV

The rest energy of ^{4}He (Example 7-15) is: 3727.409 MeV

The rest energy of ^{3}He is: 2808.432 MeV

Substitute numerical values to find the difference in the rest energy of ^{4}He and the sum of the rest energies of ^{3}He and n:

$$E = [3727.409 - (2808.41 + 939.573)]\text{ MeV} = \boxed{20.574\,\text{MeV}}$$

59 •

Picture the Problem The energy required for this reaction is the difference between the rest energy of a neutron and the sum of the rest energies of a proton and an electron.

The rest energy of a proton (Table 7-1) is: 938.280 MeV

The rest energy of an electron (Table 7-1) is: 0.511 MeV

The rest energy of a neutron (Table 7-1) is: 939.573 MeV

Substitute numerical values to find the difference in the rest energy of a neutron and the sum of the rest energies of a positron and an electron:

$$E = [939.573 - (938.280 + 0.511)]\text{ MeV} = \boxed{0.782\,\text{MeV}}$$

60 ••

Picture the Problem The reaction is ${}^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{4}\text{He} + E$. The energy released in this reaction is the difference between twice the rest energy of ^{2}H and the rest energy of ^{4}He.

The number of reactions that must take place to produce a given amount of energy is the ratio of the energy per second (power) to the energy released per reaction.

(*a*) The rest energy of ^{4}He (Example 7-14) is:

$$3727.409\text{ MeV}$$

The rest energy of a deuteron, ^{2}H, (Table 7-1) is:

$$1875.628\text{ MeV}$$

The energy released in the reaction is:

$$E = [2(1875.628) - 3727.409]\text{ MeV}$$
$$= \boxed{23.847\text{ MeV} = 3.816\times10^{-12}\text{ J}}$$

(*b*) The number of reactions per second is:

$$\frac{1000\text{ J/s}}{3.816\times10^{-12}\text{ J/reaction}}$$
$$= \boxed{2.62\times10^{14}\text{ reactions/s}}$$

61 ••

Picture the Problem The annual consumption of matter by the fission plant is the ratio of its annual energy output to the square of the speed of light. The annual consumption of coal in a coal-burning power plant is the ratio of its annual energy output to energy per unit mass of the coal.

(*a*) Express *m* in terms of *E*:

$$m = \frac{E}{c^2}$$

Assuming an efficiency of 33 percent, find the energy produced annually:

$$E = 3P\Delta t = 3(3\times10^9\text{ J/s})(1\text{ y})$$
$$= 3(3\times10^9\text{ J/s})(3600\text{ s/h})$$
$$\times(24\text{ h/d})(365.24\text{ d})$$
$$= 2.84\times10^{17}\text{ J}$$

Substitute to obtain:

$$m = \frac{2.84\times10^{17}\text{ J}}{(3\times10^8\text{ m/s})^2} = \boxed{3.16\text{ kg}}$$

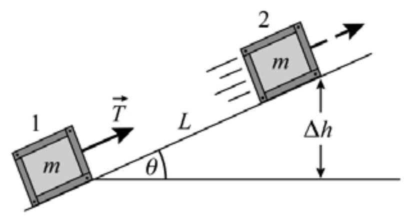
(*b*) Assuming an efficiency of 38 percent, express the mass of coal required in terms of the annual energy production and the energy released per kilogram:

$$m_{\text{coal}} = \frac{E_{\text{annual}}}{0.38(E/m)} = \frac{9.47\times10^{16}\text{ J}}{0.38(3.1\times10^7\text{ J/kg})}$$
$$= \boxed{8.04\times10^9\text{ kg}}$$

General Problems

***62 ••**

Picture the Problem Let the system consist of the block, the earth, and the incline. Then the tension in the string is an external force that will do work to change the energy of the system. Because the incline is frictionless; the work done by the tension in the string as it displaces the block on the incline is equal to the sum of the changes in the kinetic and gravitational potential energies.



Relate the work done by the tension force to the changes in the kinetic and gravitational potential energies of the block:

$$W_{\text{tension force}} = W_{\text{ext}} = \Delta U + \Delta K$$

Referring to the figure, express the change in the potential energy of the block as it moves from position 1 to position 2:

$$\Delta U = mg\Delta h = mgL\sin\theta$$

Because the block starts from rest:

$$\Delta K = K_2 = \tfrac{1}{2}mv^2$$

Substitute to obtain:

$$W_{\text{tension force}} = mgL\sin\theta + \tfrac{1}{2}mv^2$$

and $\boxed{\text{(c) is correct.}}$

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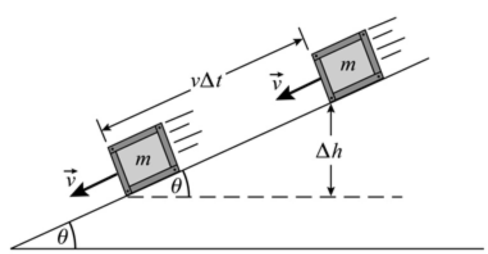
Picture the Problem Let the system include the earth, the block, and the inclined plane. Then there are no external forces to do work on the system and $W_{\text{ext}} = 0$. Apply the work-energy theorem with friction to find an expression for the energy dissipated by friction.



Express the work-energy theorem with friction:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

Because the velocity of the block is constant, $\Delta K = 0$ and:

$$\Delta E_{\text{therm}} = -\Delta U = -mg\Delta h$$

In time Δt the block slides a distance $v\Delta t$. From the figure:

$$\Delta h = v\Delta t \sin\theta$$

Substitute to obtain:

$$\Delta E_{\text{therm}} = -mgv\Delta t \sin\theta$$

and $\boxed{(b) \text{ is correct.}}$

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Picture the Problem Let the system include the earth and the box. Then the applied force is external to the system and does work on the system in compressing the spring. This work is stored in the spring as potential energy.

Express the work-energy theorem:

$$W_{\text{ext}} = \Delta K + \Delta U_{\text{g}} + \Delta U_{\text{s}} + \Delta E_{\text{therm}}$$

Because $\Delta K = \Delta U_{\text{g}} = \Delta E_{\text{therm}} = 0$:

$$W_{\text{ext}} = \Delta U_{\text{s}}$$

Substitute for W_{ext} and ΔU_{s}:

$$Fx = \tfrac{1}{2}kx^2$$

Solve for x:

$$x = \frac{2F}{k}$$

Substitute numerical values and evaluate x:

$$x = \frac{2(70\text{ N})}{6800\text{ N/m}} = \boxed{2.06\text{ cm}}$$

***65** •

Picture the Problem The solar constant is the average energy per unit area and per unit time reaching the upper atmosphere. This physical quantity can be thought of as the power per unit area and is known as *intensity*.

Letting I_{surface} represent the intensity of the solar radiation at the surface of the earth, express I_{surface} as a function of power and the area on which this energy is incident:

$$I_{\text{surface}} = \frac{P}{A} = \frac{\Delta E/\Delta t}{A}$$

Solve for ΔE:

$$\Delta E = I_{\text{surface}} A\Delta t$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = (1\,\text{kW/m}^2)(2\,\text{m}^2)(8\,\text{h})(3600\,\text{s/h}) = \boxed{57.6\,\text{MJ}}$$

66 ••

Picture the Problem The luminosity of the sun (or of any other object) is the product of the power it radiates per unit area and its surface area. If we let L represent the sun's luminosity, I the power it radiates per unit area (also known as the solar constant or the intensity of its radiation), and A its surface area, then $L = IA$. We can estimate the solar lifetime by dividing the number of hydrogen nuclei in the sun by the rate at which they are being transformed into energy.

(*a*) Express the total energy the sun radiates every second in terms of the solar constant:

$$L = IA$$

Letting R represent its radius, express the surface area of the sun:

$$A = 4\pi R^2$$

Substitute to obtain:

$$L = 4\pi R^2 I$$

Substitute numerical values and evaluate L:

$$L = 4\pi (1.5\times10^{11}\,\text{m})^2 (1.35\,\text{kW/m}^2) = \boxed{3.82\times10^{26}\,\text{watt}}$$

Note that this result is in good agreement with the value given in the text of 3.9×10^{26} watt.

(*b*) Express the solar lifetime in terms of the mass of the sun and the rate at which its mass is being converted to energy:

$$t_{\text{solar}} = \frac{N_{\text{H nuclei}}}{\Delta n/\Delta t} = \frac{M/m}{\Delta n/\Delta t}$$

where M is the mass of the sun, m the mass of a hydrogen nucleus, and n is the number of nuclei used up.

Substitute numerical values to obtain:

$$t_{\text{solar}} = \frac{\dfrac{1.99\times10^{30}\,\text{kg}}{1.67\times10^{-27}\,\text{kg/H nucleus}}}{\Delta n/\Delta t} = \frac{1.19\times10^{57}\,\text{H nuclei}}{\Delta n/\Delta t}$$

For each reaction, 4 hydrogen nuclei are "used up"; so:

$$\frac{\Delta n}{\Delta t} = \frac{4(3.82\times10^{26}\,\text{J/s})}{4.27\times10^{-12}\,\text{J}} = 3.57\times10^{38}\,\text{s}^{-1}$$

Because we've assumed that the sun will continue burning until roughly 10% of its hydrogen fuel is used up, the total solar lifetime should be:

$$t_{\text{solar}} = 0.1\left(\frac{1.19\times10^{57}\ \text{H nuclei}}{3.57\times10^{38}\ \text{s}^{-1}}\right)$$
$$= 3.33\times10^{17}\ \text{s} = \boxed{1.06\times10^{10}\ \text{y}}$$

67 •

Picture the Problem Let the system include the earth and the *Spirit of America*. Then there are no external forces to do work on the car and $W_{\text{ext}} = 0$. We can use the work-energy theorem to relate the coefficient of kinetic friction to the given information. A constant-acceleration equation will yield the car's velocity when 60 s have elapsed.

(*a*) Apply the work-energy theorem with friction to relate the coefficient of kinetic friction μ_k to the initial and final kinetic energies of the car:

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 + \mu_k mg\Delta s = 0$$

or, because $v = 0$,

$$-\tfrac{1}{2}mv_0^2 + \mu_k mg\Delta s = 0$$

Solve for μ_k:

$$\mu_k = \frac{v^2}{2g\Delta s}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{\left[(708\,\text{km/h})(1\,\text{h}/3600\,\text{s})\right]^2}{2\left(9.81\,\text{m/s}^2\right)(9.5\,\text{km})} = \boxed{0.208}$$

(*b*) Express the kinetic energy of the car:

$$K = \tfrac{1}{2}mv^2 \qquad (1)$$

Using a constant-acceleration equation, relate the speed of the car to its acceleration, initial speed, and the elapsed time:

$$v = v_0 + a\Delta t$$

Express the braking force acting on the car:

$$F_{\text{net}} = -f_k = -\mu_k mg = ma$$

Solve for a:

$$a = -\mu_k g$$

Substitute for a to obtain:

$$v = v_0 - \mu_k g\Delta t$$

Substitute in equation (1) to obtain:

$$K = \tfrac{1}{2}m\left(v_0 - \mu_k g\Delta t\right)^2$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}(1250\,\text{kg})\left[708\times10^{3}\ \text{m/h} - (0.208)(9.81\,\text{m/s}^2)(60\,\text{s})\right]^2 = \boxed{3.45\,\text{MJ}}$$

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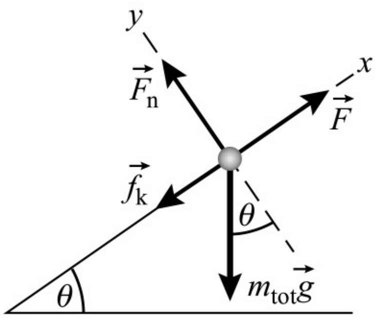
Picture the Problem The free-body diagram shows the forces acting on the skiers as they are towed up the slope at constant speed. Because the power required to move them is $\vec{F}\cdot\vec{v}$, we need to find F as a function of m_{tot}, θ, and μ_k. We can apply Newton's 2nd law to obtain such a function.



Express the power required as a function of force on the skiers and their speed:

$$P = Fv \qquad (1)$$

Apply $\sum\vec{F} = m\vec{a}$ to the skiers:

$$\sum F_x = F - f_k - m_{tot}g\sin\theta = 0$$

and

$$\sum F_y = F_n - m_{tot}g\cos\theta = 0$$

Eliminate $f_k = \mu_k F_n$ and F_n between the two equations and solve for F:

$$F = m_{tot}g\sin\theta + \mu_k m_{tot}g\cos\theta$$

Substitute in equation (1) to obtain:

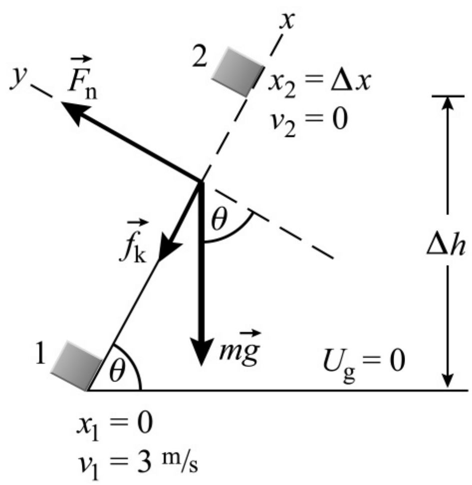
$$P = (m_{tot}g\sin\theta + \mu_k m_{tot}g\cos\theta)v$$
$$= m_{tot}gv(\sin\theta + \mu_k\cos\theta)$$

Substitute numerical values and evaluate P:

$$P = 80(75\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m/s})[\sin15° + (0.06)\cos15°] = \boxed{46.6\,\text{kW}}$$

69 ••

Picture the Problem The free-body diagram for the box is superimposed on the pictorial representation shown to the right. The work done by friction slows and momentarily stops the box as it slides up the incline. The box's speed when it returns to bottom of the incline will be less than its speed when it started up the incline due to the energy dissipated by friction while it was in motion. Let the system include the box, the earth, and the incline. Then $W_{ext} = 0$. We can use the work-energy theorem with friction to solve the several parts of this problem.



(*a*) From the FBD we can see that the forces acting on the box are the normal force exerted by the inclined plane, a kinetic friction force, and the gravitational force (the weight of the box) exerted by the earth.

(*b*) Apply the work-energy theorem with friction to relate the distance Δx the box slides up the incline to its initial kinetic energy, its final potential energy, and the work done against friction:

$$-\tfrac{1}{2}mv_1^2 + mg\Delta h + \mu_k mg\Delta x\cos\theta = 0$$

Referring to the figure, relate Δh to Δx to obtain:

$$\Delta h = \Delta x\sin\theta$$

Substitute for Δh to obtain:

$$-\tfrac{1}{2}mv_1^2 + mg\Delta x\sin\theta + \mu_k mg\Delta x\cos\theta = 0$$

Solve for Δx:

$$\Delta x = \frac{v_1^2}{2g(\sin\theta + \mu_k\cos\theta)}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(3\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)[\sin 60° + (0.3)\cos 60°]} = \boxed{0.451\,\text{m}}$$

(*c*) Express and evaluate the energy dissipated by friction:

$$\Delta E_{\text{therm}} = f_k \Delta x = \mu_k mg\Delta x\cos\theta$$
$$= (0.3)(2\,\text{kg})(9.81\,\text{m/s}^2)(0.451\,\text{m})\cos 60° = \boxed{1.33\,\text{J}}$$

(*d*) Use the work-energy theorem with friction to obtain:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$
or
$$K_1 - K_2 + U_1 - U_2 + \Delta E_{\text{therm}} = 0$$

Because $K_2 = U_1 = 0$ we have:

$$K_1 - U_2 + \Delta E_{\text{therm}} = 0$$
or
$$\tfrac{1}{2}mv_1^2 - mg\Delta x\sin\theta + \mu_k mg\Delta x\cos\theta = 0$$

Solve for v_1:

$$v_1 = \sqrt{2g\Delta x(\sin\theta - \mu_k\cos\theta)}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.451\,\text{m})[\sin 60° - (0.3)\cos 60°]} = \boxed{2.52\,\text{m/s}}$$

***70 •**

Picture the Problem The power provided by a motor that is delivering sufficient energy to exert a force *F* on a load which it is moving at a speed *v* is *Fv*.

The power provided by the motor is given by:

$$P = Fv$$

Because the elevator is ascending with constant speed, the tension in the support cable(s) is:

$$F = (m_{\text{elev}} + m_{\text{load}})g$$

Substitute for *F* to obtain:

$$P = (m_{\text{elev}} + m_{\text{load}})gv$$

Substitute numerical values and evaluate *P*:

$$P = (2000\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s}) = \boxed{45.1\,\text{kW}}$$

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Picture the Problem The power a motor must provide to exert a force F on a load that it is moving at a speed v is Fv. The counterweight does negative work and the power of the motor is reduced from that required with no counterbalance.

The power provided by the motor is given by:

$$P = Fv$$

Because the elevator is counterbalanced and ascending with constant speed, the tension in the support cable(s) is:

$$F = (m_{\text{elev}} + m_{\text{load}} - m_{\text{cw}})g$$

Substitute and evaluate P:

$$P = (m_{\text{elev}} + m_{\text{load}} - m_{\text{cw}})gv$$

Substitute numerical values and evaluate P:

$$P = (500\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s}) = \boxed{11.3\,\text{kW}}$$

Without a load:

$$F = (m_{\text{elev}} - m_{\text{cw}})g$$

and

$$P = (m_{\text{elev}} - m_{\text{cw}})gv = (-300\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s}) = \boxed{-6.77\,\text{kW}}$$

72 ••

Picture the Problem We can use the work-energy theorem with friction to describe the energy transformation within the dart-spring-air-earth system. With this choice of the system, there are no external forces to do work on the system; i.e., $W_{\text{ext}} = 0$. Choose $U_{\text{g}} = 0$ at the elevation of the dart on the compressed spring. The energy initially stored in the spring is transformed into gravitational potential energy and thermal energy. During the dart's descent, its gravitational potential energy is transformed into kinetic energy and thermal energy.

Apply conservation of energy during the dart's ascent:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

or, because $\Delta K = 0$,

$$U_{\text{g,f}} - U_{\text{g,i}} + U_{\text{s,f}} - U_{\text{s,i}} + \Delta E_{\text{therm}} = 0$$

Because $U_{\text{g,i}} = U_{\text{s,f}} = 0$:

$$U_{\text{g,f}} - U_{\text{s,i}} + \Delta E_{\text{therm}} = 0$$

Substitute for $U_{g,i}$ and $U_{s,f}$ and solve for ΔE_{therm}:

$$\Delta E_{therm} = U_{s,i} - U_{g,f} = \tfrac{1}{2}kx^2 - mgh$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\begin{aligned}\Delta E_{therm} &= \tfrac{1}{2}(5000\,\text{N/m})(0.03\,\text{m})^2 \\ &\quad - (0.007\,\text{kg})(9.81\,\text{m/s}^2)(24\,\text{m}) \\ &= \boxed{0.602\,\text{J}}\end{aligned}$$

Apply conservation of energy during the dart's descent:

$$W_{ext} = \Delta K + \Delta U + \Delta E_{therm} = 0$$

or, because $K_i = U_{g,f} = 0$,

$$K_f - U_{g,i} + \Delta E_{therm} = 0$$

Substitute for K_f and $U_{g,i}$ to obtain:

$$\tfrac{1}{2}mv_f^2 - mgh + \Delta E_{therm} = 0$$

Solve for v_f:

$$v_f = \sqrt{\frac{2(mgh - \Delta E_{therm})}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2\left[(0.007\,\text{kg})(9.81\,\text{m/s}^2)(24\,\text{m}) - 0.602\,\text{J}\right]}{0.007\,\text{kg}}} = \boxed{17.3\,\text{m/s}}$$

***73 ••**

Picture the Problem Let the system consist of the earth, rock and air. Given this choice, there are no external forces to do work on the system and $W_{ext} = 0$. Choose $U_g = 0$ to be where the rock begins its upward motion. The initial kinetic energy of the rock is partially transformed into potential energy and partially dissipated by air resistance as the rock ascends. During its descent, its potential energy is partially transformed into kinetic energy and partially dissipated by air resistance.

(*a*) Using the definition of kinetic energy, calculate the initial kinetic energy of the rock:

$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(2\,\text{kg})(40\,\text{m/s})^2 = \boxed{1.60\,\text{kJ}}$$

(*b*) Apply the work-energy theorem with friction to relate the energies of the system as the rock ascends:

$$\Delta K + \Delta U + \Delta E_{therm} = 0$$

Because $K_f = 0$:

$$-K_i + \Delta U + \Delta E_{therm} = 0$$

and

$$\Delta E_{therm} = K_i - \Delta U$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = 1600\,\text{J} - (2\,\text{kg})(9.81\,\text{m/s}^2)(50\,\text{m}) = \boxed{619\,\text{J}}$$

(*c*) Apply the work-energy theorem with friction to relate the energies of the system as the rock descends:

$$\Delta K + \Delta U + 0.7\Delta E_{\text{therm}} = 0$$

Because $K_\text{i} = U_\text{f} = 0$:

$$K_\text{f} - U_\text{i} + 0.7\Delta E_{\text{therm}} = 0$$

Substitute for the energies to obtain:

$$\tfrac{1}{2}mv_\text{f}^2 - mgh + 0.7\Delta E_{\text{therm}} = 0$$

Solve for v_f:

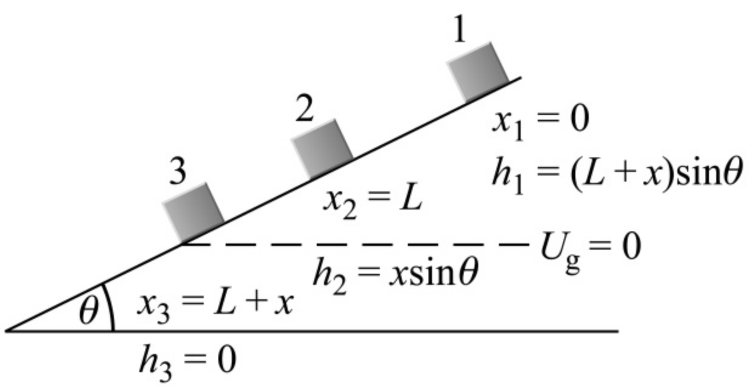
$$v_\text{f} = \sqrt{2gh - \frac{1.4\Delta E_{\text{therm}}}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_\text{f} = \sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m}) - \frac{1.4(619\,\text{J})}{2\,\text{kg}}} = \boxed{23.4\,\text{m/s}}$$

74 ••

Picture the Problem Let the distance the block slides before striking the spring be *L*. The pictorial representation shows the block at the top of the incline (1), just as it strikes the spring (2), and the block against the fully compressed spring (3). Let the block, spring, and the earth comprise the system. Then $W_{\text{ext}} = 0$. Let $U_g = 0$ where the spring is at maximum compression. We can apply the work-energy theorem to relate the energies of the system as it evolves from state 1 to state 3.



Express the work-energy theorem:

$$\Delta K + \Delta U_\text{g} + \Delta U_\text{s} = 0$$

or

$$\Delta K + U_{\text{g},3} - U_{\text{g},1} + U_{\text{s},3} - U_{\text{s},1} = 0$$

Because $\Delta K = U_{g,3} = U_{s,1} = 0$: $$-U_{g,1} + U_{s,3} = 0$$

Substitute for each of these energy terms to obtain: $$-mgh_1 + \tfrac{1}{2}kx^2 = 0$$

Substitute for h_3 and h_1: $$-mg(L + x)\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Rewrite this equation explicitly as a quadratic equation: $$x^2 - \frac{2mg\sin\theta}{k}x - \frac{2mgL\sin\theta}{k} = 0$$

Solve this quadratic equation to obtain:

$$x = \boxed{\frac{mg}{k}\sin\theta + \sqrt{\left(\frac{mg}{k}\right)^2 \sin^2\theta + \frac{2mgL}{k}\sin\theta}}$$

Note that the negative sign between the two terms leads to a non-physical solution.

***75** •

Picture the Problem We can find the work done by the girder on the slab by calculating the change in the potential energy of the slab.

(*a*) Relate the work the girder does on the slab to the change in potential energy of the slab: $$W = \Delta U = mg\Delta h$$

Substitute numerical values and evaluate *W*: $$W = (1.5\times10^4\ \text{kg})(9.81\ \text{m/s}^2)(0.001\ \text{m}) = \boxed{147\ \text{J}}$$

(*b*) The energy is transferred to the girder from its surroundings, which are warmer than the girder. As the temperature of the girder rises, the atoms in the girder vibrate with a greater average kinetic energy, leading to a larger average separation, which causes the girder's expansion.

76 ••

Picture the Problem The average power delivered by the car's engine is the rate at which it changes the car's energy. Because the car is slowing down as it climbs the hill, its potential energy increases and its kinetic energy decreases.

Express the average power delivered by the car's engine: $$P_{av} = \frac{\Delta E}{\Delta t}$$

Express the increase in the car's mechanical energy:

$$\Delta E = \Delta K + \Delta U$$
$$= K_{top} - K_{bot} + U_{top} - U_{bot}$$
$$= \tfrac{1}{2}mv_{top}^2 - \tfrac{1}{2}mv_{bot}^2 + mg\Delta h$$
$$= \tfrac{1}{2}m\left(v_{top}^2 - v_{bot}^2 + 2g\Delta h\right)$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \tfrac{1}{2}(1500\,\text{kg})\left[(10\,\text{m/s})^2 - (24\,\text{m/s})^2 + 2(9.81\,\text{m/s}^2)(120\,\text{m})\right] = 1.41\,\text{MJ}$$

Assuming that the acceleration of the car is constant, find its average speed during this climb:

$$v_{av} = \frac{v_{top} + v_{bot}}{2} = 17\,\text{m/s}$$

Using the v_{av}, find the time it takes the car to climb the hill:

$$\Delta t = \frac{\Delta s}{v_{av}} = \frac{2000\,\text{m}}{17\,\text{m/s}} = 118\,\text{s}$$

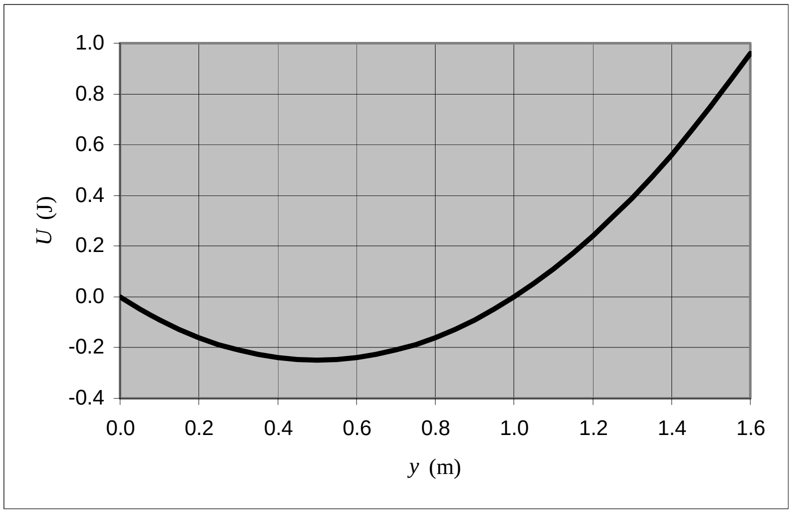
Substitute to determine P_{av}:

$$P_{av} = \frac{1.41\,\text{MJ}}{118\,\text{s}} = \boxed{11.9\,\text{kW}}$$

***77 ••**

Picture the Problem Given the potential energy function as a function of *y*, we can find the net force acting on a given system from $F = -dU/dy$. The maximum extension of the spring; i.e., the lowest position of the mass on its end, can be found by applying the work-energy theorem. The equilibrium position of the system can be found by applying the work-energy theorem with friction … as can the amount of thermal energy produced as the system oscillates to its equilibrium position.

(*a*) The graph of *U* as a function of *y* is shown to the right. Because *k* and *m* are not specified, *k* has been set equal to 2 and *mg* to 1. The spring is unstretched when $y = y_0 = 0$. Note that the minimum value of *U* (a position of stable equilibrium) occurs near $y = 5$ m.



(*b*) Evaluate the negative of the derivative of U with respect to y:

$$F = -\frac{dU}{dy} = -\frac{d}{dy}\left(\tfrac{1}{2}ky^2 - mgy\right)$$
$$= \boxed{-ky + mg}$$

(*c*) Apply conservation of energy to the movement of the mass from y = 0 to $y = y_{max}$:

$$\Delta K + \Delta U + \Delta E_{therm} = 0$$

Because $\Delta K = 0$ (the object starts from rest and is momentarily at rest at $y = y_{max}$) and $\Delta E_{therm} = 0$ (no friction), it follows that:

$$\Delta U = U(y_{max}) - U(0) = 0$$

Because $U(0) = 0$:

$$U(y_{max}) = 0 \Rightarrow \tfrac{1}{2}ky_{max}^2 - mgy_{max} = 0$$

Solve for y_{max}:

$$y_{max} = \boxed{\frac{2mg}{k}}$$

(*d*) Express the condition of F at equilibrium and solve for y_{eq}:

$$F_{eq} = 0 \Rightarrow -ky_{eq} + mg = 0$$

and

$$y_{eq} = \boxed{\frac{mg}{k}}$$

(*e*) Apply the conservation of energy to the movement of the mass from y = 0 to $y = y_{eq}$ and solve for ΔE_{therm}:

$$\Delta K + \Delta U + \Delta E_{therm} = 0$$

or, because $\Delta K = 0$.

$$\Delta E_{therm} = -\Delta U = U_i - U_f$$

Because $U_i = U(0) = 0$: $\Delta E_{therm} = -U_f = -\left(\frac{1}{2}ky_{eq}^2 - mgy_{eq}\right)$

Substitute for y_{eq} and simplify to obtain: $\Delta E_{therm} = \boxed{\dfrac{m^2g^2}{2k}}$

78 ••

Picture the Problem The energy stored in the compressed spring is initially transformed into the kinetic energy of the signal flare and then into gravitational potential energy and thermal energy as the flare climbs to its maximum height. Let the system contain the earth, the air, and the flare so that $W_{ext} = 0$. We can use the work-energy theorem with friction in the analysis of the energy transformations during the motion of the flare.

(*a*) The work done on the spring in compressing it is equal to the kinetic energy of the flare at launch. Therefore: $W_s = K_{i,\,flare} = \boxed{\frac{1}{2}mv_0^2}$

(*b*) Ignoring changes in gravitational potential energy (i.e., assume that the compression of the spring is small compared to the maximum elevation of the flare), apply the conservation of energy to the transformation that takes place as the spring decompresses and gives the flare its launch speed:

$$\Delta K + \Delta U_s = 0$$

or

$$K_f - K_i + U_{s,f} - U_{s,i} = 0$$

Because $K_i = \Delta U_g = U_{s,f}$: $K_f - U_{s,i} = 0$

Substitute for K_f and $U_{s,i}$: $\frac{1}{2}mv_0^2 - \frac{1}{2}kd^2 = 0$

Solve for k to obtain: $k = \boxed{\dfrac{mv_0^2}{d^2}}$

(*c*) Apply the work-energy theorem with friction to the upward trajectory of the flare:

$$\Delta K + \Delta U_g + \Delta E_{therm} = 0$$

Solve for ΔE_{therm}:

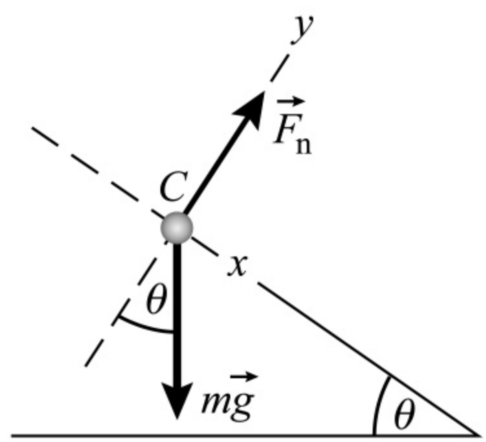
$$\Delta E_{\text{therm}} = -\Delta K - \Delta U_g = K_i - K_f + U_i - U_f$$

Because $K_f = U_i = 0$:

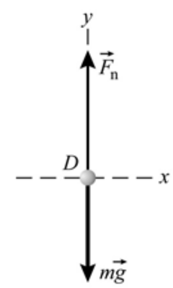
$$\Delta E_{\text{therm}} = \boxed{\tfrac{1}{2}mv_0^2 - mgh}$$

79 ••

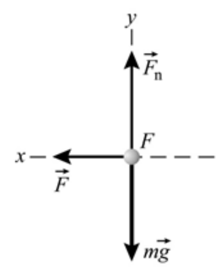
Picture the Problem Let $U_D = 0$. Choose the system to include the earth, the track, and the car. Then there are no external forces to do work on the system and change its energy and we can use Newton's 2nd law and the work-energy theorem to describe the system's energy transformations to point G … and then the work-energy theorem with friction to determine the braking force that brings the car to a stop. The free-body diagram for point C is shown to the right.



The free-body diagram for point D is shown to the right.



The free-body diagram for point F is shown to the right.



(*a*) Apply the work-energy theorem to the system's energy transformations between A and B:

$$\Delta K + \Delta U = 0$$

or

$$K_B - K_A + U_B - U_A = 0$$

If we assume that the car arrives at point B with $v_B = 0$, then:

$$-\tfrac{1}{2}mv_A^2 + mg\Delta h = 0$$

where Δh is the difference in elevation between A and B.

Solve for and evaluate Δh:

$$\Delta h = \frac{v_A^2}{2g} = \frac{(12\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 7.34\,\text{m}$$

The height above the ground is:

$$h + \Delta h = 10\,\text{m} + 7.34\,\text{m} = \boxed{17.3\,\text{m}}$$

(*b*) If the car just makes it to point B; i.e., if it gets there with $v_B = 0$, then the force exerted by the track on the car will be the normal force:

$$F_{\text{track on car}} = F_n = mg = (500\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{4.91\,\text{kN}}$$

(*c*) Apply $\sum F_x = ma_x$ to the car at point C (see the FBD) and solve for a:

$$mg\sin\theta = ma$$

and

$$a = g\sin\theta = (9.81\,\text{m/s}^2)\sin 30° = \boxed{4.91\,\text{m/s}^2}$$

(*d*) Apply $\sum F_y = ma_y$ to the car at point D (see the FBD) and solve for F_n:

$$F_n - mg = m\frac{v_D^2}{R}$$

and

$$F_n = mg + m\frac{v_D^2}{\text{R}}$$

Apply the work-energy theorem to the system's energy transformations between B and D:

$$\Delta K + \Delta U = 0$$

or

$$K_D - K_B + U_D - U_B = 0$$

Because $K_B = U_D = 0$:

$$K_D - U_B = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_D^2 - mg(h + \Delta h) = 0$$

Solve for v_D^2:

$$v_D^2 = 2g(h + \Delta h)$$

Substitute to find F_n:

$$F_n = mg + m\frac{v_D^2}{\text{R}} = mg + m\frac{2g(h+\Delta h)}{R} = mg\left[1 + \frac{2(h+\Delta h)}{R}\right]$$

Substitute numerical values and evaluate F_n:

$$F_n = (500\,\text{kg})(9.81\,\text{m/s}^2)\left[1 + \frac{2(17.3\,\text{m})}{20\,\text{m}}\right]$$
$$= \boxed{13.4\,\text{kN, directed upward.}}$$

(*e*) *F* has two components at point F; one horizontal (the inward force that the track exerts) and the other vertical (the normal force). Apply $\sum \vec{F} = m\vec{a}$ to the car at point F:

$$\sum F_y = F_n - mg = 0 \Rightarrow F_n = mg$$
and
$$\sum F_x = F_c = m\frac{v_F^2}{R}$$

Express the resultant of these two forces:

$$F = \sqrt{F_c^2 + F_n^2}$$
$$= \sqrt{\left(m\frac{v_F^2}{R}\right)^2 + (mg)^2}$$
$$= m\sqrt{\frac{v_F^4}{R^2} + g^2}$$

Substitute numerical values and evaluate *F*:

$$F = (500\,\text{kg})\sqrt{\frac{(12\,\text{m/s})^4}{(30\,\text{m})^2} + (9.81\,\text{m/s}^2)^2}$$
$$= \boxed{5.46\,\text{kN}}$$

Find the angle the resultant makes with the *x* axis:

$$\theta = \tan^{-1}\left(\frac{F_n}{F_c}\right) = \tan^{-1}\left(\frac{gR}{v_F^2}\right)$$
$$= \tan^{-1}\left[\frac{(9.81\,\text{m/s}^2)(30\,\text{m})}{(12\,\text{m/s})^2}\right] = \boxed{63.9°}$$

(*f*) Apply the work-energy theorem with friction to the system's energy transformations between F and the car's stopping position:

$$-K_G + \Delta E_{\text{therm}} = 0$$
and
$$\Delta E_{\text{therm}} = K_G = \tfrac{1}{2}mv_G^2$$

The work done by friction is also given by:

$$\Delta E_{\text{therm}} = f\Delta s = F_{\text{brake}}d$$
where *d* is the stopping distance.

Equate the two expressions for ΔE_{therm} and solve for F_{brake}:

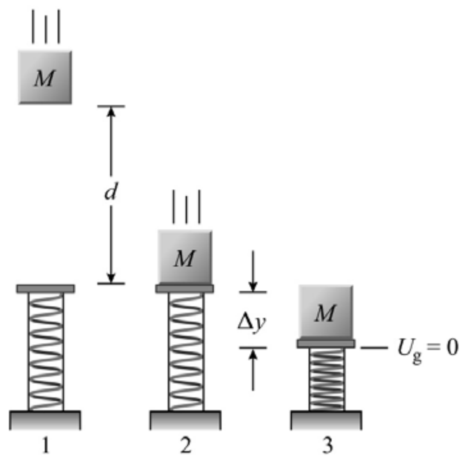
$$F_{\text{brake}} = \frac{mv_F^2}{2d}$$

Substitute numerical values and evaluate F_{brake}:

$$F_{brake} = \frac{(500\,\text{kg})(12\,\text{m/s})^2}{2(25\,\text{m})} = \boxed{1.44\,\text{kN}}$$

***80** •

Picture the Problem The rate of conversion of mechanical energy can be determined from $P = \vec{F} \cdot \vec{v}$. The pictorial representation shows the elevator moving downward just as it goes into freefall as state 1. In state 2 the elevator is moving faster and is about to strike the relaxed spring. The momentarily at rest elevator on the compressed spring is shown as state 3. Let $U_g = 0$ where the spring has its maximum compression and the system consist of the earth, the elevator, and the spring. Then $W_{ext} = 0$ and we can apply the conservation of mechanical energy to the analysis of the falling elevator and compressing spring.



(*a*) Express the rate of conversion of mechanical energy to thermal energy as a function of the speed of the elevator and braking force acting on it:

$$P = F_{braking} v_0$$

Because the elevator is moving with constant speed, the net force acting on it is zero and:

$$F_{braking} = Mg$$

Substitute for $F_{braking}$ and evaluate P:

$$\begin{aligned} P &= Mgv_0 \\ &= (2000\,\text{kg})(9.81\,\text{m/s}^2)(1.5\,\text{m/s}) \\ &= \boxed{29.4\,\text{kW}} \end{aligned}$$

(*b*) Apply the conservation of energy to the falling elevator and compressing spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_3 = U_{g,3} = U_{s,1} = 0$:

$$-\tfrac{1}{2} M v_0^2 - Mg(d + \Delta y) + \tfrac{1}{2} k(\Delta y)^2 = 0$$

Rewrite this equation as a quadratic equation in Δy, the maximum compression of the spring:

$$(\Delta y)^2 - \left(\frac{2Mg}{k}\right)\Delta y - \frac{M}{k}\left(2gd + v_0^2\right) = 0$$

Solve for Δy to obtain:

$$\Delta y = \frac{Mg}{k} \pm \sqrt{\frac{M^2 g^2}{k^2} + \frac{M}{k}\left(2gd + v_0^2\right)}$$

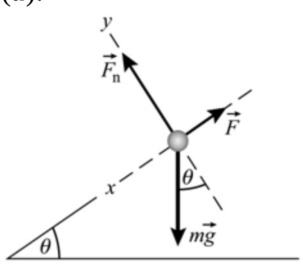
Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{(2000\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\times10^4\ \text{N/m}} + \sqrt{\frac{(2000\,\text{kg})^2(9.81\,\text{m/s}^2)^2}{(1.5\times10^4\ \text{N/m})^2} + \frac{2000\,\text{kg}}{1.5\times10^4\ \text{N/m}}\left[2(9.81\,\text{m/s}^2)(5\,\text{m}) + (1.5\,\text{m/s})^2\right]}$$
$$= \boxed{5.19\,\text{m}}$$

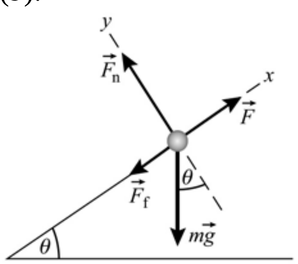
81 •

Picture the Problem We can use Newton's 2nd law to determine the force of friction as a function of the angle of the hill for a given constant speed. The power output of the engine is given by $P = \vec{F}_\text{f}\cdot\vec{v}$.

FBD for (*a*):



FBD for (*b*):



(*a*) Apply $\sum F_x = ma_x$ to the car:

$$mg\sin\theta - F_\text{f} = 0 \Rightarrow F_\text{f} = mg\sin\theta$$

Evaluate F_f for the two speeds:

$$F_{20} = (1000\,\text{kg})(9.81\,\text{m/s}^2)\sin 2.87° = \boxed{491\,\text{N}}$$

and

$$F_{30} = (1000\,\text{kg})(9.81\,\text{m/s}^2)\sin 5.74° = \boxed{981\,\text{N}}$$

(*b*) Express the power an engine must deliver on a level road in order

$$P = F_f v$$

$$P_{20} = (491\,\text{N})(20\,\text{m/s}) = \boxed{9.82\,\text{kW}}$$

to overcome friction loss and evaluate this expression for v = 20 m/s and 30 m/s:

and

$$P_{30} = (981\,\text{N})(30\,\text{m/s}) = \boxed{29.4\,\text{kW}}$$

(c) Apply $\sum F_x = ma_x$ to the car:

$$\sum F_x = F - mg\sin\theta - F_f = 0$$

Relate F to the power output of the engine and the speed of the car:

Since $P = Fv$, $F = \dfrac{P}{v}$

Substitute for F and solve for θ:

$$\theta = \sin^{-1}\left[\frac{\frac{P}{v} - F_{20}}{mg}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{\frac{40\,\text{kW}}{20\,\text{m/s}} - 491\,\text{N}}{(1000\,\text{kg})(9.81\,\text{m/s}^2)}\right] = \boxed{8.85°}$$

(d) Express the equivalence of the work done by the engine in driving the car at the two speeds:

$$W_{\text{engine}} = F_{20}(\Delta s)_{20} = F_{30}(\Delta s)_{30}$$

Let ΔV represent the volume of fuel consumed by the engine driving the car on a level road and divide both sides of the work equation by ΔV to obtain:

$$F_{20}\frac{(\Delta s)_{20}}{\Delta V} = F_{30}\frac{(\Delta s)_{30}}{\Delta V}$$

Solve for $\dfrac{(\Delta s)_{30}}{\Delta V}$:

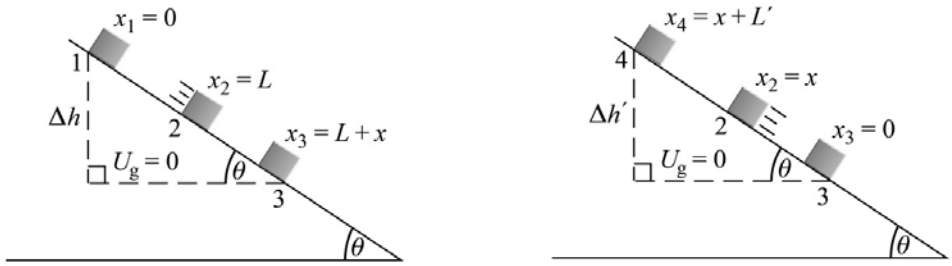
$$\frac{(\Delta s)_{30}}{\Delta V} = \frac{F_{20}}{F_{30}}\frac{(\Delta s)_{20}}{\Delta V}$$

Substitute numerical values and evaluate $\dfrac{(\Delta s)_{30}}{\Delta V}$:

$$\frac{(\Delta s)_{30}}{\Delta V} = \frac{491\,\text{N}}{981\,\text{N}}(12.7\,\text{km/L}) = \boxed{6.36\,\text{km/L}}$$

82 ••

Picture the Problem Let the system include the earth, block, spring, and incline. Then W_{ext} = 0. The pictorial representation to the left shows the block sliding down the incline and compressing the spring. Choose $U_g = 0$ at the elevation at which the spring is fully compressed. We can use the conservation of mechanical energy to determine the maximum compression of the spring. The pictorial representation to the right shows the block sliding up the rough incline after being accelerated by the fully compressed spring. We can use the work-energy theorem with friction to determine how far up the incline the block slides before stopping.



(*a*) Apply conservation of mechanical energy to the system as it evolves from state 1 to state 3:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_3 = K_1 = U_{g,3} = U_{s,1} = 0$:

$$-U_{g,1} + U_{s,3} = 0$$

or

$$-mg\Delta h + \tfrac{1}{2}kx^2 = 0$$

Relate Δh to $L + x$ and θ and substitute to obtain:

$$\Delta h = (L + x)\sin\theta$$

$$\therefore \tfrac{1}{2}kx^2 - mg(L + x)\sin\theta = 0$$

Rewrite this equation in the form of an explicit quadratic equation:

$$\tfrac{1}{2}kx^2 - (mg\sin\theta)x - mgL\sin\theta = 0$$

Substitute for *k*, *m*, *g*, θ and *L* to obtain:

$$\left(50\,\frac{\text{N}}{\text{m}}\right)x^2 - (9.81\,\text{N})x - 39.24\,\text{J} = 0$$

Solve for the physically meaningful (i.e., positive) root:

$$x = \boxed{0.989\,\text{m}}$$

(*b*) Proceed as in (*a*) but include energy dissipated by friction:

$$-U_{g,1} + U_{s,3} + \Delta E_{\text{therm}} = 0$$

The mechanical energy transformed to thermal energy is given by:

$$\Delta E_{\text{therm}} = F_f(L + x) = \mu_k F_n(L + x) = \mu_k mg\cos\theta(L + x)$$

Substitute for Δh and ΔE_{therm} to obtain:

$$-mg(L+x)\sin\theta + \tfrac{1}{2}kx^2 + \mu_k mg\cos\theta(L+x) = 0$$

Substitute for k, m, g, θ, μ_k and L to obtain:

$$\left(50\,\frac{\text{N}}{\text{m}}\right)x^2 - (6.41\,\text{N})x - 25.65\,\text{J} = 0$$

Solve for the positive root:

$$x = \boxed{0.783\,\text{m}}$$

(*c*) Apply the work-energy theorem with friction to the system as it evolves from state 3 to state 4:

$$K_4 - K_3 + U_{g,4} - U_{g,3} + U_{s,4} - U_{s,3} + \Delta E_{\text{therm}} = 0$$

Because $K_4 = K_1 = U_{g,3} = U_{s,4} = 0$:

$$U_{g,4} - U_{s,3} + \Delta E_{\text{therm}} = 0$$

or

$$-mg\Delta h' + \tfrac{1}{2}kx^2 + \Delta E_{\text{therm}} = 0$$

Substitute for $\Delta h'$ and ΔE_{therm} to obtain:

$$-mg(L'+x)\sin\theta + \tfrac{1}{2}kx^2 + \mu_k mg\cos\theta(L'+x) = 0$$

Solve for L' with $x = 0.783$ m:

$$L' = \boxed{1.54\,\text{m}}$$

83 ••

Picture the Problem The work done by the engines maintains the kinetic energy of the cars and overcomes the work done by frictional forces. Let the system include the earth, track, and the cars *but not the engines*. Then the engines will do external work on the system and we can use this work to find the power output of the train's engines.

(*a*) Use the definition of kinetic energy:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\times10^6\,\text{kg})\left(15\,\frac{\text{km}}{\text{h}}\cdot\frac{1\,\text{h}}{3600\,\text{s}}\right)^2 = \boxed{17.4\,\text{MJ}}$$

(*b*) The change in potential energy of the train is:

$$\Delta U = mg\Delta h = (2\times10^6\,\text{kg})(9.81\,\text{m/s}^2)(707\,\text{m}) = \boxed{1.39\times10^{10}\,\text{J}}$$

(*c*) Express the energy dissipated by kinetic friction:

$$\Delta E_{\text{therm}} = f\Delta s$$

Express the frictional force: $f = 0.008mg$

Substitute for f and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = 0.008mg\Delta s = 0.008\left(2\times10^{6}\text{ kg}\right)\left(9.81\text{m/s}^{2}\right)\left(62\text{ km}\right) = \boxed{9.73\times10^{9}\text{ J}}$$

(*d*) Express the power output of the train's engines in terms of the work done by them:

$$P = \frac{\Delta W}{\Delta t}$$

Use the work-energy theorem with friction to find the work done by the train's engines:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}}$$

or, because $\Delta K = 0$,

$$W_{\text{ext}} = \Delta U + \Delta E_{\text{therm}}$$

Find the time during which the engines do this work:

$$\Delta t = \frac{\Delta s}{v}$$

Substitute in the expression for P to obtain:

$$P = \frac{\left(\Delta U + \Delta E_{\text{therm}}\right)v}{\Delta s}$$

Substitute numerical values and evaluate P:

$$P = \left(1.39\times10^{10}\text{ J} + 9.73\times10^{9}\text{ J}\right)\frac{\left(15\dfrac{\text{km}}{\text{h}}\cdot\dfrac{1\text{h}}{3600\text{s}}\right)}{62\text{ km}} = \boxed{1.59\text{MW}}$$

***84 ••**

Picture the Problem While on a horizontal surface, the work done by an automobile engine changes the kinetic energy of the car and does work against friction. These energy transformations are described by the work-energy theorem with friction. Let the system include the earth, the roadway, and the car *but not the car's engine*.

(*a*) The required energy equals the change in the kinetic energy of the car:

$$\Delta K = \tfrac{1}{2}mv^{2} = \tfrac{1}{2}\left(1200\text{ kg}\right)\left(50\frac{\text{km}}{\text{h}}\cdot\frac{1\text{h}}{3600\text{s}}\right)^{2} = \boxed{116\text{ kJ}}$$

(*b*) The required energy equals the

$$\Delta E_{\text{therm}} = f\Delta s$$

work done against friction:

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (300\,\text{N})(300\,\text{m}) = \boxed{90.0\,\text{kJ}}$$

(*c*) Apply the work-energy theorem with friction to express the required energy:

$$E' = W_{\text{ext}} = \Delta K + \Delta E_{\text{therm}} = \Delta K + 0.75E$$

Divide both sides of the equation by *E* to express the ratio of the two energies:

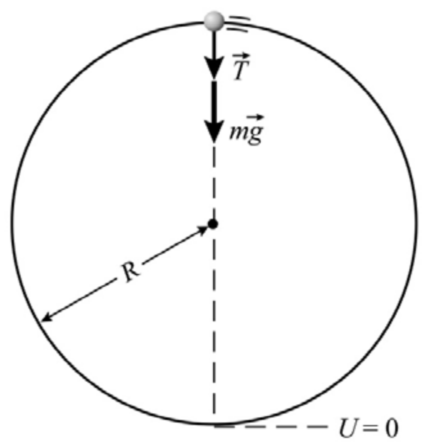
$$\frac{E'}{E} = \frac{\Delta K}{E} + 0.75$$

Substitute numerical values and evaluate E'/E:

$$\frac{E'}{E} = \frac{116\,\text{kJ}}{90\,\text{kJ}} + 0.75 = \boxed{2.04}$$

***85 •••**

Picture the Problem Assume that the bob is moving with speed *v* as it passes the top vertical point when looping around the peg. There are two forces acting on the bob: the tension in the string (if any) and the force of gravity, *Mg*; both point downward when the ball is in the topmost position. The minimum possible speed for the bob to pass the vertical occurs when the tension is 0; from this, gravity must supply the centripetal force required to keep the ball moving in a circle. We can use conservation of energy to relate *v* to *L* and *R*.



Express the condition that the bob swings around the peg in a full circle:

$$M\frac{v^2}{R} > Mg$$

Simplify to obtain:

$$\frac{v^2}{R} > g$$

Use conservation of energy to relate the kinetic energy of the bob at the bottom of the loop to its potential energy at the top of its swing:

$$\tfrac{1}{2}Mv^2 = Mg(L - 2R)$$

Solve for v^2:

$$v^2 = 2g(L - 2R)$$

Substitute to obtain:

$$\frac{2g(L - 2R)}{R} > g$$

Solve for R:

$$R < \boxed{\frac{2}{5}L}$$

86 ••

Picture the Problem If the wood exerts an average force F on the bullet, the work it does has magnitude FD. This must be equal to the change in the kinetic energy of the bullet, or because the final kinetic energy of the bullet is zero, to the negative of the initial kinetic energy. We'll let m be the mass of the bullet and v its initial speed and apply the work-kinetic energy theorem to relate the penetration depth to v.

Apply the work-kinetic energy theorem to relate the penetration depth to the change in the kinetic energy of the bullet:

$$W_{total} = \Delta K = K_f - K_i$$

or, because $K_f = 0$,

$$W_{total} = -K_i$$

Substitute for W_{total} and K_i to obtain:

$$FD = -\tfrac{1}{2}mv^2$$

Solve for D to obtain:

$$D = -\frac{mv^2}{2F}$$

For an identical bullet with twice the speed we have:

$$FD' = -\tfrac{1}{2}m(2v)^2$$

Solve for D' to obtain:

$$D' = 4\left(-\frac{mv^2}{2F}\right) = 4D$$

and $\boxed{(c)\text{ is correct.}}$

87 ••

Picture the Problem For part (a), we'll let the system include the glider, track, weight, and the earth. The speeds of the glider and the falling weight will be the same while they are in motion. Let their common speed when they have moved a distance Y be v and let the zero of potential energy be at the elevation of the weight when it has fallen the distance Y. We can use conservation of energy to relate the speed of the glider (and the weight) to the distance the weight has fallen. In part (b), we'll let the direction of motion be the x direction, the tension in the connecting string be T, and apply Newton's 2^{nd} law to the glider and the weight to find their common acceleration. Because this acceleration is constant, we can use a constant-acceleration equation to find their common speed when they have moved a distance Y.

(a) Use conservation of energy to relate the kinetic and potential energies of the system:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because the system starts from rest and $U_f = 0$:

$$K_f - U_i = 0$$

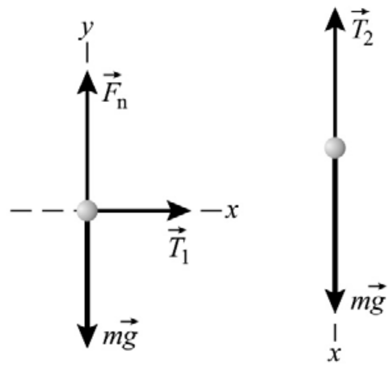
Substitute to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2 - mgY = 0$$

Solve for v:

$$v = \boxed{\sqrt{\frac{2mgY}{M+m}}}$$

(b) The free-body diagrams for the glider and the weight are shown to the right:



Apply Newton's 3rd law to obtain:

$$\left|\vec{\boldsymbol{T}}_1\right| = \left|\vec{\boldsymbol{T}}_2\right| = T$$

Apply $\sum F_x = ma$ to the glider:

$$T = Ma$$

Apply $\sum F_x = ma$ to the weight:

$$mg - T = ma$$

Add these equations to eliminate T and obtain:

$$mg = Ma + ma$$

Solve for a to obtain:

$$a = g\frac{m}{m+M}$$

Using a constant-acceleration equation, relate the speed of the glider to its initial speed and to the distance that the weight has fallen:

$$v^2 = v_0^2 + 2aY$$

or, because $v_0 = 0$,

$$v^2 = 2aY$$

Substitute for a and solve for v to obtain:

$$v = \boxed{\sqrt{\frac{2mgY}{M+m}}}$$, the same result we obtained in part (a).

***88 ••**

Picture the Problem We're given $P = dW/dt$ and are asked to evaluate it under the assumed conditions.

Express the rate of energy expenditure by the man:

$$P = 3mv^2 = 3(10\,\text{kg})(3\,\text{m/s})^2 = 270\,\text{W}$$

Express the rate of energy expenditure P' assuming that his

$$P = \tfrac{1}{5}P'$$

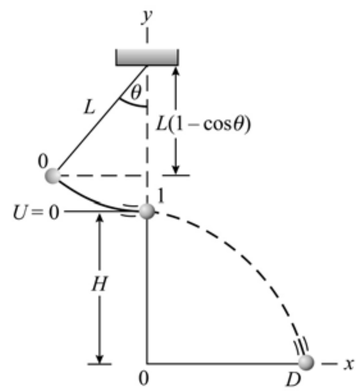
muscles have an efficiency of 20%:

Solve for and evaluate P':

$$P' = 5P = 5(270\,\text{W}) = \boxed{1.35\,\text{kW}}$$

89 ••

Picture the Problem The pictorial representation shows the bob swinging through an angle θ before the thread is cut and it is launched horizontally. Let its speed at position 1 be v. We can use conservation of energy to relate v to the change in the potential energy of the bob as it swings through the angle θ. We can find its flight time Δt from a constant-acceleration equation and then express D as the product of v and Δt.



Relate the distance D traveled horizontally by the bob to its launch speed v and time of flight Δt:

$$D = v\Delta t \qquad (1)$$

Use conservation of energy to relate its launch speed v to the length of the pendulum L and the angle θ:

$$K_1 - K_0 + U_1 - U_0 = 0$$

or, because $U_1 = K_0 = 0$,

$$K_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv^2 - mgL(1-\cos\theta) = 0$$

Solving for v yields:

$$v = \sqrt{2gL(1-\cos\theta)}$$

In the absence of air resistance, the horizontal and vertical motions of the bob are independent of each other and we can use a constant-acceleration equation to express the time of flight (the time to fall a distance H):

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $\Delta y = -H$, $a_y = -g$, and $v_{0y} = 0$,

$$-H = -\tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt to obtain:

$$\Delta t = \sqrt{2H/g}$$

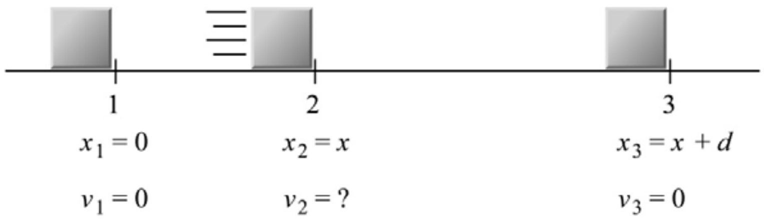
Substitute in equation (1) and simplify to obtain:

$$D = \sqrt{2gL(1-\cos\theta)}\sqrt{\frac{2H}{g}} = \boxed{2\sqrt{HL(1-\cos\theta)}}$$

which shows that, while D depends on θ, it is independent of g.

90 ••

Picture the Problem The pictorial representation depicts the block in its initial position against the compressed spring (1), as it separates from the spring with its maximum kinetic energy (2), and when it has come to rest after moving a distance $x + d$. Let the system consist of the earth, the block, and the surface on which the block slides. With this choice, $W_{ext} = 0$. We can use the work-energy theorem with friction to determine how far the block will slide before coming to rest.



(*a*) The work done by the spring on the block is given by:

$$W_{spring} = \Delta U_{spring} = \tfrac{1}{2}kx^2$$

Substitute numerical values and evaluate W_{spring}:

$$W_{spring} = \tfrac{1}{2}(20\ \text{N/cm})(3\ \text{cm})^2 = \boxed{0.900\ \text{J}}$$

(*b*) The energy dissipated by friction is given by:

$$\Delta E_{therm} = f\Delta s = \mu_k F_n \Delta x = \mu_k mg\Delta x$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{therm} = (0.2)(5\ \text{kg})(9.81\ \text{m/s}^2)(0.03\ \text{m}) = \boxed{0.294\ \text{J}}$$

(*c*) Apply the conservation of energy between points 1 and 2:

$$K_2 - K_1 + U_{s,2} - U_{s,1} + \Delta E_{therm} = 0$$

Because $K_1 = U_{s,2} = 0$:

$$K_2 - U_{s,1} + \Delta E_{therm} = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}kx^2 + \Delta E_{therm} = 0$$

Solve for v_2:

$$v_2 = \sqrt{\frac{kx^2 - 2\Delta E_{therm}}{m}}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{\frac{(20\ \text{N/cm})(3\ \text{cm})^2 - 2(0.294\ \text{J})}{5\ \text{kg}}} = \boxed{0.492\ \text{m/s}}$$

(*d*) Apply the conservation of energy between points 1 and 3:

$$\Delta K + U_{s,3} - U_{s,1} + \Delta E_{therm} = 0$$

Because $\Delta K = U_{s,3} = 0$:

$$-U_{s,1} + \Delta E_{therm} = 0$$

or

$$-\tfrac{1}{2}kx^2 + \mu_k mg(x+d) = 0$$

Solve for *d*:

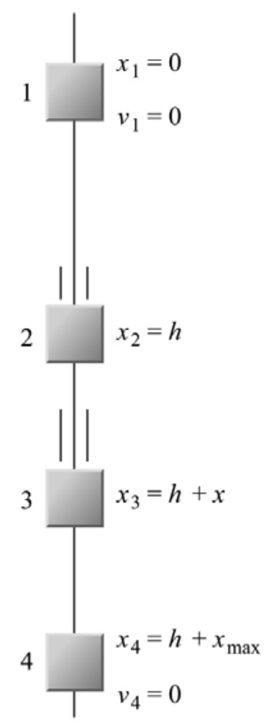
$$d = \frac{kx^2}{2\mu_k mg} - x$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(20\,\text{N/cm})(3\,\text{cm})^2}{2(0.2)(5\,\text{kg})(9.81\,\text{m/s}^2)} - 0.03\,\text{m} = \boxed{6.17\,\text{cm}}$$

91 ••

Picture the Problem The pictorial representation shows the block initially at rest at point 1, falling under the influence of gravity to point 2, partially compressing the spring as it continues to gain kinetic energy at point 3, and finally coming to rest at point 4 with the spring fully compressed. Let the system consist of the earth, the block, and the spring so that $W_{ext} = 0$. Let $U_g = 0$ at point 3 for part (*a*) and at point 4 for part (*b*). We can use the work-energy theorem to express the kinetic energy of the system as a function of the block's position and then use this function to maximize *K* as well as determine the maximum compression of the spring and the location of the block when the system has half its maximum kinetic energy.



(*a*) Apply conservation of mechanical energy to describe the energy transformations between state 1 and state 3:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_1 = U_{g,3} = U_{s,1} = 0$:

$$K_3 - U_{g,1} + U_{s,3} = 0$$

and
$$K_3 = K = mg(h+x) - \tfrac{1}{2}kx^2$$

Differentiate K with respect to x and set this derivative equal to zero to identify extreme values:

$$\frac{dK}{dx} = mg - kx = 0 \text{ for extreme values.}$$

Solve for x:

$$x = \frac{mg}{k}$$

Evaluate the second derivative of K with respect to x:

$$\frac{d^2K}{dt^2} = -k < 0$$
$$\Rightarrow x = \frac{mg}{k} \text{ maximizes } K.$$

Evaluate K for $x = mg/k$:

$$K_{\max} = mgh + mg\left(\frac{mg}{k}\right) - \tfrac{1}{2}k\left(\frac{mg}{k}\right)^2$$
$$= \boxed{mgh + \frac{m^2g^2}{2k}}$$

(b) The spring will have its maximum compression at point 4 where $K = 0$:

$$mg(h + x_{\max}) - \tfrac{1}{2}kx_{\max}^2 = 0$$
or
$$x_{\max}^2 - \frac{2mg}{k}x_{\max} - \frac{2mgh}{k} = 0$$

Solve for x and keep the physically meaningful root:

$$x_{\max} = \boxed{\frac{mg}{k} + \sqrt{\frac{m^2g^2}{k^2} + \frac{2mgh}{k}}}$$

(c) Apply conservation of mechanical energy to the system as it evolves from state 1 to the state in which $K = \tfrac{1}{2}K_{\max}$:

$$\Delta K + \Delta U_{\text{g}} + \Delta U_{\text{s}} = 0$$
or
$$K - K_1 + U_{\text{g},3} - U_{\text{g},1} + U_{\text{s},3} - U_{\text{s},1} = 0$$

Because $K_1 = U_{\text{g},3} = U_{\text{s},1} = 0$:

$$K - U_{\text{g},1} + U_{\text{s},3} = 0$$
and
$$K = mg(h+x) - \tfrac{1}{2}kx^2$$

Substitute for K to obtain:

$$\tfrac{1}{2}\left(mgh+\frac{m^2g^2}{2k}\right)=mg(h+x)-\tfrac{1}{2}kx^2$$

Express this equation in quadratic form:

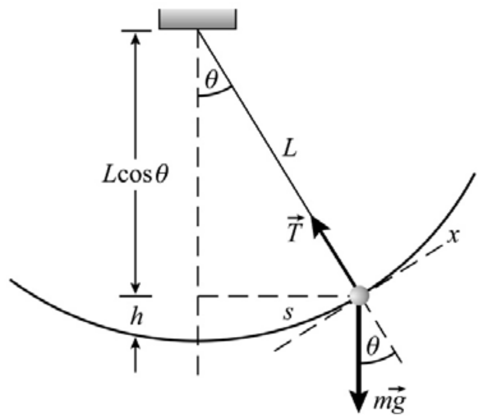
$$x^2-\frac{2mg}{k}x+\left(\frac{m^2g^2}{2k^2}-\frac{mgh}{k}\right)=0$$

Solve for the positive value of x:

$$\boxed{x=\frac{mg}{k}+\sqrt{\frac{2m^2g^2}{k^2}+\frac{4mgh}{k}}}$$

92 •••

Picture the Problem The free-body diagram shows the forces acting on the pendulum bob. The application of Newton's 2nd law leads directly to the required expression for the tangential acceleration. Recall that, provided θ is in radian measure, $s = L\theta$. Differentiation with respect to time produces the result called for in part (*b*). The remaining parts of the problem simply require following the directions for each part.



(*a*) Apply $\sum F_x = ma_x$ to the bob:

$$F_{\tan}=-mg\sin\theta=ma_{\tan}$$

Solve for $a_{\tan}$:

$$\boxed{a_{\tan}=dv/dt=-g\sin\theta}$$

(*b*) Relate the arc distance s to the length of the pendulum L and the angle θ:

$$s=L\theta$$

Differentiate with respect to time:

$$ds/dt=\boxed{v=Ld\theta/dt}$$

(*c*) Multiply $\frac{dv}{dt}$ by $\frac{d\theta}{d\theta}$ and substitute for $\frac{d\theta}{dt}$ from part (*b*):

$$\frac{dv}{dt}=\frac{dv}{dt}\frac{d\theta}{d\theta}=\frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$=\boxed{\frac{dv}{d\theta}\left(\frac{v}{L}\right)}$$

(*d*) Equate the expressions for *dv*/*dt* from (*a*) and (*c*):

$$\frac{dv}{d\theta}\left(\frac{v}{L}\right) = -g\sin\theta$$

Separate the variables to obtain:

$$\boxed{vdv = -gL\sin\theta\, d\theta}$$

(*e*) Integrate the left side of the equation in part (*d*) from $v = 0$ to the final speed v and the right side from $\theta = \theta_0$ to $\theta = 0$:

$$\int_0^v v'dv' = \int_{\theta_0}^0 -gL\sin\theta' d\theta'$$

Evaluate the limits of integration to obtain:

$$\tfrac{1}{2}v^2 = gL(1-\cos\theta_0)$$

Note, from the figure, that $h = L(1 - \cos\theta_0)$. Substitute and solve for v:

$$v = \boxed{\sqrt{2gh}}$$

93 •••

Picture the Problem The potential energy of the climber is the sum of his gravitational potential energy and the potential energy stored in the spring-like bungee cord. Let θ be the angle which the position of the rock climber on the cliff face makes with a vertical axis and choose the zero of gravitational potential energy to be at the bottom of the cliff. We can use the definitions of U_g and U_{spring} to express the climber's total potential energy.

(*a*) Express the total potential energy of the climber:

$$U(s) = U_{\text{bungee cord}} + U_g$$

Substitute to obtain:

$$\begin{aligned} U(s) &= \tfrac{1}{2}k(s-L)^2 + Mgy \\ &= \tfrac{1}{2}k(s-L)^2 + MgH\cos\theta \\ &= \tfrac{1}{2}k(s-L)^2 + MgH\cos\left(\frac{s}{H}\right) \end{aligned}$$

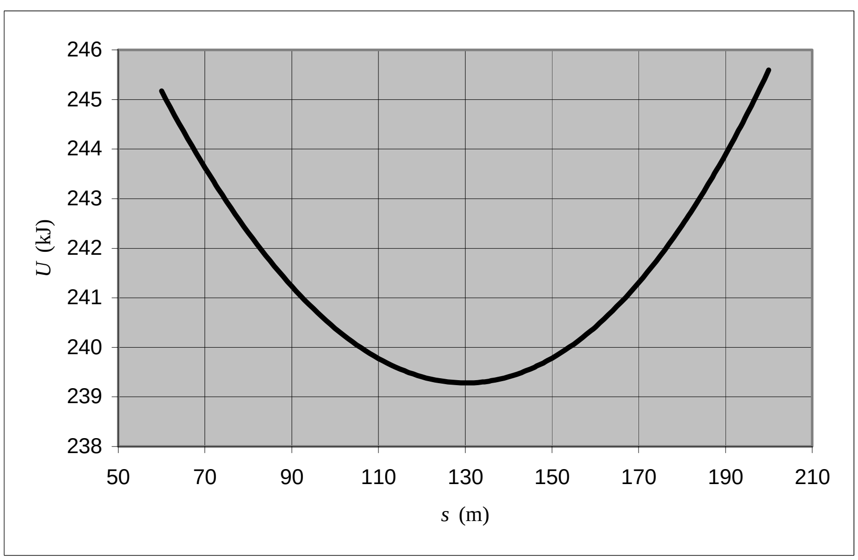
A spreadsheet solution is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B3	300	H
B4	5	k
B5	60	L
B6	85	M
B7	9.81	g
D11	60	s
D12	D11+1	$s + 1$

E11	0.5*$B$4*(D11–$B$5)^2 +$B$6*$B$7*$B$3*(cos(D11/$B$3))	$\frac{1}{2}k(s-L)^2+MgH\cos\left(\frac{s}{H}\right)$
G11	E11–E61	$U(60\,\text{m})-U(110\,\text{m})$

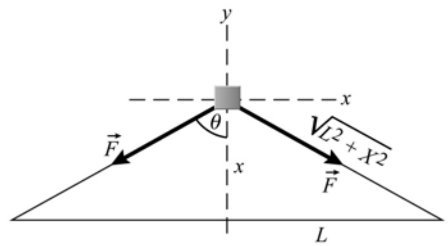
	A	B	C	D	E
1					
2					
3	H =	300	m		
4	k =	5	N/m		
5	L =	60	m		
6	m =	85	kg		
7	g =	9.81	m/s^2		
8					
9					
10				s	U(s)
11				60	2.45E+05
12				61	2.45E+05
13				62	2.45E+05
14				63	2.45E+05
15				64	2.45E+05
147				196	2.45E+05
148				197	2.45E+05
149				198	2.45E+05
150				199	2.45E+05
151				200	2.46E+05

The following graph was plotted using the data from columns D (s) and E ($U(s)$).



***94 •••**

Picture the Problem The diagram shows the forces each of the springs exerts on the block. The change in the potential energy stored in the springs is due to the elongation of both springs when the block is displaced a distance x from its equilibrium position and we can find ΔU using $\frac{1}{2}k(\Delta L)^2$. We can find the magnitude of the force pulling the block back toward its equilibrium position by finding the sum of the magnitudes of the y components of the forces exerted by the springs. In Part (d) we can use conservation of energy to find the speed of the block as it passes through its equilibrium position.



(a) Express the change in the potential energy stored in the springs when the block is displaced a distance x:

$$\Delta U = 2\left[\tfrac{1}{2}k(\Delta L)^2\right] = k(\Delta L)^2$$

where ΔL is the change in length of a spring.

Referring to the force diagram, express ΔL:

$$\Delta L = \sqrt{L^2 + x^2} - L$$

Substitute to obtain:

$$\Delta U = \boxed{k\left(\sqrt{L^2 + x^2} - L\right)^2}$$

(b) Sum the forces acting on the block to express $F_{\text{restoring}}$:

$$F_{\text{restoring}} = 2F\cos\theta = 2k\Delta L\cos\theta$$

$$= 2k\Delta L\frac{x}{\sqrt{L^2 + x^2}}$$

Substitute for ΔL to obtain:

$$F_{\text{restoring}} = 2k\left(\sqrt{L^2 + x^2} - L\right)\frac{x}{\sqrt{L^2 + x^2}}$$

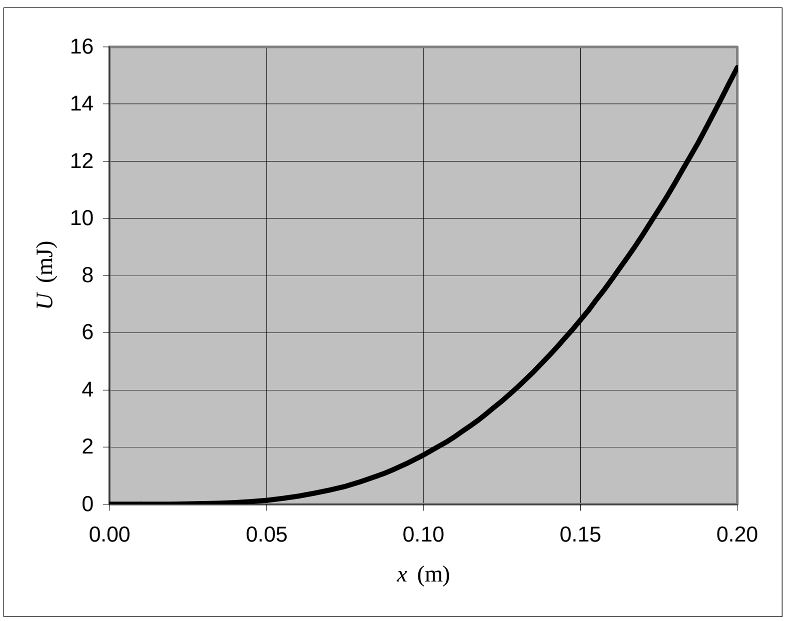
$$= \boxed{2kx\left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right)}$$

(c) A spreadsheet program to calculate $U(x)$ is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B1	1	L
B2	1	k
B3	1	M
C8	C7+0.01	x
D7	$B$2*((C7^2+$B$1^2)^0.5–$B$1)^2	$U(x)$

	A	B	C	D
1	L =	0.1	m	
2	k =	1	N/m	
3	M =	1	kg	
4				
5				
6			*x*	*U*(*x*)
7			0	0
8			0.01	2.49E–07
9			0.02	3.92E–06
10			0.03	1.94E–05
11			0.04	5.93E–05
12			0.05	1.39E–04
23			0.16	7.86E–03
24			0.17	9.45E–03
25			0.18	1.12E–02
26			0.19	1.32E–02
27			0.20	1.53E–02

The following graph was plotted using the data from columns C (*x*) and D (*U*(*x*)).



(*d*) Use conservation of energy to relate the kinetic energy of the block as it passes through the equilibrium position to the change in its potential energy as it returns to its equilibrium position:

$$K_{\text{equilibrium}} = \Delta U$$

or

$$\tfrac{1}{2}Mv^2 = \Delta U$$

Solve for *v* to obtain:

$$v=\sqrt{\frac{2\Delta U}{M}}=\sqrt{\frac{2k\left(\sqrt{L^2+x^2}-L\right)^2}{M}}$$
$$=\left(\sqrt{L^2+x^2}-L\right)\sqrt{\frac{2k}{M}}$$

Substitute numerical values and evaluate *v*:

$$v=\left(\sqrt{(0.1\,\text{m})^2+(0.1\,\text{m})^2}-0.1\,\text{m}\right)\sqrt{\frac{2(1\,\text{N/m})}{1\,\text{kg}}}=\boxed{5.86\,\text{cm/s}}$$

Chapter 8
Systems of Particles and Conservation of Momentum

Conceptual Problems

1 •

Determine the Concept A doughnut. The definition of the center of mass of an object does not require that there be any matter at its location. Any hollow sphere (such as a basketball) or an empty container with any geometry are additional examples of three-dimensional objects that have no mass at their center of mass.

***2** •

Determine the Concept The center of mass is midway between the two balls and is in free-fall along with them (all forces can be thought to be concentrated at the center of mass.) The center of mass will initially rise, then fall.

Because the initial velocity of the center of mass is half of the initial velocity of the ball thrown upwards, the mass thrown upwards will rise for twice the time that the center of mass rises. Also, the center of mass will rise until the velocities of the two balls are equal but opposite. $\boxed{(b)\text{ is correct.}}$

3 •

Determine the Concept The acceleration of the center of mass of a system of particles is described by $\vec{F}_{\text{net,ext}} = \sum_{\text{i}} \vec{F}_{\text{i,ext}} = M\vec{a}_{\text{cm}}$, where M is the total mass of the system.

Express the acceleration of the center of mass of the two pucks:

$$a_{\text{cm}} = \frac{F_{\text{net,ext}}}{M} = \frac{F_1}{m_1 + m_2}$$

and $\boxed{(b)\text{ is correct.}}$

4 •

Determine the Concept The acceleration of the center of mass of a system of particles is described by $\vec{F}_{\text{net,ext}} = \sum_{\text{i}} \vec{F}_{\text{i,ext}} = M\vec{a}_{\text{cm}}$, where M is the total mass of the system.

Express the acceleration of the center of mass of the two pucks:

$$a_{\text{cm}} = \frac{F_{\text{net,ext}}}{M} = \frac{F_1}{m_1 + m_2}$$

because the spring force is an internal force.

$\boxed{(b)\text{ is correct.}}$

***5** •

Determine the Concept No. Consider a 1-kg block with a speed of 1 m/s and a 2- kg block with a speed of 0.707 m/s. The blocks have equal kinetic energies but momenta of magnitude 1 kg·m /s and 1.414 kg·m/s, respectively.

6 •

(*a*) True. The momentum of an object is the product of its mass and velocity. Therefore, if we are considering just the magnitudes of the momenta, the momentum of a heavy object is greater than that of a light object moving at the same speed.

(*b*) True. Consider the collision of two objects of equal mass traveling in opposite directions with the same speed. Assume that they collide inelastically. The mechanical energy of the system is not conserved (it is transformed into other forms of energy), but the momentum of the system is the same after the collision as before the collision, i.e., zero. Therefore, for any inelastic collision, the momentum of a system may be conserved even when mechanical energy is not.

(*c*) True. This is a restatement of the expression for the total momentum of a system of particles.

7 •

Determine the Concept To the extent that the system in which the rifle is being fired is an isolated system, i.e., the net external force is zero, momentum is conserved during its firing.

Apply conservation of momentum to the firing of the rifle:

$$\vec{p}_{\text{rifle}} + \vec{p}_{\text{bullet}} = 0$$

or

$$\boxed{\vec{p}_{\text{rifle}} = -\vec{p}_{\text{bullet}}}$$

***8** •

Determine the Concept When she jumps from a boat to a dock, she must, in order for momentum to be conserved, give the boat a recoil momentum, i.e., her forward momentum must be the same as the boat's backward momentum. The energy she imparts to the boat is $E_{\text{boat}} = p_{\text{boat}}^2 / 2m_{\text{boat}}$.

When she jumps from one dock to another, the mass of the dock plus the earth is so large that the energy she imparts to them is essentially zero.

***9** ••

Determine the Concept Conservation of momentum requires only that the net external force acting on the system be zero. It does not require the presence of a medium such as air.

10 •

Determine the Concept The kinetic energy of the sliding ball is $\frac{1}{2}mv_{\text{cm}}^2$. The kinetic energy of the rolling ball is $\frac{1}{2}mv_{\text{cm}}^2 + K_{\text{rel}}$, where its kinetic energy relative to its center of mass is K_{rel}. Because the bowling balls are identical and have the same velocity, the rolling ball has more energy.

11 •

Determine the Concept Think of someone pushing a box across a floor. Her push on the box is equal but opposite to the push of the box on her, but the action and reaction forces act on *different objects*. You can only add forces when they act on the same object.

12 •

Determine the Concept It's not possible for both to remain at rest after the collision, as that wouldn't satisfy the requirement that momentum is conserved. It is possible for one to remain at rest: This is what happens for a one-dimensional collision of two identical particles colliding elastically.

13 •

Determine the Concept It violates the conservation of momentum! To move forward requires pushing something backwards, which Superman doesn't appear to be doing when flying around. In a similar manner, if Superman picks up a train and throws it at Lex Luthor, he (Superman) ought to be tossed backwards at a pretty high speed to satisfy the conservation of momentum.

***14** ••

Determine the Concept There is only one force which can cause the car to move forward–the friction of the road! The car's engine causes the tires to rotate, but if the road were frictionless (as is closely approximated by icy conditions) the wheels would simply spin without the car moving anywhere. Because of friction, the car's tire pushes backwards against the road–from Newton's third law, the frictional force acting on the tire must then push it forward. This may seem odd, as we tend to think of friction as being a retarding force only, but true.

15 ••

Determine the Concept The friction of the tire against the road causes the car to slow down. This is rather subtle, as the tire is in contact with the ground without slipping at all times, and so as you push on the brakes harder, the force of static friction of the road against the tires must increase. Also, of course, the brakes heat up, and not the tires.

16 •

Determine the Concept Because $\Delta p = F\Delta t$ is constant, a safety net reduces the force acting on the performer by increasing the time Δt during which the slowing force acts.

17 •

Determine the Concept Assume that the ball travels at 80 mi/h ≈ 36 m/s. The ball stops in a distance of about 1 cm. So the distance traveled is about 2 cm at an average speed of about 18 m/s. The collision time is $\frac{0.02\,\text{m}}{18\,\text{m/s}} \approx 1\,\text{ms}$.

18 •

Determine the Concept The average force on the glass is less when falling on a carpet because Δt is longer.

19 •

(*a*) False. In a perfectly inelastic collision, the colliding bodies stick together but may or may not continue moving, depending on the momentum each brings to the collision.

(*b*) True. In a head-on elastic collision both kinetic energy and momentum are conserved and the relative speeds of approach and recession are equal.

(*c*) True. This is the definition of an elastic collision.

***20** ••

Determine the Concept All the initial kinetic energy of the isolated system is lost in a perfectly inelastic collision in which the velocity of the center of mass is zero.

21 ••

Determine the Concept We can find the loss of kinetic energy in these two collisions by finding the initial and final kinetic energies. We'll use conservation of momentum to find the final velocities of the two masses in each perfectly elastic collision.

(*a*) Letting *V* represent the velocity of the masses after their perfectly inelastic collision, use conservation of momentum to determine *V*:

$$p_{\text{before}} = p_{\text{after}}$$

or

$$mv - mv = 2mV \Rightarrow V = 0$$

Express the loss of kinetic energy for the case in which the two objects have oppositely directed velocities of magnitude *v*/2:

$$\Delta K = K_\text{f} - K_\text{i} = 0 - 2\left(\tfrac{1}{2}m\left(\frac{v}{2}\right)^2\right)$$

$$= -\frac{mv^2}{4}$$

Letting *V* represent the velocity of the masses after their perfectly inelastic collision, use conservation of momentum to determine *V*:

$$p_{\text{before}} = p_{\text{after}}$$

or

$$mv = 2mV \Rightarrow V = \tfrac{1}{2}v$$

Express the loss of kinetic energy for the case in which the one object is initially at rest and the other has an initial velocity *v*:

$$\Delta K = K_f - K_i = \tfrac{1}{2}(2m)\left(\frac{v}{2}\right)^2 - \tfrac{1}{2}mv^2 = -\frac{mv^2}{4}$$

The loss of kinetic energy is the same in both cases.

(*b*) Express the percentage loss for the case in which the two objects have oppositely directed velocities of magnitude *v*/2:

$$\frac{\Delta K}{K_{\text{before}}} = \frac{\frac{1}{4}mv^2}{\frac{1}{4}mv^2} = 100\%$$

Express the percentage loss for the case in which the one object is initially at rest and the other has an initial velocity *v*:

$$\frac{\Delta K}{K_{\text{before}}} = \frac{\frac{1}{4}mv^2}{\frac{1}{2}mv^2} = 50\%$$

The percentage loss is greatest for the case in which the two objects have oppositely directed velocities of magnitude *v*/2.

***22 ••**

Determine the Concept A will travel farther. Both peas are acted on by the same force, but pea A is acted on by that force for a longer time. By the impulse-momentum theorem, its momentum (and, hence, speed) will be higher than pea B's speed on leaving the shooter.

23 ••

Determine the Concept Refer to the particles as particle 1 and particle 2. Let the direction particle 1 is moving before the collision be the positive *x* direction. We'll use both conservation of momentum and conservation of mechanical energy to obtain an expression for the velocity of particle 2 after the collision. Finally, we'll examine the ratio of the final kinetic energy of particle 2 to that of particle 1 to determine the condition under which there is maximum energy transfer from particle 1 to particle 2.

Use conservation of momentum to obtain one relation for the final velocities:

$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of

$$v_{2,f} - v_{1,f} = -(v_{2,i} - v_{1,i}) = v_{1,i} \qquad (2)$$

recession equal to the negative of the velocity of approach:

To eliminate $v_{1,f}$, solve equation (2) for $v_{1,f}$, and substitute the result in equation (1):

$$v_{1,f} = v_{2,f} + v_{1,i}$$
$$m_1 v_{1,i} = m_1\left(v_{2,f} - v_{1,i}\right) + m_2 v_{2,f}$$

Solve for $v_{2,f}$:

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i}$$

Express the ratio R of $K_{2,f}$ to $K_{1,i}$ in terms of m_1 and m_2:

$$R = \frac{K_{2,f}}{K_{1,i}} = \frac{\frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 v_{1,i}^2}{\frac{1}{2} m_1 v_{1,i}^2}$$
$$= \frac{m_2}{m_1} \frac{4m_1^2}{\left(m_1 + m_2\right)^2}$$

Differentiate this ratio with respect to m_2, set the derivative equal to zero, and obtain the quadratic equation:

$$-\frac{m_2^2}{m_1^2} + 1 = 0$$

Solve this equation for m_2 to determine its value for maximum energy transfer:

$$m_2 = m_1$$

(b) is correct because all of 1's kinetic energy is transferred to 2 when $m_2 = m_1$.

24 •

Determine the Concept In the center-of-mass reference frame the two objects approach with equal but opposite momenta and remain at rest after the collision.

25 •

Determine the Concept The water is changing direction when it rounds the corner in the nozzle. Therefore, the nozzle must exert a force on the stream of water to change its direction, and, from Newton's 3rd law, the water exerts an equal but opposite force on the nozzle.

26 •

Determine the Concept The collision usually takes place in such a short period of time that the impulse delivered by gravity or friction is negligible.

27 •

Determine the Concept No. $\vec{F}_{\text{ext,net}} = d\vec{p}/dt$ defines the relationship between the net force acting on a system and the rate at which its momentum changes. The net external force acting on the pendulum bob is the sum of the force of gravity and the tension in the string and these forces do not add to zero.

***28** ••

Determine the Concept We can apply conservation of momentum and Newton's laws of motion to the analysis of these questions.

(*a*) Yes, the car should slow down. An easy way of seeing this is to imagine a "packet" of grain being dumped into the car all at once: This is a completely inelastic collision, with the packet having an initial horizontal velocity of 0. After the collision, it is moving with the same horizontal velocity that the car does, so the car must slow down.

(*b*) When the packet of grain lands in the car, it initially has a horizontal velocity of 0, so it must be accelerated to come to the same speed as the car of the train. Therefore, the train must exert a force on it to accelerate it. By Newton's 3rd law, the grain exerts an equal but opposite force on the car, slowing it down. In general, this is a frictional force which causes the grain to come to the same speed as the car.

(*c*) No it doesn't speed up. Imagine a packet of grain being "dumped" out of the railroad car. This can be treated as a collision, too. It has the same horizontal speed as the railroad car when it leaks out, so the train car doesn't have to speed up or slow down to conserve momentum.

***29** ••

Determine the Concept Think of the stream of air molecules hitting the sail. Imagine that they bounce off the sail elastically–their net change in momentum is then roughly twice the change in momentum that they experienced going through the fan. Another way of looking at it: Initially, the air is at rest, but after passing through the fan and bouncing off the sail, it is moving backward–therefore, the boat must exert a net force on the air pushing it backward, and there must be a force on the boat pushing it forward.

Estimation and Approximation

30 ••

Picture the Problem We can estimate the time of collision from the average speed of the car and the distance traveled by the center of the car during the collision. We'll assume a car length of 6 m. We can calculate the average force exerted by the wall on the car from the car's change in momentum and it's stopping time.

(*a*) Relate the stopping time to the assumption that the center of the car travels halfway to the wall with constant deceleration:

$$\Delta t = \frac{d_{\text{stopping}}}{v_{\text{av}}} = \frac{\frac{1}{2}\left(\frac{1}{2}L_{\text{car}}\right)}{v_{\text{av}}} = \frac{\frac{1}{4}L_{\text{car}}}{v_{\text{av}}}$$

Express and evaluate v_{av}:

$$v_{av}=\frac{v_i+v_f}{2}=\frac{0+90\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}}}{2}=12.5\,\text{m/s}$$

Substitute for v_{av} and evaluate Δt:

$$\Delta t=\frac{\frac{1}{4}(6\,\text{m})}{12.5\,\text{m/s}}=\boxed{0.120\,\text{s}}$$

(b) Relate the average force exerted by the wall on the car to the car's change in momentum:

$$F_{av}=\frac{\Delta p}{\Delta t}=\frac{(2000\,\text{kg})\left(90\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}}\right)}{0.120\,\text{s}}=\boxed{417\,\text{kN}}$$

31 ••

Picture the Problem Let the direction the railcar is moving be the positive *x* direction and the system include the earth, the pumpers, and the railcar. We'll also denote the railcar with the letter c and the pumpers with the letter p. We'll use conservation of momentum to relate the center of mass frame velocities of the car and the pumpers and then transform to the earth frame of reference to find the time of fall of the car.

(a) Relate the time of fall of the railcar to the distance it falls and its velocity as it leaves the bank:

$$\Delta t=\frac{\Delta y}{v_c}$$

Use conservation of momentum to find the speed of the car relative to the velocity of its center of mass:

$$\vec{p}_i=\vec{p}_f$$

or

$$m_c u_c+m_p u_p=0$$

Relate u_c to u_p and solve for u_c:

$$u_c-u_p=4\,\text{m/s}$$

$$\therefore u_p=u_c-4\,\text{m/s}$$

Substitute for u_p to obtain:

$$m_c u_c+m_p(u_c-4\,\text{m/s})=0$$

Solve for and evaluate u_c:

$$u_c = \frac{4\,\text{m/s}}{1+\dfrac{m_c}{m_p}} = \frac{4\,\text{m/s}}{1+\dfrac{350\,\text{kg}}{4(75\,\text{kg})}} = 1.85\,\text{m/s}$$

Relate the speed of the car to its speed relative to the center of mass of the system:

$$v_c = u_c + v_{cm} = 1.85\frac{\text{m}}{\text{s}} + 32\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}} = 10.74\,\text{m/s}$$

Substitute and evaluate Δt:

$$\Delta t = \frac{25\,\text{m}}{10.74\,\text{m/s}} = \boxed{2.33\,\text{s}}$$

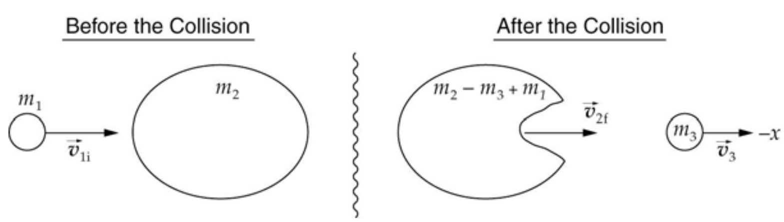
(*b*) Find the speed with which the pumpers hit the ground:

$$v_p = v_c - u_p = 10.74\,\text{m/s} - 4\,\text{m/s} = \boxed{6.74\,\text{m/s}}$$

Hitting the ground at this speed, they may be injured.

***32** ••

Picture the Problem The diagram depicts the bullet just before its collision with the melon and the motion of the melon-and-bullet-less-jet and the jet just after the collision. We'll assume that the bullet stays in the watermelon after the collision and use conservation of momentum to relate the mass of the bullet and its initial velocity to the momenta of the melon jet and the melon less the plug after the collision.



Apply conservation of momentum to the collision to obtain:

$$m_1v_{1i} = (m_2 - m_3 + m_1)v_{2f} + \sqrt{2m_3K_3}$$

Solve for v_{2f}:

$$v_{2f} = \frac{m_1v_{1i} - \sqrt{2m_3K_3}}{m_2 - m_3 + m_1}$$

Express the kinetic energy of the jet of melon in terms of the initial kinetic energy of the bullet:

$$K_3 = \tfrac{1}{10}K_1 = \tfrac{1}{10}\left(\tfrac{1}{2}m_1v_{1i}^2\right) = \tfrac{1}{20}m_1v_{1i}^2$$

Substitute and simplify to obtain:

$$v_{2f} = \frac{m_1 v_{1i} - \sqrt{2m_3\left(\frac{1}{20} m_1 v_{1i}^2\right)}}{m_2 - m_3 + m_1} = \frac{v_{1i}\left(m_1 - \sqrt{0.1 m_1 m_3}\right)}{m_2 - m_3 + m_1}$$

Substitute numerical values and evaluate v_{2f}:

$$v_{2f} = \left(1800\,\frac{\text{ft}}{\text{s}} \times \frac{1\,\text{m}}{3.281\ \text{ft}}\right)\frac{\left(0.0104\,\text{kg} - \sqrt{0.1(0.0104\,\text{kg})(0.14\,\text{kg})}\right)}{2.50\,\text{kg} - 0.14\,\text{kg} + 0.0104\,\text{kg}} = -0.386\,\text{m/s}$$

$$= \boxed{-1.27\,\text{ft/s}}$$

Note that this result is in reasonably good agreement with experimental results.

Finding the Center of Mass

33 •

Picture the Problem We can use its definition to find the center of mass of this system.

Apply its definition to find x_{cm}:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(2\,\text{kg})(0) + (2\,\text{kg})(0.2\,\text{m}) + (2\,\text{kg})(0.5\,\text{m})}{2\,\text{kg} + 2\,\text{kg} + 2\,\text{kg}} = 0.233\,\text{m}$$

Because the point masses all lie along the x axis:

$y_{cm} = 0$ and the center of mass of this system of particles is at $\boxed{(0.233\,\text{m}, 0)}$.

***34** •

Picture the Problem Let the left end of the handle be the origin of our coordinate system. We can disassemble the club-ax, find the center of mass of each piece, and then use these coordinates and the masses of the handle and stone to find the center of mass of the club-ax.

Express the center of mass of the handle plus stone system:

$$x_{cm} = \frac{m_{stick} x_{cm,stick} + m_{stone} x_{cm,stone}}{m_{stick} + m_{stone}}$$

Assume that the stone is drilled and the stick passes through it. Use symmetry considerations to locate the center of mass of the stick:

$$x_{cm,stick} = 45.0\,\text{cm}$$

Use symmetry considerations to locate the center of mass of the stone:

$$x_{\text{cm,stone}} = 89.0\,\text{cm}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{(2.5\,\text{kg})(45\,\text{cm}) + (8\,\text{kg})(89\,\text{cm})}{2.5\,\text{kg} + 8\,\text{kg}} = \boxed{78.5\text{cm}}$$

35 •

Picture the Problem We can treat each of balls as though they are point objects and apply the definition of the center of mass to find (x_{cm}, y_{cm}).

Use the definition of x_{cm}:

$$x_{\text{cm}} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(3\,\text{kg})(2\,\text{m}) + (1\,\text{kg})(1\,\text{m}) + (1\,\text{kg})(3\,\text{m})}{3\,\text{kg} + 1\,\text{kg} + 1\,\text{kg}} = 2.00\,\text{m}$$

Use the definition of y_{cm}:

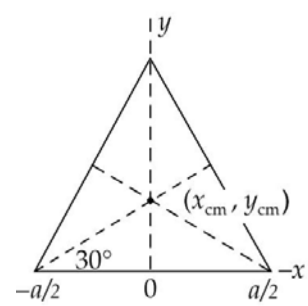
$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(3\,\text{kg})(2\,\text{m}) + (1\,\text{kg})(1\,\text{m}) + (1\,\text{kg})(0)}{3\,\text{kg} + 1\,\text{kg} + 1\,\text{kg}} = 1.40\,\text{m}$$

The center of mass of this system of particles is at:

$$\boxed{(2.00\,\text{m}, 1.40\,\text{m})}$$

36 •

Picture the Problem The figure shows an equilateral triangle with its y-axis vertex above the x axis. The bisectors of the vertex angles are also shown. We can find x coordinate of the center-of-mass by inspection and the y coordinate using trigonometry.



From symmetry considerations:

$$x_{\text{cm}} = 0$$

Express the trigonometric relationship between $a/2$, 30°, and y_{cm}:

$$\tan 30° = \frac{y_{cm}}{a/2}$$

Solve for y_{cm}:

$$y_{cm} = \tfrac{1}{2} a \tan 30° = 0.289a$$

The center of mass of an equilateral triangle oriented as shown above is at $\boxed{(0, 0.289a)}$.

***37 ••**

Picture the Problem Let the subscript 1 refer to the 3-m by 3-m sheet of plywood before the 2-m by 1-m piece has been cut from it. Let the subscript 2 refer to 2-m by 1-m piece that has been removed and let σ be the area density of the sheet. We can find the center-of-mass of these two regions; treating the missing region as though it had negative mass, and then finding the center-of-mass of the U-shaped region by applying its definition.

Express the coordinates of the center of mass of the sheet of plywood:

$$x_{cm} = \frac{m_1 x_{cm,1} - m_2 x_{cm,2}}{m_1 - m_2}$$

$$y_{cm} = \frac{m_1 y_{cm,1} - m_2 y_{cm,2}}{m_1 - m_2}$$

Use symmetry to find $x_{cm,1}$, $y_{cm,1}$, $x_{cm,2}$, and $y_{cm,2}$:

$$x_{cm,1} = 1.5\,\text{m},\ y_{cm,1} = 1.5\,\text{m}$$

and

$$x_{cm,2} = 1.5\,\text{m},\ y_{cm,2} = 2.0\,\text{m}$$

Determine m_1 and m_2:

$$m_1 = \sigma A_1 = 9\sigma\,\text{kg}$$

and

$$m_2 = \sigma A_2 = 2\sigma\,kg$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(1.5\,\text{kg})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}} = 1.50\,\text{m}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(2\,\text{m})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}} = 1.36\,\text{m}$$

The center of mass of the U-shaped sheet of plywood is at $\boxed{(1.50\,\text{m}, 1.36\,\text{m})}$.

38 ••

Picture the Problem We can use its definition to find the center of mass of the can plus water. By setting the derivative of this function equal to zero, we can find the value of x that corresponds to the minimum height of the center of mass of the water as it drains out and then use this extreme value to express the minimum height of the center of mass.

(*a*) Using its definition, express the location of the center of mass of the can + water:

$$x_{cm} = \frac{M\left(\frac{H}{2}\right) + m\left(\frac{x}{2}\right)}{M + m}$$

Let the cross-sectional area of the cup be A and use the definition of density to relate the mass m of water remaining in the can at any given time to its depth x:

$$\rho = \frac{M}{AH} = \frac{m}{Ax}$$

Solve for m to obtain:

$$m = \frac{x}{H}M$$

Substitute to obtain:

$$x_{cm} = \frac{M\left(\frac{H}{2}\right) + \left(\frac{x}{H}M\right)\left(\frac{x}{2}\right)}{M + \frac{x}{H}M}$$

$$= \boxed{\frac{H}{2}\left(\frac{1 + \left(\frac{x}{H}\right)^2}{1 + \frac{x}{H}}\right)}$$

(*b*) Differentiate x_{cm} with respect to x and set the derivative equal to zero for extrema:

$$\frac{dx_{cm}}{dx} = \frac{H}{2}\frac{d}{dx}\left(\frac{1 + \left(\frac{x}{H}\right)^2}{1 + \frac{x}{H}}\right) = \frac{H}{2}\left\{\frac{\left(1 + \frac{x}{H}\right)\frac{d}{dx}\left[1 + \left(\frac{x}{H}\right)^2\right]}{\left(1 + \frac{x}{H}\right)^2} - \frac{\left[1 + \left(\frac{x}{H}\right)^2\right]\frac{d}{dx}\left(1 + \frac{x}{H}\right)}{\left(1 + \frac{x}{H}\right)^2}\right\}$$

$$= \frac{H}{2}\left\{\frac{\left(1 + \frac{x}{H}\right)2\left(\frac{x}{H}\right)\left(\frac{1}{H}\right)}{\left(1 + \frac{x}{H}\right)^2} - \frac{\left[1 + \left(\frac{x}{H}\right)^2\right]\left(\frac{1}{H}\right)}{\left(1 + \frac{x}{H}\right)^2}\right\} = 0$$

Simplify this expression to obtain:

$$\left(\frac{x}{H}\right)^2 + 2\left(\frac{x}{H}\right) - 1 = 0$$

Solve for x/H to obtain:

$$x = H\left(\sqrt{2}-1\right) \approx 0.414H$$

where we've kept the positive solution because a negative value for x/H would make no sense.

Use your graphing calculator to convince yourself that the graph of x_{cm} as a function of x is concave upward at $x \approx 0.414H$ and that, therefore, the minimum value of x_{cm} occurs at $x \approx 0.414H$.

Evaluate x_{cm} at $x = H\left(\sqrt{2}-1\right)$ to obtain:

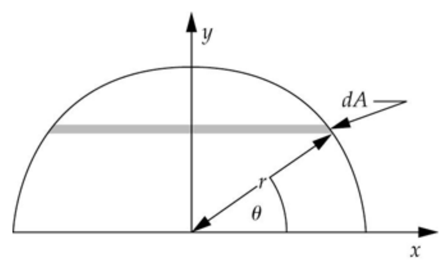
$$x_{cm}\Big|_{x=H\left(\sqrt{2}-1\right)} = \frac{H}{2}\left(\frac{1+\left(\frac{H\left(\sqrt{2}-1\right)}{H}\right)^2}{1+\frac{H\left(\sqrt{2}-1\right)}{H}}\right)$$

$$= \boxed{H\left(\sqrt{2}-1\right)}$$

Finding the Center of Mass by Integration

***39** ••

Picture the Problem A semicircular disk and a surface element of area dA is shown in the diagram. Because the disk is a continuous object, we'll use $M\vec{r}_{cm} = \int \vec{r}\,dm$ and symmetry to find its center of mass.



Express the coordinates of the center of mass of the semicircular disk:

$$x_{cm} = 0 \text{ by symmetry.}$$

$$y_{cm} = \frac{\int y\sigma\,dA}{M}$$

Express y as a function of r and θ:

$$y = r\sin\theta$$

Express dA in terms of r and θ:

$$dA = r\,d\theta\,dr$$

Express M as a function of r and θ:

$$M = \sigma A_{\text{half disk}} = \tfrac{1}{2}\sigma\pi R^2$$

Substitute and evaluate y_{cm}:

$$y_{\text{cm}} = \frac{\sigma \int_0^R \int_0^\pi r^2 \sin\theta \, d\theta \, dr}{M} = \frac{2\sigma}{M} \int_0^R r^2 dr$$

$$= \frac{2\sigma}{3M} R^3 = \boxed{\frac{4}{3\pi} R}$$

40 •••

Picture the Problem Because a solid hemisphere is a continuous object, we'll use $M\vec{r}_{\text{cm}} = \int \vec{r} dm$ to find its center of mass. The volume element for a sphere is $dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$, where θ is the polar angle and ϕ the azimuthal angle.

Let the base of the hemisphere be the xy plane and ρ be the mass density. Then:

$$z = r\cos\theta$$

Express the z coordinate of the center of mass:

$$z_{\text{cm}} = \frac{\int r\rho dV}{\int \rho dV}$$

Evaluate $M = \int \rho dV$:

$$M = \int \rho dV = \tfrac{1}{2} \rho V_{\text{sphere}}$$

$$= \tfrac{1}{2}\rho\left(\tfrac{4}{3}\pi R^3\right) = \tfrac{2}{3}\pi\rho R^3$$

Evaluate $\int r\rho dV$:

$$\int r\rho dV = \int_0^R \int_0^{\pi/2} \int_0^{2\pi} r^3 \sin\theta\cos\theta d\theta d\phi dr$$

$$= \frac{\pi\rho R^4}{2}\left[\tfrac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \frac{\pi\rho R^4}{4}$$

Substitute and simplify to find z_{cm}:

$$z_{\text{cm}} = \frac{\frac{1}{4}\pi\rho R^4}{\frac{2}{3}\pi\rho R^3} = \boxed{\tfrac{3}{8}R}$$

41 •••

Picture the Problem Because a thin hemisphere shell is a continuous object, we'll use $M\vec{r}_{\text{cm}} = \int \vec{r} dm$ to find its center of mass. The element of area on the shell is $dA = 2\pi R^2 \sin\theta \, d\theta$, where R is the radius of the hemisphere.

Let σ be the surface mass density and express the z coordinate of the center of mass:

$$z_{\text{cm}} = \frac{\int z\sigma \, dA}{\int \sigma \, dA}$$

Evaluate $M=\int\sigma\,dA$:

$$M=\int\sigma\,dA=\tfrac{1}{2}\sigma A_{\text{spherical shell}}$$
$$=\tfrac{1}{2}\sigma\left(4\pi R^2\right)=2\pi\sigma R^2$$

Evaluate $\int z\sigma\,dA$:

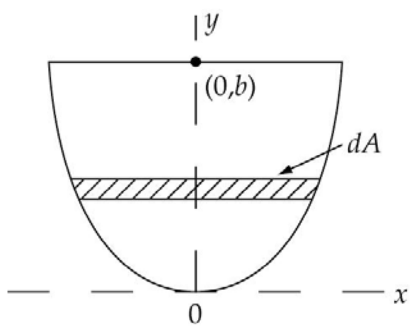
$$\int z\sigma\,dA=2\pi R^3\sigma\int_0^{\pi/2}\sin\theta\cos\theta\,d\theta$$
$$=\pi R^3\sigma\int_0^{\pi/2}\sin 2\theta\,d\theta$$
$$=\pi R^3\sigma$$

Substitute and simplify to find z_{cm}:

$$z_{\text{cm}}=\frac{\pi R^3\sigma}{2\pi\sigma R^2}=\boxed{\tfrac{1}{2}R}$$

42 •••

Picture the Problem The parabolic sheet is shown to the right. Because the area of the sheet is distributed symmetrically with respect to the y axis, $x_{\text{cm}}=0$. We'll integrate the element of area dA (= xdy) to obtain the total area of the sheet and $yxdy$ to obtain the numerator of the definition of the center of mass.



Express y_{cm}:

$$y_{\text{cm}}=\frac{\int_0^b xydy}{\int_0^b xdy}$$

Evaluate $\int_0^b xydy$:

$$\int_0^b xydy=\int_0^b\frac{y^{1/2}}{\sqrt{a}}ydy=\frac{1}{\sqrt{a}}\int_0^b y^{3/2}dy$$
$$=\frac{2}{5\sqrt{a}}b^{5/2}$$

Evaluate $\int_0^b xdy$:

$$\int_0^b xdy=\int_0^b\frac{y^{1/2}}{\sqrt{a}}dy=\frac{1}{\sqrt{a}}\int_0^b y^{1/2}dy$$
$$=\frac{2}{3\sqrt{a}}b^{3/2}$$

Substitute and simplify to determine y_{cm}:

$$y_{cm} = \frac{\frac{2}{5\sqrt{a}}b^{5/2}}{\frac{2}{3\sqrt{a}}b^{3/2}} = \tfrac{3}{5}b$$

Note that, by symmetry:

$$x_{cm} = 0$$

The center of mass of the parabolic sheet is at:

$$\boxed{\left(0, \tfrac{3}{5}b\right)}$$

Motion of the Center of Mass

43 •

Picture the Problem The velocity of the center of mass of a system of particles is related to the total momentum of the system through $\vec{\boldsymbol{P}} = \sum_{i} m_i\vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_{cm}$.

Use the expression for the total momentum of a system to relate the velocity of the center of mass of the two-particle system to the momenta of the individual particles:

$$\vec{\boldsymbol{v}}_{cm} = \frac{\sum_{i} m_i\vec{\boldsymbol{v}}_i}{M} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{cm}$:

$$\vec{\boldsymbol{v}}_{cm} = \frac{(3\,\text{kg})(\vec{\boldsymbol{v}}_1 + \vec{\boldsymbol{v}}_2)}{6\,\text{kg}} = \tfrac{1}{2}(\vec{\boldsymbol{v}}_1 + \vec{\boldsymbol{v}}_2)$$
$$= \tfrac{1}{2}\left[(6\,\text{m/s})\hat{\boldsymbol{i}} - (3\,\text{m/s})\hat{\boldsymbol{j}}\right]$$
$$= \boxed{(3\,\text{m/s})\hat{\boldsymbol{i}} - (1.5\,\text{m/s})\hat{\boldsymbol{j}}}$$

***44** •

Picture the Problem Choose a coordinate system in which east is the positive x direction and use the relationship $\vec{\boldsymbol{P}} = \sum_{i} m_i\vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_{cm}$ to determine the velocity of the center of mass of the system.

Use the expression for the total momentum of a system to relate the velocity of the center of mass of the two-vehicle system to the momenta of the individual vehicles:

$$\vec{\boldsymbol{v}}_{cm} = \frac{\sum_{i} m_i\vec{\boldsymbol{v}}_i}{M} = \frac{m_t\vec{\boldsymbol{v}}_t + m_c\vec{\boldsymbol{v}}_c}{m_t + m_c}$$

Express the velocity of the truck:

$$\vec{\boldsymbol{v}}_t = (16\,\text{m/s})\hat{\boldsymbol{i}}$$

Express the velocity of the car: $\vec{v}_c = (-20\,\text{m/s})\hat{i}$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3000\,\text{kg})(16\,\text{m/s})\,\hat{i} + (1500\,\text{kg})(-20\,\text{m/s})\,\hat{i}}{3000\,\text{kg} + 1500\,\text{kg}} = \boxed{(4.00\,\text{m/s})\,\hat{i}}$$

45 •

Picture the Problem The acceleration of the center of mass of the ball is related to the net external force through Newton's 2nd law: $\vec{F}_{\text{net,ext}} = M\vec{a}_{cm}$.

Use Newton's 2nd law to express the acceleration of the ball:

$$\vec{a}_{cm} = \frac{\vec{F}_{\text{net,ext}}}{M}$$

Substitute numerical values and evaluate $\vec{a}_{cm}$:

$$\vec{a}_{cm} = \frac{(12\,\text{N})\hat{i}}{3\,\text{kg} + 1\,\text{kg} + 1\,\text{kg}} = \boxed{(2.4\,\text{m/s}^2)\hat{i}}$$

46 ••

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can use Newton's 2nd law $\vec{F}_{\text{net,ext}} = M\vec{a}_{cm}$ to find the acceleration of the center of mass of this two-body system.

(*a*) $\boxed{\text{Yes; initially the scale reads } (M+m)g\text{; while } m \text{ is in free fall, the reading is } Mg.}$

(*b*) Using Newton's 2nd law, express the acceleration of the center of mass of the system:

$$\vec{a}_{cm} = \frac{\vec{F}_{\text{net,ext}}}{m_{\text{tot}}}$$

Substitute to obtain:

$$\vec{a}_{cm} = \boxed{-\frac{mg}{M+m}\,\hat{j}}$$

(*c*) Use Newton's 2nd law to express the net force acting on the scale while the object of mass m is falling:

$$F_{\text{net,ext}} = (M+m)g - (M+m)a_{cm}$$

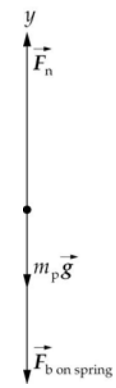
Substitute and simplify to obtain:

$$F_{\text{net,ext}} = (M+m)g - (M+m)\left(\frac{mg}{M+m}\right) = \boxed{Mg}$$

as expected, given our answer to part (*a*).

***47** ••

Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n. We can use Newton's 2nd law to determine the scale reading in part (*a*) and the work-energy theorem in conjunction with Newton's 2nd law in parts (*b*) and (*c*).



(*a*) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance *d*:

$$\sum F_y = F_n - m_p g - F_{\text{ball on spring}} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{\text{ball on spring}}$$
$$= m_p g + kd = m_p g + k\left(\frac{m_b g}{k}\right)$$
$$= \boxed{m_p g + m_b g = (m_p + m_b)g}$$

(*b*) Use conservation of mechanical energy, with $U_g = 0$ at the position at which the spring is fully compressed, to relate the gravitational potential energy of the system to the energy stored in the fully compressed spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = U_{g,f} = U_{s,i} = 0$,

$$U_{g,i} - U_{s,f} = 0$$

or

$$m_b g d - \tfrac{1}{2}kd^2 = 0$$

Solve for *d*:

$$d = \frac{2m_b g}{k}$$

Evaluate our force equation in (*a*) with $d = \dfrac{2m_b g}{k}$:

$$F_n = m_p g + F_{\text{ball on spring}}$$
$$= m_p g + kd = m_p g + k\left(\frac{2m_b g}{k}\right)$$
$$= \boxed{m_p g + 2m_b g = (m_p + 2m_b)g}$$

(*c*) When the ball is in its original position, the spring is relaxed and exerts no force on the ball. Therefore:

$$F_n = \text{scale reading} = \boxed{m_p g}$$

***48 ••**

Picture the Problem Assume that the object whose mass is m_1 is moving downward and take that direction to be the positive direction. We'll use Newton's 2nd law for a system of particles to relate the acceleration of the center of mass to the acceleration of the individual particles.

(*a*) Relate the acceleration of the center of mass to m_1, m_2, m_c and their accelerations:

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_c\vec{a}_c$$

Because m_1 and m_2 have a common acceleration a and $a_c = 0$:

$$a_{cm} = a\frac{m_1 - m_2}{m_1 + m_2 + m_c}$$

From Problem 4-81 we have:

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Substitute to obtain:

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}g\right)\left(\frac{m_1 - m_2}{m_1 + m_2 + m_c}\right) = \boxed{\frac{(m_1 - m_2)^2}{(m_1 + m_2)(m_1 + m_2 + m_c)}g}$$

(*b*) Use Newton's 2nd law for a system of particles to obtain:

$$F - Mg = -Ma_{cm}$$

where $M = m_1 + m_2 + m_c$ and F is positive upwards.

Solve for F and substitute for a_{cm} from part (*a*):

$$F = Mg - Ma_{cm} = Mg - \frac{(m_1 - m_2)^2}{m_1 + m_2}g = \boxed{\left[\frac{4m_1m_2}{m_1 + m_2} + m_c\right]g}$$

(*c*) From Problem 4-81:

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

Substitute in our result from part (*b*) to obtain:

$$F = \left[2\frac{2m_1m_2}{m_1+m_2} + m_c\right]g = \left[2\frac{T}{g} + m_c\right]g = \boxed{2T + m_c g}$$

49 ••

Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n. We can use Newton's 2nd law to determine the scale reading in part (*a*) and the result of Problem 7-96 part (*b*) to obtain the scale reading when the ball is dropped from a height *h* above the cup.

(*a*) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance *d*:

$$\sum F_y = F_n - m_p g - F_{\text{ball on spring}} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{\text{ball on spring}} = m_p g + kd = m_p g + k\left(\frac{m_b g}{k}\right) = \boxed{m_p g + m_b g = (m_p + m_b)g}$$

(*b*) From Problem 7-96, part (*b*):

$$x_{max} = \frac{m_b g}{k}\left(1 + \sqrt{1 + \frac{2kh}{m_b g}}\right)$$

From part (*a*):

$$F_n = m_p g + F_{\text{ball on spring}} = m_p g + kx_{max} = \boxed{m_p g + m_b g\left(1 + \sqrt{1 + \frac{2kh}{m_b g}}\right)}$$

The Conservation of Momentum

50 •

Picture the Problem Let the system include the woman, the canoe, and the earth. Then the *net* external force is zero and linear momentum is conserved as she jumps off the canoe. Let the direction she jumps be the positive *x* direction.

Apply conservation of momentum to the system:

$$\sum m_i \vec{v}_i = m_{girl}\vec{v}_{girl} + m_{canoe}\vec{v}_{canoe} = 0$$

Substitute to obtain:

$$(55\,\text{kg})(2.5\,\text{m/s})\hat{i} + (75\,\text{kg})\vec{v}_{canoe} = 0$$

Solve for $\vec{v}_{canoe}$:

$$\vec{v}_{canoe} = \boxed{(-1.83\,\text{m/s})\,\hat{i}}$$

51 •

Picture the Problem If we include the earth in our system, then the net external force is zero and linear momentum is conserved as the spring delivers its energy to the two objects.

Apply conservation of momentum to the system:

$$\sum m_i \vec{v}_i = m_5\vec{v}_5 + m_{10}\vec{v}_{10} = 0$$

Substitute numerical values to obtain:

$$(5\,\text{kg})(-8\,\text{m/s})\hat{i} + (10\,\text{kg})\vec{v}_{10} = 0$$

Solve for $\vec{v}_{10}$:

$$\vec{v}_{10} = \boxed{(4\,\text{m/s})\hat{i}}$$

***52** •

Picture the Problem This is an explosion-like event in which linear momentum is conserved. Thus we can equate the initial and final momenta in the *x* direction and the initial and final momenta in the *y* direction. Choose a coordinate system in the positive *x* direction is to the right and the positive *y* direction is upward.

Equate the momenta in the *y* direction before and after the explosion:

$$\sum p_{y,i} = \sum p_{y,f} = mv_2 - 2mv_1$$
$$= m(2v_1) - 2mv_1 = 0$$

We can conclude that the momentum was entirely in the *x* direction before the particle exploded.

Equate the momenta in the *x* direction before and after the explosion:

$$\sum p_{x,i} = \sum p_{x,f}$$
$$\therefore 4mv_i = mv_3$$

Solve for v_3:

$v_i = \frac{1}{4}v_3$ and $\boxed{(c)\text{ is correct.}}$

53 •

Picture the Problem Choose the direction the shell is moving just before the explosion to be the positive x direction and apply conservation of momentum.

Use conservation of momentum to relate the masses of the fragments to their velocities:

$$\vec{p}_\text{i} = \vec{p}_\text{f}$$

or

$$mv\hat{i} = \tfrac{1}{2}mv\hat{j} + \tfrac{1}{2}m\vec{v}\,'$$

Solve for $\vec{v}\,'$:

$$\vec{v}\,' = \boxed{2v\hat{i} - v\hat{j}}$$

***54** ••

Picture the Problem Let the system include the earth and the platform, gun and block. Then $\vec{F}_\text{net,ext} = 0$ and momentum is conserved within the system.

(*a*) Apply conservation of momentum to the system just before and just after the bullet leaves the gun:

$$\vec{p}_\text{before} = \vec{p}_\text{after}$$

or

$$0 = \vec{p}_\text{bullet} + \vec{p}_\text{platform}$$

Substitute for $\vec{p}_\text{bullet}$ and $\vec{p}_\text{platform}$ and solve for $\vec{v}_\text{platform}$:

$$0 = m_\text{b}v_\text{b}\hat{i} + m_\text{p}\,\vec{v}_\text{platform}$$

and

$$\vec{v}_\text{platform} = \boxed{-\frac{m_\text{b}}{m_\text{p}}v_\text{b}\hat{i}}$$

(*b*) Apply conservation of momentum to the system just before the bullet leaves the gun and just after it comes to rest in the block:

$$\vec{p}_\text{before} = \vec{p}_\text{after}$$

or

$$0 = \vec{p}_\text{platform} \Rightarrow \vec{v}_\text{platform} = 0$$

(*c*) Express the distance Δs traveled by the platform:

$$\Delta s = v_\text{platform}\Delta t$$

Express the velocity of the bullet relative to the platform:

$$v_\text{rel} = v_\text{b} - v_\text{platform} = v_\text{b} + \frac{m_\text{b}}{m_\text{p}}v_\text{b}$$

$$= \left(1 + \frac{m_\text{b}}{m_\text{p}}\right)v_\text{b} = \frac{m_\text{p} + m_\text{b}}{m_\text{p}}v_\text{b}$$

Relate the time of flight Δt to L and v_rel:

$$\Delta t = \frac{L}{v_\text{rel}}$$

Substitute to find the distance Δs moved by the platform in time Δt:

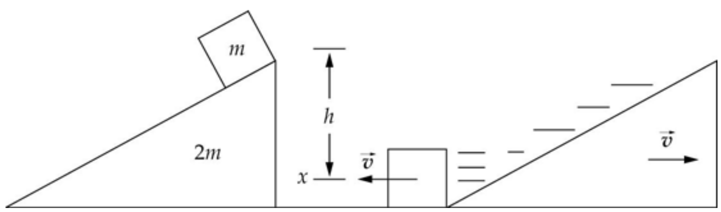
$$\Delta s = v_{\text{platform}}\Delta t = \left(\frac{m_{\text{b}}}{m_{\text{p}}}v_{\text{b}}\right)\left(\frac{L}{v_{\text{rel}}}\right)$$

$$= \left(\frac{m_{\text{b}}}{m_{\text{p}}}v_{\text{b}}\right)\left(\frac{L}{\frac{m_{\text{p}}+m_{\text{b}}}{m_{\text{p}}}v_{\text{b}}}\right)$$

$$= \boxed{\frac{m_{\text{b}}}{m_{\text{p}}+m_{\text{b}}}L}$$

55 ••

Picture the Problem The pictorial representation shows the wedge and small object, initially at rest, to the left, and, to the right, both in motion as the small object leaves the wedge. Choose the direction the small object is moving when it leaves the wedge be the positive *x* direction and the zero of potential energy to be at the surface of the table. Let the speed of the small object be *v* and that of the wedge *V*. We can use conservation of momentum to express *v* in terms of *V* and conservation of energy to express *v* in terms of *h*.



Apply conservation of momentum to the small object and the wedge:

$$\vec{p}_{\text{i},x} = \vec{p}_{\text{f},x}$$

or

$$0 = mv\hat{i} + 2m\vec{V}$$

Solve for $\vec{V}$:

$$\vec{V} = -\tfrac{1}{2}v\hat{i} \qquad (1)$$

and

$$V = \tfrac{1}{2}v$$

Use conservation of energy to determine the speed of the small object when it exits the wedge:

$$\Delta K + \Delta U = 0$$

or

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

Because $U_{\text{f}} = K_{\text{i}} = 0$:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}(2m)V^2 - mgh = 0$$

Substitute for V to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}(2m)\left(\tfrac{1}{2}v\right)^2 - mgh = 0$$

Solve for v to obtain:

$$v = 2\sqrt{\frac{gh}{3}}$$

Substitute in equation (1) to determine $\vec{V}$:

$$\vec{V} = -\tfrac{1}{2}\left(2\sqrt{\frac{gh}{3}}\right)\hat{i} = \boxed{-\sqrt{\frac{gh}{3}}\,\hat{i}}$$

i.e., the wedge moves in the direction opposite to that of the small object with a speed of $\sqrt{\frac{gh}{3}}$.

***56 ••**

Picture the Problem Because no external forces act on either cart, the center of mass of the two-cart system can't move. We can use the data concerning the masses and separation of the gliders initially to calculate its location and then apply the definition of the center of mass a second time to relate the positions X_1 and X_2 of the centers of the carts when they first touch. We can also use the separation of the centers of the gliders when they touch to obtain a second equation in X_1 and X_2 that we can solve simultaneously with the equation obtained from the location of the center of mass.

(*a*) Apply its definition to find the center of mass of the 2-glider system:

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(0.1\,\text{kg})(0.1\,\text{m}) + (0.2\,\text{kg})(1.6\,\text{m})}{0.1\,\text{kg} + 0.2\,\text{kg}} = 1.10\,\text{m}$$

from the left end of the air track.

Use the definition of the center of mass to relate the coordinates of the centers of the two gliders when they first touch to the location of the center of mass:

$$1.10\,\text{m} = \frac{m_1X_1 + m_2X_2}{m_1 + m_2} = \frac{(0.1\,\text{kg})X_1 + (0.2\,\text{kg})X_2}{0.1\,\text{kg} + 0.2\,\text{kg}} = \tfrac{1}{3}X_1 + \tfrac{2}{3}X_2$$

Also, when they first touch, their centers are separated by half their combined lengths:

$$X_2 - X_1 = \tfrac{1}{2}(10\,\text{cm} + 20\,\text{cm}) = 0.15\,\text{m}$$

Thus we have:

$$0.333X_1 + 0.667X_2 = 1.10\,\text{m}$$

and

$$X_2 - X_1 = 0.15\,\text{m}$$

Solve these equations simultaneously to obtain:

$$X_1 = \boxed{1.00\,\text{m}} \text{ and } X_2 = \boxed{1.15\,\text{m}}$$

(*b*)

No. The initial momentum of the system is zero, so it must be zero after the collision.

Kinetic Energy of a System of Particles

***57** •

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right. Use the expression for the total momentum of a system to find the velocity of the center of mass and the definition of relative velocity to express the sum of the kinetic energies relative to the center of mass.

(*a*) Find the sum of the kinetic energies:

$$\begin{aligned} K &= K_1 + K_2 \\ &= \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \\ &= \tfrac{1}{2}(3\,\text{kg})(5\,\text{m/s})^2 + \tfrac{1}{2}(3\,\text{kg})(2\,\text{m/s})^2 \\ &= \boxed{43.5\,\text{J}} \end{aligned}$$

(*b*) Relate the velocity of the center of mass of the system to its total momentum:

$$M\vec{\boldsymbol{v}}_{\text{cm}} = m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2$$

Solve for $\vec{\boldsymbol{v}}_{\text{cm}}$:

$$\vec{\boldsymbol{v}}_{\text{cm}} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\text{cm}}$:

$$\begin{aligned} \vec{\boldsymbol{v}}_{\text{cm}} &= \frac{(3\,\text{kg})(5\,\text{m/s})\hat{\boldsymbol{i}} - (3\,\text{kg})(2\,\text{m/s})\hat{\boldsymbol{i}}}{3\,\text{kg} + 3\,\text{kg}} \\ &= \boxed{(1.50\,\text{m/s})\hat{\boldsymbol{i}}} \end{aligned}$$

(*c*) The velocity of an object relative to the center of mass is given by:

$$\vec{\boldsymbol{v}}_{\text{rel}} = \vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_{\text{cm}}$$

Substitute numerical values to obtain:

$$\vec{v}_{1,\text{rel}} = (5\,\text{m/s})\hat{i} - (1.5\,\text{m/s})\hat{i} = \boxed{(3.50\,\text{m/s})\hat{i}}$$

$$\vec{v}_{2,\text{rel}} = (-2\,\text{m/s})\hat{i} - (1.5\,\text{m/s})\hat{i} = \boxed{(-3.50\,\text{m/s})\hat{i}}$$

(*d*) Express the sum of the kinetic energies relative to the center of mass:

$$K_{\text{rel}} = K_{1,\text{rel}} + K_{2,\text{rel}} = \tfrac{1}{2}m_1v_{1,\text{rel}}^2 + \tfrac{1}{2}m_2v_{2,\text{rel}}^2$$

Substitute numerical values and evaluate K_{rel}:

$$K_{\text{rel}} = \tfrac{1}{2}(3\,\text{kg})(3.5\,\text{m/s})^2 + \tfrac{1}{2}(3\,\text{kg})(-3.5\,\text{m/s})^2 = \boxed{36.75\,\text{J}}$$

(*e*) Find K_{cm}:

$$K_{\text{cm}} = \tfrac{1}{2}m_{\text{tot}}v_{\text{cm}}^2 = \tfrac{1}{2}(6\,\text{kg})(1.5\,\text{m/s})^2 = 6.75\,\text{J} = 43.5\,\text{J} - 36.75\,\text{J} = \boxed{K - K_{\text{rel}}}$$

58 •

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right. Use the expression for the total momentum of a system to find the velocity of the center of mass and the definition of relative velocity to express the sum of the kinetic energies relative to the center of mass.

(*a*) Express the sum of the kinetic energies:

$$K = K_1 + K_2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{1}{2}(3\,\text{kg})(5\,\text{m/s})^2 + \tfrac{1}{2}(5\,\text{kg})(3\,\text{m/s})^2 = \boxed{60.0\,\text{J}}$$

(*b*) Relate the velocity of the center of mass of the system to its total momentum:

$$M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Solve for $\vec{v}_{\text{cm}}$:

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3\,\text{kg})(5\,\text{m/s})\hat{\boldsymbol{i}} + (5\,\text{kg})(3\,\text{m/s})\hat{\boldsymbol{i}}}{3\,\text{kg} + 5\,\text{kg}} = \boxed{(3.75\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*c*) The velocity of an object relative to the center of mass is given by:

$$\vec{v}_{rel} = \vec{v} - \vec{v}_{cm}$$

Substitute numerical values and evaluate the relative velocities:

$$\vec{v}_{1,rel} = (5\,\text{m/s})\hat{\boldsymbol{i}} - (3.75\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(1.25\,\text{m/s})\hat{\boldsymbol{i}}}$$

and

$$\vec{v}_{2,rel} = (3\,\text{m/s})\hat{\boldsymbol{i}} - (3.75\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(-0.750\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*d*) Express the sum of the kinetic energies relative to the center of mass:

$$K_{rel} = K_{1,rel} + K_{2,rel} = \tfrac{1}{2}m_1 v_{1,rel}^2 + \tfrac{1}{2}m_2 v_{2,rel}^2$$

Substitute numerical values and evaluate K_{rel}:

$$K_{rel} = \tfrac{1}{2}(3\,\text{kg})(1.25\,\text{m/s})^2 + \tfrac{1}{2}(5\,\text{kg})(-0.75\,\text{m/s})^2 = \boxed{3.75\,\text{J}}$$

(*e*) Find K_{cm}:

$$K_{cm} = \tfrac{1}{2}m_{tot}v_{cm}^2 = \tfrac{1}{2}(8\,\text{kg})(3.75\,\text{m/s})^2 = 56.3\,\text{J} = \boxed{K - K_{rel}}$$

Impulse and Average Force

59 •

Picture the Problem The impulse imparted to the ball by the kicker equals the *change* in the ball's momentum. The impulse is also the product of the average force exerted on the ball by the kicker and the time during which the average force acts.

(*a*) Relate the impulse delivered to the ball to its change in momentum:

$$I = \Delta p = p_f - p_i = mv_f \;\text{ since } v_i = 0$$

Substitute numerical values and evaluate *I*:

$$I = (0.43\,\text{kg})(25\,\text{m/s}) = \boxed{10.8\,\text{N}\cdot\text{s}}$$

(*b*) Express the impulse delivered to the ball as a function of the average force acting on it and solve for and evaluate F_{av}:

$$I = F_{av}\Delta t$$

and

$$F_{av} = \frac{I}{\Delta t} = \frac{10.8\,\text{N}\cdot\text{s}}{0.008\,\text{s}} = \boxed{1.34\,\text{kN}}$$

60 •

Picture the Problem The impulse exerted by the ground on the brick equals the *change* in momentum of the brick and is also the product of the average force exerted by the ground on the brick and the time during which the average force acts.

(*a*) Express the impulse exerted by the ground on the brick:

$$I = \left|\Delta p_{brick}\right| = \left|p_{f,brick} - p_{i,brick}\right|$$

Because $p_{f,brick} = 0$:

$$I = p_{i,brick} = m_{brick}v \qquad (1)$$

Use conservation of energy to determine the speed of the brick at impact:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $U_f = K_i = 0$:

$$K_f - U_i = 0$$

or

$$\tfrac{1}{2}m_{brick}v^2 - m_{brick}gh = 0$$

Solve for v:

$$v = \sqrt{2gh}$$

Substitute in equation (1) to obtain:

$$I = m_{brick}\sqrt{2gh}$$

Substitute numerical values and evaluate I:

$$I = (0.3\,\text{kg})\sqrt{2(9.81\,\text{m/s}^2)(8\,\text{m})} = \boxed{3.76\,\text{N}\cdot\text{s}}$$

(*c*) Express the impulse delivered to the brick as a function of the average force acting on it and solve for and evaluate F_{av}:

$$I = F_{av}\Delta t$$

and

$$F_{av} = \frac{I}{\Delta t} = \frac{3.76\,\text{N}\cdot\text{s}}{0.0013\,\text{s}} = \boxed{2.89\,\text{kN}}$$

***61** •

Picture the Problem The impulse exerted by the ground on the meteorite equals the *change* in momentum of the meteorite and is also the product of the average force exerted by the ground on the meteorite and the time during which the average force acts.

Express the impulse exerted by the ground on the meteorite:

$$I = \Delta p_{\text{meteorite}} = p_{\text{f}} - p_{\text{i}}$$

Relate the kinetic energy of the meteorite to its initial momentum and solve for its initial momentum:

$$K_{\text{i}} = \frac{p_{\text{i}}^2}{2m} \Rightarrow p_{\text{i}} = \sqrt{2mK_{\text{i}}}$$

Express the ratio of the initial and final kinetic energies of the meteorite:

$$\frac{K_{\text{i}}}{K_{\text{f}}} = \frac{\dfrac{p_{\text{i}}^2}{2m}}{\dfrac{p_{\text{f}}^2}{2\text{m}}} = \frac{p_{\text{i}}^2}{p_{\text{f}}^2} = 2$$

Solve for p_{f}:

$$p_{\text{f}} = \frac{p_{\text{i}}}{\sqrt{2}}$$

Substitute in our expression for I and simplify:

$$I = \frac{p_{\text{i}}}{\sqrt{2}} - p_{\text{i}} = p_{\text{i}}\left(\frac{1}{\sqrt{2}} - 1\right)$$
$$= \sqrt{2mK_{\text{i}}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

Because our interest is in its magnitude, evaluate $|I|$:

$$|I| = \left|\sqrt{2(30.8\times10^3\ \text{kg})(617\times10^6\ \text{J})}\left(\frac{1}{\sqrt{2}} - 1\right)\right| = \boxed{1.81\,\text{MN}\cdot\text{s}}$$

Express the impulse delivered to the meteorite as a function of the average force acting on it and solve for and evaluate F_{av}:

$$I = F_{\text{av}}\Delta t$$

and

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{1.81\,\text{MN}\cdot\text{s}}{3\text{s}} = \boxed{0.602\,\text{MN}}$$

62 ••

Picture the Problem The impulse exerted by the bat on the ball equals the *change* in momentum of the ball and is also the product of the average force exerted by the bat on the ball and the time during which the bat and ball were in contact.

(*a*) Express the impulse exerted by the bat on the ball in terms of the change in momentum of the ball:

$$\vec{\boldsymbol{I}} = \Delta\vec{\boldsymbol{p}}_{\text{ball}} = \vec{\boldsymbol{p}}_{\text{f}} - \vec{\boldsymbol{p}}_{\text{i}}$$
$$= mv_{\text{f}}\hat{\boldsymbol{i}} - \left(-mv_{\text{i}}\hat{\boldsymbol{i}}\right) = 2mv\hat{\boldsymbol{i}}$$

where $v = v_{\text{f}} = v_{\text{i}}$

Substitute for *m* and *v* and evaluate *I*:

$$I = 2(0.15\,\text{kg})(20\,\text{m/s}) = \boxed{6.00\,\text{N}\cdot\text{s}}$$

(*b*) Express the impulse delivered to the ball as a function of the average force acting on it and solve for and evaluate F_{av}:

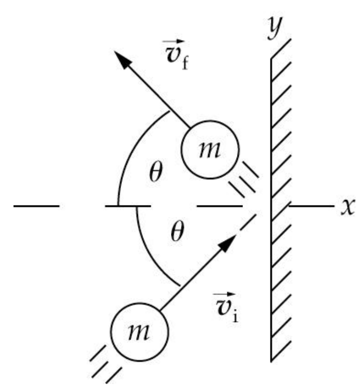
$$I = F_{\text{av}}\Delta t$$

and

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{6.00\,\text{N}\cdot\text{s}}{1.3\,\text{ms}} = \boxed{4.62\,\text{kN}}$$

***63 ••**

Picture the Problem The figure shows the handball just before and immediately after its collision with the wall. Choose a coordinate system in which the positive *x* direction is to the right. The wall changes the momentum of the ball by exerting a force on it during the ball's collision with it. The reaction to this force is the force the ball exerts on the wall. Because these action and reaction forces are equal in magnitude, we can find the average force exerted on the ball by finding the change in momentum of the ball.



Using Newton's 3rd law, relate the average force exerted by the ball on the wall to the average force exerted by the wall on the ball:

$$\vec{F}_{\text{av on wall}} = -\vec{F}_{\text{av on ball}}$$

and

$$F_{\text{av on wall}} = F_{\text{av on ball}} \qquad (1)$$

Relate the average force exerted by the wall on the ball to its change in momentum:

$$\vec{F}_{\text{av on ball}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t}$$

Express $\Delta\vec{v}_x$ for the ball:

$$\Delta\vec{v}_x = v_{\text{f},x}\hat{i} - v_{\text{i},x}\hat{i}$$

or, because $v_{\text{i},x} = v\cos\theta$ and $v_{\text{f},x} = -v\cos\theta$,

$$\Delta\vec{v}_x = -v\cos\theta\,\hat{i} - v\cos\theta\,\hat{i} = -2v\cos\theta\,\hat{i}$$

Substitute in our expression for $\vec{F}_{\text{av on ball}}$:

$$\vec{F}_{\text{av on ball}} = \frac{m\Delta\vec{v}}{\Delta t} = -\frac{2mv\cos\theta}{\Delta t}\hat{i}$$

Evaluate the magnitude of $\vec{F}_{\text{av on ball}}$:

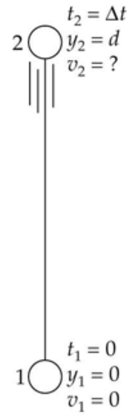
$$F_{\text{av on ball}} = \frac{2mv\cos\theta}{\Delta t} = \frac{2(0.06\,\text{kg})(5\,\text{m/s})\cos 40°}{2\,\text{ms}} = 230\,\text{N}$$

Substitute in equation (1) to obtain:

$$F_{\text{av on wall}} = \boxed{230\,\text{N}}$$

64 ••

Picture the Problem The pictorial representation shows the ball during the interval of time you are exerting a force on it to accelerate it upward. The average force you exert can be determined from the change in momentum of the ball. The change in the velocity of the ball can be found by applying conservation of mechanical energy to its rise in the air once it has left your hand.



(*a*) Relate the average force exerted by your hand on the ball to the change in momentum of the ball:

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{p_2 - p_1}{\Delta t} = \frac{mv_2}{\Delta t}$$

because v_1 and, hence, $p_1 = 0$.

Letting $U_g = 0$ at the initial elevation of your hand, use conservation of mechanical energy to relate the initial kinetic energy of the ball to its potential energy when it is at its highest point:

$$\Delta K + \Delta U = 0$$

or

$$-K_i + U_f = 0$$

since $K_f = U_i = 0$

Substitute for K_f *and* U_i and solve for v_2:

$$-\tfrac{1}{2}mv_2^2 + mgh = 0$$

and

$$v_2 = \sqrt{2gh}$$

Relate Δt to the average speed of the ball while you are throwing it upward:

$$\Delta t = \frac{d}{v_{\text{av}}} = \frac{d}{\frac{v_2}{2}} = \frac{2d}{v_2}$$

Substitute for Δt and v_2 in the expression for F_{av} to obtain:

$$F_{av} = \frac{mgh}{d}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \frac{(0.15\,\text{kg})(9.81\,\text{m/s}^2)(40\,\text{m})}{0.7\,\text{m}} = \boxed{84.1\,\text{N}}$$

(*b*) Express the ratio of the weight of the ball to the average force acting on it:

$$\frac{w}{F_{av}} = \frac{mg}{F_{av}} = \frac{(0.15\,\text{kg})(9.81\,\text{m/s}^2)}{84.1\,\text{N}} < 2\%$$

$$\boxed{\text{Because the weight of the ball is less than 2\% of the average force exerted on the ball, it is reasonable to have neglected its weight.}}$$

65 ••

Picture the Problem Choose a coordinate system in which the direction the ball is moving *after* its collision with the wall is the positive *x* direction. The impulse delivered to the wall or received by the player equals the change in the momentum of the ball. We can find the average forces from the rate of change in the momentum of the ball.

(*a*) Relate the impulse delivered to the wall to the change in momentum of the handball:

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = (0.06\,\text{kg})(8\,\text{m/s})\hat{i} - \left[-(0.06\,\text{kg})(10\,\text{m/s})\hat{i}\right] = \boxed{(1.08\,\text{N}\cdot\text{s})\hat{i}\text{ directed into wall.}}$$

(*b*) Find F_{av} from the change in the ball's momentum:

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{1.08\,\text{N}\cdot\text{s}}{0.003\,\text{s}} = \boxed{360\,\text{N, into wall.}}$$

(*c*) Find the impulse received by the player from the change in momentum of the ball:

$$I = \Delta p_{ball} = m\Delta v = (0.06\,\text{kg})(8\,\text{m/s}) = \boxed{0.480\,\text{N}\cdot\text{s, away from wall.}}$$

(*d*) Relate F_{av} to the change in the ball's momentum:

$$F_{av} = \frac{\Delta p_{ball}}{\Delta t}$$

Express the stopping time in terms of the average speed v_{av} of the ball

$$\Delta t = \frac{d}{v_{av}}$$

and its stopping distance *d*:

Substitute to obtain:

$$F_{av} = \frac{v_{av}\Delta p_{ball}}{d}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \frac{(4\,\text{m/s})(0.480\,\text{N}\cdot\text{s})}{0.5\,\text{m}} = \boxed{3.84\,\text{N, away from wall.}}$$

66 •••

Picture the Problem The average force exerted on the limestone by the droplets of water equals the rate at which momentum is being delivered to the floor. We're given the number of droplets that arrive per minute and can use conservation of mechanical energy to determine their velocity as they reach the floor.

(*a*) Letting *N* represent the rate at which droplets fall, relate F_{av} to the change in the droplet's momentum:

$$F_{av} = \frac{\Delta p_{droplets}}{\Delta t} = N\frac{m\Delta v}{\Delta t}$$

Find the mass of the droplets:

$$m = \rho V = (1\,\text{kg/L})(0.03\,\text{mL}) = 3\times10^{-5}\,\text{kg}$$

Letting $U_g = 0$ at the point of impact of the droplets, use conservation of mechanical energy to relate their speed at impact to their fall distance:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_i = U_f = 0$:

$$\tfrac{1}{2}mv_f^2 - mgh = 0$$

Solve for and evaluate $v = v_f$:

$$v = \sqrt{2gh} = \sqrt{2(9.81\,\text{m/s}^2)(5\,\text{m})} = 9.90\,\text{m/s}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \left(\frac{N}{\Delta t}\right)m\Delta v$$

$$= \left(10\frac{\text{droplets}}{\text{min}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)\times(3\times10^{-5}\,\text{kg})(9.90\,\text{m/s})$$

$$= \boxed{4.95\times10^{-5}\,\text{N}}$$

(*b*) Calculate the ratio of the weight of a droplet to F_{av}:

$$\frac{w}{F_{\text{av}}}=\frac{mg}{F_{\text{av}}}=\frac{(3\times10^{-5}\text{ kg})(9.81\text{ m/s}^2)}{4.95\times10^{-5}\text{ N}}\approx\boxed{6}$$

Collisions in One Dimension

***67** •

Picture the Problem We can apply conservation of momentum to this perfectly inelastic collision to find the after-collision speed of the two cars. The ratio of the transformed kinetic energy to kinetic energy before the collision is the fraction of kinetic energy lost in the collision.

(*a*) Letting *V* be the velocity of the two cars after their collision, apply conservation of momentum to their perfectly inelastic collision:

$$p_{\text{initial}}=p_{\text{final}}$$

or

$$mv_1+mv_2=(m+m)V$$

Solve for and evaluate *V*:

$$V=\frac{v_1+v_2}{2}=\frac{30\text{ m/s}+10\text{ m/s}}{2}=\boxed{20.0\text{ m/s}}$$

(*b*) Express the ratio of the kinetic energy that is lost to the kinetic energy of the two cars before the collision and simplify:

$$\frac{\Delta K}{K_{\text{initial}}}=\frac{K_{\text{final}}-K_{\text{initial}}}{K_{\text{initial}}}=\frac{K_{\text{final}}}{K_{\text{initial}}}-1=\frac{\frac{1}{2}(2m)V^2}{\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2}-1=\frac{2V^2}{v_1^2+v_2^2}-1$$

Substitute numerical values to obtain:

$$\frac{\Delta K}{K_{\text{initial}}}=\frac{2(20\text{ m/s})^2}{(30\text{ m/s})^2+(10\text{ m/s})^2}-1=-0.200$$

$$\boxed{\text{20\% of the initial kinetic energy is transformed into heat, sound, and the deformation of metal.}}$$

68 •

Picture the Problem We can apply conservation of momentum to this perfectly inelastic collision to find the after-collision speed of the two players.

Letting the subscript 1 refer to the running back and the subscript 2 refer to the linebacker, apply conservation of momentum to their perfectly inelastic collision:

$$p_i = p_f$$

or

$$m_1 v_1 = (m_1 + m_2)V$$

Solve for V:

$$V = \frac{m_1}{m_1 + m_2} v_1$$

Substitute numerical values and evaluate V:

$$V = \frac{85\,\text{kg}}{85\,\text{kg} + 105\,\text{kg}}(7\,\text{m/s}) = \boxed{3.13\,\text{m/s}}$$

69 •

Picture the Problem We can apply conservation of momentum to this collision to find the after-collision speed of the 5-kg object. Let the direction the 5-kg object is moving before the collision be the positive direction. We can decide whether the collision was elastic by examining the initial and final kinetic energies of the system.

(*a*) Letting the subscript 5 refer to the 5-kg object and the subscript 2 refer to the 10-kg object, apply conservation of momentum to obtain:

$$p_i = p_f$$

or

$$m_5 v_{i,5} - m_{10} v_{i,10} = m_5 v_{f,5}$$

Solve for $v_{f,5}$:

$$v_{f,5} = \frac{m_5 v_{i,5} - m_{10} v_{i,10}}{m_5}$$

Substitute numerical values and evaluate $v_{f,5}$:

$$v_{f,5} = \frac{(5\,\text{kg})(4\,\text{m/s}) - (10\,\text{kg})(3\,\text{m/s})}{5\,\text{kg}}$$

$$= \boxed{-2.00\,\text{m/s}}$$

where the minus sign means that the 5-kg object is moving to the left after the collision.

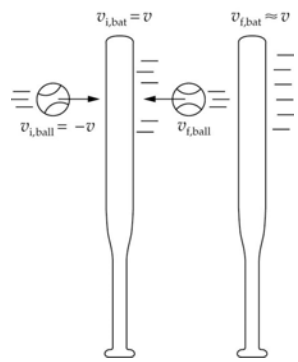
(*b*) Evaluate ΔK for the collision:

$$\Delta K = K_f - K_i = \tfrac{1}{2}(5\,\text{kg})(2\,\text{m/s})^2 - \left[\tfrac{1}{2}(5\,\text{kg})(4\,\text{m/s})^2 + \tfrac{1}{2}(10\,\text{kg})(3\,\text{m/s})^2\right] = -75.0\,\text{J}$$

$$\boxed{\text{Because } \Delta K \neq 0, \text{ the collision was inelastic.}}$$

70 •

Picture the Problem The pictorial representation shows the ball and bat just before and just after their collision. Take the direction the bat is moving to be the positive direction. Because the collision is elastic, we can equate the speeds of recession and approach, with the approximation that $v_{i,bat} \approx v_{f,bat}$ to find $v_{f,ball}$.



Express the speed of approach of the bat and ball:

$$v_{f,bat} - v_{f,ball} = -(v_{i,bat} - v_{i,ball})$$

Because the mass of the bat is much greater than that of the ball:

$$v_{i,bat} \approx v_{f,bat}$$

Substitute to obtain:

$$v_{f,bat} - v_{f,ball} = -(v_{f,bat} - v_{i,ball})$$

Solve for and evaluate $v_{f,ball}$:

$$v_{f,ball} = v_{f,bat} + (v_{f,bat} - v_{i,ball}) = -v_{i,ball} + 2v_{f,bat} = v + 2v = \boxed{3v}$$

***71** ••

Picture the Problem Let the direction the proton is moving before the collision be the positive *x* direction. We can use both conservation of momentum and conservation of mechanical energy to obtain an expression for velocity of the proton after the collision.

(*a*) Use the expression for the total momentum of a system to find v_{cm}:

$$\vec{P} = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

and

$$\vec{v}_{cm} = \frac{m\vec{v}_{p,i}}{m + 12m} = \tfrac{1}{13}(300\,\text{m/s})\hat{i} = \boxed{(23.1\,\text{m/s})\hat{i}}$$

(*b*) Use conservation of momentum to obtain one relation for the final velocities:

$$m_p v_{p,i} = m_p v_{p,f} + m_{nuc} v_{nuc,f} \quad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{nuc,f} - v_{p,f} = -(v_{nuc,i} - v_{p,i}) = v_{p,i} \quad (2)$$

To eliminate $v_{nuc,f}$, solve equation (2) for $v_{nuc,f}$, and substitute the result in equation (1):

$$v_{nuc,f} = v_{p,i} + v_{p,f}$$

$$m_p v_{p,i} = m_p v_{p,f} + m_{nuc}(v_{p,i} + v_{p,f})$$

Solve for and evaluate $v_{p,f}$:

$$v_{p,f} = \frac{m_p - m_{nuc}}{m_p + m_{nuc}} v_{p,i}$$

$$= \frac{m - 12m}{13m}(300\,\text{m/s}) = \boxed{-254\,\text{m/s}}$$

72 ••

Picture the Problem We can use conservation of momentum and the definition of an elastic collision to obtain two equations in v_{2f} and v_{3f} that we can solve simultaneously.

Use conservation of momentum to obtain one relation for the final velocities:

$$m_3 v_{3i} = m_3 v_{3f} + m_2 v_{2f} \quad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{2f} - v_{3f} = -(v_{2i} - v_{3i}) = v_{3i} \quad (2)$$

Solve equation (2) for v_{3f} , substitute in equation (1) to eliminate v_{3f}, and solve for and evaluate v_{2f}:

$$v_{2f} = \frac{2m_3 v_{3i}}{m_2 + m_3} = \frac{2(3\,\text{kg})(4\,\text{m/s})}{2\,\text{kg} + 3\,\text{kg}}$$

$$= \boxed{4.80\,\text{m/s}}$$

Use equation (2) to find v_{3f}:

$$v_{3f} = v_{2f} - v_{3i} = 4.80\,\text{m/s} - 4.00\,\text{m/s}$$

$$= \boxed{0.800\,\text{m/s}}$$

Evaluate K_i and K_f:

$$K_i = K_{3i} = \tfrac{1}{2} m_3 v_{3i}^2 = \tfrac{1}{2}(3\,\text{kg})(4\,\text{m/s})^2$$

$$= 24.0\,\text{J}$$

and

$$K_f = K_{3f} + K_{2f} = \tfrac{1}{2}m_3v_{3f}^2 + \tfrac{1}{2}m_2v_{2f}^2$$
$$= \tfrac{1}{2}(3\,\text{kg})(0.8\,\text{m/s})^2 + \tfrac{1}{2}(2\,\text{kg})(4.8\,\text{m/s})^2$$
$$= 24.0\,\text{J}$$

Because $K_i = K_f$, we can conclude that the values obtained for v_{2f} and v_{3f} are consistent with the collision having been elastic.

73 ••

Picture the Problem We can find the velocity of the center of mass from the definition of the total momentum of the system. We'll use conservation of energy to find the maximum compression of the spring and express the initial (i.e., before collision) and final (i.e., at separation) velocities. Finally, we'll transform the velocities from the center of mass frame of reference to the table frame of reference.

(*a*) Use the definition of the total momentum of a system to relate the initial momenta to the velocity of the center of mass:

$$\vec{P} = \sum_i m_i\vec{v}_i = M\vec{v}_{cm}$$

or

$$m_1v_{1i} = (m_1 + m_2)v_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(2\,\text{kg})(10\,\text{m/s}) + (5\,\text{kg})(3\,\text{m/s})}{2\,\text{kg} + 5\,\text{kg}} = \boxed{5.00\,\text{m/s}}$$

(*b*) Find the kinetic energy of the system at maximum compression ($u_1 = u_2 = 0$):

$$K = K_{cm} = \tfrac{1}{2}Mv_{cm}^2 = \tfrac{1}{2}(7\,\text{kg})(5\,\text{m/s})^2 = 87.5\,\text{J}$$

Use conservation of energy to relate the kinetic energy of the system to the potential energy stored in the spring at maximum compression:

$$\Delta K + \Delta U_s = 0$$

or

$$K_f - K_i + U_{sf} - U_{si} = 0$$

Because $K_f = K_{cm}$ and $U_{si} = 0$:

$$K_{cm} - K_i + \tfrac{1}{2}k(\Delta x)^2 = 0$$

Solve for Δx:

$$\Delta x = \sqrt{\frac{2(K_\text{i} - K_\text{cm})}{k}} = \sqrt{\frac{2\left[\frac{1}{2}m_1v_{1\text{i}}^2 + \frac{1}{2}m_2v_{2\text{i}}^2 - K_\text{cm}\right]}{k}} = \sqrt{\frac{m_1v_{1\text{i}}^2 + m_2v_{2\text{i}}^2 - 2K_\text{cm}}{k}}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \sqrt{\left[\frac{(2\,\text{kg})(10\,\text{m/s})^2 + (5\,\text{kg})(3\,\text{m/s})^2}{1120\,\text{N/m}} - \frac{2(87.5\,\text{J})}{1120\,\text{N/m}}\right]} = \boxed{0.250\,\text{m}}$$

(*c*) Find $u_{1\text{i}}$, $u_{2\text{i}}$, and $u_{1\text{f}}$ for this elastic collision:

$$u_{1\text{i}} = v_{1\text{i}} - v_\text{cm} = 10\,\text{m/s} - 5\,\text{m/s} = 5\,\text{m/s},$$
$$u_{2\text{i}} = v_{2\text{i}} - v_\text{cm} = 3\,\text{m/s} - 5\,\text{m/s} = -2\,\text{m/s},$$
and
$$u_{1\text{f}} = v_{1\text{f}} - v_\text{cm} = 0 - 5\,\text{m/s} = -5\,\text{m/s}$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach and solve for $u_{2\text{f}}$:

$$u_{2\text{f}} - u_{1\text{f}} = -(u_{2\text{i}} - u_{1\text{i}})$$
and
$$u_{2\text{f}} = -u_{2\text{i}} + u_{1\text{i}} + u_{1\text{f}} = -(-2\,\text{m/s}) + 5\,\text{m/s} - 5\,\text{m/s} = 2\,\text{m/s}$$

Transform $u_{1\text{f}}$ and $u_{2\text{f}}$ to the table frame of reference:

$$v_{1\text{f}} = u_{1\text{f}} + v_\text{cm} = -5\,\text{m/s} + 5\,\text{m/s} = \boxed{0}$$
and
$$v_{2\text{f}} = u_{2\text{f}} + v_\text{cm} = 2\,\text{m/s} + 5\,\text{m/s} = \boxed{7.00\,\text{m/s}}$$

***74 ••**

Picture the Problem Let the system include the earth, the bullet, and the sheet of plywood. Then $W_\text{ext} = 0$. Choose the zero of gravitational potential energy to be where the bullet enters the plywood. We can apply both conservation of energy and conservation of momentum to obtain the various physical quantities called for in this problem.

(*a*) Use conservation of mechanical energy after the bullet exits the sheet of plywood to relate its exit speed to the height to which it rises:

$$\Delta K + \Delta U = 0$$
or, because $K_\text{f} = U_\text{i} = 0$,
$$-\tfrac{1}{2}mv_m^2 + mgh = 0$$

Solve for v_m:

$$v_m = \boxed{\sqrt{2gh}}$$

Proceed similarly to relate the initial velocity of the plywood to the height to which it rises:

$$v_M = \boxed{\sqrt{2gH}}$$

(b) Apply conservation of momentum to the collision of the bullet and the sheet of plywood:

$$\vec{\boldsymbol{p}}_\text{i} = \vec{\boldsymbol{p}}_\text{f}$$

or

$$mv_{m\text{i}} = mv_m + Mv_M$$

Substitute for v_m and v_M and solve for $v_{m\text{i}}$:

$$v_{m\text{i}} = \boxed{\sqrt{2gh} + \frac{M}{m}\sqrt{2gH}}$$

(c) Express the initial mechanical energy of the system (i.e., just before the collision):

$$E_\text{i} = \tfrac{1}{2}mv_{m\text{i}}^2$$

$$= \boxed{mg\left[h + \frac{2M}{m}\sqrt{hH} + \left(\frac{M}{m}\right)^2 H\right]}$$

Express the final mechanical energy of the system (i.e., when the bullet and block have reached their maximum heights):

$$E_\text{f} = mgh + MgH = \boxed{g(mh + MH)}$$

(d) Use the work-energy theorem with $W_\text{ext} = 0$ to find the energy dissipated by friction in the inelastic collision:

$$E_\text{f} - E_\text{i} + W_\text{friction} = 0$$

and

$$W_\text{friction} = E_\text{i} - E_\text{f}$$

$$= \boxed{gMH\left[2\sqrt{\frac{h}{H}} + \frac{M}{m} - 1\right]}$$

75 ••

Picture the Problem We can find the velocity of the center of mass from the definition of the total momentum of the system. We'll use conservation of energy to find the speeds of the particles when their separation is least and when they are far apart.

(a) Noting that when the distance between the two particles is least, both move at the same speed, namely v_cm, use the definition of the total momentum of a system to relate the initial momenta to the velocity of

$$\vec{\boldsymbol{P}} = \sum_i m_i \vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_\text{cm}$$

or

$$m_\text{p}v_\text{pi} = (m_\text{p} + m_\alpha)v_\text{cm}\,.$$

the center of mass:

Solve for and evaluate v_{cm}:

$$v_{cm} = v' = \frac{m_p v_{pi} + m_\alpha v_{\alpha i}}{m_1 + m_2} = \frac{mv_0 + 0}{m + 4m}$$
$$= \boxed{0.200\,v_0}$$

(*b*) Use conservation of momentum to obtain one relation for the final velocities:

$$m_p v_0 = m_p v_{pf} + m_\alpha v_{\alpha f} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{pf} - v_{\alpha f} = -\left(v_{pi} - v_{\alpha i}\right) = -v_{pi} \qquad (2)$$

Solve equation (2) for v_{pf} , substitute in equation (1) to eliminate v_{pf}, and solve for $v_{\alpha f}$:

$$v_{\alpha f} = \frac{2m_p v_0}{m_p + m_\alpha} = \frac{2mv_0}{m + 4m} = \boxed{0.400\,v_0}$$

76 •

Picture the Problem Let the numeral 1 denote the electron and the numeral 2 the hydrogen atom. We can find the final velocity of the electron and, hence, the fraction of its initial kinetic energy that is transferred to the atom, by transforming to the center-of-mass reference frame, calculating the post-collision velocity of the electron, and then transforming back to the laboratory frame of reference.

Express *f*, the fraction of the electron’s initial kinetic energy that is transferred to the atom:

$$f = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i}$$
$$= 1 - \frac{\frac{1}{2}m_1 v_{1f}^2}{\frac{1}{2}m_1 v_{1i}^2} = 1 - \left(\frac{v_{1f}}{v_{1i}}\right)^2 \qquad (1)$$

Find the velocity of the center of mass:

$$v_{cm} = \frac{m_1 v_{1i}}{m_1 + m_2}$$

or, because $m_2 = 1840m_1$,

$$v_{cm} = \frac{m_1 v_{1i}}{m_1 + 1840m_1} = \frac{1}{1841}v_{1i}$$

Find the initial velocity of the electron in the center-of-mass reference frame:

$$u_{1i} = v_{1i} - v_{cm} = v_{1i} - \frac{1}{1841}v_{1i}$$
$$= \left(1 - \frac{1}{1841}\right)v_{1i}$$

Find the post-collision velocity of the electron in the center-of-mass reference frame by reversing its velocity:

$$u_{1f} = -u_{1i} = \left(\frac{1}{1841} - 1\right)v_{1i}$$

To find the final velocity of the electron in the original frame, add v_{cm} to its final velocity in the center-of-mass reference frame:

$$v_{1f} = u_{1f} + v_{cm} = \left(\frac{2}{1841} - 1\right)v_{1i}$$

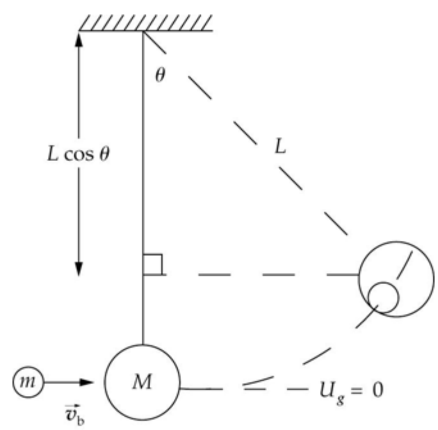
Substitute in equation (1) to obtain:

$$f = 1 - \left(\frac{\left(\frac{2}{1841} - 1\right)v_{1i}}{v_{1i}}\right)^2 = 1 - \left(\frac{2}{1841} - 1\right)^2$$

$$= 2.17\times10^{-3} = \boxed{0.217\%}$$

77 ••

Picture the Problem The pictorial representation shows the bullet about to imbed itself in the bob of the ballistic pendulum and then, later, when the bob plus bullet have risen to their maximum height. We can use conservation of momentum during the collision to relate the speed of the bullet to the initial speed of the bob plus bullet (V). The initial kinetic energy of the bob plus bullet is transformed into gravitational potential energy when they reach their maximum height. Hence we apply conservation of mechanical energy to relate V to the angle through which the bullet plus bob swings and then solve the momentum and energy equations simultaneously for the speed of the bullet.



Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob plus bullet:

$$mv_b = (m+M)V$$

Solve for the speed of the bullet:

$$v_b = \left(1 + \frac{M}{m}\right)V \qquad (1)$$

Use conservation of energy to relate

$$\Delta K + \Delta U = 0$$

the initial kinetic energy of the bullet to the final potential energy of the system:

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f and solve for V:

$$-\tfrac{1}{2}(m+M)V^2 + (m+M)gL(1-\cos\theta) = 0$$

and

$$V = \sqrt{2gL(1-\cos\theta)}$$

Substitute for V in equation (1) to obtain:

$$v_b = \left(1+\frac{M}{m}\right)\sqrt{2gL(1-\cos\theta)}$$

Substitute numerical values and evaluate v_b:

$$v_b = \left(1+\frac{1.5\,\text{kg}}{0.016\,\text{kg}}\right)\sqrt{2(9.81\,\text{m/s}^2)(2.3\,\text{m})(1-\cos 60°)} = \boxed{450\,\text{m/s}}$$

***78 ••**

Picture the Problem We can apply conservation of momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the colliding objects that we can solve for v_{1f} and v_{2f}.

Apply conservation of momentum to the elastic collision of the particles to obtain:

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i} \qquad (1)$$

Relate the initial and final kinetic energies of the particles in an elastic collision:

$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 = \tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2(v_{2f}-v_{2i})(v_{2f}+v_{2i}) = m_1(v_{1i}-v_{1f})(v_{1i}+v_{1f}) \qquad (2)$$

Rearrange equation (1) to obtain:

$$m_2(v_{2f}-v_{2i}) = m_1(v_{1i}-v_{1f}) \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \qquad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$(m_1+m_2)v_{1f} = (m_1-m_2)v_{1i} + 2m_2v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f} = \boxed{\frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}}$$

Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$(m_1 + m_2) v_{2f} = (m_2 - m_1) v_{2i} + 2m_1 v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f} = \boxed{\frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}}$$

Remarks: Note that the velocities satisfy the condition that $\boldsymbol{v}_{2f} - \boldsymbol{v}_{1f} = -(\boldsymbol{v}_{2i} - \boldsymbol{v}_{1i})$. This verifies that the speed of recession equals the speed of approach.

79 ••

Picture the Problem As in this problem, Problem 78 involves an elastic, one-dimensional collision between two objects. Both solutions involve using the conservation of momentum equation $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$ and the elastic collision equation $v_{1f} - v_{2f} = v_{2i} - v_{1i}$. In part (*a*) we can simply set the masses equal to each other and substitute in the equations in Problem 78 to show that the particles "swap" velocities. In part (*b*) we can divide the numerator and denominator of the equations in Problem 78 by m_2 and use the condition that $m_2 >> m_1$ to show that $v_{1f} \approx -v_{1i} + 2v_{2i}$ and $v_{2f} \approx v_{2i}$.

(*a*) From Problem 78 we have:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \quad (1)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \quad (2)$$

Set $m_1 = m_2 = m$ to obtain:

$$v_{1f} = \frac{2m}{m + m} v_{2i} = \boxed{v_{2i}}$$

and

$$v_{2f} = \frac{2m}{m + m} v_{1i} = \boxed{v_{1i}}$$

(*b*) Divide the numerator and denominator of both terms in equation (1) by m_2 to obtain:

$$v_{1f} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} v_{1i} + \frac{2}{\frac{m_1}{m_2} + 1} v_{2i}$$

If $m_2 >> m_1$:

$$v_{1f} \approx \boxed{-v_{1i} + 2v_{2i}}$$

Divide the numerator and denominator of both terms in equation (2) by m_2 to obtain:

$$v_{2f} = \frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}v_{1i} + \frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}v_{2i}$$

If $m_2 >> m_1$:

$$v_{2f} \approx \boxed{v_{2i}}$$

Remarks: Note that, in both parts of this problem, the velocities satisfy the condition that $v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$. This verifies that the speed of recession equals the speed of approach.

Perfectly Inelastic Collisions and the Ballistic Pendulum

80 ••

Picture the Problem Choose $U_g = 0$ at the bob's equilibrium position. Momentum is conserved in the collision of the bullet with bob and the initial kinetic energy of the bob plus bullet is transformed into gravitational potential energy as it swings up to the top of the circle. If the bullet plus bob just makes it to the top of the circle with zero speed, it will swing through a complete circle.

Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob plus bullet:

$$m_1 v = (m_1 + m_2)V$$

Solve for the speed of the bullet:

$$v = \left(1 + \frac{m_2}{m_1}\right)V \qquad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bob plus bullet to their potential energy at the top of the circle:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f:

$$-\tfrac{1}{2}(m_1 + m_2)V^2 + (m_1 + m_2)g(2L) = 0$$

Solve for V:

$$V = \sqrt{gL}$$

Substitute for V in equation (1) and simplify to obtain:

$$v = \boxed{\left(1 + \frac{m_2}{m_1}\right)\sqrt{gL}}$$

***81 ••**

Picture the Problem Choose $U_g = 0$ at the equilibrium position of the ballistic pendulum. Momentum is conserved in the collision of the bullet with the bob and kinetic energy is transformed into gravitational potential energy as the bob swings up to its maximum height.

Letting V represent the initial speed of the bob as it begins its upward swing, use conservation of momentum to relate this speed to the speeds of the bullet just before and after its collision with the bob:

$$m_1 v = m_1\left(\tfrac{1}{2}v\right) + m_2 V$$

Solve for the speed of the bob:

$$V = \frac{m_1}{2m_2}v \qquad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bob to its potential energy at its maximum height:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f:

$$-\tfrac{1}{2}m_2 V^2 + m_2 gh = 0$$

Solve for h:

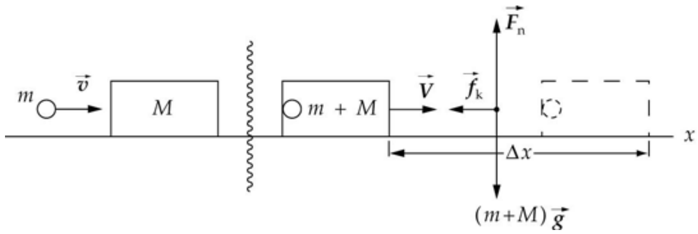
$$h = \frac{V^2}{2g} \qquad (2)$$

Substitute V from equation (1) in equation (2) and simplify to obtain:

$$h = \frac{\left(\frac{m_1}{2m_2}v\right)^2}{2g} = \boxed{\frac{v^2}{8g}\left(\frac{m_1}{m_2}\right)^2}$$

82 •

Picture the Problem Let the mass of the bullet be m, that of the wooden block M, the pre-collision velocity of the bullet v, and the post-collision velocity of the block+bullet be V. We can use conservation of momentum to find the velocity of the block with the bullet imbedded in it just after their perfectly inelastic collision. We can use Newton's 2^{nd} law to find the acceleration of the sliding block and a constant-acceleration equation to find the distance the block slides.



Using a constant-acceleration equation, relate the velocity of the block+bullet just after their collision to their acceleration and displacement before stopping:

$$0 = V^2 + 2a\Delta x$$

because the final velocity of the block+bullet is zero.

Solve for the distance the block slides before coming to rest:

$$\Delta x = -\frac{V^2}{2a} \quad (1)$$

Use conservation of momentum to relate the pre-collision velocity of the bullet to the post-collision velocity of the block+bullet:

$$mv = (m+M)V$$

Solve for *V*:

$$V = \frac{m}{m+M}v$$

Substitute in equation (1) to obtain:

$$\Delta x = -\frac{1}{2a}\left(\frac{m}{m+M}v\right)^2 \quad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the block+bullet (see the FBD in the diagram):

$$\sum F_x = -f_k = (m+M)a \quad (3)$$

and

$$\sum F_y = F_n - (m+M)g = 0 \quad (4)$$

Use the definition of the coefficient of kinetic friction and equation (4) to obtain:

$$f_k = \mu_k F_n = \mu_k (m+M)g$$

Substitute in equation (3):

$$-\mu_k (m+M)g = (m+M)a$$

Solve for *a* to obtain:

$$a = -\mu_k g$$

Substitute in equation (2) to obtain:

$$\Delta x = \frac{1}{2\mu_k g}\left(\frac{m}{m+M}v\right)^2$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{1}{2(0.22)(9.81\,\text{m/s}^2)}\left(\frac{0.0105\,\text{kg}}{0.0105\,\text{kg}+10.5\,\text{kg}}(750\,\text{m/s})\right)^2 = \boxed{0.130\,\text{m}}$$

83 ••

Picture the Problem The collision of the ball with the box is perfectly inelastic and we can find the speed of the box-and-ball immediately after their collision by applying conservation of momentum. If we assume that the kinetic friction force is constant, we can use a constant-acceleration equation to find the acceleration of the box and ball combination and the definition of μ_k to find its value.

Using its definition, express the coefficient of kinetic friction of the table:

$$\mu_k = \frac{f_k}{F_n} = \frac{(M+m)|a|}{(M+m)g} = \frac{|a|}{g} \quad (1)$$

Use conservation of momentum to relate the speed of the ball just before the collision to the speed of the ball+box immediately after the collision:

$$MV = (m+M)v$$

Solve for v:

$$v = \frac{MV}{m+M} \quad (2)$$

Use a constant-acceleration equation to relate the sliding distance of the ball+box to its initial and final velocities and its acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x$$

or, because $v_f = 0$ and $v_i = v$,

$$0 = v^2 + 2a\Delta x$$

Solve for a:

$$a = -\frac{v^2}{2\Delta x}$$

Substitute in equation (1) to obtain:

$$\mu_k = \frac{v^2}{2g\Delta x}$$

Use equation (2) to eliminate v:

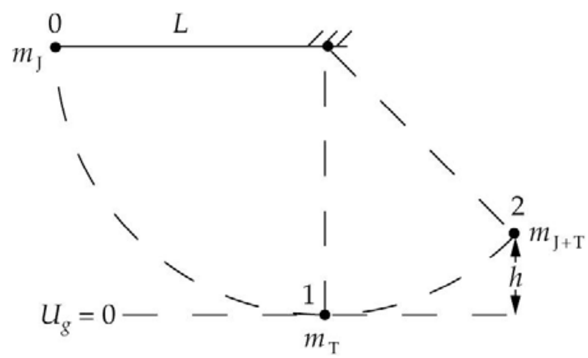
$$\mu_k = \frac{1}{2g\Delta x}\left(\frac{MV}{m+M}\right)^2 = \frac{1}{2g\Delta x}\left(\frac{V}{\frac{m}{M}+1}\right)^2$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{1}{2(9.81\,\text{m/s}^2)(0.52\,\text{m})}\left(\frac{1.3\,\text{m/s}}{\frac{0.327\,\text{kg}}{0.425\,\text{kg}}+1}\right)^2 = \boxed{0.0529}$$

***84 ••**

Picture the Problem Jane's collision with Tarzan is a perfectly inelastic collision. We can find her speed v_1 just before she grabs Tarzan from conservation of energy and their speed V just after she grabs him from conservation of momentum. Their kinetic energy just after their collision will be transformed into gravitational potential energy when they have reached their greatest height h.



Use conservation of energy to relate the potential energy of Jane and Tarzan at their highest point (2) to their kinetic energy immediately after Jane grabbed Tarzan:

$$U_2 = K_1$$

or

$$m_{J+T}gh = \tfrac{1}{2}m_{J+T}V^2$$

Solve for h to obtain:

$$h = \frac{V^2}{2g} \qquad (1)$$

Use conservation of momentum to relate Jane's velocity just before she collides with Tarzan to their velocity just after their perfectly inelastic collision:

$$m_J v_1 = m_{J+T}V$$

Solve for V:

$$V = \frac{m_J}{m_{J+T}}v_1 \qquad (2)$$

Apply conservation of energy to relate Jane's kinetic energy at 1 to her potential energy at 0:

$$K_1 = U_0$$

or

$$\tfrac{1}{2}m_J v_1^2 = m_J gL$$

Solve for v_1:

$$v_1 = \sqrt{2gL}$$

Substitute in equation (2) to obtain:

$$V = \frac{m_J}{m_{J+T}}\sqrt{2gL}$$

Substitute in equation (1) and simplify:

$$h = \frac{1}{2g}\left(\frac{m_J}{m_{J+T}}\right)^2 2gL = \left(\frac{m_J}{m_{J+T}}\right)^2 L$$

Substitute numerical values and evaluate h:

$$h = \left(\frac{54\,\text{kg}}{54\,\text{kg} + 82\,\text{kg}}\right)^2 (25\,\text{m}) = \boxed{3.94\,\text{m}}$$

Exploding Objects and Radioactive Decay

85 ••

Picture the Problem This nuclear reaction is $^4\text{Be} \rightarrow 2\alpha + 1.5{\times}10^{-14}$ J. In order to conserve momentum, the alpha particles will have move in opposite directions with the same velocities. We'll use conservation of energy to find their speeds.

Letting E represent the energy released in the reaction, express conservation of energy for this process:

$$2K_\alpha = 2\left(\tfrac{1}{2}m_\alpha v_\alpha^2\right) = E$$

Solve for v_α:

$$v_\alpha = \sqrt{\frac{E}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$v_\alpha = \sqrt{\frac{1.5\times10^{-14}\text{ J}}{6.68\times10^{-27}\text{ kg}}} = \boxed{1.50\times10^{6}\text{ m/s}}$$

86 ••

Picture the Problem This nuclear reaction is $^5\text{Li} \rightarrow \alpha + \text{p} + 3.15 \times 10^{-13}$ J. To conserve momentum, the alpha particle and proton must move in opposite directions. We'll apply both conservation of energy and conservation of momentum to find the speeds of the proton and alpha particle.

Use conservation of momentum in this process to express the alpha particle's velocity in terms of the proton's:

$$p_i = p_f = 0$$

and

$$0 = m_p v_p - m_\alpha v_\alpha$$

Solve for v_α and substitute for m_α to obtain:

$$v_\alpha = \frac{m_p}{m_\alpha} v_p = \frac{m_p}{4m_p} v_p = \tfrac{1}{4} v_p$$

Letting E represent the energy released in the reaction, apply conservation of energy to the process:

$$K_p + K_\alpha = E$$

or

$$\tfrac{1}{2} m_p v_p^2 + \tfrac{1}{2} m_\alpha v_\alpha^2 = E$$

Substitute for v_α:

$$\tfrac{1}{2} m_p v_p^2 + \tfrac{1}{2} m_\alpha \left(\tfrac{1}{4} v_p\right)^2 = E$$

Solve for v_p and substitute for m_α to obtain:

$$v_p = \sqrt{\frac{32E}{16m_p + m_\alpha}} = \sqrt{\frac{32E}{16m_p + 4m_p}}$$

Substitute numerical values and evaluate v_p:

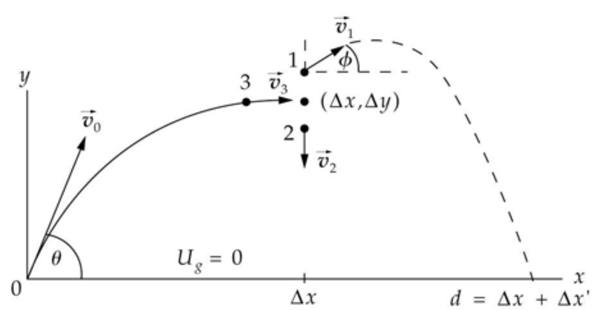
$$v_p = \sqrt{\frac{32\left(3.15\times10^{-13}\text{ J}\right)}{20\left(1.67\times10^{-27}\text{ kg}\right)}} = \boxed{1.74\times10^{7}\text{ m/s}}$$

Use the relationship between v_p and v_α to obtain v_α:

$$v_\alpha = \tfrac{1}{4} v_p = \tfrac{1}{4}\left(1.74\times10^{7}\text{ m/s}\right) = \boxed{4.34\times10^{6}\text{ m/s}}$$

87 •••

Picture the Problem The pictorial representation shows the projectile at its maximum elevation and is moving horizontally. It also shows the two fragments resulting from the explosion. We chose the system to include the projectile and the earth so that no external forces act to change the momentum of the system during the explosion. With this choice of system we can also use conservation of energy to determine the elevation of the projectile when it explodes. We'll also find it useful to use constant-acceleration equations in our description of the motion of the projectile and its fragments.



(*a*) Use conservation of momentum to relate the velocity of the projectile before its explosion to the velocities of its two parts after the explosion:

$$\vec{p}_i = \vec{p}_f$$
$$m_3\vec{v}_3 = m_1\vec{v}_1 + m_2\vec{v}_2$$
$$m_3v_3\hat{i} = m_1v_{x1}\hat{i} + m_1v_{y1}\hat{j} - m_2v_{y2}\hat{j}$$

The only way this equality can hold is if:

$$m_3v_3 = m_1v_{x1}$$
and
$$m_1v_{y1} = m_2v_{y2}$$

Express v_3 in terms of v_0 and substitute for the masses to obtain:

$$v_{x1} = 3v_3 = 3v_0\cos\theta$$
$$= 3(120\,\text{m/s})\cos 30° = 312\,\text{m/s}$$
and
$$v_{y1} = 2v_{y2} \quad (1)$$

Using a constant-acceleration equation with the downward direction positive, relate v_{y2} to the time it takes the 2-kg fragment to hit the ground:

$$\Delta y = v_{y2}\Delta t + \tfrac{1}{2}g(\Delta t)^2$$
$$v_{y2} = \frac{\Delta y - \frac{1}{2}g(\Delta t)^2}{\Delta t} \quad (2)$$

With $U_g = 0$ at the launch site, apply conservation of energy to the climb of the projectile to its maximum elevation:

$$\Delta K + \Delta U = 0$$
Because $K_f = U_i = 0$, $-K_i + U_f = 0$
or
$$-\tfrac{1}{2}m_3v_{y0}^2 + m_3g\Delta y = 0$$

Solve for Δy:

$$\Delta y = \frac{v_{y0}^2}{2g} = \frac{(v_0\sin 30°)^2}{2g}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{[(120\,\text{m/s})\sin 30°]^2}{2(9.81\,\text{m/s}^2)} = 183.5\,\text{m}$$

Substitute in equation (2) and evaluate v_{y2}:

$$v_{y2} = \frac{183.5\,\text{m} - \frac{1}{2}(9.81\,\text{m/s}^2)(3.6\,\text{s})^2}{3.6\,\text{s}}$$
$$= 33.3\,\text{m/s}$$

Substitute in equation (1) and evaluate v_{y1}:

$$v_{y1} = 2(33.3\,\text{m/s}) = 66.6\,\text{m/s}$$

Express $\vec{v}_1$ in vector form:

$$\vec{v}_1 = v_{x1}\hat{i} + v_{y1}\hat{j} = \boxed{(312\,\text{m/s})\hat{i} + (66.6\,\text{m/s})\hat{j}}$$

(*b*) Express the total distance *d* traveled by the 1-kg fragment:

$$d = \Delta x + \Delta x' \qquad (3)$$

Relate Δx to v_0 and the time-to-explosion:

$$\Delta x = (v_0 \cos\theta)(\Delta t_{\text{exp}}) \qquad (4)$$

Using a constant-acceleration equation, express Δt_{exp}:

$$\Delta t_{\text{exp}} = \frac{v_{y0}}{g} = \frac{v_0 \sin\theta}{g}$$

Substitute numerical values and evaluate Δt_{exp}:

$$\Delta t_{\text{exp}} = \frac{(120\,\text{m/s})\sin 30°}{9.81\,\text{m/s}^2} = 6.12\,\text{s}$$

Substitute in equation (4) and evaluate Δx:

$$\Delta x = (120\,\text{m/s})(\cos 30°)(6.12\,\text{s}) = 636.5\,\text{m}$$

Relate the distance traveled by the 1-kg fragment after the explosion to the time it takes it to reach the ground:

$$\Delta x' = v_{x1}\Delta t'$$

Using a constant-acceleration equation, relate the time $\Delta t'$ for the 1-kg fragment to reach the ground to its initial speed in the *y* direction and the distance to the ground:

$$\Delta y = v_{y1}\Delta t' - \tfrac{1}{2} g (\Delta t')^2$$

Substitute to obtain the quadratic equation:

$$(\Delta t')^2 - (13.6\,\text{s})\Delta t' - 37.4\,\text{s}^2 = 0$$

Solve the quadratic equation to find $\Delta t'$:

$$\Delta t' = 15.9\text{ s}$$

Substitute in equation (3) and evaluate *d*:

$$d = \Delta x + \Delta x' = \Delta x + v_{x1}\Delta t' = 636.5\,\text{m} + (312\,\text{m/s})(15.9\,\text{s}) = \boxed{5.61\,\text{km}}$$

(*c*) Express the energy released in the explosion:

$$E_{exp} = \Delta K = K_f - K_i \quad (5)$$

Find the kinetic energy of the fragments after the explosion:

$$K_f = K_1 + K_2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$
$$= \tfrac{1}{2}(1\,\text{kg})\left[(312\,\text{m/s})^2 + (66.6\,\text{m/s})^2\right]$$
$$+ \tfrac{1}{2}(2\,\text{kg})(33.3\,\text{m/s})^2$$
$$= 52.0\,\text{kJ}$$

Find the kinetic energy of the projectile before the explosion:

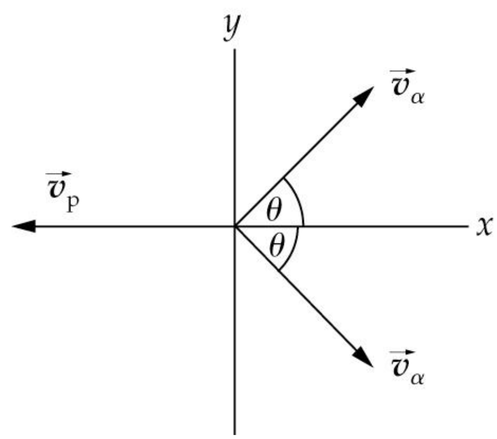
$$K_i = \tfrac{1}{2}m_3v_3^2 = \tfrac{1}{2}m_3(v_0\cos\theta)^2$$
$$= \tfrac{1}{2}(3\,\text{kg})\left[(120\,\text{m/s})\cos 30°\right]^2$$
$$= 16.2\,\text{kJ}$$

Substitute in equation (5) to determine the energy released in the explosion:

$$E_{exp} = K_f - K_i = 52.0\,\text{kJ} - 16.2\,\text{kJ}$$
$$= \boxed{35.8\,\text{kJ}}$$

***88** •••

Picture the Problem This nuclear reaction is $^9\text{B} \rightarrow 2\alpha + \text{p} + 4.4\times10^{-14}$ J. Assume that the proton moves in the –*x* direction as shown in the figure. The sum of the kinetic energies of the decay products equals the energy released in the decay. We'll use conservation of momentum to find the angle between the velocities of the proton and the alpha particles. Note that $v_\alpha = v_\alpha{}'$.



Express the energy released to the kinetic energies of the decay products:

$$K_p + 2K_\alpha = E_{rel}$$

or

$$\tfrac{1}{2}m_pv_p^2 + 2\left(\tfrac{1}{2}m_\alpha v_\alpha^2\right) = E_{rel}$$

Solve for v_α:

$$v_\alpha = \sqrt{\frac{E_{rel} - \tfrac{1}{2}m_pv_p^2}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$v_\alpha = \sqrt{\frac{4.4\times10^{-14}\text{ J}}{6.68\times10^{-27}\text{ kg}} - \frac{\frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)\left(6\times10^{6}\text{ m/s}\right)^2}{6.68\times10^{-27}\text{ kg}}} = \boxed{1.44\times10^{6}\text{ m/s}}$$

Given that the boron isotope was at rest prior to the decay, use conservation of momentum to relate the momenta of the decay products:

$$\vec{p}_\text{f} = \vec{p}_\text{i} = 0 \Rightarrow p_{x\text{f}} = 0$$
$$\therefore 2\left(m_\alpha v_\alpha \cos\theta\right) - m_\text{p} v_\text{p} = 0$$
or
$$2\left(4m_\text{p} v_\alpha \cos\theta\right) - m_\text{p} v_\text{p} = 0$$

Solve for θ:

$$\theta = \cos^{-1}\left[\frac{v_\text{p}}{8v_\alpha}\right]$$
$$= \cos^{-1}\left[\frac{6\times10^{6}\text{ m/s}}{8\left(1.44\times10^{6}\text{ m/s}\right)}\right] = \pm58.7°$$

Let θ' equal the angle the velocities of the alpha particles make with that of the proton:

$$\theta' = \pm\left(180° - 58.7°\right)$$
$$= \boxed{\pm121°}$$

Coefficient of Restitution

89 •

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which the ball was dropped and the height to which it rebounded by using conservation of mechanical energy.

Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_\text{rec}}{v_\text{app}}$$

Letting $U_\text{g} = 0$ at the surface of the steel plate, apply conservation of energy to express the velocity of approach:

$$\Delta K + \Delta U = 0$$
Because $K_\text{i} = U_\text{f} = 0$,
$$K_\text{f} - U_\text{i} = 0$$
or
$$\tfrac{1}{2} m v_\text{app}^2 - mgh_\text{app} = 0$$

Solve for v_app:

$$v_\text{app} = \sqrt{2gh_\text{app}}$$

In like manner, show that: $v_{rec} = \sqrt{2gh_{rec}}$

Substitute in the equation for *e* to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute numerical values and evaluate *e*:

$$e = \sqrt{\frac{2.5\,\text{m}}{3\,\text{m}}} = \boxed{0.913}$$

***90** •

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which an object was dropped and the height to which it rebounded by using conservation of mechanical energy.

Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{rec}}{v_{app}}$$

Letting $U_g = 0$ at the surface of the steel plate, apply conservation of energy to express the velocity of approach:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

or

$$\tfrac{1}{2}mv_{app}^2 - mgh_{app} = 0$$

Solve for v_{app}:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute in the equation for *e* to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Find e_{min}:

$$e_{min} = \sqrt{\frac{173\,\text{cm}}{254\,\text{cm}}} = 0.825$$

Find e_{max}:

$$e_{max} = \sqrt{\frac{183\,\text{cm}}{254\,\text{cm}}} = 0.849$$

and $\boxed{0.825 \le e \le 0.849}$

91 •

Picture the Problem Because the rebound kinetic energy is proportional to the rebound height, the percentage of mechanical energy lost in one bounce can be inferred from knowledge of the rebound height. The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which an object was dropped and the height to which it rebounded by using conservation of mechanical energy.

(*a*) We know, from conservation of energy, that the kinetic energy of an object dropped from a given height *h* is proportional to *h*:

$K \propto h.$

If, for each bounce of the ball, $h_{rec} = 0.8h_{app}$:

$\boxed{20\% \text{ of its mechanical energy is lost.}}$

(*b*) Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{rec}}{v_{app}}$$

Letting $U_g = 0$ at the surface from which the ball is rebounding, apply conservation of energy to express the velocity of approach:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

or

$$\tfrac{1}{2}mv_{app}^2 - mgh_{app} = 0$$

Solve for v_{app}:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute in the equation for *e* to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute for $\frac{h_{rec}}{h_{app}}$ to obtain:

$$e = \sqrt{0.8} = \boxed{0.894}$$

92 ••

Picture the Problem Let the numeral 2 refer to the 2-kg object and the numeral 4 to the 4-kg object. Choose a coordinate system in which the direction the 2-kg object is moving before the collision is the positive *x* direction and let the system consist of the earth, the surface on which the objects slide, and the objects. Then we can use conservation of momentum to find the velocity of the recoiling 4-kg object. We can find the energy transformed in the collision by calculating the difference between the kinetic energies before and after the collision and the coefficient of restitution from its definition.

(*a*) Use conservation of momentum in one dimension to relate the initial and final momenta of the participants in the collision:

$$\vec{\boldsymbol{p}}_\text{i} = \vec{\boldsymbol{p}}_\text{f}$$

or

$$m_2 v_{2\text{i}} = m_4 v_{4\text{f}} - m_2 v_{2\text{f}}$$

Solve for and evaluate the final velocity of the 4-kg object:

$$v_{4\text{f}} = \frac{m_2 v_{2\text{i}} + m_2 v_{2\text{f}}}{m_4} = \frac{(2\,\text{kg})(6\,\text{m/s} + 1\,\text{m/s})}{4\,\text{kg}} = \boxed{3.50\,\text{m/s}}$$

(*b*) Express the energy lost in terms of the kinetic energies before and after the collision:

$$\begin{aligned} E_\text{lost} &= K_\text{i} - K_\text{f} \\ &= \tfrac{1}{2} m_2 v_{2\text{i}}^2 - \left(\tfrac{1}{2} m_2 v_{2\text{f}}^2 + \tfrac{1}{2} m_4 v_{4\text{f}}^2\right) \\ &= \tfrac{1}{2}\left[m_2\left(v_{2\text{i}}^2 - v_{2\text{f}}^2\right) - m_4 v_{4\text{f}}^2\right] \end{aligned}$$

Substitute numerical values and evaluate E_lost:

$$E_\text{lost} = \tfrac{1}{2}\left[\left((2\,\text{kg})\left\{(6\,\text{m/s})^2 - (1\,\text{m/s})^2\right\}\right) - (4\,\text{kg})(3.5\,\text{m/s})^2\right] = \boxed{10.5\,\text{J}}$$

(*c*) Use the definition of the coefficient of restitution:

$$e = \frac{v_\text{rec}}{v_\text{app}} = \frac{v_{4\text{f}} - v_{2\text{f}}}{v_{2\text{i}}} = \frac{3.5\,\text{m/s} - (-1\,\text{m/s})}{6\,\text{m/s}} = \boxed{0.750}$$

93 ••

Picture the Problem Let the numeral 2 refer to the 2-kg block and the numeral 3 to the 3-kg block. Choose a coordinate system in which the direction the blocks are moving before the collision is the positive *x* direction and let the system consist of the earth, the surface on which the blocks move, and the blocks. Then we can use conservation of momentum find the velocity of the 2-kg block after the collision. We can find the coefficient of restitution from its definition.

(*a*) Use conservation of momentum in one dimension to relate the initial and final momenta of the participants in the collision:

$$\vec{p}_i = \vec{p}_f$$

or

$$m_2 v_{2i} + m_3 v_{3i} = m_2 v_{2f} + m_3 v_{3f}$$

Solve for the final velocity of the 2-kg object:

$$v_{2f} = \frac{m_2 v_{2i} + m_3 v_{3i} - m_3 v_{3f}}{m_2}$$

Substitute numerical values and evaluate v_{2f}:

$$v_{2f} = \frac{(2\,\text{kg})(5\,\text{m/s}) + (3\,\text{kg})(2\,\text{m/s} - 4.2\,\text{m/s})}{2\,\text{kg}} = \boxed{1.70\,\text{m/s}}$$

(*b*) Use the definition of the coefficient of restitution:

$$e = \frac{v_{rec}}{v_{app}} = \frac{v_{3f} - v_{2f}}{v_{2i} - v_{3i}} = \frac{4.2\,\text{m/s} - 1.7\,\text{m/s}}{5\,\text{m/s} - 2\,\text{m/s}}$$

$$= \boxed{0.833}$$

Collisions in Three Dimensions

***94 ••**

Picture the Problem We can use the definition of the magnitude of a vector and the definition of the dot product to establish the result called for in (*a*). In part (*b*) we can use the result of part (*a*), the conservation of momentum, and the definition of an elastic collision (kinetic energy is conserved) to show that the particles separate at right angles.

(*a*) Find the dot product of $\vec{B} + \vec{C}$ with itself:

$$\left(\vec{B} + \vec{C}\right) \cdot \left(\vec{B} + \vec{C}\right) = B^2 + C^2 + 2\vec{B} \cdot \vec{C}$$

Because $\vec{A} = \vec{B} + \vec{C}$:

$$A^2 = \left|\vec{B} + \vec{C}\right|^2 = \left(\vec{B} + \vec{C}\right) \cdot \left(\vec{B} + \vec{C}\right)$$

Substitute to obtain:

$$\boxed{A^2 = B^2 + C^2 + 2\vec{B} \cdot \vec{C}}$$

(*b*) Apply conservation of momentum to the collision of the particles:

$$\vec{p}_1 + \vec{p}_2 = \vec{P}$$

Form the dot product of each side of this equation with itself to obtain:

$$\left(\vec{p}_1 + \vec{p}_2\right) \cdot \left(\vec{p}_1 + \vec{p}_2\right) = \vec{P} \cdot \vec{P}$$

or

$$p_1^2 + p_2^2 + 2\vec{p}_1 \cdot \vec{p}_2 = P^2 \qquad (1)$$

Apply the definition of an elastic collision to obtain:

$$\frac{p_1^2}{2m} + \frac{p_2^2}{2m} = \frac{P^2}{2m}$$

or

$$p_1^2 + p_2^2 = P^2 \qquad (2)$$

Subtract equation (1) from equation (2) to obtain:

$$2\vec{p}_1 \cdot \vec{p}_2 = 0 \text{ or } \boxed{\vec{p}_1 \cdot \vec{p}_2 = 0}$$

i.e., the particles move apart along paths that are at right angles to each other.

95 •

Picture the Problem Let the initial direction of motion of the cue ball be the positive *x* direction. We can apply conservation of energy to determine the angle the cue ball makes with the positive *x* direction and the conservation of momentum to find the final velocities of the cue ball and the eight ball.

(*a*) Use conservation of energy to relate the velocities of the collision participants before and after the collision:

$$\tfrac{1}{2}mv_{\text{ci}}^2 = \tfrac{1}{2}mv_{\text{cf}}^2 + \tfrac{1}{2}mv_8^2$$

or

$$v_{\text{ci}}^2 = v_{\text{cf}}^2 + v_8^2$$

This Pythagorean relationship tells us that $\vec{v}_{\text{ci}}$, $\vec{v}_{\text{cf}}$, and $\vec{v}_8$ form a right triangle. Hence:

$$\theta_{\text{cf}} + \theta_8 = 90°$$

and

$$\theta_{\text{cf}} = \boxed{60°}$$

(*b*) Use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$\vec{p}_{\text{xi}} = \vec{p}_{\text{xf}}$$

or

$$mv_{\text{ci}} = mv_{\text{cf}}\cos\theta_{\text{cf}} + mv_8\cos\theta_8$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$\vec{p}_{\text{yi}} = \vec{p}_{\text{yf}}$$

or

$$0 = mv_{\text{cf}}\sin\theta_{\text{cf}} + mv_8\sin\theta_8$$

Solve these equations simultaneously to obtain:

$$v_{\text{cf}} = \boxed{2.50\,\text{m/s}}$$

and

$$v_8 = \boxed{4.33\,\text{m/s}}$$

96 ••

Picture the Problem We can find the final velocity of the object whose mass is M_1 by using the conservation of momentum. Whether the collision was elastic can be decided by examining the difference between the initial and final kinetic energy of the interacting objects.

(*a*) Use conservation of momentum to relate the initial and final velocities of the two objects:

$$\vec{p}_i = \vec{p}_f$$

or

$$mv_0\hat{i} + 2m\left(\tfrac{1}{2}v_0\hat{j}\right) = 2m\left(\tfrac{1}{4}v_0\hat{i}\right) + m\vec{v}_{1f}$$

Simplify to obtain:

$$v_0\hat{i} + v_0\hat{j} = \tfrac{1}{2}v_0\hat{i} + \vec{v}_{1f}$$

Solve for $\vec{v}_{1f}$:

$$\vec{v}_{1f} = \boxed{\tfrac{1}{2}v_0\hat{i} + v_0\hat{j}}$$

(*b*) Express the difference between the kinetic energy of the system before the collision and its kinetic energy after the collision:

$$\begin{aligned}\Delta E &= K_i - K_f = K_{1i} + K_{2i} - (K_{1f} + K_{2f}) = \tfrac{1}{2}\left[M_1v_{1i}^2 + M_2v_{2i}^2 - M_1v_{1f}^2 - M_2v_{2f}^2\right] \\ &= \tfrac{1}{2}\left[mv_{1i}^2 + 2mv_{2i}^2 - mv_{1f}^2 - 2mv_{2f}^2\right] = \tfrac{1}{2}m\left[v_{1i}^2 + 2v_{2i}^2 - v_{1f}^2 - 2v_{2f}^2\right] \\ &= \tfrac{1}{2}m\left[v_0^2 + 2\left(\tfrac{1}{4}v_0^2\right) - \tfrac{5}{4}v_0^2 - 2\left(\tfrac{1}{16}v_0^2\right)\right] = \boxed{\tfrac{1}{16}mv_0^2}\end{aligned}$$

$$\boxed{\text{Because } \Delta E \neq 0 \text{, the collision is } \textit{inelastic}.}$$

***97** ••

Picture the Problem Let the direction of motion of the puck that is moving before the collision be the positive *x* direction. Applying conservation of momentum to the collision in both the *x* and *y* directions will lead us to two equations in the unknowns v_1 and v_2 that we can solve simultaneously. We can decide whether the collision was elastic by either calculating the system's kinetic energy before and after the collision or by determining whether the angle between the final velocities is 90°.

(*a*) Use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$p_{xi} = p_{xf}$$

or

$$mv = mv_1\cos 30° + mv_2\cos 60°$$

or

$$v = v_1\cos 30° + v_2\cos 60°$$

Use conservation of momentum in the y direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$p_{yi} = p_{yf}$$

or

$$0 = mv_1 \sin 30° - mv_2 \sin 60°$$

or

$$0 = v_1 \sin 30° - v_2 \sin 60°$$

Solve these equations simultaneously to obtain:

$$v_1 = \boxed{1.73\,\text{m/s}} \text{ and } v_2 = \boxed{1.00\,\text{m/s}}$$

(*b*) $\boxed{\text{Because the angle between } \vec{v}_1 \text{ and } \vec{v}_2 \text{ is } 90°\text{, the collision was } \textit{elastic}.}$

98 ••

Picture the Problem Let the direction of motion of the object that is moving before the collision be the positive x direction. Applying conservation of momentum to the motion in both the x and y directions will lead us to two equations in the unknowns v_2 and θ_2 that we can solve simultaneously. We can show that the collision was elastic by showing that the system's kinetic energy before and after the collision is the same.

(*a*) Use conservation of momentum in the x direction to relate the velocities of the collision participants before and after the collision:

$$p_{xi} = p_{xf}$$

or

$$3mv_0 = \sqrt{5}mv_0 \cos\theta_1 + 2mv_2 \cos\theta_2$$

or

$$3v_0 = \sqrt{5}v_0 \cos\theta_1 + 2v_2 \cos\theta_2$$

Use conservation of momentum in the y direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$p_{yi} = p_{yf}$$

or

$$0 = \sqrt{5}mv_0 \sin\theta_1 - 2mv_2 \sin\theta_2$$

or

$$0 = \sqrt{5}v_0 \sin\theta_1 - 2v_2 \sin\theta_2$$

Note that if $\tan\theta_1 = 2$, then:

$$\cos\theta_1 = \frac{1}{\sqrt{5}} \text{ and } \sin\theta_1 = \frac{2}{\sqrt{5}}$$

Substitute in the momentum equations to obtain:

$$3v_0 = \sqrt{5}v_0 \frac{1}{\sqrt{5}} + 2v_2 \cos\theta_2$$

or

$$v_0 = v_2 \cos\theta_2$$

and

$$0 = \sqrt{5}v_0 \frac{2}{\sqrt{5}} - 2v_2 \sin\theta_2$$

or

$$0 = v_0 - v_2 \sin\theta_2$$

Solve these equations simultaneously for θ_2:

$$\theta_2 = \tan^{-1}1 = \boxed{45.0°}$$

Substitute to find v_2:

$$v_2 = \frac{v_0}{\cos\theta_2} = \frac{v_0}{\cos 45°} = \boxed{\sqrt{2}v_0}$$

(*b*) To show that the collision was elastic, find the before-collision and after-collision kinetic energies:

$$K_\text{i} = \tfrac{1}{2}m(3v_0)^2 = 4.5mv_0^2$$

and

$$K_\text{f} = \tfrac{1}{2}m\left(\sqrt{5}v_0\right)^2 + \tfrac{1}{2}(2m)\left(\sqrt{2}v_0\right)^2 = 4.5mv_0^2$$

$$\boxed{\text{Because } K_\text{i} = K_\text{f}, \text{ the collision is elastic.}}$$

***99** ••

Picture the Problem Let the direction of motion of the ball that is moving before the collision be the positive *x* direction. Let *v* represent the velocity of the ball that is moving before the collision, v_1 its velocity after the collision and v_2 the velocity of the initially-at-rest ball after the collision. We know that because the collision is elastic and the balls have the same mass, v_1 and v_2 are 90° apart. Applying conservation of momentum to the collision in both the *x* and *y* directions will lead us to two equations in the unknowns v_1 and v_2 that we can solve simultaneously.

Noting that the angle of deflection for the recoiling ball is 60°, use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$p_\text{xi} = p_\text{xf}$$

or

$$mv = mv_1 \cos 30° + mv_2 \cos 60°$$

or

$$v = v_1 \cos 30° + v_2 \cos 60°$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$p_\text{yi} = p_\text{yf}$$

or

$$0 = mv_1 \sin 30° - mv_2 \sin 60°$$

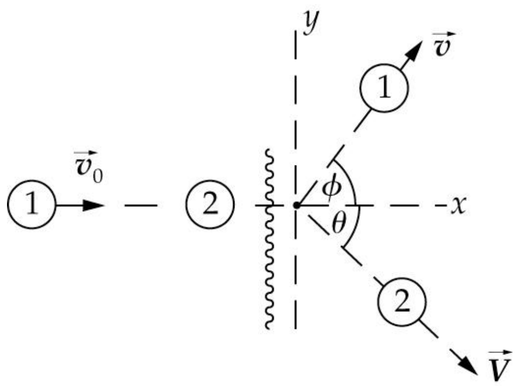
or

$$0 = v_1 \sin 30° - v_2 \sin 60°$$

Solve these equations simultaneously to obtain: $v_1 = \boxed{8.66\,\text{m/s}}$ and $v_2 = \boxed{5.00\,\text{m/s}}$

100 ••

Picture the Problem Choose the coordinate system shown in the diagram below with the *x*-axis the axis of initial approach of the first particle. Call *V* the speed of the target particle after the collision. In part (*a*) we can apply conservation of momentum in the *x* and *y* directions to obtain two equations that we can solve simultaneously for tan*θ*. In part (*b*) we can use conservation of momentum in vector form and the elastic-collision equation to show that $v = v_0\cos\phi$.



(*a*) Apply conservation of momentum in the *x* direction to obtain:

$$v_0 = v\cos\phi + V\cos\theta \quad (1)$$

Apply conservation of momentum in the *y* direction to obtain:

$$v\sin\phi = V\sin\theta \quad (2)$$

Solve equation (1) for $V\cos\theta$:

$$V\cos\theta = v_0 - v\cos\phi \quad (3)$$

Divide equation (2) by equation (3) to obtain:

$$\frac{V\sin\theta}{V\cos\theta} = \frac{v\sin\phi}{v_0 - v\cos\phi}$$

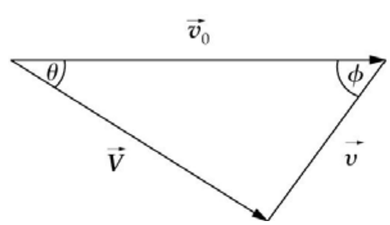
or

$$\tan\theta = \boxed{\frac{v\sin\phi}{v_0 - v\cos\phi}}$$

(*b*) Apply conservation of momentum to obtain:

$$\vec{v}_0 = \vec{v} + \vec{V}$$

Draw the vector diagram representing this equation:



Use the definition of an elastic

$$v_0^2 = v^2 + V^2$$

collision to obtain:

If this Pythagorean condition is to hold, the third angle of the triangle must be a right angle and, using the definition of the cosine function:

$$v = \boxed{v_0 \cos\phi}$$

Center-of-Mass Frame

101 ••

Picture the Problem The total kinetic energy of a system of particles is the sum of the kinetic energy of the center of mass and the kinetic energy relative to the center of mass. The kinetic energy of a particle of mass m is related to momentum according to $K = p^2/2m$.

Express the total kinetic energy of the system:

$$K = K_{\text{rel}} + K_{\text{cm}} \qquad (1)$$

Relate the kinetic energy relative to the center of mass to the momenta of the two particles:

$$K_{\text{rel}} = \frac{p_1^2}{2m_1} + \frac{p_1^2}{2m_2} = \frac{p_1^2(m_1 + m_2)}{2m_1m_2}$$

Express the kinetic energy of the center of mass of the two particles:

$$K_{\text{cm}} = \frac{(2p_1)^2}{2(m_1 + m_2)} = \frac{2p_1^2}{m_1 + m_2}$$

Substitute in equation (1) and simplify to obtain:

$$K = \frac{p_1^2(m_1 + m_2)}{2m_1m_2} + \frac{2p_1^2}{m_1 + m_2}$$

$$= \frac{p_1^2}{2}\left[\frac{m_1^2 + 6m_1m_2 + m_2^2}{m_1^2m_2 + m_1m_2^2}\right]$$

In an elastic collision:

$$K_{\text{i}} = K_{\text{f}}$$

$$= \boxed{\frac{p_1^2}{2}\left[\frac{m_1^2 + 6m_1m_2 + m_2^2}{m_1^2m_2 + m_1m_2^2}\right]}$$

$$= \boxed{\frac{p_1'^2}{2}\left[\frac{m_1^2 + 6m_1m_2 + m_2^2}{m_1^2m_2 + m_1m_2^2}\right]}$$

Simplify to obtain:

$$(p_1')^2 = (p_1)^2 \Rightarrow \boxed{p_1' = \pm p_1}$$

and

$$\boxed{\text{If } p_1' = +p_1, \text{the particles do not collide.}}$$

***102** ••

Picture the Problem Let the numerals 3 and 1 denote the blocks whose masses are 3 kg and 1 kg respectively. We can use $\sum_i m_i\vec{v}_i = M\vec{v}_{\rm cm}$ to find the velocity of the center-of-mass of the system and simply follow the directions in the problem step by step.

(*a*) Express the total momentum of this two-particle system in terms of the velocity of its center of mass:

$$\vec{P} = \sum_i m_i\vec{v}_i = m_1\vec{v}_1 + m_3\vec{v}_3 = M\vec{v}_{\rm cm} = (m_1 + m_3)\vec{v}_{\rm cm}$$

Solve for $\vec{v}_{\rm cm}$:

$$\vec{v}_{\rm cm} = \frac{m_3\vec{v}_3 + m_1\vec{v}_1}{m_3 + m_1}$$

Substitute numerical values and evaluate $\vec{v}_{\rm cm}$:

$$\vec{v}_{\rm cm} = \frac{(3\,\text{kg})(-5\,\text{m/s})\hat{i} + (1\,\text{kg})(3\,\text{m/s})\hat{i}}{3\,\text{kg} + 1\,\text{kg}} = \boxed{(-3.00\,\text{m/s})\hat{i}}$$

(*b*) Find the velocity of the 3-kg block in the center of mass reference frame:

$$\vec{u}_3 = \vec{v}_3 - \vec{v}_{\rm cm} = (-5\,\text{m/s})\hat{i} - (-3\,\text{m/s})\hat{i} = \boxed{(-2.00\,\text{m/s})\hat{i}}$$

Find the velocity of the 1-kg block in the center of mass reference frame:

$$\vec{u}_1 = \vec{v}_1 - \vec{v}_{\rm cm} = (3\,\text{m/s})\hat{i} - (-3\,\text{m/s})\hat{i} = \boxed{(6.00\,\text{m/s})\hat{i}}$$

(*c*) Express the after-collision velocities of both blocks in the center of mass reference frame:

$$\vec{u}_3' = \boxed{(2.00\,\text{m/s})\hat{i}}$$

and

$$\vec{u}_1' = \boxed{(-6.00\,\text{m/s})\hat{i}}$$

(*d*) Transform the after-collision velocity of the 3-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_3' = \vec{u}_3' + \vec{v}_{\rm cm} = (2\,\text{m/s})\hat{i} + (-3\,\text{m/s})\hat{i} = \boxed{(-1.00\,\text{m/s})\hat{i}}$$

Transform the after-collision velocity of the 1-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_1' = \vec{u}_1' + \vec{v}_{\rm cm} = (-6\,\text{m/s})\hat{i} + (-3\,\text{m/s})\hat{i} = \boxed{(-9.00\,\text{m/s})\hat{i}}$$

(*e*) Express $K_{\rm i}$ in the original frame of

$$K_{\rm i} = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_1v_1^2$$

reference:

Substitute numerical values and evaluate K_i:

$$K_i = \tfrac{1}{2}\left[(3\,\text{kg})(5\,\text{m/s})^2 + (1\,\text{kg})(3\,\text{m/s})^2\right] = \boxed{42.0\,\text{J}}$$

Express K_f in the original frame of reference:

$$K_f = \tfrac{1}{2}m_3 v_3'^2 + \tfrac{1}{2}m_1 v_1'^2$$

Substitute numerical values and evaluate K_f:

$$K_f = \tfrac{1}{2}\left[(3\,\text{kg})(1\,\text{m/s})^2 + (1\,\text{kg})(9\,\text{m/s})^2\right] = \boxed{42.0\,\text{J}}$$

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Picture the Problem Let the numerals 3 and 1 denote the blocks whose masses are 3 kg and 1 kg respectively. We can use $\sum_i m_i \vec{v}_i = M\vec{v}_{cm}$ to find the velocity of the center-of-mass of the system and simply follow the directions in the problem step by step.

(*a*) Express the total momentum of this two-particle system in terms of the velocity of its center of mass:

$$\vec{P} = \sum_i m_i \vec{v}_i = m_3\vec{v}_3 + m_5\vec{v}_5 = M\vec{v}_{cm} = (m_3 + m_5)\vec{v}_{cm}$$

Solve for $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{m_3\vec{v}_3 + m_5\vec{v}_5}{m_3 + m_5}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3\,\text{kg})(-5\,\text{m/s})\hat{\boldsymbol{i}} + (5\,\text{kg})(3\,\text{m/s})\hat{\boldsymbol{i}}}{3\,\text{kg} + 5\,\text{kg}} = \boxed{0}$$

(*b*) Find the velocity of the 3-kg block in the center of mass reference frame:

$$\vec{u}_3 = \vec{v}_3 - \vec{v}_{cm} = (-5\,\text{m/s})\,\hat{\boldsymbol{i}} - 0 = \boxed{(-5\,\text{m/s})\,\hat{\boldsymbol{i}}}$$

Find the velocity of the 5-kg block in the center of mass reference frame:

$$\vec{u}_5 = \vec{v}_5 - \vec{v}_{cm} = (3\,\text{m/s})\hat{\boldsymbol{i}} - 0 = \boxed{(3\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*c*) Express the after-collision velocities of both blocks in the center of mass reference frame:

$$\vec{u}_3' = \boxed{(5\,\text{m/s})\hat{\boldsymbol{i}}}$$

and

$$u_5' = \boxed{0.75\,\text{m/s}}$$

(*d*) Transform the after-collision velocity of the 3-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_3' = \vec{u}_3' + \vec{v}_{cm} = (5\,\text{m/s})\hat{i} + 0 = \boxed{(5\,\text{m/s})\hat{i}}$$

Transform the after-collision velocity of the 5-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_5' = \vec{u}_5' + \vec{v}_{cm} = (-3\,\text{m/s})\hat{i} + 0 = \boxed{(-3\,\text{m/s})\hat{i}}$$

(*e*) Express K_i in the original frame of reference:

$$K_i = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_5v_5^2$$

Substitute numerical values and evaluate K_i:

$$K_i = \tfrac{1}{2}\left[(3\,\text{kg})(5\,\text{m/s})^2 + (5\,\text{kg})(3\,\text{m/s})^2\right] = \boxed{60.0\,\text{J}}$$

Express K_f in the original frame of reference:

$$K_f = \tfrac{1}{2}m_3v_3'^2 + \tfrac{1}{2}m_5v_5'^2$$

Substitute numerical values and evaluate K_f:

$$K_f = \tfrac{1}{2}\left[(3\,\text{kg})(5\,\text{m/s})^2 + (5\,\text{kg})(3\,\text{m/s})^2\right] = \boxed{60.0\,\text{J}}$$

Systems With Continuously Varying Mass: Rocket Propulsion

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Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel *dm/dt* and the relative speed of its exhaust gases u_{ex} according to $F_{th} = |dm/dt|u_{ex}$.

Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{th} = \left|\frac{dm}{dt}\right|u_{ex}$$

Substitute numerical values and evaluate F_{th}:

$$F_{th} = (200\,\text{kg/s})(6\,\text{km/s}) = \boxed{1.20\,\text{MN}}$$

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Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel dm/dt and the relative speed of its exhaust gases u_{ex} according to $F_{th} = |dm/dt|u_{ex}$. The final velocity v_f of a rocket depends on the relative speed of its exhaust gases u_{ex}, its payload to initial mass ratio m_f/m_0 and its burn time according to $v_f = -u_{ex}\ln(m_f/m_0) - gt_b$.

(*a*) Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{th} = \left|\frac{dm}{dt}\right|u_{ex}$$

Substitute numerical values and evaluate F_{th}:

$$F_{th} = (200\,\text{kg/s})(1.8\,\text{km/s}) = \boxed{360\,\text{kN}}$$

(*b*) Relate the time to burnout to the mass of the fuel and its burn rate:

$$t_b = \frac{m_{fuel}}{dm/dt} = \frac{0.8m_0}{dm/dt}$$

Substitute numerical values and evaluate t_b:

$$t_b = \frac{0.8(30{,}000\,\text{kg})}{200\,\text{kg/s}} = \boxed{120\,\text{s}}$$

(*c*) Relate the final velocity of a rocket to its initial mass, exhaust velocity, and burn time:

$$v_f = -u_{ex}\ln\left(\frac{m_f}{m_0}\right) - gt_b$$

Substitute numerical values and evaluate v_f:

$$v_f = -(1.8\,\text{km/s})\ln\left(\frac{1}{5}\right) - (9.81\,\text{m/s}^2)(120\,\text{s}) = \boxed{1.72\,\text{km/s}}$$

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Picture the Problem We can use the dimensions of thrust, burn rate, and acceleration to show that the dimension of specific impulse is time. Combining the definitions of rocket thrust and specific impulse will lead us to $u_{ex} = gI_{sp}$.

(*a*) Express the dimension of specific impulse in terms of the dimensions of F_{th}, R, and g:

$$[I_{sp}] = \frac{[F_{th}]}{[R][g]} = \frac{\frac{\text{M}\cdot\text{L}}{\text{T}^2}}{\frac{\text{M}}{\text{T}}\cdot\frac{\text{L}}{\text{T}^2}} = \boxed{\text{T}}$$

(*b*) From the definition of rocket thrust we have:

$$F_{th} = Ru_{ex}$$

Solve for u_{ex}:

$$u_{ex} = \frac{F_{th}}{R}$$

Substitute for F_{th} to obtain:

$$u_{ex} = \frac{RgI_{sp}}{R} = \boxed{gI_{sp}} \qquad (1)$$

(*c*) Solve equation (1) for I_{sp} and substitute for u_{ex} to obtain:

$$I_{sp} = \frac{F_{th}}{Rg}$$

From Example 8-21 we have:

$R = 1.384\times10^4$ kg/s and $F_{th} = 3.4\times10^6$ N

Substitute numerical values and evaluate I_{sp}:

$$I_{sp} = \frac{3.4\times10^6\ \text{N}}{(1.384\times10^4\ \text{kg/s})(9.81\ \text{m/s}^2)} = \boxed{25.0\ \text{s}}$$

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Picture the Problem We can use the rocket equation and the definition of rocket thrust to show that $\tau_0 = 1 + a_0/g$. In part (*b*) we can express the burn time t_b in terms of the initial and final masses of the rocket and the rate at which the fuel burns, and then use this equation to express the rocket's final velocity in terms of I_{sp}, τ_0, and the mass ratio m_0/m_f. In part (*d*) we'll need to use trial-and-error methods or a graphing calculator to solve the transcendental equation giving v_f as a function of m_0/m_f.

(*a*) Express the rocket equation:

$$-mg + Ru_{ex} = ma$$

From the definition of rocket thrust we have:

$$F_{th} = Ru_{ex}$$

Substitute to obtain:

$$-mg + F_{th} = ma$$

Solve for F_{th} at takeoff:

$$F_{th} = m_0 g + m_0 a_0$$

Divide both sides of this equation by $m_0 g$ to obtain:

$$\frac{F_{th}}{m_0 g} = 1 + \frac{a_0}{g}$$

Because $\tau_0 = F_{th}/(m_0 g)$:

$$\tau_0 = \boxed{1 + \frac{a_0}{g}}$$

(*b*) Use equation 8-42 to express the final speed of a rocket that starts from rest with mass m_0:

$$v_f = u_{ex}\ln\frac{m_0}{m_f} - gt_b, \qquad (1)$$

where t_b is the burn time.

Express the burn time in terms of the burn rate R (assumed constant):

$$t_b = \frac{m_0 - m_f}{R} = \frac{m_0}{R}\left(1 - \frac{m_f}{m_0}\right)$$

Multiply t_b by one in the form gT/gT and simplify to obtain:

$$t_b = \frac{gF_{th}}{gF_{th}} \frac{m_0}{R}\left(1-\frac{m_f}{m_0}\right) = \frac{gm_0}{F_{th}} \frac{F_{th}}{gR}\left(1-\frac{m_f}{m_0}\right) = \frac{I_{sp}}{\tau_0}\left(1-\frac{m_f}{m_0}\right)$$

Substitute in equation (1):

$$v_f = u_{ex} \ln \frac{m_0}{m_f} - \frac{gI_{sp}}{\tau_0}\left(1-\frac{m_f}{m_0}\right)$$

From Problem 32 we have:

$$u_{ex} = gI_{sp},$$

where u_{ex} is the exhaust velocity of the propellant.

Substitute and factor to obtain:

$$v_f = gI_{sp} \ln \frac{m_0}{m_f} - \frac{gI_{sp}}{\tau_0}\left(1-\frac{m_f}{m_0}\right) = \boxed{gI_{sp}\left[\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau_0}\left(1-\frac{m_f}{m_0}\right)\right]}$$

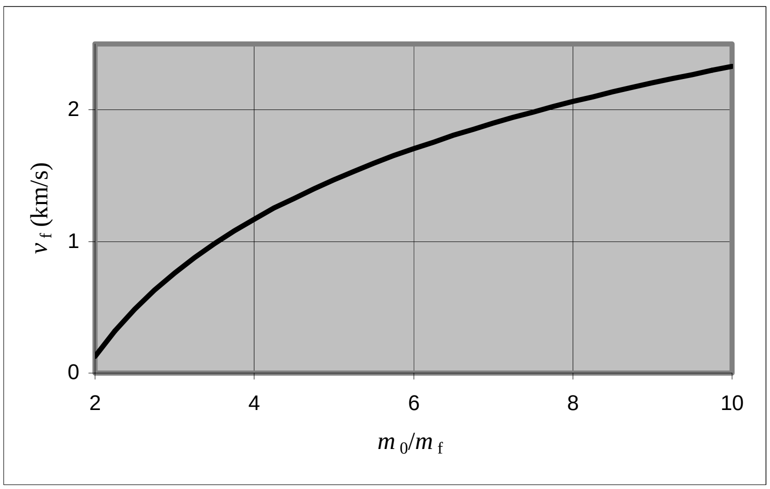
(*c*) A spreadsheet program to calculate the final velocity of the rocket as a function of the mass ratio m_0/m_f is shown below. The constants used in the velocity function and the formulas used to calculate the final velocity are as follows:

Cell	Content/Formula	Algebraic Form
B1	250	I_{sp}
B2	9.81	g
B3	2	τ
D9	D8 + 0.25	m_0/m_f
E8	\$B\$2*\$B\$1*(LOG(D8) – (1/\$B\$3)*(1/D8))	$gI_{sp}\left[\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau_0}\left(1-\frac{m_f}{m_0}\right)\right]$

	A	B	C	D	E
1	Isp =	250	s		
2	g =	9.81	m/s^2		
3	tau =	2			
4					
5					
6					
7				mass ratio	vf
8				2.00	1.252E+02
9				2.25	3.187E+02

10				2.50	4.854E+02
11				2.75	6.316E+02
12				3.00	7.614E+02
36				9.00	2.204E+03
37				9.25	2.237E+03
38				9.50	2.269E+03
39				9.75	2.300E+03
40				10.00	2.330E+03
41				725.00	7.013E+03

A graph of final velocity as a function of mass ratio is shown below.



(d) Substitute the data given in part (c) in the equation derived in part (b) to obtain:

$$7\,\text{km/s} = \left(9.81\,\text{m/s}^2\right)\left(250\,\text{s}\right)\left(\ln\frac{m_0}{m_\text{f}} - \frac{1}{2}\left(1 - \frac{m_\text{f}}{m_0}\right)\right)$$

or

$$2.854 = \ln x - 0.5 + \frac{0.5}{x} \text{ where } x = m_0/m_\text{f}.$$

Use trial-and-error methods or a graphing calculator to solve this transcendental equation for the root greater than 1:

$$x = \boxed{28.1},$$

a value considerably larger than the practical limit of 10 for single-stage rockets.

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Picture the Problem We can use the velocity-at-burnout equation from Problem 106 to find v_f and constant-acceleration equations to approximate the maximum height the rocket will reach and its total flight time.

(a) Assuming constant acceleration, relate the maximum height reached

$$h = \tfrac{1}{2} g t_\text{top}^2 \qquad (1)$$

by the model rocket to its time-to-top-of-trajectory:

From Problem 106 we have:

$$v_f = gI_{sp}\left(\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau}\left(1 - \frac{m_f}{m_0}\right)\right)$$

Evaluate the velocity at burnout v_f for I_{sp} = 100 s, m_0/m_f = 1.2, and τ = 5:

$$v_f = \left(9.81\,\text{m/s}^2\right)\left(100\,\text{s}\right) \times\left[\ln(1.2) - \frac{1}{5}\left(1 - \frac{1}{1.2}\right)\right] = 146\,\text{m/s}$$

Assuming that the time for the fuel to burn up is short compared to the total flight time, find the time to the top of the trajectory:

$$t_{top} = \frac{v_f}{g} = \frac{146\,\text{m/s}}{9.81\,\text{m/s}^2} = 14.9\,\text{s}$$

Substitute in equation (1) and evaluate h:

$$h = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(14.9\,\text{s}\right)^2 = \boxed{1.09\,\text{km}}$$

(*b*) Find the total flight time from the time it took the rocket to reach its maximum height:

$$t_{flight} = 2t_{top} = 2(14.9\,\text{s}) = \boxed{29.8\,\text{s}}$$

(*c*) Express and evaluate the fuel burn time t_b:

$$t_b = \frac{I_{sp}}{\tau}\left(1 - \frac{m_f}{m_0}\right) = \frac{100\,\text{s}}{5}\left(1 - \frac{1}{1.2}\right) = 3.33\,\text{s}$$

Because this burn time is approximately 1/5 of the total flight time, we can't expect the answer we obtained in Part (*b*) to be very accurate. It should, however, be good to about 30% accuracy, as the maximum distance the model rocket could possibly move in this time is $\frac{1}{2}vt_b = 243\,\text{m}$, assuming constant acceleration until burnout.

General Problems

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Picture the Problem Let the direction of motion of the 250-g car before the collision be the positive *x* direction. Let the numeral 1 refer to the 250-kg car, the numeral 2 refer to the 400-kg car, and *V* represent the velocity of the linked cars. Let the system include the earth and the cars. We can use conservation of momentum to find their speed after they have linked together and the definition of kinetic energy to find their initial and final kinetic energies.

Use conservation of momentum to relate the speeds of the cars immediately before and immediately after their collision:

$$p_{ix} = p_{fx}$$
or
$$m_1v_1 = (m_1 + m_2)V$$

Solve for V:

$$V = \frac{m_1v_1}{m_1 + m_2}$$

Substitute numerical values and evaluate V:

$$V = \frac{(0.250\,\text{kg})(0.50\,\text{m/s})}{0.250\,\text{kg} + 0.400\,\text{kg}} = \boxed{0.192\,\text{m/s}}$$

Find the initial kinetic energy of the cars:

$$K_i = \tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}(0.250\,\text{kg})(0.50\,\text{m/s})^2 = \boxed{31.3\,\text{mJ}}$$

Find the final kinetic energy of the coupled cars:

$$K_f = \tfrac{1}{2}(m_1 + m_2)V^2 = \tfrac{1}{2}(0.250\,\text{kg} + 0.400\,\text{kg})(0.192\,\text{m/s})^2 = \boxed{12.0\,\text{mJ}}$$

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Picture the Problem Let the direction of motion of the 250-g car before the collision be the positive x direction. Let the numeral 1 refer to the 250-kg car and the numeral 2 refer to the 400-g car and the system include the earth and the cars. We can use conservation of momentum to find their speed after they have linked together and the definition of kinetic energy to find their initial and final kinetic energies.

(*a*) Express and evaluate the initial kinetic energy of the cars:

$$K_i = \tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}(0.250\,\text{kg})(0.50\,\text{m/s})^2 = \boxed{31.3\,\text{mJ}}$$

(*b*) Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{\boldsymbol{P}} = \sum_i m_i\vec{\boldsymbol{v}}_i = m\vec{\boldsymbol{v}}_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(0.250\,\text{kg})(0.50\,\text{m/s})}{0.250\,\text{kg} + 0.400\,\text{kg}} = 0.192\,\text{m/s}$$

Find the initial velocity of the 250-g car relative to the velocity of the center of mass:

$$u_1 = v_1 - v_{cm} = 0.50\,\text{m/s} - 0.192\,\text{m/s}$$
$$= \boxed{0.308\,\text{m/s}}$$

Find the initial velocity of the 400-g car relative to the velocity of the center of mass:

$$u_2 = v_2 - v_{cm} = 0\,\text{m/s} - 0.192\,\text{m/s}$$
$$= \boxed{-0.192\,\text{m/s}}$$

Express the initial kinetic energy of the system relative to the center of mass:

$$K_{i,rel} = \tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2$$

Substitute numerical values and evaluate $K_{i,rel}$:

$$K_{i,rel} = \tfrac{1}{2}(0.250\,\text{kg})(0.308\,\text{m/s})^2$$
$$+ \tfrac{1}{2}(0.400\,\text{kg})(-0.192\,\text{m/s})^2$$
$$= \boxed{19.2\,\text{mJ}}$$

(*c*) Express the kinetic energy of the center of mass:

$$K_{cm} = \tfrac{1}{2} M v_{cm}^2$$

Substitute numerical values and evaluate K_{cm}:

$$K_{cm} = \tfrac{1}{2}(0.650\,\text{kg})(0.192\,\text{m/s})^2$$
$$= \boxed{12.0\,\text{mJ}}$$

(*d*) Relate the initial kinetic energy of the system to its initial kinetic energy relative to the center of mass and the kinetic energy of the center of mass:

$$K_i = K_{i,rel} + K_{cm}$$
$$= 19.2\,\text{mJ} + 12.0\,\text{mJ}$$
$$= 31.2\,\text{mJ}$$

$$\therefore \boxed{K_i = K_{i,rel} + K_{cm}}$$

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Picture the Problem Let the direction the 4-kg fish is swimming be the positive *x* direction and the system include the fish, the water, and the earth. The velocity of the larger fish immediately after its lunch is the velocity of the center of mass in this perfectly inelastic collision.

Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{P} = \sum_i m_i \vec{v}_i = m\vec{v}_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_4 v_4 - m_{1.2} v_{1.2}}{m_4 + m_{1.2}}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(4\,\text{kg})(1.5\,\text{m/s}) - (1.2\,\text{kg})(3\,\text{m/s})}{4\,\text{kg} + 1.2\,\text{kg}} = \boxed{0.462\,\text{m/s}}$$

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Picture the Problem Let the direction the 3-kg block is moving be the positive x direction and include both blocks and the earth in the system. The total kinetic energy of the two-block system is the sum of the kinetic energies of the blocks. We can relate the momentum of the system to the velocity of its center of mass and use this relationship to find v_{cm}. Finally, we can use the definition of kinetic energy to find the kinetic energy relative to the center of mass.

(*a*) Express the total kinetic energy of the system in terms of the kinetic energy of the blocks:

$$K_{tot} = \tfrac{1}{2} m_3 v_3^2 + \tfrac{1}{2} m_6 v_6^2$$

Substitute numerical values and evaluate K_{tot}:

$$K_{tot} = \tfrac{1}{2}(3\,\text{kg})(6\,\text{m/s})^2 + \tfrac{1}{2}(6\,\text{kg})(3\,\text{m/s})^2 = \boxed{81.0\,\text{J}}$$

(*b*) Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{\boldsymbol{P}} = \sum_i m_i \vec{\boldsymbol{v}}_i = m\vec{\boldsymbol{v}}_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_3 v_3 + m_6 v_6}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(3\,\text{kg})(6\,\text{m/s}) + (6\,\text{kg})(3\,\text{m/s})}{3\,\text{kg} + 6\,\text{kg}} = \boxed{4.00\,\text{m/s}}$$

(*c*) Find the center of mass kinetic energy from the velocity of the center of mass:

$$K_{cm} = \tfrac{1}{2} M v_{cm}^2 = \tfrac{1}{2}(9\,\text{kg})(4\,\text{m/s})^2 = \boxed{72.0\,\text{J}}$$

(*d*) Relate the initial kinetic energy of the system to its initial kinetic energy relative to the center of mass and the kinetic energy of the center of mass:

$$K_{\text{rel}} = K_{\text{tot}} - K_{\text{cm}} = 81.0\,\text{J} - 72.0\,\text{J} = \boxed{9.00\,\text{J}}$$

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Picture the Problem Let east be the positive *x* direction and north the positive *y* direction. Include both cars and the earth in the system and let the numeral 1 denote the 1500-kg car and the numeral 2 the 2000-kg car. Because the net external force acting on the system is zero, momentum is conserved in this perfectly inelastic collision.

(*a*) Express the total momentum of the system:

$$\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2 = m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2 = m_1 v_1\hat{\boldsymbol{j}} - m_2 v_2\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{p}}$:

$$\vec{\boldsymbol{p}} = (1500\,\text{kg})(70\,\text{km/h})\hat{\boldsymbol{j}} - (2000\,\text{kg})(55\,\text{km/h})\hat{\boldsymbol{i}}$$
$$= \boxed{-\left(1.10\times10^5\,\text{kg}\cdot\text{km/h}\right)\hat{\boldsymbol{i}} + \left(1.05\times10^5\,\text{kg}\cdot\text{km/h}\right)\hat{\boldsymbol{j}}}$$

(*b*) Express the velocity of the wreckage in terms of the total momentum of the system:

$$\vec{\boldsymbol{v}}_{\text{f}} = \vec{\boldsymbol{v}}_{\text{cm}} = \frac{\vec{\boldsymbol{p}}}{M}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\text{f}}$:

$$\vec{\boldsymbol{v}}_{\text{f}} = \frac{-\left(1.10\times10^5\,\text{kg}\cdot\text{km/h}\right)\hat{\boldsymbol{i}}}{1500\,\text{kg} + 2000\,\text{kg}} + \frac{\left(1.05\times10^5\,\text{kg}\cdot\text{km/h}\right)\hat{\boldsymbol{j}}}{1500\,\text{kg} + 2000\,\text{kg}}$$
$$= -(31.4\,\text{km/h})\,\hat{\boldsymbol{i}} + (30.0\,\text{km/h})\hat{\boldsymbol{j}}$$

Find the magnitude of the velocity of the wreckage:

$$v_{\text{f}} = \sqrt{(31.4\,\text{km/h})^2 + (30.0\,\text{km/h})^2} = \boxed{43.4\,\text{km/h}}$$

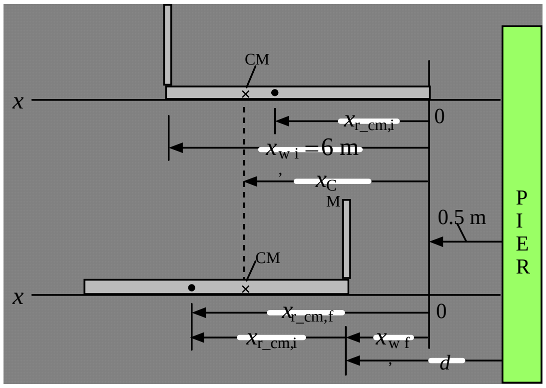
Find the direction of the velocity of the wreckage:

$$\theta = \tan^{-1}\left[\frac{30.0\,\text{km/h}}{-31.4\,\text{km/h}}\right] = -43.7°$$

$$\boxed{\text{The direction of the wreckage is } 46.3° \text{ west of north.}}$$

***114** ••

Picture the Problem Take the origin to be at the initial position of the right-hand end of raft and let the positive x direction be to the left. Let "w" denote the woman and "r" the raft, d be the distance of the end of the raft from the pier after the woman has walked to its front. The raft moves to the left as the woman moves to the right; with the center of mass of the woman-raft system remaining fixed (because $F_{\text{ext,net}} = 0$). The diagram shows the initial ($x_{\text{w,i}}$) and final ($x_{\text{w,f}}$) positions of the woman as well as the initial ($x_{\text{r\_cm,i}}$) and final ($x_{\text{r\_cm,f}}$) positions of the center of mass of the raft both before and after the woman has walked to the front of the raft.



(a) Express the distance of the raft from the pier after the woman has walked to the front of the raft:

$$d = 0.5\,\text{m} + x_{\text{f,w}} \qquad (1)$$

Express x_{cm} before the woman has walked to the front of the raft:

$$x_{\text{cm}} = \frac{m_{\text{w}} x_{\text{w,i}} + m_{\text{r}} x_{\text{r\_cm,i}}}{m_{\text{w}} + m_{\text{r}}}$$

Express x_{cm} after the woman has walked to the front of the raft:

$$x_{\text{cm}} = \frac{m_{\text{w}} x_{\text{w,f}} + m_{\text{r}} x_{\text{r\_cm,f}}}{m_{\text{w}} + m_{\text{r}}}$$

Because $F_{\text{ext,net}} = 0$, the center of mass remains fixed and we can equate these two expressions for x_{cm} to obtain:

$$m_{\text{w}} x_{\text{w,i}} + m_{\text{r}} x_{\text{r\_cm,i}} = m_{\text{w}} x_{\text{w,f}} + m_{\text{r}} x_{\text{r\_cm,f}}$$

Solve for $x_{\text{w,f}}$:

$$x_{\text{w,f}} = x_{\text{w,i}} - \frac{m_{\text{r}}}{m_{\text{w}}}\left(x_{\text{r\_cm,f}} - x_{\text{r\_cm,i}}\right)$$

From the figure it can be seen that $x_{\text{r\_cm,f}} - x_{\text{r\_cm,i}} = x_{\text{w,f}}$. Substitute $x_{\text{w,f}}$

$$x_{\text{w,f}} = \frac{m_{\text{w}} x_{\text{w,i}}}{m_{\text{w}} + m_{\text{r}}}$$

for $x_{r\_cm,f} - x_{r\_cm,i}$ and to obtain:

Substitute numerical values and evaluate $x_{w,f}$:

$$x_{w,f} = \frac{(60\,\text{kg})(6\,\text{m})}{60\,\text{kg} + 120\,\text{kg}} = 2.00\,\text{m}$$

Substitute in equation (1) to obtain:

$$d = 2.00\,\text{m} + 0.5\,\text{m} = \boxed{2.50\,\text{m}}$$

(*b*) Express the total kinetic energy of the system:

$$K_{tot} = \tfrac{1}{2} m_w v_w^2 + \tfrac{1}{2} m_r v_r^2$$

Noting that the elapsed time is 2 s, find v_w and v_r:

$$v_w = \frac{x_{w,f} - x_{w,i}}{\Delta t} = \frac{2\,\text{m} - 6\,\text{m}}{2\,\text{s}} = -2\,\text{m/s}$$

relative to the dock, and

$$v_r = \frac{x_{r,f} - x_{r,i}}{\Delta t} = \frac{2.50\,\text{m} - 0.5\,\text{m}}{2\,\text{s}} = 1\,\text{m/s},$$

also relative to the dock.

Substitute numerical values and evaluate K_{tot}:

$$K_{tot} = \tfrac{1}{2}(60\,\text{kg})(-2\,\text{m/s})^2 + \tfrac{1}{2}(120\,\text{kg})(1\,\text{m/s})^2 = \boxed{180\,\text{J}}$$

Evaluate K with the raft tied to the pier:

$$K_{tot} = \tfrac{1}{2} m_w v_w^2 = \tfrac{1}{2}(60\,\text{kg})(3\,\text{m/s})^2 = \boxed{270\,\text{J}}$$

(*c*) All the kinetic energy derives from the chemical energy of the woman and, assuming she stops via static friction, the kinetic energy is transformed into her internal energy.

(*d*) After the shot leaves the woman's hand, the raft - woman system constitutes an inertial reference frame. In that frame the shot has the same initial velocity as did the shot that had a range of 6 m in the reference frame of the land. Thus, in the raft - woman frame, the shot also has a range of 6 m and lands at the front of the raft.

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Picture the Problem Let the zero of gravitational potential energy be at the elevation of the 1-kg block. We can use conservation of energy to find the speed of the bob just before its perfectly elastic collision with the block and conservation of momentum to find the speed of the block immediately after the collision. We'll apply Newton's 2nd law to find the acceleration of the sliding block and use a constant-acceleration equation to find how far it slides before coming to rest.

(*a*) Use conservation of energy to find the speed of the bob just before its collision with the block:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_i = U_f = 0$:

$$\tfrac{1}{2} m_{ball} v_{ball}^2 + m_{ball} g \Delta h = 0$$

and

$$v_{ball} = \sqrt{2g\Delta h}$$

Substitute numerical values and evaluate v_{ball}:

$$v_{ball} = \sqrt{2(9.81\,\text{m/s}^2)(2\,\text{m})} = 6.26 \text{ m/s}$$

Because the collision is perfectly elastic and the ball and block have the same mass:

$$v_{block} = v_{ball} = \boxed{6.26\,\text{m/s}}$$

(*b*) Using a constant-acceleration equation, relate the displacement of the block to its acceleration and initial speed and solve for its displacement:

$$v_f^2 = v_i^2 + 2a_{block}\Delta x$$

Since $v_f = 0$,

$$\Delta x = \frac{-v_i^2}{2a_{block}} = \frac{-v_{block}^2}{2a_{block}}$$

Apply $\sum \vec{F} = m\vec{a}$ to the sliding block:

$$\sum F_x = -f_k = ma_{block}$$

and

$$\sum F_y = F_n - m_{block} g = 0$$

Using the definition of f_k ($\mu_k F_n$) eliminate f_k and F_n between the two equations and solve for a_{block}:

$$a_{block} = -\mu_k g$$

Substitute for a_{block} to obtain:

$$\Delta x = \frac{-v_{block}^2}{-2\mu_k g} = \frac{v_{block}^2}{2\mu_k g}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(6.26\,\text{m/s})^2}{2(0.1)(9.81\,\text{m/s}^2)} = \boxed{20.0\,\text{m}}$$

***116** ••

Picture the Problem We can use conservation of momentum in the horizontal direction to find the recoil velocity of the car along the track after the firing. Because the shell will neither rise as high nor be moving as fast at the top of its trajectory as it would be in the absence of air friction, we can apply the work-energy theorem to find the amount of thermal energy produced by the air friction.

(*a*) No. The vertical reaction force of the rails is an external force and so the momentum of the system will not be conserved.

(*b*) Use conservation of momentum in the horizontal (x) direction to obtain:

$$\Delta p_x = 0$$

or

$$mv\cos 30° - Mv_{\text{recoil}} = 0$$

Solve for and evaluate v_{recoil}:

$$v_{\text{recoil}} = \frac{mv\cos 30°}{M}$$

Substitute numerical values and evaluate v_{recoil}:

$$v_{\text{recoil}} = \frac{(200\,\text{kg})(125\,\text{m/s})\cos 30°}{5000\,\text{kg}} = \boxed{4.33\,\text{m/s}}$$

(*c*) Using the work-energy theorem, relate the thermal energy produced by air friction to the change in the energy of the system:

$$W_{\text{ext}} = W_{\text{f}} = \Delta E_{\text{sys}} = \Delta U + \Delta K$$

Substitute for ΔU and ΔK to obtain:

$$W_{\text{ext}} = mgy_{\text{f}} - mgy_{\text{i}} + \tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_i^2 = mg(y_{\text{f}} - y_{\text{i}}) + \tfrac{1}{2}m(v_{\text{f}}^2 - v_i^2)$$

Substitute numerical values and evaluate W_{ext}:

$$W_{\text{ext}} = (200\,\text{kg})(9.81\,\text{m/s}^2)(180\,\text{m}) + \tfrac{1}{2}(200\,\text{kg})\left[(80\,\text{m/s})^2 - (125\,\text{m/s})^2\right] = \boxed{-569\,\text{kJ}}$$

117 ••

Picture the Problem Because this is a perfectly inelastic collision, the velocity of the block after the collision is the same as the velocity of the center of mass before the collision. The distance the block travels before hitting the floor is the product of its velocity and the time required to fall 0.8 m; which we can find using a constant-acceleration equation.

Relate the distance D to the velocity of the center of mass and the time for the block to fall to the floor:

$$D = v_{\rm cm}\Delta t$$

Relate the velocity of the center of mass to the total momentum of the system and solve for $v_{\rm cm}$:

$$\vec{P} = \sum_{\rm i} m_{\rm i}\vec{v}_{\rm i} = M\vec{v}_{\rm cm}$$

and

$$v_{\rm cm} = \frac{m_{\rm bullet}v_{\rm bullet} + m_{\rm block}v_{\rm block}}{m_{\rm bullet} + m_{\rm block}}$$

Substitute numerical values and evaluate $v_{\rm cm}$:

$$v_{\rm cm} = \frac{(0.015\,{\rm kg})(500\,{\rm m/s})}{0.015\,{\rm kg} + 0.8\,{\rm kg}} = 9.20\,{\rm m/s}$$

Using a constant-acceleration equation, find the time for the block to fall to the floor:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

$$\text{Because } v_0 = 0,\ \Delta t = \sqrt{\frac{2\Delta y}{g}}$$

Substitute to obtain:

$$D = v_{\rm cm}\sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate D:

$$D = (9.20\,{\rm m/s})\sqrt{\frac{2(0.8\,{\rm m})}{9.81\,{\rm m/s^2}}} = \boxed{3.72\,{\rm m}}$$

118 ••

Picture the Problem Let the direction the particle whose mass is m is moving initially be the positive x direction and the direction the particle whose mass is $4m$ is moving initially be the negative y direction. We can determine the impulse delivered by $\vec{F}$ and, hence, the change in the momentum of the system from the change in the momentum of the particle whose mass is m. Knowing $\Delta\vec{p}$, we can express the final momentum of the particle whose mass is $4m$ and solve for its final velocity.

Express the impulse delivered by the force $\vec{F}$:

$$\vec{I} = \vec{F}T = \Delta\vec{p} = \vec{p}_{\rm f} - \vec{p}_{\rm i}$$
$$= m(4v)\hat{i} - mv\hat{i} = 3mv\hat{i}$$

Express $\vec{p}'_{4m}$:

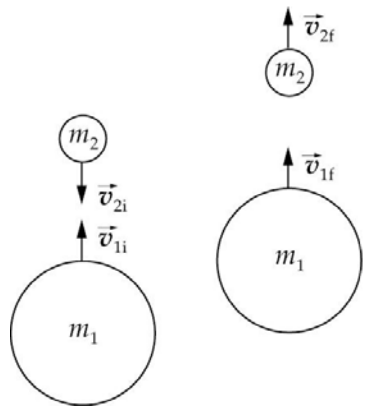
$$\vec{p}'_{4m} = 4m\vec{v}' = \vec{p}_{4m}(0) + \Delta\vec{p} = -4mv\,\hat{j} + 3mv\,\hat{i}$$

Solve for $\vec{v}'$:

$$\vec{v}' = \boxed{\tfrac{3}{4}v\,\hat{i} - v\,\hat{j}}$$

119 ••

Picture the Problem Let the numeral 1 refer to the basketball and the numeral 2 to the baseball. The left-hand side of the diagram shows the balls after the basketball's elastic collision with the floor and just before they collide. The right-hand side of the diagram shows the balls just after their collision. We can apply conservation of momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the masses of the colliding objects that we can solve for v_{1f} and v_{2f}.



(*a*) Because both balls are in free-fall, and both are in the air for the same amount of time, they have the same velocity just before the basketball rebounds. After the basketball rebounds elastically, its velocity will have the same magnitude, but the opposite direction than just before it hit the ground.

The velocity of the basketball will be equal in magnitude but opposite in direction to the velocity of the baseball.

(*b*) Apply conservation of momentum to the collision of the balls to obtain:

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i} \qquad (1)$$

Relate the initial and final kinetic energies of the balls in their elastic collision:

$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 = \tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) \qquad (2)$$

Rearrange equation (1) to obtain:

$$m_2\left(v_{2f} - v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right) \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \quad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$(m_1 + m_2)v_{1f} = (m_1 - m_2)v_{1i} + 2m_2v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$$

or, because $v_{2i} = -v_{1i}$,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} - \frac{2m_2}{m_1 + m_2}v_{1i} = \frac{m_1 - 3m_2}{m_1 + m_2}v_{1i}$$

For $m_1 = 3m_2$ and $v_{1i} = v$:

$$v_{1f} = \frac{3m_2 - 3m_2}{3m_2 + m_2}v = \boxed{0}$$

(*c*) Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$(m_1 + m_2)v_{2f} = (m_2 - m_1)v_{2i} + 2m_1v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

or, because $v_{2i} = -v_{1i}$,

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} - \frac{m_2 - m_1}{m_1 + m_2}v_{1i} = \frac{3m_1 - m_2}{m_1 + m_2}v_{1i}$$

For $m_1 = 3m_2$ and $v_{1i} = v$:

$$v_{2f} = \frac{3(3m_2) - m_2}{3m_2 + m_2}v = \boxed{2v}$$

120 •••

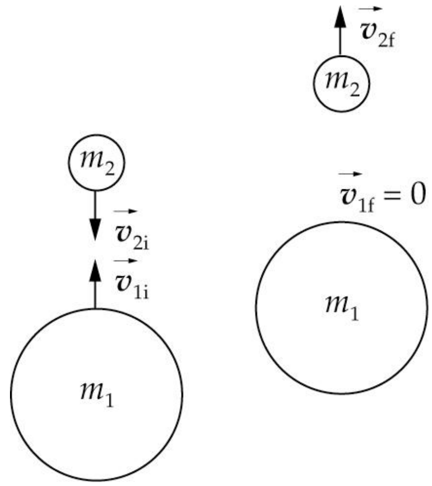
Picture the Problem In Problem 119 only two balls are dropped. They collide head on, each moving at speed v, and the collision is elastic. In this problem, as it did in Problem 119, the solution involves using the conservation of momentum equation

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}$$

and the elastic collision equation

$$v_{1f} - v_{2f} = v_{2i} - v_{1i},$$

where the numeral 1 refers to the baseball, and the numeral 2 to the top ball. The diagram shows the balls just before and just after their collision. From Problem 119 we know that that $v_{1i} = 2v$ and $v_{2i} = -v$.



(*a*) Express the final speed v_{1f} of the baseball as a function of its initial speed v_{1i} and the initial speed of the top ball v_{2i} (see Problem 78):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$$

Substitute for v_{1i} and , v_{2i} to obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}(2v) + \frac{2m_2}{m_1 + m_2}(-v)$$

Divide the numerator and denominator of each term by m_2 to introduce the mass ratio of the upper ball to the lower ball:

$$v_{1f} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}(2v) + \frac{2}{\frac{m_1}{m_2} + 1}(-v)$$

Set the final speed of the baseball v_{1f} equal to zero, let x represent the mass ratio m_1/m_2, and solve for x:

$$0 = \frac{x - 1}{x + 1}(2v) + \frac{2}{x + 1}(-v)$$

and

$$x = \frac{m_1}{m_2} = \boxed{\frac{1}{2}}$$

(*b*) Apply the second of the two equations in Problem 78 to the collision between the top ball and the baseball:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

Substitute $v_{1i} = 2v$ and are given that $v_{2i} = -v$ to obtain:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}(2v) + \frac{m_2 - m_1}{m_1 + m_2}(-v)$$

In part (*a*) we showed that $m_2 = 2m_1$. Substitute and simplify:

$$v_{3f} = \frac{2(2m_1)}{m_1 + 2m_1}(2v) - \frac{2m_1 - m_1}{m_1 + 2m_1}v$$
$$= \frac{4m_1}{3m_1}(2v) - \frac{m_1}{3m_1}v = \tfrac{8}{3}v - \tfrac{1}{3}v$$
$$= \boxed{\tfrac{7}{3}v}$$

***121 ••**

Picture the Problem Let the direction the probe is moving after its elastic collision with Saturn be the positive direction. The probe gains kinetic energy at the expense of the kinetic energy of Saturn. We'll relate the velocity of approach relative to the center of mass to u_{rec} and then to v.

(*a*) Relate the velocity of recession to the velocity of recession relative to the center of mass:

$$v = u_{rec} + v_{cm}$$

Find the velocity of approach:

$$u_{app} = -9.6\,\text{km/s} - 10.4\,\text{km/s} = -20.0\,\text{km/s}$$

Relate the relative velocity of approach to the relative velocity of recession for an elastic collision:

$$u_{rec} = -u_{app} = 20.0\,\text{km/s}$$

Because Saturn is so much more massive than the space probe:

$$v_{cm} = v_{Saturn} = 9.6\,\text{km/s}$$

Substitute and evaluate v:

$$v = u_{rec} + v_{cm} = 20\,\text{km/s} + 9.6\,\text{km/s} = \boxed{29.6\,\text{km/s}}$$

(*b*) Express the ratio of the final kinetic energy to the initial kinetic energy:

$$\frac{K_f}{K_i} = \frac{\tfrac{1}{2}Mv_{rec}^2}{\tfrac{1}{2}Mv_i^2} = \left(\frac{v_{rec}}{v_i}\right)^2$$
$$= \left(\frac{29.6\,\text{km/s}}{10.4\,\text{km/s}}\right)^2 = \boxed{8.10}$$

$$\boxed{\text{The energy comes from an immeasurably small slowing of Saturn.}}$$

***122 ••**

Picture the Problem We can use the relationships $P = c\Delta m$ and $\Delta E = \Delta mc^2$ to show that $P = \Delta E/c$. We can then equate this expression with the change in momentum of the flashlight to find the latter's final velocity.

(*a*) Express the momentum of the mass lost (i.e., carried away by the light) by the flashlight:

$$P = c\Delta m$$

Relate the energy carried away by the light to the mass lost by the flashlight:

$$\Delta m = \frac{\Delta E}{c^2}$$

Substitute to obtain:

$$P = c\frac{\Delta E}{c^2} = \boxed{\frac{\Delta E}{c}}$$

(*b*) Relate the final momentum of the flashlight to ΔE:

$$\frac{\Delta E}{c} = \Delta p = mv$$

because the flashlight is initially at rest.

Solve for *v*:

$$v = \frac{\Delta E}{mc}$$

Substitute numerical values and evaluate *v*:

$$v = \frac{1.5\times10^3\text{ J}}{(1.5\text{ kg})(2.998\times10^8\text{ m/s})} = 3.33\times10^{-6}\text{ m/s} = \boxed{3.33\,\mu\text{m/s}}$$

123 •

Picture the Problem We can equate the change in momentum of the block to the momentum of the beam of light and relate the momentum of the beam of light to the mass converted to produce the beam. Combining these expressions will allow us to find the speed attained by the block.

Relate the change in momentum of the block to the momentum of the beam:

$$(M - m)v = P_{\text{beam}}$$

because the block is initially at rest.

Express the momentum of the mass converted into a well-collimated beam of light:

$$P_{\text{beam}} = mc$$

Substitute to obtain:

$$(M - m)v = mc$$

Solve for *v*:

$$v = \frac{mc}{M - m}$$

Substitute numerical values and evaluate v:

$$v = \frac{(0.001\,\text{kg})(2.998\times10^8\ \text{m/s})}{1\,\text{kg}-0.001\,\text{kg}} = \boxed{3.00\times10^5\ \text{m/s}}$$

124 ••

Picture the Problem Let the origin of the coordinate system be at the end of the boat at which your friend is sitting prior to changing places. If we let the system include you and your friend, the boat, the water and the earth, then $F_{\text{ext,net}} = 0$ and the center of mass is at the same location after you change places as it was before you shifted.

Express the center of mass of the system prior to changing places:

$$x_{\text{cm}} = \frac{m_{\text{boat}}x_{\text{boat}} + m_{\text{you}}x_{\text{you}} + mx_{\text{friend}}}{m_{\text{boat}} + m_{\text{you}} + m_{\text{friend}}} = \frac{x_{\text{you}}(m_{\text{boat}} + m_{\text{you}}) + mx_{\text{friend}}}{m_{\text{boat}} + m_{\text{you}} + m}$$

Substitute numerical values and simplify to obtain an expression for x_{cm} in terms of m:

$$x_{\text{cm}} = \frac{(2\,\text{m})(60\,\text{kg}+80\,\text{kg})+(0)m}{60\,\text{kg}+80\,\text{kg}+m} = \frac{280\,\text{kg}\cdot\text{m}}{140\,\text{kg}+m}$$

Find the center of mass of the system after changing places:

$$x'_{\text{cm}} = \frac{m_{\text{boat}}x_{\text{boat}} + m_{\text{you}}x_{\text{you}} + mx_{\text{friend}}}{m_{\text{boat}} + m_{\text{you}} + m_{\text{friend}}} = \frac{(m_{\text{boat}}+m)(2\,\text{m}\pm0.2\,\text{m})}{m_{\text{boat}} + m_{\text{you}} + m} + \frac{m_{\text{you}}(\pm0.2\,\text{m})}{m_{\text{boat}} + m_{\text{you}} + m}$$

Substitute numerical values and simplify to obtain:

$$x'_{\text{cm}} = \frac{(60\,\text{kg}+m)(2\,\text{m}\pm0.2\,\text{m})}{60\,\text{kg}+80\,\text{kg}+m} + \frac{(80\,\text{kg})(\pm0.2\,\text{m})}{60\,\text{kg}+80\,\text{kg}+m} = \frac{120\,\text{kg}\cdot\text{m}\pm12\,\text{kg}\cdot\text{m}}{140\,\text{kg}+m} + \frac{(2\,\text{m})m\pm0.2m\,\text{m}\pm16\,\text{kg}\cdot\text{m}}{140\,\text{kg}+m}$$

Because $F_{\text{ext,net}} = 0$, $x'_{\text{cm}} = x_{\text{cm}}$. Equate the two expressions and solve for m to obtain:

$$m = \frac{(160\pm28)}{(2\pm0.2)}\,\text{kg}$$

Calculate the largest possible mass for your friend:

$$m = \frac{(160+28)}{(2-0.2)}\,\text{kg} = \boxed{104\,\text{kg}}$$

Calculate the smallest possible mass for your friend:

$$m = \frac{(160 - 28)}{(2 + 0.2)}\text{kg} = \boxed{60.0\,\text{kg}}$$

125 ••

Picture the Problem Let the system include the woman, both vehicles, and the earth. Then $F_{\text{ext,net}} = 0$ and $a_{\text{cm}} = 0$. Include the mass of the man in the mass of the truck. We can use Newton's 2nd and 3rd laws to find the acceleration of the truck and net force acting on both the car and the truck.

(*a*) Relate the action and reaction forces acting on the car and truck:

$$F_{\text{car}} = F_{\text{truck}}$$

or

$$m_{\text{car}}a_{\text{car}} = m_{\text{truck+woman}}a_{\text{truck}}$$

Solve for the acceleration of the truck:

$$a_{\text{truck}} = \frac{m_{\text{car}}a_{\text{car}}}{m_{\text{truck+woman}}}$$

Substitute numerical values and evaluate a_{truck}:

$$a_{\text{truck}} = \frac{(800\,\text{kg})(1.2\,\text{m/s}^2)}{1600\,\text{kg}} = \boxed{0.600\,\text{m/s}^2}$$

(*b*) Apply Newton's 2nd law to either vehicle to obtain:

$$F_{\text{net}} = m_{\text{car}}a_{\text{car}}$$

Substitute numerical values and evaluate F_{net}:

$$F_{\text{net}} = (800\,\text{kg})(1.2\,\text{m/s}^2) = \boxed{960\,\text{N}}$$

126 ••

Picture the Problem Let the system include the block, the putty, and the earth. Then $F_{\text{ext,net}} = 0$ and momentum is conserved in this perfectly inelastic collision. We'll use conservation of momentum to relate the after-collision velocity of the block plus blob and conservation of energy to find their after-collision velocity.

Noting that, because this is a perfectly elastic collision, the final velocity of the block plus blob is the velocity of the center of mass, use conservation of momentum to relate the velocity of the center of mass to the velocity of the glob before the collision:

$$p_{\text{i}} = p_{\text{f}}$$

or

$$m_{\text{gl}}v_{\text{gl}} = Mv_{\text{cm}}$$

where $M = m_{\text{gl}} + m_{\text{bl}}$.

Solve for v_{gl} to obtain:

$$v_{gl} = \frac{M}{m_{gl}} v_{cm} \qquad (1)$$

Use conservation of energy to find the initial energy of the block plus glob:

$$\Delta K + \Delta U + W_f = 0$$

Because $\Delta U = K_f = 0$,

$$-\tfrac{1}{2} M v_{cm}^2 + f_k \Delta x = 0$$

Use $f_k = \mu_k Mg$ to eliminate f_k and solve for v_{cm}:

$$v_{cm} = \sqrt{2\mu_k g \Delta x}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \sqrt{2(0.4)(9.81\,\text{m/s}^2)(0.15\,\text{m})} = 1.08\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate v_{gl}:

$$v_{gl} = \frac{13\,\text{kg} + 0.4\,\text{kg}}{0.4\,\text{kg}}(1.08\,\text{m/s}) = \boxed{36.2\,\text{m/s}}$$

***127 ••**

Picture the Problem Let the direction the moving car was traveling before the collision be the positive *x* direction. Let the numeral 1 denote this car and the numeral 2 the car that is stopped at the stop sign and the system include both cars and the earth. We can use conservation of momentum to relate the speed of the initially-moving car to the speed of the meshed cars immediately after their perfectly inelastic collision and conservation of energy to find the initial speed of the meshed cars.

Using conservation of momentum, relate the before-collision velocity to the after-collision velocity of the meshed cars:

$$p_i = p_f$$

or

$$m_1 v_1 = (m_1 + m_2)V$$

Solve for v_1:

$$v_1 = \frac{m_1 + m_2}{m_1} V = \left(1 + \frac{m_2}{m_1}\right) V$$

Using conservation of energy, relate the initial kinetic energy of the meshed cars to the work done by friction in bringing them to a stop:

$$\Delta K + \Delta E_{thermal} = 0$$

or, because $K_f = 0$ and $\Delta E_{thermal} = f\Delta s$,

$$-K_i + f_k \Delta s = 0$$

Substitute for *Ki* and, using $f_k = \mu_k F_n = \mu_k Mg$, eliminate f_k to

$$-\tfrac{1}{2} M V^2 + \mu_k Mg \Delta x = 0$$

obtain:

Solve for V: $V = \sqrt{2\mu_k g\Delta x}$

Substitute to obtain: $v_1 = \left(1 + \frac{m_2}{m_1}\right)\sqrt{2\mu_k g\Delta x}$

Substitute numerical values and evaluate v_1:

$$v_1 = \left(1 + \frac{900\,\text{kg}}{1200\,\text{kg}}\right)\sqrt{2(0.92)(9.81\,\text{m/s}^2)(0.76\,\text{m})} = 6.48\,\text{m/s} = 23.3\,\text{km/h}$$

$$\boxed{\text{The driver was not telling the truth. He was traveling at 23.3 km/h.}}$$

128 ••

Picture the Problem Let the zero of gravitational potential energy be at the lowest point of the bob's swing and note that the bob can swing either forward or backward after the collision. We'll use both conservation of momentum and conservation of energy to relate the velocities of the bob and the block before and after their collision.

Express the kinetic energy of the block in terms of its after-collision momentum: $K_{\text{m}} = \frac{p_m^2}{2m}$

Solve for m to obtain: $m = \frac{p_m^2}{2K_m}$ (1)

Use conservation of energy to relate K_{m} to the change in the potential energy of the bob:

$\Delta K + \Delta U = 0$

or, because $K_{\text{i}} = 0$,

$K_m + U_{\text{f}} - U_{\text{i}} = 0$

Solve for K_m:

$$\begin{aligned} K_m &= -U_{\text{f}} + U_{\text{i}} \\ &= m_{\text{bob}} g\left[L(1-\cos\theta_{\text{i}}) - L(1-\cos\theta_{\text{f}})\right] \\ &= m_{\text{bob}} gL\left[\cos\theta_{\text{f}} - \cos\theta_{\text{i}}\right] \end{aligned}$$

Substitute numerical values and evaluate K_m:

$$K_m = (0.4\,\text{kg})(9.81\,\text{m/s}^2)(1.6\,\text{m})[\cos 5.73° - \cos 53°] = 2.47\,\text{J}$$

Use conservation of energy to find the velocity of the bob just before its collision with the block:

$\Delta K + \Delta U = 0$

or, because $K_i = U_f = 0$,

$K_f - U_i = 0$

$\therefore \frac{1}{2} m_{bob} v^2 - m_{bob} gL(1 - \cos\theta_i) = 0$

or

$v = \sqrt{2gL(1 - \cos\theta_i)}$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(1.6\,\text{m})(1 - \cos 53°)} = 3.544\,\text{m/s}$$

Use conservation of energy to find the velocity of the bob just after its collision with the block:

$\Delta K + \Delta U = 0$

or, because $K_f = U_i = 0$,

$-K_i + U_f = 0$

Substitute for K_i and U_f to obtain:

$-\frac{1}{2} m_{bob} v'^2 + m_{bob} gL(1 - \cos\theta_f) = 0$

Solve for v':

$v' = \sqrt{2gL(1 - \cos\theta_f)}$

Substitute numerical values and evaluate v':

$$v' = \sqrt{2(9.81\,\text{m/s}^2)(1.6\,\text{m})(1 - \cos 5.73°)} = 0.396\,\text{m/s}$$

Use conservation of momentum to relate p_m after the collision to the momentum of the bob just before and just after the collision:

$p_i = p_f$

or

$m_{bob} v = m_{bob} v' \pm p_m$

Solve for and evaluate p_m:

$$p_m = m_{bob} v \pm m_{bob} v' = (0.4\,\text{kg})(3.544\,\text{m/s} \pm 0.396\,\text{m/s}) = 1.418\,\text{kg}\cdot\text{m/s} \pm 0.158\,\text{kg}\cdot\text{m/s}$$

Find the larger value for p_m:

$$p_m = 1.418\,\text{kg}\cdot\text{m/s} + 0.158\,\text{kg}\cdot\text{m/s} = 1.576\,\text{kg}\cdot\text{m/s}$$

Find the smaller value for p_m:

$$p_m = 1.418\,\text{kg}\cdot\text{m/s} - 0.158\,\text{kg}\cdot\text{m/s} = 1.260\,\text{kg}\cdot\text{m/s}$$

Substitute in equation (1) to determine the two values for m:

$$m = \frac{(1.576\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.503\,\text{kg}}$$

or

$$m = \frac{(1.260\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.321\,\text{kg}}$$

129 ••

Picture the Problem Choose the zero of gravitational potential energy at the location of the spring's maximum compression. Let the system include the spring, the blocks, and the earth. Then the net external force is zero as is work done against friction. We can use conservation of energy to relate the energy transformations taking place during the evolution of this system.

Apply conservation of energy:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = 0$:

$$\Delta U_g + \Delta U_s = 0$$

Express the change in the gravitational potential energy:

$$\Delta U_g = -mg\Delta h - Mgx\sin\theta$$

Express the change in the potential energy of the spring:

$$\Delta U_s = \tfrac{1}{2}kx^2$$

Substitute to obtain:

$$-mg\Delta h - Mgx\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Solve for M:

$$M = \frac{\tfrac{1}{2}kx^2 - mg\Delta h}{gx\sin 30°} = \frac{kx}{g} - \frac{2m\Delta h}{x}$$

Relate Δh to the initial and rebound positions of the block whose mass is m:

$$\Delta h = (4\,\text{m} - 2.56\,\text{m})\sin 30° = 0.720\,\text{m}$$

Substitute numerical values and evaluate M:

$$M = \frac{(11\times10^3\,\text{N/m})(0.04\,\text{m})}{9.81\,\text{m/s}^2} - \frac{2(1\,\text{kg})(0.72\,\text{m})}{0.04\,\text{m}} = \boxed{8.85\,\text{kg}}$$

***130** ••

Picture the Problem By symmetry, $x_{cm} = 0$. Let σ be the mass per unit area of the disk. The mass of the modified disk is the difference between the mass of the whole disk and the mass that has been removed.

Start with the definition of y_{cm}:

$$y_{cm}=\frac{\sum_i m_i y_i}{M-m_{hole}}=\frac{m_{disk}y_{disk}-m_{hole}y_{hole}}{M-m_{hole}}$$

Express the mass of the complete disk:

$$M=\sigma A=\sigma\pi r^2$$

Express the mass of the material removed:

$$m_{hole}=\sigma\pi\left(\frac{r}{2}\right)^2=\tfrac{1}{4}\sigma\pi r^2=\tfrac{1}{4}M$$

Substitute and simplify to obtain:

$$y_{cm}=\frac{M(0)-\left(\frac{1}{4}M\right)\left(-\frac{1}{2}r\right)}{M-\frac{1}{4}M}=\boxed{\tfrac{1}{6}r}$$

131 ••

Picture the Problem Let the horizontal axis by the *y* axis and the vertical axis the *z* axis. By symmetry, $x_{cm}=y_{cm}=0$. Let ρ be the mass per unit volume of the sphere. The mass of the modified sphere is the difference between the mass of the whole sphere and the mass that has been removed.

Start with the definition of y_{cm}:

$$z_{cm}=\frac{\sum_i m_i y_i}{M-m_{hole}}=\frac{m_{sphere}y_{sphere}-m_{hole}y_{hole}}{M-m_{hole}}$$

Express the mass of the complete sphere:

$$M=\rho V=\tfrac{4}{3}\rho\pi r^3$$

Express the mass of the material removed:

$$m_{hole}=\tfrac{4}{3}\rho\pi\left(\frac{r}{2}\right)^3=\tfrac{1}{8}\left(\tfrac{4}{3}\rho\pi r^3\right)=\tfrac{1}{8}M$$

Substitute and simplify to obtain:

$$z_{cm}=\frac{M(0)-\left(\frac{1}{8}M\right)\left(-\frac{1}{2}r\right)}{M-\frac{1}{8}M}=\boxed{\tfrac{1}{14}r}$$

***132 ••**

Picture the Problem In this elastic head-on collision, the kinetic energy of recoiling nucleus is the difference between the initial and final kinetic energies of the neutron. We can derive the indicated results by using both conservation of energy and conservation of momentum and writing the kinetic energies in terms of the momenta of the particles before and after the collision.

(*a*) Use conservation of energy to relate the kinetic energies of the particles before and after the collision:

$$\frac{p_{ni}^2}{2m} = \frac{p_{nf}^2}{2m} + \frac{p_{nucleus}^2}{2M} \qquad (1)$$

Apply conservation of momentum to obtain a second relationship between the initial and final momenta:

$$p_{ni} = p_{nf} + p_{nucleus} \qquad (2)$$

Eliminate p_{nf} in equation (1) using equation (2):

$$\frac{p_{nucleus}}{2M} + \frac{p_{nucleus}}{2m} - \frac{p_{ni}}{m} = 0 \qquad (3)$$

Use equation (3) to write $p_{ni}^2/2m$ in terms of $p_{nucleus}$:

$$\frac{p_{ni}^2}{2m} = K_n = \frac{p_{nucleus}^2(M+m)^2}{8M^2m} \qquad (4)$$

Use equation (4) to express $K_{nucleus} = p_{nucleus}^2/2M$ in terms of K_n:

$$K_{nucleus} = \boxed{K_n\left[\frac{4Mm}{(M+m)^2}\right]} \qquad (5)$$

(*b*) Relate the *change* in the kinetic energy of the neutron to the after-collision kinetic energy of the nucleus:

$$\Delta K_n = -K_{nucleus}$$

Using equation (5), express the fraction of the energy lost in the collision:

$$\frac{-\Delta K_n}{K_n} = \boxed{\frac{4Mm}{(M+m)^2} = \frac{4\dfrac{m}{M}}{\left(1+\dfrac{m}{M}\right)^2}}$$

133 ••

Picture the Problem Problem 132 (*b*) provides an expression for the fractional loss of energy per collision.

(*a*) Using the result of Problem 132 (*b*), express the fractional loss of energy per collision:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{(M-m)^2}{(M+m)^2}$$

Evaluate this fraction to obtain:

$$\frac{K_{nf}}{E_0} = \frac{(12m-m)^2}{(12m+m)^2} = 0.716$$

Express the kinetic energy of one

$$K_{nf} = \boxed{0.716^N E_0}$$

neutron after *N* collisions:

(*b*) Substitute for K_{nf} and E_0 to obtain:

$$0.716^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$N = \frac{-8}{\log 0.716} \approx \boxed{55}$$

134 ••

Picture the Problem We can relate the number of collisions needed to reduce the energy of a neutron from 2 MeV to 0.02 eV to the fractional energy loss per collision and solve the resulting exponential equation for *N*.

(*a*) Using the result of Problem 132 (*b*), express the fractional loss of energy per collision:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{K_{ni} - 0.63K_{ni}}{K_{ni}} = 0.37$$

Express the kinetic energy of one neutron after *N* collisions:

$$K_{nf} = \boxed{0.37^N E_0}$$

Substitute for K_{nf} and E_0 to obtain:

$$0.37^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$N = \frac{-8}{\log 0.37} \approx \boxed{19}$$

(*b*) Proceed as in (*a*) to obtain:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{K_{ni} - 0.11K_{ni}}{K_{ni}} = 0.89$$

Express the kinetic energy of one neutron after *N* collisions:

$$K_{nf} = \boxed{0.89^N E_0}$$

Substitute for K_{nf} and E_0 to obtain:

$$0.89^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$N = \frac{-8}{\log 0.89} \approx \boxed{158}$$

135 ••

Picture the Problem Let $\lambda = M/L$ be the mass per unit length of the rope, the subscript 1 refer to the portion of the rope that is being supported by the force F at any given time, and the subscript 2 refer to the rope that is still on the table at any given time. We can find the height h_{cm} of the center of mass as a function of time and then differentiate this expression twice to find the acceleration of the center of mass.

(*a*) Apply the definition of the center of mass to obtain:

$$h_{cm} = \frac{m_1 h_{1,cm} + m_2 h_{2,cm}}{M} \qquad (1)$$

From the definition of λ we have:

$$\frac{M}{L} = \frac{m_1}{vt} \Rightarrow m_1 = \frac{M}{L}vt$$

$h_{1,cm}$ and $h_{2,cm}$ are given by :

$$h_{1,cm} = \frac{1}{2}vt \text{ and } h_{2,cm} = 0$$

Substitute for m_1, $h_{1,cm}$, and $h_{2,cm}$ in equation (1) and simplify to obtain:

$$h_{cm} = \frac{\left(\frac{M}{L}vt\right)h_{1,cm} + m_2(0)}{M} = \boxed{\frac{v^2}{2L}t^2}$$

(*b*) Differentiate h_{cm} twice to obtain a_{cm}:

$$\frac{dh_{cm}}{dt} = 2\left(\frac{v^2}{2L}\right)t = \frac{v^2}{L}t$$

and

$$\frac{d^2h_{cm}}{dt^2} = a_{cm} = \boxed{\frac{v^2}{L}}$$

(*c*) Letting N represent the normal force that the table exerts on the rope, apply $\sum F_y = ma_{cm}$ to the rope to obtain:

$$F + N - Mg = Ma_{cm}$$

Solve for F, substitute for a_{cm} and N to obtain:

$$F = Mg + Ma_{cm} - N$$
$$= Mg + M\frac{v^2}{L} - m_2 g$$

Use the definition of λ again to obtain:

$$\frac{m_2}{L - vt} = \frac{M}{L} \Rightarrow m_2 = M\left(1 - \frac{vt}{L}\right)$$

Substitute for m_2 and simplify:

$$F = Mg + M\frac{v^2}{L} - M\left(1 - \frac{vt}{L}\right)g = M\left(g + \frac{v^2}{L} - g + \frac{vt}{L}g\right) = Mg\left(\frac{v^2}{gL} + \frac{vt}{L}\right)$$

$$= \boxed{\frac{vt}{L}\left(\frac{v}{gt} + 1\right)Mg}$$

136 ••

Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n. We can use Newton's 2nd law to determine the scale reading in part (*a*). We'll use both conservation of energy and momentum to obtain the scale reading when the ball collides inelastically with the cup.

(*a*) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance *d*:

$$F_n - m_p g - F_{\text{ball on spring}} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{\text{ball on spring}}$$
$$= m_p g + kd$$
$$= m_p g + k\left(\frac{m_b g}{k}\right)$$
$$= \boxed{m_p g + m_b g = (m_p + m_b)g}$$

(*b*) Letting the zero of gravitational energy be at the initial elevation of the cup and v_{bi} represent the velocity of the ball just before it hits the cup, use conservation of energy to find this velocity:

$$\Delta K + \Delta U_g = 0 \text{ where } K_i = U_{gf} = 0$$
$$\therefore \tfrac{1}{2}m_b v_{bi}^2 - mgh = 0$$
and
$$v_{bi} = \sqrt{2gh}$$

Use conservation of momentum to

$$\vec{p}_i = \vec{p}_f$$

find the velocity of the center of mass:

$$\therefore v_{\text{cm}} = \frac{m_{\text{b}}v_{\text{bi}}}{m_{\text{b}} + m_{\text{c}}} = \sqrt{2gh}\left[\frac{m_{\text{b}}}{m_{\text{b}} + m_{\text{c}}}\right]$$

Apply conservation of energy to the collision to obtain:

$$\Delta K_{\text{cm}} + \Delta U_{\text{s}} = 0$$

or, with $K_{\text{f}} = U_{\text{si}} = 0$,

$$-\tfrac{1}{2}(m_{\text{b}} + m_{\text{c}})v_{\text{cm}}^2 + \tfrac{1}{2}kx^2 = 0$$

Substitute for v_{cm} and solve for kx^2:

$$\begin{aligned} kx^2 &= (m_{\text{b}} + m_{\text{c}})v_{\text{cm}}^2 \\ &= 2gh(m_{\text{b}} + m_{\text{c}})\left[\frac{m_{\text{b}}}{m_{\text{b}} + m_{\text{c}}}\right]^2 \\ &= \frac{2ghm_{\text{b}}^2}{m_{\text{b}} + m_{\text{c}}} \end{aligned}$$

Solve for x:

$$x = m_{\text{b}}\sqrt{\frac{2gh}{k(m_{\text{b}} + m_{\text{c}})}}$$

From part (a):

$$\begin{aligned} F_{\text{n}} &= m_{\text{p}}g + kx \\ &= m_{\text{p}}g + km_{\text{b}}\sqrt{\frac{2gh}{k(m_{\text{b}} + m_{\text{c}})}} \\ &= \boxed{g\left(m_{\text{p}} + m_{\text{b}}\sqrt{\frac{2kh}{g(m_{\text{b}} + m_{\text{c}})}}\right)} \end{aligned}$$

(c) Because the collision is inelastic, the ball never returns to its original height.

137 ••

Picture the Problem Let the direction that astronaut 1 first throws the ball be the positive direction and let v_{b} be the initial speed of the ball in the laboratory frame. Note that each collision is perfectly inelastic. We can apply conservation of momentum and the definition of the speed of the ball relative to the thrower to each of the perfectly inelastic collisions to express the final speeds of each astronaut after one throw and one catch.

Use conservation of momentum to relate the speeds of astronaut 1 and the ball after the first throw:

$$m_1v_1 + m_{\text{b}}v_{\text{b}} = 0 \quad (1)$$

Relate the speed of the ball in the laboratory frame to its speed relative

$$v = v_{\text{b}} - v_1 \quad (2)$$

to astronaut 1:

Eliminate v_b between equations (1) and (2) and solve for v_1:

$$v_1 = -\frac{m_b}{m_1 + m_b} v \quad (3)$$

Substitute equation (3) in equation (2) and solve for v_b:

$$v_b = \frac{m_1}{m_1 + m_b} v \quad (4)$$

Apply conservation of momentum to express the speed of astronaut 2 and the ball after the first catch:

$$0 = m_b v_b = (m_2 + m_b) v_2 \quad (5)$$

Solve for v_2:

$$v_2 = \frac{m_b}{m_2 + m_b} v_b \quad (6)$$

Express v_2 in terms of v by substituting equation (4) in equation (6):

$$v_2 = \frac{m_b}{m_2 + m_b} \frac{m_1}{m_1 + m_b} v = \left[\frac{m_b m_1}{(m_2 + m_b)(m_1 + m_b)}\right] v \quad (7)$$

Use conservation of momentum to express the speed of astronaut 2 and the ball after she throws the ball:

$$(m_2 + m_b) v_2 = m_b v_{bf} + m_2 v_{2f} \quad (8)$$

Relate the speed of the ball in the laboratory frame to its speed relative to astronaut 2:

$$v = v_{2f} - v_{bf} \quad (9)$$

Eliminate v_{bf} between equations (8) and (9) and solve for v_{2f}:

$$v_{2f} = \boxed{\left(\frac{m_b}{m_2 + m_b}\right)\left[1 + \frac{m_1}{m_1 + m_b}\right] v} \quad (10)$$

Substitute equation (10) in equation (9) and solve for v_{bf}:

$$v_{bf} = -\left[1 - \frac{m_b}{m_2 + m_b}\right] \times \left[1 + \frac{m_1}{m_1 + m_b}\right] v \quad (11)$$

Apply conservation of momentum to express the speed of astronaut 1 and the ball after she catches the ball:

$$(m_1 + m_b) v_{1f} = m_b v_{bf} + m_1 v_1 \quad (12)$$

Using equations (3) and (11), eliminate v_{bf} and v_1 in equation (12) and solve for v_{1f}:

$$v_{1f} = \boxed{-\frac{m_2 m_b (2m_1 + m_b)}{(m_1 + m_b)^2 (m_2 + m_b)} v}$$

***138** ••

Picture the Problem We can use the definition of the center of mass of a system containing multiple objects to locate the center of mass of the earth–moon system. Any object external to the system will exert accelerating forces on the system.

(*a*) Express the center of mass of the earth–moon system relative to the center of the earth:

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

or

$$r_{cm} = \frac{M_e(0) + m_m r_{em}}{M_e + m_m} = \frac{m_m r_{em}}{M_e + m_m} = \frac{r_{em}}{\frac{M_e}{m_m} + 1}$$

Substitute numerical values and evaluate r_{cm}:

$$r_{cm} = \frac{3.84 \times 10^5 \text{ km}}{81.3 + 1} = \boxed{4670 \text{ km}}$$

Because this distance is less than the radius of the earth, the position of the center of mass of the earth – moon system is below the surface of the earth.

(*b*) Any object not in the earth – moon system exerts forces on the system, e.g., the sun and other planets.

(*c*) Because the sun exerts the dominant external force on the earth – moon system, the acceleration of the system is toward the sun.

(*d*) Because the center of mass is at a fixed distance from the sun, the distance d moved by the earth in this time interval is:

$$d = 2r_{em} = 2(4670 \text{ km}) = \boxed{9340 \text{ km}}$$

139 ••

Picture the Problem Let the numeral 2 refer to you and the numeral 1 to the water leaving the hose. Apply conservation of momentum to the system consisting of yourself, the water, and the earth and then differentiate this expression to relate your recoil acceleration to your mass, the speed of the water, and the rate at which the water is leaving the hose.

Use conservation of momentum to relate your recoil velocity to the velocity of the water leaving the hose:

$$\vec{p}_1 + \vec{p}_2 = 0$$

or

$$m_1 v_1 + m_2 v_2 = 0$$

Differentiate this expression with respect to t:

$$m_1 \frac{dv_1}{dt} + v_1 \frac{dm_1}{dt} + m_2 \frac{dv_2}{dt} + v_2 \frac{dm_2}{dt} = 0$$

or

$$m_1 a_1 + v_1 \frac{dm_1}{dt} + m a_2 + v_2 \frac{dm_2}{dt} = 0$$

Because the acceleration of the water leaving the hose, a_1, is zero … as is $\frac{dm_2}{dt}$, the rate at which you are losing mass:

$$v_1 \frac{dm_1}{dt} + m_2 a_2 = 0$$

and

$$a_2 = -\frac{v_1}{m_2}\frac{dm_1}{dt}$$

Substitute numerical values and evaluate a_2:

$$a_2 = -\frac{30\,\text{m/s}}{75\,\text{kg}}(2.4\,\text{kg/s})$$

$$= \boxed{-0.960\,\text{m/s}^2}$$

***140 •••**

Picture the Problem Take the zero of gravitational potential energy to be at the elevation of the pan and let the system include the balance, the beads, and the earth. We can use conservation of energy to find the vertical component of the velocity of the beads as they hit the pan and then calculate the net downward force on the pan from Newton's 2nd law.

Use conservation of energy to relate the y component of the bead's velocity as it hits the pan to its height of fall:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2} m v_y^2 - mgh = 0$$

Solve for v_y:

$$v_y = \sqrt{2gh}$$

Substitute numerical values and evaluate v_y:

$$v_y = \sqrt{2(9.81\,\text{m/s}^2)(0.5\,\text{m})} = 3.13\,\text{m/s}$$

Express the change in momentum in the y direction per bead:

$$\Delta p_y = p_{yf} - p_{yi} = m v_y - (-m v_y) = 2 m v_y$$

Use Newton's 2nd law to express the net force in the y direction exerted on the pan by the beads:

$$F_{\text{net},y} = N\frac{\Delta p_y}{\Delta t}$$

Letting M represent the mass to be placed on the other pan, equate its weight to the net force exerted by the beads, substitute for Δp_y, and solve for M:

$$Mg = N\frac{\Delta p_y}{\Delta t}$$

and

$$M = \frac{N}{\Delta t}\left(\frac{2mv_y}{g}\right)$$

Substitute numerical values and evaluate M:

$$M = (100/\text{s})\frac{[2(0.0005\,\text{kg})(3.13\,\text{m/s})]}{9.81\,\text{m/s}^2}$$

$$= \boxed{31.9\,\text{g}}$$

141 •••

Picture the Problem Assume that the connecting rod goes halfway through both balls, i.e., the centers of mass of the balls are separated by L. Let the system include the dumbbell, the wall and floor, and the earth. Let the zero of gravitational potential be at the center of mass of the lower ball and use conservation of energy to relate the speeds of the balls to the potential energy of the system. By symmetry, the speeds will be equal when the angle with the vertical is 45°.

Use conservation of energy to express the relationship between the initial and final energies of the system:

$$E_\text{i} = E_\text{f}$$

Express the initial energy of the system:

$$E_\text{i} = mgL$$

Express the energy of the system when the angle with the vertical is 45°:

$$E_\text{f} = mgL\sin 45° + \tfrac{1}{2}(2m)v^2$$

Substitute to obtain:

$$gL = gL\left(\frac{1}{\sqrt{2}}\right) + v^2$$

Solve for v:

$$v = \sqrt{gL\left(1 - \frac{1}{\sqrt{2}}\right)}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(9.81\,\text{m/s}^2)L\left(1-\frac{1}{\sqrt{2}}\right)}$$

$$= \boxed{(1.70\,\text{m}^{1/2}/\text{s})\sqrt{L}}$$

Chapter 9
Rotation

Conceptual Problems

***1** •

Determine the Concept Because r is greater for the point on the rim, it moves the greater distance. Both turn through the same angle. Because r is greater for the point on the rim, it has the greater speed. Both have the same angular velocity. Both have zero tangential acceleration. Both have zero angular acceleration. Because r is greater for the point on the rim, it has the greater centripetal acceleration.

2 •

(*a*) False. Angular velocity has the dimensions $\left[\frac{1}{T}\right]$ whereas linear velocity has dimensions $\left[\frac{L}{T}\right]$.

(*b*) True. The angular velocity of all points on the wheel is $d\theta/dt$.

(*c*) True. The angular acceleration of all points on the wheel is $d\omega/dt$.

3 ••

Picture the Problem The constant-acceleration equation that relates the given variables is $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$. We can set up a proportion to determine the number of revolutions required to double ω and then subtract to find the number of additional revolutions to accelerate the disk to an angular speed of 2ω.

Using a constant-acceleration equation, relate the initial and final angular velocities to the angular acceleration:

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

or, because $\omega_0^2 = 0$,

$$\omega^2 = 2\alpha\Delta\theta$$

Let $\Delta\theta_{10}$ represent the number of revolutions required to reach an angular velocity ω:

$$\omega^2 = 2\alpha\Delta\theta_{10} \qquad (1)$$

Let $\Delta\theta_{2\omega}$ represent the number of revolutions required to reach an angular velocity ω:

$$(2\omega)^2 = 2\alpha\Delta\theta_{2\omega} \qquad (2)$$

Divide equation (2) by equation (1) and solve for $\Delta\theta_{2\omega}$:

$$\Delta\theta_{2\omega} = \frac{(2\omega)^2}{\omega^2}\Delta\theta_{10} = 4\Delta\theta_{10}$$

The number of *additional* revolutions is: $4\Delta\theta_{10} - \Delta\theta_{10} = 3\Delta\theta_{10} = 3(10\,\text{rev}) = 30\,\text{rev}$

and $\boxed{(c)\text{ is correct.}}$

***4 •**

Determine the Concept Torque has the dimension $\left[\frac{ML^2}{T^2}\right]$.

(*a*) Impulse has the dimension $\left[\frac{ML}{T}\right]$.

(*b*) Energy has the dimension $\left[\frac{ML^2}{T^2}\right]$. $\boxed{(b)\text{ is correct.}}$

(*c*) Momentum has the dimension $\left[\frac{ML}{T}\right]$.

5 •

Determine the Concept The moment of inertia of an object is the product of a constant that is characteristic of the object's distribution of matter, the mass of the object, and the square of the distance from the object's center of mass to the axis about which the object is rotating. Because both (*b*) and (*c*) are correct $\boxed{(d)\text{ is correct.}}$

***6 •**

Determine the Concept Yes. A net torque is required to *change* the rotational state of an object. In the absence of a net torque an object continues in whatever state of rotational motion it was at the instant the net torque became zero.

7 •

Determine the Concept No. A net torque is required to *change* the rotational state of an object. A net torque may decrease the angular speed of an object. All we can say for sure is that a net torque will *change* the angular speed of an object.

8 •

(*a*) False. The net torque acting on an object determines the angular acceleration of the object. At any given instant, the angular velocity may have any value including zero.

(*b*) True. The moment of inertia of a body is *always* dependent on one's choice of an axis of rotation.

(*c*) False. The moment of inertia of an object is the product of a constant that is characteristic of the object's distribution of matter, the mass of the object, and the square of the distance from the object's center of mass to the axis about which the object is rotating.

9 •

Determine the Concept The angular acceleration of a rotating object is proportional to the *net* torque acting on it. The net torque is the product of the tangential force and its lever arm.

Express the angular acceleration of the disk as a function of the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Fd}{I} = \frac{F}{I}d$$

i.e., $\alpha \propto d$

Because $\alpha \propto d$, doubling d will double the angular acceleration.

$\boxed{(b)\text{ is correct.}}$

***10** •

Determine the Concept From the parallel-axis theorem we know that $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes. Therefore, I is always greater than I_{cm} by Mh^2. $\boxed{(d)\text{ is correct.}}$

11 •

Determine the Concept The power delivered by the constant torque is the product of the torque and the angular velocity of the merry-go-round. Because the constant torque causes the merry-go-round to accelerate, neither the power input nor the angular velocity of the merry-go-round is constant. $\boxed{(b)\text{ is correct.}}$

12 •

Determine the Concept Let's make the simplifying assumption that the object and the surface do not deform when they come into contact, i.e., we'll assume that the system is rigid. A force does no work if and only if it is perpendicular to the velocity of an object, and exerts no torque on an extended object if and only if it's directed toward the center of the object. Because neither of these conditions is satisfied, the statement is *false*.

13 •

Determine the Concept For a given applied force, this increases the torque about the hinges of the door, which increases the door's angular acceleration, leading to the door being opened more quickly. It is clear that putting the knob far from the hinges means that the door can be opened with less effort (force). However, it also means that the hand on the knob must move through the greatest distance to open the door, so it may not be the quickest way to open the door. Also, if the knob were at the center of the door, you would have to walk around the door after opening it, assuming the door is opening toward you.

***14 •**

Determine the Concept If the wheel is rolling without slipping, a point at the top of the wheel moves with a speed twice that of the center of mass of the wheel, but the bottom of the wheel is momentarily at rest. $\boxed{(c)\text{ is correct.}}$

15 ••

Picture the Problem The kinetic energies of both objects is the sum of their translational and rotational kinetic energies. Their speed dependence will differ due to the differences in their moments of inertia. We can express the total kinetic of both objects and equate them to decide which of their translational speeds is greater.

Express the kinetic energy of the cylinder:

$$K_{\text{cyl}} = \tfrac{1}{2}I_{\text{cyl}}\omega_{\text{cyl}}^2 + \tfrac{1}{2}mv_{\text{cyl}}^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v_{\text{cyl}}^2}{r^2} + \tfrac{1}{2}mv_{\text{cyl}}^2 = \tfrac{3}{4}mv_{\text{cyl}}^2$$

Express the kinetic energy of the sphere:

$$K_{\text{sph}} = \tfrac{1}{2}I_{\text{sph}}\omega_{\text{sphl}}^2 + \tfrac{1}{2}mv_{\text{sph}}^2 = \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v_{\text{sph}}^2}{r^2} + \tfrac{1}{2}mv_{\text{sph}}^2 = \tfrac{7}{10}mv_{\text{sph}}^2$$

Equate the kinetic energies and simplify to obtain:

$$v_{\text{cyl}} = \sqrt{\tfrac{14}{15}}v_{\text{sph}} < v_{\text{sph}}$$

and $\boxed{(b)\text{ is correct.}}$

***16 •**

Determine the Concept You could spin the pipes about their center. The one which is easier to spin has its mass concentrated closer to the center of mass and, hence, has a smaller moment of inertia.

17 ••

Picture the Problem Because the coin and the ring begin from the same elevation, they will have the same kinetic energy at the bottom of the incline. The kinetic energies of both objects is the sum of their translational and rotational kinetic energies. Their speed dependence will differ due to the differences in their moments of inertia. We can express the total kinetic of both objects and equate them to their common potential energy loss to decide which of their translational speeds is greater at the bottom of the incline.

Express the kinetic energy of the coin at the bottom of the incline:

$$K_{\text{coin}} = \tfrac{1}{2} I_{\text{cyl}} \omega_{\text{coin}}^2 + \tfrac{1}{2} m_{\text{coin}} v_{\text{coin}}^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m_{\text{coin}} r^2\right)\frac{v_{\text{coin}}^2}{r^2} + \tfrac{1}{2} m_{\text{coin}} v_{\text{coin}}^2 = \tfrac{3}{4} m_{\text{coin}} v_{\text{coin}}^2$$

Express the kinetic energy of the ring at the bottom of the incline:

$$K_{\text{ring}} = \tfrac{1}{2} I_{\text{ring}} \omega_{\text{ring}}^2 + \tfrac{1}{2} m_{\text{ring}} v_{\text{ring}}^2 = \tfrac{1}{2}\left(m_{\text{ring}} r^2\right)\frac{v_{\text{ring}}^2}{r^2} + \tfrac{1}{2} m_{\text{ring}} v_{\text{ring}}^2 = m_{\text{ring}} v_{\text{ring}}^2$$

Equate the kinetic of the coin to its change in potential energy as it rolled down the incline and solve for v_{coin}:

$$\tfrac{3}{4} m_{\text{coin}} v_{\text{coin}}^2 = m_{\text{coin}} gh$$

and

$$v_{\text{coin}}^2 = \tfrac{4}{3} gh$$

Equate the kinetic of the ring to its change in potential energy as it rolled down the incline and solve for v_{ring}:

$$m_{\text{ring}} v_{\text{ring}}^2 = m_{\text{ring}} gh$$

and

$$v_{\text{ring}}^2 = gh$$

Therefore, $v_{\text{coin}} > v_{\text{ring}}$ and (b) is correct.

18 ••

Picture the Problem We can use the definitions of the translational and rotational kinetic energies of the hoop and the moment of inertia of a hoop (ring) to express and compare the kinetic energies.

Express the translational kinetic energy of the hoop:

$$K_{\text{trans}} = \tfrac{1}{2} mv^2$$

Express the rotational kinetic energy of the hoop:

$$K_{\text{rot}} = \tfrac{1}{2} I_{\text{hoop}} \omega^2 = \tfrac{1}{2}\left(mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{2} mv^2$$

Therefore, the translational and rotational kinetic energies are the same and (c) is correct.

19 ••

Picture the Problem We can use the definitions of the translational and rotational kinetic energies of the disk and the moment of inertia of a disk (cylinder) to express and compare the kinetic energies.

Express the translational kinetic energy of the disk:

$$K_{trans} = \tfrac{1}{2}mv^2$$

Express the rotational kinetic energy of the disk:

$$K_{rot} = \tfrac{1}{2}I_{hoop}\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{4}mv^2$$

Therefore, the translational kinetic energy is greater and $\boxed{(a)\text{ is correct.}}$

20 ••

Picture the Problem Let us assume that $f \neq 0$ and acts along the direction of motion. Now consider the acceleration of the center of mass and the angular acceleration about the point of contact with the plane. Because $F_{net} \neq 0$, $a_{cm} \neq 0$. However, $\tau = 0$ because $\ell = 0$, so $\alpha = 0$. But $\alpha = 0$ is not consistent with $a_{cm} \neq 0$. Consequently, $f = 0$.

21 •

Determine the Concept True. If the sphere is slipping, then there is kinetic friction which dissipates the mechanical energy of the sphere.

22 •

Determine the Concept Because the ball is struck high enough to have topspin, the frictional force is forward; reducing ω until the nonslip condition is satisfied.

$\boxed{(a)\text{ is correct.}}$

Estimation and Approximation

23 ••

Picture the Problem Assume the wheels are hoops, i.e., neglect the mass of the spokes, and express the total kinetic energy of the bicycle and rider. Let M represent the mass of the rider, m the mass of the bicycle, m_w the mass of each bicycle wheel, and r the radius of the wheels.

Express the ratio of the kinetic energy associated with the rotation of the wheels to that associated with the total kinetic energy of the bicycle and rider:

$$\frac{K_{rot}}{K_{tot}} = \frac{K_{rot}}{K_{trans} + K_{rot}} \quad (1)$$

Express the translational kinetic energy of the bicycle and rider:

$$K_{\text{trans}} = K_{\text{bicycle}} + K_{\text{rider}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2$$

Express the rotational kinetic energy of the bicycle wheels:

$$K_{\text{rot}} = 2K_{\text{rot, 1 wheel}} = 2\left(\tfrac{1}{2}I_{\text{w}}\omega^2\right) = \left(m_{\text{w}}r^2\right)\frac{v^2}{r^2} = m_{\text{w}}v^2$$

Substitute in equation (1) to obtain:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{m_{\text{w}}v^2}{\tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2 + m_{\text{w}}v^2} = \frac{m_{\text{w}}}{\tfrac{1}{2}m + \tfrac{1}{2}M + m_{\text{w}}} = \frac{2}{2 + \dfrac{m+M}{m_{\text{w}}}}$$

Substitute numerical values and evaluate $K_{\text{rot}}/K_{\text{tot}}$:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{2}{2 + \dfrac{14\,\text{kg} + 38\,\text{kg}}{3\,\text{kg}}} = \boxed{10.3\,\%}$$

24 ••

Picture the Problem We can apply the definition of angular velocity to find the angular orientation of the slice of toast when it has fallen a distance of 0.5 m from the edge of the table. We can then interpret the orientation of the toast to decide whether it lands jelly-side up or down.

Relate the angular orientation θ of the toast to its initial angular orientation, its angular velocity ω, and time of fall Δt:

$$\theta = \theta_0 + \omega\Delta t \qquad (1)$$

Use the equation given in the problem statement to find the angular velocity corresponding to this length of toast:

$$\omega = 0.956\sqrt{\frac{9.81\,\text{m/s}^2}{0.1\,\text{m}}} = 9.47\,\text{rad/s}$$

Using a constant-acceleration equation, relate the distance the toast falls Δy to its time of fall Δt:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(0.5\,\text{m})}{9.81\,\text{m/s}^2}} = 0.319\,\text{s}$$

$\frac{(v_f')^2}{2gL} + \cos\theta_0$ Substitute in equation (1) to find θ:

$$\theta = \frac{\pi}{6} + (9.47\,\text{rad/s})(0.319\,\text{s})$$
$$= 3.54\,\text{rad} \times \frac{180°}{\pi\,\text{rad}} = 203°$$

The orientation of the slice of toast will therefore be at an angle of 203° with respect to the ground, i.e. with the jelly-side down.

***25 ••**

Picture the Problem Assume that the mass of an average adult male is about 80 kg, and that we can model his body when he is standing straight up with his arms at his sides as a cylinder. From experience in men's clothing stores, a man's average waist circumference seems to be about 34 inches, and the average chest circumference about 42 inches. We'll also assume that about 20% of the body's mass is in the two arms, and each has a length L = 1 m, so that each arm has a mass of about m = 8 kg.

Letting I_{out} represent his moment of inertia with his arms straight out and I_{in} his moment of inertia with his arms at his side, the ratio of these two moments of inertia is:

$$\frac{I_{out}}{I_{in}} = \frac{I_{body} + I_{arms}}{I_{in}} \qquad (1)$$

Express the moment of inertia of the "man as a cylinder":

$$I_{in} = \tfrac{1}{2}MR^2$$

Express the moment of inertia of his arms:

$$I_{arms} = 2\left(\tfrac{1}{3}\right)mL^2$$

Express the moment of inertia of his body-less-arms:

$$I_{body} = \tfrac{1}{2}(M - m)R^2$$

Substitute in equation (1) to obtain:

$$\frac{I_{out}}{I_{in}} = \frac{\tfrac{1}{2}(M - m)R^2 + 2\left(\tfrac{1}{3}\right)mL^2}{\tfrac{1}{2}MR^2}$$

Assume the circumference of the cylinder to be the average of the average waist circumference and the average chest circumference:

$$c_{av} = \frac{34\,\text{in} + 42\,\text{in}}{2} = 38\,\text{in}$$

Find the radius of a circle whose circumference is 38 in:

$$R = \frac{c_{av}}{2\pi} = \frac{38\,\text{in} \times \frac{2.54\,\text{cm}}{\text{in}} \times \frac{1\,\text{m}}{100\,\text{cm}}}{2\pi}$$
$$= 0.154\,\text{m}$$

Substitute numerical values and evaluate I_{out}/I_{in}:

$$\frac{I_{out}}{I_{in}} = \frac{\frac{1}{2}(80\,\text{kg} - 16\,\text{kg})(0.154\,\text{m})^2 + \frac{2}{3}(8\,\text{kg})(1\,\text{m})^2}{\frac{1}{2}(80\,\text{kg})(0.154\,\text{m})^2} = \boxed{6.42}$$

Angular Velocity and Angular Acceleration

26 •

Picture the Problem The tangential and angular velocities of a particle moving in a circle are directly proportional. The number of revolutions made by the particle in a given time interval is proportional to both the time interval and its angular speed.

(*a*) Relate the angular velocity of the particle to its speed along the circumference of the circle:

$$v = r\omega$$

Solve for and evaluate ω:

$$\omega = \frac{v}{r} = \frac{25\,\text{m/s}}{90\,\text{m}} = \boxed{0.278\,\text{rad/s}}$$

(*b*) Using a constant-acceleration equation, relate the number of revolutions made by the particle in a given time interval to its angular velocity:

$$\Delta\theta = \omega\Delta t = \left(0.278\frac{\text{rad}}{\text{s}}\right)(30\,\text{s})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)$$

$$= \boxed{1.33\,\text{rev}}$$

27 •

Picture the Problem Because the angular acceleration is constant; we can find the various physical quantities called for in this problem by using constant-acceleration equations.

(*a*) Using a constant-acceleration equation, relate the angular velocity of the wheel to its angular acceleration and the time it has been accelerating:

$$\omega = \omega_0 + \alpha\Delta t$$

or, when $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

Evaluate ω when $\Delta t = 6$ s:

$$\omega = \left(2.6\,\text{rad/s}^2\right)(6\,\text{s}) = \boxed{15.6\,\text{rad/s}}$$

(*b*) Using another constant-acceleration equation, relate the angular displacement to the wheel's angular acceleration and the time it

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, when $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

has been accelerating:

Evaluate $\Delta\theta$ when Δt = 6 s:

$$\Delta\theta(6\,\text{s}) = \tfrac{1}{2}(2.6\,\text{rad/s}^2)(6\,\text{s})^2 = \boxed{46.8\,\text{rad}}$$

(*c*) Convert $\Delta\theta(6\,\text{s})$ from rad to revolutions:

$$\Delta\theta(6\,\text{s}) = 46.8\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{7.45\,\text{rev}}$$

(*d*) Relate the angular velocity of the particle to its tangential speed and evaluate the latter when Δt = 6 s:

$$v = r\omega = (0.3\,\text{m})(15.6\,\text{rad/s}) = \boxed{4.68\,\text{m/s}}$$

Relate the resultant acceleration of the point to its tangential and centripetal accelerations when Δt = 6 s:

$$a = \sqrt{a_{\text{t}}^2 + a_{\text{c}}^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Substitute numerical values and evaluate a:

$$a = (0.3\,\text{m})\sqrt{(2.6\,\text{rad/s}^2)^2 + (15.6\,\text{rad/s})^4} = \boxed{73.0\,\text{m/s}^2}$$

***28** •

Picture the Problem Because we're assuming constant angular acceleration; we can find the various physical quantities called for in this problem by using constant-acceleration equations.

(*a*) Using its definition, express the angular acceleration of the turntable:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{0 - 33\tfrac{1}{3}\dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}} \times \dfrac{1\,\text{min}}{60\,\text{s}}}{26\,\text{s}} = \boxed{0.134\,\text{rad/s}^2}$$

(*b*) Because the angular acceleration is constant, the average angular velocity is the average of its initial and final values:

$$\omega_{av} = \frac{\omega_0 + \omega}{2}$$

$$= \frac{33\frac{1}{3}\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}}{2}$$

$$= \boxed{1.75\,\text{rad/s}}$$

(*c*) Using the definition of ω_{av}, find the number or revolutions the turntable makes before stopping:

$$\Delta\theta = \omega_{av}\Delta t = (1.75\,\text{rad/s})(26\,\text{s})$$

$$= 45.5\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{7.24\,\text{rev}}$$

29 •

Picture the Problem Because the angular acceleration of the disk is constant, we can use a constant-acceleration equation to relate its angular velocity to its acceleration and the time it has been accelerating. We can find the tangential and centripetal accelerations from their relationships to the angular velocity and angular acceleration of the disk.

(*a*) Using a constant-acceleration equation, relate the angular velocity of the disk to its angular acceleration and time during which it has been accelerating:

$$\omega = \omega_0 + \alpha\,\Delta t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha\,\Delta t$$

Evaluate ω when $t = 5$ s:

$$\omega(5\,\text{s}) = (8\,\text{rad/s}^2)(5\,\text{s}) = \boxed{40.0\,\text{rad/s}}$$

(*b*) Express a_t in terms of α:

$$a_t = r\alpha$$

Evaluate a_t when $t = 5$ s:

$$a_t(5\,\text{s}) = (0.12\,\text{m})(8\,\text{rad/s}^2)$$

$$= \boxed{0.960\,\text{m/s}^2}$$

Express a_c in terms of ω:

$$a_c = r\omega^2$$

Evaluate a_c when $t = 5$ s:

$$a_c(5\,\text{s}) = (0.12\,\text{m})(40.0\,\text{rad/s})^2$$

$$= \boxed{192\,\text{m/s}^2}$$

30 •

Picture the Problem We can find the angular velocity of the Ferris wheel from its definition and the linear speed and centripetal acceleration of the passenger from the relationships between those quantities and the angular velocity of the Ferris wheel.

(*a*) Find ω from its definition:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{27\,\text{s}} = \boxed{0.233\,\text{rad/s}}$$

(*b*) Find the linear speed of the passenger from his/her angular speed:

$$v = r\omega = (12\,\text{m})(0.233\,\text{rad/s}) = \boxed{2.79\,\text{m/s}}$$

Find the passenger's centripetal acceleration from his/her angular velocity:

$$a_c = r\omega^2 = (12\,\text{m})(0.233\,\text{rad/s})^2 = \boxed{0.651\,\text{m/s}^2}$$

31 •

Picture the Problem Because the angular acceleration of the wheels is constant, we can use constant-acceleration equations in rotational form to find their angular acceleration and their angular velocity at any given time.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the wheel to its angular acceleration and the time it has been accelerating:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

Solve for α:

$$\alpha = \frac{2\Delta\theta}{(\Delta t)^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(3\,\text{rev})\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)}{(8\,\text{s})^2} = \boxed{0.589\,\text{rad/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the angular velocity of the wheel to its angular acceleration and the time it has been accelerating:

$$\omega = \omega_0 + \alpha\Delta t$$

or, when $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

Evaluate ω when $\Delta t = 8$ s:

$$\omega(8\,\text{s}) = (0.589\,\text{rad/s}^2)(8\,\text{s}) = \boxed{4.71\,\text{rad/s}}$$

32 •

Picture the Problem The earth rotates through 2π radians every 24 hours.

Find ω using its definition:

$$\omega \equiv \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{24\text{ h}\times\frac{3600\text{ s}}{\text{h}}}$$

$$= \boxed{7.27\times10^{-5}\text{ rad/s}}$$

33 •

Picture the Problem When the angular acceleration of a wheel is constant, its average angular velocity is the average of its initial and final angular velocities. We can combine this relationship with the always applicable definition of angular velocity to find the initial angular velocity of the wheel.

Express the average angular velocity of the wheel in terms of its initial and final angular speeds:

$$\omega_{\text{av}} = \frac{\omega_0 + \omega}{2}$$

or, because $\omega = 0$,

$$\omega_{\text{av}} = \tfrac{1}{2}\omega_0$$

Express the definition of the average angular velocity of the wheel:

$$\omega_{\text{av}} \equiv \frac{\Delta\theta}{\Delta t}$$

Equate these two expressions and solve for ω_0:

$$\omega_0 = \frac{2\Delta\theta}{\Delta t} = \frac{2(5\text{ rad})}{2.8\text{ s}} = 3.57\text{ s and}$$

$$\boxed{(d)\text{ is correct.}}$$

34 •

Picture the Problem The tangential and angular accelerations of the wheel are directly proportional to each other with the radius of the wheel as the proportionality constant. Provided there is no slippage, the acceleration of a point on the rim of the wheel is the same as the acceleration of the bicycle. We can use its defining equation to determine the acceleration of the bicycle.

Relate the tangential acceleration of a point on the wheel (equal to the acceleration of the bicycle) to the wheel's angular acceleration and solve for its angular acceleration:

$$a = a_{\text{t}} = r\alpha$$

and

$$\alpha = \frac{a}{r}$$

Use its definition to express the acceleration of the wheel:

$$a=\frac{\Delta v}{\Delta t}=\frac{v-v_0}{\Delta t}$$

or, because $v_0 = 0$,

$$a=\frac{v}{\Delta t}$$

Substitute in the expression for α to obtain:

$$\alpha=\frac{v}{r\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha=\frac{\left(24\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\left(\frac{1000\,\text{m}}{\text{km}}\right)}{(0.6\,\text{m})(14.0\,\text{s})}=\boxed{0.794\,\text{rad/s}^2}$$

***35 ••**

Picture the Problem The two tapes will have the same tangential and angular velocities when the two reels are the same size, i.e., have the same area. We can calculate the tangential speed of the tape from its length and running time and relate the angular velocity to the constant tangential speed and the radius of the reels when they are turning with the same angular velocity.

Relate the angular velocity of the tape to its tangential speed:

$$\omega=\frac{v}{r} \qquad (1)$$

Letting R_f represent the outer radius of the reel when the reels have the same area, express the condition that they have the same speed:

$$\pi R_\text{f}^2-\pi r^2=\tfrac{1}{2}\left(\pi R^2-\pi r^2\right)$$

Solve for R_f:

$$R_\text{f}=\sqrt{\frac{R^2+r^2}{2}}$$

Substitute numerical values and evaluate R_f:

$$R_\text{f}=\sqrt{\frac{(45\,\text{mm})^2+(12\,\text{mm})^2}{2}}=32.9\,\text{mm}$$

Find the tangential speed of the tape from its length and running time:

$$v=\frac{L}{\Delta t}=\frac{246\,\text{m}\times\frac{100\,\text{cm}}{\text{m}}}{2\,\text{h}\times\frac{3600\,\text{s}}{\text{h}}}=3.42\,\text{cm/s}$$

Substitute in equation (1) and evaluate ω:

$$\omega = \frac{v}{R_f} = \frac{3.42\,\text{cm/s}}{32.9\,\text{mm}\times\frac{1\,\text{cm}}{10\,\text{mm}}} = \boxed{1.04\,\text{rad/s}}$$

Convert 1.04 rad/s to rev/min:

$$1.04\,\text{rad/s} = 1.04\frac{\text{rad}}{\text{s}}\times\frac{1\,\text{rev}}{2\pi\,\text{rad}}\times\frac{60\,\text{s}}{\text{min}} = \boxed{9.93\,\text{rev/min}}$$

Torque, Moment of Inertia, and Newton's Second Law for Rotation

36 •

Picture the Problem The force that the woman exerts through her axe, because it does not act at the axis of rotation, produces a net torque that changes (decreases) the angular velocity of the grindstone.

(*a*) From the definition of angular acceleration we have:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

or, because $\omega = 0$,

$$\alpha = \frac{-\omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = -\frac{730\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}}{9\,\text{s}} = \boxed{-8.49\,\text{rad/s}^2}$$

where the minus sign means that the grindstone is slowing down.

(*b*) Use Newton's 2nd law in rotational form to relate the angular acceleration of the grindstone to the net torque slowing it:

$$\tau_{\text{net}} = I\alpha$$

Express the moment of inertia of disk with respect to its axis of rotation:

$$I = \tfrac{1}{2}MR^2$$

Substitute to obtain: $\tau_{net} = \frac{1}{2}MR\alpha$

Substitute numerical values and evaluate τ_{net}:

$$\tau_{net} = \frac{1}{2}(1.7\,\text{kg})(0.08\,\text{m})^2(8.49\,\text{rad/s}^2) = \boxed{0.0462\,\text{N}\cdot\text{m}}$$

***37 •**

Picture the Problem We can find the torque exerted by the 17-N force from the definition of torque. The angular acceleration resulting from this torque is related to the torque through Newton's 2nd law in rotational form. Once we know the angular acceleration, we can find the angular velocity of the cylinder as a function of time.

(*a*) Calculate the torque from its definition:

$$\tau = F\ell = (17\,\text{N})(0.11\,\text{m}) = \boxed{1.87\,\text{N}\cdot\text{m}}$$

(*b*) Use Newton's 2nd law in rotational form to relate the acceleration resulting from this torque to the torque:

$$\alpha = \frac{\tau}{I}$$

Express the moment of inertia of the cylinder with respect to its axis of rotation:

$$I = \frac{1}{2}MR^2$$

Substitute to obtain:

$$\alpha = \frac{2\tau}{MR^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(1.87\,\text{N}\cdot\text{m})}{(2.5\,\text{kg})(0.11\,\text{m})^2} = \boxed{124\,\text{rad/s}^2}$$

(*c*) Using a constant-acceleration equation, express the angular velocity of the cylinder as a function of time:

$$\omega = \omega_0 + \alpha t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha t$$

Evaluate ω (5 s):

$$\omega(5\,\text{s}) = (124\,\text{rad/s}^2)(5\,\text{s}) = \boxed{620\,\text{rad/s}}$$

38 ••

Picture the Problem We can find the angular acceleration of the wheel from its definition and the moment of inertia of the wheel from Newton's 2nd law.

(*a*) Express the moment of inertia of the wheel in terms of the angular acceleration produced by the applied torque:

$$I = \frac{\tau}{\alpha}$$

Find the angular acceleration of the wheel:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{600\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}}{20\,\text{s}}$$
$$= 3.14\,\text{rad/s}^2$$

Substitute and evaluate *I*:

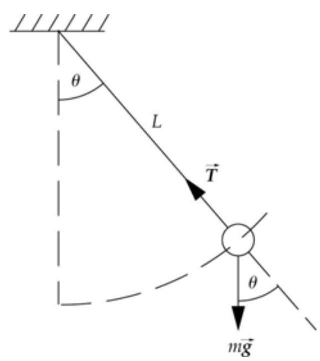
$$I = \frac{50\,\text{N}\cdot\text{m}}{3.14\,\text{rad/s}^2} = \boxed{15.9\,\text{kg}\cdot\text{m}^2}$$

(*b*) Because the wheel takes 120 s to slow to a stop (it took 20 s to acquire an angular velocity of 600 rev/min) and its angular acceleration is directly proportional to the accelerating torque:

$$\tau_{\text{fr}} = \tfrac{1}{6}\tau = \tfrac{1}{6}(50\,\text{N}\cdot\text{m}) = \boxed{8.33\,\text{N}\cdot\text{m}}$$

39 ••

Picture the Problem The pendulum and the forces acting on it are shown in the free-body diagram. Note that the tension in the string is radial, and so exerts no tangential force on the ball. We can use Newton's 2nd law in both translational and rotational form to find the tangential component of the acceleration of the bob.



(*a*) Referring to the FBD, express the component of $m\vec{g}$ that is tangent to the circular path of the bob:

$$F_{\text{t}} = mg\sin\theta$$

Use Newton's 2nd law to express the tangential acceleration of the bob:

$$a_{\text{t}} = \frac{F_{\text{t}}}{m} = \boxed{g\sin\theta}$$

(*b*) Noting that, because the line-of-action of the tension passes through the pendulum's pivot point, its lever arm is zero and the net torque is due

$$\sum \tau_{\text{pivot point}} = \boxed{mgL\sin\theta}$$

to the weight of the bob, sum the torques about the pivot point to obtain:

$$\tau_{\text{net}} = mgL\sin\theta = I\alpha$$

(*c*) Use Newton's 2nd law in rotational form to relate the angular acceleration of the pendulum to the net torque acting on it:

Solve for α to obtain:

$$\alpha = \frac{mgL\sin\theta}{I}$$

Express the moment of inertia of the bob with respect to the pivot point:

$$I = mL^2$$

Substitute to obtain:

$$\alpha = \frac{mgL\sin\theta}{mL^2} = \frac{g\sin\theta}{L}$$

Relate α to a_t:

$$a_t = r\alpha = L\left(\frac{g\sin\theta}{L}\right) = \boxed{g\sin\theta}$$

***40** •••

Picture the Problem We can express the velocity of the center of mass of the rod in terms of its distance from the pivot point and the angular velocity of the rod. We can find the angular velocity of the rod by using Newton's 2nd law to find its angular acceleration and then a constant-acceleration equation that relates ω to α. We'll use the impulse-momentum relationship to derive the expression for the force delivered to the rod by the pivot. Finally, the location of the *center of percussion* of the rod will be verified by setting the force exerted by the pivot to zero.

(*a*) Relate the velocity of the center of mass to its distance from the pivot point:

$$v_{\text{cm}} = \frac{L}{2}\omega \qquad (1)$$

Express the torque due to F_0:

$$\tau = F_0 x = I_{\text{pivot}}\alpha$$

Solve for α:

$$\alpha = \frac{F_0 x}{I_{\text{pivot}}}$$

Express the moment of inertia of the rod with respect to an axis through

$$I_{\text{pivot}} = \tfrac{1}{3}ML^2$$

its pivot point:

Substitute to obtain:

$$\alpha = \frac{3F_0 x}{ML^2}$$

Express the angular velocity of the rod in terms of its angular acceleration:

$$\omega = \alpha\,\Delta t = \frac{3F_0 x\Delta t}{ML^2}$$

Substitute in equation (1) to obtain:

$$v_{\mathrm{cm}} = \boxed{\frac{3F_0 x\Delta t}{2ML}}$$

(*b*) Let I_{P} be the impulse exerted by the pivot on the rod. Then the total impulse (equal to the change in momentum of the rod) exerted on the rod is:

$$I_{\mathrm{P}} + F_0\Delta t = Mv_{\mathrm{cm}}$$

and

$$I_{\mathrm{P}} = Mv_{\mathrm{cm}} - F_0\Delta t$$

Substitute our result from (a) to obtain:

$$I_{\mathrm{P}} = \frac{3F_0 x\Delta t}{2L} - F_0\Delta t = F_0\Delta t\left(\frac{3x}{2L} - 1\right)$$

Because $I_{\mathrm{P}} = F_{\mathrm{P}}\Delta t$:

$$F_{\mathrm{P}} = \boxed{F_0\left(\frac{3x}{2L} - 1\right)}$$

In order for F_{P} to be zero:

$$\frac{3x}{2L} - 1 = 0 \Rightarrow x = \boxed{\frac{2L}{3}}$$

41 •••

Picture the Problem We'll first express the torque exerted by the force of friction on the elemental disk and then integrate this expression to find the torque on the entire disk. We'll use Newton's 2nd law to relate this torque to the angular acceleration of the disk and then to the stopping time for the disk.

(*a*) Express the torque exerted on the elemental disk in terms of the friction force and the distance to the elemental disk:

$$d\tau_{\mathrm{f}} = rdf_{\mathrm{k}} \qquad (1)$$

Using the definition of the coefficient of friction, relate the

$$df_{\mathrm{k}} = \mu_{\mathrm{k}} g dm \qquad (2)$$

force of friction to μ_k and the weight of the circular element:

Letting σ represent the mass per unit area of the disk, express the mass of the circular element:

$$dm = 2\pi r\sigma\, dr \qquad (3)$$

Substitute equations (2) and (3) in (1) to obtain:

$$d\tau_f = 2\pi \mu_k \sigma\, g r^2 dr \qquad (4)$$

Because $\sigma = \dfrac{M}{\pi R^2}$:

$$d\tau_f = \boxed{\frac{2\mu_k M g}{R^2} r^2 dr}$$

(*b*) Integrate $d\tau_f$ to obtain the total torque on the elemental disk:

$$\tau_f = \frac{2\mu_k M g}{R^2}\int_0^R r^2 dr = \boxed{\tfrac{2}{3} MR\mu_k g}$$

(*c*) Relate the disk's stopping time to its angular velocity and acceleration:

$$\Delta t = \frac{\omega}{\alpha}$$

Using Newton's 2nd law, express α in terms of the net torque acting on the disk:

$$\alpha = \frac{\tau_f}{I}$$

The moment of inertia of the disk, with respect to its axis of rotation, is:

$$I = \tfrac{1}{2} MR^2$$

Substitute and simplify to obtain:

$$\Delta t = \boxed{\frac{3R\omega}{4\mu_k g}}$$

Calculating the Moment of Inertia

42 •

Picture the Problem One can find the formula for the moment of inertia of a thin spherical shell in Table 9-1.

The moment of inertia of a thin spherical shell about its diameter is:

$$I = \tfrac{2}{3} MR^2$$

Substitute numerical values and evaluate I:

$$I = \tfrac{2}{3}(0.057\,\text{kg})(0.035\,\text{m})^2 = \boxed{4.66\times10^{-5}\,\text{kg}\cdot\text{m}^2}$$

***43 •**

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

Use the definition of the moment of inertia of a system of particles to obtain:

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I:

$$I = (3\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})\left(2\sqrt{2}\,\text{m}\right)^2 + (4\,\text{kg})(0)^2 + (3\,\text{kg})(2\,\text{m})^2 = \boxed{56.0\,\text{kg}\cdot\text{m}^2}$$

44 •

Picture the Problem Note, from symmetry considerations, that the center of mass of the system is at the intersection of the diagonals connecting the four masses. Thus the distance of each particle from the axis through the center of mass is $\sqrt{2}$ m. According to the parallel-axis theorem, $I = I_{cm} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes.

Express the parallel axis theorem:

$$I = I_{cm} + Mh^2$$

Solve for I_{cm} and substitute from Problem 44:

$$I_{cm} = I - Mh^2 = 56.0\,\text{kg}\cdot\text{m}^2 - (14\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

Use the definition of the moment of inertia of a system of particles to express I_{cm}:

$$I_{cm} = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I_{cm}:

$$I_{cm} = (3\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 + (4\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 + (4\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 + (3\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

45 •

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

(*a*) Apply the definition of the moment of inertia of a system of particles to express I_x:

$$I_x = \sum_{\rm i} m_{\rm i} r_{\rm i}^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I_x:

$$I_x = (3\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})(0) + (3\,\text{kg})(0) = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

(*b*) Apply the definition of the moment of inertia of a system of particles to express I_y:

$$I_y = \sum_{\rm i} m_{\rm i} r_{\rm i}^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I_y:

$$I_y = (3\,\text{kg})(0) + (4\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})(0) + (3\,\text{kg})(2\,\text{m})^2 = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

Remarks: We could also use a symmetry argument to conclude that $I_y = I_x$.

46 •

Picture the Problem According to the parallel-axis theorem, $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes.

Use Table 9-1 to find the moment of inertia of a sphere with respect to an axis through its center of mass:

$$I_{\text{cm}} = \tfrac{2}{5} MR^2$$

Express the parallel axis theorem:

$$I = I_{\text{cm}} + Mh^2$$

Substitute for I_{cm} and simplify to obtain:

$$I = \tfrac{2}{5} MR^2 + MR^2 = \boxed{\tfrac{7}{5} MR^2}$$

47 ••

Picture the Problem The moment of inertia of the wagon wheel is the sum of the moments of inertia of the rim and the six spokes.

Express the moment of inertia of the wagon wheel as the sum of the moments of inertia of the rim and the spokes:

$$I_{\text{wheel}} = I_{\text{rim}} + I_{\text{spokes}}$$

Using Table 9-1, find formulas for the moments of inertia of the rim and spokes:

$$I_{\text{rim}} = M_{\text{rim}}R^2$$

and

$$I_{\text{spoke}} = \tfrac{1}{3}M_{\text{spoke}}L^2$$

Substitute to obtain:

$$I_{\text{wheel}} = M_{\text{rim}}R^2 + 6\left(\tfrac{1}{3}M_{\text{spoke}}L^2\right) = M_{\text{rim}}R^2 + 2M_{\text{spoke}}L^2$$

Substitute numerical values and evaluate I_{wheel}:

$$I_{\text{wheel}} = (8\,\text{kg})(0.5\,\text{m})^2 + 2(1.2\,\text{kg})(0.5\,\text{m})^2 = \boxed{2.60\,\text{kg}\cdot\text{m}^2}$$

***48** ••

Picture the Problem The moment of inertia of a system of particles depends on the axis with respect to which it is calculated. Once this choice is made, the moment of inertia is the sum of the products of the mass of each particle and the square of its distance from the chosen axis.

(*a*) Apply the definition of the moment of inertia of a system of particles:

$$I = \sum_{\text{i}} m_{\text{i}}r_{\text{i}}^2 = \boxed{m_1x^2 + m_2(L-x)^2}$$

(*b*) Set the derivative of *I* with respect to *x* equal to zero in order to identify values for *x* that correspond to either maxima or minima:

$$\frac{dI}{dx} = 2m_1x + 2m_2(L-x)(-1) = 2(m_1x + m_2x - m_2L) = 0 \text{ for extrema}$$

If $\frac{dI}{dx} = 0$, then:

$$m_1x + m_2x - m_2L = 0$$

Solve for x:

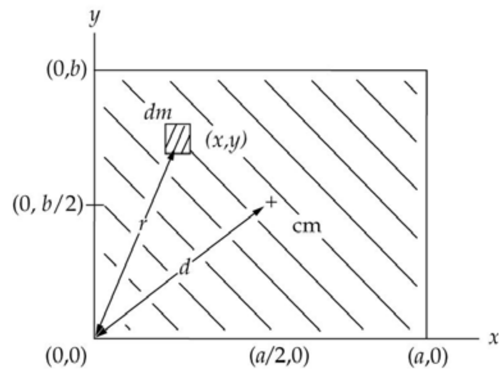
$$x = \frac{m_2 L}{m_1 + m_2}$$

Convince yourself that you've found a minimum by showing that $\frac{d^2 I}{dx^2}$ is positive at this point.

$x = \frac{m_2 L}{m_1 + m_2}$ is, by definition, the distance of the center of mass from m.

49 ••

Picture the Problem Let σ be the mass per unit area of the uniform rectangular plate. Then the elemental unit has mass $dm = \sigma\, dxdy$. Let the corner of the plate through which the axis runs be the origin. The distance of the element whose mass is dm from the corner r is related to the coordinates of dm through the Pythagorean relationship $r^2 = x^2 + y^2$.



(*a*) Express the moment of inertia of the element whose mass is dm with respect to an axis perpendicular to it and passing through one of the corners of the uniform rectangular plate:

$$dI = \sigma\left(x^2 + y^2\right)dxdy$$

Integrate this expression to find I:

$$I = \sigma \int_0^a \int_0^b \left(x^2 + y^2\right)dxdy$$

$$= \tfrac{1}{3}\sigma\left(a^3 b + ab^3\right) = \boxed{\tfrac{1}{3}m\left(a^2 + b^3\right)}$$

(*b*) Letting d represent the distance from the origin to the center of mass of the plate, use the parallel axis theorem to relate the moment of inertia found in (*a*) to the moment of inertia with respect to an axis through the center of mass:

$$I = I_{\text{cm}} + md^2$$

or

$$I_{\text{cm}} = I - md^2 = \tfrac{1}{3}m\left(a^2 + b^2\right) - md^2$$

Using the Pythagorean theorem, relate the distance d to the center of

$$d^2 = \left(\tfrac{1}{2}a\right)^2 + \left(\tfrac{1}{2}b\right)^2 = \tfrac{1}{4}\left(a^2 + b^2\right)$$

mass to the lengths of the sides of the plate:

Substitute for d^2 in the expression for I_{cm} and simplify to obtain:

$$I_{cm} = \tfrac{1}{3}m\left(a^2+b^2\right)-\tfrac{1}{4}m\left(a^2+b^2\right)^2 = \boxed{\tfrac{1}{12}m\left(a^2+b^2\right)}$$

***50 ••**

Picture the Problem Corey will use the point-particle relationship $I_{app}=\sum_i m_i r_i^2 = m_1r_1^2+m_2r_2^2$ for his calculation whereas Tracey's calculation will take into account not only the rod but also the fact that the spheres are not point particles.

(*a*) Using the point-mass approximation and the definition of the moment of inertia of a system of particles, express I_{app}:

$$I_{app}=\sum_i m_i r_i^2 = m_1r_1^2+m_2r_2^2$$

Substitute numerical values and evaluate I_{app}:

$$I_{app}=(0.5\,\text{kg})(0.2\,\text{m})^2+(0.5\,\text{kg})(0.2\,\text{m})^2 = \boxed{0.0400\,\text{kg}\cdot\text{m}^2}$$

Express the moment of inertia of the two spheres and connecting rod system:

$$I = I_{spheres}+I_{rod}$$

Use Table 9-1 to find the moments of inertia of a sphere (with respect to its center of mass) and a rod (with respect to an axis through its center of mass):

$$I_{sphere}=\tfrac{2}{5}M_{sphere}R^2$$

and

$$I_{rod}=\tfrac{1}{12}M_{rod}L^2$$

Because the spheres are not on the axis of rotation, use the parallel axis theorem to express their moment of inertia with respect to the axis of rotation:

$$I_{sphere}=\tfrac{2}{5}M_{sphere}R^2+M_{sphere}h^2$$

where h is the distance from the center of mass of a sphere to the axis of rotation.

Substitute to obtain:

$$I = 2\left\{\tfrac{2}{5}M_{sphere}R^2+M_{sphere}h^2\right\}+\tfrac{1}{12}M_{rod}L^2$$

Substitute numerical values and evaluate I:

$$I = 2\left\{\tfrac{2}{5}(0.5\,\text{kg})(0.05\,\text{m})^2 + (0.5\,\text{kg})(0.2\,\text{m})^2\right\} + \tfrac{1}{12}(0.06\,\text{kg})(0.3\,\text{m})^2$$
$$= \boxed{0.0415\,\text{kg}\cdot\text{m}^2}$$

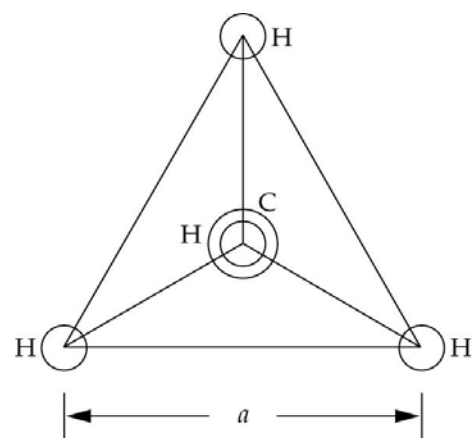
Compare I and I_{app} by taking their ratio:

$$\frac{I_{app}}{I} = \frac{0.0400\,\text{kg}\cdot\text{m}^2}{0.0415\,\text{kg}\cdot\text{m}^2} = \boxed{0.964}$$

(*b*) The rotational inertia would increase because I_{cm} of a hollow sphere is greater than I_{cm} of a solid sphere.

51 ••

Picture the Problem The axis of rotation passes through the center of the base of the tetrahedron. The carbon atom and the hydrogen atom at the apex of the tetrahedron do not contribute to I because the distance of their nuclei from the axis of rotation is zero. From the geometry, the distance of the three H nuclei from the rotation axis is $a/\sqrt{3}$, where a is the length of a side of the tetrahedron.



Apply the definition of the moment of inertia for a system of particles to obtain:

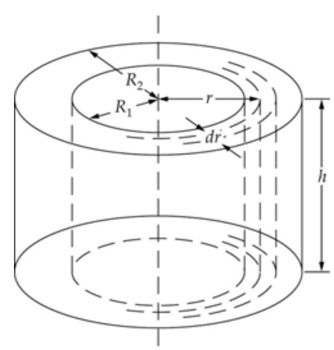
$$I = \sum_i m_i r_i^2 = m_H r_1^2 + m_H r_2^2 + m_H r_3^2$$
$$= 3m_H\left(\frac{a}{\sqrt{3}}\right)^2 = m_H a^2$$

Substitute numerical values and evaluate I:

$$I = (1.67\times10^{-27}\,\text{kg})(0.18\times10^{-9}\,\text{m})^2$$
$$= \boxed{5.41\times10^{-47}\,\text{kg}\cdot\text{m}^2}$$

52 ••

Picture the Problem Let the mass of the element of volume dV be $dm = \rho dV = 2\pi\rho hrdr$ where h is the height of the cylinder. We'll begin by expressing the moment of inertia dI for the element of volume and then integrating it between R_1 and R_2.



Express the moment of inertia of the element of mass dm:

$$dI = r^2 dm = 2\pi\rho\, h r^3 dr$$

Integrate dI from R_1 to R_2 to obtain:

$$I = 2\pi\rho\, h\int_{R_1}^{R_2} r^3 dr = \tfrac{1}{2}\pi\rho\, h\left(R_2^4 - R_1^4\right)$$
$$= \tfrac{1}{2}\pi\rho\, h\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)$$

The mass of the hollow cylinder is $m = \pi\,\rho\, h\left(R_2^2 - R_1^2\right)$, so:

$$\rho = \frac{m}{\pi\, h\left(R_2^2 - R_1^2\right)}$$

Substitute for ρ and simplify to obtain:

$$I = \tfrac{1}{2}\pi\left(\frac{m}{\pi\, h\left(R_2^2 - R_1^2\right)}\right) h\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right) = \boxed{\tfrac{1}{2}m\left(R_2^2 + R_1^2\right)}$$

53 •••

Picture the Problem We can derive the given expression for the moment of inertia of a spherical shell by following the procedure outlined in the problem statement.

Find the moment of inertia of a sphere, with respect to an axis through a diameter, in Table 9-1:

$$I = \tfrac{2}{5}mR^2$$

Express the mass of the sphere as a function of its density and radius:

$$m = \tfrac{4}{3}\pi\,\rho\, R^3$$

Substitute to obtain:

$$I = \tfrac{8}{15}\pi\,\rho\, R^5$$

Express the differential of this expression:

$$dI = \tfrac{8}{3}\pi\,\rho\, R^4 dR \qquad (1)$$

Express the increase in mass dm as the radius of the sphere increases by dR:

$$dm = 4\pi\,\rho\, R^2 dR \qquad (2)$$

Eliminate dR between equations (1) and (2) to obtain:

$$dI = \tfrac{2}{3}R^2 dm$$

Therefore, the moment of inertia of the spherical shell of mass m is $\tfrac{2}{3}mR^2$.

***54** •••

Picture the Problem We can find C in terms of M and R by integrating a spherical shell of mass dm with the given density function to find the mass of the earth as a function of M and then solving for C. In part (b), we'll start with the moment of inertia of the same spherical shell, substitute the earth's density function, and integrate from 0 to R.

(a) Express the mass of the earth using the given density function:

$$M = \int dm = \int_0^R 4\pi\,\rho\, r^2 dr$$

$$= 4\pi\, C\int_0^R 1.22r^2 dr - \frac{4\pi\, C}{R}\int_0^R r^3 dr$$

$$= \frac{4\pi}{3}1.22CR^3 - \pi\, CR^3$$

Solve for C as a function of M and R to obtain:

$$C = \boxed{0.508\frac{M}{R^3}}$$

(b) From Problem 9-40 we have:

$$dI = \tfrac{8}{3}\pi\,\rho\, r^4 dr$$

Integrate to obtain:

$$I = \tfrac{8}{3}\pi\int_0^R \rho\, r^4 dr$$

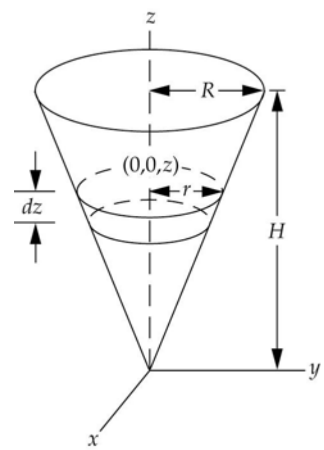
$$= \frac{8\pi(0.508)M}{3R^3}\left[\int_0^R 1.22r^4 dr - \frac{1}{R}\int_0^R r^5 dr\right]$$

$$= \frac{4.26M}{R^3}\left[\frac{1.22}{5}R^5 - \frac{1}{6}R^5\right]$$

$$= \boxed{0.329MR^2}$$

55 •••

Picture the Problem Let the origin be at the apex of the cone, with the z axis along the cone's symmetry axis. Then the radius of the elemental ring, at a distance z from the apex, can be obtained from the proportion $\frac{r}{z}=\frac{R}{H}$. The mass dm of the elemental disk is $\rho dV = \rho\pi r^2 dz$. We'll integrate $r^2 dm$ to find the moment of inertia of the disk in terms of R and H and then integrate dm to obtain a second equation in R and H that we can use to eliminate H in our expression for I.



Express the moment of inertia of the cone in terms of the moment of inertia of the elemental disk:

$$I=\tfrac{1}{2}\int r^2 dm$$
$$=\tfrac{1}{2}\int_0^H \frac{R^2}{H^2}z^2\left(\rho\pi\frac{R^2}{H^2}z^2\right)dz$$
$$=\frac{\pi\rho R^4}{2H^4}\int_0^H z^4 dz=\frac{\pi\rho R^4 H}{10}$$

Express the total mass of the cone in terms of the mass of the elemental disk:

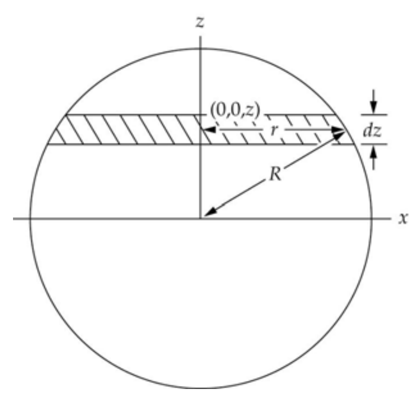
$$M=\pi\rho\int_0^H r^2 dz=\pi\rho\int_0^H \frac{R^2}{H^2}z^2 dz$$
$$=\tfrac{1}{3}\pi\rho R^2 H$$

Divide I by M, simplify, and solve for I to obtain:

$$I=\boxed{\tfrac{3}{10}MR^2}$$

56 •••

Picture the Problem Let the axis of rotation be the x axis. The radius r of the elemental area is $\sqrt{R^2-z^2}$ and its mass, dm, is $\sigma\, dA=2\sigma\sqrt{R^2-z^2}\,dz$. We'll integrate $z^2\, dm$ to determine I in terms of σ and then divide this result by M in order to eliminate σ and express I in terms of M and R.



Express the moment of inertia about the x axis:

$$I = \int z^2 dm = \int z^2 \sigma\, dA = \int_{-R}^{R} z^2 \left(2\sigma\sqrt{R^2 - z^2}\, dz\right) = \tfrac{1}{4}\sigma\pi R^4$$

The mass of the thin uniform disk is:

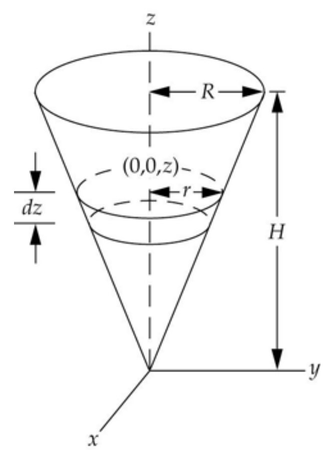
$$M = \sigma\pi R^2$$

Divide I by M, simplify, and solve for I to obtain:

$$I = \boxed{\tfrac{1}{4}MR^2}$$, a result in agreement with the expression given in Table 9-1 for a cylinder of length $L = 0$.

57 •••

Picture the Problem Let the origin be at the apex of the cone, with the z axis along the cone's symmetry axis, and the axis of rotation be the x rotation. Then the radius of the elemental disk, at a distance z from the apex, can be obtained from the proportion $\frac{r}{z} = \frac{R}{H}$. The mass dm of the elemental disk is $\rho dV = \rho\pi r^2 dz$. Each elemental disk rotates about an axis that is parallel to its diameter but removed from it by a distance z. We can use the result from Problem 9-57 for the moment of inertia of the elemental disk with respect to a diameter and then use the parallel axis theorem to express the moment of inertia of the cone with respect to the x axis.



Using the parallel axis theorem, express the moment of inertia of the elemental disk with respect to the x axis:

$$dI_x = dI_{\text{disk}} + dm\, z^2 \qquad (1)$$

where

$$dm = \rho\, dV = \rho\pi\, r^2 dz$$

In Problem 9-57 it was established that the moment of inertia of a thin uniform disk of mass M and radius R rotating about a diameter is $\frac{1}{4}MR^2$. Express this result in

$$dI_{\text{disk}} = \tfrac{1}{4}\left(\rho\pi\, r^2 dz\right)r^2 = \tfrac{1}{4}\rho\pi\left(\frac{R^2}{H^2}z^2\right)^2 dz$$

terms of our elemental disk:

Substitute in equation (1) to obtain:

$$dI_x = \pi\rho\left[\frac{1}{4}\left(\frac{R^2}{H^2}z^2\right)^2\right]dz + \left(\pi\rho\left(\frac{R}{H}z\right)^2 dz\right)z^2$$

Integrate from 0 to H to obtain:

$$I_x = \pi\rho\int_0^H\left[\frac{1}{4}\left(\frac{R^2}{H^2}z^2\right)^2 + \frac{R^2}{H^2}z^4\right]dz = \pi\rho\left(\frac{R^4H}{20} + \frac{R^2H^3}{5}\right)$$

Express the total mass of the cone in terms of the mass of the elemental disk:

$$M = \pi\rho\int_0^H r^2dz = \pi\rho\int_0^H \frac{R^2}{H^2}z^2dz = \tfrac{1}{3}\pi\rho R^2H$$

Divide I_x by M, simplify, and solve for I_x to obtain:

$$I_x = \boxed{3M\left(\frac{H^2}{5} + \frac{R^2}{20}\right)}$$

Remarks: Because both *H* and *R* appear in the numerator, the larger the cones are, the greater their moment of inertia and the greater the energy consumption required to set them into motion.

Rotational Kinetic Energy

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Picture the Problem The kinetic energy of this rotating system of particles can be calculated either by finding the tangential velocities of the particles and using these values to find the kinetic energy or by finding the moment of inertia of the system and using the expression for the rotational kinetic energy of a system.

(a) Use the relationship between v and ω to find the speed of each particle:

$$v_3 = r_3\omega = (0.2\,\text{m})(2\,\text{rad/s}) = 0.4\,\text{m/s}$$

and

$$v_1 = r_1\omega = (0.4\,\text{m})(2\,\text{rad/s}) = 0.8\,\text{m/s}$$

Find the kinetic energy of the system:

$$K = 2K_3 + 2K_1 = m_3v_3^2 + m_1v_1^2$$
$$= (3\,\text{kg})(0.4\,\text{m/s})^2 + (1\,\text{kg})(0.8\,\text{m/s})^2$$
$$= \boxed{1.12\,\text{J}}$$

(*b*) Use the definition of the moment of inertia of a system of particles to obtain:

$$I = \sum_i m_i r_i^2$$
$$= m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$$

Substitute numerical values and evaluate *I*:

$$I = (1\,\text{kg})(0.4\,\text{m})^2 + (3\,\text{kg})(0.2\,\text{m})^2$$
$$+ (1\,\text{kg})(0.4\,\text{m})^2 + (3\,\text{kg})(0.2\,\text{m})^2$$
$$= 0.560\,\text{kg}\cdot\text{m}^2$$

Calculate the kinetic energy of the system of particles:

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(0.560\,\text{kg}\cdot\text{m}^2)(2\,\text{rad/s})^2$$
$$= \boxed{1.12\,\text{J}}$$

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Picture the Problem We can find the kinetic energy of this rotating ball from its angular speed and its moment of inertia. We can use the same relationship to find the new angular speed of the ball when it is supplied with additional energy.

(*a*) Express the kinetic energy of the ball:

$$K = \tfrac{1}{2}I\omega^2$$

Express the moment of inertia of ball with respect to its diameter:

$$I = \tfrac{2}{5}MR^2$$

Substitute for *I*:

$$K = \tfrac{1}{5}MR^2\omega^2$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{1}{5}(1.4\,\text{kg})(0.075\,\text{m})^2$$
$$\times\left(70\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)^2$$
$$= \boxed{84.6\,\text{mJ}}$$

(*b*) Express the new kinetic energy with K' = 2.0846 J:

$$K' = \tfrac{1}{2}I\omega'^2$$

Express the ratio of *K* to *K'*:

$$\frac{K'}{K} = \frac{\frac{1}{2}I\omega'^2}{\frac{1}{2}I\omega'^2} = \left(\frac{\omega'}{\omega}\right)^2$$

Solve for ω':

$$\omega' = \omega\sqrt{\frac{K'}{K}}$$

Substitute numerical values and evaluate ω':

$$\omega' = (70\,\text{rev/min})\sqrt{\frac{2.0846\,\text{J}}{0.0846\,\text{J}}} = \boxed{347\,\text{rev/min}}$$

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Picture the Problem The power delivered by an engine is the product of the torque it develops and the angular speed at which it delivers the torque.

Express the power delivered by the engine as a function of the torque it develops and the angular speed at which it delivers this torque:

$$P = \tau\omega$$

Substitute numerical values and evaluate P:

$$P = (400\,\text{N}\cdot\text{m})\left(3700\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right) = \boxed{155\,\text{kW}}$$

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Picture the Problem Let r_1 and r_2 be the distances of m_1 and m_2 from the center of mass. We can use the definition of rotational kinetic energy and the definition of the center of mass of the two point masses to show that $K_1/K_2 = m_2/m_1$.

Use the definition of rotational kinetic energy to express the ratio of the rotational kinetic energies:

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}I\omega_1^2}{\frac{1}{2}I\omega_2^2} = \frac{m_1r_1^2\omega^2}{m_2r_2^2\omega^2} = \frac{m_1r_1^2}{m_2r_2^2}$$

Use the definition of the center of mass to relate m_1, m_2, r_1, and r_2:

$$r_1m_1 = r_2m_2$$

Solve for $\frac{r_1}{r_2}$, substitute and simplify to obtain:

$$\frac{K_1}{K_2} = \frac{m_1}{m_2}\left(\frac{m_2}{m_1}\right)^2 = \boxed{\frac{m_2}{m_1}}$$

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Picture the Problem The earth's rotational kinetic energy is given by $K_{\text{rot}} = \frac{1}{2}I\omega^2$ where I is its moment of inertia with respect to its axis of rotation. The center of mass of the earth-sun system is so close to the center of the sun and the earth-sun distance so large that we can use the earth-sun distance as the separation of their centers of mass and assume each to be point mass.

Express the rotational kinetic energy of the earth:

$$K_{rot} = \tfrac{1}{2} I\omega^2 \qquad (1)$$

Find the angular speed of the earth's rotation using the definition of ω:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{24\,\text{h}\times\dfrac{3600\,\text{s}}{\text{h}}} = 7.27\times10^{-5}\,\text{rad/s}$$

From Table 9-1, for the moment of inertia of a homogeneous sphere, we find:

$$I = \tfrac{2}{5} MR^2 = \tfrac{2}{5}\left(6.0\times10^{24}\,\text{kg}\right)\left(6.4\times10^{6}\,\text{m}\right)^2 = 9.83\times10^{37}\,\text{kg}\cdot\text{m}^2$$

Substitute numerical values in equation (1) to obtain:

$$K_{rot} = \tfrac{1}{2}\left(9.83\times10^{37}\,\text{kg}\cdot\text{m}^2\right)\times\left(7.27\times10^{-5}\,\text{rad/s}\right)^2 = \boxed{2.60\times10^{29}\,\text{J}}$$

Express the earth's orbital kinetic energy:

$$K_{orb} = \tfrac{1}{2} I\omega_{orb}^2 \qquad (2)$$

Find the angular speed of the center of mass of the earth-sun system:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{365.25\,\text{days}\times 24\dfrac{\text{h}}{\text{day}}\times\dfrac{3600\,\text{s}}{\text{h}}} = 1.99\times10^{-7}\,\text{rad/s}$$

Express and evaluate the orbital moment of inertia of the earth:

$$I = M_E R_{orb}^2 = \left(6.0\times10^{24}\,\text{kg}\right)\left(1.50\times10^{11}\,\text{m}\right)^2 = 1.35\times10^{47}\,\text{kg}\cdot\text{m}^2$$

Substitute in equation (2) to obtain:

$$K_{orb} = \tfrac{1}{2}\left(1.35\times10^{47}\,\text{kg}\cdot\text{m}^2\right)\times\left(1.99\times10^{-7}\,\text{rad/s}\right)^2 = 2.67\times10^{33}\,\text{J}$$

Evaluate the ratio $\frac{K_{\text{orb}}}{K_{\text{rot}}}$:

$$\frac{K_{\text{orb}}}{K_{\text{rot}}} = \frac{2.67\times10^{33}\ \text{J}}{2.60\times10^{29}\ \text{J}} \approx \boxed{10^4}$$

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Picture the Problem Because the load is not being accelerated, the tension in the cable equals the weight of the load. The role of the massless pulley is to change the direction the force (tension) in the cable acts.

(*a*) Because the block is lifted at constant speed:

$$T = mg = (2000\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{19.6\,\text{kN}}$$

(*b*) Apply the definition of torque at the winch drum:

$$\tau = Tr = (19.6\,\text{kN})(0.30\,\text{m}) = \boxed{5.89\,\text{kN}\cdot\text{m}}$$

(*c*) Relate the angular speed of the winch drum to the rate at which the load is being lifted (the tangential speed of the cable on the drum):

$$\omega = \frac{v}{r} = \frac{0.08\,\text{m/s}}{0.30\,\text{m}} = \boxed{0.267\,\text{rad/s}}$$

(*d*) Express the power developed by the motor in terms of the tension in the cable and the speed with which the load is being lifted:

$$P = Tv = (19.6\,\text{kN})(0.08\,\text{m/s}) = \boxed{1.57\,\text{kW}}$$

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Picture the Problem Let the zero of gravitational potential energy be at the lowest point of the small particle. We can use conservation of energy to find the angular velocity of the disk when the particle is at its lowest point and Newton's 2nd law to find the force the disk will have to exert on the particle to keep it from falling off.

(*a*) Use conservation of energy to relate the initial potential energy of the system to its rotational kinetic energy when the small particle is at its lowest point:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2}\left(I_{\text{disk}} + I_{\text{particle}}\right)\omega_f^2 - mg\Delta h = 0$$

Solve for ω_f:

$$\omega_f = \sqrt{\frac{2mg\Delta h}{I_{\text{disk}} + I_{\text{particle}}}}$$

Substitute for I_{disk}, $I_{particle}$, and Δh and simplify to obtain:

$$\omega_f = \sqrt{\frac{2mg(2R)}{\frac{1}{2}MR^2 + mR^2}} = \boxed{\sqrt{\frac{8mg}{R(2m+M)}}}$$

(*b*) The mass is in uniform circular motion at the bottom of the disk, so the sum of the force *F* exerted by the disk and the gravitational force must be the centripetal force:

$$F - mg = mR\omega_f^2$$

Solve for *F* and simplify to obtain:

$$F = mg + mR\omega_f^2$$
$$= mg + mR\left(\frac{8mg}{R(2mM)}\right)$$
$$= \boxed{mg\left(1 + \frac{8m}{2m+M}\right)}$$

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Picture the Problem Let the zero of gravitational potential energy be at the center of mass of the ring when it is directly below the point of support. We'll use conservation of energy to relate the maximum angular velocity and the initial angular velocity required for a complete revolution to the changes in the potential energy of the ring.

(*a*) Use conservation of energy to relate the initial potential energy of the ring to its rotational kinetic energy when its center of mass is directly below the point of support:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2}I_P\omega_{max}^2 - mg\Delta h = 0 \qquad (1)$$

Use the parallel axis theorem and Table 9-1 to express the moment of inertia of the ring with respect to its pivot point *P*:

$$I_P = I_{cm} + mR^2$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_{max}^2 - mgR = 0$$

Solve for ω_{max}:

$$\omega_{max} = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_{max}:

$$\omega_{max} = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.62\,\text{rad/s}}$$

(*b*) Use conservation of energy to relate the final potential energy of the ring to its initial rotational kinetic energy:

$$\Delta K + \Delta U = 0$$

or, because $U_\text{i} = K_\text{f} = 0$,

$$-\tfrac{1}{2} I_P \omega_\text{i}^2 + mg\Delta h = 0$$

Noting that the center of mass must rise a distance R if the ring is to make a complete revolution, substitute for I_P and Δh to obtain:

$$-\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_\text{i}^2 + mgR = 0$$

Solve for ω_i:

$$\omega_i = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_i:

$$\omega_i = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.62\,\text{rad/s}}$$

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Picture the Problem We can find the energy that must be stored in the flywheel and relate this energy to the radius of the wheel and use the definition of rotational kinetic energy to find the wheel's radius.

Relate the kinetic energy of the flywheel to the energy it must deliver:

$$K_\text{rot} = \tfrac{1}{2} I_\text{cyl}\omega^2 = (2\,\text{MJ/km})(300\,\text{km}) = 600\,\text{MJ}$$

Express the moment of inertia of the flywheel:

$$I_\text{cyl} = \tfrac{1}{2} MR^2$$

Substitute for I_cyl and solve for ω:

$$R = \frac{2}{\omega}\sqrt{\frac{K_\text{rot}}{M}}$$

Substitute numerical values and evaluate R:

$$R = \frac{2}{400\,\dfrac{\text{rev}}{\text{s}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}}}\sqrt{\frac{600\,\text{MJ} \times \dfrac{10^6\,\text{J}}{\text{MJ}}}{100\,\text{kg}}} = \boxed{1.95\,\text{m}}$$

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Picture the Problem We'll solve this problem for the general case of a ladder of length L, mass M, and person of mass m. Let the zero of gravitational potential energy be at floor level and include you, the ladder, and the earth in the system. We'll use conservation of energy to relate your impact speed falling freely to your impact speed riding the ladder to the ground.

Use conservation of energy to relate the speed with which a person will strike the ground to the fall distance L:

$$\Delta K + \Delta U = 0$$

or, because $K_\text{i} = U_\text{f} = 0$,

$$\tfrac{1}{2} m v_\text{f}^2 - mgL = 0$$

Solve for v_f^2:

$$v_\text{f}^2 = 2gL$$

Letting ω_r represent the angular velocity of the ladder+person system as it strikes the ground, use conservation of energy to relate the initial and final momenta of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_\text{i} = U_\text{f} = 0$,

$$\tfrac{1}{2}\left(I_\text{person} + I_\text{ladder}\right)\omega_\text{r}^2 - \left(mgL + Mg\frac{L}{2}\right) = 0$$

Substitute for the moments of inertia to obtain:

$$\tfrac{1}{2}\left(m + \frac{1}{3}M\right)L^2\omega_\text{f}^2 - \left(mgL + Mg\frac{L}{2}\right) = 0$$

Substitute v_r for $L\omega_\text{f}$ and solve for v_r^2:

$$v_\text{r}^2 = \frac{2gL\left(m + \frac{M}{2}\right)}{m + \frac{M}{3}}$$

Express the ratio $\dfrac{v_\text{r}^2}{v_\text{f}^2}$:

$$\frac{v_\text{r}^2}{v_\text{f}^2} = \frac{m + \frac{M}{2}}{m + \frac{M}{3}}$$

Solve for v_r to obtain:

$$v_\text{r} = v_\text{f}\sqrt{\frac{6m + 3M}{6m + 2M}}$$

Unless M, the mass of the ladder, is zero, $v_\text{r} > v_\text{f}$. It is better to let go and fall to the ground.

Pulleys, Yo-Yos, and Hanging Things

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Picture the Problem We'll solve this problem for the general case in which the mass of the block on the ledge is M, the mass of the hanging block is m, and the mass of the pulley is M_p, and R is the radius of the pulley. Let the zero of gravitational potential energy be 2.5 m below the initial position of the 2-kg block and R represent the radius of the pulley. Let the system include both blocks, the shelf and pulley, and the earth. The initial potential energy of the 2-kg block will be transformed into the translational kinetic energy of both blocks plus rotational kinetic energy of the pulley.

(*a*) Use energy conservation to relate the speed of the 2 kg block when it has fallen a distance Δh to its initial potential energy and the kinetic energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}I_{\text{pulley}}\omega^2 - mgh = 0$$

Substitute for I_{pulley} and ω to obtain:

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2} - mgh = 0$$

Solve for v:

$$v = \sqrt{\frac{2mgh}{M + m + \frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(2\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m})}{4\text{kg} + 2\,\text{kg} + \frac{1}{2}(0.6\,\text{kg})}} = \boxed{3.95\,\text{m/s}}$$

(*b*) Find the angular velocity of the pulley from its tangential speed:

$$\omega = \frac{v}{R} = \frac{3.95\,\text{m/s}}{0.08\,\text{m}} = \boxed{49.3\,\text{rad/s}}$$

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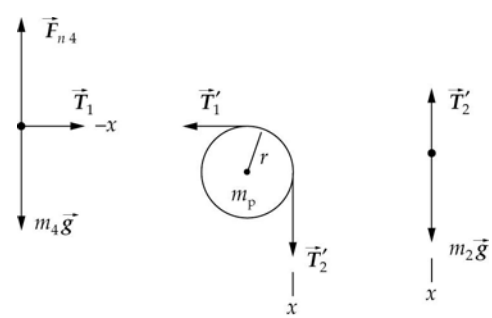
Picture the Problem The diagrams show the forces acting on each of the masses and the pulley. We can apply Newton's 2nd law to the two blocks and the pulley to obtain three equations in the unknowns T_1, T_2, and a.



Apply Newton's 2nd law to the two blocks and the pulley:

$$\sum F_x = T_1 = m_4 a\,, \qquad (1)$$

$$\sum \tau_p = (T_2 - T_1)r = I_p\alpha\,, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Eliminate α in equation (2) to obtain:

$$T_2 - T_1 = \tfrac{1}{2}M_p a \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a:

$$a = \frac{m_2 g}{m_2 + m_4 + \frac{1}{2}M_p}$$

Substitute numerical values and evaluate a:

$$a = \frac{(2\,\text{kg})(9.81\,\text{m/s}^2)}{2\,\text{kg} + 4\,\text{kg} + \frac{1}{2}(0.6\,\text{kg})} = \boxed{3.11\,\text{m/s}^2}$$

Using equation (1), evaluate T_1:

$$T_1 = (4\,\text{kg})(3.11\,\text{m/s}^2) = \boxed{12.5\,\text{N}}$$

Solve equation (3) for T_2:

$$T_2 = m_2(g - a)$$

Substitute numerical values and evaluate T_2:

$$T_2 = (2\,\text{kg})(9.81\,\text{m/s}^2 - 3.11\,\text{m/s}^2) = \boxed{13.4\,\text{N}}$$

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Picture the Problem We'll solve this problem for the general case in which the mass of the block on the ledge is M, the mass of the hanging block is m, the mass of the pulley is M_p, and R is the radius of the pulley. Let the zero of gravitational potential energy be 2.5 m below the initial position of the 2-kg block. The initial potential energy of the 2-kg block will be transformed into the translational kinetic energy of both blocks plus rotational kinetic energy of the pulley plus work done against friction.

(*a*) Use energy conservation to relate the speed of the 2 kg block when it has fallen a distance Δh to its initial potential energy, the kinetic energy of the system and the work done against friction:

$$\Delta K + \Delta U + W_f = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}I_{\text{pulley}}\omega^2 - mgh + \mu_k Mgh = 0$$

Substitute for I_{pulley} and ω to obtain:

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}M_p\right)\frac{v^2}{R^2} - mgh + \mu_k Mgh = 0$$

Solve for v:

$$v = \sqrt{\frac{2gh(m - \mu_k M)}{M + m + \frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9.81\,\text{m/s}^2)(2.5\,\text{m})[2\,\text{kg} - (0.25)(4\,\text{kg})]}{4\text{kg} + 2\,\text{kg} + \frac{1}{2}(0.6\,\text{kg})}} = \boxed{2.79\,\text{m/s}}$$

(b) Find the angular velocity of the pulley from its tangential speed:

$$\omega = \frac{v}{R} = \frac{2.79\,\text{m/s}}{0.08\,\text{m}} = \boxed{34.9\,\text{rad/s}}$$

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Picture the Problem Let the zero of gravitational potential energy be at the water's surface and let the system include the winch, the car, and the earth. We'll apply energy conservation to relate the car's speed as it hits the water to its initial potential energy. Note that some of the car's initial potential energy will be transformed into rotational kinetic energy of the winch and pulley.

Use energy conservation to relate the car's speed as it hits the water to its initial potential energy:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_w\omega_w^2 + \tfrac{1}{2}I_p\omega_p^2 - mg\Delta h = 0$$

Express ω_w and ω_p in terms of the speed v of the rope, which is the same throughout the system:

$$\omega_w = \frac{v^2}{r_w^2} \text{ and } \omega_p = \frac{v^2}{r_p^2}$$

Substitute to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_w\frac{v^2}{r_w^2} + \tfrac{1}{2}I_p\frac{v^2}{r_p^2} - mg\Delta h = 0$$

Solve for v:

$$v = \sqrt{\frac{2mg\Delta h}{m + \frac{I_w}{r_w^2} + \frac{I_p}{r_p^2}}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(1200\,\text{kg})(9.81\,\text{m/s}^2)(5\,\text{m})}{1200\,\text{kg} + \frac{320\,\text{kg}\cdot\text{m}^2}{(0.8\,\text{m})^2} + \frac{4\,\text{kg}\cdot\text{m}^2}{(0.3\,\text{m})^2}}}$$

$$= \boxed{8.21\,\text{m/s}}$$

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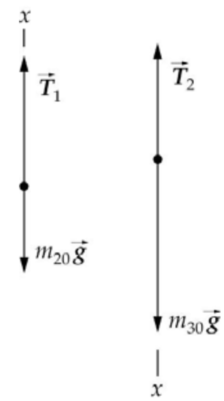
Picture the Problem Let the system include the blocks, the pulley and the earth. Choose the zero of gravitational potential energy to be at the ledge and apply energy conservation to relate the impact speed of the 30-kg block to the initial potential energy of the system. We can use a constant-acceleration equations and Newton's 2nd law to find the tensions in the strings and the descent time.



(*a*) Use conservation of energy to relate the impact speed of the 30-kg block to the initial potential energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}m_{30}v^2 + \tfrac{1}{2}m_{20}v^2 + \tfrac{1}{2}I_p\omega_p^2 + m_{20}g\Delta h - m_{30}g\Delta h = 0$$

Substitute for ω_p and I_p to obtain:

$$\tfrac{1}{2}m_{30}v^2 + \tfrac{1}{2}m_{20}v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}M_p r^2\right)\left(\frac{v^2}{r^2}\right) + m_{20}g\Delta h - m_{30}g\Delta h = 0$$

Solve for v:

$$v = \sqrt{\frac{2g\Delta h(m_{30} - m_{20})}{m_{20} + m_{30} + \tfrac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9.81\,\text{m/s}^2)(2\,\text{m})(30\,\text{kg} - 20\,\text{kg})}{20\,\text{kg} + 30\,\text{kg} + \tfrac{1}{2}(5\,\text{kg})}} = \boxed{2.73\,\text{m/s}}$$

(*b*) Find the angular speed at impact from the tangential speed at impact and the radius of the pulley:

$$\omega = \frac{v}{r} = \frac{2.73\,\text{m/s}}{0.1\,\text{m}} = \boxed{27.3\,\text{rad/s}}$$

(*c*) Apply Newton's 2nd law to the blocks:

$$\sum F_x = T_1 - m_{20}g = m_{20}a \qquad (1)$$

$$\sum F_x = m_{30}g - T_2 = m_{30}a \qquad (2)$$

Using a constant-acceleration equation, relate the speed at impact to the fall distance and the

$$v^2 = v_0^2 + 2a\Delta h$$

or, because $v_0 = 0$,

acceleration and solve for and evaluate a:

$$a = \frac{v^2}{2\Delta h} = \frac{(2.73\,\text{m/s})^2}{2(2\,\text{m})} = 1.87\,\text{m/s}^2$$

Substitute in equation (1) to find T_1:

$$T_1 = m_{20}(g + a) = (20\,\text{kg})(9.81\,\text{m/s}^2 + 1.87\,\text{m/s}^2) = \boxed{234\,\text{N}}$$

Substitute in equation (2) to find T_2:

$$T_2 = m_{30}(g - a) = (30\,\text{kg})(9.81\,\text{m/s}^2 - 1.87\,\text{m/s}^2) = \boxed{238\,\text{N}}$$

(*d*) Noting that the initial speed of the 30-kg block is zero, express the time-of-fall in terms of the fall distance and the block's average speed:

$$\Delta t = \frac{\Delta h}{v_{\text{av}}} = \frac{\Delta h}{\frac{1}{2}v} = \frac{2\Delta h}{v}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(2\,\text{m})}{2.73\,\text{m/s}} = \boxed{1.47\,\text{s}}$$

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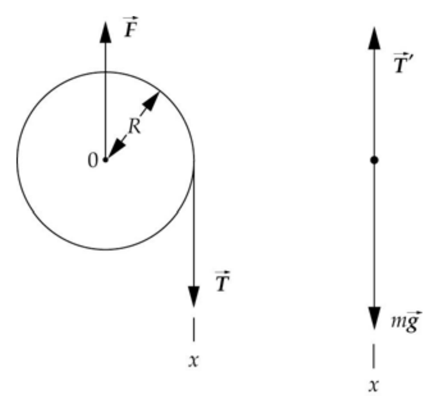
Picture the Problem The force diagram shows the forces acting on the sphere and the hanging object. The tension in the string is responsible for the angular acceleration of the sphere and the difference between the weight of the object and the tension is the net force acting on the hanging object. We can use Newton's 2nd law to obtain two equations in a and T that we can solve simultaneously.



(*a*)Apply Newton's 2nd law to the sphere and the hanging object:

$$\sum \tau_0 = TR = I_{\text{sphere}}\alpha \qquad (1)$$

and

$$\sum F_x = mg - T = ma \qquad (2)$$

Substitute for I_{sphere} and α in equation (1) to obtain:

$$TR = \left(\tfrac{2}{5}MR^2\right)\frac{a}{R} \qquad (3)$$

Eliminate T between equations (2) and (3) and solve for a to obtain:

$$a = \boxed{\frac{g}{1+\frac{2M}{5m}}}$$

(*b*) Substitute for a in equation (2) and solve for T to obtain:

$$T = \boxed{\frac{2mMg}{5m+2M}}$$

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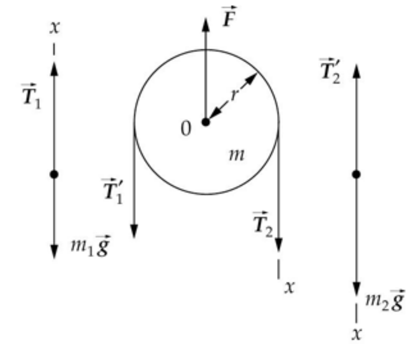
Picture the Problem The diagram shows the forces acting on both objects and the pulley. By applying Newton's 2nd law of motion, we can obtain a system of three equations in the unknowns T_1, T_2, and a that we can solve simultaneously.



(*a*) Apply Newton's 2nd law to the pulley and the two objects:

$$\sum F_x = T_1 - m_1 g = m_1 a\,, \qquad (1)$$

$$\sum \tau_0 = (T_2 - T_1)r = I_0\alpha\,, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Substitute for $I_0 = I_{\text{pulley}}$ and α in equation (2) to obtain:

$$(T_2 - T_1)r = \left(\tfrac{1}{2}mr^2\right)\frac{a}{r} \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a to obtain:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{1}{2}m}$$

Substitute numerical values and evaluate a:

$$a = \frac{(510\,\text{g} - 500\,\text{g})(981\,\text{cm/s}^2)}{500\,\text{g} + 510\,\text{g} + \frac{1}{2}(50\,\text{g})} = \boxed{9.478\,\text{cm/s}^2}$$

(*b*) Substitute for a in equation (1) and solve for T_1 to obtain:

$$T_1 = m_1(g + a) = (0.500\,\text{kg})(9.81\,\text{m/s}^2 + 0.09478\,\text{m/s}^2) = \boxed{4.9524\,\text{N}}$$

Substitute for a in equation (3) and solve for T_2 to obtain:

$$T_2 = m_2(g-a)$$
$$= (0.510\,\text{kg})(9.81\,\text{m/s}^2 - 0.09478\,\text{m/s}^2)$$
$$= \boxed{4.9548\,\text{N}}$$

Find ΔT:

$$\Delta T = T_2 - T_1 = 4.9548\,\text{N} - 4.9524\,\text{N}$$
$$= \boxed{0.0024\,\text{N}}$$

(c) If we ignore the mass of the pulley, our acceleration equation is:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Substitute numerical values and evaluate a:

$$a = \frac{(510\,\text{g} - 500\,\text{g})(981\,\text{cm/s}^2)}{500\,\text{g} + 510\,\text{g}}$$
$$= \boxed{9.713\,\text{cm/s}^2}$$

Substitute for a in equation (1) and solve for T_1 to obtain:

$$T_1 = m_1(g+a)$$

Substitute numerical values and evaluate T_1:

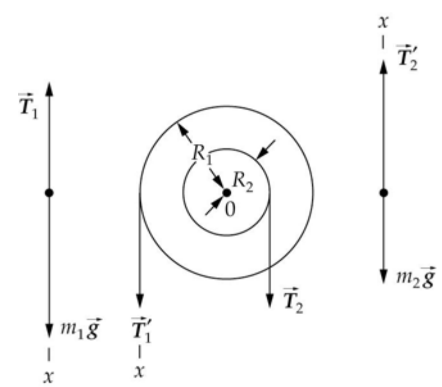
$$T_1 = (0.500\,\text{kg})(9.81\,\text{m/s}^2 + 0.09713\,\text{m/s}^2) = \boxed{4.9536\,\text{N}}$$

From equation (4), if m = 0:

$$\boxed{T_1 = T_2}$$

***75 ••**

Picture the Problem The diagram shows the forces acting on both objects and the pulley. By applying Newton's 2nd law of motion, we can obtain a system of three equations in the unknowns T_1, T_2, and α that we can solve simultaneously.



(a) Express the condition that the system does not accelerate:

$$\tau_{\text{net}} = m_1 g R_1 - m_2 g R_2 = 0$$

Solve for m_2:

$$m_2 = m_1 \frac{R_1}{R_2}$$

Substitute numerical values and evaluate m_2:

$$m_2 = (24\,\text{kg})\frac{1.2\,\text{m}}{0.4\,\text{m}} = \boxed{72.0\,\text{kg}}$$

(*b*) Apply Newton's 2nd law to the objects and the pulley:

$$\sum F_x = m_1 g - T_1 = m_1 a\,, \qquad (1)$$

$$\sum \tau_0 = T_1 R_1 - T_2 R_2 = I_0 \alpha\,, \qquad (2)$$

and

$$\sum F_x = T_2 - m_2 g = m_2 a \qquad (3)$$

Eliminate a in favor of α in equations (1) and (3) and solve for T_1 and T_2:

$$T_1 = m_1(g - R_1\alpha) \qquad (4)$$

and

$$T_2 = m_2(g + R_2\alpha) \qquad (5)$$

Substitute for T_1 and T_2 in equation (2) and solve for α to obtain:

$$\alpha = \frac{(m_1 R_1 - m_2 R_2)g}{m_1 R_1^2 + m_2 R_2^2 + I_0}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{[(36\,\text{kg})(1.2\,\text{m}) - (72\,\text{kg})(0.4\,\text{m})](9.81\,\text{m/s}^2)}{(36\,\text{kg})(1.2\,\text{m})^2 + (72\,\text{kg})(0.4\,\text{m})^2 + 40\,\text{kg}\cdot\text{m}^2} = \boxed{1.37\,\text{rad/s}^2}$$

Substitute in equation (4) to find T_1:

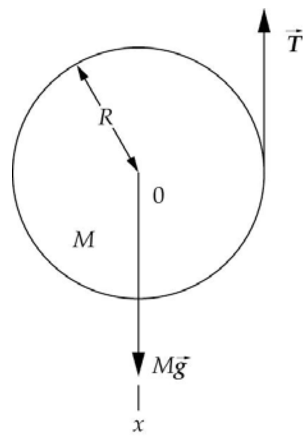
$$T_1 = (36\,\text{kg})\left[9.81\,\text{m/s}^2 - (1.2\,\text{m})(1.37\,\text{rad/s}^2)\right] = \boxed{294\,\text{N}}$$

Substitute in equation (5) to find T_2:

$$T_2 = (72\,\text{kg})\left[9.81\,\text{m/s}^2 + (0.4\,\text{m})(1.37\,\text{rad/s}^2)\right] = \boxed{746\,\text{N}}$$

76 ••

Picture the Problem Choose the coordinate system shown in the diagram. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T and a. In (b) we can use the torque equation from (a) and our value for T to find α. In (c) we use the condition that the acceleration of a point on the rim of the cylinder is the same as the acceleration of the hand, together with the angular acceleration of the cylinder, to find the acceleration of the hand.



(a) Apply Newton's 2nd law to the cylinder about an axis through its center of mass:

$$\sum \tau_0 = TR = I_0 \frac{a}{R} \quad (1)$$

and

$$\sum F_x = Mg - T = 0 \quad (2)$$

Solve for T to obtain:

$$T = \boxed{Mg}$$

(b) Rewrite equation (1) in terms of α:

$$TR = I_0 \alpha$$

Solve for α:

$$\alpha = \frac{TR}{I_0}$$

Substitute for T and I_0 to obtain:

$$\alpha = \frac{MgR}{\frac{1}{2}MR^2} = \boxed{\frac{2g}{R}}$$

(c) Relate the acceleration a of the hand to the angular acceleration of the cylinder:

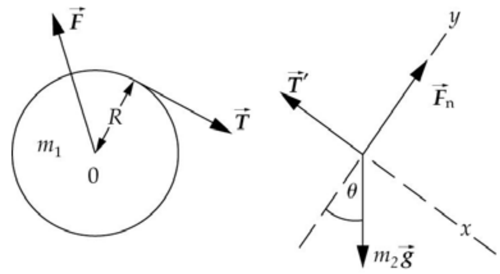
$$a = R\alpha$$

Substitute for α to obtain:

$$a = R\left(\frac{2g}{R}\right) = \boxed{2g}$$

77 ••

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. By applying Newton's 2nd law to the cylinder and the block we can obtain simultaneous equations in a, T, and α from which we can express a and T. By applying the conservation of energy, we can derive an expression for the speed of the block when it reaches the bottom of the incline.



(*a*) Apply Newton's 2nd law to the cylinder and the block:

$$\sum \tau_0 = TR = I_0\alpha \qquad (1)$$

and

$$\sum F_x = m_2 g\sin\theta - T = m_2 a \qquad (2)$$

Substitute for α in equation (1), solve for T, and substitute in equation (2) and solve for a to obtain:

$$a = \boxed{\frac{g\sin\theta}{1+\dfrac{m_1}{2m_2}}}$$

(*b*) Substitute for a in equation (2) and solve for T:

$$T = \boxed{\frac{\frac{1}{2}m_1 g\sin\theta}{1+\dfrac{m_1}{2m_2}}}$$

(*c*) Noting that the block is released from rest, express the total energy of the system when the block is at height h:

$$E = U + K = \boxed{m_2 gh}$$

(*d*) Use the fact that this system is conservative to express the total energy at the bottom of the incline:

$$E_{\text{bottom}} = \boxed{m_2 gh}$$

(*e*) Express the total energy of the system when the block is at the bottom of the incline in terms of its kinetic energies:

$$E_{\text{bottom}} = K_{\text{tran}} + K_{\text{rot}} = \tfrac{1}{2}m_2v^2 + \tfrac{1}{2}I_0\omega^2$$

Substitute for ω and I_0 to obtain:

$$\tfrac{1}{2}m_2v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_1r^2\right)\frac{v^2}{r^2} = m_2gh$$

Solve for v to obtain:

$$v = \boxed{\sqrt{\frac{2gh}{1+\frac{m_1}{2m_2}}}}$$

(f) For $\theta = 0$:

$$\boxed{a = T = 0}$$

For $\theta = 90°$:

$$a = \boxed{\frac{g}{1+\frac{m_1}{2m_2}}},$$

$$T = \frac{\frac{1}{2}m_1g}{1+\frac{m_1}{2m_2}} = \boxed{\tfrac{1}{2}m_1a},$$

and

$$v = \boxed{\sqrt{\frac{2gh}{1+\frac{m_1}{2m_2}}}}$$

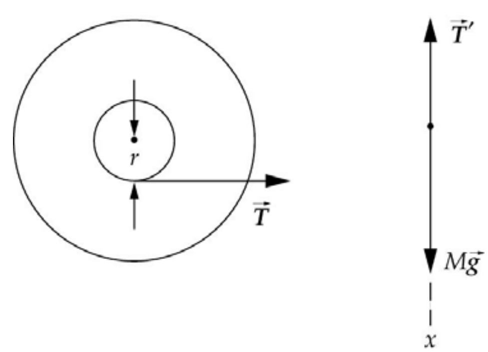
For $m_1 = 0$:

$a = \boxed{g\sin\theta}$, $T = \boxed{0}$, and

$v = \boxed{\sqrt{2gh}}$

***78 ••**

Picture the Problem Let r be the radius of the concentric drum (10 cm) and let I_0 be the moment of inertia of the drum plus platform. We can use Newton's 2nd law in both translational and rotational forms to express I_0 in terms of a and a constant-acceleration equation to express a and then find I_0. We can use the same equation to find the total moment of inertia when the object is placed on the platform and then subtract to find its moment of inertia.



(a) Apply Newton's 2nd law to the platform and the weight:

$$\sum \tau_0 = Tr = I_0\alpha \qquad (1)$$

$$\sum F_x = Mg - T = Ma \qquad (2)$$

Substitute a/r for α in equation (1) and solve for T:

$$T = \frac{I_0}{r^2} a$$

Substitute for T in equation (2) and solve for a to obtain:

$$I_0 = \frac{Mr^2(g - a)}{a} \qquad (3)$$

Using a constant-acceleration equation, relate the distance of fall to the acceleration of the weight and the time of fall and solve for the acceleration:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2$$

or, because $v_0 = 0$ and $\Delta x = D$,

$$a = \frac{2D}{(\Delta t)^2}$$

Substitute for a in equation (3) to obtain:

$$I_0 = Mr^2\left(\frac{g}{a} - 1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D} - 1\right)$$

Substitute numerical values and evaluate I_0:

$$I_0 = (2.5\,\text{kg})(0.1\,\text{m})^2 \times \left[\frac{(9.81\,\text{m/s}^2)(4.2\,\text{s})^2}{2(1.8\,\text{m})} - 1\right] = \boxed{1.177\,\text{kg}\cdot\text{m}^2}$$

(b) Relate the moments of inertia of the platform, drum, shaft, and pulley (I_0) to the moment of inertia of the object and the total moment of inertia:

$$I_{\text{tot}} = I_0 + I = Mr^2\left(\frac{g}{a} - 1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D} - 1\right)$$

Substitute numerical values and evaluate I_{tot}:

$$I_{\text{tot}} = (2.5\,\text{kg})(0.1\,\text{m})^2 \times \left[\frac{(9.81\,\text{m/s}^2)(6.8\,\text{s})^2}{2(1.8\,\text{m})} - 1\right] = \boxed{3.125\,\text{kg}\cdot\text{m}^2}$$

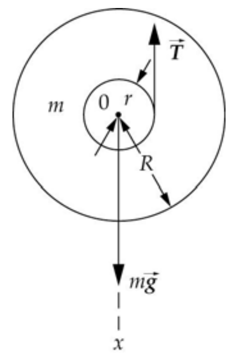
Solve for and evaluate I:

$$I = I_{\text{tot}} - I_0 = 3.125\,\text{kg}\cdot\text{m}^2 - 1.177\,\text{kg}\cdot\text{m}^2 = \boxed{1.948\,\text{kg}\cdot\text{m}^2}$$

Objects Rolling Without Slipping

***79 ••**

Picture the Problem The forces acting on the yo-yo are shown in the figure. We can use a constant-acceleration equation to relate the velocity of descent at the end of the fall to the yo-yo's acceleration and Newton's 2nd law in both translational and rotational form to find the yo-yo's acceleration.



Using a constant-acceleration equation, relate the yo-yo's final speed to its acceleration and fall distance:

$$v^2 = v_0^2 + 2a\Delta h$$

or, because $v_0 = 0$,

$$v = \sqrt{2a\Delta h} \quad (1)$$

Use Newton's 2nd law to relate the forces that act on the yo-yo to its acceleration:

$$\sum F_x = mg - T = ma \quad (2)$$

and

$$\sum \tau_0 = Tr = I_0\alpha \quad (3)$$

Use $a = r\alpha$ to eliminate α in equation (3)

$$Tr = I_0 \frac{a}{r} \quad (4)$$

Eliminate T between equations (2) and (4) to obtain:

$$mg - \frac{I_0}{r^2}a = ma \quad (5)$$

Substitute $\frac{1}{2}mR^2$ for I_0 in equation (5):

$$mg - \frac{\frac{1}{2}mR^2}{r^2}a = ma$$

Solve for a:

$$a = \frac{g}{1 + \frac{R^2}{2r^2}}$$

Substitute numerical values and evaluate a:

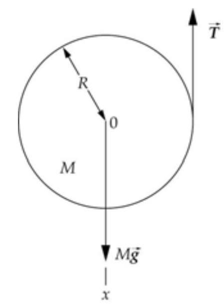
$$a = \frac{9.81\,\text{m/s}^2}{1 + \frac{(1.5\,\text{m})^2}{2(0.1\,\text{m})^2}} = 0.0864\,\text{m/s}^2$$

Substitute in equation (1) and evaluate v:

$$v = \sqrt{2(0.0864\,\text{m/s}^2)(57\,\text{m})} = \boxed{3.14\,\text{m/s}}$$

80 ••

Picture the Problem The diagram shows the forces acting on the cylinder. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T, a, and α that we can solve simultaneously.



(*a*) Apply Newton's 2nd law to the cylinder:

$$\sum \tau_0 = TR = I_0\alpha \quad (1)$$

and

$$\sum F_x = Mg - T = Ma \quad (2)$$

Substitute for α and I_0 in equation (1) to obtain:

$$TR = \left(\tfrac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

Solve for T:

$$T = \tfrac{1}{2}Ma \quad (3)$$

Substitute for T in equation (2) and solve for a to obtain:

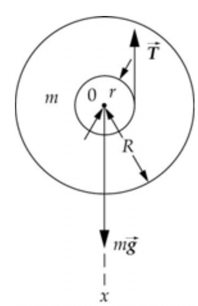
$$a = \boxed{\tfrac{2}{3}g}$$

(*b*) Substitute for a in equation (3) to obtain:

$$T = \tfrac{1}{2}M\left(\tfrac{2}{3}g\right) = \boxed{\tfrac{1}{3}Mg}$$

81 ••

Picture the Problem The forces acting on the yo-yo are shown in the figure. Apply Newton's 2nd law in both translational and rotational form to obtain simultaneous equations in T, a, and α from which we can eliminate α and solve for T and a.



Apply Newton's 2nd law to the yo-yo:

$$\sum F_x = mg - T = ma \quad (1)$$

and

$$\sum \tau_0 = Tr = I_0\alpha \quad (2)$$

Use $a = r\alpha$ to eliminate α in equation (2)

$$Tr = I_0\frac{a}{r} \quad (3)$$

Eliminate T between equations (1) and (3) to obtain:

$$mg - \frac{I_0}{r^2}a = ma \quad (4)$$

Substitute $\frac{1}{2}mR^2$ for I_0 in equation (4):

$$mg - \frac{\frac{1}{2}mR^2}{r^2}a = ma$$

Solve for a:

$$a = \frac{g}{1 + \frac{R^2}{2r^2}}$$

Substitute numerical values and evaluate a:

$$a = \frac{9.81\,\text{m/s}^2}{1 + \frac{(0.1\,\text{m})^2}{2(0.01\,\text{m})^2}} = \boxed{0.192\,\text{m/s}^2}$$

Use equation (1) to solve for and evaluate T:

$$T = m(g - a)$$
$$= (0.1\,\text{kg})(9.81\,\text{m/s}^2 - 0.192\,\text{m/s}^2)$$
$$= \boxed{0.962\,\text{N}}$$

***82 •**

Picture the Problem We can determine the kinetic energy of the cylinder that is due to its rotation about its center of mass by examining the ratio K_{rot}/K.

Express the rotational kinetic energy of the homogeneous solid cylinder:

$$K_{\text{rot}} = \tfrac{1}{2}I_{\text{cyl}}\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{4}mv^2$$

Express the total kinetic energy of the homogeneous solid cylinder:

$$K = K_{\text{rot}} + K_{\text{trans}} = \tfrac{1}{4}mv^2 + \tfrac{1}{2}mv^2 = \tfrac{3}{4}mv^2$$

Express the ratio $\frac{K_{\text{rot}}}{K}$:

$$\frac{K_{\text{rot}}}{K} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \tfrac{1}{3} \text{ and } \boxed{(b)\text{ is correct.}}$$

83 •

Picture the Problem Any work done on the cylinder by a net force will change its kinetic energy. Therefore, the work needed to give the cylinder this motion is equal to its kinetic energy.

Express the relationship between the work needed to stop the cylinder and its kinetic energy:

$$|W| = |\Delta K| = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Because the cylinder is rolling without slipping, its translational and angular speeds are related according to:

$$v = r\omega$$

Substitute for I (see Table 9-1) and ω and simplify to obtain:

$$|W| = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v^2}{r^2} = \tfrac{3}{4}mv^2$$

Substitute for m and v to obtain:

$$|W| = \tfrac{3}{4}(60\,\text{kg})(5\,\text{m/s})^2 = \boxed{1.13\,\text{kJ}}$$

84 •

Picture the Problem The total kinetic energy of any object that is rolling without slipping is given by $K = K_{\text{trans}} + K_{\text{rot}}$. We can find the percentages associated with each motion by expressing the moment of inertia of the objects as kmr^2 and deriving a general expression for the ratios of rotational kinetic energy to total kinetic energy and translational kinetic energy to total kinetic energy and substituting the appropriate values of k.

Express the total kinetic energy associated with a rotating and translating object:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmr^2\right)\frac{v^2}{r^2} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 = \tfrac{1}{2}mv^2(1+k)$$

Express the ratio $\dfrac{K_{\text{rot}}}{K}$:

$$\frac{K_{\text{rot}}}{K} = \frac{\tfrac{1}{2}kmv^2}{\tfrac{1}{2}mv^2(1+k)} = \frac{k}{1+k} = \frac{1}{1+\dfrac{1}{k}}$$

Express the ratio $\dfrac{K_{\text{trans}}}{K}$:

$$\frac{K_{\text{trans}}}{K} = \frac{\tfrac{1}{2}mv^2}{\tfrac{1}{2}mv^2(1+k)} = \frac{1}{1+k}$$

(*a*) Substitute $k = 2/5$ for a uniform sphere to obtain:

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\dfrac{1}{0.4}} = 0.286 = \boxed{28.6\%}$$

and

$$\frac{K_{\text{trans}}}{K} = \frac{1}{1+0.4} = 0.714 = \boxed{71.4\%}$$

(*b*) Substitute $k = 1/2$ for a uniform cylinder to obtain:

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\frac{1}{0.5}} = \boxed{33.3\%}$$

and

$$\frac{K_{\text{trans}}}{K} = \frac{1}{1+0.5} = \boxed{66.7\%}$$

(*c*) Substitute $k = 1$ for a hoop to obtain:

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\frac{1}{1}} = \boxed{50.0\%}$$

and

$$\frac{K_{\text{trans}}}{K} = \frac{1}{1+1} = \boxed{50.0\%}$$

85 •

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. As the hoop rolls up the incline its translational and rotational kinetic energies are transformed into gravitational potential energy. We can use energy conservation to relate the distance the hoop rolls up the incline to its total kinetic energy at the bottom of the incline.

Using energy conservation, relate the distance the hoop will roll up the incline to its kinetic energy at the bottom of the incline:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0 \qquad (1)$$

Express K_i as the sum of the translational and rotational kinetic energies of the hoop:

$$K_i = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

When a rolling object moves with speed v, its outer surface turns with a speed v also. Hence $\omega = v/r$. Substitute for I and ω to obtain:

$$K_i = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(mr^2\right)\frac{v^2}{r^2} = mv^2$$

Letting Δh be the change in elevation of the hoop as it rolls up the incline and ΔL the distance it rolls along the incline, express U_f:

$$U_f = mg\Delta h = mg\Delta L\sin\theta$$

Substitute in equation (1) to obtain:

$$-mv^2 + mg\Delta L\sin\theta = 0$$

Solve for ΔL:

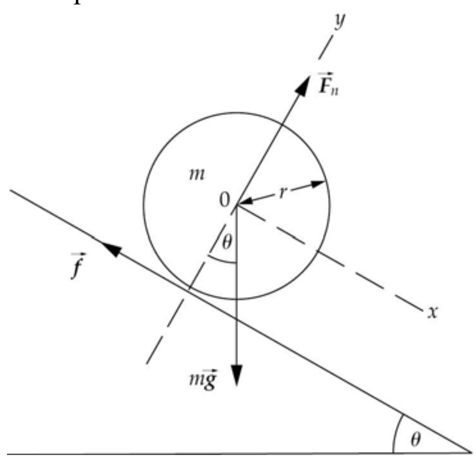
$$\Delta L = \frac{v^2}{g\sin\theta}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(15\,\text{m/s})^2}{(9.81\,\text{m/s}^2)\sin 30°} = \boxed{45.9\,\text{m}}$$

***86 ••**

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass equals the net force divided by the mass. The forces acting on the sphere are its weight $m\vec{g}$ downward, the normal force $\vec{F}_n$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the sphere accelerates down the incline, the angular velocity of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the sphere to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_n$ act through the center of mass. Choose the positive direction to be down the incline.



Apply $\sum\vec{F} = m\vec{a}$ to the sphere:

$$mg\sin\theta - f = ma_{cm} \qquad (1)$$

Apply $\sum\tau = I_{cm}\alpha$ to the sphere:

$$fr = I_{cm}\alpha$$

Use the nonslip condition to eliminate α and solve for f:

$$fr = I_{cm}\frac{a_{cm}}{r}$$

and

$$f = \frac{I_{cm}}{r^2}a_{cm}$$

Substitute this result for f in equation (1) to obtain:

$$mg\sin\theta - \frac{I_{cm}}{r^2}a_{cm} = ma_{cm}$$

From Table 9-1 we have, for a solid sphere:

$$I_{cm} = \tfrac{2}{5}mr^2$$

Substitute in equation (1) and simplify to obtain:

$$mg\sin\theta - \tfrac{2}{5}a_{cm} = ma_{cm}$$

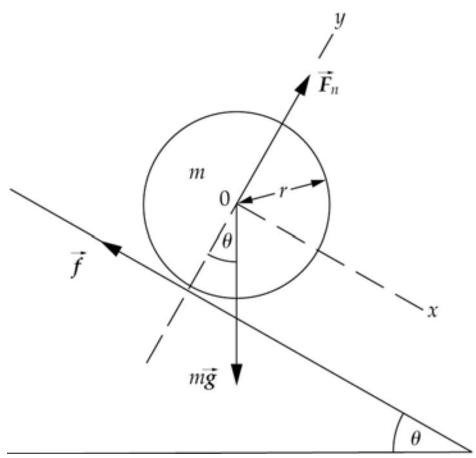
Solve for and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{7a_{cm}}{5g}\right)$$

$$= \sin^{-1}\left[\frac{7(0.2g)}{5g}\right] = \boxed{16.3°}$$

87 ••

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass equals the net force divided by the mass. The forces acting on the thin spherical shell are its weight $m\vec{g}$ downward, the normal force $\vec{F}_n$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the spherical shell accelerates down the incline, the angular velocity of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the sphere to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_n$ act through the center of mass. Choose the positive direction to be down the incline.



Apply $\sum\vec{F} = m\vec{a}$ to the thin spherical shell:

$$mg\sin\theta - f = ma_{cm} \qquad (1)$$

Apply $\sum\tau = I_{cm}\alpha$ to the thin spherical shell:

$$fr = I_{cm}\alpha$$

Use the nonslip condition to eliminate α and solve for f:

$$fr = I_{cm}\frac{a_{cm}}{r} \text{ and } f = \frac{I_{cm}}{r^2}a_{cm}$$

Substitute this result for f in equation (1) to obtain:

$$mg\sin\theta - \frac{I_{cm}}{r^2}a_{cm} = ma_{cm}$$

From Table 9-1 we have, for a thin

$$I_{cm} = \tfrac{2}{3}mr^2$$

spherical shell:

Substitute in equation (1) and simplify to obtain:

$$mg\sin\theta - \tfrac{2}{3}a_{cm} = ma_{cm}$$

Solve for and evaluate θ:

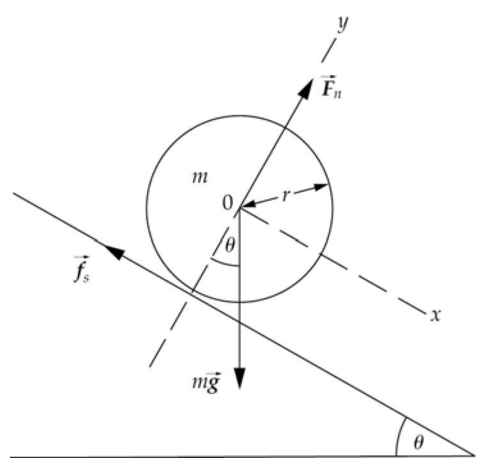
$$\theta = \sin^{-1}\frac{5a_{cm}}{3g}$$

$$= \sin^{-1}\frac{5(0.2g)}{3g} = \boxed{19.5°}$$

Remarks: This larger angle makes sense, as the moment of inertia for a given mass is larger for a hollow sphere than for a solid one.

88 ••

Picture the Problem The three forces acting on the basketball are the weight of the ball, the normal force, and the force of friction. Because the weight can be assumed to be acting at the center of mass, and the normal force acts through the center of mass, the only force which exerts a torque about the center of mass is the frictional force. We can use Newton's 2nd law to find a system of simultaneous equations that we can solve for the quantities called for in the problem statement.



(*a*) Apply Newton's 2nd law in both translational and rotational form to the ball:

$$\sum F_x = mg\sin\theta - f_s = ma, \quad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

and

$$\sum \tau_0 = f_s r = I_0\alpha \quad (3)$$

Because the basketball is rolling without slipping we know that:

$$\alpha = \frac{a}{r}$$

Substitute in equation (3) to obtain:

$$f_s r = I_0\frac{a}{r} \quad (4)$$

From Table 9-1 we have:

$$I_0 = \tfrac{2}{3}mr^2$$

Substitute for I_0 and α in equation (4) and solve for f_s:

$$f_s r = \left(\tfrac{2}{3}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{2}{3}ma \quad (5)$$

Substitute for f_s in equation (1) and solve for a: $a = \boxed{\frac{3}{5} g \sin\theta}$

(*b*) Find f_s using equation (5): $f_s = \frac{2}{3} m\left(\frac{3}{5} g \sin\theta\right) = \boxed{\frac{2}{5} mg \sin\theta}$

(*c*) Solve equation (2) for F_n: $F_n = mg\cos\theta$

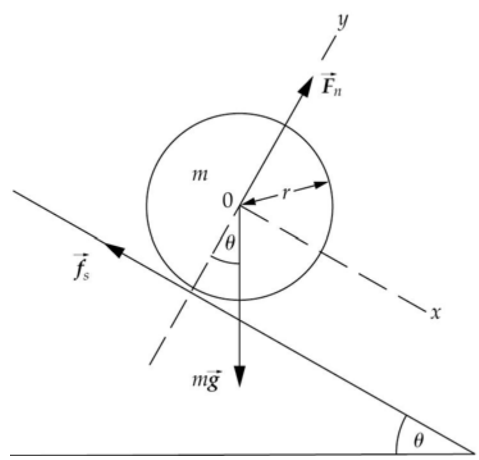
Use the definition of $f_{s,max}$ to obtain: $f_{s,max} = \mu_s F_n = \mu_s mg\cos\theta_{max}$

Use the result of part (*b*) to obtain: $\frac{2}{5} mg \sin\theta_{max} = \mu_s mg\cos\theta_{max}$

Solve for θ_{max}: $\theta_{max} = \boxed{\tan^{-1}\left(\frac{5}{2}\mu_s\right)}$

89 ••

Picture the Problem The three forces acting on the cylinder are the weight of the cylinder, the normal force, and the force of friction. Because the weight can be assumed to be acting at the center of mass, and the normal force acts through the center of mass, the only force which exerts a torque about the center of mass is the frictional force. We can use Newton's 2nd law to find a system of simultaneous equations that we can solve for the quantities called for in the problem statement.



(*a*) Apply Newton's 2nd law in both translational and rotational form to the cylinder:

$$\sum F_x = mg\sin\theta - f_s = ma, \qquad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0 \qquad (2)$$

and

$$\sum \tau_0 = f_s r = I_0\alpha \qquad (3)$$

Because the cylinder is rolling without slipping we know that: $\alpha = \frac{a}{r}$

Substitute in equation (3) to obtain:

$$f_s r = I_0 \frac{a}{r} \qquad (4)$$

From Table 9-1 we have: $I_0 = \frac{1}{2} mr^2$

Substitute for I_0 and α in equation (4) and solve for f_s:

$$f_s r = \left(\tfrac{1}{2}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{1}{2}ma \qquad (5)$$

Substitute for f_s in equation (1) and solve for a:

$$a = \boxed{\tfrac{2}{3}g\sin\theta}$$

(*b*) Find f_s using equation (5):

$$f_s = \tfrac{1}{2}m\left(\tfrac{2}{3}g\sin\theta\right) = \boxed{\tfrac{1}{3}mg\sin\theta}$$

(*c*) Solve equation (2) for F_n:

$$F_n = mg\cos\theta$$

Use the definition of $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s F_n = \mu_s mg\cos\theta_{max}$$

Use the result of part (*b*) to obtain:

$$\tfrac{1}{3}mg\sin\theta_{max} = \mu_s mg\cos\theta_{max}$$

Solve for θ_{max}:

$$\theta_{max} = \boxed{\tan^{-1}(3\mu_s)}$$

***90** ••

Picture the Problem Let the zero of gravitational potential energy be at the elevation where the spheres leave the ramp. The distances the spheres will travel are directly proportional to their speeds when they leave the ramp.

Express the ratio of the distances traveled by the two spheres in terms of their speeds when they leave the ramp:

$$\frac{L'}{L} = \frac{v'\Delta t}{v\Delta t} = \frac{v'}{v} \qquad (1)$$

Use conservation of mechanical energy to find the speed of the spheres when they leave the ramp:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0 \qquad (2)$$

Express K_f for the spheres:

$$\begin{aligned} K_f &= K_{trans} + K_{rot} \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{cm}\omega^2 \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmR^2\right)\frac{v^2}{R^2} \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 \\ &= (1+k)\tfrac{1}{2}mv^2 \end{aligned}$$

where k is 2/3 for the spherical shell and 2/5 for the uniform sphere.

Substitute in equation (2) to obtain:

$$(1+k)\tfrac{1}{2}mv^2 = mgH$$

Solve for *v*:

$$v=\sqrt{\frac{2gH}{1+k}}$$

Substitute in equation (1) to obtain:

$$\frac{L'}{L}=\sqrt{\frac{1+k}{1+k'}}=\sqrt{\frac{1+\frac{2}{3}}{1+\frac{2}{5}}}=1.09$$

or

$$L'=\boxed{1.09L}$$

91 ••

Picture the Problem Let the subscripts u and h refer to the uniform and thin-walled spheres, respectively. Because the cylinders climb to the same height, their kinetic energies at the bottom of the incline must be equal.

Express the total kinetic energy of the thin-walled cylinder at the bottom of the inclined plane:

$$K_h=K_{trans}+K_{rot}=\tfrac{1}{2}m_hv^2+\tfrac{1}{2}I_h\omega^2$$
$$=\tfrac{1}{2}m_hv^2+\tfrac{1}{2}\left(m_hr^2\right)\frac{v^2}{r^2}=m_hv^2$$

Express the total kinetic energy of the solid cylinder at the bottom of the inclined plane:

$$K_u=K_{trans}+K_{rot}=\tfrac{1}{2}m_uv'^2+\tfrac{1}{2}I_u\omega^2$$
$$=\tfrac{1}{2}m_uv'^2+\tfrac{1}{2}\left(\tfrac{1}{2}m_ur^2\right)\frac{v'^2}{r^2}=\tfrac{3}{4}m_uv'^2$$

Because the cylinders climb to the same height:

$$\tfrac{3}{4}m_uv'^2=m_ugh$$

and

$$m_hv^2=m_hgh$$

Divide the first of these equations by the second:

$$\frac{\tfrac{3}{4}m_uv'^2}{m_hv^2}=\frac{m_ugh}{m_hgh}$$

Simplify to obtain:

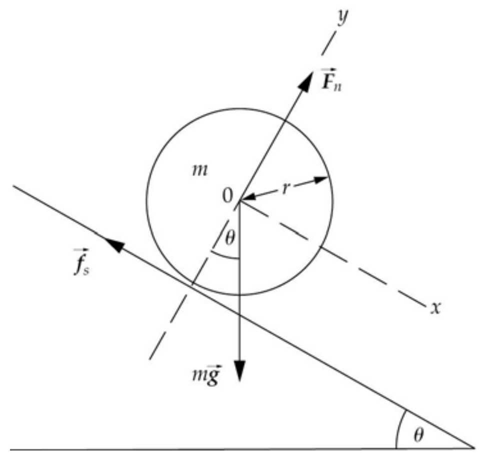
$$\frac{3v'^2}{4v^2}=1$$

Solve for *v*′:

$$v'=\boxed{\sqrt{\frac{4}{3}}v}$$

92 ••

Picture the Problem Let the subscripts s and c refer to the solid sphere and thin-walled cylinder, respectively. Because the cylinder and sphere descend from the same height, their kinetic energies at the bottom of the incline must be equal. The force diagram shows the forces acting on the solid sphere. We'll use Newton's 2nd law to relate the accelerations to the angle of the incline and use a constant acceleration to relate the accelerations to the distances traveled down the incline.



Apply Newton's 2nd law to the sphere:

$$\sum F_x = mg\sin\theta - f_s = ma_s\,, \qquad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0\,, \qquad (2)$$

and

$$\sum \tau_0 = f_s r = I_0\alpha \qquad (3)$$

Substitute for I_0 and α in equation (3) and solve for f_s:

$$f_s r = \left(\tfrac{2}{5}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{2}{5}ma_s$$

Substitute for f_s in equation (1) and solve for a:

$$a_s = \tfrac{5}{7}g\sin\theta$$

Proceed as above for the thin-walled cylinder to obtain:

$$a_c = \tfrac{1}{2}g\sin\theta$$

Using a constant-acceleration equation, relate the distance traveled down the incline to its acceleration and the elapsed time:

$$\Delta s = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta s = \tfrac{1}{2}a(\Delta t)^2 \qquad (4)$$

Because Δs is the same for both objects:

$$a_s t_s^2 = a_c t_c^2$$

where

$$t_c^2 = (t_s + 2.4)^2 = t_s^2 + 4.8t_s + 5.76$$

provided t_c and t_s are in seconds.

Substitute for a_s and a_c to obtain the quadratic equation:

$$t_s^2 + 4.8t_s + 5.76 = \tfrac{10}{7}t_s^2$$

Solve for the positive root to obtain: $t_s = 12.3\,\text{s}$

Substitute in equation (4), simplify, and solve for θ:

$$\theta = \sin^{-1}\left[\frac{14\Delta s}{5gt_s^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{14(3\,\text{m})}{5(9.81\,\text{m/s}^2)(12.3\,\text{s})^2}\right] = \boxed{0.324°}$$

93 •••

Picture the Problem The kinetic energy of the wheel is the sum of its translational and rotational kinetic energies. Because the wheel is a composite object, we can model its moment of inertia by treating the rim as a cylindrical shell and the spokes as rods.

Express the kinetic energy of the wheel:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2 = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}I_{\text{cm}}\frac{v^2}{R^2}$$

where $M_{\text{tot}} = M_{\text{rim}} + 4M_{\text{spoke}}$

Express the moment of inertia of the wheel:

$$I_{\text{cm}} = I_{\text{rim}} + I_{\text{spokes}} = M_{\text{rim}}R^2 + 4\left(\tfrac{1}{3}M_{\text{spoke}}R^2\right) = \left(M_{\text{rim}} + \tfrac{4}{3}M_{\text{spoke}}\right)R^2$$

Substitute for I_{cm} in the equation for K:

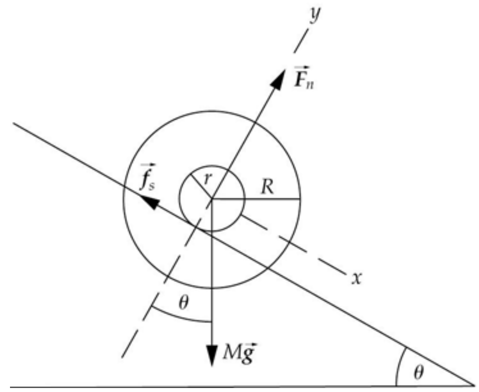
$$K = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}\left[\left(M_{\text{rim}} + \tfrac{4}{3}M_{\text{spoke}}\right)R^2\right]\frac{v^2}{R^2} = \left[\tfrac{1}{2}\left(M_{\text{tot}} + M_{\text{rim}}\right) + \tfrac{2}{3}M_{\text{spoke}}\right]v^2$$

Substitute numerical values and evaluate K:

$$K = \left[\tfrac{1}{2}(7.8\,\text{kg} + 3\,\text{kg}) + \tfrac{2}{3}(1.2\,\text{kg})\right](6\,\text{m/s})^2 = \boxed{223\,\text{J}}$$

94 •••

Picture the Problem Let M represent the combined mass of the two disks and their connecting rod and I their moment of inertia. The object's initial potential energy is transformed into translational and rotational kinetic energy as it rolls down the incline. The force diagram shows the forces acting on this composite object as it rolls down the incline. Application of Newton's 2nd law will allow us to derive an expression for the acceleration of the object.



(a) Apply Newton's 2nd law to the disks and rod:

$$\sum F_x = Mg\sin\theta - f_s = Ma\,, \qquad (1)$$

$$\sum F_y = F_n - Mg\cos\theta = 0\,, \qquad (2)$$

and

$$\sum \tau_0 = f_s r = I\alpha \qquad (3)$$

Eliminate f_s and α between equations (1) and (3) and solve for a to obtain:

$$a = \frac{Mg\sin\theta}{M + \dfrac{I}{r^2}} \qquad (4)$$

Express the moment of inertia of the two disks plus connecting rod:

$$I = 2I_{\text{disk}} + I_{\text{rod}} = 2\left(\tfrac{1}{2}m_{\text{disk}}R^2\right) + \tfrac{1}{2}m_{\text{rod}}r^2 = m_{\text{disk}}R^2 + \tfrac{1}{2}m_{\text{rod}}r^2$$

Substitute numerical values and evaluate I:

$$I = (20\,\text{kg})(0.3\,\text{m})^2 + \tfrac{1}{2}(1\,\text{kg})(0.02\,\text{m})^2 = 1.80\,\text{kg}\cdot\text{m}^2$$

Substitute in equation (4) and evaluate a:

$$a = \frac{(41\,\text{kg})(9.81\,\text{m/s}^2)\sin 30°}{41\,\text{kg} + \dfrac{1.80\,\text{kg}\cdot\text{m}^2}{(0.02\,\text{m})^2}} = \boxed{0.0443\,\text{m/s}^2}$$

(b) Find α from a:

$$\alpha = \frac{a}{r} = \frac{0.0443\,\text{m/s}^2}{0.02\,\text{m}} = \boxed{2.21\,\text{rad/s}^2}$$

(*c*) Express the kinetic energy of translation of the disks-plus-rod when it has rolled a distance Δs down the incline:

$$K_{\text{trans}} = \tfrac{1}{2}Mv^2$$

Using a constant-acceleration equation, relate the speed of the disks-plus-rod to their acceleration and the distance moved:

$$v^2 = v_0^2 + 2a\Delta s$$

or, because $v_0 = 0$,

$$v^2 = 2a\Delta s$$

Substitute to obtain:

$$K_{\text{trans}} = Ma\Delta s = (41\,\text{kg})(0.0443\,\text{m/s}^2)(2\,\text{m}) = \boxed{3.63\,\text{J}}$$

(*d*) Express the rotational kinetic energy of the disks after rolling 2 m in terms of their initial potential energy and their translational kinetic energy:

$$K_{\text{rot}} = U_{\text{i}} - K_{\text{trans}} = Mgh - K_{\text{trans}}$$

Substitute numerical values and evaluate K_{rot}:

$$K_{\text{rot}} = (41\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m})\sin 30° - 3.63\,\text{J} = \boxed{399\,\text{J}}$$

95 •••

Picture the Problem We can express the coordinates of point *P* as the sum of the coordinates of the center of the wheel and the coordinates, relative to the center of the wheel, of the tip of the vector $\vec{r}_0$. Differentiation of these expressions with respect to time will give us the *x* and *y* components of the velocity of point *P*.

(*a*) Express the coordinates of point *P* relative to the center of the wheel:

$$x = r_0 \cos\theta$$

and

$$y = r_0 \sin\theta$$

Because the coordinates of the center of the circle are *X* and *R*:

$$(x_P, y_P) = \boxed{(X + r_0\cos\theta, R + r_0\sin\theta)}$$

(b) Differentiate x_P to obtain:

$$v_{Px} = \frac{d}{dt}\left(X + r_0 \cos\theta\right) = \frac{dX}{dt} - r_0 \sin\theta \cdot \frac{d\theta}{dt}$$

Note that $\dfrac{dX}{dt} = V$ and $\dfrac{d\theta}{dt} = -\omega = -\dfrac{V}{R}$ so:

$$v_{Px} = \boxed{V + \frac{r_0 V}{R}\sin\theta}$$

Differentiate y_P to obtain:

$$v_{Py} = \frac{d}{dt}\left(R + r_0 \sin\theta\right) = r_0 \cos\theta \cdot \frac{d\theta}{dt}$$

Because $\dfrac{d\theta}{dt} = -\omega = -\dfrac{V}{R}$:

$$v_{Py} = \boxed{-\frac{r_0 V}{R}\cos\theta}$$

(c) Calculate $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{r}}$:

$$\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{r}} = v_{Px} r_x + v_{Py} r_y = \left(V + \frac{r_0 V}{R}\sin\theta\right)\left(r_0 \cos\theta\right) - \left(\frac{r_0 V}{R}\cos\theta\right)\left(R + r_0 \sin\theta\right) = \boxed{0}$$

(d) Express v in terms of its components:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(V + \frac{r_0 V}{R}\sin\theta\right)^2 + \left(-\frac{r_0 V}{R}\cos\theta\right)^2} = V\sqrt{1 + 2\frac{r_0}{R}\sin\theta + \frac{r_0^2}{R^2}}$$

Express r in terms of its components:

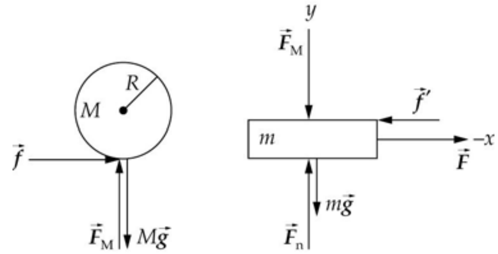
$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{\left(r_0 \cos\theta\right)^2 + \left(R + r_0 \sin\theta\right)^2} = R\sqrt{1 + 2\frac{r_0}{R}\sin\theta + \frac{r_0^2}{R^2}}$$

Divide v by r to obtain:

$$\omega = \frac{v}{r} = \boxed{\frac{V}{R}}$$

***96** •••

Picture the Problem Let the letter B identify the block and the letter C the cylinder. We can find the accelerations of the block and cylinder by applying Newton's 2nd law and solving the resulting equations simultaneously.



Apply $\sum F_x = ma_x$ to the block: $F - f' = ma_B$ (1)

Apply $\sum F_x = ma_x$ to the cylinder: $f = Ma_C$, (2)

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder: $fR = I_{CM}\alpha$ (3)

Substitute for I_{CM} in equation (3) and solve for $f = f'$ to obtain: $f = \frac{1}{2}MR\alpha$ (4)

Relate the acceleration of the block to the acceleration of the cylinder:

$a_C = a_B + a_{CB}$

or, because $a_{CB} = -R\alpha$ is the acceleration of the cylinder relative to the block,

$a_C = a_B - R\alpha$

and

$R\alpha = a_B - a_C$ (5)

Equate equations (2) and (4) and substitute from (5) to obtain: $a_B = 3a_C$

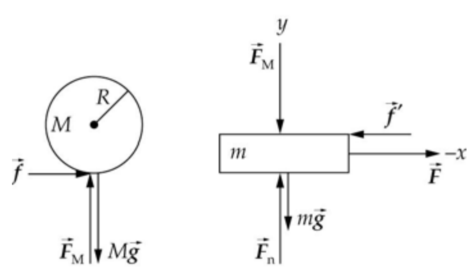
Substitute equation (4) in equation (1) and substitute for a_C to obtain: $F - \frac{1}{3}Ma_B = ma_B$

Solve for a_B:

$$a_B = \boxed{\frac{3F}{M + 3m}}$$

97 •••

Picture the Problem Let the letter B identify the block and the letter C the cylinder. In this problem, as in Problem 97, we can find the accelerations of the block and cylinder by applying Newton's 2nd law and solving the resulting equations simultaneously.



Apply $\sum F_x = ma_x$ to the block: $F - f = ma_\text{B}$ (1)

Apply $\sum F_x = ma_x$ to the cylinder: $f = Ma_\text{C}$, (2)

Apply $\sum \tau_\text{CM} = I_\text{CM}\alpha$ to the cylinder: $fR = I_\text{CM}\alpha$ (3)

Substitute for I_CM in equation (3) and solve for f: $f = \frac{1}{2}MR\alpha$ (4)

Relate the acceleration of the block to the acceleration of the cylinder:

$a_\text{C} = a_\text{B} + a_\text{CB}$

or, because $a_\text{CB} = -R\alpha$,

$a_\text{C} = a_\text{B} - R\alpha$

and

$R\alpha = a_\text{B} - a_\text{C}$ (5)

(*a*) Solve for α and substitute for a_B to obtain:

$$\alpha = \frac{a_\text{B} - a_\text{C}}{R} = \frac{3a_\text{C} - a_\text{C}}{R} = \frac{2a_\text{C}}{R} = \boxed{\frac{2F}{R(M + 3m)}}$$

From the force diagram it is evident that the torque and, therefore, α is in the counterclockwise direction.

(*b*) Equate equations (2) and (4) and substitute (5) to obtain: $a_\text{B} = 3a_\text{C}$

From equations (1) and (4) we obtain: $F - \frac{1}{3}Ma_\text{B} = ma_\text{B}$

Solve for a_B:

$$a_\text{B} = \frac{3F}{M + 3m}$$

Substitute to obtain the linear acceleration of the cylinder relative to the table:

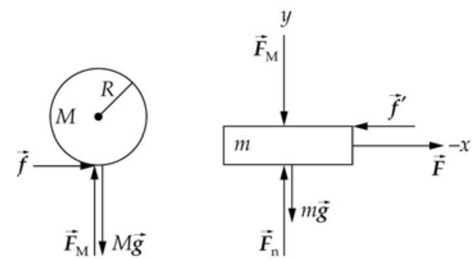
$$a_\text{C} = \frac{1}{3}a_B = \boxed{\frac{F}{M + 3m}}$$

(*c*) Express the acceleration of the cylinder relative to the block:

$$a_{CB} = a_C - a_B = a_C - 3a_C = -2a_C$$
$$= \boxed{-\frac{2F}{M+3m}}$$

98 •••

Picture the Problem Let the system include the earth, the cylinder, and the block. Then $\vec{F}$ is an external force that changes the energy of the system by doing work on it. We can find the kinetic energy of the block from its speed when it has traveled a distance *d*. We can find the kinetic energy of the cylinder from the sum of its translational and rotational kinetic energies. In part (c) we can add the kinetic energies of the block and the cylinder to show that their sum is the work done by $\vec{F}$ in displacing the system a distance *d*.



(*a*) Express the kinetic energy of the block: $K_B = W_{\text{on block}} = \frac{1}{2}mv_B^2$

Using a constant-acceleration equation, relate the velocity of the block to its acceleration and the distance traveled:

$$v_B^2 = v_0^2 + 2a_Bd$$

or, because the block starts from rest,

$$v_B^2 = 2a_Bd$$

Substitute to obtain:

$$K_B = \tfrac{1}{2}m(2a_Bd) = ma_Bd \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$F - f = ma_B \qquad (2)$$

Apply $\sum F_x = ma_x$ to the cylinder:

$$f = Ma_C\,, \qquad (3)$$

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder:

$$fR = I_{CM}\alpha \qquad (4)$$

Substitute for I_{CM} in equation (4) and solve for *f*:

$$f = \tfrac{1}{2}MR\alpha \qquad (5)$$

Relate the acceleration of the block to the acceleration of the cylinder:

$$a_C = a_B + a_{CB}$$

or, because $a_{CB} = -R\alpha$,

$$a_C = a_B - R\alpha$$
and
$$R\alpha = a_B - a_C \qquad (6)$$

Equate equations (3) and (5) and substitute in (6) to obtain:

$$a_B = 3a_C$$

Substitute equation (5) in equation (2) and use $a_B = 3a_C$ to obtain:

$$F - Ma_C = ma_B$$
or
$$F - \tfrac{1}{3}Ma_B = ma_B$$

Solve for a_B:

$$a_B = \frac{F}{m + \frac{1}{3}M}$$

Substitute in equation (1) to obtain:

$$K_B = \boxed{\frac{mFd}{m + \frac{1}{3}M}}$$

(*b*) Express the total kinetic energy of the cylinder:

$$K_{cyl} = K_{trans} + K_{rot} = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_{CM}\omega^2$$
$$= \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_{CM}\frac{v_{CB}^2}{R^2} \qquad (7)$$
where $v_{CB} = v_C - v_B$.

In part (*a*) it was established that:

$$a_B = 3a_C$$

Integrate both sides of the equation with respect to time to obtain:

$$v_B = 3v_C + \text{constant}$$
where the constant of integration is determined by the initial conditions that $v_C = 0$ when $v_B = 0$.

Substitute the initial conditions to obtain:

$$\text{constant} = 0$$
and
$$v_B = 3v_C$$

Substitute in our expression for v_{CB} to obtain:

$$v_{CB} = v_C - v_B = v_C - 3v_C = -2v_C$$

Substitute for I_{CM} and v_{CB} in equation (7) to obtain:

$$K_{cyl} = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{(-2v_C)^2}{R^2} \qquad (8)$$
$$= \tfrac{3}{2}Mv_C^2$$

Because $v_C = \frac{1}{3}v_B$:

$$v_C^2 = \frac{1}{9}v_B^2$$

It part (*a*) it was established that:

$$v_B^2 = 2a_B d$$

and

$$a_B = \frac{F}{m + \frac{1}{3}M}$$

Substitute to obtain:

$$v_C^2 = \frac{1}{9}(2a_B d) = \frac{2}{9}\left(\frac{F}{m + \frac{1}{3}M}\right)d$$

$$= \frac{2Fd}{9\left(m + \frac{1}{3}M\right)}$$

Substitute in equation (8) to obtain:

$$K_{cyl} = \frac{3}{2}M\left(\frac{2Fd}{9\left(m + \frac{1}{3}M\right)}\right)$$

$$= \boxed{\frac{MFd}{3\left(m + \frac{1}{3}M\right)}}$$

(*c*) Express the total kinetic energy of the system and simplify to obtain:

$$K_{tot} = K_B + K_{cyl}$$

$$= \frac{mFd}{m + \frac{1}{3}M} + \frac{MFd}{3\left(m + \frac{1}{3}M\right)}$$

$$= \frac{(3m + M)}{3\left(m + \frac{1}{3}M\right)}Fd = \boxed{Fd}$$

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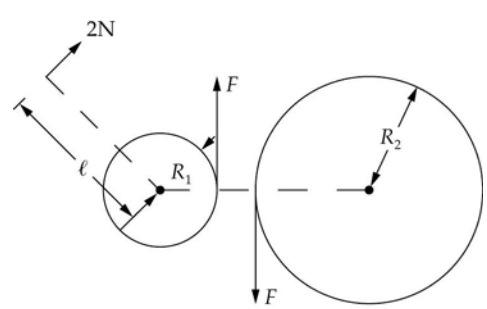
Picture the Problem The forces responsible for the rotation of the gears are shown in the diagram to the right. The forces acting through the centers of mass of the two gears have been omitted because they produce no torque. We can apply Newton's 2nd law in rotational form to obtain the equations of motion of the gears and the not slipping condition to relate their angular accelerations.



(*a*) Apply $\sum \tau = I\alpha$ to the gears to obtain their equations of motion:

$$2\text{ N}\cdot\text{m} - FR_1 = I_1\alpha_1 \quad (1)$$

and

$$FR_2 = I_2\alpha_2 \quad (2)$$

where F is the force keeping the gears from slipping with respect to each other.

Because the gears do not slip

$$R_1\alpha_1 = R_2\alpha_2$$

relative to each other, the tangential accelerations of the points where they are in contact must be the same:

or

$$\alpha_2 = \frac{R_1}{R_2}\alpha_1 = \tfrac{1}{2}\alpha_1 \qquad (3)$$

Divide equation (1) by R_1 to obtain:

$$\frac{2\ \text{N}\cdot\text{m}}{R_1} - F = \frac{I_1}{R_1}\alpha_1$$

Divide equation (2) by R_2 to obtain:

$$F = \frac{I_2}{R_2}\alpha_2$$

Add these equations to obtain:

$$\frac{2\ \text{N}\cdot\text{m}}{R_1} = \frac{I_1}{R_1}\alpha_1 + \frac{I_2}{R_2}\alpha_2$$

Use equation (3) to eliminate α_2:

$$\frac{2\ \text{N}\cdot\text{m}}{R_1} = \frac{I_1}{R_1}\alpha_1 + \frac{I_2}{2R_2}\alpha_1$$

Solve for α_1 to obtain:

$$\alpha_1 = \frac{2\ \text{N}\cdot\text{m}}{I_1 + \frac{R_1}{2R_2}I_2}$$

Substitute numerical values and evaluate α_1:

$$\alpha_1 = \frac{2\ \text{N}\cdot\text{m}}{1\,\text{kg}\cdot\text{m}^2 + \frac{0.5\,\text{m}}{2(1\,\text{m})}\left(16\,\text{kg}\cdot\text{m}^2\right)} = \boxed{0.400\,\text{rad/s}^2}$$

Use equation (3) to evaluate α_2:

$$\alpha_2 = \tfrac{1}{2}\left(0.400\,\text{rad/s}^2\right) = \boxed{0.200\,\text{rad/s}^2}$$

(*b*) To counterbalance the 2-N·m torque, a counter torque of 2 N·m must be applied to the first gear. Use equation (2) with $\alpha_1 = 0$ to find F:

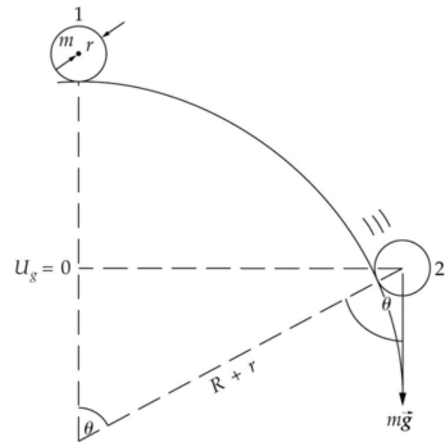
$$2\ \text{N}\cdot\text{m} - FR_1 = 0$$

and

$$F = \frac{2\ \text{N}\cdot\text{m}}{R_1} = \frac{2\ \text{N}\cdot\text{m}}{0.5\,\text{m}} = \boxed{4.00\,\text{N}}$$

***100** ••

Picture the Problem Let r be the radius of the marble, m its mass, R the radius of the large sphere, and v the speed of the marble when it breaks contact with the sphere. The numeral 1 denotes the initial configuration of the sphere-marble system and the numeral 2 is configuration as the marble separates from the sphere. We can use conservation of energy to relate the initial potential energy of the marble to the sum of its translational and rotational kinetic energies as it leaves the sphere. Our choice of the zero of potential energy is shown on the diagram.



(a) Apply conservation of energy:

$$\Delta U + \Delta K = 0$$

or

$$U_2 - U_1 + K_2 - K_1 = 0$$

Because $U_2 = K_1 = 0$:

$$-mg[R + r - (R + r)\cos\theta] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = 0$$

or

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = 0$$

Use the rolling-without-slipping condition to eliminate ω:

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\frac{v^2}{r^2} = 0$$

From Table 9-1 we have:

$$I = \tfrac{2}{5}mr^2$$

Substitute to obtain:

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2}{r^2} = 0$$

or

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{5}mv^2 = 0$$

Solve for v^2 to obtain:

$$v^2 = \frac{10}{7}g(R + r)(1 - \cos\theta)$$

Apply $\sum F_r = ma_r$ to the marble as it separates from the sphere:

$$mg\cos\theta = m\frac{v^2}{R + r}$$

or

$$\cos\theta = \frac{v^2}{g(R+r)}$$

Substitute for v^2:

$$\cos\theta = \frac{1}{g(R+r)}\left[\frac{10}{7}g(R+r)(1-\cos\theta)\right] = \left[\frac{10}{7}(1-\cos\theta)\right]$$

Solve for and evaluate θ:

$$\theta = \cos^{-1}\left(\frac{10}{17}\right) = \boxed{54.0°}$$

(*b*) The force of friction is always less than μ_s multiplied by the normal force on the marble. However, the normal force decreases to 0 at the point where the ball leaves the sphere, meaning that the force of friction must be less than the force needed to keep the ball rolling without slipping before it leaves the sphere.

Rolling With Slipping

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Picture the Problem Part (*a*) of this problem is identical to Example 9-16. In part (*b*) we can use the definitions of translational and rotational kinetic energy to find the ratio of the final and initial kinetic energies.

(*a*) From Example 9-16:

$$s_1 = \boxed{\frac{12}{49}\frac{v_0^2}{\mu_k g}},$$

$$t_1 = \boxed{\frac{2}{7}\frac{v_0}{\mu_k g}}, \text{ and}$$

$$v_1 = \tfrac{5}{2}\mu_k g t_1 = \boxed{\frac{5}{7}v_0}$$

(*b*) When the ball rolls without slipping, $v_1 = r\omega$. Express the final kinetic energy of the ball:

$$K_f = K_{trans} + K_{rot}$$
$$= \tfrac{1}{2}Mv_1^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}Mv_1^2 + \tfrac{1}{2}\left(\tfrac{2}{5}Mr^2\right)\frac{v_1^2}{r^2}$$
$$= \tfrac{7}{10}Mv_1^2 = \tfrac{5}{14}Mv_0^2$$

Express the ratio of the final and initial kinetic energies:

$$\frac{K_f}{K_i}=\frac{\frac{5}{14}Mv_0^2}{\frac{1}{2}Mv_0^2}=\boxed{\frac{5}{7}}$$

(*c*) Substitute in the expressions in (*a*) to obtain:

$$s_1=\frac{12}{49}\frac{(8\,\text{m/s})^2}{(0.06)(9.81\,\text{m/s}^2)}=\boxed{26.6\,\text{m}}$$

$$t_1=\frac{2}{7}\frac{8\,\text{m/s}}{(0.06)(9.81\,\text{m/s}^2)}=\boxed{3.88\,\text{s}}$$

$$v_1=\frac{5}{7}(8\,\text{m/s})=\boxed{5.71\,\text{m/s}}$$

***102 ••**

Picture the Problem The cue stick's blow delivers a rotational impulse as well as a translational impulse to the cue ball. The rotational impulse changes the angular momentum of the ball and the translational impulse changes its linear momentum.

Express the rotational impulse P_{rot} as the product of the average torque and the time during which the rotational impulse acts:

$$P_{rot}=\tau_{av}\Delta t$$

Express the average torque it produces about an axis through the center of the ball:

$$\tau_{av}=P_0(h-r)\sin\theta=P_0(h-r)$$

where θ (= 90°) is the angle between F and the lever arm $h-r$.

Substitute in the expression for P_{rot} to obtain:

$$P_{rot}=P_0(h-r)\Delta t=(P_0\Delta t)(h-r)$$
$$=P_{trams}(h-r)=\Delta L=I\omega_0$$

The translational impulse is also given by:

$$P_{trans}=P_0\Delta t=\Delta p=mv_0$$

Substitute to obtain:

$$mv_0(h-r)=\tfrac{2}{5}mr^2\omega_0$$

Solve for ω_0:

$$\omega_0=\boxed{\frac{5v_0(h-r)}{2r^2}}$$

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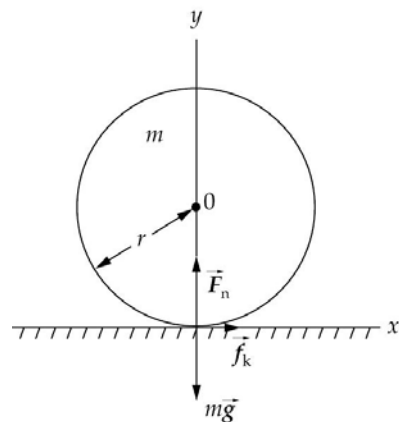
Picture the Problem The angular velocity of the rotating sphere will decrease until the condition for rolling without slipping is satisfied and then it will begin to roll. The force diagram shows the forces acting on the sphere. We can apply Newton's 2nd law to the sphere and use the condition for rolling without slipping to find the speed of the center of mass when the sphere begins to roll without slipping.



Relate the velocity of the sphere when it begins to roll to its acceleration and the elapsed time:

$$v = a\Delta t \quad (1)$$

Apply Newton's 2nd law to the sphere:

$$\sum F_x = f_k = ma\ , \quad (2)$$

$$\sum F_y = F_n - mg = 0\,, \quad (3)$$

and

$$\sum \tau_0 = f_k r = I_0\alpha \quad (4)$$

Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_k g$$

Substitute in equation (1) to obtain:

$$v = a\Delta t = \mu_k g\Delta t \quad (5)$$

Solve for α in equation (4):

$$\alpha = \frac{f_k r}{I_0} = \frac{mar}{\frac{2}{5}mr^2} = \frac{5}{2}\frac{\mu_k g}{r}$$

Express the angular speed of the sphere when it has been moving for a time Δt:

$$\omega = \omega_0 - \alpha\,\Delta t = \omega_0 - \frac{5\mu_k g}{2r}\Delta t \quad (6)$$

Express the condition that the sphere rolls without slipping:

$$v = r\omega$$

Substitute from equations (5) and (6) and solve for the elapsed time until the sphere begins to roll:

$$\Delta t = \frac{2}{7}\frac{r\omega_0}{\mu_k g}$$

Use equation (4) to find v when the sphere begins to roll:

$$v = \mu_k g\Delta t = \frac{2}{7}\frac{r\omega_0\mu_k g}{\mu_k g} = \boxed{\frac{2r\omega_0}{7}}$$

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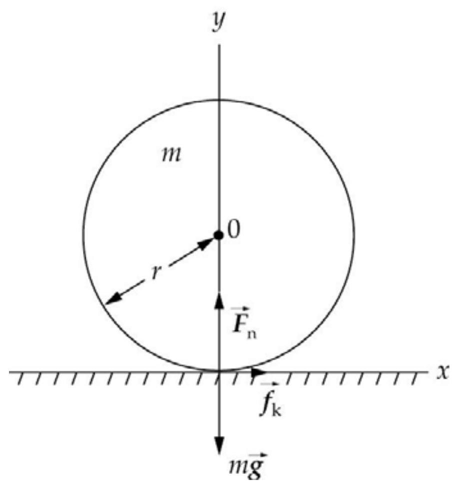
Picture the Problem The sharp force delivers a rotational impulse as well as a translational impulse to the ball. The rotational impulse changes the angular momentum of the ball and the translational impulse changes its linear momentum. In parts (*c*) and (*d*) we can apply Newton's 2nd law to the ball to obtain equations describing both the translational and rotational motion of the ball. We can then solve these equations to find the constant accelerations that allow us to apply constant-acceleration equations to find the velocity of the ball when it begins to roll and its sliding time.



(*a*) Relate the translational impulse delivered to the ball to its change in its momentum:

$$P_{trans} = F_{av}\Delta t = \Delta p = mv_0$$

Solve for v_0:

$$v_0 = \frac{F_{av}\Delta t}{m}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{(20\,kN)(2\times10^{-4}\,s)}{0.02\,kg} = \boxed{200\,m/s}$$

(*b*) Express the rotational impulse P_{rot} as the product of the average torque and the time during which the rotational impulse acts:

$$P_{rot} = \tau_{av}\Delta t$$

Letting h be the height at which the impulsive force is delivered, express the average torque it produces about an axis through the center of the ball:

$$\tau_{av} = F\ell\sin\theta$$

where θ is the angle between F and the lever arm ℓ.

Substitute $h - r$ for ℓ and 90° for θ

$$\tau_{av} = F(h - r)$$

to obtain:

Substitute in the expression for P_{rot} to obtain:

$$P_{\text{rot}} = F(h-r)\Delta t$$

Because $P_{\text{trans}} = F\Delta t$:

$$P_{\text{rot}} = P_{\text{trans}}(h-r) = \Delta L = I\omega_0 = \tfrac{2}{5}mr^2\omega_0$$

Express the translational impulse delivered to the cue ball:

$$P_{\text{trans}} = P_0\Delta t = \Delta p = mv_0$$

Substitute for P_{trans} to obtain:

$$\tfrac{2}{5}mr^2\omega_0 = mv_0$$

Solve for ω_0:

$$\omega_0 = \frac{5v_0(h-r)}{2r^2}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{5(200\,\text{m/s})(0.09\,\text{m}-0.05\,\text{m})}{2(.05\,\text{m})^2} = \boxed{8000\,\text{rad/s}}$$

(*c*) and (*d*) Relate the velocity of the ball when it begins to roll to its acceleration and the elapsed time:

$$v = a\Delta t \qquad (1)$$

Apply Newton's 2nd law to the ball:

$$\sum F_x = f_k = ma\,, \qquad (2)$$

$$\sum F_y = F_n - mg = 0\,, \qquad (3)$$

and

$$\sum \tau_0 = f_k r = I_0\alpha \qquad (4)$$

Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_k g$$

Substitute in equation (1) to obtain:

$$v = a\Delta t = \mu_k g\Delta t \qquad (5)$$

Solve for α in equation (4):

$$\alpha = \frac{f_k r}{I_0} = \frac{mar}{\tfrac{2}{5}mr^2} = \frac{5}{2}\frac{\mu_k g}{r}$$

Express the angular speed of the ball when it has been moving for a time Δt:

$$\omega = \omega_0 - \alpha\,\Delta t = \omega_0 - \frac{5\mu_k g}{2r}\Delta t \quad (6)$$

Express the speed of the ball when it has been moving for a time Δt:

$$v = v_0 + \mu_k g \Delta t \quad (7)$$

Express the condition that the ball rolls without slipping:

$$v = r\omega$$

Substitute from equations (6) and (7) and solve for the elapsed time until the ball begins to roll:

$$\Delta t = \frac{2}{7}\frac{r\omega_0 - v_0}{\mu_k g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2}{7}\left[\frac{(0.05\,\text{m})(8000\,\text{rad/s}) - 200\,\text{m/s}}{(0.5)(9.81\,\text{m/s}^2)}\right] = \boxed{11.6\,\text{s}}$$

Use equation (4) to express v when the ball begins to roll:

$$v = v_0 + \mu_k g \Delta t$$

Substitute numerical values and evaluate v:

$$v = 200\,\text{m/s} + (0.5)(9.81\,\text{m/s}^2)(11.6\,\text{s}) = \boxed{257\,\text{m/s}}$$

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Picture the Problem Because the impulse is applied through the center of mass, $\omega_0 = 0$. We can use the results of Example 9-16 to find the rolling time without slipping, the distance traveled to rolling without slipping, and the velocity of the ball once it begins to roll without slipping.

(*a*) From Example 9-16 we have:

$$t_1 = \frac{2}{7}\frac{v_0}{\mu_k g}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \frac{2}{7}\frac{4\,\text{m/s}}{(0.6)(9.81\,\text{m/s}^2)} = \boxed{0.194\,\text{s}}$$

(*b*) From Example 9-16 we have:

$$s_1 = \frac{12}{49}\frac{v_0^2}{\mu_k g}$$

Substitute numerical values and evaluate s_1:

$$s_1 = \frac{12}{49}\frac{(4\,\text{m/s})^2}{(0.6)(9.81\,\text{m/s}^2)} = \boxed{0.666\,\text{m}}$$

(*c*) From Example 9-16 we have:

$$v_1 = \frac{5}{7}v_0$$

Substitute numerical values and evaluate v_1:

$$v_1 = \frac{5}{7}(4\,\text{m/s}) = \boxed{2.86\,\text{m/s}}$$

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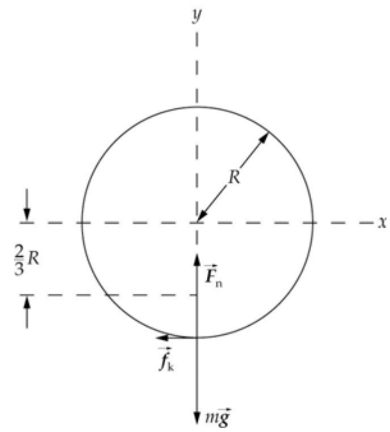
Picture the Problem Because the impulsive force is applied below the center line, the spin is backward, i.e., the ball will slow down. We'll use the impulse-momentum theorem and Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the elapsed time to rolling without slipping.



(*a*) Express the rotational impulse delivered to the ball:

$$P_{\text{rot}} = mv_0 r = mv_0\frac{2R}{3} = I_{\text{cm}}\omega_0 = \left(\tfrac{2}{5}mR^2\right)\omega_0$$

Solve for ω_0:

$$\omega_0 = \boxed{\frac{5}{3}\frac{v_0}{R}}$$

(*b*) Apply Newton's 2nd law to the ball to obtain:

$$\sum \tau_0 = f_k R = I_{\text{cm}}\alpha\,, \qquad (1)$$

$$\sum F_y = F_n - mg = 0\,, \qquad (2)$$

and

$$\sum F_x = -f_k = ma \qquad (3)$$

Using the definition of f_k and F_n from equation (2), solve for α:

$$\alpha = \frac{\mu_k mgR}{I_{\text{cm}}} = \frac{\mu_k mgR}{\frac{2}{5}mR^2} = \frac{5\mu_k g}{2R}$$

Using a constant-acceleration equation, relate the angular speed of the ball to its acceleration:

$$\omega = \omega_0 + \alpha\Delta t = \omega_0 + \frac{5\mu_k g}{2R}\Delta t$$

Using the definition of f_k and F_n from equation (2), solve equation (3) for a:

$$a = -\mu_k g$$

Using a constant-acceleration equation, relate the speed of the ball to its acceleration:

$$v = v_0 + a\Delta t = v_0 - \mu_k g\Delta t \qquad (4)$$

Impose the condition for rolling without slipping to obtain:

$$R\left(\omega_0 + \frac{5\mu_k g}{2R}\Delta t\right) = v_0 - \mu_k g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{16}{21}\frac{v_0}{\mu_k g}$$

Substitute in equation (4) to obtain:

$$v = v_0 - \mu_k g\left(\frac{16}{21}\frac{v_0}{\mu_k}g\right) = \frac{5}{21}v_0$$

$$= \boxed{0.238v_0}$$

(c) Express the initial kinetic energy of the ball:

$$K_i = K_{trans} + K_{rot} = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2$$

$$= \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\left(\frac{5v_0}{3R}\right)^2 = \frac{19}{18}mv_0^2$$

$$= \boxed{1.056mv_0^2}$$

(d) Express the work done by friction in terms of the initial and final kinetic energies of the ball:

$$W_{fr} = K_i - K_f$$

Express the final kinetic energy of the ball:

$$K_f = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{cm}\omega^2$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\frac{v^2}{R^2} = \tfrac{7}{10}mv^2$$

$$= \tfrac{7}{10}m(0.238v_0)^2 = 0.0397mv_0^2$$

Substitute to find W_{fr}:

$$W_{fr} = 1.056mv_0^2 - 0.0397mv_0^2$$

$$= \boxed{1.016mv_0^2}$$

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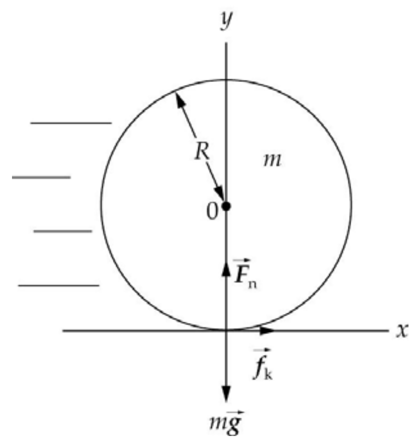
Picture the Problem The figure shows the forces acting on the bowling during the sliding phase of its motion. Because the ball has a forward spin, the friction force is in the direction of motion and will cause the ball's translational speed to increase. We'll apply Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the elapsed time to rolling without slipping.



(*a*) and (*b*) Relate the velocity of the ball when it begins to roll to its acceleration and the elapsed time:

$$v = v_0 + a\Delta t \qquad (1)$$

Apply Newton's 2nd law to the ball:

$$\sum F_x = f_k = ma\,, \qquad (2)$$

$$\sum F_y = F_n - mg = 0\,, \qquad (3)$$

and

$$\sum \tau_0 = f_k R = I_0\alpha \qquad (4)$$

Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_k g$$

Substitute in equation (1) to obtain:

$$v = v_0 + a\Delta t = v_0 + \mu_k g\Delta t \qquad (5)$$

Solve for α in equation (4):

$$\alpha = \frac{f_k R}{I_0} = \frac{maR}{\frac{2}{5}mR^2} = \frac{5}{2}\frac{\mu_k g}{R}$$

Relate the angular speed of the ball to its acceleration:

$$\omega = \omega_0 - \frac{5}{2}\frac{\mu_k g}{R}\Delta t$$

Apply the condition for rolling without slipping:

$$v = R\omega = R\left(\omega_0 - \frac{5}{2}\frac{\mu_k g}{R}\Delta t\right)$$

$$= R\left(\frac{3v_0}{R} - \frac{5}{2}\frac{\mu_k g}{R}\Delta t\right)$$

$$\therefore v = 3v_0 - \frac{5}{2}\mu_k g\Delta t \qquad (6)$$

Equate equations (5) and (6) and solve Δt:

$$\Delta t = \boxed{\frac{4}{7}\frac{v_0}{\mu_k g}}$$

Substitute for Δt in equation (6) to obtain:

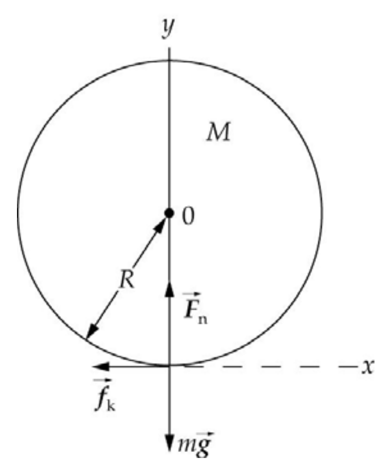
$$v = \frac{11}{7}v_0 = \boxed{1.57v_0}$$

(c) Relate Δx to the average speed of the ball and the time it moves before beginning to roll without slipping:

$$\Delta x = v_{av}\Delta t = \tfrac{1}{2}(v_0 + v)\Delta t$$
$$= \tfrac{1}{2}\left(v_0 + \frac{11}{7}v_0\right)\left(\frac{4v_0}{7\mu_k g}\right)$$
$$= \frac{36}{49}\frac{v_0^2}{\mu_k g} = \boxed{0.735\frac{v_0^2}{\mu_k g}}$$

***108 ••**

Picture the Problem The figure shows the forces acting on the cylinder during the sliding phase of its motion. The friction force will cause the cylinder's translational speed to decrease and eventually satisfy the condition for rolling without slipping. We'll use Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the distance traveled and the elapsed time until the satisfaction of the condition for rolling without slipping.



(a) Apply Newton's 2nd law to the cylinder:

$$\sum F_x = -f_k = Ma, \qquad (1)$$
$$\sum F_y = F_n - Mg = 0, \qquad (2)$$

and

$$\sum \tau_0 = f_k R = I_0\alpha \qquad (3)$$

Use $f_k = \mu_k F_n$ to eliminate F_n between equations (1) and (2) and solve for a:

$$a = -\mu_k g$$

Using a constant-acceleration equation, relate the speed of the cylinder to its acceleration and the elapsed time:

$$v = v_0 + a\Delta t = v_0 - \mu_k g\Delta t$$

Similarly, eliminate f_k between equations (2) and (3) and solve for α:

$$\alpha = \frac{2\mu_k g}{R}$$

Using a constant-acceleration equation, relate the angular speed of the cylinder to its acceleration and the elapsed time:

$$\omega = \omega_0 + \alpha\Delta t = \frac{2\mu_k g}{R}\Delta t$$

Apply the condition for rolling without slipping:

$$v = v_0 - \mu_k g\Delta t = R\omega = R\left(\frac{2\mu_k g}{R}\Delta t\right)$$
$$= 2\mu_k g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_0}{3\mu_k g}$$

Substitute for Δt in the expression for v:

$$v = v_0 - \mu_k g\frac{v_0}{3\mu_k g} = \boxed{\frac{2}{3}v_0}$$

(*b*) Relate the distance the cylinder travels to its average speed and the elapsed time:

$$\Delta x = v_{av}\Delta t = \tfrac{1}{2}\left(v_0 + \tfrac{2}{3}v_0\right)\left(\frac{v_0}{3\mu_k g}\right)$$
$$= \boxed{\frac{5}{18}\frac{v_0^2}{\mu_k g}}$$

(*c*) Express the ratio of the energy dissipated in friction to the cylinder's initial mechanical energy:

$$\frac{W_{fr}}{K_i} = \frac{K_i - K_f}{K_i}$$

Express the kinetic energy of the cylinder as it begins to roll without slipping:

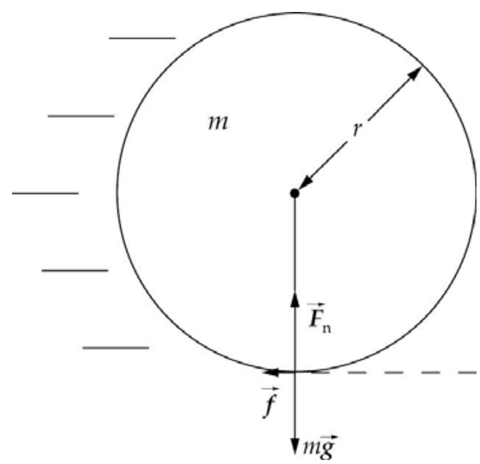
$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{cm}\omega^2$$
$$= \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2}$$
$$= \frac{3}{4}Mv^2 = \frac{3}{4}M\left(\frac{2}{3}v_0\right)^2 = \frac{1}{3}Mv_0^2$$

Substitute for K_i and K_f and simplify to obtain:

$$\frac{W_{fr}}{K_i} = \frac{\frac{1}{2}Mv_0^2 - \frac{1}{3}Mv_0^2}{\frac{1}{2}Mv_0^2} = \boxed{\frac{1}{3}}$$

109 ••

Picture the Problem The forces acting on the ball as it slides across the floor are its weight $m\vec{g}$, the normal force $\vec{F}_n$ exerted by the floor, and the friction force $\vec{f}$. Because the weight and normal force act through the center of mass of the ball and are equal in magnitude, the friction force is the net (decelerating) force. We can apply Newton's 2nd law in both translational and rotational form to obtain a set of equations that we can solve for the acceleration of the ball. Once we have determined the ball's acceleration, we can use constant-acceleration equations to obtain its velocity when it begins to roll without slipping.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the ball:

$$\sum F_x = -f = ma \quad (1)$$

and

$$\sum F_y = F_n - mg = 0 \quad (2)$$

From the definition of the coefficient of kinetic friction we have:

$$f = \mu_k F_n \quad (3)$$

Solve equation (2) for F_n:

$$F_n = mg$$

Substitute in equation (3) to obtain:

$$f = \mu_k mg$$

Substitute in equation (1) to obtain:

$$-\mu_k mg = ma$$

or

$$a = -\mu_k g$$

Apply $\sum \tau = I\alpha$ to the ball:

$$fr = I\alpha$$

Solve for α to obtain:

$$\alpha = \frac{fr}{I} = \frac{\mu_k mgr}{I}$$

Assuming that the coefficient of kinetic friction is constant*, we can use constant-acceleration equations to describe how long it will take the ball to begin

$$v_f - v = a\Delta t = -\mu_k g\Delta t \quad (4)$$

and

$$\omega_f = \frac{\mu_k gmr}{I}\Delta t \quad (5)$$

rolling without slipping:

Once rolling without slipping has been established, we also have:

$$\omega_{\text{f}} = \frac{v_{\text{f}}}{r} \qquad (6)$$

Equate equations (5) and (6):

$$\frac{v_{\text{f}}}{r} = \frac{\mu_k gmr}{I}\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_{\text{f}} I}{\mu_k gmr^2}$$

Substitute in equation (4) to obtain:

$$v_{\text{f}} - v = -\mu_k g\left(\frac{v_{\text{f}} I}{\mu_k gmr^2}\right)$$

$$= -\frac{I}{mr^2}v_{\text{f}}$$

Solve for v_{f}:

$$v_f = \boxed{\frac{1}{1+\frac{I}{mr^2}}v}$$

(*b*) Express the total kinetic energy of the ball:

$$K = \frac{1}{2}mv_{\text{f}}^2 + \frac{1}{2}I\omega_{\text{f}}^2$$

Because the ball is now rolling without slipping, $v = r\omega_{\text{f}}$ and:

$$K = \frac{1}{2}m\left(\frac{1}{1+I/mr^2}\right)^2 v^2 + \frac{1}{2}I\left(\frac{1}{1+I/mr^2}\right)^2 \frac{v^2}{r^2} = \frac{1}{2}mv^2\left(\left(1+I/mr^2\right)\left(\frac{1}{1+I/mr^2}\right)^2\right)$$

$$= \boxed{\frac{1}{2}mv^2\left(\frac{1}{1+I/mr^2}\right)}$$

*** Remarks: This assumption is not necessary. One can use the impulse-momentum theorem and the related theorem for torque and change in angular momentum to prove that the result holds for an *arbitrary* frictional force acting on the ball, so long as the ball moves along a straight line and the force is directed opposite to the direction of motion of the ball.**

General Problems

***110** •

Picture the Problem The angular velocity of an object is the ratio of the number of revolutions it makes in a given period of time to the elapsed time.

The moon's angular velocity is:

$$\omega = \frac{1\,\text{rev}}{27.3\,\text{days}}$$
$$= \frac{1\,\text{rev}}{27.3\,\text{days}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{day}}{24\,\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}$$
$$= \boxed{2.66\times10^{-6}\ \text{rad/s}}$$

111 •

Picture the Problem The moment of inertia of the hoop, about an axis perpendicular to the plane of the hoop and through its edge, is related to its moment of inertia with respect to an axis through its center of mass by the parallel axis theorem.

Apply the parallel axis theorem:

$$I = I_{\text{cm}} + Mh^2 = MR^2 + MR^2 = \boxed{2mR^2}$$

112 ••

Picture the Problem The force you exert on the rope results in a net torque that accelerates the merry-go-round. The moment of inertia of the merry-go-round, its angular acceleration, and the torque you apply are related through Newton's 2nd law.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the merry-go-round to its angular acceleration and acceleration time:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

Solve for and evaluate α:

$$\alpha = \frac{2\Delta\theta}{(\Delta t)^2} = \frac{2(2\pi\,\text{rad})}{(12\,\text{s})^2} = \boxed{0.0873\,\text{rad/s}^2}$$

(*b*) Use the definition of torque to obtain:

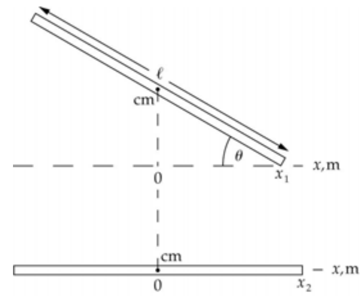
$$\tau = Fr = (260\,\text{N})(2.2\,\text{m}) = \boxed{572\,\text{N}\cdot\text{m}}$$

(*c*) Use Newton's 2nd law to find the moment of inertia of the merry-go-round:

$$I = \frac{\tau_{\text{net}}}{\alpha} = \frac{572\,\text{N}\cdot\text{m}}{0.0873\,\text{rad/s}^2}$$
$$= \boxed{6.55\times10^3\ \text{kg}\cdot\text{m}^2}$$

113 •

Picture the Problem Because there are no horizontal forces acting on the stick, the center of mass of the stick will not move in the horizontal direction. Choose a coordinate system in which the origin is at the horizontal position of the center of mass. The diagram shows the stick in its initial raised position and when it has fallen to the ice.



Express the displacement of the right end of the stick Δx as the difference between the position coordinates x_2 and x_2:

$$\Delta x = x_2 - x_1$$

Using trigonometry, find the initial coordinate of the right end of the stick:

$$x_1 = \ell\cos\theta = (1\,\text{m})\cos 30° = 0.866\,\text{m}$$

Because the center of mass has not moved horizontally:

$$x_2 = \ell = 1\,\text{m}$$

Substitute to find the displacement of the right end of the stick:

$$\Delta x = 1\,\text{m} - 0.866\,\text{m} = \boxed{0.134\,\text{m}}$$

114 ••

Picture the Problem The force applied to the string results in a torque about the center of mass of the disk that accelerates it. We can relate these quantities to the moment of inertia of the disk through Newton's 2nd law and then use constant-acceleration equations to find the disk's angular velocity the angle through which it has rotated in a given period of time. The disk's rotational kinetic energy can be found from its definition.

(*a*) Use the definition of torque to obtain:

$$\tau \equiv FR = (20\,\text{N})(0.12\,\text{m}) = \boxed{2.40\,\text{N}\cdot\text{m}}$$

(*b*) Use Newton's 2nd law to express the angular acceleration of the disk in terms of the net torque acting on it and its moment of inertia:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{\tau_{\text{net}}}{\frac{1}{2}MR^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(2.40\,\text{N}\cdot\text{m})}{(5\,\text{kg})(0.12\,\text{m})^2} = \boxed{66.7\,\text{rad/s}^2}$$

(*c*) Using a constant-acceleration equation, relate the angular velocity of the disk to its angular

$$\omega = \omega_0 + \alpha\Delta t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

acceleration and the elapsed time:

Substitute numerical values and evaluate ω:

$$\omega = \left(66.7\,\text{rad/s}^2\right)\left(5\,\text{s}\right) = \boxed{333\,\text{rad/s}}$$

(d) Use the definition of rotational kinetic energy to obtain:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2$$

Substitute numerical values and evaluate K_{rot}:

$$K_{\text{rot}} = \tfrac{1}{4}\left(5\,\text{kg}\right)\left(0.12\,\text{m}\right)^2\left(333\,\text{rad/s}\right)^2 = \boxed{2.00\,\text{kJ}}$$

(e) Using a constant-acceleration equation, relate the angle through which the disk turns to its angular acceleration and the elapsed time:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

Substitute numerical values and evaluate $\Delta\theta$:

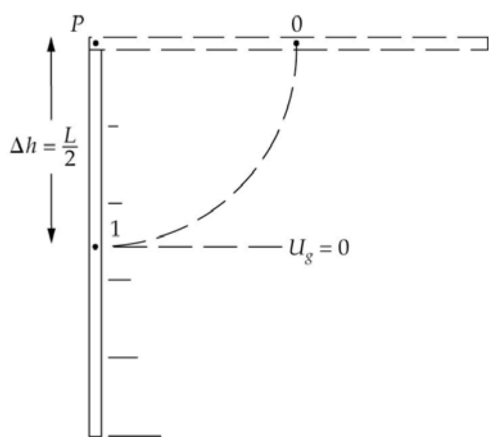
$$\Delta\theta = \tfrac{1}{2}\left(66.7\,\text{rad/s}^2\right)\left(5\,\text{s}\right)^2 = \boxed{834\,\text{rad}}$$

(f) Express K in terms of τ and θ:

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\frac{\tau}{\alpha}\right)\left(\alpha\Delta t\right)^2 = \tfrac{1}{2}\alpha\tau\left(\Delta t\right)^2 = \boxed{\tau\,\Delta\theta}$$

115 ••

Picture the Problem The diagram shows the rod in its initial horizontal position and then, later, as it swings through its vertical position. The center of mass is denoted by the numerals 0 and 1. Let the length of the rod be represented by L and its mass by m. We can use Newton's 2nd law in rotational form to find, first, the angular acceleration of the rod and then, from α, the acceleration of any point on the rod. We can use conservation of energy to find the angular velocity of the center of mass of the rod when it is vertical and then use this value to find its linear velocity.



(a) Relate the acceleration of the center of the rod to the angular

$$a = \ell\alpha = \frac{L}{2}\alpha$$

acceleration of the rod:

Use Newton's 2nd law to relate the torque about the suspension point of the rod (exerted by the weight of the rod) to the rod's angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{Mg\frac{L}{2}}{\frac{1}{3}ML^2} = \frac{3g}{2L}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{3(9.81\,\text{m/s}^2)}{2(0.8\,\text{m})} = 18.4\,\text{rad/s}^2$$

Substitute numerical values and evaluate a:

$$a = \tfrac{1}{2}(0.8\,\text{m})(18.4\,\text{rad/s}^2) = \boxed{7.36\,\text{m/s}^2}$$

(*b*) Relate the acceleration of the end of the rod to α:

$$a_{\text{end}} = L\alpha = (0.8\,\text{m})(18.4\,\text{rad/s}^2) = \boxed{14.7\,\text{m/s}^2}$$

(*c*) Relate the linear velocity of the center of mass of the rod to its angular velocity as it passes through the vertical:

$$v = \omega\Delta h = \tfrac{1}{2}\omega L$$

Use conservation of energy to relate the changes in the kinetic and potential energies of the rod as it swings from its initial horizontal orientation through its vertical orientation:

$$\Delta K + \Delta U = K_1 - K_0 + U_1 - U_0 = 0$$

or, because $K_0 = U_1 = 0$,

$$K_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2} I_P\omega^2 = mg\Delta h$$

Substitute for Δh and solve for ω:

$$\omega = \sqrt{\frac{3g}{L}}$$

Substitute to obtain:

$$v = \tfrac{1}{2}L\sqrt{\frac{3g}{L}} = \tfrac{1}{2}\sqrt{3gL}$$

Substitute numerical values and evaluate v:

$$v = \tfrac{1}{2}\sqrt{3(9.81\,\text{m/s}^2)(0.8\,\text{m})} = \boxed{2.43\,\text{m/s}}$$

116 ••

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the track. The initial potential energy of the marble is transformed into translational and rotational kinetic energy as it rolls down the track to its lowest point and then, because the portion of the track to the right is frictionless, into translational kinetic energy and, eventually, into gravitational potential energy.

Using conservation of energy, relate h_2 to the kinetic energy of the marble at the bottom of the track:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f to obtain:

$$-\tfrac{1}{2}Mv^2 - Mgh_2 = 0$$

Solve for h_2:

$$h_2 = \frac{v^2}{2g} \qquad (1)$$

Using conservation of energy, relate h_1 to the kinetic energy of the marble at the bottom of the track:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

Substitute for K_f and U_i to obtain:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 - Mgh_1 = 0$$

Substitute for I and solve for v^2 to obtain:

$$v^2 = \tfrac{10}{7}gh_1$$

Substitute in equation (1) to obtain:

$$h_2 = \frac{\tfrac{10}{7}gh_1}{2g} = \boxed{\tfrac{5}{7}h_1}$$

***117** ••

Picture the Problem To stop the wheel, the tangential force will have to do an amount of work equal to the initial rotational kinetic energy of the wheel. We can find the stopping torque and the force from the average power delivered by the force during the slowing of the wheel. The number of revolutions made by the wheel as it stops can be found from a constant-acceleration equation.

(*a*) Relate the work that must be done to stop the wheel to its kinetic energy:

$$W = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\omega^2 = \tfrac{1}{4}mr^2\omega^2$$

Substitute numerical values and evaluate W:

$$W = \tfrac{1}{4}(120\,\text{kg})(1.4\,\text{m})^2 \times \left[1100\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}\right]^2 = \boxed{780\,\text{kJ}}$$

(*b*) Express the stopping torque is terms of the average power required:

$$P_{\text{av}} = \tau\omega_{\text{av}}$$

Solve for τ:

$$\tau = \frac{P_{\text{av}}}{\omega_{\text{av}}}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{\dfrac{780\,\text{kJ}}{(2.5\,\text{min})(60\,\text{s/min})}}{\dfrac{(1100\,\text{rev/min})(2\pi\,\text{rad/rev})(1\,\text{min}/60\,\text{s})}{2}} = \boxed{90.3\,\text{N}\cdot\text{m}}$$

Relate the stopping torque to the magnitude of the required force and solve for F:

$$F = \frac{\tau}{R} = \frac{90.3\,\text{N}\cdot\text{m}}{0.6\,\text{m}} = \boxed{151\text{ N}}$$

(*c*) Using a constant-acceleration equation, relate the angular displacement of the wheel to its average angular velocity and the stopping time:

$$\Delta\theta = \omega_{\text{av}}\Delta t$$

Substitute numerical values and evaluate $\Delta\theta$:

$$\Delta\theta = \left(\frac{1100\,\text{rev/min}}{2}\right)(2.5\,\text{min}) = \boxed{1380\,\text{rev}}$$

118 ••

Picture the Problem The work done by the four children on the merry-go-round will change its kinetic energy. We can use the work-energy theorem to relate the work done by the children to the distance they ran and Newton's 2^{nd} law to find the angular acceleration of the merry-go-round.

(*a*) Use the work-kinetic energy theorem to relate the work done by the children to the kinetic energy of the merry-go-round:

$$W_{\text{net force}} = \Delta K = K_f$$

or

$$4F\Delta s = \tfrac{1}{2}I\omega^2$$

Substitute for I and solve for Δs to obtain:

$$\Delta s = \frac{I\omega^2}{8F} = \frac{\frac{1}{2}mr^2\omega^2}{8F} = \frac{mr^2\omega^2}{16F}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(240\,\text{kg})(2\,\text{m})^2\left[\frac{1\,\text{rev}}{2.8\,\text{s}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\right]^2}{16(26\,\text{N})} = \boxed{11.6\,\text{m}}$$

(*b*) Apply Newton's 2nd law to express the angular acceleration of the merry-go-round:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{4Fr}{\frac{1}{2}mr^2} = \frac{8F}{mr}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{8(26\,\text{N})}{(240\,\text{kg})(2\,\text{m})} = \boxed{0.433\,\text{rad/s}^2}$$

(*c*) Use the definition of work to relate the force exerted by each child to the distance over which that force is exerted:

$$W = F\Delta s = (26\,\text{N})(11.6\,\text{m}) = \boxed{302\,\text{J}}$$

(*d*) Relate the kinetic energy of the merry-go-round to the work that was done on it:

$$W_{\text{net force}} = \Delta K = K_f - 0 = 4F\Delta s$$

Substitute numerical values and evaluate $W_{\text{net force}}$:

$$W_{\text{net force}} = 4(26\,\text{N})(11.6\,\text{m}) = \boxed{1.21\,\text{kJ}}$$

119 ••

Picture the Problem Because the center of mass of the hoop is at its center, we can use Newton's second law to relate the acceleration of the hoop to the net force acting on it. The distance moved by the center of the hoop can be determined using a constant-acceleration equation, as can the angular velocity of the hoop.

(*a*) Using a constant-acceleration equation, relate the distance the

$$\Delta s = \tfrac{1}{2}a_{\text{cm}}(\Delta t)^2$$

center of the travels in 3 s to the acceleration of its center of mass:

Relate the acceleration of the center of mass of the hoop to the net force acting on it:

$$a_{\text{cm}} = \frac{F_{\text{net}}}{m}$$

Substitute to obtain:

$$\Delta s = \frac{F(\Delta t)^2}{2m}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(5\,\text{N})(3\,\text{s})^2}{2(1.5\,\text{kg})} = \boxed{15.0\,\text{m}}$$

(*b*) Relate the angular velocity of the hoop to its angular acceleration and the elapsed time:

$$\omega = \alpha\,\Delta t$$

Use Newton's 2nd law to relate the angular acceleration of the hoop to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{FR}{mR^2} = \frac{F}{mR}$$

Substitute to obtain:

$$\omega = \frac{F\Delta t}{mR}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{(5\,\text{N})(3\,\text{s})}{(1.5\,\text{kg})(0.65\,\text{m})} = \boxed{15.4\,\text{rad/s}}$$

120 ••

Picture the Problem Let R represent the radius of the grinding wheel, M its mass, r the radius of the handle, and m the mass of the load attached to the handle. In the absence of information to the contrary, we'll treat the 25-kg load as though it were concentrated at a point. Let the zero of gravitational potential energy be where the 25-kg load is at its lowest point. We'll apply Newton's 2nd law and the conservation of mechanical energy to determine the initial angular acceleration and the maximum angular velocity of the wheel.

(*a*) Use Newton's 2nd law to relate the acceleration of the wheel to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{mgr}{\frac{1}{2}MR^2 + mr^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)(0.65\,\text{m})}{\frac{1}{2}(60\,\text{kg})(0.45\,\text{m})^2 + (25\,\text{kg})(0.65\,\text{m})^2} = \boxed{9.58\,\text{rad/s}^2}$$

(*b*) Use the conservation of mechanical energy to relate the initial potential energy of the load to its kinetic energy and the rotational kinetic energy of the wheel when the load is directly below the center of mass of the wheel:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_{\text{f,trans}} + K_{\text{f,rot}} - U_i = 0\,.$$

Substitute and solve for ω:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 - mgr = 0\,,$$

$$\tfrac{1}{2}mr^2\omega^2 + \tfrac{1}{4}MR^2\omega^2 - mgr = 0\,,$$

and

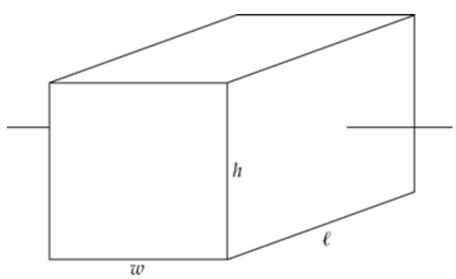
$$\omega = \sqrt{\frac{4mgr}{2mr^2 + MR^2}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{4(25\,\text{kg})(9.81\,\text{m/s}^2)(0.65\,\text{m})}{2(25\,\text{kg})(0.65\,\text{m})^2 + (60\,\text{kg})(0.45\,\text{m})^2}} = \boxed{4.38\,\text{rad/s}}$$

***121 ••**

Picture the Problem Let the smaller block have the dimensions shown in the diagram. Then the length, height, and width of the larger block are $S\ell$, Sh, and Sw, respectively. Let the numeral 1 denote the smaller block and the numeral 2 the larger block and express the ratios of the surface areas, masses, and moments of inertia of the two blocks.



(*a*) Express the ratio of the surface areas of the two blocks:

$$\frac{A_2}{A_1} = \frac{2(Sw)(S\ell) + 2(S\ell)(Sh) + 2(Sw)(Sh)}{2w\ell + 2\ell h + 2wh} = \frac{S^2(2w\ell + 2\ell h + 2wh)}{2w\ell + 2\ell h + 2wh} = \boxed{S^2}$$

(*b*) Express the ratio of the masses of the two blocks:

$$\frac{M_2}{M_1}=\frac{\rho V_2}{\rho V_1}=\frac{V_2}{V_1}=\frac{(Sw)(S\ell)(Sh)}{w\ell h}$$

$$=\frac{S^3(w\ell h)}{w\ell h}=\boxed{S^3}$$

(*c*) Express the ratio of the moments of inertia, about the axis shown in the diagram, of the two blocks:

$$\frac{I_2}{I_1}=\frac{\frac{1}{12}M_2\left[(S\ell)^2+(Sh)^2\right]}{\frac{1}{12}M_1\left[\ell^2+h^2\right]}$$

$$=\frac{M_2}{M_1}\frac{S^2\left[\ell^2+h^2\right]}{\left[\ell^2+h^2\right]}=\left(\frac{M_2}{M_1}\right)\left(S^2\right)$$

In part (*b*) we showed that:

$$\frac{M_2}{M_1}=S^3$$

Substitute to obtain:

$$\frac{I_2}{I_1}=\left(S^3\right)\left(S^2\right)=\boxed{S^5}$$

122 ••

Picture the Problem We can derive the perpendicular-axis theorem for planar objects by following the step-by-step procedure outlined in the problem.

(*a*) and (*b*)

$$I_z=\int r^2dm=\int\left(x^2+y^2\right)dm$$

$$=\int x^2dm+\int y^2dm$$

$$=\boxed{I_x+I_y}$$

(*c*) Let the *z* axis be the axis of rotation of the disk. By symmetry:

$$I_x=I_y$$

Express I_z in terms of I_x:

$$I_z=2I_x$$

Letting *M* represent the mass of the disk, solve for I_x:

$$I_x=\tfrac{1}{2}I_z=\tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)=\boxed{\tfrac{1}{4}MR^2}$$

123 ••

Picture the Problem Let the zero of gravitational potential energy be at the center of the disk when it is directly below the pivot. The initial gravitational potential energy of the disk is transformed into rotational kinetic energy when its center of mass is directly below the pivot. We can use Newton's 2nd law to relate the force exerted by the pivot to the weight of the disk and the centripetal force acting on it at its lowest point.

(*a*) Use the conservation of mechanical energy to relate the initial potential energy of the disk to its kinetic energy when its center of mass is directly below the pivot:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_{f,rot} - U_i = 0$$

Substitute for $K_{f,rot}$ and U_i :

$$\tfrac{1}{2} I\omega^2 - Mgr = 0 \qquad (1)$$

Use the parallel-axis theorem to relate the moment of inertia of the disk about the pivot to its moment of inertia with respect to an axis through its center of mass:

$$I = I_{cm} + Mh^2$$

or

$$I = \tfrac{1}{2} Mr^2 + Mr^2 = \tfrac{3}{2} Mr^2$$

Solve equation (1) for ω and substitute for I to obtain:

$$\omega = \boxed{\sqrt{\frac{4g}{3r}}}$$

(*b*) Letting F represent the force exerted by the pivot, use Newton's 2nd law to express the net force acting on the swinging disk as it passes through its lowest point:

$$F_{net} = F - Mg = Mr\omega^2$$

Solve for F and simplify to obtain:

$$F = Mg + Mr\omega^2 = Mg + Mr\frac{4g}{3r}$$

$$= Mg + \tfrac{4}{3} Mg = \boxed{\tfrac{7}{3} Mg}$$

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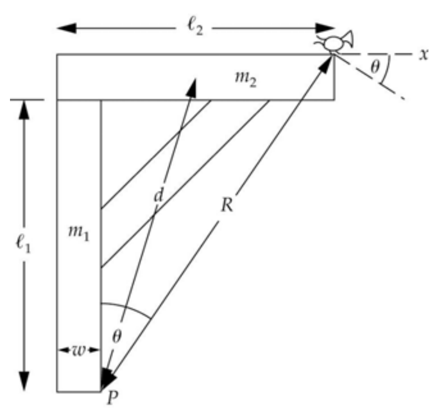
Picture the Problem The diagram shows a vertical cross-piece. Because we'll need to take moments about the point of rotation (point *P*), we'll need to use the parallel-axis theorem to find the moments of inertia of the two parts of this composite structure. Let the numeral 1 denote the vertical member and the numeral 2 the horizontal member. We can apply Newton's 2nd law in rotational form to the structure to express its angular acceleration in terms of the net torque causing it to fall and its moment of inertia with respect to point *P*.



(*a*) Taking clockwise rotation to be positive (this is the direction the structure is going to rotate), apply $\sum \tau = I_P \alpha$:

$$m_2 g\left(\frac{\ell_2}{2}\right) - m_1 g\left(\frac{w}{2}\right) = I_P \alpha$$

Solve for α to obtain:

$$\alpha = \frac{m_2 g \ell_2 - m_1 g w}{2I_P}$$

or

$$\alpha = \frac{g(m_2 \ell_2 - m_1 w)}{2(I_{1P} + I_{2P})} \qquad (1)$$

Convert $\ell_1, \ell_2,$ and w to SI units:

$$\ell_1 = 12\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 3.66\,\text{m},$$

$$\ell_2 = 6\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 1.83\,\text{m}, \text{ and}$$

$$w = 2\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 0.610\,\text{m}$$

Using Table 9-1 and the parallel-axis theorem, express the moment of inertia of the vertical member about an axis through point *P*:

$$I_{1P} = \tfrac{1}{3} m_1 \ell_1^2 + m_1 \left(\frac{w}{2}\right)^2 = m_1\left(\tfrac{1}{3}\ell_1^2 + \tfrac{1}{4} w^2\right)$$

Substitute numerical values and evaluate I_{1P}:

$$I_{1P} = (350\,\text{kg})\left[\tfrac{1}{3}(3.66\,\text{m})^2 + \tfrac{1}{4}(0.610\,\text{m})^2\right] = 1.60 \times 10^3\,\text{kg}\cdot\text{m}^2$$

Using the parallel-axis theorem, express the moment of inertia of the horizontal member about an axis through point *P*:

$$I_{2P} = I_{2,\text{cm}} + m_2 d^2 \qquad (2)$$

where

$$d^2 = \left(\ell_1 + \tfrac{1}{2} w\right)^2 + \left(\tfrac{1}{2}\ell_2 - w\right)^2$$

Solve for *d*:

$$d = \sqrt{\left(\ell_1 + \tfrac{1}{2} w\right)^2 + \left(\tfrac{1}{2}\ell_2 - w\right)^2}$$

Substitute numerical values and evaluate *d*:

$$d = \sqrt{\left[3.66\,\text{m} + \tfrac{1}{2}(0.610\,\text{m})\right]^2 + \left[\tfrac{1}{2}(1.83\,\text{m}) - 0.610\,\text{m}\right]^2} = 3.86\,\text{m}$$

From Table 9-1 we have:

$$I_{2,\text{cm}} = \tfrac{1}{12} m_2 \ell_2^2$$

Substitute in equation (2) to obtain:

$$I_{2P} = \tfrac{1}{12} m_2 \ell_2^2 + m_2 d^2 = m_2\left(\tfrac{1}{12}\ell_2^2 + d^2\right)$$

Evaluate I_{2P}:

$$I_{2P} = (175\,\text{kg})\left[\tfrac{1}{12}(1.83\,\text{m})^2 + (3.86\,\text{m})^2\right] = 2.66\times10^3\ \text{kg}\cdot\text{m}^2$$

Substitute in equation (1) and evaluate α:

$$\alpha = \frac{(9.81\,\text{m/s}^2)[(175\,\text{kg})(1.83\,\text{m}) - (350\,\text{kg})(0.61\,\text{m})]}{2(1.60 + 2.66)\times10^3\ \text{kg}\cdot\text{m}^2} = \boxed{0.123\,\text{rad/s}^2}$$

(*b*) Express the magnitude of the acceleration of the sparrow:

$a = \alpha R$

where R is the distance of the sparrow from the point of rotation and

$$R^2 = (\ell_1 + w)^2 + (\ell_2 - w)^2$$

Solve for R:

$$R = \sqrt{(\ell_1 + w)^2 + (\ell_2 - w)^2}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{(3.66\,\text{m} + 0.610\,\text{m})^2 + (1.83\,\text{m} - 0.610\,\text{m})^2} = 4.44\,\text{m}$$

Substitute numerical values and evaluate a:

$$a = (0.123\,\text{rad/s}^2)(4.44\,\text{m}) = \boxed{0.546\,\text{m/s}^2}$$

(*c*) Refer to the diagram to express a_x in terms of a:

$$a_x = a\cos\theta = a\frac{\ell_1 + w}{R}$$

Substitute numerical values and evaluate a_x:

$$a_x = (0.546\,\text{m/s}^2)\frac{3.66\,\text{m} + 0.61\,\text{m}}{4.44\,\text{m}} = \boxed{0.525\,\text{m/s}^2}$$

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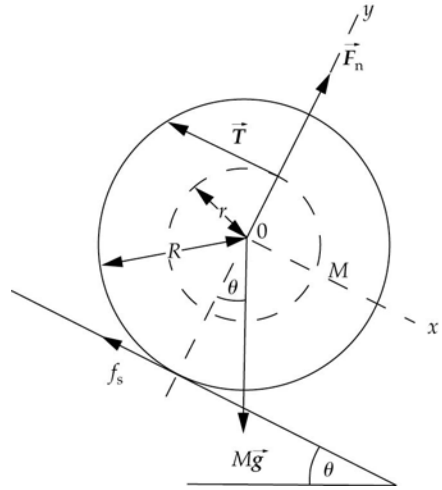
Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. The initial potential energy of the spool is transformed into rotational and translational kinetic energy when the spool reaches the bottom of the incline. We can apply the conservation of mechanical energy to find an expression for its speed at that location. The force diagram shows the forces acting on the spool when there is enough friction to keep it from slipping. We'll use Newton's 2nd law in both translational and rotational form to derive an expression for the static friction force.



(*a*) In the absence of friction, the forces acting on the spool will be its weight, the normal force exerted by the incline, and the tension in the string. A component of its weight will cause the spool to accelerate down the incline and the tension in the string will exert a torque that will cause counterclockwise rotation of the spool.

The spool will move down the plane at constant acceleration, spinning in a counterclockwise direction as string unwinds.

Use the conservation of mechanical energy to relate the speed of the center of mass of the spool at the bottom of the slope to its initial potential energy:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_{f,trans} + K_{f,rot} - U_i = 0.$$

Substitute for $K_{f,trans}$, $K_{f,rot}$ and U_i:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 - MgD\sin\theta = 0 \quad (1)$$

Substitute for ω and solve for v to obtain:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\frac{v^2}{r^2} - MgD\sin\theta = 0$$

and

$$\boxed{v = \sqrt{\frac{2MgD\sin\theta}{M + \frac{I}{r^2}}}}$$

(*b*) Apply Newton's 2nd law to the spool:

$$\sum F_x = Mg\sin\theta - T - f_s = 0$$
$$\sum \tau_0 = Tr - f_s R = 0$$

Eliminate T between these equations to obtain:

$$f_s = \boxed{\frac{Mg\sin\theta}{1+\frac{R}{r}}}$$, up the incline.

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Picture the Problem While the angular acceleration of the rod is the same at each point along its length, the linear acceleration and, hence, the force exerted on each coin by the rod, varies along its length. We can relate this force the linear acceleration of the rod through Newton's 2nd law and the angular acceleration of the rod.

Letting x be the distance from the pivot, use Newton's 2nd law to express the force F acting on a coin:

$$F_{net} = mg - F(x) = ma(x)$$
or
$$F(x) = m(g - a(x)) \quad (1)$$

Use Newton's 2nd law to relate the angular acceleration of the system to the net torque acting on it:

$$\alpha = \frac{\tau_{net}}{I} = \frac{Mg\frac{L}{2}}{\frac{1}{3}ML^2} = \frac{3g}{2L}$$

Relate $a(x)$ and α:

$$a(x) = x\alpha = x\frac{3g}{2(1.5\,\text{m})} = gx$$

Substitute in equation (1) to obtain:

$$F(x) = m(g - gx) = mg(1 - x)$$

Evaluate $F(0.25\text{ m})$:

$$F(0.25\,\text{m}) = mg(1 - 0.25\,\text{m}) = \boxed{0.75mg}$$

Evaluate $F(0.5\text{ m})$:

$$F(0.5\,\text{m}) = mg(1 - 0.5\,\text{m}) = \boxed{0.5mg}$$

Evaluate $F(0.75\text{ m})$:

$$F(0.75\,\text{m}) = mg(1 - 0.75\,\text{m}) = \boxed{0.25mg}$$

Evaluate $F(1\text{ m})$:

$$F(1\,\text{m}) = F(1.25\,\text{m}) = F(1.5\,\text{m}) = \boxed{0}$$

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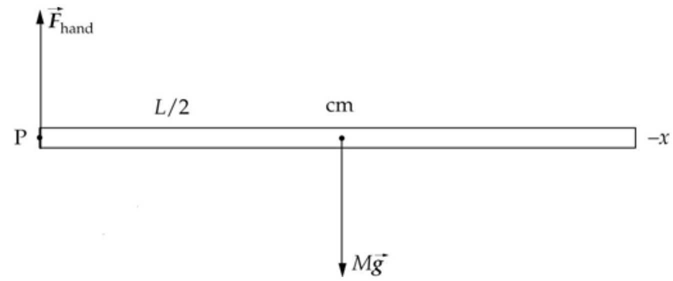
Picture the Problem The diagram shows the force the hand supporting the meterstick exerts at the pivot point and the force the earth exerts on the meterstick acting at the center of mass. We can relate the angular acceleration to the acceleration of the end of the meterstick using $a = L\alpha$ and use Newton's 2nd law in rotational form to relate α to the moment of inertia of the meterstick.



(*a*) Relate the acceleration of the far end of the meterstick to the angular acceleration of the meterstick:

$$a = L\alpha \qquad (1)$$

Apply $\sum \tau_P = I_P\alpha$ to the meterstick:

$$Mg\left(\frac{L}{2}\right) = I_P\alpha$$

Solve for α:

$$\alpha = \frac{MgL}{2I_P}$$

From Table 9-1, for a rod pivoted at one end, we have:

$$I_P = \frac{1}{3}ML^2$$

Substitute to obtain:

$$\alpha = \frac{3MgL}{2ML^2} = \frac{3g}{2L}$$

Substitute in equation (1) to obtain:

$$a = \frac{3g}{2}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{3(9.81\,\text{m/s}^2)}{2} = \boxed{14.7\,\text{m/s}^2}$$

(*b*) Express the acceleration of a point on the meterstick a distance *x* from the pivot point:

$$a = \alpha x = \frac{3g}{2L}x$$

Express the condition that the meterstick leaves the penny behind:

$$a > g$$

Substitute to obtain:

$$\frac{3g}{2L}x > g$$

Solve for and evaluate *x*:

$$x > \frac{2L}{3} = \frac{2(1\,\text{m})}{3} = \boxed{66.7\,\text{cm}}$$

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Picture the Problem Let m represent the 0.2-kg mass, M the 0.8-kg mass of the cylinder, L the 1.8-m length, and $x + \Delta x$ the distance from the center of the objects whose mass is m. We can use Newton's 2nd law to relate the radial forces on the masses to the spring's stiffness constant and use the work-energy theorem to find the work done as the system accelerates to its final angular speed.

(*a*) Express the net inward force acting on each of the 0.2-kg masses:

$$\sum F_{\text{radial}} = k\Delta x = m(x+\Delta x)\omega^2$$

Solve for k:

$$k = \frac{m(x+\Delta x)\omega^2}{\Delta x}$$

Substitute numerical values and evaluate k:

$$k = \frac{(0.2\,\text{kg})(0.8\,\text{m})(24\,\text{rad/s})^2}{0.4\,\text{m}} = \boxed{230\,\text{N/m}}$$

(*b*) Using the work-energy theorem, relate the work done to the change in energy of the system:

$$W = K_{\text{rot}} + \Delta U_{\text{spring}} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}k(\Delta x)^2 \qquad (1)$$

Express I as the sum of the moments of inertia of the cylinder and the masses:

$$I = I_M + I_{2m} = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2I_m$$

From Table 9-1 we have, for a solid cylinder about a diameter through its center:

$$I = \tfrac{1}{4}mr^2 + \tfrac{1}{12}mL^2$$

where L is the length of the cylinder.

For a disk (thin cylinder), L is small and:

$$I = \tfrac{1}{4}mr^2$$

Apply the parallel-axis theorem to obtain:

$$I_m = \tfrac{1}{4}mr^2 + mx^2$$

Substitute to obtain:

$$I = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2\left(\tfrac{1}{4}mr^2 + mx^2\right) = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2m\left(\tfrac{1}{4}r^2 + x^2\right)$$

Substitute numerical values and evaluate I:

$$I = \tfrac{1}{2}(0.8\,\text{kg})(0.2\,\text{m})^2 + \tfrac{1}{12}(0.8\,\text{kg})(1.8\,\text{m})^2 + 2(0.2\,\text{kg})\left[\tfrac{1}{4}(0.2\,\text{m})^2 + (0.8\,\text{m})^2\right]$$
$$= 0.492\,\text{N}\cdot\text{m}^2$$

Substitute in equation (1) to obtain:

$$W = \tfrac{1}{2}(0.492\,\text{N}\cdot\text{m}^2)(24\,\text{rad/s})^2 + \tfrac{1}{2}(230\,\text{N/m})(0.4\,\text{m})^2 = \boxed{160\,\text{J}}$$

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Picture the Problem Let m represent the 0.2-kg mass, M the 0.8-kg mass of the cylinder, L the 1.8-m length, and $x + \Delta x$ the distance from the center of the objects whose mass is m. We can use Newton's 2nd law to relate the radial forces on the masses to the spring's stiffness constant and use the work-energy theorem to find the work done as the system accelerates to its final angular speed.

Using the work-energy theorem, relate the work done to the change in energy of the system:

$$W = K_{\text{rot}} + \Delta U_{\text{spring}} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}k(\Delta x)^2 \qquad (1)$$

Express I as the sum of the moments of inertia of the cylinder and the masses:

$$I = I_M + I_{2m} = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2I_m$$

From Table 9-1 we have, for a solid cylinder about a diameter through its center:

$$I = \tfrac{1}{4}mr^2 + \tfrac{1}{12}mL^2$$

where L is the length of the cylinder.

For a disk (thin cylinder), L is small and:

$$I = \tfrac{1}{4}mr^2$$

Apply the parallel-axis theorem to obtain:

$$I_m = \tfrac{1}{4}mr^2 + mx^2$$

Substitute to obtain:

$$I = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2\left(\tfrac{1}{4}mr^2 + mx^2\right)$$
$$= \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2m\left(\tfrac{1}{4}r^2 + x^2\right)$$

Substitute numerical values and evaluate I:

$$I = \tfrac{1}{2}(0.8\,\text{kg})(0.2\,\text{m})^2 + \tfrac{1}{12}(0.8\,\text{kg})(1.8\,\text{m})^2 + 2(0.2\,\text{kg})\left[\tfrac{1}{4}(0.2\,\text{m})^2 + (0.8\,\text{m})^2\right]$$
$$= 0.492\,\text{N}\cdot\text{m}^2$$

Express the net inward force acting on each of the 0.2-kg masses:

$$\sum F_{\text{radial}} = k\Delta x = m(x+\Delta x)\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{k\Delta x}{m(x+\Delta x)}}$$

Substitute numerical values and evaluate ω:

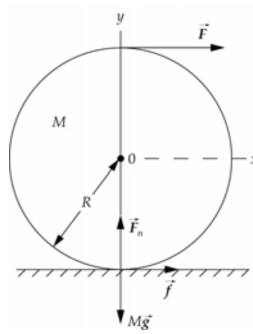
$$\omega = \sqrt{\frac{(60\,\text{N/m})(0.4\,\text{m})}{(0.2\,\text{kg})(0.8\,\text{m})}} = 12.2\,\text{rad/s}$$

Substitute numerical values in equation (1) to obtain:

$$W = \tfrac{1}{2}\left(0.492\,\text{N}\cdot\text{m}^2\right)(12.2\,\text{rad/s})^2 + \tfrac{1}{2}(60\,\text{N/m})(0.4\,\text{m})^2 = \boxed{41.4\,\text{J}}$$

130 ••

Picture the Problem The force diagram shows the forces acting on the cylinder. Because F causes the cylinder to rotate clockwise, f, which opposes this motion, is to the right. We can use Newton's 2nd law in both translational and rotational forms to relate the linear and angular accelerations to the forces acting on the cylinder.



(*a*) Use Newton's 2nd law to relate the angular acceleration of the center of mass of the cylinder to F:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

Use Newton's 2nd law to relate the acceleration of the center of mass of the cylinder to F:

$$a_{\text{cm}} = \frac{F_{\text{net}}}{M} = \frac{F}{M}$$

Express the rolling-without-slipping condition to the accelerations:

$$\alpha' = \frac{a_{\text{cm}}}{R} = \frac{F}{MR} = \boxed{2\alpha}$$

(*b*) Take the point of contact with the floor as the "pivot" point, express the net torque about that point, and solve for α:

$$\tau_{\text{net}} = 2FR = I\alpha$$

and

$$\alpha = \frac{2FR}{I}$$

Express the moment of inertia of the cylinder with respect to the pivot point:

$$I = \tfrac{1}{2}MR^2 + MR^2 = \tfrac{3}{2}MR^2$$

Substitute to obtain:

$$\alpha = \frac{2FR}{\tfrac{3}{2}MR^2} = \frac{4F}{3MR}$$

Express the linear acceleration of the cylinder:

$$a_{cm} = R\alpha = \boxed{\frac{4F}{3M}}$$

Apply Newton's 2nd law to the forces acting on the cylinder:

$$\sum F_x = F + f = Ma_{cm}$$

Solve for f:

$$f = Ma_{cm} - F = \frac{4F}{3} - F$$
$$= \boxed{\tfrac{1}{3}F \text{ in the positive } x \text{ direction.}}$$

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Picture the Problem As the load falls, mechanical energy is conserved. As in Example 9-7, choose the initial potential energy to be zero. Apply conservation of mechanical energy to obtain an expression for the speed of the bucket as a function of its position and use the given expression for t to determine the time required for the bucket to travel a distance y.

Apply conservation of mechanical energy:

$$U_f + K_f = U_i + K_i = 0 + 0 = 0 \quad (1)$$

Express the total potential energy when the bucket has fallen a distance y:

$$U_f = U_{bf} + U_{cf} + U_{wf}$$
$$= -mgy - m_c'g\left(\frac{y}{2}\right)$$

where m_c' is the mass of the hanging part of the cable.

Assume the cable is uniform and express m_c' in terms of m_c, y, and L:

$$\frac{m_c'}{y} = \frac{m_c}{L} \text{ or } m_c' = \frac{m_c}{L}y$$

Substitute to obtain:

$$U_f = -mgy - \frac{m_c g y^2}{2L}$$

Noting that bucket, cable, and rim of the winch have the same speed v, express the total kinetic energy when the bucket is falling with speed v:

$$K_f = K_{bf} + K_{cf} + K_{wf} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}I\omega_f^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{4}Mv^2$$

Substitute in equation (1) to obtain:

$$-mgy - \frac{m_c gy^2}{2L} + \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{4}Mv^2 = 0$$

Solve for v:

$$v = \sqrt{\frac{4mgy + \dfrac{2m_c gy^2}{L}}{M + 2m + 2m_c}}$$

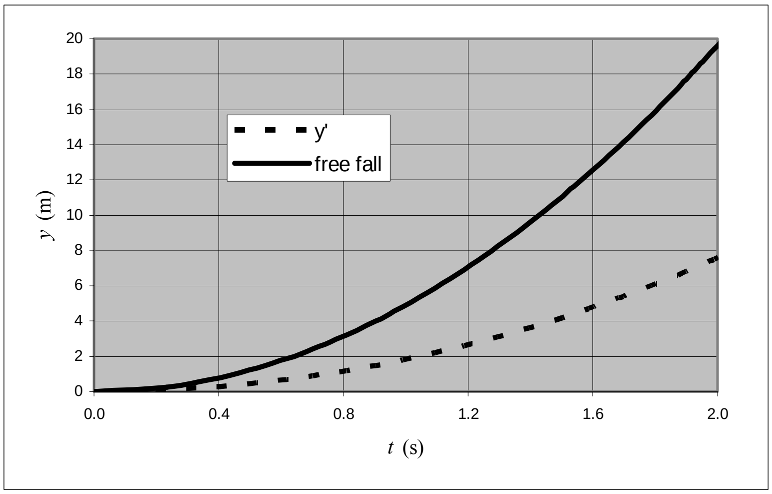
A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
D9	0	y_0
D10	D9+$B$8	$y + \Delta y$
E9	0	v_0
E10	((4*$B$3*$B$7*D10+2*$B$7*D10^2/(2*$B$5))/ ($B$1+2*$B$3+2*$B$4))^0.5	$\sqrt{\frac{4mgy + \frac{2m_c gy^2}{L}}{M + 2m + 2m_c}}$
F10	F9+$B$8/((E10+E9)/2)	$t_{n-1} + \left(\frac{v_{n-1} + v_n}{2}\right)\Delta y$
J9	0.5*$B$7*H9^2	$\frac{1}{2}gt^2$

	A	B	C	D	E	F	G	H	I	J
1	M=	10	kg							
2	R=	0.5	m							
3	m=	5	kg							
4	mc=	3.5	kg							
5	L=	10	m							
6										
7	g=	9.81	m/s^2							
8	dy=	0.1	m	y	v(y)	t(y)		t(y)	y	1/2gt^2
9				0.0	0.00	0.00		0.00	0.0	0.00
10				0.1	0.85	0.23		0.23	0.1	0.27
11				0.2	1.21	0.33		0.33	0.2	0.54
12				0.3	1.48	0.41		0.41	0.3	0.81
13				0.4	1.71	0.47		0.47	0.4	1.08
15				0.5	1.91	0.52		0.52	0.5	1.35

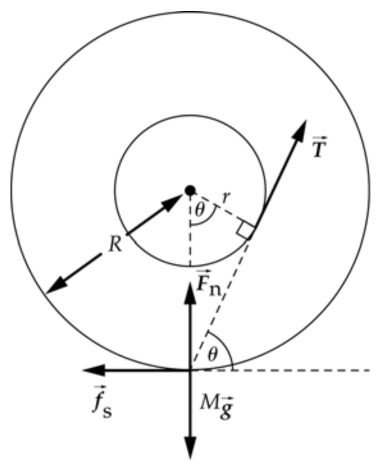
105				9.6	9.03	2.24		2.24	9.6	24.61
106				9.7	9.08	2.25		2.25	9.7	24.85
107				9.8	9.13	2.26		2.26	9.8	25.09
108				9.9	9.19	2.27		2.27	9.9	25.34
109				10.0	9.24	2.28		2.28	10.0	25.58

The solid line on the graph shown below shows the position y of the bucket when it is in free fall and the dashed line shows y under the conditions modeled in this problem.



132 ••

Picture the Problem The pictorial representation shows the forces acting on the cylinder when it is stationary. First, we note that if the tension is small, then there can be no slipping, and the system must roll. Now consider the point of contact of the cylinder with the surface as the "pivot" point. If τ about that point is zero, the system will not roll. This will occur if the line of action of the tension passes through the pivot point.

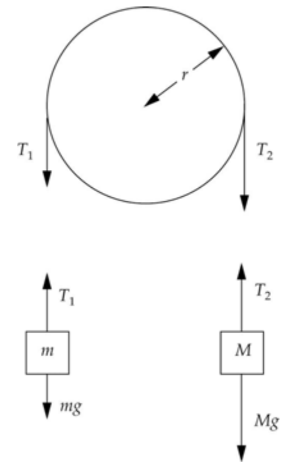


From the diagram we see that:

$$\theta = \boxed{\cos^{-1}\left(\frac{r}{R}\right)}$$

***133** ••

Picture the Problem Free-body diagrams for the pulley and the two blocks are shown to the right. Choose a coordinate system in which the direction of motion of the block whose mass is M (downward) is the positive y direction. We can use the given relationship $T'_{\text{max}} = Te^{\mu_s \Delta\theta}$ to relate the tensions in the rope on either side of the pulley and apply Newton's 2nd law in both rotational form (to the pulley) and translational form (to the blocks) to obtain a system of equations that we can solve simultaneously for a, T_1, T_2, and M.



(*a*) Use $T'_{\text{max}} = Te^{\mu_s \Delta\theta}$ to evaluate the maximum tension required to prevent the rope from slipping on the pulley:

$$T'_{\text{max}} = (10\,\text{N})e^{(0.3)\pi} = \boxed{25.7\,\text{N}}$$

(*c*) Given that the angle of wrap is π radians, express T_2 in terms of T_1:

$$T_2 = T_1 e^{0.3\pi} = 2.57T_1 \qquad (1)$$

Because the rope doesn't slip, we can relate the angular acceleration, α, of the pulley to the acceleration, a, of the hanging masses by:

$$\alpha = \frac{a}{r}$$

Apply $\sum F_y = ma_y$ to the two blocks to obtain:

$$T_1 - mg = ma \qquad (2)$$

and

$$Mg - T_2 = Ma \qquad (3)$$

Apply $\sum \tau = I\alpha$ to the pulley to obtain:

$$(T_2 - T_1)r = I\frac{a}{r} \qquad (4)$$

Substitute for T_2 from equation (1) in equation (4) to obtain:

$$(2.57T_1 - T_1)r = I\frac{a}{r}$$

Solve for T_1 and substitute numerical values to obtain:

$$T_1 = \frac{I}{1.57r^2}a = \frac{0.35\,\text{kg}\cdot\text{m}^2}{1.57(0.15\,\text{m})^2}a \qquad (5)$$
$$= (9.91\,\text{kg})a$$

Substitute in equation (2) to obtain:

$$(9.91\,\text{kg})a - mg = ma$$

Solve for and evaluate a:

$$a = \frac{mg}{9.91\,\text{kg} - m} = \frac{g}{\frac{9.91\,\text{kg}}{m} - 1}$$

$$= \frac{9.81\,\text{m/s}^2}{\frac{9.91\,\text{kg}}{1\,\text{kg}} - 1} = \boxed{1.10\,\text{m/s}^2}$$

(b) Solve equation (3) for M:

$$M = \frac{T_2}{g - a}$$

Substitute in equation (5) to find T_1:

$$T_1 = (9.91\,\text{kg})(1.10\,\text{m/s}^2) = 10.9\,\text{N}$$

Substitute in equation (1) to find T_2:

$$T_2 = (2.57)(10.9\,\text{N}) = 28.0\,\text{N}$$

Evaluate M:

$$M = \frac{28.0\,\text{N}}{9.81\,\text{m/s}^2 - 1.10\,\text{m/s}^2} = \boxed{3.21\,\text{kg}}$$

134 •••

Picture the Problem When the tension is horizontal, the cylinder will roll forward and the friction force will be in the direction of $\vec{T}$. We can use Newton's 2nd law to obtain equations that we can solve simultaneously for a and f.

(a) Apply Newton's 2nd law to the cylinder:

$$\sum F_x = T + f = ma \qquad (1)$$

and

$$\sum \tau = Tr - fR = I\alpha \qquad (2)$$

Substitute for I and α in equation (2) to obtain:

$$Tr - fR = \tfrac{1}{2} mR^2 \frac{a}{R} = \tfrac{1}{2} mRa \qquad (3)$$

Solve equation (3) for f:

$$f = \frac{Tr}{R} - \tfrac{1}{2} ma \qquad (4)$$

Substitute equation (4) in equation (1) and solve for a:

$$a = \frac{2T}{3m}\left(1 + \frac{r}{R}\right) \qquad (5)$$

Substitute equation (5) in equation (4) to obtain:

$$f = \boxed{\frac{T}{3}\left(\frac{2r}{R} - 1\right)}$$

(b) Equation (4) gives the acceleration of the center of mass:

$$a = \boxed{\frac{2T}{3m}\left(1 + \frac{r}{R}\right)}$$

(*c*) Express the condition that $a > \frac{T}{m}$:

$$\frac{2T}{3m}\left(1+\frac{r}{R}\right) > \frac{T}{m} \Rightarrow \frac{2}{3}\left(1+\frac{r}{R}\right) > 1$$

or

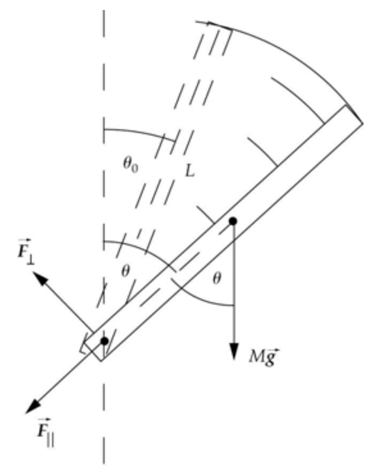
$$r > \boxed{\tfrac{1}{2}R}$$

(*d*) If $r > \frac{1}{2}R$:

$$\boxed{f > 0\text{, i.e., in the direction of } \vec{\boldsymbol{T}}.}$$

135 •••

Picture the Problem The system is shown in the drawing in two positions, with angles θ_0 and θ with the vertical. The drawing also shows all the forces that act on the stick. These forces result in a rotation of the stick—and its center of mass—about the pivot, and a tangential acceleration of the center of mass. We'll apply the conservation of mechanical energy and Newton's 2nd law to relate the radial and tangential forces acting on the stick.



Use the conservation of mechanical energy to relate the kinetic energy of the stick when it makes an angle θ with the vertical and its initial potential energy:

$$K_\text{f} - K_\text{i} + U_\text{f} - U_\text{i} = 0$$

or, because $K_\text{f} = 0$,

$$-\tfrac{1}{2}I\omega^2 + Mg\frac{L}{2}\cos\theta - Mg\frac{L}{2}\cos\theta_0 = 0$$

Substitute for I and solve for ω^2:

$$\omega^2 = \frac{3g}{L}\left(\cos\theta - \cos\theta_0\right)$$

Express the centripetal force acting on the center of mass:

$$F_\text{c} = M\frac{L}{2}\omega^2$$
$$= M\frac{L}{2}\frac{3g}{L}\left(\cos\theta - \cos\theta_0\right)$$
$$= \frac{3Mg}{2}\left(\cos\theta - \cos\theta_0\right)$$

Express the radial component of $M\vec{\boldsymbol{g}}$:

$$(Mg)_\text{radial} = Mg\cos\theta$$

Express the total radial force at the hinge:

$$F_{||} = F_\text{c} + (Mg)_\text{radial}$$

$$= \frac{3Mg}{2}\left(\cos\theta - \cos\theta_0\right) + Mg\cos\theta$$

$$= \boxed{\tfrac{1}{2}Mg\left(5\cos\theta - 3\cos\theta_0\right)}$$

Relate the tangential acceleration of the center of mass to its angular acceleration:

$$a_\perp = \tfrac{1}{2}L\alpha$$

Use Newton's 2nd law to relate the angular acceleration of the stick to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Mg\frac{L}{2}\sin\theta}{\frac{1}{3}ML^2} = \frac{3g\sin\theta}{2L}$$

Express $a_\perp$ in terms of α:

$$a_\perp = \tfrac{1}{2}L\alpha = \tfrac{3}{4}g\sin\theta = g\sin\theta + F_\perp/M$$

Solve for $F_\perp$ to obtain:

$F_\perp = \boxed{-\tfrac{1}{4}Mg\sin\theta}$ where the minus sign indicates that the force is directed oppositely to the tangential component of $M\vec{\boldsymbol{g}}$.

Chapter 10
Conservation of Angular Momentum

Conceptual Problems

***1** •

(*a*) True. The cross product of the vectors $\vec{A}$ and $\vec{B}$ is defined to be $\vec{A}\times\vec{B} = AB\sin\phi\,\hat{n}$. If $\vec{A}$ and $\vec{B}$ are parallel, $\sin\phi = 0$.

(*b*) True. By definition, $\vec{\omega}$ is along the axis.

(*c*) True. The direction of a torque exerted by a force is determined by the definition of the cross product.

2 •

Determine the Concept The cross product of the vectors $\vec{A}$ and $\vec{B}$ is defined to be $\vec{A}\times\vec{B} = AB\sin\phi\,\hat{n}$. Hence, the cross product is a maximum when $\sin\phi = 1$. This condition is satisfied provided $\vec{A}$ and $\vec{B}$ are *perpendicular*. $\boxed{(c)\text{ is correct.}}$

3 •

Determine the Concept $\vec{L}$ and $\vec{p}$ are related according to $\vec{L} = \vec{r}\times\vec{p}$. From this definition of the cross product, $\vec{L}$ and $\vec{p}$ are perpendicular; i.e., the angle between them is 90°.

4 •

Determine the Concept $\vec{L}$ and $\vec{p}$ are related according to $\vec{L} = \vec{r}\times\vec{p}$. Because the motion is along a line that passes through point *P*, $r = 0$ and so is L. $\boxed{(b)\text{ is correct.}}$

***5** ••

Determine the Concept $\vec{L}$ and $\vec{p}$ are related according to $\vec{L} = \vec{r}\times\vec{p}$.

(*a*) Because $\vec{L}$ is directly proportional to $\vec{p}$:

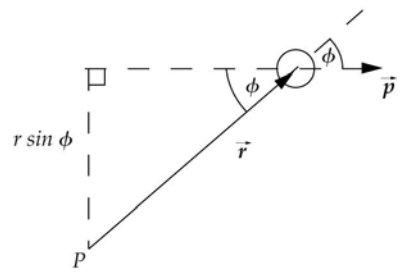
$$\boxed{\text{Doubling } \vec{p} \text{ doubles } \vec{L}.}$$

(*b*) Because $\vec{L}$ is directly proportional to $\vec{r}$:

$$\boxed{\text{Doubling } \vec{r} \text{ doubles } \vec{L}.}$$

6 ••

Determine the Concept The figure shows a particle moving with constant speed in a straight line (i.e., with constant velocity and constant linear momentum). The magnitude of L is given by $rp\sin\phi = mv(r\sin\phi)$.



Referring to the diagram, note that the distance $r\sin\phi$ from P to the line along which the particle is moving is constant. Hence, $mv(r\sin\phi)$ is constant and so $\boxed{\vec{L} \text{ is constant.}}$

7 •

False. The net torque acting on a rotating system equals the change in the system's angular momentum; i.e., $\tau_{\text{net}} = dL/dt$, where $L = I\omega$. Hence, if τ_{net} is zero, all we can say for sure is that the angular momentum (the product of I and ω) is constant. If I changes, so must ω.

***8 ••**

Determine the Concept Yes, you can. Imagine rotating the top half of your body with arms flat at sides through a (roughly) 90° angle. Because the net angular momentum of the system is 0, the bottom half of your body rotates in the opposite direction. Now extend your arms out and rotate the top half of your body back. Because the moment of inertia of the top half of your body is larger than it was previously, the angle which the bottom half of your body rotates through will be smaller, leading to a net rotation. You can repeat this process as necessary to rotate through any arbitrary angle.

9 •

Determine the Concept If L is constant, we know that the *net* torque acting on the system is zero. There may be multiple constant or time-dependent torques acting on the system as long as the net torque is zero. $\boxed{(e) \text{ is correct.}}$

10 ••

Determine the Concept No. In order to do work, a force must act over some distance. In each "inelastic collision" the force of static friction does not act through any distance.

11 ••

Determine the Concept It is easier to crawl radially outward. In fact, a radially inward force is required just to prevent you from sliding outward.

***12 ••**

Determine the Concept The pull that the student exerts on the block is at right angles to its motion and exerts no torque (recall that $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$ and $\tau = rF\sin\theta$). Therefore, we can conclude that the angular momentum of the block is conserved. The student does, however, do work in displacing the block in the direction of the radial force and so the block's energy increases. $\boxed{(b) \text{ is correct.}}$

***13 ••**

Determine the Concept The hardboiled egg is solid inside, so everything rotates with a uniform velocity. By contrast, it is difficult to get the viscous fluid inside a raw egg to start rotating; however, once it is rotating, stopping the shell will not stop the motion of the interior fluid, and the egg may start rotating again after momentarily stopping for this reason.

14 •

False. The relationship $\vec{\boldsymbol{\tau}} = d\vec{\boldsymbol{L}}/dt$ describes the motion of a gyroscope independently of whether it is spinning.

15 •

Picture the Problem We can divide the expression for the kinetic energy of the object by the expression for its angular momentum to obtain an expression for K as a function of I and L.

Express the rotational kinetic energy of the object:

$$K = \tfrac{1}{2} I\omega^2$$

Relate the angular momentum of the object to its moment of inertia and angular velocity:

$$L = I\omega$$

Divide the first of these equations by the second and solve for K to obtain:

$$K = \frac{L^2}{2I} \text{ and so } \boxed{(b) \text{ is correct.}}$$

16 •

Determine the Concept The purpose of the second smaller rotor is to prevent the body of the helicopter from rotating. If the rear rotor fails, the body of the helicopter will tend to rotate on the main axis due to angular momentum being conserved.

17 ••

Determine the Concept One can use a right-hand rule to determine the direction of the torque required to turn the angular momentum vector from east to south. Letting the fingers of your right hand point east, rotate your wrist until your fingers point south. Note that your thumb points downward. $\boxed{(b) \text{ is correct.}}$

18 ••

Determine the Concept In turning east, the man redirects the angular momentum vector from north to east by exerting a clockwise torque (viewed from above) on the gyroscope. As a consequence of this torque, the front end of the suitcase will dip downward. $\boxed{(d)\text{ is correct.}}$

19 ••

(a) The lifting of the nose of the plane rotates the angular momentum vector upward. It veers to the right in response to the torque associated with the lifting of the nose.

(b) The angular momentum vector is rotated to the right when the plane turns to the right. In turning to the right, the torque points down. The nose will move downward.

20 ••

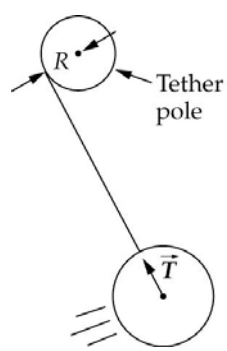
Determine the Concept If $\vec{L}$ points up and the car travels over a hill or through a valley, the force on the wheels on one side (or the other) will increase and car will tend to tip. If $\vec{L}$ points forward and car turns left or right, the front (or rear) of the car will tend to lift. These problems can be averted by having two identical flywheels that rotate on the same shaft in opposite directions.

21 ••

Determine the Concept The rotational kinetic energy of the woman-plus-stool system is given by $K_{\text{rot}} = \frac{1}{2}I\omega^2 = L^2/2I$. Because L is constant (angular momentum is conserved) and her moment of inertia is greater with her arms extended, $\boxed{(b)\text{ is correct.}}$

***22** ••

Determine the Concept Consider the overhead view of a tether pole and ball shown in the adjoining figure. The ball rotates counterclockwise. The torque about the center of the pole is clockwise and of magnitude RT, where R is the pole's radius and T is the tension. So L must decrease and $\boxed{(e)\text{ is correct.}}$



23 ••

Determine the Concept The center of mass of the rod-and-putty system moves in a straight line, and the system rotates about its center of mass.

24 •

(*a*) True. The net external torque acting a system equals the rate of change of the angular momentum of the system; i.e., $\sum_i \vec{\boldsymbol{\tau}}_{\text{i,ext}} = \frac{d\vec{\boldsymbol{L}}}{dt}$.

(*b*) False. If the net torque on a body is zero, its angular momentum is *constant* but not necessarily zero.

Estimation and Approximation

***25** ••

Picture the Problem Because we have no information regarding the mass of the skater, we'll assume that her body mass (not including her arms) is 50 kg and that each arm has a mass of 4 kg. Let's also assume that her arms are 1 m long and that her body is cylindrical with a radius of 20 cm. Because the net external torque acting on her is zero, her angular momentum will remain constant during her pirouette.

Express the conservation of her angular momentum during her pirouette:

$$L_{\text{i}} = L_{\text{f}}$$

or

$$I_{\text{arms out}}\omega_{\text{arms out}} = I_{\text{arms in}}\omega_{\text{arms in}} \qquad (1)$$

Express her total moment of inertia with her arms out:

$$I_{\text{arms out}} = I_{\text{body}} + I_{\text{arms}}$$

Treating her body as though it is cylindrical, calculate its moment of inertia of her body, minus her arms:

$$I_{\text{body}} = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(50\,\text{kg})(0.2\,\text{m})^2 = 1.00\,\text{kg}\cdot\text{m}^2$$

Modeling her arms as though they are rods, calculate their moment of inertia when she has them out:

$$I_{\text{arms}} = 2\left[\tfrac{1}{3}(4\,\text{kg})(1\,\text{m})^2\right] = 2.67\,\text{kg}\cdot\text{m}^2$$

Substitute to determine her total moment of inertia with her arms out:

$$I_{\text{arms out}} = 1.00\,\text{kg}\cdot\text{m}^2 + 2.67\,\text{kg}\cdot\text{m}^2 = 3.67\,\text{kg}\cdot\text{m}^2$$

Express her total moment of inertia with her arms in:

$$I_{\text{arms}\,in} = I_{\text{body}} + I_{\text{arms}} = 1.00\,\text{kg}\cdot\text{m}^2 + 2\left[(4\,\text{kg})(0.2\,\text{m})^2\right] = 1.32\,\text{kg}\cdot\text{m}^2$$

Solve equation (1) for $\omega_{\text{arms in}}$ and substitute to obtain:

$$\omega_{\text{arms in}} = \frac{I_{\text{arms out}}}{I_{\text{arms in}}}\omega_{\text{arms out}} = \frac{3.67\,\text{kg}\cdot\text{m}^2}{1.32\,\text{kg}\cdot\text{m}^2}(1.5\,\text{rev/s}) = \boxed{4.17\,\text{rev/s}}$$

26 ••

Picture the Problem We can express the period of the earth's rotation in terms of its angular velocity of rotation and relate its angular velocity to its angular momentum and moment of inertia with respect to an axis through its center. We can differentiate this expression with respect to *I* and then use differentials to approximate the changes in *I* and *T*.

Express the period of the earth's rotation in terms of its angular velocity of rotation:

$$T = \frac{2\pi}{\omega}$$

Relate the earth's angular velocity of rotation to its angular momentum and moment of inertia:

$$\omega = \frac{L}{I}$$

Substitute to obtain:

$$T = \frac{2\pi I}{L}$$

Find dT/dI:

$$\frac{dT}{dI} = \frac{2\pi}{L} = \frac{T}{I}$$

Solve for dT/T and approximate ΔT:

$$\frac{dT}{T} = \frac{dI}{I} \text{ or } \Delta T \approx \frac{\Delta I}{I}T$$

Substitute for ΔI and I to obtain:

$$\Delta T \approx \frac{\frac{2}{3}mr^2}{\frac{2}{5}M_E R_E^2}T = \frac{5m}{3M_E}T$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{5(2.3\times10^{19}\,\text{kg})}{3(6\times10^{24}\,\text{kg})}(1\,\text{d}) = 6.39\times10^{-6}\,\text{d} = 6.39\times10^{-6}\,\text{d}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\,\text{s}}{\text{h}} = \boxed{0.552\,\text{s}}$$

27 •

Picture the Problem We can use $L = mvr$ to find the angular momentum of the particle. In (*b*) we can solve the equation $L = \sqrt{\ell(\ell+1)}\hbar$ for $\ell(\ell+1)$ and the approximate value of ℓ.

(*a*) Use the definition of angular momentum to obtain:

$$L = mvr = (2\times10^{-3}\ \text{kg})(3\times10^{-3}\ \text{m/s})(4\times10^{-3}\ \text{m}) = \boxed{2.40\times10^{-8}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(*b*) Solve the equation $L = \sqrt{\ell(\ell+1)}\hbar$ for $\ell(\ell+1)$:

$$\ell(\ell+1) = \frac{L^2}{\hbar^2}$$

Substitute numerical values and evaluate $\ell(\ell+1)$:

$$\ell(\ell+1) = \left(\frac{2.40\times10^{-8}\ \text{kg}\cdot\text{m}^2/\text{s}}{1.05\times01^{-34}\ \text{J}\cdot\text{s}}\right)^2 = \boxed{5.22\times10^{52}}$$

Because $\ell >> 1$, approximate its value with the square root of $\ell(\ell+1)$:

$$\ell \approx \boxed{2.29\times10^{26}}$$

(*c*) The quantization of angular momentum is not noticed in macroscopic physics because no experiment can differentiate between $\ell = 2\times10^{26}$ and $\ell = 2\times10^{26} + 1$.

***28** ••

Picture the Problem We can use conservation of angular momentum in part (*a*) to relate the before-and-after collapse rotation rates of the sun. In part (*b*), we can express the fractional change in the rotational kinetic energy of the sun as it collapses into a neutron star to decide whether its rotational kinetic energy is greater initially or after the collapse.

(*a*) Use conservation of angular momentum to relate the angular momenta of the sun before and after its collapse:

$$I_b\omega_b = I_a\omega_a \qquad (1)$$

Using the given formula, approximate the moment of inertia I_b of the sun before collapse:

$$I_b = 0.059MR_{sun}^2 = 0.059(1.99\times10^{30}\ \text{kg})(6.96\times10^5\ \text{km})^2 = 5.69\times10^{46}\ \text{kg}\cdot\text{m}^2$$

Find the moment of inertia I_a of the sun when it has collapsed into a spherical neutron star of radius 10 km and uniform mass distribution:

$$I_a = \tfrac{2}{5}MR^2 = \tfrac{2}{5}\left(1.99\times10^{30}\text{ kg}\right)\left(10\text{ km}\right)^2 = 7.96\times10^{37}\text{ kg}\cdot\text{m}^2$$

Substitute in equation (1) and solve for ω_a to obtain:

$$\omega_a = \frac{I_b}{I_a}\omega_b = \frac{5.69\times10^{46}\text{ kg}\cdot\text{m}^2}{7.96\times10^{37}\text{ kg}\cdot\text{m}^2}\omega_b = 7.15\times10^{8}\,\omega_b$$

Given that ω_b = 1 rev/25 d, evaluate ω_a:

$$\omega_a = 7.15\times10^{8}\left(\frac{1\text{ rev}}{25\text{ d}}\right) = \boxed{2.86\times10^{7}\text{ rev/d}}$$

The additional rotational kinetic energy comes at the expense of gravitational potential energy, which decreases as the sun gets smaller.

Note that the rotational period decreases by the same factor of I_b/I_a and becomes:

$$T_a = \frac{2\pi}{\omega_a} = \frac{2\pi}{2.86\times10^{7}\,\dfrac{\text{rev}}{\text{d}}\times\dfrac{2\pi\text{ rad}}{\text{rev}}\times\dfrac{1\text{d}}{24\text{h}}\times\dfrac{1\text{h}}{3600\text{s}}} = 3.02\times10^{-3}\text{ s}$$

(*b*) Express the fractional change in the sun's rotational kinetic energy as a consequence of its collapse and simplify to obtain:

$$\frac{\Delta K}{K_b} = \frac{K_a - K_b}{K_b} = \frac{K_a}{K_b} - 1 = \frac{\tfrac{1}{2}I_a\omega_a^2}{\tfrac{1}{2}I_b\omega_b^2} - 1 = \frac{I_a\omega_a^2}{I_b\omega_b^2} - 1$$

Substitute numerical values and evaluate $\Delta K/K_b$:

$$\frac{\Delta K}{K_b} = \left(\frac{1}{7.15\times10^{8}}\right)\left(\frac{2.86\times10^{7}\text{ rev/d}}{1\text{ rev/25 d}}\right)^2 - 1 = \boxed{7.15\times10^{8}}$$ (i.e., the rotational kinetic energy *increases* by a factor of approximately 7×10^{8}.)

29 ••

Picture the Problem We can solve $I = CMR^2$ for C and substitute numerical values in order to determine an experimental value of C for the earth. We can then compare this value to those for a spherical shell and a sphere in which the mass is uniformly distributed to decide whether the earth's mass density is greatest near its core or near its crust.

(*a*) Express the moment of inertia of the earth in terms of the constant *C*:

$$I = CMR^2$$

Solve for *C* to obtain:

$$C = \frac{I}{MR^2}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{8.03\times10^{37}\ \text{kg}\cdot\text{m}^2}{\left(5.98\times10^{24}\ \text{kg}\right)\left(6370\ \text{km}\right)^2} = \boxed{0.331}$$

(*b*) If all of the mass were in the crust, the moment of inertia of the earth would be that of a thin spherical shell:

$$I_{\text{spherical shell}} = \tfrac{2}{3}MR^2$$

If the mass of the earth were uniformly distributed throughout its volume, its moment of inertia would be:

$$I_{\text{solid sphere}} = \tfrac{2}{5}MR^2$$

Because experimentally $C < 2/5 = 0.4$, the mass density must be greater near the center of the earth.

***30 ••**

Picture the Problem Let's estimate that the diver with arms extended over head is about 2.5 m long and has a mass $M = 80$ kg. We'll also assume that it is reasonable to model the diver as a uniform stick rotating about its center of mass. From the photo, it appears that he sprang about 3 m in the air, and that the diving board was about 3 m high. We can use these assumptions and estimated quantities, together with their definitions, to estimate ω and *L*.

Express the diver's angular velocity ω and angular momentum *L*:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (1)$$

and

$$L = I\omega \qquad (2)$$

Using a constant-acceleration equation, express his time in the air:

$$\Delta t = \Delta t_{\text{rise 3 m}} + \Delta t_{\text{fall 6 m}} = \sqrt{\frac{2\Delta y_{\text{up}}}{g}} + \sqrt{\frac{2\Delta y_{\text{down}}}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(3\ \text{m})}{9.81\ \text{m/s}^2}} + \sqrt{\frac{2(6\ \text{m})}{9.81\ \text{m/s}^2}} = 1.89\ \text{s}$$

Estimate the angle through which he rotated in 1.89 s:

$$\Delta\theta \approx 0.5\ \text{rev} = \pi\ \text{rad}$$

Substitute in equation (1) and evaluate ω:

$$\omega = \frac{\pi\ \text{rad}}{1.89\ \text{s}} = \boxed{1.66\ \text{rad/s}}$$

Use the "stick rotating about an axis through its center of mass" model to approximate the moment of inertia of the diver:

$$I = \tfrac{1}{12}ML^2$$

Substitute in equation (2) to obtain:

$$L = \tfrac{1}{12}ML^2\omega$$

Substitute numerical values and evaluate L:

$$L = \tfrac{1}{12}(80\ \text{kg})(2.5\ \text{m})^2(1.66\ \text{rad/s})$$
$$= 69.2\ \text{kg}\cdot\text{m}^2/\text{s} \approx \boxed{70\ \text{kg}\cdot\text{m}^2/\text{s}}$$

Remarks: We can check the reasonableness of this estimation in another way. Because he rose about 3 m in the air, the initial impulse acting on him must be about 600 kg·m/s (i.e., $I = \Delta p = Mv_i$). If we estimate that the lever arm of the force is roughly ℓ = 1.5 m, and the angle between the force exerted by the board and a line running from his feet to the center of mass is about 5°, we obtain $L = I\ell\sin 5° \approx 78$ kg·m^2/s, which is not too bad considering the approximations made here.

31 ••

Picture the Problem First we assume a spherical diver whose mass M = 80 kg and whose diameter, when curled into a ball, is 1 m. We can estimate his angular velocity when he has curled himself into a ball from the ratio of his angular momentum to his moment of inertia. To estimate his angular momentum, we'll guess that the lever arm ℓ of the force that launches him from the diving board is about 1.5 m and that the angle between the force exerted by the board and a line running from his feet to the center of mass is about 5°.

Express the diver's angular velocity ω when he curls himself into a ball in mid-dive:

$$\omega = \frac{L}{I} \qquad (1)$$

Using a constant-acceleration equation, relate the speed with which he left the diving board v_0 to his maximum height Δy and our estimate of his angle with the vertical direction:

$$0 = v_{0y}^2 + 2a_y\Delta y$$
where
$$v_{0y} = v_0\cos 5°$$

Solve for v_0:

$$v_0 = \sqrt{\frac{2g\Delta y}{\cos^2 5°}}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{\sqrt{2(9.81\ \text{m/s}^2)(3\ \text{m})}}{\cos 5°} = 7.70\ \text{m/s}$$

Approximate the impulse acting on the diver to launch him with the speed v_0:

$$I = \Delta p = Mv_0$$

Letting ℓ represent the lever arm of the force acting on the diver as he leaves the diving board, express his angular momentum:

$$L = I\ell \sin 5° = Mv_0\ell \sin 5°$$

Use the "uniform sphere" model to approximate the moment of inertia of the diver:

$$I = \tfrac{2}{5}MR^2$$

Substitute in equation (1) to obtain:

$$\omega = \frac{Mv_0\ell \sin 5°}{\tfrac{2}{5}MR^2} = \frac{5v_0\ell \sin 5°}{2R^2}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{5(7.70\,\text{m/s})(1.5\,\text{m})\sin 5°}{2(0.5\,\text{m})^2} = \boxed{10.1\,\text{rad/s}}$$

***32 ••**

Picture the Problem We'll assume that he launches himself at an angle of 45° with the horizontal with his arms spread wide, and then pulls them in to increase his rotational speed during the jump. We'll also assume that we can model him as a 2-m long cylinder with an average radius of 0.15 m and a mass of 60 kg. We can then find his take-off speed and "air time" using constant-acceleration equations, and use the latter, together with the definition of rotational velocity, to find his initial rotational velocity. Finally, we can apply conservation of angular momentum to find his initial angular momentum.

Using a constant-acceleration equation, relate his takeoff speed v_0 to his maximum elevation Δy:

$$v^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $v_{0y} = v_0\sin 45°$, $v = 0$, and $a_y = -g$,

$$0 = v_0^2 \sin^2 45° - 2g\Delta y$$

Solve for v_0 to obtain:

$$v_0 = \sqrt{\frac{2g\Delta y}{\sin^2 45°}} = \frac{\sqrt{2g\Delta y}}{\sin 45°}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{\sqrt{2(9.81\,\text{m/s}^2)(0.6\,\text{m})}}{\sin 45°} = \boxed{4.85\,\text{m/s}}$$

Use its definition to express Goebel's angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Use a constant-acceleration equation to express Goebel's "air time" Δt:

$$\Delta t = 2\Delta t_{\text{rise}\,0.6\,\text{m}} = 2\sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = 2\sqrt{\frac{2(0.6\,\text{m})}{9.81\,\text{m/s}^2}} = 0.699\,\text{s}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{4\,\text{rev}}{0.699\,\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}} = \boxed{36.0\,\text{rad/s}}$$

Use conservation of angular momentum to relate his take-off angular velocity ω_0 to his average angular velocity ω as he performs a quadruple Lutz:

$$I_0\omega_0 = I\omega$$

Assuming that he can change his angular momentum by a factor of 2 by pulling his arms in, solve for and evaluate ω_0:

$$\omega_0 = \frac{I}{I_0}\omega = \frac{1}{2}(36\,\text{rad/s}) = \boxed{18.0\,\text{rad/s}}$$

Express his take-off angular momentum:

$$L_0 = I_0\omega_0$$

Assuming that we can model him as a solid cylinder of length ℓ with an average radius r and mass m, express his moment of inertia with arms drawn in (his take-off configuration):

$$I_0 = 2\left(\tfrac{1}{2}mr^2\right) = mr^2$$

where the factor of 2 represents our assumption that he can double his moment of inertia by extending his arms.

Substitute to obtain:

$$L_0 = mr^2\omega_0$$

Substitute numerical values and evaluate L_0:

$$L_0 = (60\,\text{kg})(0.15\,\text{m})^2(18\,\text{rad/s}) = \boxed{24.3\,\text{kg}\cdot\text{m}^2/\text{s}}$$

Vector Nature of Rotation

33 •

Picture the Problem We can express $\vec{F}$ and $\vec{r}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$ and then use the definition of the cross product to find $\vec{\tau}$.

Express $\vec{F}$ in terms of F and the unit vector $\hat{i}$:

$$\vec{F} = -F\hat{i}$$

Express $\vec{r}$ in terms of R and the unit vector $\hat{j}$:

$$\vec{r} = R\hat{j}$$

Calculate the cross product of $\vec{r}$ and $\vec{F}$:

$$\vec{\tau} = \vec{r} \times \vec{F} = FR\left(\hat{j} \times -\hat{i}\right) = FR\left(\hat{i} \times \hat{j}\right) = \boxed{FR\hat{k}}$$

34 •

Picture the Problem We can find the torque is the cross product of $\vec{r}$ and $\vec{F}$.

Compute the cross product of $\vec{r}$ and $\vec{F}$:

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(x\hat{i} + y\hat{j}\right)\left(-mg\hat{j}\right) = -mgx\left(\hat{i} \times \hat{j}\right) - mgy\left(\hat{j} \times \hat{j}\right) = \boxed{-mgx\hat{k}}$$

35 •

Picture the Problem The cross product of the vectors $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$ is given by

$$\vec{A} \times \vec{B} = A_xB_x\left(\hat{i} \times \hat{i}\right) + A_xB_y\left(\hat{i} \times \hat{j}\right) + A_yB_x\left(\hat{j} \times \hat{i}\right) + A_yB_y\left(\hat{j} \times \hat{j}\right)$$
$$= A_xB_x(0) + A_xB_y\left(\hat{k}\right) + A_yB_x\left(-\hat{k}\right) + A_yB_y(0)$$
$$= A_xB_y\left(\hat{k}\right) + A_yB_x\left(-\hat{k}\right)$$

(*a*) Find $\vec{A} \times \vec{B}$ for $\vec{A} = 4\hat{i}$ and $\vec{B} = 6\hat{i} + 6\hat{j}$:

$$\vec{A} \times \vec{B} = 4\hat{i} \times \left(6\hat{i} + 6\hat{j}\right) = 24\left(\hat{i} \times \hat{i}\right) + 24\left(\hat{i} \times \hat{j}\right) = 24(0) + 24\hat{k} = \boxed{24\hat{k}}$$

(*b*) Find $\vec{A} \times \vec{B}$ for $\vec{A} = 4\hat{i}$ and $\vec{B} = 6\hat{i} + 6\hat{k}$:

$$\vec{A} \times \vec{B} = 4\hat{i} \times \left(6\hat{i} + 6\hat{k}\right) = 24\left(\hat{i} \times \hat{i}\right) + 24\left(\hat{i} \times \hat{k}\right) = 24(0) + 24\left(-\hat{j}\right) = \boxed{-24\hat{j}}$$

(*c*) Find $\vec{A} \times \vec{B}$ for $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 2\hat{j}$:

$$\vec{A} \times \vec{B} = \left(2\hat{i} + 3\hat{j}\right) \times \left(3\hat{i} + 2\hat{j}\right) = 6\left(\hat{i} \times \hat{i}\right) + 4\left(\hat{i} \times \hat{j}\right) + 9\left(\hat{j} \times \hat{i}\right) + 6\left(\hat{j} \times \hat{j}\right)$$
$$= 6(0) + 4\left(\hat{k}\right) + 9\left(-\hat{k}\right) + 6(0) = \boxed{-5\hat{k}}$$

***36 •**

Picture the Problem The magnitude of $\vec{A}\times\vec{B}$ is given by $|AB\sin\theta|$.

Equate the magnitudes of $\vec{A}\times\vec{B}$ and $\vec{A}\cdot\vec{B}$:

$$|AB\sin\theta| = |AB\cos\theta|$$

$$\therefore |\sin\theta| = |\cos\theta|$$

or

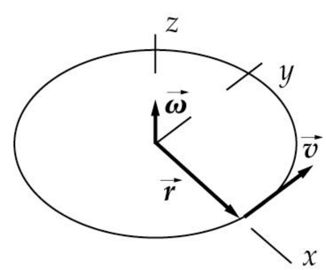
$$\tan\theta = \pm 1$$

Solve for θ to obtain:

$$\theta = \tan^{-1}\pm 1 = \boxed{\pm 45^\circ \text{ or } \pm 135^\circ}$$

37 ••

Picture the Problem Let $\vec{r}$ be in the *xy* plane. Then $\vec{\omega}$ points in the positive *z* direction. We can establish the results called for in this problem by forming the appropriate cross products and by differentiating $\vec{v}$.



(*a*) Express $\vec{\omega}$ using unit vectors:

$$\vec{\omega} = \omega\hat{k}$$

Express $\vec{r}$ using unit vectors:

$$\vec{r} = r\hat{i}$$

Form the cross product of $\vec{\omega}$ and $\vec{r}$:

$$\vec{\omega}\times\vec{r} = \omega\hat{k}\times r\hat{i} = r\omega\left(\hat{k}\times\hat{i}\right) = r\omega\,\hat{j}$$
$$= v\hat{j}$$

$$\therefore \boxed{\vec{v} = \vec{\omega}\times\vec{r}}$$

(*b*) Differentiate $\vec{v}$ with respect to *t* to express $\vec{a}$:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{\omega}\times\vec{r}\right)$$
$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$
$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}$$
$$= \vec{a}_t + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$
$$= \vec{a}_t + \vec{a}_c$$

where $\vec{a}_c = \boxed{\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)}$

and $\vec{a}_t$ and $\vec{a}_c$ are the tangential and centripetal accelerations, respectively.

38 ••

Picture the Problem Because $B_z = 0$, we can express $\vec{B}$ as $\vec{B} = B_x\hat{i} + B_y\hat{j}$ and form its cross product with $\vec{A}$ to determine B_x and B_y.

Express $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} \qquad (1)$$

Express $\vec{A}\times\vec{B}$:

$$\vec{A}\times\vec{B} = 4\hat{i}\times\left(B_x\hat{i} + B_y\hat{j}\right) = 4B_y\hat{k} = 12\hat{k}$$

Solve for B_y:

$$B_y = 3$$

Relate B to B_x and B_y:

$$B^2 = B_x^2 + B_y^2$$

Solve for and evaluate B_x:

$$B_x = \sqrt{B^2 - B_y^2} = \sqrt{5^2 - 3^2} = 4$$

Substitute in equation (1):

$$\vec{B} = \boxed{4\hat{i} + 3\hat{j}}$$

39 •

Picture the Problem We can write $\vec{B}$ in the form $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ and use the dot product of $\vec{A}$ and $\vec{B}$ to find B_y and their cross product to find B_x and B_z.

Express $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (1)$$

Evaluate $\vec{A}\cdot\vec{B}$:

$$\vec{A}\cdot\vec{B} = 3B_y = 12$$

and

$B_y = 4$

Evaluate $\vec{A}\times\vec{B}$:

$$\vec{A}\times\vec{B} = 3\hat{j}\times\left(B_x\hat{i} + 4\hat{j} + B_z\hat{k}\right)$$
$$= -3B_x\hat{k} + 3B_z\hat{i}$$

Because $\vec{A}\times\vec{B} = 9\hat{i}$:

$B_x = 0$ and $B_z = 3$.

Substitute in equation (1) to obtain:

$$\vec{B} = \boxed{4\hat{j} + 3\hat{k}}$$

40 ••

Picture the Problem The dot product of $\vec{A}$ with the cross product of $\vec{B}$ and $\vec{C}$ is a scalar quantity and can be expressed in determinant form as $\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$. We can expand this determinant by minors to show that it is equivalent to $\vec{A}\cdot(\vec{B}\times\vec{C})$, $\vec{C}\cdot(\vec{A}\times\vec{B})$, and $\vec{B}\cdot(\vec{C}\times\vec{A})$.

The dot product of $\vec{A}$ with the cross product of $\vec{B}$ and $\vec{C}$ is a scalar quantity and can be expressed in determinant form as:

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Expand the determinant by minors to obtain:

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = a_x b_y c_z - a_x b_z c_y + a_y b_z c_x - a_y b_x c_z + a_z b_x c_y - a_z b_y c_x \quad (1)$$

Evaluate the cross product of $\vec{B}$ and $\vec{C}$ to obtain:

$$\vec{B}\times\vec{C} = \left(b_y c_z - b_z c_y\right)\hat{i} + \left(b_z c_x - b_x c_z\right)\hat{j} + \left(b_x c_y - b_y c_x\right)\hat{k}$$

Form the dot product of $\vec{A}$ with $\vec{B}\times\vec{C}$ to obtain:

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = a_x b_y c_z - a_x b_z c_y + a_y b_z c_x - a_y b_x c_z + a_z b_x c_y - a_z b_y c_x \quad (2)$$

Because (1) and (2) are the same, we can conclude that:

$$\boxed{\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}}$$

Proceed as above to establish that:

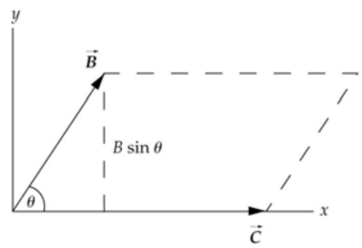
$$\boxed{\vec{C}\cdot(\vec{A}\times\vec{B}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}}$$

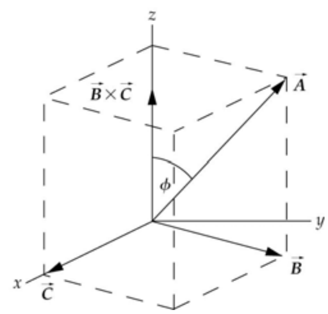
and

$$\boxed{\vec{B}\cdot(\vec{C}\times\vec{A})=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}}$$

41 ••

Picture the Problem Let, without loss of generality, the vector $\vec{C}$ lie along the *x* axis and the vector $\vec{B}$ lie in the *xy* plane as shown below to the left. The diagram to the right shows the parallelepiped spanned by the three vectors. We can apply the definitions of the cross- and dot-products to show that $\vec{A}\cdot(\vec{B}\times\vec{C})$ is the volume of the parallelepiped.





Express the cross-product of $\vec{B}$ and $\vec{C}$:

$$\vec{B}\times\vec{C}=(BC\sin\theta)(-\hat{k})$$

and

$$\left|\vec{B}\times\vec{C}\right|=(B\sin\theta)C$$

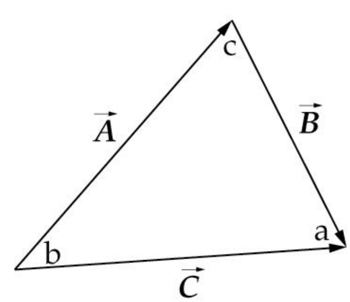
$$=\text{area of the parallelogram}$$

Form the dot-product of $\vec{A}$ with the cross-product of $\vec{B}$ and $\vec{C}$ to obtain:

$$\begin{aligned}\vec{A}\cdot(\vec{B}\times\vec{C})&=A(B\sin\theta)C\cos\phi\\&=(BC\sin\theta)(A\cos\phi)\\&=(\text{area of base})(\text{height})\\&=\boxed{V_{\text{parallelepiped}}}\end{aligned}$$

***42 ••**

Picture the Problem Draw the triangle using the three vectors as shown below. Note that $\vec{A}+\vec{B}=\vec{C}$. We can find the magnitude of the cross product of $\vec{A}$ and $\vec{B}$ and of $\vec{A}$ and $\vec{C}$ and then use the cross product of $\vec{A}$ and $\vec{C}$, using $\vec{A}+\vec{B}=\vec{C}$, to show that $AC\sin b=AB\sin c$ or $B/\sin b=C/\sin c$. Proceeding similarly, we can extend the law of sines to the third side of the triangle and the angle opposite it.



Express the magnitude of the cross product of $\vec{A}$ and $\vec{B}$: $\left|\vec{A}\times\vec{B}\right| = AB\sin c$

Express the magnitude of the cross product of $\vec{A}$ and $\vec{C}$: $\left|\vec{A}\times\vec{C}\right| = AC\sin b$

Form the cross product of $\vec{A}$ with $\vec{C}$ to obtain:

$$\vec{A}\times\vec{C} = \vec{A}\times\left(\vec{A}+\vec{B}\right) = \vec{A}\times\vec{A}+\vec{A}\times\vec{B} = \vec{A}\times\vec{B}$$

because $\vec{A}\times\vec{A} = 0$.

Because $\vec{A}\times\vec{C} = \vec{A}\times\vec{B}$:

$$\left|\vec{A}\times\vec{C}\right| = \left|\vec{A}\times\vec{B}\right|$$

and

$$AC\sin b = AB\sin c$$

Simplify and rewrite this expression to obtain:

$$\boxed{\frac{B}{\sin b} = \frac{C}{\sin c}}$$

Proceed similarly to extend this result to the law of sines:

$$\boxed{\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}}$$

Angular Momentum

43 •

Picture the Problem $\vec{L}$ and $\vec{p}$ are related according to $\vec{L} = \vec{r}\times\vec{p}$. If $\vec{L} = 0$, then examination of the magnitude of $\vec{r}\times\vec{p}$ will allow us to conclude that $\sin\phi = 0$ and that the particle is moving either directly toward the point, directly away from the point, or through the point.

Because $\vec{L} = 0$:

$$\vec{r}\times\vec{p} = \vec{r}\times m\vec{v} = m\vec{r}\times\vec{v} = 0$$

or

$$\vec{r}\times\vec{v} = 0$$

Express the magnitude of $\vec{r}\times\vec{v}$: $\left|\vec{r}\times\vec{v}\right| = rv\sin\phi = 0$

Because neither r nor v is zero: $\sin\phi = 0$

where ϕ is the angle between $\vec{r}$ and $\vec{v}$.

Solve for ϕ: $\phi = \sin^{-1}0 = \boxed{0° \text{ or } 180°}$

44 •

Picture the Problem We can use their definitions to calculate the angular momentum and moment of inertia of the particle and the relationship between L, I, and ω to determine its angular speed.

(*a*) Express and evaluate the magnitude of $\vec{L}$:

$$L = mvr = (2\,\text{kg})(3.5\,\text{m/s})(4\,\text{m}) = \boxed{28.0\,\text{kg}\cdot\text{m}^2/\text{s}}$$

(*b*) Express the moment of inertia of the particle with respect to an axis through the center of the circle in which it is moving:

$$I = mr^2 = (2\,\text{kg})(4\,\text{m})^2 = \boxed{32\,\text{kg}\cdot\text{m}^2}$$

(*c*) Relate the angular speed of the particle to its angular momentum and solve for and evaluate ω:

$$\omega = \frac{L}{I} = \frac{28.0\,\text{kg}\cdot\text{m}^2/\text{s}}{32\,\text{kg}\cdot\text{m}^2} = \boxed{0.875\,\text{rad/s}^2}$$

45 •

Picture the Problem We can use the definition of angular momentum to calculate the angular momentum of this particle and the relationship between its angular momentum and angular speed to describe the variation in its angular speed with time.

(*a*) Express the angular momentum of the particle as a function of its mass, speed, and distance of its path from the reference point:

$$L = rmv\sin\theta = (6\,\text{m})(2\,\text{kg})(4.5\,\text{m/s})\sin 90° = \boxed{54.0\,\text{kg}\cdot\text{m}^2/\text{s}}$$

(*b*) Because $L = mr^2\omega$:

$$\omega \propto \frac{1}{r^2}$$ and

$\boxed{\omega \text{ increases as the particle approaches the point and decreases as it recedes.}}$

***46** ••

Picture the Problem We can use the formula for the area of a triangle to find the area swept out at $t = t_1$, add this area to the area swept out in time dt, and then differentiate this expression with respect to time to obtain the given expression for dA/dt.

Express the area swept out at $t = t_1$:

$$A_1 = \tfrac{1}{2}br_1\cos\theta_1 = \tfrac{1}{2}bx_1$$

where θ_1 is the angle between $\vec{r}_1$ and $\vec{v}$ and

x_1 is the component of $\vec{r}_1$ in the direction of $\vec{v}$.

Express the area swept out at $t = t_1 + dt$:

$$A = A_1 + dA = \tfrac{1}{2}b(x_1 + dx) = \tfrac{1}{2}b(x_1 + vdt)$$

Differentiate with respect to t:

$$\frac{dA}{dt} = \tfrac{1}{2}b\frac{dx}{dt} = \tfrac{1}{2}bv = \text{constant}$$

Because $r\sin\theta = b$:

$$\tfrac{1}{2}bv = \tfrac{1}{2}(r\sin\theta)v = \frac{1}{2m}(rp\sin\theta) = \boxed{\frac{L}{2m}}$$

47 ••

Picture the Problem We can find the total angular momentum of the coin from the sum of its spin and orbital angular momenta.

(*a*) Express the spin angular momentum of the coin:

$$L_{\text{spin}} = I_{\text{cm}}\omega_{\text{spin}}$$

From Problem 9-44:

$$I = \tfrac{1}{4}MR^2$$

Substitute for I to obtain:

$$L_{\text{spin}} = \tfrac{1}{4}MR^2\omega_{\text{spin}}$$

Substitute numerical values and evaluate L_{spin}:

$$L_{\text{spin}} = \tfrac{1}{4}(0.015\,\text{kg})(0.0075\,\text{m})^2 \times \left(10\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}\right) = \boxed{1.33\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

(*b*) Express and evaluate the total angular momentum of the coin:

$$L = L_{\text{orbit}} + L_{\text{spin}} = 0 + L_{\text{spin}} = \boxed{1.33\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

(*c*) From Problem 10-14:

$$L_{\text{orbit}} = 0$$

and

$$L = \boxed{1.33\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

(*d*) Express the total angular momentum of the coin:

$$L = L_{\text{orbit}} + L_{\text{spin}}$$

Find the orbital momentum of the coin:

$$L_{\text{orbit}} = \pm MvR = \pm(0.015\,\text{kg})(0.05\,\text{m/s})(0.1\,\text{m}) = \pm 7.50\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}$$

where the ± is a consequence of the fact that the coin's direction is not specified.

Substitute to obtain:

$$L = \pm 7.50\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s} + 1.33\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}$$

The possible values for L are:

$$L = \boxed{8.83\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

or

$$L = \boxed{-6.17\times10^{-5}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

48 ••

Picture the Problem Both the forces acting on the particles exert torques with respect to an axis perpendicular to the page and through point O and the net torque about this axis is their vector sum.

Express the net torque about an axis perpendicular to the page and through point O:

$$\vec{\boldsymbol{\tau}}_{\text{net}} = \sum_i \vec{\boldsymbol{\tau}}_i = \vec{\mathbf{r}}_1\times\vec{\mathbf{F}}_1 + \vec{\mathbf{r}}_2\times\vec{\mathbf{F}}_2 = (\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2)\times\vec{\mathbf{F}}_1$$

because $\vec{F}_2 = -\vec{F}_1$

Because $\vec{r}_1 - \vec{r}_2$ points along $-\vec{F}_1$:

$$\boxed{(\vec{r}_1 - \vec{r}_2)\times\vec{F}_1 = 0}$$

Torque and Angular Momentum

49 •

Picture the Problem The angular momentum of the particle changes because a *net* torque acts on it. Because we know how the angular momentum depends on time, we can find the net torque acting on the particle by differentiating its angular momentum. We can use a constant-acceleration equation and Newton's 2nd law to relate the angular speed of the particle to its angular acceleration.

(*a*) Relate the magnitude of the torque acting on the particle to the rate at which its angular momentum changes:

$$\tau_{\text{net}} = \frac{dL}{dt} = \frac{d}{dt}\left[(4\,\text{N}\cdot\text{m})t\right] = \boxed{4.00\,\text{N}\cdot\text{m}}$$

(*b*) Using a constant-acceleration equation, relate the angular speed of the particle to its acceleration and time-in-motion:

$$\omega = \omega_0 + \alpha t$$

where $\omega_0 = 0$

Use Newton's 2nd law to relate the angular acceleration of the particle to
the net torque acting on it:

$$\alpha = \frac{\tau_{net}}{I} = \frac{\tau_{net}}{mr^2}$$

Substitute to obtain:

$$\omega = \frac{\tau_{net}}{mr^2} t$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{(4\ \text{N} \cdot \text{m})t}{(1.8\,\text{kg})(3.4\,\text{m})^2} = \boxed{(0.192\,\text{rad/s}^2)t}$$

provided t is in seconds.

50 ••

Picture the Problem The angular momentum of the cylinder changes because a *net* torque acts on it. We can find the angular momentum at $t = 25$ s from its definition and the *net* torque acting on the cylinder from the rate at which the angular momentum is changing. The magnitude of the frictional force acting on the rim can be found using the definition of torque.

(*a*) Use its definition to express the angular momentum of the cylinder:

$$L = I\omega = \tfrac{1}{2}mr^2\omega$$

Substitute numerical values and evaluate L:

$$L = \tfrac{1}{2}(90\,\text{kg})(0.4\,\text{m})^2 \times \left(500\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}\right) = \boxed{377\,\text{kg} \cdot \text{m}^2/\text{s}}$$

(*b*) Express and evaluate $\frac{dL}{dt}$:

$$\frac{dL}{dt} = \frac{(377\,\text{kg} \cdot \text{m}^2/\text{s})}{25\,\text{s}} = \boxed{15.1\,\text{kg} \cdot \text{m}^2/\text{s}^2}$$

(*c*) Because the torque acting on the uniform cylinder is constant, the rate

$$\tau = \frac{dL}{dt} = \boxed{15.1\,\text{kg} \cdot \text{m}^2/\text{s}^2}$$

of change of the angular momentum is constant and hence the instantaneous rate of change of the angular momentum at any instant is equal to the average rate of change over the time during which the torque acts:

(*d*) Using the definition of torque that relates the applied force to its lever arm, express the magnitude of the frictional force *f* acting on the rim:

$$f = \frac{\tau}{\ell} = \frac{15.1\,\text{kg}\cdot\text{m}^2/\text{s}^2}{0.4\,\text{m}} = \boxed{37.7\,\text{N}}$$

***51** ••

Picture the Problem Let the system include the pulley, string, and the blocks and assume that the mass of the string is negligible. The angular momentum of this system changes because a *net* torque acts on it.

(*a*) Express the net torque about the center of mass of the pulley:

$$\tau_{\text{net}} = Rm_2 g\sin\theta - Rm_1 g = \boxed{Rg(m_2\sin\theta - m_1)}$$

where we have taken clockwise to be positive to be consistent with a positive upward velocity of the block whose mass is m_1 as indicated in the figure.

(*b*) Express the total angular momentum of the system about an axis through the center of the pulley:

$$L = I\omega + m_1 vR + m_2 vR = \boxed{vR\left(\frac{I}{R^2} + m_1 + m_2\right)}$$

(*c*) Express τ as the time derivative of the angular momentum:

$$\tau = \frac{dL}{dt} = \frac{d}{dt}\left[vR\left(\frac{I}{R^2} + m_1 + m_2\right)\right] = aR\left(\frac{I}{R^2} + m_1 + m_2\right)$$

Equate this result to that of part (*a*) and solve for *a* to obtain:

$$a = \boxed{\frac{g(m_2\sin\theta - m_1)}{\dfrac{I}{R^2} + m_1 + m_2}}$$

52 ••

Picture the Problem The forces resulting from the release of gas from the jets will exert a torque on the spaceship that will slow and eventually stop its rotation. We can relate this net torque to the angular momentum of the spaceship and to the time the jets must fire.

Relate the firing time of the jets to the desired change in angular momentum:

$$\Delta t = \frac{\Delta L}{\tau_{\text{net}}} = \frac{I\Delta\omega}{\tau_{\text{net}}}$$

Express the magnitude of the net torque exerted by the jets:

$$\tau_{\text{net}} = 2FR$$

Letting $\Delta m/\Delta t'$ represent the mass of gas per unit time exhausted from the jets, relate the force exerted by the gas on the spaceship to the rate at which the gas escapes:

$$F = \frac{\Delta m}{\Delta t'}v$$

Substitute and solve for Δt to obtain:

$$\Delta t = \frac{I\Delta\omega}{2\dfrac{\Delta m}{\Delta t'}vR}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{\left(4000\,\text{kg}\cdot\text{m}^2\right)\left(6\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\,\text{rad}}{\text{rev}}\times\dfrac{1\,\text{min}}{60\,\text{s}}\right)}{2\left(10^{-2}\,\text{kg/s}\right)\left(800\,\text{m/s}\right)\left(3\,\text{m}\right)} = \boxed{52.4\,\text{s}}$$

53 ••

Picture the Problem We can use constant-acceleration equations to express the projectile's position and velocity coordinates as functions of time. We can use these coordinates to express the particle's position and velocity vectors $\vec{r}$ and $\vec{v}$. Using its definition, we can express the projectile's angular momentum $\vec{L}$ as a function of time and then differentiate this expression to obtain $d\vec{L}/dt$. Finally, we can use the definition of the torque, relative to an origin located at the launch position, the gravitational force exerts on the projectile to express $\vec{\tau}$ and complete the demonstration that $d\vec{L}/dt = \vec{\tau}$.

Using its definition, express the angular momentum vector $\vec{L}$ of the projectile:

$$\vec{L} = \vec{r}\times m\vec{v} \qquad (1)$$

Using constant-acceleration

$$x = v_{0x}t = \left(V\cos\theta\right)t$$

equations, express the position coordinates of the projectile as a function of time:

and

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = (V\sin\theta)t - \tfrac{1}{2}gt^2$$

Express the projectile's position vector $\vec{r}$:

$$\vec{r} = [(V\cos\theta)t]\hat{i} + \left[(V\sin\theta)t - \tfrac{1}{2}gt^2\right]\hat{j}$$

Using constant-acceleration equations, express the velocity of the projectile as a function of time:

$$v_x = v_{0x} = V\cos\theta$$

and

$$v_y = v_{0y} + a_y t = V\sin\theta - gt$$

Express the projectile's velocity vector $\vec{v}$:

$$\vec{v} = [V\cos\theta]\hat{i} + [V\sin\theta - gt]\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{L} = \left\{[(V\cos\theta)t]\hat{i} + \left[(V\sin\theta)t - \tfrac{1}{2}gt^2\right]\hat{j}\right\} \times m\left\{[V\cos\theta]\hat{i} + [V\sin\theta - gt]\hat{j}\right\} = \left(-\tfrac{1}{2}mgt^2V\cos\theta\right)\hat{k}$$

Differentiate $\vec{L}$ with respect to t to obtain:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(-\tfrac{1}{2}mgt^2V\cos\theta\right)\hat{k} = (-mgtV\cos\theta)\hat{k} \quad (2)$$

Using its definition, express the torque acting on the projectile:

$$\vec{\tau} = \vec{r}\times(-mg)\hat{j} = [(V\cos\theta)t]\hat{i} + \left[(V\sin\theta)t - \tfrac{1}{2}gt^2\right]\hat{j} \times (-mg)\hat{j}$$

or

$$\vec{\tau} = (-mgtV\cos\theta)\hat{k} \quad (3)$$

Comparing equations (2) and (3) we see that:

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

Conservation of Angular Momentum

***54** •

Picture the Problem Let m represent the mass of the planet and apply the definition of torque to find the torque produced by the gravitational force of attraction. We can use Newton's 2nd law of motion in the form $\vec{\tau} = d\vec{L}/dt$ to show that $\vec{L}$ is constant and apply conservation of angular momentum to the motion of the planet at points A and B.

(*a*) Express the torque produced by the gravitational force of attraction of the sun for the planet:

$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = \boxed{0}$ because $\vec{\boldsymbol{F}}$ acts along the direction of $\vec{\boldsymbol{r}}$.

(*b*) Because $\vec{\boldsymbol{\tau}} = 0$:

$$\frac{d\vec{\boldsymbol{L}}}{dt} = 0 \Rightarrow \vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times m\vec{\boldsymbol{v}} = \text{constant}$$

Noting that at points *A* and *B* $|\vec{\boldsymbol{r}} \times \vec{\boldsymbol{v}}| = rv$, express the relationship between the distances from the sun and the speeds of the planets:

$$r_1 v_1 = r_2 v_2$$

or

$$\frac{v_1}{v_2} = \boxed{\frac{r_2}{r_1}}$$

55 ••

Picture the Problem Let the system consist of you, the extended weights, and the platform. Because the net external torque acting on this system is zero, its angular momentum remains constant during the pulling in of the weights.

(*a*) Using conservation of angular momentum, relate the initial and final angular speeds of the system to its initial and final moments of inertia:

$$I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f}$$

Solve for ω_f:

$$\omega_\text{f} = \frac{I_\text{i}}{I_\text{f}}\omega_\text{i}$$

Substitute numerical values and evaluate ω_f:

$$\omega_\text{f} = \frac{6\,\text{kg}\cdot\text{m}^2}{1.8\,\text{kg}\cdot\text{m}^2}(1.5\,\text{rev/s}) = \boxed{5.00\,\text{rev/s}}$$

(*b*) Express the change in the kinetic energy of the system:

$$\Delta K = K_\text{f} - K_\text{i} = \tfrac{1}{2}I_\text{f}\omega_\text{f}^2 - \tfrac{1}{2}I_\text{i}\omega_\text{i}^2$$

Substitute numerical values and evaluate ΔK:

$$\Delta K = \tfrac{1}{2}(1.8\,\text{kg}\cdot\text{m}^2)\left(5\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}\right)^2 - \tfrac{1}{2}(6\,\text{kg}\cdot\text{m}^2)\left(1.5\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}\right)^2 = \boxed{622\,\text{J}}$$

(*c*) Because no external agent does work on the system, the energy comes from the internal energy of the man.

***56 ••**

Picture the Problem Let the system consist of the blob of putty and the turntable. Because the net external torque acting on this system is zero, its angular momentum remains constant when the blob of putty falls onto the turntable.

(*a*) Using conservation of angular momentum, relate the initial and final angular speeds of the turntable to its initial and final moments of inertia and solve for ω_f:

$$I_0\omega_i = I_f\omega_f$$

and

$$\omega_f = \frac{I_0}{I_f}\omega_i$$

Express the final rotational inertia of the turntable-plus-blob:

$$I_f = I_0 + I_{blob} = I_0 + mR^2$$

Substitute and simplify to obtain:

$$\omega_f = \frac{I_0}{I_0 + mR^2}\omega_i = \boxed{\frac{1}{1+\frac{mR^2}{I_0}}\omega_i}$$

(*b*) If the blob flies off tangentially to the turntable, its angular momentum doesn't change (with respect to an axis through the center of turntable). Because there is no external torque acting on the blob-turntable system, the total angular momentum of the system will remain constant and the angular momentum of the turntable will not change. Because the moment of inertia of the table hasn't changed either, the turntable will continue to spin at $\boxed{\omega' = \omega_f}$.

57 ••

Picture the Problem Because the net external torque acting on the Lazy Susan-cockroach system is zero, the net angular momentum of the system is constant (equal to zero because the Lazy Susan is initially at rest) and we can use conservation of angular momentum to find the angular velocity ω of the Lazy Susan. The speed of the cockroach relative to the floor v_f is the difference between its speed with respect to the Lazy Susan and the speed of the Lazy Susan at the location of the cockroach with respect to the floor.

Relate the speed of the cockroach with respect to the floor v_f to the speed of the Lazy Susan at the location of the cockroach:

$$v_f = v - \omega r \qquad (1)$$

Use conservation of angular momentum to obtain:

$$L_{LS} - L_C = 0$$

Express the angular momentum of the Lazy Susan:

$$L_{\mathrm{LS}} = I_{\mathrm{LS}}\omega = \tfrac{1}{2}MR^2\omega$$

Express the angular momentum of the cockroach:

$$L_{\mathrm{C}} = I_{\mathrm{C}}\omega_{\mathrm{C}} = mr^2\left(\frac{v}{r} - \omega\right)$$

Substitute to obtain:

$$\tfrac{1}{2}MR^2\omega - mr^2\left(\frac{v}{r} - \omega\right) = 0$$

Solve for ω to obtain:

$$\omega = \frac{2mrv}{MR^2 + 2mr^2}$$

Substitute in equation (1):

$$v_{\mathrm{f}} = v - \frac{2mr^2 v}{MR^2 + 2mr^2}$$

Substitute numerical values and evaluate v_{f}:

$$v_{\mathrm{f}} = 0.01\,\mathrm{m/s} - \frac{2(0.015\,\mathrm{kg})(0.08\,\mathrm{m})^2(0.01\,\mathrm{m/s})}{(0.25\,\mathrm{m})(0.15\,\mathrm{m})^2 + 2(0.015\,\mathrm{kg})(0.08\,\mathrm{m})^2} = \boxed{9.67\,\mathrm{mm/s}}$$

***58 ••**

Picture the Problem The net external torque acting on this system is zero and so we know that angular momentum is conserved as these disks are brought together. Let the numeral 1 refer to the disk to the left and the numeral 2 to the disk to the right. Let the angular momentum of the disk with the larger radius be positive.

Using conservation of angular momentum, relate the initial angular speeds of the disks to their common final speed and to their moments of inertia:

$$I_{\mathrm{i}}\omega_{\mathrm{i}} = I_{\mathrm{f}}\omega_{\mathrm{f}}$$

or

$$I_1\omega_0 - I_2\omega_0 = (I_1 + I_2)\omega_{\mathrm{f}}$$

Solve for ω_{f}:

$$\omega_{\mathrm{f}} = \frac{I_1 - I_2}{I_1 + I_2}\omega_0$$

Express I_1 and I_2:

$$I_1 = \tfrac{1}{2}m(2r)^2 = 2mr^2$$

and

$$I_2 = \tfrac{1}{2}mr^2$$

Substitute and simplify to obtain:

$$\omega_{\mathrm{f}} = \frac{2mr^2 - \frac{1}{2}mr^2}{2mr^2 + \frac{1}{2}mr^2}\omega_0 = \boxed{\tfrac{3}{5}\omega_0}$$

59 ••

Picture the Problem We can express the angular momentum and kinetic energy of the block directly from their definitions. The tension in the string provides the centripetal force required for the uniform circular motion and can be expressed using Newton's 2nd law. Finally, we can use the work-kinetic energy theorem to express the work required to reduce the radius of the circle by a factor of two.

(*a*) Express the initial angular momentum of the block:

$$L_0 = \boxed{r_0 m v_0}$$

(*b*) Express the initial kinetic energy of the block:

$$K_0 = \boxed{\tfrac{1}{2} m v_0^2}$$

(*c*) Using Newton's 2nd law, relate the tension in the string to the centripetal force required for the circular motion:

$$T = F_c = \boxed{m\frac{v_0^2}{r_0}}$$

Use the work-kinetic energy theorem to relate the required work to the change in the kinetic energy of the block:

$$W = \Delta K = K_f - K_0 = \frac{L_f^2}{2I_f} - \frac{L_0^2}{2I_0}$$
$$= \frac{L_0^2}{2I_f} - \frac{L_0^2}{2I_0} = \frac{L_0^2}{2}\left(\frac{1}{I_f - I_0}\right)$$
$$= \frac{L_0^2}{2}\left(\frac{1}{m\left(\frac{1}{2}r_0\right)^2 - mr_0^2}\right) = -\frac{2}{3}\frac{L_0^2}{mr_0^2}$$

Substitute the result from part (*a*) and simplify to obtain:

$$W = \boxed{-\tfrac{2}{3} m v_0^2}$$

***60** ••

Picture the Problem Because the force exerted by the rubber band is parallel to the position vector of the point mass, the net external torque acting on it is zero and we can use the conservation of angular momentum to determine the speeds of the ball at points *B* and *C*. We'll use mechanical energy conservation to find *b* by relating the kinetic and elastic potential energies at *A* and *B*.

(*a*) Use conservation of momentum to relate the angular momenta at points *A*, *B* and *C*:

$$L_A = L_B = L_C$$

or

$$mv_A r_A = mv_B r_B = mv_C r_C$$

Solve for v_B in terms of v_A:

$$v_B = v_A \frac{r_A}{r_B}$$

Substitute numerical values and evaluate v_B:

$$v_B = (4\,\text{m/s})\frac{0.6\,\text{m}}{1\,\text{m}} = \boxed{2.40\,\text{m/s}}$$

Solve for v_C in terms of v_A:

$$v_C = v_A \frac{r_A}{r_C}$$

Substitute numerical values and evaluate v_C:

$$v_C = (4\,\text{m/s})\frac{0.6\,\text{m}}{0.6\,\text{m}} = \boxed{4.00\,\text{m/s}}$$

(*b*) Use conservation of mechanical energy between points *A* and *B* to relate the kinetic energy of the point mass and the energy stored in the stretched rubber band:

$$E_A = E_B$$

or

$$\tfrac{1}{2}mv_A^2 + \tfrac{1}{2}br_A^2 = \tfrac{1}{2}mv_B^2 + \tfrac{1}{2}br_B^2$$

Solve for b:

$$b = \frac{m\left(v_B^2 - v_A^2\right)}{r_A^2 - r_B^2}$$

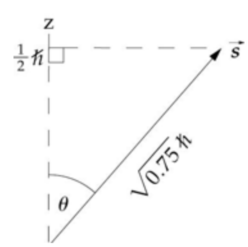
Substitute numerical values and evaluate b:

$$b = \frac{(0.2\,\text{kg})\left[(2.4\,\text{m/s})^2 - (4\,\text{m/s})^2\right]}{(0.6\,\text{m})^2 - (1\,\text{m})^2} = \boxed{3.20\,\text{N/m}}$$

Quantization of Angular Momentum

***61** •

Picture the Problem The electron's spin angular momentum vector is related to its *z* component as shown in the diagram.



Using trigonometry, relate the magnitude of $\vec{s}$ to its z component:

$$\theta = \cos^{-1}\frac{\tfrac{1}{2}\hbar}{\sqrt{0.75}\hbar} = \boxed{54.7°}$$

62 ••

Picture the Problem Equation 10-27*a* describes the quantization of rotational energy. We can show that the energy difference between a given state and the next higher state is proportional to $\ell + 1$ by using Equation 10-27*a* to express the energy difference.

From Equation 10-27*a* we have:

$$K_\ell = \ell(\ell+1)E_{0\mathrm{r}}$$

Using this equation, express the difference between one rotational state and the next higher state:

$$\Delta E = (\ell+1)(\ell+2)E_{0\mathrm{r}} - \ell(\ell+1)E_{0\mathrm{r}}$$
$$= \boxed{2(\ell+1)E_{0\mathrm{r}}}$$

63 ••

Picture the Problem The rotational energies of HBr molecule are related to ℓ and $E_{0\mathrm{r}}$ according to $K_\ell = \ell(\ell+1)E_{0\mathrm{r}}$ where $E_{0\mathrm{r}} = \hbar^2/2I$.

(*a*) Express and evaluate the moment of inertia of the H atom:

$$I = m_\mathrm{p}r^2$$
$$= (1.67\times10^{-27}\ \mathrm{kg})(0.144\times10^{-9}\ \mathrm{m})^2$$
$$= \boxed{3.46\times10^{-47}\ \mathrm{kg\cdot m^2}}$$

(*b*) Relate the rotational energies to ℓ and $E_{0\mathrm{r}}$:

$$K_\ell = \ell(\ell+1)E_{0\mathrm{r}}$$

Evaluate $E_{0\mathrm{r}}$:

$$E_{0\mathrm{r}} = \frac{\hbar^2}{2I} = \frac{(1.05\times10^{-34}\ \mathrm{J\cdot s})^2}{2(3.46\times10^{-47}\ \mathrm{kg\cdot m^2})}$$
$$= 1.59\times10^{-22}\ \mathrm{J}\times\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}$$
$$= 0.996\,\mathrm{meV}$$

Evaluate E for $\ell = 1$:

$$E_1 = (1+1)(0.996\,\mathrm{meV}) = \boxed{1.99\,\mathrm{meV}}$$

Evaluate E for $\ell = 2$:

$$E_2 = 2(2+1)(0.996\,\mathrm{meV})$$
$$= \boxed{5.98\,\mathrm{meV}}$$

Evaluate E for $\ell = 3$:

$$E_3 = 3(3+1)(0.996\,\mathrm{meV})$$
$$= \boxed{12.0\,\mathrm{meV}}$$

64 ••

Picture the Problem We can use the definition of the moment of inertia of point particles to calculate the rotational inertia of the nitrogen molecule. The rotational energies of nitrogen molecule are related to ℓ and $E_{0\mathrm{r}}$ according to $K_\ell = \ell(\ell+1)E_{0\mathrm{r}}$ where $E_{0\mathrm{r}} = \hbar^2/2I$.

(*a*) Using a rigid dumbbell model, express and evaluate the moment of inertia of the nitrogen molecule about its center of mass:

$$I=\sum_{\text{i}} m_{\text{i}}r_{\text{i}}^2 = m_{\text{N}}r^2+m_{\text{N}}r^2$$
$$=2m_{\text{N}}r^2$$

Substitute numerical values and evaluate *I*:

$$I=2(14)\left(1.66\times10^{-27}\text{ kg}\right)\left(5.5\times10^{-11}\text{ m}\right)^2$$
$$=\boxed{1.41\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

(*b*) Relate the rotational energies to ℓ and $E_{0\text{r}}$:

$$E_\ell=\ell(\ell+1)E_{0\text{r}}$$

Evaluate $E_{0\text{r}}$:

$$E_{0\text{r}}=\frac{\hbar^2}{2I}=\frac{\left(1.05\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.41\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}$$
$$=3.91\times10^{-23}\text{ J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{ J}}$$
$$=0.244\text{ meV}$$

Substitute to obtain:

$$E_\ell=\boxed{0.244\ell(\ell+1)\text{ meV}}$$

***65 ••**

Picture the Problem We can obtain an expression for the speed of the nitrogen molecule by equating its translational and rotational kinetic energies and solving for *v*. Because this expression includes the moment of inertia *I* of the nitrogen molecule, we can use the definition of the moment of inertia to express *I* for a dumbbell model of the nitrogen molecule. The rotational energies of a nitrogen molecule depend on the quantum number ℓ according to $E_\ell=L^2/2I=\ell(\ell+1)\hbar^2/2I$.

Equate the rotational kinetic energy of the nitrogen molecule in its $\ell=1$ quantum state and its translational kinetic energy:

$$E_1=\tfrac{1}{2}m_{\text{N}}v^2 \qquad (1)$$

Express the rotational energy levels of the nitrogen molecule:

$$E_\ell=\frac{L^2}{2I}=\frac{\ell(\ell+1)\hbar^2}{2I}$$

For $\ell=1$:

$$E_1=\frac{1(1+1)\hbar^2}{2I}=\frac{\hbar^2}{I}$$

Substitute in equation (1):

$$\frac{\hbar^2}{I}=\tfrac{1}{2}m_{\mathrm{N}}v^2$$

Solve for v to obtain:

$$v=\sqrt{\frac{2\hbar^2}{m_{\mathrm{N}}I}} \qquad (2)$$

Using a rigid dumbbell model, express the moment of inertia of the nitrogen molecule about its center of mass:

$$I=\sum_{\mathrm{i}}m_{\mathrm{i}}r_{\mathrm{i}}^2=m_{\mathrm{N}}r^2+m_{\mathrm{N}}r^2=2m_{\mathrm{N}}r^2$$

and

$$m_{\mathrm{N}}I=2m_{\mathrm{N}}^2r^2$$

Substitute in equation (2):

$$v=\sqrt{\frac{2\hbar^2}{2m_{\mathrm{N}}^2r^2}}=\frac{\hbar}{m_{\mathrm{N}}r}$$

Substitute numerical values and evaluate v:

$$v=\frac{1.055\times10^{-34}\ \mathrm{J\cdot s}}{14\left(1.66\times10^{-27}\ \mathrm{kg}\right)\left(5.5\times10^{-11}\ \mathrm{m}\right)}=\boxed{82.5\ \mathrm{m/s}}$$

Collision Problems

66 ••

Picture the Problem Let the zero of gravitational potential energy be at the elevation of the rod. Because the net external torque acting on this system is zero, we know that angular momentum is conserved in the collision. We'll use the definition of angular momentum to express the angular momentum just after the collision and conservation of mechanical energy to determine the speed of the ball just before it makes its perfectly inelastic collision with the rod.

Use conservation of angular momentum to relate the angular momentum before the collision to the angular momentum just after the perfectly inelastic collision:

$$L_{\mathrm{f}}=L_{\mathrm{i}}$$
$$=mvr$$

Use conservation of mechanical energy to relate the kinetic energy of the ball just before impact to its initial potential energy:

$$K_{\mathrm{f}}-K_{\mathrm{i}}+U_{\mathrm{f}}-U_{\mathrm{i}}=0$$

or, because $K_{\mathrm{i}} = U_{\mathrm{f}} = 0$,

$$K_{\mathrm{f}}-U_{\mathrm{i}}=0$$

Letting h represent the distance the

$$v=\sqrt{2gh}$$

ball falls, substitute for K_f and U_i and solve for v to obtain:

Substitute for v to obtain:

$$L_f = mr\sqrt{2gh}$$

Substitute numerical values and evaluate L_f:

$$L_f = (3.2\,\text{kg})(0.9\,\text{m})\sqrt{2(9.81\,\text{m/s}^2)(1.2\,\text{m})}$$
$$= \boxed{14.0\,\text{J}\cdot\text{s}}$$

***67 ••**

Picture the Problem Because there are no external forces or torques acting on the system defined in the problem statement, both linear and angular momentum are conserved in the collision and the velocity of the center of mass after the collision is the same as before the collision. Let the direction the blob of putty is moving initially be the positive x direction and toward the top of the page in the figure be the positive y direction.

Using its definition, express the location of the center of mass relative to the center of the bar:

$$y_{\text{cm}} = \frac{md}{M+m} \text{ below the center of the bar.}$$

Using its definition, express the velocity of the center of mass:

$$v_{\text{cm}} = \boxed{\frac{mv}{M+m}}$$

Using the definition of L in terms of I and ω, express ω:

$$\omega = \frac{L_{\text{cm}}}{I_{\text{cm}}} \qquad (1)$$

Express the angular momentum about the center of mass:

$$L_{\text{cm}} = mv(d - y_{\text{cm}})$$
$$= mv\left(d - \frac{md}{M+m}\right) = \frac{mMvd}{M+m}$$

Using the parallel axis theorem, express the moment of inertia of the system relative to its center of mass:

$$I_{\text{cm}} = \tfrac{1}{12}ML^2 + My_{\text{cm}}^2 + m(d - y_{\text{cm}})^2$$

Substitute for y_{cm} and simplify to obtain:

$$I_{\rm cm} = \tfrac{1}{12}ML^2 + M\left(\frac{md}{M+m}\right)^2 + m\left(d - \frac{md}{M+m}\right)^2$$

$$= \tfrac{1}{12}ML^2 + \frac{Mm^2d^2}{(M+m)^2} + m\left(\frac{d(M+m)-md}{M+m}\right)^2$$

$$= \tfrac{1}{12}ML^2 + \frac{Mm^2d^2}{(M+m)^2} + \frac{mM^2d^2}{(M+m)^2} = \tfrac{1}{12}ML^2 + \frac{(M+m)mMd^2}{(M+m)^2}$$

$$= \tfrac{1}{12}ML^2 + \frac{mMd^2}{M+m}$$

Substitute for $I_{\rm cm}$ and $L_{\rm cm}$ in equation (1) and simplify to obtain:

$$\boxed{\omega = \frac{mMvd}{\frac{1}{12}ML^2(M+m) + Mmd^2}}$$

Remarks: You can verify the expression for $I_{\rm cm}$ by letting $m \to 0$ to obtain $I_{cm} = \frac{1}{12}ML^2$ and letting $M \to 0$ to obtain $I_{\rm cm} = 0$.

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Picture the Problem Because there are no external forces or torques acting on the system defined in the statement of Problem 67, both linear and angular momentum are conserved in the collision and the velocity of the center of mass after the collision is the same as before the collision. Kinetic energy is also conserved as the collision of the hard sphere with the bar is elastic. Let the direction the sphere is moving initially be the positive x direction and toward the top of the page in the figure be the positive y direction and v' and V' be the final velocities of the objects whose masses are m and M, respectively.

Apply conservation of linear momentum to obtain:

$$p_{\rm i} = p_{\rm f}$$
or
$$mv = mv' + MV' \qquad (1)$$

Apply conservation of angular momentum to obtain:

$$L_{\rm i} = L_{\rm f}$$
or
$$mvd = mv'd + \tfrac{1}{12}ML^2\omega \qquad (2)$$

Set $v' = 0$ in equation (1) and solve for V':

$$V' = \frac{mv}{M} \qquad (3)$$

Use conservation of mechanical energy to relate the kinetic energies of translation and rotation before

$$K_{\rm i} = K_{\rm f}$$
or
$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}MV'^2 + \tfrac{1}{2}\left(\tfrac{1}{12}ML^2\right)\omega^2 \qquad (4)$$

and after the elastic collision:

Substitute (2) and (3) in (4) and simplify to obtain:

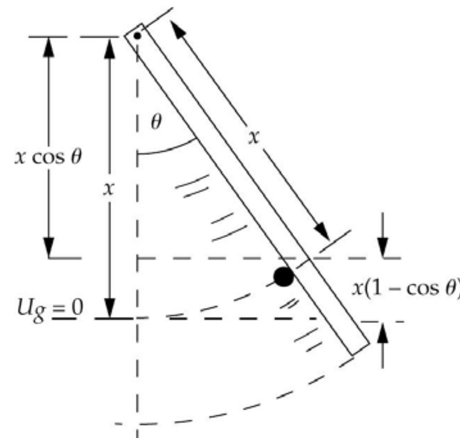
$$1=\frac{m}{M}+\frac{12m}{M}\left(\frac{d^2}{L^2}\right)$$

Solve for d:

$$d=\boxed{L\sqrt{\frac{M-m}{12m}}}$$

69 ••

Picture the Problem Let the zero of gravitational potential energy be a distance *x* below the pivot as shown in the diagram. Because the net external torque acting on the system is zero, angular momentum is conserved in this perfectly inelastic collision. We can also use conservation of mechanical energy to relate the initial kinetic energy of the system after the collision to its potential energy at the top of its swing.



Using conservation of mechanical energy, relate the rotational kinetic energy of the system just after the collision to its gravitational potential energy when it has swung through an angle θ:

$$\Delta K+\Delta U=0$$

or, because $K_f=U_i=0$,

$$-K_i+U_f=0$$

and

$$\tfrac{1}{2}I\omega^2=\left(Mg\frac{d}{2}+mgx\right)(1-\cos\theta)\quad(1)$$

Apply conservation of momentum to the collision:

$$L_i=L_f$$

or

$$0.8dmv=I\omega=\left[\tfrac{1}{3}Md^2+(0.8d)^2m\right]\omega$$

Solve for ω to obtain:

$$\omega=\frac{0.8dmv}{\tfrac{1}{3}Md^2+0.64md^2}\quad(2)$$

Express the moment of inertia of the system about the pivot:

$$I=m(0.8d)^2+\tfrac{1}{3}Md^2$$
$$=0.64md^2+\tfrac{1}{3}Md^2\quad(3)$$

Substitute equations (2) and (3) in equation (1) and simplify to obtain:

$$\left(Mg\frac{d}{2}+mgd\right)(1-\cos\theta)=\frac{0.32(dmv)^2}{\frac{1}{3}Md^2+0.64md^2}$$

Solve for *v*:

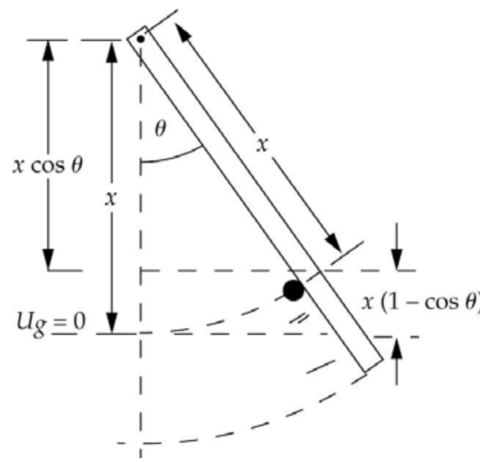
$$v=\sqrt{\frac{(0.5M+0.8m)\left(\frac{1}{3}Md^2+0.64md^2\right)g(1-\cos\theta)}{0.32dm^2}}$$

Evaluate *v* for $\theta = 90°$ to obtain:

$$\boxed{v=\sqrt{\frac{(0.5M+0.8m)\left(\frac{1}{3}Md^2+0.64md^2\right)g}{0.32dm^2}}}$$

70 ••

Picture the Problem Let the zero of gravitational potential energy be a distance *x* below the pivot as shown in the diagram. Because the net external torque acting on the system is zero, angular momentum is conserved in this perfectly inelastic collision. We can also use conservation of mechanical energy to relate the initial kinetic energy of the system after the collision to its potential energy at the top of its swing.



Using conservation of mechanical energy, relate the rotational kinetic energy of the system just after the collision to its gravitational potential energy when it has swung through an angle θ:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

and

$$\tfrac{1}{2}I\omega^2=\left(Mg\frac{d}{2}+mgx\right)(1-\cos\theta) \quad (1)$$

Apply conservation of momentum to the collision:

$$L_i = L_f$$

or

$$0.8dmv = I\omega = \left[\tfrac{1}{3}Md^2+(0.8d)^2m\right]\omega$$

Solve for ω to obtain:

$$\omega = \frac{0.8dmv}{\frac{1}{2}Md^2 + 0.64md^2} \quad (2)$$

Express the moment of inertia of the system about the pivot:

$$\begin{aligned} I &= m(0.8d)^2 + \tfrac{1}{3}Md^2 \\ &= \left(0.64m + \tfrac{1}{3}M\right)d^2 \\ &= \left[0.64(0.3\,\text{kg}) + \tfrac{1}{3}(0.8\,\text{kg})\right](1.2\,\text{m})^2 \\ &= 0.660\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Substitute equation (2) in equation (1) and simplify to obtain:

$$\left(Mg\frac{d}{2} + 0.8dmg\right)(1 - \cos\theta) = \frac{0.32(dmv)^2}{I}$$

Solve for v:

$$v = \sqrt{\frac{g(0.5M + 0.8m)(1 - \cos\theta)I}{0.32dm^2}}$$

Substitute numerical values and evaluate v for $\theta = 60°$ to obtain:

$$v = \sqrt{\frac{(9.81\,\text{m/s}^2)\left[0.5(0.8\,\text{kg}) + 0.8(0.3\,\text{kg})\right](0.5)(0.660\,\text{kg}\cdot\text{m}^2)}{0.32(1.2\,\text{m})(0.3\,\text{kg})^2}} = \boxed{7.74\,\text{m/s}}$$

71 ••

Picture the Problem Let the length of the uniform stick be ℓ. We can use the impulse-change in momentum theorem to express the velocity of the center of mass of the stick. By expressing the velocity V of the end of the stick in terms of the velocity of the center of mass and applying the angular impulse-change in angular momentum theorem we can find the angular velocity of the stick and, hence, the velocity of the end of the stick.

(*a*) Apply the impulse-change in momentum theorem to obtain:

$$K = \Delta p = p - p_0 = p$$

or, because $p_0 = 0$ and $p = Mv_{cm}$,

$$K = Mv_{cm}$$

Solve for v_{cm} to obtain:

$$v_{cm} = \boxed{\frac{K}{M}}$$

(*b*) Relate the velocity V of the end of the stick to the velocity of the center of mass v_{cm}:

$$V = v_{cm} + v_{\text{rel to c of m}} = v_{cm} + \omega\left(\tfrac{1}{2}\ell\right) \quad (1)$$

Relate the angular impulse to the change in the angular momentum of the stick:

$$K\left(\tfrac{1}{2}\ell\right) = \Delta L = L - L_0 = I_{cm}\omega$$

or, because $L_0 = 0$,

$$K\left(\tfrac{1}{2}\ell\right) = I_{cm}\omega$$

Refer to Table 9-1 to find the moment of inertia of the stick with respect to its center of mass:

$$I_{cm} = \tfrac{1}{12}M\ell^2$$

Substitute to obtain:

$$K\left(\tfrac{1}{2}\ell\right) = \tfrac{1}{12}M\ell^2\omega$$

Solve for ω:

$$\omega = \frac{6K}{M\ell}$$

Substitute in equation (1) to obtain:

$$V = \frac{K}{M} + \left(\frac{6K}{M\ell}\right)\frac{\ell}{2} = \boxed{\frac{4K}{M}}$$

(*c*) Relate the velocity V' of the other end of the stick to the velocity of the center of mass v_{cm}:

$$V = v_{cm} - v_{\text{rel to c of m}} = v_{cm} - \omega\left(\tfrac{1}{2}\ell\right)$$
$$= \frac{K}{M} - \left(\frac{6K}{M\ell}\right)\frac{\ell}{2} = \boxed{-\frac{2K}{M}}$$

(*d*) Letting x be the distance from the center of mass toward the end not struck, express the condition that the point at x is at rest:

$$v_{cm} - \omega x = 0$$

Solve for x to obtain:

$$\frac{K}{M} - \frac{6K}{M\ell}x = 0$$

Solve for x to obtain:

$$x = \frac{\frac{K}{M}}{\frac{6K}{M\ell}} = \boxed{\tfrac{1}{6}\ell}$$

Note that for a meter stick struck at the 100-cm mark, the stationary point would be at the 33.3-cm mark.

Remarks: You can easily check this result by placing a meterstick on the floor and giving it a sharp blow at the 100-cm mark.

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Picture the Problem Because the net external torque acting on the system is zero, angular momentum is conserved in this perfectly inelastic collision.

(*a*) Use its definition to express the total angular momentum of the disk and projectile just before impact:

$$L_0 = \boxed{m_p v_0 b}$$

(*b*) Use conservation of angular momentum to relate the angular momenta just before and just after the collision:

$$L_0 = L = I\omega \text{ and } \omega = \frac{L_0}{I}$$

Express the moment of inertia of the disk + projectile:

$$I = \tfrac{1}{2}MR^2 + m_p b^2$$

Substitute for I in the expression for ω to obtain:

$$\omega = \boxed{\frac{2m_p v_0 b}{MR^2 + 2m_p b^2}}$$

(*c*) Express the kinetic energy of the system after impact in terms of its angular momentum:

$$K_f = \frac{L^2}{2I} = \frac{(m_p v_0 b)^2}{2\left(\frac{1}{2}MR^2 + m_p b^2\right)}$$

$$= \boxed{\frac{(m_p v_0 b)^2}{MR^2 + 2m_p b^2}}$$

(*d*) Express the difference between the initial and final kinetic energies, substitute, and simplify to obtain:

$$\Delta E = K_i - K_f$$

$$= \tfrac{1}{2}m_p v_0^2 - \frac{(m_p v_0 b)^2}{MR^2 + 2m_p b^2}$$

$$= \boxed{\tfrac{1}{2}m_p v_0^2\left(1 - \frac{m_p b^2}{MR^2 + 2m_p b^2}\right)}$$

***73 ••**

Picture the Problem Because the net external torque acting on the system is zero, angular momentum is conserved in this perfectly inelastic collision. The rod, on its downward swing, acquires rotational kinetic energy. Angular momentum is conserved in the perfectly inelastic collision with the particle and the rotational kinetic of the after-collision system is then transformed into gravitational potential energy as the rod-plus-particle swing upward. Let the zero of gravitational potential energy be at a distance L_1 below the pivot and use both angular momentum and mechanical energy conservation to relate the distances L_1 and L_2 and the masses M and m.

Use conservation of energy to relate the initial and final potential energy of the rod to its rotational kinetic energy just before it collides with the particle:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_i = 0$,

$$K_f + U_f - U_i = 0$$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega^2 + Mg\frac{L_1}{2} - MgL_1 = 0$$

Solve for ω:

$$\omega = \sqrt{\frac{3g}{L_1}}$$

Letting ω' represent the angular speed of the rod-and-particle system just after impact, use conservation of angular momentum to relate the angular momenta before and after the collision:

$$L_i = L_f$$

or

$$\left(\tfrac{1}{3}ML_1^2\right)\omega = \left(\tfrac{1}{3}ML_1^2 + mL_2^2\right)\omega'$$

Solve for ω':

$$\omega' = \frac{\tfrac{1}{3}ML_1^2}{\tfrac{1}{3}ML_1^2 + mL_2^2}\omega$$

Use conservation of energy to relate the rotational kinetic energy of the rod-plus-particle just after their collision to their potential energy when they have swung through an angle θ_{max}:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-\tfrac{1}{2}I\omega'^2 + Mg\left(\tfrac{1}{2}L_1\right)\left(1-\cos\theta_{max}\right) + mgL_2\left(1-\cos\theta_{max}\right) = 0 \qquad (1)$$

Express the moment of inertia of the system with respect to the pivot:

$$I = \tfrac{1}{3}ML_1^2 + mL_2^2$$

Substitute for θ_{max}, I and ω' in equation (1):

$$\frac{3\frac{g}{L_1}\left(\tfrac{1}{3}ML_1^2\right)^2}{\tfrac{1}{3}ML_1^2 + mL_2^2} = Mg\left(\tfrac{1}{2}L_1\right) + mgL_2$$

Simplify to obtain:

$$L_1^3 = 2\frac{m}{M}L_1^2L_2 + 3L_2^2L_1 + 6\frac{m}{M}L_2^3 \qquad (2)$$

Simplify equation (2) by letting $\alpha = m/M$ and $\beta = L_2/L_1$ to obtain:

$$6\alpha^2\beta^3 + 3\beta^2 + 2\alpha\beta - 1 = 0$$

Substitute for α and simplify to obtain the cubic equation in β:

$$12\beta^3 + 9\beta^2 + 4\beta - 3 = 0$$

Use the solver function* of your calculator to find the only real value

$$\beta = \boxed{0.349}$$

of β:

***Remarks**: **Most graphing calculators have a ″solver″ feature. One can solve the cubic equation using either the ″graph″ and ″trace″ capabilities or the ″solver″ feature. The root given above was found using SOLVER on a TI-85.**

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Picture the Problem Because the net external torque acting on the system is zero, angular momentum is conserved in this perfectly inelastic collision. The rod, on its downward swing, acquires rotational kinetic energy. Angular momentum is conserved in the perfectly inelastic collision with the particle and the rotational kinetic energy of the after-collision system is then transformed into gravitational potential energy as the rod-plus-particle swing upward. Let the zero of gravitational potential energy be at a distance L_1 below the pivot and use both angular momentum and mechanical energy conservation to relate the distances L_1 and L_2 and the mass M to m.

(*a*) Use conservation of energy to relate the initial and final potential energy of the rod to its rotational kinetic energy just before it collides with the particle:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_i = 0$,

$$K_f + U_f - U_i = 0$$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega^2 + Mg\frac{L_1}{2} - MgL_1 = 0$$

Solve for ω:

$$\omega = \sqrt{\frac{3g}{L_1}}$$

Letting ω' represent the angular speed of the system after impact, use conservation of angular momentum to relate the angular momenta before and after the collision:

$$L_i = L_f$$

or

$$\left(\tfrac{1}{3}ML_1^2\right)\omega = \left(\tfrac{1}{3}ML_1^2 + mL_2^2\right)\omega' \qquad (1)$$

Solve for ω':

$$\omega' = \frac{\tfrac{1}{3}ML_1^2}{\tfrac{1}{3}ML_1^2 + mL_2^2}\,\omega = \frac{\tfrac{1}{3}ML_1^2}{\tfrac{1}{3}ML_1^2 + mL_2^2}\sqrt{\frac{3g}{L_1}}$$

Substitute numerical values to obtain:

$$\omega' = \frac{\frac{1}{3}(2\,\text{kg})(1.2\,\text{m})^2}{\frac{1}{3}(2\,\text{kg})(1.2\,\text{m})^2 + m(0.8\,\text{m})^2} \times \sqrt{\frac{3(9.81\,\text{m/s}^2)}{1.2\,\text{m}}}$$
$$= \frac{4.75\,\text{kg}\cdot\text{m}^2/\text{s}}{0.960\,\text{kg}\cdot\text{m}^2 + (0.64\,\text{m}^2)m}$$
$$= \frac{4.75\,\text{kg/s}}{0.960\,\text{kg} + 0.64m}$$

Use conservation of energy to relate the rotational kinetic energy of the rod-plus-particle just after their collision to their potential energy when they have swung through an angle θ_{max}:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-K_i + U_f - U_i = 0$$

Substitute for K_i, U_f, and U_i to obtain:

$$-\tfrac{1}{2}I\omega'^2 + Mg\left(\tfrac{1}{2}L_1\right)(1-\cos\theta_{max}) + mgL_2(1-\cos\theta_{max}) = 0$$

Express the moment of inertia of the system with respect to the pivot:

$$I = \tfrac{1}{3}ML_1^2 + mL_2^2$$

Substitute for θ_{max}, I and ω' in equation (1) and simplify to obtain:

$$\frac{\frac{1}{2}(4.75\,\text{kg/s})^2}{0.960\,\text{kg} + 0.64m} = 0.2g(ML_1 + mL_2)$$

Substitute for M, L_1 and L_2 and simplify to obtain:

$$m^2 + 3.00m - 8.901 = 0$$

Solve the quadratic equation for its positive root:

$$m = \boxed{1.84\,\text{kg}}$$

(*b*) The energy dissipated in the inelastic collision is:

$$\Delta E = U_i - U_f \qquad (2)$$

Express U_i:

$$U_i = Mg\frac{L_1}{2}$$

Express U_f:

$$U_f = (1-\cos\theta_{max})g\left(M\frac{L_1}{2} + mL_2\right)$$

Substitute in equation (2) to obtain:

$$\Delta E = Mg\frac{L_1}{2} - (1-\cos\theta_{max})g\left(M\frac{L_1}{2} + mL_2\right)$$

Substitute numerical values and evaluate ΔE:

$$U_f = \frac{(2\,\text{kg})(9.81\,\text{m/s}^2)(1.2\,\text{m})}{2} - (1-\cos37°)(9.81\,\text{m/s}^2)\left(\frac{(2\,\text{kg})(1.2\,\text{m})}{2} + (1.85\,\text{kg})(0.8\,\text{m})\right) = \boxed{6.51\,\text{J}}$$

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Picture the Problem Let ω_i and ω_f be the angular velocities of the rod immediately before and immediately after the inelastic collision with the mass m. Let ω_0 be the initial angular velocity of the rod. Choose the zero of gravitational potential energy be at a distance L_1 below the pivot. We apply energy conservation to determine ω_f and conservation of angular momentum to determine ω_i. We'll apply energy conservation to determine ω_0. Finally, we'll find the energies of the system immediately before and after the collision and the energy dissipated.

Express the energy dissipated in the inelastic collision:

$$\Delta E = U_i - U_f \quad (1)$$

Use energy conservation to relate the kinetic energy of the system immediately after the collision to its potential energy after a 180° rotation:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = K_{top} = 0$ and $K_i = K_{bottom}$,

$$-K_{bottom} + U_{top} - U_{bottom} = 0$$

Substitute for K_{bottom}, U_{top}, and U_{bottom} to obtain:

$$-\tfrac{1}{2}I\omega_f^2 + \tfrac{3}{2}MgL_1 + mg(L_1 + L_2) - \tfrac{1}{2}MgL_1 - mg(L_1 - L_2) = 0$$

Simplify to obtain:

$$-\tfrac{1}{2}I\omega_f^2 + MgL_1 + 2mgL_2 = 0 \quad (2)$$

Express I:

$$I = \tfrac{1}{3}ML_1^2 + mL_2^2$$

Substitute for I in equation (2) and solve for ω_f to obtain:

$$\omega_f = \sqrt{\frac{2g(ML_1 + 2mL_2)}{\tfrac{1}{3}ML_1^2 + mL_2^2}}$$

Substitute numerical values and evaluate ω_f:

$$\omega_f = \sqrt{\frac{2(9.81\,\text{m/s}^2)[(0.75\,\text{kg})(1.2\,\text{m}) + 2(0.4\,\text{kg})(0.8\,\text{m})]}{\frac{1}{3}(0.75\,\text{kg})(1.2\,\text{m})^2 + (0.4\,\text{kg})(0.8\,\text{m})^2}} = 7.00\,\text{rad/s}$$

Use conservation of angular momentum to relate the angular momentum of the system just before the collision to its angular momentum just after the collision:

$$L_i = L_f$$

or

$$I_i\omega_i = I_f\omega_f$$

Substitute for I_i and I_f and solve for ω_i:

$$\left(\tfrac{1}{3}ML_1^2\right)\omega_i = \left(\tfrac{1}{3}ML_1^2 + mL_2^2\right)\omega_f$$

and

$$\omega_i = \left[1 + \frac{3m}{M}\left(\frac{L_2}{L_1}\right)^2\right]\omega_f$$

Substitute numerical values and evaluate ω_i:

$$\omega_i = \left[1 + \frac{3(0.4\,\text{kg})}{0.75\,\text{kg}}\left(\frac{0.8\,\text{m}}{1.2\,\text{m}}\right)^2\right](7.00\,\text{rad/s})$$
$$= 12.0\,\text{rad/s}$$

Apply conservation of mechanical energy to relate the initial rotational kinetic energy of the rod to its rotational kinetic energy just before its collision with the particle:

$$K_f - K_i + U_f - U_i = 0$$

Substitute to obtain:

$$\tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega_i^2 - \tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega_0^2 + Mg\frac{L_1}{2} - MgL_1 = 0$$

Solve for ω_0:

$$\omega_0 = \sqrt{\omega_i^2 - \frac{3g}{L_1}}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \sqrt{(12\,\text{rad/s})^2 - \frac{3(9.81\,\text{m/s}^2)}{1.2\,\text{m}}}$$
$$= \boxed{10.9\,\text{rad/s}}$$

Substitute in equation (1) to express the energy dissipated in the collision:

$$\Delta E = \tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega_i^2 - MgL_1 + 2mgL_2$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \tfrac{1}{6}(0.75\,\text{kg})(1.2\,\text{m})^2(12\,\text{rad/s})^2 - \left(9.81\,\text{m/s}^2\right)[(0.75\,\text{kg})(1.2\,\text{m}) + 2(0.4\,\text{kg})(0.8\,\text{m})]$$
$$= \boxed{10.8\,\text{J}}$$

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Picture the Problem Let v be the speed of the particle immediately after the collision and ω_i and ω_f be the angular velocities of the rod immediately before and immediately after the elastic collision with the mass m. Choose the zero of gravitational potential energy be at a distance L_1 below the pivot. Because the net external torque acting on the system is zero, angular momentum is conserved in this elastic collision. The rod, on its downward swing, acquires rotational kinetic energy. Angular momentum is conserved in the elastic collision with the particle and the kinetic energy of the after-collision system is then transformed into gravitational potential energy as the rod-plus-particle swing upward. Let the zero of gravitational potential energy be at a distance L_1 below the pivot and use both angular momentum and mechanical energy conservation to relate the distances L_1 and L_2 and the mass M to m.

Use energy conservation to relate the energies of the system immediately before and after the elastic collision:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_i = 0$,

$$K_f + U_f - U_i = 0$$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}mv^2 + Mg\frac{L_1}{2}\left(1 - \cos\theta_{max}\right) - Mg\frac{L_1}{2} = 0$$

Solve for mv^2:

$$mv^2 = MgL_1\cos\theta_{max} \qquad (1)$$

Apply conservation of energy to express the angular speed of the rod just before the collision:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_i = 0$,

$$K_f + U_f - U_i = 0$$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega_i^2 + Mg\frac{L_1}{2} - MgL_1 = 0$$

Solve for ω_i:

$$\omega_i = \sqrt{\frac{3g}{L_1}}$$

Apply conservation of energy to the rod after the collision:

$$\tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega_f^2 - Mg\frac{L_1}{2}\left(1-\cos\theta_{max}\right)=0$$

Solve for ω_f:

$$\omega_f = \sqrt{\frac{0.6g}{L_1}}$$

Apply conservation of angular momentum to the collision:

$$L_i = L_f$$

or

$$\left(\tfrac{1}{3}ML_1^2\right)\omega_i = \left(\tfrac{1}{3}ML_1^2\right)\omega_f + mvL_2$$

Solve for mv:

$$mv = \frac{\tfrac{1}{3}ML_1^2\left(\omega_i-\omega_f\right)}{L_2}$$

Substitute for ω_f and ω_I to obtain:

$$mv = \frac{ML_1^2\left(\sqrt{\frac{3g}{L_1}}-\sqrt{\frac{0.6g}{L_1}}\right)}{3L_2} \quad (2)$$

Divide equation (1) by equation (2) to eliminate m and solve for v:

$$v = \frac{MgL_1\cos\theta_{max}}{\frac{ML_1^2\left(\sqrt{\frac{3g}{L_1}}-\sqrt{\frac{0.6g}{L_1}}\right)}{3L_2}} = \frac{3gL_2\cos\theta_{max}}{\sqrt{3gL_1}-\sqrt{0.6gL_1}}$$

Substitute numerical values and evaluate v:

$$v = \frac{3\left(9.81\,\text{m/s}^2\right)\left(0.8\,\text{m}\right)\cos 37°}{\sqrt{3\left(9.81\,\text{m/s}^2\right)\left(1.2\,\text{m}\right)}-\sqrt{0.6\left(9.81\,\text{m/s}^2\right)\left(1.2\,\text{m}\right)}} = 5.72\,\text{m/s}$$

Solve equation (1) for m:

$$m = \frac{MgL_1\cos\theta_{max}}{v^2}$$

Substitute for v in the expression for mv and solve for m:

$$m = \frac{\left(2\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)\left(1.2\,\text{m}\right)\cos 37°}{\left(5.72\,\text{m/s}\right)^2} = \boxed{0.575\,\text{kg}}$$

Because the collision was elastic:

$$\Delta E = \boxed{0}$$

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Picture the Problem We can determine the angular momentum of the wheel and the angular velocity of its precession from their definitions. The period of the precessional motion can be found from its angular velocity and the angular momentum associated with the motion of the center of mass from its definition.

(*a*) Using the definition of angular momentum, express the angular momentum of the spinning wheel:

$$L = I\omega = MR^2\omega = \frac{w}{g}R^2\omega$$

Substitute numerical values and evaluate *L*:

$$L = \left(\frac{30\,\text{N}}{9.81\,\text{m/s}^2}\right)(0.28\,\text{m})^2 \times\left(12\frac{\text{rev}}{\text{s}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\right) = \boxed{18.1\ \text{J}\cdot\text{s}}$$

(*b*) Using its definition, express the angular velocity of precession:

$$\omega_{\text{p}} = \frac{d\phi}{dt} = \frac{MgD}{L}$$

Substitute numerical values and evaluate ω_{p}:

$$\omega_{\text{p}} = \frac{(30\,\text{N})(0.25\,\text{m})}{18.1\,\text{J}\cdot\text{s}} = \boxed{0.414\,\text{rad/s}}$$

(*c*) Express the period of the precessional motion as a function of the angular velocity of precession:

$$T = \frac{2\pi}{\omega_{\text{p}}} = \frac{2\pi}{0.414\,\text{rad/s}} = \boxed{15.2\,\text{s}}$$

(*d*) Express the angular momentum of the center of mass due to the precession:

$$L_{\text{p}} = I_{\text{cm}}\omega_{\text{p}} = MD^2\omega_{\text{p}}$$

Substitute numerical values and evaluate L_{p}:

$$L_{\text{p}} = \left(\frac{30\,\text{N}}{9.81\,\text{m/s}^2}\right)(0.25\,\text{m})^2(0.414\,\text{rad/s}) = \boxed{0.0791\,\text{J}\cdot\text{s}}$$

The direction of L_{p} is either up or down, depending on the direction of *L*.

***78** ••

Picture the Problem The angular velocity of precession can be found from its definition. Both the speed and acceleration of the center of mass during precession are related to the angular velocity of precession. We can use Newton's 2nd law to find the vertical and horizontal components of the force exerted by the pivot.

(*a*) Using its definition, express the angular velocity of precession:

$$\omega_p = \frac{d\phi}{dt} = \frac{MgD}{I_s\omega_s} = \frac{MgD}{\frac{1}{2}MR^2\omega_s} = \frac{2gD}{R^2\omega_s}$$

Substitute numerical values and evaluate ω_p:

$$\omega_p = \frac{2(9.81\,\text{m/s}^2)(0.05\,\text{m})}{(0.064\,\text{m})^2\left(700\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)} = \boxed{3.27\,\text{rad/s}}$$

(*b*) Express the speed of the center of mass in terms of its angular velocity of precession:

$$v_{cm} = D\omega_p = (0.05\,\text{m})(3.27\,\text{rad/s}) = \boxed{0.164\,\text{m/s}}$$

(*c*) Relate the acceleration of the center of mass to its angular velocity of precession:

$$a_{cm} = D\omega_p^2 = (0.05\,\text{m})(3.27\,\text{rad/s})^2 = \boxed{0.535\,\text{m/s}^2}$$

(*d*) Use Newton's 2nd law to relate the vertical component of the force exerted by the pivot to the weight of the disk:

$$F_v = Mg = (2.5\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{24.5\,\text{N}}$$

Relate the horizontal component of the force exerted by the pivot to the acceleration of the center of mass:

$$F_v = Ma_{cm} = (2.5\,\text{kg})(0.535\,\text{m/s}^2) = \boxed{1.34\,\text{N}}$$

General Problems

79 •

Picture the Problem While the 3-kg particle is moving in a straight line, it has angular momentum given by $\vec{L} = \vec{r}\times\vec{p}$ where $\vec{r}$ is its position vector and $\vec{p}$ is its linear momentum. The torque due to the applied force is given by $\vec{\tau} = \vec{r}\times\vec{F}$.

(*a*) Express the angular momentum of the particle:

$$\vec{L} = \vec{r}\times\vec{p}$$

Express the vectors $\vec{r}$ and $\vec{p}$:

$$\vec{r} = (12\,\text{m})\hat{i} + (5.3\,\text{m})\hat{j}$$

and

$$\vec{p} = mv\hat{i} = (3\,\text{kg})(3\,\text{m/s})\hat{i} = (9\,\text{kg}\cdot\text{m/s})\hat{i}$$

Substitute and simplify to find $\vec{L}$:

$$\vec{L} = [(12\,\text{m})\hat{i} + (5.3\,\text{m})\hat{j}] \times (9\,\text{kg}\cdot\text{m/s})\hat{i} = (47.7\,\text{kg}\cdot\text{m}^2/\text{s})(\hat{j}\times\hat{i}) = \boxed{-(47.7\,\text{kg}\cdot\text{m}^2/\text{s})\hat{k}}$$

(*b*) Using its definition, express the torque due to the force:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Substitute and simplify to find $\vec{\tau}$:

$$\vec{\tau} = [(12\,\text{m})\hat{i} + (5.3\,\text{m})\hat{j}] \times (-3\,\text{N})\hat{i} = -(15.9\,\text{N}\cdot\text{m})(\hat{j}\times\hat{i}) = \boxed{(15.9\,\text{N}\cdot\text{m})\hat{k}}$$

80 •

Picture the Problem The angular momentum of the particle is given by $\vec{L} = \vec{r} \times \vec{p}$ where $\vec{r}$ is its position vector and $\vec{p}$ is its linear momentum. The torque acting on the particle is given by $\vec{\tau} = d\vec{L}/dt$.

Express the angular momentum of the particle:

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m\vec{r} \times \vec{v} = m\vec{r} \times \frac{d\vec{r}}{dt}$$

Evaluate $\frac{d\vec{r}}{dt}$:

$$\frac{d\vec{r}}{dt} = 6t\hat{j}$$

Substitute and simplify to find $\vec{L}$:

$$\vec{L} = [(3\,\text{kg})\{(4\,\text{m})\hat{i} + (3t^2\,\text{m/s}^2)\hat{j}\}] \times (6t\,\text{m/s})\hat{j} = \boxed{(72.0t\,\text{J}\cdot\text{s})\hat{k}}$$

Find the torque due to the force:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}[(72.0t\,\text{J}\cdot\text{s})\hat{k}] = \boxed{(72.0\,\text{N}\cdot\text{m})\hat{k}}$$

81 ••

Picture the Problem The ice skaters rotate about their center of mass; a point we can locate using its definition. Knowing the location of the center of mass we can determine their moment of inertia with respect to an axis through this point. The angular momentum of the system is then given by $L = I_{cm}\omega$ and its kinetic energy can be found from $K = L^2/2I_{cm}$.

(*a*) Express the angular momentum of the system about the center of mass of the skaters:

$$L = I_{cm}\omega$$

Using its definition, locate the center of mass, relative to the 85-kg skater, of the system:

$$x_{cm} = \frac{(55\,\text{kg})(1.7\,\text{m}) + (85\,\text{kg})(0)}{55\,\text{kg} + 85\,\text{kg}} = 0.668\,\text{m}$$

Calculate I_{cm}:

$$I_{cm} = (55\,\text{kg})(1.7\,\text{m} - 0.668\,\text{m})^2 + (85\,\text{kg})(0.668\,\text{m})^2 = 96.5\,\text{kg}\cdot\text{m}^2$$

Substitute to determine *L*:

$$L = \left(96.5\,\text{kg}\cdot\text{m}^2\right)\left(\frac{1\,\text{rev}}{2.5\,\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}\right) = \boxed{243\,\text{J}\cdot\text{s}}$$

(*b*) Relate the total kinetic energy of the system to its angular momentum and evaluate *K*:

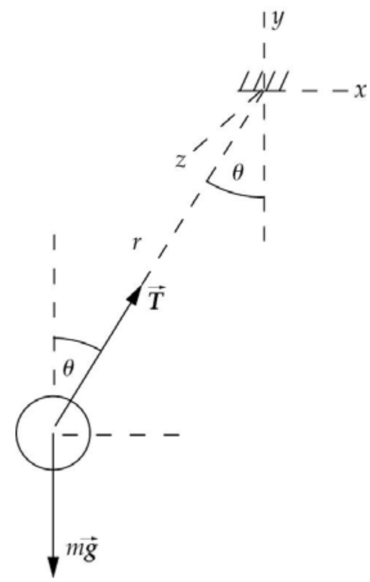
$$K = \frac{L^2}{2I_{cm}}$$

Substitute numerical values and evaluate *K*:

$$K = \frac{(243\,\text{J}\cdot\text{s})^2}{2\left(96.5\,\text{kg}\cdot\text{m}^2\right)} = \boxed{306\,\text{J}}$$

***82** ••

Picture the Problem Let the origin of the coordinate system be at the pivot (point *P*). The diagram shows the forces acting on the ball. We'll apply Newton's 2nd law to the ball to determine its speed. We'll then use the derivative of its position vector to express its velocity and the definition of angular momentum to show that $\vec{L}$ has both horizontal and vertical components. We can use the derivative of $\vec{L}$ with respect to time to show that the rate at which the angular momentum of the ball changes is equal to the torque, relative to the pivot point, acting on it.



(*a*) Express the angular momentum of the ball about the point of support:

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} \qquad (1)$$

Apply Newton's 2nd law to the ball:

$$\sum F_x = T\sin\theta = m\frac{v^2}{r\sin\theta}$$

and

$$\sum F_z = T\cos\theta - mg = 0$$

Eliminate *T* between these equations and solve for *v*:

$$v = \sqrt{rg\sin\theta\tan\theta}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(1.5\,\text{m})(9.81\,\text{m/s}^2)\sin 30°\tan 30°} = 2.06\,\text{m/s}$$

Express the position vector of the ball:

$$\vec{r} = (1.5\,\text{m})\sin 30°(\cos\omega t\,\hat{i} + \sin\omega t\hat{j}) - (1.5\,\text{m})\cos 30°\hat{k}$$

where $\omega = \omega\hat{k}$.

Find the velocity of the ball:

$$\vec{v} = \frac{d\vec{r}}{dt} = (0.75\omega\,\text{m/s})(-\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})$$

Evaluate ω:

$$\omega = \frac{2.06\,\text{m/s}}{(1.5\,\text{m})\sin 30°} = 2.75\,\text{rad/s}$$

Substitute for ω to obtain: $\vec{v} = (2.06\,\text{m/s})\left(-\sin\omega t\,\hat{\boldsymbol{i}} + \cos\omega t\hat{\boldsymbol{j}}\right)$

Substitute in equation (1) and evaluate $\vec{L}$:

$$\begin{aligned}\vec{L} &= (2\,\text{kg})\left[(1.5\,\text{m})\sin 30°\left(\cos\omega t\,\hat{\boldsymbol{i}} + \sin\omega t\,\hat{\boldsymbol{j}}\right) - (1.5\,\text{m})\cos 30°\hat{\boldsymbol{k}}\right] \\ &\quad \times\left[(2.06\,\text{m/s})\left(-\sin\omega t\,\hat{\boldsymbol{i}} + \cos\omega t\,\hat{\boldsymbol{j}}\right)\right] \\ &= \left[5.36\left(\cos\omega t\,\hat{\boldsymbol{i}} + \sin\omega t\,\hat{\boldsymbol{j}}\right) + 3.09\,\hat{\boldsymbol{k}}\right]\text{J}\cdot\text{s}\end{aligned}$$

The horizontal component of $\vec{L}$ is: $\boxed{5.36\left(\cos\omega t\,\hat{\boldsymbol{i}} + \sin\omega t\,\hat{\boldsymbol{j}}\right)\text{J}\cdot\text{s}}$

The vertical component of $\vec{L}$ is: $\boxed{3.09\,\hat{\boldsymbol{k}}\,\text{J}\cdot\text{s}}$

(*b*) Evaluate $\dfrac{d\vec{L}}{dt}$: $\dfrac{d\vec{L}}{dt} = \left[5.36\omega\left(-\sin\omega t\,\hat{\boldsymbol{i}} + \cos\omega t\hat{\boldsymbol{j}}\right)\right]\text{J}$

Evaluate the magnitude of $\dfrac{d\vec{L}}{dt}$:

$$\begin{aligned}\left|\frac{d\vec{L}}{dt}\right| &= (5.36\,\text{N}\cdot\text{m}\cdot\text{s})(2.75\,\text{rad/s}) \\ &= \boxed{14.7\,\text{N}\cdot\text{m}}\end{aligned}$$

Express the magnitude of the torque exerted by gravity about the point of support: $\tau = mgr\sin\theta$

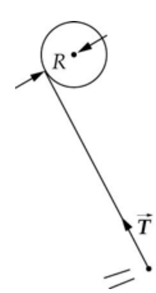
Substitute numerical values and evaluate τ:

$$\begin{aligned}\tau &= (2\,\text{kg})\left(9.81\,\text{m/s}^2\right)(1.5\,\text{m})\sin 30° \\ &= \boxed{14.7\,\text{N}\cdot\text{m}}\end{aligned}$$

83 ••

Picture the Problem In part (*a*) we need to decide whether a net torque acts on the object. In part (*b*) the issue is whether any external forces act on the object. In part (*c*) we can apply the definition of kinetic energy to find the speed of the object when the unwrapped length has shortened to *r*/2.

(*a*) Consider the overhead view of the cylindrical post and the object shown in the adjoining figure. The object rotates counterclockwise. The torque about the center of the cylinder is clockwise and of magnitude *RT*, where *R* is the radius of the cylinder and *T* is the tension. So



L must decrease. $\boxed{\text{No, } L \text{ decreases.}}$

(*b*) Because, in this frictionless environment, no net external forces act on the object: $\boxed{\text{Its kinetic energy is constant.}}$

(*c*) Express the kinetic energy of the object as it spirals inward:

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{2}mv^2$$

$\boxed{v_0\text{. (The kinetic energy remains constant.)}}$

84 ••

Picture the Problem Because the net torque acting on the system is zero; we can use conservation of angular momentum to relate the initial and final angular velocities of the system.

Using conservation of angular momentum, relate the initial and final angular velocities to the initial and final moments of inertia:

$$L_i = L_f$$

or

$$I_i\omega_i = I_f\omega_f$$

Solve for ω_i:

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_i}{I_f}\omega$$

Express I_i:

$$I_i = \tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}m\ell^2\right)$$

Express I_f:

$$I_f = \tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}mL^2\right)$$

Substitute to express ω_f in terms of ω:

$$\omega_f = \frac{\tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}m\ell^2\right)}{\tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}mL^2\right)}\omega = \boxed{\frac{M + 5m\dfrac{\ell^2}{L^2}}{M + 5m}\omega}$$

Express the initial kinetic energy of the system:

$$K_i = \tfrac{1}{2}I_i\omega^2 = \tfrac{1}{2}\left[\tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}m\ell^2\right)\right]\omega^2 = \boxed{\tfrac{1}{20}\left(ML^2 + 5m\ell^2\right)\omega^2}$$

Express the final kinetic energy of the system and simplify to obtain:

$$K_f = \tfrac{1}{2} I_f \omega_f^2 = \tfrac{1}{2}\left[\tfrac{1}{10} ML^2 + 2\left(\tfrac{1}{4} mL^2\right)\right]\omega_f^2 = \tfrac{1}{20}\left(ML^2 + 5mL^2\right)\omega_f^2$$

$$= \tfrac{1}{20}\left(ML^2 + 5mL^2\right)\left(\frac{M + 5m\frac{\ell^2}{L^2}}{M + 5m}\omega\right)^2 = \tfrac{1}{20}\left[\frac{\left(ML + 5m\frac{\ell^2}{L}\right)^2}{M + 5m}\right]\omega^2$$

$$= \boxed{\tfrac{1}{20}\left[\frac{\left(ML^2 + 5m\ell^2\right)^2}{ML^2 + 5mL^2}\right]\omega^2}$$

85 ••

Determine the Concept Yes. The net external torque is zero and angular momentum is conserved as the system evolves from its initial to its final state. Because the disks come to the same final position, the initial and final configurations are the same as in Problem 84. Therefore, the answers are the same as for Problem 84.

86 ••

Picture the Problem Because the net torque acting on the system is zero; we can use conservation of angular momentum to relate the initial and final angular velocities of the system.

Using conservation of angular momentum, relate the initial and final angular velocities to the initial and final moments of inertia:

$$L_i = L_f$$

or

$$I_i\omega_i = I_f\omega_f$$

Solve for ω_f:

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_i}{I_f}\omega \qquad (1)$$

Relate the tension in the string to the angular speed of the system and solve for and evaluate ω:

$$T = mr\omega^2 = m\frac{\ell}{2}\omega^2$$

and

$$\omega = \sqrt{\frac{2T}{m\ell}} = \sqrt{\frac{2(108\,\text{N})}{(0.4\,\text{kg})(0.6\,\text{m})}} = \boxed{30.0\,\text{rad/s}}$$

Express and evaluate I_i:

$$\begin{aligned} I_i &= \tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}m\ell^2\right) \\ &= \tfrac{1}{10}(0.8\,\text{kg})(2\,\text{m})^2 + \tfrac{1}{2}(0.4\,\text{kg})(0.6\,\text{m})^2 \\ &= 0.392\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Express and evaluate I_f:

$$\begin{aligned} I_f &= \tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}mL^2\right) \\ &= \tfrac{1}{10}(0.8\,\text{kg})(2\,\text{m})^2 + \tfrac{1}{2}(0.4\,\text{kg})(2\,\text{m})^2 \\ &= 1.12\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Substitute in equation (1) and solve for ω_f:

$$\begin{aligned} \omega_f &= \frac{I_i}{I_f}\omega = \frac{0.392\,\text{kg}\cdot\text{m}^2}{1.12\,\text{kg}\cdot\text{m}^2}(30.0\,\text{rad/s}) \\ &= \boxed{10.5\,\text{rad/s}} \end{aligned}$$

Express and evaluate the initial kinetic energy of the system:

$$\begin{aligned} K_i &= \tfrac{1}{2}I_i\omega^2 \\ &= \tfrac{1}{2}\left(0.392\,\text{kg}\cdot\text{m}^2\right)(30.0\,\text{rad/s})^2 \\ &= \boxed{176\,\text{J}} \end{aligned}$$

Express and evaluate the final kinetic energy of the system:

$$\begin{aligned} K_f &= \tfrac{1}{2}I_f\omega_f^2 \\ &= \tfrac{1}{2}\left(1.12\,\text{kg}\cdot\text{m}^2\right)(10.5\,\text{rad/s})^2 \\ &= \boxed{61.7\,\text{J}} \end{aligned}$$

87 ••

Picture the Problem Until the inelastic collision of the cylindrical objects at the ends of the cylinder, both angular momentum and energy are conserved. Let *K*' represent the kinetic energy of the system just before the disks reach the end of the cylinder and use conservation of energy to relate the initial and final kinetic energies to the final radial velocity.

Using conservation of mechanical energy, relate the initial and final kinetic energies of the disks:

$$K_i = K'$$

or

$$\tfrac{1}{2}I_i\omega^2 = \tfrac{1}{2}I_f\omega_f^2 + \tfrac{1}{2}\left(2mv_r^2\right)$$

Solve for v_r:

$$v_r = \sqrt{\frac{I_i\omega^2 - I_f\omega_f^2}{2m}} \qquad (1)$$

Using conservation of angular momentum, relate the initial and final angular velocities to the initial

$$L_i = L_f$$

or

and final moments of inertia:

$$I_i\omega_i = I_f\omega_f$$

Solve for ω_f:

$$\omega_f = \frac{I_i}{I_f}\omega = \frac{I_i}{I_f}\omega$$

Express I_i:

$$I_i = \tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}m\ell^2\right)$$

Express I_f:

$$I_f = \tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}mL^2\right)$$

Substitute to obtain ω_f in terms of ω:

$$\omega_f = \frac{\tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}m\ell^2\right)}{\tfrac{1}{10}ML^2 + 2\left(\tfrac{1}{4}mL^2\right)}\omega = \frac{ML^2 + 5m\ell^2}{ML^2 + 5mL^2}\omega$$

Substitute in equation (1) and simplify to obtain:

$$v_r = \boxed{\frac{\ell\omega}{2L}\sqrt{\left(L^2 - \ell^2\right)}}$$

88 ••

Picture the Problem Because the net torque acting on the system is zero, we can use conservation of angular momentum to relate the initial and final angular velocities and the initial and final kinetic energy of the system.

Using conservation of angular momentum, relate the initial and final angular velocities to the initial and final moments of inertia:

$$L_i = L_f$$

or

$$I_i\omega_i = I_f\omega_f$$

Solve for ω_f:

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_i}{I_f}\omega \qquad (1)$$

Relate the tension in the string to the angular speed of the system:

$$T = mr\omega^2 = m\frac{\ell}{2}\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{2T}{m\ell}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{2(108\,\text{N})}{(0.4\,\text{kg})(0.6\,\text{m})}} = \boxed{30.0\,\text{rad/s}}$$

Express and evaluate I_i:

$$I_i = \frac{1}{10}ML^2 + 2\left(\frac{1}{4}m\ell^2\right) = \frac{1}{10}(0.8\,\text{kg})(2\,\text{m})^2 + \frac{1}{2}(0.4\,\text{kg})(0.6\,\text{m})^2 = 0.392\,\text{kg}\cdot\text{m}^2$$

Letting L' represent the final separation of the disks, express and evaluate I_f:

$$I_f = \frac{1}{10}ML^2 + 2\left(\frac{1}{4}mL'^2\right) = \frac{1}{10}(0.8\,\text{kg})(2\,\text{m})^2 + \frac{1}{2}(0.4\,\text{kg})(1.6\,\text{m})^2 = 0.832\,\text{kg}\cdot\text{m}^2$$

Substitute in equation (1) and solve for ω_f:

$$\omega_f = \frac{I_i}{I_f}\omega = \frac{0.392\,\text{kg}\cdot\text{m}^2}{0.832\,\text{kg}\cdot\text{m}^2}(30.0\,\text{rad/s}) = 14.1\,\text{rad/s}$$

Express and evaluate the initial kinetic energy of the system:

$$K_i = \frac{1}{2}I_i\omega^2 = \frac{1}{2}(0.392\,\text{kg}\cdot\text{m}^2)(30.0\,\text{rad/s})^2 = \boxed{176\,\text{J}}$$

Express and evaluate the final kinetic energy of the system:

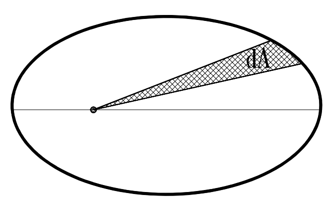
$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(0.832\,\text{kg}\cdot\text{m}^2)(14.1\,\text{rad/s})^2 = \boxed{82.7\,\text{J}}$$

The energy dissipated in friction is:

$$\Delta E = K_i - K_f = 176\,\text{J} - 82.7\,\text{J} = \boxed{93.3\,\text{J}}$$

***89 ••**

Picture the Problem The drawing shows an elliptical orbit. The triangular element of the area is $dA = \frac{1}{2}r(rd\theta) = \frac{1}{2}r^2 d\theta$.



Differentiate dA with respect to t to obtain:

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

Because the gravitational force acts along the line joining the two objects, $\tau = 0$ and:

$$L = mr^2\omega = \text{constant}$$

Eliminate $r^2\omega$ between the two equations to obtain:

$$\frac{dA}{dt}=\boxed{\frac{L}{2m}=\text{constant}}$$

90 ••

Picture the Problem Let x be the radial distance each disk moves outward. Because the net torque acting on the system is zero, we can use conservation of angular momentum to relate the initial and final angular velocities to the initial and final moments of inertia. We'll assume that the disks are thin enough so that we can ignore their lengths in expressing their moments of inertia.

Use conservation of angular momentum to relate the initial and final angular velocities of the disks:

$$L_\text{i}=L_\text{f}$$

or

$$I_\text{i}\omega_\text{i}=I_\text{f}\omega_\text{f}$$

Solve for ω_f:

$$\omega_\text{f}=\frac{I_\text{i}}{I_\text{f}}\omega_\text{i} \qquad (1)$$

Express the initial moment of inertia of the system:

$$I_\text{i}=I_\text{cyl}+2I_\text{disk}$$

Express the moment of inertia of the cylinder:

$$\begin{aligned}I_\text{cyl}&=\tfrac{1}{12}ML^2+\tfrac{1}{2}MR^2\\&=\tfrac{1}{12}M\left(L^2+6R^2\right)\\&=\tfrac{1}{12}(0.8\,\text{kg})\left[(1.8\,\text{m})^2+6(0.2\,\text{m})^2\right]\\&=0.232\,\text{kg}\cdot\text{m}^2\end{aligned}$$

Letting ℓ represent the distance of the clamped disks from the center of rotation and ignoring the thickness of each disk (we're told they are thin), use the parallel-axis theorem to express the moment of inertia of each disk:

$$\begin{aligned}I_\text{disk}&=\tfrac{1}{4}mr^2+m\ell^2\\&=\tfrac{1}{4}m\left(r^2+4\ell^2\right)\\&=\tfrac{1}{4}(0.2\,\text{kg})\left[(0.2\,\text{m})^2+4(0.4\,\text{m})^2\right]\\&=0.0340\,\text{kg}\cdot\text{m}^2\end{aligned}$$

With the disks clamped:

$$\begin{aligned}I_\text{i}&=I_\text{cyl}+2I_\text{disk}\\&=0.232\,\text{kg}\cdot\text{m}^2+2\left(0.0340\,\text{kg}\cdot\text{m}^2\right)\\&=0.300\,\text{kg}\cdot\text{m}^2\end{aligned}$$

With the disks unclamped, $\ell = 0.6$ m and:

$$I_{\text{disk}} = \tfrac{1}{4}m\left(r^2 + 4\ell^2\right) = \tfrac{1}{4}(0.2\,\text{kg})\left[(0.2\,\text{m})^2 + 4(0.6\,\text{m})^2\right] = 0.0740\,\text{kg}\cdot\text{m}^2$$

Express and evaluate the final moment of inertia of the system:

$$I_{\text{f}} = I_{\text{cyl}} + 2I_{\text{disk}} = 0.232\,\text{kg}\cdot\text{m}^2 + 2\left(0.0740\,\text{kg}\cdot\text{m}^2\right) = 0.380\,\text{kg}\cdot\text{m}^2$$

Substitute in equation (1) to determine ω_{f}:

$$\omega_{\text{f}} = \frac{0.300\,\text{kg}\cdot\text{m}^2}{0.380\,\text{kg}\cdot\text{m}^2}(8\,\text{rad/s}) = \boxed{6.32\,\text{rad/s}}$$

Express the energy dissipated in friction:

$$\Delta E = E_{\text{i}} - E_{\text{f}} = \tfrac{1}{2}I_{\text{i}}\omega_{\text{i}}^2 - \left(\tfrac{1}{2}I_{\text{f}}\omega_{\text{f}}^2 + \tfrac{1}{2}kx^2\right)$$

Apply Newton's 2nd law to each disk when they are in their final positions:

$$\sum F_{\text{radial}} = kx = mr\omega^2$$

Solve for k:

$$k = \frac{mr\omega^2}{x}$$

Substitute numerical values and evaluate k:

$$k = \frac{(0.2\,\text{kg})(0.6\,\text{m})(6.32\,\text{rad/s})^2}{0.2\,\text{m}} = 24.0\,\text{N/m}$$

Express the energy dissipated in friction:

$$W_{\text{fr}} = E_{\text{i}} - E_{\text{f}} = \tfrac{1}{2}I_{\text{i}}\omega_{\text{i}}^2 - \left(\tfrac{1}{2}I_{\text{f}}\omega_{\text{f}}^2 + \tfrac{1}{2}kx^2\right)$$

Substitute numerical values and evaluate W_{fr}:

$$W_{\text{fr}} = \tfrac{1}{2}\left(0.300\,\text{kg}\cdot\text{m}^2\right)(8\,\text{rad/s})^2 - \tfrac{1}{2}\left(0.380\,\text{kg}\cdot\text{m}^2\right)(6.32\,\text{rad/s})^2 - \tfrac{1}{2}(24\,\text{N/m})(0.2\,\text{m})^2 = \boxed{1.53\,\text{J}}$$

91 ••

Picture the Problem Let the letters *d*, *m*, and *r* denote the disk and the letters *t*, *M*, and *R* the turntable. We can use conservation of angular momentum to relate the final angular speed of the turntable to the initial angular speed of the Euler disk and the moments of inertia of the turntable and the disk. In part (*b*) we'll need to use the parallel-axis theorem to express the moment of inertia of the disk with respect to the rotational axis of the turntable. You can find the moments of inertia of the disk in its two orientations and that of the turntable in Table 9-1.

(*a*) Use conservation of angular momentum to relate the initial and final angular momenta of the system:

$$I_{\rm di}\omega_{\rm di} = I_{\rm df}\omega_{\rm df} + I_{\rm tf}\omega_{\rm tf}$$

Because $\omega_{\rm tf} = \omega_{\rm df}$:

$$I_{\rm di}\omega_{\rm di} = I_{\rm df}\omega_{\rm tf} + I_{\rm tf}\omega_{\rm tf}$$

Solve for $\omega_{\rm tf}$:

$$\omega_{\rm tf} = \frac{I_{\rm di}}{I_{\rm df} + I_{\rm tf}}\omega_{\rm di} \qquad (1)$$

Ignoring the negligible thickness of the disk, express its initial moment of inertia:

$$I_{\rm di} = \tfrac{1}{4}mr^2$$

Express the final moment of inertia of the disk:

$$I_{\rm df} = \tfrac{1}{2}mr^2$$

Express the final moment of inertia of the turntable:

$$I_{\rm tf} = \tfrac{1}{2}MR^2$$

Substitute in equation (1) to obtain:

$$\omega_{\rm tf} = \frac{\tfrac{1}{4}mr^2}{\tfrac{1}{2}mr^2 + \tfrac{1}{2}MR^2}\omega_{\rm di} = \frac{1}{2 + 2\dfrac{MR^2}{mr^2}}\omega_{\rm di} \qquad (2)$$

Express $\omega_{\rm di}$ in rad/s:

$$\omega_{\rm di} = 30\,\frac{\rm rev}{\rm min}\times\frac{2\pi\,\rm rad}{\rm rev}\times\frac{1\,\rm min}{60\,\rm s} = \pi\,\rm rad/s$$

Substitute numerical values in equation (2) and evaluate $\omega_{\rm tf}$:

$$\omega_{\rm tf} = \frac{\pi\,\rm rad/s}{2 + 2\dfrac{(0.735\,\rm kg)(0.25\,\rm m)^2}{(0.5\,\rm kg)(0.125\,\rm m)^2}} = \boxed{0.228\,\rm rad/s}$$

(*b*) Use the parallel-axis theorem to express the final moment of inertia of the disk when it is a distance L from the center of the turntable:

$$I_{\rm df} = \tfrac{1}{2}mr^2 + mL^2 = m\left(\tfrac{1}{2}r^2 + L^2\right)$$

Substitute in equation (1) to obtain:

$$\omega_{\text{tf}} = \frac{\frac{1}{4}mr^2}{m\left(\frac{1}{2}r^2 + L^2\right) + \frac{1}{2}MR^2}\omega_{\text{di}} = \frac{1}{2 + 4\dfrac{L^2}{r^2} + 2\dfrac{MR^2}{mr^2}}\omega_{\text{di}}$$

Substitute numerical values and evaluate ω_{tf}:

$$\omega_{\text{tf}} = \frac{\pi\ \text{rad/s}}{2 + 4\dfrac{(0.1\,\text{m})^2}{(0.125\,\text{m})^2} + 2\dfrac{(0.735\,\text{kg})(0.25\,\text{m})^2}{(0.5\,\text{kg})(0.125\,\text{m})^2}} = \boxed{0.192\,\text{rad/s}}$$

92 ••

Picture the Problem We can express the period of the earth's rotation in terms of its angular velocity of rotation and relate its angular velocity to its angular momentum and moment of inertia with respect to an axis through its center. We can differentiate this expression with respect to *T* and then use differentials to approximate the changes in *r* and *T*.

(*a*) Express the period of the earth's rotation in terms of its angular velocity of rotation:

$$T = \frac{2\pi}{\omega}$$

Relate the earth's angular velocity of rotation to its angular momentum and moment of inertia:

$$\omega = \frac{L}{I} = \frac{L}{\frac{2}{5}mr^2}$$

Substitute and simplify to obtain:

$$T = \frac{2\pi\left(\frac{2}{5}mr^2\right)}{L} = \boxed{\frac{4\pi m}{5L}r^2}$$

(*b*) Find dT/dr:

$$\frac{dT}{dr} = 2\left(\frac{4\pi m}{5L}\right)r = 2\left(\frac{T}{r^2}\right)r = \frac{2T}{r}$$

Solve for dT/T:

$$\frac{dT}{T} = 2\frac{dr}{r} \text{ or } \boxed{\frac{\Delta T}{T} \approx 2\frac{\Delta r}{r}}$$

(*c*) Using the equation we just derived, substitute for the change in the period of the earth:

$$\frac{\Delta T}{T} = \frac{\frac{1}{4}\text{d}}{\text{y}} \times \frac{1\,\text{y}}{365.24\,\text{d}} = \frac{1}{1460} = 2\frac{\Delta r}{r}$$

Solve for and evaluate Δr:

$$\Delta r = \frac{r}{2(1460)} = \frac{6.37\times10^{3}\text{ km}}{2(1460)} = \boxed{2.18\,\text{km}}$$

***93** ••

Picture the Problem Let ω_P be the angular velocity of precession of the earth-as-gyroscope, ω_s its angular velocity about its spin axis, and I its moment of inertia with respect to an axis through its poles, and relate ω_P to ω_s and I using its definition.

Use its definition to express the precession rate of the earth as a giant gyroscope:

$$\omega_P = \frac{\tau}{L}$$

Substitute for I and solve for τ:

$$\tau = L\omega_P = I\omega\omega_P$$

Express the angular velocity ω_s of the earth about its spin axis:

$\omega = \frac{2\pi}{T}$ where T is the period of rotation of the earth.

Substitute to obtain:

$$\tau = \frac{2\pi I\omega_P}{T}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{2\pi\left(8.03\times10^{37}\text{ kg}\cdot\text{m}^2\right)\left(7.66\times10^{-12}\text{ s}^{-1}\right)}{1\,\text{d}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\,\text{s}}{\text{h}}} = \boxed{4.47\times10^{22}\text{ N}\cdot\text{m}}$$

94 •••

Picture the Problem The applied torque accelerates the system and increases the tension in the string until it breaks. The work done before the string breaks is the change in the kinetic energy of the system. We can use Newton's 2nd law to relate the breaking tension to the angular velocity of the system at the instant the string breaks. Once the applied torque is removed, angular momentum is conserved.

Express the work done before the string breaks:

$$W = \Delta K = K_f = \tfrac{1}{2}I_f\omega_f^2 \qquad (1)$$

Express the moment of inertia of the system (see Table 9-1):

$$\begin{aligned} I &= I_{cyl} + 2I_m = I(x) = \tfrac{1}{12}M_{cyl}L_{cyl}^2 + 2mx^2 \\ &= \tfrac{1}{12}(1.2\,\text{kg})(1.6\,\text{m})^2 + 2(0.4\,\text{kg})x^2 \\ &= 0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2 \end{aligned}$$

Evaluate $I_f = I(0.4\text{ m})$:

$$I_f = I(0.4\,\text{m}) = 0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})(0.4\,\text{m})^2 = 0.384\,\text{kg}\cdot\text{m}^2$$

Using Newton's 2nd law, relate the forces acting on a disk to its angular velocity:

$$\sum F_{\text{radial}} = T = mr\omega_f^2$$

where T is the tension in the string at which it breaks.

Solve for ω_f:

$$\omega_f = \sqrt{\frac{T}{mr}}$$

Substitute numerical values and evaluate ω_f:

$$\omega_f = \sqrt{\frac{100\,\text{N}}{(0.4\,\text{kg})(0.4\,\text{m})}} = 25.0\,\text{rad/s}$$

Substitute in equation (1) to express the work done before the string breaks:

$$W = \tfrac{1}{2}I_f\omega_f^2$$

Substitute numerical values and evaluate W:

$$W = \tfrac{1}{2}(0.384\,\text{kg}\cdot\text{m}^2)(25\,\text{rad/s})^2 = \boxed{120\,\text{J}}$$

With the applied torque removed, angular momentum is conserved and we can express the angular momentum as a function of x:

$$L = I_f\omega_f = I(x)\omega(x)$$

Solve for $\omega(x)$:

$$\omega(x) = \frac{I_f\omega_f}{I(x)}$$

Substitute numerical values to obtain:

$$\omega(x) = \frac{(0.384\,\text{kg}\cdot\text{m}^2)(25\,\text{rad/s})}{0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2} = \boxed{\frac{9.60\,\text{J}\cdot\text{s}}{0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2}}$$

95 •••

Picture the Problem The applied torque accelerates the system and increases the tension in the string until it breaks. The work done before the string breaks is the change in the kinetic energy of the system. We can use Newton's 2nd law to relate the breaking tension to the angular velocity of the system at the instant the string breaks. Once the applied torque is removed, angular momentum is conserved.

Express the work done before the string breaks:

$$W = \Delta K = K_f = \tfrac{1}{2} I_f \omega_f^2 \qquad (1)$$

Express the moment of inertia of the system (see Table 9-1):

$$I = I_{cyl} + 2I_m = I(x) = \tfrac{1}{12} M_{cyl} L_{cyl}^2 + 2mx^2$$

Substitute numerical values to obtain:

$$I = \tfrac{1}{12}(1.2\,\text{kg})(1.6\,\text{m})^2 + 2(0.4\,\text{kg})x^2$$
$$= 0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2$$

Evaluate $I_f = I(0.4\text{ m})$:

$$I_f = I(0.4\,\text{m})$$
$$= 0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})(0.4\,\text{m})^2$$
$$= 0.384\,\text{kg}\cdot\text{m}^2$$

Using Newton's 2nd law, relate the forces acting on a disk to its angular velocity:

$$\sum F_{rad} = T = mr\omega_f^2$$

where T is the tension in the string at which it breaks.

Solve for ω_f:

$$\omega_f = \sqrt{\frac{T}{mr}}$$

Substitute numerical values and evaluate ω_f:

$$\omega_f = \sqrt{\frac{100\,\text{N}}{(0.4\,\text{kg})(0.4\,\text{m})}} = 25.0\,\text{rad/s}$$

With the applied torque removed, angular momentum is conserved and we can express the angular momentum as a function of x:

$$L = I_f \omega_f$$
$$= I(x)\omega(x)$$

Solve for $\omega(x)$:

$$\omega(x) = \frac{I_f \omega_f}{I(x)}$$

Substitute numerical values and simplify to obtain:

$$\omega(x) = \frac{(0.384\,\text{kg}\cdot\text{m}^2)(25\,\text{rad/s})}{0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2} = \frac{9.60\,\text{J}\cdot\text{s}}{0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2}$$

Evaluate $\omega(0.8\,\text{m})$:

$$\omega(0.8\,\text{m}) = \frac{9.60\,\text{J}\cdot\text{s}}{0.256\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})(0.8\,\text{m})^2} = \boxed{12.5\,\text{rad/s}}$$

Remarks: Note that this is the angular velocity in both instances. Because the disks leave the cylinder with a tangential velocity of $\frac{1}{2}L\omega$, the angular momentum of the system remains constant.

96 •••

Picture the Problem The applied torque accelerates the system and increases the tension in the string until it breaks. The work done before the string breaks is the change in the kinetic energy of the system. We can use Newton's 2nd law to relate the breaking tension to the angular velocity of the system at the instant the string breaks. Once the applied torque is removed, angular momentum is conserved.

Express the work done before the string breaks:

$$W = \Delta K = K_\text{f} = \tfrac{1}{2}I_\text{f}\omega_\text{f}^2 \qquad (1)$$

Using the parallel axis theorem and treating the disks as thin disks, express the moment of inertia of the system (see Table 9-1):

$$\begin{aligned} I(x) &= I_\text{cyl} + 2I_\text{m} \\ &= \tfrac{1}{12}ML^2 + \tfrac{1}{2}MR^2 + 2\left(\tfrac{1}{4}mR^2 + mx^2\right) \\ &= \tfrac{1}{12}M\left(L^2 + 6R^2\right) + 2m\left(\tfrac{1}{4}R^2 + x^2\right) \end{aligned}$$

Substitute numerical values to obtain:

$$\begin{aligned} I(x) &= \tfrac{1}{12}(1.2\,\text{kg})\left[(1.6\,\text{m})^2 + 6(0.4\,\text{m})^2\right] \\ &\quad + 2(0.4\,\text{kg})\left[\tfrac{1}{4}(0.4\,\text{m})^2 + x^2\right] \\ &= 0.384\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2 \end{aligned}$$

Evaluate $I_\text{f} = I(0.4\text{ m})$:

$$\begin{aligned} I_\text{f} &= I(0.4\,\text{m}) \\ &= 0.384\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})(0.4\,\text{m})^2 \\ &= 0.512\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Using Newton's 2nd law, relate the forces acting on a disk to its angular velocity:

$$\sum F_\text{rad} = T = mr\omega_\text{f}^2$$

where T is the tension in the string at which it breaks.

Solve for ω_f:

$$\omega_\text{f} = \sqrt{\frac{T}{mr}}$$

Substitute numerical values and evaluate ω_f:

$$\omega_\text{f} = \sqrt{\frac{100\,\text{N}}{(0.4\,\text{kg})(0.4\,\text{m})}} = 25.0\,\text{rad/s}$$

Substitute in equation (1) to express the work done before the string breaks:

$$W = \tfrac{1}{2} I_f \omega_f^2$$

Substitute numerical values and evaluate W:

$$W = \tfrac{1}{2}\left(0.512\,\text{kg}\cdot\text{m}^2\right)\left(25\,\text{rad/s}\right)^2 = \boxed{160\,\text{J}}$$

With the applied torque removed, angular momentum is conserved and we can express the angular momentum as a function of x:

$$L = I_f \omega_f = I(x)\omega(x)$$

Solve for $\omega(x)$:

$$\omega(x) = \frac{I_f \omega_f}{I(x)}$$

Substitute numerical values to obtain:

$$\omega(x) = \frac{\left(0.512\,\text{kg}\cdot\text{m}^2\right)\left(25\,\text{rad/s}\right)}{0.384\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2} = \boxed{\frac{12.8\,\text{J}\cdot\text{s}}{0.384\,\text{kg}\cdot\text{m}^2 + (0.8\,\text{kg})x^2}}$$

***97 •••**

Picture the Problem Let the origin of the coordinate system be at the center of the pulley with the upward direction positive. Let λ be the linear density (mass per unit length) of the rope and L_1 and L_2 the lengths of the hanging parts of the rope. We can use conservation of mechanical energy to find the angular velocity of the pulley when the difference in height between the two ends of the rope is 7.2 m.

(*a*) Apply conservation of energy to relate the final kinetic energy of the system to the change in potential energy:

$$\Delta K + \Delta U = 0$$

or, because $K_i = 0$,

$$K + \Delta U = 0 \qquad (1)$$

Express the change in potential energy of the system:

$$\begin{aligned}\Delta U &= U_f - U_i \\ &= -\tfrac{1}{2}L_{1f}(L_{1f}\lambda)g - \tfrac{1}{2}L_{2f}(L_{2f}\lambda)g \\ &\quad -\left[-\tfrac{1}{2}L_{1i}(L_{1i}\lambda)g - \tfrac{1}{2}L_{2i}(L_{2i}\lambda)g\right] \\ &= -\tfrac{1}{2}\left(L_{1f}^2 + L_{2f}^2\right)\lambda g + \tfrac{1}{2}\left(L_{1i}^2 + L_{2i}^2\right)\lambda g \\ &= -\tfrac{1}{2}\lambda g\left[\left(L_{1f}^2 + L_{2f}^2\right) - \left(L_{1i}^2 + L_{2i}^2\right)\right]\end{aligned}$$

Because $L_1 + L_2 = 7.4$ m, $L_{2i} - L_{1i} = 0.6$ m, and $L_{2f} - L_{1f} = 7.2$ m, we obtain:

$L_{1i} = 3.4$ m, $L_{2i} = 4.0$ m, $L_{1f} = 0.1$ m, and $L_{2f} = 7.3$ m.

Substitute numerical values and evaluate ΔU:

$$\begin{aligned}\Delta U &= -\tfrac{1}{2}(0.6\,\text{kg/m})(9.81\,\text{m/s}^2)\\ &\quad\times\left[(0.1\,\text{m})^2 + (7.3\,\text{m})^2 - (3.4\,\text{m})^2 - (4\,\text{m})^2\right]\\ &= -75.75\,\text{J}\end{aligned}$$

Express the kinetic energy of the system when the difference in height between the two ends of the rope is 7.2 m:

$$\begin{aligned}K &= \tfrac{1}{2}I_\text{p}\omega^2 + \tfrac{1}{2}Mv^2\\ &= \tfrac{1}{2}\left(\tfrac{1}{2}M_\text{p}R^2\right)\omega^2 + \tfrac{1}{2}MR^2\omega^2\\ &= \tfrac{1}{2}\left(\tfrac{1}{2}M_\text{p} + M\right)R^2\omega^2\end{aligned}$$

Substitute numerical values and simplify:

$$\begin{aligned}K &= \tfrac{1}{2}\left[\tfrac{1}{2}(2.2\,\text{kg}) + 4.8\,\text{kg}\right]\left(\frac{1.2\,\text{m}}{2\pi}\right)^2\omega^2\\ &= \left(0.1076\,\text{kg}\cdot\text{m}^2\right)\omega^2\end{aligned}$$

Substitute in equation (1) and solve for ω:

$$\left(0.1076\,\text{kg}\cdot\text{m}^2\right)\omega^2 - 75.75\,\text{J} = 0$$

and

$$\omega = \sqrt{\frac{75.75\,\text{J}}{0.1076\,\text{kg}\cdot\text{m}^2}} = \boxed{26.5\,\text{rad/s}}$$

(*b*) Noting that the moment arm of each portion of the rope is the same, express the total angular momentum of the system:

$$\begin{aligned}L &= L_\text{p} + L_\text{r} = I_\text{p}\omega + M_\text{r}R^2\omega\\ &= \left(\tfrac{1}{2}M_\text{p}R^2 + M_\text{r}R^2\right)\omega \qquad (2)\\ &= \left(\tfrac{1}{2}M_\text{p} + M_\text{r}\right)R^2\omega\end{aligned}$$

Letting θ be the angle through which the pulley has turned, express $U(\theta)$:

$$U(\theta) = -\tfrac{1}{2}\left[(L_{1i} - R\theta)^2 + (L_{2i} + R\theta)^2\right]\lambda g$$

Express ΔU and simplify to obtain:

$$\begin{aligned}\Delta U &= U_\text{f} - U_\text{i} = U(\theta) - U(0)\\ &= -\tfrac{1}{2}\left[(L_{1i} - R\theta)^2 + (L_{2i} + R\theta)^2\right]\lambda g\\ &\quad + \tfrac{1}{2}\left(L_{1i}^2 + L_{2i}^2\right)\lambda g\\ &= -R^2\theta^2\lambda g + (L_{1i} - L_{2i})R\theta\lambda g\end{aligned}$$

Assuming that, at $t = 0$, $L_{1i} \approx L_{2i}$:

$$\Delta U \approx -R^2\theta^2\lambda g$$

Substitute for K and ΔU in equation (1) to obtain:

$$\left(0.1076\,\text{kg}\cdot\text{m}^2\right)\omega^2 - R^2\theta^2\lambda g = 0$$

Solve for ω:

$$\omega = \sqrt{\frac{R^2\theta^2\lambda g}{0.1076\,\text{kg}\cdot\text{m}^2}}$$

Substitute numerical values to obtain:

$$\omega = \sqrt{\frac{\left(\frac{1.2\,\text{m}}{2\pi}\right)^2\left(0.6\,\text{kg/m}\right)\left(9.81\,\text{m/s}^2\right)}{0.1076\,\text{kg}\cdot\text{m}^2}}\,\theta = \left(1.41\,\text{s}^{-1}\right)\theta$$

Express ω as the rate of change of θ:

$$\frac{d\theta}{dt} = \left(1.41\,\text{s}^{-1}\right)\theta \Rightarrow \frac{d\theta}{\theta} = \left(1.41\,\text{s}^{-1}\right)dt$$

Integrate θ from 0 to θ to obtain:

$$\ln\theta = \left(1.41\,\text{s}^{-1}\right)t$$

Transform from logarithmic to exponential form to obtain:

$$\theta(t) = e^{\left(1.41\,\text{s}^{-1}\right)t}$$

Differentiate to express ω as a function of time:

$$\omega(t) = \frac{d\theta}{dt} = \left(1.41\,\text{s}^{-1}\right)e^{\left(1.41\,\text{s}^{-1}\right)t}$$

Substitute for ω in equation (2) to obtain:

$$L = \left(\tfrac{1}{2}M_\text{p} + M_\text{r}\right)R^2\left(1.41\,\text{s}^{-1}\right)e^{\left(1.41\,\text{s}^{-1}\right)t}$$

Substitute numerical values and evaluate L:

$$L = \left[\tfrac{1}{2}\left(2.2\,\text{kg}\right) + \left(4.8\,\text{kg}\right)\right]\left(\frac{1.2\,\text{m}}{2\pi}\right)^2\left[\left(1.41\,\text{s}^{-1}\right)e^{\left(1.41\,\text{s}^{-1}\right)t}\right] = \boxed{\left(0.303\,\text{kg}\cdot\text{m}^2/\text{s}\right)e^{\left(1.41\,\text{s}^{-1}\right)t}}$$

Chapter 11
Gravity

Conceptual Problems

***1** •

(a) False. Kepler's law of equal areas is a consequence of the fact that the gravitational force acts along the line joining two bodies but is independent of the manner in which the force varies with distance.

(b) True. The periods of the planets vary with the three-halves power of their distances from the sun. So the shorter the distance from the sun, the shorter the period of the planet's motion.

2 •

Determine the Concept We can apply Newton's 2nd law and the law of gravity to the satellite to obtain an expression for its speed as a function of the radius of its orbit.

Apply Newton's 2nd law to the satellite to obtain:

$$\sum F_{\text{radial}} = \frac{GMm}{r^2} = m\frac{v^2}{r}$$

where M is the mass of the object the satellite is orbiting and m is the mass of the satellite.

Solve for v to obtain:

$$v = \sqrt{\frac{GM}{r}}$$

Thus the speed of the satellite is independent of its mass and:

$$\boxed{(c)\text{ is correct.}}$$

3 ••

Picture the Problem The acceleration due to gravity varies inversely with the square of the distance from the center of the moon.

Express the dependence of the acceleration due to the gravity of the moon on the distance from its center:

$$a' \propto \frac{1}{r^2}$$

Express the dependence of the acceleration due to the gravity of the moon at its surface on its radius:

$$a \propto \frac{1}{R_{\text{M}}^2}$$

Divide the first of these expressions by the second to obtain:

$$\frac{a'}{a}=\frac{R_M^2}{r^2}$$

Solve for a':

$$a'=\frac{R_M^2}{r^2}a=\frac{R_M^2}{(4R_M)^2}a=\tfrac{1}{16}a$$

and $\boxed{(d)\text{ is correct.}}$

4 •

Determine the Concept Measurement of G is difficult because masses accessible in the laboratory are very small compared to the mass of the earth.

5 •

Determine the Concept The escape speed for a planet is given by $v_e=\sqrt{2Gm/R}$.

Between v_e depends on the square root of M, doubling M increases the escape speed by a factor of $\sqrt{2}$ and $\boxed{(a)\text{ is correct.}}$

6 ••

Determine the Concept We can take careful measurements of its position in order to determine whether its trajectory is an ellipse, a hyperbola, or a parabola. If the path is an ellipse, it will return; if its path is hyperbolic or parabolic, it will not return.

7 ••

Determine the Concept The gravitational field is proportional to the mass within the sphere of radius r *and* inversely proportional to the square of r, i.e., proportional to $r^3/r^2=r$.

***8 •**

Determine the Concept Let m represent the mass of Mercury, M_S the mass of the sun, v the orbital speed of Mercury, and R the mean orbital radius of Mercury. We can use Newton's 2nd law of motion to relate the gravitational force acting on the Mercury to its orbital speed.

Use Newton's 2nd law to relate the gravitational force acting on Mercury to its orbital speed:

$$F_{net}=\frac{GM_Sm}{R^2}=m\frac{v^2}{R}$$

Simplify to obtain:

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}\frac{GM_Sm}{R}=-\tfrac{1}{2}\left(-\frac{GM_Sm}{R}\right)$$
$$=-\tfrac{1}{2}U$$

or $\boxed{K = -\frac{1}{2}U}$

9 ••

Picture the Problem We can use the definition of the gravitational field to express the ratio of the student's weight at an elevation of two earth radii to her weight at the surface of the earth.

Express the weight of the student at the surface of the earth:

$$w = mg = \frac{GM_E m}{R_E^2}$$

Express the weight of the student at an elevation of two earth radii:

$$w' = mg' = \frac{GM_E m}{(3R_E)^2}$$

Express the ratio of w' to w:

$$\frac{w'}{w} = \frac{\frac{GM_E m}{(3R_E)^2}}{\frac{GM_E m}{R_E^2}} = \frac{1}{9} \text{ and } \boxed{(d) \text{ is correct.}}$$

10 ••

Determine the Concept One such machine would be a balance wheel with weights attached to the rim with half of them shielded using Cavourite. The weights on one side would be pulled down by the force of gravity, while the other side would not, leading to rotation, which can be converted into useful work, in violation of the law of the conservation of energy.

Estimation and Approximation

11 •

Picture the Problem To approximate the mass of the galaxy we'll assume the galactic center to be a point mass with the sun in orbit about it and apply Kepler's 3rd law.

Using Kepler's 3rd law, relate the period of the sun T to its mean distance r from the center of the galaxy:

$$T^2 = \frac{4\pi^2}{GM_{galaxy}}r^3 = \frac{\frac{4\pi^2}{M_s}}{G\frac{M_{galaxy}}{M_s}}r^3$$

Solve for $\frac{r^3}{T^2}$ to obtain:

$$\frac{r^3}{T^2} = \frac{G\frac{M_{galaxy}}{M_s}}{\frac{4\pi^2}{M_s}} = \frac{\frac{M_{galaxy}}{M_s}}{\frac{4\pi^2}{GM_s}}$$

If we measure distances in AU and times in years:

$$\frac{4\pi^2}{GM_s}=1 \text{ and } \frac{r^3}{T^2}=\frac{M_{galaxy}}{M_s}$$

Substitute numerical values and evaluate M_{galaxy}/M_s:

$$\frac{M_{galaxy}}{M_s}=\frac{\left(3\times10^4\,\text{LY}\times\frac{6.3\times10^4\,\text{AU}}{\text{LY}}\right)^3}{\left(250\times10^6\,\text{y}\right)^2}$$
$$=1.08\times10^{11}$$

or

$$M_{galaxy}=\boxed{1.08\times10^{11}M_s}$$

***12 •••**

Picture the Problem We can use Kepler's 3rd law to find the size of the semi-major axis of the planet's orbit and the conservation of momentum to find its mass.

(*a*) Using Kepler's 3rd law, relate the period of this planet T to the length r of its semi-major axis:

$$T^2=\frac{4\pi^2}{GM_{\text{Iota Draconis}}}r^3$$
$$=\frac{\frac{4\pi^2}{M_s}}{G\frac{M_{\text{Iota Draconis}}}{M_s}}r^3$$
$$=\frac{\frac{4\pi^2}{GM_s}}{\frac{M_{\text{Iota Draconis}}}{M_s}}r^3$$

If we measure time in years, distances in AU, and masses in terms of the mass of the sun:

$$\frac{4\pi^2}{MG_s}=1 \text{ and } T^2=\frac{1}{\frac{M_{\text{Iota Draconis}}}{M_s}}r^3$$

Solve for r to obtain:

$$r=\sqrt[3]{\frac{M_{\text{Iota Draconis}}}{M_s}T^2}$$

Substitute numerical values and evaluate r:

$$r=\sqrt[3]{\left(\frac{1.05M_s}{M_s}\right)(1.5\,\text{y})^2}=\boxed{1.33\,\text{AU}}$$

(*b*) Apply conservation of momentum to the planet (mass m and speed v) and the star (mass $M_{\text{Iota Draconis}}$ and speed V) to obtain:

$$mv=M_{\text{Iota Draconis}}V$$

Solve for m to obtain:

$$m = M_{\text{Iota Draconis}} \frac{V}{v}$$

Use its definition to find the speed of the orbiting planet:

$$v = \frac{\Delta d}{\Delta t} = \frac{2\pi r}{T}$$

$$= \frac{2\pi\left(1.33\,\text{AU} \times \frac{1.5\times10^{11}\,\text{m}}{\text{AU}}\right)}{1.50\,\text{y} \times \frac{365.25\,\text{d}}{\text{y}} \times \frac{24\,\text{h}}{\text{d}} \times \frac{3600\,\text{s}}{\text{h}}}$$

$$= 2.65\times10^{4}\ \text{m/s}$$

Substitute numerical values and evaluate v:

$$m = M_{\text{Iota Draconis}}\left(\frac{296\,\text{m/s}}{2.65\times10^{4}\,\text{m/s}}\right)$$

$$= 0.0112 M_{\text{Iota Draconis}}$$

$$= 0.0112(1.05 M_{\text{sun}})$$

$$= 0.0112(1.05)(1.99\times10^{30}\,\text{kg})$$

$$= 2.34\times10^{28}\,\text{kg}$$

Express m in terms of the mass M_J of Jupiter:

$$\frac{m}{M_J} = \frac{2.34\times10^{28}\,\text{kg}}{1.90\times10^{27}\,\text{kg}} = 12.3$$

or

$$m = \boxed{12.3 M_J}$$

Remarks: A more sophisticated analysis, using the eccentricity of the orbit, leads to a lower bound of 8.7 Jovian masses. (Only a lower bound can be established, as the plane of the orbit is not known.)

13 •••

Picture the Problem We can apply Newton's law of gravity to estimate the maximum angular velocity which the sun can have if it is to stay together and use the definition of angular momentum to find the orbital angular momenta of Jupiter and Saturn. In part (*c*) we can relate the final angular velocity of the sun to its initial angular velocity, its moment of inertia, and the orbital angular momenta of Jupiter and Saturn.

(*a*) Gravity must supply the centripetal force which keeps an element of the sun's mass m rotating around it. Letting the radius of the sun be R, apply Newton's law of gravity to an element of mass m to obtain:

$$m\omega^2 R < \frac{GMm}{R^2}$$

or

$$\omega^2 R < \frac{GM}{R^2}$$

where we've used the inequality because we're estimating the *maximum* angular velocity which the sun can have if it is to stay together.

Solve for ω:

$$\omega < \sqrt{\frac{GM}{R^3}}$$

Substitute numerical values and evaluate ω:

$$\omega < \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(6.96\times10^{8}\ \text{m})^3}} = \boxed{6.28\times10^{-4}\ \text{rad/s}}$$

Calculate the period of this motion from its angular velocity:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6.28\times10^{-4}\ \text{rad/s}}$$

$$= 1.00\times10^{4}\ \text{s}\times\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{2.78\,\text{h}}$$

(b) Express the orbital angular momenta of Jupiter and Saturn:

$$L_J = m_J r_J v_J \text{ and } L_S = m_S r_S v_S$$

Express the orbital speeds of Jupiter and Saturn in terms of their periods and distances from the sun:

$$v_J = \frac{2\pi\, r_J}{T_J} \text{ and } v_S = \frac{2\pi\, r_S}{T_S}$$

Substitute to obtain:

$$L_J = \frac{2\pi\, m_J r_J^2}{T_J} \text{ and } L_S = \frac{2\pi\, m_S r_S^2}{T_S}$$

Substitute numerical values and evaluate L_J and L_S:

$$L_J = \frac{2\pi(318M_E)r_J^2}{T_J} = \frac{2\pi(318)(5.98\times10^{24}\ \text{kg})(778\times10^{9}\ \text{m})^2}{11.9\,\text{y}\times\frac{365.25\,\text{d}}{\text{y}}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\,\text{s}}{\text{h}}}$$

$$= \boxed{1.93\times10^{43}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

and

$$L_S = \frac{2\pi(95.1M_E)r_S^2}{T_S} = \frac{2\pi(95.1)(5.98\times10^{24}\ \text{kg})(1430\times10^{9}\ \text{m})^2}{29.5\,\text{y}\times\frac{365.25\,\text{d}}{\text{y}}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\,\text{s}}{\text{h}}}$$

$$= \boxed{7.85\times10^{42}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

Express the angular momentum of the sun as a fraction of the sum of the angular momenta of Jupiter and Saturn:

$$f = \frac{L_{\text{sun}}}{L_J + L_S}$$

$$= \frac{1.91\times10^{41}\ \text{kg}\cdot\text{m}^2/\text{s}}{(19.3+7.85)\times10^{42}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

$$= \boxed{0.703\%}$$

(*c*) Relate the final angular momentum of the sun to its initial angular momentum and the angular momenta of Jupiter and Saturn:

$$L_f = L_i + L_J + L_S$$

or

$$I_{sun}\omega_f = I_{sun}\omega_i + L_J + L_S$$

Solve for ω_f to obtain:

$$\omega_f = \omega_i + \frac{L_J + L_S}{I_{sun}}$$

Substitute for ω_i and I_{sun}:

$$\omega_f = \frac{2\pi}{T_{sun}} + \frac{L_J + L_S}{0.059 M_{sun} R_{sun}^2}$$

Substitute numerical values and evaluate ω_f:

$$\omega_f = \frac{2\pi}{30\,\text{d} \times \frac{24\,\text{h}}{\text{d}} \times \frac{3600\,\text{s}}{\text{h}}} + \frac{(19.3 + 7.85)\times 10^{42}\ \text{kg}\cdot\text{m}^2/\text{s}}{0.059(1.99\times 10^{30}\ \text{kg})(6.96\times 10^{8}\ \text{m})^2}$$

$$= \boxed{4.80\times 10^{-4}\ \text{rad/s}}$$

Note that this result is about 76% of the maximum possible rotation allowed by gravity that we calculated in part (*a*).

Kepler's Laws

14 •

Picture the Problem We can use the relationship between the semi-major axis and the distances of closest approach and greatest separation, together with Kepler's 3rd law, to find the greatest separation of Alex-Casey from the sun.

Letting x represent the greatest distance from the sun, express the relationship between x, the distance of closest approach, and its semi-major axis R:

$$R = \frac{x + 0.1\,\text{AU}}{2}$$

Solve for x to obtain:

$$x = 2R - 0.1\,\text{AU} \qquad (1)$$

Apply Kepler's 3rd law, with the period T measured in years and R in AU to obtain:

$$T^2 = R^3$$

Solve for R:

$$R = \sqrt[3]{T^2}$$

Substitute numerical values and evaluate R:

$$R = \sqrt[3]{(127.4\,\text{y})^2} = 25.3\,\text{AU}$$

Substitute in equation (1) and evaluate R:

$$x = 2(25.3\,\text{AU}) - 0.1\,\text{AU} = \boxed{50.5\,\text{AU}}$$

15 •

Picture the Problem We can use Kepler's 3rd law to relate the period of Uranus to its mean distance from the sun.

Using Kepler's 3rd law, relate the period of Uranus to its mean distance from the sun:

$$T^2 = Cr^3$$

where $C = \frac{4\pi^2}{GM_s} = 2.973\times10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for T:

$$T = \sqrt{Cr^3}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\left(2.973\times10^{-19}\ \text{s}^2/\text{m}^3\right)\left(2.87\times10^{12}\ \text{m}\right)^3}$$
$$= 2.651\times10^{9}\ \text{s}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1\,\text{d}}{24\,\text{h}}\times\frac{1\,\text{y}}{365.25\,\text{d}} = \boxed{84.0\,\text{y}}$$

16 •

Picture the Problem We can use Kepler's 3rd law to relate the period of Hektor to its mean distance from the sun.

Using Kepler's 3rd law, relate the period of Hektor to its mean distance from the sun:

$$T^2 = Cr^3$$

where $C = \frac{4\pi^2}{GM_s} = 2.973\times10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for T:

$$T = \sqrt{Cr^3}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\left(2.973\times10^{-19}\ \text{s}^2/\text{m}^3\right)\left(5.16\,\text{AU}\times\frac{1.50\times10^{11}\ \text{m}}{\text{AU}}\right)^3}$$
$$= 3.713\times10^{8}\ \text{s}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1\,\text{d}}{24\,\text{h}}\times\frac{1\,\text{y}}{365.25\,\text{d}} = \boxed{11.8\,\text{y}}$$

17 ••

Picture the Problem Kepler's 3rd law relates the period of Icarus to the length of its semimajor axis. The aphelion distance r_a is related to the perihelion distance r_p and the semimajor axis by $r_a + r_p = 2a$.

(*a*) Using Kepler's 3rd law, relate the period of Icarus to the length of its semimajor axis:

$$T^2 = Ca^3$$

where $C = \frac{4\pi^2}{GM_s} = 2.973\times10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for *a*:

$$a = \sqrt[3]{\frac{T^2}{C}}$$

Substitute numerical values and evaluate *a*:

$$a = \sqrt[3]{\frac{\left(1.1\,\text{y}\times\frac{365.25\,\text{d}}{\text{y}}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\,\text{s}}{\text{h}}\right)^2}{2.973\times10^{-19}\ \text{s}^2/\text{m}^3}} = \boxed{1.59\times10^{11}\ \text{m}}$$

(*b*) Use the definition of the eccentricity of an ellipse to determine the perihelion distance of Icarus:

$$r_\text{p} = a(1-e) = \left(1.59\times10^{11}\ \text{m}\right)(1-0.83) = \boxed{2.71\times10^{10}\ \text{m}}$$

Express the relationship between r_p and r_a for an ellipse:

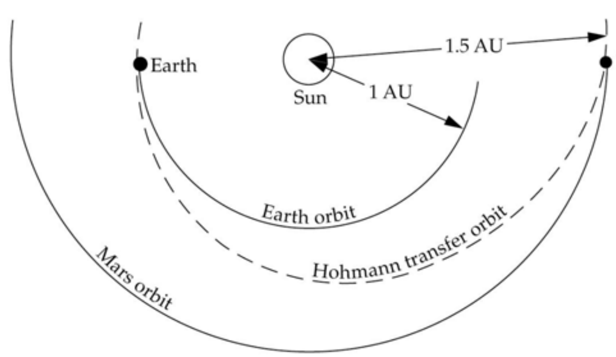
$$r_\text{a} + r_\text{p} = 2a$$

Solve for and evaluate r_a:

$$r_\text{a} = 2a - r_\text{p} = 2\left(1.59\times10^{11}\ \text{m}\right) - 2.71\times10^{10}\ \text{m} = \boxed{2.91\times10^{11}\ \text{m}}$$

18 ••

Picture the Problem The Hohmann transfer orbit is shown in the diagram. We can apply Kepler's 3rd law to relate the time-in-orbit to the period of the spacecraft in its Hohmann Earth-to-Mars orbit. The period of this orbit is, in turn, a function of its semi-major axis which we can find from the average of the lengths of the semi-major axes of the Earth and Mars orbits.



Using Kepler's 3rd law, relate the period T of the spacecraft to the semi-major axis of its orbit:

$$T^2 = R^3$$

Solve for T to obtain:

$$T = \sqrt{R^3}$$

Relate the transit time to the period of this orbit:

$$t_{\text{trip}} = \tfrac{1}{2}T = \tfrac{1}{2}\sqrt{R^3}$$

Express the semi-major axis of the Hohmann transfer orbit in terms of the mean sun-Mars and sun-Earth distances:

$$R = \frac{1.52\,\text{AU} + 1.00\,\text{AU}}{2} = 1.26\,\text{AU}$$

Substitute numerical values and evaluate t_{trip}:

$$t_{\text{trip}} = \tfrac{1}{2}\sqrt{(1.26\,\text{AU})^3}$$
$$= 0.707\,\text{y} \times \frac{365.24\,\text{d}}{1\,\text{y}} = \boxed{258\ \text{d}}$$

***19 ••**

Picture the Problem We can use a property of lines tangent to a circle and radii drawn to the point of contact to show that $b = 90°$. Once we've established that b is a right angle we can use the definition of the sine function to relate the distance from the sun to Venus to the distance from the sun to the earth.

(a) The line from earth to Venus′ orbit is tangent to the orbit of Venus at the point of maximum extension. Venus will appear closer to the sun in earth's sky when it passes the line drawn from earth and tangent to its orbit. Hence:

$$b = \boxed{90°}$$

(b) Using trigonometry, relate the distance from the sun to Venus d_{SV} to the angle a:

$$\sin a = \frac{d_{\text{SV}}}{d_{\text{SE}}}$$

Solve for d_{SV}:

$$d_{\text{SV}} = d_{\text{SE}} \sin a$$

Substitute numerical values and evaluate d_{SV}:

$$d_{\text{SV}} = (1\,\text{AU})\sin 47° = \boxed{0.731\,\text{AU}}$$

Remarks: The correct distance from the sun to Venus is closer to 0.723 AU.

20 ••

Picture the Problem Because the gravitational force the Earth exerts on the moon is along the line joining their centers, the net torque acting on the moon is zero and its angular momentum is conserved in its orbit about the Earth. Because energy is also conserved, we can combine these two expressions to solve for either v_p or v_a initially and then substitute in the conservation of angular momentum equation to find the other.

Letting m be the mass of the moon, apply conservation of angular momentum to the moon at apogee and perigee to obtain:

$$mv_p r_p = mv_a r_a$$

or

$$v_p r_p = v_a r_a$$

Solve for v_a:

$$v_a = \frac{r_p}{r_a} v_p \qquad (1)$$

Apply conservation of energy to the moon-earth system to obtain:

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

or

$$\frac{1}{2}v_p^2 - \frac{GM}{r_p} = \frac{1}{2}v_a^2 - \frac{GM}{r_a}$$

Substitute for v_a to obtain:

$$\frac{1}{2}v_p^2 - \frac{GM}{r_p} = \frac{1}{2}\left(\frac{r_p}{r_a}v_p\right)^2 - \frac{GM}{r_a} = \frac{1}{2}\left(\frac{r_p}{r_a}\right)^2 v_p^2 - \frac{GM}{r_a}$$

Solve for v_p to obtain:

$$v_p = \sqrt{\frac{2GM}{r_p}\left(\frac{1}{1+r_p/r_a}\right)}$$

Substitute numerical values and evaluate v_p:

$$v_p = \sqrt{\frac{2\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{3.576\times10^{8}\ \text{m}}\left(\frac{1}{1+\frac{3.576\times10^{8}\ \text{m}}{4.064\times10^{8}\ \text{m}}}\right)} = \boxed{1.09\ \text{km/s}}$$

Substitute numerical values in equation (1) and evaluate v_a:

$$v_a = \frac{3.576\times10^{8}\ \text{m}}{4.064\times10^{8}\ \text{m}}(1.09\ \text{km/s}) = \boxed{0.959\ \text{km/s}}$$

Newton's Law of Gravity

***21** ••

Picture the Problem We can use Kepler's 3rd law to find the mass of Jupiter in part (*a*). In part (*b*) we can express the centripetal accelerations of Europa and Callisto and compare their ratio to the square of the ratio of their distances from the center of Jupiter to show that the given data is consistent with an inverse square law for gravity.

(*a*) Assuming a circular orbit, apply Kepler's 3rd law to the motion of Europa to obtain:

$$T_E^2 = \frac{4\pi^2}{GM_J} R_E^3$$

Solve for the mass of Jupiter:

$$M_J = \frac{4\pi^2}{GT_E^2} R_E^3$$

Substitute numerical values and evaluate M_J:

$$M_J = \frac{4\pi^2}{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)} \times \frac{\left(6.71\times10^8\ \text{m}\right)^3}{\left(3.55\ \text{d}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}\right)^2}$$

$$= \boxed{1.90\times10^{27}\ \text{kg}}$$, a result in excellent agreement with the accepted value of 1.902×10^{27} kg.

(*b*) Express the centripetal acceleration of both of the moons to obtain:

$$\frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$$

where *R* and *T* are the radii and periods of their motion.

Using this result, express the centripetal accelerations of Europa and Callisto:

$$a_E = \frac{4\pi^2 R_E}{T_E^2} \text{ and } a_C = \frac{4\pi^2 R_C}{T_C^2}$$

Substitute numerical values and evaluate a_E:

$$a_E = \frac{4\pi^2\left(6.71\times10^8\ \text{m}\right)}{\left[(3.55\ \text{d})(24\ \text{h/d})(3600\ \text{s/h})\right]^2} = \boxed{0.282\ \text{m/s}^2}$$

Substitute numerical values and evaluate a_C:

$$a_C = \frac{4\pi^2\left(18.8\times10^8\ \text{m}\right)}{\left[(16.7\ \text{d})(24\ \text{h/d})(3600\ \text{s/h})\right]^2} = \boxed{0.0356\ \text{m/s}^2}$$

Evaluate the ratio of these accelerations:

$$\frac{a_E}{a_C} = \frac{0.282\ \text{m/s}^2}{0.0356\ \text{m/s}^2} = 7.91$$

Evaluate the square of the ratio of the distance of Callisto divided by the distance of Europa to obtain:

$$\left(\frac{R_C}{R_E}\right)^2 = \left(\frac{18.8\times10^8\text{ m}}{6.71\times10^8\text{ m}}\right)^2 = 7.85$$

The close agreement (within 1%) of our last two calculations strongly supports the conclusion that the gravitational force varies inversely with the square of the distance.

***22 •**

Determine the Concept The weight of anything, including astronauts, is the reading of a scale from which the object is suspended or on which it rests. If the scale reads zero, then we say the object is ″weightless.″ The pull of the earth's gravity, on the other hand, depends on the local value of the acceleration of gravity and we can use Newton's law of gravity to find this acceleration at the elevation of the shuttle.

(*a*) Apply Newton's law of gravitation to an astronaut of mass *m* in a shuttle at a distance *h* above the surface of the earth:

$$mg_{\text{shuttle}} = \frac{GmM_E}{(h+R_E)^2}$$

Solve for g_{shuttle}:

$$g_{\text{shuttle}} = \frac{GM_E}{(h+R_E)^2}$$

Substitute numerical values and evaluate g_{shuttle}:

$$g_{\text{shuttle}} = \frac{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(400\text{ km}+6370\text{ km})^2} = \boxed{8.71\text{ m/s}^2}$$

(*b*) Because they are in "free fall" everything on the shuttle is falling toward the center of the earth with exactly the same acceleration, so the astronauts will seem to be "weightless."

23 •

Picture the Problem We can use Kepler's 3rd law to relate the periods of the moons of Saturn to their mean distances from its center.

(*a*) Using Kepler's 3rd law, relate the period of Mimas to its mean distance from the center of Saturn:

$$T_M^2 = \frac{4\pi^2}{GM_S}r_M^3$$

Solve for T_M:

$$T_M = \sqrt{\frac{4\pi^2}{GM_S}r_M^3}$$

(*b*) Using Kepler's 3rd law, relate the period of Titan to its mean distance from the center of Saturn:

$$T_T^2 = \frac{4\pi^2}{GM_S} r_T^3$$

Substitute numerical values and evaluate T_M:

$$T_M = \sqrt{\frac{4\pi^2 (1.86\times10^8\ \text{m})^3}{(5.69\times10^{26}\ \text{kg})(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}} = \boxed{8.18\times10^4\ \text{s}}$$

Solve for r_T:

$$r_T = \sqrt[3]{\frac{T_T^2 GM_S}{4\pi^2}}$$

Substitute numerical values and evaluate r_T:

$$r_T = \sqrt[3]{\frac{(1.38\times10^6\ \text{s})^2 (6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.69\times10^{26}\ \text{kg})}{4\pi^2}} = \boxed{1.22\times10^9\ \text{m}}$$

24 •

Picture the Problem We can use Kepler's 3rd law to relate the period of the moon to the mass of the earth and the mean earth-moon distance.

(*a*) Using Kepler's 3rd law, relate the period of the moon to its mean orbital radius:

$$T_m^2 = \frac{4\pi^2}{GM_E} r_m^3$$

Solve for M_E:

$$M_E = \frac{4\pi^2}{GT_m^2} r_m^3$$

Substitute numerical values and evaluate M_E:

$$M_E = \frac{4\pi^2 (3.84\times10^8\ \text{m})^3}{(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(27.3\,\text{d}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}\right)^2} = \boxed{6.02\times10^{24}\ \text{kg}}$$

Remarks: This analysis neglects the mass of the moon; consequently the mass calculated here is slightly too great.

25 •

Picture the Problem We can use Kepler's 3rd law to relate the period of the earth to the mass of the sun and the mean earth-sun distance.

(*a*) Using Kepler's 3rd law, relate the period of the earth to its mean orbital radius:

$$T_E^2 = \frac{4\pi^2}{GM_S} r_E^3$$

Solve for M_S:

$$M_S = \frac{4\pi^2}{GT_E^2} r_E^3$$

Substitute numerical values and evaluate M_S:

$$M_S = \frac{4\pi^2 \left(1.496\times10^{11}\text{ m}\right)^3}{\left(6.6726\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1\text{ y}\times\frac{365.25\text{ d}}{\text{y}}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}\right)^2}$$

$$= \boxed{1.99\times10^{30}\text{ kg}}$$

***26 •**

Picture the Problem We can relate the acceleration of an object at any elevation to its acceleration at the surface of the earth through the law of gravity and Newton's 2nd law of motion.

Letting a represent the acceleration due to gravity at this altitude (R_E) and m the mass of the object, apply Newton's 2nd law and the law of gravity to obtain:

$$\sum F_{\text{radial}} = \frac{GmM_E}{(2R_E)^2} = ma$$

and

$$a = \frac{GM_E}{(2R_E)^2} \qquad (1)$$

Apply Newton's 2nd law to the same object when it is at the surface of the earth:

$$\sum F_{\text{radial}} = \frac{GmM_E}{R_E^2} = mg$$

and

$$g = \frac{GM_E}{R_E^2} \qquad (2)$$

Divide equation (1) by equation (2) and solve for a:

$$\frac{a}{g} = \frac{R_E^2}{4R_E^2}$$

and

$$a = \tfrac{1}{4}g = \tfrac{1}{4}\left(9.81\text{ m/s}^2\right) = \boxed{2.45\text{ m/s}^2}$$

27 •

Picture the Problem Your weight is the local gravitational force exerted on you. We can use the definition of density to relate the mass of the planet to the mass of earth and the law of gravity to relate your weight on the planet to your weight on earth.

Using the definition of density, relate the mass of the earth to its radius:

$$M_E = \rho V_E = \tfrac{4}{3}\rho\pi R_E^3$$

Relate the mass of the planet to its radius:

$$M_P = \rho V_P = \tfrac{4}{3}\rho\pi R_P^3 = \tfrac{4}{3}\rho\pi(10R_E)^3$$

Divide the second of these equations by the first to express M_P in terms of M_E:

$$\frac{M_P}{M_E} = \rho\frac{\tfrac{4}{3}\rho\pi(10R_E)^3}{\tfrac{4}{3}\rho\pi R_E^3}$$

and

$$M_P = 10^3 M_E$$

Letting w' represent your weight on the planet, use the law of gravity to relate w' to your weight on earth:

$$w' = \frac{GmM_P}{R_P^2} = \frac{Gm(10^3 M_E)}{(10R_E)^2} = 10\frac{GmM_E}{R_E^2} = \boxed{10w}$$

where w is your weight on earth.

28 •

Picture the Problem We can relate the acceleration due to gravity of a test object at the surface of the new planet to the acceleration due to gravity at the surface of the earth through use of the law of gravity and Newton's 2nd law of motion.

Letting a represent the acceleration due to gravity at the surface of this new planet and m the mass of a test object, apply Newton's 2nd law and the law of gravity to obtain:

$$\sum F_{\text{radial}} = \frac{GmM_E}{\left(\tfrac{1}{2}R_E\right)^2} = ma$$

and

$$a = \frac{GM_E}{\left(\tfrac{1}{2}R_E\right)^2}$$

Simplify this expression to obtain:

$$a = 4\frac{GM_E}{R_E^2} = 4g = \boxed{39.2\,\text{m/s}^2}$$

29 •

Picture the Problem We can use conservation of angular momentum to relate the planet's speeds at aphelion and perihelion.

Using conservation of angular

$$L_a = L_p$$

momentum, relate the angular momenta of the planet at aphelion and perihelion:

or

$$mv_p r_p = mv_a r_a$$

Solve for the planet's speed at aphelion:

$$v_a = \frac{v_p r_p}{r_a}$$

Substitute numerical values and evaluate v_a:

$$v_a = \frac{(5\times10^4\ \text{m/s})(1.0\times10^{15}\ \text{m})}{2.2\times10^{15}\ \text{m}} = \boxed{2.27\times10^4\ \text{m/s}}$$

30 •

Picture the Problem We can use Newton's law of gravity to express the gravitational force acting on an object at the surface of the neutron star in terms of the weight of the object. We can then simplify this expression be dividing out the mass of the object … leaving an expression for the acceleration due to gravity at the surface of the neutron star.

Apply Newton's law of gravity to an object of mass m at the surface of the neutron star to obtain:

$$\frac{GM_{\text{Neutron Star}}m}{R^2_{\text{Neutron Star}}} = mg$$

where g represents the acceleration due to gravity at the surface of the neutron star.

Solve for g and substitute for the mass of the neutron star:

$$g = \frac{GM_{\text{Neutron Star}}}{R^2_{\text{Neutron Star}}} = \frac{G(1.60M_{\text{sun}})}{R^2_{\text{Neutron Star}}}$$

Substitute numerical values and evaluate g:

$$g = \frac{1.60(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(10.5\ \text{km})^2} = \boxed{1.93\times10^{12}\ \text{m/s}^2}$$

***31** ••

Picture the Problem We can use conservation of angular momentum to relate the asteroid's aphelion and perihelion distances.

Using conservation of angular momentum, relate the angular momenta of the asteroid at aphelion and perihelion:

$$L_a = L_p$$

or

$$mv_p r_p = mv_a r_a$$

Solve for and evaluate the ratio of the asteroid's aphelion and perihelion distances:

$$\frac{r_a}{r_p} = \frac{v_p}{v_a} = \frac{20\,\text{km/s}}{14\,\text{km/s}} = \boxed{1.43}$$

32 ••

Picture the Problem We'll use the law of gravity to find the gravitational force acting on the satellite. The application of Newton's 2nd law will lead us to the speed of the satellite and its period can be found from its definition.

(*a*) Letting *m* represent the mass of the satellite and *h* its elevation, use the law of gravity to express the gravitational force acting on it:

$$F_g = \frac{GmM_E}{(R_E + h)^2} = \frac{mR_E^2 g}{(R_E + h)^2} = \frac{mg}{\left(1 + \frac{h}{R_E}\right)^2}$$

Substitute numerical values and evaluate F_g:

$$F_g = \frac{mg}{\left(1 + \frac{h}{R_E}\right)^2} = \frac{(300\,\text{kg})(9.81\,\text{N/kg})}{\left(1 + \frac{5\times10^7\,\text{m}}{6.37\times10^6\,\text{m}}\right)^2} = \boxed{37.6\,\text{N}}$$

(*b*) Using Newton's 2nd law, relate the gravitational force acting on the satellite to its centripetal acceleration:

$$F_g = m\frac{v^2}{r}$$

Solve for *v*:

$$v = \sqrt{\frac{F_g r}{m}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{(37.6\,\text{N})(6.37\times10^6\,\text{m} + 5\times10^7\,\text{m})}{300\,\text{kg}}} = \boxed{2.66\ \text{km/s}}$$

(*c*) Express the period of the satellite:

$$T = \frac{2\pi r}{v}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{2\pi(6.37\times10^6\,\text{m} + 5\times10^7\,\text{m})}{2.66\times10^3\,\text{m/s}} = 1.33\times10^5\,\text{s}\times\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{36.9\,\text{h}}$$

***33 ••**

Picture the Problem We can determine the maximum range at which an object with a given mass can be detected by substituting the equation for the gravitational field in the expression for the resolution of the meter and solving for the distance. Differentiating $g(r)$ with respect to r, separating variables to obtain dg/g, and approximating Δr with dr will allow us to determine the vertical change in the position of the gravity meter in the earth's gravitational field is detectable.

(*a*) Express the gravitational field of the earth:

$$g_E = \frac{GM_E}{R_E^2}$$

Express the gravitational field due to the mass m (assumed to be a point mass) of your friend and relate it to the resolution of the meter:

$$g(r) = \frac{Gm}{r^2} = 10^{-11} g_E = 10^{-11}\frac{GM_E}{R_E^2}$$

Solve for r:

$$r = R_E\sqrt{\frac{10^{11} m}{M_E}}$$

Substitute numerical values and evaluate r:

$$r = \left(6.37\times10^6\ \text{m}\right)\sqrt{\frac{10^{11}(80\ \text{kg})}{5.98\times10^{24}\ \text{kg}}} = \boxed{7.37\ \text{m}}$$

(*b*) Differentiate $g(r)$ and simplify to obtain:

$$\frac{dg}{dr} = \frac{-2Gm}{r^3} = -\frac{2}{r}\left(\frac{Gm}{r^2}\right) = -\frac{2}{r}g$$

Separate variables to obtain:

$$\frac{dg}{g} = -2\frac{dr}{r} = 10^{-11}$$

Approximating dr with Δr, evaluate Δr with $r = R_E$:

$$\Delta r = \left|-\tfrac{1}{2}\left(10^{-11}\right)\left(6.37\times10^6\ \text{m}\right)\right| = 3.19\times10^{-5}\ \text{m} = \boxed{0.0319\ \text{mm}}$$

34 ••

Picture the Problem We can use the law of gravity and Newton's 2nd law to relate the force exerted on the planet by the star to its orbital speed and the definition of the period to relate it to the radius of the orbit.

Using the law of gravity and Newton's 2nd law, relate the force exerted on the planet by the star to its centripetal acceleration:

$$F_{\text{net}} = \frac{KMm}{r} = m\frac{v^2}{r}$$

Solve for v^2 to obtain:

$$v^2 = KM$$

Express the period of the planet:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{KM}} = \frac{2\pi}{\sqrt{KM}} r$$

or

$$\boxed{T \propto r}$$

***35 ••**

Picture the Problem We can use the definitions of the gravitational fields at the surfaces of the earth and the moon to express the accelerations due to gravity at these locations in terms of the average densities of the earth and the moon. Expressing the ratio of these accelerations will lead us to the ratio of the densities.

Express the acceleration due to gravity at the surface of the earth in terms of the earth's average density:

$$g_{\text{E}} = \frac{GM_{\text{E}}}{R_{\text{E}}^2} = \frac{G\rho_{\text{E}}V_{\text{E}}}{R_{\text{E}}^2} = \frac{G\rho_{\text{E}}\,\frac{4}{3}\pi R_{\text{E}}^3}{R_{\text{E}}^2}$$
$$= \tfrac{4}{3}G\rho_{\text{E}}\pi R_{\text{E}}$$

Express the acceleration due to gravity at the surface of the moon in terms of the moon's average density:

$$g_{\text{M}} = \tfrac{4}{3}G\rho_{\text{M}}\pi R_{\text{M}}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_{\text{M}}}{g_{\text{E}}} = \frac{\rho_{\text{M}}\,R_{\text{M}}}{\rho_{\text{E}}\,R_{\text{E}}}$$

Solve for $\frac{\rho_{\text{M}}}{\rho_{\text{E}}}$:

$$\frac{\rho_{\text{M}}}{\rho_{\text{E}}} = \frac{g_{\text{M}}R_{\text{E}}}{g_{\text{E}}R_{\text{M}}}$$

Substitute numerical values and evaluate $\frac{\rho_{\text{M}}}{\rho_{\text{E}}}$:

$$\frac{\rho_{\text{M}}}{\rho_{\text{E}}} = \frac{(1.62\,\text{m/s}^2)(6.37\times10^6\,\text{m})}{(9.81\,\text{m/s}^2)(1.738\times10^6\,\text{m})}$$
$$= \boxed{0.605}$$

Measurement of G

36 •

Picture the Problem We can use the law of gravity to find the forces of attraction between the two masses and the definition of torque to determine the balancing torque required.

(*a*) Use the law of gravity to express the force of attraction between the two masses:

$$F=\frac{Gm_1m_2}{r^2}$$

Substitute numerical values and evaluate *F*:

$$F=\frac{\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(10\,\text{kg}\right)\left(0.01\,\text{kg}\right)}{\left(0.06\,\text{m}\right)^2}=\boxed{1.85\times10^{-9}\ \text{N}}$$

(*b*) Use its definition to find the torque exerted by the suspension to balance these forces:

$$\tau=2Fr=2\left(1.85\times10^{-9}\ \text{N}\right)\left(0.1\,\text{m}\right)=\boxed{3.70\times10^{-10}\ \text{N}\cdot\text{m}}$$

Gravitational and Inertial Mass

37 •

Picture the Problem Newton's 2nd law of motion relates the masses and accelerations of these objects to their common accelerating force.

(*a*) Apply Newton's 2nd law to the standard object:

$$F=m_1a_1$$

Apply Newton's 2nd law to the object of unknown mass:

$$F=m_2a_2$$

Eliminate *F* between these two equations and solve for m_2:

$$m_2=\frac{a_1}{a_2}m_1$$

Substitute numerical values and evaluate m_2:

$$m_2=\frac{2.6587\,\text{m/s}^2}{1.1705\,\text{m/s}^2}\left(1\,\text{kg}\right)=\boxed{2.27\,\text{kg}}$$

(*b*)

$$\boxed{\text{It is the } \textit{inertial} \text{ mass of } m_2.}$$

38 •

Picture the Problem Newton's 2nd law of motion relates the weights of these two objects to their masses and the acceleration due to gravity.

(*a*) Apply Newton's 2nd law to the standard object:

$$F_{net} = w_1 = m_1 g$$

Apply Newton's 2nd law to the object of unknown mass:

$$F_{net} = w_2 = m_2 g$$

Eliminate *g* between these two equations and solve for m_2:

$$m_2 = \frac{w_2}{w_1} m_1$$

Substitute numerical values and evaluate m_2:

$$m_2 = \frac{56.6\text{ N}}{9.81\text{ N}}(1\text{ kg}) = \boxed{5.77\text{ kg}}$$

(*b*) Since this result is determined by the effect on m_2 of the earth's gravitational field, it is the *gravitational* mass of m_2.

***39** •

Picture the Problem Noting that $g_1 \sim g_2 \sim g$, let the acceleration of gravity on the first object be g_1, and on the second be g_2. We can use a constant-acceleration equation to express the difference in the distances fallen by each object and then relate the average distance fallen by the two objects to obtain an expression from which we can approximate the distance they would have to fall before we might measure a difference in their fall distances greater than 1 mm.

Express the difference Δd in the distances fallen by the two objects in time *t*:

$$\Delta d = d_1 - d_2$$

Express the distances fallen by each of the objects in time *t*:

$$d_1 = \tfrac{1}{2} g_1 t^2$$

and

$$d_2 = \tfrac{1}{2} g_2 t^2$$

Substitute to obtain:

$$\Delta d = \tfrac{1}{2} g_1 t^2 - \tfrac{1}{2} g_2 t^2 = \tfrac{1}{2}(g_1 - g_2)\, t^2$$

Relate the average distance *d* fallen by the two objects to their time of fall:

$$d = \tfrac{1}{2} g t^2$$

or

$$t^2 = \frac{2d}{g}$$

Substitute to obtain:

$$\Delta d \approx \tfrac{1}{2}\Delta g \frac{2d}{g} = d\frac{\Delta g}{g}$$

Solve for d to obtain:

$$d = \Delta d \frac{g}{\Delta g}$$

Substitute numerical values and evaluate d:

$$d = \left(10^{-3}\text{ m}\right)\left(10^{12}\right) = \boxed{10^{9}\text{ m}}$$

Gravitational Potential Energy

40 •

Picture the Problem Choosing the zero of gravitational potential energy to be at infinite separation yields, as the potential energy of a two-body system in which the objects are separated by a distance r, $U(r) = -GMm/r$, where M and m are the masses of the two bodies. In order for an object to just escape a gravitational field from a particular location, it must have enough kinetic energy so that its total energy is zero.

(*a*) Letting $U(\infty) = 0$, express the gravitational potential energy of the earth-object system:

$$U(r) = -\frac{GM_{\text{E}}m}{r} \qquad (1)$$

Substitute for GM_{E} and simplify to obtain:

$$U(R_{\text{E}}) = -\frac{GM_{\text{E}}m}{R_{\text{E}}} = -\frac{gR_{\text{E}}^2 m}{R_{\text{E}}} = -mgR_{\text{E}}$$

Substitute numerical values and evaluate $U(R_{\text{E}})$:

$$U(R_{\text{E}}) = -(100\,\text{kg})(9.81\,\text{N/kg})\left(6.37\times10^{6}\text{ m}\right) = \boxed{-6.25\times10^{9}\text{ J}}$$

(*b*) Evaluate equation (1) with $r = 2R_{\text{E}}$:

$$U(2R_{\text{E}}) = -\frac{GM_{\text{E}}m}{2R_{\text{E}}} = -\frac{gR_{\text{E}}^2 m}{2R_{\text{E}}} = -\tfrac{1}{2}mgR_{\text{E}}$$

Substitute numerical values and evaluate $U(2R_{\text{E}})$:

$$U(2R_{\text{E}}) = -\tfrac{1}{2}(100\,\text{kg})(9.81\,\text{N/kg})\left(6.37\times10^{6}\text{ m}\right) = \boxed{-3.12\times10^{9}\text{ J}}$$

(*c*) Express the condition that an object must satisfy in order to escape from the earth's gravitational

$$K_{\text{esc}}(2R_{\text{E}}) + U(2R_{\text{E}}) = 0$$

or

$$\tfrac{1}{2}mv_{\text{esc}}^2 + U(2R_{\text{E}}) = 0$$

field from a height R_E above its surface:

Solve for v_{esc}:

$$v_{esc} = \sqrt{\frac{-2U(2R_E)}{m}}$$

Substitute numerical values and evaluate v_{esc}:

$$v_{esc} = \sqrt{\frac{-2(-3.12\times10^9\ \text{J})}{100\,\text{kg}}} = \boxed{7.90\,\text{km/s}}$$

41 •

Picture the Problem In order for an object to just escape a gravitational field from a particular location, an amount of work must be done on it that is equal to its potential energy in its initial position.

Express the work needed to remove the point mass from the surface of the sphere to a point a very large distance away:

$$W = \Delta U = U_f - U_i$$

or, because $U_f = 0$,

$$W = \Delta U = -U_i \qquad (1)$$

Express the initial potential energy of the system:

$$U_i = -\frac{GMm_0}{R}$$

Substitute in equation (1) to obtain:

$$W = \boxed{\frac{GMm_0}{R}}$$

42 •

Picture the Problem Let the zero of gravitational potential energy be at infinity and let m represent the mass of the spacecraft. We'll use conservation of energy to relate the initial kinetic and potential energies to the final potential energy of the earth-spacecraft system.

Use conservation of energy to relate the initial kinetic and potential energies of the system to its final energy when the spacecraft is one earth radius above the surface of the planet:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-K(R_E) + U(2R_E) - U(R_E) = 0 \qquad (1)$$

Express the potential energy of the spacecraft-and-earth system when the spacecraft is at a distance r from the surface of the earth:

$$U(r) = -\frac{GM_{\text{E}}m}{r}$$

Substitute in equation (1) to obtain:

$$-\tfrac{1}{2}mv^2 - \frac{GM_{\text{E}}m}{2R_{\text{E}}} + \frac{GM_{\text{E}}m}{R_{\text{E}}} = 0$$

Solve for v:

$$v = \sqrt{\frac{GM_{\text{E}}}{R_{\text{E}}}} = \sqrt{\frac{gR_{\text{E}}^2}{R_{\text{E}}}} = \sqrt{gR_{\text{E}}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(9.81\,\text{m/s}^2)(6.37\times10^6\,\text{m})} = \boxed{7.91\,\text{km/s}}$$

***43 ••**

Picture the Problem Let the zero of gravitational potential energy be at infinity and let m represent the mass of the object. We'll use conservation of energy to relate the initial potential energy of the object-earth system to the final potential and kinetic energies.

Use conservation of energy to relate the initial potential energy of the system to its energy as the object is about to strike the earth:

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

or, because $K_{\text{i}} = 0$,

$$K(R_{\text{E}}) + U(R_{\text{E}}) - U(R_{\text{E}} + h) = 0 \quad (1)$$

where h is the initial height above the earth's surface.

Express the potential energy of the object-earth system when the object is at a distance r from the surface of the earth:

$$U(r) = -\frac{GM_{\text{E}}m}{r}$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv^2 - \frac{GM_{\text{E}}m}{R_{\text{E}}} + \frac{GM_{\text{E}}m}{R_{\text{E}} + h} = 0$$

Solve for v:

$$v = \sqrt{2\left(\frac{GM_{\text{E}}}{R_{\text{E}}} - \frac{GM_{\text{E}}}{R_{\text{E}} + h}\right)} = \sqrt{2gR_{\text{E}}\left(\frac{h}{R_{\text{E}} + h}\right)}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9.81\,\text{m/s}^2)(6.37\times10^6\,\text{m})(4\times10^6\,\text{m})}{6.37\times10^6\,\text{m} + 4\times10^6\,\text{m}}} = \boxed{6.94\,\text{km/s}}$$

44 ••

Picture the Problem Let the zero of gravitational potential energy be at infinity, *m* represent the mass of the object, and *h* the maximum height reached by the object. We'll use conservation of energy to relate the initial potential and kinetic energies of the object-earth system to the final potential energy.

Use conservation of energy to relate the initial potential energy of the system to its energy as the object is about to strike the earth:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$K(R_E) + U(R_E) - U(R_E + h) = 0 \quad (1)$$

where h is the initial height above the earth's surface.

Express the potential energy of the object-earth system when the object is at a distance r from the surface of the earth:

$$U(r) = -\frac{GM_E m}{r}$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv^2 - \frac{GM_E m}{R_E} + \frac{GM_E m}{R_E + h} = 0$$

Solve for h:

$$h = \frac{R_E}{\dfrac{2gR_E}{v^2} - 1}$$

Substitute numerical values and evaluate h:

$$h = \frac{6.37\times10^6\,\text{m}}{\dfrac{2(9.81\,\text{m/s}^2)(6.37\times10^6\,\text{m})}{(4\times10^3\,\text{m})^2} - 1} = \boxed{935\,\text{km}}$$

45 ••

Picture the Problem When the point mass is inside the spherical shell, there is no mass between it and the center of the shell. On the other hand, when the point mass is outside the spherical shell we can use the law of gravity to express the force acting on it. In (*b*) we can derive $U(r)$ from $F(r)$.

(*a*) The force exerted by the shell on a point mass m_0 when m_0 is inside the shell is:

$$\vec{F}_{\text{inside}} = \boxed{0}$$

The force exerted by the shell on a point mass m_0 when m_0 is outside the shell is:

$$\vec{F}_{\text{outside}} = m_0\vec{g} = \boxed{-\frac{GMm_0}{r^2}\hat{r}}$$

where $\hat{r}$ is radially outward from the center of the spherical shell.

(*b*) Use its definition to express $U(r)$ for $r > R$:

$$U(r) = -\int_\infty^r F_r dr = GMm_0\int_\infty^r r^{-2}dr$$

$$= \boxed{-\frac{GMm_0}{r}}$$

When $r = R$:

$$U(R) = \boxed{-\frac{GMm_0}{R}}$$

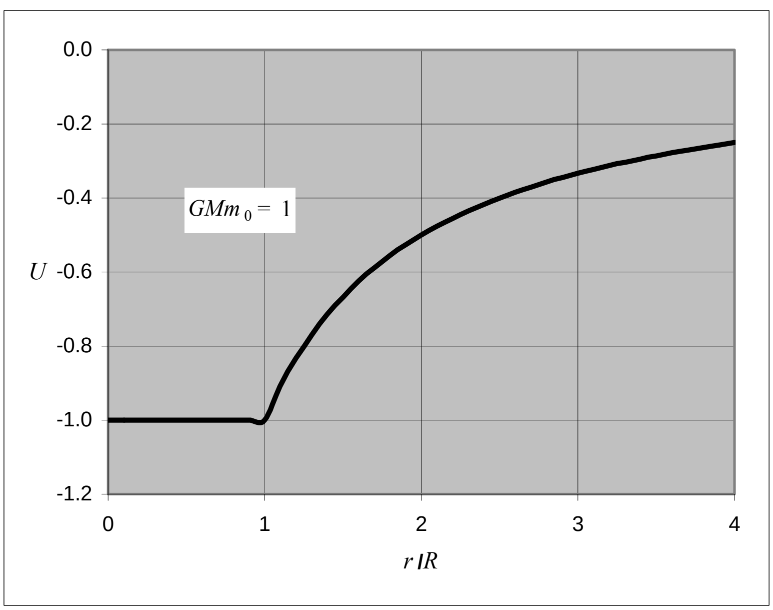
(*c*) For $r < R$, $F = 0$ and:

$$\frac{dU}{dr} = 0 \Rightarrow \boxed{U = \text{constant}}$$

(*d*) Because U is continuous, then for $r < R$:

$$U(r) = U(R) = \boxed{-\frac{GMm_0}{R}}$$

(*e*) A sketch of U as a function of r/R (with $GMm_0 = 1$) is shown below:



46 •

Picture the Problem The escape speed from a planet is related to its mass according to $v_e = \sqrt{2GM/R}$, where M and R represent the mass and radius of the planet, respectively.

Express the escape speed from Saturn:

$$v_{e,S} = \sqrt{\frac{2GM_S}{R_S}} \qquad (1)$$

Express the escape speed from Earth:

$$v_{e,E} = \sqrt{\frac{2GM_E}{R_E}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{v_{e,S}}{v_{e,E}} = \frac{\sqrt{\frac{2GM_S}{R_S}}}{\sqrt{\frac{2GM_E}{R_E}}} = \sqrt{\frac{R_E}{R_S} \cdot \frac{M_S}{M_E}}$$

Substitute numerical values and evaluate $\frac{v_{e,S}}{v_{e,E}}$:

$$\frac{v_{e,S}}{v_{e,E}} = \sqrt{\frac{1}{9.47} \times \frac{95.2}{1}} = 3.17$$

Solve for and evaluate $v_{e,S}$:

$$v_{e,S} = 3.17 v_{e,E} = 3.17(11.2\,\text{km/s}) = \boxed{35.5\,\text{km/s}}$$

47 •

Picture the Problem The escape speed from the moon or the earth is given by $v_e = \sqrt{2GM/R}$, where M and R represent the masses and radii of the moon or the earth.

Express the escape speed from the moon:

$$v_{e,S} = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{2g_m R_m} \qquad (1)$$

Express the escape speed from earth:

$$v_{e,E} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2g_E R_E} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{v_{e,m}}{v_{e,E}} = \frac{\sqrt{g_m R_m}}{\sqrt{g_E R_E}} = \sqrt{\frac{g_m R_m}{g_E R_E}}$$

Solve for $v_{e,m}$:

$$v_{\text{e.m}} = \sqrt{\frac{g_{\text{m}} R_{\text{m}}}{g_{\text{E}} R_{\text{E}}}} v_{\text{e.E}}$$

Substitute numerical values and evaluate $v_{e,m}$:

$$v_{\text{e.m}} = \sqrt{(0.166)(0.273)}(11.2\,\text{km/s}) = \boxed{2.38\,\text{km/s}}$$

***48** •

Picture the Problem We'll consider a rocket of mass *m* which is initially on the surface of the earth (mass *M* and radius *R*) and compare the kinetic energy needed to get the rocket to its escape velocity with its kinetic energy in a low circular orbit around the earth. We can use conservation of energy to find the escape kinetic energy and Newton's law of gravity to derive an expression for the low earth-orbit kinetic energy.

Apply conservation of energy to relate the initial energy of the rocket to its escape kinetic energy:

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

Letting the zero of gravitational potential energy be at infinity we have $U_f = K_f = 0$ and:

$$-K_{\text{i}} - U_{\text{i}} = 0$$

or

$$K_{\text{e}} = -U_{\text{i}} = \frac{GMm}{R}$$

Apply Newton's law of gravity to the rocket in orbit at the surface of the earth to obtain:

$$\frac{GMm}{R^2} = m\frac{v^2}{R}$$

Rewrite this equation to express the low-orbit kinetic energy E_o of the rocket:

$$K_{\text{o}} = \tfrac{1}{2}mv^2 = \frac{GMm}{2R}$$

Express the ratio of K_o to K_e:

$$\frac{K_{\text{o}}}{K_{\text{e}}} = \frac{\frac{GMm}{2R}}{\frac{GMm}{R}} = \frac{1}{2} \Rightarrow K_{\text{e}} = \boxed{2K_{\text{o}}}$$, as asserted by Heinlein.

49 ••

Picture the Problem Let the zero of gravitational potential energy be at infinity, *m* represent the mass of the particle, and the subscript E refer to the earth. When the particle is very far from the earth, the gravitational potential energy of the earth-particle system will be zero. We'll use conservation of energy to relate the initial potential and kinetic energies of the particle-earth system to the final kinetic energy of the particle.

Use conservation of energy to relate the initial energy of the system to its energy when the particle is very

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

or, because $U_f = 0$,

$$K(\infty) - K(R_{\text{E}}) - U(R_{\text{E}}) = 0 \qquad (1)$$

from the earth:

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}m(2v_e)^2 + \frac{GM_E m}{R_E} = 0$$

or, because $GM_E = gR_E^2$,

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}mv^2 + mgR_E = 0$$

Solve for v_∞:

$$v_\infty = \sqrt{2\left(2v_e^2 - gR_E\right)}$$

Substitute numerical values and evaluate v_∞:

$$v_\infty = \sqrt{2\left[2\left(11.2\times10^3\ \text{m/s}\right)^2 - \left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)\right]} = \boxed{19.4\,\text{km/s}}$$

50 ••

Picture the Problem Let the zero of gravitational potential energy be at infinity, *m* represent the mass of the particle, and the subscript E refer to the earth. When the particle is very far from the earth, the gravitational potential energy of the earth-particle system will be zero. We'll use conservation of energy to relate the initial potential and kinetic energies of the particle-earth system to the final kinetic energy of the particle.

Use conservation of energy to relate the initial energy of the system to its energy when the particle is very far away:

$$K_f - K_i + U_f - U_i = 0$$

or, because $U_f = 0$,

$$K(\infty) - K(R_E) - U(R_E) = 0 \qquad (1)$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}mv_i^2 + \frac{GM_E m}{R_E} = 0$$

or, because $GM_E = gR_E^2$,

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}mv_i^2 + mgR_E = 0$$

Solve for v_i:

$$v_i = \sqrt{v_\infty^2 + 2gR_E}$$

Substitute numerical values and evaluate v_i:

$$v_i = \sqrt{\left(11.2\times10^3\ \text{m/s}\right)^2 + 2\left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)} = \boxed{15.8\,\text{km/s}}$$

51 ••

Picture the Problem We can use the definition of kinetic energy to find the energy necessary to launch a 1-kg object from the earth at escape speed.

(*a*) Using the definition of kinetic energy, find the energy required to launch a 1-kg object from the surface of the earth at escape speed:

$$K = \tfrac{1}{2}mv_e^2$$
$$= \tfrac{1}{2}(1\,\text{kg})(11.2\times10^3\ \text{m/s})^2$$
$$= \boxed{62.7\,\text{MJ}}$$

(*b*) Using the conversion factor 1 kW·h = 3.6 MJ, convert 62.7 MJ to kW·h:

$$K = 62.7\,\text{MJ}\times\frac{1\,\text{kW}\cdot\text{h}}{3.6\,\text{MJ}}$$
$$= \boxed{17.4\,\text{kW}\cdot\text{h}}$$

(*c*) Express the cost of this project in terms of the mass of the astronaut:

$$\text{Cost} = \text{rate}\times\frac{\text{required energy}}{\text{kg}}\times\text{mass}$$

Substitute numerical values and find the cost:

$$\text{Cost} = \frac{\$0.10}{\text{kW}\cdot\text{h}}\times\frac{17.4\,\text{kW}\cdot\text{h}}{\text{kg}}(80\,\text{kg})$$
$$= \boxed{\$139}$$

52 ••

Picture the Problem Let *m* represent the mass of the body that is projected vertically from the surface of the earth. We'll begin by using conservation of energy under the assumption that the gravitational field is constant to determine H'. We'll apply conservation of energy a second time, with the zero of gravitational potential energy at infinity, to express *H*. Finally, we'll solve these two equations simultaneously to express *H* in terms of H'.

Assuming the gravitational field to be constant and letting the zero of potential energy be at the surface of the earth, apply conservation of energy to relate the initial kinetic energy and the final potential energy of the object-earth system:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f and solve for H':

$$-\tfrac{1}{2}mv^2 + mgH' = 0$$

and

$$H' = \frac{v^2}{2g} \qquad (1)$$

Letting the zero of gravitational potential energy be at infinity, use conservation of energy to relate the initial kinetic energy and the final

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-K_i + U_f - U_i = 0$$

potential energy of the object-earth system:

Substitute to obtain:

$$-\tfrac{1}{2}mv^2 - \frac{GMm}{R_E + H} + \frac{GMm}{R_E} = 0$$

or

$$-\tfrac{1}{2}v^2 - \frac{gR_E^2}{R_E + H} + \frac{gR_E^2}{R_E} = 0$$

Solve for v^2:

$$v^2 = 2gR_E^2\left(\frac{1}{R_E} - \frac{1}{R_E + H}\right) = 2gR_E\left(\frac{H}{R_E + H}\right)$$

Substitute in equation (1) to obtain:

$$H' = R_E\left(\frac{H}{R_E + H}\right)$$

Solve for H:

$$H = \boxed{\frac{H'R_E}{R_E - H'}}$$

Orbits

53 ••

Picture the Problem We can use its definition to express the period of the spacecraft's motion and apply Newton's 2nd law to the spacecraft to determine its orbital velocity. We can then use this orbital velocity to calculate the kinetic energy of the spacecraft. We can relate the spacecraft's angular momentum to its kinetic energy and moment of inertia.

(*a*) Express the period of the spacecraft's orbit about the earth:

$$T = \frac{2\pi R}{v} = \frac{2\pi(3R_E)}{v} = \frac{6\pi R_E}{v}$$

where v is the orbital speed of the spacecraft.

Use Newton's 2nd law to relate the gravitational force acting on the spacecraft to its orbital speed:

$$F_{radial} = \frac{GM_E m}{(3R_E)^2} = m\frac{v^2}{3R_E}$$

Solve for v to obtain:

$$v = \sqrt{\frac{gR_E}{3}}$$

Substitute for v in our expression for T to obtain:

$$T = 6\sqrt{3}\pi\sqrt{\frac{R_E}{g}}$$

Substitute numerical values and evaluate T:

$$T = 6\sqrt{3}\pi\sqrt{\frac{6.37\times10^6\ \text{m}}{9.81\ \text{m/s}^2}}$$
$$= 2.631\times10^4\ \text{s}\times\frac{1\ \text{h}}{3600\ \text{s}} = \boxed{7.31\ \text{h}}$$

(b) Using its definition, express the spacecraft's kinetic energy:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\tfrac{1}{3}gR_E\right)$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{6}(100\ \text{kg})(9.81\ \text{m/s}^2)(6.37\times10^6\ \text{m})$$
$$= \boxed{1.04\ \text{GJ}}$$

(c) Express the kinetic energy of the spacecraft in terms of its angular momentum:

$$K = \frac{L^2}{2I}$$

Solve for L:

$$L = \sqrt{2IK}$$

Express the moment of inertia of the spacecraft with respect to an axis through the center of the earth:

$$I = m(3R_E)^2$$
$$= 9mR_E^2$$

Substitute and solve for L:

$$L = \sqrt{18mR_E^2K} = 3R_E\sqrt{2mK}$$

Substitute numerical values and evaluate L:

$$L = 3(6.37\times10^6\ \text{m})\sqrt{2(100\ \text{kg})(1.04\times10^9\ \text{J})} = \boxed{8.72\times10^{12}\ \text{J}\cdot\text{s}}$$

***54** •

Picture the Problem Let the origin of our coordinate system be at the center of the earth and let the positive x direction be toward the moon. We can apply the definition of center of mass to find the center of mass of the earth-moon system and find the "orbital" speed of the earth using x_{cm} as the radius of its motion and the period of the moon as the period of this motion of the earth.

(a) Using its definition, express the x coordinate of the center of mass of the earth-moon system:

$$x_{cm} = \frac{M_E x_E + m_{moon}x_{moon}}{M_E + m_{moon}}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{M_E(0) + (7.36\times10^{22}\text{ kg})(3.82\times10^{8}\text{ m})}{5.98\times10^{24}\text{ kg} + 7.36\times10^{22}\text{ kg}} = \boxed{4.64\times10^{6}\text{ m}}$$

Note that, because the radius of the earth is 6.37×10^6 m, the center of mass is actually located about 1700 km *below* the surface of the earth.

(*b*) Express the "orbital" speed of the earth in terms of the radius of its circular orbit and its period of rotation:

$$v = \frac{2\pi x_{cm}}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(4.64\times10^{6}\text{ m})}{27.3\text{ d}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}} = \boxed{12.4\text{ m/s}}$$

55 ••

Picture the Problem We can express the energy difference between these two orbits in terms of the total energy of a satellite at each elevation. The application of Newton's 2nd law to the force acting on a satellite will allow us to express the total energy of each satellite as function of its mass, the radius of the earth, and its orbital radius.

Express the energy difference:

$$\Delta E = E_{geo} - E_{1000} \quad (1)$$

Express the total energy of an orbiting satellite:

$$E_{tot} = K + U = \tfrac{1}{2}mv^2 - \frac{GM_E m}{R} \quad (2)$$

where R is the orbital radius.

Apply Newton's 2nd law to a satellite to relate the gravitational force to the orbital speed:

$$F_{radial} = \frac{GM_E m}{R^2} = m\frac{v^2}{R}$$

or

$$\frac{gR_E^2}{R^2} = \frac{v^2}{R}$$

Simplify and solve for v^2:

$$v^2 = \frac{gR_E^2}{R}$$

Substitute in equation (2) to obtain:

$$E_{tot} = \tfrac{1}{2}m\frac{gR_E^2}{R} - \frac{gR_E^2 m}{R} = -\frac{mgR_E^2}{2R}$$

Substitute in equation (1) and simplify to obtain:

$$\Delta E = -\frac{mgR_E^2}{2R_{geo}} + \frac{mgR_E^2}{2R_{1000}} = \frac{mgR_E^2}{2}\left(\frac{1}{R_{1000}} - \frac{1}{R_{geo}}\right)$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \tfrac{1}{2}(500\,\text{kg})(9.81\,\text{N/kg})(6.37\times10^6\,\text{m})^2\left(\frac{1}{7.37\times10^6\,\text{m}} - \frac{1}{4.22\times10^7\,\text{m}}\right) = \boxed{11.1\,\text{GJ}}$$

56 ••

Picture the Problem We can use Kepler's 3rd law to relate the periods of the moon and Earth, in their orbits about the earth and the sun, to their mean distances from the objects about which they are in orbit. We can solve these equations for the masses of the sun and the earth and then divide one by the other to establish a value for the ratio of the mass of the sun to the mass of the earth.

Using Kepler's 3rd law, relate the period of the moon to its mean distance from the earth:

$$T_m^2 = \frac{4\pi^2}{GM_E} r_m^3 \quad (1)$$

where r_m is the distance between the centers of the earth and the moon.

Using Kepler's 3rd law, relate the period of the earth to its mean distance from the sun:

$$T_E^2 = \frac{4\pi^2}{GM_s} r_E^3 \quad (2)$$

where r_E is the distance between the centers of the earth and the sun.

Solve equation (1) for M_E:

$$M_E = \frac{4\pi^2}{GT_m^2} r_m^3 \quad (3)$$

Solve equation (2) for M_s:

$$M_s = \frac{4\pi^2}{GT_E^2} r_E^3 \quad (4)$$

Divide equation (4) by equation (3) and simplify to obtain:

$$\frac{M_s}{M_E} = \left(\frac{r_E}{r_m}\right)^3\left(\frac{T_m}{T_E}\right)^2$$

Substitute numerical values and evaluate M_s/M_E:

$$\frac{M_s}{M_E} = \left(\frac{1.5\times10^{11}\,\text{m}}{3.82\times10^8\,\text{m}}\right)^3\left(\frac{27.3\,\text{d}}{365.24\,\text{d}}\right)^2 = \boxed{3.38\times10^5}$$

Express the difference between this value and the measured value of 3.33×10^5:

$$\%\,\text{diff} = \frac{3.38\times10^5 - 3.33\times10^5}{3.33\times10^5} = \boxed{1.50\,\%}$$

The Gravitational Field

57 •

Picture the Problem The gravitational field at any point is defined by $\vec{g} = \vec{F}/m$.

Using its definition, express the gravitational field at a point in space:

$$\vec{g} = \frac{\vec{F}}{m} = \frac{(12\,\text{N})\hat{i}}{3\,\text{kg}} = \boxed{(4\,\text{N/kg})\hat{i}}$$

***58** •

Picture the Problem The gravitational field at any point is defined by $\vec{g} = \vec{F}/m$.

Using its definition, express the gravitational field at a point in space:

$$\vec{g} = \frac{\vec{F}}{m}$$

Solve for $\vec{F}$ and substitute for m and $\vec{g}$ to obtain:

$$\vec{F} = m\vec{g} = (0.004\,\text{kg})(2.5\times10^{-6}\,\text{N/kg})\hat{j} = \boxed{(10^{-8}\,N)\hat{j}}$$

59 ••

Picture the Problem We can use the definition of the gravitational field due to a point mass to find the x and y components of the field at the origin and then add these components to find the resultant field. We can find the magnitude of the field from its components using the Pythagorean theorem.

(*a*) Express the gravitational field due to the point mass at $x = L$:

$$\vec{g}_x = \frac{Gm}{L^2}\hat{i}$$

Express the gravitational field due to the point mass at $y = L$:

$$\vec{g}_y = \frac{Gm}{L^2}\hat{j}$$

Add the two fields to obtain:

$$\vec{g} = \vec{g}_x + \vec{g}_y = \boxed{\frac{Gm}{L^2}\hat{i} + \frac{Gm}{L^2}\hat{j}}$$

(*b*) Find the magnitude of $\vec{g}$:

$$|\vec{g}| = \sqrt{g_x^2 + g_y^2} = \sqrt{\frac{Gm}{L^2} + \frac{Gm}{L^2}}$$
$$= \boxed{\sqrt{2}\frac{Gm}{L^2}}$$

60 ••

Picture the Problem We can find the net force acting on *m* by superposition of the forces due to each of the objects arrayed on the circular arc. Once we have expressed the net force, we can find the gravitational field at the center of curvature from its definition.

(*a*) Express the net force acting on *m*:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} \quad (1)$$

Express F_x:

$$F_x = \frac{GMm}{R^2} - \frac{GMm}{R^2} + \frac{GMm}{R^2}\cos 45° - \frac{GMm}{R^2}\cos 45° = 0$$

Express F_y:

$$F_y = \frac{GMm}{R^2} + \frac{GMm}{R^2}\sin 45° + \frac{GMm}{R^2}\sin 45° = \frac{GMm}{R^2}(2\sin 45° + 1)$$

Substitute numerical values and evaluate F_y:

$$F_y = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}{(0.1\ \text{m})^2}\times(3\ \text{kg})(2\ \text{kg})(2\sin 45° + 1) = 9.67\times10^{-8}\ \text{N}$$

Substitute in equation (1) to obtain:

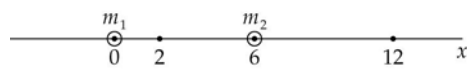
$$\vec{F} = \boxed{0\hat{i} + (9.67\times10^{-8}\ \text{N})\hat{j}}$$

(*b*) Using its definition, express $\vec{g}$ at the center of curvature of the arc:

$$\vec{g} = \frac{\vec{F}}{m} = \frac{0\hat{i} + (9.67\times10^{-8}\ \text{N})\hat{j}}{2\ \text{kg}}$$
$$= \boxed{(4.83\times10^{-8}\ \text{N/kg})\hat{j}}$$

61 ••

Picture the Problem The configuration of point masses is shown to the right. The gravitational field at any point can be found by superimposing the fields due to each of the point masses.



(*a*) Express the gravitational field at $x = 2$ m as the sum of the fields due to the point masses m_1 and m_2:

$$\vec{g} = \vec{g}_1 + \vec{g}_2 \qquad (1)$$

Express $\vec{g}_1$ and $\vec{g}_2$:

$$\vec{g}_1 = -\frac{Gm_1}{x_1^2}\hat{i} \text{ and } \vec{g}_2 = \frac{Gm_2}{x_2^2}\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{g} = -\frac{Gm_1}{x_1^2}\hat{i} + \frac{Gm_2}{x_2^2}\hat{i} = -\frac{Gm_1}{x_1^2}\hat{i} + \frac{Gm_2}{(2x_1)^2}\hat{i} = -\frac{G}{x_1^2}\left(m_1 - \tfrac{1}{4}m_2\right)\hat{i}$$

Substitute numerical values and evaluate $\vec{g}$:

$$\vec{g} = -\frac{6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2}{(2\,\text{m})^2} \times\left[2\,\text{kg} - \tfrac{1}{4}(4\,\text{kg})\right]\hat{i} = \boxed{\left(-1.67\times10^{-11}\,\text{N/kg}\right)\hat{i}}$$

(*b*) Express $\vec{g}_1$ and $\vec{g}_2$:

$$\vec{g}_1 = -\frac{Gm_1}{x_1^2}\hat{i} \text{ and } \vec{g}_2 = -\frac{Gm_2}{x_2^2}\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{g} = -\frac{Gm_1}{x_1^2}\hat{i} - \frac{Gm_2}{x_2^2}\hat{i} = -\frac{Gm_1}{(2x_2)^2}\hat{i} - \frac{Gm_2}{x_2^2}\hat{i} = -\frac{G}{x_2^2}\left(\tfrac{1}{4}m_1 + m_2\right)\hat{i}$$

Substitute numerical values and evaluate $\vec{g}$:

$$\vec{g} = -\frac{6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{(6\,\text{m})^2}\times\left[\tfrac{1}{4}(2\,\text{kg})+4\,\text{kg}\right]\hat{i} = \boxed{\left(-8.34\times10^{-12}\ \text{N/kg}\right)\hat{i}}$$

(*c*) Express the condition that $\vec{g} = 0$:

$$\frac{Gm_1}{x^2}-\frac{Gm_2}{(6-x)^2}=0$$

or

$$\frac{2}{x^2}-\frac{4}{(6-x)^2}=0$$

Express this quadratic equation in standard form:

$x^2+12x-36=0$, where x is in meters.

Solve the equation to obtain:

$x = 2.48\,\text{m}$ and $x = -14.5\,\text{m}$

From the diagram it is clear that the physically meaningful root is the positive one at:

$$x = \boxed{2.48\,\text{m}}$$

62 ••

Picture the Problem To show that the maximum value of $|g_x|$ for the field of Example 11-7 occurs at the points $x = \pm a/\sqrt{2}$, we can differentiate g_x with respect to x and set the derivative equal to zero.

From Example 11-7:

$$g_x = -\frac{2GMx}{\left(x^2+a^2\right)^{3/2}}$$

Differentiate g_x with respect to x and set the derivative equal to zero to find extreme values:

$$\frac{dg_x}{dx} = -2GM\left[\left(x^2+a^2\right)^{-3/2}-3x^2\left(x^2+a^2\right)^{-5/2}\right]=0 \text{ for extrema.}$$

Solve for x to obtain:

$$x = \boxed{\pm\frac{a}{\sqrt{2}}}$$

Remarks: To establish that this value for x corresponds to a relative maximum, we need to either evaluate the second derivative of g_x at $x = \pm a/\sqrt{2}$ or examine the graph of $|g_x|$ at $x = \pm a/\sqrt{2}$ for concavity downward.

63 ••

Picture the Problem We can find the mass of the rod by integrating dm over its length. The gravitational field at $x_0 > L$ can be found by integrating $d\vec{g}$ at x_0 over the length of the rod.

(*a*) Express the total mass of the stick:

$$M = \int_0^L \lambda\, dx = C\int_0^L x dx = \boxed{\tfrac{1}{2}CL^2}$$

(*b*) Express the gravitational field due to an element of the stick of mass dm:

$$d\vec{g} = -\frac{Gdm}{(x_0 - x)^2}\hat{i} = -\frac{G\lambda\, dx}{(x_0 - x)^2}\hat{i}$$
$$= -\frac{GCxdx}{(x_0 - x)^2}\hat{i}$$

Integrate this expression over the length of the stick to obtain:

$$\vec{g} = -GC\int_0^L \frac{xdx}{(x_0 - x)^2}\hat{i}$$
$$= \boxed{\frac{2GM}{L^2}\left[\ln\left(\frac{x_0}{x_0 - L}\right) - \left(\frac{L}{x_0 - L}\right)\right]\hat{i}}$$

64 •••

Picture the Problem Choose a mass element dm of the rod of thickness dx at a distance x from the origin. All such elements produce a gravitational field at a point P located a distance $x_0 > \frac{1}{2}L$ from the origin. We can calculate the total field by integrating the magnitude of the field produced by dm from $x = -L/2$ to $x = +L/2$.

(*a*) Express the gravitational field at P due to the element dm:

$$d\vec{g}_x = -\frac{Gdm}{r^2}\hat{i}$$

Relate dm to dx:

$$dm = \frac{M}{L}dx$$

Express the distance r between dm and point P in terms of x and x_0:

$$r = x_0 - x$$

Substitute these results to express $d\vec{g}_x$ in terms of x and x_0:

$$d\vec{g}_x = \boxed{\left\{-\frac{GM}{L(x_0 - x)^2}dx\right\}\hat{i}}$$

(*b*) Integrate to find the total field:

$$\vec{g}_x = -\frac{GM}{L}\int_{-L/2}^{L/2}\frac{dx}{(x_0 - x)^2}\hat{i} = \left\{-\frac{GM}{L}\left[\frac{1}{x_0 - x}\right]_{-L/2}^{L/2}\right\}\hat{i} = \boxed{-\frac{GM}{x_0^2 - \frac{1}{4}L^2}\hat{i}}$$

(*c*) Use the definition of $\vec{g}$ to express $\vec{F}$:

$$\vec{F} = m_0\vec{g} = \boxed{-\frac{GMm_0}{x_0^2 - \frac{1}{4}L^2}\hat{i}}$$

(*d*) Factor x_0^2 from the denominator of our expression for $\vec{g}_x$ to obtain:

$$\vec{g}_x = -\frac{GM}{x_0^2\left(1 - \frac{L^2}{4x_0^2}\right)}\hat{i}$$

For $x_0 >> L$ the second term in parentheses is very small and:

$$\vec{g}_x \approx \boxed{-\frac{GM}{x_0^2}\hat{i}}$$

which is the gravitational field of a point mass M located at the origin.

$\vec{g}$ due to Spherical Objects

65 •

Picture the Problem The gravitational field inside a spherical shell is zero and the field at the surface of and outside the shell is given by $g = GM/r^2$.

(*a*) Because 0.5 m < R:

$$g = \boxed{0}$$

(*b*) Because 1.9 m < R:

$$g = \boxed{0}$$

(*c*) Because 2.5 m > R:

$$g = \frac{GM}{r^2} = \frac{(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(300\,\text{kg})}{(2.5\,\text{m})^2} = \boxed{3.20\times10^{-9}\,\text{N/kg}}$$

66 •

Determine the Concept The gravitational attraction is zero. The gravitational field inside the 2 m shell due to that shell is zero; therefore, it exerts no force on the 1 m shell, and, by Newton's 3rd law, that shell exerts no force on the larger shell.

***67 •**

Picture the Problem The gravitational field and acceleration of gravity at the surface of a sphere given by $g = GM/R^2$, where R is the radius of the sphere and M is its mass.

Express the acceleration of gravity on the surface of S_1:

$$g_1 = \frac{GM}{R^2}$$

Express the acceleration of gravity on the surface of S_2:

$$g_2 = \frac{GM}{R^2}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_2}{g_1} = \frac{\frac{GM}{R^2}}{\frac{GM}{R^2}} = 1 \text{ or } \boxed{g_1 = g_2}$$

68 ••

Picture the Problem The gravitational field and acceleration of gravity at the surface of a sphere given by $g = GM/R^2$, where R is the radius of the sphere and M is its mass.

Express the acceleration of gravity on the surface of S_1:

$$g_1 = \frac{GM}{R_1^2}$$

Express the acceleration of gravity on the surface of S_2:

$$g_2 = \frac{GM}{R_2^2}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_2}{g_1} = \frac{\frac{GM}{R_2^2}}{\frac{GM}{R_1^2}} = \frac{R_1^2}{R_2^2}$$

Solve for g_2:

$$g_2 = \boxed{\frac{R_1^2}{R_2^2} g_1}$$

Remarks: The accelerations depend only on the masses and radii because the points of interest are outside spherically symmetric distributions of mass.

69 ••

Picture the Problem The magnitude of the gravitational force is $F = mg$ where g inside a spherical shell is zero and outside is given by $g = GM/r^2$.

(*a*) At $r = 3a$, the masses of both spheres contribute to g:

$$F = mg = m\frac{G(M_1 + M_2)}{(3a)^2} = \boxed{\frac{Gm(M_1 + M_2)}{9a^2}}$$

(*b*) At $r = 1.9a$, g due to $M_2 = 0$:

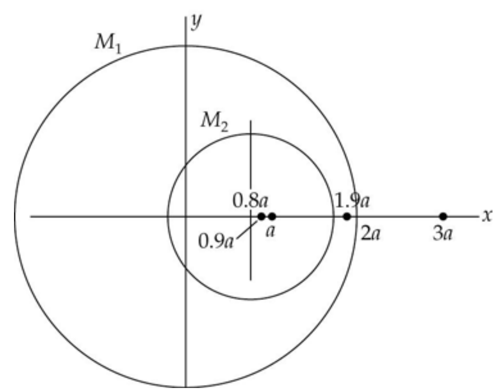
$$F = mg = m\frac{GM_1}{(1.9a)^2} = \boxed{\frac{GmM_1}{3.61a^2}}$$

(*c*) At $r = 0.9a$, $g = 0$:

$$F = \boxed{0}$$

70 ••

Picture the Problem The configuration is shown on the right. The centers of the spheres are indicated by the center-lines. The x coordinates of the mass m for parts (*a*), (*b*), and (*c*) are indicated along the x axis. The magnitude of the gravitational force is $F = mg$ where g inside a spherical shell is zero and outside is given by $g = \frac{GM}{r^2}$.



(*a*) Express the force acting on the object whose mass is m:

$$F = m(g_{1x} + g_{2x})$$

Find g_{1x} at $x = 3a$:

$$g_{1x} = \frac{GM_1}{(3a)^2} = \frac{GM_1}{9a^2}$$

Find g_{2x} at $x = 3a$:

$$g_{2x} = \frac{GM_2}{(3a - 0.8a)^2} = \frac{GM_2}{4.84a^2}$$

Substitute to obtain:

$$F = m\left(\frac{GM_1}{9a^2} + \frac{GM_2}{4.84a^2}\right) = \boxed{\frac{Gm}{a^2}\left(\frac{M_1}{9} + \frac{M_2}{4.84}\right)}$$

(*b*) Find g_{2x} at $x = 1.9a$:

$$g_{2x} = \frac{GM_2}{(1.9a - 0.8a)^2} = \frac{GM_2}{1.21a^2}$$

Find g_{1x} at $x = 1.9a$:

$$g_{1x} = 0$$

Substitute to obtain:

$$F = mg = \boxed{\frac{GmM_2}{1.21a^2}}$$

(*c*) At $x = 0.9a$, $g_{1x} = g_{2x} = 0$:

$$F = \boxed{0}$$

$\vec{g}$ Inside Solid Spheres

***71 ••**

Picture the Problem The "weight" as measured by a spring scale will be the normal force which the spring scale presses up against you. There are two forces acting on you as you stand at a distance r from the center of the planet: the normal force (F_N) and the force of gravity (mg). Because you are in equilibrium under the influence of these forces, your weight (the scale reading or normal force) will be equal to the gravitational force acting on you. We can use Newton's law of gravity to express this force.

(*a*) Express the force of gravity acting on you when you are a distance r from the center of the earth:

$$F_g = \frac{GM(r)m}{r^2} \qquad (1)$$

Using the definition of density, express the density of the earth between you and the center of the earth and the density of the earth as a whole:

$$\rho = \frac{M(r)}{V(r)} = \frac{M(r)}{\frac{4}{3}\pi r^3}$$

and

$$\rho = \frac{M_E}{V_E} = \frac{M_E}{\frac{4}{3}\pi R^3}$$

Because we're assuming the earth to of uniform-density and perfectly spherical:

$$\frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{M_E}{\frac{4}{3}\pi R^3}$$

or

$$M(r) = M_E\left(\frac{r}{R}\right)^3$$

Substitute in equation (1) and simplify to obtain:

$$F_g = \frac{GM_E\left(\frac{r}{R}\right)^3 m}{r^2} = \frac{GM_E m}{R^2}\frac{r}{R}$$

Apply Newton's law of gravity to yourself at the surface of the earth to obtain:

$$mg = \frac{GM_{\text{E}}m}{R^2}$$

or

$$g = \frac{GM_{\text{E}}}{R^2}$$

where g is the magnitude of free-fall acceleration at the surface of the earth.

Substitute to obtain:

$$F_g = \boxed{\frac{mg}{R}r}$$

i.e., the force of gravity on you is proportional to your distance from the center of the earth.

(*b*) Apply Newton's 2nd law to your body to obtain:

$$F_{\text{N}} - mg\frac{r}{R} = -mr\omega^2$$

Solve for your "effective weight" (i.e., what a spring scale will measure) F_{N}:

$$F_{\text{N}} = \frac{mg}{R}r - mr\omega^2 = \boxed{\left(\frac{mg}{R} - m\omega^2\right)r}$$

Note that this equation tells us that your effective weight increases linearly with distance from the center of the earth. The second term can be interpreted as a "centrifugal force" pushing out, which increases the farther you get from the center of the earth.

(*c*) We can decide whether the change in mass with distance from the center of the earth or the rotational effect is more important by examining the ratio of the two terms in the expression for your effective weight:

$$\frac{\frac{mg}{R}r}{mr\omega^2} = \frac{\frac{g}{R}}{\omega^2} = \frac{g}{R\left(\frac{2\pi}{T}\right)^2} = \frac{gT^2}{4\pi^2 R}$$

Substitute numerical values and evaluate this ratio:

$$\frac{gT^2}{4\pi^2 R} = \frac{\left(9.81\,\text{m/s}^2\right)\left(24\,\text{h} \times \frac{3600\,\text{s}}{\text{h}}\right)^2}{4\pi^2\left(6370\,\text{km}\right)} = 291$$

The change in the mass between you and the center of the earth as you move away from the center is 291 times more important than the rotational effect.

72 ••

Picture the Problem We can find the loss in weight at this depth by taking the difference between the weight of the student at the surface of the earth and her weight at a depth d = 15 km. To find the gravitational field at depth d, we'll use its definition and the mass of the earth that is between the bottom of the shaft and the center of the earth. We'll assume (incorrectly) that the density of the earth is constant.

Express the loss in weight:

$$\Delta w = w(R_E) - w(R) \quad (1)$$

Express the mass M inside $R = R_E - d$:

$$M = \rho V = \tfrac{4}{3}\rho\pi(R_E - d)^3$$

Express the mass of the earth:

$$M_E = \rho V_E = \tfrac{4}{3}\rho\pi R_E^3$$

Divide the first of these equations by the second to obtain:

$$\frac{M}{M_E} = \frac{\tfrac{4}{3}\rho\pi(R_E - d)^3}{\tfrac{4}{3}\rho\pi R_E^3} = \frac{(R_E - d)^3}{R_E^3}$$

Solve for M:

$$M = M_E \frac{(R_E - d)^3}{R_E^3}$$

Express the gravitational field at $R = R_E - d$:

$$g = \frac{GM}{R^2} = \frac{GM_E(R_E - d)^3}{(R_E - d)^2 R_E^2} \quad (2)$$

Express the gravitational field at $R = R_E$:

$$g_E = \frac{GM_E}{R_E^2} \quad (3)$$

Divide equation (2) by equation (3) to obtain:

$$\frac{g}{g_E} = \frac{\dfrac{GM_E(R_E - d)^3}{(R_E - d)^2 R_E^2}}{\dfrac{GM_E}{R_E^2}} = \frac{R_E - d}{R_E}$$

Solve for g:

$$g = \frac{R_E - d}{R_E} g_E$$

Express the weight of the student at $R = R_E - d$:

$$w(R) = mg(R) = \frac{R_E - d}{R_E} mg_E = \left(1 - \frac{d}{R_E}\right) mg_E$$

Substitute in equation (1) to obtain:

$$\Delta w = mg_{\text{E}} - \left(1 - \frac{d}{\text{R}_{\text{E}}}\right)mg_{\text{E}} = \frac{mg_{\text{E}}d}{R_{\text{E}}}$$

Substitute numerical values and evaluate Δw:

$$\Delta w = \frac{(800\,\text{N})(15\,\text{km})}{6370\,\text{km}} = \boxed{1.88\,\text{N}}$$

73 ••

Picture the Problem We can use the hint to find the gravitational field along the x axis.

Using the hint, express $g(x)$:

$$g(x) = g_{\text{solid sphere}} + g_{\text{hollow sphere}}$$

Substitute for $g_{\text{solid sphere}}$ and $g_{\text{hollow sphere}}$ and simplify to obtain:

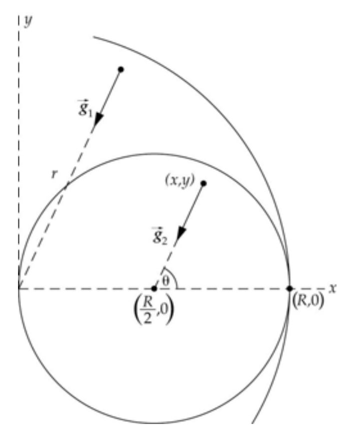
$$g(x) = \frac{GM_{\text{solid sphere}}}{x^2} + \frac{GM_{\text{hollow sphere}}}{\left(x - \frac{1}{2}R\right)^2}$$

$$= \frac{G\rho_0\left(\frac{4}{3}\pi R^3\right)}{x^2} + \frac{G\rho_0\left[-\frac{4}{3}\pi\left(\frac{1}{2}R\right)^3\right]}{\left(x - \frac{1}{2}R\right)^2}$$

$$= \boxed{G\left(\frac{4\pi\rho_0 R^3}{3}\right)\left[\frac{1}{x^2} - \frac{1}{8\left(x - \frac{1}{2}R\right)^2}\right]}$$

74 •••

Picture the Problem The diagram shows the portion of the solid sphere in which the hollow sphere is embedded. $\vec{g}_1$ is the field due to the solid sphere of radius R and density ρ_0 and $\vec{g}_2$ is the field due to the sphere of radius ½R and negative density ρ_0 centered at ½R. We can find the resultant field by adding the x and y components of $\vec{g}_1$ and $\vec{g}_2$.



Use its definition to express $|\vec{g}_1|$:

$$|\vec{g}_1| = \frac{GM}{r^2} = \frac{G\rho_0 V}{r^2} = \frac{4\pi\rho_0 r^3 G}{3r^2}$$

$$= \frac{4\pi\rho_0 rG}{3}$$

Find the x and y components of $\vec{g}_1$:

$$g_{1x} = -g_1\cos\theta = -g_1\left(\frac{x}{r}\right) = -\frac{4\pi\rho_0 Gx}{3}$$

and

$$g_{1y} = -g_1 \sin\theta = -g_1\left(\frac{y}{r}\right) = -\frac{4\pi\rho_0 G y}{3}$$

where the negative signs indicate that the field points inward.

Use its definition to express $|\vec{g}_2|$:

$$|\vec{g}_2| = \frac{GM_2}{r^2} = \frac{G\rho_0 V_2}{r_2^2} = \frac{4\pi\rho_0 r_2^3 G}{3r_2^2}$$
$$= \frac{4\pi\rho_0 r_2 G}{3}$$

where $r_2 = \sqrt{\left(x - \frac{1}{2}R\right)^2 + y^2}$

Express the x and y components of $\vec{g}_2$:

$$g_{2x} = g_2\left(\frac{x - \frac{1}{2}R}{r_2}\right) = \frac{4\pi\rho_0 G\left(x - \frac{1}{2}R\right)}{3}$$

$$g_{2y} = g_2\left(\frac{y}{r_2}\right) = \frac{4\pi\rho_0 G y}{3}$$

Add the x components to obtain the x component of the resultant field:

$$g_x = g_{1x} + g_{2x}$$
$$= -\frac{4\pi\rho_0 G x}{3} + \frac{4\pi\rho_0 G\left(x - \frac{1}{2}R\right)}{3}$$
$$= -\frac{2\pi\rho_0 G R}{3}$$

where the negative sign indicates that the field points inward.

Add the y components to obtain the y component of the resultant field:

$$g_y = g_{1y} + g_{2y}$$
$$= -\frac{4\pi\rho_0 G y}{3} + \frac{4\pi\rho_0 G y}{3} = 0$$

Express $\vec{g}$ in vector form and evaluate $|\vec{g}|$:

$$\vec{g} = g_x\hat{i} + G_y\hat{j} = \boxed{\left(-\frac{2\pi\rho_0 G R}{3}\right)\hat{i}}$$

and

$$|\vec{g}| = \boxed{\frac{2\pi\rho_0 G R}{3}}$$

75 •••

Picture the Problem The gravitational field will exert an inward radial force on the objects in the tunnel. We can relate this force to the angular velocity of the planet by using Newton's 2nd law of motion.

Letting r be the distance from the objects to the center of the planet, use Newton's 2nd law to relate the gravitational force acting on the objects to their angular velocity:

$$F_{net} = F_g = mr\omega^2$$

or

$$mg = mr\omega^2$$

Solve for ω to obtain:

$$\omega = \sqrt{\frac{g}{r}} \quad (1)$$

Use its definition to express g:

$$g = \frac{GM}{r^2} = \frac{G\rho_0 V}{r^2} = \frac{4\pi\rho_0 r^3 G}{3r^2} = \frac{4\pi\rho_0 rG}{3}$$

Substitute in equation (1) and simplify:

$$\omega = \sqrt{\frac{\frac{4\pi\rho_0 rG}{3}}{r}} = \boxed{\sqrt{\frac{4\pi\rho_0 G}{3}}}$$

76 •••

Picture the Problem Because we're given the mass of the sphere, we can find C by expressing the mass of the sphere in terms of C. We can use its definition to find the gravitational field of the sphere both inside and outside its surface.

(a) Express the mass of a differential element of the sphere:

$$dm = \rho\, dV = \rho\left(4\pi\, r^2 dr\right)$$

Integrate to express the mass of the sphere in terms of C:

$$M = 4\pi\, C \int_0^{5\,\text{m}} r dr = \left(50\,\text{m}^2\right)\pi\, C$$

Solve for C:

$$C = \frac{M}{\left(50\,\text{m}^2\right)\pi}$$

Substitute numerical values and evaluate C:

$$C = \frac{1011\,\text{kg}}{\left(50\,\text{m}^2\right)\pi} = \boxed{6.436\,\text{kg/m}^2}$$

(b) Use its definition to express the gravitational field of the sphere at a distance from its center greater than its radius:

$$g = \frac{GM}{r^2}$$

(1) For $r > 5$ m:

$$g = \frac{\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(1011\,\text{kg})}{r^2} = \boxed{\frac{6.75\times10^{-8}\ \text{N/kg}}{r^2}}$$

Use its definition to express the gravitational field of the sphere at a distance from its center less than its radius:

$$g = G\frac{\int_0^r 4\pi r^2\rho\,dr}{r^2} = G\frac{\int_0^r 4\pi r^2\frac{C}{r}\,dr}{r^2}$$

$$= G\frac{4\pi C\int_0^r r\,dr}{r^2} = 2\pi GC$$

(2) For $r < 5$ m:

$$g = 2\pi\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\times\left(6.436\,\text{kg/m}^2\right) = \boxed{2.70\times10^{-9}\ \text{N/kg}}$$

Remarks: Note that *g* is continuous at *r* = 5 m.

***77** •••

Picture the Problem We can use conservation of energy to relate the work done by the gravitational field to the speed of the small object as it strikes the bottom of the hole. Because we're given the mass of the sphere, we can find *C* by expressing the mass of the sphere in terms of *C*. We can then use its definition to find the gravitational field of the sphere inside its surface. The work done by the field equals the negative of the change in the potential energy of the system as the small object falls in the hole.

Use conservation of energy to relate the work done by the gravitational field to the speed of the small object as it strikes the bottom of the hole:

$$K_f - K_i + \Delta U = 0$$

or, because $K_i = 0$ and $W = -\Delta U$,

$$W = \tfrac{1}{2}mv^2$$

where *v* is the speed with which the object strikes the bottom of the hole and *W* is the work done by the gravitational field.

Solve for *v*:

$$v = \sqrt{\frac{2W}{m}} \qquad (1)$$

Express the mass of a differential element of the sphere:

$$dm = \rho\,dV = \rho\left(4\pi r^2 dr\right)$$

Integrate to express the mass of the sphere in terms of C:

$$M = 4\pi C \int_0^{5\,\text{m}} r dr = \left(50\,\text{m}^2\right)\pi C$$

Solve for and evaluate C:

$$C = \frac{M}{\left(50\,\text{m}^2\right)\pi} = \frac{1011\,\text{kg}}{\left(50\,\text{m}^2\right)\pi} = 6.436\,\text{kg/m}^2$$

Use its definition to express the gravitational field of the sphere at a distance from its center less than its radius:

$$g = G\frac{\int_0^r 4\pi r^2 \rho\, dr}{r^2} = G\frac{\int_0^r 4\pi r^2 \frac{C}{r} dr}{r^2} = G\frac{4\pi C\int_0^r r\, dr}{r^2} = 2\pi GC$$

Express the work done on the small object by the gravitational force acting on it:

$$W = -\int_{5\,\text{m}}^{3\,\text{m}} mgdr = (2\,\text{m})mg$$

Substitute in equation (1) and simplify to obtain:

$$v = \sqrt{\frac{2(2\,\text{m})m(2\pi GC)}{m}} = \sqrt{(8\,\text{m})\pi GC}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(8\,\text{m})\pi\left(6.6726\times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.436\,\text{kg/m}^2\right)} = \boxed{0.104\,\text{mm/s}}$$

78 •••

Picture the Problem The spherical deposit of heavy metals will increase the gravitational field at the surface of the earth. We can express this increase in terms of the difference in densities of the deposit and the earth and then form the quotient $\Delta g/g$.

Express Δg due to the spherical deposit:

$$\Delta g = \frac{G\Delta M}{r^2} \qquad (1)$$

Express the mass of the spherical deposit:

$$M = \Delta\rho V = \Delta\rho\left(\tfrac{4}{3}\pi R^3\right) = \tfrac{4}{3}\pi\Delta\rho R^3$$

Substitute in equation (1):

$$\Delta g = \frac{\tfrac{4}{3}G\pi\Delta\rho R^3}{r^2}$$

Express $\Delta g/g$:

$$\frac{\Delta g}{g}=\frac{\frac{\frac{4}{3}G\pi\,\Delta\rho\,R^3}{r^2}}{g}=\frac{\frac{4}{3}G\pi\,\Delta\rho\,R^3}{gr^2}$$

Substitute numerical values and evaluate $\Delta g/g$:

$$\frac{\Delta g}{g}=\frac{\frac{4}{3}\pi\left(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5000\,\text{kg/m}^3\right)\left(1000\,\text{m}\right)^3}{\left(9.81\,\text{N/kg}\right)\left(2000\,\text{m}\right)^2}=\boxed{3.56\times10^{-5}}$$

***79 •••**

Picture the Problem The force of attraction of the small sphere of mass m to the lead sphere is the sum of the forces due to the solid sphere ($\vec{F}_S$) and the cavities ($\vec{F}_C$) of negative mass.

(*a*) Express the force of attraction:

$$\vec{F}=\vec{F}_S+\vec{F}_C \qquad (1)$$

Use the law of gravity to express the force due to the solid sphere:

$$\vec{F}_S=-\frac{GMm}{d^2}\hat{i}$$

Express the magnitude of the force acting on the small sphere due to one cavity:

$$F_C=\frac{GM'm}{d^2+\left(\frac{R}{2}\right)^2}$$

where M' is the negative mass of a cavity.

Relate the negative mass of a cavity to the mass of the sphere before hollowing:

$$M'=-\rho V=-\rho\left[\tfrac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]$$
$$=-\tfrac{1}{8}\left(\tfrac{4}{3}\pi\rho R^3\right)=-\tfrac{1}{8}M$$

Letting θ be the angle between the x axis and the line joining the center of the small sphere to the center of either cavity, use the law of gravity to express the force due to the two cavities:

$$\vec{F}_C=2\frac{GMm}{8\left(d^2+\frac{R^2}{4}\right)}\cos\theta\,\hat{i}$$

because, by symmetry, the y components add to zero.

Express $\cos\theta$:

$$\cos\theta=\frac{d}{\sqrt{d^2+\frac{R^2}{4}}}$$

Substitute to obtain:

$$\vec{F}_C = \frac{GMm}{4\left(d^2+\frac{R^2}{4}\right)}\frac{d}{\sqrt{d^2+\frac{R^2}{4}}}\hat{i} = \frac{GMmd}{4\left(d^2+\frac{R^2}{4}\right)^{3/2}}\hat{i}$$

Substitute in equation (1) and simplify:

$$\vec{F} = -\frac{GMm}{d^2}\hat{i} + \frac{GMmd}{4\left(d^2+\frac{R^2}{4}\right)^{3/2}}\hat{i}$$

$$= \boxed{-\frac{GMm}{d^2}\left[1-\frac{\frac{d^3}{4}}{\left\{d^2+\frac{R^2}{4}\right\}^{3/2}}\right]\hat{i}}$$

(*b*) Evaluate $\vec{F}$ at $d = R$:

$$\vec{F}(R) = -\frac{GMm}{R^2}\left[1-\frac{\frac{R^3}{4}}{\left\{R^2+\frac{R^2}{4}\right\}^{3/2}}\right]\hat{i}$$

$$= \boxed{-0.821\frac{GMm}{R^2}\hat{i}}$$

80 ••

Picture the Problem Let *R* be the size of the cluster, and *N* the total number of stars in it. We can apply Newton's law of gravity and the 2nd law of motion to relate the net force (which depends on the number of stars *N*(*r*) in a sphere whose radius is equal to the distance between the star of interest and the center of the cluster) acting on a star at a distance *r* from the center of the cluster to its speed. We can use the definition of density, in conjunction with the assumption of uniform distribution of the starts within the cluster, to find *N*(*r*) and, ultimately, express the orbital speed *v* of a star in terms of the total mass of the cluster.

Using Newton's law of gravity and 2nd law, express the force acting on a star at a distance *r* from the center of the cluster:

$$F(r) = \frac{GN(r)M^2}{r^2} = M\frac{v^2}{r}$$

where *N*(*r*) is the number of stars within a distance *r* of the center of the cluster and *M* is the mass of an individual star.

Using the uniform distribution assumption and the definition of density, relate the number of stars $N(r)$ within a distance r of the center of the cluster to the total number N of stars in the cluster:

$$\rho = \frac{N(r)M}{\frac{4}{3}\pi r^3} = \frac{NM}{\frac{4}{3}\pi R^3}$$

or

$$N(r) = N\frac{r^3}{R^3}$$

Substitute to obtain:

$$\frac{GNM^2}{r^2}\frac{r^3}{R^3} = M\frac{v^2}{r}$$

or

$$GNM\frac{r^2}{R^3} = v^2$$

Solve for v to obtain:

$$v = r\sqrt{\frac{GNM}{R^3}} \Rightarrow v \propto r$$

The mean velocity v of a star in a circular orbit around the center of the cluster increases linearly with distance r from the center.

General Problems

***81 •**

Picture the Problem We can use Kepler's 3rd law to relate Pluto's period to its mean distance from the sun.

Using Kepler's 3rd law, relate the period of Pluto to its mean distance from the sun:

$$T^2 = Cr^3$$

where $C = \frac{4\pi^2}{GM_s} = 2.973\times10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for T:

$$T = \sqrt{Cr^3}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\left(2.973\times10^{-19}\ \text{s}^2/\text{m}^3\right)\left(39.5\ \text{AU}\times\frac{1.50\times10^{11}\ \text{m}}{\text{AU}}\right)^3}$$

$$= 7.864\times10^9\ \text{s}\times\frac{1\ \text{h}}{3600\ \text{s}}\times\frac{1\ \text{d}}{24\ \text{h}}\times\frac{1\ \text{y}}{365.25\ \text{d}}$$

$$= \boxed{249\ \text{y}}$$

82 •

Picture the Problem Consider an object of mass m at the surface of the earth. We can relate the weight of this object to the gravitational field of the earth and to the mass of the earth.

Using Newton's 2nd law, relate the weight of an object at the surface of the earth to the gravitational force acting on it:

$$w = mg = \frac{GM_E m}{R_E^2}$$

Solve for M_E:

$$M_E = \frac{gR_E^2}{G}$$

Substitute numerical values and evaluate M_E:

$$M_E = \frac{(9.81\,\text{N/kg})(6.37\times10^6\,\text{m})^2}{6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2} = \boxed{5.97\times10^{24}\,\text{kg}}$$

83 ••

Picture the Problem The work you must do against gravity to move the particle from a distance r_1 to r_2 is the negative of the change in the particle's gravitational potential energy.

(*a*) Relate the work you must do to the change in the gravitational potential energy of the earth-particle system:

$$W = -\Delta U = -\int_{r_1}^{r_2} F_g dr = GM_E m\int_{r_1}^{r_2}\frac{dr}{r^2}$$

$$= -GM_E m\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

$$= \boxed{GM_E m\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

(*b*) Substitute gR_E^2 for GM_E, R_E for r_1, and $R_E + h$ for r_2 to obtain:

$$W = \boxed{mgR_E^2\left(\frac{1}{R_E}-\frac{1}{R_E+h}\right)} \quad (1)$$

(*c*) Rewrite equation (1) with a common denominator and simplify to obtain:

$$W = mgR_E^2\left(\frac{R_E + h - R_E}{R_E(R_E + h)}\right)$$

$$= mgh\left(\frac{R_E}{R_E + h}\right) = mgh\left(\frac{1}{1+\frac{h}{R_E}}\right)$$

$$\approx \boxed{mgh}$$

when $h << R_E$.

84 ••

Picture the Problem The gravitational field outside a uniform sphere is given by $g = -GM/r^2$ and the field inside the sphere by $g = -(GM/R^3)r$.

(*a*) Express *g* outside the sphere:

$$g = -\frac{GM}{r^2}$$

Find the mass of the sphere:

$$M = \rho V = \rho\left(\tfrac{4}{3}\pi R^3\right)$$

Substitute and simplify to obtain:

$$g = -\frac{G\rho\left(\tfrac{4}{3}\pi R^3\right)}{r^2} = -\tfrac{4}{3}\frac{G\rho R^3}{r^2}$$

Substitute numerical values and evaluate *g*:

$$g = -\tfrac{4}{3}\frac{\left(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2000\,\text{kg/m}^3\right)(100\,\text{m})^3}{r^2} = \boxed{-\frac{0.559\,\text{N}\cdot\text{m}^2/\text{kg}}{r^2}}$$

(*b*) Express the gravitational field inside the uniform sphere:

$$g = -\frac{GM}{R^3}r = -\frac{G\rho\left(\tfrac{4}{3}\pi R^3\right)}{R^3}r$$

$$= -\tfrac{4}{3}\pi\rho Gr$$

Substitute numerical values and evaluate *g*:

$$g = -\tfrac{4}{3}\pi\left(2000\,\text{kg/m}^3\right)\left(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)r = \boxed{-\left(5.59\times10^{-7}\,\text{N/kg}\cdot\text{m}\right)r}$$

85 ••

Picture the Problem We can use Kepler's 3rd law to relate the period of the satellite to its mean distance from the center of Jupiter.

Use Kepler's 3rd law to relate the period of the satellite to its mean distance from the center of Jupiter:

$$T^2 = \frac{4\pi^2}{GM_J}(R_J + h)^3$$

Solve for h:

$$h = \sqrt[3]{\frac{T^2 GM_J}{4\pi^2}} - R_J \qquad (1)$$

Express the mass of Jupiter in terms of the mass of the earth:

$$M_J = 320M_E$$

Express the volume of Jupiter in terms of the mass of the earth:

$$V_J = 1320V_E$$

Express the volumes of Jupiter and Earth in terms of their radii and solve for R_J:

$$R_J = \sqrt[3]{1320}R_E$$

Substitute in equation (1) to obtain:

$$h = \sqrt[3]{\frac{T^2 G\{320M_E\}}{4\pi^2}} - \sqrt[3]{1320}R_E$$

Express the period of the satellite in seconds:

$$\begin{aligned} T &= 9\,\text{h} + 50\,\text{min} \\ &= 9\,\text{h} \times \frac{3600\,\text{s}}{\text{h}} + 50\,\text{min} \times \frac{60\,\text{s}}{\text{min}} \\ &= 3.54 \times 10^4\,\text{s} \end{aligned}$$

Substitute numerical values and evaluate h:

$$\begin{aligned} h &= \sqrt[3]{\frac{(3.54\times10^4\,\text{s})^2(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)\{320(5.98\times10^{24}\,\text{kg})\}}{4\pi^2}} \\ &\quad - \sqrt[3]{1320}(6.37\times10^6\,\text{m}) \\ &= \boxed{8.96\times10^7\,\text{m}} \end{aligned}$$

86 ••

Picture the Problem Let m represent the mass of the spacecraft. From Kepler's 3rd law we know that its period will be a minimum when it is in orbit just above the surface of the moon. We'll use Newton's 2nd law to relate the angular velocity of the spacecraft to the gravitational force acting on it.

Relate the period of the spacecraft to its angular velocity:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Using Newton's 2nd law of motion, relate the gravitational force acting on the spacecraft when it is in orbit at the surface of the moon to the angular velocity of the spacecraft:

$$\sum F_{\text{radial}} = \frac{GM_{\text{M}}m}{R_{\text{M}}^2} = mR_{\text{M}}\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{GM_{\text{M}}}{R_{\text{M}}^3}} = \sqrt{\frac{G\left(\frac{4}{3}\pi\rho R_{\text{M}}^3\right)}{R_{\text{M}}^3}} = \sqrt{\tfrac{4}{3}G\pi\rho}$$

Substitute in equation (1) and simplify to obtain:

$$T_{\text{min}} = \frac{2\pi}{\sqrt{\frac{4}{3}G\pi\rho}} = \sqrt{\frac{3\pi}{\rho G}}$$

Substitute numerical values and evaluate T_{min}:

$$T_{\text{min}} = \sqrt{\frac{3\pi}{\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(3340\ \text{kg/m}^3\right)}} = 6503\ \text{s} = \boxed{1\ \text{h}\ 48\ \text{min}}$$

87 ••

Picture the Problem We can use conservation of energy to establish a relationship between the height h to which the projectile will rise and its initial speed. The application of Newton's 2nd law will relate the orbital speed, which is equal to the initial speed of the projectile, to the mass and radius of the moon.

Use conservation of energy to relate the initial energies of the projectile to its final energy:

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

or, because $K_{\text{f}} = 0$,

$$-\tfrac{1}{2}mv^2 - \frac{GM_{\text{M}}m}{R_{\text{M}} + h} + \frac{GM_{\text{M}}m}{R_{\text{M}}} = 0$$

Solve for h:

$$h = R\left(\frac{1}{1 - \dfrac{v^2 R_{\text{M}}}{2GM_{\text{M}}}} - 1\right) \qquad (1)$$

Use Newton's 2nd law to relate velocity of the satellite to the

$$\sum F_{\text{radial}} = \frac{GM_{\text{M}}m}{R_{\text{M}}^2} = m\frac{v^2}{R_{\text{M}}}$$

gravitational force acting on it:

Solve for v^2:

$$v^2 = \frac{GM_M}{R_M}$$

Substitute for v^2 in equation (1) and simplify to obtain:

$$h = R\left(\frac{1}{1-\frac{1}{2}} - 1\right) = R = \boxed{1.70\,\text{Mm}}$$

***88** ••

Picture the Problem If we assume the astronauts experience a constant acceleration in the barrel of the cannon, we can use a constant-acceleration equation to relate their exit speed (the escape speed from the earth) to the acceleration they would need to undergo in order to reach that speed. We can use conservation of energy to express their escape speed in terms of the mass and radius of the earth and then substitute in the constant-acceleration equation to find their acceleration. To find the balance point between the earth and the moon we can equate the gravitational forces exerted by the earth and the moon at that point.

(*a*) Assuming constant acceleration down the cannon barrel, relate the ship's speed as it exits the barrel to the length of the barrel and the acceleration required to get the ship to escape speed:

$$v_e^2 = 2a\Delta\ell$$

where ℓ is the length of the cannon.

Solve for the acceleration:

$$a = \frac{v_e^2}{2\Delta\ell} \qquad (1)$$

Use conservation of energy to relate the initial energy of astronaut's ship to its energy when it has escaped the earth's gravitational field:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

When the ship has escaped the earth's gravitational field:

$$K_f = U_f = 0$$

and

$$-K_i - U_i = 0$$

or

$$-\tfrac{1}{2}mv_e^2 - \left(-\frac{GM_E m}{R}\right) = 0$$

where m is the mass of the spaceship.

Solve for v_e^2 to obtain:

$$v_e^2 = \frac{2GM_E}{R}$$

Substitute in equation (1) to obtain:

$$a = \frac{GM_E}{\Delta \ell R}$$

Substitute numerical values and evaluate a:

$$a = \left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\times\frac{\left(5.98\times10^{24}\ \text{kg}\right)}{(274\ \text{m})(6370\ \text{km})}$$
$$= 2.29\times10^{5}\ \text{m/s}^2$$
$$\approx 23{,}300g$$
$$\boxed{\text{Survival is extremely unlikely!}}$$

(*b*) Let the distance from the center of the earth to the center of the moon be R, and the distance from the center of the spaceship to the earth be x. If M is the mass of the earth and m the mass of the moon, the forces will balance out when:

$$\frac{GM}{x^2} = \frac{Gm}{(R-x)^2}$$
or
$$\frac{x}{\sqrt{M}} = \frac{R-x}{\sqrt{m}}$$
where we've ignored the negative solution, as it doesn't indicate a point between the two bodies.

Solve for x to obtain:

$$x = \frac{R}{1+\sqrt{\dfrac{m}{M}}}$$

Substitute numerical values and evaluate x:

$$x = \frac{3.84\times10^{8}\ \text{m}}{1+\sqrt{\dfrac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}}} = \boxed{3.46\times10^{8}\ \text{m}}$$

(*c*) No it is not. During the entire trip, the astronauts would be in free-fall, and so would not seem to weigh anything.

89 ••

Picture the Problem Let the origin of our coordinate system be at the center of mass of the binary star system and let the distances of the stars from their center of mass be r_1 and r_2. The period of rotation is related to the angular velocity of the star system and we can use Newton's 2nd law of motion to relate this velocity to the separation of the stars.

Relate the square of the period of the motion of the stars to their angular velocity:

$$T^2 = \frac{4\pi^2}{\omega^2} \qquad (1)$$

Using Newton's 2nd law of motion, relate the gravitational force acting on the star whose mass is m_2 to the angular velocity of the system:

$$\sum F_{\text{radial}} = \frac{Gm_1m_2}{(r_1+r_2)^2} = m_2r_2\omega^2$$

Solve for ω^2:

$$\omega^2 = \frac{Gm_1}{r_2(r_1+r_2)^2} \qquad (2)$$

From the definition of the center of mass we have:

$$m_1r_1 = m_2r_2 \qquad (3)$$

where $r = r_1 + r_2 \qquad (4)$

Eliminate r_1 from equations (3) and (4) and solve for r_2:

$$r_2 = \frac{rm_1}{m_1+m_2}$$

Eliminate r_2 from equations (3) and (4) and solve for r_1:

$$r_1 = \frac{rm_2}{m_1+m_2}$$

Substitute for r_1 and r_2 in equation (2) to obtain:

$$\omega^2 = \frac{G(m_1+m_2)}{r^3}$$

Finally, substitute in equation (1) and simplify:

$$T^2 = \frac{4\pi^2}{\frac{G(m_1+m_2)}{r^3}} = \boxed{\frac{4\pi^2r^3}{G(m_1+m_2)}}$$

90 ••

Picture the Problem Because the two-particle system has zero initial energy and zero initial linear momentum; we can use energy and momentum conservation to obtain simultaneous equations in the variables r, v_1 and v_2. We'll assume that initial separation distance of the particles and their final separation r *is large compared to the size of the particles* so that we can treat them as though they are point particles.

Use conservation of energy to relate the speeds of the particles when their separation distance is r:

$$E_i = E_f$$

or

$$0 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{r} \qquad (1)$$

Use conservation of linear momentum to obtain a second relationship between the speeds of the particles and their masses:

$$p_i = p_f$$

or

$$0 = m_1v_1 + m_2v_2 \qquad (2)$$

Solve equation (2) for v_1 and substitute in equation (1) to obtain:

$$v_2^2\left(m_2+\frac{m_2^2}{m_1}\right)=\frac{2Gm_1m_2}{r} \qquad (3)$$

Solve equation (3) for v_2:

$$\boxed{v_2=\sqrt{\frac{2Gm_1^2}{r(m_1+m_2)}}}$$

Solve equation (2) for v_1 and substitute for v_2 to obtain:

$$\boxed{v_1=\sqrt{\frac{2Gm_2^2}{r(m_1+m_2)}}}$$

***91 ••**

Picture the Problem We can find the orbital speeds of the planets from their distance from the center of mass of the system and the period of their motion. Application of Kepler's 3rd law will allow us to express the period of their motion T in terms of the effective mass of the system … which we can find from its definition.

Express the orbital speeds of the planets in terms of their period T:

$$v=\frac{2\pi R}{T}$$

where R is the distance to the center of mass of the four-planet system.

Apply Kepler's 3rd law to express the period of the planets:

$$T=\sqrt{\frac{4\pi^2}{GM_{\text{eff}}}R^3}$$

where M_{eff} is the effective mass of the four planets.

Substitute to obtain:

$$v=\frac{2\pi R}{\sqrt{\frac{4\pi^2}{GM_{\text{eff}}}R^3}}=\sqrt{\frac{GM_{\text{eff}}}{R}}$$

The distance of each planet from the effective mass is:

$$R=\frac{a}{\sqrt{2}}$$

Find M_{eff} from its definition:

$$\frac{1}{M_{\text{eff}}}=\frac{1}{M}+\frac{1}{M}+\frac{1}{M}+\frac{1}{M}$$

and

$$M_{\text{eff}}=\tfrac{1}{4}M$$

Substitute for R and M_{eff} to obtain:

$$\boxed{v=\sqrt{\frac{\sqrt{2}GM}{4a}}}$$

92 ••

Picture the Problem Let *r* represent the separation of the particle from the center of the earth and assume a uniform density for the earth. The work required to lift the particle from the center of the earth to its surface is the integral of the gravitational force function. This function can be found from the law of gravity and by relating the mass of the earth between the particle and the center of the earth to the earth's mass. We can use the work-kinetic energy theorem to find the speed with which the particle, when released from the surface of the earth, will strike the center of the earth. Finally, the energy required for the particle to escape the earth from the center of the earth is the sum of the energy required to get it to the surface of the earth and the kinetic energy it must have to escape from the surface of the earth.

(*a*) Express the work required to lift the particle from the center of the earth to the earth's surface:

$$W = \int_0^{R_E} F\,dr \qquad (1)$$

where *F* is the gravitational force acting on the particle.

Using the law of gravity, express the force acting on the particle as a function of its distance from the center of the earth:

$$F = \frac{GmM}{r^2} \qquad (2)$$

where *M* is the mass of a sphere whose radius is *r*.

Express the ratio of M to M_E:

$$\frac{M}{M_E} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\rho\left(\frac{4}{3}\pi R_E^3\right)} \Rightarrow M = M_E\frac{r^3}{R_E^3}$$

Substitute for *M* in equation (2) to obtain:

$$F = \frac{GmM_E}{R_E^3}r = \frac{mgR_E^2}{R_E^3}r = \frac{mg}{R_E}r$$

Substitute for *F* in equation (1) and evaluate the integral:

$$W = \frac{mg}{R_E}\int_0^{R_E} r\,dr = \boxed{\frac{gmR_E}{2}}$$

(*b*) Use the work-kinetic energy theorem to relate the kinetic energy of the particle as it reaches the center of the earth to the work done on it in moving it to the surface of the earth:

$$W = \Delta K = \tfrac{1}{2}mv^2$$

Substitute for *W* and solve for *v*:

$$v = \boxed{\sqrt{gR_E}}$$

(*c*) Express the total energy required for the particle to escape when projected from the center of the earth:

$$E_{esc} = W + \tfrac{1}{2}mv_e^2 = \tfrac{1}{2}mv_{esc}^2$$

where v_e is the escape speed from the surface of the earth.

Substitute for W and solve for v_{esc}:

$$v_{esc} = \sqrt{3gR_E}$$

Substitute numerical values and evaluate v_{esc}:

$$v_{esc} = \sqrt{3(9.81\,\text{N/kg})(6.37\times10^6\,\text{m})} = \boxed{13.7\,\text{km/s}}$$

93 ••

Picture the Problem We need to find the gravitational field in three regions: $r < R_1$, $R_1 < r < R_2$, and $r > R_2$.

For $r < R_1$:

$$g = \boxed{0}$$

For $r > R_2$, $g(r)$ is the field of a mass M centered at the origin:

$$g(r) = \boxed{\frac{GM}{r^2}}$$

For $R_1 < r < R_2$, $g(r)$ is determined by the mass within the shell of radius r:

$$g(r) = \frac{Gm}{r^2} \qquad (1)$$

where $m = \tfrac{4}{3}\pi\rho\left(r^3 - R_1^3\right) \qquad (2)$

Express the density of the spherical shell:

$$\rho = \frac{M}{V} = \frac{M}{\tfrac{4}{3}\pi\left(R_2^3 - R_1^3\right)}$$

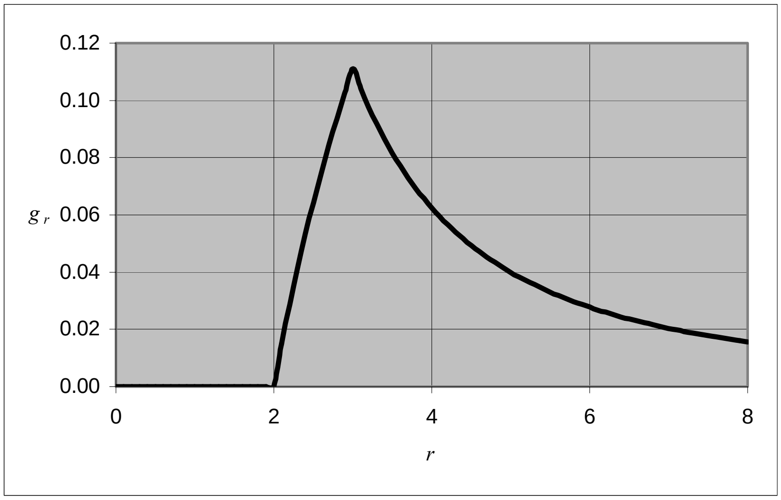
Substitute for ρ in equation (2) and simplify to obtain:

$$m = \frac{M\left(r^3 - R_1^3\right)}{R_2^3 - R_1^3}$$

Substitute for m in equation (1) to obtain:

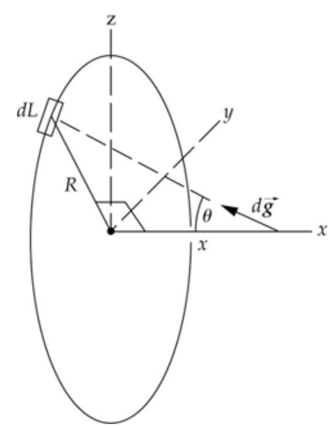
$$g(r) = \boxed{\frac{GM\left(r^3 - R_1^3\right)}{r^2\left(R_2^3 - R_1^3\right)}}$$

A graph of g_r with $R_1 = 2$, $R_2 = 3$, and $GM = 1$ follows:



94 ••

Picture the Problem A ring of radius R is shown to the right. Choose a coordinate system in which the origin is at the center of the ring and x axis is as shown. An element of length dL and mass dm is responsible for the field dg at a distance x from the center of the ring. We can express the x component of dg and then integrate over the circumference of the ring to find the total field as a function of x.



(*a*) Express the differential gravitational field at a distance x from the center of the ring in terms of the mass of elemental length dL:

$$dg = \frac{Gdm}{R^2 + x^2}$$

Relate the mass of the element to its length:

$$dm = \lambda\, dL$$

where λ is the linear density of the ring.

Substitute to obtain:

$$dg = \frac{G\lambda\, dL}{R^2 + x^2}$$

By symmetry, the y and z components of g vanish. Express the x component of dg:

$$dg_x = dg\cos\theta$$

$$= \frac{G\lambda\, dL}{R^2 + x^2}\cos\theta$$

Referring to the figure, express $\cos\theta$:

$$\cos\theta = \frac{x}{\sqrt{R^2 + x^2}}$$

Substitute to obtain:

$$dg_x = \frac{G\lambda\, dL}{R^2 + x^2} \times \frac{x}{\sqrt{R^2 + x^2}} = \frac{G\lambda\, x dL}{\left(R^2 + x^2\right)^{3/2}}$$

Because $\lambda = \dfrac{M}{2\pi R}$:

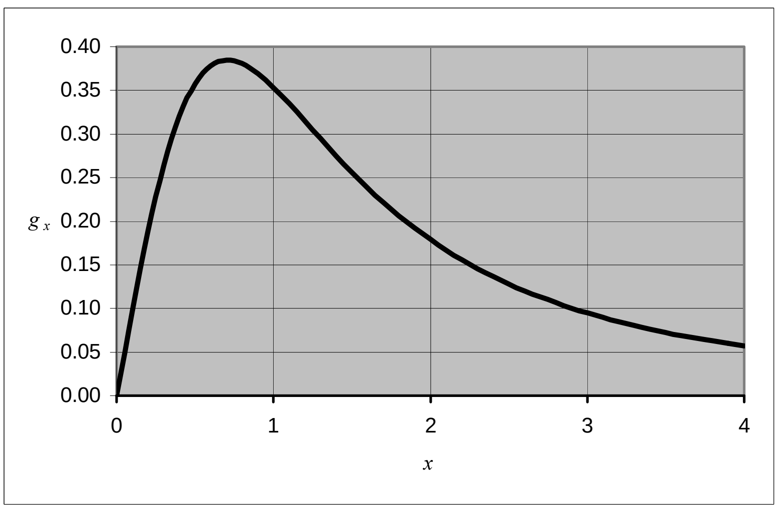
$$dg_x = \frac{GM\, x dL}{2\pi R\left(R^2 + x^2\right)^{3/2}}$$

Integrate to find $g(x)$:

$$g(x) = \frac{GM\, x}{2\pi R\left(R^2 + x^2\right)^{3/2}} \int_0^{2\pi R} dL$$

$$= \boxed{\frac{GM}{\left(R^2 + x^2\right)^{3/2}}\, x}$$

A plot of g_x is shown below. The curve is normalized for $R = 1$ and $GM = 1$.



(*b*) Differentiate $g(x)$ with respect to x and set the derivative equal to zero to identify extreme values:

$$\frac{dg}{dx} = GM\left[\frac{\left(x^2 + R^2\right)^{3/2} - x\left(\frac{3}{2}\right)\left(x^2 + R^2\right)^{1/2}(2x)}{\left(R^2 + x^2\right)^3}\right] = 0 \text{ for extrema}$$

Simplify to obtain:

$$\left(x^2 + R^2\right)^{3/2} - 3x^2\left(x^2 + R^2\right)^{1/2} = 0$$

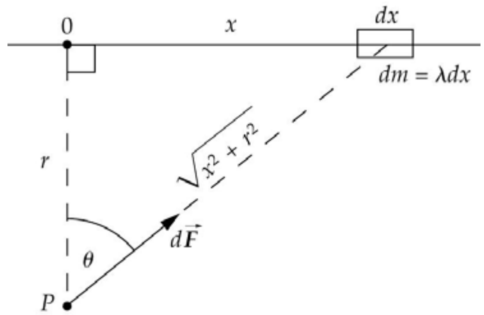
Solve for x to obtain:

$$x = \boxed{\pm\frac{R}{\sqrt{2}}}$$

Because the curve is concave downward, we can conclude that this result corresponds to a maximum. Note that this result agrees with our graphical maximum.

95 •••

Picture the Problem The diagram shows a segment of the wire of length dx and mass $dm = \lambda dx$ at a distance x from the origin of our coordinate system. We can find the magnitude of the gravitational field at a distance r from the wire from the resultant gravitational force acting on a particle of mass m' located at point P and then integrating over the length of the wire.



Express the gravitational force acting on a particle of mass m' at a distance r from the wire due to the segment of the wire of length dx:

$$dF = m'dg$$

or

$$dg = \frac{dF}{m'}$$

Using Newton's law of gravity, express dF:

$$dF = \frac{Gm'\lambda dx}{R^2}$$

or, because $R^2 = x^2 + r^2$,

$$dF = \frac{Gm'\lambda dx}{x^2 + r^2}$$

Substitute and simplify to express the gravitational field due to the segment of the wire of length dx:

$$dg = \frac{G\lambda dx}{x^2 + r^2}$$

By symmetry, the segment on the opposite side of the origin at the same distance from the origin will cancel out all but the radial component of the field, so the gravitational field will be given by:

$$\begin{aligned} dg &= \frac{G\lambda dx}{x^2 + r^2}\cos\theta \\ &= \frac{G\lambda dx}{x^2 + r^2}\frac{r}{\sqrt{x^2 + r^2}} \\ &= \frac{G\lambda r}{\left(x^2 + r^2\right)^{3/2}}dx \end{aligned}$$

Integrate dg from $x' = -\infty$ to $x' = +\infty$ to obtain:

$$g = \int_{-\infty}^{\infty}\frac{G\lambda r}{\left(x'^2 + r^2\right)^{3/2}}dx' = 2G\lambda\int_{0}^{\infty}\frac{r}{\left(x'^2 + r^2\right)^{3/2}}dx' = \frac{2G\lambda}{r}\left[\frac{x}{\sqrt{x'^2 + r^2}}\right]_0^{\infty} = \boxed{\frac{2G\lambda}{r}}$$

96 •••

Picture the Problem We can use the relationship between the angular velocity of an orbiting object and its tangential velocity to express the speeds v_{in} and v_{out} of the innermost and outermost portions of the ring. In part (*b*) we can use Newton's law of gravity, in conjunction with the 2nd law of motion, to relate the tangential speed of a chunk of the ring to the gravitational force acting on it. As in part (*a*), once we know v_{in} and v_{out}, we can express the difference between them to obtain the desired results.

(*a*) Express the speed of a point in the ring at a distance R' from the center of the planet under the assumption that the ring is solid and rotates with an angular velocity ω:

$$v(R') = \omega R$$

Express the speeds v_{in} and v_{out} of the innermost and outermost portions of the ring:

$$v_{in} = \left(R - \tfrac{1}{2}r\right)\omega$$

and

$$v_{out} = \left(R + \tfrac{1}{2}r\right)\omega$$

Express the difference between v_{out} and v_{in}:

$$v_{out} - v_{in} = \left(R + \tfrac{1}{2}r\right)\omega - \left(R - \tfrac{1}{2}r\right)\omega$$

$$= \omega r = \frac{v}{R}r = \boxed{v\frac{r}{R}}$$

(*b*) Assume that a chunk of the ring is moving in a circular orbit around the center of the planet under the force of gravity. Then, we can find its velocity by equating the force of gravity to the centripetal force needed to keep it in orbit:

$$\frac{GMm}{R'^2} = \frac{mv^2}{R'}$$

or

$$v = \sqrt{\frac{GM}{R'}}$$

where M is the mass of the planet and R' the distance from the center.

Substitute for R' to express v_{out}:

$$v_{out} = \sqrt{\frac{GM}{R + \frac{1}{2}r}} = \sqrt{\frac{GM}{R\left(1 + \frac{1}{2}\frac{r}{R}\right)}}$$

$$= \sqrt{\frac{GM}{R}}\left(1 + \frac{1}{2}\frac{r}{R}\right)^{-1/2}$$

Expand binomially to obtain:

$$v_{out} = \sqrt{\frac{GM}{R}}\left(1 - \frac{1}{2}\frac{1}{2}\frac{r}{R} + \text{higher order terms}\right)$$

$$\approx \sqrt{\frac{GM}{R}}\left(1 - \frac{1}{4}\frac{r}{R}\right)$$

Proceed similarly to obtain, for v_{in}:

$$v_{\text{in}} \approx \sqrt{\frac{GM}{R}}\left(1+\frac{1}{4}\frac{r}{R}\right)$$

Express the difference between v_{out} and v_{in}:

$$v_{\text{out}} - v_{\text{in}} \approx \sqrt{\frac{GM}{R}}\left(1-\frac{1}{4}\frac{r}{R}\right) - \sqrt{\frac{GM}{R}}\left(1+\frac{1}{4}\frac{r}{R}\right) = \sqrt{\frac{GM}{R}}\left(-\frac{1}{2}\frac{r}{R}\right)$$

and, because $v = \sqrt{\dfrac{GM}{R}}$, $\boxed{v_{\text{out}} - v_{\text{in}} \approx -\dfrac{1}{2}\dfrac{r}{R}v}$

97 •••

Picture the Problem Let $U = 0$ at $x = \infty$. The potential energy of an element of the stick dm and the point mass m_0 is given by the definition of gravitational potential energy: $dU = -Gm_0dm/r$ where r is the separation of dm and m_0.

(*a*) Express the potential energy of the masses m_0 and dm:

$$dU = -\frac{Gm_0dm}{x_0 - x}$$

The mass dm is proportional to the size of the element dx:

$$dm = \lambda\, dx$$

where $\lambda = \dfrac{M}{L}$.

Substitute these results to express dU in terms of x:

$$dU = -\frac{Gm_0\lambda\, dx}{x_0 - x} = \boxed{-\frac{GMm_0\, dx}{L(x_0 - x)}}$$

(*b*) Integrate to find the total potential energy for the system:

$$U = -\frac{GMm_0}{L}\int_{-L/2}^{L/2}\frac{dx}{x_0 - x} = \frac{GMm_0}{L}\left[\ln\left(x_0 - \frac{L}{2}\right) - \ln\left(x_0 + \frac{L}{2}\right)\right]$$

$$= \boxed{-\frac{GMm_0}{L}\ln\left(\frac{x_0 + L/2}{x_0 - L/2}\right)}$$

(*c*) Because x_0 is a general point along the x axis:

$$F(x_0) = -\frac{dU}{dx_0} = \frac{Gmm_0}{L}\left[\frac{1}{x_0 + \frac{L}{2}} - \frac{1}{x_0 - \frac{L}{2}}\right]$$

Simplify this expression to obtain:

$$F(x_0) = \boxed{-\frac{Gmm_0}{x^2 - L^2/4}}$$

in agreement with the result of Example 11-8.

***98 •••**

Picture the Problem Choose a mass element *dm* of the rod of thickness *dx* at a distance *x* from the origin. All such elements of the rod experience a gravitational force *dF* due to presence of the sphere centered at the origin. We can find the total gravitational force of attraction experienced by the rod by integrating *dF* from *x* = *a* to *x* = *a* + *L*.

Express the gravitational force *dF* acting on the element of the rod of mass *dm*:

$$dF = \frac{GMdm}{x^2}$$

Express *dm* in terms of the mass *m* and length *L* of the rod:

$$dm = \frac{m}{L}dx$$

Substitute to obtain:

$$dF = \frac{GMm}{L}\frac{dx}{x^2}$$

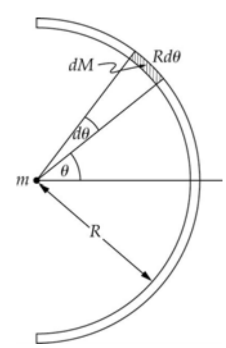
Integrate *dF* from *x* = *a* to *x* = *a* + *L* to find the total gravitational force acting on the rod:

$$F = \frac{GMm}{L}\int_a^{a+L} x^{-2}dx = -\frac{GMm}{L}\left[\frac{1}{x}\right]_a^{a+L}$$

$$= \boxed{\frac{GMm}{a(a+L)}}$$

99 •••

Picture the Problem The semicircular rod is shown in the figure. We'll use an element of length $Rd\theta = \frac{L}{\pi}d\theta$ whose mass *dM* is $\frac{M}{\pi}d\theta$. By symmetry, $F_y = 0$. We'll first find dF_x and then integrate over θ from $-\pi/2$ to $\pi/2$.



Express dF_x:

$$dF_x = \frac{GmdM}{R^2} = \frac{GMm}{\pi\left(\frac{L}{\pi}\right)^2}d\theta\cos\theta$$

Integrate dF_x over θ from $-\pi/2$ to $\pi/2$:

$$F_x = \frac{\pi\, GMm}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{2\pi\, GMm}{L^2}$$

Substitute numerical values and evaluate F_x:

$$F_x = \frac{2\pi\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(20\,\text{kg}\right)\left(0.1\,\text{kg}\right)}{\left(5\,\text{m}\right)^2} = \boxed{33.5\,\text{pN}}$$

***100** •••

Picture the Problem We can begin by expressing the forces exerted by the sun and the moon on a body of water of mass *m* and taking the ratio of these forces. In (*b*) we'll simply follow the given directions and in (*c*) we can approximate differential quantities with finite quantities to establish the given ratio.

(*a*) Express the force exerted by the sun on a body of water of mass *m*:

$$F_\text{S} = \frac{GM_\text{S}m}{r_\text{S}^2}$$

Express the force exerted by the moon on a body of water of mass *m*:

$$F_\text{m} = \frac{GM_\text{m}m}{r_\text{m}^2}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{F_\text{S}}{F_\text{m}} = \boxed{\frac{M_\text{S}r_\text{m}^2}{M_\text{m}r_\text{S}^2}}$$

Substitute numerical values and evaluate this ratio:

$$\frac{F_\text{S}}{F_\text{m}} = \frac{\left(1.99\times10^{30}\ \text{kg}\right)\left(3.84\times10^{8}\ \text{m}\right)^2}{\left(7.36\times10^{22}\ \text{kg}\right)\left(1.50\times10^{11}\ \text{m}\right)^2} = \boxed{177}$$

(*b*) Find $\dfrac{dF}{dr}$:

$$\frac{dF}{dr} = -\frac{2Gm_1m_2}{r^3} = -2\frac{F}{r}$$

Solve for the ratio $\dfrac{dF}{F}$:

$$\frac{dF}{F} = \boxed{-2\frac{dr}{r}}$$

(*c*) Express the change in force ΔF for a small change in distance Δr:

$$\Delta F = -2\frac{F}{r}\Delta r$$

Express ΔF_S:

$$\Delta F_S = -2\frac{\frac{GmM_S}{r_S^2}}{r_S}\Delta r_S = -2\frac{GmM_S}{r_S^3}\Delta r_S$$

Express ΔF_m:

$$\Delta F_m = -2\frac{GmM_m}{r_m^3}\Delta r_m$$

Divide the first of these equations by the second and simplify:

$$\frac{\Delta F_S}{\Delta F_m} = \frac{\frac{M_S}{r_S^3}\Delta r_S}{\frac{M_m}{r_m^3}\Delta r_m} = \frac{M_S r_m^3}{M_m r_S^3}\frac{\Delta r_S}{\Delta r_m} = \boxed{\frac{M_S r_m^3}{M_m r_S^3}}$$

because $\frac{\Delta r_S}{\Delta r_m} = 1$.

Substitute numerical values and evaluate this ratio:

$$\frac{\Delta F_S}{\Delta F_m} = \frac{(1.99\times10^{30}\text{ kg})(3.84\times10^{8}\text{ m})^3}{(7.36\times10^{22}\text{ kg})(1.50\times10^{11}\text{ m})^3} = \boxed{0.454}$$

101 ••

Picture the Problem Let M_{NS} be the mass of the Neutron Star and m the mass of each robot. We can use Newton's law of gravity to express the difference in the tidal-like forces acting on the coupled robots. Expanding the expression for the force on the robot further from the Neutron Star binomially will lead us to an expression for the distance at which the breaking tension in the connecting cord will be exceeded.

(*a*) The gravitational force is greater on the lower robot, so if it were not for the cable its acceleration would be greater than that of the upper robot, and they would separate. In opposing this separation the cable is stressed.

(*b*) Letting the separation of the two robots be Δr, and the distance from the center of the star to the lower robot be r, use Newton's law of gravity to express the difference in the forces acting on the robots:

$$F_{\text{tide}} = \frac{GM_{\text{NS}}m}{r^2} - \frac{GM_{\text{NS}}m}{(r+\Delta r)^2}$$

$$= GM_{\text{NS}}m\left[\frac{1}{r^2} - \frac{1}{r^2\left(1+\frac{\Delta r}{r}\right)^2}\right]$$

$$= \frac{GM_{\text{NS}}m}{r^2}\left[1-\left(1+\frac{\Delta r}{r}\right)^{-2}\right]$$

Expand the expression in the square brackets binomially to obtain:

$$1-\left(1+\frac{\Delta r}{r}\right)^{-2} \approx 1-\left(1-2\frac{\Delta r}{r}\right)$$

$$= 2\frac{\Delta r}{r}$$

Substitute to obtain:

$$F_{\text{tide}} \approx \frac{2GM_{\text{NS}}m}{r^3}\Delta r$$

Letting F_B be the breaking tension of the cord, substitute for F_{tide} and solve for the value of r corresponding to the breaking strain being exceeded:

$$r = \sqrt[3]{\frac{2GM_{\text{NS}}m}{F_B}\Delta r}$$

Substitute numerical values and evaluate r:

$$r = \sqrt[3]{\frac{2\left(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\,\text{kg}\right)(1\,\text{kg})}{25\,\text{kN}}(1\,\text{m})} = \boxed{220\text{ km}}$$

Chapter 12
Static Equilibrium and Elasticity

Conceptual Problems

1 •

(*a*) False. The conditions $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$ must be satisfied.

(*b*) True. The necessary and sufficient conditions for static equilibrium are $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$.

(*c*) True. The conditions $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$ must be satisfied.

(*d*) False. An object is in equilibrium provided the conditions $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$ are satisfied.

2 •

False. The location of the center of gravity depends on the mass distribution.

3 •

No. The definition of the center of gravity does not require that there be any material at its location.

4 •

Determine the Concept When the acceleration of gravity is not constant over an object, the center of gravity is the pivot point for balance.

5 ••

Determine the Concept This technique works because the center of mass must be directly under the balance point. Thus, a line drawn straight downward will pass through the center of mass, and another line drawn straight downward when the figure is hanging from another point will also pass through the center of mass. The center of mass is where the lines cross.

***6** •

Determine the Concept No. Because the floor can exert no horizontal force, neither can the wall. Consequently, the friction force between the wall and the ladder is zero regardless of the coefficient of friction between the wall and the ladder.

7 •

Determine the Concept We know that equal lengths of aluminum and steel wire of the same diameter will stretch different amounts when subjected to the same tension. Also, because we are neglecting the mass of the wires, the tension in them is independent of which one is closer to the roof and depends only on W. $\boxed{(b) \text{ is correct.}}$

8 •

Determine the Concept Yes; if it were otherwise, angular momentum conservation would depend on the choice of coordinates.

***9** •

Determine the Concept The condition that the bar is in rotational equilibrium is that the net torque acting on it be zero; i.e., $R_1M_1 = R_2M_2$. This condition is satisfied provided $R_1 = R_2$ and $M_1 = M_2$. $\boxed{(c) \text{ is correct.}}$

10 ••

Determine the Concept You cannot stand up because your body's center of gravity must be above your feet.

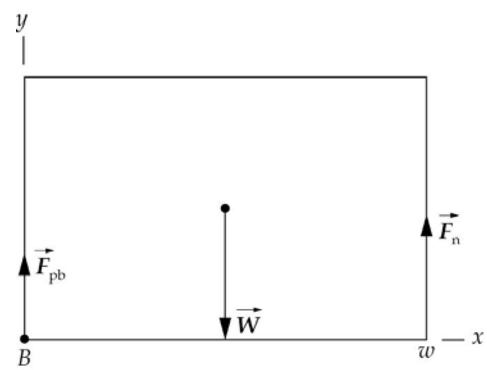
***11** ••

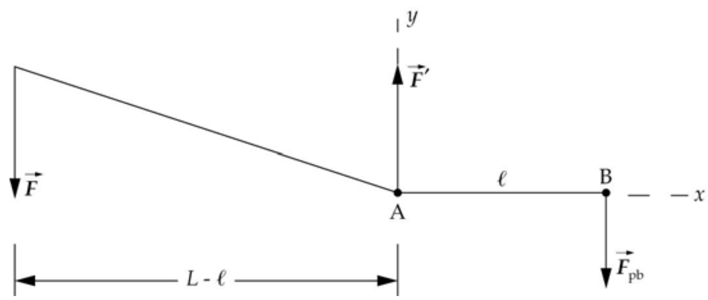
Determine the Concept The tensile strengths of stone and concrete are at least an order of magnitude lower than their compressive strengths, so you want to build compressive structures to match their properties.

Estimation and Approximation

12 ••

Picture the Problem The diagram to the right shows the forces acting on the crate as it is being lifted at its left end. Note that when the crowbar lifts the crate, only half the weight of the crate is supported by the bar. Choose the coordinate system shown and let the subscript ″pb″ refer to the pry bar. The diagram below shows the forces acting on the pry bar as it is being used to lift the end of the crate.





Assume that the maximum force F' you can apply is 500 N (about 110 lb). Let ℓ be the distance between the points of contact of the steel bar with the floor and the crate, and let L be the total length of the bar. Lacking information regarding the bend in pry bar at the fulcrum, we'll assume that it is small enough to be negligible. We can apply the condition for rotational equilibrium to the pry bar and a condition for translational equilibrium to the crate when its left end is on the verge of lifting.

Apply $\sum F_y = 0$ to the crate: $$F_{\text{pb}} - W + F_{\text{n}} = 0 \quad (1)$$

Apply $\sum \vec{\tau} = 0$ to the crate about an axis through point B and perpendicular to the plane of the page to obtain: $$wF_{\text{n}} - \tfrac{1}{2}wW = 0$$

Solve for F_{n}: $$F_{\text{n}} = \tfrac{1}{2}W$$
as noted in Picture the Problem.

Solve equation (1) for F_{pb} and substitute for F_{n} to obtain: $$F_{\text{pb}} = W - \tfrac{1}{2}W = \tfrac{1}{2}W$$

Apply $\sum \vec{\tau} = 0$ to the pry bar about an axis through point A and perpendicular to the plane of the page to obtain: $$F(L-\ell) - \ell F_{\text{pb}} = 0$$

Solve for L: $$L = \ell\left(1 + \frac{F_{\text{pb}}}{F}\right)$$

Substitute for F_{pb} to obtain: $$L = \ell\left(1 + \frac{W}{2F}\right)$$

Substitute numerical values and evaluate L:

$$L = (0.1\,\text{m})\left(1 + \frac{4500\,\text{N}}{2(500\,\text{N})}\right) = \boxed{55.0\,\text{cm}}$$

***13 ••**

Picture the Problem We can derive this expression by imagining that we pull on an area A of the given material, expressing the force each spring will experience, finding the fractional change in length of the springs, and substituting in the definition of Young's modulus.

(*a*) Express Young's modulus:

$$Y = \frac{F/A}{\Delta L/L} \qquad (1)$$

Express the elongation ΔL of each spring:

$$\Delta L = \frac{F_s}{k} \qquad (2)$$

Express the force F_s each spring will experience as a result of a force F acting on the area A:

$$F_s = \frac{F}{N}$$

Express the number of springs N in the area A:

$$N = \frac{A}{a^2}$$

Substitute to obtain:

$$F_s = \frac{Fa^2}{A}$$

Substitute in equation (2) to obtain, for the extension of one spring:

$$\Delta L = \frac{Fa^2}{kA}$$

Assuming that the springs extend/compress linearly, express the fractional extension of the springs:

$$\frac{\Delta L_{tot}}{L} = \frac{\Delta L}{a} = \frac{1}{a}\frac{Fa^2}{kA} = \frac{Fa}{kA}$$

Substitute in equation (1) and simplify:

$$Y = \frac{\frac{F}{A}}{\frac{Fa}{kA}} = \boxed{\frac{k}{a}}$$

(*b*) From our result in part (*a*):

$$k = Ya$$

From Table 12-1:

$$Y = 200\text{GN/m}^2 = 2\times10^{11}\ \text{N/m}^2$$

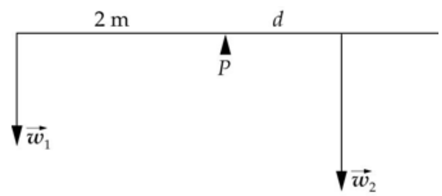
Assuming that $a \sim 1$ nm, evaluate k:

$$k = \left(2\times10^{11}\ \text{N/m}^2\right)\left(10^{-9}\ \text{m}\right) = \boxed{200\,\text{N/m}}$$

Conditions for Equilibrium

14 •

Picture the Problem Let w_1 represent the weight of the 28-kg child sitting at the left end of the board, w_2 the weight of the 40-kg child, and d the distance of the 40-kg child from the pivot point. We can apply the condition for rotational equilibrium to find d.



Apply $\sum \vec{\tau} = 0$ about an axis through the pivot point P:

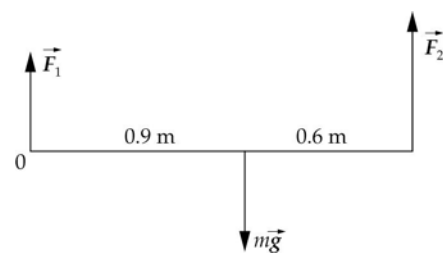
$$w_1(2\,m) - w_2 d = 0$$

Solve for and evaluate d:

$$d = \frac{w_1(2\,\text{m})}{w_2} = \frac{(28\,\text{kg})g(2\,\text{m})}{(40\,\text{kg})g} = \boxed{1.4\,\text{m}}$$

15 •

Picture the Problem Let F_1 represent the force exerted by the floor on Misako's feet, F_2 the force exerted on her hands, and m her mass. We can apply the condition for rotational equilibrium to find F_2.



Apply $\sum \vec{\tau} = 0$ about an axis through point 0:

$$F_2(1.5\,\text{m}) - mg(0.9\,\text{m}) = 0$$

Solve for F_2:

$$F_2 = \frac{mg(0.9\,\text{m})}{1.5\,\text{m}}$$

Substitute numerical values and evaluate F_2:

$$F_2 = \frac{(54\,\text{kg})(9.81\,\text{m/s}^2)(0.9\,\text{m})}{1.5\,\text{m}}$$

$$= \boxed{318\,\text{N}}$$

***16** •

Picture the Problem Let F represent the force exerted by Misako's biceps. To find F we apply the condition for rotational equilibrium about a pivot chosen at the tip of her elbow.

Apply $\sum \vec{\tau} = 0$ about an axis

$$(5\,\text{cm})F - (28\,\text{cm})(18\,\text{N}) = 0$$

through the pivot:

Solve for F:

$$F = \frac{(28\,\text{cm})(18\,\text{N})}{5\,\text{cm}} = \boxed{101\,\text{N}}$$

17 •

Picture the Problem Choose a coordinate system in which upward is the positive y direction and to the right is the positive x direction and use the conditions for translational equilibrium.

(a) Apply $\sum \vec{F} = 0$ to the forces acting on the tip of the crutch:

$$\sum F_x = -f_s + F_c \sin\theta = 0 \quad (1)$$

and

$$\sum F_y = F_n - F_c \cos\theta = 0 \quad (2)$$

Solve equation (2) for F_n and assuming that $f_s = f_{s,max}$, obtain:

$$f_s = f_{s,max} = \mu_s F_n = \mu_s F_c \cos\theta$$

Substitute in equation (1) and solve for μ_s:

$$\mu_s = \boxed{\tan\theta}$$

(b) Taking long strides requires a large coefficient of static friction because θ is large for long strides.

(c) If μ_s is small, i.e., there is ice on the surface, θ must be small to avoid slipping.

The Center of Gravity

18 •

Picture the Problem Let the weight of the automobile be w. Choose a coordinate system in which the origin is at the point of contact of the front wheels with the ground and the positive x axis includes the point of contact of the rear wheels with the ground. Apply the definition of the center of gravity to find its location.

Use the definition of the center of gravity:

$$x_{cg}W = \sum_i w_i x_i$$
$$= 0.58w(0) + 0.42w(2\,\text{m})$$
$$= (0.84\,\text{m})w$$

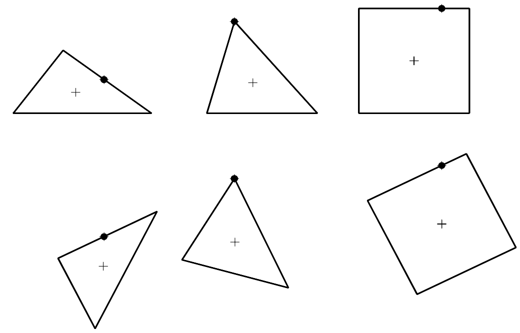
or, because $W = w$, $x_{cg}(w) = (0.84\,\text{m})w$

Solve for x_{cg}:

$$x_{cg} = \boxed{0.84\,\text{m}}$$

***19 •**

Picture the Problem The figures are shown on the right. The center of mass for each is indicated by a small +. At static equilibrium, the center of gravity is directly below the point of support.



20 ••

Picture the Problem Using the coordinate system indicated in the figure, we can apply the definition of the center of gravity to determine x_{cg} and y_{cg}.

Apply the definition of the center of gravity to find x_{cg}:

$$\begin{aligned} x_{cg}W &= \sum_i w_i x_i \\ &= (40\,\text{N})\left(\tfrac{1}{2}a\right) + (60\,\text{N})\left(\tfrac{1}{2}a\right) \\ &\quad + (30\,\text{N})\left(\tfrac{3}{2}a\right) + (50\,\text{N})\left(\tfrac{3}{2}a\right) \\ &= (170\,\text{N})a \end{aligned}$$

or, because $W = 180$ N,

$$x_{cg}(180\,\text{N}) = (170\,\text{N})a$$

Solve for x_{cg}:

$$x_{cg} = \frac{170\,\text{N}}{180\,\text{N}}a = 0.944a$$

Apply the definition of the center of gravity to find y_{cg}:

$$\begin{aligned} y_{cg}W &= \sum_i w_i y_i \\ &= (40\,\text{N})\left(\tfrac{1}{2}a\right) + (60\,\text{N})\left(\tfrac{3}{2}a\right) \\ &\quad + (30\,\text{N})\left(\tfrac{3}{2}a\right) + (50\,\text{N})\left(\tfrac{1}{2}a\right) \\ &= (180\,\text{N})a \end{aligned}$$

or, because $W = 180$ N,

$$y_{cg}(180\,\text{N}) = (180\,\text{N})a$$

Solve for y_{cg}:

$$y_{cg} = a$$

The coordinates of the center of gravity are:

$$(x_{cg}, y_{cg}) = \boxed{(0.944a, a)}$$

21 ••

Picture the Problem Let the origin of the coordinate system be at the lower left corner of the plate and the positive x direction be to the right. Let a and b be the length and width of the plate. Let σ be the mass per unit area of the plate. Then the weight of the plate is given by $w = ab\sigma g$ and that of the matter missing from the hole is $-\pi R^2\sigma g$. Noting that, by symmetry, $y_{cg} = b/2$, we can apply the definition of the center of gravity to find x_{cg}.

Apply the definition of the center of gravity to find x_{cg}:

$$x_{cg}W = \sum_i w_i x_i = (ab\sigma g)\left(\tfrac{1}{2}a\right) - \left(\pi R^2\sigma g\right)(a - R)$$

or, because

$$W = w_{plate} - w_{hole} = ab\sigma g - \pi R^2\sigma g,$$

$$x_{cg}\left(ab\sigma g - \pi R^2\sigma g\right) = (ab\sigma g)\left(\tfrac{1}{2}a\right) - \left(\pi R^2\sigma\right)(a - R)$$

Solve for x_{cg}:

$$x_{cg} = \frac{\frac{1}{2}a^2b - \pi aR^2 + \pi R^3}{ab - \pi R^2}$$

The coordinates of the center of gravity are:

$$\left(x_{cg}, y_{cg}\right) = \boxed{\left(\frac{\frac{1}{2}a^2b - \pi aR^2 + \pi R^3}{ab - \pi R^2}, \tfrac{1}{2}b\right)}$$

Some Examples of Static Equilibrium

22 •

Picture the Problem We can use the given definition of the mechanical advantage of a lever and the condition for rotational equilibrium to show that $M = x/X$.

(*a*) Express the definition of mechanical advantage for a lever:

$$M = \frac{F}{f}$$

Apply the condition for rotational equilibrium to the lever:

$$xf - XF = 0$$

Solve for the ratio of F to f to obtain:

$$\frac{F}{f} = \frac{x}{X}$$

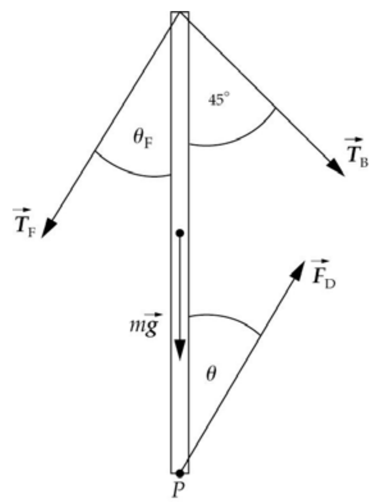
Substitute to obtain:

$$M = \boxed{\frac{x}{X}}$$

(*b*) A shorter moment arm for the applied force is useful when one wishes to move the load over a large distance using a short movement of the applied force.

23 •

Picture the Problem The force diagram shows the tension in the forestay, $\vec{T}_F$, the tension in the backstay, $\vec{T}_B$, the gravitational force on the mast $m\vec{g}$, and the force exerted by the deck, $\vec{F}_D$. Let the origin of the coordinate system be at the foot of the mast with the positive *x* direction to the right and the positive *y* direction upward. Because the mast is in equilibrium, we can apply the conditions for both translational and rotational equilibrium to find the tension in the backstay and the force that the deck exerts on the mast.



Apply $\sum\vec{\tau} = 0$ to the mast about an axis through its foot and solve for T_B:

$$(4.88\,\text{m})(1000\,\text{N})\sin\theta_F - (4.88\,\text{m})T_B\sin 45° = 0$$

and

$$T_B = \frac{(1000\,\text{N})\sin\theta_F}{\sin 45°}$$

Find θ_F, the angle of the forestay with the vertical:

$$\theta_F = \tan^{-1}\left(\frac{2.74\,\text{m}}{4.88\,\text{m}}\right) = 29.3°$$

Substitute to obtain:

$$T_B = \frac{(1000\,\text{N})\sin 29.3°}{\sin 45°} = \boxed{692\,\text{N}}$$

Apply the condition for translational equilibrium in the *x* direction to the mast:

$$\sum F_x = F_D\cos\theta + T_B\sin 45° - T_F\sin\theta_F = 0$$

or

$$F_D\cos\theta = (1000\,\text{N})\sin 29.3° - (692\,\text{N})\sin 45° \approx 0$$

Apply the condition for translational equilibrium in the *y* direction to the mast:

$$\sum F_y = F_D\sin\theta - T_F\cos\theta_F - T_B\cos 45° - mg = 0$$

or

$$F_D \sin\theta = (1000\,\text{N})\cos 29.3° + (692\,\text{N})\cos 45° + (120\,\text{kg})(9.81\,\text{m/s}^2) = 2539\,\text{N}$$

Because $F_D\cos\theta = 0$:

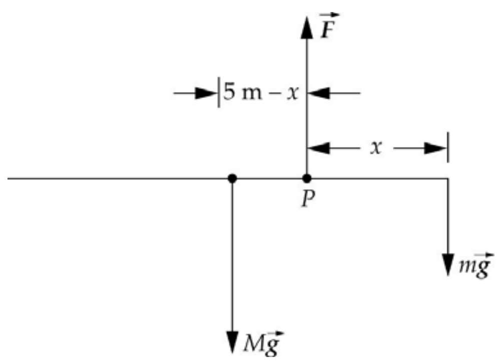
$$\theta = \boxed{90°},\ F_D = \boxed{2.54\,\text{kN}}$$

and

no block is required to prevent the mast from moving.

24 ••

Picture the Problem The diagram shows $M\vec{g}$, the weight of the beam, $m\vec{g}$, the weight of the student, and the force the ledge exerts $\vec{F}$, acting on the beam. Because the beam is in equilibrium, we can apply the condition for rotational equilibrium to the beam to find the location of the pivot point P that will allow the student to walk to the end of the beam.



Apply $\sum\vec{\tau} = 0$ about an axis through the pivot point P:

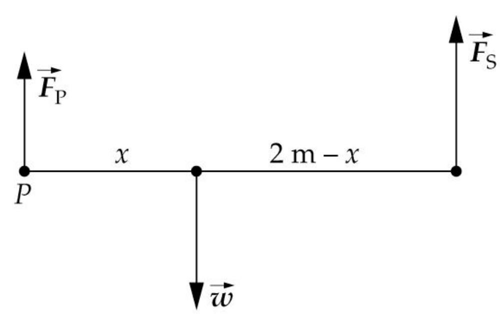
$$Mg(5\,\text{m} - x) - mgx = 0$$

Solve for x:

$$x = \frac{5M}{M+m} = \frac{5(300\,\text{kg})}{300\,\text{kg} + 60\,\text{kg}} = \boxed{4.17\,\text{m}}$$

***25** ••

Picture the Problem The diagram shows $\vec{w}$, the weight of the student, $\vec{F}_P$, the force exerted by the board at the pivot, and $\vec{F}_s$, the force exerted by the scale, acting on the student. Because the student is in equilibrium, we can apply the condition for rotational equilibrium to the student to find the location of his center of gravity.



Apply $\sum\vec{\tau} = 0$ about an axis

$$F_s(2\,\text{m}) - wx = 0$$

through the pivot point *P*:

Solve for *x*:

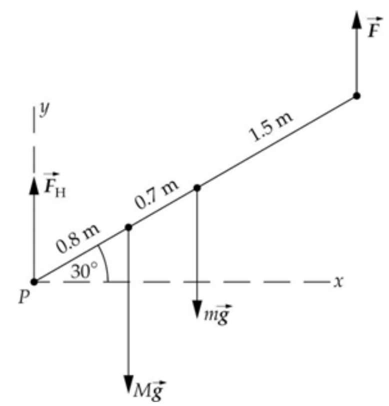
$$x = \frac{(2\,\text{m})F_s}{w}$$

Substitute numerical values and evaluate *x*:

$$x = \frac{(2\,\text{m})(250\,\text{N})}{(70\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.728\,\text{m}}$$

26 ••

Picture the Problem The diagram shows $m\vec{g}$, the weight of the board, $\vec{F}_H$, the force exerted by the hinge, $M\vec{g}$, the weight of the block, and $\vec{F}$, the force acting vertically at the right end of the board. Because the board is in equilibrium, we can apply the condition for rotational equilibrium to it to find the magnitude of $\vec{F}$.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the hinge:

$$F[(3\,\text{m})\cos 30°] - mg[(1.5\,\text{m})\cos 30°] - Mg[(0.8\,\text{m})\cos 30°] = 0$$

Solve for *F*:

$$F = \frac{m(1.5\,\text{m}) + M(0.8\,\text{m})}{3\,\text{m}}g$$

Substitute numerical values and evaluate *F*:

$$F = \frac{(5\,\text{kg})(1.5\,\text{m}) + (60\,\text{kg})(0.8\,\text{m})}{3\,\text{m}} \times (9.81\,\text{m/s}^2) = \boxed{181\,\text{N}}$$

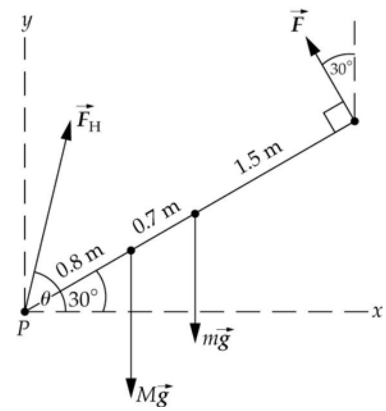
(*b*) Apply $\sum F_y = 0$ to the board:

$$F_H - Mg - mg + F = 0$$

Solve for and evaluate F_H:

$$F_H = Mg + mg - F = (M + m)g - F = (60\,\text{kg} + 5\,\text{kg})(9.81\,\text{m/s}^2) - 181\,\text{N} = \boxed{457\,\text{N}}$$

(*c*) The force diagram showing the force $\vec{F}$ acting at right angles to the board is shown to the right:



Apply $\sum \vec{\tau} = 0$ about the hinge:

$$F(3\,\text{m}) - mg[(1.5\,\text{m})\cos 30°] - Mg[(0.8\,\text{m})\cos 30°] = 0$$

Solve for *F*:

$$F = \frac{m(1.5\,\text{m}) + M(0.8\,\text{m})}{3\,\text{m}} g\cos 30°$$

Substitute numerical values and evaluate *F*:

$$F = \frac{(5\,\text{kg})(1.5\,\text{m}) + (60\,\text{kg})(0.8\,\text{m})}{3\,\text{m}} \times (9.81\,\text{m/s}^2)\cos 30° = \boxed{157\,\text{N}}$$

Apply $\sum F_y = 0$ to the board:

$$F_\text{H}\sin\theta - Mg - mg + F\cos 30° = 0$$

or

$$F_\text{H}\sin\theta = (M + m)g - F\cos 30° \quad (1)$$

Apply $\sum F_x = 0$ to the board:

$$F_\text{H}\cos\theta - F\sin 30° = 0$$

or

$$F_\text{H}\cos\theta = F\sin 30° \quad (2)$$

Divide the first of these equations by the second to obtain:

$$\frac{F_\text{H}\sin\theta}{F_\text{H}\cos\theta} = \frac{(M + m)g - F\cos 30°}{F\sin 30°}$$

Solve for θ:

$$\theta = \tan^{-1}\left[\frac{(M + m)g - F\cos 30°}{F\sin 30°}\right]$$

Substitute numerical values and evaluate θ:

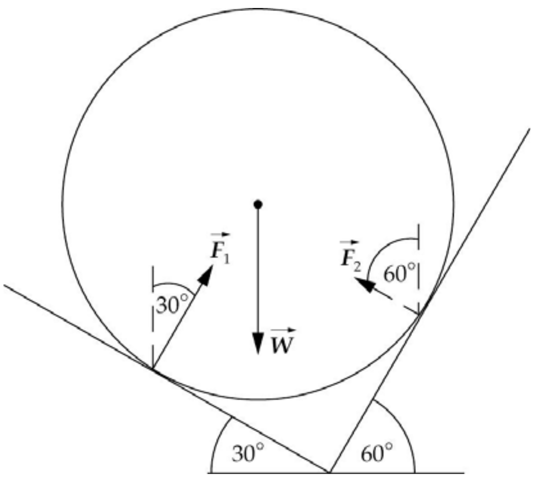
$$\theta = \tan^{-1}\left[\frac{(65\,\text{kg})(9.81\,\text{m/s}^2) - (157\,\text{N})\cos 30°}{(157\,\text{N})\sin 30°}\right] = 81.1°$$

Substitute numerical values in equation (2) and evaluate F_H:

$$F_H = \frac{(157\,\text{N})\sin 30°}{\cos 81.1°} = \boxed{507\,\text{N}}$$

***27** •

Picture the Problem The planes are frictionless; therefore, the force exerted by each plane must be perpendicular to that plane. Let $\vec{F}_1$ be the force exerted by the 30° plane, and let $\vec{F}_2$ be the force exerted by the 60° plane. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the cylinder is in equilibrium, we can use the conditions for translational equilibrium to find the magnitudes of $\vec{F}_1$ and $\vec{F}_2$.



Apply $\sum F_x = 0$ to the cylinder:

$$F_1 \sin 30° - F_2 \sin 60° = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the cylinder:

$$F_1 \cos 30° + F_2 \cos 60° - W = 0 \qquad (2)$$

Solve equation (1) for F_1:

$$F_1 = \sqrt{3}F_2 \qquad (3)$$

Substitute in equation (2) to obtain:

$$\sqrt{3}F_2 \cos 30° + F_2 \cos 60° - W = 0$$

Solve for F_2:

$$\left(\sqrt{3}\cos 30° + \cos 60°\right)F_2 = W$$

or

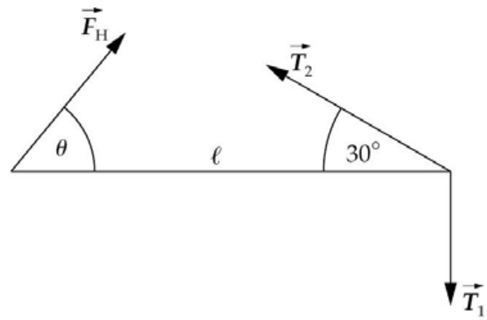
$$F_2 = \frac{W}{\sqrt{3}\cos 30° + \cos 60°} = \boxed{\tfrac{1}{2}W}$$

Substitute in equation (3):

$$F_1 = \sqrt{3}\left(\tfrac{1}{2}W\right) = \boxed{\tfrac{\sqrt{3}}{2}W}$$

28 ••

Picture the Problem The force diagram shows the forces $\vec{F}_H$, $\vec{T}_2$, and $\vec{T}_1$ acting on the strut. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the strut is in equilibrium, we can apply the conditions for translational and rotational equilibrium to it.



(*a*) The forces acting on the strut are the tensions $\vec{T}_1$ and $\vec{T}_2$ and $\vec{F}_H$, the force exerted on the strut by the hinge.

(*b*) Apply $\sum \vec{\tau} = 0$ about an axis through the hinge:

$$T_2 \ell \sin 30° - T_1 \ell = 0$$

Solve for T_1:

$$T_{2v} = T_2 \sin 30° = T_1$$

or, because T_1 = 80 N,

$$T_{2v} = \boxed{80\text{ N}}$$

(*c*) Apply $\sum F_x = 0$ to the beam:

$$F_H \cos\theta - T_2 \cos 30° = 0$$

or

$$F_H \cos\theta = T_2 \cos 30° \quad (1)$$

Apply $\sum F_y = 0$ to the beam:

$$F_H \sin\theta + T_2 \sin 30° - T_1 = 0$$

or

$$F_H \sin\theta = T_1 - T_2 \sin 30° = 80\text{ N} - T_2 \sin 30° \quad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\tan\theta = \frac{80\text{ N} - T_2 \sin 30°}{T_2 \cos 30°}$$

Solve for θ:

$$\theta = \tan^{-1}\left[\frac{80\text{ N} - T_2 \sin 30°}{T_2 \cos 30°}\right]$$

Express T_2 in terms of T_{2v}:

$$T_2 = \frac{T_{2v}}{\sin 30°} = \frac{80\text{ N}}{\sin 30°} = 160\text{ N}$$

Evaluate θ:

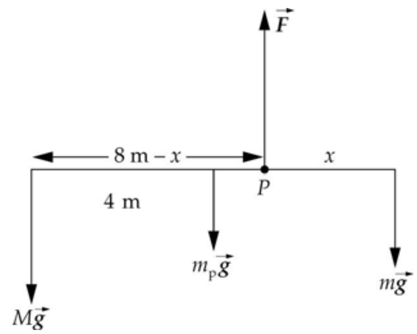
$$\theta = \tan^{-1}\left[\frac{80\,\text{N} - (160\,\text{N})\sin 30°}{(160\,\text{N})\cos 30°}\right] = 0°$$

Substitute numerical values in equation (1) and evaluate F_H:

$$F_H = \frac{(160\,\text{N})\cos 30°}{\cos 0°} = \boxed{139\,\text{N}}$$ to the right.

29 ••

Picture the Problem The force diagram shows the weight of the pirate, $M\vec{g}$, the weight of the victim, $m\vec{g}$, and the force the deck exerts at the edge of the ship, $\vec{F}$ acting at the fulcrum P. The diagram also shows, for part (b), the weight of the plank acting through the plank's center of gravity.



(a) Apply $\sum\vec{\tau} = 0$ at the pivot point P:

$$Mg(8\,\text{m} - x) - mgx = 0$$

or

$$M(8\,\text{m} - x) - mx = 0$$

Solve for x:

$$x = \frac{8M}{M+m} = \frac{8(105\,\text{kg})}{105\,\text{kg} + 63\,\text{kg}} = \boxed{5.00\,\text{m}}$$

(b) Apply $\sum\vec{\tau} = 0$ about an axis through the pivot point P:

$$Mg(8\,\text{m} - x) + m_p g(4\,\text{m} - x) - mgx = 0$$

or

$$M(8\,\text{m} - x) + m_p(4\,\text{m} - x) - mx = 0$$

Solve for x:

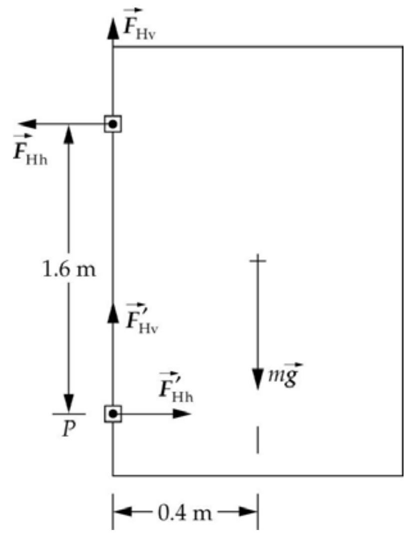
$$x = \frac{8M + 4m_p}{M + m + m_p}$$

Substitute numerical values and evaluate x:

$$x = \frac{8(105\,\text{kg}) + 4(25\,\text{kg})}{105\,\text{kg} + 63\,\text{kg} + 25\,\text{kg}} = \boxed{4.87\,\text{m}}$$

30 ••

Picture the Problem The drawing shows the door and its two supports. The center of gravity of the door is 0.8 m above (and below) the hinge, and 0.4 m from the hinges horizontally. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Denote the horizontal and vertical components of the hinge force by F_{Hh} and F_{Hv}. Because the door is in equilibrium, we can use the conditions for translational and rotational equilibrium to determine the horizontal forces exerted by the hinges.



Apply $\sum \vec{\tau} = 0$ about an axis through the lower hinge:

$$F_{Hh}(1.6\,\text{m}) - mg(0.4\,\text{m}) = 0$$

Solve for F_{Hh}:

$$F_{Hh} = \frac{mg(0.4\,\text{m})}{1.6\,\text{m}}$$

Substitute numerical values and evaluate F_{Hh}:

$$F_{Hh} = \frac{(18\,\text{kg})(9.81\,\text{m/s}^2)(0.4\,\text{m})}{1.6\,\text{m}} = \boxed{44.1\,\text{N}}$$

Apply $\sum F_x = 0$ to the door and solve for F'_{Hh}:

$$F'_{Hh} - F_{Hh} = 0$$

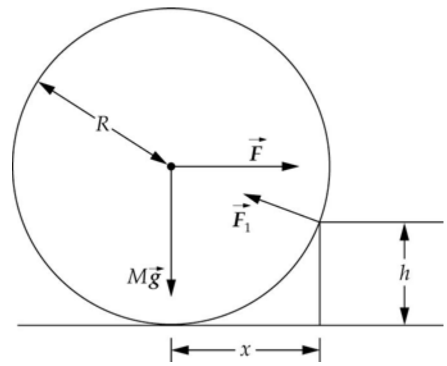
and

$$F'_{Hh} = \boxed{44.1\,\text{N}}$$

Note that the upper hinge pulls on the door and the lower hinge pushes on it.

31 ••

Picture the Problem The figure shows the wheel on the verge of rolling over the edge of the step. Note that, under this condition, the normal force the floor exerts on the wheel is zero. Choose the coordinate system shown in the figure and apply the conditions for translational equilibrium and the result for F from Example 12-4 to the wheel.



Apply $\sum \vec{F} = 0$ to the wheel:

$$\sum F_x = F - F_{1x} = 0$$

and

$$\sum F_y = F_{1y} - Mg = 0$$

Write $\vec{F}_1$ in vector form:

$$\vec{F}_1 = -F_{1x}\hat{\boldsymbol{i}} + F_{1y}\hat{\boldsymbol{j}} = -F\hat{\boldsymbol{i}} + Mg\hat{\boldsymbol{j}}$$

From Example 12-4 we have:

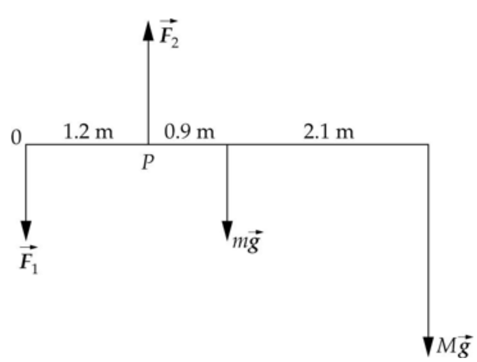
$$F = \frac{Mg\sqrt{h(2R-h)}}{R-h}$$

Substitute to obtain:

$$\vec{F}_1 = -\frac{Mg\sqrt{h(2R-h)}}{R-h}\hat{\boldsymbol{i}} + Mg\hat{\boldsymbol{j}} = \boxed{\frac{Mg\sqrt{h(2R-h)}}{h-R}\hat{\boldsymbol{i}} + Mg\hat{\boldsymbol{j}}}$$

32 ••

Picture the Problem The diagram shows the forces $\vec{F}_1$ and $\vec{F}_2$ acting at the supports, the weight of the board $m\vec{g}$, acting at its center of gravity, and the weight of the diver $M\vec{g}$ acting at the end of the diving board. Because the board is in equilibrium, we can apply the condition for rotational equilibrium to find the forces at the supports.



Apply $\sum \vec{\tau} = 0$ about an axis through the left support:

$$(1.2\,\text{m})F_2 - (2.1\,\text{m})mg - (4.2\,\text{m})Mg = 0$$

Solve for F_2:

$$F_2 = \frac{(2.1\,\text{m})m + (4.2\,\text{m})M}{(1.2\,\text{m})}g$$

Substitute numerical values and evaluate F_2:

$$F_2 = \frac{(2.1\,\text{m})(30\,\text{kg}) + (4.2\,\text{m})(70\,\text{kg})}{(1.2\,\text{m})}(9.81\,\text{m/s}^2) = \boxed{2.92\,\text{kN, compression}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the right support:

$$(1.2\,\text{m})F_1 - (0.9\,\text{m})mg - (3\,\text{m})Mg = 0$$

Solve for F_1:

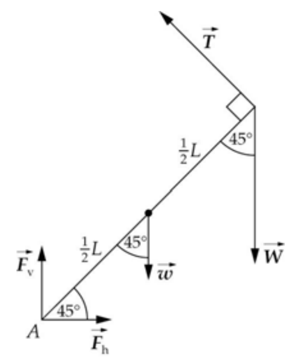
$$F_1 = \frac{(0.9\,\text{m})m + (3\,\text{m})M}{(1.2\,\text{m})}g$$

Substitute numerical values and evaluate F_1:

$$F_1 = \frac{(0.9\,\text{m})(30\,\text{kg}) + (3\,\text{m})(70\,\text{kg})}{(1.2\,\text{m})}(9.81\,\text{m/s}^2) = \boxed{1.94\,\text{kN, tension}}$$

33 ••

Picture the Problem Let T be the tension in the line attached to the wall and L be the length of the strut. The figure includes w, the weight of the strut, for part (b). Because the strut is in equilibrium, we can use the conditions for both rotational and translational equilibrium to find the force exerted on the strut by the hinge.



(a) Express the force exerted on the strut at the hinge:

$$\vec{F} = F_h\hat{i} + F_v\hat{j} \qquad (1)$$

Ignoring the weight of the strut, apply $\sum \vec{\tau} = 0$ at the hinge:

$$LT - (L\cos 45°)W = 0$$

Solve for the tension in the line:

$$T = W\cos 45° = (60\,\text{N})\cos 45° = 42.43\,\text{N}$$

Apply $\sum \vec{F} = 0$ to the strut:

$$\sum F_x = F_h - T\cos 45° = 0$$

and

$$\sum F_y = F_\text{v} + T\cos 45° - Mg = 0$$

Solve for F_h:

$$T_\text{h} = T\cos 45° = (42.43\,\text{N})\cos 45°$$
$$= 30.0\,\text{N}$$

Solve for F_v:

$$F_\text{v} = Mg - T\cos 45°$$
$$= 60\,\text{N} - (42.43\,\text{N})\cos 45°$$
$$= 30.0\,\text{N}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \boxed{(30.0\,\text{N})\hat{\boldsymbol{i}} + (30.0\,\text{N})\hat{\boldsymbol{j}}}$$

(*b*) Including the weight of the strut, apply $\sum\vec{\tau} = 0$ at the hinge:

$$LT - (L\cos 45°)W - \left(\frac{L}{2}\cos 45\right)w = 0$$

Solve for the tension in the line:

$$T = (\cos 45°)W + \left(\frac{1}{2}\cos 45°\right)w$$
$$= (\cos 45°)(60\,\text{N}) + \left(\frac{1}{2}\cos 45°\right)(20\,\text{N})$$
$$= 49.5\,\text{N}$$

Apply $\sum\vec{F} = 0$ to the strut:

$$\sum F_x = F_\text{h} - T\cos 45° = 0$$

and

$$\sum F_y = F_\text{v} + T\cos 45° - W - w = 0$$

Solve for F_h:

$$T_\text{h} = T\cos 45° = (49.5\,\text{N})\cos 45°$$
$$= 35.0\,\text{N}$$

Solve for F_v:

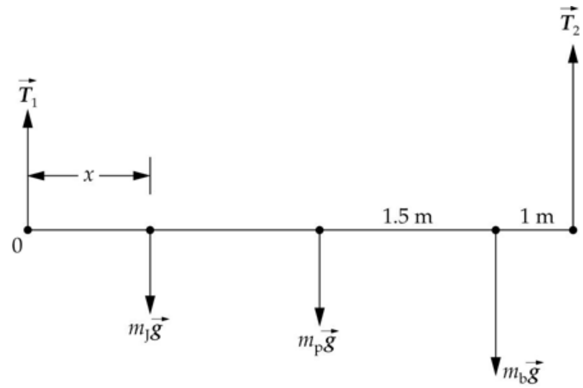
$$F_\text{v} = W + w - T\cos 45°$$
$$= 60\,\text{N} + 20\,\text{N} - (49.5\,\text{N})\cos 45°$$
$$= 45.0\,\text{N}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \boxed{(35.0\,\text{N})\hat{\boldsymbol{i}} + (45.0\,\text{N})\hat{\boldsymbol{j}}}$$

34 ••

Picture the Problem Note that if the 60-kg mass is at the far left end of the plank, T_1 and T_2 are less than 1 kN. Let x be the distance of the 60-kg mass from T_1. Because the plank is in equilibrium, we can apply the condition for rotational equilibrium to relate the distance x to the other distances and forces.



Apply $\sum\vec{\tau} = 0$ about an axis through the left end of the plank:

$$(5\,\text{m})T_2 - (4\,\text{m})m_b g - (2.5\,\text{m})m_p g - m_J g x = 0$$

Solve for x:

$$x = \frac{(5\,\text{m})T_2 - (4\,\text{m})m_b g - (2\,\text{m})m_p g}{m_J g}$$

Substitute numerical values and simplify to obtain:

$$x = \frac{(5\,\text{m})T_2 - 3.63\,\text{kN}\cdot\text{m}}{0.5886\,\text{kN}}$$

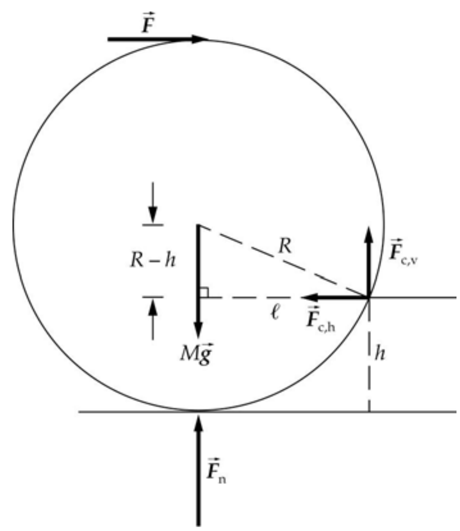
Set T_2 = 1 kN and evaluate x:

$$x = \frac{(5\,\text{m})(1\,\text{kN}) - 3.63\,\text{kN}\cdot\text{m}}{0.5886\,\text{kN}} = 2.33\,\text{m}$$

and $\boxed{\text{Julie is safe for } 0 < x < 2.33\,\text{m.}}$

35 ••

Picture the Problem The figure to the right shows the forces acting on the cylinder. Choose a coordinate system in which the positive *x* direction is to the right and the positive *y* direction is upward. Because the cylinder is in equilibrium, we can apply the conditions for translational and
rotational equilibrium to find F_n and the horizontal and vertical components of the force the corner of the step exerts on the cylinder.



(*a*) Apply $\sum\vec{\tau}=0$ to the cylinder about the step's corner:

$$Mg\ell - F_n\ell - F(2R-h)=0$$

Solve for F_n:

$$F_n = Mg - \frac{F(2R-h)}{\ell}$$

Express ℓ as a function of *R* and *h*:

$$\ell = \sqrt{R^2-(R-h)^2} = \sqrt{2Rh-h^2}$$

$$F_n = Mg - \frac{F(2R-h)}{\sqrt{2Rh-h^2}} = \boxed{Mg - F\sqrt{\frac{2R-h}{h}}}$$

(*b*) Apply $\sum F_x = 0$ to the cylinder:

$$-F_{c,h} + F = 0$$

Solve for $F_{c,h}$:

$$F_{c,h} = \boxed{F}$$

(*c*) Apply $\sum F_y = 0$ to the cylinder:

$$F_n - Mg + F_{c,v} = 0$$

Solve for $F_{c,v}$:

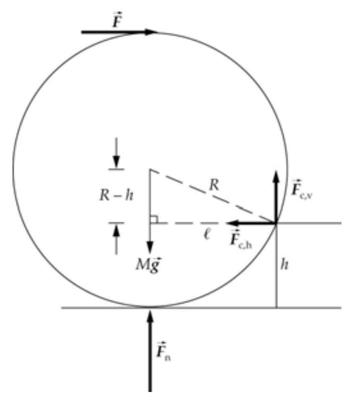
$$F_{c,v} = Mg - F_n$$

Substitute the result from part (*a*):

$$F_{c,v} = Mg - \left\{Mg - F\sqrt{\frac{2R-h}{h}}\right\} = \boxed{F\sqrt{\frac{2R-h}{h}}}$$

36 ••

Picture the Problem The figure to the right shows the forces acting on the cylinder. Because the cylinder is in equilibrium, we can use the condition for rotational equilibrium to express F_n in terms of F. Because, to roll over the step, the cylinder must lift off the floor, we can set $F_n = 0$ in our expression relating F_n and F and solve for F.



Apply $\sum \vec{\tau} = 0$ about the step's corner:

$$Mg\ell - F_n\ell - F(2R-h) = 0$$

Solve for F_n:

$$F_n = Mg - \frac{F(2R-h)}{\ell}$$

Express ℓ as a function of R and h:

$$\ell = \sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$$

Substitute to obtain:

$$F_n = Mg - \frac{F(2R-h)}{\sqrt{2Rh-h^2}} = Mg - F\sqrt{\frac{2R-h}{h}}$$

To roll over the step, the cylinder must lift off the floor, i.e., $F_n = 0$:

$$0 = Mg - F\sqrt{\frac{2R-h}{h}}$$

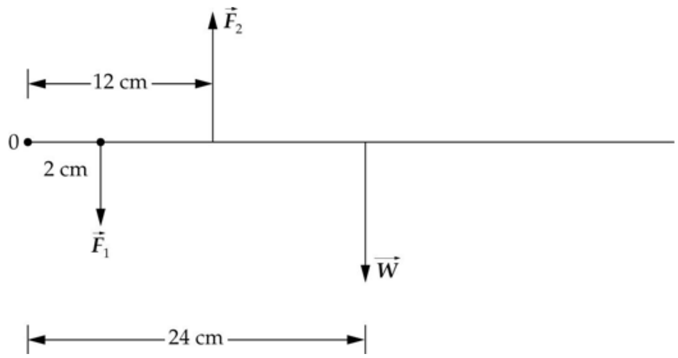
Solve for F:

$$F = \boxed{Mg\sqrt{\frac{h}{2R-h}}}$$

***37** ••

Picture the Problem The diagram shows the forces F_1 and F_2 that the fencer's hand exerts on the epee. We can use a condition for translational equilibrium to find the upward force the fencer must exert on the epee when it is in equilibrium and the definition of torque to determine the total torque exerted. In part (*c*) we can use the conditions for translational and rotational equilibrium to obtain two equations in F_1 and

F_2 that we can solve simultaneously. In part (*d*) we can apply Newton's 2nd law in rotational form and the condition for translational equilibrium to obtain two equations in F_1 and F_2 that, again, we can solve simultaneously.



(*a*) Letting the upward force exerted by the fencer's hand be *F*, apply $\sum F_y = 0$ to the epee to obtain:

$$F - W = 0$$

Solve for and evaluate *F*:

$$F = mg = (0.7\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{6.87\,\text{N}}$$

(*b*) Express the torque due to the weight about the left end of the epee:

$$\tau = \ell w = (0.24\,\text{m})(6.87\,\text{N}) = \boxed{1.65\,\text{N}\cdot\text{m}}$$

(*c*) Apply $\sum F_y = 0$ to the epee to obtain:

$$-F_1 + F_2 - 6.87\,\text{N} = 0 \qquad (1)$$

Apply $\sum \tau_0 = 0$ to obtain:

$$-(0.02\,\text{m})F_1 + (0.12\,\text{m})F_2 - 1.65\,\text{N}\cdot\text{m} = 0$$

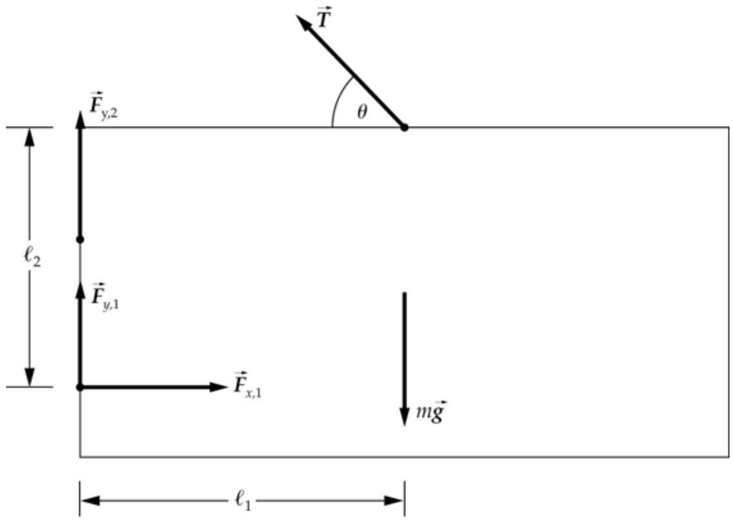
Solve these equations simultaneously to obtain:

$F_1 = \boxed{8.26\,\text{N}}$ and $F_2 = \boxed{15.1\,\text{N}}$.

Note that the force nearest the butt of the epee is directed downward and the force nearest the hand guard is directed upward.

38 ••

Picture the Problem In the force diagram, the forces exerted by the hinges are $\vec{F}_{y,2}$, $\vec{F}_{y,1}$, and $\vec{F}_{x,1}$ where the subscript 1 refers to the lower hinge. Because the gate is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find the tension in the wire and the forces at the hinges.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the lower hinge and perpendicular to the plane of the page:

$$\ell_1 T\sin\theta + \ell_2 T\cos\theta - \ell_1 mg = 0$$

Solve for *T*:

$$T = \frac{\ell_1 mg}{\ell_1 \sin\theta + \ell_2 \cos\theta}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{(1.5\,\text{m})(200\,\text{N})}{(1.5\,\text{m})\sin 45° + (1.5\,\text{m})\cos 45°} = \boxed{141\,\text{N}}$$

(*b*) Apply $\sum F_x = 0$ to the gate:

$$F_{x,1} - T\cos 45° = 0$$

Solve for and evaluate $F_{x,1}$:

$$F_{x,1} = T\cos 45° = (141\,\text{N})\cos 45° = \boxed{99.7\,\text{N}}$$

(*c*) Apply $\sum F_y = 0$ to the gate:

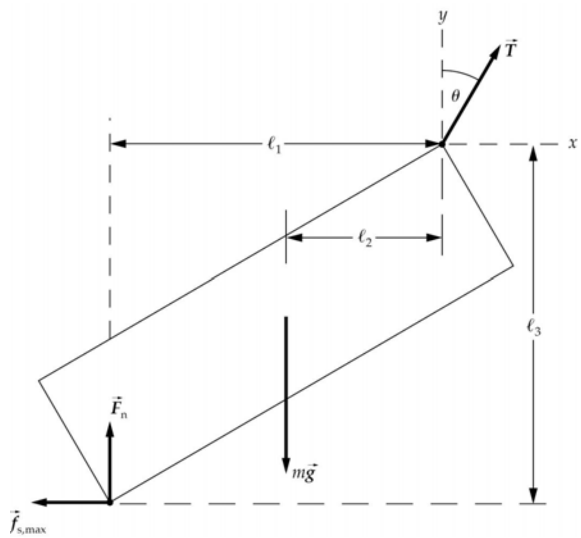
$$F_{y,1} + F_{y,2} + T\sin 45° - mg = 0$$

Because $F_{y,1}$ and $F_{y,2}$ cannot be determined independently, solve for and evaluate their sum:

$$F_{y,1} + F_{y,2} = mg - T\sin 45° = 200\,\text{N} - 99.7\,\text{N} = \boxed{100\,\text{N}}$$

39 •••

Picture the Problem Let T = the tension in the wire; F_n = the normal force of the surface; and $f_{s,max} = \mu_s F_n$ the maximum force of static friction. Letting the point at which the wire is attached to the log be the origin, the center of mass of the log is at (–1.838 m, –0.797 m) and the point of contact with the floor is at (–3.676 m, –1.594 m). Because the log is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the log: $T\sin\theta - f_{s,max} = 0$

or

$$T\sin\theta = f_{s,max} = \mu_s F_n \qquad (1)$$

Apply $\sum F_y = 0$ to the log: $T\cos\theta + F_n - mg = 0$

or

$$T\cos\theta = mg - F_n \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\mu_s F_n}{mg - F_n}$$

or

$$\theta = \tan^{-1}\frac{\mu_s}{\dfrac{mg}{F_n} - 1} \qquad (3)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the origin:

$$\ell_2 mg - \ell_1 F_n - \ell_3 \mu_s F_n = 0$$

Solve for F_n:

$$F_n = \frac{\ell_2 mg}{\ell_1 + \ell_3 \mu_s}$$

Substitute numerical values and evaluate F_n:

$$F_n = \frac{1.838(100\,\text{kg})(9.81\,\text{m/s}^2)}{3.676 + 1.594(0.6)} = 389\,\text{N}$$

Substitute in equation (3) and evaluate θ:

$$\theta = \tan^{-1}\frac{0.6}{\dfrac{(100\,\text{kg})(9.81\,\text{m/s}^2)}{389\,\text{N}} - 1}$$

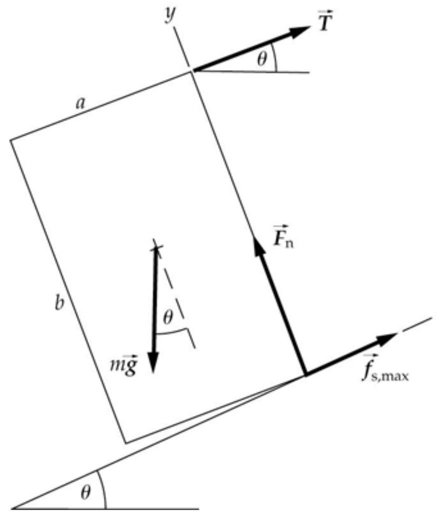
$$= \boxed{21.5^\circ}$$

Substitute numerical values in equation (1) and evaluate T:

$$T = \frac{(0.6)(389\,\text{N})}{\sin 21.5^\circ} = \boxed{636\,\text{N}}$$

40 •••

Picture the Problem Consider what happens just as θ increases beyond θ_{max}. Because the top of the block is fixed by the cord, the block will in fact rotate with only the lower right edge of the block remaining in contact with the plane. It follows that just prior to this slipping, F_n and $f_s = \mu_s F_n$ act at the lower right edge of the block. Choose a coordinate system in which up the incline is the positive x direction and the direction of $\vec{F}_n$ is the positive y direction. Because the block is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the block:

$$T + \mu_s F_n - mg\sin\theta = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the block:

$$F_n - mg\cos\theta = 0 \qquad (2)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the lower right edge of the block:

$$\tfrac{1}{2}a(mg\cos\theta) + \tfrac{1}{2}b(mg\sin\theta) - bT = 0 \qquad (3)$$

Eliminate F_n between equations (1) and (2) and solve for T:

$$T = mg(\sin\theta - \mu_s\cos\theta)$$

Substitute for T in equation (3):

$$\tfrac{1}{2}a(mg\cos\theta)+\tfrac{1}{2}b(mg\sin\theta)-b[mg(\sin\theta-\mu_s\cos\theta)]=0$$

Substitute $4a$ for b:

$$\tfrac{1}{2}a(mg\cos\theta)+\tfrac{1}{2}(4a)(mg\sin\theta)-(4a)[mg(\sin\theta-\mu_s\cos\theta)]=0$$

Simplify to obtain:

$$(1+8\mu_s)\cos\theta-4\sin\theta=0$$

Solve for θ:

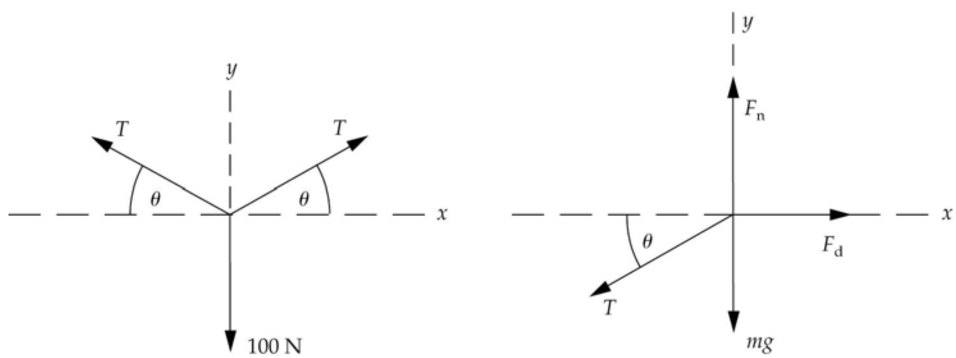
$$\theta=\tan^{-1}\frac{1+8\mu_s}{4}$$

Substitute numerical values and evaluate θ:

$$\theta=\tan^{-1}\frac{1+8(0.8)}{4}=\boxed{61.6°}$$

***41 ••**

Picture the Problem The free-body diagram shown to the left below is for the weight and the diagram to the right is for the boat. Because both are in equilibrium under the influences of the forces acting on them, we can apply a condition for translational equilibrium to find the tension in the chain.



(*a*) Apply $\sum F_x=0$ to the boat:

$$F_d-T\cos\theta=0$$

Solve for T:

$$T=\frac{F_d}{\cos\theta}$$

Apply $\sum F_y=0$ to the weight:

$$2T\sin\theta-100\text{ N}=0 \qquad (1)$$

Substitute for T to obtain:

$$2F_d\tan\theta-100\text{ N}=0$$

Solve for θ:

$$\theta=\tan^{-1}\frac{100\text{ N}}{2F_d}$$

Substitute for F_d and evaluate θ:

$$\theta = \tan^{-1}\frac{100\text{ N}}{2(50\text{ N})} = 45°$$

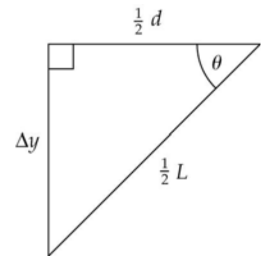
Solve equation (1) for T:

$$T = \frac{100\text{ N}}{2\sin\theta}$$

Substitute for θ and evaluate T:

$$T = \frac{100\text{ N}}{2\sin 45°} = \boxed{70.7\text{ N}}$$

(*b*) Use the diagram to the right to relate the sag Δy in the chain to the angle θ the chain makes with the horizontal:



$$\sin\theta = \frac{\Delta y}{\frac{1}{2}L}$$

where L is the length of the chain.

Solve for Δy:

$$\Delta y = \tfrac{1}{2}L\sin\theta$$

Because the horizontal and vertical forces in the chain are equal, $\theta =$ 45° and:

$$\Delta y = \tfrac{1}{2}(5\text{ m})\sin 45° = \boxed{1.77\text{ m}}$$

(*c*) Relate the distance d of the boat from the dock to the angle θ the chain makes with the horizontal:

$$\cos\theta = \frac{\frac{1}{2}d}{\frac{1}{2}L} = \frac{d}{L}$$

Solve for and evaluate d:

$$d = L\cos\theta = (5\text{ m})\cos 45° = \boxed{3.54\text{ m}}$$

(*d*) Relate the resultant tension in the chain to the vertical component of the tension F_v and the maximum drag force exerted on the boat by the water $F_{d,max}$:

$$F_v^2 + F_{d,max}^2 = (500\text{ N})^2$$

Solve for $F_{d,max}$:

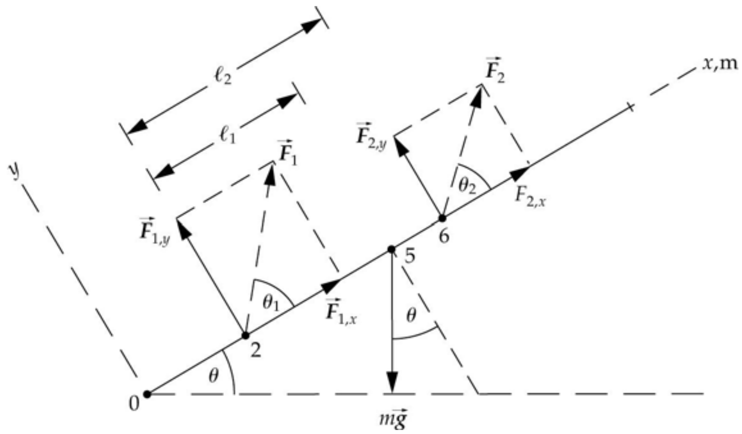
$$F_{d,max} = \sqrt{(500\text{ N})^2 - F_v^2}$$

Because the vertical component of the tension is 50 N:

$$F_{d,max} = \sqrt{(500\text{ N})^2 - (50\text{ N})^2} = \boxed{497\text{ N}}$$

42 ••

Picture the Problem Choose a coordinate system in which the positive x axis is along the rod and the positive y direction is normal to the rod. The rod and the forces acting on it are shown in the free-body diagram. The forces acting at the supports are denoted by the numerals 1 and 2. The resultant forces at the supports are shown as dashed lines. We'll assume that the rod is on the verge of sliding. Because the *x* components of the forces at the supports are friction forces, they are proportional to the normal, i.e., *y*, components of the forces at the supports. Because the rod is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum\vec{\tau}=0$ about an axis through the support at x = 2 m:

$$\ell_2 F_{2,y}-\ell_1 mg\cos\theta=0$$

Solve for $F_{2,y}$:

$$F_{2,y}=\frac{\ell_1 mg\cos\theta}{\ell_2}$$

Substitute numerical values and evaluate $F_{2,y}$:

$$F_{2,y}=\frac{(3\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)\cos 30°}{4\,\text{m}}=127.4\,\text{N}$$

Apply $\sum\vec{\tau}=0$ about an axis through the support at x = 6 m:

$$(\ell_2-\ell_1)mg\cos\theta-\ell_2 F_{1,y}=0$$

Solve for $F_{1,y}$:

$$F_{1,y}=\frac{(\ell_2-\ell_1)mg\cos\theta}{\ell_2}$$

Substitute numerical values and evaluate $F_{1,y}$:

$$F_{1,y}=\frac{(4\,\text{m}-3\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)}{4\,\text{m}}\times\cos 30°=42.48\,\text{N}$$

Apply $\sum F_x = 0$ to the rail: $F_{1,x} + F_{2,x} - mg\sin 30° = 0$ (1)

Assuming that the rod is on the verge of sliding and that the coefficient of static friction is the same for both supports:

$$F_{1,\mathrm{x}} = \mu_{\mathrm{s}} F_{1,y}$$
and
$$F_{2,\mathrm{x}} = \mu_{\mathrm{s}} F_{2,y}$$

Divide the first of these equations by the second and evaluate this ratio to obtain:

$$\frac{F_{1,x}}{F_{2,x}} = \frac{F_{1,y}}{F_{2,y}} = \frac{42.48\,\mathrm{N}}{127.4\,\mathrm{N}} = \frac{1}{3}$$

Solve for $F_{2,x}$:

$$F_{2,x} = 3F_{1,x}$$

Substitute in equation (1):

$$F_{1,x} + 3F_{1,x} - mg\sin\theta = 0$$

Solve for $F_{1,x}$:

$$F_{1,x} = \tfrac{1}{4} mg\sin\theta$$

Substitute numerical values and evaluate $F_{1,x}$:

$$F_{1,x} = \tfrac{1}{4}(20\,\mathrm{kg})(9.81\,\mathrm{m/s^2})\sin 30° = 24.53\,\mathrm{N}$$

Evaluate $F_{2,x}$:

$$F_{2,x} = 3(24.53\,\mathrm{N}) = 73.58\,\mathrm{N}$$

Find the angle θ_1 the force at support 1 (x = 2 m) makes with the rod:

$$\theta_1 = \tan^{-1}\frac{F_{1,y}}{F_{1,x}} = \tan^{-1}\frac{42.48\,\mathrm{N}}{24.53\,\mathrm{N}} = \boxed{60.0°}$$

Find the angle θ_2 the force at support 2 makes with the rod:

$$\theta_2 = \tan^{-1}\frac{F_{2,y}}{F_{2,x}} = \tan^{-1}\frac{127.4\,\mathrm{N}}{73.58\,\mathrm{N}} = \boxed{60.0°}$$

Find the magnitude of $\vec{F}_1$:

$$F_1 = \sqrt{F_{1,x}^2 + F_{1,y}^2} = \sqrt{(24.53\,\mathrm{N})^2 + (42.48\,\mathrm{N})^2} = \boxed{49.1\,\mathrm{N}}$$

Find the magnitude of $\vec{F}_2$:

$$F_2 = \sqrt{F_{2,x}^2 + F_{2,y}^2}$$
$$= \sqrt{(73.58\,\text{N})^2 + (127.4\,\text{N})^2}$$
$$= \boxed{147\,\text{N}}$$

43 •

Picture the Problem The forces shown in the figure constitute a couple and will cause the plate to experience a counterclockwise angular acceleration. We can find this net torque by expressing the torque about either of the corners of the plate.

Sum the torques about an axis through the upper left corner of the plate to obtain:

$$\tau_{\text{net}} = b[(80\,\text{N})\cos 30°] - a[(80\,\text{N})\sin 30°]$$
$$= \boxed{(69.3\,\text{N})b - (40.0\,\text{N})a}$$

44 •

Picture the Problem We can use the condition for translational equilibrium and the definition of a couple to show that the force of static friction exerted by the surface and the applied force constitute a couple. We can use the definition of torque to find the torque exerted by the couple. We can use our result from (*b*) to find the effective point of application of the normal force when $F = Mg/3$ and the condition for rotational equilibrium to find the greatest magnitude of $\vec{F}$ for which the cube will not tip.

(*a*) Apply $\sum \vec{F}_x = 0$ to the stationary cube:

$$\vec{F} + \vec{f}_s = 0$$

$$\boxed{\therefore \vec{F} = -\vec{f}_s \text{ and this pair of equal, parallel, and oppositely directed forces constitute a couple.}}$$

The torque of the couple is:

$$\tau_{\text{couple}} = \boxed{Fa}$$

(*b*) Let x = the distance from the point of application of F_n to the center of the cube. Now, $F_n = Mg$, so applying $\sum \vec{\tau} = 0$ to the cube yields:

$$Mgx - Fa = 0 \qquad (1)$$

or

$$x = \frac{Fa}{Mg}$$

Substitute for $F = Mg/3$ to obtain:

$$x = \frac{\frac{Mg}{3}a}{Mg} = \boxed{\frac{a}{3}}$$

(*c*) Solve equation (1) for F:

$$F = \frac{Mgx}{a}$$

Noting that $x_{max} = a/2$, substitute to express the condition that the cube will tip:

$$F > \frac{Mgx_{max}}{a} = \frac{Mg\frac{a}{2}}{a} = \boxed{\frac{Mg}{2}}$$

45 ••

Picture the Problem We can find the perpendicular distance between the lines of action of the two forces by following the outline given in the problem statement.

Express the vertical components of the forces:

$$F\cos 30° = \frac{\sqrt{3}}{2}F$$

Express the horizontal components of the forces:

$$F\sin 30° = \frac{F}{2}$$

Express the net torque acting on the plate:

$$\tau_{net} = \frac{\sqrt{3}}{2}Fb - \frac{1}{2}Fa = \frac{1}{2}F\left(\sqrt{3}b - a\right)$$

Letting D be the moment arm of the couple, express the net torque acting on the plate:

$$\tau_{net} = FD$$

Equate these two expressions for τ_{net}:

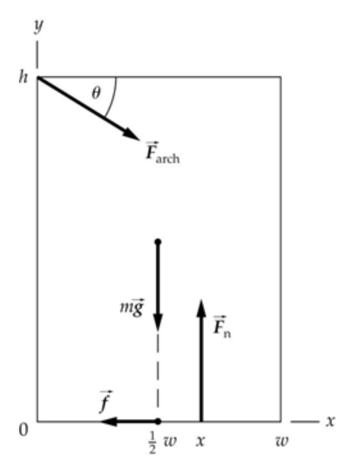
$$FD = \tfrac{1}{2}F\left(\sqrt{3}b - a\right)$$

Solve for D:

$$D = \boxed{\tfrac{1}{2}\left(\sqrt{3}b - a\right)}$$

***46 ••**

Picture the Problem Choose the coordinate system shown in the diagram and let x be the coordinate of the thrust point. The diagram to the right shows the forces acting on the wall. The normal force must balance out the weight of the wall and the vertical component of the thrust from the arch and the frictional force must balance out the horizontal component of the thrust. We can apply the conditions for translational equilibrium to find f and F_n and the condition for rotational equilibrium to find the distance x from the origin of our coordinate system at which F_n acts.



(*a*) Apply the conditions for translational equilibrium to the wall to obtain:

$$\sum F_x = -f + F_{\text{arch}}\cos\theta = 0 \qquad (1)$$

and

$$\sum F_y = F_{\text{n}} - mg - F_{\text{arch}}\sin\theta = 0 \qquad (2)$$

Solve equation (1) for and evaluate *f*:

$$f = F_{\text{arch}}\cos\theta = \left(2\times10^4\ \text{N}\right)\cos 30° = \boxed{17.3\,\text{kN}}$$

Solve equation (2) for F_{n}:

$$F_{\text{n}} = mg + F_{\text{arch}}\sin\theta$$

Substitute numerical values and evaluate F_{n}:

$$F_{\text{n}} = \left(3\times10^4\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right) + \left(2\times10^4\ \text{N}\right)\sin 30° = \boxed{304\,\text{kN}}$$

Apply $\sum \tau_{z\,\text{axis}} = 0$ to the to the wall:

$$xF_{\text{n}} - \tfrac{1}{2}wmg - hF_{\text{arch}}\cos\theta = 0$$

Solve for *x*:

$$x = \frac{\tfrac{1}{2}wmg + hF_{\text{arch}}\cos\theta}{F_{\text{n}}}$$

Substitute numerical values and evaluate *x*:

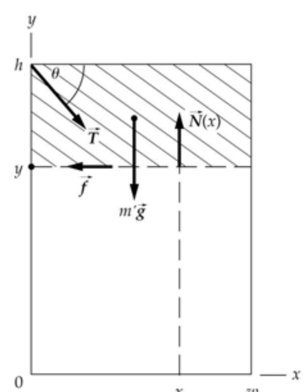
$$x = \frac{\tfrac{1}{2}(1.25\,\text{m})\left(3\times10^4\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right) + (10\,\text{m})\left(2\times10^4\ \text{N}\right)\cos 30°}{304\,\text{kN}} = \boxed{0.570\,\text{m}}$$

(*b*)

If there were no thrust on the side of the wall, the normal force would act through the center of mass, so making the weight larger compared to the thrust must move the point of action of the normal force closer to the center.

47 ••

Picture the Problem Let *h* be the height of the structure, *T* be the thrust, θ the angle from the horizontal of the thrust, $m'g$ the weight of the wall above height *y*, *N*(*x*) the normal force, *f* the friction force the lower part of the wall exerts on the upper part, and *w* the width of the structure. We can apply the conditions for translational and rotational equilibrium to the portion of the wall above the point at which the thrust is applied to obtain two equations that we can solve simultaneously for *x*.



Apply $\sum F_y = 0$ to that fraction of the wall above height *y*:

$$N(x) - T\sin\theta - m'g = 0$$

Assuming the wall is of uniform density, express $m'g$ in terms of *mg*:

$$\frac{m'g}{h-y} = \frac{mg}{h}$$

and

$$m'g = mg\left(1 - \frac{y}{h}\right)$$

Substitute to obtain:

$$N(x) - T\sin\theta - mg\left(1 - \frac{y}{h}\right) = 0$$

Solve for *N*(*x*):

$$N(x) = T\sin\theta + mg\left(1 - \frac{y}{h}\right)$$

Apply $\sum \vec{\tau} = 0$ about an axis through (0,*y*) and perpendicular to the *xy* plane to obtain:

$$xN(x) - (h-y)T\cos\theta - \tfrac{1}{2}mgw\left(1 - \frac{y}{h}\right) = 0$$

Solve for *x* to obtain:

$$x = \frac{\tfrac{1}{2}mgw + hT\cos\theta}{N(x)}\left(1 - \frac{y}{h}\right)$$

Substitute for *N*(*x*) to obtain:

$$x = \frac{\left(\tfrac{1}{2}mgw + hT\cos\theta\right)\left(1 - \frac{y}{h}\right)}{T\sin\theta + mg\left(1 - \frac{y}{h}\right)}$$

Substitute numerical values and simplify to obtain:

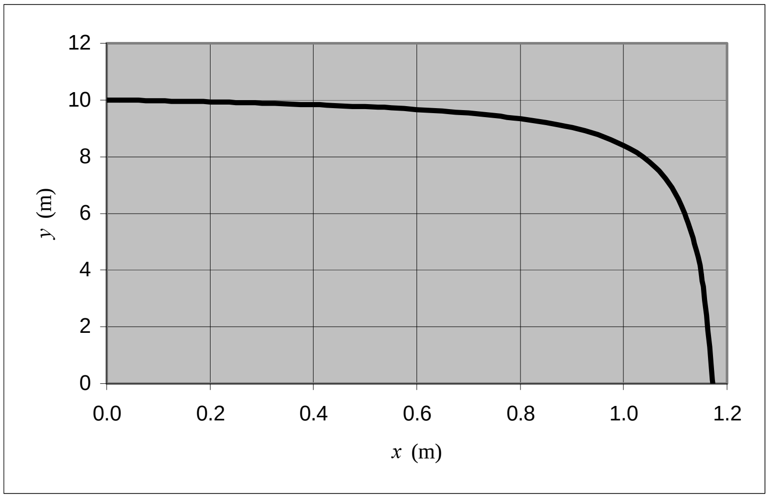
$$x = \frac{\left[\tfrac{1}{2}\left(3\times10^4\text{ kg}\right)\left(9.81\text{m/s}^2\right)\left(1.25\text{m}\right) + \left(10\text{m}\right)\left(2\times10^4\text{ N}\right)\cos 30°\right]\left(1 - \frac{y}{10\text{m}}\right)}{\left(2\times10^4\text{ N}\right)\sin 30° + \left(3\times10^4\text{ kg}\right)\left(9.81\text{m/s}^2\right)\left(1 - \frac{y}{10\text{m}}\right)}$$

$$= \frac{35.71\text{m} - 3.571y}{30.43 - \left(2.943\text{m}^{-1}\right)y}$$

Solve for *y*:

$$y = \boxed{\frac{35.71\text{m} - 30.43x}{3.571 - \left(2.943\text{m}^{-1}\right)x}}$$

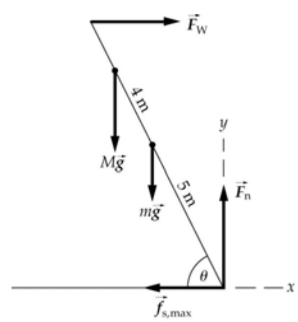
The graph shown below was plotted using a spreadsheet program:



Ladder Problems

***48 ••**

Picture the Problem The ladder and the forces acting on it at the critical moment of slipping are shown in the diagram. Use the coordinate system shown. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Using its definition, express μ_s:

$$\mu_s = \frac{f_{s,max}}{F_n} \qquad (1)$$

Apply $\sum \vec{\tau} = 0$ about the bottom of the ladder:

$$[(9\,\text{m})\cos\theta\,]Mg + [(5\,\text{m})\cos\theta]mg - [(10\,\text{m})\sin\theta]F_W = 0$$

Solve for F_W:

$$F_W = \frac{(9\,\text{m})M + (5\,\text{m})m}{(10\,\text{m})\sin\theta} g\cos\theta$$

Find the angle θ:

$$\theta = \cos^{-1}\frac{2.8\,\text{m}}{10\,\text{m}} = 73.74°$$

Evaluate F_W:

$$F_W = \frac{(9\,\text{m})(70\,\text{kg}) + (5\,\text{m})(22\,\text{kg})}{(10\,\text{m})\sin 73.74°} \times (9.81\,\text{m/s}^2)\cos 73.74° = 211.7\,\text{N}$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0$$

and

$$f_{s,max} = F_W = 211.7\,\text{N}$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - Mg - mg = 0$$

Solve for F_n:

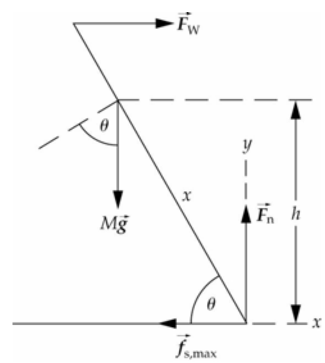
$$F_n = (M + m)g = (70\,\text{kg} + 22\,\text{kg})(9.81\,\text{m/s}^2) = 902.5\,\text{N}$$

Substitute numerical values in equation (1) and evaluate μ_s:

$$\mu_s = \frac{211.7\,\text{N}}{902.5\,\text{N}} = \boxed{0.235}$$

49 ••

Picture the Problem The ladder and the forces acting on it are shown in the diagram. Because the wall is smooth, the force the wall exerts on the ladder must be horizontal. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium to it.



Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - Mg = 0 \Rightarrow F_n = Mg$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0 \Rightarrow f_{s,max} = F_W$$

Apply $\sum \vec{\tau} = 0$ about the bottom of the ladder:

$$Mgx\cos\theta - F_W L\sin\theta = 0$$

Solve for x:

$$x = \frac{F_W L\sin\theta}{Mg\cos\theta} = \frac{f_{s,max}L}{Mg}\tan\theta$$
$$= \frac{\mu_s F_n L}{Mg}\tan\theta = \mu_s L\tan\theta$$

Referring to the figure, relate x to h and solve for h:

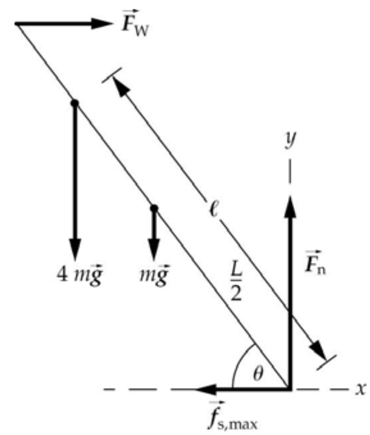
$$\sin\theta = \frac{h}{x}$$

and

$$h = x\sin\theta = \boxed{\mu_s L\tan\theta\sin\theta}$$

50 ••

Picture the Problem The ladder and the forces acting on it are shown in the drawing. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the wall is smooth, the force the wall exerts on the ladder must be horizontal. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - mg - 4mg = 0$$

and

$$F_n = 5mg$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0$$

and

$$f_{s,max} = F_W$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta + 4mg\ell\cos\theta - F_W L\sin\theta = 0$$

Substitute for F_W and then $f_{s,max}$ and solve for ℓ:

$$\ell = \frac{5\mu_s mgL\sin\theta - \frac{1}{2}mgL\cos\theta}{4mg\cos\theta}$$

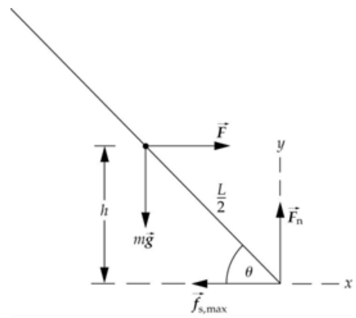
Simplify to obtain:

$$\ell = \left(\frac{5\mu_s}{4}\tan\theta - \frac{1}{8}\right)L = \left(\frac{5(0.45)}{4}\tan 60° - \frac{1}{8}\right)L = \boxed{0.849L}$$

i.e., you can climb about 85% of the way to the top of the ladder.

51 ••

Picture the Problem The ladder and the forces acting on it are shown in the figure. Because the ladder is separating from the wall, the force the wall exerts on the ladder is zero. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



To find the force required to pull the ladder away from the wall, apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta - \frac{L}{2}F\sin\theta = 0$$

or, because $\frac{L}{2}\cos\theta = \frac{h}{\tan\theta}$,

$$\frac{mgh}{\tan\theta} - \frac{L}{2}F\sin\theta = 0$$

Solve for F:

$$F = \frac{2mgh}{L\tan\theta\sin\theta} \quad (1)$$

Apply $\sum F_x = 0$ to the ladder:

$$F - f_{s,max} = 0 \Rightarrow F = f_{s,max} = \mu_s F_n \quad (2)$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Equate equations (1) and (2) and substitute for F_n to obtain:

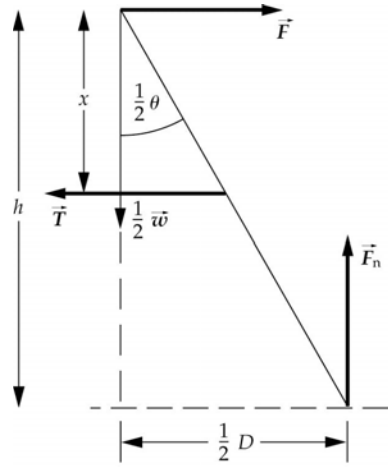
$$\mu_s mg = \frac{2mgh}{L\tan\theta\sin\theta}$$

Solve for μ_s:

$$\mu_s = \boxed{\frac{2h}{L\tan\theta\sin\theta}}$$

52 ••

Picture the Problem Assume that half the man's weight acts on each side of the ladder. The force exerted by the frictionless floor must be vertical. D is the separation between the legs at the bottom and x is the distance of the cross brace from the apex. Because each leg of the ladder is in equilibrium, we can apply the condition for rotational equilibrium the right leg to relate the tension in the cross brace to its distance from the apex.



(*a*) By symmetry, each leg carries half the total weight. So the force on each leg is:

$$\boxed{450\text{ N}}$$

(*b*) Consider one of the ladder's legs and apply $\sum\vec{\tau} = 0$ about the apex:

$$F_n\frac{D}{2} - Tx = 0$$

Solve for T:

$$T = \frac{F_n D}{2x}$$

Using trigonometry, relate h and θ through the tangent function:

$$\tan\tfrac{1}{2}\theta = \frac{D/2}{h}$$

Solve for D to obtain:

$$D = 2h\tan\tfrac{1}{2}\theta$$

Substitute and simplify to obtain:

$$T = \frac{2F_n h\tan\frac{1}{2}\theta}{2x} = \frac{F_n h\tan\frac{1}{2}\theta}{x}$$

Substitute numerical values and evaluate T:

$$T = \frac{F_n h\tan\frac{1}{2}\theta}{x}$$

Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - \tfrac{1}{2}w = 0 \text{ and } F_n = \tfrac{1}{2}w$$

Substitute to obtain:

$$T = \frac{wh\tan\frac{1}{2}\theta}{2x} \qquad (1)$$

Substitute numerical values and evaluate T:

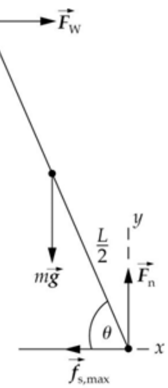
$$T = \frac{(900\,\text{N})(4\,\text{m})\tan 15°}{2(2\,\text{m})} = \boxed{241\,\text{N}}$$

(c) From equation (1) we can see that, if x is increased, i.e., the brace moved lower:

$$\boxed{T \text{ will decrease.}}$$

53 ••

Picture the Problem The figure shows the forces acting on the ladder. Because the wall is frictionless, the force the wall exerts on the ladder is perpendicular to the wall. Because the ladder is on the verge of slipping, the static friction force is $f_{s,max}$. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the ladder:

$$F_W - f_{s,max} = 0 \Rightarrow F_W = f_{s,max} = \mu_s F_n$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta - LF_W \sin\theta = 0$$

Substitute for F_W and F_n and simplify to obtain:

$$\tfrac{1}{2}\cos\theta - \mu_s \sin\theta = 0$$

Solve for and evaluate θ:

$$\theta = \tan^{-1}\frac{1}{2\mu_s} = \tan^{-1}\frac{1}{2(0.3)} = \boxed{59.0°}$$

Stress and Strain

***54** •

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$. We can use Table 12-1 to find the numerical value of Young's modulus for steel.

Find the amount the wire is stretched from Young's modulus:

$$Y = \frac{F/A}{\Delta L/L}$$

Solve for ΔL:

$$\Delta L = \frac{FL}{YA}$$

Substitute for F and A to obtain:

$$\Delta L = \frac{mgL}{Y\pi r^2}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(50\,\text{kg})(9.81\,\text{m/s}^2)(5\,\text{m})}{2\pi\times10^{11}\,\text{N/m}^2(2\times10^{-3}\,\text{m})^2} = \boxed{0.976\,\text{mm}}$$

55 •

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = \text{stress/strain} = (F/A)/(\Delta L/L)$.

(*a*) Express the maximum load in terms of the wire's breaking stress:

$$F_{\text{max}} = \text{breaking stress} \times A = \text{breaking stress} \times \pi r^2$$

Substitute numerical values and evaluate F_{max}:

$$F_{\text{max}} = (3\times10^8\,\text{N/m}^2)\pi(0.21\times10^{-3}\,\text{m})^2 = \boxed{41.6\,\text{N}}$$

(*b*) Using the definition of Young's modulus, express the fractional change in length of the copper wire:

$$\Delta L/L = \frac{F/A}{Y} = \frac{1.5\times10^8\,\text{N/m}^2}{1.1\times10^{11}\,\text{N/m}^2} = 1.36\times10^{-3} = \boxed{0.136\%}$$

56 •

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$. We can use Table 12-1 to find the numerical value of Young's modulus for steel.

Find the amount the wire is stretched from Young's modulus:

$$Y = \frac{F/A}{\Delta L/L}$$

Solve for ΔL:

$$\Delta L = \frac{FL}{YA}$$

Substitute for F and A to obtain:

$$\Delta L = \frac{mgL}{Y\pi r^2}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(4\,\text{kg})(9.81\,\text{m/s}^2)(1.2\,\text{m})}{2\pi \times 10^{11}\,\text{N/m}^2\,(0.3\times 10^{-3}\,\text{m})^2} = \boxed{0.833\,\text{mm}}$$

***57** •

Picture the Problem The shear stress, defined as the ratio of the shearing force to the area over which it is applied, is related to the shear strain through the definition of the shear modulus; $M_s = \dfrac{\text{shear stress}}{\text{shear strain}} = \dfrac{F_s/A}{\tan\theta}$.

Using the definition of shear modulus, relate the angle of shear, θ to the shear force and shear modulus:

$$\tan\theta = \frac{F_s}{M_s A}$$

Solve for θ:

$$\theta = \tan^{-1}\frac{F_s}{M_s A}$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\frac{25\,\text{N}}{(1.9\times 10^5\,\text{N/m}^2)(15\times 10^{-4}\,\text{m}^2)} = \boxed{5.01°}$$

58 ••

Picture the Problem The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$, where L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. For a composite wire, the length under stress is the unstressed length plus the sum of the elongations of the components of the wire.

Express the length of the composite wire when it is supporting a mass of 5 kg:

$$L = 3.00\,\text{m} + \Delta L \qquad (1)$$

Express the change in length of the composite wire:

$$\Delta L = \Delta L_{\text{steel}} + \Delta L_{\text{Al}} = \frac{F}{A}\frac{L_{\text{steel}}}{Y_{\text{steel}}} + \frac{F}{A}\frac{L_{\text{Al}}}{Y_{\text{Al}}} = \frac{F}{A}\left(\frac{L_{\text{steel}}}{Y_{\text{steel}}} + \frac{L_{\text{Al}}}{Y_{\text{Al}}}\right)$$

Find the stress in each wire:

$$\frac{F}{A}=\frac{(5\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(0.5\times10^{-3}\,\text{m})^2}=6.245\times10^{7}\ \text{N/m}^2$$

Substitute numerical values and evaluate ΔL:

$$\Delta L=(6.245\times10^{7}\ \text{N/m}^2)\left(\frac{1.5\,\text{m}}{2\times10^{11}\ \text{N/m}^2}+\frac{1.5\,\text{m}}{0.7\times10^{11}\ \text{N/m}^2}\right)=1.81\times10^{-3}\ \text{m}$$

Substitute in equation (1) and evaluate L:

$$L=3.00\,\text{m}+1.81\times10^{-3}\ \text{m}=\boxed{3.0018\,\text{m}}$$

59 ••

Picture the Problem We can use Hooke's law and Young's modulus to show that, if the wire is considered to be a spring, the force constant k is given by $k = AY/L$. By treating the wire as a spring we can show the energy stored in the wire is $U = ½F\Delta L$.

Express the relationship between the stretching force, the stiffness constant, and the elongation of a spring:

$$F=k\Delta L$$

or

$$k=\frac{F}{\Delta L}$$

Using the definition of Young's modulus, express the ratio of the stretching force to the elongation of the wire:

$$\frac{F}{\Delta L}=\frac{AY}{L}\qquad(1)$$

Equate these two expressions for $F/\Delta L$ to obtain:

$$k=\boxed{\frac{AY}{L}}$$

Treating the wire as a spring, express its stored energy:

$$U=\tfrac{1}{2}k(\Delta L)^2=\tfrac{1}{2}\frac{AY}{L}(\Delta L)^2$$

Solve equation (1) for F:

$$F=\frac{AY\Delta L}{L}$$

Substitute in our expression for U to obtain:

$$U=\tfrac{1}{2}\frac{AY\Delta L}{L}\Delta L=\boxed{\tfrac{1}{2}F\Delta L}$$

60 ••

Picture the Problem Let L' represent the stretched and L the unstretched length of the wire. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$, where F is the force acting on it, and A is its cross-sectional area. In problem 58 we showed that the energy stored in the wire is $U = \frac{1}{2}F\Delta L$, where Y is Young's modulus and ΔL is the amount the wire has stretched.

(*a*) Express the stretched length of the wire:

$$L' = L + \Delta L$$

Using the definition of Young's modulus, express ΔL:

$$\Delta L = \frac{LF}{AY}$$

Substitute and simplify:

$$L' = L + \frac{LF}{AY} = L\left(1 + \frac{F}{AY}\right)$$

Solve for L:

$$L = \frac{L'}{1 + \frac{F}{AY}}$$

Substitute numerical values and evaluate L:

$$L = \frac{0.35\,\text{m}}{1 + \frac{53\,\text{N}}{\pi\left(0.1\times10^{-3}\,\text{m}\right)^2\left(2\times10^{11}\,\text{N/m}^2\right)}} = \boxed{0.347\,\text{m}}$$

(*b*) Using the expression from Problem 59, express the work done in stretching the wire:

$$W = \Delta U = \tfrac{1}{2}F\Delta L = \tfrac{1}{2}(53\,\text{N})(0.35\,\text{m} - 0.347\,\text{m}) = \boxed{0.0795\,\text{J}}$$

***61** ••

Picture the Problem The table to the right summarizes the ratios $\Delta L/F$ for the student's data. Note that this ratio is constant, to three significant figures, for loads less than or equal to 200 g. We can use this ratio to calculate Young's modulus for the rubber strip.

Load (g)	F (N)	ΔL (m)	$\Delta L/F$ (m/N
100	0.981	0.006	6.12×10^{-3}
200	1.962	0.012	6.12×10^{-3}
300	2.943	0.019	6.46×10^{-3}
400	3.924	0.028	7.14×10^{-3}
500	4.905	0.05	10.2×10^{-3}

(*a*) Referring to the table, we see that for loads ≤ 200 g:

$$\frac{\Delta L}{F}=6.12\times10^{-3}\ \text{m/N}$$

Use the definition of Young's modulus to express *Y*:

$$Y=\frac{FL}{A\Delta L}=\frac{L}{A\frac{\Delta L}{F}}$$

Substitute numerical values and evaluate *Y*:

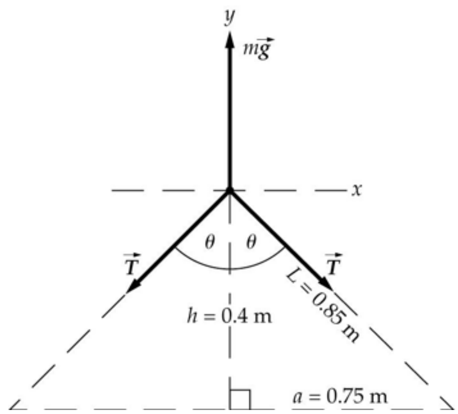
$$Y=\frac{5\times10^{-2}\ \text{m}}{\left(3\times10^{-3}\ \text{m}\right)\left(1.5\times10^{-3}\ \text{m}\right)\left(6.12\times10^{-3}\ \text{m/N}\right)}=\boxed{1.82\times10^{6}\ \text{N/m}^2}$$

(*b*) Interpolate to determine the stretch when the load is 150 g, and use the expression from Problem 58, to express the energy stored in the strip:

$$U=\tfrac{1}{2}F\Delta L$$
$$=\tfrac{1}{2}(0.15\,\text{kg})\left(9.81\,\text{m/s}^2\right)\left(9\times10^{-3}\ \text{m}\right)$$
$$=\boxed{6.62\,\text{mJ}}$$

62 ••

Picture the Problem The figure shows the forces acting on the wire where it passes over the nail. *m* represents the mass of the mirror and *T* is the tension in the supporting wires. The figure also shows the geometry of the right triangle defined by the support wires and the top of the mirror frame. The distance *a* is fixed by the geometry while *h* and *L* will change as the mirror is suspended from the nail.



Express the distance between the nail and the top of the frame when the wire is under tension:

$$h'=h+\Delta h$$
$$=0.4\,\text{m}+\Delta h \qquad (1)$$

Apply $\sum F_y=0$ to the wire where it passes over the supporting nail:

$$mg-2T\cos\theta=0$$

Solve for the tension in the wire:

$$T=\frac{mg}{2\cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(2.4\,\text{kg})(9.81\,\text{m/s}^2)}{2\left(\dfrac{0.4\,\text{m}}{0.85\,\text{m}}\right)} = 25.0\,\text{N}$$

Using its definition, find the stress in the wire:

$$\text{stress} = \frac{T}{A} = \frac{25.0\,\text{N}}{\pi(0.1\times10^{-3}\,\text{m})^2} = 7.96\times10^{8}\ \text{N/m}^2$$

Using the definition of Young's modulus, find the strain in the hypotenuse of the right triangle shown in the figure:

$$\text{strain} = \frac{\Delta L}{L} = \frac{\text{stress}}{Y} = \frac{7.96\times10^{8}\ \text{N/m}^2}{2\times10^{11}\ \text{N/m}^2} = 3.98\times10^{-3}$$

Using the Pythagorean theorem, express the relationship between the sides of the right triangle in the figure:

$$a^2 + h^2 = L^2$$

Express the differential of this equation:

$$2a\Delta a + 2h\Delta h = 2L\Delta L$$

or, because $\Delta a = 0$,

$$h\Delta h = L\Delta L$$

Solve for and evaluate Δh:

$$\Delta h = \frac{L\Delta L}{h} = \frac{L^2}{h}\cdot\frac{\Delta L}{L}$$

Substitute numerical values and evaluate Δh:

$$\Delta h = \frac{(0.85\,\text{m})^2}{0.4\,\text{m}}(3.98\times10^{-3}) = 7.19\,\text{mm}$$

Substitute in equation (1) to obtain:

$$h' = 0.4\,\text{m} + 7.19\,\text{mm} = \boxed{40.72\,\text{cm}}$$

63 ••

Picture the Problem Let the numeral 1 denote the aluminum wire and the numeral 2 the steel wire. Because their initial lengths and amount they stretch are the same, we can use the definition of Young's modulus to express the change in the lengths of each wire and then equate these expressions to obtain an equation solvable for the ratio M_1/M_2.

Using the definition of Young's modulus, express the change in

$$\Delta L_1 = \frac{M_1 g L_1}{A_1 Y_{\text{Al}}}$$

length of the aluminum wire:

Using the definition of Young's modulus, express the change in length of the steel wire:

$$\Delta L_2 = \frac{M_2 g L_2}{A_2 Y_{\text{steel}}}$$

Because the two wires stretch by the same amount, equate ΔL_1 and ΔL_2 and simplify:

$$\frac{M_1}{A_1 Y_{\text{Al}}} = \frac{M_2}{A_2 Y_{\text{steel}}}$$

Solve for the ratio M_1/M_2:

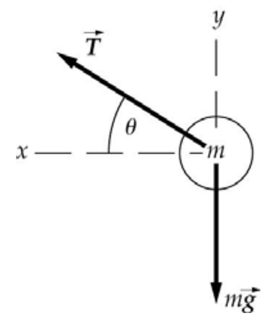
$$\frac{M_1}{M_2} = \frac{A_1 Y_{\text{Al}}}{A_2 Y_{\text{steel}}}$$

Substitute numerical values and evaluate M_1/M_2:

$$\frac{M_1}{M_2} = \frac{\frac{\pi}{4}(0.7\,\text{mm})^2(0.7\times10^{11}\ \text{N/m}^2)}{\frac{\pi}{4}(0.5\,\text{mm})^2(2\times10^{11}\ \text{N/m}^2)} = \frac{(0.7\,\text{mm})^2(0.7\times10^{11}\ \text{N/m}^2)}{(0.5\,\text{mm})^2(2\times10^{11}\ \text{N/m}^2)} = \boxed{0.686}$$

64 ••

Picture the Problem The free-body diagram shows the forces acting on the ball as it rotates around the post in a horizontal plane. We can apply Newton's 2nd law to find the tension in the wire and use the definition of Young's modulus to find the amount by which the aluminum wire stretches.



Express the length of the wire under tension to its unstretched length:

$$L = L_0 + \Delta L = 0.7\,\text{m} + \Delta L \qquad (1)$$

Apply $\sum F_y = 0$ to the ball:

$$T\sin\theta - mg = 0$$

Solve for the tension in the wire:

$$T = \frac{mg}{\sin\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(0.5\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 5°} = \boxed{56.3\,\text{N}}$$

Using the definition of Young's modulus, express ΔL:

$$\Delta L = \frac{FL}{AY}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(56.3\,\text{N})(0.7\,\text{m})}{\frac{\pi}{4}(1.6\times10^{-3}\,\text{m})^2(0.7\times10^{11}\,\text{N/m}^2)} = 0.280\,\text{mm}$$

Substitute in equation (1) to obtain:

$$L = 0.7\,\text{m} + 0.280\,\text{mm} = \boxed{70.03\,\text{cm}}$$

***65 ••**

Picture the Problem We can use the definition of stress to calculate the failing stress of the cable and the stress on the elevator cable. Note that the failing stress of the composite cable is the same as the failing stress of the test sample.

Express the stress on the elevator cable:

$$\text{Stress}_{\text{cable}} = \frac{F}{A} = \frac{20\,\text{kN}}{1.2\times10^{-6}\,\text{m}^2} = 1.67\times10^{10}\,\text{N/m}^2$$

Express the failing stress of the sample:

$$\text{Stress}_{\text{failing}} = \frac{F}{A} = \frac{1\text{kN}}{0.2\times10^{-6}\,\text{m}^2} = 0.500\times10^{10}\,\text{N/m}^2$$

$$\boxed{\text{Because Stress}_{\text{failing}} < \text{Stress}_{\text{cable}}\text{, it will not support the elevator.}}$$

***66 •••**

Picture the Problem Let the length of the sides of the rectangle be x, y and z. Then the volume of the rectangle will be $V = xyz$ and we can express the new volume V' resulting from the pulling in the x direction and the change in volume ΔV in terms of Δx, Δy, and Δz. Discarding the higher order terms in ΔV and dividing our equation by V and using the given condition that $\Delta y/y = \Delta z/z$ will lead us to the given expression for $\Delta y/y$.

Express the new volume of the rectangular box when its sides change in length by Δx, Δy, and Δz:

$$V' = (x+\Delta x)(y+\Delta y)(z+\Delta z) = xyz + \Delta x(yz) + \Delta y(xz) + \Delta z(xy) + \{z\Delta x\Delta y + y\Delta x\Delta z + x\Delta y\Delta z + \Delta x\Delta y\Delta z\}$$

where the terms in brackets are very small (i.e., second order or higher).

Discard the second order and higher terms to obtain:

$$V' = V + \Delta x(yz) + \Delta y(xz) + \Delta z(xy)$$

or

$$\Delta V = V' - V = \Delta x(yz) + \Delta y(xz) + \Delta z(xy)$$

Because $\Delta V = 0$:

$$\Delta x(yz) = -[\Delta y(xz) + \Delta z(xy)]$$

Divide both sides of this equation by $V = xyz$ to obtain:

$$\frac{\Delta x}{x} = -\left[\frac{\Delta y}{y} + \frac{\Delta z}{z}\right]$$

Because $\Delta y/y = \Delta z/z$, our equation becomes:

$$\frac{\Delta x}{x} = -2\frac{\Delta y}{y} \text{ or } \frac{\Delta y}{y} = \boxed{-\frac{1}{2}\frac{\Delta x}{x}}$$

67 ••

Picture the Problem We can evaluate the differential of the volume of the wire and, using the assumptions that the volume of the wire does not change under stretching and that the change in its length is small compared to its length, show that $\Delta r/r = -(1/2)\,\Delta L/L$.

Express the volume of the wire:

$$V = \pi r^2 L$$

Evaluate the differential of V to obtain:

$$dV = \pi r^2 dL + 2\pi rLdr$$

Because $dV = 0$:

$$0 = rdL + 2Ldr \Rightarrow \frac{dr}{r} = -\frac{1}{2}\frac{dL}{L}$$

Because $\Delta L << L$, we can approximate the differential changes dr and dL with small changes Δr and ΔL to obtain:

$$\frac{\Delta r}{r} = \boxed{-\frac{1}{2}\frac{\Delta L}{L}}$$

***68** •••

Picture the Problem Because the volume of the thread remains constant during the stretching process, we can equate the initial and final volumes to express r_0 in terms of r. We can also use Young's modulus to express the tension needed to break the thread in terms of Y and r_0.

(*a*) Express the conservation of volume during the stretching of the spider's silk:

$$\pi r^2 L = \pi r_0^2 L_0$$

Solve for r:

$$r = r_0\sqrt{\frac{L_0}{L}}$$

Substitute for L to obtain:

$$r = r_0\sqrt{\frac{L_0}{10L_0}} = \boxed{0.316 r_0}$$

(*b*) Express Young's modulus in terms of the breaking tension T:

$$Y = \frac{T/A}{\Delta L/L} = \frac{T/\pi r^2}{\Delta L/L} = \frac{10T/\pi r_0^2}{\Delta L/L}$$

Solve for T to obtain:

$$T = \frac{1}{10}\pi r_0^2 Y \frac{\Delta L}{L}$$

Because $\Delta L/L = 9$:

$$T = \boxed{\frac{9\pi r_0^2 Y}{10}}$$

General Problems

69 •

Picture the Problem Because the board is in equilibrium, we can apply the conditions for translational and rotational equilibrium to determine the forces exerted by the supports.

Apply $\sum_i \vec{\tau}_i = 0$ about the right support:

$$(2\,\text{m})(360\,\text{N}) + (5\,\text{m})(90\,\text{N}) - (10\,\text{m})F_\text{L} = 0$$

Solve for and evaluate F_L:

$$F_\text{L} = \frac{(2\,\text{m})(360\,\text{N}) + (5\,\text{m})(90\,\text{N})}{10\,\text{m}} = \boxed{117\,\text{N}}$$

Apply $\sum F_y = 0$ to the board:

$$F_\text{L} + F_\text{R} - 90\,\text{N} - 360\,\text{N} = 0$$

Solve for and evaluate F_R:

$$F_\text{R} = -F_\text{L} + 90\,\text{N} + 360\,\text{N} = -117\,\text{N} + 90\,\text{N} + 360\,\text{N} = \boxed{333\,\text{N}}$$

Remarks: **We could just as easily found F_R by applying $\sum \vec{\tau} = 0$ about the left support.**

70 •

Picture the Problem Because the man-and-board system is in equilibrium, we can apply the conditions for translational and rotational equilibrium to determine the forces exerted by the supports. Let d represent the distance from the man's feet to his center of gravity.

Apply $\sum \vec{\tau} = 0$ about an axis through the man's feet and perpendicular to the page:

$$(845\,\text{N})d - (1.88\,\text{m})(445\,\text{N}) = 0$$

Solve for and evaluate d:

$$d = \frac{(1.88\,\text{m})(445\,\text{N})}{845\,\text{N}} = 0.990\,\text{m}$$

$$= \boxed{99.0\,\text{cm}}$$

No. Holding his head slightly above the board would not change the location of his center of mass and so the scale readings would not change.

***71** •

Picture the Problem We can apply the balance condition $\sum \vec{\tau} = 0$ successively, starting with the lowest part of the mobile, to find the value of each of the unknown weights.

Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension of the lowest part of the mobile:

$$(3\,\text{cm})(2\,\text{N}) - (4\,\text{cm})w_1 = 0$$

Solve for and evaluate w_1:

$$w_1 = \frac{(3\,\text{cm})(2\,\text{N})}{4\,\text{cm}} = \boxed{1.50\,\text{N}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension of the middle part of the mobile:

$$(2\,\text{cm})w_2 - (4\,\text{cm})(2\,\text{N} + 1.5\,\text{N}) = 0$$

Solve for and evaluate w_2:

$$w_2 = \frac{(4\,\text{cm})(2\,\text{N} + 1.5\,\text{N})}{2\,\text{cm}} = \boxed{7.00\,\text{N}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension of the top part of the mobile:

$$(2\,\text{cm})(10.5\,\text{N}) - (6\,\text{cm})w_3 = 0$$

Solve for and evaluate w_3:

$$w_3 = \frac{(2\,\text{cm})(10.5\,\text{N})}{6\,\text{cm}} = \boxed{3.50\,\text{N}}$$

72 •

Picture the Problem We can determine the ratio of L to h by noting the number of ropes supporting the load whose mass is M.

(*a*) Noting that three ropes support the pulley to which the object whose mass is M is fastened we can

$$\frac{L}{h} = \boxed{3}$$

conclude that:

(*b*) Apply the work-energy principle to the block-tackle object to obtain:

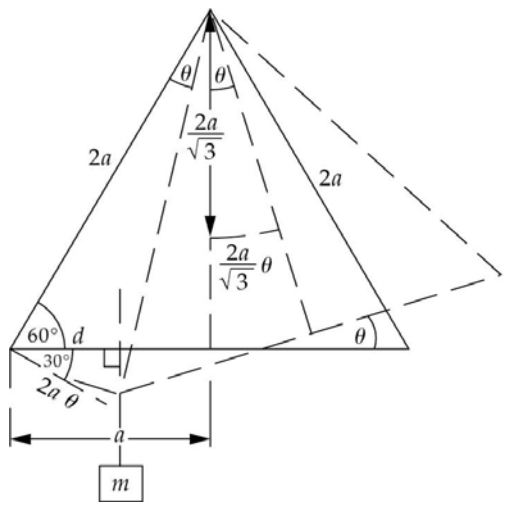
$$W_{ext} = \Delta E_{system} = \Delta U_{block\text{-}tackle}$$

or

$$\boxed{FL = Mgh}$$

73 ••

Picture the Problem The figure shows the equilateral triangle without the mass *m*, and then the same triangle with the mass *m* and rotated through an angle θ. Let the side length of the triangle to be $2a$. Then the center of mass of the triangle is at a distance of $\dfrac{2a}{\sqrt{3}}$ from each vertex. As the triangle rotates, its center of mass shifts by $\dfrac{2a}{\sqrt{3}}\theta$, for $\theta << 1$. Also, the vertex to which *m* is attached moves toward the plumb line by the distance $d = 2a\theta\cos 30° = \sqrt{3}a\theta$ (see the drawing).



Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension:

$$mg\left(a - \sqrt{3}a\theta\right) - Mg\frac{2a}{\sqrt{3}}\theta = 0$$

Solve for m/M:

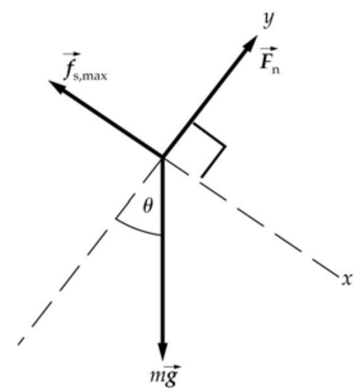
$$\frac{m}{M} = \frac{2\theta}{\sqrt{3}\left(1 - \sqrt{3}\theta\right)}$$

Substitute numerical values and evaluate m/M:

$$\frac{m}{M} = \frac{2(6°)\left(\dfrac{\pi\text{ rad}}{180°}\right)}{\sqrt{3}\left[1 - \sqrt{3}(6°)\left(\dfrac{\pi\text{ rad}}{180°}\right)\right]} = \boxed{0.148}$$

74 ••

Picture the Problem If the hexagon is to roll rather than slide, the incline's angle must be such that the center of mass falls just beyond the support base. From the geometry of the hexagon, one can see that the critical angle is 30°. The free-body diagram shows the forces acting on the hexagonal pencil when it is on the verge of sliding. We can use Newton's 2nd law to relate the coefficient of static friction to the angle of the incline for which rolling rather than sliding occurs.



Apply $\sum \vec{F} = 0$ to the pencil:

$$\sum F_x = mg\sin\theta - f_{s,max} = 0 \quad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

Substitute $f_{s,max} = \mu_s F_n$ in equation (1):

$$mg\sin\theta - \mu_s F_n = 0 \quad (3)$$

Divide equation (3) by equation (2) to obtain:

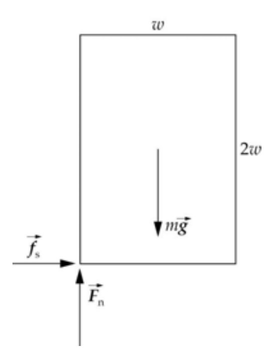
$$\tan\theta = \mu_s$$

Thus, if the pencil is to roll rather than slide when the pad is inclined:

$$\mu_s \geq \tan 30° = \boxed{0.577}$$

75 ••

Picture the Problem The box and the forces acting on it are shown in the figure. When the box is about to tip, F_n acts at its edge, as indicated in the drawing. We can use the definition of μ_s and apply the condition for rotational equilibrium in an accelerated frame to relate f_s to the weight of the box and, hence, to the normal force.



Using its definition, express μ_s:

$$\mu_s \geq \frac{f_s}{F_n}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the box's center of mass:

$$wf_s - \tfrac{1}{2} wF_n = 0$$

Solve for the ratio $\frac{f_s}{F_n}$:

$$\frac{f_s}{F_n} = \frac{1}{2}$$

Substitute to obtain the condition for tipping:

$$\mu_s \geq 0.500$$

Therefore, if the box is to slide:

$$\mu_s < \boxed{0.500}$$

76 ••

Picture the Problem Because the balance is in equilibrium, we can use the condition for rotational equilibrium to relate the masses of the blocks to the lever arms of the balance in the two configurations described in the problem statement.

Apply $\sum \vec{\tau} = 0$ about an axis through the fulcrum:

$$(1.5\,\text{kg})L_1 = (1.95\,\text{kg})L_2$$

Solve for the ratio L_1/L_2:

$$\frac{L_1}{L_2} = \frac{1.95\,\text{kg}}{1.5\,\text{kg}} = 1.30$$

Apply $\sum \vec{\tau} = 0$ about an axis through the fulcrum with 1.5 kg at L_2:

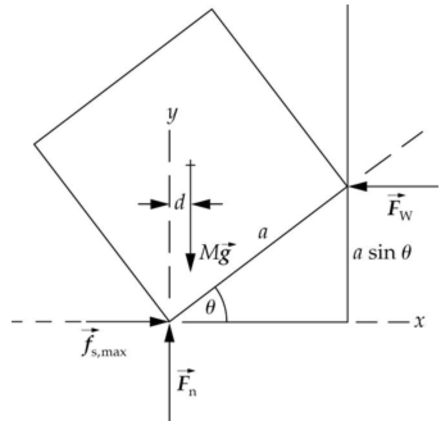
$$(1.5\,\text{kg})L_2 = ML_1$$

Solve for and evaluate M:

$$M = \frac{(1.5\,\text{kg})L_2}{L_1} = \frac{1.5\,\text{kg}}{L_1/L_2} = \frac{1.5\,\text{kg}}{1.30} = \boxed{1.15\,\text{kg}}$$

***77 ••**

Picture the Problem The figure shows the location of the cube's center of mass and the forces acting on the cube. The opposing couple is formed by the friction force $f_{s,max}$ and the force exerted by the wall. Because the cube is in equilibrium, we can use the condition for translational equilibrium to establish that $f_{s,max} = F_W$ and $F_n = Mg$ and the condition for rotational equilibrium to relate the opposing couples.



Apply $\sum \vec{F} = 0$ to the cube:

$$\sum F_y = F_n - Mg = 0 \Rightarrow F_n = Mg$$

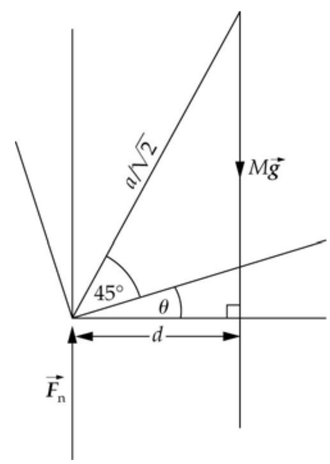
and

$$\sum F_x = f_s - F_W = 0 \Rightarrow F_W = f_s$$

Noting that $\vec{f}_{s,max}$ and $\vec{F}_W$ form a couple, as do $\vec{F}_n$ and $M\vec{g}$, apply $\sum \vec{\tau} = 0$ about an axis though the center of mass of the cube:

$$f_{s,max} a \sin\theta - Mgd = 0$$

Referring to the diagram to the right, note that $d = \dfrac{a}{\sqrt{2}} \sin(45° + \theta)$.



Substitute for d and $f_{s,max}$ to obtain:

$$\mu_s Mga \sin\theta - Mg \frac{a}{\sqrt{2}} \sin(45° + \theta) = 0$$

or

$$\mu_s \sin\theta - \frac{1}{\sqrt{2}} \sin(45° + \theta) = 0$$

Solve for μ_s and simplify to obtain:

$$\mu_s = \frac{1}{\sqrt{2}\sin\theta}\sin(45° + \theta) = \frac{1}{\sqrt{2}\sin\theta}(\sin 45° \cos\theta + \cos 45° \sin\theta)$$

$$= \frac{1}{\sqrt{2}\sin\theta}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right) = \boxed{\frac{1}{2}(\cot\theta + 1)}$$

78 ••

Picture the Problem Because the meter stick is in equilibrium, we can apply the condition for rotational equilibrium to find the maximum distance from the hinge at which the block can be suspended.

Apply $\sum \vec{\tau} = 0$ about an axis through the hinge to obtain:

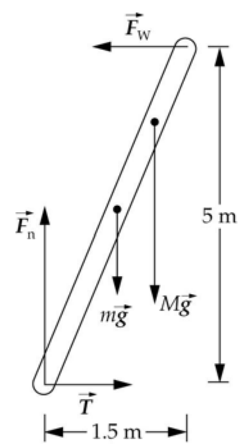
$$(1\,\text{m})(75\,\text{N}) - (0.5\,\text{m})(5\,\text{kg})(9.81\,\text{m/s}^2)\cos 45° - d(10\,\text{kg})(9.81\,\text{m/s}^2)\cos 45° = 0$$

Solve for and evaluate d:

$$d = \frac{(1\,\text{m})(75\,\text{N}) - (0.5\,\text{m})(5\,\text{kg})(9.81\,\text{m/s}^2)}{(10\,\text{kg})(9.81\,\text{m/s}^2)\cos 45°}\cos 45° = \boxed{0.831\,\text{m}}$$

79 ••

Picture the Problem Let m represent the mass of the ladder and M the mass of the person. The force diagram shows the forces acting on the ladder for part (b). From the condition for translational equilibrium, we can conclude that $T = F_W$, a result we'll need in part (b). Because the ladder is also in rotational equilibrium, summing the torques about the bottom of the ladder will eliminate both F_n and T.



(a) Apply $\sum_i \vec{\tau}_i = 0$ about an axis through the bottom of the ladder:

$$(5\,\text{m})F_W - (0.75\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2) - (0.75\,\text{m})(80\,\text{kg})(9.81\,\text{m/s}^2) = 0$$

Solve for and evaluate F_W:

$$F_W = \frac{(0.75\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)}{5\,\text{m}} + \frac{(0.75\,\text{m})(80\,\text{kg})(9.81\,\text{m/s}^2)}{5\,\text{m}} = \boxed{147\,\text{N}}$$

(*b*) Solve for and evaluate f:

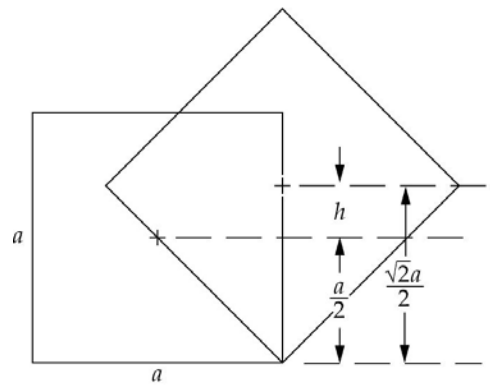
$$f = \frac{(5\,\text{m})(200\,\text{N}) - (0.75\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)}{(1.5\,\text{m})(80\,\text{kg})(9.81\,\text{m/s}^2)} = 0.724$$

Find the distance the 80-kg person can climb the ladder:

$$d = f(5\,\text{m}) = (0.724)(5\,\text{m}) = \boxed{3.62\,\text{m}}$$

***80 ••**

Picture the Problem To ″roll″ the cube one must raise its center of mass from $y = a/2$ to $y = \sqrt{2}a/2$, where a is the cube length. During this process the work done is the change in the gravitational potential energy of the cube. No additional work is done on the cube as it ″flops″ down. We can also use the definition of work to express the work done in sliding the cube a distance a along a horizontal surface and then equate the two expressions to determine μ_k.



Express the work done in moving the cube a distance a by raising its center of mass from $y = a/2$ to $y = \sqrt{2}a/2$ and then letting the cube flop down:

$$W = mg\left(\frac{\sqrt{2}a}{2} - \frac{a}{2}\right) = \frac{mga}{2}\left(\sqrt{2} - 1\right)$$
$$= 0.207mga$$

Letting f_k represent the kinetic friction force, express the work done in dragging the cube a distance a along the surface at constant speed:

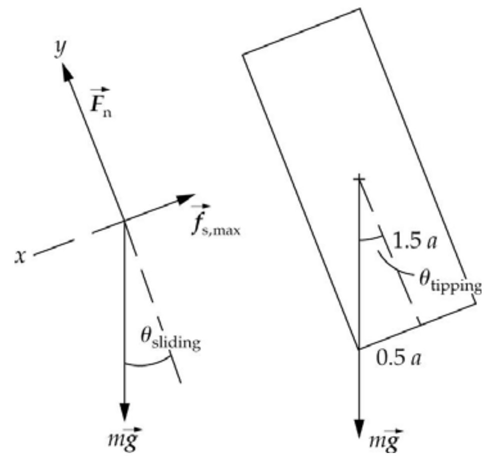
$$W = f_k a = \mu_k mga$$

Equate these two expressions to obtain:

$$\mu_k = \boxed{0.207}$$

81 ••

Picture the Problem The free-body diagram shows the forces acting on the block when it is on the verge of sliding. Because the block is in equilibrium, we can use the conditions for translational equilibrium to determine the minimum angle for which the block will slide. The diagram to the right of the FBD shows that the condition for tipping is that the plumb line from the center of mass pass outside of the base. We can determine the tipping angle from the geometry of the block under this condition.



Apply $\sum \vec{F} = 0$ to the block:

$$\sum F_x = mg\sin\theta_{sliding} - f_{s,max} \geq 0$$

if the block is to slide, and

$$\sum F_y = F_n - mg\cos\theta_{sliding} = 0$$

Substitute for $f_{s,max}$ and eliminate F_n between these equations to obtain:

$$\mu_s \leq \tan\theta_{sliding}$$

Solve for the condition for sliding:

$$\theta_{sliding} \geq \tan^{-1}(\mu_s) = \tan^{-1}(0.4) = 21.8°$$

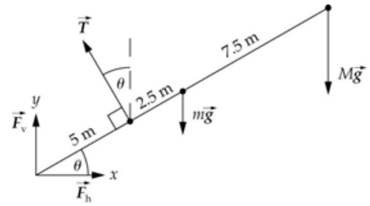
Using the geometry of the block, express the condition on θ that must be satisfied if the block is to tip:

$$\theta_{tipping} \geq \tan^{-1}\left(\frac{0.5a}{1.5a}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.4°$$

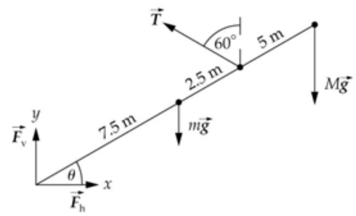
$$\boxed{\text{Because } \theta_{tipping} < \theta_{sliding}\text{, the block will tip before it slides.}}$$

82 ••

Picture the Problem Let m represent the mass of the bar, M the mass of the suspended object, F_v the vertical component of the force the wall exerts on the bar, F_h the horizontal component of the force exerted the wall exerts on the bar, and T the tension in the cable. The free-body diagrams show these forces and their points of application on the bar for parts (a) and (b) of the problem. Because the bar is in equilibrium, we can apply the conditions for translational and rotational equilibrium to relate the various forces and distances.



(a)



(b)

(a) Apply $\sum_i \vec{\tau}_i = 0$ about an axis through the hinge:

$$(5\,\text{m})T - (7.5\,\text{m})mg\cos 30° - (15\,\text{m})Mg\cos 30° = 0$$

Solve for T:

$$T = \frac{[(7.5\,\text{m})m + (15\,\text{m})M]g\cos 30°}{5\,\text{m}}$$

Substitute numerical values and evaluate T:

$$T = \frac{(7.5\,\text{m})(85\,\text{kg}) + (15\,\text{m})(360\,\text{kg})}{5\,\text{m}} \times (9.81\,\text{m/s}^2)\cos 30° = \boxed{10.3\,\text{kN}}$$

Apply $\sum_i \vec{F}_i = 0$ to the bar:

$$\sum F_y = F_\text{v} + T\sin 60° - mg - Mg = 0$$

and

$$\sum F_x = F_\text{h} - T\cos 60° = 0$$

Solve the y equation for F_v:

$$F_\text{v} = -T\sin 60° + (m+M)g = -(10.3\,\text{kN})\sin 60° + (85\,\text{kg} + 360\,\text{kg})(9.81\,\text{m/s}^2) = -4.55\,\text{kN}$$

Solve the x equation for F_h:

$$F_\text{h} = T\cos 60° = (10.3\,\text{kN})\cos 60° = 5.15\,\text{kN}$$

Find the magnitude of the force exerted by the wall on the bar:

$$F = \sqrt{F_{\mathrm{v}}^2 + F_{\mathrm{h}}^2} = \sqrt{(-4.55\,\mathrm{kN})^2 + (5.15\,\mathrm{kN})^2} = \boxed{6.87\,\mathrm{kN}}$$

Find the direction of the force exerted by the wall on the bar:

$$\theta = \tan^{-1}\left(\frac{F_{\mathrm{v}}}{F_{\mathrm{h}}}\right) = \tan^{-1}\left(\frac{-4.55\,\mathrm{kN}}{5.15\,\mathrm{kN}}\right) = \boxed{-41.5°}$$

i.e., 41.5° below the horizontal.

(*b*) Apply $\sum \vec{\tau} = 0$ about the hinge:

$$[(10\,\mathrm{m})\sin 60°]T - (7.5\,\mathrm{m})mg\cos 30° - (15\,\mathrm{m})Mg\cos 30° = 0$$

Solve for *T*:

$$T = \frac{(7.5\,\mathrm{m})m + (15\,\mathrm{m})M}{(10\,\mathrm{m})\sin 60°}g\cos 30°$$

Substitute numerical values and evaluate *T*:

$$T = \frac{(7.5\,\mathrm{m})(85\,\mathrm{kg}) + (15\,\mathrm{m})(360\,\mathrm{kg})}{(10\,\mathrm{m})\sin 60°} \times (9.81\,\mathrm{m/s^2})\cos 30° = \boxed{5.92\,\mathrm{kN}}$$

Apply $\sum \vec{F} = 0$ to the bar:

$$\sum F_y = F_{\mathrm{v}} + T\cos 60° - (85\,\mathrm{kg})g - (360\,\mathrm{kg})g = 0$$

and

$$\sum F_x = F_{\mathrm{h}} - T\sin 60° = 0$$

Solve the *y* equation for F_{v}:

$$F_{\mathrm{v}} = -(5.92\,\mathrm{kN})\cos 60° + (85\,\mathrm{kg} + 360\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1.41\,\mathrm{kN}$$

Solve the *x* equation for F_{h}:

$$F_{\mathrm{h}} = T\sin 60° = (5.92\,\mathrm{kN})\sin 60° = 5.13\,\mathrm{kN}$$

Find the magnitude of the force exerted by the wall on the bar:

$$F = \sqrt{F_{\mathrm{v}}^2 + F_{\mathrm{h}}^2} = \sqrt{(1.41\,\mathrm{kN})^2 + (5.13\,\mathrm{kN})^2} = \boxed{5.32\,\mathrm{kN}}$$

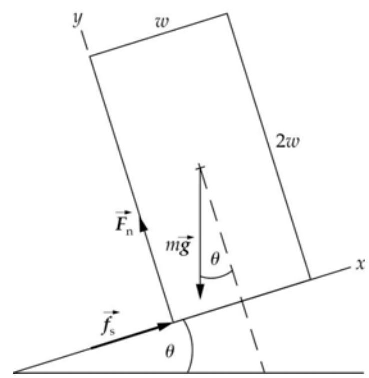
Find the direction of the force exerted by the wall on the bar:

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{1.41\,\text{kN}}{5.13\,\text{kN}}\right)$$
$$= \boxed{15.4°}$$

i.e., 15.4° above the horizontal.

83 ••

Picture the Problem The box and the forces acting on it are shown in the figure. When the box is about to tip, F_n acts at its edge, as indicated in the drawing. We can use the definition of μ_s and apply the condition for rotational equilibrium in an accelerated frame to relate f_s to the weight of the box and, hence, to the normal force.



Using its definition, express μ_s:

$$\mu_s \geq \frac{f_s}{F_n}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the box's center of mass:

$$wf_s - \tfrac{1}{2}wF_n = 0$$

Solve for the ratio $\frac{f_s}{F_n}$:

$$\frac{f_s}{F_n} = \frac{1}{2}$$

Substitute to obtain the condition for tipping:

$$\mu_s \geq 0.500$$

Therefore, if the box is to slide:

$\mu_s < \boxed{0.500}$, as in Problem 75.

Remarks: The difference between problems 75 and 83 is that in 75 the maximum acceleration before slipping is 0.5g, whereas in 88 it is (0.5 cos9°– sin9°) = 0.337g.

***84** ••

Picture the Problem Let the mass of the rod be represented by M. Because the rod is in equilibrium, we can apply the condition for rotational equilibrium to relate the masses of the objects placed on it to its mass.

Apply $\sum\vec{\tau}=0$ about an axis through the pivot for the initial condition:

$$(20\,\text{cm})(2m+2\,\text{g})-(40\,\text{cm})m-(10\,\text{cm})M=0$$

Solve for and evaluate M:

$$M=\frac{(20\,\text{cm})(2m+2\,\text{g})-(40\,\text{cm})m}{10\,\text{cm}}=\boxed{4.00\,\text{g}}$$

Apply $\sum\vec{\tau}=0$ about an axis through the pivot for the second condition:

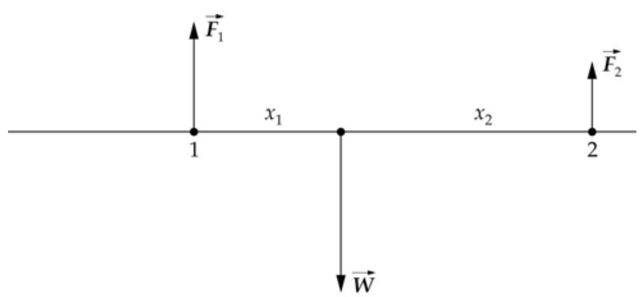
$$(20\,\text{cm})m-(10\,\text{cm})M=0$$

Solve for and evaluate m:

$$m=\frac{(10\,\text{cm})M}{20\,\text{cm}}=\tfrac{1}{2}M=\boxed{2.00\,\text{g}}$$

***85 ••**

Picture the Problem Let the distance from the center of the meterstick of either finger be x_1 and x_2 and W the weight of the stick. Because the meterstick is in equilibrium, we can apply the condition for rotational equilibrium to obtain expressions for the forces one's fingers exert on the meterstick as functions of the distances x_1 and x_2 and the weight of the meterstick W. We can then explain the stop-and-start motion of one's fingers as they are brought closer together by considering the magnitudes of these forces in relationship the coefficients of static and kinetic friction.



(*a*) The stick remains balanced as long as the center of mass is between the two fingers. For a balanced stick the normal force exerted by the finger nearest the center of mass is greater than that exerted by the other finger. Consequently, a larger static-frictional force can be exerted by the finger closer to the center of mass, which means the slipping occurs at the other finger.

(*b*) Apply $\sum \vec{\tau} = 0$ about an axis through point 1 to obtain:

$$F_2(x_1 + x_2) - Wx_1 = 0$$

Solve for F_2 to obtain:

$$F_2 = W\frac{x_1}{x_1 + x_2}$$

Apply $\sum \vec{\tau} = 0$ about an axis through point 2 to obtain:

$$-F_1(x_1 + x_2) + Wx_2 = 0$$

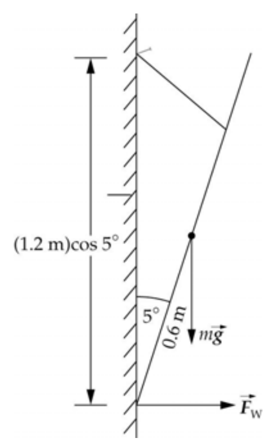
Solve for F_1 to obtain:

$$F_1 = W\frac{x_2}{x_1 + x_2}$$

The finger farthest from the center of mass will slide inward until the normal force it exerts on the stick is sufficiently large to produce a kinetic - frictional force exceeding the maximum static - frictional force exerted by the other finger. At that point the finger that was not sliding begins to slide, the finger that was sliding stops sliding, and the process is reversed. When one finger is is slipping the other is not.

86 ••

Picture the Problem The drawing shows a side view of the wall-and-picture system. Because the frame's width is not specified, we assume it to be negligible. Note that 0.75, 0.4, and 0.85 form a Pythagorean triad. Thus, the nail will be at the same level as the top of the frame. We can apply the condition for rotational equilibrium to determine the force exerted by the wall.



(*a*) Because the center of gravity of the picture is in front of the wall, the torque due to *mg* about the nail must be balanced by an opposing torque due to the force of the wall on the picture, acting horizontally. So that $\sum F_x = 0$, the tension in the wire must have a horizontal component, and the picture must therefore tilt forward.

(*b*) Apply $\sum \vec{\tau} = 0$ about an axis through the nail and parallel to the

$$-[(0.6\,\text{m})\sin 5°](8\,\text{kg})(9.81\,\text{m/s}^2) + [(1.2\,\text{m})\cos 5°]F_W = 0$$

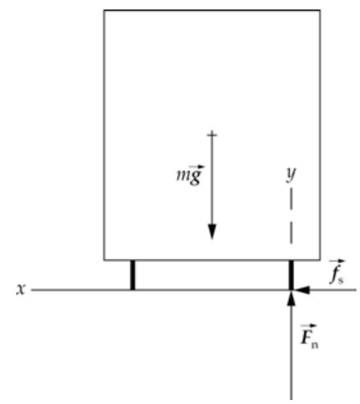
wall to obtain:

Solve for and evaluate F_W:

$$F_W = \frac{[(0.6\,\text{m})\sin 5°](8\,\text{kg})(9.81\,\text{m/s}^2)}{(1.2\,\text{m})\cos 5°} = \boxed{3.43\,\text{N}}$$

87 ••

Picture the Problem The box car and rail are shown in the drawing. At the critical speed, the normal force is entirely on the outside rail. The center of gravity is 0.775 m from that rail and 2.15 m above it. Choose the coordinate system shown in the figure. To find the speed at which this situation prevails, we can apply the conditions for static equilibrium in an accelerated frame.



Apply $\sum \vec{\tau} = 0$ about an axis through the center of gravity of the box car:

$$(0.775\,\text{m})F_n - (2.15\,\text{m})f_s = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the box car and solve for F_n:

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Apply $\sum F_x = ma_{cm}$ to the box:

$$f_s = m\frac{v^2}{R}$$

Substitute in equation (1) to obtain:

$$(0.775\,\text{m})mg - (2.15\,\text{m})m\frac{v^2}{R} = 0$$

Solve for v:

$$v = \sqrt{0.360Rg}$$

(a) Evaluate v for R = 150 m:

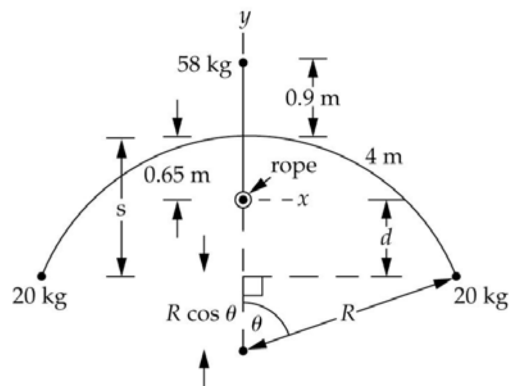
$$v = \sqrt{0.360(150\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{23.0\,\text{m/s}}$$

(*b*) Evaluate v for R = 240 m:

$$v=\sqrt{0.360(240\,\text{m})(9.81\,\text{m/s}^2)}=\boxed{29.1\,\text{m/s}}$$

88 ••

Picture the Problem For neutral equilibrium, the center of mass of the system must be at the same height as the feet of the tightrope walker. The system is shown in the drawing. Let the origin of the coordinate system be at the rope. We'll determine the distance d such that y_{cm} = 0. We'll then determine the angle θ subtended by one half the long rod.



Express the y coordinate of the center of mass of the system:

$$y_{cm}=\frac{(58\,\text{kg})(0.9\,\text{m})-2(20\,\text{kg})d}{58\,\text{kg}+40\,\text{kg}}$$

Set y_{cm} = 0 and solve for d:

d = 1.305 m

Relate the distances s and d and solve for s:

s = 0.65 m + d = 1.955 m

Relate s to R and θ:

$$s=R(1-\cos\theta) \qquad (1)$$

Relate R and θ to the half-length of the rod:

$$R\theta=4\,\text{m} \qquad (2)$$

Substitute in equation (1) to obtain:

$$1.955\,\text{m}=(4\,\text{m})\frac{1-\cos\theta}{\theta}$$

or

$$\frac{1-\cos\theta}{\theta}=0.489$$

Use graphical or trial-and-error methods to solve for θ:

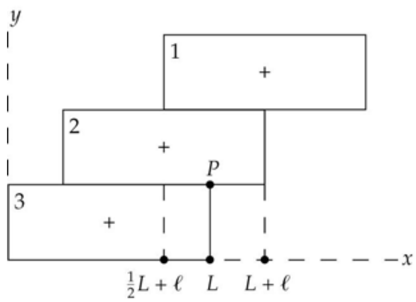
$$\theta=1.08\,\text{rad}$$

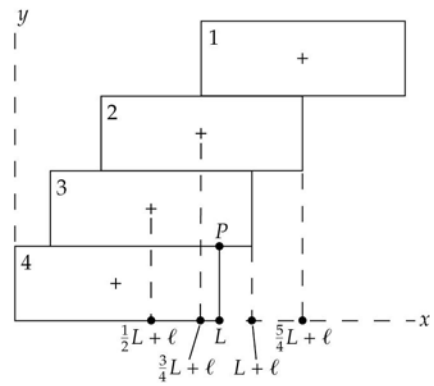
Substitute in equation (2) to obtain:

$$R=\frac{4\,\text{m}}{1.08\,\text{rad}}=\boxed{3.70\,\text{m}}$$

***89** •••

Picture the Problem Let the mass of each brick be m and number them as shown in the diagrams for 3 bricks and 4 bricks below. Let ℓ denote the maximum offset of the nth brick. Choose the coordinate system shown and apply the condition for rotational equilibrium about an axis parallel to the z axis and passing through the point P at the supporting edge of the nth brick.





(*a*) Apply $\sum\vec{\tau}=0$ about an axis through P and parallel to the z axis to bricks 1 and 2 for the 3-brick arrangement shown above on the left:

$$mg\left[L-\left(\tfrac{1}{2}L+\ell\right)\right]-mg\ell=0$$

Solve for ℓ to obtain:

$$\ell=\boxed{\tfrac{1}{4}L}$$

(*b*) Apply $\sum\vec{\tau}=0$ about an axis through P and parallel to the z axis to bricks 1 and 2 for the 4-brick arrangement shown above on the right:

$$mg\left[L-\left(\tfrac{1}{2}L+\ell\right)\right]+mg\left[L-\left(\tfrac{3}{4}L+\ell\right)\right]-mg\left(\tfrac{5}{4}L+\ell-L\right)=0$$

Solve for ℓ to obtain:

$$\ell=\tfrac{1}{6}L$$

Continuing in this manner we obtain, as the successive offsets, the sequence:

$$\boxed{\frac{L}{2},\frac{L}{4},\frac{L}{6},\frac{L}{8},\ldots\frac{L}{2n}}$$

where n = 1, 2, 3, … N.

(c) Express the offset of the (n +1)st brick in terms of the offset of the nth brick:

$$\ell_{n+1}=\ell_n+\frac{L}{2n}$$

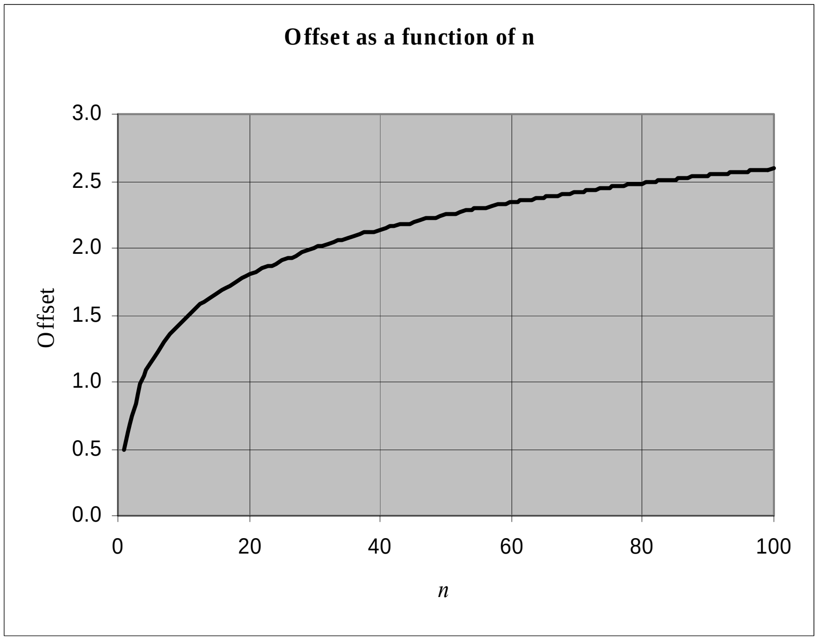
A spreadsheet program to calculate the sum of the offsets as a function of *n* is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B5	B4+1	$n + 1$
C5	C4+$B$1/(2*B5)	$\ell_n + \frac{L}{2n}$

	A	B	C	D
1	L=	1	m	
2				
3		n	offset	
4		1	0.500	
5		2	0.750	
6		3	0.917	
7		4	1.042	
8		5	1.142	
9		6	1.225	
10		7	1.296	
11		8	1.359	
12		9	1.414	
13		10	1.464	
98		95	2.568	
99		96	2.573	
100		97	2.579	
101		98	2.584	
102		99	2.589	
103		100	2.594	

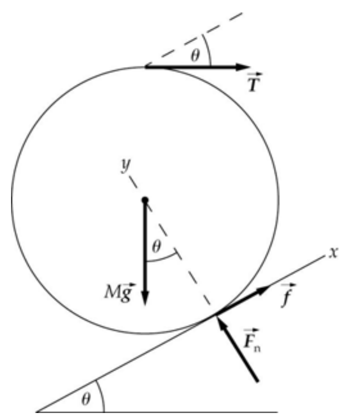
From the table we see that $\ell_5 = \boxed{1.142\text{ m,}}$ $\ell_{10} = \boxed{1.464\text{ m,}}$ and $\ell_{100} = \boxed{2.594\text{ m.}}$

(*d*) Increasing *N* in the spreadsheet solution suggests that the sum of the individual offsets continues to grow as *N* increases without bound. The series is, in fact, divergent and the stack of bricks has no maximum offset or length.



90 •••

Picture the Problem The four forces acting on the sphere: its weight, mg; the normal force of the plane, F_n; the frictional force, f, acting parallel to the plane; and the tension in the string, T, are shown in the figure. Choose the coordinate system shown. Because the sphere is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find f, F_n, and T.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the center of the sphere:

$$fR - TR = 0 \Rightarrow T = f$$

Apply $\sum F_x = 0$ to the sphere:

$$f + T\cos\theta - Mg\sin\theta = 0$$

Substitute for f and solve for T:

$$T = \frac{Mg\sin\theta}{1+\cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(3\text{ kg})(9.81\text{ m/s}^2)\sin 30°}{1+\cos 30°} = \boxed{7.89\text{ N}}$$

(*b*) Apply $\sum F_y = 0$ to the sphere:

$$F_n - T\sin\theta - Mg\cos\theta = 0$$

Solve for F_n:

$$F_n = T\sin\theta + Mg\cos\theta$$

Substitute numerical values and evaluate F_n:

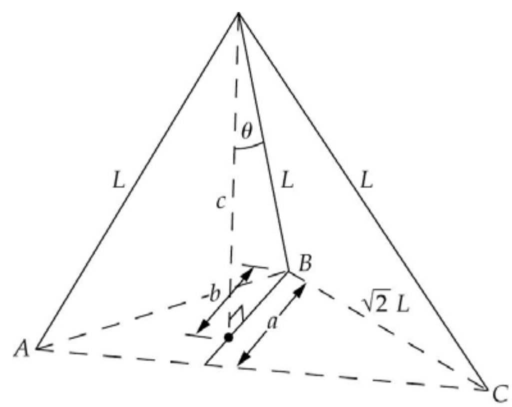
$$F_n = (7.89\,\text{N})\sin 30° + (3\,\text{kg})(9.81\,\text{m/s}^2)\cos 30° = \boxed{29.4\,\text{N}}$$

(*c*) In part (*a*) we showed that $f = T$:

$$f = \boxed{7.89\,\text{N}}$$

91 •••

Picture the Problem Let L be the length of each leg of the tripod. Applying the Pythagorean theorem leads us to conclude that the distance a shown in the figure is $\sqrt{\frac{3}{2}}L$ and the distance b, the distance to the centroid of the triangle ABC is $\frac{2}{3}\sqrt{\frac{3}{2}}L$, and the distance c is $\frac{L}{\sqrt{3}}$. These results allow us to conclude that $\cos\theta = \frac{L}{\sqrt{3}}$. Because the tripod is in equilibrium, we can apply the condition for translational equilibrium to find the compressional forces in each leg.



Letting F_C represent the compressional force in a leg of the tripod, apply $\sum \vec{F} = 0$ to the apex of the tripod:

$$3F_C\cos\theta - mg = 0$$

Solve for F_C:

$$F_C = \frac{mg}{3\cos\theta}$$

Solve for F_C:

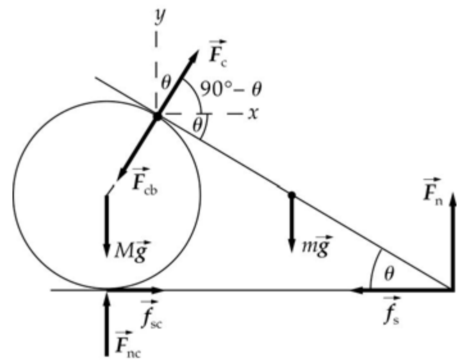
$$F_C = \frac{mg}{3\times\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{3}mg$$

Substitute numerical values and evaluate F_C:

$$F_C = \frac{\sqrt{3}}{3}(100\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{566\,\text{N}}$$

92 ••

Picture the Problem The forces that act on the beam are its weight, mg; the force of the cylinder, F_c, acting along the radius of the cylinder; the normal force of the ground, F_n; and the friction force $f_s = \mu_s F_n$. The forces acting on the cylinder are its weight, Mg; the force of the beam on the cylinder, $F_{cb} = F_c$ in magnitude, acting radially inward; the normal force of the ground on the cylinder, F_{nc}; and the force of friction, $f_{sc} = \mu_{sc} F_{nc}$. Choose the coordinate system shown in the figure and apply the conditions for rotational and translational equilibrium.



Express $\mu_{s,\text{beam-floor}}$ in terms of f_s and F_n:

$$\mu_{s,\text{beam-floor}} = \frac{f_s}{F_n} \quad (1)$$

Express $\mu_{s,\text{cylinder-floor}}$ in terms of f_{sc} and F_{nc}:

$$\mu_{s,\text{cylinder-floor}} = \frac{f_{sc}}{F_{nc}} \quad (2)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the right end of the beam:

$$[(10\,\text{cm})\cos\theta]mg - (15\,\text{cm})F_c = 0$$

Solve for and evaluate F_c:

$$F_c = \frac{[(10\,\text{cm})\cos\theta]mg}{15\,\text{cm}} = \frac{[10\cos 30°](5\,\text{kg})(9.81\,\text{m/s}^2)}{15} = 28.3\,\text{N}$$

Apply $\sum F_y = 0$ to the beam:

$$F_n + F_c\cos(90° - \theta) - mg = 0$$

Solve for F_n:

$$F_n = mg - F_c\cos\theta = (5\,\text{kg})(9.81\,\text{m/s}^2) - (28.3\,\text{N})\cos 30° = 24.5\,\text{N}$$

Apply $\sum F_x = 0$ to the beam:

$$-f_s + F_c\cos(90° - \theta) = 0$$

Solve for and evaluate f_s:

$$f_s = F_c\cos(90° - \theta) = (28.3\,\text{N})\cos 60° = 14.2\,\text{N}$$

$\vec{F}_{cb}$ is the reaction force to $\vec{F}_c$:

$F_{cb} = F_c = 28.3\,\text{N}$ radially inward.

Apply $\sum F_y = 0$ to the cylinder:

$$F_{nc} - F_{cb}\cos\theta - Mg = 0$$

Solve for and evaluate F_{nc}:

$$F_{nc} = F_{cb}\cos\theta + Mg = (28.3\,\text{N})\cos 30° + (8\,\text{kg})(9.81\,\text{m/s}^2) = 103\,\text{N}$$

Apply $\sum F_x = 0$ to the cylinder:

$$f_{sc} - F_{cb}\cos(90° - \theta) = 0$$

Solve for and evaluate f_{sc}:

$$f_{sc} = F_{cb}\cos(90° - \theta) = (28.3\,\text{N})\cos 60° = 14.2\,\text{N}$$

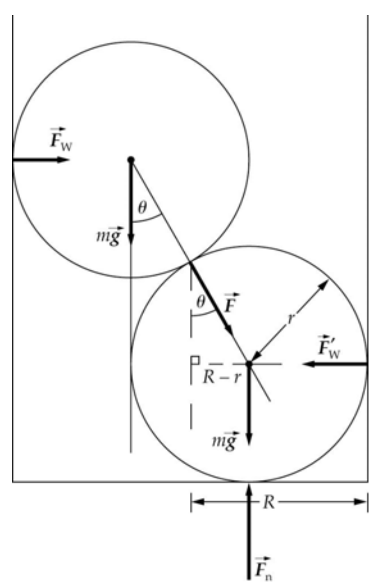
Substitute numerical values in equations (1) and (2) and evaluate $\mu_{s,\text{beam-floor}}$ and $\mu_{s,\text{cylinder-floor}}$:

$$\mu_{s,\text{beam-floor}} = \frac{14.2\,\text{N}}{24.5\,\text{N}} = \boxed{0.580}$$

and

$$\mu_{s,\text{cylinder-floor}} = \frac{14.2\,\text{N}}{103\,\text{N}} = \boxed{0.138}$$

93 •••



Picture the Problem The geometry of the system is shown in the drawing. Let upward be the positive y direction and to the right be the positive x direction. Let the angle between the vertical center line and the line joining the two centers be θ. Then $\sin\theta = \dfrac{R-r}{r}$ and $\tan\theta = \dfrac{R-r}{\sqrt{R(2r-R)}}$.

The force exerted by the bottom of the cylinder is just $2mg$. Let F be the force that the top sphere exerts on the lower sphere. Because the spheres are in equilibrium, we can apply the condition for translational equilibrium.

Apply $\sum F_y = 0$ to the spheres:

$$F_n - mg - mg = 0$$

Solve for F_n: $$F_n = \boxed{2mg}$$

Because the cylinder wall is smooth, $F\cos\theta = mg$, and: $$F = \boxed{\frac{mg}{\cos\theta}}$$

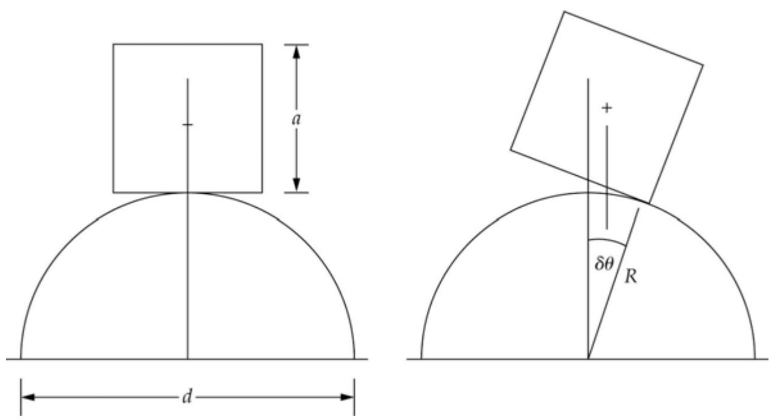
Express the x component of F: $$F_x = F\sin\theta = mg\tan\theta$$

Express the force that the wall of the cylinder exerts: $$F_W = \boxed{mg\frac{R-r}{\sqrt{R(2r-R)}}}$$

Remarks: Note that as r approaches $R/2$, $F_w \to \infty$.

***94 •••**

Picture the Problem Consider a small rotational displacement, $\delta\theta$ of the cube from equilibrium. This shifts the point of contact between cube and cylinder by $R\delta\theta$, where $R = d/2$. As a result of that motion, the cube itself is rotated through the same angle $\delta\theta$, and so its center is shifted in the same direction by the amount $(a/2)\delta\theta$, neglecting higher order terms in $\delta\theta$.



If the displacement of the cube's center of mass is less than that of the point of contact, the torque about the point of contact is a restoring torque, and the cube will return to its equilibrium position. If, on the other hand, $(a/2)\delta\theta > (d/2)\delta\theta$, then the torque about the point of contact due to mg is in the direction of $\delta\theta$, and will cause the displacement from equilibrium to increase. We see that the minimum value of d/a for stable equilibrium is $d/a = 1$.

Chapter 13
Fluids

Conceptual Problems

1 •

Determine the Concept The absolute pressure is related to the gauge pressure according to $P = P_{\text{gauge}} + P_{\text{at}}$. While doubling the gauge pressure will increase the absolute pressure, we do not have enough information to say what the resulting absolute pressure will be. $\boxed{(e)\text{ is correct.}}$

***2** •

Determine the Concept No. In an environment where $g_{\text{eff}} = F_{\text{g}} - m\dfrac{v^2}{r} = 0$, there is no buoyant force; there is no ″up″ or ″down.″

3 ••

Determine the Concept As you lower the rock into the water, the upward force you exert on the rock plus the upward buoyant force on the rock balance its weight. When the thread breaks, there will be an additional downward force on the scale equal to the buoyant force on the rock (the water exerts the upward buoyant force on the rock and the reaction force is the force the rock exerts on the water … and hence on the scale). Let ρ represent the density of the water, V the volume of the rock, and w_{f} the weight of the displaced water. Then the density of the rock is 3ρ. We can use Archimedes' principle to find the additional force on the scale.

Apply Archimedes' principle to the rock: $B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{f}}V_{\text{f}}g$

Because $V_{\text{f}} = V_{\text{rock}}$:

$$B = \rho\frac{M}{\rho_{\text{rock}}}g = \rho\frac{M}{3\rho}g = \tfrac{1}{3}Mg$$

and $\boxed{(d)\text{ is correct.}}$

4 ••

Determine the Concept The density of water increases with depth and the buoyant force on the rock equals the weight of the displaced water. Because the weight of the displaced water depends on the density of the water, it follows that the buoyant force on the rock increases as it sinks. $\boxed{(b)\text{ is correct.}}$

5 ••

Determine the Concept Nothing. The fish is in neutral buoyancy (that is, its density equals that water), so the upward acceleration of the fish is balanced by the downward acceleration of the displaced water.

***6 ••**

Determine the Concept Yes. Because the volumes of the two objects are equal, the downward force on each side is reduced by the same amount when they are submerged, not in proportion to their masses. That is, if $m_1L_1 = m_2L_2$ and $L_1 \neq L_2$, then $(m_1 - c)L_1 \neq (m_2 - c)L_2$.

7 ••

Determine the Concept The buoyant forces acting on these submerged objects are equal to the weight of the water each displaces. The weight of the displaced water, in turn, is directly proportional to the volume of the submerged object. Because $\rho_{Pb} > \rho_{Cu}$, the volume of the copper must be greater than that of the lead and, hence, the buoyant force on the copper is greater than that on the lead. $\boxed{(b)\text{ is correct.}}$

8 ••

Determine the Concept The buoyant forces acting on these submerged objects are equal to the weight of the water each displaces. The weight of the displaced water, in turn, is directly proportional to the volume of the submerged object. Because their volumes are the same, the buoyant forces on them must be the same. $\boxed{(c)\text{ is correct.}}$

9 •

Determine the Concept It blows over the ball, reducing the pressure above the ball to below atmospheric pressure.

10 •

Determine the Concept From the equation of continuity ($I_V = Av =$ constant), we can conclude that, as the pipe narrows, the velocity of the fluid must increase. Using Bernoulli's equation for constant elevation ($P + \frac{1}{2}\rho v^2 = \text{constant}$), we can conclude that as the velocity of the fluid increases, the pressure must decrease. $\boxed{(b)\text{ is correct.}}$

***11 •**

Determine the Concept False. The buoyant force on a submerged object depends on the weight of the displaced fluid which, in turn, depends on the volume of the displaced fluid.

12 •

Determine the Concept When the bottle is squeezed, the force is transmitted equally through the fluid, leading to a pressure increase on the air bubble in the diver. The air bubble shrinks, and the loss in buoyancy is enough to sink the diver.

13 •

Determine the Concept The buoyant force acting on the ice cubes equals the weight of the water they displace, i.e., $B = w_\mathrm{f} = \rho_\mathrm{f} V_\mathrm{f} g$. When the ice melts the volume of water displaced by the ice cubes will occupy the space previously occupied by the submerged part of the ice cubes. Therefore, the water level remains constant.

14 •

Determine the Concept The density of salt water is greater than that of fresh water and so the buoyant force exerted on one in salt water is greater than in fresh water.

15 ••

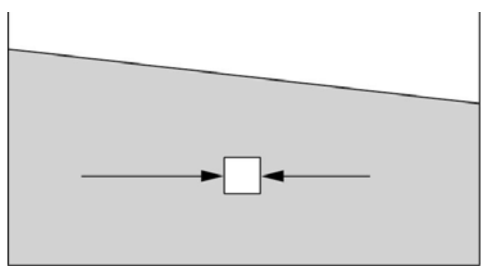
Determine the Concept Because the pressure increases with depth, the object will be compressed and its density will increase. Its volume will decrease. Thus, it will sink to the bottom.

16 ••

Determine the Concept The force acting on the fluid is the difference in pressure between the wide and narrow parts times the area of the narrow part.

17 ••

Determine the Concept The drawing shows the beaker and a strip within the water. As is readily established by a simple demonstration, the surface of the water is not level while the beaker is accelerated, showing that there is a pressure gradient. That pressure gradient results in a net force on the small element shown in the figure.



***18** ••

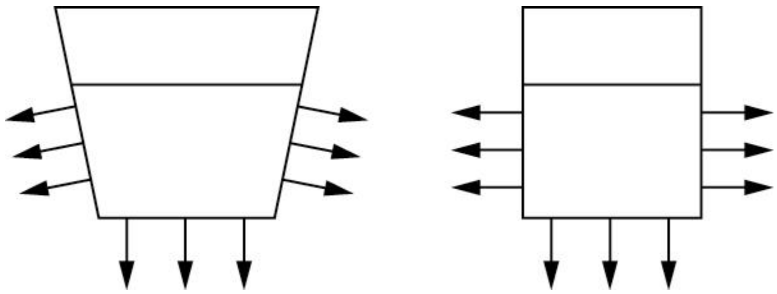
Determine the Concept The water level in the pond will drop slightly. When the anchor is in the boat, the boat displaces enough water so that the buoyant force on it equals the sum of the weight of the boat, your weight, and the weight of the anchor. When you put the anchor overboard, it will displace its volume and the volume of water displaced by the boat will decrease.

19 ••

Determine the Concept From Bernoulli's principle, the opening above which the air flows faster will be at a lower pressure than the other one, which will cause a circulation of air in the tunnel from opening 1 toward opening 2. It has been shown that enough air will circulate inside the tunnel even with the slightest breeze outside.

***20 •**

Determine the Concept The diagram that follows shows the forces exerted by the pressure of the liquid on the two cups to the left.



Because the force is normal to the surface of the cup, there is a larger downward component to the net force on the cup on the left. Similarly, there will be less total force exerted by the fluid in the cup on the far right in the diagram in the problem statement.

Density

21 •

Picture the Problem The mass of the cylinder is the product of its density and volume. The density of copper can be found in Figure 13-1.

Using the definition of density, express the mass of the cylinder:

$$m = \rho V = \rho\left(\pi R^2 h\right)$$

Substitute numerical values and evaluate m:

$$m = \pi\left(8.93\times10^3\ \text{kg/m}^3\right)\left(2\times10^{-2}\ \text{m}\right)^2 \times\left(6\times10^{-2}\ \text{m}\right) = \boxed{0.673\,\text{kg}}$$

22 •

Picture the Problem The mass of the sphere is the product of its density and volume. The density of lead can be found in Figure 13-1.

Using the definition of density, express the mass of the sphere:

$$m = \rho V = \rho\left(\tfrac{4}{3}\pi R^3\right)$$

Substitute numerical values and evaluate m:

$$m = \tfrac{4}{3}\pi\left(11.3\times10^3\ \text{kg/m}^3\right)\left(2\times10^{-2}\ \text{m}\right)^3 = \boxed{0.379\,\text{kg}}$$

23 •

Picture the Problem The mass of the air in the room is the product of its density and volume. The density of air can be found in Figure 13-1.

Using the definition of density, express the mass of the air:

$$m = \rho V = \rho LWH$$

Substitute numerical values and evaluate m:

$$m = (1.293\,\text{kg/m}^3)(4\,\text{m})(5\,\text{m})(4\,\text{m}) = \boxed{103\,\text{kg}}$$

***24** •

Picture the Problem Let ρ_0 represent the density of mercury at 0°C and ρ' its density at 80°C, and let m represent the mass of our sample at 0°C and m' its mass at 80°C. We can use the definition of density to relate its value at the higher temperature to its value at the lower temperature and the amount spilled.

Using its definition, express the density of the mercury at 80°C:

$$\rho' = \frac{m'}{V}$$

Express the mass of the mercury at 80°C in terms of its mass at 0°C and the amount spilled at the higher temperature:

$$\rho' = \frac{m - \Delta m}{V} = \frac{m}{V} - \frac{\Delta m}{V} = \rho_0 - \frac{\Delta m}{V}$$

Substitute numerical values and evaluate ρ':

$$\rho' = 1.3645\times10^4\,\text{kg/m}^3 - \frac{1.47\times10^{-3}\,\text{kg}}{60\times10^{-6}\,\text{m}^3} = \boxed{1.3621\times10^4\,\text{kg/m}^3}$$

Pressure

25 •

Picture the Problem The pressure due to a column of height h of a liquid of density ρ is given by $P = \rho gh$.

Letting h represent the height of the column of mercury, express the pressure at its base:

$$\rho_{\text{Hg}} gh = 101\,\text{kPa}$$

Solve for h:

$$h = \frac{101\,\text{kPa}}{\rho_{\text{Hg}} g}$$

Substitute numerical values and evaluate h:

$$h = \frac{1.01\times10^5\ \text{N/m}^2}{(13.6\times10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= 0.757\,\text{m}\times\frac{1\,\text{in}}{2.54\times10^{-2}\ \text{m}}$$
$$= \boxed{29.8\,\text{in of Hg}}$$

26 •

Picture the Problem The pressure due to a column of height h of a liquid of density ρ is given by $P = \rho gh$.

(a) Express the pressure as a function of depth in the lake:

$$P = P_{\text{at}} + \rho_{\text{water}}gh$$

Solve for and evaluate h:

$$h = \frac{P - P_{\text{at}}}{\rho_{\text{water}}g} = \frac{2P_{\text{at}} - P_{\text{at}}}{\rho_{\text{water}}g} = \frac{P_{\text{at}}}{\rho_{\text{water}}g}$$

Substitute numerical values and evaluate h:

$$h = \frac{1.01\times10^5\ \text{N/m}^2}{(10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= \boxed{10.3\,\text{m}}$$

(b) Proceed as in (a) with ρ_{water} replaced by ρ_{Hg} to obtain:

$$h = \frac{2P_{\text{at}} - P_{\text{at}}}{\rho_{\text{Hg}}g} = \frac{P_{\text{at}}}{\rho_{\text{Hg}}g}$$

Substitute numerical values and evaluate h:

$$h = \frac{1.01\times10^5\ \text{N/m}^2}{(13.6\times10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= \boxed{75.7\,\text{cm}}$$

***27** •

Picture the Problem The pressure applied to an enclosed liquid is transmitted undiminished to every point in the fluid and to the walls of the container. Hence we can equate the pressure produced by the force applied to the piston to the pressure due to the weight of the automobile and solve for F.

Express the pressure the weight of the automobile exerts on the shaft of the lift:

$$P_{\text{auto}} = \frac{w_{\text{auto}}}{A_{\text{shaft}}}$$

Express the pressure the force applied to the piston produces:

$$P = \frac{F}{A_{\text{piston}}}$$

Because the pressures are the same, we can equate them to obtain:

$$\frac{w_{auto}}{A_{shaft}} = \frac{F}{A_{piston}}$$

Solve for F:

$$F = w_{auto}\frac{A_{piston}}{A_{shaft}} = m_{auto}g\frac{A_{piston}}{A_{shaft}}$$

Substitute numerical values and evaluate F:

$$F = (1500\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{cm}}{8\,\text{cm}}\right)^2$$

$$= \boxed{230\,\text{N}}$$

28 ••

Picture the Problem The pressure exerted by the woman's heel on the floor is her weight divided by the area of her heel.

Using its definition, express the pressure exerted on the floor by the woman's heel:

$$P = \frac{F}{A} = \frac{w}{A} = \frac{mg}{A}$$

Substitute numerical values and evaluate P:

$$P = \frac{(56\,\text{kg})(9.81\,\text{m/s}^2)}{10^{-4}\,\text{m}^2}$$

$$= 5.49\times10^{6}\,\text{N/m}^2 \times \frac{1\,\text{atm}}{101.3\,\text{kPa}}$$

$$= \boxed{54.2\,\text{atm}}$$

***29** •

Picture the Problem The required pressure ΔP is related to the change in volume ΔV and the initial volume V through the definition of the bulk modulus B; $B = -\frac{\Delta P}{\Delta V/V}$.

Using the definition of the bulk modulus, relate the change in volume to the initial volume and the required pressure:

$$B = -\frac{\Delta P}{\Delta V/V}$$

Solve for ΔP:

$$\Delta P = -B\frac{\Delta V}{V}$$

Substitute numerical values and evaluate ΔP:

$$\Delta P = -2.0\times10^{9}\ \text{Pa}\times\left(\frac{-0.01\,\text{L}}{1\,\text{L}}\right) = 2.00\times10^{7}\ \text{Pa}\times\frac{1\,\text{atm}}{101.325\,\text{kPa}} = \boxed{198\,\text{atm}}$$

30 •

Picture the Problem The area of contact of each tire with the road is related to the weight on each tire and the pressure in the tire through the definition of pressure.

Using the definition of gauge pressure, relate the area of contact to the pressure and the weight of the car:

$$A = \frac{\frac{1}{4}w}{P_{\text{gauge}}}$$

Substitute numerical values and evaluate A:

$$A = \frac{\frac{1}{4}(1500\,\text{kg})(9.81\,\text{m/s}^2)}{200\,\text{kPa}} = \frac{\frac{1}{4}(1500\,\text{kg})(9.81\,\text{m/s}^2)}{200\times10^{3}\ \text{N/m}^2} = 1.84\times10^{-2}\ \text{m}^2 = \boxed{184\,\text{cm}^2}$$

31 ••

Picture the Problem The force on the lid is related to pressure exerted by the water and the cross-sectional area of the column of water through the definition of density. We can find the mass of the water from the product of its density and volume.

(*a*) Using the definition of pressure, express the force exerted on the lid:

$$F = PA$$

Express the pressure due to a column of water of height h:

$$P = \rho_{\text{water}}gh$$

Substitute for P and A to obtain:

$$F = \rho_{\text{water}}gh\pi r^2$$

Substitute numerical values:

$$F = (10^{3}\ \text{kg/m}^3)(9.81\,\text{m/s}^2)\times(12\,\text{m})\pi(0.2\,\text{m})^2 = \boxed{14.8\,\text{kN}}$$

(*b*) Relate the mass of the water to its density and volume:

$$m = \rho_{\text{water}}V = \rho_{\text{water}}h\pi r^2$$

Substitute numerical values and evaluate *m*:

$$m = \left(10^3\ \text{kg/m}^3\right)(12\ \text{m})\pi\left(3\times10^{-3}\ \text{m}\right)^2 = \boxed{0.339\ \text{kg}}$$

32 ••

Picture the Problem The minimum elevation of the bag *h* that will produce a pressure of at least 12 mmHg is related to this pressure and the density of the blood plasma through $P = \rho_{\text{blood}}gh$.

Using the definition of the pressure due to a column of liquid, relate the pressure at its base to its height:

$$P = \rho_{\text{blood}}gh$$

Solve for *h*:

$$h = \frac{P}{\rho_{\text{blood}}g}$$

Substitute numerical values and evaluate *h*:

$$h = \frac{12\ \text{mmHg}\times\frac{133.32\ \text{Pa}}{1\ \text{mmHg}}}{\left(1.03\times10^3\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)} = 0.158\ \text{m} = \boxed{15.8\ \text{cm}}$$

33 ••

Picture the Problem The depth *h* below the surface at which you would be able to breath is related to the pressure at that depth and the density of water ρ_{w} through $P = \rho_{\text{w}}gh$.

Express the pressure at a depth *h* and solve for *h*:

$$P = \rho_{\text{w}}gh$$

and

$$h = \frac{P}{\rho_{\text{w}}g}$$

Express the pressure at depth *h* in terms of the weight on your chest:

$$P = \frac{F}{A}$$

Substitute to obtain:

$$h = \frac{F}{A\rho_{\text{w}}g}$$

Substitute numerical values and evaluate h:

$$h = \frac{400\,\text{N}}{(0.09\,\text{m}^2)(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} = \boxed{0.453\,\text{m}}$$

34 ••

Picture the Problem Let A_1 and A_2 represent the cross-sectional areas of the large piston and the small piston, and F_1 and F_2 the forces exerted by the large and on the small piston, respectively. The work done by the large piston is $W_1 = F_1h_1$ and that done on the small piston is $W_2 = F_2h_2$. We'll use Pascal's principle and the equality of the volume of the displaced liquid in both pistons to show that W_1 and W_2 are equal.

Express the work done in lifting the car a distance h:

$$W_1 = F_1h_1$$

where F is the weight of the car.

Using the definition of pressure, relate the forces F_1 (= w) and F_2 to the areas A_1 and A_2:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Solve for F_1:

$$F_1 = F_2\frac{A_1}{A_2}$$

Equate the volumes of the displaced fluid in the two pistons:

$$h_1A_1 = h_2A_2$$

Solve for h_1:

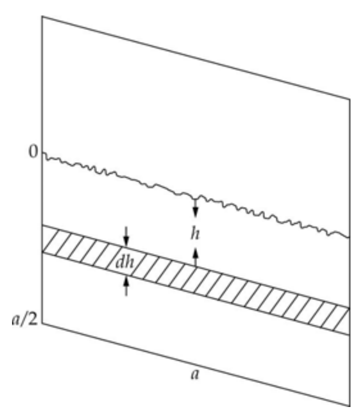
$$h_1 = h_2\frac{A_2}{A_1}$$

Substitute in the expression for W_1 and simplify to obtain:

$$W_1 = F_2\frac{A_1}{A_2}h_2\frac{A_2}{A_1} = F_2h_2 = \boxed{W_2}$$

35 •

Picture the Problem Because the pressure varies with depth, we cannot simply multiply the pressure times the half-area of a side of the cube to find the force exerted by the water. We therefore consider the force exerted on a strip of width a, height dh, and area $dA = adh$ at a depth h and integrate from $h = 0$ to $h = a/2$. The water pressure at depth h is $P_{at} + \rho gh$. We can



omit the atmospheric pressure because it is exerted on both sides of the wall of the cube.

Express the force dF on the element of length a and height dh in terms of the net pressure ρgh:

$$dF = PdA = \rho ghadh$$

Integrate from $h = 0$ to $h = a/2$:

$$F = \int_0^{a/2} dF = \rho ga\int_0^{a/2} hdh = \frac{1}{2}\rho ga\left(\frac{a^2}{4}\right)$$
$$= \boxed{\frac{\rho ga^3}{8}}$$

***36** •••

Picture the Problem The weight of the water in the vessel is the product of its mass and the gravitational field. Its mass, in turn, is related to its volume through the definition of density. The force the water exerts on the base of the container can be determined from the product of the pressure it creates and the area of the base.

(*a*) Using the definition of density, relate the weight of the water to the volume it occupies:

$$w = mg = \rho Vg$$

Substitute for V to obtain:

$$w = \tfrac{1}{3}\pi\rho r^2 hg$$

Substitute numerical values and evaluate w:

$$w = \tfrac{1}{3}\pi\left(10^3\ \text{kg/m}^3\right)\left(15\times10^{-2}\ \text{m}\right)^2\left(25\times10^{-2}\ \text{m}\right)\left(9.81\,\text{m/s}^2\right) = \boxed{57.8\,\text{N}}$$

(*b*) Using the definition of pressure, relate the force exerted by the water on the base of the vessel to the pressure it exerts and the area of the base:

$$F = PA = \rho gh\pi r^2$$

Substitute numerical values and evaluate F:

$$F = \left(10^3\ \text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)\left(25\times10^{-2}\ \text{m}\right)\pi\left(15\times10^{-2}\ \text{m}\right)^2 = \boxed{173\,\text{N}}$$

This occurs in the same way that the force on Pascals barrel is much greater than the weight of the water in the tube. The downward force on the base is also the result of the downward component of the force exerted by the slanting walls of the cone on the water.

Buoyancy

***37 •**

Picture the Problem The scale's reading will be the difference between the weight of the piece of copper in air and the buoyant force acting on it.

Express the apparent weight w' of the piece of copper:

$$w' = w - B$$

Using the definition of density and Archimedes' principle, substitute for w and B to obtain:

$$w' = \rho_{\mathrm{Cu}}Vg - \rho_{\mathrm{w}}Vg = (\rho_{\mathrm{Cu}} - \rho_{\mathrm{w}})Vg$$

Express w in terms of ρ_{Cu} and V and solve for Vg:

$$w = \rho_{\mathrm{Cu}}Vg \Rightarrow Vg = \frac{w}{\rho_{\mathrm{Cu}}}$$

Substitute to obtain:

$$w' = (\rho_{\mathrm{Cu}} - \rho_{\mathrm{w}})\frac{w}{\rho_{\mathrm{Cu}}} = \left(1 - \frac{\rho_{\mathrm{w}}}{\rho_{\mathrm{Cu}}}\right)w$$

Substitute numerical values and evaluate w':

$$w' = \left(1 - \frac{1}{9}\right)(0.5\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = \boxed{4.36\,\mathrm{N}}$$

38 •

Picture the Problem We can use the definition of density and Archimedes' principle to find the density of the stone. The difference between the weight of the stone in air and in water is the buoyant force acting on the stone.

Using its definition, express the density of the stone:

$$\rho_{\mathrm{stone}} = \frac{m_{\mathrm{stone}}}{V_{\mathrm{stone}}} \qquad (1)$$

Apply Archimedes' principle to obtain:

$$B = w_{\mathrm{f}} = m_{\mathrm{f}}g = \rho_{\mathrm{f}}V_{\mathrm{f}}g$$

Solve for V_f:

$$V_f = \frac{B}{\rho_f g}$$

Because $V_f = V_{stone}$ and $\rho_f = \rho_{water}$:

$$V_{stone} = \frac{B}{\rho_{water} g}$$

Substitute in equation (1) and simplify to obtain:

$$\rho_{stone} = \frac{m_{stone} g}{B}\rho_{water} = \frac{w_{stone}}{B}\rho_{water}$$

Substitute numerical values and evaluate ρ_{stone}:

$$\rho_{stone} = \frac{60\,\text{N}}{60\,\text{N} - 20\,\text{N}}\left(10^3\ \text{kg/m}^3\right) = \boxed{3.00\times10^3\ \text{kg/m}^3}$$

39 •

Picture the Problem We can use the definition of density and Archimedes' principle to find the density of the unknown object. The difference between the weight of the object in air and in water is the buoyant force acting on the object.

(*a*) Using its definition, express the density of the object:

$$\rho_{object} = \frac{m_{object}}{V_{object}} \qquad (1)$$

Apply Archimedes' principle to obtain:

$$B = w_f = m_f g = \rho_f V_f g$$

Solve for V_f:

$$V_f = \frac{B}{\rho_f g}$$

Because $V_f = V_{object}$ and $\rho_f = \rho_{water}$:

$$V_{object} = \frac{B}{\rho_{water} g}$$

Substitute in equation (1) and simplify to obtain:

$$\rho_{object} = \frac{m_{object} g}{B}\rho_{water} = \frac{w_{object}}{B}\rho_{water}$$

Substitute numerical values and evaluate ρ_{object}:

$$\rho_{object} = \frac{5\,\text{N}}{5\,\text{N} - 4.55\,\text{N}}\left(10^3\ \text{kg/m}^3\right) = \boxed{11.1\times10^3\ \text{kg/m}^3}$$

(*b*) From Figure 13-1, we see that the unknown material has a density close to that of lead.

40 •

Picture the Problem We can use the definition of density and Archimedes' principle to find the density of the unknown object. The difference between the weight of the object in air and in water is the buoyant force acting on it.

Using its definition, express the density of the metal:

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}}{V_{\text{metal}}} \quad (1)$$

Apply Archimedes' principle to obtain:

$$B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{f}}V_{\text{f}}g$$

Solve for V_{f}:

$$V_{\text{f}} = \frac{B}{\rho_{\text{f}}g}$$

Because $V_{\text{f}} = V_{\text{metal}}$ and $\rho_{\text{f}} = \rho_{\text{water}}$:

$$V_{\text{metal}} = \frac{B}{\rho_{\text{water}}g}$$

Substitute in equation (1) and simplify to obtain:

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}g}{B}\rho_{\text{water}} = \frac{w_{\text{metal}}}{B}\rho_{\text{water}}$$

Substitute numerical values and evaluate ρ_{metal}:

$$\rho_{\text{metal}} = \frac{90\text{N}}{90\,\text{N} - 56.6\,\text{N}}\left(10^3\ \text{kg/m}^3\right) = \boxed{2.69 \times 10^3\ \text{kg/m}^3}$$

41 ••

Picture the Problem Let V be the volume of the object and V' be the volume that is submerged when it floats. The weight of the object is ρVg and the buoyant force due to the water is $\rho_{\text{w}}V'g$. Because the floating object is translational equilibrium, we can use $\sum F_y = 0$ to relate the buoyant forces acting on the object in the two liquids to its weight.

Apply $\sum F_y = 0$ to the object floating in water:

$$\rho_{\text{w}}V'g - mg = \rho_{\text{w}}V'g - \rho Vg = 0 \quad (1)$$

Solve for ρ:

$$\rho = \rho_{\text{w}}\frac{V'}{V}$$

Substitute numerical values and evaluate ρ:

$$\rho = \left(10^3\ \text{kg/m}^3\right)\frac{0.8V}{V} = \boxed{800\,\text{kg/m}^3}$$

Apply $\sum F_y = 0$ to the object floating in the second liquid and solve for *mg*:

$$mg = 0.72V\rho_L g$$

Solve equation (1) for *mg*:

$$mg = 0.8\rho_w Vg$$

Equate these two expressions to obtain:

$$0.72\rho_L = 0.8\rho_w$$

Substitute in the definition of specific gravity to obtain:

$$\text{specific gravity} = \frac{\rho_L}{\rho_w} = \frac{0.8}{0.72} = \boxed{1.11}$$

***42 ••**

Picture the Problem We can use Archimedes' principle to find the density of the unknown object. The difference between the weight of the block in air and in the fluid is the buoyant force acting on the block.

Apply Archimedes' principle to obtain:

$$B = w_f = m_f g = \rho_f V_f g$$

Solve for ρ_f:

$$\rho_f = \frac{B}{V_f g}$$

Because $V_f = V_{\text{Fe block}}$:

$$\rho_f = \frac{B}{V_{\text{Fe block}}g} = \frac{B}{m_{\text{Fe block}}g}\rho_{\text{Fe}}$$

Substitute numerical values and evaluate ρ_f:

$$\rho_f = \frac{(5\,\text{kg})(9.81\,\text{m/s}^2) - 6.16\,\text{N}}{(5\,\text{kg})(9.81\,\text{m/s}^2)}\left(7.96\times10^3\,\text{kg/m}^3\right) = \boxed{6.96\times10^3\,\text{kg/m}^3}$$

43 ••

Picture the Problem The forces acting on the cork are *B*, the upward force due to the displacement of water, *mg*, the weight of the piece of cork, and F_s, the force exerted by the spring. The piece of cork is in equilibrium under the influence of these forces.

Apply $\sum F_y = 0$ to the piece of cork:

$$B - w - F_s = 0 \qquad (1)$$

or

$$B - \rho_{\text{cork}}Vg - F_s = 0 \qquad (2)$$

Express the buoyant force as a function of the density of water:

$$B = w_f = \rho_w Vg$$

Solve for Vg:

$$Vg = \frac{B}{\rho_w}$$

Substitute for Vg in equation (2):

$$B - \rho_{cork}\frac{B}{\rho_w} - F_s = 0 \qquad (3)$$

Solve equation (1) for B:

$$B = w + F_s$$

Substitute in equation (3) to obtain:

$$w + F_s - \rho_{cork}\frac{w + F_s}{\rho_w} - F_s = 0$$

or

$$w - \rho_{cork}\frac{w + F_s}{\rho_w} = 0$$

Solve for ρ_{cork}:

$$\rho_{cork} = \rho_w \frac{w}{w + F_s}$$

Substitute numerical values and evaluate ρ_{cork}:

$$\rho_{cork} = \left(10^3\ \text{kg/m}^3\right)\frac{0.285\ \text{N}}{0.285\ \text{N} + 0.855\ \text{N}} = \boxed{250\ \text{kg/m}^3}$$

44 ••

Picture the Problem Under minimum-volume conditions, the balloon will be in equilibrium. Let B represent the buoyant force acting on the balloon, w_{tot} represent its total weight, and V its volume. The total weight is the sum of the weights of its basket, cargo, and helium in its balloon.

Apply $\sum F_y = 0$ to the balloon:

$$B - w_{tot} = 0$$

Express the total weight of the balloon:

$$w_{tot} = 2000\ \text{N} + \rho_{He} V g$$

Express the buoyant force due to the displaced air:

$$B = w_f = \rho_{air} V g$$

Substitute to obtain:

$$\rho_{air} V g - 2000\ \text{N} - \rho_{He} V g = 0$$

Solve for V:

$$V = \frac{2000\ \text{N}}{\left(\rho_{air} - \rho_{He}\right)g}$$

Substitute numerical values and evaluate V:

$$V = \frac{2000\,\text{N}}{\left(1.29\,\text{kg/m}^3 - 0.178\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)} = \boxed{183\,\text{m}^3}$$

***45 ••**

Picture the Problem Let V = volume of diver, ρ_D the density of the diver, V_{Pb} the volume of added lead, and m_{Pb} the mass of lead. The diver is in equilibrium under the influence of his weight, the weight of the lead, and the buoyant force of the water.

Apply $\sum F_y = 0$ to the diver:

$$B - w_D - w_{Pb} = 0$$

Substitute to obtain:

$$\rho_w V_{D+Pb} g - \rho_D V_D g - m_{Pb} g = 0$$

or

$$\rho_w V_D + \rho_w V_{Pb} - \rho_D V_D - m_{Pb} = 0$$

Rewrite this expression in terms of masses and densities:

$$\rho_w \frac{m_D}{\rho_D} + \rho_w \frac{m_{Pb}}{\rho_{Pb}} - \rho_D \frac{m_D}{\rho_D} - m_{Pb} = 0$$

Solve for the mass of the lead:

$$m_{Pb} = \frac{\rho_{Pb}\left(\rho_w - \rho_D\right) m_D}{\rho_D\left(\rho_{Pb} - \rho_w\right)}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{Pb} = \frac{\left(11.3\times10^3\,\text{kg/m}^3\right)\left(10^3\,\text{kg/m}^3 - 0.96\times10^3\,\text{kg/m}^3\right)\left(85\,\text{kg}\right)}{\left(0.96\times10^3\,\text{kg/m}^3\right)\left(11.3\times10^3\,\text{kg/m}^3 - 10^3\,\text{kg/m}^3\right)} = \boxed{3.89\,\text{kg}}$$

46 ••

Picture the Problem The scale's reading w' is the difference between the weight of the aluminum block in air w and the buoyant force acting on it. The buoyant force is equal to the weight of the displaced fluid, which, in turn, is the product of its density and mass. We can apply a condition for equilibrium to relate the reading of the bottom scale to the weight of the beaker and its contents and the buoyant force acting on the block.

Express the apparent weight w' of the aluminum block:

$$w' = w - B \qquad (1)$$

Letting F be the reading of the bottom scale and choosing upward to be the positive y direction, apply

$$F + w' - M_{tot} g = 0 \qquad (2)$$

$\sum F_y = 0$ to the scale to obtain:

Using the definition of density and Archimedes' principle, substitute for *w* and *B* in equation (1) to obtain:

$$w' = \rho_{\text{Al}} Vg - \rho_{\text{w}} Vg = (\rho_{\text{Al}} - \rho_{\text{w}})Vg$$

Express *w* in terms of ρ_{Al} and *V* and solve for *Vg*:

$$w = \rho_{\text{Al}} Vg \Rightarrow Vg = \frac{w}{\rho_{\text{Al}}}$$

Substitute to obtain:

$$w' = (\rho_{\text{Al}} - \rho_{\text{w}})\frac{w}{\rho_{\text{Al}}} = \left(1 - \frac{\rho_{\text{w}}}{\rho_{\text{Al}}}\right)w$$

Substitute numerical values and evaluate *w′*:

$$w' = \left(1 - \frac{10^3\ \text{kg/m}^3}{2.7\times10^3\ \text{kg/m}^3}\right) \times (2\ \text{kg})(9.81\ \text{m/s}^2) = \boxed{12.4\ \text{N}}$$

Solve equation (2) for *F*:

$$F = M_{\text{tot}} g - w'$$

Substitute numerical values and evaluate the reading of the bottom scale:

$$F = (5\ \text{kg})(9.81\ \text{m/s}^2) - 12.4\ \text{N} = \boxed{36.7\ \text{N}}$$

47 •••

Picture the Problem Let *V* = displacement of ship in the two cases, *m* be the mass of ship without load, and Δm be the load. The ship is in equilibrium under the influence of the buoyant force exerted by the water and its weight. We'll apply the condition for floating in the two cases and solve the equations simultaneously to determine the loaded mass of the ship.

Apply $\sum F_y = 0$ to the ship in fresh water: $\rho_{\text{w}} Vg - mg = 0$ (1)

Apply $\sum F_y = 0$ to the ship in salt water: $\rho_{\text{sw}} Vg - (m + \Delta m)g = 0$ (2)

Solve equation (1) for *Vg*:

$$Vg = \frac{mg}{\rho_{\text{w}}}$$

Substitute in equation (2) to obtain:

$$\rho_{\text{sw}} \frac{mg}{\rho_{\text{w}}} - (m + \Delta m)g = 0$$

Solve for m:

$$m = \frac{\rho_w \Delta m}{\rho_{sw} - \rho_w}$$

Add Δm to both sides of the equation and simplify to obtain:

$$m + \Delta m = \frac{\rho_w \Delta m}{\rho_{sw} - \rho_w} + \Delta m$$
$$= \Delta m\left(\frac{\rho_w}{\rho_{sw} - \rho_w} + 1\right)$$
$$= \frac{\Delta m \rho_{sw}}{\rho_{sw} - \rho_w}$$

Substitute numerical values and evaluate $m + \Delta m$:

$$m + \Delta m = \frac{(6\times 10^5\ \text{kg})(1.025\rho_w)}{1.025\rho_w - \rho_w}$$
$$= \frac{(6\times 10^5\ \text{kg})(1.025)}{1.025 - 1}$$
$$= \boxed{2.46\times 10^7\ \text{kg}}$$

***48 •••**

Picture the Problem For minimum liquid density, the bulb and its stem will be submerged. For maximum liquid density, only the bulb is submerged. In both cases the hydrometer will be in equilibrium under the influence of its weight and the buoyant force exerted by the liquids.

(*a*) Apply $\sum F_y = 0$ to the hydrometer:

$$B - w = 0$$

Using Archimedes' principle to express B, substitute to obtain:

$$\rho_{min} V g - m_{tot} g = 0$$

or

$$\rho_{min}(V_{bulb} + V_{stem}) = m_{glass} + m_{Pb}$$

Solve for m_{Pb}:

$$m_{Pb} = \rho_{min}(V_{bulb} + V_{stem}) - m_{glass}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{Pb} = (0.9\,\text{kg/L})\left[0.020\,\text{L} + \frac{\pi}{4}(0.15\,\text{m})(0.005\,\text{m})^2\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right)\right] - (6\times 10^{-3}\,\text{kg})$$
$$= \boxed{14.7\,\text{g}}$$

(*b*) Apply $\sum F_y = 0$ to the hydrometer:

$$\rho_{max} V g - m_{tot} g = 0$$

or

$$\rho_{max}V_{bulb} = m_{glass} + m_{Pb}$$

Solve for ρ_{max}:

$$\rho_{max} = \frac{m_{glass} + m_{Pb}}{V_{bulb}}$$

Substitute numerical values and evaluate ρ_{max}:

$$\rho_{max} = \frac{6\,g + 14.7\,g}{20\,mL} = \boxed{1.04\,kg/L}$$

49 •

Picture the Problem We can relate the upward force exerted on the dam wall to the area over which it acts using $F = P_g A$ and express P_g in terms of the depth of the water using $P_g = \rho g h$.

Using the definition of pressure, express the upward force exerted on the dam wall:

$$F = P_g A$$

Express the gauge pressure P_g of the water 5 m below the surface of the dam:

$$P_g = \rho g h$$

Substitute to obtain:

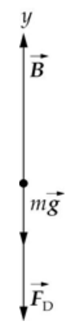
$$F = \rho g h A$$

Substitute numerical values and evaluate F:

$$F = (10^3\,kg/m^3)(9.81\,m/s^2)(5\,m)(10\,m^2) = \boxed{491\,kN}$$

50 ••

Picture the Problem The forces acting on the balloon are the buoyant force B, its weight mg, and a drag force F_D. We can find the initial upward acceleration of the balloon by applying Newton's 2nd law at the instant it is released. We can find the terminal velocity of the balloon by recognizing that when $a_y = 0$, the net force acting on the balloon will be zero.



(*a*) Apply $\sum F_y = ma_y$ to the balloon at the instant of its release to obtain:

$$B - m_{balloon}g = m_{balloon}a_y$$

Solve for a_y:

$$a_y = \frac{B - m_{balloon}g}{m_{balloon}} = \frac{B}{m_{balloon}} - g$$

Using Archimedes principle, express the buoyant force B acting on the balloon:

$$B = w_{\text{f}} = m_{\text{f}} g = \rho_{\text{f}} V_{\text{f}} g = \rho_{\text{air}} V_{\text{balloon}} g = \tfrac{4}{3}\pi \rho_{\text{air}} r^3 g$$

Substitute to obtain:

$$\tfrac{4}{3}\pi \rho_{\text{air}} r^3 g - m_{\text{balloon}} g = m_{\text{balloon}} a_y$$

Solve for a_y:

$$a_y = \left(\frac{\tfrac{4}{3}\pi \rho_{\text{air}} r^3}{m_{\text{balloon}}} - 1\right) g$$

Substitute numerical values and evaluate a_y:

$$a_y = \left[\frac{\tfrac{4}{3}\pi \left(1.29\,\text{kg/m}^3\right)\left(2.5\,\text{m}\right)^3}{15\,\text{kg}} - 1\right] \times \left(9.81\,\text{m/s}^2\right) = \boxed{45.4\,\text{m/s}^2}$$

(*b*) Apply $\sum F_y = ma_y$ to the balloon under terminal-speed conditions to obtain:

$$B - mg - \tfrac{1}{2}\pi r^2 \rho v_{\text{t}}^2 = 0$$

Substitute for B:

$$\tfrac{4}{3}\pi \rho_{\text{air}} r^3 g - mg - \tfrac{1}{2}\pi r^2 \rho v_{\text{t}}^2 = 0$$

Solve for v_{t}:

$$v_{\text{t}} = \sqrt{\frac{2\left(\tfrac{4}{3}\pi \rho_{\text{air}} r^3 - m\right) g}{\pi r^2 \rho}}$$

Substitute numerical values and evaluate v:

$$v_{\text{t}} = \sqrt{\frac{2\left[\tfrac{4}{3}\pi \left(1.29\,\text{kg/m}^3\right)\left(2.5\,\text{m}\right)^3 - 15\,\text{kg}\right]\left(9.81\,\text{m/s}^2\right)}{\pi \left(2.5\,\text{m}\right)^2 \left(1.29\,\text{kg/m}^3\right)}} = \boxed{7.33\,\text{m/s}}$$

(*c*) Relate the time required for the balloon to rise to 10 km to its terminal speed:

$$\Delta t = \frac{h}{v_t} = \frac{10\,\text{km}}{7.33\,\text{m/s}} = 1364\,\text{s} = \boxed{22.7\,\text{min}}$$

Continuity and Bernoulli's Equation

***51 ••**

Picture the Problem Let J represent the flow rate of the water. Then we can use $J = Av$ to relate the flow rate to the cross-sectional area of the circular tap and the velocity of the water. In (*b*) we can use the equation of continuity to express the diameter of the stream 7.5 cm below the tap and a constant-acceleration equation to find the velocity of the water at this distance. In (*c*) we can use a constant-acceleration equation to express the distance-to-turbulence in terms of the velocity of the water at turbulence v_{t} and the definition of Reynolds number N_{R} to relate v_{t} to N_{R}.

(*a*) Express the flow rate of the water in terms of the cross-sectional area A of the circular tap and the velocity v of the water:

$$J = Av = \pi r^2 v = \tfrac{1}{4}\pi d^2 v \qquad (1)$$

Solve for v:

$$v = \frac{J}{\frac{1}{4}\pi d^2}$$

Substitute numerical values and evaluate v:

$$v = \frac{10.5\,\text{cm}^3/\text{s}}{\frac{1}{4}\pi(1.2\,\text{cm})^2} = \boxed{9.28\,\text{cm/s}}$$

(*b*) Apply the equation of continuity to the stream of water:

$$v_\text{f} A_\text{f} = v_\text{i} A_\text{i} = vA_\text{i}$$

or

$$v_\text{f}\frac{\pi}{4}d_\text{f}^2 = v\frac{\pi}{4}d_\text{i}^2$$

Solve for d_f:

$$d_\text{f} = \sqrt{\frac{v}{v_\text{f}}}d_\text{i} \qquad (2)$$

Use a constant-acceleration equation to relate v_f and v to the distance Δh fallen by the water:

$$v_\text{f}^2 = v^2 + 2g\Delta h$$

Solve for v_f to obtain:

$$v_\text{f} = \sqrt{v^2 + 2g\Delta h}$$

Substitute numerical values and evaluate v_f:

$$v_\text{f} = \sqrt{(9.28\,\text{cm/s})^2 + 2(981\,\text{cm/s}^2)(7.5\,\text{cm})} = 122\,\text{cm/s}$$

Substitute in equation (2) and evaluate d_f:

$$d_\text{f} = (1.2\,\text{cm})\sqrt{\frac{9.28\,\text{cm/s}}{122\,\text{cm/s}}} = \boxed{0.331\,\text{cm}}$$

(*c*) Using a constant-acceleration equation, relate the fall-distance-to-turbulence Δd to its initial speed v and its speed v_t when its flow becomes turbulent:

$$v_\text{t}^2 = v^2 + 2g\Delta d$$

Solve for Δd to obtain:

$$\Delta d = \frac{v_\text{t}^2 - v^2}{2g} \qquad (3)$$

Express Reynolds number N_R for turbulent flow:

$$N_\text{R} = \frac{2r\rho v_\text{t}}{\eta}$$

From equation (1):

$$r = \sqrt{\frac{J}{\pi v_t}}$$

Substitute to obtain:

$$N_R = \frac{2\rho v_t}{\eta}\sqrt{\frac{J}{\pi v_t}}$$

Solve for v_t:

$$v_t = \frac{\pi N_R^2 \eta^2}{4\rho^2 J}$$

Substitute numerical values (see Figure 13-1 for the density of water and Table 13-1 for the coefficient of viscosity for water) and evaluate v_t:

$$v_t = \frac{\pi(2300)^2(1.8\times10^{-3}\ \text{Pa}\cdot\text{s})^2}{4(10^3\ \text{kg/m}^3)^2(10.5\,\text{cm}^3/\text{s})}$$
$$= 1.28\,\text{m/s}$$

Substitute in equation (3) and evaluate the fall-distance-to turbulence:

$$\Delta d = \frac{(128\,\text{cm/s})^2 - (9.28\,\text{cm/s})^2}{2(981\,\text{cm/s}^2)}$$
$$= \boxed{8.31\,\text{cm}}$$

in reasonable agreement with everyday experience.

52 •

Picture the Problem Let A_1 represent the cross-sectional area of the hose, A_2 the cross-sectional area of the nozzle, v_1 the velocity of the water in the hose, and v_2 the velocity of the water as it passes through the nozzle. We can use the continuity equation to find v_2 and Bernoulli's equation for constant elevation to find the pressure at the pump.

(*a*) Using the continuity equation, relate the speeds of the water to the diameter of the hose and the diameter of the nozzle:

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$$

Solve for v_2:

$$v_2 = \frac{d_1^2}{d_2^2} v_1$$

Substitute numerical values and evaluate v_2:

$$v_2 = \left(\frac{3\,\text{cm}}{0.3\,\text{cm}}\right)^2 (0.65\,\text{m/s}) = \boxed{65.0\,\text{m/s}}$$

(*b*) Using Bernoulli's equation for constant elevation, relate the pressure at the pump P_P to the

$$P_P + \tfrac{1}{2}\rho v_1^2 = P_{at} + \tfrac{1}{2}\rho v_2^2$$

atmospheric pressure and the velocities of the water in the hose and the nozzle:

Solve for the pressure at the pump:

$$P_P = P_{at} + \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

Substitute numerical values and evaluate P_P:

$$P_P = 101\,\text{kPa} + \tfrac{1}{2}\left(10^3\,\text{kg/m}^3\right)\left[(65\,\text{m/s})^2 - (0.65\,\text{m/s})^2\right]$$
$$= 2.21\times10^6\,\text{Pa}\times\frac{1\,\text{atm}}{101.325\,\text{kPa}} = \boxed{21.9\,\text{atm}}$$

53 •

Picture the Problem Let A_1 represent the cross-sectional area of the larger-diameter pipe, A_2 the cross-sectional area of the smaller-diameter pipe, v_1 the velocity of the water in the larger-diameter pipe, and v_2 the velocity of the water in the smaller-diameter pipe. We can use the continuity equation to find v_2 and Bernoulli's equation for constant elevation to find the pressure in the smaller-diameter pipe.

(*a*) Using the continuity equation, relate the velocities of the water to the diameters of the pipe:

$$A_1v_1 = A_2v_2$$
or
$$\frac{\pi d_1^2}{4}v_1 = \frac{\pi d_2^2}{4}v_2$$

Solve for and evaluate v_2:

$$v_2 = \frac{d_1^2}{d_2^2}v_1$$

Substitute numerical values and evaluate v_2:

$$v_2 = \left(\frac{d_1}{\tfrac{1}{2}d_1}\right)^2(3\,\text{m/s}) = \boxed{12.0\,\text{m/s}}$$

(*b*) Using Bernoulli's equation for constant elevation, relate the pressures in the two segments of the pipe to the velocities of the water in these segments:

$$P_1 + \tfrac{1}{2}\rho_w v_1^2 = P_2 + \tfrac{1}{2}\rho_w v_2^2$$

Solve for P_2:

$$P_2 = P_1 + \tfrac{1}{2}\rho_w v_1^2 - \tfrac{1}{2}\rho_w v_2^2$$
$$= P_1 + \tfrac{1}{2}\rho_w\left(v_1^2 - v_2^2\right)$$

Substitute numerical values and evaluate P_2:

$$P_2 = 200\,\text{kPa} + \tfrac{1}{2}\left(10^3\ \text{kg/m}^3\right) \times \left[(3\,\text{m/s})^2 - (12\,\text{m/s})^2\right] = \boxed{133\,\text{kPa}}$$

(*c*) Using the continuity equation, evaluate I_{V1}:

$$I_{V1} = A_1 v_1 = \frac{\pi\, d_1^2}{4} v_1 = \frac{\pi\, d_1^2}{4}(3\,\text{m/s})$$

Using the continuity equation, express I_{V2}:

$$I_{V2} = A_2 v_2 = \frac{\pi\, d_2^2}{4} v_2$$

Substitute numerical values and evaluate I_{V2}:

$$I_{V2} = \frac{\pi\left(\frac{d_1}{2}\right)^2}{4}(12\,\text{m/s}) = \frac{\pi\, d_1^2}{4}(3\,\text{m/s})$$

Thus, as we expected would be the case:

$$\boxed{I_{V1} = I_{V2}}$$

54 •

Picture the Problem Let A_1 represent the cross-sectional area of the 2-cm diameter pipe, A_2 the cross-sectional area of the constricted pipe, v_1 the velocity of the water in the 2-cm diameter pipe, and v_2 the velocity of the water in the constricted pipe. We can use the continuity equation to express d_2 in terms of d_1 and to find v_1 and Bernoulli's equation for constant elevation to find the velocity of the water in the constricted pipe.

Using the continuity equation, relate the volume flow rate in the 2-cm diameter pipe to the volume flow rate in the constricted pipe:

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$$

Solve for d_2:

$$d_2 = d_1 \sqrt{\frac{v_1}{v_2}}$$

Using the continuity equation, relate v_1 to the volume flow rate I_V:

$$v_1 = \frac{I_V}{A_1} = \frac{2.80\,\text{L/s}}{\frac{\pi(0.02\,\text{m})^2}{4}} = 8.91\,\text{m/s}$$

Using Bernoulli's equation for constant elevation, relate the pressures in the two segments of the pipe to the velocities of the water in

$$P_1 + \tfrac{1}{2}\rho_w v_1^2 = P_2 + \tfrac{1}{2}\rho_w v_2^2$$

these segments:

Solve for v_2:

$$v_2 = \sqrt{\frac{2(P_1 - P)}{\rho_w} + v_1^2}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{\frac{2(142\,\text{kPa} - 101\,\text{kPa})}{10^3\,\text{kg/m}^3} + (8.91\,\text{m/s})^2}$$
$$= 12.7\,\text{m/s}$$

Substitute and evaluate d_2:

$$d_2 = (2\,\text{cm})\sqrt{\frac{8.91\,\text{m/s}}{12.7\,\text{m/s}}} = \boxed{1.68\,\text{cm}}$$

***55 ••**

Picture the Problem We can use the definition of the volume flow rate to find the volume flow rate of blood in an aorta and to find the total cross-sectional area of the capillaries.

(*a*) Use the definition of the volume flow rate to find the volume flow rate through an aorta:

$$I_V = Av$$

Substitute numerical values and evaluate I_V:

$$I_V = \pi(9\times10^{-3}\,\text{m}^3)(0.3\,\text{m/s})$$
$$= 7.63\times10^{-5}\,\frac{\text{m}^3}{\text{s}}\times\frac{60\,\text{s}}{\text{min}}\times\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}$$
$$= \boxed{4.58\,\text{L/min}}$$

(*b*) Use the definition of the volume flow rate to express the volume flow rate through the capillaries:

$$I_V = A_{\text{cap}}v_{\text{cap}}$$

Solve for the total cross-sectional area of the capillaries:

$$A_{\text{cap}} = \frac{I_V}{v_{\text{cap}}}$$

Substitute numerical values and evaluate A_{cap}:

$$A_{\text{cap}} = \frac{7.63\times10^{-5}\,\text{m}^3/\text{s}}{0.001\,\text{m/s}}$$
$$= 7.63\times10^{-2}\,\text{m}^2 = \boxed{763\,\text{cm}^2}$$

56 ••

Picture the Problem We can apply Bernoulli's equation to points *a* and *b* to determine the rate at which the water exits the tank. Because the diameter of the small pipe is much smaller than the diameter of the tank, we can neglect the velocity of the water at the point *a*. The distance the water travels once it exits the pipe is the product of its speed and the time required to fall the distance $H - h$.

Express the distance x as a function of the exit speed of the water and the time to fall the distance $H - h$:

$$x = v_b \Delta t \qquad (1)$$

Apply Bernoulli's equation to the water at points *a* and *b*:

$$P_a + \rho_w gH + \tfrac{1}{2}\rho_w v_a^2 = P_b + \rho_w g(H-h) + \tfrac{1}{2}\rho_w v_b^2$$

or, because $v_a \approx 0$ and $P_a = P_b = P_{at}$,

$$gH = g(H-h) + \tfrac{1}{2}v_b^2$$

Solve for v_b:

$$v_b = \sqrt{2gh}$$

Using a constant-acceleration equation, relate the time of fall to the distance of fall:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$H - h = \tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2(H-h)}{g}}$$

Substitute in equation (1) to obtain:

$$x = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = \boxed{2\sqrt{h(H-h)}}$$

57 ••

Picture the Problem Let the subscript 60 denote the 60-cm-radius pipe and the subscript 30 denote the 30-cm-radius pipe. We can use Bernoulli's equation for constant elevation to express P_{30} in terms of v_{60} and v_{30}, the definition of volume flow rate to find v_{60} and the continuity equation to find v_{30}.

Using Bernoulli's equation for constant elevation, relate the pressures in the two pipes to the velocities of the oil:

$$P_{60} + \tfrac{1}{2}\rho v_{60}^2 = P_{30} + \tfrac{1}{2}\rho v_{30}^2$$

Solve for P_{30}:

$$P_{30} = P_{60} + \tfrac{1}{2}\rho\left(v_{60}^2 - v_{30}^2\right) \qquad (1)$$

Use the definition of volume flow rate to find v_{60}:

$$v_{60} = \frac{I_V}{A_{60}} = \frac{2.4\times10^5\,\frac{\text{m}^3}{\text{day}}\times\frac{1\,\text{day}}{24\,\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}}{\pi(0.6\,\text{m})^2} = 2.456\,\text{m/s}$$

Using the continuity equation, relate the velocity of the oil in the half-standard pipe to its velocity in the standard pipe:

$$A_{60}v_{60} = A_{30}v_{30}$$

Solve for and evaluate v_{30}:

$$v_{30} = \frac{A_{60}}{A_{30}}v_{60} = \frac{\pi(0.6\,\text{m})^2}{\pi(0.3\,\text{m})^2}(2.456\,\text{m/s}) = 9.824\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate P_{30}:

$$P_{30} = 180\,\text{kPa} + \tfrac{1}{2}\left(800\,\text{kg/m}^3\right)\left[(2.456\,\text{m/s})^2 - (9.824\,\text{m/s})^2\right] = \boxed{144\,\text{kPa}}$$

***58 ••**

Picture the Problem We'll use its definition to relate the volume flow rate in the pipe to the velocity of the water and the result of Example 13-9 to find the velocity of the water.

Using its definition, express the volume flow rate:

$$I_V = A_1v_1 = \pi r^2 v_1$$

Using the result of Example 13-9, find the velocity of the water upstream from the Venturi meter:

$$v_1 = \sqrt{\frac{2\rho_{Hg}gh}{\rho_w\left(\frac{R_1^2}{R_2^2} - 1\right)}}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \sqrt{\frac{2\left(13.6\times10^3\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)(0.024\,\text{m})}{\left(10^3\,\text{kg/m}^3\right)\left[\left(\frac{0.095\,\text{m}}{0.056\,\text{m}}\right)^2 - 1\right]}} = 1.847\,\text{m/s}$$

Substitute numerical values and evaluate I_V:

$$I_V = \frac{\pi}{4}(0.095\,\text{m})^2(1.847\,\text{m/s})$$
$$= 1.309\times10^{-2}\,\text{m}^3/\text{s}$$
$$= \boxed{13.1\,\text{L/s}}$$

59 ••

Picture the Problem We can apply the definition of the volume flow rate to find the mass of water emerging from the hose in 1 s and the definition of momentum to find the momentum of the water. The force exerted on the water by the hose can be found from the rate at which the momentum of the water changes.

(*a*) Using its definition, express the volume flow rate of the water emerging from the hose:

$$I_V = \frac{\Delta V}{\Delta t} = \frac{\Delta m}{\rho_w \Delta t} = Av$$

Solve for Δm:

$$\Delta m = Av\rho_w\Delta t$$

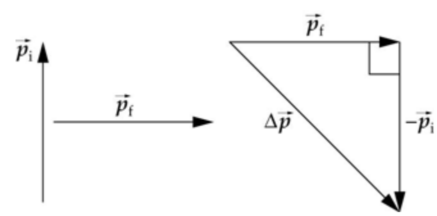
Substitute numerical values and evaluate Δm:

$$\Delta m = \pi(0.015\,\text{m})^2(30\,\text{m/s})(10^3\,\text{kg/m}^3)(1\,\text{s})$$
$$= \boxed{21.2\,\text{kg/s}}$$

(*b*) Using its definition, express and evaluate the momentum of the water:

$$p = \Delta m v = (21.2\,\text{kg/s})(30\,\text{m/s})$$
$$= \boxed{636\,\text{kg}\cdot\text{m/s}}$$

(*c*) The vector diagrams are to the right:



Express the change in momentum of the water:

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Substitute numerical values and evaluate Δp:

$$\Delta p = \sqrt{(636\,\text{kg}\cdot\text{m/s})^2 + (636\,\text{kg}\cdot\text{m/s})^2}$$
$$= (636\,\text{kg}\cdot\text{m/s})\sqrt{2}$$
$$= \boxed{899\,\text{kg}\cdot\text{m/s}}$$

Relate the force exerted on the water by the hose to the rate at which the water's momentum changes and evaluate F:

$$F = \frac{\Delta p}{\Delta t} = \frac{899\,\text{kg}\cdot\text{m/s}}{1\,\text{s}} = \boxed{899\,\text{N}}$$

60 ••

Picture the Problem Let the letter P denote the pump and the 2-cm diameter pipe and the letter N the 1-cm diameter nozzle. We'll use Bernoulli's equation to express the necessary pump pressure, the continuity equation to relate the velocity of the water coming out of the pump to its velocity at the nozzle, and a constant-acceleration equation to relate its velocity at the nozzle to the height to which the water rises.

Using Bernoulli's equation, relate the pressures, areas, and velocities in the pipe and nozzle:

$$P_P + \rho_w g h_P + \tfrac{1}{2}\rho_w v_P^2 = P_N + \rho_w g h_N + \tfrac{1}{2}\rho_w v_N^2$$

or, because $P_N = P_{at}$ and $h_P = 0$,

$$P_P + \tfrac{1}{2}\rho_w v_P^2 = P_N + \rho_w g h_N + \tfrac{1}{2}\rho_w v_N^2$$

Solve for the pump pressure:

$$P_P = P_{at} + \rho_w g h_N + \tfrac{1}{2}\rho_w\left(v_N^2 - v_P^2\right) \quad (1)$$

Use the continuity equation to relate v_P and v_N to the cross-sectional areas of the pipe from the pump and the nozzle:

$$A_P v_P = A_N v_N$$

and

$$v_P = \frac{A_N}{A_P} v_N = \frac{\frac{1}{4}\pi d_N^2}{\frac{1}{4}\pi d_P^2} v_N = \left(\frac{1\,\text{cm}}{2\,\text{cm}}\right)^2 v_N = \tfrac{1}{4} v_N$$

Using a constant-acceleration equation, express the velocity of the water at the nozzle in terms of the desired height Δh:

$$v^2 = v_N^2 - 2g\Delta h$$

or, because $v = 0$,

$$v_N^2 = 2g\Delta h$$

Substitute in equation (1) to obtain:

$$\begin{aligned} P_P &= P_{at} + \rho_w g h_N + \tfrac{1}{2}\rho_w\left[2g\Delta h - \tfrac{1}{16}(2g\Delta h)\right] = P_{at} + \rho_w g h_N + \tfrac{1}{2}\rho_w\left(\tfrac{15}{8}g\Delta h\right) \\ &= P_{at} + \rho_w g\left(h_N + \tfrac{15}{16}\Delta h\right) \end{aligned}$$

Substitute numerical values and evaluate P_P:

$$P_P = 101\,\text{kPa} + \left(10^3\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)\left[3\,\text{m} + \tfrac{15}{16}(12\,\text{m})\right] = \boxed{241\,\text{kPa}}$$

61 •••

Picture the Problem We can apply Bernoulli's equation to points *a* and *b* to determine the rate at which the water exits the tank. Because the diameter of the small pipe is much smaller than the diameter of the tank, we can neglect the velocity of the water at the point *a*. The distance the water travels once it exits the pipe is the product of its velocity and the time required to fall the distance $H - h$. That there are two values of *h* that are equidistant from the point $h = \frac{1}{2}H$ can be shown by solving the quadratic equation that relates *x* to *h* and *H*. That *x* is a maximum for this value of *h* can be established by treating $x = f(h)$ as an extreme-value problem.

(*a*) Express the distance *x* as a function of the exit speed of the water and the time to fall the distance $H - h$:

$$x = v_b \Delta t \qquad (1)$$

Apply Bernoulli's equation to the water at points *a* and *b*:

$$P_a + \rho_w gH + \tfrac{1}{2}\rho_w v_a^2 = P_b + \rho_w g(H-h) + \tfrac{1}{2}\rho_w v_b^2$$

or, because $v_a \approx 0$ and $P_a = P_b = P_{at}$,

$$gH = g(H-h) + \tfrac{1}{2}v_b^2$$

Solve for v_b:

$$v_b = \sqrt{2gh}$$

Using a constant-acceleration equation, relate the time of fall to the distance of fall:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$H - h = \tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2(H-h)}{g}}$$

Substitute in equation (1) to obtain:

$$x = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = \boxed{2\sqrt{h(H-h)}}$$

(*b*) Square both sides of this equation and simplify to obtain:

$$x^2 = 4hH - 4h^2 \text{ or } 4h^2 - 4Hh + x^2 = 0$$

Solve this quadratic equation to obtain:

$$h = \boxed{\tfrac{1}{2}H \pm \tfrac{1}{2}\sqrt{H^2 - x^2}}$$

Find the average of these two values for h:

$$h_{av} = \frac{\frac{1}{2}H + \sqrt{H^2 - x^2} + \frac{1}{2}H - \sqrt{H^2 - x^2}}{2}$$

$$= \boxed{\tfrac{1}{2}H}$$

(c) Differentiate $x = 2\sqrt{h(H-h)}$ with respect to h:

$$\frac{dx}{dh} = 2\left(\frac{1}{2}\right)\left[h(H-h)\right]^{-\frac{1}{2}}(H-2h)$$

$$= \frac{H-2h}{\sqrt{h(H-h)}}$$

Set the derivative equal to zero for extrema:

$$\frac{H-2h}{\sqrt{h(H-h)}} = 0$$

Solve for h to obtain:

$$h = \boxed{\tfrac{1}{2}H}$$

Evaluate $x = 2\sqrt{h(H-h)}$ with $h = \frac{1}{2}H$:

$$x_{max} = 2\sqrt{\tfrac{1}{2}H\left(H - \tfrac{1}{2}H\right)}$$

$$= \boxed{H}$$

Remarks: To show that this value for *h* corresponds to a *maximum*, one can either show that $\frac{d^2x}{dh^2} < 0$ at $h = \frac{1}{2}H$ or confirm that the graph of *f*(*h*) at $h = \frac{1}{2}H$ is concave downward.

***62 ••**

Picture the Problem Let the numeral 1 denote the opening in the end of the inner pipe and the numeral 2 to one of the holes in the outer tube. We can apply Bernoulli's principle at these locations and solve for the pressure difference between them. By equating this pressure difference to the pressure difference due to the height h of the liquid column we can express v as a function of ρ, ρ_g, g, and h.

Apply Bernoulli's principle at locations 1 and 2 to obtain:

$$P_1 + \tfrac{1}{2}\rho_g v_1^2 = P_2 + \tfrac{1}{2}\rho_g v_2^2$$

where we've ignored the difference in elevation between the two openings.

Solve for the pressure difference $\Delta P = P_1 - P_2$:

$$\Delta P = P_1 - P_2 = \tfrac{1}{2}\rho_g v_2^2 - \tfrac{1}{2}\rho_g v_1^2$$

Express the velocity of the gas at 1:

$v_1 = 0$ because the gas is brought to a halt (i.e., is stagnant) at the opening to the inner pipe.

Express the velocity of the gas at 2:

$v_2 = v$ because the gas flows freely past the holes in the outer ring.

Substitute to obtain:

$$\Delta P = \tfrac{1}{2}\rho_g v^2$$

Letting A be the cross-sectional area of the tube, express the pressure at a depth h in the column of liquid whose density is ρ_1:

$$P_1 = P_2 + \frac{w_{\text{displaced liquid}}}{A} - \frac{B}{A}$$

where $B = \rho_g Ahg$ is the buoyant force acting on the column of liquid of height h.

Substitute to obtain:

$$P_1 = P_2 + \frac{\rho ghA}{A} - \frac{\rho_g ghA}{A}$$
$$= P_2 + (\rho - \rho_g)gh$$

or

$$\Delta P = P_1 - P_2 = (\rho - \rho_g)gh$$

Equate these two expressions for ΔP:

$$\tfrac{1}{2}\rho_g v^2 = (\rho - \rho_g)gh$$

Solve for v^2 to obtain:

$$\boxed{v^2 = \frac{2gh(\rho - \rho_g)}{\rho_g}}$$

Note that the correction for buoyant force due to the displaced gas is very small and that, to a good approximation,

$$v = \sqrt{\frac{2gh\rho}{\rho_g}}.$$

Remarks: Pitot tubes are used to measure the airspeed of airplanes.

63 ••

Picture the Problem Let the letter ″a″ denote the entrance to the siphon tube and the letter ″b″ denote its exit. Assuming streamline flow between these points, we can apply Bernoulli's equation to relate the entrance and exit speeds of the water flowing in the siphon to the pressures at either end, the density of the water, and the difference in elevation between the entrance and exit points. We can use the expression for the pressure as a function of depth in an incompressible fluid to find the pressure at the entrance to the tube in terms of its distance below the surface. We'll also use the equation of continuity to argue that, provided the surface area of the beaker is large compared to the area of the opening of the tube, the entrance speed of the water is approximately zero.

(*a*) Apply Bernoulli's equation at the entrance to the siphon tube (point a) and at its exit (point b):

$$P_a + \tfrac{1}{2}\rho v_a^2 + \rho g(H - h)$$
$$= P_b + \tfrac{1}{2}\rho v_b^2 + \rho g(H - h - d) \quad (1)$$

where H is the height of the containers.

Apply the continuity equation to a point at the surface of the liquid in the container to the left and to point a:

$$v_a A_a = v_{surface} A_{surface}$$

or, because $A_a << A_{surface}$,

$$v_a = v_{surface} = 0$$

Express the pressure at the inlet (point a) and the outlet (point b):

$$P_a = P_{atm} + \rho g(H - h)$$

and

$$P_b = P_{atm} + \rho g(H - h - d)$$

Letting $v_b = v$, substitute in equation (1) to obtain:

$$P_{atm} + \rho g(H - h) + \rho g H = P_{atm} + \rho g(H - h - d) + \tfrac{1}{2}\rho v^2 + \rho g(H - h - d)$$

or, upon simplification,

$$g(H - h) + gH = g(H - h - d) + \tfrac{1}{2}v^2 + g(H - h - d)$$

Solve for v:

$$v = \boxed{\sqrt{2gd}}$$

(*b*) Relate the pressure at the highest part of the tube P_{top} to the pressure at point b:

$$P_{top} + \rho g(H - h) + \tfrac{1}{2}\rho v_h^2 = P_{atm} + \rho g(H - h - d) + \tfrac{1}{2}\rho v_b^2$$

or, because $v_h = v_b$,

$$P_{top} = \boxed{P_{atm} - \rho g d}$$

Remarks: If we let P_{top} = 0, we can use this result to find the maximum theoretical height a siphon can lift water.

Viscous Flow

64 •

Picture the Problem The required pressure difference can be found by applying Poiseuille's law to the viscous flow of water through the horizontal tube.

Using Poiseuille's law, relate the pressure difference between the two ends of the tube to its length, radius, and the volume flow rate of the water:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Substitute numerical values and evaluate ΔP:

$$\Delta P = \frac{8(1\,\text{mPa}\cdot\text{s})(0.25\,\text{m})}{\pi(0.6\times10^{-3}\,\text{m})^4}(0.3\,\text{mL/s}) = \boxed{1.47\,\text{kPa}}$$

65 •

Picture the Problem Because the pressure difference is unchanged, we can equate the expressions of Poiseuille's law for the two tubes and solve for the diameter of the tube that would double the flow rate.

Using Poiseuille's law, express the pressure difference required for the radius and volume flow rate of Problem 64:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Express the pressure difference required for the radius r' that would double the volume flow rate of Problem 57:

$$\Delta P = \frac{8\eta L}{\pi r'^4}(2I_V)$$

Equate these equations and simplify to obtain:

$$\frac{8\eta L}{\pi r'^4}(2I_V) = \frac{8\eta L}{\pi r^4} I_V$$

or

$$\frac{2}{r'^4} = \frac{1}{r^4}$$

Solve for r':

$$r' = \sqrt[4]{2}r$$

Express d':

$$d' = 2r' = 2\sqrt[4]{2}r = \sqrt[4]{2}d$$

Substitute numerical values and evaluate d':

$$d' = \sqrt[4]{2}(1.2\,\text{mm}) = \boxed{1.43\,\text{mm}}$$

***66** •

Picture the Problem We can apply Poiseuille's law to relate the pressure drop across the capillary tube to the radius and length of the tube, the rate at which blood is flowing through it, and the viscosity of blood.

Using Poiseuille's law, relate the pressure drop to the length and diameter of the capillary tube, the volume flow rate of the blood, and the viscosity of the blood:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Solve for the viscosity of the blood:

$$\eta = \frac{\pi r^4 \Delta P}{8 L I_V}$$

Using its definition, express the volume flow rate of the blood:

$$I_V = A_{cap}v = \pi r^2 v$$

Substitute and simplify:

$$\eta = \frac{r^2 \Delta P}{8Lv}$$

Substitute numerical values to obtain:

$$\eta = \frac{(3.5\times10^{-6}\,\text{m})^2(2.60\,\text{kPa})}{8(10^{-3}\,\text{m})\left(\dfrac{10^{-3}\,\text{m}}{1\,\text{s}}\right)} = \boxed{3.98\,\text{mPa}\cdot\text{s}}$$

***67** •

Picture the Problem We can use the definition of Reynolds number to find the velocity of a baseball at which the drag crisis occurs.

Using its definition, relate Reynolds number to the velocity v of the baseball:

$$N_R = \frac{2r\rho v}{\eta}$$

Solve for v:

$$v = \frac{\eta N_R}{2r\rho}$$

Substitute numerical values (see Figure 13-1 for the density of air and Table 13-1 for the coefficient of viscosity for air) and evaluate v:

$$v = \frac{(0.018\,\text{mPa}\cdot\text{s})(3\times10^5)}{2(0.05\,\text{m})(1.293\,\text{kg/m}^3)} = 41.8\,\text{m/s}\times\frac{1\,\text{mi/h}}{0.447\,\text{m/s}} = \boxed{93.4\,\text{mi/h}}$$

Because most major league pitchers can throw a fastball in the low - to mid - 90s, this drag crisis may very well play a role in the game.

Remarks: This is a topic which has been fiercely debated by people who study the physics of baseball.

68 •••

Picture the Problem Let the subscripts ″f″ refer to ″displaced fluid”, ″s″ to ″soda″, and ″g″ to the ″gas″ in the bubble. The free-body diagram shows the forces acting on the bubble prior to reaching its terminal velocity. We can apply Newton’s 2nd law, Stokes′ law, and Archimedes principle to express the terminal velocity of the bubble in terms of its radius, and the viscosity and density of water.

Apply $\sum F_y = ma_y$ to the bubble to obtain:

$$B - m_g g - F_D = ma_y$$

Under terminal speed conditions:

$$B - m_g g - F_D = 0$$

Using Archimedes principle, express the buoyant force B acting on the bubble:

$$B = w_f = m_f g = \rho_f V_f g = \rho_s V_{bubble} g$$

Express the mass of the gas bubble:

$$m_g = \rho_g V_g = \rho_g V_{bubble}$$

Substitute to obtain:

$$\rho_w V_{bubble} g - \rho_g V_{bubble} g - 6\pi\eta a v_t = 0$$

Solve for v_t:

$$v_t = \frac{V_{bubble} g(\rho_s - \rho_g)}{6\pi\eta a}$$

Substitute for V_{bubble} and simplify:

$$v_t = \frac{\frac{4}{3}\pi a^3 g(\rho_s - \rho_g)}{6\pi\eta a} = \frac{2a^2 g(\rho_s - \rho_g)}{9\eta}$$

$$\approx \frac{2a^2 g\rho_s}{9\eta}, \text{ since } \rho_s >> \rho_g.$$

Substitute numerical values and evaluate v_t:

$$v_t = \frac{2(0.5\times10^{-3}\,\text{m})^2(9.81\,\text{m/s}^2)}{9(1.8\times10^{-3}\,\text{Pa}\cdot\text{s})} \times (1.1\times10^3\,\text{kg/m}^3) = \boxed{0.333\,\text{m/s}}$$

Express the rise time Δt in terms of the height of the soda glass h and the terminal speed of the bubble:

$$\Delta t = \frac{h}{v_t}$$

Assuming that a "typical" soda glass has a height of about 15 cm, evaluate Δt:

$$\Delta t = \frac{0.15\,\text{m}}{0.333\,\text{m/s}} = \boxed{0.450\,\text{s}}$$

Remarks: About half a second seems reasonable for the rise time of the bubble.

General Problems

***69 ••**

Picture the Problem We can solve the given equation for the coefficient of roundness C and substitute estimates/assumptions of typical masses and heights for adult males and females.

Express the mass of a person as a function of C, ρ, and h:

$$M = C\rho h^3$$

Solve for C:

$$C = \frac{M}{\rho h^3}$$

Assuming that a "typical" adult male stands 5' 10" (1.78 m) and weighs 170 lbs (77 kg), then:

$$C = \frac{77\,\text{kg}}{(10^3\,\text{kg/m}^3)(1.78\,\text{m})^3} = \boxed{0.0137}$$

Assuming that a "typical" adult female stands 5' 4" (1.63 m) and weighs 110 lbs (50 kg), then:

$$C = \frac{50\,\text{kg}}{(10^3\,\text{kg/m}^3)(1.63\,\text{m})^3} = \boxed{0.0115}$$

70 •

Picture the Problem Let the letter "s" denote the shorter of the two men and the letter "t" the taller man. We can find the difference in weight of the two men using the relationship $M = C\rho h^3$ from Problem 69.

Express the difference in weight of the two men:

$$\Delta w = w_t - w_s = M_t g - M_s g = (M_t - M_s)g$$

Express the masses of the two men:

$$M_s = C\rho h_s^3$$
and
$$M_t = C\rho h_t^3$$

Substitute to obtain:

$$\Delta w = (C\rho h_t^3 - C\rho h_s^3)g = (h_t^3 - h_s^3)C\rho g$$

Assuming that a "typical" adult male stands 5' 10" (1.78 m) and weighs 170 lbs (77 kg), then:

$$C = \frac{77\,\text{kg}}{(10^3\,\text{kg/m}^3)(1.78\,\text{m})^3} = 0.0137$$

Express the heights of the two men in SI units:

$$h_t = 72\,\text{in} \times 2.54\,\text{cm/in} = 1.83\,\text{m}$$
and

$$h_s = 69\,\text{in} \times 2.54\,\text{cm/in} = 1.75\,\text{m}$$

Substitute numerical values (assume that $\rho = 10^3$ kg/m^3) and evaluate Δw:

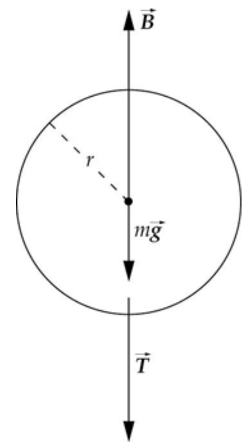
$$\Delta w = \left[(1.83\,\text{m})^3 - (1.75\,\text{m})^3\right] \times (0.0137)\left(10^3\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)$$
$$= 103\,\text{N} \times \frac{1\,\text{lb}}{4.4482\,\text{N}} = \boxed{23.2\,\text{lb}}$$

71 •

Determine the Concept The net force is zero. Neglecting the thickness of the table, the atmospheric pressure is the same above and below the surface of the table.

72 •

Picture the Problem The forces acting on the Ping-Pong ball, shown in the free-body diagram, are the buoyant force, the weight of the ball, and the tension in the string. Because the ball is in equilibrium under the influence of these forces, we can apply the condition for translational equilibrium to establish the relationship between them. We can also apply Archimedes' principle to relate the buoyant force on the ball to its diameter.



Apply $\sum F_y = 0$ to the ball:

$$B - mg - T = 0$$

Using Archimedes' principle, relate the buoyant force on the ball to its diameter:

$$B = w_f = m_f g = \rho_w V_{ball} g = \tfrac{1}{6}\pi\rho_w d^3$$

Substitute to obtain:

$$\tfrac{1}{6}\pi\rho_w d^3 - mg - T = 0$$

Solve for d:

$$d = \sqrt[3]{\frac{6(T + mg)}{\pi\rho_w}}$$

Substitute numerical values and evaluate d:

$$d = \sqrt[3]{\frac{6\left[2.8\times10^{-2}\,\text{N} + (0.004\,\text{kg})\left(9.81\,\text{m/s}^2\right)\right]}{\pi\left(10^3\,\text{kg/m}^3\right)}} = \boxed{5.05\,\text{cm}}$$

73 •

Picture the Problem Let ρ_0 represent the density of seawater at the surface. We can use the definition of density and the fact that mass is constant to relate the fractional change in the density of water to its fractional change in volume. We can also use the definition of bulk modulus to relate the fractional change in density to the increase in pressure with depth and solve the resulting equation for the density at the depth at which the pressure is 800 atm.

Using the definition of density, relate the mass of a given volume of seawater to its volume:

$$m = \rho V$$

Noting that the mass does not vary with depth, evaluate its differential:

$$\rho dV + V d\rho = 0$$

Solve for $d\rho/\rho$:

$$\frac{d\rho}{\rho} = -\frac{dV}{V} \text{ or } \frac{\Delta\rho}{\rho} \approx -\frac{\Delta V}{V}$$

Using the definition of the bulk modulus, relate ΔP to $\Delta\rho/\rho_0$:

$$B = -\frac{\Delta P}{\Delta V/V} = \frac{\Delta P}{\Delta\rho/\rho_0}$$

Solve $\Delta\rho$:

$$\Delta\rho = \rho - \rho_0 = \frac{\rho_0 \Delta P}{B}$$

Solve for ρ:

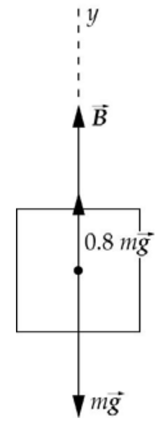
$$\rho = \rho_0 + \frac{\rho_0 \Delta P}{B} = \rho_0\left(1 + \frac{\Delta P}{B}\right)$$

Substitute numerical values and evaluate ρ:

$$\rho = \left(1025\,\text{kg/m}^3\right)\left(1 + \frac{800\,\text{atm} \times \frac{1.01\times10^5\ \text{Pa}}{1\,\text{atm}}}{2.3\times10^9\ \text{N/m}^2}\right) = \boxed{1061\ \text{kg/m}^3}$$

74 •

Picture the Problem When it is submerged, the block is in equilibrium under the influence of the buoyant force due to the water, the force exerted by the spring balance, and its weight. We can use the condition for translational equilibrium to relate the buoyant force to the weight of the block and the definition of density to express the weight of the block in terms of its density.



Apply $\sum F_y = 0$ to the block: $B + 0.8mg - mg = 0 \Rightarrow B = 0.2mg$

Substitute for B and m to obtain: $\rho_w V_{block} g = 0.2\rho_{block} V_{block} g$

Solve for and evaluate ρ_{block}:

$$\rho_{block} = \frac{\rho_w}{0.2} = 5\left(10^3\ \text{kg/m}^3\right) = \boxed{5.00\times10^3\ \text{kg/m}^3}$$

***75** •

Picture the Problem When the copper block is floating on a pool of mercury, it is in equilibrium under the influence of its weight and the buoyant force acting on it. We can apply the condition for translational equilibrium to relate these forces. We can find the fraction of the block that is submerged by applying Archimedes' principle and the definition of density to express the forces in terms of the volume of the block and the volume of the displaced mercury. Let V represent the volume of the copper block, V' the volume of the displaced mercury. Then the fraction submerged when the material is floated on water is V'/V. Choose the upward direction to be the positive y direction.

Apply $\sum F_y = 0$ to the block: $B - w = 0$, where B is the buoyant force and w is the weight of the block.

Apply Archimedes' principle and the definition of density to obtain: $\rho_{Hg} V' g - \rho_{Cu} V g = 0$

Solve for V'/V:

$$\frac{V'}{V} = \frac{\rho_{Cu}}{\rho_{Hg}}$$

Substitute numerical values and evaluate V'/V:

$$\frac{V'}{V}=\frac{8.93\times10^{3}\ \text{kg/m}^{3}}{13.6\times10^{3}\ \text{kg/m}^{3}}=0.657=\boxed{65.7\%}$$

76 •

Picture the Problem When the block is floating on a pool of ethanol, it is in equilibrium under the influence of its weight and the buoyant force acting on it. We can apply the condition for translational equilibrium to relate these forces. We can find the fraction of the block that is submerged by applying Archimedes' principle and the definition of density to express the forces in terms of the volume of the block and the volume of the displaced ethanol. Let V represent the volume of the copper block, V' the volume of the displaced ethanol. Then the fraction of the volume of the block that will be submerged when the material is floated on water is V'/V. Choose the upward direction to be the positive y direction.

Apply $\sum F_y=0$ to the block floating on ethanol:

$B_{\text{eth}}-w=0$, where B_{eth} is the buoyant force due to the ethanol and w is the weight of the block.

Apply Archimedes' principle to obtain:

$$w=\rho_{\text{eth}}(0.9V)g$$

Apply $\sum F_y=0$ to the block floating on water:

$B_{\text{w}}-w=0$, where B_{w} is the buoyant force due to the water and w is the weight of the block.

Apply Archimedes' principle to obtain:

$w=\rho_{\text{w}}V'g$, where V' is the volume of the displaced water.

Equate the two expressions for w and solve for V'/V:

$$\frac{V'}{V}=\frac{0.9\rho_{\text{eth}}}{\rho_{\text{w}}}$$

Substitute numerical values and evaluate V'/V:

$$\frac{V'}{V}=\frac{0.9\left(0.806\times10^{3}\ \text{kg/m}^{3}\right)}{10^{3}\ \text{kg/m}^{3}}$$

$$=0.725=\boxed{72.5\%}$$

77 •

Determine the Concept If you are floating, the density (or specific gravity) of the liquid in which you are floating is immaterial as you are in translational equilibrium under the influence of your weight and the buoyant force on your body. Thus, the buoyant force on your body is your weight in both (*a*) and (*b*).

78 •

Picture the Problem Let m and V represent the mass and volume of your body. Because you are in translational equilibrium when you are floating, we can apply the condition for translational equilibrium and Archimedes' principle to your body to express the dependence of the volume of water it displaces when it is fully submerged on your weight. Let the upward direction be the positive y direction.

Apply $\sum F_y = 0$ to your floating body:

$$B - mg = 0$$

Use Archimedes' principle to relate the density of water to your volume:

$$B = w_f = m_f g = \rho_w (0.96V)g$$

Substitute to obtain:

$$\rho_w (0.96V)g - mg = 0$$

Solve for V:

$$V = \boxed{\frac{m}{0.96\rho_w}}$$

79 ••

Picture the Problem Let m and V represent the mass and volume of the block of wood. Because the block is in translational equilibrium when it is floating, we can apply the condition for translational equilibrium and Archimedes' principle to express the dependence of the volume of water it displaces when it is fully submerged on its weight. We'll repeat this process for the situation in which the lead block is resting on the wood block with the latter fully submerged. Let the upward direction be the positive y direction.

Apply $\sum F_y = 0$ to floating block:

$$B - mg = 0$$

Use Archimedes' principle to relate the density of water to the volume of the block of wood:

$$B = w_f = m_f g = \rho_w (0.68V)g$$

Using the definition of density, express the weight of the block in terms of its density:

$$mg = \rho_{wood} V g$$

Substitute to obtain:

$$\rho_w (0.68V)g - \rho_{wood} V g = 0$$

Solve for and evaluate the density of the wood block:

$$\rho_{wood} = 0.68\rho_w = 0.68(10^3 \text{ kg/m}^3) = 680 \text{ kg/m}^3$$

Use the definition of density to find the volume of the wood:

$$V = \frac{m_{\text{wood}}}{\rho_{\text{wood}}} = \frac{1.5\,\text{kg}}{680\,\text{kg/m}^3} = 2.206\times10^{-3}\,\text{m}^3$$

Apply $\sum F_y = 0$ to the floating block when the lead block is placed on it:

$B' - m'g = 0$, where B' is the new buoyant force on the block and m' is the combined mass of the wood block and the lead block.

Use Archimedes' principle and the definition of density to obtain:

$$\rho_{\text{w}} Vg - (m_{\text{Pb}} + m_{\text{block}})g = 0$$

Solve for the mass of the lead block:

$$m_{\text{Pb}} = \rho_{\text{w}} V - m_{\text{block}}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{\text{Pb}} = (10^3\,\text{kg/m}^3)(2.206\times10^{-3}\,\text{m}^3) - 1.5\,\text{kg} = \boxed{0.706\,\text{kg}}$$

***80 ••**

Picture the Problem The true mass of the Styrofoam cube is greater than that indicated by the balance due to the buoyant force acting on it. The balance is in rotational equilibrium under the influence of the buoyant and gravitational forces acting on the Styrofoam cube and the brass masses. Neglect the buoyancy of the brass masses. Let *m* and *V* represent the mass and volume of the cube and *L* the lever arm of the balance.

Apply $\sum \vec{\tau} = 0$ to the balance:

$$(mg - B)L - m_{\text{brass}} gL = 0$$

Use Archimedes' principle to express the buoyant force on the Styrofoam cube as a function of volume and density of the air it displaces:

$$B = \rho_{\text{air}} Vg$$

Substitute and simplify to obtain:

$$m - \rho_{\text{air}} V - m_{\text{brass}} = 0$$

Solve for *m*:

$$m = \rho_{\text{air}} V + m_{\text{brass}}$$

Substitute numerical values and evaluate *m*:

$$m = (1.293\,\text{kg/m}^3)(0.25\,\text{m})^3 + 20\times10^{-3}\,\text{kg} = 4.02\times10^{-2}\,\text{kg} = \boxed{40.2\,\text{g}}$$

81 ••

Picture the Problem Let d_{in} and d_{out} represent the inner and outer diameters of the copper shell and V' the volume of the sphere that is submerged. Because the spherical shell is floating, it is in translational equilibrium and we can apply a condition for translational equilibrium to relate the buoyant force B due to the displaced water and its weight w.

Apply $\sum F_y = 0$ to the spherical shell:

$$B - w = 0$$

Using Archimedes' principle and the definition of w, substitute to obtain:

$$\rho_w V'g - mg = 0$$

or

$$\rho_w V' - m = 0 \qquad (1)$$

Express V' as a function d_{out}:

$$V' = \frac{1}{2}\frac{\pi}{6}d_{out}^3 = \frac{\pi}{12}d_{out}^3$$

Express m in terms of d_{in} and d_{out}:

$$m = \rho_{Cu}\left(V_{out} - V_{in}\right) = \rho_{Cu}\left(\frac{\pi}{6}d_{out}^3 - \frac{\pi}{6}d_{in}^3\right)$$

Substitute in equation (1) to obtain:

$$\rho_w\frac{\pi}{12}d_{out}^3 - \rho_{Cu}\left(\frac{\pi}{6}d_{out}^3 - \frac{\pi}{6}d_{in}^3\right) = 0$$

Simplify:

$$\frac{1}{2}\rho_w d_{out}^3 - \rho_{Cu}\left(d_{out}^3 - d_{in}^3\right) = 0$$

Solve for d_{in}:

$$d_{in} = d_{out}\sqrt[3]{1 - \frac{\rho_w}{2\rho_{Cu}}}$$

Substitute numerical values and evaluate d_{in}:

$$d_{in} = (12\,\text{cm})\sqrt[3]{1 - \frac{1}{2(8.93)}} = \boxed{11.8\,\text{cm}}$$

82 ••

Determine the Concept The additional weight on the beaker side equals the weight of the displaced water, i.e., 64 g. This is the mass that must be placed on the other cup to maintain balance.

***83** ••

Picture the Problem We can use the definition of Reynolds number and assume a value for N_R of 1000 (well within the laminar flow range) to obtain a trial value for the radius of the pipe. We'll then use Poiseuille's law to determine the pressure difference between the ends of the pipe that would be required to maintain a volume flow rate of 500 L/s.

Use the definition of Reynolds number to relate N_R to the radius of the pipe:

$$N_R = \frac{2r\rho v}{\eta}$$

Use the definition of I_V to relate the volume flow rate of the pipe to its radius:

$$I_V = Av = \pi r^2 v \Rightarrow v = \frac{I_V}{\pi r^2}$$

Substitute to obtain:

$$N_R = \frac{2\rho I_V}{\eta\pi r}$$

Solve for r:

$$r = \frac{2\rho I_V}{\eta\pi N_R}$$

Substitute numerical values and evaluate r:

$$r = \frac{2(700\,\text{kg/m}^3)(0.500\,\text{m}^3/\text{s})}{\pi(0.8\,\text{Pa}\cdot\text{s})(1000)} = 27.9\,\text{cm}$$

Using Poiseuille's law, relate the pressure difference between the ends of the pipe to its radius:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Substitute numerical values and evaluate ΔP:

$$\Delta P = \frac{8(0.8\,\text{Pa}\cdot\text{s})(50\,\text{km})}{\pi(0.279\,\text{m})^4}(0.500\text{m}^3/\text{s})$$
$$= 8.41\times10^6\ \text{Pa}$$
$$= 8.41\times10^6\ \text{Pa}\times\frac{1\,\text{atm}}{1.01325\times10^5\ \text{Pa}}$$
$$= 83.0\,\text{atm}$$

This pressure is too large to maintain in the pipe.

Evaluate ΔP for a pipe of 50 cm radius:

$$\Delta P = \frac{8(0.8\,\text{Pa}\cdot\text{s})(50\,\text{km})}{\pi(0.50\,\text{m})^4}(0.500\,\text{m}^3/\text{s})$$
$$= 8.15\times10^5\ \text{Pa}$$
$$= 8.15\times10^5\ \text{Pa}\times\frac{1\,\text{atm}}{1.01325\times10^5\ \text{Pa}}$$
$$= 8.04\,\text{atm}$$

1 m is a reasonable diameter for the pipeline.

84 ••

Picture the Problem We'll measure the height of the liquid–air interfaces relative to the centerline of the pipe. We can use the definition of the volume flow rate in a pipe to find the speed of the water at point A and the relationship between the gauge pressures at points A and C to determine the level of the liquid-air interface at A. We can use the continuity equation to express the speed of the water at B in terms of its speed at A and Bernoulli's equation for constant elevation to find the gauge pressure at B. Finally, we can use the relationship between the gauge pressures at points A and B to find the level of the liquid-air interface at B.

Relate the gauge pressure in the pipe at A to the height of the liquid-air interface at A:

$$P_{\text{gauge,A}} = \rho g h_A$$

where h_A is measured from the center of the pipe.

Solve for h_A:

$$h_A = \frac{P_{\text{gauge,A}}}{\rho g}$$

Substitute numerical values and evaluate h_A:

$$h_A = \frac{(1.22\,\text{atm})(1.01\times10^5\ \text{Pa/atm})}{(10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)} = \boxed{12.6\,\text{m}}$$

Determine the velocity of the water at A:

$$v_A = \frac{I_V}{A_A} = \frac{0.8\times10^{-3}\ \text{m}^3/\text{s}}{\frac{\pi}{4}(0.02\,\text{m})^2} = 2.55\,\text{m/s}$$

Apply Bernoulli's equation for constant elevation to relate P_B and P_A:

$$P_A + \tfrac{1}{2}\rho v_A^2 = P_B + \tfrac{1}{2}\rho v_B^2 \qquad (1)$$

Use the continuity equation to relate v_B and v_A:

$$A_A v_A = A_B v_B$$

Solve for v_B:

$$v_B = \frac{A_A}{A_B} v_A = \frac{d_A^2}{d_B^2} v_A = \frac{(2\,\text{cm})^2}{(1\,\text{cm})^2} v_A = 4 v_A$$

Substitute in equation (1) to obtain:

$$P_A + \tfrac{1}{2}\rho v_A^2 = P_B + 8\rho v_A^2$$

Solve for P_B:

$$P_B = P_A - \tfrac{15}{2}\rho v_A^2$$

Substitute numerical values and evaluate P_B:

$$\begin{aligned} P_B &= (2.22\,\text{atm})(1.01\times10^5\,\text{Pa/atm}) \\ &\quad - \tfrac{15}{2}(10^3\,\text{kg/m}^3)(2.55\,\text{m/s})^2 \\ &= 1.75\times10^5\,\text{Pa} \\ &= 1.75\times10^5\,\text{Pa}\times\frac{1\,\text{atm}}{1.01\times10^5\,\text{Pa}} \\ &= 1.733\,\text{atm} \end{aligned}$$

Relate the gauge pressure in the pipe at B to the height of the liquid-air interface at B:

$$P_{\text{gauge,B}} = \rho g h_B$$

Solve for h_B:

$$h_B = \frac{P_{\text{gauge,B}}}{\rho g}$$

Substitute numerical values and evaluate h_B:

$$h_B = \frac{[(1.733-1)\text{atm}]\left(1.01\times10^5\,\frac{\text{Pa}}{\text{atm}}\right)}{(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} = \boxed{7.55\,\text{m}}$$

85 ••

Picture the Problem We'll measure the height of the liquid–air interfaces relative to the centerline of the pipe. We can use the definition of the volume flow rate in a pipe to find the speed of the water at point A and the relationship between the gauge pressures at points A and C to determine the level of the liquid-air interface at A. We can use the continuity equation to express the speed of the water at B in terms of its speed at A and Bernoulli's equation for constant elevation to find the gauge pressure at B. Finally, we can use the relationship between the gauge pressures at points A and B to find the level of the liquid-air interface at B.

Relate the gauge pressure in the pipe at A to the height of the liquid-air interface at A:

$$P_{\text{gauge,A}} = \rho g h_A$$

where h_A is measured from the center of the pipe.

Solve for h_A:

$$h_A = \frac{P_{\text{gauge,A}}}{\rho g}$$

Substitute numerical values and evaluate h_A:

$$h_A = \frac{(1.22\,\text{atm})(1.01\times10^5\ \text{Pa/atm})}{(10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)} = \boxed{12.6\,\text{m}}$$

Determine the velocity of the water at A:

$$v_A = \frac{I_V}{A_A} = \frac{0.6\times10^{-3}\ \text{m}^3/\text{s}}{\frac{\pi}{4}(0.02\,\text{m})^2} = 1.91\,\text{m/s}$$

Use the continuity equation to relate v_B and v_A:

$$A_A v_A = A_B v_B$$

Solve for v_B:

$$v_B = \frac{A_A}{A_B}v_A = \frac{d_A^2}{d_B^2}v_A = \frac{(2\,\text{cm})^2}{(1\,\text{cm})^2}v_A = 4v_A$$

Apply Bernoulli's equation for constant elevation to relate P_B and P_A:

$$P_A + \tfrac{1}{2}\rho v_A^2 = P_B + \tfrac{1}{2}\rho v_B^2 \qquad (1)$$

Substitute in equation (1) to obtain:

$$P_A + \tfrac{1}{2}\rho v_A^2 = P_B + 8\rho v_A^2$$

Solve for P_B:

$$P_B = P_A - \tfrac{15}{2}\rho v_A^2$$

Substitute numerical values and evaluate P_B:

$$\begin{aligned} P_B &= (2.22\,\text{atm})(1.01\times10^5\ \text{Pa/atm}) \\ &\quad - \tfrac{15}{2}(10^3\ \text{kg/m}^3)(1.91\,\text{m/s})^2 \\ &= 1.969\times10^5\ \text{Pa} \\ &= 1.969\times10^5\ \text{Pa}\times\frac{1\,\text{atm}}{1.01\times10^5\ \text{Pa}} \\ &= 1.95\,\text{atm} \end{aligned}$$

Relate the gauge pressure in the pipe at B to the height of the liquid-air interface at B:

$$P_{\text{gauge,B}} = \rho g h_B$$

Solve for h_B:

$$h_B = \frac{P_{\text{gauge,B}}}{\rho g}$$

Substitute numerical values and evaluate h_B:

$$h_B = \frac{[(1.95-1)\text{atm}]\left(1.01\times10^5\,\frac{\text{Pa}}{\text{atm}}\right)}{(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)} = \boxed{9.78\text{ m}}$$

***86 ••**

Picture the Problem Because it is not given, we'll neglect the difference in height between the centers of the pipes at A and B. We can use the definition of the volume flow rate to find the speed of the water at A and Bernoulli's equation for constant elevation to find its speed at B. Once we know the speed of the water at B, we can use the equation of continuity to find the diameter of the constriction at B.

Use the definition of the volume flow rate to find v_A:

$$v_A = \frac{I_V}{A_A} = \frac{0.5\times10^{-3}\text{ m}^3/\text{s}}{\frac{\pi}{4}(0.02\text{ m})^2} = 1.59\text{ m/s}$$

Use Bernoulli's equation for constant elevation to relate the pressures and velocities at A and B:

$$P_B + \tfrac{1}{2}\rho v_B^2 = P_A + \tfrac{1}{2}\rho v_A^2$$

Solve for v_B^2:

$$v_B^2 = \frac{2(P_A - P_B)}{\rho} + v_A^2$$

Substitute numerical values and evaluate v_B^2:

$$v_B^2 = \frac{2[(1.187-0.1)\text{atm}(1.01\times10^5\text{ Pa/atm})]}{10^3\text{ kg/m}^3} + (1.59\text{ m/s})^2 = 222\text{ m}^2/\text{s}^2$$

Using the continuity equation, relate the volume flow rate to the radius at B:

$$I_V = A_B v_B = \pi r_B^2 v_B$$

Solve for and evaluate r_B:

$$r_B = \sqrt{\frac{I_V}{\pi v_B}} = \sqrt{\frac{0.5\times10^{-3}\text{ m}^3/\text{s}}{\pi(14.9\text{ m/s})}} = 3.27\text{ mm}$$

and

$$d_B = 2r_B = \boxed{6.54\text{ mm}}$$

87 ••

Picture the Problem Let *V*′ represent the volume of the buoy that is submerged and *h*′ the height of the submerged portion of the cylinder. We can find the fraction of the cylinder's volume that is submerged by applying the condition for translational equilibrium to the buoy and using Archimedes' principle. When the buoy is submerged it is in equilibrium under the influence of the tension *T* in the cable, the buoyant force due to the displaced water, and its weight. When the cable breaks, the net force acting on the buoy will accelerate it and we can use Newton's 2nd law to find its acceleration.

(*a*) Apply $\sum F_y = 0$ to the cylinder:

$$B - w = 0$$

Using Archimedes' principle and the definition of weight, substitute for *B* and *w*:

$$\rho_{sw} V' g - mg = 0$$

or

$$\rho_{sw} h' A g - mg = 0$$

where *A* is the cross-sectional area of the buoy.

Solve for and evaluate *h*′:

$$h' = \frac{m}{\rho_{sw} A}$$

Substitute numerical values and evaluate *h*′:

$$h' = \frac{600\,\text{kg}}{\left(1.025\times10^3\,\text{kg/m}^3\right)\frac{\pi}{4}\left(0.9\,\text{m}\right)^2} = 0.920\,\text{m}$$

Use *h*′ to find the height *h* of the buoy:

$$h - h' = 2.6\,\text{m} - 0.920\,\text{m} = \boxed{1.68\,\text{m}}$$

Express the fraction of the volume of the cylinder that is above water:

$$\frac{V - V'}{V} = 1 - \frac{V'}{V} = 1 - \frac{\frac{\pi}{4} d^2 h'}{\frac{\pi}{4} d^2 h} = 1 - \frac{h'}{h}$$

Substitute numerical values to obtain:

$$\frac{V - V'}{V} = 1 - \frac{0.920\,\text{m}}{2.6\,\text{m}} = \boxed{64.6\%}$$

(*b*) Apply $\sum F_y = 0$ to the submerged buoy:

$$B - T - w = 0$$

Solve for T and substitute for B and w to obtain:

$$T = B - w = \rho_{sw} V g - mg = (\rho_{sw} V - m)g$$

Substitute numerical values and evaluate T:

$$T = \left[\left(1.025\times10^3\ \text{kg/m}^3\right)\frac{\pi}{4}(0.9\ \text{m})^2(2.6\ \text{m}) - 600\ \text{kg}\right]\left(9.81\ \text{m/s}^2\right) = \boxed{10.7\ \text{kN}}$$

(*c*) Apply $\sum F_y = 0$ to the buoy:

$$B - w = ma$$

Substitute for $B - w$ and solve for a to obtain:

$$a = \frac{B-w}{m} = \frac{T}{m}$$

Substitute numerical values and evaluate a:

$$a = \frac{10.75\ \text{kN}}{600\ \text{kg}} = \boxed{17.9\ \text{m/s}^2}$$

88 ••

Picture the Problem Because the floating object is in equilibrium under the influence of the buoyant force acting on it and its weight; we can apply the condition for translational equilibrium to relate B and w. Let Δh represent the change in elevation of the liquid level and V_f the volume of the displaced fluid.

Apply $\sum F_y = 0$ to the floating object:

$$B - w = 0$$

Using Archimedes' principle and the definition of weight, substitute for B and w:

$$\rho_0 g V_f - mg = 0$$

The volume of fluid displaced is the sum of the volume displaced in the two vessels:

$$V_f = \Delta V_A + \Delta V_{3A} = A\Delta h + 3A\Delta h = 4A\Delta h$$

Substitute for V_f to obtain:

$$4\rho_0 g A\Delta h - mg = 0$$

Solve for Δh:

$$\Delta h = \boxed{\frac{m}{4A\rho_0}}$$

89 ••

Picture the Problem We can calculate the smallest pressure change ΔP that can be detected from the reading Δh from $\Delta P = \rho g \Delta h$.

Express and evaluate the pressure difference between the two columns of the manometer:

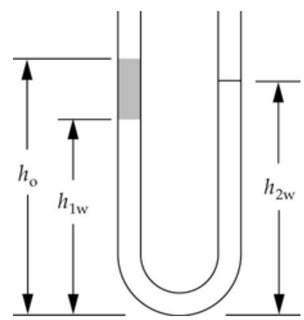
$$\Delta P = \rho g \Delta h = (900\,\text{kg/m}^3)(9.81\,\text{m/s}^2) \times (0.05\times10^{-3}\,\text{m}) = 0.4415\,\text{Pa}$$

Express this pressure in mmHg and μmHg:

$$\Delta P = 0.4415\,\text{Pa} \times \frac{1\,\text{atm}}{1.01325\times10^{5}\,\text{Pa}} \times \frac{760\,\text{mmHg}}{1\,\text{atm}} = \boxed{3.31\times10^{-3}\,\text{mmHg}} = \boxed{3.31\,\mu\text{mHg}}$$

90 ••

Picture the Problem We can use the equality of the pressure at the bottom of the U-tube due to the water on one side and that due to the oil and water on the other to relate the various heights. Let h represent the height of the oil above the water. Then $h_o = h_{1w} + h$.



Using the constancy of the amount of water, express the relationship between h_{1w} and h_{2w}:

$$h_{1w} + h_{2w} = 56\text{ cm}$$

Find the height of the oil-water interface:

$$h_{1w} = 56\,\text{cm} - 34\,\text{cm} = \boxed{22.0\,\text{cm}}$$

Express the equality of the pressure at the bottom of the two arms of the U tube:

$$\rho_w g(34\,\text{cm}) = \rho_w g(22\,\text{cm}) + 0.78\rho_w g h_{oil}$$

Solve for and evaluate h_{oil}:

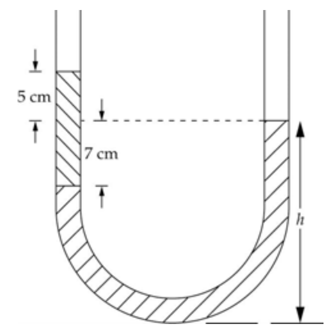
$$h_{oil} = \frac{\rho_w g(34\,\text{cm}) - \rho_w g(22\,\text{cm})}{0.78\rho_w g} = \frac{(34\,\text{cm}) - (22\,\text{cm})}{0.78} = 15.4\,\text{cm}$$

Find the height of the air-oil interface h_o:

$$h_o = 22\text{cm} + 15.4\,\text{cm} = \boxed{37.4\,\text{cm}}$$

91 ••

Picture the Problem Let σ_L represent the specific gravity of the liquid. The specific gravity of the oil is $\sigma_o = 0.8$. We can use the equality of the pressure at the bottom of the U-tube due to the water on one side and that due to the oil and water on the other to relate the various heights.



Express the equality of the pressure at the bottom of the two arms of the U tube:

$$\sigma_L gh = \sigma_L g(h - 7\,\text{cm}) + 0.8\sigma_w g(12\,\text{cm})$$

Solve for and evaluate σ_L:

$$\sigma_L = \frac{0.8\sigma_w(12\,\text{cm})}{7\,\text{cm}} = \frac{0.8(12\,\text{cm})}{7\,\text{cm}} = \boxed{1.37}$$

92 ••

Picture the Problem The block of wood is in translational equilibrium under the influence of the buoyant force due to the displaced water acting on it and on the lead block, its weight, and the weight of the lead block. We can use a condition for translational equilibrium and Archimedes' principle to obtain a relationship between the mass of the lead block and the densities of water, wood, and lead and the mass of the wood block.

Apply $\sum F_y = 0$ to the block of wood:

$$B_{wood} + B_{Pb} - m_{wood}g - m_{Pb}g = 0$$

Use Archimedes' principle to express the buoyant force on the block of wood:

$$B_{wood} = \rho_w V_{wood} g$$

Use Archimedes' principle to

$$B_{Pb} = \rho_w V_{Pb} g$$

express the buoyant force on the lead block:

Substitute and simplify to obtain:

$$\rho_w V_{wood} + \rho_w V_{Pb} - m_{wood} - m_{Pb} = 0$$

Express the volume of the wood block in terms of its density and mass:

$$V_{wood} = \frac{m_{wood}}{\rho_{wood}}$$

Express the volume of the lead block in terms of its density and mass:

$$V_{Pb} = \frac{m_{Pb}}{\rho_{Pb}}$$

Substitute for V_{wood} and V_{Pb}:

$$\rho_w \frac{m_{wood}}{\rho_{wood}} + \rho_w \frac{m_{Pb}}{\rho_{Pb}} - m_{wood} - m_{Pb} = 0$$

Solve for m_{Pb}:

$$m_{Pb} = \frac{\left(\frac{\rho_w}{\rho_{wood}} - 1\right) m_{wood}}{1 - \frac{\rho_w}{\rho_{Pb}}}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{Pb} = \frac{\left(\frac{1}{0.7} - 1\right)(0.5\,\text{kg})}{1 - \frac{1}{11.3}} = \boxed{0.235\,\text{kg}}$$

***93 ••**

Picture the Problem Because the balloon is in equilibrium under the influence of the buoyant force exerted by the air, the weight of its basket and load *w*, the weight of the skin of the balloon, and the weight of the helium. Choose upward to be the positive *y* direction and apply the condition for translational equilibrium to relate these forces. Archimedes' principle relates the buoyant force on the balloon to the density of the air it displaces and the volume of the balloon.

(*a*) Apply $\sum F_y = 0$ to the balloon:

$$B - m_{skin}g - m_{He}g - w = 0$$

Letting *V* represent the volume of the balloon, use Archimedes' principle to express the buoyant force:

$$\rho_{air}Vg - m_{skin}g - m_{He}g - w = 0$$

Substitute for m_{He}:

$$\rho_{air}Vg - m_{skin}g - \rho_{He}Vg - w = 0$$

Solve for V:

$$V = \frac{m_{\text{skin}} g + w}{(\rho_{\text{air}} - \rho_{\text{He}})g}$$

Substitute numerical values and evaluate V:

$$V = \frac{(1.5\,\text{kg})(9.81\,\text{m/s}^2) + 750\,\text{N}}{(1.293 - 0.1786)(\text{kg/m}^3)(9.81\,\text{m/s}^2)} = \boxed{70.0\,\text{m}^3}$$

(*b*) Apply $\sum F_y = ma$ to the balloon:

$$B - m_{\text{tot}} g = m_{\text{tot}} a$$

Solve for a:

$$a = \frac{B}{m_{\text{tot}}} - g$$

Assuming that the mass of the skin has not changed and letting V' represent the doubled volume of the balloon, express m_{tot}:

$$m_{\text{tot}} = m_{\text{load}} + m_{\text{He}} + m_{\text{skin}} = \frac{w_{\text{load}}}{g} + \rho_{\text{He}} V' + m_{\text{skin}}$$

Substitute numerical values and evaluate m_{tot}:

$$m_{\text{tot}} = \frac{900\,\text{N}}{9.81\,\text{m/s}^2} + (0.1786\,\text{kg/m}^3)(140\,\text{m}^3) + 1.5\,\text{kg} = 118\,\text{kg}$$

Express the buoyant force acting on the balloon:

$$B = w_{\text{displaced fluid}} = \rho_{\text{air}} V' g$$

Substitute numerical values and evaluate B:

$$B = (1.293\,\text{kg/m}^3)(140\,\text{m}^3)(9.81\,\text{m/s}^2) = 1.78\,\text{kN}$$

Substitute and evaluate a:

$$a = \frac{1.78\,\text{kN}}{118\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{5.27\,\text{m/s}^2}$$

94 ••

Picture the Problem When the hollow sphere is completely submerged but floating, it is in translational equilibrium under the influence of a buoyant force and its weight. The buoyant force is given by Archimedes' principle and the weight of the sphere is the sum of the weights of the hollow sphere and the material filling its center.

Apply $\sum F_y = 0$ to the hollow sphere:

$$B - w = 0$$

Express the buoyant force acting on the hollow sphere:

$$B = 2\rho_0 V_{\text{sphere}} g = 2\rho_0\left[\tfrac{4}{3}\pi(2R)^3\right]g = \tfrac{64}{3}\rho_0\pi R^3 g$$

Express the weight of the sphere when it's hollow is filled with a material of density ρ':

$$w = \rho_0 V_{\text{hollow sphere}} g + \rho' V_{\text{hollow}} g = \rho_0\left[\tfrac{4}{3}\pi\left\{(2R)^3 - R^3\right\}\right]g + \rho'\left[\tfrac{4}{3}\pi R^3\right]g = \tfrac{28}{3}\rho_0\pi R^3 g + \tfrac{4}{3}\rho'\pi R^3 g$$

Substitute to obtain:

$$\tfrac{64}{3}\rho_0\pi R^3 g - \tfrac{28}{3}\rho_0\pi R^3 g - \tfrac{4}{3}\rho'\pi R^3 g = 0$$

Solve for ρ':

$$\rho' = \boxed{9\rho_0}$$

***95 ••**

Picture the Problem We can differentiate the function $P(h)$ to show that it satisfies the differential equation $dP/P = -C\,dh$ and in part (*b*) we can use the approximation $e^{-x} \approx 1 - x$ and $\Delta h << h_0$ to establish the given result.

(*a*) Differentiate $P(h) = P_0 e^{-Ch}$:

$$\frac{dP}{dh} = -CP_0 e^{-Ch} = -CP$$

Separate variables to obtain:

$$\boxed{\frac{dP}{P} = -C\,dh}$$

(*b*) Express $P(h + \Delta h)$:

$$P(h+\Delta h) = P_0 e^{-C(h+\Delta h)} = P_0 e^{-Ch} e^{-C\Delta h} = P(h) e^{-C\Delta h}$$

For $\Delta h << h_0$:

$$\frac{\Delta h}{h_0} << 1$$

Let $h_0 = 1/C$. Then:

$$C\Delta h << 1$$

and

$$e^{-C\Delta h} \approx 1 - C\Delta h = 1 - \frac{\Delta h}{h_0}$$

Substitute to obtain:

$$P(h+\Delta h) = \boxed{P(h)\left(1 - \frac{\Delta h}{h_0}\right)}$$

(*c*) Take the logarithm of both sides of the function $P(h)$:

$$\ln P = \ln P_0 e^{-Ch} = \ln P_0 + \ln e^{-Ch} = \ln P_0 - Ch$$

Solve for C:

$$C = \frac{1}{h}\ln\left(\frac{P_0}{P}\right)$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{5.5\,\text{km}}\ln\left(\frac{P_0}{\frac{1}{2}P_0}\right) = \frac{1}{5.5\,\text{km}}\ln 2 = \boxed{0.126\,\text{km}^{-1}}$$

96 ••

Picture the Problem Let V represent the volume of the submarine and V' the volume of seawater it displaces when it is on the surface. The submarine is in equilibrium in both parts of the problem. Hence we can apply the condition for translational equilibrium (neutral buoyancy) to the submarine to relate its weight to the buoyant force acting on it. We'll also use Archimedes' principle to connect the buoyant forces to the volume of seawater the submarine displaces. Let upward be the positive y direction.

(*a*) Express f, the fraction of the submarine's volume above the surface when the tanks are filled with air:

$$f = \frac{V - V'}{V} = 1 - \frac{V'}{V} \quad (1)$$

Apply $\sum F_y = 0$ to the submarine when its tanks are full of air:

$$B - w = 0$$

Use Archimedes' principle to express the buoyant force on the submarine in terms of the volume of the displaced water:

$$B = \rho_{sw} V' g$$

Substitute and solve for V':

$$V' = \frac{m}{\rho_{sw}}$$

Substitute in equation (1) to obtain:

$$f = 1 - \frac{m}{\rho_{sw} V}$$

Substitute numerical values and evaluate *f*:

$$f = 1 - \frac{2.4\times10^{6}\ \text{kg}}{\left(1.025\times10^{3}\ \text{kg/m}^{3}\right)\left(2.4\times10^{3}\ \text{m}^{3}\right)} = 2.44\times10^{-2} = \boxed{2.44\%}$$

(*b*) Express the volume of seawater in terms of its mass and density:

$$V_{sw} = \frac{m_{sw}}{\rho_{sw}} \qquad (2)$$

Apply $\sum F_y = 0$, the condition for neutral buoyancy, to the submarine:

$$B - w_{sub} - w_{sw} = 0$$

Use Archimedes' principle to express the buoyant force on the submarine in terms of the volume of the displaced water:

$$B = \rho_{sw} V g$$

Substitute to obtain:

$$\rho_{sw} V g - m_{sub} g - m_{sw} g = 0$$

Solve for m_{sw}:

$$m_{sw} = \rho_{sw} V - m_{sub}$$

Substitute for V_{sw} in equation (2) to obtain:

$$V_{sw} = \frac{\rho_{sw} V - m_{sub}}{\rho_{sw}} = V - \frac{m_{sub}}{\rho_{sw}}$$

Substitute numerical values and evaluate V_{sw}:

$$V_{sw} = 2.4\times10^{3}\ \text{m}^{3} - \frac{2.4\times10^{6}\ \text{kg}}{1.025\times10^{3}\ \text{kg/m}^{3}} = \boxed{58.5\ \text{m}^{3}}$$

97 ••

Picture the Problem While the loaded crate is under the surface, it is in equilibrium under the influence of the tension in the cable, the buoyant force acting on the gold, and the gravitational force acting on the gold. The empty crate has neutral buoyancy. When the crate is out of the water, the buoyant force of the air is negligible and the tension in the cable is the sum of the weights of the crate, the gold bullion, and the seawater.

(*a*) Apply $\sum F_y = 0$ to the crate while it is below the surface:

$$T + B_{Au} - w_{Au} = 0$$

Solve for the tension in the cable:

$$T = w_{Au} - B_{Au}$$

Using Archimedes' principle, relate the buoyant force acting on the gold

$$B_{Au} = \rho_{sw} V_{Au} g$$

to its density and volume:

Substitute for B_{Au} and simplify to obtain: $T = (\rho_{\text{Au}} - \rho_{\text{sw}})V_{\text{Au}}g$

Substitute numerical values and evaluate T:

$$T = [(19.3\times10^3\ \text{kg/m}^3) - (1.025\times10^3\ \text{kg/m}^3)](0.36)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m})(9.81\,\text{m/s}^2) = \boxed{33.9\,\text{kN}}$$

(*b*) 1. Apply $\sum F_y = 0$ to the crate while it is being lifted to the deck of the ship with none of the seawater leaking out: $T - w_{\text{Au}} - w_{\text{crate}} - w_{\text{sw}} = 0$

Substitute for the weights of the gold, crate, and seawater and solve for the tension in the cable and express:

$$\begin{aligned} T &= w_{\text{Au}} + w_{\text{crate}} + w_{\text{sw}} \\ &= \rho_{\text{Au}}V_{\text{Au}}g + m_{\text{crate}}g + \rho_{\text{sw}}V_{\text{sw}}g \\ &= (\rho_{\text{Au}}V_{\text{Au}} + m_{\text{crate}} + \rho_{\text{sw}}V_{\text{sw}})g \end{aligned}$$

Substitute numerical values and evaluate T:

$$\begin{aligned} T &= [(19.3\times10^3\ \text{kg/m}^3)(0.36)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m}) + 32\,\text{kg} + (1.025\times10^3\ \text{kg/m}^3) \\ &\quad\times(0.64)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m})](9.81\,\text{m/s}^2) \\ &= \boxed{39.8\,\text{kN}} \end{aligned}$$

2. With the seawater term missing, the expression for the tension is:

$$\begin{aligned} T &= w_{\text{Au}} + w_{\text{crate}} \\ &= \rho_{\text{Au}}V_{\text{Au}}g + m_{\text{crate}}g \\ &= (\rho_{\text{Au}}V_{\text{Au}} + m_{\text{crate}})g \end{aligned}$$

Substitute numerical values and evaluate T:

$$T = [(19.3\times10^3\ \text{kg/m}^3)(0.36)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m}) + 32\,\text{kg}](9.81\,\text{m/s}^2) = \boxed{36.1\,\text{kN}}$$

98 •••

Picture the Problem In the three situations described in the problem the hydrometer will be in equilibrium under the influence of its weight and the buoyant force exerted by the liquids. We can use Archimedes' principle to relate the buoyant force acting on the hydrometer to the density of the liquid in which it is floating and to its weight.

(*a*) Find the volume of the bulb:

$$V_{\text{bulb}} = \tfrac{1}{6}\pi d^3 = \tfrac{1}{6}\pi(2.4\,\text{cm})^3 = 7.238\,\text{cm}^3$$

Find the volume of the tube:

$$V_{\text{tube}} = \tfrac{1}{4}\pi d^2 L = \tfrac{1}{4}\pi(0.75\,\text{cm})^2(20\,\text{cm}) = 8.836\,\text{cm}^2$$

Apply $\sum F_y = 0$ to the hydrometer just floating in the liquid:

$$B - w_{\text{hyd}} - m_{\text{Pb}}g = 0$$

Substitute for B and w_{glass}:

$$\rho_{\text{liq}}V_{\text{hyd}}g - m_{\text{glass}}g - m_{\text{Pb}}g = 0$$

Solve for m_{Pb}:

$$m_{\text{Pb}} = \rho_{\text{liq}}V_{\text{hyd}} - m_{\text{hyd}}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{\text{Pb}} = 0.78(1\,\text{g/cm}^3) \times (7.238\,\text{cm}^3 + 8.836\,\text{cm}^3) - 7.28\,\text{g} = \boxed{5.26\,\text{g}}$$

(*b*) Letting V represent the volume of the hydrometer that is submerged, apply $\sum F_y = 0$ to the hydrometer just floating in the liquid:

$$\rho_{\text{w}}Vg - mg = 0$$

Solve for V:

$$V = \frac{m}{\rho_{\text{w}}} = \frac{m_{\text{hyd}} + m_{\text{Pb}}}{\rho_{\text{w}}}$$

Substitute numerical values and evaluate V:

$$V = \frac{7.28\,\text{g} + 5.26}{1\,\text{g/cm}^3} = 12.54\,\text{cm}^3$$

Relate the volume of the hydrometer that is submerged to the volume of the bulb and the volume of the tube that is submerged:

$$V = \tfrac{1}{4}\pi\, d_{\text{tube}}^2 h' + V_{\text{bulb}}$$

Solve for h':

$$h' = \frac{V - V_{\text{bulb}}}{\tfrac{1}{4}\pi\, d_{\text{tube}}^2}$$

Substitute numerical values and evaluate h':

$$h' = \frac{12.54\,\text{cm}^3 - 7.238\,\text{cm}^3}{\tfrac{1}{4}\pi(0.75\,\text{cm})^2} = 12.0\,\text{cm}$$

Find the length of the tube that shows above the surface of the water:

$$h = 20\,\text{cm} - h' = 20\,\text{cm} - 12.0\,\text{cm} = \boxed{8.00\,\text{cm}}$$

(*c*) Apply $\sum F_y = 0$ to the hydrometer floating in the liquid of unknown specific gravity:

$$\rho_\text{L} V_\text{L} g - m_\text{hyd} g = 0$$

Solve for the density of the liquid:

$$\rho_\text{L} = \frac{m_\text{hyd}}{V_\text{L}}$$

Express the volume of the displaced liquid:

$$V_\text{L} = V_\text{bulb} + \tfrac{1}{4}\pi d_\text{tube}^2 h'$$

Substitute numerical values and evaluate V_L:

$$V_\text{L} = 7.238\,\text{cm}^3 + \tfrac{1}{4}\pi(0.75\,\text{cm})^2 \times (20\,\text{cm} - 12.2\,\text{cm}) = 10.68\,\text{cm}^3$$

Substitute for V_L and m_hyd and evaluate ρ_L:

$$\rho_\text{L} = \frac{12.54\,\text{g}}{10.68\,\text{cm}^3} = 1.174\,\text{g/cm}^3$$

Express and evaluate the specific gravity of the liquid:

$$\text{specific gravity}_\text{liquid} = \frac{\rho_\text{w}}{\rho_\text{L}} = \boxed{1.17}$$

99 •••

Picture the Problem We can apply Bernoulli's equation to the top of the keg and to the spigot opening to determine the rate at which the root beer exits the tank. Because the area of the spigot is much smaller than that of the keg, we can neglect the velocity of the root beer at the top of the keg. We'll use the continuity equation to obtain an expression for the rate of change of the height of the root beer in the keg as a function of the its height and integrate this function to find *h* as a function of time.

(*a*) Apply Bernoulli's equation to the beer at the top of the keg and at the spigot:

$$P_1 + \rho_\text{beer} gh + \tfrac{1}{2}\rho_\text{beer} v_1^2 = P_2 + \rho_\text{beer} gh_2 + \tfrac{1}{2}\rho_\text{beer} v_2^2$$

or, because $v_1 \approx 0$, $h_2 = 0$, $P_1 = P_2 = P_\text{at}$, and $h_1 = h$,

$$gh = \tfrac{1}{2}v_2^2$$

Solve for v_2:

$$v_2 = \boxed{\sqrt{2gh}}$$

(*b*) Use the continuity equation to relate v_1 and v_2:

$$A_1 v_1 = A_2 v_2$$

Substitute $-dh/dt$ for v_1:

$$-A_1 \frac{dh}{dt} = A_2 v_2$$

Substitute for v_2 and solve for dh/dt to obtain:

$$\frac{dh}{dt} = \boxed{-\frac{A_2}{A_1}\sqrt{2gh}}$$

(*c*) Separate the variables in the differential equation:

$$-\frac{A_1/A_2}{\sqrt{2g}}\frac{dh}{\sqrt{h}} = dt$$

Express the integral from $h' = H$ to h and $t' = 0$ to t:

$$-\frac{A_1/A_2}{\sqrt{2g}}\int_H^h \frac{dh'}{\sqrt{h'}} = \int_0^t dt'$$

Evaluate the integral to obtain:

$$-\frac{A_1/A_2}{\sqrt{2g}}\left(\sqrt{H}-\sqrt{h}\right) = t$$

Solve for h:

$$h = \boxed{\left(\sqrt{H} - \frac{A_2}{2A_1}\sqrt{2g}\,t\right)^2}$$

(*d*) Solve $h(t)$ for the time-to-drain t':

$$t' = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}$$

Substitute numerical values and evaluate t'

$$t' = \frac{A_1}{10^{-4}A_1}\sqrt{\frac{2(2\,\text{m})}{9.81\,\text{m/s}^2}} = 6.39\times10^3\ \text{s}$$

$$= \boxed{1\,\text{h}\ 46\ \text{min}}$$

Chapter 14
Oscillations

Conceptual Problems

1 •

Determine the Concept The acceleration of an oscillator of amplitude A and frequency f is zero when it is passing through its equilibrium position and is a maximum when it is at its turning points.

When $v = v_{max}$: $a = \boxed{0}$

When $x = x_{max}$: $a = \omega^2 A = \boxed{4\pi^2 f^2 A}$

2 •

Determine the Concept The condition for simple harmonic motion is that there be a linear restoring force; i.e., that $F = -kx$. Thus, the acceleration and displacement (when they are not zero) are always oppositely directed. v and a can be in the same direction, as can v and x.

3 •

(*a*) False. In simple harmonic motion, the period is independent of the amplitude.

(*b*) True. In simple harmonic motion, the frequency is the reciprocal of the period which, in turn, is independent of the amplitude.

(*c*) True. The condition that the acceleration of a particle is proportional to the displacement and oppositely directed is equivalent to requiring that there be a linear restoring force; i.e., $F = -kx \Leftrightarrow ma = -kx$ or $a = -(k/m)x$.

***4 •**

Determine the Concept The energy of a simple harmonic oscillator varies as the square of the amplitude of its motion. Hence, tripling the amplitude increases the energy by a factor of 9.

5 ••

Picture the Problem The total energy of an object undergoing simple harmonic motion is given by $E_{tot} = \frac{1}{2}kA^2$, where k is the stiffness constant and A is the amplitude of the motion. The potential energy of the oscillator when it is a distance x from its equilibrium position is $U(x) = \frac{1}{2}kx^2$.

Express the ratio of the potential energy of the object when it is 2 cm from the equilibrium position to its total energy:

$$\frac{U(x)}{E_{tot}}=\frac{\frac{1}{2}kx^2}{\frac{1}{2}kA^2}=\frac{x^2}{A^2}$$

Evaluate this ratio for $x = 2$ cm and $A = 4$ cm:

$$\frac{U(2\,\text{cm})}{E_{tot}}=\frac{(2\,\text{cm})^2}{(4\,\text{cm})^2}=\frac{1}{4}$$

$$\frac{U(2\,\text{cm})}{E_{tot}}=\frac{(2\,\text{cm})^2}{(4\,\text{cm})^2}=\frac{1}{4}$$

and $\boxed{(a)\text{ is correct.}}$

6 •

(*a*) True. The factors determining the period of the object, i.e., its mass and the spring constant, are independent of the oscillator's orientation.

(*b*) True. The factors determining the maximum speed of the object, i.e., its amplitude and angular frequency, are independent of the oscillator's orientation.

7 •

False. In order for a simple pendulum to execute simple harmonic motion, the restoring force must be linear. This condition is satisfied, at least approximately, for small initial angular displacements.

8 •

True. In order for a simple pendulum to execute periodic motion, the restoring force must be linear. This condition is satisfied for any initial angular displacement.

***9** ••

Determine the Concept Assume that the first cart is given an initial velocity *v* by the blow. After the initial blow, there are no external forces acting on the carts, so their center of mass moves at a constant velocity *v*/2. The two carts will oscillate about their center of mass in simple harmonic motion where the amplitude of their velocity is *v*/2. Therefore, when one cart has velocity *v*/2 with respect to the center of mass, the other will have velocity –*v*/2. The velocity with respect to the laboratory frame of reference will be +*v* and 0, respectively. Half a period later, the situation is reversed; one cart will move as the other stops, and vice-versa.

***10** ••

Determine the Concept The period of a simple pendulum depends on the reciprocal of the length of the pendulum. Increasing the length of the pendulum will decrease its period and the clock would run slow.

11 •

True. The mechanical energy of a damped, undriven oscillator varies with time according to $E = E_0 e^{-t/\tau}$ where E_0 is the oscillator's energy at $t = 0$ and τ is the time constant.

12 •

(*a*) True. The amplitude of the motion of a driven oscillator depends on the driving (ω) and natural (ω_0) frequencies according to $A = F_0 \big/ \sqrt{m^2\left(\omega_0^2 - \omega^2\right)^2 + b^2\omega^2}$. When $\omega = \omega_0$, the amplitude of the motion is a maximum and is given by $A = F_0 \big/ \sqrt{b^2\omega^2}$.

(*b*) True. The width of the resonance curve ($\Delta\omega$) depends on the Q value according to $\Delta\omega/\omega_0 = 1/Q$. Thus when Q is large, $\Delta\omega$ is small and the resonance is sharp.

13 •

Determine the Concept Examples of driven oscillators include the pendulum of a clock, a bowed violin string, and the membrane of any loudspeaker.

14 •

Determine the Concept The shattering of a crystal wineglass is a consequence of the glass being driven at or near its resonant frequency. $\boxed{(a)\text{ is correct.}}$

***15** •

Determine the Concept We can use the expression for the frequency of a spring-and-mass oscillator to determine the effect of the mass of the spring.

If m represents the mass of the object attached to the spring in a spring-and-mass oscillator, the frequency is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If the mass of the spring is taken into account, the effective mass is greater than the mass of the object alone.

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m_{\text{eff}}}}$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{f'}{f} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{m_{\text{eff}}}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m}}} = \sqrt{\frac{m}{m_{\text{eff}}}}$$

Solve for f':

$$f' = f\sqrt{\frac{m}{m_{\text{eff}}}}$$

Because f' varies inversely with the square root of m, taking into account the effective mass of the spring predicts that the frequency will be reduced.

16 ••

Determine the Concept The period of the lamp varies inversely with the square root of the effective value of the local gravitational field.

1. greater than T_0 when — B. the train rounds a curve of radius R with speed v.

2. less than T_0 when — D. the train goes over the crest of a hill of radius of curvature R with constant speed.

3. equal to T_0 when — A. the train moves horizontally with constant velocity.

C. the train climbs a hill of inclination θ at constant speed.

17 ••

Picture the Problem We can use $f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$ to express the frequencies of the two mass-spring systems in terms of their masses. Dividing one of the equations by the other will allow us to express M_A in terms of M_B.

Express the frequency of mass-spring system A as a function of its mass:

$$f_A = \frac{1}{2\pi}\sqrt{\frac{k}{M_A}}$$

Express the frequency of mass-spring system B as a function of its mass:

$$f_B = \frac{1}{2\pi}\sqrt{\frac{k}{M_B}}$$

Divide the second of these equations by the first to obtain:

$$\frac{f_B}{f_A} = \sqrt{\frac{M_A}{M_B}}$$

Solve for M_A:

$$M_A = \left(\frac{f_B}{f_A}\right)^2 M_B = \left(\frac{f_B}{2f_B}\right)^2 M_B = \tfrac{1}{4}M_B$$

and $\boxed{(d)\text{ is correct.}}$

18 ••

Picture the Problem We can relate the energies of the two mass-spring systems through either $E = \frac{1}{2}kA^2$ or $E = \frac{1}{2}M\omega^2 A^2$ and investigate the relationship between their amplitudes by equating the expressions, substituting for M_A, and expressing A_A in terms of A_B.

Express the energy of mass-spring system A:

$$E_A = \frac{1}{2}k_A A_A^2 = \frac{1}{2}M_A\omega_A^2 A_A^2$$

Express the energy of mass-spring system B:

$$E_B = \frac{1}{2}k_B A_B^2 = \frac{1}{2}M_B\omega_B^2 A_B^2$$

Divide the first of these equations by the second to obtain:

$$\frac{E_A}{E_B} = 1 = \frac{\frac{1}{2}M_A\omega_A^2 A_A^2}{\frac{1}{2}M_B\omega_B^2 A_B^2}$$

Substitute for M_A and simplify:

$$1 = \frac{2M_B\omega_A^2 A_A^2}{M_B\omega_B^2 A_B^2} = \frac{2\omega_A^2 A_A^2}{\omega_B^2 A_B^2}$$

Solve for A_A:

$$A_A = \frac{\omega_B}{\sqrt{2}\omega_A}A_B$$

Without knowing how ω_A and ω_B, or k_A and k_B, are related, we cannot simplify this expression further. $\boxed{(d)\text{ is correct.}}$

19 ••

Picture the Problem We can express the energy of each system using $E = \frac{1}{2}kA^2$ and, because the energies are equal, equate them and solve for A_A.

Express the energy of mass-spring system A in terms of the amplitude of its motion:

$$E_A = \frac{1}{2}k_A A_A^2$$

Express the energy of mass-spring system B in terms of the amplitude of its motion:

$$E_B = \frac{1}{2}k_B A_B^2$$

Because the energies of the two systems are equal we can equate them to obtain:

$$\tfrac{1}{2}k_A A_A^2 = \tfrac{1}{2}k_B A_B^2$$

Solve for A_A:

$$A_A = \sqrt{\frac{k_B}{k_A}}A_B$$

Substitute for k_A and simplify to obtain:

$$A_A = \sqrt{\frac{k_B}{2k_B}}A_B = \frac{A_B}{\sqrt{2}}$$

and $\boxed{(b)\text{ is correct.}}$

20 ••

Picture the Problem The period of a simple pendulum is independent of the mass of its bob and is given by $T = 2\pi\sqrt{L/g}$.

Express the period of pendulum A:

$$T_A = 2\pi\sqrt{\frac{L_A}{g}}$$

Express the period of pendulum B:

$$T_B = 2\pi\sqrt{\frac{L_B}{g}}$$

Divide the first of these equations by the second and solve for L_A/L_B:

$$\frac{L_A}{L_B} = \left(\frac{T_A}{T_B}\right)^2$$

Substitute for T_A and solve for L_B to obtain:

$$L_A = \left(\frac{2T_B}{T_B}\right)^2 L_B = 4L_B$$

and $\boxed{(c)\text{ is correct.}}$

Estimation and Approximation

21 ••

Picture the Problem The Q factor for this system is related to the decay constant τ through $Q = \omega_0\tau = 2\pi\tau/T$ and the amplitude of the child's damped motion varies with time according to $A = A_0 e^{-t/2\tau}$. We can set the ratio of two displacements separated by eight periods equal to $1/e$ to determine τ in terms of T.

Express Q as a function of τ:

$$q = \omega_0\tau = \frac{2\pi\tau}{T} \qquad (1)$$

The amplitude of the oscillations varies with time according to:

$$A = A_0 e^{-t/2\tau}$$

The amplitude after eight periods is:

$$A_8 = A_0 e^{-(t+8T)/2\tau}$$

Express and simplify the ratio A_8/A:

$$\frac{A_8}{A} = \frac{A_0 e^{-(t+8T)/2\tau}}{A_0 e^{-t/2\tau}} = e^{-4T/\tau}$$

Set this ratio equal to $1/e$ and solve for τ:

$$e^{-4T/\tau} = e^{-1} \Rightarrow \tau = 4T$$

Substitute in equation (1) and evaluate Q:

$$Q = \frac{2\pi(4T)}{T} = \boxed{8\pi}$$

***22 ••**

Picture the Problem Assume that an average length for an arm is about 0.8 m, and that it can be treated as a uniform stick, pivoted at one end. We can use the expression for the period of a physical pendulum to derive an expression for the period of the swinging arm. When carrying a heavy briefcase, the mass is concentrated mostly at the end of the pivot (i.e., in the briefcase), so we can treat the arm-plus-briefcase as a simple pendulum.

(*a*) Express the period of a uniform rod pivoted at one end:

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

where I is the moment of inertia of the stick about an axis through one end, M is the mass of the stick, and D (= $L/2$) is the distance from the end of the stick to its center of mass.

Express the moment of inertia of the stick with respect to an axis through its end:

$$I = \tfrac{1}{3}ML^2$$

Substitute the values for I and D to find T:

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\left(\frac{1}{2}L\right)}} = 2\pi\sqrt{\frac{2L}{3g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{2(0.8\,\text{m})}{3(9.81\,\text{m/s}^2)}} = \boxed{1.47\,\text{s}}$$

(*b*) Express the period of a simple pendulum:

$$T' = 2\pi\sqrt{\frac{L'}{g}}$$

where L' is slightly longer than the arm length due to the size of the briefcase.

Assuming $L' = 1$ m, evaluate the period of the simple pendulum:

$$T' = 2\pi\sqrt{\frac{1\,\text{m}}{9.81\,\text{m/s}^2}} = \boxed{2.01\,\text{s}}$$

$$\boxed{\text{From observation of people as they walk, these estimates seem reasonable.}}$$

Simple Harmonic Motion

23 •

Picture the Problem The position of the particle is given by $x = A\cos(\omega t + \delta)$ where A is the amplitude of the motion, ω is the angular frequency, and δ is a phase constant.

(*a*) Use the definition of ω to determine f:

$$f = \frac{\omega}{2\pi} = \frac{6\pi\,\text{s}^{-1}}{2\pi} = \boxed{3.00\,\text{Hz}}$$

(*b*) Evaluate the reciprocal of the frequency:

$$T = \frac{1}{f} = \frac{1}{3.00\,\text{Hz}} = \boxed{0.333\,\text{s}}$$

(*c*) Compare $x = (7\text{ cm})\cos 6\pi t$ to $x = A\cos(\omega t + \delta)$:

$$A = \boxed{7.00\,\text{cm}}$$

(*d*) $x = 0$ when $\cos\omega t = 0$:

$$\omega t = \cos^{-1} 0 = \frac{\pi}{2}$$

Solve for t:

$$t = \frac{\pi}{2\omega} = \frac{\pi}{2(6\pi)} = \boxed{0.0833\,\text{s}}$$

Differentiate x to find $v(t)$:

$$v = \frac{d}{dt}[(7\,\text{cm})\cos 6\pi t] = -(42\pi\,\text{cm/s})\sin 6\pi t$$

Evaluate $v(0.0833\text{ s})$:

$$v(0.0833\,\text{s}) = -(42\pi\,\text{cm/s})\sin 6\pi(0.0833\,\text{s}) < 0$$

$$\boxed{\text{Because } v < 0\text{, the particle is moving in the negative direction at } t = 0.0833\,\text{s.}}$$

24 •

Picture the Problem The initial position of the oscillating particle is related to the amplitude and phase constant of the motion by $x_0 = A\cos\delta$ where $0 \le \delta < 2\pi$.

(*a*) For $x_0 = 0$:

$$\cos\delta = 0$$

and

$$\delta = \cos^{-1} 0 = \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$$

(*b*) For $x_0 = -A$:

$$-A = A\cos\delta$$

and

$$\delta = \cos^{-1}(-1) = \boxed{\pi}$$

(*c*) For $x_0 = A$:

$$A = A\cos\delta$$

and

$$\delta = \cos^{-1}(1) = \boxed{0}$$

(*d*) When $x = A/2$:

$$\frac{A}{2} = A\cos\delta$$

and

$$\delta = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$$

***25 •**

Picture the Problem The position of the particle as a function of time is given by $x = A\cos(\omega t + \delta)$. Its velocity as a function of time is given by $v = -A\omega\sin(\omega t + \delta)$ and its acceleration by $a = -A\omega^2\cos(\omega t + \delta)$. The initial position and velocity give us two equations from which to determine the amplitude A and phase constant δ.

(*a*) Express the position, velocity, and acceleration of the particle as a function of t:

$$x = A\cos(\omega t + \delta) \quad (1)$$

$$v = -A\omega\sin(\omega t + \delta) \quad (2)$$

$$a = -A\omega^2\cos(\omega t + \delta) \quad (3)$$

Find the angular frequency of the particle's motion:

$$\omega = \frac{2\pi}{T} = \frac{4\pi}{3}\,\text{s}^{-1} = 4.19\,\text{s}^{-1}$$

Relate the initial position and velocity to the amplitude and phase constant:

$$x_0 = A\cos\delta$$

and

$$v_0 = -\omega A\sin\delta$$

Divide these equations to eliminate A:

$$\frac{v_0}{x_0} = \frac{-\omega A\sin\delta}{A\cos\delta} = -\omega\tan\delta$$

Solve for δ and substitute numerical values to obtain:

$$\delta = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) = \tan^{-1}\left(-\frac{0}{x_0\omega}\right) = 0$$

Substitute in equation (1) to obtain:

$$x = (25\,\text{cm})\cos\left[\left(\frac{4\pi}{3}\text{s}^{-1}\right)t\right]$$
$$= \boxed{(25\,\text{cm})\cos\left[\left(4.19\,\text{s}^{-1}\right)t\right]}$$

(*b*) Substitute in equation (2) to obtain:

$$v = -(25\,\text{cm})\left(\frac{4\pi}{3}\text{s}^{-1}\right)\sin\left[\left(\frac{4\pi}{3}\text{s}^{-1}\right)t\right]$$
$$= \boxed{-(105\,\text{cm/s})\sin\left[\left(4.19\,\text{s}^{-1}\right)t\right]}$$

(*c*) Substitute in equation (3) to obtain:

$$a = -(25\,\text{cm})\left(\frac{4\pi}{3}\text{s}^{-1}\right)^2\cos\left[\left(\frac{4\pi}{3}\text{s}^{-1}\right)t\right]$$
$$= \boxed{-\left(439\,\text{cm/s}^2\right)\cos\left[\left(4.19\,\text{s}^{-1}\right)t\right]}$$

26 •

Picture the Problem The maximum speed and maximum acceleration of the particle in are given by $v_{max} = A\omega$ and $a_{max} = A\omega^2$. The particle's position is given by $x = A\cos(\omega t + \delta)$ where $A = 7$ cm, $\omega = 6\pi\,\text{s}^{-1}$, and $\delta = 0$, and its velocity is given by $v = -A\omega\sin(\omega t + \delta)$.

(*a*) Express v_{max} in terms of A and ω:

$$v_{max} = A\omega = (7\,\text{cm})\left(6\pi\,\text{s}^{-1}\right)$$
$$= 42\pi\,\text{cm/s} = \boxed{1.32\,\text{m/s}}$$

(*b*) Express a_{max} in terms of A and ω:

$$a_{max} = A\omega^2 = (7\,\text{cm})\left(6\pi\,\text{s}^{-1}\right)^2$$
$$= 252\pi^2\,\text{cm/s}^2 = \boxed{24.9\,\text{m/s}^2}$$

(*c*) When $x = 0$:

$$\cos\omega t = 0$$

and

$$\omega t = \cos^{-1}0 = \frac{\pi}{2}, \frac{3\pi}{2}$$

Evaluate v at $\omega t = \dfrac{\pi}{2}$:

$$v = -A\omega\sin\left(\frac{\pi}{2}\right) = -A\omega$$

i.e., the particle is moving to the left.

Evaluate v at $\omega t = \frac{3\pi}{2}$:

$$v = -A\omega \sin\left(\frac{3\pi}{2}\right) = A\omega$$

i.e., the particle is moving to the right.

Solve for t:

$$t = \frac{3\pi}{2\omega} = \frac{3\pi}{2(6\pi\,\text{s}^{-1})} = \boxed{0.250\,\text{s}}$$

27 ••

Picture the Problem The position of the particle as a function of time is given by $x = A\cos(\omega t + \delta)$. Its velocity as a function of time is given by $v = -A\omega\sin(\omega t + \delta)$ and its acceleration by $a = -A\omega^2\cos(\omega t + \delta)$. The initial position and velocity give us two equations from which to determine the amplitude A and phase constant δ.

(*a*) Express the position, velocity, and acceleration of the particle as functions of t:

$$x = A\cos(\omega t + \delta) \quad (1)$$
$$v = -A\omega\sin(\omega t + \delta) \quad (2)$$
$$a = -A\omega^2\cos(\omega t + \delta) \quad (3)$$

Find the angular frequency of the particle's motion:

$$\omega = \frac{2\pi}{T} = \frac{4\pi}{3}\text{s}^{-1} = 4.19\,\text{s}^{-1}$$

Relate the initial position and velocity to the amplitude and phase constant:

$$x_0 = A\cos\delta$$

and

$$v_0 = -\omega A\sin\delta$$

Divide these equations to eliminate A:

$$\frac{v_0}{x_0} = \frac{-\omega A\sin\delta}{A\cos\delta} = -\omega\tan\delta$$

Solve for δ:

$$\delta = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right)$$

Substitute numerical values and evaluate δ:

$$\delta = \tan^{-1}\left(-\frac{50\,\text{cm/s}}{(25\,\text{cm})(4.192\,\text{s}^{-1})}\right) = -0.445\,\text{rad}$$

Use either the x_0 or v_0 equation (x_0 is used here) to find the amplitude:

$$A = \frac{x_0}{\cos\delta} = \frac{25\,\text{cm}}{\cos(-0.445\,\text{rad})} = 27.7\,\text{cm}$$

Substitute in equation (1) to obtain:

$$x = \boxed{(27.7\,\text{cm})\cos\left[(4.19\,\text{s}^{-1})t - 0.445\right]}$$

(*b*) Substitute in equation (2) to obtain:

$$v = -(27.7\,\text{cm})\left(\frac{4\pi}{3}\,\text{s}^{-1}\right) \times \sin\left[\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)t - 0.445\right] = \boxed{-(116\,\text{cm/s})\sin\left[\left(4.19\,\text{s}^{-1}\right)t - 0.445\right]}$$

(*c*) Substitute in equation (3) to obtain:

$$a = -(27.7\,\text{cm})\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)^2 \cos\left[\left(\frac{4\pi}{3}\,\text{s}^{-1}\right)t - 0.445\right] = \boxed{-\left(486\,\text{cm/s}^2\right)\cos\left[\left(4.19\,\text{s}^{-1}\right)t - 0.445\right]}$$

28 ••

Picture the Problem The position of the particle as a function of time is given by $x = A\cos(\omega t + \delta)$. We're given the amplitude A of the motion and can use the initial position of the particle to determine the phase constant δ. Once we've determined these quantities, we can express the distance traveled Δx during any interval of time.

Express the position of the particle as a function of t:

$$x = (12\,\text{cm})\cos(\omega t + \delta) \qquad (1)$$

Find the angular frequency of the particle's motion:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8\,\text{s}} = \frac{\pi}{4}\,\text{s}^{-1}$$

Relate the initial position of the particle to the amplitude and phase constant:

$$x_0 = A\cos\delta$$

Solve for δ:

$$\delta = \cos^{-1}\frac{x_0}{A} = \cos^{-1}\frac{0}{A} = \frac{\pi}{2}$$

Substitute in equation (1) to obtain:

$$x = (12\,\text{cm})\cos\left[\left(\frac{\pi}{4}\,\text{s}^{-1}\right)t + \frac{\pi}{2}\right]$$

Express the distance the particle travels in terms of t_f and t_i:

$$\Delta x = \left|(12\,\text{cm})\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_f + \frac{\pi}{2}\right] - (12\,\text{cm})\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_i + \frac{\pi}{2}\right]\right|$$

$$= \left|(12\,\text{cm})\left\{\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_f + \frac{\pi}{2}\right] - \cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_i + \frac{\pi}{2}\right]\right\}\right|$$

(*a*) Evaluate Δx for $t_f = 2$ s, $t_i = 1$ s:

$$\Delta x = \left|(12\,\text{cm})\left\{\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(2\,\text{s}) + \frac{\pi}{2}\right] - \cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(0) + \frac{\pi}{2}\right]\right\}\right|$$

$$= \left|(12\,\text{cm})\{-1 - 0\}\right|$$

$$= \boxed{12.0\,\text{cm}}$$

(*b*) Evaluate Δx for $t_f = 4$ s, $t_i = 2$ s:

$$\Delta x = \left|(12\,\text{cm})\left\{\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(4\,\text{s}) + \frac{\pi}{2}\right] - \cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(2\,\text{s}) + \frac{\pi}{2}\right]\right\}\right|$$

$$= \left|(12\,\text{cm})\{0 - 1\}\right|$$

$$= \boxed{12.0\,\text{cm}}$$

(*c*) Evaluate Δx for $t_f = 1$ s, $t_i = 0$:

$$\Delta x = \left|(12\,\text{cm})\left\{\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(1\,\text{s}) + \frac{\pi}{2}\right] - \cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(0) + \frac{\pi}{2}\right]\right\}\right|$$

$$= \left|(12\,\text{cm})\{-0.7071 - 0\}\right|$$

$$= \boxed{8.49\,\text{cm}}$$

(*d*) Evaluate Δx for $t_f = 2$ s, $t_i = 1$ s:

$$\Delta x = \left|(12\,\text{cm})\left\{\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(2\,\text{s})+\frac{\pi}{2}\right] - \cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)(1\,\text{s})+\frac{\pi}{2}\right]\right\}\right|$$
$$= |(12\,\text{cm})\{-1+0.7071\}|$$
$$= \boxed{3.51\,\text{cm}}$$

29 ••

Picture the Problem The position of the particle as a function of time is given by $x = (10\,\text{cm})\cos(\omega t + \delta)$. We can determine the angular frequency ω from the period of the motion and the phase constant δ from the initial position and velocity. Once we've determined these quantities, we can express the distance traveled Δx during any interval of time.

Express the position of the particle as a function of *t*:

$$x = (10\,\text{cm})\cos(\omega t + \delta) \qquad (1)$$

Find the angular frequency of the particle's motion:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8\,\text{s}} = \frac{\pi}{4}\text{s}^{-1}$$

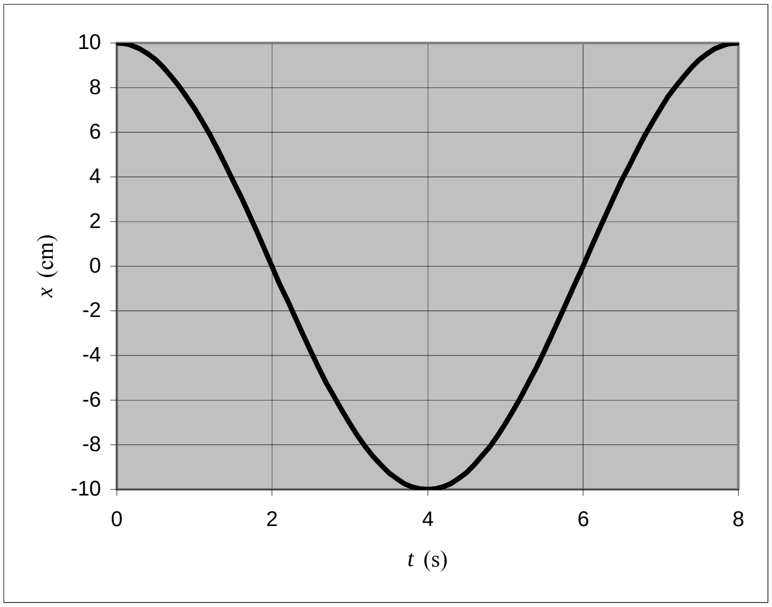
Find the phase constant of the motion:

$$\delta = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) = \tan^{-1}\left(-\frac{0}{x_0\omega}\right) = 0$$

Substitute in equation (1) to obtain:

$$x = (10\,\text{cm})\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t\right]$$

(*a*) A graph of $x=(10\,\text{cm})\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t\right]$ follows:



(*b*) Express the distance the particle travels in terms of t_f and t_i:

$$\Delta x=\left|(10\,\text{cm})\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_f\right]-(10\,\text{cm})\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_i\right]\right|$$
$$=\left|(10\,\text{cm})\left\{\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_f\right]-\cos\left[\left(\frac{\pi}{4}\text{s}^{-1}\right)t_i\right]\right\}\right| \quad (2)$$

Substitute numerical values in equation (2) and evaluate Δx in each of the given time intervals to obtain:

t_f	t_i	Δx
(s)	(s)	(cm)
1	0	2.93
2	1	7.07
3	2	7.07
4	3	2.93

***30 ••**

Picture the Problem We can use the expression for the maximum acceleration of an oscillator to relate the 10*g* military specification to the compliance frequency.

Express the maximum acceleration of an oscillator:

$$a_{max}=A\omega^2$$

Express the relationship between the angular frequency and the frequency of the vibrations:

$$\omega = 2\pi f$$

Substitute to obtain:

$$a_{max} = 4\pi^2 A f^2$$

Solve for f:

$$f = \frac{1}{2\pi}\sqrt{\frac{a_{max}}{A}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{98.1\,\text{m/s}^2}{1.5\times10^{-2}\,\text{m}}} = \boxed{12.9\,\text{Hz}}$$

31 ••

Picture the Problem The maximum speed and acceleration of the particle are given by $v_{max} = A\omega$ and $a_{max} = A\omega^2$. The velocity and acceleration of the particle are given by $v = -A\omega\sin\omega t$ and $a = -A\omega^2\cos\omega t$.

(*a*) Find v_{max} from A and ω:

$$v_{max} = A\omega = (2.5\,\text{m})(\pi\,\text{s}^{-1}) = \boxed{7.85\,\text{m/s}}$$

Find a_{max} from A and ω:

$$a_{max} = A\omega^2 = (2.5\,\text{m})(\pi\,\text{s}^{-1})^2 = \boxed{24.7\,\text{m/s}^2}$$

(*b*) Use the equation for the position of the particle to relate its position at $x = 1.5$ m to the time t' to reach this position:

$$1.5\,\text{m} = (2.5\,\text{m})\cos\pi t'$$

Solve for $\pi t'$:

$$\pi t' = \cos^{-1}0.6 = 0.9273\,\text{rad}$$

Evaluate v when $\pi t = \pi t'$:

$$v = -(2.5\,\text{m})(\pi\,\text{s}^{-1})\sin(0.9273\,\text{rad}) = \boxed{-6.28\,\text{m/s}}$$

where the minus sign indicates that the particle is moving in the negative direction.

Evaluate a when $\pi t = \pi t'$:

$$a = -(2.5\,\text{m})(\pi\,\text{s}^{-1})^2\cos(0.9273\,\text{rad}) = \boxed{-14.8\,\text{m/s}^2}$$

where the minus sign indicates that the particle's acceleration is in the negative direction.

***32 ••**

Picture the Problem We can use the formula for the cosine of the sum of two angles to write $x = A_0 \cos(\omega t + \delta)$ in the desired form. We can then evaluate x and dx/dt at $t = 0$ to relate A_c and A_s to the initial position and velocity of a particle undergoing simple harmonic motion.

(*a*) Apply the trigonometric identity $\cos(\omega t + \delta) = \cos\omega t\cos\delta - \sin\omega t\sin\delta$ to obtain:

$$x = A_0\cos(\omega t + \delta) = A_0[\cos\omega t\cos\delta - \sin\omega t\sin\delta]$$
$$= -A_0\sin\delta\sin\omega t + A_0\cos\delta\cos\omega t$$
$$= \boxed{A_s\sin\omega t + A_c\cos\omega t}$$

provided

$A_s = -A_0\sin\delta$ and $A_c = A_0\cos\delta$

(*b*) At $t = 0$:

$$x(0) = \boxed{A_0\cos\delta = A_c}$$

Evaluate dx/dt:

$$v = \frac{dx}{dt} = \frac{d}{dt}[A_s\sin\omega t + A_c\cos\omega t]$$
$$= A_s\omega\cos\omega t - A_c\omega\sin\omega t$$

Evaluate $v(0)$ to obtain:

$$v(0) = \omega A_s = \boxed{-\omega A_0\sin\delta}$$

Simple Harmonic Motion and Circular Motion

33 •

Picture the Problem We can find the period of the motion from the time required for the particle to travel completely around the circle. The frequency of the motion is the reciprocal of its period and the *x*-component of the particle's position is given by $x = A\cos(\omega t + \delta)$.

(*b*) Use the definition of speed to find the period of the motion:

$$T = \frac{2\pi r}{v} = \frac{2\pi(0.4\,\text{m})}{0.8\,\text{m/s}} = \boxed{3.14\,\text{s}}$$

(*a*) Because the frequency and the period are reciprocals of each other:

$$f = \frac{1}{T} = \frac{1}{3.14\,\text{s}} = \boxed{0.318\,\text{Hz}}$$

(*c*) Express the *x* component of the

$$x = A\cos(\omega t + \delta)$$

position of the particle:

Assuming that the particle is on the positive x axis at time $t = 0$:

$$A = A\cos\delta \Rightarrow \delta = \cos^{-1}1 = 0$$

Substitute for A, ω, and δ to obtain:

$$x = A\cos(2\pi ft) = \boxed{(40\,\text{cm})\cos\left[\left(2\,\text{s}^{-1}\right)t\right]}$$

***34** •

Picture the Problem We can find the period of the motion from the time required for the particle to travel completely around the circle. The angular frequency of the motion is 2π times the reciprocal of its period and the x-component of the particle's position is given by $x = A\cos(\omega t + \delta)$.

(*a*) Use the definition of speed to express and evaluate the speed of the particle:

$$v = \frac{2\pi r}{T} = \frac{2\pi(15\,\text{cm})}{3\,\text{s}} = \boxed{31.4\,\text{cm/s}}$$

(*b*) Express the angular velocity of the particle:

$$\omega = \frac{2\pi}{T} = \boxed{\frac{2\pi}{3}\,\text{rad/s}}$$

(*c*) Express the x component of the position of the particle:

$$x = A\cos(\omega t + \delta)$$

Assuming that the particle is on the positive x axis at time $t = 0$:

$$A = A\cos\delta \Rightarrow \delta = \cos^{-1}1 = 0$$

Substitute to obtain:

$$x = \boxed{(15\,\text{cm})\cos\left(\frac{2\pi}{3}\,\text{s}^{-1}\right)t}$$

Energy in Simple Harmonic Motion

35 •

Picture the Problem The total energy of the object is given by $E_{\text{tot}} = \frac{1}{2}kA^2$, where A is the amplitude of the object's motion.

Express the total energy of the system:

$$E_{\text{tot}} = \tfrac{1}{2}kA^2$$

Substitute numerical values and evaluate E_{tot}:

$$E_{\text{tot}} = \tfrac{1}{2}(4.5\,\text{kN/m})(0.1\,\text{m})^2 = \boxed{22.5\,\text{J}}$$

36 •

Picture the Problem The total energy of an oscillating object can be expressed in terms of its kinetic energy as it passes through its equilibrium position: $E_{tot} = \frac{1}{2}mv_{max}^2$. Its maximum speed, in turn, can be expressed in terms of its angular frequency and the amplitude of its motion.

Express the total energy of the object in terms of its maximum kinetic energy:

$$E = \tfrac{1}{2}mv_{max}^2$$

Express v_{max}:

$$v_{max} = A\omega = 2\pi Af$$

Substitute to obtain:

$$E = \tfrac{1}{2}m(2\pi Af)^2 = 2mA^2\pi^2 f^2$$

Substitute numerical values and evaluate E:

$$E = 2(3\,\text{kg})(0.1\,\text{m})^2\pi^2\left(2.4\,\text{s}^{-1}\right)^2 = \boxed{3.41\,\text{J}}$$

37 •

Picture the Problem The total mechanical energy of the oscillating object can be expressed in terms of its kinetic energy as it passes through its equilibrium position: $E_{tot} = \frac{1}{2}mv_{max}^2$. Its total energy is also given by $E_{tot} = \frac{1}{2}kA^2$. We can equate these expressions to obtain an expression for A.

(*a*) Express the total mechanical energy of the object in terms of its maximum kinetic energy:

$$E = \tfrac{1}{2}mv_{max}^2$$

Substitute numerical values and evaluate E:

$$E = \tfrac{1}{2}(1.5\,\text{kg})(0.7\,\text{m/s})^2 = \boxed{0.368\,\text{J}}$$

(*b*) Express the total energy of the object in terms of the amplitude of its motion:

$$E_{tot} = \tfrac{1}{2}kA^2$$

Solve for A:

$$A = \sqrt{\frac{2E_{tot}}{k}}$$

Substitute numerical values and evaluate A:

$$A = \sqrt{\frac{2(0.368\,\text{J})}{500\,\text{N/m}}} = \boxed{3.84\,\text{cm}}$$

38 •

Picture the Problem The total energy of the oscillating object can be expressed in terms of its kinetic energy as it passes through its equilibrium position: $E_{tot} = \frac{1}{2}mv_{max}^2$. Its total energy is also given by $E_{tot} = \frac{1}{2}kA^2$. We can solve the latter equation to find *A* and solve the former equation for v_{max}.

(*a*) Express the total energy of the object as a function of the amplitude of its motion:

$$E_{tot} = \frac{1}{2}kA^2$$

Solve for *A*:

$$A = \sqrt{\frac{2E_{tot}}{k}}$$

Substitute numerical values and evaluate *A*:

$$A = \sqrt{\frac{2(0.9\,\text{J})}{2000\,\text{N/m}}} = \boxed{3.00\,\text{cm}}$$

(*b*) Express the total energy of the object in terms of its maximum speed:

$$E_{tot} = \frac{1}{2}mv_{max}^2$$

Solve for v_{max}:

$$v_{max} = \sqrt{\frac{2E_{tot}}{m}}$$

Substitute numerical values and evaluate v_{max}:

$$v_{max} = \sqrt{\frac{2(0.9\,\text{J})}{3\,\text{kg}}} = \boxed{0.775\,\text{m/s}}$$

39 •

Picture the Problem The total energy of the object is given by $E_{tot} = \frac{1}{2}kA^2$. We can solve this equation for the force constant *k* and substitute the numerical data to determine its value.

Express the total energy of the oscillator as a function of the amplitude of its motion:

$$E_{tot} = \frac{1}{2}kA^2$$

Solve for *k*:

$$k = \frac{2E_{tot}}{A^2}$$

Substitute numerical values and evaluate *k*:

$$k = \frac{2(1.4\,\text{J})}{(0.045\,\text{m})^2} = \boxed{1.38\,\text{kN/m}}$$

***40 ••**

Picture the Problem The total energy of the object is given, in terms of its maximum kinetic energy by $E_{tot} = \frac{1}{2}mv_{max}^2$. We can express v_{max} in terms of A and ω and, in turn, express ω in terms of a_{max} to obtain an expression for E_{tot} in terms of a_{max}.

Express the total energy of the object in terms of its maximum kinetic energy:

$$E_{tot} = \tfrac{1}{2}mv_{max}^2$$

Relate the maximum speed of the object to its angular frequency:

$$v_{max} = A\omega$$

Substitute to obtain:

$$E_{tot} = \tfrac{1}{2}m(A\omega)^2 = \tfrac{1}{2}mA^2\omega^2$$

Relate the maximum acceleration of the object to its angular frequency:

$$a_{max} = A\omega^2$$

or

$$\omega^2 = \frac{a_{max}}{A}$$

Substitute and simplify to obtain:

$$E_{tot} = \tfrac{1}{2}mA^2\frac{a_{max}}{A} = \tfrac{1}{2}mAa_{max}$$

Substitute numerical values and evaluate E_{tot}:

$$E_{tot} = \tfrac{1}{2}(3\,\text{kg})(0.08\,\text{m})(3.50\,\text{m/s}^2) = \boxed{0.420\,\text{J}}$$

Springs

41 •

Picture the Problem The frequency of the object's motion is given by $f = \frac{1}{2\pi}\sqrt{k/m}$.

Its period is the reciprocal of its frequency. The maximum velocity and acceleration of an object executing simple harmonic motion are $v_{max} = A\omega$ and $a_{max} = A\omega^2$, respectively.

(*a*) The frequency of the motion is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{4.5\,\text{kN/m}}{2.4\,\text{kg}}} = \boxed{6.89\,\text{Hz}}$$

(*b*) The period of the motion to is the reciprocal of its frequency:

$$T=\frac{1}{f}=\frac{1}{6.89\,\text{s}^{-1}}=\boxed{0.145\,\text{s}}$$

(*c*) Because the object is released from rest after the spring to which it is attached is stretched 10 cm:

$$A=\boxed{0.100\,\text{m}}$$

(*d*) Express the object's maximum speed:

$$v_{\text{max}}=A\omega=2\pi fA$$

Substitute numerical values and evaluate v_{max}:

$$v_{\text{max}}=2\pi\left(6.89\,\text{s}^{-1}\right)(0.1\,\text{m})=\boxed{4.33\,\text{m/s}}$$

(*e*) Express the object's maximum acceleration:

$$a_{\text{max}}=A\omega^2=\omega v_{\text{max}}=2\pi f v_{\text{max}}$$

Substitute numerical values and evaluate a_{max}:

$$a_{\text{max}}=2\pi\left(6.89\,\text{s}^{-1}\right)(4.33\,\text{m/s})=\boxed{187\,\text{m/s}^2}$$

(*f*) The object first reaches its equilibrium when:

$$t=\tfrac{1}{4}T=\tfrac{1}{4}(0.145\,\text{s})=\boxed{36.3\,\text{ms}}$$

Because the resultant force acting on the object as it passes through its equilibrium point is zero, the acceleration of the object is:

$$a=\boxed{0}$$

42 •

Picture the Problem The frequency of the object's motion is given by $f=\frac{1}{2\pi}\sqrt{k/m}$. Its period is the reciprocal of its frequency. The maximum velocity and acceleration of an object executing simple harmonic motion are $v_{\text{max}}=A\omega$ and $a_{\text{max}}=A\omega^2$, respectively.

(*a*) The frequency of the motion is given by:

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Substitute numerical values and evaluate *f*:

$$f=\frac{1}{2\pi}\sqrt{\frac{700\,\text{N/m}}{5\,\text{kg}}}=\boxed{1.88\,\text{Hz}}$$

(*b*) The period of the motion is the reciprocal of its frequency:

$$T=\frac{1}{f}=\frac{1}{1.88\,\text{s}^{-1}}=\boxed{0.531\,\text{s}}$$

(*c*) Because the object is released from rest after the spring to which it is attached is stretched 8 cm:

$$A=\boxed{0.0800\,\text{m}}$$

(*d*) Express the object's maximum speed:

$$v_{max}=A\omega=2\pi fA$$

Substitute numerical values and evaluate v_{max}:

$$v_{max}=2\pi\left(1.88\,\text{s}^{-1}\right)\left(0.08\,\text{m}\right)=\boxed{0.945\,\text{m/s}}$$

(*e*) Express the object's maximum acceleration:

$$a_{max}=A\omega^2=\omega v_{max}=2\pi f v_{max}$$

Substitute numerical values and evaluate a_{max}:

$$a_{max}=2\pi\left(1.88\,\text{s}^{-1}\right)\left(0.945\,\text{m/s}\right)=\boxed{11.2\,\text{m/s}^2}$$

(*f*) The object first reaches its equilibrium when:

$$t=\tfrac{1}{4}T=\tfrac{1}{4}\left(0.531\,\text{s}\right)=\boxed{0.133\,\text{s}}$$

Because the resultant force acting on the object as it passes through its equilibrium point is zero, the acceleration of the object is:

$$a=\boxed{0}$$

43 •

Picture the Problem The angular frequency, in terms of the force constant of the spring and the mass of the oscillating object, is given by $\omega^2=k/m$. The period of the motion is the reciprocal of its frequency. The maximum velocity and acceleration of an object executing simple harmonic motion are $v_{max}=A\omega$ and $a_{max}=A\omega^2$, respectively.

(*a*) Relate the angular frequency of the motion to the force constant of the spring:

$$\omega^2=\frac{k}{m}$$

or

$$k=m\omega^2=4\pi^2f^2m$$

Substitute numerical values to obtain:

$$k=4\pi^2\left(2.4\,\text{s}^{-1}\right)^2\left(3\,\text{kg}\right)=\boxed{682\,\text{N/m}}$$

(*b*) Relate the period of the motion to its frequency:

$$T = \frac{1}{f} = \frac{1}{2.4\,\text{s}^{-1}} = \boxed{0.417\,\text{s}}$$

(*c*) Express the maximum speed of the object:

$$v_{\max} = A\omega = 2\pi f A$$

Substitute numerical values and evaluate $v_{\max}$:

$$v_{\max} = 2\pi\left(2.4\,\text{s}^{-1}\right)(0.1\,\text{m}) = \boxed{1.51\,\text{m/s}}$$

(*d*) Express the maximum acceleration of the object:

$$a_{\max} = A\omega^2 = 4\pi^2 f^2 A$$

Substitute numerical values and evaluate $a_{\max}$:

$$a_{\max} = 4\pi^2\left(2.4\,\text{s}^{-1}\right)^2(0.1\,\text{m}) = \boxed{22.7\,\text{m/s}^2}$$

***44 •**

Picture the Problem We can find the frequency of vibration of the car-and-passenger system using $f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$, where *M* is the total mass of the system. The spring constant can be determined from the compressing force and the amount of compression.

Express the frequency of the car-and-passenger system:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

Express the spring constant:

$$k = \frac{F}{\Delta x} = \frac{mg}{\Delta x}$$

where *m* is the person's mass.

Substitute to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{mg}{M\Delta x}}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{1}{2\pi}\sqrt{\frac{(85\,\text{kg})\left(9.81\,\text{m/s}^2\right)}{(2485\,\text{kg})\left(2.35\times10^{-2}\,\text{m}\right)}} = \boxed{0.601\,\text{Hz}}$$

45 •

Picture the Problem We can relate the force constant *k* to the maximum acceleration by eliminating ω^2 between $\omega^2 = k/m$ and $a_{\max} = A\omega^2$. We can also express the frequency f

of the motion by substituting ma_{max}/A for k in $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(*a*) Relate the angular frequency of the motion to the force constant and the mass of the oscillator:

$$\omega^2=\frac{k}{m} \text{ or } k=\omega^2 m$$

Relate the object's maximum acceleration to its angular frequency and amplitude and solve for the square of the angular frequency:

$$a_{max}=A\omega^2$$

or

$$\omega^2=\frac{a_{max}}{A} \qquad (1)$$

Substitute to obtain:

$$k=\frac{ma_{max}}{A}$$

Substitute numerical values and evaluate k:

$$k=\frac{(4.5\,\text{kg})(26\,\text{m/s}^2)}{3.8\times10^{-2}\,\text{m}}=\boxed{3.08\,\text{kN/m}}$$

(*b*) Replace ω in equation (1) by $2\pi f$ and solve for f to obtain:

$$f=\frac{1}{2\pi}\sqrt{\frac{a_{max}}{A}}$$

Substitute numerical values and evaluate f:

$$f=\frac{1}{2\pi}\sqrt{\frac{26\,\text{m/s}^2}{3.8\times10^{-2}\,\text{m}}}=\boxed{4.16\,\text{Hz}}$$

(*c*) The period of the motion is the reciprocal of its frequency:

$$T=\frac{1}{f}=\frac{1}{4.16\,\text{s}^{-1}}=\boxed{0.240\,\text{s}}$$

46 •

Picture the Problem We can find the frequency of the motion from its maximum speed and the relationship between frequency and angular frequency. The mass of the object can be found by eliminating ω between $\omega^2=k/m$ and $v_{max}=A\omega$.

(*b*) Express the object's maximum speed as a function of the frequency of its motion:

$$v_{max}=A\omega=2\pi fA \qquad (1)$$

Solve for f:

$$f=\frac{v_{max}}{2\pi A}$$

Substitute numerical values and evaluate f:

$$f=\frac{2.2\,\text{m/s}}{2\pi(5.8\times10^{-2}\,\text{m})}=\boxed{6.04\,\text{Hz}}$$

(*a*) Relate the square of the angular frequency of the motion to the force constant and the mass of the object:

$$\omega^2 = \frac{k}{m} \Rightarrow m = \frac{k}{\omega^2} \qquad (2)$$

Eliminate ω between equations (1) and (2) to obtain:

$$m = \frac{kA^2}{v_{max}^2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{(1.8\times10^3\ \text{N/m})(5.8\times10^{-2}\ \text{m})^2}{(2.2\ \text{m/s})^2} = \boxed{1.25\ \text{kg}}$$

(*c*) The period of the motion is the reciprocal of its frequency:

$$T = \frac{1}{f} = \frac{1}{6.04\ \text{s}^{-1}} = \boxed{0.166\ \text{s}}$$

47 ••

Picture the Problem The maximum speed of the block is given by $v_{max} = A\omega$ and the angular frequency of the motion is $\omega = \sqrt{k/m} = 5.48\ \text{rad/s}$. We'll assume that the position of the block is given by $x = A\cos\omega t$ and solve for ωt for *x* = 4 cm and *x* = 0. We can use these values for ωt to find the time for the block to travel from *x* = 4 cm to its equilibrium position.

(*a*) Express the maximum speed of the block as a function of the system's angular frequency:

$$v_{max} = A\omega$$

Substitute numerical values and evaluate v_{max}:

$$v_{max} = (0.08\ \text{m})(5.48\ \text{rad/s}) = \boxed{0.438\ \text{m/s}}$$

(*b*) Assuming that $x = A\cos\omega t$, evaluate ωt for *x* = 4 cm = *A*/2:

$$\frac{A}{2} = A\cos\omega t \Rightarrow \omega t = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

Evaluate *v* for $\omega t = \pi/3$:

$$v = v_{max}\sin\omega t = (0.438\ \text{m/s})\sin\frac{\pi}{3} = (0.438\ \text{m/s})\frac{\sqrt{3}}{2} = \boxed{0.379\ \text{m/s}}$$

Express *a* as a function of v_{max} and ω:

$$a = A\omega^2\cos\omega t = v_{max}\,\omega\cos\omega t$$

Substitute numerical values and evaluate a:

$$a = (0.438\,\text{m/s})(5.48\,\text{rad/s})\cos\frac{\pi}{3}$$
$$= \boxed{1.20\,\text{m/s}^2}$$

(*c*) Evaluate ωt for $x = 0$:

$$0 = A\cos\omega t \Rightarrow \omega t = \cos^{-1}0 = \frac{\pi}{2}$$

Let Δt = time to go from $\omega t = \pi/3$ to $\omega t = \pi/2$. Then:

$$\omega\Delta t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

Solve for and evaluate Δt:

$$\Delta t = \frac{\pi}{6\omega} = \frac{\pi}{6(5.48\,\text{rad/s})} = \boxed{95.5\,\text{ms}}$$

***48 ••**

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can find the mass of the object using $m = k/\omega^2$. We can apply a condition for translational equilibrium to the object when it is at its equilibrium position to determine the amount the spring has stretched from its natural length. Finally, we can use the initial conditions to determine A and δ and express $x(t)$ and then differentiate this expression to obtain $v(t)$ and $a(t)$.

(*a*) Express the angular frequency of the system in terms of the mass of the object fastened to the vertical spring and solve for the mass of the object:

$$\omega^2 = \frac{k}{m} \Rightarrow m = \frac{k}{\omega^2}$$

Express ω^2 in terms of f:

$$\omega^2 = 4\pi^2 f^2$$

Substitute to obtain:

$$m = \frac{k}{4\pi^2 f^2}$$

Substitute numerical values and evaluate m:

$$m = \frac{1800\,\text{N/m}}{4\pi^2(5.5\,\text{s}^{-1})^2} = \boxed{1.51\,\text{kg}}$$

(*b*) Letting Δx represent the amount the spring is stretched from its natural length when the object is in equilibrium, apply $\sum F_y = 0$ to the object when it is in equilibrium:

$$k\Delta x - mg = 0$$

Solve for Δx:

$$\Delta x = \frac{mg}{k}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(1.51\,\text{kg})(9.81\,\text{m/s}^2)}{1800\text{ N/m}} = \boxed{8.23\,\text{mm}}$$

(*c*) Express the position of the object as a function of time:

$$x = A\cos(\omega t + \delta)$$

Use the initial conditions ($x_0 = -2.5$ cm and $v_0 = 0$) to find δ:

$$\delta = \tan^{-1}\left(-\frac{v_0}{\omega x_0}\right) = \tan^{-1} 0 = \pi$$

Evaluate ω:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800\,\text{N/m}}{1.51\,\text{kg}}} = 34.5\,\text{rad/s}$$

Substitute to obtain:

$$x = (2.5\,\text{cm})\cos[(34.5\,\text{rad/s})t + \pi]$$
$$= \boxed{-(2.5\,\text{cm})\cos[(34.5\,\text{rad/s})t]}$$

Differentiate $x(t)$ to obtain v:

$$v = \boxed{(86.4\,\text{cm/s})\sin[(34.5\,\text{rad/s})t]}$$

Differentiate $v(t)$ to obtain a:

$$a = \boxed{(29.8\,\text{m/s}^2)\cos[(34.5\,\text{rad/s})t]}$$

49 ••

Picture the Problem Let the system include the object and the spring. Then, the net external force acting on the system is zero. Choose $E_\text{i} = 0$ and apply the conservation of mechanical energy to the system.

Express the period of the motion in terms of its angular frequency:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply conservation of energy to the system:

$$E_\text{i} = E_\text{f} \text{ or } 0 = U_\text{g} + U_\text{spring}$$

Substitute for U_g and U_spring:

$$0 = -mg\Delta x + \tfrac{1}{2}k(\Delta x)^2$$

Solve for $\omega^2 = k/m$:

$$\omega^2 = \frac{k}{m} = \frac{2g}{\Delta x}$$

Substitute numerical values and evaluate ω^2:

$$\omega^2 = \frac{2(9.81\,\text{m/s}^2)}{3.42\times10^{-2}\text{ m}} = 574\,\text{rad/s}^2$$

Substitute in equation (1) to obtain:

$$T=\frac{2\pi}{\sqrt{574\,\text{rad/s}^2}}=\boxed{0.262\,\text{s}}$$

50 ••

Picture the Problem Let the system include the object and the spring. Then the net external force acting on the system is zero. Because the net force acting on the object when it is at its equilibrium position is zero, we can apply a condition for translational equilibrium to determine the distance from the starting point to the equilibrium position. Letting $E_i = 0$, we can apply conservation of energy to the system to determine how far down the object moves before coming momentarily to rest. We can find the period of the motion and the maximum speed of the object from $T=2\pi\sqrt{m/k}$ and $v_{max}=A\sqrt{k/m}$.

(*a*) Apply $\sum F_y=0$ to the object when it is at the equilibrium position:

$$ky_0-mg=0$$

Solve for y_0:

$$y_0=\frac{mg}{k}$$

Substitute numerical values and evaluate y_0:

$$y_0=\frac{(1\,\text{kg})(9.81\,\text{m/s}^2)}{250\,\text{N/m}}=\boxed{3.92\,\text{cm}}$$

(*b*) Apply conservation of energy to the system:

$$E_i=E_f$$

or

$$0=U_g+U_{spring}$$

Substitute for U_g and U_{spring}:

$$0=-mgy_f+\tfrac{1}{2}ky_f^2$$

Solve for y_f:

$$y_f=\frac{2mg}{k}$$

Substitute numerical values and evaluate y_f:

$$y_f=\frac{2(1\,\text{kg})(9.81\,\text{m/s}^2)}{250\,\text{N/m}}=\boxed{7.85\,\text{cm}}$$

(*c*) Express the period T of the motion in terms of the mass of the object and the spring constant:

$$T=2\pi\sqrt{\frac{m}{k}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{1\,\text{kg}}{250\,\text{N/m}}} = \boxed{0.397\,\text{s}}$$

(*d*) The object will be moving with its maximum speed when it reaches its equilibrium position:

$$v_{\max} = A\omega = A\sqrt{\frac{k}{m}}$$

Substitute numerical values and evaluate $v_{\max}$:

$$v_{\max} = (3.92\,\text{cm})\sqrt{\frac{250\,\text{N/m}}{1\,\text{kg}}} = \boxed{62.0\,\text{cm/s}}$$

(*e*) The time required for the object to reach equilibrium is one-fourth of its period:

$$t = \tfrac{1}{4}T = \tfrac{1}{4}(0.397\,\text{s}) = \boxed{99.3\,\text{ms}}$$

51 ••

Picture the Problem The stunt woman's kinetic energy, after 2 s of flight, is $K_{2\text{s}} = \frac{1}{2}mv_{2\text{s}}^2$. We can evaluate this quantity as soon as we know how fast she is moving after two seconds. Because her motion is oscillatory, her velocity as a function of time is $v(t) = -A\omega\sin(\omega t + \delta)$. We can find the amplitude of her motion from her distance of fall and the angular frequency of her motion by applying conservation of energy to her fall to the ground.

Express the kinetic energy of the stunt woman when she has fallen for 2 s:

$$K_{2\text{s}} = \tfrac{1}{2}mv_{2\text{s}}^2 \qquad (1)$$

Express her velocity as a function of time:

$$v(t) = -A\omega\sin(\omega t + \delta)$$

where $\delta = 0$ (she starts from rest with positive displacement) and

$$A = \tfrac{1}{2}(192\,\text{m}) = 96\,\text{m}$$

$$\therefore v(t) = -(96\,\text{m})\omega\sin(\omega t) \qquad (2)$$

Letting $E_i = 0$, use conservation of energy to find the force constant of the elastic band:

$$0 = U_g + U_{\text{elastic}}$$

or

$$0 = -mgh + \tfrac{1}{2}kh^2 = 0$$

Solve for k:

$$k = \frac{2mg}{h}$$

Substitute numerical values and evaluate k:

$$k = \frac{2(60\,\text{kg})(9.81\,\text{m/s}^2)}{192\,\text{m}} = 6.13\,\text{N/m}$$

Express the angular frequency of her motion:

$$\omega = \sqrt{\frac{k}{m}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{6.13\,\text{N/m}}{60\,\text{kg}}} = 0.320\,\text{rad/s}$$

Substitute in equation (2) to obtain:

$$\begin{aligned} v(t) &= -(96\,\text{m})(0.320\,\text{rad/s}) \\ &\quad \times \sin[(0.320\,\text{rad/s})t] \\ &= (30.7\,\text{m/s})\sin[(0.320\,\text{rad/s})t] \end{aligned}$$

Evaluate $v(2\text{ s})$:

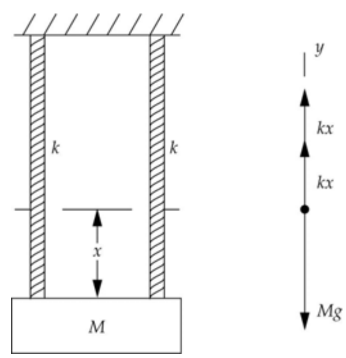
$$\begin{aligned} v(2\,\text{s}) &= (30.7\,\text{m/s})\sin[(0.320\,\text{rad/s})(2\,\text{s})] \\ &= 18.3\,\text{m/s} \end{aligned}$$

Substitute in equation (1) and evaluate $K(2\text{ s})$:

$$K(2\,\text{s}) = \tfrac{1}{2}(60\,\text{kg})(18.3\,\text{m/s})^2 = \boxed{10.1\,\text{kJ}}$$

***52 ••**

Picture the Problem The diagram shows the stretched bungie cords supporting the suitcase under equilibrium conditions. We can use $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{M}}$ to express the frequency of the suitcase in terms of the effective "spring" constant k_{eff} and apply a condition for translational equilibrium to the suitcase to find k_{eff}.



Express the frequency of the suitcase oscillator:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{M}}$$

Apply $\sum F_y = 0$ to the suitcase to obtain:

$$kx + kx - Mg = 0$$

or

$$2kx - Mg = 0$$

or

$$k_{\text{eff}}x - Mg = 0$$

where $k_{\text{eff}} = 2k$

Solve for k_{eff} to obtain:

$$k_{\text{eff}} = \frac{Mg}{x}$$

Substitute to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{x}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{0.05\,\text{m}}} = \boxed{2.23\,\text{Hz}}$$

53 ••

Picture the Problem The frequency of the motion of the stone and block depends on the force constant of the spring and the mass of the stone plus block. The force constant can be determined from the equilibrium of the system when the spring is stretched additionally by the addition of the stone to the mass. When the block is at the point of maximum upward displacement, it is momentarily at rest and the net force acting on it is its weight.

(*a*) Express the frequency of the motion in terms of k and m:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{\text{tot}}}}$$

where m_{tot} is the total mass suspended from the spring.

Apply $\sum F_y = 0$ to the stone when it is at its equilibrium position:

$$k\Delta y - mg = 0$$

Solve for k:

$$k = \frac{mg}{\Delta y}$$

Substitute numerical values and evaluate k:

$$k = \frac{(0.03\,\text{kg})(9.81\,\text{m/s}^2)}{0.05\,\text{m}} = 5.89\,\text{N/m}$$

Substitute and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{5.89\,\text{N/m}}{0.15\,\text{kg}}} = \boxed{0.997\,\text{Hz}}$$

(*b*) The time to travel from its lowest point to its highest point is one-half its period:

$$t = \tfrac{1}{2}T = \frac{1}{2f} = \frac{1}{2(0.997\,\text{s}^{-1})} = \boxed{0.502\,\text{s}}$$

(*c*) When the stone is at a point of maximum upward displacement:

$$F_{\text{net}} = mg = (0.03\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{0.294\,\text{N}}$$

54 ••

Picture the Problem We can use the maximum acceleration of the oscillator $a_{\max} = A\omega^2$ to express $a_{\max}$ in terms of A, k, and m. k can be determined from the equilibrium of the system when the spring is stretched additionally by the addition of the stone to the mass. If the stone is to remain in contact with the block, the block's maximum downward acceleration must not exceed g.

Express the maximum acceleration in terms of the angular frequency and amplitude of the motion:

$$a_{\max} = A\omega^2$$

Relate ω^2 to the force constant and the mass of the stone:

$$\omega^2 = \frac{k}{m}$$

Substitute to obtain:

$$a_{\max} = A\frac{k}{m}$$

Apply $\sum F_y = 0$ to the stone when it is at its equilibrium position:

$$k\Delta y - mg = 0$$

Solve for k:

$$k = \frac{mg}{\Delta y}$$

Substitute numerical values and evaluate k:

$$k = \frac{(0.03\,\text{kg})(9.81\,\text{m/s}^2)}{0.05\,\text{m}} = 5.89\,\text{N/m}$$

Substitute numerical values to express $a_{\max}$ in terms of A:

$$a_{\max} = A\frac{5.89\,\text{N/m}}{0.15\,\text{kg}} = (39.3\,\text{s}^{-2})A$$

Set $a_{\max} = g$ and solve for $A_{\max}$:

$$A_{\max} = \frac{g}{39.3\,\text{s}^{-2}}$$

Substitute for g and evaluate $A_{\max}$:

$$A_{\max} = \frac{9.81\,\text{m/s}^2}{39.3\,\text{s}^{-2}} = \boxed{25.0\,\text{cm}}$$

55 ••

Picture the Problem The maximum height above the floor to which the object rises is the sum of its initial distance from the floor and the amplitude of its motion. We can find the amplitude of its motion by relating it to the object's maximum speed. Because the object initially travels downward, it will be three-fourths of the way through its cycle when it first reaches its maximum height. We can find the minimum initial speed the object would need to be given in order for the spring to become uncompressed by applying conservation of energy.

(*a*) Relate *h,* the maximum height above the floor to which the object rises, to the amplitude of its motion:

$$h = A + 5.0\text{ cm} \qquad (1)$$

Relate the maximum speed of the object to the angular frequency and amplitude of its motion and solve for the amplitude:

$$v_{max} = A\omega$$

or

$$A = v_{max}\sqrt{\frac{m}{k}} \qquad (2)$$

Using its definition, express and evaluate the force constant of the spring:

$$k = \frac{mg}{\Delta y} = \frac{(2\text{ kg})(9.81\text{ m/s}^2)}{0.03\text{ m}} = 654\text{ N/m}$$

Substitute numerical values in equation (2) and evaluate *A*:

$$A = 0.3\text{ m/s}\sqrt{\frac{2\text{ kg}}{654\text{ N/m}}} = 1.66\text{ cm}$$

Substitute in equation (1) to obtain:

$$h = 1.66\text{ cm} + 5.00\text{ cm} = \boxed{6.66\text{ cm}}$$

(*b*) Express the time required for the object to reach its maximum height the first time:

$$t = \tfrac{3}{4}T$$

Express the period of the motion:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Substitute numerical values and evaluate *T*:

$$T = 2\pi\sqrt{\frac{2\text{ kg}}{654\text{ N/m}}} = 0.347\text{ s}$$

Substitute to obtain:

$$t = \tfrac{3}{4}(0.347\text{ s}) = \boxed{0.261\text{ s}}$$

(c) Because $h < 8.0$ cm: $\boxed{\text{the spring is never uncompressed.}}$

Using conservation of energy and letting U_g be zero 5 cm above the floor, relate the height to which the object rises, Δy, to its initial kinetic energy:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or, because $K_f = U_i = 0$,

$$\tfrac{1}{2}mv_i^2 - mg\Delta y + \tfrac{1}{2}k(\Delta y)^2 - \tfrac{1}{2}k(L - y_i)^2 = 0$$

Because $\Delta y = L - y_i$:

$$\tfrac{1}{2}mv_i^2 - mg\Delta y + \tfrac{1}{2}k(\Delta y)^2 - \tfrac{1}{2}k(\Delta y)^2 = 0$$

and

$$\tfrac{1}{2}mv_i^2 - mg\Delta y = 0$$

Solve for and evaluate v_i for $\Delta y = 3$ cm:

$$v_i = \sqrt{2g\Delta y} = \sqrt{2(9.81\,\text{m/s}^2)(3\,\text{cm})} = 0.767\,\text{m/s}$$

i.e., the minimum initial velocity that must be given to the object for the spring to be uncompressed at some time is $\boxed{0.767\,\text{m/s}}$

***56 ••**

Picture the Problem We can relate the elongation of the cable to the load on it using the definition of Young's modulus and use the expression for the frequency of a spring and mass oscillator to find the oscillation frequency of the engine block at the end of the wire.

(a) Using the definition of Young's modulus, relate the elongation of the cable to the applied stress:

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta\ell/\ell}$$

Solve for $\Delta\ell$:

$$\Delta\ell = \frac{F\ell}{AY} = \frac{Mg\ell}{AY}$$

Substitute numerical values and evaluate $\Delta\ell$:

$$\Delta\ell = \frac{(950\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m})}{(1.5\,\text{cm}^2)(150\,\text{GN/m}^2)} = \boxed{1.04\,\text{mm}}$$

(b) Express the oscillation frequency of the wire-engine block system:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{M}}$$

Express the effective "spring" constant of the cable:

$$k_{eff} = \frac{F}{\Delta\ell} = \frac{Mg}{\Delta\ell}$$

Substitute to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta\ell}}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{1.04\,\text{mm}}} = \boxed{15.5\,\text{Hz}}$$

Energy of an Object on a Vertical Spring

57 ••

Picture the Problem Let the origin of our coordinate system be at y_0, where y_0 is the equilibrium position of the object and let $U_g = 0$ at this location. Because $F_{net} = 0$ at equilibrium, the extension of the spring is then $y_0 = mg/k$, and the potential energy stored in the spring is $U_s = \frac{1}{2}ky_0^2$. A further extension of the spring by an amount y increases U_s to $\frac{1}{2}k(y + y_0)^2 = \frac{1}{2}ky^2 + kyy_0 + \frac{1}{2}ky_0^2 = \frac{1}{2}ky^2 + mgy + \frac{1}{2}ky_0^2$. Consequently, if we set $U = U_g + U_s = 0$, a further extension of the spring by y increases U_s by $\frac{1}{2}ky^2 + mgy$ while decreasing U_g by mgy. Therefore, if $U = 0$ at the equilibrium position, the change in U is given by $\frac{1}{2}k(y')^2$, where $y' = y - y_0$.

(*a*) Express the total energy of the system:

$$E = \tfrac{1}{2}kA^2$$

Substitute numerical values and evaluate *E*:

$$E = \tfrac{1}{2}(600\,\text{N/m})(0.03\,\text{m})^2 = \boxed{0.270\,\text{J}}$$

(*b*) Express and evaluate U_g when the object is at its maximum downward displacement:

$$\begin{aligned} U_g &= -mgA \\ &= -(2.5\,\text{kg})(9.81\,\text{m/s}^2)(0.03\,\text{m}) \\ &= \boxed{-0.736\,\text{J}} \end{aligned}$$

(*c*) When the object is at its maximum downward displacement:

$$\begin{aligned} U_s &= \tfrac{1}{2}kA^2 + mgA \\ &= \tfrac{1}{2}(600\,\text{N/m})(0.03\,\text{m})^2 \\ &\quad + (2.5\,\text{kg})(9.81\,\text{m/s}^2)(0.03\,\text{m}) \\ &= \boxed{1.01\,\text{J}} \end{aligned}$$

(*d*) The object has its maximum kinetic energy when it is passing through its equilibrium position:

$$K_{max} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(600\,\text{N/m})(0.03\,\text{m})^2 = \boxed{0.270\,\text{J}}$$

58 ••

Picture the Problem Let the origin of our coordinate system be at y_0, where y_0 is the equilibrium position of the object and let $U_g = 0$ at this location. Because $F_{net} = 0$ at equilibrium, the extension of the spring is then $y_0 = mg/k$, and the potential energy stored in the spring is $U_s = \tfrac{1}{2}ky_0^2$. A further extension of the spring by an amount y increases U_s to $\tfrac{1}{2}k(y+y_0)^2 = \tfrac{1}{2}ky^2 + kyy_0 + \tfrac{1}{2}ky_0^2 = \tfrac{1}{2}ky^2 + mgy + \tfrac{1}{2}ky_0^2$. Consequently, if we set $U = U_g + U_s = 0$, a further extension of the spring by y increases U_s by $\tfrac{1}{2}ky^2 + mgy$ while decreasing U_g by mgy. Therefore, if $U = 0$ at the equilibrium position, the change in U is given by $\tfrac{1}{2}k(y')^2$, where $y' = y - y_0$.

(*a*) Express the total energy of the system:

$$E = \tfrac{1}{2}kA^2$$

Letting Δy represent the amount the spring is stretched from its natural length by the 1.5-kg object, apply $\sum F_y = ma_y$ to the object when it is in its equilibrium position:

$$k\Delta y - mg = 0$$

Solve for k:

$$k = \frac{mg}{\Delta y}$$

Substitute for k to obtain:

$$E = \frac{mgA^2}{2\Delta y}$$

Substitute numerical values and evaluate E:

$$E = \frac{(1.5\,\text{kg})(9.81\,\text{m/s}^2)(0.022\,\text{m})^2}{2(0.028\,\text{m})} = \boxed{0.127\,\text{J}}$$

(*b*) Express U_g when the object is at its maximum downward displacement:

$$U_g = -mgA$$

Substitute numerical values and evaluate U_g:

$$U_g = -(1.5\,\text{kg})(9.81\,\text{m/s}^2)(0.022\,\text{m}) = \boxed{-0.324\,\text{J}}$$

(*c*) When the object is at its maximum downward displacement:

$$U_s = \tfrac{1}{2}kA^2 + mgA$$

Substitute numerical values and evaluate U_s:

$$U_s = \tfrac{1}{2}(526\,\text{N/m})(0.022\,\text{m})^2 + (1.5\,\text{kg})(9.81\,\text{m/s}^2)(0.022\,\text{m}) = \boxed{0.451\,\text{J}}$$

(*d*) The object has its maximum kinetic energy when it is passing through its equilibrium position:

$$K_{max} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(526\,\text{N/m})(0.022\,\text{m})^2 = \boxed{0.127\,\text{J}}$$

***59 ••**

Picture the Problem We can find the amplitude of the motion by relating it to the maximum speed of the object. Let the origin of our coordinate system be at y_0, where y_0 is the equilibrium position of the object and let $U_g = 0$ at this location. Because $F_{net} = 0$ at equilibrium, the extension of the spring is then $y_0 = mg/k$, and the potential energy stored in the spring is $U_s = \tfrac{1}{2}ky_0^2$. A further extension of the spring by an amount y increases U_s to $\tfrac{1}{2}k(y+y_0)^2 = \tfrac{1}{2}ky^2 + kyy_0 + \tfrac{1}{2}ky_0^2 = \tfrac{1}{2}ky^2 + mgy + \tfrac{1}{2}ky_0^2$. Consequently, if we set $U = U_g + U_s = 0$, a further extension of the spring by y increases U_s by $\tfrac{1}{2}ky^2 + mgy$ while decreasing U_g by mgy. Therefore, if $U = 0$ at the equilibrium position, the change in U is given by $\tfrac{1}{2}k(y')^2$, where $y' = y - y_0$.

(*a*) Relate the maximum speed of the object to the amplitude of its motion:

$$v_{max} = A\omega$$

Solve for A:

$$A = \frac{v_{max}}{\omega} = v_{max}\sqrt{\frac{m}{k}}$$

Substitute numerical values and evaluate A:

$$A = (0.3\,\text{m/s})\sqrt{\frac{1.2\,\text{kg}}{300\,\text{N/m}}} = \boxed{1.90\,\text{cm}}$$

(*b*) Express the energy of the object at maximum displacement:

$$E = \tfrac{1}{2}kA^2$$

Substitute numerical values and evaluate E:

$$E = \tfrac{1}{2}(300\,\text{N/m})(0.019\,\text{m})^2 = \boxed{0.0542\,\text{J}}$$

(*c*) At maximum displacement from equilibrium:

$$U_g = -mgA$$

Substitute numerical values and evaluate U_g:

$$U_g = -(1.2\,\text{kg})(9.81\,\text{m/s}^2)(0.019\,\text{m}) = \boxed{-0.224\,\text{J}}$$

(*d*) Express the potential energy in the spring when the object is at its maximum downward displacement:

$$U_s = \tfrac{1}{2}kA^2 + mgA$$

Substitute numerical values and evaluate U_s:

$$U_s = \tfrac{1}{2}(300\,\text{N/m})(0.019\,\text{m})^2 + (1.2\,\text{kg})(9.81\,\text{m/s}^2)(0.019\,\text{m}) = \boxed{0.278\,\text{J}}$$

Simple Pendulums

60 •

Picture the Problem We can determine the required length of the pendulum from the expression for the period of a simple pendulum.

Express the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solve for *L*:

$$L = \frac{T^2 g}{4\pi^2}$$

Substitute numerical values and evaluate *L*:

$$L = \frac{(5\,\text{s})^2(9.81\,\text{m/s}^2)}{4\pi^2} = \boxed{6.21\,\text{m}}$$

61 •

Picture the Problem We can find the period of the pendulum from $T = 2\pi\sqrt{L/g_{\text{moon}}}$ where $g_{\text{moon}} = \frac{1}{6}g$ and $L = 6.21$ m.

Express the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g_{\text{moon}}}}$$

Substitute numerical values and evaluate *T*:

$$T = 2\pi\sqrt{\frac{6.21\,\text{m}}{\frac{1}{6}(9.81\,\text{m/s}^2)}} = \boxed{12.2\,\text{s}}$$

62 •

Picture the Problem We can find the value of g at the location of the pendulum by solving the equation $T = 2\pi\sqrt{L/g}$ for g and evaluating it for the given length and period.

Express the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solve for g:

$$g = \frac{4\pi^2 L}{T^2}$$

Substitute numerical values and evaluate g:

$$g = \frac{4\pi^2(0.7\,\text{m})}{(1.68\,\text{s})^2} = \boxed{9.79\,\text{m/s}^2}$$

***63** •

Picture the Problem We can use $T = 2\pi\sqrt{L/g}$ to find the period of this pendulum.

Express the period of a simple pendulum:

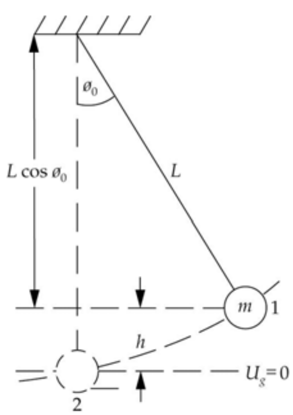
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{34\,\text{m}}{9.81\,\text{m/s}^2}} = \boxed{11.7\,\text{s}}$$

64 ••

Picture the Problem The figure shows the simple pendulum at maximum angular displacement ϕ_0. The total energy of the simple pendulum is equal to its initial gravitational potential energy. We can apply the definition of gravitational potential energy and use the small-angle approximation to show that $E \approx \frac{1}{2}mgL\phi_0^2$.



Express the total energy of the simple pendulum at maximum displacement:

$$E = U_{\text{max displacement}} = mgh = mgL[1 - \cos\phi_0]$$

For $\phi << 1$:

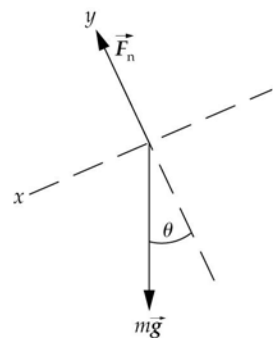
$$\cos\phi \approx 1 - \tfrac{1}{2}\phi^2$$

Substitute and simplify to obtain:

$$E = mgL\left[1-\left(1-\tfrac{1}{2}\phi_0^2\right)\right] = \boxed{\tfrac{1}{2}mgL\phi_0^2}$$

65 ••

Picture the Problem Because the cart is accelerating down the incline, the period of the simple pendulum will be given by $T = 2\pi\sqrt{L/g_{\text{eff}}}$ where g_{eff} is less than g by the acceleration of the cart. We can apply Newton's 2nd law to the cart to find its acceleration down the incline and then subtract this acceleration from g to find g_{eff}.



Express the period of a simple pendulum in terms of its length and the effective value of the acceleration of gravity:

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

Relate g_{eff} to the acceleration of the cart:

$$g_{\text{eff}} = g - a$$

Apply $\sum F_x = ma_x$ to the cart and solve for its acceleration:

$$mg\sin\theta = ma$$

and

$$a = g\sin\theta$$

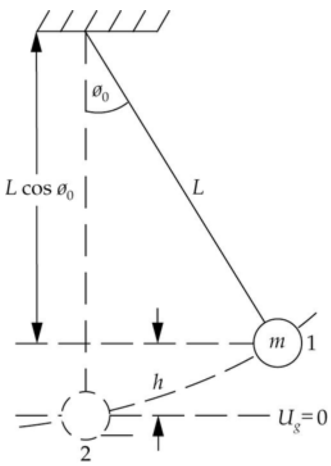
Substitute to obtain:

$$T = 2\pi\sqrt{\frac{L}{g-a}} = 2\pi\sqrt{\frac{L}{g-g\sin\theta}}$$

$$= \boxed{2\pi\sqrt{\frac{L}{g(1-\sin\theta)}}}$$

66 ••

Picture the Problem The figure shows the simple pendulum at maximum angular displacement ϕ_0. We can express the angular position of the pendulum's bob in terms of its initial angular position and time and differentiate this expression to find the maximum speed of the bob. We can use conservation of energy to find an exact value for v_{max} and the approximation $\cos\phi \approx 1 - \frac{1}{2}\phi^2$ to show that this value reduces to the former value for small ϕ.



(*a*) Relate the speed of the pendulum's bob to its angular speed:

$$v = L\frac{d\phi}{dt} \qquad (1)$$

Express the angular position of the pendulum as a function of time:

$$\phi = \phi_0 \cos\omega t$$

Differentiate this expression to express the angular speed of the pendulum:

$$\frac{d\phi}{dt} = -\phi_0\omega\sin\omega t$$

Substitute in equation (1) to obtain:

$$v = -L\phi_0\omega\sin\omega t = -v_{max}\sin\omega t$$

Simplify v_{max} to obtain:

$$v_{max} = L\phi_0\sqrt{\frac{g}{L}} = \boxed{\phi_0\sqrt{gL}}$$

(*b*) Use conservation of energy to relate the potential energy of the pendulum at point 1 to its kinetic energy at point 2:

$$\Delta K + \Delta U = 0$$

or, because $K_1 = U_2 = 0$,

$$K_2 - U_1 = 0$$

Substitute for K_2 and U_1:

$$\tfrac{1}{2}mv_2^2 - mgh = 0$$

Express h in terms of L and ϕ_0:

$$h = L(1-\cos\phi_0)$$

Substitute for h and solve for $v_2 = v_{max}$ to obtain:

$$v_{max} = \boxed{\sqrt{2gL(1-\cos\phi_0)}} \qquad (2)$$

(*c*) For $\phi_0 << 1$: $1-\cos\phi_0 \approx \frac{1}{2}\phi_0^2$

Substitute in equation (2) to obtain:

$$v_{\max} = \sqrt{2gL\left(\tfrac{1}{2}\phi_0^2\right)} = \boxed{\phi_0\sqrt{gL}}$$

in agreement with our result in part (*a*).

(*d*) Express the difference in the results from (*a*) and (*b*):

$$\Delta v = v_{\max,a} - v_{\max,b} \qquad (3)$$

Using ϕ_0 = 0.20 rad and L = 1 m, evaluate the result in (*b*):

$$v_{\max,b} = \sqrt{2\left(9.81\,\text{m/s}^2\right)(1\,\text{m})(1-\cos 0.2)} = 0.6254\,\text{m/s}$$

Using ϕ_0 = 0.20 rad and L = 1 m, evaluate the result in part (*a*):

$$v_{\max.a} = (0.20\,\text{rad})\sqrt{\left(9.81\,\text{m/s}^2\right)(1\,\text{m})} = 0.6264\,\text{m/s}$$

Substitute in equation (3) to obtain:

$$\Delta v = 0.6264\,\text{m/s} - 0.6254\,\text{m/s} = 0.001\,\text{m/s} = \boxed{1.00\,\text{mm/s}}$$

Physical Pendulums

67 •

Picture the Problem The period of this physical pendulum is given by $T = 2\pi\sqrt{I/MgD}$ where I is the moment of inertia of the thin disk with respect to an axis through its pivot point. We can use the parallel-axis theorem to express I in terms of the moment of inertia of the disk with respect to its center of mass and the distance from its center of mass to its pivot point.

Express the period of physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

Using the parallel-axis theorem, find the moment of inertia of the thin disk about an axis through the pivot point:

$$I = I_{\text{cm}} + MR^2 = \tfrac{1}{2}MR^2 + MR^2 = \tfrac{3}{2}MR^2$$

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{\tfrac{3}{2}MR^2}{MgR}} = 2\pi\sqrt{\frac{3R}{2g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{3(0.2\,\text{m})}{2\left(9.81\,\text{m/s}^2\right)}} = \boxed{1.10\,\text{s}}$$

68 •

Picture the Problem The period of this physical pendulum is given by $T = 2\pi\sqrt{I/MgD}$ where I is the moment of inertia of the circular hoop with respect to an axis through its pivot point. We can use the parallel-axis theorem to express I in terms of the moment of inertia of the hoop with respect to its center of mass and the distance from its center of mass to its pivot point.

Express the period of the physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

Using the parallel-axis theorem, find the moment of inertia of the circular hoop about an axis through the pivot point:

$$I = I_{\text{cm}} + MR^2 = MR^2 + MR^2 = 2MR^2$$

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{2MR^2}{MgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{2(0.5\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{2.01\,\text{s}}$$

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Picture the Problem The period of a physical pendulum is given by $T = 2\pi\sqrt{I/MgD}$ where I is its moment of inertia with respect to an axis through its pivot point. We can solve this equation for I and evaluate it using the given numerical data.

Express the period of the physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

Solve for I:

$$I = \frac{MgDT^2}{4\pi^2}$$

Substitute numerical values and evaluate I:

$$I = \frac{(3\,\text{kg})(9.81\,\text{m/s}^2)(0.1\,\text{m})(2.6\,\text{s})^2}{4\pi^2} = \boxed{0.504\,\text{kg}\cdot\text{m}^2}$$

***70 ••**

Picture the Problem We can use the expression for the period of a simple pendulum to find the period of the clock.

(*a*) Express the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Substitute numerical values and evaluate *T*:

$$T = 2\pi\sqrt{\frac{4\,\text{m}}{9.81\,\text{m/s}^2}} = \boxed{4.01\,\text{s}}$$

(*b*) By effectively raising the center of mass of the pendulum, placing coins in the tray shortens the period.

71 ••

Picture the Problem Let *x* be the distance of the pivot from the center of the rod, *m* the mass at each end of the rod, and *L* the length of the rod. We can express the period of the physical pendulum as a function of the distance *x* and then differentiate this expression with respect to *x* to show that, when *x* = *L*/2, the period is a minimum.

(*a*) Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}} \qquad (1)$$

Express the moment of inertia of the dumbbell with respect to an axis through its center of mass:

$$I_{\text{cm}} = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \tfrac{1}{2}mL^2$$

Using the parallel-axis theorem, express the moment of inertia of the dumbbell with respect to an axis through the pivot point:

$$I = I_{\text{cm}} + 2mx^2 = \tfrac{1}{2}mL^2 + 2mx^2$$

Substitute in equation (1) to obtain:

$$T = 2\pi\sqrt{\frac{\frac{1}{2}mL^2 + 2mx^2}{2mgx}} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{\frac{1}{4}L^2 + x^2}{x}} \qquad (2)$$

$$= C\sqrt{\frac{\frac{1}{4}L^2 + x^2}{x}}$$

where $C = \dfrac{2\pi}{\sqrt{g}}$

Set $dT/dx = 0$ to find the condition for minimum T:

$$\frac{dT}{dx} = C \cdot \frac{d}{dx}\sqrt{\frac{\frac{1}{4}L^2 + x^2}{x}} = 0 \text{ for extrema}$$

Evaluate the derivative to obtain:

$$\frac{2x^2 - \left(\frac{1}{4}L^2 + x^2\right)}{x^2\sqrt{\frac{\frac{1}{4}L^2 + x^2}{x}}} = 0$$

Because the denominator of this expression cannot be zero, it must be true that:

$$2x^2 - \left(\frac{1}{4}L^2 + x^2\right) = 0$$

Solve for x to obtain:

$$x = \boxed{\frac{1}{2}L}$$

i.e., the period is a minimum when the pivot point is at one of the masses.

(*b*) Substitute $x = L/4$ in equation (2) and simplify to obtain:

$$T = 2\pi\sqrt{\frac{\frac{1}{4}L^2 + \left(\frac{1}{4}L\right)^2}{\frac{1}{2}gL}} = \pi\sqrt{\frac{5L}{g}}$$

Substitute numerical values and evaluate T:

$$T = \pi\sqrt{\frac{5(2\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{3.17\,\text{s}}$$

Remarks: In (*a*), we've shown that $x = L/2$ corresponds to an *extreme* value; i.e., to either a maximum or a minimum. To complete the demonstration that this value of x corresponds to a minimum, we can either (1) show that d^2T/dx^2 evaluated at $x = L/2$ is positive, or (2) graph T as a function of x and note that the graph is a minimum at $x = L/2$.

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Picture the Problem Let x be the distance of the pivot from the center of the rod. We'll express the period of the physical pendulum as a function of the distance x and then differentiate this expression with respect to x to find the location of the pivot point that minimizes the period of the physical pendulum.

Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}} \qquad (1)$$

Express the moment of inertia of the dumbbell with respect to an axis through its center of mass:

$$I_{\text{cm}} = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 + \tfrac{1}{12}(2m)L^2 = \tfrac{2}{3}mL^2$$

Using the parallel-axis theorem, express the moment of inertia of the dumbbell with respect to an axis through the pivot point:

$$I = I_{\text{cm}} + 4mx^2 = \tfrac{2}{3}mL^2 + 4mx^2$$

Substitute in equation (1) to obtain:

$$T = 2\pi\sqrt{\frac{\tfrac{2}{3}mL^2 + 4mx^2}{4mgx}} = \frac{\pi}{\sqrt{g}}\sqrt{\frac{\tfrac{2}{3}L^2 + 4x^2}{x}}$$

or

$$T = C\sqrt{\frac{\tfrac{2}{3}L^2 + 4x^2}{x}} \text{ where } C = \frac{\pi}{\sqrt{g}}$$

Set $dT/dx = 0$ to find the condition for minimum T:

$$\frac{dT}{dx} = C \times \frac{d}{dx}\sqrt{\frac{\tfrac{2}{3}L^2 + 4x^2}{x}} = 0 \text{ for extrema}$$

Evaluate the derivative to obtain:

$$\frac{8x^2 - \left(\tfrac{2}{3}L^2 + 4x^2\right)}{2x^2\sqrt{\frac{\tfrac{2}{3}L^2 + 4x^2}{x}}} = 0$$

Because the denominator of this expression cannot be zero, it follows that:

$$8x^2 - \left(\tfrac{2}{3}L^2 + 4x^2\right) = 0$$

Solve for x to obtain:

$$x = \frac{L}{\sqrt{6}}$$

The distance to the pivot point from the nearer mass is:

$$d = \frac{L}{2} - \frac{L}{\sqrt{6}} = \boxed{0.0918L}$$

Remarks: We've shown that $x = L/\sqrt{6}$ corresponds to an *extreme* value; i.e., to either a maximum or a minimum. To complete the demonstration that this value of x corresponds to a minimum, we can either (1) show that d^2T/dx^2 evaluated at

$x = L/\sqrt{6}$ is positive, or (2) graph T as a function of x and note that the graph is a minimum at $x = L/\sqrt{6}$.

***73 ••**

Picture the Problem Let *x* be the distance of the pivot from the center of the meter stick, *m* the mass of the meter stick, and *L* its length. We'll express the period of the meter stick as a function of the distance *x* and then differentiate this expression with respect to *x* to determine where the hole should be drilled to minimize the period.

Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}} \qquad (1)$$

Express the moment of inertia of the meter stick with respect to its center of mass:

$$I_{\text{cm}} = \tfrac{1}{12}mL^2$$

Using the parallel-axis theorem, express the moment of inertia of the meter stick with respect to the pivot point:

$$I = I_{\text{cm}} + mx^2 = \tfrac{1}{12}mL^2 + mx^2$$

Substitute in equation (1) to obtain:

$$T = 2\pi\sqrt{\frac{\frac{1}{12}mL^2 + mx^2}{mgx}} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{\frac{1}{12}L^2 + x^2}{x}} = C\sqrt{\frac{\frac{1}{12}L^2 + x^2}{x}}$$

where $C = \dfrac{2\pi}{\sqrt{g}}$

Set $dT/dx = 0$ to find the condition for minimum T:

$$\frac{dT}{dx} = C \times \frac{d}{dx}\sqrt{\frac{\frac{1}{12}L^2 + x^2}{x}} = 0 \text{ for extrema}$$

Evaluate the derivative to obtain:

$$\frac{2x^2 - \left(\frac{1}{12}L^2 + x^2\right)}{x^2\sqrt{\frac{\frac{1}{12}L^2 + x^2}{x}}} = 0$$

Because the denominator of this expression cannot be zero, it follows

$$2x^2 - \left(\tfrac{1}{12}L^2 + x^2\right) = 0$$

that:

Solve for and evaluate x to obtain:

$$x = \frac{L}{\sqrt{12}} = \frac{100\,\text{cm}}{\sqrt{12}} = 28.9\,\text{cm}$$

The hole should be drilled at a distance:

$$d = 50\,\text{cm} - 28.9\,\text{cm} = \boxed{21.1\,\text{cm}}$$

from the center of the meter stick.

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Picture the Problem Let *m* represent the mass and *r* the radius of the uniform disk. We'll use the expression for the period of a physical pendulum and the parallel-axis theorem to obtain a quadratic equation that we can solve for *d*. We will then treat our expression for the period of the pendulum as an extreme-value problem, setting its derivative equal to zero in order to determine the value for *d* that will minimize the period.

(*a*) Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

Using the parallel-axis theorem, relate the moment of inertia with respect to an axis through the hole to the moment of inertia with respect to the disk's center of mass:

$$I = I_{\text{cm}} + md^2 = \tfrac{1}{2}mR^2 + md^2$$

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{\frac{1}{2}mR^2 + md^2}{mgd}} = 2\pi\sqrt{\frac{\frac{1}{2}R^2 + d^2}{gd}} \qquad (1)$$

Square both sides of this equation, simplify, and substitute numerical values to obtain:

$$d^2 - \frac{gT^2}{4\pi^2}d + \frac{R^2}{2} = 0$$

or

$$d^2 - (1.553\,\text{m})d + 0.320\,\text{m}^2 = 0$$

Solve the quadratic equation to obtain:

$$d = \boxed{0.245\,\text{m}}$$

The second root, $d = 1.31$ m, is too large to be physically meaningful.

(*b*) Set the derivative of equation (1) equal to zero to find relative maxima and minima:

$$\frac{dT}{dd}=\frac{2\pi}{\sqrt{g}}\cdot\frac{d}{dd}\sqrt{\frac{\frac{1}{2}R^2+d^2}{d}}$$
$$=0\text{ for extrema}$$

Evaluate the derivative to obtain:

$$\frac{2d^2-\left(\frac{1}{2}R^2+d^2\right)}{2d^2\sqrt{\frac{\frac{1}{2}R^2+d^2}{d}}}=0$$

Because the denominator of this fraction cannot be zero:

$$2d^2-\left(\frac{1}{2}R^2+d^2\right)=0$$

Solve this equation to obtain:

$$d=\boxed{\frac{R}{\sqrt{2}}}$$

Evaluate equation (1) with $d=R/\sqrt{2}$ to obtain an expression for the shortest possible period of this physical pendulum:

$$T=2\pi\sqrt{\frac{\frac{1}{2}R^2+\frac{1}{2}R^2}{g\frac{R}{\sqrt{2}}}}=2\pi\sqrt{\frac{\sqrt{2}R}{g}}$$

Substitute numerical values and evaluate *T*:

$$T=2\pi\sqrt{\frac{\sqrt{2}(0.8\,\text{m})}{9.81\,\text{m/s}^2}}=\boxed{2.13\,\text{s}}$$

Remarks: We've shown that $d=R/\sqrt{2}$ corresponds to an *extreme* value; i.e., to either a maximum or a minimum. To complete the demonstration that this value of *d* corresponds to a minimum, we can either (1) show that d^2T/dd^2 evaluated at $d=R/\sqrt{2}$ is positive, or (2) graph *T* as a function of *d* and note that the graph is a minimum at $d=R/\sqrt{2}$.

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Picture the Problem We can use the equation for the period of a physical pendulum and the parallel-axis theorem to show that $h_1+h_2=gT^2/4\pi^2$.

Express the period of the physical pendulum:

$$T=2\pi\sqrt{\frac{I}{mgd}}$$

Using the parallel-axis theorem, relate the moment of inertia with respect to an axis through P_1 to the

$$I=I_{\text{cm}}+mh_1^2$$

moment of inertia with respect to the disk's center of mass:

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{I_{\text{cm}} + mh_1^2}{mgh_1}}$$

Square both sides of this equation and rearrange to obtain:

$$\frac{mgT^2}{4\pi^2} = \frac{I_{\text{cm}}}{h_1} + mh_1 \qquad (1)$$

Because the period of oscillation is the same for point P_2:

$$\frac{I_{\text{cm}}}{h_1} + mh_1 = \frac{I_{\text{cm}}}{h_2} + mh_2$$

Solve this equation for I_{cm}:

$$I_{\text{cm}} = mh_1h_2$$

Substitute in equation (1) to obtain:

$$\frac{mgT^2}{4\pi^2} = \frac{mh_1h_2}{h_1} + mh_1$$

or

$$\boxed{h_2 + h_1 = \frac{gT^2}{4\pi^2}}$$

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Picture the Problem We can find the period of the physical pendulum in terms of the period of a simple pendulum by starting with $T = 2\pi\sqrt{I/mgL}$ and applying the parallel-axis theorem. Performing a binomial expansion for $r << L$ on the radicand of our expression for T will lead to $T \approx T_0(1 + r^2/5L^2)$.

(a) Express the period of the physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

Using the parallel-axis theorem, relate the moment of inertia of the pendulum about an axis through its center of mass to its moment of inertia with respect to an axis through its point of support:

$$I = I_{\text{cm}} + mL^2 = \tfrac{2}{5}mr^2 + mL^2$$

Substitute and simplify to obtain:

$$T = 2\pi\sqrt{\frac{\frac{2}{5}mr^2 + mL^2}{mgL}} = 2\pi\sqrt{\frac{\frac{2}{5}r^2 + L^2}{gL}}$$

$$= 2\pi\sqrt{\frac{L}{g}\left(1+\frac{2r^2}{5L^2}\right)} = 2\pi\sqrt{\frac{L}{g}}\sqrt{1+\frac{2r^2}{5L^2}}$$

$$= \boxed{T_0\sqrt{1+\frac{2r^2}{5L^2}}}$$

(*b*) Using the binomial expansion, expand $\left(1+\frac{2r^2}{5L^2}\right)^{1/2}$:

$$\left(1+\frac{2r^2}{5L^2}\right)^{1/2} = 1+\frac{1}{2}\left(\frac{2r^2}{5L^2}\right)+\frac{1}{8}\left(\frac{2r^2}{5L^2}\right)^2 + \text{higher-order terms}$$

$$\approx 1+\frac{r^2}{5L^2}$$

provided $r << L$

Substitute in our result from (*a*) to obtain:

$$T \approx \boxed{T_0\left(1+\frac{r^2}{5L^2}\right)}$$

(*c*) Express the fractional error when the approximation $T = T_0$ is used for this pendulum:

$$\frac{\Delta T}{T} \approx \frac{T - T_0}{T_0} = \frac{T}{T_0} - 1$$

$$= 1 + \frac{r^2}{5L^2} - 1 = \frac{r^2}{5L^2}$$

Substitute numerical values and evaluate $\Delta T/T$:

$$\frac{\Delta T}{T} \approx \frac{(2\,\text{cm})^2}{5(100\,\text{cm})^2} = \boxed{0.008\%}$$

For an error of 1%:

$$\frac{r^2}{5L^2} = 0.01$$

Solve for and evaluate r with L = 100 cm:

$$r = L\sqrt{0.05} = (100\,\text{cm})\sqrt{0.05}$$

$$= \boxed{22.4\,\text{cm}}$$

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Picture the Problem The period of this physical pendulum is given by $T = 2\pi\sqrt{I/MgD}$. We can express its period as a function of the distance d by using the definition of the center of mass of the pendulum to find D in terms of d and the parallel-axis theorem to express I in terms of d. Solving the resulting quadratic equation yields d.

In (*b*), because the clock is losing 5 minutes per day, one would reposition the disk so that the clock runs faster; i.e., so the pendulum has a shorter period. We can determine the appropriate correction to make in the position of the disk by relating the fractional time loss to the fractional change in its position.

(*a*) Express the period of the physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{m_{\text{tot}}gx_{\text{cm}}}}$$

Solve for $\frac{I}{x_{\text{cm}}}$:

$$\frac{I}{x_{\text{cm}}} = \frac{T^2 g m_{\text{tot}}}{4\pi^2} \qquad (1)$$

Express the moment of inertia of the physical pendulum, relative to an axis through the pivot point, as a function of *d*:

$$I = I_{\text{cm}} + Md^2 = \tfrac{1}{3}mL^2 + \tfrac{1}{2}Mr^2 + Md^2$$

Substitute numerical values and evaluate *I*:

$$\begin{aligned} I &= \tfrac{1}{3}(0.8\,\text{kg})(2\,\text{m})^2 + \tfrac{1}{2}(1.2\,\text{kg})(0.15\,\text{m})^2 \\ &\quad + (1.2\,\text{kg})d^2 \\ &= 1.0802\,\text{kg}\cdot\text{m}^2 + (1.2\,\text{kg})d^2 \end{aligned}$$

Locate the center of mass of the physical pendulum relative to the pivot point:

$$(2\,\text{kg})x_{\text{cm}} = (0.8\,\text{kg})(1\,\text{m}) + (1.2\,\text{kg})d$$

and

$$x_{\text{cm}} = 0.4\,\text{m} + 0.6d$$

Substitute in equation (1) to obtain:

$$\frac{1.0802\,\text{kg}\cdot\text{m}^2 + (1.2\,\text{kg})d^2}{0.4\,\text{m} + 0.6d} = \frac{T^2(9.81\,\text{m/s}^2)(2\,\text{kg})}{4\pi^2} = (0.49698\,\text{kg}\cdot\text{m/s}^2)T^2 \qquad (2)$$

Setting $T = 2.5$ s and solving for *d* yields:

$$d = \boxed{1.63572\,\text{m}}$$

where we have kept more than three significant figures for use in part (*b*).

(*b*) There are 1440 minutes per day. If the clock loses 5 minutes per day, then the period of the clock is related to the perfect period of the clock by:

$$1435T = 1440T_{\text{perfect}}$$

where $T_{\text{perfect}} = 3.5$ s.

Solve for and evaluate T:

$$T = \frac{1440}{1435} T_{\text{perfect}} = \frac{1440}{1435}(3.5\,\text{s}) = 3.51220\,\text{s}$$

Substitute $T = 3.51220$ s in equation (2) and solve for d to obtain:

$$d = 3.40140\,\text{m}$$

Substitute $T = 3.50$ s in equation (2) and solve for d' to obtain:

$$d' = 3.37825\,\text{m}$$

Express the distance the disk needs to be moved upward to correct the period:

$$\Delta d = d - d' = 3.40140\,\text{m} - 3.37825\,\text{m} = \boxed{2.32\,\text{cm}}$$

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Picture the Problem The period of a simple pendulum depends on its amplitude ϕ_0 according to $T = 2\pi\sqrt{\frac{L}{g}}\left[1 + \frac{1}{2^2}\sin^2\frac{1}{2}\phi_0 + \frac{1}{2^2}\left(\frac{3}{4}\right)^2\sin^4\frac{1}{2}\phi_0 + \ldots\right]$. We can approximate T to the second-order term and express $\Delta T/T = (T_{\text{slow}} - T_{\text{accurate}})/T$. Equating this expression to $\Delta T/T$ calculated from the fractional daily loss of time will allow us to solve for and evaluate the amplitude of the pendulum that corresponds to keeping perfect time.

Express the fractional daily loss of time:

$$\frac{\Delta T}{T} = \frac{48\,\text{s}}{\text{day}} \times \frac{1\,\text{day}}{24\,\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} = \frac{48}{86400}$$

Approximate the period of the clock to the second-order term:

$$T = 2\pi\sqrt{\frac{L}{g}}\left[1 + \frac{1}{2^2}\sin^2\frac{1}{2}\phi_0\right]$$

Express the difference in the periods of the slow and accurate clocks:

$$\begin{aligned}\Delta T &= T_{\text{slow}} - T_{\text{accurate}} \\ &= 2\pi\sqrt{\frac{L}{g}}\left\{\left[1 + \frac{1}{2^2}\sin^2\frac{1}{2}(8.4°)\right] - \left[1 + \frac{1}{2^2}\sin^2\frac{1}{2}\phi_0\right]\right\} \\ &= 2\pi\sqrt{\frac{L}{g}}\left[\frac{1}{2^2}\sin^2\frac{1}{2}(8.4°) - \frac{1}{2^2}\sin^2\frac{1}{2}\phi_0\right]\end{aligned}$$

Divide both sides of this equation by T to obtain:

$$\frac{\Delta T}{T}=\frac{1}{4}\sin^2 4.2°-\frac{1}{4}\sin^2\frac{1}{2}\phi_0$$

Substitute for $\frac{\Delta T}{T}$ and simplify to obtain:

$$\frac{1}{4}\sin^2 4.2°-\frac{1}{4}\sin^2\frac{1}{2}\phi_0=\frac{48}{86400}$$

and

$$\sin\frac{1}{2}\phi_0=0.05605$$

Solve for ϕ_0:

$$\phi_0=\boxed{6.43°}$$

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Picture the Problem The period of a simple pendulum depends on its amplitude ϕ_0 according to $T=2\pi\sqrt{\frac{L}{g}}\left[1+\frac{1}{2^2}\sin^2\frac{1}{2}\phi_0+\frac{1}{2^2}\left(\frac{3}{4}\right)^2\sin^4\frac{1}{2}\phi_0+...\right]$. We'll approximate T to the second-order term and express $\Delta T/T=(T_{slow}-T_{correct})/T$. Equating this expression to $\Delta T/T$ calculated from the fractional daily loss of time will allow us to solve for and evaluate the amplitude of the pendulum that corresponds to keeping correct time.

Express the fractional daily loss of time:

$$\frac{\Delta T}{T}=\frac{5\,\text{min}}{\text{day}}\times\frac{1\,\text{day}}{24\,\text{h}}\times\frac{1\,\text{h}}{60\,\text{min}}=\frac{5}{1440}$$

Approximate the period of the clock to the second-order term:

$$T=2\pi\sqrt{\frac{L}{g}}\left[1+\frac{1}{2^2}\sin^2\frac{1}{2}\phi_0\right]$$

Assuming that the amplitude of the slow-running clock's pendulum is small enough to ignore, express the difference in the periods of the slow and corrected clocks:

$$\begin{aligned}\Delta T&=T_{slow}-T_{correct}\\&=2\pi\sqrt{\frac{L}{g}}\left\{1-\left[1+\frac{1}{2^2}\sin^2\frac{1}{2}\phi_0\right]\right\}\\&=2\pi\sqrt{\frac{L}{g}}\left[-\frac{1}{2^2}\sin^2\frac{1}{2}\phi_0\right]\end{aligned}$$

Divide both sides of this expression by T to obtain:

$$\frac{\Delta T}{T}=-\frac{1}{4}\sin^2\frac{1}{2}\phi_0$$

Substitute for $\frac{\Delta T}{T}$ and simplify to obtain:

$$-\frac{1}{4}\sin^2\frac{1}{2}\phi_0=\frac{-5}{1440}$$

and

$$\sin\frac{1}{2}\phi_0 = 0.1178$$

Solve for ϕ_0: $\phi_0 = \boxed{13.5°}$

Damped Oscillations

80 •

Picture the Problem We can use the definition of the damping constant and its dimensions to show that it has units of kg/s.

Using its definition, relate the decay constant τ to the damping constant b:

$$\tau = \frac{m}{b} \Rightarrow b = \frac{m}{\tau}$$

Substitute the units of m and τ to obtain:

$$\text{Dimensionally, } b = \frac{[M]}{[T]} = \boxed{\frac{\text{kg}}{\text{s}}}$$

81 •

Picture the Problem For small damping, $Q = \dfrac{2\pi}{\left(|\Delta E|/E\right)_{\text{cycle}}}$ where $\Delta E/E$ is the fractional energy loss per cycle.

Relate the Q factor to the fractional energy loss per cycle:

$$Q = \frac{2\pi}{\left(|\Delta E|/E\right)_{\text{cycle}}}$$

Solve for and evaluate the fractional energy loss per cycle:

$$\left(|\Delta E|/E\right)_{\text{cycle}} = \frac{2\pi}{Q} = \frac{2\pi}{200} = \boxed{3.14\%}$$

82 •

Picture the Problem We can find the period of the oscillator from $T = 2\pi\sqrt{m/k}$ and its total initial energy from $E_0 = \frac{1}{2}kA^2$. The Q factor can be found from its definition $Q = 2\pi/\left(|\Delta E|/E\right)_{\text{cycle}}$ and the damping constant from $Q = \omega_0 m/b$.

(*a*) The period of the oscillator is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{2\,\text{kg}}{400\,\text{N/m}}} = \boxed{0.444\,\text{s}}$$

(*b*) Relate the initial energy of the

$$E_0 = \tfrac{1}{2}kA^2$$

oscillator to its amplitude:

Substitute numerical values and evaluate E_0:

$$E_0 = \tfrac{1}{2}(400\,\text{N/m})(0.03\,\text{m})^2 = \boxed{0.180\,\text{J}}$$

(*c*) Relate the fractional rate at which the energy decreases to the Q value and evaluate Q:

$$Q = \frac{2\pi}{(|\Delta E|/E)_{\text{cycle}}} = \frac{2\pi}{0.01} = \boxed{628}$$

Express the Q value in terms of b:

$$Q = \frac{\omega_0 m}{b}$$

Solve for the damping constant b:

$$b = \frac{\omega_0 m}{Q} = \frac{2\pi m}{TQ}$$

Substitute numerical values and evaluate b:

$$b = \frac{2\pi(2\,\text{kg})}{(0.444\,\text{s})(628)} = \boxed{0.0451\,\text{kg/s}}$$

83 ••

Picture the Problem The amplitude of the oscillation at time t is $A(t) = A_0 e^{-t/2\tau}$ where $\tau = m/b$ is the decay constant. We'll express the amplitudes one period apart and then show that their ratio is constant.

Relate the amplitude of a given oscillation peak to the time at which the peak occurs:

$$A(t) = A_0 e^{-t/2\tau}$$

Express the amplitude of the oscillation peak at $t' = t + T$:

$$A(t+T) = A_0 e^{-(t+T)/2\tau}$$

Express the ratio of these consecutive peaks:

$$\frac{A(t)}{A(t+T)} = \frac{A_0 e^{-t/2\tau}}{A_0 e^{-(t+T)/2\tau}} = e^{-T/2\tau} = \boxed{\text{constant}}$$

84 ••

Picture the Problem We can relate the fractional change in the energy of the oscillator each cycle to the fractional change in its amplitude. Both the Q value and the decay constant τ can be found from their definitions.

(*a*) Relate the energy of the oscillator to its amplitude:

$$E = \tfrac{1}{2}kA^2$$

Take the differential of this relationship to obtain:

$$dE = kAdA$$

Divide both sides of this equation by E:

$$\frac{dE}{E} = \frac{kAdA}{\frac{1}{2}kA^2} = 2\frac{dA}{A}$$

Approximate dE and dA by ΔE and ΔA and evaluate $\Delta E/E$:

$$\frac{\Delta E}{E} = 2(5\%) = \boxed{10\%}$$

(*b*) For small damping:

$$\frac{|\Delta E|}{E} = \frac{T}{\tau}$$

and

$$\tau = \frac{T}{|\Delta E|/E} = \frac{3\,\text{s}}{0.01} = \boxed{30\,\text{s}}$$

(*c*) Using its definition, express and evaluate Q:

$$A = \omega_0\tau = \frac{2\pi}{T}\tau = \frac{2\pi}{3\,\text{s}}(30\,\text{s}) = \boxed{62.8}$$

85 ••

Picture the Problem We can use the physical interpretation of Q for small damping $Q = \frac{2\pi}{\left(|\Delta E|/E\right)_{\text{cycle}}}$ to find the fractional decrease in the energy of the oscillator each cycle.

(*a*) Express the fractional decrease in energy each cycle as a function of the Q factor and evaluate $|\Delta E|/E$:

$$\frac{|\Delta E|}{E} = \frac{2\pi}{Q} = \frac{2\pi}{20} = \boxed{0.314}$$

(*b*) Using the definition of the Q factor, use Equation 14-35 to express ω' as a function of Q:

$$\omega' = \omega_0\left[1 - \frac{1}{4}\left(\frac{b^2}{m^2\omega_0^2}\right)\right]^{1/2}$$

$$= \omega_0\left[1 - \frac{1}{4Q^2}\right]^{1/2}$$

Use the approximation $(1 + x)^{½} \approx 1 + ½x$ for small x to obtain:

$$\omega' = \omega_0\left[1 - \frac{1}{8Q^2}\right]$$

Express and evaluate $\omega' - \omega_0$:

$$\omega' - \omega_0 = \omega_0\left[1 - \frac{1}{8Q^2}\right] - \omega_0 = -\frac{1}{8Q^2}$$

$$= -\frac{1}{8(20)^2}$$

$$= \boxed{-3.13\times10^{-2}\,\text{percent}}$$

86 ••

Picture the Problem The amplitude of the spring-and-mass oscillator varies with time according to $A = A_0 e^{-t/2\tau}$ and its energy according to $E = E_0 e^{-t/\tau}$.

(*a*) Express the amplitude of the oscillations as a function of time:

$$A = (6\,\text{cm})e^{-t/4\text{s}}$$

Evaluate the amplitude when t = 2 s:

$$A(2\,\text{s}) = (6\,\text{cm})e^{-2\,\text{s}/4\,\text{s}} = (6\,\text{cm})e^{-1/2}$$

$$= \boxed{3.64\,\text{cm}}$$

Evaluate the amplitude when t = 4 s:

$$A(4\,\text{s}) = (6\,\text{cm})e^{-4\,\text{s}/4\,\text{s}} = (6\,\text{cm})e^{-1}$$

$$= \boxed{2.21\,\text{cm}}$$

(*b*) Express the energy of the system at t = 0:

$$E(0) = E_0 e^{-0/2\,\text{s}} = E_0 = 60\,\text{J}$$

Express the energy in the system at t =2 s:

$$E(2\,\text{s}) = E_0 e^{-2\,\text{s}/2\,\text{s}} = E_0 e^{-1}$$

The energy dissipated in the first 2 s is:

$$\Delta E_{0-2\text{s}} = E(0) - E(2\,\text{s})$$

$$= E_0\left(1 - e^{-1}\right)$$

$$= (60\,\text{J})\left(1 - e^{-1}\right)$$

$$= \boxed{37.9\,\text{J}}$$

The energy dissipated in the second 2-s interval is:

$$\Delta E_{2-4\,\text{s}} = E_{2\,\text{s}}\left(1 - e^{-2\text{s}/2\,\text{s}}\right)$$

$$= (37.9\,\text{J})\left(1 - e^{-1}\right) = \boxed{24.0\,\text{J}}$$

***87** ••

Picture the Problem We can find the fractional loss of energy per cycle from the physical interpretation of Q for small damping. We will also find a general expression for the earth's vibrational energy as a function of the number of cycles it has completed. We can then solve this equation for the earth's vibrational energy after any number of days.

(*a*) Express the fractional change in energy as a function of *Q*:

$$\frac{\Delta E}{E}=\frac{2\pi}{Q}=\frac{2\pi}{400}=\boxed{1.57\%}$$

(*b*) Express the energy of the damped oscillator after one cycle:

$$E_1=E_0\left(1-\frac{\Delta E}{E}\right)$$

Express the energy after two cycles:

$$E_2=E_1\left(1-\frac{\Delta E}{E}\right)=E_0\left(1-\frac{\Delta E}{E}\right)^2$$

Generalizing to *n* cycles:

$$E_n=E_0\left(1-\frac{\Delta E}{E}\right)^n=E_0(1-0.0157)^n$$

$$=\boxed{E_0(0.9843)^n}$$

(*c*) Express 2 d in terms of the number of cycles; i.e., the number of vibrations the earth will have experienced:

$$2\,\text{d}=2\,\text{d}\times\frac{24\,\text{h}}{\text{d}}\times\frac{60\,\text{m}}{\text{h}}$$

$$=2880\,\text{min}\times\frac{1T}{54\,\text{min}}$$

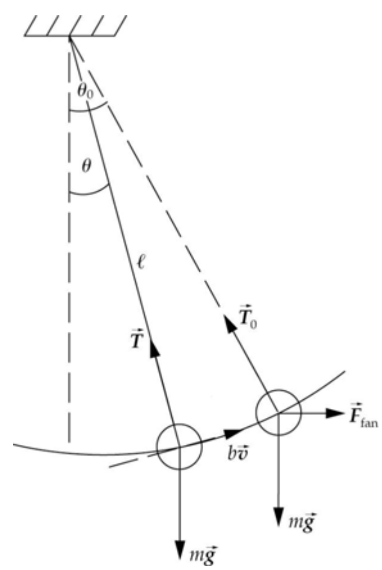
$$=53.3T$$

Evaluate $E(2\text{ d})$:

$$E(2\,\text{d})=E_0(0.9843)^{53.3}=\boxed{0.430E_0}$$

88 ••

Picture the Problem The diagram shows 1) the pendulum bob displaced through an angle θ_0 and held in equilibrium by the force exerted on it by the air from the fan and 2) the bob accelerating, under the influence of gravity, tension force, and drag force, toward its equilibrium position. We can apply Newton's 2nd law to the bob to obtain the differential equation of motion of the damped pendulum and then use its solution to find the decay time constant and the time required for the amplitude of oscillation to decay to 1°.



(*a*) Apply $\sum\tau=I\alpha$ to the pendulum to obtain:

$$-mg\ell\sin\theta+\ell F_{\text{d}}=I\frac{d^2\theta}{dt^2}$$

Express the moment of inertia of the pendulum with respect to an axis through its point of support:

$$I = m\ell^2$$

Substitute for I and F_d to obtain:

$$m\ell^2 \frac{d^2\theta}{dt^2} + \ell bv + mg\ell \sin\theta = 0$$

Because $\theta << 1$ and $v = \ell\omega = \ell d\theta/dt$:

$$m\ell^2 \frac{d^2\theta}{dt^2} + \ell^2 b\frac{d\theta}{dt} + mg\ell\theta = 0$$

or

$$m\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + \frac{mg}{\ell}\theta = 0$$

The solution to this second-order homogeneous differential equation with constant coefficients is:

$$\theta = \theta_0 e^{-t/2\tau}\cos(\omega' t + \delta) \qquad (1)$$

where θ_0 is the maximum amplitude, $\tau = m/b$ is the time constant, and the frequency $\omega' = \omega_0\sqrt{1-(b/2m\omega_0)^2}$.

Apply $\sum \vec{F} = m\vec{a}$ to the bob when it is at its maximum angular displacement to obtain:

$$\sum F_x = F_{\text{fan}} - T\sin\theta_0 = 0$$

and

$$\sum F_y = T\cos\theta_0 - mg = 0$$

Divide the x equation by the y equation to obtain:

$$\frac{F_{\text{fan}}}{mg} = \frac{T\sin\theta_0}{T\cos\theta_0} = \tan\theta_0$$

or

$$F_{\text{fan}} = mg\tan\theta_0$$

When the bob is in equilibrium, the drag force on it equals F_{fan}:

$$bv = mg\tan\theta_0$$

Solve for m/b in the definition of τ to obtain:

$$\tau = \frac{m}{b} = \frac{v}{g\tan\theta_0}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{7\text{ m/s}}{(9.81\text{ m/s}^2)\tan 5°} = \boxed{8.16\text{ s}}$$

(*b*) From equation (1) we have:

$$\theta = \theta_0 e^{-t/2\tau}$$

When the amplitude has decreased to 1°:

$$5° e^{-t/2\tau} = 1° \text{ or } e^{-t/2\tau} = 0.2$$

Take the natural logarithm of both sides of the equation to obtain:

$$-\frac{t}{2\tau} = \ln(0.2)$$

Solve for t: $t = -2\tau \ln(0.2)$

Substitute for τ and evaluate t: $t = -2(8.16\,\text{s})\ln(0.2) = \boxed{26.3\,\text{s}}$

Driven Oscillations and Resonance

89 •

Picture the Problem The resonant frequency of a vibrating system depends on the mass of the system and on a "stiffness" constant according to $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ or, in the case of a simple pendulum oscillating with small-amplitude vibrations, $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$.

(*a*) For this spring-and-mass oscillator we have:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{400\,\text{N/m}}{10\,\text{kg}}} = \boxed{1.01\,\text{Hz}}$$

(*b*) For this spring-and-mass oscillator we have:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{800\,\text{N/m}}{5\,\text{kg}}} = \boxed{2.01\,\text{Hz}}$$

(*c*) For this simple pendulum we have:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{2\,\text{m}}} = \boxed{0.352\,\text{Hz}}$$

90 •

Picture the Problem We can use the physical interpretation of Q for small damping to find the Q factor for this damped oscillator. The width of the resonance curve depends on the Q factor according to $\Delta\omega = \omega_0/Q$.

(*a*) Using the physical interpretation of Q for small damping, relate Q to the fractional loss of energy of the damped oscillator per cycle:

$$Q = \frac{2\pi}{\left(|\Delta E|/E\right)_{\text{cycle}}}$$

Evaluate this expression for $\left(|\Delta E|/E\right)_{\text{cycle}} = 2\%$:

$$Q = \frac{2\pi}{0.02} = \boxed{314}$$

(*b*) Relate the width of the resonance curve to the Q value of the oscillatory system:

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{2\pi f_0}{Q}$$

Substitute numerical values and evaluate $\Delta\omega$:

$$\Delta\omega = \frac{2\pi(300\,\text{s}^{-1})}{3.14} = \boxed{6.00\,\text{rad/s}}$$

91 ••

Picture the Problem The amplitude of the damped oscillations is related to the damping constant, mass of the system, the amplitude of the driving force, and the natural and driving frequencies through $A = \dfrac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}}$. Resonance occurs when $\omega = \omega_0$. At resonance, the amplitude of the oscillations is $A = F_0\big/\sqrt{b^2\omega^2}$ and the width of the resonance curve is related to the damping constant and the mass of the system according to $\Delta\omega = b/m$.

(*a*) Express the amplitude of the oscillations as a function of the driving frequency:

$$A = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}}$$

Determine ω_0:

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{400\,\text{N/m}}{2\,\text{kg}}} = 14.14\,\text{rad/s}$$

Evaluate the radicand in the expression for *A* to obtain:

$$(2\,\text{kg})^2\left[(14.14\,\text{rad/s})^2 - (10\,\text{rad/s})^2\right]^2 + (2\,\text{kg/s})^2(10\,\text{rad/s})^2 = 4.04\times10^4\,\text{kg}^2/\text{s}^4$$

Substitute numerical values and evaluate *A*:

$$A = \frac{10\,\text{N}}{\sqrt{4.04\times10^4\,\text{kg}^2/\text{s}^4}} = \boxed{4.98\,\text{cm}}$$

(*b*) Resonance occurs when:

$$\omega = \omega_0 = \boxed{14.1\,\text{rad/s}}$$

(*c*) Express the amplitude of the motion at resonance:

$$A = \frac{F_0}{\sqrt{b^2\omega_0^2}}$$

Substitute numerical values and evaluate *A*:

$$A = \frac{10\,\text{N}}{\sqrt{(2\,\text{kg/s})^2(14.14\,\text{rad/s})^2}} = \boxed{35.4\,\text{cm}}$$

(*d*) The width of the resonance curve is:

$$\Delta\omega = \frac{b}{m} = \frac{2\,\text{kg/s}}{2\,\text{kg}} = \boxed{1.00\,\text{rad/s}}$$

92 ••

Picture the Problem We'll find a general expression for the damped oscillator's energy as a function of the number of cycles it has completed. We can then solve this equation for the number of cycles corresponding to the loss of half the oscillator's energy. The Q factor is related to the fractional energy loss per cycle through $\Delta E/E = 2\pi/Q$ and the width of the resonance curve is $\Delta\omega = \omega_0/Q$ where ω_0 is the oscillator's natural angular frequency.

(*a*) Express the energy of the damped oscillator after one cycle:

$$E_1 = E_0\left(1 - \frac{\Delta E}{E}\right)$$

Express the energy after two cycles:

$$E_2 = E_1\left(1 - \frac{\Delta E}{E}\right) = E_0\left(1 - \frac{\Delta E}{E}\right)^2$$

Generalizing to n cycles:

$$E_n = E_0\left(1 - \frac{\Delta E}{E}\right)^n$$

Substitute numerical values:

$$0.5E_0 = E_0(1 - 0.035)^n$$

or

$$0.5 = (0.965)^n$$

Solve for n to obtain:

$$n = \frac{\ln 0.5}{\ln 0.965} = 19.5$$

$$\approx \boxed{20 \text{ complete cycles.}}$$

(*b*) Apply the physical interpretation of Q for small damping to obtain:

$$Q = \frac{2\pi}{\Delta E/E} = \frac{2\pi}{0.035} = \boxed{180}$$

(*c*) The width of the resonance curve is given by:

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{2\pi f_0}{Q} = \frac{2\pi(100\,\text{Hz})}{180}$$

$$= \boxed{3.49\,\text{rad/s}}$$

Collisions

93 •••

Picture the Problem Let the system include the spring-and-mass oscillator and the second object of mass m. Because the net external force acting on this system is zero, momentum is conserved during the collision of the second object with the oscillator.

Because the collision is elastic, we can also apply conservation of energy. Let the subscript 1 refer to the object attached to the spring and the subscript 2 identify the second object.

(*a*) Using momentum conservation, relate the speeds of the objects before and after their collision:

$$mv_{1i} + mv_{2i} = mv_{2f}$$

or

$$v_{1i} + v_{2i} = v_{2f} \qquad (1)$$

Using conservation of energy, obtain a second relationship between the speeds of the objects before and after their collision:

$$\tfrac{1}{2}mv_{1i}^2 + \tfrac{1}{2}mv_{2i}^2 = \tfrac{1}{2}mv_{2f}^2$$

or

$$v_{1i}^2 + v_{2i}^2 = v_{2f}^2 \qquad (2)$$

Solve equation (2) for v_{2i}^2:

$$v_{2i}^2 = v_{2f}^2 - v_{1i}^2 = (v_{2f} + v_{1i})(v_{2f} - v_{1i})$$

Substitute for v_{2f} from equation (1):

$$v_{2i}^2 = (v_{1i} + v_{2i} + v_{1i})(v_{1i} + v_{2i} - v_{1i})$$
$$= (2v_{1i} + v_{2i})(v_{2i}) = 2v_{1i}v_{2i} + v_{2i}^2$$

or

$$2v_{1i}v_{2i} = 0$$

Because $v_{1i} \neq 0$, it follows that:

$$v = v_{2i} = \boxed{0}$$

i.e., the second object must be initially at rest.

(*b*) Because $v_{2i} = 0$, we have, from equation (1):

$$v_{2f} = v_{1i}$$

Because the object connected to the spring was moving through its equilibrium position at the time of collision:

$$v_{1i} = v_{max} = A\omega = (0.1\,\text{m})(40\,\text{s}^{-1})$$
$$= \boxed{4.00\,\text{m/s}}$$

94 •••

Picture the Problem Let the system include the spring-and-mass oscillator and the second object of mass *m*. Because the net external force acting on this system is zero, momentum is conserved during the collision of the second object with the oscillator. Because the collision is elastic, we can also apply conservation of energy. Let the subscript 1 refer to the object attached to the spring and the subscript 2 identify the second object.

Using momentum conservation, relate the speeds of the objects

$$mv_{1i} + mv_{2i} = mv_{2f}$$

or

before and after their collision: $v_{1i} + v_{2i} = v_{2f}$ (1)

Using conservation of energy, obtain a second relationship between the speeds of the objects before and after their collision:

$$\tfrac{1}{2}mv_{1i}^2 + \tfrac{1}{2}mv_{2i}^2 = \tfrac{1}{2}mv_{2f}^2$$

or

$$v_{1i}^2 + v_{2i}^2 = v_{2f}^2 \quad (2)$$

Solve equation (2) for v_{2i}^2:

$$v_{2i}^2 = v_{2f}^2 - v_{1i}^2 = (v_{2f} + v_{1i})(v_{2f} - v_{1i})$$

Substitute for v_{2f} from equation (1):

$$v_{2i}^2 = (v_{1i} + v_{2i} + v_{1i})(v_{1i} + v_{2i} - v_{1i})$$
$$= (2v_{1i} + v_{2i})(v_{2i}) = 2v_{1i}v_{2i} + v_{2i}^2$$

or

$$2v_{1i}v_{2i} = 0$$

Because $v_{1i} \neq 0$, it follows that:

$$v = v_{2i} = 0$$

i.e., the second object must be initially at rest.

Because the object connected to the spring was moving through its equilibrium position at the time of collision:

$$v_{1i} = v_{max} = A\omega = (0.1\,\text{m})(40\,\text{s}^{-1})$$
$$= 4\,\text{m/s}$$

Express the total energy of the system just before the collision:

$$E = \tfrac{1}{2}mv_{1i}^2$$

Solve for *m*:

$$m = \frac{2E}{v_{1i}^2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{2(8\,\text{J})}{(4\,\text{m/s})^2} = \boxed{1.00\,\text{kg}}$$

Relate the spring constant to the angular frequency of the oscillator:

$$k = m\omega^2$$

Substitute numerical values and evaluate *k*:

$$k = (1\,\text{kg})(40\,\text{s}^{-1})^2 = \boxed{1.60\,\text{kN/m}}$$

95 •••

Picture the Problem Let the system include the spring-and-mass oscillator and the 1-kg object. Because the net external force acting on this system is zero, momentum is conserved during the collision of the second object with the oscillator. Let the subscript 1 refer to the 1-kg object and the subscript 2 to the 2-kg object. We can relate the amplitude of the motion to the maximum speed of the oscillator (which we can find from conservation of momentum) and the angular frequency of the oscillator, which we can determine from its definition. Once we have found the amplitudes and angular frequencies for both collisions, we express the position of each as a function of time, using the initial conditions to find the phase constants.

(*a*) Relate the amplitude of the motion to the angular frequency and maximum speed of the oscillator:

$$A = \frac{v_{\text{max}}}{\omega} \qquad (1)$$

Because the 2-kg object is initially at rest, the maximum speed of the oscillator will be its speed immediately after the collision. Use conservation of momentum to relate this maximum speed to the speed of the 1-kg object before the collision:

$$m_1 v_{1\text{i}} = (m_1 + m_2) v_{\text{max}}$$

Solve for v_{max}:

$$v_{\text{max}} = \frac{m_1}{m_1 + m_2} v_{1\text{i}}$$

Substitute numerical values and evaluate v_{max}:

$$v_{\text{max}} = \frac{1\,\text{kg}}{1\,\text{kg} + 2\,\text{kg}} (6\,\text{m/s}) = 2\,\text{m/s}$$

Express the angular frequency of the oscillator:

$$\omega = \sqrt{\frac{k}{m_1 + m_2}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{600\,\text{N/m}}{3\,\text{kg}}} = 14.14\,\text{rad/s}$$

Substitute in equation (1) and evaluate A:

$$A = \frac{2\,\text{m/s}}{14.14\,\text{s}^{-1}} = \boxed{14.1\,\text{cm}}$$

Express and evaluate the period of the oscillator's period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{14.14\,\text{s}^{-1}} = \boxed{0.444\,\text{s}}$$

(*b*) For an elastic collision:

$$v_{\text{max}} = v_{2\text{f}} = \frac{2m_1}{m_1 + m_2} v_{1\text{i}}$$

Substitute numerical values and evaluate v_{max}:

$$v_{max} = \frac{2(1\,\text{kg})}{3\,\text{kg}}(6\,\text{m/s}) = 4\,\text{m/s}$$

Using its definition, evaluate the angular frequency of the oscillator:

$$\omega = \sqrt{\frac{k}{m_2}} = \sqrt{\frac{600\,\text{N/m}}{2\,\text{kg}}} = 17.32\,\text{rad/s}$$

Substitute in equation (1) and evaluate A:

$$A = \frac{4\,\text{m/s}}{17.32\,\text{s}^{-1}} = \boxed{23.1\,\text{cm}}$$

Express and evaluate the period of the oscillator's period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{17.32\,\text{s}^{-1}} = \boxed{0.363\,\text{s}}$$

(*c*) For the perfectly inelastic collision:

$$x(t) = (14.1\,\text{cm})\cos\left[\left(14.1\,\text{s}^{-1}\right)t + \delta\right] \quad (2)$$

Use the initial conditions to evaluate δ:

$$\delta = \tan^{-1}\left(-\frac{v_0}{\omega x_0}\right) = \tan^{-1}\left(-\frac{v_0}{\omega(0)}\right) = -\frac{\pi}{2}$$

Substitute in equation (2) to obtain:

$$x(t) = (14.1\,\text{cm})\cos\left[\left(14.1\,\text{s}^{-1}\right)t - \frac{\pi}{2}\right] = \boxed{(14.1\,\text{cm})\sin\left[\left(14.1\,\text{s}^{-1}\right)t\right]}$$

For the elastic collision:

$$x(t) = (23.1\,\text{cm})\cos\left[\left(17.3\,\text{s}^{-1}\right)t + \delta\right] \quad (3)$$

Use the initial conditions to evaluate δ:

$$\delta = \tan^{-1}\left(-\frac{v_0}{\omega x_0}\right) = \tan^{-1}\left(-\frac{v_0}{\omega(0)}\right) = -\frac{\pi}{2}$$

Substitute in equation (3) to obtain:

$$x(t) = (23.1\,\text{cm})\cos\left[\left(17.3\,\text{s}^{-1}\right)t - \frac{\pi}{2}\right] = \boxed{(23.1\,\text{cm})\sin\left[\left(17.3\,\text{s}^{-1}\right)t\right]}$$

General Problems

96 •

Picture the Problem The particle's displacement is of the form $x = A\cos(\omega t + \delta)$. Thus, we have $A = 0.4$ m, $\omega = 3$ rad/s, and $\delta = \pi/4$. We can find the frequency of the motion from its angular frequency and the period from the frequency. The particle's position at $t = 0$ and $t = 0.5$ s can be found directly from its displacement function.

(*a*) Express and evaluate the frequency of the particle's motion:

$$f=\frac{\omega}{2\pi}=\frac{3\,\text{rad/s}}{2\pi}=\boxed{0.477\,\text{Hz}}$$

Use the relationship between the frequency and the period of the particle's motion to find its period:

$$T=\frac{1}{f}=\frac{1}{0.477\,\text{s}^{-1}}=\boxed{2.09\,\text{s}}$$

(*b*) Using the expression for the particle's displacement, find its position at $t = 0$:

$$x(0)=(0.4\,\text{m})\cos\left[(3\,\text{rad/s})(0)+\frac{\pi}{4}\right]$$
$$=(0.4\,\text{m})\cos\left[\frac{\pi}{4}\right]=\boxed{0.283\,\text{m}}$$

(*c*) Using the expression for the particle's displacement, find its position at $t = 0.5$ s:

$$x(0)=(0.4\,\text{m})\cos\left[(3\,\text{rad/s})(0.5\,\text{s})+\frac{\pi}{4}\right]$$
$$=(0.4\,\text{m})\cos[2.29\,\text{rad}]$$
$$=\boxed{-0.264\,\text{m}}$$

97 •

Picture the Problem We can express the velocity of the particle by differentiating its displacement with respect to time.

(*a*) Differentiate the particle's displacement to obtain:

$$v=\frac{dx}{dt}$$
$$=\frac{d}{dt}\left\{(0.4\,\text{m})\sin\left[(3\,\text{rad/s})t+\frac{\pi}{4}\right]\right\}$$
$$=\boxed{-(1.2\,\text{m/s})\sin\left[(3\,\text{rad/s})t+\frac{\pi}{4}\right]}$$

(*b*) Evaluate the result in part (*a*) at $t = 0$:

$$v(0)=-(1.2\,\text{m/s})\sin\left[(3\,\text{rad/s})(0)+\frac{\pi}{4}\right]$$
$$=-(1.2\,\text{m/s})\sin\left[\frac{\pi}{4}\right]$$
$$=\boxed{-0.849\,\text{m/s}}$$

(*c*) By inspection of the result in part (*a*) (or from $v_{max}=A\omega$):

$$v_{max}=\boxed{1.20\,\text{m/s}}$$

(*d*) Substitute v_{max} for v to obtain:

$$1.2\,\text{m/s}=-(1.2\,\text{m/s})\sin\left[(3\,\text{rad/s})t'+\frac{\pi}{4}\right]$$

or

$$(3\,\text{rad/s})t' + \frac{\pi}{4} = \sin^{-1}(-1) = \frac{3\pi}{2}$$

Solve for t' to obtain:

$$t' = \boxed{1.31\,\text{s}}$$

98 •

Picture the Problem Let Δy represent the amount by which the spring stretches. We'll apply a condition for equilibrium to the object to relate the amount the spring has stretched to the angular frequency of its motion and then solve this equation for Δy.

Apply $\sum_i F_y = 0$ to the object when it is in its equilibrium position and solve for the elongation of the spring:

$$k\Delta y - mg = 0$$

or

$$\Delta y = \frac{m}{k}g = \frac{g}{\omega^2}$$

Relate the angular frequency of the object's motion to its period:

$$\omega = \frac{2\pi}{T}$$

Substitute to obtain:

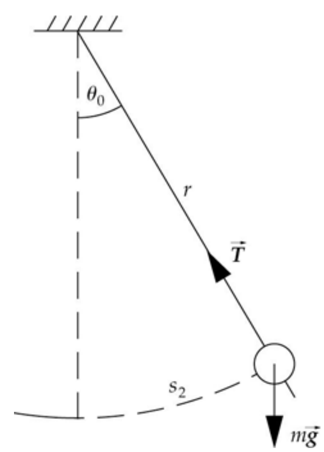
$$\Delta y = \left(\frac{T}{2\pi}\right)^2 g$$

Substitute numerical values and evaluate Δx:

$$\Delta y = \left(\frac{4.5\,\text{s}}{2\pi}\right)^2 (9.81\,\text{m/s}^2) = \boxed{5.03\,\text{m}}$$

***99** ••

Picture the Problem Compare the forces acting on the particle to the right in Figure 14-36 with the forces shown acting on the bob of the simple pendulum shown in the free-body diagram to the right. Because there is no friction, the only forces acting on the particle are *mg* and the normal force acting radially inward. In (*b*), we can think of the particles as the bobs of simple pendulums of equal length.

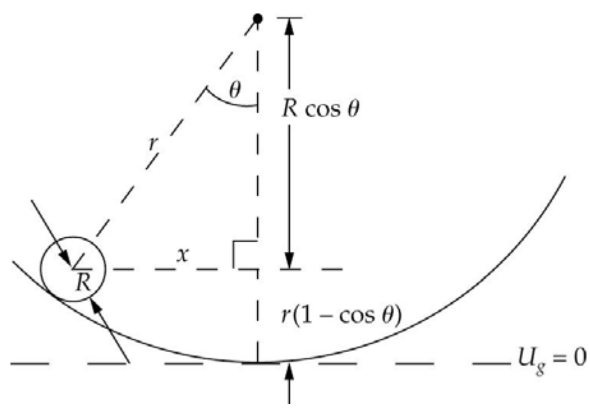


(*a*) The normal force is identical to the tension in a string of length *r* that keeps the particle moving in a circular path and a component of *mg* provides, for small displacements θ_0 or s_2, the linear restoring force required for oscillatory motion.

(*b*) The particles meet at the bottom. Because s_1 and s_2 are both much smaller than *r*, the particles behave like the bobs of simple pendulums of equal length; therefore they have the same periods.

100 ••

Picture the Problem The diagram shows the ball when it is a horizontal distance *x* from the bottom of the bowl. Note that we've chosen the zero of gravitational potential energy to be at the bottom of the bowl. The total energy of the ball is the sum of its potential energy and kinetic energies due to translation and rotation. Once we've obtained an expression for the total energy of the rolling ball, we can require, because the surface is frictionless, that the total energy of the sliding object be the same as that of the rolling ball. Because the motion of the ball is simple harmonic motion, we can assume a solution to its differential equation of motion and express the total energy of the ball in terms of this assumed solution. Doing so will lead us to an expression that we can solve for the oscillation frequency of the ball.



(*a*) Express the total energy *E* of the ball:

$$E = U + K = U + K_{\text{trans}} + K_{\text{rot}} \qquad (1)$$

Referring to the diagram shown above and assuming that $R << r$, express the potential energy of the ball when it is a horizontal distance *x* from the bottom of the bowl:

$$U(x) = mgr(1 - \cos\theta)$$

Express $\cos\theta$ as a power series:

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \ldots$$

For $\theta << 1$:

$$\cos\theta \approx 1 - \frac{\theta^2}{2!}$$

Substitute to obtain:

$$U(x) \approx mgr\left[1 - \left(1 - \frac{\theta^2}{2!}\right)\right] = \tfrac{1}{2} mgr\theta^2$$

For $R << r$:

$$\theta \approx \frac{x}{r}$$

Substitute to obtain:

$$U(x) = \frac{mgx^2}{2r}$$

Substitute in equation (1):

$$E = \frac{mgx^2}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Because the ball is rolling without slipping, $v = R\omega$. Substitute for ω and I to obtain:

$$E = \frac{mgx^2}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

Simplify to obtain:

$$E = \boxed{\frac{mgx^2}{2r} + \frac{7}{10}mv^2}$$

(*b*) Because energy is conserved if the side of the bowl is frictionless:

$$E = \frac{mgx^2}{2r} + \frac{7}{10}mv^2 = \text{constant}$$

Because the motion is simple harmonic motion, assume a solution of the form:

$$x = x_0 \cos(\omega t + \delta)$$

Differentiate this assumed solution with respect to time to obtain:

$$v = -\omega x_0 \sin(\omega t + \delta)$$

Substitute to obtain:

$$\begin{aligned} E &= \frac{mg}{2r}\left(x_0 \cos(\omega t + \delta)\right)^2 \\ &\quad + \frac{7}{10}m\left(-\omega x_0 \sin(\omega t + \delta)\right)^2 \\ &= \frac{mgx_0^2}{2r}\cos^2(\omega t + \delta) \\ &\quad + \frac{7m\omega^2 x_0^2}{10}\sin^2(\omega t + \delta) \end{aligned}$$

Express the condition the E = constant:

$$\frac{mgx_0^2}{2r} = \frac{7m\omega^2 x_0^2}{10} \text{ or } \frac{g}{r} = \frac{7\omega^2}{5}$$

Solve for ω to obtain:

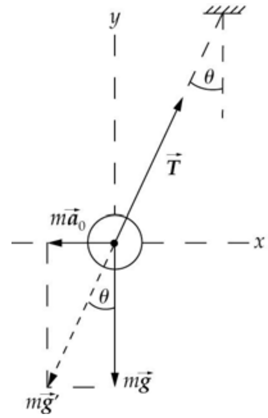
$$\omega = \boxed{\sqrt{\frac{5g}{7r}}}$$

101 ••

Picture the Problem Assume that the plane is accelerating to the right with an acceleration a_0. The free-body diagram shows the forces on the bob as seen in the accelerated frame of the airplane. Let g' represent the effective value of the acceleration due to gravity. The period of the yo-yo is given by

$$T = 2\pi\sqrt{L/g'}$$

where g' is the effective value of the acceleration due to gravity.



Express the period of your yo-yo pendulum as a function of the effective value for the acceleration due to gravity:

$$T = 2\pi\sqrt{\frac{L}{g'}}$$

Using the FBD, relate g' and g:

$$mg = mg'\cos\theta \Rightarrow g' = \frac{g}{\cos\theta}$$

Substitute to obtain:

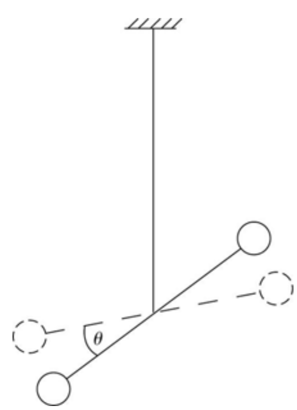
$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{(0.7\,\text{m})\cos 22°}{9.81\,\text{m/s}^2}} = \boxed{1.62\,\text{s}}$$

102 ••

Picture the Problem The diagram shows the wire described in the problem statement with an object of moment of inertia I suspended from its end. We can apply Newton's 2nd law to the suspended object to obtain its differential equation of motion. By comparing this equation to the equation of a simple harmonic oscillator, we can show that $\omega = \sqrt{\kappa/I}$.



Apply $\sum\tau = I\alpha$ to the object hung from the wire to obtain:

$$-\kappa\theta = I\alpha = I\frac{d^2\theta}{dt^2}$$

Divide both sides of this differential equation by I to obtain:

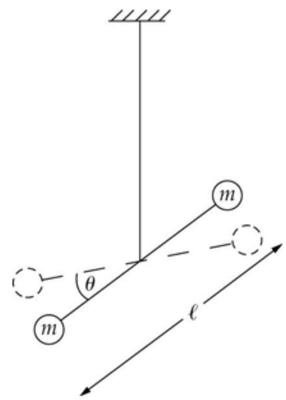
$$\frac{d^2\theta}{dt^2}+\frac{\kappa}{I}\theta=0$$

This equation can be written as:

$$\frac{d^2\theta}{dt^2}+\omega^2\theta=0 \text{ where } \boxed{\omega=\sqrt{\frac{\kappa}{I}}}$$

103 ••

Picture the Problem The diagram shows the torsion balance described in the problem statement. We can apply Newton's 2nd law to the suspended object to obtain its differential equation of motion. By comparing this equation and its solution to that of a simple harmonic oscillator, we can obtain an equation that we can solve for the torsion constant κ.



Apply $\sum\tau=I\alpha$ to the torsion pendulum:

$$-\kappa\theta=I\alpha=I\frac{d^2\theta}{dt^2}$$

or

$$\frac{d^2\theta}{dt^2}+\frac{\kappa}{I}\theta=0 \qquad (1)$$

The differential equation of simple harmonic motion is:

$$\frac{d^2x}{dt^2}+\omega^2x=0$$

where

$$x(t)=x_0\cos(\omega t+\delta) \text{ and } \omega=\frac{2\pi}{T}$$

The solution to equation (1) is:

$$\theta(t)=\theta_0\cos(\omega t+\delta)$$

where

$$\omega=\sqrt{\frac{\kappa}{I}}$$

Solve for κ to obtain:

$$\kappa=\omega^2I$$

Express the moment of inertia of the torsion pendulum:

$$I = 2m\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{2}$$

Substitute to obtain:

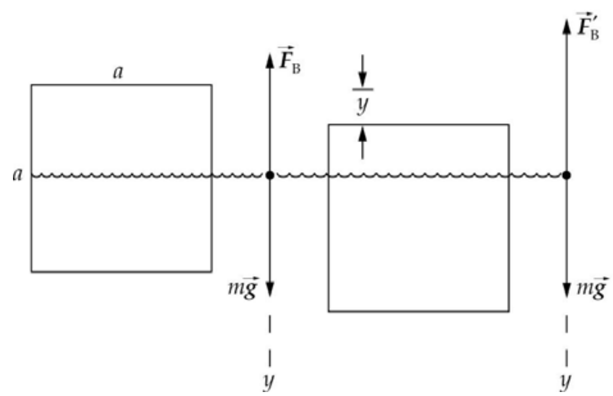
$$\kappa = \frac{\omega^2 m\ell^2}{2} = \frac{4\pi^2 m\ell^2}{2T^2} = \frac{2\pi^2 m\ell^2}{T^2}$$

Substitute numerical values and evaluate κ:

$$\kappa = \frac{2\pi^2(0.050\,\text{kg})(0.05\,\text{m})^2}{(80\,\text{s})^2} = \boxed{3.86\times10^{-7}\,\text{N}\cdot\text{m/rad}}$$

***104 ••**

Picture the Problem Choose a coordinate system in which the direction the cube is initially displaced (downward) is the positive *y* direction. The figure shows the forces acting on the cube when it is in equilibrium floating in the water and when it has been pushed down a small distance *y*. We can find the period of its oscillatory motion from its angular frequency. By applying Newton's 2nd law to the cube, we can obtain its equation of motion; from this equation we can determine the angular frequency of the cube's small-amplitude oscillations.



Express the period of oscillation in terms of the angular frequency of the oscillations:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply $\sum F_y = 0$ to the cube when it is floating in the water:

$$mg - F_B = 0$$

Apply $\sum T_y = ma_y$ to the cube when it is pushed down a small distance *y*:

$$mg - F_B' = ma_y$$

Eliminate *mg* between these equations to obtain:

$$F_B - F_B^{'} = ma_y$$

or

$$\Delta F_B = F_B - F_B^{'} = ma_y$$

For *y* << 1:

$$\Delta F_B \approx dF_B = -\rho Vg = -a^2 \rho gy = m\frac{d^2 y}{dt^2}$$

Rewrite the equation of motion as:

$$m\frac{d^2 y}{dt^2} = -a^2 \rho gy$$

or

$$\frac{d^2 y}{dt^2} = -\frac{a^2 \rho g}{m} y = -\omega^2 y$$

where $\omega^2 = \frac{a^2 \rho g}{m}$

Solve for ω:

$$\omega = a\sqrt{\frac{\rho g}{m}}$$

Substitute in equation (1) to obtain:

$$T = \frac{2\pi}{a\sqrt{\frac{\rho g}{m}}} = \boxed{\frac{2\pi}{a}\sqrt{\frac{m}{\rho g}}}$$

105 ••

Picture the Problem Assume that the density of the earth ρ is constant and let *m* represent the mass of the clock. We can decide the question of where the clock is more accurate by applying the law of gravitation to the clock at a depth *h* below/above the surface of the earth and at the earth's surface and expressing the ratios of the acceleration due to gravity below/above the surface of the earth to its value at the surface of the earth.

Express the gravitational force acting on the clock when it is at a depth *h* in a mine:

$$mg' = \frac{GM'm}{(R_E - h)^2}$$

where *M′* is the mass between the location of the clock and the center of the earth.

Express the gravitational force acting on the clock at the surface of the earth:

$$mg = \frac{GM_E m}{R_E^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{g'}{g}=\frac{\dfrac{GM'}{(R_E-h)^2}}{\dfrac{GM_E}{R_E^2}}=\frac{M'}{M_E}\frac{R_E^2}{(R_E-h)^2}$$

Express M':

$$M'=\rho V'=\tfrac{4}{3}\pi\rho(R_E-h)^3$$

Express M_E:

$$M_E=\rho V=\tfrac{4}{3}\pi\rho R_E^3$$

Substitute to obtain:

$$\frac{g'}{g}=\frac{\frac{4}{3}\pi\rho(R_E-h)^3}{\frac{4}{3}\pi\rho R_E^3}\frac{R_E^2}{(R_E-h)^2}$$

Simplify and solve for g':

$$g'=g\left(\frac{R_E-h}{R_E}\right)=g\left(1-\frac{h}{R_E}\right)$$

or

$$g'=g\left(1-\frac{h}{R_E}\right) \qquad (1)$$

Express the gravitational force acting on the clock when it is at an elevation h:

$$mg''=\frac{GM_Em}{(R_E+h)^2}$$

Express the gravitational force acting on the clock at the surface of the earth:

$$mg=\frac{GM_Em}{R_E^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{g''}{g}=\frac{\dfrac{GM_E}{(R_E+h)^2}}{\dfrac{GM_E}{R_E^2}}=\frac{R_E^2}{(R_E-h)^2}$$

$$=\frac{1}{\left(1+\dfrac{h}{R_E}\right)^2}$$

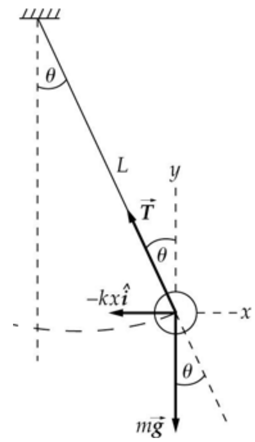
Solve for g'':

$$g''=g\left(1+\frac{h}{R_E}\right)^{-2}$$

Comparing equations (1) and (2), we see that g' is closer to g than is g''. Thus, the error is greater if the clock is elevated.

106 ••

Picture the Problem The figure shows this system when it has an angular displacement θ. The period of the system is related to its angular frequency according to $T = 2\pi/\omega$. We can find the equation of motion of the system by applying Newton's 2nd law. By writing this equation in terms of θ and using a small-angle approximation, we'll find an expression for ω that we can use to express T.



(*a*) Express the period of the system in terms of its angular frequency:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply $\sum \vec{F} = m\vec{a}$ to the bob:

$$\sum F_x = -kx - T\sin\theta = Ma_x$$

and

$$\sum F_y = T\cos\theta - Mg = 0$$

Eliminate T between the two equations to obtain:

$$-kx - Mg\tan\theta = Ma_x$$

Noting that $x = L\theta$ and $a_x = L\alpha = L\frac{d^2\theta}{dt^2}$, eliminate the variable x in favor of θ:

$$ML\frac{d^2\theta}{dt^2} = -kL\theta - Mg\tan\theta$$

For $\theta << 1$, $\tan\theta \approx \theta$:

$$ML\frac{d^2\theta}{dt^2} = -kL\theta - Mg\theta = -(kL + Mg)\theta$$

or

$$\frac{d^2\theta}{dt^2} = -\left(\frac{k}{M} + \frac{g}{L}\right)\theta = -\omega^2\theta$$

where

$$\omega = \sqrt{\frac{k}{M} + \frac{g}{L}}$$

Substitute in equation (1) to obtain:

$$T = \boxed{\dfrac{2\pi}{\sqrt{\dfrac{k}{M} + \dfrac{g}{L}}}}$$

(*b*) When $T = 2$ s and $M = 1$ kg we have:

$$2 = \frac{2\pi}{\sqrt{\dfrac{g}{L}}}$$

When $T = 1$ s we have:

$$1 = \frac{2\pi}{\sqrt{k + \dfrac{g}{L}}}$$

Solve these equations simultaneously to obtain:

$$k = \boxed{29.6\,\text{N/m}}$$

107 ••

Picture the Problem Applying Newton's 2nd law to the first object as it is about to slip will allow us to express μ_s in terms of the maximum acceleration of the system which, in turn, depends on the amplitude and angular frequency of the oscillatory motion.

(*a*) Apply $\sum F_x = ma_x$ to the second object as it is about to slip:

$$f_{s,\max} = m_2 a_{\max}$$

Apply $\sum F_y = 0$ to the second object:

$$F_n - m_2 g = 0$$

Use $f_{s,\max} = \mu_s F_n$ to eliminate $f_{s,\max}$ and F_n between the two equations:

$$\mu_s m_2 g = m_2 a_{\max}$$

and

$$\mu_s = \frac{a_{\max}}{g}$$

Relate the maximum acceleration of the oscillator to its amplitude and angular frequency:

$$a_{\max} = A\omega^2 = A\frac{k}{m_1 + m_2}$$

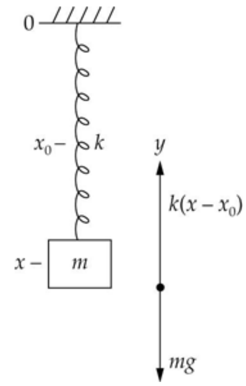
Substitute for $a_{\max}$ to obtain:

$$\mu_s = \boxed{\frac{Ak}{(m_1 + m_2)g}}$$

(*b*) A is unchanged. E is unchanged because $E=\frac{1}{2}kA^2$. ω is reduced by increasing the total mass of the system and T is increased.

108 ••

Picture the Problem The diagram shows the box hanging from the stretched spring and the free-body diagram when the box is in equilibrium. We can apply $\sum F_y = 0$ to the box to derive an expression for *x*. In (*b*) and (*c*), we can proceed similarly to obtain expressions for the effective spring constant, the new equilibrium position of the box, and frequency of oscillations when the box is released.



(*a*) Apply $\sum F_y = 0$ to the box to obtain:

$$k(x-x_0)-mg=0$$

Solve for *x*:

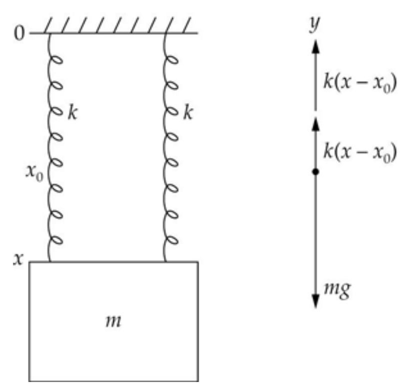
$$x=\frac{mg}{k}+x_0$$

Substitute numerical values and evaluate *x*:

$$x=\frac{(100\,\text{kg})(9.81\,\text{m/s}^2)}{500\,\text{N/m}}+0.5\text{m}$$

$$=\boxed{2.46\,\text{m}}$$

(*b*) Draw the free-body diagram for the block with the two springs exerting equal upward forces on it:



Apply $\sum F_y = 0$ to the box to obtain:

$$k(x-x_0)+k(x-x_0)-mg=0$$

or

$$k_{\text{eff}}(x-x_0)-mg=0 \qquad (1)$$

where

$$k_{\text{eff}}=2k$$

When the box is displaced from this equilibrium position and released, its motion is simple harmonic motion and its frequency is given by:

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{2k}{m}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{2(500\,\text{N/m})}{100\,\text{kg}}} = \boxed{3.16\,\text{rad/s}}$$

(c) Solve equation (1) for x:

$$x = \frac{mg}{2k} + x_0$$

Substitute numerical values and evaluate x:

$$x = \frac{(100\,\text{kg})(9.81\,\text{m/s}^2)}{2(500\,\text{N/m})} + 0.5\,\text{m} = \boxed{1.48\,\text{m}}$$

109 ••

Picture the Problem We'll differentiate the expression for the period of simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$ with respect to g, separate the variables, and use a differential approximation to establish that $\frac{\Delta T}{T} \approx -\frac{1}{2}\frac{\Delta g}{g}$.

(a) Express the period of a simple pendulum in terms of its length and the local value of the acceleration due to gravity:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Differentiate this expression with respect to g to obtain:

$$\frac{dT}{dg} = \frac{d}{dg}\left[2\pi\sqrt{L}g^{-1/2}\right] = -\pi\sqrt{L}g^{-3/2} = -\frac{T}{2g}$$

Separate the variables to obtain:

$$\frac{dT}{T} = -\frac{1}{2}\frac{dg}{g}$$

Approximate dT and dg by ΔT and Δg for $\Delta g << g$:

$$\frac{\Delta T}{T} \approx \boxed{-\frac{1}{2}\frac{\Delta g}{g}}$$

(b) Solve the result in part (a) for Δg:

$$\Delta g = -2g\frac{\Delta T}{T}$$

Express $\Delta T/T$:

$$\frac{\Delta T}{T}=-90\frac{\text{s}}{\text{d}}\times\frac{1\,\text{d}}{24\,\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}=-1.04\times10^{-3}$$

Substitute and evaluate Δg:

$$\Delta g=-2\left(9.81\,\text{m/s}^2\right)\left(-1.04\times10^{-3}\right)=0.0204\,\text{m/s}^2=\boxed{2.04\,\text{cm/s}^2}$$

110 ••

Picture the Problem We can find the frequency of the vibrating system from its angular frequency; this depends on the spring constant and the total mass involved in the motion. The energy of the system can be found from the amplitude of its motion.

(*a*) Relate the frequency of the vibrating system to its angular frequency:

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$$

Substitute numerical values and evaluate *f*:

$$f=\frac{1}{2\pi}\sqrt{\frac{240\,\text{N/m}}{2(0.6\,\text{kg})}}=\boxed{2.25\,\text{Hz}}$$

Express the total energy of the system:

$$E=\tfrac{1}{2}kA^2$$

Substitute numerical values and evaluate *E*:

$$E=\tfrac{1}{2}(240\,\text{N/m})(0.6\,\text{m})^2=\boxed{43.2\,\text{J}}$$

(*b*) (1) The glue dissolves when the spring is at maximum compression:

Relate the frequency to the system's new angular frequency:

$$f_1=\frac{\omega_1}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Substitute numerical values and evaluate f_1:

$$f_1=\frac{1}{2\pi}\sqrt{\frac{240\,\text{N/m}}{0.6\,\text{kg}}}=\boxed{3.18\,\text{Hz}}$$

Express the system's new amplitude as a function of the oscillator's maximum speed and its new angular frequency:

$$A_1=\frac{v_{\text{max}}}{\omega_1}=v_{\text{max}}\sqrt{\frac{m}{k}}$$

Find the maximum speed of the oscillator:

$$v_{\text{max}}=A\omega=2\pi fA=2\pi\left(2.25\,\text{s}^{-1}\right)(0.6\,\text{m})=8.48\,\text{m/s}$$

Substitute and evaluate A_1:

$$A_1 = (8.48\,\text{m/s})\sqrt{\frac{0.6\,\text{kg}}{240\,\text{N/m}}} = \boxed{42.4\,\text{cm}}$$

Express and evaluate the energy of the system:

$$E_1 = \tfrac{1}{2}kA_1^2 = \tfrac{1}{2}(240\,\text{N/m})(0.424\,\text{m})^2 = \boxed{21.6\,\text{J}}$$

(*b*) (2) The glue dissolves when the spring is at maximum extension and f_2 is the same as f_1:

$$f_2 = \boxed{3.18\,\text{Hz}}$$

Because the second object is at rest, the amplitude and energy of the system are unchanged:

$$A_2 = A = \boxed{0.600\,\text{m}}$$

and

$$E_2 = E = \boxed{43.2\,\text{J}}$$

111 ••

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right and assume that the object is displaced to the right. In case (*a*), note that the two springs undergo the same displacement whereas in (*b*) they experience the same force.

(*a*) Express the net force acting on the object:

$$F_{\text{net}} = -k_1x - k_2x = -(k_1 + k_2)x = -k_{\text{eff}}x$$

where $k_{\text{eff}} = \boxed{k_1 + k_2}$

(*b*) Express the force acting on each spring and solve for x_2:

$$F = -k_1x_1 = -k_2x_2$$

or

$$x_2 = \frac{k_1}{k_2}x_1$$

Express the total extension of the springs:

$$x_1 + x_2 = -\frac{F}{k_{\text{eff}}}$$

Solve for k_{eff}:

$$k_{\text{eff}} = -\frac{F}{x_1 + x_2} = -\frac{-k_1x_1}{x_1 + x_2} = \frac{k_1x_1}{x_1 + \frac{k_1}{k_2}x_1} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

Take the reciprocal of both sides of the equation to obtain:

$$\frac{1}{k_{\text{eff}}} = \boxed{\frac{1}{k_1} + \frac{1}{k_2}}$$

***112 ••**

Picture the Problem If the displacement of the block is $y = A \sin \omega t$, its acceleration is $a = -\omega^2 A\sin\omega t$.

(*a*) At maximum upward extension, the block is momentarily at rest. Its downward acceleration is g. The downward acceleration of the piston is $\omega^2 A$. Therefore, if $\omega^2 A > g$, the block will separate from the piston.

(*b*) Express the acceleration of the small block:

$$a = -A\omega^2 \sin \omega t$$

For $\omega^2 A = 3g$ and $A = 15$ cm:

$$a = -3g \sin \omega t = -g$$

Solve for t:

$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{1}{3}\right) = \sqrt{\frac{A}{3g}}\sin^{-1}\left(\frac{1}{3}\right)$$

Substitute numerical values and evaluate t:

$$t = \sqrt{\frac{0.15\,\text{m}}{3(9.81\,\text{m/s}^2)}}\sin^{-1}\frac{1}{3} = \boxed{0.0243\,\text{s}}$$

113 ••

Picture the Problem The plunger and ball are moving with their maximum speed as they pass through their equilibrium position ($x = 0$). Once it has passed its equilibrium position, the acceleration of the plunger becomes negative; therefore it begins to slow down and the ball, continuing with speed v_s, separates from the plunger. We can find this separation speed by equating it to the maximum speed of the plunger. Application of conservation of energy to the motion of the plunger will allow us to express the distance at which the plunger comes momentarily to rest.

(*a*) The ball will leave the plunger when the plunger is moving with its maximum speed; i.e., at its equilibrium position:

$$x = \boxed{0}$$

(*b*) Express the speed of the ball upon separation in terms of the maximum speed of the plunger:

$$v_s = v_{\max} = A\omega = x_0\omega$$

The angular frequency is given by:

$$\omega = \sqrt{\frac{k}{m_b + m_p}}$$

Substitute to obtain:

$$v_s = \boxed{x_0\sqrt{\frac{k}{m_b + m_p}}}$$

(*c*) Apply conservation of energy to the plunger:

$$K_f - K_i + U_{f,s} - U_{i,s} = 0$$

or, because $K_f = U_i = 0$,

$$-\tfrac{1}{2}m_p v_s^2 + \tfrac{1}{2}kx_f^2 = 0$$

Solve for x_f:

$$x_f = \sqrt{\frac{m_p}{k}}v_s$$

Substitute for v_s and simplify to obtain:

$$x_f = \boxed{x_0\sqrt{\frac{m_p}{m_b + m_p}}}$$

114 ••

Picture the Problem Applying Newton's 2nd law to the box as it is about to slip will allow us to express μ_s in terms of the maximum acceleration of the platform which, in turn, depends on the amplitude and angular frequency of the oscillatory motion.

(*a*) Apply $\sum F_x = ma_x$ to the box as it is about to slip:

$$f_{s,max} = ma_{max}$$

Apply $\sum F_y = 0$ to the box:

$$F_n - mg = 0$$

Use $f_{s,max} = \mu_s F_n$ to eliminate $f_{s,max}$ and F_n between the two equations:

$$\mu_s mg = ma_{max}$$

and

$$\mu_s = \frac{a_{max}}{g}$$

Relate the maximum acceleration of the oscillator to its amplitude and angular frequency:

$$a_{max} = A\omega^2$$

Substitute for a_{max} :

$$\mu_s = \frac{A\omega^2}{g} = \frac{4\pi^2 A}{T^2 g}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{4\pi^2(0.4\,\text{m})}{(0.8\,\text{s})^2(9.81\,\text{m/s}^2)} = \boxed{2.52}$$

(*b*) Solve the equation derived above for A_{max}:

$$A_{max} = \frac{\mu_s g}{\omega^2} = \frac{\mu_s g T^2}{4\pi^2}$$

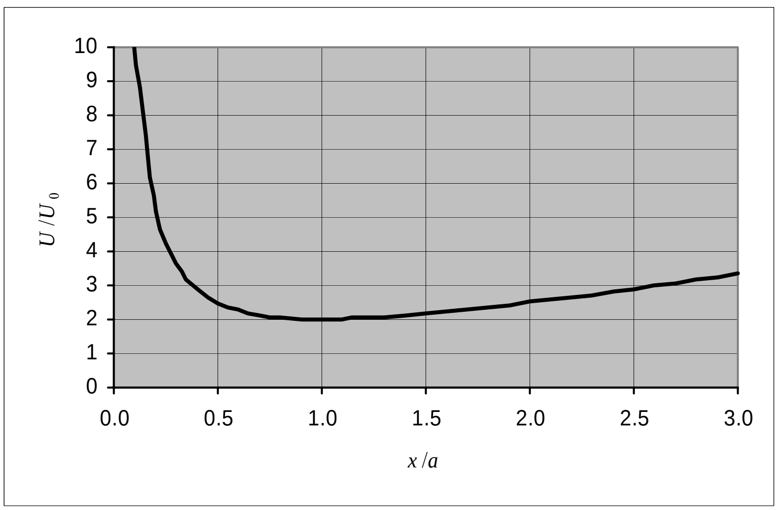
Substitute numerical values and evaluate A_{max}:

$$A_{max} = \frac{(0.4)(9.81\,\text{m/s}^2)(0.8\,\text{s})^2}{4\pi^2} = \boxed{6.36\,\text{cm}}$$

115 •••

Picture the Problem In (*b*), we can use the condition $F_{net} = dU/dx = 0$ for stable equilibrium to find the value of $x = x_0$ at stable equilibrium. In (*c*) and (*d*), we can simply follow the outline provided in the problem statement. In (*e*), we can obtain the frequency from $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ using the value for *k* from the potential function.

(*a*) A graph of $U(x)$ follows:



(*b*) Express the condition for equilibrium:

$$F = \frac{dU}{dx} = 0$$

Differentiate U with respect to x:

$$\frac{dU}{dx} = \frac{d}{dx}\left[U_0\left(\frac{x}{a} + \frac{a}{x}\right)\right] = U_0\left[\frac{1}{a} - \frac{a}{x^2}\right] = \frac{U_0}{a}\left(1 - \frac{a^2}{x^2}\right)$$

Set this derivative equal to zero and solve for *x*:

$$\frac{U_0}{a}\left(1-\frac{a^2}{x_0^2}\right)=0$$

and

$$x_0 = \boxed{a} \text{ or } \alpha = \boxed{1}$$

(*c*) Express $U(x_0 + \varepsilon)$:

$$U(x_0+\varepsilon)=U_0\left[\frac{x_0+\varepsilon}{a}+\frac{a}{x_0+\varepsilon}\right]$$

$$=U_0\left[\frac{x_0}{a}+\frac{\varepsilon}{a}+\frac{1}{\frac{x_0}{a}+\frac{\varepsilon}{a}}\right]$$

or, because $x_0 = a$,

$$U(x_0+\varepsilon)=U_0\left[1+\frac{\varepsilon}{a}+\frac{1}{1+\frac{\varepsilon}{a}}\right]$$

$$=\boxed{U_0\left[1+\beta+(1+\beta)^{-1}\right]}$$

where $\beta=\frac{\varepsilon}{a}$

(*d*) Expand $(1+\beta)^{-1}$ to obtain:

$$(1+\beta)^{-1}=1+(-1)\beta+\frac{(-1)(-2)}{2\times 1}\beta^2+...$$

$$\approx 1-\beta+\beta^2$$

Substitute in $U(x_0 + \varepsilon)$:

$$U(x_0+\varepsilon)=U_0\left[1+\beta+1-\beta+\beta^2\right]$$

$$=U_0\left[2+\beta^2\right]$$

$$=2U_0+U_0\frac{\varepsilon^2}{a^2}$$

$$=\boxed{\text{constant}+U_0\frac{\varepsilon^2}{a^2}}$$

(*e*) Express the potential energy of a simple harmonic oscillator:

$$U=\text{constant}+\tfrac{1}{2}k\varepsilon^2$$

If the particle whose potential energy is given in part (*d*) is to undergo simple harmonic motion:

$$k=\frac{2U_0}{a^2}$$

Express the frequency of the simple harmonic motion, substitute for *k*, and simplify to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2U_0}{a^2 m}}$$
$$= \boxed{\frac{1}{2\pi a}\sqrt{\frac{2U_0}{m}}}$$

116 •••

Picture the Problem Let *m* represent the mass of the cylindrical drum, *R* its radius, and *k* the stiffness constant of the spring. We can find the angular frequency of the oscillations by equating the maximum kinetic energy of the drum and the maximum energy stored in the spring. We can then express the frequency of the system in terms of its angular frequency. The application of Newton's 2nd law, under on-the-verge-of-sliding conditions, together with the introduction of the oscillator's total energy, will lead us to an expression for the minimum value of the coefficient of static friction.

(*a*) Express the frequency of oscillation of the system for small displacements from equilibrium:

$$f = \frac{\omega}{2\pi} \qquad (1)$$

Express the kinetic energy of the drum and simplify to obtain:

$$K = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + \tfrac{1}{2}mv^2$$
$$= \tfrac{3}{4}mv^2$$

Apply conservation of energy to obtain:

$$K_{max} = \tfrac{3}{4}mv_{max}^2 = \tfrac{1}{2}kA^2$$

Substitute $A\omega$ for v_{max}:

$$\tfrac{3}{4}m(A\omega)^2 = \tfrac{1}{2}kA^2$$

Solve for ω:

$$\omega = \sqrt{\frac{2k}{3m}}$$

Substitute in equation (1) to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{1}{2\pi}\sqrt{\frac{2(4000\,\text{N/m})}{3(6\,\text{kg})}} = \boxed{3.36\,\text{Hz}}$$

(*b*) Apply $\sum F_x = 0$ to the drum to

$$kA - f_{s,max} = 0$$

or

establish the condition that governs slipping:

$$kA - \mu_s F_n = 0$$

Using $F_n = mg$, solve for μ_s:

$$\mu_s = \frac{kA}{mg} \qquad (2)$$

Express the oscillator's total energy in terms of the amplitude of its motion:

$$E = \tfrac{1}{2}kA^2 \Rightarrow kA = \sqrt{2Ek}$$

Substitute in equation (2) to obtain:

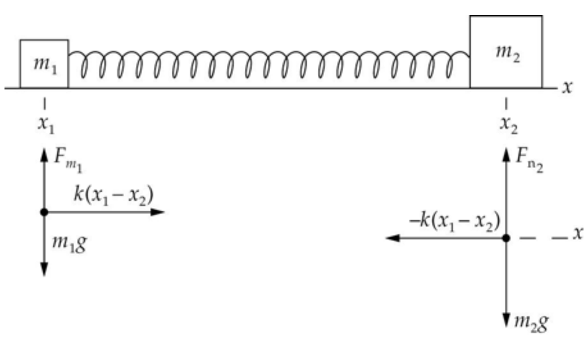
$$\mu_s = \frac{\sqrt{2Ek}}{mg}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{\sqrt{2(5\,\text{J})(4000\,\text{N/m})}}{(6\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{3.40}$$

***117** •••

Picture the Problem The pictorial representation shows the two blocks connected by the spring and displaced from their equilibrium positions. We can apply Newton's 2nd law to each of these coupled oscillators and solve the resulting equations simultaneously to obtain the differential equation of motion of the coupled oscillators. We can then compare this differential equation and its solution to the differential equation of motion of the simple harmonic oscillator and its solution to show that the oscillation frequency is $\omega = (k/\mu)^{1/2}$ where $\mu = m_1m_2/(m_1 + m_2)$ is the reduced mass of the system.



Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1 and solve for its acceleration:

$$k(x_1 - x_2) = m_1a_1 = m_1\frac{d^2x_1}{dt^2}$$

or

$$a_1 = \frac{d^2x_1}{dt^2} = \frac{k}{m_1}(x_1 - x_2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_2 and solve for its

$$-k(x_1 - x_2) = m_2a_2 = m_1\frac{d^2x_2}{dt^2}$$

acceleration: or

$$a_2 = \frac{d^2x_2}{dt^2} = \frac{k}{m_2}(x_2 - x_1)$$

Subtract the first equation from the second to obtain:

$$\frac{d^2(x_2 - x_1)}{dt^2} = \frac{d^2x}{dt^2} = -k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)x$$

where $x = x_2 - x_1$

The reduced mass of the system is:

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \text{ or } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Substitute to obtain:

$$\frac{d^2x}{dt^2} = -\frac{k}{\mu}x \qquad (1)$$

Compare this differential equation with the differential equation of the simple harmonic oscillator:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

The solution to this equation is:

$$x = x_0 \cos(\omega t + \delta)$$

where $\omega = \sqrt{\frac{k}{m}}$

Express the solution to equation (1):

$$x = x_0 \cos(\omega t + \delta)$$

where $\omega = \boxed{\sqrt{\frac{k}{\mu}}}$

118 ••

Picture the Problem We can use $\omega = (k/\mu)^{1/2}$ and $\mu = m_1m_2/(m_1 + m_2)$ from Problem 117 to find the spring constant for the HCl molecule.

Use the result of Problem 118 to relate the oscillation frequency to the spring constant and reduced mass of the HCl molecule:

$$\omega = \sqrt{\frac{k}{\mu}}$$

Solve for k to obtain:

$$k = \mu\omega^2$$

Express the reduced mass of the HCl molecule:

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Substitute to obtain:

$$k = \frac{m_1 m_2 \omega^2}{m_1 + m_2}$$

Express the masses of the hydrogen and Cl atoms:

$m_1 = 1\text{ amu} = 1.67\times10^{-27}$ kg
and
$m_2 = 35.45\text{ amu} = 5.92\times10^{-26}$ kg

Substitute numerical values and evaluate k:

$$k = \frac{(1.67\times10^{-27}\text{ kg})(5.92\times10^{-26}\text{ kg})(8.969\times10^{13}\text{ s}^{-1})^2}{1.67\times10^{-27}\text{ kg} + 5.92\times10^{-26}\text{ kg}} = \boxed{13.1\text{ N/m}}$$

119 ••

Picture the Problem In Problem 117, we derived an expression for the oscillation frequency of a spring-and-two-block system as a function of the stiffness constant of the spring and the reduced mass of the two blocks. We can solve this problem, assuming that the "spring constant" does not change, by using the result of Problem 117 and the reduced mass of a deuterium atom and a Cl atom in the equation for the oscillation frequency.

Use the result of Problem 117 to relate the oscillation frequency to the spring constant and reduced mass of the HCl molecule:

$$\omega = \sqrt{\frac{k}{\mu}}$$

Express the reduced mass of the HCl molecule:

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Express the masses of the deuterium and Cl atoms:

$m_1 = 2\text{ amu} = 3.34\times10^{-27}$ kg
and
$m_2 = 35.45\text{ amu} = 5.92\times10^{-26}$ kg

Evaluate the reduced mass of the molecule:

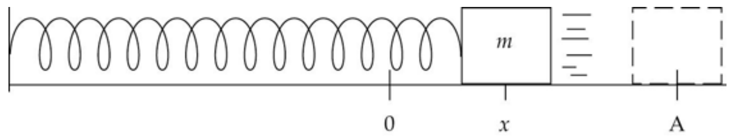
$$\mu = \frac{(3.34\times10^{-27}\text{ kg})(5.92\times10^{-26}\text{ kg})}{3.34\times10^{-27}\text{ kg} + 5.92\times10^{-26}\text{ kg}} = 3.16\times10^{-27}\text{ kg}$$

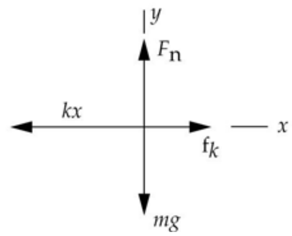
Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{13.1\text{ N/m}}{3.16\times10^{-27}\text{ kg}}} = \boxed{6.44\times10^{13}\text{ rad/s}}$$

120 •••

Picture the Problem The pictorial representation shows the block moving from right to left with an instantaneous displacement x from its equilibrium position. The free-body diagram shows the forces acting on the block during the half-cycles that it moves from right to left. When the block is moving from left to right, the directions of the kinetic friction force and the restoring force exerted by the spring are reversed. We can apply Newton's 2nd law to these motions to obtain the differential equations given in the problem statements and then use their solutions to plot the graph called for in (*c*).





(*a*) Apply $\sum F_x = ma_x$ to the block while it is moving to the left to obtain:

$$f_k - kx = m\frac{d^2x}{dt^2}$$

Using $f_k = \mu_k F_n = \mu_k mg$, eliminate f_k in the differential equation of motion:

$$m\frac{d^2x}{dt^2} = -kx + \mu_k mg$$

or

$$m\frac{d^2x}{dt^2} = -k\left(x - \frac{\mu_k mg}{k}\right)$$

Let $x_0 = \dfrac{\mu_k mg}{k}$ to obtain:

$$m\frac{d^2x}{dt^2} = -k(x - x_0)$$

or

$$\boxed{\frac{d^2x'}{dt^2} = -\frac{k}{m}x' = -\omega^2 x'}$$

provided $x' = x - x_0$ and

$$x_0 = \boxed{\frac{\mu_k mg}{k} = \frac{\mu_k g}{\omega^2}}$$

The solution to the differential equation is:

$$x' = x_0' \cos(\omega t + \delta)$$

and its derivative is

$$v' = -\omega x_0' \sin(\omega t + \delta)$$

The initial conditions are:

$$x'(0) = x - x_0 \text{ and } v'(0) = 0$$

Apply these conditions to obtain:

$$x'(0) = x_0' \cos\delta = x - x_0$$

and

$$v'(0) = -\omega x_0' \sin\delta = 0$$

Solve these equations simultaneously to obtain:

$$\delta = 0 \text{ and } x_0' = x - x_0$$

and

$$x' = (x - x_0)\cos\omega t$$

or

$$\boxed{x = (x - x_0)\cos\omega t + x_0} \qquad (1)$$

(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the block while it is moving to the right to obtain:

$$-f_k - kx = m\frac{d^2x}{dt^2}$$

Using $f_k = \mu_k F_n = \mu_k mg$, eliminate f_k in the differential equation of motion:

$$m\frac{d^2x}{dt^2} = -kx - \mu_k mg$$

or

$$m\frac{d^2x}{dt^2} = -k\left(x + \frac{\mu_k mg}{k}\right)$$

Let $x_0 = \dfrac{\mu_k mg}{k}$ to obtain:

$$m\frac{d^2x}{dt^2} = -k(x + x_0)$$

or

$$\boxed{\frac{d^2x''}{dt^2} = -\frac{k}{m}x'' = -\omega^2 x''}$$

provided $x'' = x + x_0$ and

$$x_0 = \boxed{\frac{\mu_k mg}{k} = \frac{\mu_k g}{\omega^2}}\,.$$

The solution to the differential equation is:

$$x'' = x_0''\cos(\omega t + \delta)$$

and its derivative is

$$v'' = -\omega x_0''\sin(\omega t + \delta)$$

The initial conditions are:

$$x''(0) = x + x_0 \text{ and } v''(0) = 0$$

Apply these conditions to obtain:

$$x''(0) = x_0''\cos\delta = x + x_0$$

and

$$v''(0) = -\omega x_0''\sin\delta = 0$$

Solve these equations simultaneously to obtain:

$$\delta = 0 \text{ and } x_0'' = x + x_0$$

and

$$x'' = (x + x_0)\cos\omega t$$

or

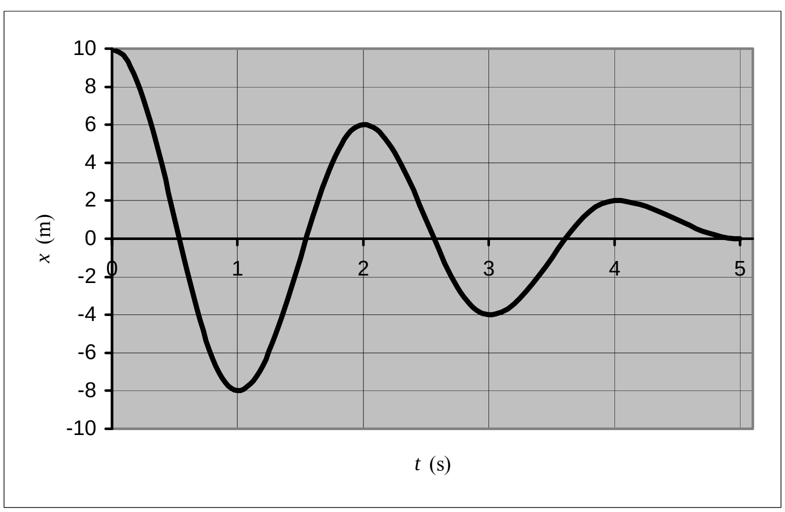
$$\boxed{x = (x + x_0)\cos\omega t - x_0} \qquad (2)$$

(*c*) A spreadsheet program to calculate the position of the oscillator as a function of time (equations (1) and (2)) is shown below. The constants used in the position functions (x_0 = 1 m and T = 2 s were used for simplicity) and the formulas used to calculate the positions are shown in the table. *After each half-period, one must compute a new amplitude for the oscillation, using the final value of the position from the last half-period.*

Cell	Content/Formula	Algebraic Form
B1	1	x_0
B2	10	A
C7	C6 + 0.1	$t+\Delta t$
D7	($B$2–$B$1)*COS(PI()*C7)+$B$1	$\left(A-x_0\right)\cos\pi t+x_0$
D17	(ABS($D$6+$B$1))*COS(PI()*C17)–$B$1	$\left\|x+x_0\right\|\cos\pi t-x_0$
D27	(ABS($D$6–$B$1))*COS(PI()*C27)+$B$1	$\left\|x-x_0\right\|\cos\pi t+x_0$
D37	(ABS($D$36+$B$1))*COS(PI()*C37)–$B$1	$\left\|x+x_0\right\|\cos\pi t-x_0$
D47	($D$46–$B$1)*COS(PI()*C47)+$B$1	$\left(x-x_0\right)\cos\pi t+x_0$

	A	B	C	D
1	x0=	1	m	
2	A=	10		
3				
4			*t*	*x*
5			(s)	(m)
6			0.0	10.00
7			0.1	9.56
8			0.2	8.28
9			0.3	6.29
10			0.4	3.78
53			4.7	0.41
54			4.8	0.19
55			4.9	0.05
56			5.0	0.00

The graph shown below was plotted using the data from columns C (t) and D (x). Note that the motion of the block ceases after five half - cycles.



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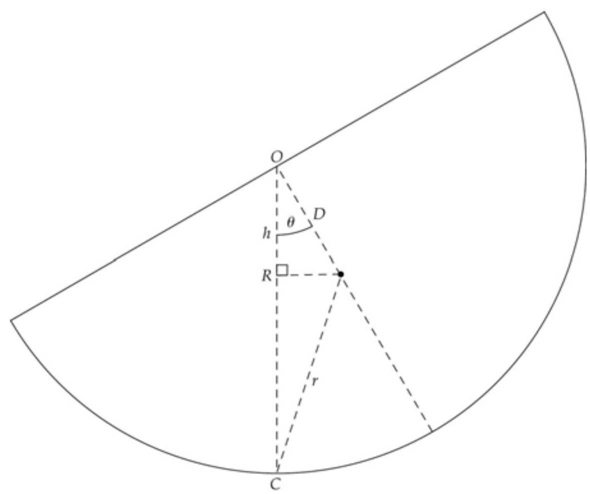
Picture the Problem The diagram shows the half-cylinder displaced from its equilibrium position through an angle θ. The frequency of its motion will be found by expressing the mechanical energy E in terms of θ and $d\theta/dt$. For small θ we will obtain an equation of the form $E = \frac{1}{2}\kappa\theta^2 + \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2$. Differentiating both sides of this equation with respect to time will lead to $0 = \left(\kappa\theta + I\frac{d^2\theta}{dt^2}\right)\frac{d\theta}{dt}$, an equation that must be valid at all times. Because the situation of interest to us requires that $d\theta/dt$ is not always equal to zero, we have $0 = \kappa\theta + I\frac{d^2\theta}{dt^2}$ or $\frac{d^2\theta}{dt^2} + \frac{\kappa}{I}\theta = 0$, the differential equation of simple harmonic motion with $\omega^2 = \kappa/I$. The distance from O to the center of mass D, where, from Problem 8-39, $D = (4/3\pi)R$, and the distance from the contact point C to the center of mass is r. Finally, we'll take the potential energy to be zero where θ is zero and assume that there is no slipping.



Apply conservation of energy to obtain:

$$E = U + K$$

$$= Mg(h - D) + \frac{1}{2}I_C\left(\frac{d\theta}{dt}\right)^2 \quad (1)$$

From Table 9-1, the moment of inertia of a solid cylinder about an axis perpendicular to its face and through its center is given by:

$$I_{0,\,\text{solid cylinder}} = \frac{1}{2}(2M)R^2 = MR^2$$

where M is the mass of the half-cylinder.

Express the moment of inertia of the half-cylinder about the same axis:

$$I_{0,\text{ half cylinder}} = I_0 = \frac{1}{2}\left[MR^2\right] = \frac{1}{2}MR^2$$

Use the parallel-axis theorem to relate I_{cm} to I_0:

$$I_0 = I_{cm} + MD^2$$

Substitute for I_{cm} and solve for I_{cm}:

$$I_{cm} = I_0 - D^2M = \frac{1}{2}MR^2 - D^2M$$

Apply the parallel-axis theorem a second time to obtain an expression for I_C:

$$I_C = \frac{1}{2}MR^2 - D^2M + Mr^2 = M\left(\frac{1}{2}R^2 - D^2 + r^2\right) \quad (2)$$

Apply the law of cosines to obtain:

$$r^2 = R^2 + D^2 - 2RD\cos\theta$$

Substitute for r^2 in equation (2) to obtain:

$$I_C = M\left(\frac{1}{2}R^2 - D^2 + R^2 + D^2 - 2RD\cos\theta\right) = MR^2\left(\frac{3}{2} - 2\frac{D}{R}\cos\theta\right)$$

Substitute for h and I_C in equation (1):

$$E = MgD(1-\cos\theta) + \frac{1}{2}MR^2\left(\frac{3}{2} - 2\frac{D}{R}\cos\theta\right)\left(\frac{d\theta}{dt}\right)^2$$

Use the small angle approximation $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ to obtain:

$$E = \frac{1}{2}MgD\theta^2 + \frac{1}{2}MR^2\left(\frac{3}{2} - \frac{D}{R}\left[2-\theta^2\right]\right)\left(\frac{d\theta}{dt}\right)^2$$

Because $\theta^2 << 2$, we can neglect the θ^2 in the square brackets to obtain:

$$E = \frac{1}{2}MgD\theta^2 + \frac{1}{2}MR^2\left(\frac{3}{2} - 2\frac{D}{R}\right)\left(\frac{d\theta}{dt}\right)^2$$

Differentiating both sides with respect to time yields:

$$0 = MgD\theta\frac{d\theta}{dt} + MR^2\left(\frac{3}{2} - 2\frac{D}{R}\right)\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right),$$

$$R^2\left(\frac{3}{2} - 2\frac{D}{R}\right)\left(\frac{d^2\theta}{dt^2}\right) + gD\theta = 0,$$

and

$$\frac{d^2\theta}{dt^2} + \frac{gD}{R^2\left(\frac{3}{2} - 2\frac{D}{R}\right)}\theta = 0,$$

the differential equation of simple harmonic motion with $\omega^2 = \frac{gD}{R^2\left(\frac{3}{2} - 2\frac{D}{R}\right)}$.

Substitute for D to obtain:

$$\omega^2 = \frac{\frac{4}{3\pi}}{\left(\frac{3}{2} - \frac{8}{3\pi}\right)}\frac{g}{R} = \left(\frac{8}{9\pi - 16}\right)\frac{g}{R}$$

Express the period of the motion in terms of ω and simplify to obtain:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\left(\frac{9\pi - 16}{8}\right)\frac{R}{g}}$$

$$= \boxed{7.78\sqrt{\frac{R}{g}}}$$

***122** •••

Picture the Problem The net force acting on the particle as it moves in the tunnel is the x-component of the gravitational force acting on it. We can find the period of the particle from the angular frequency of its motion. We can apply Newton's 2nd law to the particle in order to express ω in terms of the radius of the earth and the acceleration due to gravity at the surface of the earth.

(a) From the figure we see that:

$$F_x = F_r \sin\theta = -\frac{GmM_E}{R_E^3}r\frac{x}{r}$$

$$= \boxed{-\frac{GmM_E}{R_E^3}x}$$

Because this force is a linear restoring force, the motion of the particle is simple harmonic motion.

(*b*) Express the period of the particle as a function of its angular frequency:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the particle:

$$-\frac{GmM_E}{R_E^3}x = ma$$

Solve for *a*:

$$a = -\frac{GM_E}{R_E^3}x = -\omega^2 x$$

where

$$\omega = \sqrt{\frac{GM_E}{R_E^2}}$$

Use $GM_E = gR_E^2$ to simplify ω:

$$\omega = \sqrt{\frac{gR_E^2}{R_E^3}} = \sqrt{\frac{g}{R_E}}$$

Substitute in equation (1) to obtain:

$$T = \frac{2\pi}{\sqrt{\dfrac{g}{R_E}}} = \boxed{2\pi\sqrt{\frac{R_E}{g}}}$$

Substitute numerical values and evaluate *T*:

$$T = 2\pi\sqrt{\frac{6.37\times10^6\text{ m}}{9.81\text{ m/s}^2}} = 5.06\times10^3\text{ s}$$

$$= \boxed{84.4\text{ min}}$$

123 •••

Picture the Problem The amplitude of a damped oscillator decays with time according to $A = A_0e^{-(b/2m)t}$. We can find *b*/2*m* from $\omega' = \omega_0\sqrt{1-\left(\dfrac{b}{2m\omega_0}\right)^2}$ and then substitute in the amplitude equation to find the factor by which the amplitude is decreased during each oscillation. We'll use our result from (*a*), together with the dependence of the energy of the oscillator on the square of its amplitude, to find the factor by which its energy is reduced during each oscillation.

(*a*) Express the variation in amplitude with time:

$$A = A_0e^{-(b/2m)t} \qquad (1)$$

Relate the damped and undamped frequencies of the oscillator:

$$\omega' = \omega_0\sqrt{1-\left(\frac{b}{2m\omega_0}\right)^2} \qquad (14\text{-}46)$$

Solve for $b/2m$:

$$\frac{b}{2m} = \omega_0\sqrt{1-\frac{\omega'^2}{\omega_0^2}} = \omega_0\sqrt{1-(0.9)^2}$$
$$= 0.436\omega_0$$

Find the period of the damped oscillations:

$$T = \frac{2\pi}{\omega'} = \frac{2\pi}{0.9\omega_0}$$

Substitute in equation (1) with $t = T$ to obtain:

$$\frac{A}{A_0} = e^{-(0.436\omega_0)\left(\frac{2\pi}{0.9\omega_0}\right)} = e^{-3.04}$$
$$= \boxed{0.0478}$$

(*b*) Express the energy of the oscillator at time $t = 0$:

$$E_0 = \tfrac{1}{2}kA_0^2$$

Express the energy of the oscillator at time $t = T$:

$$E = \tfrac{1}{2}kA^2$$

Divide the second of these equations by the first, simplify, and substitute to evaluate E/E_0:

$$\frac{E}{E_0} = \frac{A^2}{A_0^2} = \left(\frac{A}{A_0}\right)^2 = (0.0477)^2$$
$$= \boxed{0.00228}$$

124 •••

Picture the Problem We can differentiate Equation 14-52 twice and substitute *x* and d^2x/dt^2 in Equation 14-51 to determine the condition that must be satisfied in order for Equation 14-52 to be a solution of Equation 14-51.

The differential equation of motion is Equation 14-51:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + m\omega_0^2 x = F_0\cos\omega t$$

Its proposed solution is Equation 14-52:

$$x = A\cos(\omega t - \delta)$$

Obtain the first and second derivatives of *x*:

$$\frac{dx}{dt} = -A\omega\sin(\omega t - \delta)$$

and

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t - \delta)$$

Substitute in the differential equation to obtain:

$$-mA\omega^2\cos(\omega t - \delta) - bA\omega\sin(\omega t - \delta) + m\omega_0^2 A\cos(\omega t - \delta) = F_0\cos\omega t$$

Using trigonometric identities, expand $\cos(\omega t - \delta)$ and $\sin(\omega t - \delta)$ to obtain:

$$-mA\omega^2(\cos\omega t\cos\delta + \sin\omega t\sin\delta) - bA\omega(\sin\omega t\cos\delta - \cos\omega t\sin\delta)$$
$$+ m\omega_0^2 A(\cos\omega t\cos\delta + \sin\omega t\sin\delta) = F_0\cos\omega t$$

Factor $mA(\cos\omega t\cos\delta + \sin\omega t\sin\delta)$ from the first and third terms to obtain:

$$mA(\omega_0^2 - \omega^2)(\cos\omega t\cos\delta + \sin\omega t\sin\delta) - bA\omega(\sin\omega t\cos\delta - \cos\omega t\sin\delta) = F_0\cos\omega t$$

Factor $\cos\omega t\cos\delta$ from the first term on the left-hand side of the equation and $\sin\omega t\cos\delta$ from the 2nd term:

$$mA(\omega_0^2 - \omega^2)(\cos\omega t\cos\delta)\left(1 + \frac{\sin\omega t\sin\delta}{\cos\omega t\cos\delta}\right) - bA\omega(\sin\omega t\cos\delta)\left(1 - \frac{\cos\omega t\sin\delta}{\sin\omega t\cos\delta}\right)$$
$$= F_0\cos\omega t$$

Simplify to obtain:

$$mA(\omega_0^2 - \omega^2)(\cos\omega t\cos\delta)(1 + \tan\omega t\tan\delta) - bA\omega(\sin\omega t\cos\delta)\left(1 - \frac{\tan\delta}{\tan\omega t}\right)$$
$$= F_0\cos\omega t$$

Divide both sides of the equation by $m(\omega_0^2 - \omega^2)$:

$$A(\cos\omega t\cos\delta)(1 + \tan\omega t\tan\delta) - \frac{bA\omega}{m(\omega_0^2 - \omega^2)}(\sin\omega t\cos\delta)\left(1 - \frac{\tan\delta}{\tan\omega t}\right)$$
$$= \frac{F_0}{m(\omega_0^2 - \omega^2)}\cos\omega t$$

The phase constant for a driven oscillator is given by Equation 14-54:

$$\tan\delta = \frac{b\omega}{m(\omega_0^2 - \omega^2)}$$

Substitute for $\tan\delta$:

$$A(\cos\omega t\cos\delta)\left(1+\tan\omega t\frac{b\omega}{m(\omega_0^2-\omega^2)}\right)-\frac{bA\omega}{m(\omega_0^2-\omega^2)}(\sin\omega t\cos\delta)$$
$$\times\left(1-\frac{\frac{b\omega}{m(\omega_0^2-\omega^2)}}{\tan\omega t}\right)=\frac{F_0}{m(\omega_0^2-\omega^2)}\cos\omega t$$

Simplify to obtain:

$$A(\cos\omega t\cos\delta)(1+\tan^2\delta)=\frac{F_0}{m(\omega_0^2-\omega^2)}\cos\omega t$$

Use the trigonometric identity $1+\tan^2\delta=\frac{1}{\cos^2\delta}$:

$$A(\cos\omega t\cos\delta)\frac{1}{\cos^2\delta}=\frac{F_0}{m(\omega_0^2-\omega^2)}\cos\omega t$$

Simplify to obtain:

$$A\cos\omega t=\frac{F_0\cos\delta}{m(\omega_0^2-\omega^2)}\cos\omega t$$

Thus $x=A\cos(\omega t-\delta)$ is a solution to Equation 14-51 provided:

$$\boxed{A=\frac{F_0\cos\delta}{m(\omega_0^2-\omega^2)}}$$

***125** •••

Picture the Problem We can follow the step-by-step instructions provided in the problem statement to obtain the desired results.

(*a*) Express the average power delivered by a driving force to a driven oscillator:

$P=\vec{F}\cdot\vec{v}=Fv\cos\theta$

or, because θ is 0°,

$P=Fv$

Express *F* as a function of time:

$F=F_0\cos\omega t$

Express the position of the driven oscillator as a function of time:

$x=A\cos(\omega t-\delta)$

Differentiate this expression with respect to time to express the velocity of the oscillator as a function of time:

$$v = -A\omega\sin(\omega t - \delta)$$

Substitute to express the average power delivered to the driven oscillator:

$$P = (F_0 \cos \omega t)[-A\omega\sin(\omega t - \delta)]$$
$$= \boxed{-A\omega F_0 \cos \omega t \sin(\omega t - \delta)}$$

(*b*) Expand $\sin(\omega t - \delta)$ to obtain:

$$\sin(\omega t - \delta) = \sin \omega t \cos \delta - \cos \omega t \sin \delta$$

Substitute in your result from (*a*) and simplify to obtain:

$$P = -A\omega F_0 \cos \omega t(\sin \omega t \cos \delta - \cos \omega t \sin \delta)$$
$$= \boxed{A\omega F_0 \sin \delta \cos^2 \omega t - A\omega F_0 \cos \delta \cos \omega t \sin \omega t}$$

(*c*) Integrate $\sin\theta\cos\theta$ over one period to determine $\langle \sin\theta\cos\theta \rangle$:

$$\langle \sin\theta\cos\theta \rangle = \frac{1}{2\pi}\left[\int_0^{2\pi} \sin\theta\cos\theta d\theta\right]$$
$$= \frac{1}{2\pi}\left[\frac{1}{2}\sin^2\theta\Big|_0^{2\pi}\right]$$
$$= 0$$

Integrate $\cos^2\theta$ over one period to determine $\langle \cos^2\theta \rangle$:

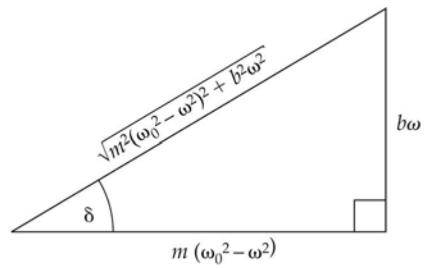
$$\langle \cos^2\theta \rangle = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\theta d\theta$$
$$= \frac{1}{2\pi}\left[\frac{1}{2}\int_0^{2\pi}(1+\cos 2\theta)d\theta\right]$$
$$= \frac{1}{2\pi}\left[\tfrac{1}{2}\int_0^{2\pi} d\theta + \tfrac{1}{2}\int_0^{2\pi}\cos 2\theta d\theta\right]$$
$$= \frac{1}{2\pi}(\pi + 0) = \frac{1}{2}$$

Substitute and simplify to express P_{av}:

$$P_{av} = A\omega F_0 \sin\delta\langle \cos^2 \omega t\rangle - A\omega F_0 \cos\delta\langle \cos\omega t \sin \omega t\rangle$$
$$= \tfrac{1}{2}A\omega F_0 \sin\delta - A\omega F_0 \cos\delta(0)$$
$$= \boxed{\tfrac{1}{2}A\omega F_0 \sin\delta}$$

(*d*) Construct a triangle that is consistent with

$$\tan\delta = \frac{b\omega}{m\left(\omega_0^2 - \omega^2\right)}:$$



Using the triangle, express sin*δ*:

$$\boxed{\sin\delta = \frac{b\omega}{\sqrt{m^2\left(\omega_0^2-\omega^2\right)^2 + b^2\omega^2}}}$$

Using equation 14-53, reduce this expression to the simpler form:

$$\boxed{\sin\delta = \frac{b\omega A}{F_0}}$$

(*e*) Solve $\sin\delta = \dfrac{b\omega A}{F_0}$ for *ω*:

$$\omega = \frac{F_0}{bA}\sin\delta$$

Substitute in the expression for P_{av} to eliminate *ω*:

$$\boxed{P_{av} = \frac{F_0^2}{2b}\sin^2\delta}$$

Substitute for $\sin\delta$ from (*d*) to obtain Equation 14-55:

$$\boxed{P_{av} = \frac{1}{2}\left[\frac{b\omega^2 F_0^2}{m^2\left(\omega_0^2-\omega^2\right)^2 + b^2\omega^2}\right]}$$

126 •••

Picture the Problem We can follow the step-by-step instructions given in the problem statement to derive the given results.

(*a*) Express the condition on the denominator of Equation 14-55 when the power input is half its maximum value:

$$m^2\left(\omega_0^2-\omega^2\right)^2 + b^2\omega^2 = 2b^2\omega_0^2$$

and, for a sharp resonance,

$$m^2\left(\omega_0^2-\omega^2\right)^2 \approx b^2\omega_0^2$$

Factor the difference of two squares to obtain:

$$m^2\left[(\omega_0-\omega)(\omega_0+\omega)\right]^2 \approx b^2\omega_0^2$$

or

$$\boxed{m^2(\omega_0-\omega)^2(\omega_0+\omega)^2 \approx b^2\omega_0^2}$$

(*b*) Use the approximation $\omega + \omega_0 \approx 2\omega_0$ to obtain:

$$m^2(\omega_0-\omega)^2(2\omega_0)^2 \approx b^2\omega_0^2$$

Solve for $\omega_0 - \omega$:

$$\omega_0 - \omega = \boxed{\pm\frac{b}{2m}} \qquad (1)$$

(c) Using its definition, express Q:

$$Q = \frac{\omega_0 m}{b}$$

Solve for b:

$$b = \boxed{\frac{\omega_0 m}{Q}}$$

(d) Substitute for b in equation (1) to obtain:

$$\omega_0 - \omega = \pm\frac{\omega_0}{2Q}$$

Solve for ω:

$$\omega = \omega_0 \pm \frac{\omega_0}{2Q}$$

Express the two values of ω:

$$\omega_+ = \boxed{\omega_0 + \frac{\omega_0}{2Q}}$$

and

$$\omega_- = \boxed{\omega_0 - \frac{\omega_0}{2Q}}$$

Remarks: Note that the width of the resonance at half-power is $\Delta\omega = \omega_+ - \omega_- = \omega_0/Q$, in agreement with Equation 14-49.

127 •••

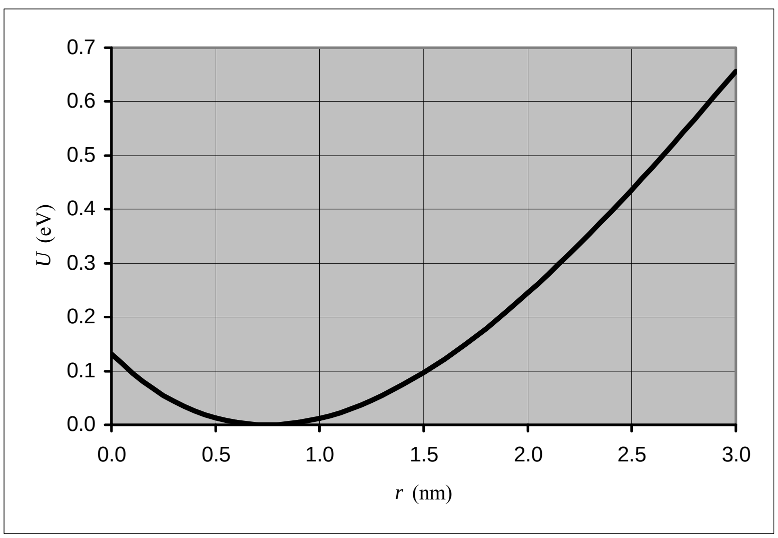
Picture the Problem We can find the equilibrium separation for the Morse potential by setting $dU/dr = 0$ and solving for r. The second derivative of U will give the "spring constant" for small displacements from equilibrium. In (c), we can use $\omega = \sqrt{k/\mu}$, where k is our result from (b) and μ is the reduced mass of a homonuclear diatomic molecule, to find the oscillation frequency of the molecule.

(a) A spreadsheet program to calculate the Morse potential as a function of r is shown below. The constants and cell formulas used to calculate the potential are shown in the table.

Cell	Content/Formula	Algebraic Form
B1	5	D
B2	0.2	β
C9	C8 + 0.1	$r + \Delta r$
D8	$B$1*(1–EXP(–$B$2*(C8–$B$3)))^2	$D\left[1 - e^{-\beta(r-r_0)}\right]^2$

	A	B	C	D
1	D=	5	eV	
2	Beta=	0.2	nm^{-1}	
3	r0=	0.75	nm	
4				
5				
6			r	U(r)
7			(nm)	(eV)
8			0.0	0.13095
9			0.1	0.09637
10			0.2	0.06760
11			0.3	0.04434
12			0.4	0.02629
235			22.7	4.87676
236			22.8	4.87919
237			22.9	4.88156
238			23.0	4.88390
239			23.1	4.88618

The graph shown below was plotted using the data from columns C (*r*) and D (*U*(*r*)).



(*b*) Differentiate the Morse potential with respect to r to obtain:

$$\frac{dU}{dr}=\frac{d}{dr}\left\{D\left[1-e^{-\beta(r-r_0)}\right]^2\right\}$$
$$=-2\beta D\left[1-e^{-\beta(r-r_0)}\right]$$

Set this derivative equal to zero for extrema:

$$-2\beta D\left[1-e^{-\beta(r-r_0)}\right]=0$$

Solve for r to obtain: $r = \boxed{r_0}$

Evaluate the second derivative of $U(r)$ to obtain:

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left\{-2\beta D\left[1 - e^{-\beta(r-r_0)}\right]\right\} = 2\beta^2 D e^{-\beta(r-r_0)}$$

Evaluate this derivative at $r = r_0$:

$$\left.\frac{d^2U}{dr^2}\right|_{r=r_0} = 2\beta^2 D \qquad (1)$$

Recall that the potential function for a simple harmonic oscillator is: $U = \frac{1}{2}kx^2$

Differentiate this expression twice to obtain:

$$\frac{d^2U}{dx^2} = k$$

By comparison with equation (1) we have: $k = \boxed{2\beta^2 D}$

(*c*) Express the oscillation frequency of the diatomic molecule:

$$\omega = \sqrt{\frac{k}{\mu}}$$

where μ is the reduced mass of the molecule.

Express the reduced mass of the homonuclear diatomic molecule:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m^2}{2m} = \frac{m}{2}$$

Substitute and simplify to obtain:

$$\omega = \sqrt{\frac{2\beta^2 D}{\frac{m}{2}}} = \boxed{2\beta\sqrt{\frac{D}{m}}}$$

Remarks: An alternative approach in (*b*) is the expand the Morse potential in a Taylor series

$$U(r) = U(r_0) + (r - r_0)U'(r_0) + \frac{1}{2!}(r - r_0)^2 U''(r_0) + \text{higher order terms}$$

to obtain $U(r) \approx \beta^2 D(r - r_0)^2$**. Comparing this expression to the energy of a spring-and-mass oscillator we see that, as was obtained above,** $k = 2\beta^2 D$**.**

Chapter 15
Wave Motion

Conceptual Problems

***1 •**

Determine the Concept The speed of a transverse wave on a rope is given by $v = \sqrt{F/\mu}$ where F is the tension in the rope and μ is its linear density. The waves on the rope move faster as they move up because the tension increases due to the weight of the rope below.

2 •

Determine the Concept The distance between successive crests is one wavelength and the time between successive crests is the period of the wave motion. Thus, T = 0.2 s and $f = 1/T = 5$ Hz. $\boxed{(b)\text{ is correct.}}$

3 •

Picture the Problem True. The energy per unit volume (the average energy density) is given by $\eta_{av} = \frac{1}{2}\rho\omega^2 s_0^2$ where s_0 is the displacement amplitude.

4 •

Determine the Concept At every point along the rope the wavelength, speed, and frequency of the wave are related by $\lambda = v/f$. The speed of the wave, in turn, is given by $v = \sqrt{F/\mu}$ where F is the tension in the rope and μ is its linear density. Due to the weight of the rope below, the tension is greater at the top and the speed of the wave is also greater at the top. Because $\lambda \propto v$, the wavelength is greater at the top.

***5 •**

Determine the Concept The speed of the wave v on the bullwhip varies with the tension F in the whip and its linear density μ according to $v = \sqrt{F/\mu}$. As the whip tapers, the wave speed in the tapered end increases due to the decrease in the mass density, so the wave travels faster.

6 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

Express the intensity of the 60-dB sound:

$$60\text{dB} = (10\,\text{dB})\log\frac{I_{60}}{I_0} \Rightarrow I_{60} = 10^6 I_0$$

Express the intensity of the 30-dB sound:

$$30\,\text{dB} = (10\,\text{dB})\log\frac{I_{30}}{I_0} \Rightarrow I_{30} = 10^3 I_0$$

Because $I_{60} = 10^3 I_{30}$:

$$\boxed{\text{The statement is false.}}$$

7 •

Determine the Concept No. Because the source and receiver are at rest relative to each other, there is no relative motion of the source and receiver and there will be no Doppler shift in frequency.

8 •

Determine the Concept Because there is no relative motion of the source and receiver, there will be no Doppler shift and the observer will hear sound of frequency f_0.

$\boxed{(a)\text{ is correct.}}$

***9** ••

Determine the Concept The light from the companion star will be shifted about its mean frequency periodically due to the relative approach to and recession from the earth of the companion star as it revolves about the black hole.

10 •

Determine the Concept In any medium, the wavelength, frequency, and speed of a wave are related through $\lambda = v/f$. Because the frequency of a wave is determined by its source and is independent of the nature of the medium, if v differs in the two media, the wavelengths will also differ. In this situation, the frequencies are the same but the speeds of propagation are different.

11 •

(*a*) True. The particles that make up the string move at right angles to the direction the wave is propagating.

(*b*) False. Sound waves in air are *longitudinal* waves of compression and rarefaction.

(*c*) False. The speed of sound in air varies with the square root of the absolute temperature. The speed of sound at 20°C is 4% greater than that at 5°C.

12 •

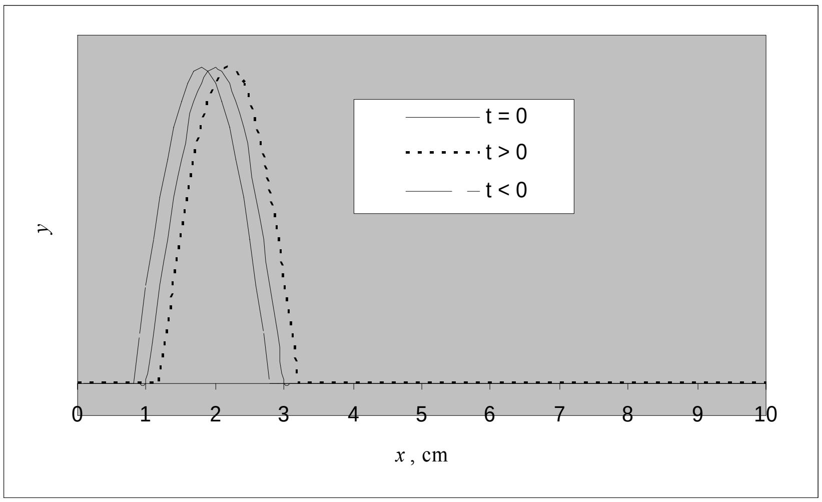
Determine the Concept In any medium, the wavelength, frequency, and speed of a sound wave are related through $\lambda = v/f$. Because the frequency of a wave is determined by its source and is independent of the nature of the medium, if v is greater in water than in air, λ will be greater in water than in air. $\boxed{(a) \text{ is correct.}}$

***13 •**

Determine the Concept There was only one explosion. Sound travels faster in water than air. Abel heard the sound wave in the water first, then, surfacing, heard the sound wave traveling through the air, which took longer to reach him.

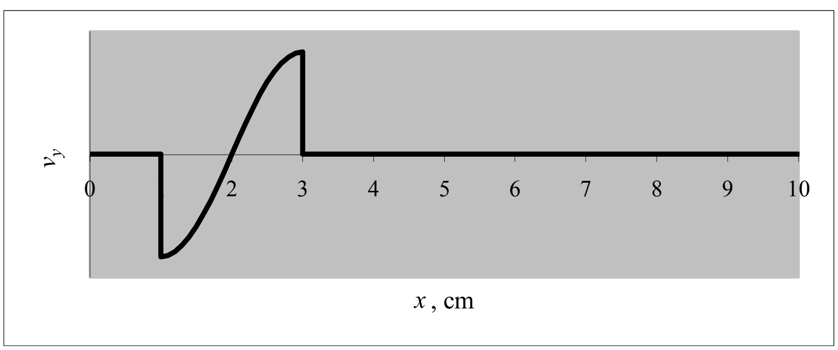
14 ••

Determine the Concept The graph shown below shows the pulse at an earlier time ($-\Delta t$) and later time (Δt). One can see that at $t = 0$, the portion of the string between 1 cm and 2 cm is moving down, the portion between 2 cm and 3 cm is moving up, and the string at $x = 2$ cm is instantaneously at rest.



15 ••

Determine the Concept The velocity of the string at $t = 0$ is shown. Note that the velocity is negative for 1 cm < x < 2 cm and is positive for 2 cm < x < 3 cm.



16 ••

Determine the Concept As the jar is evacuated, the speed of sound inside the jar decreases. Because of the mismatch between the speed of sound inside and outside of the jar, a larger fraction of the sound wave is reflected back into the jar, and a smaller fraction is transmitted through the glass of the bell jar.

***17** ••

Determine the Concept Path C. Because the wave speed is highest in the water, and more of path C is underwater than A or B, the sound wave will spend the least time on path C.

Estimation and Approximation

18 ••

Picture the Problem The rate at which energy is delivered by sound waves is the product of its intensity and the area over which the energy is delivered. We can use the definition of the intensity level of the speech at 1 m to find the intensity of the sound and the formula for the area of a sphere to find the area over which the energy is distributed.

Express the power of human speech as a function of its intensity:

$$P = IA$$

Express the area of a sphere of radius 1 m:

$$A = 4\pi r^2 = 4\pi(1\,\text{m})^2 = 4\pi\,\text{m}^2$$

Use $\beta = (10\,\text{dB})\log(I/I_0)$ to solve for the intensity of the sound at the 65-dB level:

$$65\,\text{dB} = (10\,\text{dB})\log\frac{I}{I_0}$$

and

$$I = 10^{6.5} I_0 = 10^{6.5}\left(10^{-12}\,\text{W/m}^2\right) = 3.16\times10^{-6}\,\text{W/m}^2$$

Substitute and evaluate P:

$$P = \left(3.16\times10^{-6}\,\text{W/m}^2\right)\left(4\pi\,\text{m}^2\right) = \boxed{3.97\times10^{-5}\,\text{W}}$$

19 ••

Picture the Problem Let d represent the distance from the bridge to the water under the assumption that the time for the sound to reach the man is negligible; let t be the elapsed time between dropping the stone and hearing the splash. We'll use a constant-acceleration equation to find the distance to the water in all three parts of the problem, just improving our initial value with corrections taking into account the time required for the sound of the splash to reach the man on the bridge.

(*a*) Using a constant-acceleration equation, relate the distance the stone falls to its time-of-fall:

$$d = v_0 t + \tfrac{1}{2} g t^2$$

or, because $v_0 = 0$,

$$d = \tfrac{1}{2} g t^2$$

Substitute numerical values and evaluate *d*:

$$d = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(4\,\text{s}\right)^2 = \boxed{78.5\,\text{m}}$$

(*b*) Express the actual distance to the water *d′* in terms of a time correction Δ*t*:

$$d' = \tfrac{1}{2} g\left(t - \Delta t\right)^2$$

Express Δ*t*:

$$\Delta t = \frac{d}{v_\text{s}}$$

Substitute to obtain:

$$d' = \tfrac{1}{2} g\left(t - \frac{d}{v_\text{s}}\right)^2$$

Substitute numerical values and evaluate *d′*:

$$d' = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(4\,\text{s} - \frac{78.5\,\text{m}}{340\,\text{m/s}}\right)^2$$

$$= \boxed{69.7\,\text{m}}$$

(*c*) Express the total time for the rock to fall and the sound to return to the man:

$$\Delta t = \Delta t_\text{falling rock} + \Delta t_\text{sound} = \sqrt{\frac{2d}{\text{g}}} + \frac{d}{v_\text{s}}$$

Rewrite the equation in quadratic form:

$$d^2 - 2v_\text{s}^2\left(\frac{1}{g} + \frac{\Delta t}{v_\text{s}}\right)d + v_\text{s}^2\left(\Delta t\right)^2 = 0$$

Substitute numerical values to obtain:

$$d^2 - \left(2.63\times10^4\,\text{m}\right)d + 1.85\times10^6\,\text{m}^2 = 0$$

Solve for the positive value of *d*:

$d = \boxed{70.5\,\text{m}}$ … about 1% larger than our result in part (*b*) and 11% smaller than our first approximation in (*a*).

***20 ••**

Picture the Problem You can use a protractor to measure the angle of the shock cone and then estimate the speed of the bullet using $\sin\theta = v/u$. The speed of sound in helium at room temperature (293 K) is 977 m/s.

Relate the speed of the bullet u to the speed of sound v in helium and the angle of the shock cone θ:

$$\sin\theta = \frac{v}{u}$$

Solve for u:

$$u = \frac{v}{\sin\theta}$$

Measure θ to obtain:

$$\theta \approx 70°$$

Substitute numerical values and evaluate u:

$$u = \frac{977\,\text{m/s}}{\sin 70°} = \boxed{1.04\,\text{km/s}}$$

21 ••

Picture the Problem Let d be the distance to the townhouses. We can relate the speed of sound to the distance to the townhouses to the frequency of the clapping for which no echo is heard.

Relate the speed of sound to the distance it travels to the townhouses and back to the elapsed time:

$$v = \frac{2d}{\Delta t}$$

Express d in terms of the number of strides and distance covered per stride:

$$d = (30\,\text{strides})(1.8\,\text{m/stride}) = 54\,\text{m}$$

Relate the elapsed time Δt to the frequency f of the clapping:

$$\Delta t = \frac{1}{f} = \frac{1}{2.5\,\text{claps/s}} = 0.4\,\text{s}$$

Substitute numerical values and evaluate v:

$$v = \frac{2(54\,\text{m})}{0.4\,\text{s}} = \boxed{270\,\text{m/s}}$$

Express the percent difference between this result and 340 m/s:

$$\frac{340\,\text{m/s} - 270\,\text{m/s}}{340\,\text{m/s}} = \boxed{20.6\%}$$

Speed of Waves

22 •

Picture the Problem The speed of sound in a fluid is given by $v = \sqrt{B/\rho}$ where B is the bulk modulus of the fluid and ρ is its density.

(*a*) Express the speed of sound in water in terms of its bulk modulus:

$$v = \sqrt{\frac{B}{\rho}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2.0\times10^{9}\ \text{N/m}^2}{10^{3}\ \text{kg/m}^3}} = \boxed{1.41\,\text{km/s}}$$

(*b*) Solve $v = \sqrt{B/\rho}$ for *B*:

$$B = \rho v^2$$

Substitute numerical values and evaluate *B*:

$$B = \left(13.6\times10^3\ \text{kg/m}^3\right)\left(1410\,\text{m/s}\right)^2 = \boxed{2.70\times10^{10}\ \text{N/m}^2}$$

***23** •

Picture the Problem The speed of sound in a gas is given by $v = \sqrt{\gamma RT/M}$ where *R* is the gas constant, *T* is the absolute temperature, *M* is the molecular mass of the gas, and γ is a constant that is characteristic of the particular molecular structure of the gas. Because hydrogen gas is diatomic, γ = 1.4.

Express the dependence of the speed of sound in hydrogen gas on the absolute temperature:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{1.4\left(8.314\,\text{J/mol}\cdot\text{K}\right)\left(300\,\text{K}\right)}{2\times10^{-3}\,\text{kg/mol}}} = \boxed{1.32\,\text{km/s}}$$

24 •

Picture the Problem The speed of a transverse wave pulse on a wire is given by $v = \sqrt{F/\mu}$ where *F* is the tension in the wire, *m* is its mass, *L* is its length, and μ is its mass per unit length.

Express the dependence of the speed of the pulse on the tension in the wire:

$$v = \sqrt{\frac{F}{\mu}}$$

where μ is the mass per unit length of the wire.

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{900\,\text{N}}{0.1\,\text{kg}/7\,\text{m}}} = \boxed{251\,\text{m/s}}$$

25 •

Picture the Problem The speed of transverse waves on a wire is given by $v = \sqrt{F/\mu}$ where *F* is the tension in the wire, *m* is its mass, *L* is its length, and μ is its mass per unit length.

Express the dependence of the speed of the pulse on the tension in the wire and the linear density of the wire:

$$v = \sqrt{\frac{F}{m/L}}$$

Solve for *m*:

$$m = \frac{FL}{v^2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{(550\,\text{N})(0.8\,\text{m})}{(150\,\text{m/s})^2} = \boxed{19.6\,\text{g}}$$

***26** •

Picture the Problem The speed of a wave pulse on a wire is given by $v = \sqrt{F/\mu}$ where F is the tension in the wire, m is its mass, L is its length, and μ is its mass per unit length.

(*a*) Doubling the length while keeping the mass per unit length constant does not change the linear density:

$$v = \boxed{20\,\text{m/s}}$$

(*b*) Because *v* depends of $\sqrt{F}$, doubling the tension increases *v* by a factor of $\sqrt{2}$:

$$v = \sqrt{2}(20\,\text{m/s}) = \boxed{28.3\,\text{m/s}}$$

(*c*) Because *v* depends on $1/\sqrt{\mu}$, doubling μ reduces *v* by a factor of $\sqrt{2}$:

$$v = \frac{20\,\text{m/s}}{\sqrt{2}} = \boxed{14.1\,\text{m/s}}$$

27 •

Picture the Problem The speed of a transverse wave on the piano wire is given by $v = \sqrt{F/\mu}$ where F is the tension in the wire, m is its mass, L is its length, and μ is its mass per unit length.

(*a*) The speed of transverse waves on the wire is given by:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{500\,\text{N}}{(0.005\,\text{kg})/(0.7\,\text{m})}} = \boxed{265\,\text{m/s}}$$

(*b*) Letting m' represent the mass of the wire when copper has been wrapped around the steel wire, express Δm, the amount of copper wire required:

$$\Delta m = m' - m$$

Express the new wave speed v':

$$v' = \sqrt{\frac{F}{m'/L}}$$

Express the ratio of the speed of the waves in part (*a*) to the reduced wave speed:

$$\frac{v}{v'} = 2 = \frac{\sqrt{\frac{F}{m/L}}}{\sqrt{\frac{F}{m'/L}}} = \sqrt{\frac{m'}{m}}$$

Solve for and evaluate m':

$$m' = 4m$$

Substitute to obtain:

$$\Delta m = m' - m = 4m - m = 3(5\,\text{g}) = \boxed{15.0\,\text{g}}$$

28 ••

Picture the Problem We can estimate the accuracy of this procedure by comparing the estimated distance to the actual distance. Whether a correction for the time it takes the light to reach you is important can be decided by comparing the times required for light and sound to travel a given distance.

(*a*) Convert 340 m/s to km/s:

$$v = 340\,\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{1000\,\text{m}} = \boxed{0.340\,\text{km/s}}$$

(*b*) Express the fractional error in the procedure:

$$\frac{\Delta s}{s} = \frac{s - s_{\text{estimated}}}{s} = \frac{0.340t - 0.333t}{0.340t} = \frac{0.007}{0.340} = \boxed{2.06\%}$$

(*c*) Compare the time required for light to travel 1 km to the time required for sound to travel the same distance:

$$\frac{\Delta t_{\text{light}}}{\Delta t_{\text{sound}}} = \frac{\frac{1\,\text{km}}{c}}{\frac{1\,\text{km}}{v}} = \frac{v}{c} = \frac{340\,\text{m/s}}{3\times10^{8}\,\text{m/s}} \approx 10^{-6}$$

Because this fraction is so small, a correction for the time for light to reach you is not important.

***29** ••

Picture the Problem The speed of a transverse wave on a string is given by $v=\sqrt{F/\mu}$ where F is the tension in the wire and μ is its linear density. We can differentiate this expression with respect to F and then separate the variables to show that the differentials satisfy $dv/v=\frac{1}{2}dF/F$. We'll approximate the differential quantities to determine by how much the tension must be changed to increase the speed of the wave to 312 m/s.

(*a*) Evaluate *dv*/*dF*:

$$\frac{dv}{dF}=\frac{d}{dF}\left[\sqrt{\frac{F}{\mu}}\right]=\frac{1}{2}\sqrt{\frac{1}{F\mu}}=\frac{1}{2}\cdot\frac{v}{F}$$

Separate the variables to obtain:

$$\boxed{\frac{dv}{v}=\frac{1}{2}\frac{dF}{F}}$$

(*b*) Solve for *dF*:

$$dF=2F\frac{dv}{v}$$

Approximate *dF* with Δ*F* and *dv* with Δ*v* to obtain:

$$\Delta F=2F\frac{\Delta v}{v}$$

Substitute numerical values and evaluate Δ*F*:

$$\Delta F=2(500\,\text{N})\frac{12\,\text{m/s}}{300\,\text{m/s}}=\boxed{40.0\,\text{N}}$$

30 ••

Picture the Problem The speed of sound in a gas is given by $v=\sqrt{\gamma RT/M}$ where R is the gas constant, T is the absolute temperature, M is the molecular mass of the gas, and γ is a constant that is characteristic of the particular molecular structure of the gas. We can differentiate this expression with respect to T and then separate the variables to show that the differentials satisfy $dv/v=\frac{1}{2}dT/T$. We'll approximate the differential quantities to determine the percentage change in the velocity of sound when the temperature increases from 0 to 27°C. Lacking information regarding the nature of the gas, we can express the ratio of the speeds of sound at 300 K and 273 K to obtain an expression that involves just the temperatures.

(*a*) Evaluate *dv*/*dT*:

$$\frac{dv}{dT}=\frac{d}{dT}\left[\sqrt{\frac{\gamma RT}{M}}\right]=\frac{1}{2}\sqrt{\frac{M}{\gamma RT}}\left(\frac{\gamma R}{M}\right)$$

$$=\frac{1}{2}\frac{v}{T}$$

Separate the variables to obtain:

$$\boxed{\frac{dv}{v}=\frac{1}{2}\frac{dT}{T}}$$

(*b*) Approximate *dT* with Δ*T* and *dv* with Δ*v* and substitute numerical values to obtain:

$$\frac{\Delta v}{v}=\frac{1}{2}\frac{\Delta T}{T}=\frac{1}{2}\left(\frac{27K}{273\text{K}}\right)=\boxed{4.95\%}$$

(*c*) Using a differential approximation, approximate the speed of sound at 300 K:

$$v_{300\,\text{K}}\approx v_{273\,\text{K}}+v_{273\,\text{K}}\frac{\Delta v}{v}$$

$$=v_{273\,\text{K}}\left(1+\frac{\Delta v}{v}\right)$$

Substitute numerical values and evaluate $v_{300\,\text{K}}$:

$$v_{300\,\text{K}}=(331\,\text{m/s})(1+0.0495)=\boxed{347\,\text{m/s}}$$

Use $v=\sqrt{\gamma RT/M}$ to express the speed of sound at 300 K:

$$v_{300\,\text{K}}=\sqrt{\frac{\gamma R(300\,\text{K})}{M}}$$

Use $v=\sqrt{\gamma RT/M}$ to express the speed of sound at 273 K:

$$v_{273\,\text{K}}=\sqrt{\frac{\gamma R(273\,\text{K})}{M}}$$

Divide the first of these equations by the second and solve for and evaluate $v_{300\,\text{K}}$:

$$\frac{v_{300\,\text{K}}}{v_{273\,\text{K}}}=\frac{\sqrt{\frac{\gamma R(300\,\text{K})}{M}}}{\sqrt{\frac{\gamma R(273\,\text{K})}{M}}}=\sqrt{\frac{300}{273}}$$

and

$$v_{300\,\text{K}}=(331\,\text{m/s})\sqrt{\frac{300}{273}}=\boxed{347\,\text{m/s}}$$

Note that these two results agree to three significant figures.

31 •••

Picture the Problem The speed of sound in a gas is given by $v=\sqrt{\gamma RT/M}$ where R is the gas constant, T is the absolute temperature, M is the molecular mass of the gas, and γ

is a constant that is characteristic of the particular molecular structure of the gas. Because $T = t + 273$ K, we can differentiate v with respect to t to show that $dv/dt = \frac{1}{2}(v/T)$.

Evaluate dv/dt:

$$\frac{dv}{dt} = \frac{d}{dt}\left[\sqrt{\frac{\gamma R(t + 273\,\text{K})}{M}}\right] = \frac{1}{2}\sqrt{\frac{M}{\gamma R(t + 273\,\text{K})}}\left(\frac{\gamma R}{M}\right) = \frac{1}{2}\frac{v}{T}$$

Substitute for t to obtain:

$$\frac{dv}{dt} = \frac{1}{2}\left[\frac{v}{t + 273\,\text{K}}\right]$$

Use the approximation

$$v(T) \approx v(T_0) + \left(\frac{dv}{dT}\right)_{T_0} \Delta T$$

to write:

$$v(t) \approx v(0°C) + \frac{1}{2}\left[\frac{v}{t + 273\,\text{K}}\right]t = v(0°C) + \Delta v \qquad (1)$$

where

$$\Delta v = \frac{1}{2}\left[\frac{331\,\text{m/s}}{t + 273\,\text{K}}\right]t$$

For $t << 273$ K:

$$\Delta v \approx \frac{1}{2}\left[\frac{331\,\text{m/s}}{273\,\text{K}}\right]t = (0.606\,\text{m/s}\cdot\text{K})t$$

Substitute in equation (1) to obtain:

$$v(t) = v(0°C) + (0.606\,\text{m/s}\cdot\text{K})t = \boxed{331\,\text{m/s} + (0.606\,\text{m/s}\cdot\text{K})t}$$

32 ••

Picture the Problem Let d be the distance to the munitions plant, v_1 be the speed of sound in air, v_2 be the speed of sound in rock, and Δt be the difference in the arrival times of the sound at the man's apartment. We can express Δt in terms of $t1$ and $t2$ and then express these travel times in terms of the distance d and the speeds of the sound waves in air and in rock to obtain an equation we can solve for the distance from the man's apartment to the munitions plant.

Express the difference in travel times for the sound wave transmitted through air and the sound wave transmitted through the earth:

$$t_1 - t_2 = \Delta t$$

Express the transmission times in terms of the distance traveled and the speeds in the two media:

$$t_1 = \frac{d}{v_1} \text{ and } t_2 = \frac{d}{v_2}$$

Substitute to obtain:

$$\frac{d}{v_1} - \frac{d}{v_2} = \Delta t$$

or

$$d\left(\frac{1}{v_1} - \frac{1}{v_2}\right) = d\left(\frac{v_2 - v_1}{v_1 v_2}\right) = \Delta t$$

Solve for *d*:

$$d = \frac{v_1 v_2}{v_2 - v_1}\Delta t$$

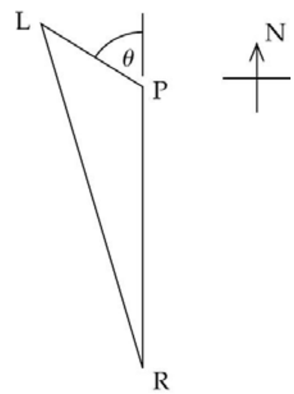
Substitute numerical values and evaluate *d*:

$$d = \frac{(340\,\text{m/s})(3000\,\text{m/s})}{3000\,\text{m/s} - 340\,\text{m/s}}(3\,\text{s})$$

$$= \boxed{1.15\,\text{km}}$$

33 •••

Picture the Problem The locations of the lightning strike (L), dorm room (R), and baseball park (P) are indicated on the diagram. We can neglect the time required for the electromagnetic pulse to reach the source of the radio transmission, which is the ballpark. The angle θ is related to the sides of the triangle through the law of cosines. We're given the distance d_{PR} and can find the other sides of the triangle using the speed of sound and the elapsed times.



Using the law of cosines, relate the angle θ to the distances that make up the sides of the triangle:

$$d_{LR}^2 = d_{LP}^2 + d_{PR}^2 - 2d_{LP}d_{PR}\cos(180° - \theta) = d_{LP}^2 + d_{PR}^2 + 2d_{LP}d_{PR}\cos\theta$$

Solve for θ:

$$\theta = \cos^{-1}\left(\frac{d_{LR}^2 - d_{LP}^2 - d_{PR}^2}{2d_{LP}d_{PR}}\right)$$

Express the distance from the lightning strike to the ball park:

$$d_{LP} = v_s \Delta t_{LP} = (340\,\text{m/s})(2\,\text{s}) = 680\,\text{m}$$

Express the distance from the lightning strike to the dorm room:

$$d_{LR} = v_s \Delta t_{LR} = (340\,\text{m/s})(6\,\text{s}) = 2040\,\text{m}$$

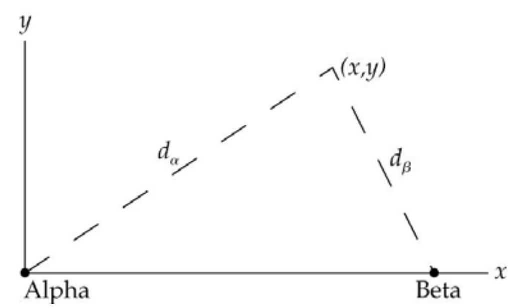
Substitute and evaluate θ:

$$\theta = \cos^{-1}\left(\frac{(2040\,\text{m})^2 - (680\,\text{m})^2 - (1600\,\text{m})^2}{2(680\,\text{m})(1600\,\text{m})}\right) = \pm 58.4°$$

The lightning struck 680 m from the ballpark, 58.4° W (or E) of north.

***34** •••

Picture the Problem Choose a coordinate system in which station Alpha is at the origin and the axes are oriented as shown in the pictorial representation. Because 0.75 mi = 1.21 km, Alpha's coordinates are (0, 0), Beta's are (1.21 km, 0), and those of the lightning strike are (x, y). We can relate the distances from the stations to the speed of sound in air and the times required to hear the thunder at the two stations.



Relate the distance d_α to the position coordinates of Alpha and the lightning strike:

$$x^2 + y^2 = d_\alpha^2 \quad (1)$$

Relate the distance d_β to the position coordinates of Beta and the lightning strike:

$$(x - 1.21\,\text{km})^2 + y^2 = d_\beta^2 \quad (2)$$

Relate the distance d_α to the speed of sound in air v and the time that elapses between seeing the lightning at Alpha and hearing the thunder:

$$d_\alpha = v\Delta t_\alpha = (340\,\text{m/s})(3.4\,\text{s}) = 1156\,\text{m}$$

Relate the distance d_β to the speed of sound in air v and the time that elapses between seeing the lightning at Beta and hearing the thunder:

$$d_\beta = v\Delta t_\beta = (340\,\text{m/s})(2.5\,\text{s}) = 850\,\text{m}$$

Substitute in equations (1) and (2) to obtain:

$$x^2 + y^2 = (1156\,\text{m})^2 = 1.336\,\text{km}^2 \quad (3)$$

and

$$(x - 1.21\,\text{km})^2 + y^2 = (850\,\text{m})^2 = 0.7225\,\text{km}^2 \quad (4)$$

Subtract equation (4) from equation (3) to obtain:

$$x^2 - (x - 1.21\,\text{km})^2 = 1.336\,\text{km}^2 - 0.7225\,\text{km}^2$$

or

$$(2.42\,\text{km})x - (1.21\,\text{km})^2 = 0.6135\,\text{km}^2$$

Solve for x to obtain:

$$x = 0.855\,\text{km}$$

Substitute in equation (3) to obtain:

$$(0.855\,\text{km})^2 + y^2 = 1.336\,\text{km}^2$$

Solve for y, keeping the positive root because the lightning strike is to the north of the stations, to obtain:

$$y = 0.778\,\text{km}$$

The coordinates of the lightning strike are:

$$\boxed{(0.855\,\text{km}, 0.778\,\text{km})}$$

or

$$\boxed{(0.531\,\text{mi}, 0.484\,\text{mi})}$$

35 •••

Picture the Problem The velocity of longitudinal waves on the Slinky is given by $v = \sqrt{B/\rho}$ where B is the bulk modulus of the material from which the slinky is constructed and ρ is its mass per unit volume. The velocity of transverse waves on the slinky is given by $v = \sqrt{F/\mu}$ where F is the tension in the slinky and μ is its mass per unit length. Substitution for B and ρ will lead to $v = L\sqrt{k/m}$ in (*a*) and similar substitutions, together with the assumption that $L_0 << L$ will yield the same result for (*b*).

(*a*) Express the velocity of longitudinal waves on the slinky:

$$v = \sqrt{\frac{B}{\rho}} \quad (1)$$

For the slinky:

$$\rho = \frac{m}{V}$$

and

$$B = -\frac{P}{\Delta V/V} \quad (2)$$

Letting A be the cross-sectional area of the slinky:

$$\rho = \frac{m}{AL} \text{ and } P = -k\frac{\Delta L}{A}$$

Substitute in equation (2) and simplify to obtain:

$$B = k\frac{L}{A}$$

Substitute in equation (1):

$$v = \sqrt{\frac{k\frac{L}{A}}{\frac{m}{AL}}} = \boxed{L\sqrt{\frac{k}{m}}}$$

(*b*) Express the velocity of transverse waves on the slinky:

$$v = \sqrt{\frac{F}{\mu}} \qquad (3)$$

For the slinky:

$$\mu = \frac{m}{L}$$

and

$$F = k\Delta L = k(L - L_0) = kL\left(1 - \frac{L_0}{L}\right)$$
$$\approx kL \text{ if } L_0 << L$$

Substitute in equation (3) to obtain:

$$v = \sqrt{\frac{kL}{\frac{m}{L}}} = \boxed{L\sqrt{\frac{k}{m}}}$$

The Wave Equation

36 •

Picture the Problem The general wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$. To show that each of the functions satisfies this equation, we'll need to find their first and second derivatives with respect to *x* and *t* and then substitute these derivatives in the wave equation.

(*a*) Find the first two spatial derivatives of $y(x,t) = k(x + vt)^3$:

$$\frac{\partial y}{\partial x} = 3k(x + vt)^2$$

and

$$\frac{\partial^2 y}{\partial x^2} = 6k(x + vt) \qquad (1)$$

Find the first two temporal derivatives of $y(x,t) = k(x + vt)^3$:

$$\frac{\partial y}{\partial t} = 3kv(x + vt)^2$$

and

$$\frac{\partial^2 y}{\partial t^2} = 6kv^2(x + vt) \qquad (2)$$

Express the ratio of equation (1) to equation (2):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}} = \frac{6k(x + vt)}{6kv^2(x + vt)} = \boxed{\frac{1}{v^2}}$$

confirming that $y(x,t) = k(x + vt)^3$ satisfies the general wave equation.

(*b*) Find the first two spatial derivatives of $y(x,t) = Ae^{ik(x-vt)}$:

$$\frac{\partial y}{\partial x} = ikAe^{ik(x-vt)}$$

and

$$\frac{\partial^2 y}{\partial x^2} = i^2k^2Ae^{ik(x-vt)}$$

or

$$\frac{\partial^2 y}{\partial x^2} = -k^2Ae^{ik(x-vt)} \qquad (3)$$

Find the first two temporal derivatives of $y(x,t) = Ae^{ik(x-vt)}$:

$$\frac{\partial y}{\partial t} = -ikvAe^{ik(x-vt)}$$

and

$$\frac{\partial^2 y}{\partial t^2} = i^2k^2v^2Ae^{ik(x-vt)}$$

or

$$\frac{\partial^2 y}{\partial t^2} = -k^2v^2Ae^{ik(x-vt)} \qquad (4)$$

Express the ratio of equation (3) to equation (4):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}} = \frac{-k^2Ae^{ik(x-vt)}}{-k^2v^2Ae^{ik(x-vt)}} = \boxed{\frac{1}{v^2}}$$

confirming that $y(x,t) = Ae^{ik(x-vt)}$ satisfies the general wave equation.

(*c*) Find the first two spatial derivatives of $y(x,\, t) = \ln k(x - vt)$:

$$\frac{\partial y}{\partial x} = \frac{k}{x - vt}$$

and

$$\frac{\partial^2 y}{\partial x^2} = -\frac{k^2}{(x - vt)^2} \qquad (5)$$

Find the first two temporal derivatives of $y(x,t)=\ln k(x-vt)$:

$$\frac{\partial y}{\partial t}=-\frac{vk}{x-vt}$$

and

$$\frac{\partial^2 y}{\partial t^2}=-\frac{v^2k^2}{(x-vt)^2} \qquad (6)$$

Express the ratio of equation (5) to equation (6):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}}=\frac{-\dfrac{k^2}{(x-vt)^2}}{-\dfrac{v^2k^2}{(x-vt)^2}}=\boxed{\frac{1}{v^2}}$$

confirming that $y(x,t)=\ln k(x-vt)$ satisfies the general wave equation.

***37 •**

Picture the Problem The general wave equation is $\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$. To show that $y=A\sin kx\cos\omega t$ satisfies this equation, we'll need to find the first and second derivatives of *y* with respect to *x* and *t* and then substitute these derivatives in the wave equation.

Find the first two spatial derivatives of $y=A\sin kx\cos\omega t$:

$$\frac{\partial y}{\partial x}=Ak\cos kx\cos\omega t$$

and

$$\frac{\partial^2 y}{\partial x^2}=-Ak^2\sin kx\cos\omega t \qquad (1)$$

Find the first two temporal derivatives of $y=A\sin kx\cos\omega t$:

$$\frac{\partial y}{\partial t}=-\omega A\sin kx\sin\omega t$$

and

$$\frac{\partial^2 y}{\partial t^2}=-\omega^2 A\sin kx\cos\omega t \qquad (2)$$

Express the ratio of equation (1) to equation (2):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}}=\frac{-Ak^2\sin kx\cos\omega t}{-A\omega^2\sin kx\cos\omega t}=\frac{k^2}{\omega^2}$$

$$=\boxed{\frac{1}{v^2}}$$

confirming that $y=A\sin kx\cos\omega t$ satisfies the general wave equation.

Harmonic Waves on a String

38 •

Picture the Problem We can find the velocity of the waves from the definition of velocity and their wavelength from $\lambda = v/f$.

Express the wavelength of the waves:

$$\lambda = \frac{v}{f}$$

Using the definition of velocity, find the wave velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{6\,\text{m}}{0.5\,\text{s}} = 12\,\text{m/s}$$

Substitute to obtain:

$$\lambda = \frac{12\,\text{m/s}}{60\,\text{s}^{-1}} = \boxed{20.0\,\text{cm}}$$

39 •

Picture the Problem Equation 15-13, $y(x,t) = A\sin(kx - \omega t)$, describes a wave traveling in the positive x direction. For a wave traveling in the negative x direction we have $y(x,t) = A\sin(kx + \omega t)$.

(a) Factor k from the argument of the sine function to obtain:

$$y(x,t) = A\sin k\left(x - \frac{\omega}{k}t\right) = \boxed{A\sin k(x - vt)}$$

(b) Substitute $k = 2\pi/\lambda$ and $\omega = 2\pi f$ to obtain:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - 2\pi ft\right) = \boxed{A\sin 2\pi\left(\frac{x}{\lambda} - ft\right)}$$

(c) Substitute $k = 2\pi/\lambda$ and $\omega = 2\pi/T$ to obtain:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda} - \frac{2\pi}{T}t\right) = \boxed{A\sin 2\pi\left(\frac{x}{\lambda} - \frac{1}{T}t\right)}$$

(*d*) Substitute $k = 2\pi/\lambda$ to obtain:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right) = A\sin\frac{2\pi}{\lambda}\left(x - \frac{\lambda\omega}{2\pi}t\right) = \boxed{A\sin\frac{2\pi}{\lambda}(x - vt)}$$

(*e*) Substitute $k = 2\pi/\lambda$ and $\omega = 2\pi f$ to obtain:

$$y(x,t) = A\sin(kx - 2\pi ft) = A\sin 2\pi f\left(\frac{k}{2\pi f}x - t\right) = \boxed{A\sin 2\pi f\left(\frac{x}{v} - t\right)}$$

For waves traveling in the negative *x* direction, we simply change the – signs to + signs.

***40 •**

Picture the Problem We can use $f = c/\lambda$ to express the frequency of any periodic wave in terms of its wavelength and velocity.

(*a*) Find the frequency of light of wavelength 4×10^{-7} m:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{4\times10^{-7}\ \text{m}} = 7.50\times10^{14}\ \text{Hz}$$

Find the frequency of light of wavelength 7×10^{-7} m:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{7\times10^{-7}\ \text{m}} = 4.28\times10^{14}\ \text{Hz}$$

Therefore the range of frequencies is:

$$\boxed{4.28\times10^{14}\ \text{Hz} \le f \le 7.50\times10^{14}\ \text{Hz}}$$

(*b*) Use the same relationship to calculate the frequency of these microwaves:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{3\times10^{-2}\ \text{m}} = \boxed{1.00\times10^{10}\ \text{Hz}}$$

41 •

Picture the Problem The average power propagated along the string by a harmonic wave is $P_{av} = \frac{1}{2}\mu\omega^2A^2v$, where *v* is the speed of the wave, and μ, ω, and *A* are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively.

Express and evaluate the power propagated along the string:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v$$

The speed of the wave on the string is given by:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute for v to obtain:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 \sqrt{\frac{F}{\mu}}$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = \tfrac{1}{2}\left(4\pi^2\right)(0.05\,\text{kg/m})\left(10\,\text{s}^{-1}\right)^2(0.05\,\text{m})^2\sqrt{\frac{80\,\text{N}}{0.05\,\text{kg/m}}} = \boxed{9.87\,\text{W}}$$

42 •

Picture the Problem The average power propagated along the rope by a harmonic wave is $P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v$, where v is the velocity of the wave, and μ, ω, and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively.

Rewrite the power equation in terms of the frequency of the wave:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v = 2\pi^2 \mu f^2 A^2 v$$

Solve for the frequency:

$$f = \sqrt{\frac{P_{av}}{2\pi^2 \mu A^2 v}} = \frac{1}{2\pi A}\sqrt{\frac{2P_{av}}{\mu v}}$$

The wave velocity is given by:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute for v and simplify to obtain:

$$f = \frac{1}{2\pi A}\sqrt{\frac{2P_{av}}{\mu\sqrt{\frac{F}{\mu}}}} = \frac{1}{2\pi A}\sqrt{\frac{2P_{av}}{\sqrt{\mu F}}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi(0.01\,\text{m})}\sqrt{\frac{2(100\,\text{W})}{\sqrt{\left(\frac{0.1\,\text{kg}}{2\,\text{m}}\right)(60\,\text{N})}}} = \boxed{171\,\text{Hz}}$$

43 ••

Picture the Problem Equation 15-13, $y(x,t) = A\sin(kx - \omega t)$, describes a wave traveling in the positive *x* direction. For a wave traveling in the negative *x* direction, we have $y(x,t) = A\sin(kx + \omega t)$. We can determine *A, k,* and ω by examination of the wave function. The wavelength, frequency, and period of the wave can, in turn, be determined from *k* and ω.

(*a*) Because the sign between the *kx* and ωt terms is positive, the wave is traveling in the negative *x* direction.

Find the speed of the wave:

$$v = \frac{\omega}{k} = \frac{314\,\text{s}^{-1}}{62.8\,\text{m}^{-1}} = \boxed{5.00\,\text{m/s}}$$

(*b*) The coefficient of *x* is *k* and:

$$k = \frac{2\pi}{\lambda}$$

Solve for and evaluate λ:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{62.8\,\text{m}^{-1}} = \boxed{10.0\,\text{cm}}$$

The coefficient of *t* is ω and:

$$f = \frac{\omega}{2\pi} = \frac{314\,\text{s}^{-1}}{2\pi} = \boxed{50.0\,\text{Hz}}$$

The period of the wave motion is the reciprocal of its frequency:

$$T = \frac{1}{f} = \frac{1}{50\,\text{s}^{-1}} = \boxed{0.0200\,\text{s}}$$

(*c*) Express and evaluate the maximum speed of any string segment:

$$v_{max} = A\omega = (0.001\,\text{m})(314\,\text{rad/s}) = \boxed{0.314\,\text{m/s}}$$

44 ••

Picture the Problem Let the positive *x* direction be to the right. Then equation 15-13, $y(x,t) = A\sin(kx - \omega t)$, describes a wave traveling in the positive *x* direction. We can find ω and *k* from the data included in the problem statement and substitute in the general equation. The maximum speed of a point on the string can be found from $v_{max} = A\omega$ and the maximum acceleration from $a_{max} = A\omega^2$.

(*a*) Express the general form of the equation of a harmonic wave traveling to the right:

$$y(x,t) = A\sin(kx - \omega t)$$

Evaluate ω:

$$\omega = 2\pi f = 2\pi\left(80\,\text{s}^{-1}\right) = 503\,\text{s}^{-1}$$

Determine k:

$$k = \frac{\omega}{v} = \frac{503\,\text{s}^{-1}}{12\,\text{m/s}} = 41.9\,\text{m}^{-1}$$

Substitute to obtain: $y(x,t) = \boxed{(0.025\,\text{m})\sin\left[\left(41.9\,\text{m}^{-1}\right)x - \left(503\,\text{s}^{-1}\right)t\right]}$

(*b*) Express and evaluate the maximum speed of a point on the string:

$$v_{\text{max}} = A\omega = (0.025\,\text{m})\left(503\,\text{s}^{-1}\right) = \boxed{12.6\,\text{m/s}}$$

(*c*) Express the maximum acceleration of a point on the string:

$$a_{\text{max}} = A\omega^2$$

Substitute numerical values and evaluate a_{max}:

$$a_{\text{max}} = (0.025\,\text{m})\left(503\,\text{s}^{-1}\right)^2 = \boxed{6.33\,\text{km/s}^2}$$

45 ••

Picture the Problem The average total energy of waves on a string is given by $\Delta E_{\text{av}} = \frac{1}{2}\mu\omega^2A^2\Delta x,$ where μ is the linear density of the string, ω is its angular frequency, A the amplitude of the wave motion, and, in this problem, Δx is the length of the string. The average power propagated along the string is $P_{\text{av}} = \frac{1}{2}\mu\omega^2A^2v.$

(*a*) ΔE_{av} is given by:

$$\Delta E_{\text{av}} = \tfrac{1}{2}\mu\omega^2A^2\Delta x = 2\pi^2\mu f^2A^2\Delta x$$

Evaluate $\Delta E_{\text{av}} = \frac{1}{2}\mu\omega^2A^2\Delta x$ with $\Delta x = L = 20$ m:

$$\Delta E_{\text{av}} = 2\pi^2\left(\frac{0.06\,\text{kg}}{20\,\text{m}}\right)\left(200\,\text{s}^{-1}\right)^2(0.012\,\text{m})^2(20\,\text{m}) = \boxed{6.82\,\text{J}}$$

(*b*) Express the power transmitted past a given point on the string:

$$P_{\text{av}} = \tfrac{1}{2}\mu\omega^2A^2v$$

The speed of the wave is given by: $v = \sqrt{\frac{F}{\mu}}$

Substitute for v to obtain: $P_{av} = \frac{1}{2}\mu\omega^2 A^2 \sqrt{\frac{F}{\mu}}$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = 2\pi^2\left(\frac{0.06\,\text{kg}}{20\,\text{m}}\right)\left(200\,\text{s}^{-1}\right)^2(0.012\,\text{m})^2\sqrt{\frac{50\,\text{N}}{\frac{0.06\,\text{kg}}{20\,\text{m}}}} = \boxed{44.0\,\text{W}}$$

***46 ••**

Picture the Problem The power propagated along the rope by a harmonic wave is $P = \frac{1}{2}\mu\omega^2 A^2 v$ where v is the velocity of the wave, and μ, ω, and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively. We can use the wave function $y = \left(A_0 e^{-bx}\right)\sin(kx - \omega t)$ to determine the amplitude of the wave at $x = 0$ and at point x.

(*a*) Express the power associated with the wave at the origin: $P = \frac{1}{2}\mu\omega^2 A^2 v$

Evaluate the amplitude at $x = 0$: $A(0) = \left(A_0 e^0\right) = A_0$

Substitute to obtain: $P(0) = \boxed{\frac{1}{2}\mu\omega^2 A_0^2 v}$

(*b*) Express the amplitude of the wave at x: $A(x) = \left(A_0 e^{-bx}\right)$

Substitute to obtain:

$$P(x) = \frac{1}{2}\mu\omega^2\left(A_0 e^{-bx}\right)^2 v = \boxed{\frac{1}{2}\mu\omega^2 A_0^2 v e^{-2bx}}$$

47 ••

Picture the Problem The average power propagated along the rope by a harmonic wave is $P_{av} = \frac{1}{2}\mu\omega^2 A^2 v$ where v is the velocity of the wave, and μ, ω, and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively.

(*a*) Express the average power transmitted along the wire:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v = 2\pi^2 \mu f^2 A^2 v$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = 2\pi^2(0.01\,\text{kg/m})(400\,\text{s}^{-1})^2 \times (0.5\times10^{-3}\,\text{m})^2(10\,\text{m/s}) = \boxed{79.0\,\text{mW}}$$

(*b*) Because $P_{av} \propto f^2$:

Increasing f by a factor of 10 would increase P_{av} by a factor of 100.

Because $P_{av} \propto A^2$:

Increasing A by a factor of 10 would increase P_{av} by a factor of 100.

Because $P_{av} \propto v$ and $v \propto \sqrt{F}$:

Increasing F by a factor of 10^4 would increase v by a factor of 100 and P_{av} by a factor of 100.

(*c*)

Depending on the adjustability of the power source, increasing f or A would be the easiest.

***48 •••**

Picture the Problem We can use the assumption that both the wave function and its first spatial derivative are continuous at $x = 0$ to establish equations relating A, B, C, k_1, and k_2. Then, we can solve these simultaneous equations to obtain expressions for B and C in terms of A, v_1, and v_2.

(*a*) Let $y_1(x, t)$ represent the wave function in the region $x < 0$, and $y_2(x, t)$ represent the wave function in the region $x > 0$. Express the continuity of the two wave functions at $x = 0$:

$$y_1(0,t) = y_2(0,t)$$

and

$$A\sin[k_1(0) - \omega t] + B\sin[k_1(0) + \omega t] = C\sin[k_2(0) - \omega t]$$

or

$$A\sin(-\omega t) + B\sin\omega t = C\sin(-\omega t)$$

Because the sine function is odd; i.e., $\sin(-\theta) = -\sin\theta$:

$$-A\sin\omega t + B\sin\omega t = -C\sin\omega t$$

and

$$A - B = C \qquad (1)$$

Differentiate the wave functions with respect to x to obtain:

$$\frac{\partial y_1}{\partial x} = Ak_1 \cos(k_1 x - \omega t) + Bk_1 \cos(k_1 x + \omega t)$$

and

$$\frac{\partial y_2}{\partial x} = Ck_2 \cos(k_2 x - \omega t)$$

Express the continuity of the slopes of the two wave functions at $x = 0$:

$$\left.\frac{\partial y_1}{\partial x}\right|_{x=0} = \left.\frac{\partial y_2}{\partial x}\right|_{x=0}$$

and

$$Ak_1 \cos[k_1(0) - \omega t] + Bk_1 \cos[k_1(0) + \omega t] = Ck_2 \cos[k_2(0) - \omega t]$$

or

$$Ak_1 \cos(-\omega t) + Bk_1 \cos \omega t = Ck_2 \cos(-\omega t)$$

Because the cosine function is even; i.e., $\cos(-\theta) = \cos\theta$:

$$Ak_1 \cos \omega t + Bk_1 \cos \omega t = Ck_2 \cos \omega t$$

and

$$k_1 A + k_1 B = k_2 C \qquad (2)$$

Multiply equation (1) by k_1 and add it to equation (2) to obtain:

$$2k_1 A = (k_1 + k_2)C$$

Solve for C:

$$C = \frac{2k_1}{k_1 + k_2} A = \frac{2}{1 + k_2/k_1} A$$

Solve for C/A and substitute ω/v_1 for k_1 and ω/v_2 for k_2 to obtain:

$$\frac{C}{A} = \frac{2}{1 + k_2/k_1} = \boxed{\frac{2}{1 + v_1/v_2}}$$

Substitute in equation (1) to obtain:

$$A - B = \left(\frac{2}{1 + v_1/v_2}\right) A$$

Solve for B/A:

$$\frac{B}{A} = \boxed{-\frac{1 - v_1/v_2}{1 + v_1/v_2}}$$

(b) We wish to show that

$$B^2 + (v_1/v_2)C^2 = A^2$$

Use the results of (a) to obtain the expressions $B = -[(1 - \alpha)/(1 + \alpha)]\,A$ and $C = 2A/(1 + \alpha)$, where $\alpha = v_1/v_2$.

Substitute these expressions into

$$B^2 + (v_1/v_2)C^2 = A^2$$

and check to see if the resulting equation is an identity:

$$B^2 + \frac{v_1}{v_2}C^2 = A^2$$

$$\left(\frac{1-\alpha}{1+\alpha}\right)^2 A^2 + \alpha\left(\frac{2}{1+\alpha}\right)^2 A^2 = A^2$$

$$\left(\frac{1-\alpha}{1+\alpha}\right)^2 + \alpha\left(\frac{2}{1+\alpha}\right)^2 = 1$$

$$\frac{(1-\alpha)^2 + 4\alpha}{(1+\alpha)^2} = 1$$

$$\frac{1 - 2\alpha + \alpha^2 + 4\alpha}{(1+\alpha)^2} = 1$$

$$\frac{1 + 2\alpha + \alpha^2}{(1+\alpha)^2} = 1$$

$$\frac{(1+\alpha)^2}{(1+\alpha)^2} = 1$$

$$1 = 1$$

The equation is an identity:

Therefore, $\boxed{B^2 + \frac{v_1}{v_2}C^2 = A^2}$

Remarks: Our result in (*a*) can be checked by considering the limit of *B*/*A* as $v_2/v_1 \to 0$. This limit gives *B*/*A* = +1, telling us that the transmitted wave has zero amplitude and the incident and reflected waves superpose to give a standing wave with a node at *x* = 0.

Harmonic Sound Waves

***49** •

Picture the Problem The pressure variation is of the form $p(x,t) = p_0 \cos k(x - vt)$ where $k = \frac{\pi}{2}$ and $v = 340\,\text{m/s}$. We can find λ from k and f from ω and k.

(a) By inspection of the equation:

$$p_0 = \boxed{0.750\,\text{Pa}}$$

(b) Because $k = \frac{2\pi}{\lambda} = \frac{\pi}{2}$:

$$\lambda = \boxed{4.00\,\text{m}}$$

(c) Solve $v = \frac{\omega}{k} = \frac{2\pi f}{k}$ for f to obtain:

$$f = \frac{kv}{2\pi} = \frac{\frac{\pi}{2}(340\,\text{m/s})}{2\pi} = \boxed{85.0\,\text{Hz}}$$

(*d*) By inspection of the equation: $v = \boxed{340\,\text{m/s}}$

50 •

Picture the Problem The frequency, wavelength, and speed of the sound waves are related by $v = f\lambda$.

(*a*) Express and evaluate the wavelength of middle C:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{262\,\text{s}^{-1}} = \boxed{1.30\,\text{m}}$$

(*b*) Double the frequency corresponding to middle C; solve for and evaluate λ:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{2\left(262\,\text{s}^{-1}\right)} = \boxed{0.649\,\text{m}}$$

51 •

Picture the Problem The pressure amplitude depends on the density of the medium ρ, the angular frequency of the sound wave μ, the speed of the wave v, and the displacement amplitude s_0 according to $p_0 = \rho\omega v s_0$.

(*a*) Solve $p_0 = \rho\omega v s_0$ for s_0:

$$s_0 = \frac{p_0}{\rho\omega v}$$

Substitute numerical values and evaluate s_0:

$$s_0 = \frac{\left(10^{-4}\,\text{atm}\right)\left(1.01325\times10^{5}\,\text{Pa/atm}\right)}{2\pi\left(1.29\,\text{kg/m}^3\right)\left(100\,\text{s}^{-1}\right)\left(340\,\text{m/s}\right)} = \boxed{3.68\times10^{-5}\,\text{m}}$$

(*b*) Use $p_0 = \rho\omega v s_0$ to find p_0:

$$p_0 = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(300\,\text{s}^{-1}\right)\left(340\,\text{m/s}\right)\left(10^{-7}\,\text{m}\right) = \boxed{8.27\times10^{-2}\,\text{Pa}}$$

52 •

Picture the Problem The pressure amplitude depends on the density of the medium ρ, the angular frequency of the sound wave μ, the speed of the wave v, and the displacement amplitude s_0 according to $p_0 = \rho\omega v s_0$.

(*a*) Solve $p_0 = \rho\omega v s_0$ for s_0:

$$s_0 = \frac{p_0}{\rho\omega v}$$

Substitute numerical values and evaluate s_0:

$$s_0 = \frac{29\,\text{Pa}}{2\pi(1.29\,\text{kg/m}^3)(1000\,\text{s}^{-1})(340\,\text{m/s})} = \boxed{2.10\times10^{-5}\,\text{m}}$$

(*b*) Proceed as in (*a*) with $f = 1$ kHz:

$$s_0 = \frac{29\,\text{Pa}}{2\pi(1.29\,\text{kg/m}^3)(1000\,\text{s}^{-1})(340\,\text{m/s})} = \boxed{1.05\times10^{-5}\,\text{m}}$$

53 •

Picture the Problem The pressure or density wave is 90° out of phase with the displacement wave. When the displacement is zero, the pressure and density changes are either a maximum or a minimum. When the displacement is a maximum or minimum, the pressure and density changes are zero. We can use $p_0 = \rho\omega v s_0$ to find the maximum value of the displacement at any time and place.

(*a*) If the pressure is a maximum at x_1 when $t = 0$:

$\boxed{\text{the displacement } s \text{ is zero.}}$

(*b*) Solve $p_0 = \rho\omega v s_0$ for s_0:

$$s_0 = \frac{p_0}{\rho\omega v}$$

Substitute numerical values and evaluate s_0:

$$s_0 = \frac{(10^{-4}\,\text{atm})(1.01325\times10^5\,\text{Pa/atm})}{2\pi(1.29\,\text{kg/m}^3)(1000\,\text{s}^{-1})(340\,\text{m/s})} = \boxed{3.68\,\mu\text{m}}$$

***54** •

Picture the Problem A human can hear sounds between roughly 20 Hz and 20 kHz; a factor of 1000. An octave represents a change in frequency by a factor of 2. We can evaluate $2^N = 1000$ to find the number of octaves heard by a person who can hear this range of frequencies.

Relate the number of octaves to the difference between 20 kHz and 20 Hz:

$$2^N = 1000$$

Take the logarithm of both sides of the equation to obtain:

$$\log 2^N = \log 10^3$$

or

$$N \log 2 = 3$$

Solve for and evaluate N:

$$N = \frac{3}{\log 2} = 9.97 \approx \boxed{10}$$

Waves in Three Dimensions: Intensity

55 •

Picture the Problem The pressure amplitude depends on the density of the medium ρ, the angular frequency of the sound wave μ, the speed of the wave v, and the displacement amplitude s_0 according to $p_0 = \rho\omega v s_0$. The intensity of the waves is given by $I = \frac{1}{2}\rho\omega^2 s_0^2 v = \frac{1}{2}\frac{p_0^2}{\rho v}$ and the rate at which energy is delivered (power) is the product of the intensity and the surface area of the piston.

(*a*) Using $p_0 = \rho\omega v s_0$, evaluate p_0:

$$p_0 = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(500\,\text{s}^{-1}\right)\times\left(340\,\text{m/s}\right)\left(0.1\times10^{-3}\,\text{m}\right) = \boxed{138\,\text{Pa}}$$

(*b*) Use $I = \frac{1}{2}\frac{p_0^2}{\rho v}$ to find the intensity of the waves:

$$I = \frac{1}{2}\frac{(138\,\text{Pa})^2}{\left(1.29\,\text{kg/m}^3\right)(340\,\text{m/s})} = \boxed{21.7\,\text{W/m}^2}$$

(*c*) Using $P_{\text{av}} = IA$ to find the power required to keep the piston oscillating:

$$P_{\text{av}} = \left(21.6\,\text{W/m}^2\right)\left(10^{-2}\,\text{m}^2\right) = \boxed{0.217\,\text{W}}$$

56 •

Picture the Problem The intensity of the sound from the spherical source varies inversely with the square of the distance from the source. The power radiated by the source is the product of the intensity of the radiation and the surface area over which it is distributed.

(*a*) Relate the intensity at 10 m to the distance from the source:

$$I = \frac{P_{\text{av}}}{4\pi r^2}$$

or

$$10^{-4}\,\text{W/m}^2 = \frac{P_{\text{av}}}{4\pi(10\,\text{m})^2}$$

Letting r' represent the distance at which the intensity is 10^{-6} W/m^2, express the intensity as in part (*a*):

$$10^{-6}\,\text{W/m}^2 = \frac{P_{\text{av}}}{4\pi r'^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{10^{-4}\ \text{W/m}^2}{10^{-6}\ \text{W/m}^2}=\frac{\dfrac{P_{\text{av}}}{4\pi(10\,\text{m})^2}}{\dfrac{P_{\text{av}}}{4\pi r'^2}}$$

Solve for and evaluate r':

$$r'=\sqrt{(10^2)(10\,\text{m})^2}=\boxed{100\,\text{m}}$$

(b) Solve $I=\dfrac{P_{\text{av}}}{4\pi r^2}$ for P_{av}:

$$P_{\text{av}}=4\pi r^2 I$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}}=4\pi(10\,\text{m})^2\left(10^{-4}\ \text{W/m}^2\right)=\boxed{0.126\,\text{W}}$$

***57 •**

Picture the Problem Because the power radiated by the loudspeaker is the product of the intensity of the sound and the surface area over which it is distributed, we can use this relationship to find the average power, the intensity of the radiation, or the distance to the speaker for a given intensity or average power.

(a) Use $P_{\text{av}}=4\pi r^2 I$ to find the total acoustic power output of the speaker:

$$P_{\text{av}}=4\pi(20\,\text{m})^2\left(10^{-2}\ \text{W/m}^2\right)=\boxed{50.3\,\text{W}}$$

(b) Relate the intensity of the sound at 20 m to the distance from the speaker:

$$10^{-2}\ \text{W/m}^2=\frac{P_{\text{av}}}{4\pi(20\,\text{m})^2}$$

Relate the threshold-of-pain intensity to the distance from the speaker:

$$1\,\text{W/m}^2=\frac{P_{\text{av}}}{4\pi r^2}$$

Divide the first of these equations by the second; solve for and evaluate r:

$$r=\sqrt{10^{-2}(20\,\text{m})^2}=\boxed{2.00\,\text{m}}$$

(c) Use $I=\dfrac{P_{\text{av}}}{4\pi r^2}$ to find the intensity at 30 m:

$$I(30\,\text{m})=\frac{50.3\,\text{W}}{4\pi(30\,\text{m})^2}=\boxed{4.45\times10^{-3}\ \text{W/m}^2}$$

58 ••

Picture the Problem We can use conservation of energy to find the acoustical energy resulting from the dropping of the pin. The power developed can then be found from the given time during which the energy was transformed from mechanical to acoustical form. We can find the range at which the dropped pin can be heard from $I = P/4\pi r^2$.

(*a*) Assuming that $I = P/4\pi r^2$, express the distance at which one can hear the dropped pin:

$$r = \sqrt{\frac{P}{4\pi I}}$$

Use conservation of energy to determine the sound energy generated when the pin falls:

$$E = (0.0005)(mgh) = (0.0005)(0.1\times10^{-3}\text{ kg})(9.81\text{ m/s}^2)\times(1\text{ m}) = 4.91\times10^{-7}\text{ J}$$

Express the power of the sound pulse:

$$P = \frac{E}{\Delta t} = \frac{4.91\times10^{-7}\text{ J}}{0.1\text{ s}} = 4.91\times10^{-6}\text{ W}$$

Substitute numerical values and evaluate *r*:

$$r = \sqrt{\frac{4.91\times10^{-6}\text{ W}}{4\pi(10^{-11}\text{ W/m}^2)}} = \boxed{198\text{ m}}$$

(*b*) Repeat the last step in (*a*) with $I = 10^{-8}$ W/m^2:

$$r = \sqrt{\frac{4.91\times10^{-6}\text{ W}}{4\pi(10^{-8}\text{ W/m}^2)}} = \boxed{6.25\text{ m}}$$

Intensity Level

59 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\text{ dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

(*a*) Using its definition, calculate the intensity level of a sound wave whose intensity is 10^{-10} W/m^2:

$$\beta = (10\text{ dB})\log\left(\frac{10^{-10}\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right) = 10\log10^2 = \boxed{20.0\text{ dB}}$$

(*b*) Proceed as in (*a*) with $I = 10^{-2}$ W/m^2:

$$\beta = (10\,\text{dB})\log\left(\frac{10^{-2}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$
$$= 10\log 10^{10} = \boxed{100\,\text{dB}}$$

60 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

(*a*) Solve $\beta = (10\,\text{dB})\log(I/I_0)$ for I to obtain:

$$I = 10^{\beta/(10\,\text{dB})}I_0$$

Evaluate I for β = 10 dB:

$$I = 10^{(10\,\text{dB})/(10\,\text{dB})}I_0 = 10I_0$$
$$= 10\left(10^{-12}\ \text{W/m}^2\right) = \boxed{10^{-11}\ \text{W/m}^2}$$

(*b*) Proceed as in (*a*) with β = 3 dB:

$$I = 10^{(3\,\text{dB})/(10\,\text{dB})}I_0 = 2I_0$$
$$= 2\left(10^{-12}\ \text{W/m}^2\right) = \boxed{2\times10^{-12}\ \text{W/m}^2}$$

***61** •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

Express the sound level of the rock concert:

$$\beta_{\text{concert}} = (10\,\text{dB})\log\left(\frac{I_{\text{concert}}}{I_0}\right) \qquad (1)$$

Express the sound level of the dog's bark:

$$50\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{\text{dog}}}{I_0}\right)$$

Solve for the intensity of the dog's bark:

$$I_{\text{dog}} = 10^5 I_0 = 10^5\left(10^{-12}\ \text{W/m}^2\right)$$
$$= 10^{-7}\ \text{W/m}^2$$

Express the intensity of the rock concert in terms of the intensity of the dog's bark:

$$I_{\text{concert}} = 10^4 I_{\text{dog}} = 10^4\left(10^{-7}\ \text{W/m}^2\right)$$
$$= 10^{-3}\ \text{W/m}^2$$

Substitute in equation (1) and evaluate β_{concert}:

$$\beta_{\text{concert}} = (10\,\text{dB})\log\left(\frac{10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = (10\,\text{dB})\log 10^9 = \boxed{90.0\,\text{dB}}$$

62 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

Express the intensity level of the louder sound:

$$\beta_{\text{L}} = (10\,\text{dB})\log\left(\frac{I_{\text{L}}}{I_0}\right)$$

Express the intensity level of the softer sound:

$$\beta_{\text{S}} = (10\,\text{dB})\log\left(\frac{I_{\text{S}}}{I_0}\right)$$

Express the difference between the intensity levels of the two sounds:

$$\beta_{\text{L}} - \beta_{\text{S}} = 30\,\text{dB} = (10\,\text{dB})\left[\log\left(\frac{I_{\text{L}}}{I_0}\right) - \log\left(\frac{I_{\text{S}}}{I_0}\right)\right] = (10\,\text{dB})\log\left(\frac{I_{\text{L}}/I_0}{I_{\text{S}}/I_0}\right) = (10\,\text{dB})\log\left(\frac{I_{\text{L}}}{I_{\text{S}}}\right)$$

Solve for and evaluate the ratio $I_{\text{L}}/I_{\text{S}}$:

$$\frac{I_{\text{L}}}{I_{\text{S}}} = 10^3 \text{ and } \boxed{(a)\text{ is correct.}}$$

63 •

Picture the Problem We can use the definition of the intensity level to express the difference in the intensity levels of two sounds whose intensities differ by a factor of 2.

Express the intensity level before the intensity is doubled:

$$\beta_1 = (10\,\text{dB})\log\left(\frac{I}{I_0}\right)$$

Express the intensity level with the intensity doubled:

$$\beta_2 = (10\,\text{dB})\log\left(\frac{2I}{I_0}\right)$$

Express and evaluate $\Delta\beta = \beta_2 - \beta_1$:

$$\begin{aligned}\Delta\beta &= \beta_2 - \beta_1 \\ &= (10\,\text{dB})\log\left(\frac{2I}{I_0}\right) - (10\,\text{dB})\log\left(\frac{I}{I_0}\right) \\ &= (10\,\text{dB})\log\left(\frac{2I}{I}\right) = (10\,\text{dB})\log 2 \\ &= 3.01\,\text{dB} \approx \boxed{3.0\,\text{dB}}\end{aligned}$$

***64 •**

Picture the Problem We can express the intensity levels at both 90 dB and 70 dB in terms of the intensities of the sound at those levels. By subtracting the two expressions, we can solve for the ratio of the intensities at the two levels and then find the fractional change in the intensity that corresponds to a decrease in intensity level from 90 dB to 70 dB.

Express the intensity level at 90 dB:

$$90\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{90}}{I_0}\right)$$

Express the intensity level at 70 dB:

$$70\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{70}}{I_0}\right)$$

Express $\Delta\beta = \beta_{90} - \beta_{70}$:

$$\begin{aligned}\Delta\beta &= 20\,\text{dB} \\ &= (10\,\text{dB})\log\left(\frac{I_{90}}{I_0}\right) - (10\,\text{dB})\log\left(\frac{I_{70}}{I_0}\right) \\ &= (10\,\text{dB})\log\left(\frac{I_{90}}{I_{70}}\right)\end{aligned}$$

Solve for I_{90}:

$$I_{90} = 100 I_{70}$$

Express the fractional change in the intensity from 90 dB to 70 dB:

$$\frac{I_{90} - I_{70}}{I_{90}} = \frac{100 I_{70} - I_{70}}{100 I_{70}} = \boxed{99\%}$$

65 ••

Picture the Problem The intensity at a distance r from a spherical source varies with distance from the source according to $I = P_{\text{av}}/4\pi r^2$. We can use this relationship to relate the intensities corresponding to an 80-dB intensity level (I_{80}) and the intensity corresponding to a 60-dB intensity level (I_{60}) to their distances from the source. We can relate the intensities to the intensity levels through $\beta = (10\,\text{dB})\log(I/I_0)$.

(*a*) Express the intensity of the sound where the intensity level is 80 dB:

$$I_{10} = \frac{P_{av}}{4\pi r_{10}^2}$$

Express the intensity of the sound where the intensity level is 60 dB:

$$I_{60} = \frac{P_{av}}{4\pi r^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{I_{80}}{I_{60}} = \frac{\frac{P_{av}}{4\pi(10\,\text{m})^2}}{\frac{P_{av}}{4\pi r^2}} = \frac{r^2}{100\,\text{m}^2}$$

Solve for *r*:

$$r = (10\,\text{m})\sqrt{\frac{I_{80}}{I_{60}}}$$

Find the intensity of the 80-dB sound level radiation:

$$80\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{80}}{I_0}\right)$$

and

$$I_{80} = 10^8 I_0 = 10^{-4}\ \text{W/m}^2$$

Find the intensity of the 60-dB sound level radiation:

$$60\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{60}}{I_0}\right)$$

and

$$I_{60} = 10^6 I_0 = 10^{-6}\ \text{W/m}^2$$

Substitute and evaluate *r*:

$$r = (10\,\text{m})\sqrt{\frac{10^{-4}\ \text{W/m}^2}{10^{-6}\ \text{W/m}^2}} = \boxed{100\,\text{m}}$$

(*b*) Using the intensity corresponding to an intensity level of 80 dB, express and evaluate the power radiated by this source:

$$P = I_{80}A = (10^{-4}\ \text{W/m}^2)\left[4\pi(10\,\text{m})^2\right] = \boxed{0.126\,\text{W}}$$

66 ••

Picture the Problem Let I_1 and I_2 be the intensities of the sound at distances r_1 and r_2. We can relate these intensities to the intensity levels through $\beta = (10\,\text{dB})\log(I/I_0)$ and to the distances through $I = P_{av}/4\pi r^2$.

Using $\beta = (10\,\text{dB})\log(I/I_0)$, express the ratio β_1/β_2:

$$\frac{\beta_2}{\beta_1} = \frac{(10\,\text{dB})\log(I_2/I_0)}{(10\,\text{dB})\log(I_1/I_0)} = \frac{\log I_2 - \log I_0}{\log I_1 - \log I_0}$$

Express the ratio of the intensities at distances r_1 and r_2 from the source and solve for I_2:

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \text{ and } I_2 = \frac{r_1^2}{r_2^2} I_1$$

Substitute and simplify to obtain:

$$\frac{\beta_2}{\beta_1} = \frac{10\log\frac{r_1^2}{r_2^2} I_1 - 10\log I_0}{\log I_1 - \log I_0} = \frac{10\log I_1 + 20\log(r_1/r_2) - 10\log I_0}{10\log I_1 - 10\log I_0}$$

$$= \frac{10\log(I_1/I_0) + 20\log(r_1/r_2)}{10\log(I_1/I_0)} = \boxed{\frac{\beta_1 + 20\log(r_1/r_2)}{\beta_1}}$$

67 ••

Picture the Problem We can use $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing, to find the intensity level at 20 m. Because the power radiated by the loudspeaker is the product of the intensity of the sound and the surface area over which it is distributed, we can use this relationship to find either the average power, the intensity of the radiation, or the distance to the speaker for a given intensity or average power.

(*a*) Relate the intensity level to the intensity at 20 m:

$$\beta = (10\,\text{dB})\log\left(\frac{I}{I_0}\right) = (10\,\text{dB})\log\left(\frac{10^{-2}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = (10\,\text{dB})\log(10^{10}) = \boxed{100\,\text{dB}}$$

(*b*) Use $P_{\text{av}} = 4\pi r^2 I$ to find the total acoustic power output of the speaker:

$$P_{\text{av}} = 4\pi(20\,\text{m})^2(10^{-2}\ \text{W/m}^2) = \boxed{50.3\,\text{W}}$$

(*c*) Relate the intensity of the sound at 20 m to the distance from the speaker:

$$10^{-2}\ \text{W/m}^2 = \frac{P_{\text{av}}}{4\pi(20\,\text{m})^2}$$

Relate the threshold-of-pain intensity to the distance from the speaker:

$$1\,\text{W/m}^2 = \frac{P_{\text{av}}}{4\pi r^2}$$

Divide the first of these equations by the second; solve for and evaluate r:

$$r = \sqrt{10^{-2}(20\,\text{m})^2} = \boxed{2.00\,\text{m}}$$

(*d*) Use $I = \dfrac{P_{\text{av}}}{4\pi r^2}$ to find the intensity at 30 m:

$$I(30\,\text{m}) = \frac{50.3\,\text{W}}{4\pi(30\,\text{m})^2} = 4.45\times10^{-3}\ \text{W/m}^2$$

Find the intensity level at 30 m:

$$\beta(30\,\text{m}) = (10\,\text{dB})\log\frac{4.45\times10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2} = (10\,\text{dB})\log 4.45\times10^{9} = \boxed{96.5\,\text{dB}}$$

68 ••

Picture the Problem Let *I′* and *I* represent the sound in consecutive years. Then, we can use $\beta = (10\,\text{dB})\log(I/I_0)$ to express the annual increase in intensity levels.

(*a*) Express the annual change in intensity level:

$$\Delta\beta = \beta' - \beta = 1\,\text{dB} = (10\,\text{dB})\log\frac{I'}{I_0} - (10\,\text{dB})\log\frac{I}{I_0} = (10\,\text{dB})\log\frac{I'}{I}$$

Solve for *I′*/*I*:

$$\frac{I'}{I} = 10^{0.1} = 1.26$$

and the annual increase in intensity is $\boxed{26\%.}$ This is not a plausible annual increase because, if it were true, the intensity level would increase by a factor of 10 in ten years.

(*b*) Because doubling of the intensity corresponds to $\Delta\beta$ = 3 dB and the intensity is increasing 1 dB annually:

$\boxed{\text{The intensity level will double in 3 years.}}$

69 ••

Picture the Problem We can find the intensities of the three sources from their intensity levels and, because their intensities are additive, find the intensity level when all three sources are acting.

(*a*) Express the sound intensity level when the three sources act at the same time:

$$\beta_{3\,\text{sources}} = (10\,\text{dB})\log\frac{I_{3\,\text{sources}}}{I_0} = (10\,\text{dB})\log\frac{I_{70}+I_{73}+I_{80}}{I_0}$$

Find the intensities of each of the three sources:

$$70\,\text{dB} = (10\,\text{dB})\log\frac{I_{70}}{I_0} \Rightarrow I_{70} = 10^7 I_0$$

$$73\,\text{dB} = (10\,\text{dB})\log\frac{I_{73}}{I_0} \Rightarrow I_{73} = 10^{7.3} I_0$$

and

$$80\,\text{dB} = (10\,\text{dB})\log\frac{I_{80}}{I_0} \Rightarrow I_{80} = 10^8 I_0$$

Substitute and evaluate $\beta_{3\text{ sources}}$:

$$\beta_{3\,\text{sources}} = (10\,\text{dB})\log\frac{10^7 I_0 + 10^{7.3} I_0 + 10^8 I_0}{I_0} = (10\,\text{dB})\log\left(10^7 + 10^{7.3} + 10^8\right) = \boxed{81.1\,\text{dB}}$$

(*b*) Find the intensity level with the two least intense sources eliminated:

$$\beta_{80} = (10\,\text{dB})\log\frac{10^8 I_0}{I_0} = (10\,\text{dB})\log\left(10^8\right) = \boxed{80.0\,\text{dB}}$$

Eliminating the two least intense sources does not reduce the intensity level significantly.

***70** ••

Picture the Problem Let *P* be the power radiated by the source of sound, and *r* be the initial distance from the source to the receiver. We can use the definition of intensity to find the ratio of the intensities before and after the distance is doubled and then use the definition of the decibel level to find the change in its level.

Relate the change in decibel level to the change in the intensity level:

$$\Delta\beta = 10\log\frac{I}{I'}$$

Using its definition, express the intensity of the sound from the source as a function of P and r:

$$I = \frac{P}{4\pi r^2}$$

Express the intensity when the distance is doubled:

$$I' = \frac{P}{4\pi(2r)^2} = \frac{P}{16\pi r^2}$$

Evaluate the ratio of I to I':

$$\frac{I}{I'} = \frac{\frac{P}{4\pi r^2}}{\frac{P}{16\pi r^2}} = 4$$

Substitute to obtain:

$$\Delta\beta = 10\log 4 = 6.02\,\text{dB}$$

and $\boxed{(c)\text{ is correct.}}$

71 •••

Picture the Problem The sound level can be found from the intensity of the sound due to the talking people. When 38 people are talking, the intensities add.

Express the sound level when all 38 people are talking:

$$\begin{aligned}\beta_{38} &= (10\,\text{dB})\log\frac{38I_1}{I_0}\\ &= (10\,\text{dB})\log 38 + (10\,\text{dB})\log\frac{I_1}{I_0}\\ &= (10\,\text{dB})\log 38 + 72\,\text{dB}\\ &= \boxed{87.8\,\text{dB}}\end{aligned}$$

An equivalent but longer solution:

Express the sound level when all 38 people are talking:

$$\beta_{38} = (10\,\text{dB})\log\frac{38I_1}{I_0}$$

Express the sound level when only one person is talking:

$$\beta_1 = 72\,\text{dB} = (10\,\text{dB})\log\frac{I_1}{I_0}$$

Solve for and evaluate I_1:

$$\begin{aligned}I_1 &= 10^{7.2} I_0 = 10^{7.2}\left(10^{-12}\ \text{W/m}^2\right)\\ &= 1.58\times10^{-5}\ \text{W/m}^2\end{aligned}$$

Express the intensity when all 38 people are talking:

$$I_{38} = 38I_1$$

The decibel level is:

$$\beta_{38} = (10\,\text{dB})\log\frac{38(1.58\times10^{-5}\ \text{W/m}^2)}{10^{-12}\ \text{W/m}^2} = \boxed{87.8\,\text{dB}}$$

***72 •••**

Picture the Problem Let η represent the efficiency with which mechanical energy is converted to sound energy. Because we're given information regarding the rate at which mechanical energy is delivered to the string and the rate at which sound energy arrives at the location of the listener, we'll take the efficiency to be the ratio of the sound power delivered to the listener divided by the power delivered to the string. We can calculate the power input directly from the given data. We'll calculate the intensity of the sound at 35 m from its intensity level at that distance and use this result to find the power output.

Express the efficiency of the conversion of mechanical energy to sound energy:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Find the power delivered by the bow to the string:

$$P_{\text{in}} = Fv = (0.6\,\text{N})(0.5\,\text{m/s}) = 0.3\,\text{W}$$

Using $\beta = (10\,\text{dB})\log(I/I_0)$, find the intensity of the sound at 35 m:

$$60\,\text{dB} = (10\,\text{dB})\log\frac{I_{35\,\text{m}}}{I_0}$$

and

$$I_{35\,\text{m}} = 10^6 I_0 = 10^{-6}\ \text{W/m}^2$$

Find the power of the sound emitted:

$$P_{\text{out}} = IA = 4\pi(10^{-6}\ \text{W/m}^2)(35\,\text{m})^2 = 0.0154\,\text{W}$$

Substitute numerical values and evaluate η:

$$\eta = \frac{0.0154\,\text{W}}{0.3\,\text{W}} = \boxed{5.13\%}$$

73 •••

Picture the Problem Because the sound intensities are additive, we'll find the intensity due to one student by subtracting the background intensity from the intensity due to the students and dividing by 100. Then, we'll use this result to calculate the intensity level due to 50 students.

Express the intensity level due to 50 students:

$$\beta_{50} = (10\,\text{dB})\log\frac{50I_1}{I_0}$$

Find the sound intensity when 100 students are writing the exam:

$$60\,\text{dB} = (10\,\text{dB})\log\frac{I_{100}}{I_0}$$

and

$$I_{100} = 10^6 I_0 = 10^{-6}\ \text{W/m}^2$$

Find the sound intensity due to the background noise:

$$40\,\text{dB} = (10\,\text{dB})\log\frac{I_{\text{background}}}{I_0}$$

and

$$I_{\text{background}} = 10^4 I_0 = 10^{-8}\ \text{W/m}^2$$

Express the sound intensity due to the 100 students:

$$I_{100} - I_{\text{background}} = 10^{-6}\ \text{W/m}^2 - 10^{-8}\ \text{W/m}^2 \approx 10^{-6}\ \text{W/m}^2$$

Find the sound intensity due to 1 student:

$$\frac{I_{100} - I_{\text{background}}}{100} = 10^{-8}\ \text{W/m}^2$$

Substitute numerical values and evaluate the intensity level due to 50 students:

$$\beta_{50} = (10\,\text{dB})\log\frac{50(10^{-8}\ \text{W/m}^2)}{10^{-12}\ \text{W/m}^2} = \boxed{57.0\,\text{dB}}$$

The Doppler Effect

74 •

Picture the Problem We can use equation 15-32 ($\lambda = \dfrac{v \pm u}{f_s}$) to find the wavelength of the sound between the source and the listener and 15-35*a* ($f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$) to find the frequency heard by the listener.

(*a*) Because the source is approaching the listener, use the minus sign in the numerator of Equation 15-32 to find the wavelength of the sound between the source and the listener:

$$\lambda = \frac{340\,\text{m/s} - 80\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.30\,\text{m}}$$

(*b*) Because the listener is at rest and the source is approaching, $u_r = 0$ and the denominator of Equation 15-35<u>*a*</u> is the difference between the two speeds:

$$f_r = \frac{v}{v - u_s} f_s = \frac{340\,\text{m/s}}{340\,\text{m/s} - 80\,\text{m/s}}\left(200\,\text{s}^{-1}\right) = \boxed{262\,\text{Hz}}$$

75 •

Picture the Problem In the reference frame described, the speed of sound from the source to the listener is reduced by the speed of the wind. We can find the wavelength of the sound in the region between the source and the listener from $v = f\lambda$. Because the sound waves in the region between the source and the listener will be compressed by the motion of the listener, the frequency of the sound heard by the listener will be higher than the frequency emitted by the source and can be calculated using $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$.

(*a*) The speed of sound in the reference frame of the source is:

$$v = 340\,\text{m/s} - 80\,\text{m/s} = \boxed{260\,\text{m/s}}$$

(*b*) Noting that the frequency is unchanged, express the wavelength of the sound:

$$\lambda = \frac{v}{f} = \frac{260\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.30\,\text{m}}$$

(*c*) Apply Equation 15-35*a* to obtain:

$$f_r = \left(\frac{v + u_r}{v}\right) f_s = \left(\frac{260\,\text{m/s} + 80\,\text{m/s}}{260\,\text{m/s}}\right)\left(200\,\text{s}^{-1}\right) = \boxed{262\,\text{Hz}}$$

76 •

Picture the Problem We can use $\lambda = (v \pm u)/f_s$ to find the wavelength of the sound in the region between the source and the listener and $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$ to find the frequency heard by the listener. Because the sound waves in the region between the source and the listener will be spread out by the motion of the listener, the frequency of the sound heard by the listener will be lower than the frequency emitted by the source.

(*a*) Because the source is moving away from the listener, use the positive sign in the numerator of Equation 15-32 to find the wavelength of the sound between the source and the listener:

$$\lambda = \frac{340\,\text{m/s} + 80\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{2.10\,\text{m}}$$

(*b*) Because the listener is at rest and the source is receding, $u_r = 0$ and the denominator of Equation 15-35*a* is the sum of the two speeds:

$$f_r = \frac{v}{v + u_s} f_s = \frac{340\,\text{m/s}}{340\,\text{m/s} + 80\,\text{m/s}}\left(200\,\text{s}^{-1}\right) = \boxed{162\,\text{Hz}}$$

77 •

Picture the Problem We can find the wavelength of the sound in the region between the source and the listener from $v = f\lambda$. Because the sound waves in the region between the source and the listener will be compressed by the motion of the listener, the frequency of the sound heard by the listener will be higher than the frequency emitted by the source and can be calculated using $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$.

(*a*) Because the wavelength is unaffected by the motion of the observer:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.70\,\text{m}}$$

(*b*) Apply Equation 15-35*a* to obtain:

$$f_r = \left(\frac{v + u_r}{v}\right) f_0 = \left(\frac{340\,\text{m/s} + 80\,\text{m/s}}{340\,\text{m/s}}\right)\left(200\,\text{s}^{-1}\right) = \boxed{247\,\text{Hz}}$$

78 •

Picture the Problem In this reference frame, the speed of sound will be increased by the speed of the listener. We can find the wavelength of the sound in the region between the source and the listener from $v = f\lambda$. Because the sound waves in the region between the source and the listener will be compressed by the motion of the listener, the frequency of the sound heard by the listener will be higher than the frequency emitted by the source and can be calculated using $v' = f_r\lambda'$.

(*a*) Moving at 80 m/s in still air: The observer experiences a wind velocity of 80 m/s.

(*b*) Use the standard Galilean transformation to obtain:

$$v' = v + u_r = 340\text{ m/s} + 80\text{ m/s} = \boxed{420\text{ m/s}}$$

(*c*) Because the distance between the wave crests is unchanged:

$$\lambda' = \frac{v'}{f} = \frac{340\text{ m/s}}{200\text{ s}^{-1}} = \boxed{1.70\text{ m}}$$

(*d*) Using the speed of sound in this reference frame, express and evaluate the frequency heard by the listener:

$$f_r = \frac{v'}{\lambda'} = \frac{420\text{ m/s}}{1.70\text{ m}} = \boxed{247\text{ Hz}}$$

79 •

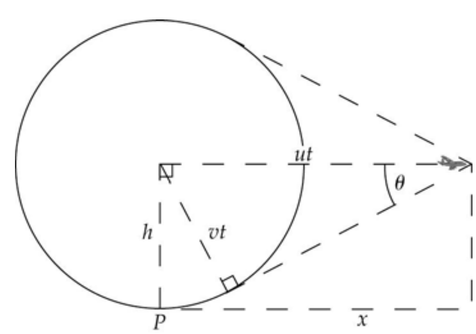
Picture the Problem Because the listener is moving away from the source, we know that the frequency he/she will hear will be less than the frequency emitted by the source. We can use $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$, with $u_s = 0$ and the minus sign in the numerator, to determine its value.

Relate the frequency heard by the listener to that of the source:

$$f_r = \left(\frac{v - u_r}{v}\right) f_s = \left(\frac{340\text{ m/s} - 80\text{ m/s}}{340\text{ m/s}}\right)\left(200\text{ s}^{-1}\right) = \boxed{153\text{ Hz}}$$

80 •

Picture the Problem The diagram shows the position of the supersonic plane at time *t* after it was directly over a person located at point *P* 5000 m below it. Let *u* represent the speed of the plane and *v* the speed of sound. We can use trigonometry to determine the angle of the shock wave as well as the location of the jet *x* when the person on the ground hears the shock wave.



(*a*) Referring to the diagram, express θ in terms of *v, u,* and *t*:

$$\theta = \sin^{-1}\left(\frac{vt}{ut}\right) = \sin^{-1}\left(\frac{1}{u/v}\right) = \sin^{-1}\left(\frac{1}{2.5}\right) = \boxed{23.6°}$$

(*b*) Using the diagram, relate the angle θ to the altitude *h* of the plane and the distance *x* and solve for *x*:

$$\tan\theta = \frac{h}{x} \Rightarrow x = \frac{h}{\tan\theta}$$

Substitute numerical values and evaluate *x*:

$$x = \frac{5000\,\text{m}}{\tan 23.6°} = \boxed{11.4\,\text{km}}$$

81 •

Picture the Problem If both u_s and u_r are much smaller than the speed of sound *v,* then the shift in frequency is given approximately by $\frac{\Delta f}{f_r} = \pm\frac{u}{v}$, where $u = u_s \pm u_r$ is the relative speed of the source and receiver. For purposes of this problem, we'll assume that you are an Olympics qualifier and can run at a top speed of approximately 10 m/s.

Express the frequency, f_r, you hear in terms of the frequency of the source f_s and your running speed *u*:

$$f_r = \left(\frac{v+u}{v}\right)f_s$$

Assuming that you can run 10 m/s , substitute numerical values and evaluate f_r:

$$f_r = \left(\frac{340\,\text{m/s} + 10\,\text{m/s}}{340\,\text{m/s}}\right)(1000\,\text{Hz}) = \boxed{1029\,\text{Hz}}$$

Using the positive sign (you are approaching the source), express and evaluate the ratio $\frac{\Delta f}{f_r}$:

$$\frac{\Delta f}{f_r} = \frac{10\,\text{m/s}}{340\,\text{m/s}} = 2.94\%$$

Because this fractional change in frequency is less than the 3% criterion for recognition of a change in frequency, it would be *impossible* to use your sense of pitch to estimate your running speed.

82 ••

Picture the Problem Because the car is moving away from the radar device, the frequency f_r it receives will be less than the frequency emitted by the device. The microwaves reflected from the car, moving away from a stationary detector, will be of a still lower frequency f_r'. We can use the Doppler shift equations to derive an expression for the speed of the car in terms of difference of these frequencies.

Express the frequency f_r received by the moving car in terms of f_s, *u,* and *v*:

$$f_r = \left(\frac{c-u}{c}\right)f_s \qquad (1)$$

Express the frequency f_r' received by the stationary source in terms of f_r, *u,* and *v*:

$$f_r' = \left(\frac{c-u}{c}\right)f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(\frac{c-u}{c}\right)^2 f_s \approx \left(1-\frac{2u}{c}\right)f_s$$

provided $u << c$

Express the frequency difference detected at the source:

$$\Delta f = f_s - f_r' = f_s - \left(1-\frac{2u}{c}\right)f_s$$

$$= \frac{2u}{c}f_s$$

Solve for *u*:

$$u = \frac{c}{2f_s}\Delta f$$

Substitute numerical values and evaluate *u*:

$$u = \frac{3\times10^8\ \text{m/s}}{2(2\ \text{GHz})}(293\ \text{Hz})$$

$$= 22.0\frac{\text{m}}{\text{s}}\times\frac{1\ \text{km}}{10^3\ \text{m}}\times\frac{3600\ \text{s}}{\text{h}}$$

$$= \boxed{79.1\ \text{km/h}}$$

***83 ••**

Picture the Problem Because the radial component *u* of the velocity of the raindrops is small compared to the speed $v = c$ of the radar pulse, we can approximate the fractional change in the frequency of the reflected radar pulse to find the speed of the winds carrying the raindrops in the storm system.

Express the shift in frequency when the speed of the source (the storm system) *u* is much smaller than the wave speed $v = c$:

$$\frac{\Delta f}{f_s} \approx \frac{u}{c}$$

Solve for u:

$$u = c\frac{\Delta f}{f_s}$$

Substitute numerical values and evaluate u:

$$u = (2.998\times10^8\ \text{m/s})\frac{325\,\text{Hz}}{625\,\text{MHz}}$$
$$= 156\,\text{m/s}\times\frac{0.6215\,\text{mi/h}}{0.2778\,\text{m/s}}$$
$$= \boxed{349\,\text{mi/h}}$$

84 ••

Picture the Problem Let the depth of the submarine be represented by D and its vertical speed by u. The submarine acts as both a receiver and source. We can apply the definition of average speed to determine the depth of the submarine and use the Doppler shift equations to derive an expression for the vertical speed of the submarine in terms of the frequency difference.

(*a*) Using the definition of average speed, relate the depth of the submarine to the time delay between the transmitted and reflected pulses:

$$2D = v\Delta t \Rightarrow D = \tfrac{1}{2}v\Delta t$$

Substitute numerical values and evaluate D:

$$D = \tfrac{1}{2}(1.54\,\text{km/s})(80\,\text{ms}) = \boxed{61.6\,\text{m}}$$

(*b*) Express the frequency f_r received by the submarine in terms of f_0, u, and c:

$$f_r = \left(\frac{c\pm u}{c}\right)f_s \qquad (1)$$

Express the frequency f_r' received by the destroyer in terms of f_r, u, and c:

$$f_r' = \left(\frac{c\pm u}{c}\right)f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(\frac{c\pm u}{c}\right)^2 f_s \approx \left(1\pm\frac{2u}{c}\right)f_s$$

provided $u << c$.

Express the frequency difference detected by the destroyer:

$$\Delta f = f_s - f_r' = f_s - \left(1\pm\frac{2u}{c}\right)f_s$$
$$= \frac{2u}{c}f_s$$

Solve for u:

$$u = \frac{c}{2f_s}\Delta f$$

Substitute numerical values and evaluate u:

$$u = \frac{1.54\,\text{km/s}}{2(40\,\text{MHz})}(0.0420\,\text{MHz}) = \boxed{0.809\,\text{m/s}}$$

where the positive speed indicates that the velocity of the submarine is downward.

85 ••

Picture the Problem Because the car is moving away from the radar unit, the frequency f_r it receives will be less than the frequency emitted by the unit. The radar waves reflected from the car, moving away from a stationary detector, will be of a still lower frequency f_r'. Let u represent the relative speed of the police car and the receding car (140 km/h) and use the Doppler shift equations to derive an expression for the difference between f_s, the transmitted signal, and f_r'.

Express the frequency f_r received by the moving car in terms of f_s, u, and c:

$$f_r = \left(\frac{c-u}{c}\right)f_s \qquad (1)$$

Express the frequency f_r' received by the stationary source in terms of f_r, u, and c:

$$f'_r = \left(\frac{c-u}{c}\right)f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(1-\frac{u}{c}\right)^2 f_s \approx \left(1-\frac{2u}{c}\right)f_s$$

provided $u \ll c$

Express the frequency difference detected at the source:

$$\Delta f = f_s - f'_r = f_s - \left(1-\frac{2u}{c}\right)f_s = \frac{2u}{c}f_s$$

Substitute numerical values and evaluate Δf:

$$\Delta f = \left[\frac{2\left(140\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)}{3\times10^8\,\text{m/s}}\right](3\times10^{10}\,\text{Hz}) = \boxed{7.78\,\text{kHz}}$$

86 ••

Picture the Problem Because the car is moving away from the radar unit, the frequency f_r it receives will be less than the frequency emitted by the unit. The radar waves reflected from the car, moving away from a stationary detector, will be of a still lower frequency f_r'. Let u represent the relative speed of the police car and the receding car (80 km/h) and use the Doppler shift equations to derive an expression for the difference between f_s, the transmitted signal, and f_r'.

Express the frequency f_r received by the moving car in terms of f_s, u, and c:

$$f_r = \left(\frac{c-u}{c}\right) f_s \qquad (1)$$

Express the frequency f_r' received by the stationary source in terms of f_r, u, and c:

$$f_r' = \left(\frac{c-u}{c}\right) f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(1-\frac{u}{c}\right)^2 f_s \approx \left(1-\frac{2u}{c}\right) f_s$$

provided $u << c$.

Express the frequency difference detected at the source:

$$\Delta f = f_s - f_r' = f_s - \left(1-\frac{2u}{c}\right) fs_0$$

$$= \frac{2u}{c} f_s$$

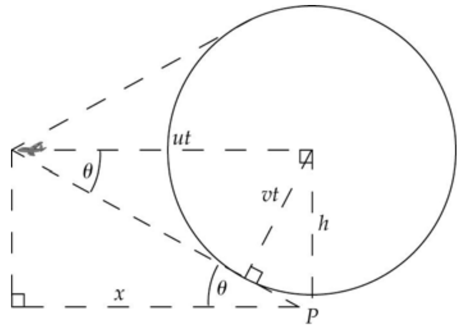
Substitute numerical values and evaluate Δf:

$$\Delta f = \left[\frac{2\left(80\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)}{3\times10^8\ \text{m/s}}\right]\left(3\times10^{10}\ \text{Hz}\right)$$

$$= \boxed{4.44\,\text{kHz}}$$

87 ••

Picture the Problem The diagram shows the position of the supersonic plane flying due west at time *t* after it was directly over point *P* 12 km below it. Let *x* be measured from point *P*, *u* represent the speed of the plane, and *v* be the speed of sound. We can use trigonometry to determine the angle of the shock wave as well as the location of the jet *x* when the person on the ground hears the shock wave.



Using the diagram to relate the distance x to the shock-wave angle θ and the elevation of the plane:

$$\tan\theta = \frac{h}{x} \text{ and } x = \frac{h}{\tan\theta}$$

Referring to the diagram, express θ in terms of *v, u,* and *t* and determine its value:

$$\theta = \sin^{-1}\left(\frac{vt}{ut}\right) = \sin^{-1}\left(\frac{1}{u/v}\right) = \sin^{-1}\left(\frac{1}{1.6}\right) = 38.7°$$

Substitute numerical values and evaluate *x*:

$$x = \frac{12\,\text{km}}{\tan 38.7°} = \boxed{15.0\,\text{km west of } P.}$$

88 ••

Picture the Problem The change in frequency of source as it oscillates on the air track is 3 Hz. We can use $\frac{\Delta f}{f_s} \approx \pm\frac{u}{v}$ to find the maximum speed of the vibrating mass-spring system in terms of this change in frequency and then use this speed to find the energy of the system. Knowing the energy of the system, we can find the amplitude of its motion. We can calculate the period of the motion from knowledge of the mass of the radio and the stiffness constant of the spring.

(*a*) Express the energy of the vibrating mass-spring system in terms of its maximum speed:

$$E = \tfrac{1}{2} m u_{\text{max}}^2 \qquad (1)$$

Relate the change in the frequency heard by the listener to the maximum speed of the oscillator:

$$\frac{\Delta f}{f_s} = \pm\frac{u}{v}$$

where $u = u_s \pm u_r$ is the relative speed of the source and receiver and v is the speed of sound.

Solve for and evaluate $u = u_{max}$:

$$u_{max} = \frac{\Delta f}{f_0} v = \frac{3\,\text{Hz}}{800\,\text{Hz}}(340\,\text{m/s}) = 1.275\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate E:

$$E = \tfrac{1}{2}(0.1\,\text{kg})(1.275\,\text{m/s}^2)^2 = \boxed{81.3\,\text{mJ}}$$

(b) Relate the energy of the oscillator to the amplitude of its motion:

$$E = \tfrac{1}{2}kA^2$$

Solve for A to obtain:

$$A = \sqrt{\frac{2E}{k}}$$

Substitute numerical values and evaluate A:

$$A = \sqrt{\frac{2(81.3\,\text{mJ})}{200\,\text{N/m}}} = \boxed{2.85\,\text{cm}}$$

89 ••

Picture the Problem The received and transmitted frequencies are related through $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$, where the variables have the meanings given in the problem statement. Because the source and receiver are moving in the same direction, we use the minus signs in both the numerator and denominator.

(a) Relate the received frequency f_r to the frequency f_0 of the source:

$$f_r = \frac{1 - u_r/v}{1 - u_s/v} f_0 = \boxed{(1 - u_r/v)(1 - u_s/v)^{-1} f_0}$$

(b) Apply the binomial expansion to $(1 - u_s/v)^{-1}$:

$$(1 - u_s/v)^{-1} \approx 1 + u_s/v$$

Substitute to obtain:

$$f_r = (1 - u_r/v)(1 + u_s/v) f_0 = [1 + u_s/v - u_r/v - (u_r/v)(u_s/v)] f_0 \approx \left(1 + \frac{u_s - u_r}{v}\right)$$

because both u_s and u_r are small compared to v.

Because $u_{rel} = u_s - u_r$

$$f_r' \approx \boxed{\left(1+\frac{u_{rel}}{v}\right)f_0}$$

90 ••

Picture the Problem Because the students are walking away from each other, the frequency f' each receives will be less than the frequency f_s = 440 Hz emitted by their tuning forks. Let u represent the speed of each student and use the Doppler shift equations to derive an expression for the difference between f_s, the frequency of the tuning fork each carries, and the frequency heard from the other's tuning fork. Because this equation will contain u, we'll be able to solve for and evaluate each student's walking speed.

Using equation 15-35*a*, express the frequency f_r received by either student, when they are walking away from each other, in terms of f_s, u, and v:

$$f_r = \frac{1-u/v}{1+u/v}f_s = (1-u/v)(1+u/v)^{-1}f_s$$

Expand $(1+u/v)^{-1}$ binomially:

$$(1+u/v)^{-1} \approx 1-u/v \text{ provided } u << v.$$

Substitute and simplify to obtain:

$$f_r = (1-u/v)^2 f_s = \left(1-2u/v+u^2/v^2\right)f_s \approx (1-2u/v)f_s$$

for $u << v$.

Express the frequency difference heard by each student:

$$\Delta f = f_0 - f_r = f_s - \left(1-\frac{2u}{v}\right)f_s = \frac{2u}{v}f_s$$

Solve for u:

$$u = \frac{\Delta f}{2f_s}v$$

Substitute numerical values and evaluate u:

$$u = \frac{2\text{ Hz}}{2(440\text{ Hz})}(340\text{ m/s}) = \boxed{0.773\text{ m/s}}$$

91 ••

Picture the Problem The student serves as a source moving toward the wall, and a moving receiver for the echo. The received and transmitted frequencies are related through $f_r = \frac{v \pm u_r}{v \pm u_s}f_s$. We'll express the frequency received by the wall and the frequency it transmits back to the moving student in order to express the difference in the frequency the student hears from the wall and frequency she hears directly from her tuning fork. Because this expression will contain the student's walking speed, we'll be able to solve for and evaluate this speed.

Relate the frequency f_r received by the wall to the frequency of the student's tuning fork f_s:

$$\Delta f = f_s - f_r' = f_s - \left(1 - \frac{2u}{c}\right) f_s = \frac{2u}{c} f_s$$

Because the source is moving toward a stationary receiver:

$$f_r = \frac{1}{1 - u_s/v} f_s = (1 - u_s/v)^{-1} f_s$$

Apply the binomial expansion to $(1 - u_s/v)^{-1}$:

$$(1 - u_s/v)^{-1} \approx 1 + u_s/v$$

because $u_s << v$.

Substitute to obtain:

$$f_r = (1 + u_s/v) f_0$$

Relate the frequency f_r' reflected from the wall and received by the student to the frequency f_r reflected from the wall:

$$f_r' = \frac{1 \pm u_r/v}{1 \pm u_s/v} f_r = \frac{1}{1 - u_s/v} f_r$$

because the source (the wall) is at rest and the receiver is approaching.

Substitute for f_r to obtain:

$$f_r' = \frac{1}{1 - u_s/v}(1 + u_s/v) f_s = (1 + u_s/v)(1 - u_s/v)^{-1} f_s$$

Expand $(1 - u_s/v)^{-1}$ binomially:

$$(1 - u_s/v)^{-1} \approx 1 + u_s/v$$

because $u_s << v$.

$$f_r' = (1 + u_s/v)(1 + u_s/v) f_s \approx (1 + 2u_s/v) f_s$$

because $u_s << v$

Express the difference between the frequency the student receives from the wall and the frequency of her tuning fork:

$$\Delta f = f_r' - f_s = (1 + 2u_s/v) f_s - f_s = \frac{2u_s}{v} f_s$$

Solve for u_s:

$$u_s = \frac{\Delta f}{2f_s}v$$

Substitute numerical values and evaluate u_s:

$$u_s = \frac{4\,\text{Hz}}{2(512\,\text{Hz})}(340\,\text{m/s}) = \boxed{1.33\,\text{m/s}}$$

***92 ••**

Picture the Problem The frequency heard by the stationary observer will vary with time as the speaker rotates on its support arm. We can use a Doppler equation to express the frequency heard by the observer as a function of the velocity of the source and find the velocity of the source from the expression for the tangential velocity of an object moving in a circular path.

Express the frequency f_r heard by a stationary observer:

$$f_r = \frac{1}{1-u_s/v}f_s = (1-u_s/v)^{-1}f_s$$

Expand $(1-u_s/v)^{-1}$ to obtain:

$$(1-u_s/v)^{-1} \approx 1+u_s/v$$

because $u_s/v << 1$

Substitute in the expression for f_r:

$$f_r = (1+u_s/v)f_s \qquad (1)$$

Express the speed of the source as a function of time:

$$\begin{aligned} u_s &= r\omega\sin\omega t \\ &= (0.8\,\text{m})(4\,\text{rad/s})\sin[(4\,\text{rad/s})t] \\ &= (3.2\,\text{m/s})\sin[(4\,\text{rad/s})t] \end{aligned}$$

Substitute in equation (1) to obtain:

$$f_r = \left(1+\frac{3.2\,\text{m/s}}{v}\sin[(4\,\text{rad/s})t]\right)f_s$$

Substitute for v and simplify:

$$\begin{aligned} f_r &= \left(1+\frac{3.2\,\text{m/s}}{340\,\text{m/s}}\sin[(4\,\text{rad/s})t]\right)(1000\,\text{s}^{-1}) \\ &= \boxed{1000\,\text{Hz}+(9.41\,\text{Hz})\sin[(4\,\text{rad/s})t]} \end{aligned}$$

93 ••

Picture the Problem The simplest way to approach this problem is to transform to a reference frame in which the balloon is at rest. In that reference frame, the speed of sound is v = 340 m/s, and u_r = 36 km/h = 10 m/s. Then, we can use the equations for a moving receiver and a moving source to find the frequencies heard at the window and on the balloon.

(*a*) Express the observed frequency in terms of the frequency of the source:

$$f_r = \left(1 + \frac{u_r}{v}\right) f_s$$

Substitute numerical values and evaluate f_r:

$$f_r = \left(1 + \frac{10\,\text{m/s}}{340\,\text{m/s}}\right)(800\,\text{Hz}) = \boxed{824\,\text{Hz}}$$

(*b*) Treating the tall building as a moving source, express the frequency of the reflected sound heard by a person riding in the balloon:

$$f_r' = \left(\frac{1}{1 - \frac{u_s}{v}}\right) f_r$$

Substitute numerical values and evaluate f_r':

$$f_r' = \left(\frac{1}{1 - \frac{10\,\text{m/s}}{340\,\text{m/s}}}\right)(824\,\text{Hz}) = \boxed{849\,\text{Hz}}$$

94 ••

Picture the Problem We can relate the frequencies f_r and f_r' heard by the stationary observer behind the car to the speed of the car *u* and the frequency of the car's horn f_s. Dividing these equations will eliminate the frequency of the car's horn and allow us to solve for the speed of the car. We can then substitute to find the frequency of the car's horn. We can find the frequency heard by the driver as a moving receiver by relating this frequency to the frequency reflected from the wall.

(*a*) Relate the frequency heard by the observer directly from the car's horn to the speed of the car:

$$f_r = \frac{1}{1 + u/v} f_s \qquad (1)$$

Relate the frequency reflected from the wall to the speed of the car:

$$f_r' = \frac{1}{1 - u/v} f_s \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{f_r'}{f_r} = \frac{1 + u/v}{1 - u/v}$$

Solve for *u*:

$$u = \frac{f_r' - f_r}{f_r' + f_r'} v$$

Substitute numerical values and evaluate u:

$$u = \frac{863\,\text{Hz} - 745\,\text{Hz}}{863\,\text{Hz} + 745\,\text{Hz}}(340\,\text{m/s})$$
$$= 24.95\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{10^3\,\text{m}} \times \frac{3600\,\text{s}}{\text{h}}$$
$$= \boxed{89.8\,\text{km/h}}$$

(*b*) Solve equation (1) for f_s:

$$f_s = (1 + u/v)f_r$$

Substitute numerical values and evaluate f_s:

$$f_s = \left(1 + \frac{24.95\,\text{m/s}}{340\,\text{m/s}}\right)(745\,\text{Hz})$$
$$= \boxed{800\,\text{Hz}}$$

(*c*) The driver is a moving receiver and so we can relate the frequency heard by the driver to the frequency reflected by the wall (the frequency heard by the stationary observer):

$$f_{\text{driver}} = \left(1 + \frac{u}{v}\right)f_r'$$

Substitute numerical values and evaluate f_{driver}:

$$f_{\text{driver}} = \left(1 + \frac{24.95\,\text{m/s}}{340\,\text{m/s}}\right)(863\,\text{Hz})$$
$$= \boxed{926\,\text{Hz}}$$

95 ••

Picture the Problem Let $t = 0$ when the driver sounds her horn and let the distance to the cliff at that instant be d. The received and transmitted frequencies are related through $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$. Solving this equation for f_s will allow us to determine the frequency of the car horn. We can use the total distance the sound travels (car-to-cliff plus cliff back to car … now closer to the cliff) to determine the distance to the cliff when the horn was briefly sounded.

Relate the frequency heard by the driver to her speed and to the frequency of her horn:

$$f_r = \frac{1 + u_r/v}{1 - u_s/v} f_s$$

Solve for f_s:

$$f_s = \frac{1 - u_s/v}{1 + u_r/v} f_r$$

Substitute numerical values and evaluate f_s:

$$f_0 = \frac{1 - \dfrac{100\dfrac{\text{km}}{\text{h}} \times \dfrac{1\,\text{h}}{3600\,\text{s}}}{340\,\text{m/s}}}{1 + \dfrac{100\dfrac{\text{km}}{\text{h}} \times \dfrac{1\,\text{h}}{3600\,\text{s}}}{340\,\text{m/s}}}(840\,\text{Hz}) = \frac{1 - \dfrac{27.78\,\text{m/s}}{340\,\text{m/s}}}{1 + \dfrac{27.78\,\text{m/s}}{340\,\text{m/s}}}(840\,\text{Hz}) = 713\,\text{Hz}$$

Relate the distance d to the cliff at $t = 0$ to the distance she travels in time $\Delta t = 1$ s, her speed u, and the speed of sound v:

$$d + (d - u\Delta t) = v\Delta t$$

Solve for d:

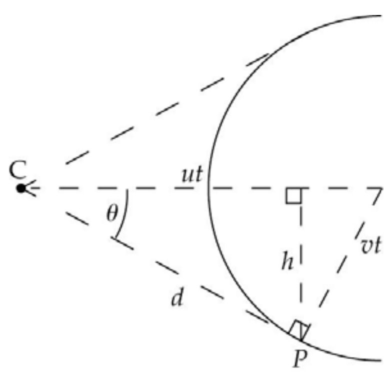
$$d = \tfrac{1}{2}(u + v)\Delta t$$

Substitute numerical values and evaluate d:

$$d = \tfrac{1}{2}(27.78\,\text{m/s} + 340\,\text{m/s})(1\,\text{s}) = \boxed{184\,\text{m}}$$

96 ••

Picture the Problem You'll year the sonic boom when the surface of its cone reaches your plane. In the diagram the Concorde is at C and your plane is at P. The distance $h = 3$ km. The distance between the planes when you hear the sonic boom is d. We can use trigonometry to determine the angle of the shock wave as well as the separation of the planes when you hear the sonic boom.



Using the Pythagorean theorem, relate the separation of the planes d, to the distance h and the angle θ:

$$d^2 = h^2 + d^2\cos^2\theta$$

Solve for d to obtain:

$$d = h\sqrt{\frac{1}{1 - \cos^2\theta}}$$

Express θ in terms of v, u, and t:

$$\theta = \sin^{-1}\left(\frac{vt}{ut}\right) = \sin^{-1}\left(\frac{1}{u/v}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{1}{1.6}\right) = \boxed{38.7°}$$

Substitute numerical values and evaluate *h*:

$$d = (3\,\text{km})\sqrt{\frac{1}{1-\cos^2 38.7°}} = \boxed{4.80\,\text{km}}$$

***97 ••**

Picture the Problem The sun and Jupiter orbit about their effective mass located at their common center of mass. We can apply Newton's 2nd law to the sun to obtain an expression for its orbital speed about the sun-Jupiter center of mass and then use this speed in the Doppler shift equation to estimate the maximum and minimum wavelengths resulting from the Jupiter-induced motion of the sun.

Letting *v* be the orbital speed of the sun about the center of mass of the sun-Jupiter system, express the Doppler shift of the light due to this motion when the sun is approaching the earth:

$$f' = \frac{c}{\lambda'} = f\sqrt{\frac{1+v/c}{1-v/c}} = \frac{c}{\lambda}\sqrt{\frac{1+v/c}{1-v/c}}$$

Solve for λ':

$$\lambda' = \lambda\sqrt{\frac{1-v/c}{1+v/c}}$$
$$= \lambda\sqrt{(1-v/c)(1+v/c)^{-1}}$$
$$= \lambda(1-v/c)^{1/2}(1+v/c)^{-1/2}$$

Because $v << c$, we can expand $(1-v/c)^{1/2}$ and $(1+v/c)^{-1/2}$ binomially to obtain:

$$(1-v/c)^{1/2} \approx 1-\frac{v}{2c}$$

and

$$(1+v/c)^{-1/2} \approx 1-\frac{v}{2c}$$

Substitute to obtain:

$$\sqrt{\frac{1-v/c}{1+v/c}} = \left(1-\frac{v}{2c}\right)^2 \approx 1-\frac{v}{c}$$

When the sun is receding from the earth:

$$\sqrt{\frac{1+v/c}{1-v/c}} = \left(1+\frac{v}{2c}\right)^2 \approx 1+\frac{v}{c}$$

Hence the motion of the sun will give an observed Doppler shift of:

$$\lambda' \approx \lambda\left(1 \pm \frac{v}{c}\right) \qquad (1)$$

Apply Newton's 2nd law to the sun:

$$\frac{GM_S M_{\text{eff}}}{r_{\text{cm}}^2} = M_S \frac{v^2}{r_{\text{cm}}}$$

Solve for v to obtain:

$$v = \sqrt{\frac{GM_{\text{eff}}}{r_{\text{cm}}}}$$

Measured from the center of the sun, the distance to the center of mass of the sun-Jupiter system is:

$$r_{\text{cm}} = \frac{(0)M_S + r_{\text{s-J}} M_J}{M_s + M_J} = \frac{r_{\text{s-J}} M_J}{M_s + M_J}$$

The effective mass is related to the masses of the sun and Jupiter according to:

$$\frac{1}{M_{\text{eff}}} = \frac{1}{M_s} + \frac{1}{M_J}$$

or

$$M_{\text{eff}} = \frac{M_s M_J}{M_s + M_J}$$

Substitute for M_{eff} and r_{cm} to obtain:

$$v = \sqrt{\frac{G\dfrac{M_s M_J}{M_s + M_J}}{\dfrac{r_{\text{s-J}} M_J}{M_s + M_J}}} = \sqrt{\frac{GM_s}{r_{\text{s-J}}}}$$

Using $r_{\text{s-J}} = 7.78\times10^{11}$ m as the mean orbital radius of Jupiter, substitute numerical values and evaluate v:

$$v = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{7.78\times10^{11}\ \text{m}}} = 1.306\times10^4\ \text{m/s}$$

Substitute in equation (1) to obtain:

$$\lambda' \approx (500\ \text{nm})\left(1 \pm \frac{1.306\times10^4\ \text{m/s}}{2.998\times10^8\ \text{m/s}}\right) = (500\ \text{nm})(1 \pm 4.36\times10^{-5})$$

The maximum and minimum wavelengths are:

$$\lambda_{\max} = \boxed{(500\ \text{nm})(1 + 4.36\times10^{-5})}$$

and

$$\lambda_{\min} = \boxed{(500\ \text{nm})(1 - 4.36\times10^{-5})}$$

98 •••

Picture the Problem Choose a coordinate system in which downward is the positive y direction. Let d represent the distance the tuning fork has fallen when the student hears a frequency of 400 Hz, t_1 the time for the source to fall that distance, and t_2 the time for the sound to travel back to the student. We can a constant-acceleration equation to express d in terms of the time that elapses between the dropping of the tuning fork and the return of the sound to the student. A Doppler-effect equation will allow us to solve for the speed of the tuning fork when the student hears a frequency of 400 Hz; we can use constant-acceleration equations to find the fall time for the fork and the return time for the sound from the tuning fork.

Using a constant-acceleration equation, relate the distance the source has fallen to the elapsed time:

$$d = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$ and $a = g$,

$$d = \tfrac{1}{2}gt^2 \qquad (1)$$

where $t = t_1 + t_2$.

Relate the frequency f_r heard by the student to the speed of the falling tuning fork:

$$f_r = \frac{1}{1 + u_s/v} f_s$$

Solve for u_s:

$$u_s = \left(\frac{f_s}{f_r} - 1\right)v$$

Substitute numerical values and evaluate u_s:

$$u_s = \left(\frac{440\ \text{Hz}}{400\ \text{Hz}} - 1\right)(340\ \text{m/s}) = 34.0\ \text{m/s}$$

Letting y be the distance the fork has fallen when its speed is u_s, use a constant-acceleration equation to relate y and u_s:

$$u_s^2 = v_0^2 + 2gy$$

or, because $v_0 = 0$,

$$u_s^2 = 2gy$$

Solve for y:

$$y = \frac{u_s^2}{2g}$$

Substitute numerical values and evaluate y:

$$y = \frac{(34\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 58.92\ \text{m}$$

Using a constant-acceleration equation, relate the speed of the falling tuning fork to its time of fall t_1:

$$u_s = v_0 + gt_1$$

or, because $v_0 = 0$,

$$u_s = gt_1$$

Solve for and evaluate t_1:

$$t_1 = \frac{u_s}{g} = \frac{34.0\ \text{m/s}}{9.81\ \text{m/s}^2} = 3.466\ \text{s}$$

Using the relationship between distance traveled, time and average speed, find the time t_2 for the sound to travel back to the student:

$$t_2 = \frac{y}{v} = \frac{58.92\,\text{m}}{340\,\text{m/s}} = 0.173\,\text{s}$$

Substitute in equation (1) and evaluate d:

$$d = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(3.466\,\text{s} + 0.173\,\text{s}\right)^2 = \boxed{65.0\,\text{m}}$$

99 ••

Picture the Problem The angle θ of the Cerenkov shock wave is related to the speed of light in water v and the speed of light in a vacuum c according to $\sin\theta = v/c$.

Relate the speed of light in water v to the angle of the Cerenkov cone:

$$\sin\theta = \frac{v}{c}$$

Solve for v:

$$v = c\sin\theta$$

Substitute numerical values and evaluate v:

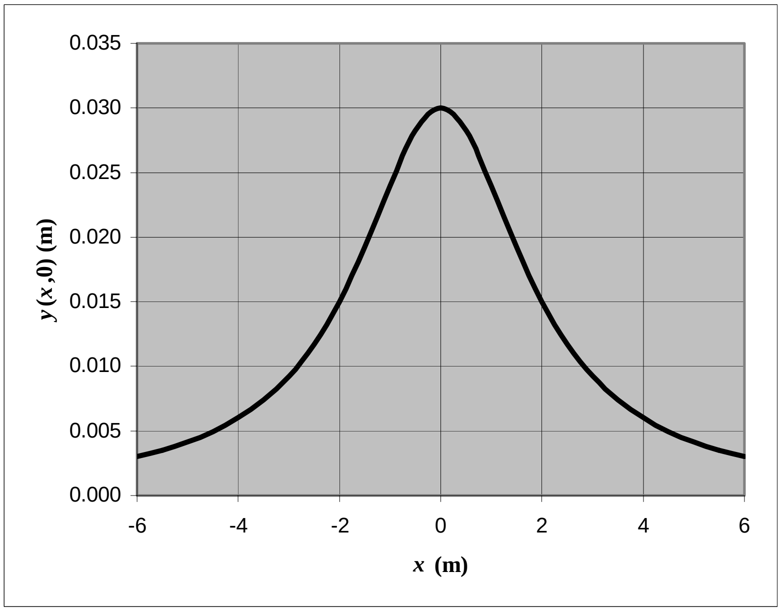
$$v = \left(2.998\times10^8\ \text{m/s}\right)\sin 48.75° = \boxed{2.25\times10^8\ \text{m/s}}$$

General Problems

100 •

Picture the Problem The equation of a wave traveling in the positive x direction is of the form $y(x,t) = f(x - vt)$ and that of a wave traveling in the negative x direction is $y(x,t) = f(x + vt)$.

(a) The pulse at $t = 0$ shown below was plotted using a spreadsheet program:



(b) The wave function must be of the form:

$$y(x,t) = f(x - vt) = f[x - (10\,\text{m/s})t]$$

because $v = 10$ m/s

Replace x with $x - (10\,\text{m/s})t$ to obtain:

$$y(x,t) = \boxed{\frac{0.12\,\text{m}^3}{(2.00\,\text{m})^2 + [x - (10\,\text{m/s})t]^2}}$$

(c) The wave function must be of the form:

$$y(x,t) = f(x + vt) = f[x + (10\,\text{m/s})t]$$

because $v = 10$ m/s

Replace x with $x + (10\,\text{m/s})t$ to obtain:

$$y(x,t) = \boxed{\frac{0.12\,\text{m}^3}{(2.00\,\text{m})^2 + [x + (10\,\text{m/s})t]^2}}$$

101 •

Picture the Problem Let the subscript 1 refer to the initial situation–a tension of 800 N and a wavelength of 24 cm. Let the subscript 2 refer to the conditions that the tension is 600 N and the wavelength unknown. We can express the wavelengths of the waves on the wire in terms of the two tensions in the wire and then eliminate the constant frequency by expressing the ratio of the two wavelengths. Finally, we can solve this equation for λ_2.

Express the wavelength in terms of the frequency and speed of the

$$\lambda = \frac{v}{f}$$

wave:

Express the speed of the wave as a function of the tension in the wire:

$$v=\sqrt{\frac{T}{\mu}}$$

Substitute to obtain:

$$\lambda=\frac{1}{f}\sqrt{\frac{T}{\mu}}$$

Express the wavelength when the tension in the wire is 600 N:

$$\lambda_2=\frac{1}{f}\sqrt{\frac{T_2}{\mu}}$$

Express the wavelength when the tension in the wire is 800 N:

$$\lambda_1=\frac{1}{f}\sqrt{\frac{T_1}{\mu}}$$

Divide the first of these equations by the second and solve for λ_2:

$$\lambda_2=\lambda_1\sqrt{\frac{T_2}{T_1}}$$

Substitute numerical values and evaluate λ_2:

$$\lambda_2=(24\,\text{cm})\sqrt{\frac{600\,\text{N}}{800\,\text{N}}}=\boxed{20.8\,\text{cm}}$$

102 •

Picture the Problem Let *m* represent the mass of the rubber tubing whose length is *L*. We can express the travel time for the pulse *t* in terms of the separation of the post and the pulley and its speed. The speed of the pulse, in turn, can be found from the tension in the rubber tubing and its linear density.

Express the time required for the pulse to travel the length of the tubing in terms of its speed and the length of the rubber tubing:

$$t=\frac{L}{v}$$

Relate the speed of the pulses to the tension in the tubing:

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{F}{m/L}}=\sqrt{\frac{FL}{m}}$$

Substitute for v and simplify to obtain:

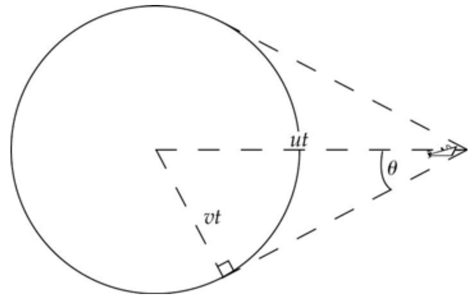
$$t=\sqrt{\frac{Lm}{F}}$$

Substitute numerical values and evaluate t:

$$t=\sqrt{\frac{(10\,\text{m})(0.7\,\text{kg})}{110\,\text{N}}}=\boxed{0.252\,\text{s}}$$

103 •

Picture the Problem The diagram shows the boat traveling on a still lake with a speed v. A bow wave generated a time t earlier is shown at an angle of θ with the direction of the boat's motion. We can use trigonometry to relate the speed of the bow wave to the speed of the boat.



Using the diagram, relate u and v to the angle θ:

$$\sin\theta = \frac{vt}{ut} = \frac{v}{u}$$

Solve for v:

$$v = u\sin\theta$$

Substitute numerical values and evaluate v:

$$v = (10\,\text{m/s})\sin 20° = \boxed{3.42\,\text{m/s}}$$

104 •

Picture the Problem The frequencies and wavelengths of the sound waves are related to the speed of sound through $f = v/\lambda$.

(*a*) Use $f = v/\lambda$ to find f:

$$f = \frac{340\,\text{m/s}}{10(0.3\,\text{m})} = \boxed{113\,\text{Hz}}$$

(*b*) Proceed as in (*a*):

$$f = \frac{340\,\text{m/s}}{0.1(0.3\,\text{m})} = \boxed{11.3\,\text{kHz}}$$

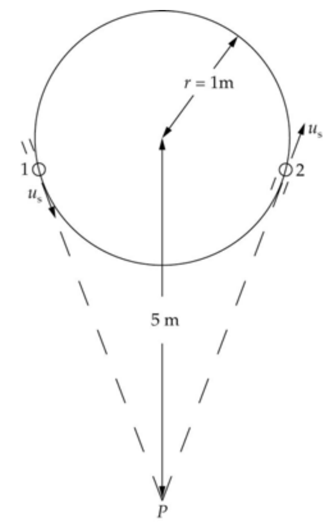
(*c*) Proceed as in (*a*) and (*b*):

$$f = \frac{340\,\text{m/s}}{10(0.06\,\text{m})} = \boxed{567\,\text{Hz}}$$

$$f = \frac{340\,\text{m/s}}{0.1(0.06\,\text{m})} = \boxed{56.7\,\text{kHz}}$$

105 •

Picture the Problem The diagram depicts the whistle traveling in a circular path of radius $r = 1$ m. The stationary listener will hear the maximum frequency when the whistle is at point 1 and the minimum frequency when it is at point 2. These maximum and minimum frequencies are determined by f_0 and the tangential speed $u_s = 2\pi r/T$. We can relate the frequencies heard at point P to the speed of the approaching whistle at point 1 and the speed of the receding whistle at point 2.



Relate the frequency heard at point P to the speed of the approaching whistle at point 1:

$$f_{max} = \frac{1}{1 - u_s/v} f_s$$

Use the relationship between translational velocity and angular velocity to find the speed u_s of the whistle:

$$u_s = r\omega = (1\,\text{m})\left(3\,\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}\right) = 18.85\,\text{m/s}$$

Substitute numerical values and evaluate f_{max}:

$$f_{max} = \frac{1}{1 - \dfrac{18.85\,\text{m/s}}{340\,\text{m/s}}}(500\,\text{Hz}) = \boxed{529\,\text{Hz}}$$

Relate the frequency heard at point P to the speed of the receding whistle at point 2:

$$f_{min} = \frac{1}{1 + u_s/v} f_s$$

Substitute numerical values and evaluate f_{min}:

$$f_{min} = \frac{1}{1 + \dfrac{18.85\,\text{m/s}}{340\,\text{m/s}}}(500\,\text{Hz}) = \boxed{474\,\text{Hz}}$$

106 •

Picture the Problem The crest-to-crest separation of the waves is their wavelength. We can find the frequency of the waves from $v = f\lambda$. When you lift anchor and head out to sea you'll become a moving receiver and we can apply $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$ to calculate the frequency you'll observe.

(*a*) Express the frequency of the ocean waves in terms of their speed and wavelength:

$$f_0 = \frac{v}{\lambda}$$

Substitute numerical values and evaluate f_s:

$$f_0 = \frac{8.9\,\text{m/s}}{15\,\text{m}} = \boxed{0.593\,\text{Hz}}$$

(*b*) Express the frequency of the waves in terms of their speed and the speed of a moving receiver:

$$f_r = (1 + u_r/v) f_s$$

Substitute numerical values and evaluate f_r:

$$f_r = \left(1 + \frac{15\,\text{m/s}}{8.9\,\text{m/s}}\right)(0.593\,\text{Hz}) = \boxed{1.59\,\text{Hz}}$$

107 ••

Picture the Problem Let t be the time of travel of the lefthand pulse and the subscripts L and R refer to the pulse coming from the left and right, respectively. Because the pulse traveling from the right starts later than the pulse from the left, its travel time is $t - \Delta t$, where $\Delta t = 25$ ms. Both pulses travel at the same speed and the sum of the distances they travel is 12 m.

Express the total distance the two pulses travel:

$$d = d_L + d_R = vt + v(t - \Delta t)$$

Solve for vt to obtain:

$$vt = \tfrac{1}{2}(d + v\Delta t)$$

The speed of the pulse is given by:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}}$$

Substitute for v to obtain:

$$vt = \tfrac{1}{2}\left(d + \sqrt{\frac{F}{m/L}}\Delta t\right)$$

Substitute numerical values and evaluate vt:

$$vt = \tfrac{1}{2}\left[12\,\text{m} + \sqrt{\frac{(180\,\text{N})(12\,\text{m})}{0.085\,\text{kg}}}\left(25\times10^{-3}\,\text{s}\right)\right] = \boxed{7.99\,\text{m}}$$

***108 ••**

Picture the Problem Let the frequency of the car's horn be f_s, the frequency you hear as the car approaches f_r, and the frequency you hear as the car recedes f_r'. We can use $f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$ to express the frequencies heard as the car approaches and recedes and then use these frequencies to express the fractional change in frequency as the car passes you.

Express the fractional change in frequency as the car passes you:

$$\frac{\Delta f}{f_r} = 0.1$$

Relate the frequency heard as the car approaches to the speed of the car:

$$f_r = \frac{1}{1 - u_s/v} f_s$$

Express the frequency heard as the car recedes in terms of the speed of the car:

$$f_r' = \frac{1}{1 + u_s/v} f_s$$

Divide the second of these frequency equations by the first to obtain:

$$\frac{f_r'}{f_r} = \frac{1 - u_s/v}{1 + u_s/v}$$

and

$$\frac{f_r}{f_r} - \frac{f_r'}{f_r} = \frac{\Delta f}{f_r} = 1 - \frac{1 - u_s/v}{1 + u_s/v} = 0.1$$

Solve u_s:

$$u_s = \frac{0.1}{1.9} v$$

Substitute numerical values and evaluate u_s:

$$u_s = \frac{0.1}{1.9}(340\,\text{m/s})$$

$$= 17.89\,\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{10^3} \times \frac{3600\,\text{s}}{\text{h}}$$

$$= \boxed{64.4\,\text{km/h}}$$

109 ••

Picture the Problem The pressure amplitude can be calculated directly from $p_0 = \rho\omega v s_0$, and the intensity from $I = \frac{1}{2}\rho\omega^2 s_0^2 v$. The power radiated is the intensity times the area of the driver.

(*a*) Relate the pressure amplitude to the displacement amplitude, angular frequency, wave velocity, and air density:

$$p_0 = \rho\omega v s_0$$

Substitute numerical values and evaluate p_0:

$$p_0 = \left(1.29\,\text{kg/m}^3\right)\left[2\pi\left(800\,\text{s}^{-1}\right)\right] \times \left(340\,\text{m/s}\right)\left(0.025\times10^{-3}\,\text{m}\right) = \boxed{55.1\,\text{N/m}^2}$$

(*b*) Relate the intensity to these same quantities:

$$I = \tfrac{1}{2}\rho\omega^2 s_0^2 v$$

Substitute numerical values and evaluate I:

$$I = \tfrac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left[2\pi\left(800\,\text{s}^{-1}\right)\right]^2 \times \left(0.025\times10^{-3}\,\text{m}\right)^2\left(340\,\text{m/s}\right) = \boxed{3.46\,\text{W/m}^2}$$

(*c*) Express the power in terms of the intensity and the area of the driver:

$$P = IA = \pi r^2 I$$

Substitute numerical values and evaluate P:

$$P = \pi\left(0.1\,\text{m}\right)^2\left(3.46\,\text{W/m}^2\right) = \boxed{0.109\,\text{W}}$$

110 ••

Picture the Problem The frequency of the sound wave is related to the density of the air, displacement amplitude, and velocity by $I = \frac{1}{2}\rho\omega^2 s_0^2 v$.

Relate the intensity of the sound wave to the density of the air, displacement amplitude, velocity, and angular frequency:

$$I = \tfrac{1}{2}\rho\omega^2 s_0^2 v$$

Solve for the angular frequency:

$$\omega = \frac{1}{s_0}\sqrt{\frac{2I}{\rho v}}$$

Solve for *f*:

$$f = \frac{1}{2\pi s_0}\sqrt{\frac{2I}{\rho v}}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{1}{2\pi(10^{-6}\text{ m})}\sqrt{\frac{2(10^{-2}\text{ W/m}^2)}{(1.29\text{ kg/m}^3)(340\text{ m/s})}} = \boxed{1.07\text{ kHz}}$$

111 ••

Picture the Problem The force exerted on the plate is due to the change in momentum of the water. We can use Newton's 2nd law in the form $F = \Delta p/\Delta t$ to relate F to the mass of water in a length of tube equal to $v_s\Delta t$ and to the speed of the water. This mass of water, in turn, is given by the product of its density and the volume of water in a length of the tube equal to $v_s\Delta t$.

Relate the force exerted on the plate to the change in momentum of the water:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta m v_w}{\Delta t}$$

Express Δm in terms of the mass of water in a length of tube equal to $v_s\Delta t$:

$$\Delta m = \rho\Delta V = \rho v_s A\Delta t$$

Substitute to obtain:

$$F = \rho v_s A v_w$$

Substitute numerical values and evaluate F:

$$F = (10^3\text{ kg/m}^3)(1.4\text{ km/s})\left[\pi(0.05\text{ m})^2\right](7\text{ m/s}) = \boxed{77.0\text{ kN}}$$

112 ••

Picture the Problem Let d be the horizontal distance from the soap bubble to the position of the microphone. The angle θ of the shock wave is related to the speed of sound in air u and the speed of the bullet c according to $\sin\theta = u/v$. We can determine θ from the given information and then use this angle to find d.

Express d in terms of the angle of the shock wave and the distance from the soap bubble to the laboratory bench:

$$d = \frac{0.35\text{ m}}{\tan\theta}$$

Relate the speed of the bullet to the angle of the shock-wave cone:

$$\sin\theta = \frac{u}{v}$$

Solve for θ to obtain:

$$\theta = \sin^{-1}\frac{u}{v}$$

Substitute to obtain:

$$d = \frac{0.35\,\text{m}}{\tan\left[\sin^{-1}\frac{u}{v}\right]}$$

Substitute numerical values and evaluate d:

$$d = \frac{0.35\,\text{m}}{\tan\left[\sin^{-1}\left(\frac{u}{1.25u}\right)\right]} = \boxed{26.3\,\text{cm}}$$

113 ••

Picture the Problem The source of the problem is that it takes a finite time for the sound to travel from the front of the line of marchers to the back. We can use the given data to determine the time required for the beat to reach the marchers in the back of the column and then use this time and the speed of sound to find the length of the column.

Express the length of the column in terms of the speed of sound and the time required for the beat to travel the length of the column:

$$L = v\Delta t$$

Calculate the time for the sound to travel the length of the column:

$$\Delta t = \frac{1}{100}\,\text{min} = 0.6\,\text{s}$$

Substitute and evaluate L:

$$L = (340\,\text{m/s})(0.6\,\text{s}) = \boxed{204\,\text{m}}$$

114 ••

Picture the Problem The interval between the arrival times of the echo pulses heard by the bat is the reciprocal of the frequency of the reflected pulses. We can use $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$ to relate the frequency of the reflected pulses to the speed of the bat and the frequency it emits.

Relate the interval between the arrival times of the echo pulses heard by the bat to frequency of the reflected pulses:

$$\Delta t = \frac{1}{f_r}$$

Relate the frequency of the pulses received by the bat to its speed and the frequency it emits:

$$f_r = \frac{1 + u_r/v}{1 - u_s/v} f_s$$

Substitute to obtain:

$$\Delta t = \frac{1 - u_s/v}{(1 + u_r/v) f_s}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1 - \dfrac{12\,\text{m/s}}{340\,\text{m/s}}}{\left(1 + \dfrac{12\,\text{m/s}}{340\,\text{m/s}}\right)\left(80\,\text{s}^{-1}\right)} = \boxed{11.6\,\text{ms}}$$

***115** ••

Picture the Problem Let *d* be the distance to the moon, *h* be the height of earth's atmosphere, and *v* be the speed of light in earth's atmosphere. We can express *d* ′*,* the distance measured when the earth's atmosphere is ignored, in terms of the time for a pulse of light to make a round-trip from the earth to the moon and solve this equation for the length of correction *d* ′ − *d*.

Express the roundtrip time for a pulse of light to reach the moon and return:

$$t = t_{\text{earth's atmosphere}} + t_{\text{out of earth's atmosphere}} = 2\frac{h}{v} + 2\frac{d-h}{c}$$

Express the "measured" distance *d* ′ when we do not account for the atmosphere:

$$d' = \frac{1}{2}ct = \frac{1}{2}c\left(2\frac{h}{v} + 2\frac{d-h}{c}\right) = \frac{c}{v}h + d - h$$

Solve for the length of correction *d* ′ − *d*:

$$d' - d = h\left(\frac{c}{v} - 1\right)$$

Substitute numerical values and evaluate *d* ′ − *d*:

$$d' - d = (8\,\text{km})\left(\frac{c}{0.99997c} - 1\right) = \boxed{24.0\,\text{cm}}$$

Remarks: This is larger than the accuracy of the measurements, which is about 3 to 4 cm.

116 ••

Picture the Problem The frequency of the waves on the wire is the same as the frequency of the tuning fork and their period is the reciprocal of the frequency. We can find the speed of the waves from the tension in the wire and its linear density. The wavelength can be determined from the frequency and the speed of the waves and the wave number from its definition. The general form of the wave function for waves on a wire is $y(x,t) = A\sin(kx \pm \omega t)$, so, once we know k and ω, because A is given, we can write a suitable wave function for the waves on this wire. The maximum speed and acceleration of a point on the wire can be found from the angular frequency and amplitude of the waves. Finally, we can use $P_{av} = \frac{1}{2}\mu\omega^2 A^2 v$ to find the average rate at which energy must be supplied to the tuning fork to keep it oscillating with a steady amplitude.

(*a*) The frequency of the waves on the wire is the same as the frequency of the tuning fork:

$$f = \boxed{400\,\text{Hz}}$$

The period of the waves on the wire is the reciprocal of their frequency:

$$T = \frac{1}{f} = \frac{1}{400\,\text{s}^{-1}} = \boxed{2.50\,\text{ms}}$$

(*b*) Relate the speed of the waves to the tension in the wire and its linear density:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{1\,\text{kN}}{0.01\,\text{kg/m}}} = \boxed{316\,\text{m/s}}$$

(*c*) Use the relationship between the wavelength, speed and frequency of a wave to find λ:

$$\lambda = \frac{v}{f} = \frac{316\,\text{m/s}}{400\,\text{s}^{-1}} = \boxed{79.0\,\text{cm}}$$

Using its definition, express and evaluate the wave number:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{79\times10^{-2}\,\text{m}} = \boxed{7.95\,\text{m}^{-1}}$$

(*d*) Determine the angular frequency of the waves:

$$\omega = 2\pi f = 2\pi\left(400\,\text{s}^{-1}\right) = 2.51\times10^{3}\,\text{s}^{-1}$$

Substitute for A, k, and ω in the general form of the wave function to obtain:

$$y(x,t) = \boxed{(0.50\,\text{mm})\sin\left[\left(7.95\,\text{m}^{-1}\right)x - \left(2.51\times10^{3}\,\text{s}^{-1}\right)t\right]}$$

(*e*) Relate the maximum speed of a point on the wire to the amplitude of the waves and the angular frequency of the tuning fork:

$$\begin{aligned} v_{max} &= A\omega \\ &= \left(0.5\times10^{-3}\,\text{m}\right)\left(2.51\times10^{3}\,\text{s}^{-1}\right) \\ &= \boxed{1.26\,\text{m/s}} \end{aligned}$$

Express the maximum acceleration of a point on the wire in terms of the amplitude of the waves and the angular frequency of the tuning fork:

$$a_{max} = A\omega^2 = (0.5\times10^{-3}\text{ m})(2.51\times10^3\text{ s}^{-1})^2 = \boxed{3.15\times10^3\text{ m/s}^2}$$

(*f*) Express the average power required to keep the tuning fork oscillating at a steady amplitude in terms of the linear density of the wire, the amplitude of its vibrations, and the speed of the waves on the wire:

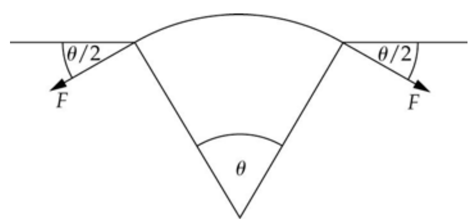
$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = \tfrac{1}{2}(0.1\text{ kg/m})(2.51\times10^3\text{ s}^{-1})^2(0.5\times10^{-3}\text{ m})^2(316\text{ m/s}) = \boxed{24.9\text{ W}}$$

117 •••

Picture the Problem Because the chain is rolling at high speed we can neglect the effect of gravity. The diagram shows a small portion of the chain. We'll assume that the angle θ is small even though it shown as a large angle in the diagram. Let Δm be the mass of the segment of the chain shown. We'll apply Newton's 2nd law to the segment in order to relate the tension in the chain to its linear density and speed.



(*a*) Apply $\sum F_{radial} = m\frac{v^2}{R}$ to a segment of the chain whose mass is Δm:

$$F_{net} = \Delta m\frac{v_0^2}{R}$$

Express Δm in terms of μ, θ, and R:

$$\Delta m = \mu d\ell = \mu R\theta$$

Express F_{net} in terms of T and θ:

$$F_{net} = 2F\sin\tfrac{1}{2}\theta$$

Substitute to obtain:

$$2F\sin\tfrac{1}{2}\theta = \mu\theta v_0^2$$

Solve for F:

$$F = \frac{\mu\theta v_0^2}{2\sin\frac{1}{2}\theta}$$

Apply the small angle approximation $\sin\frac{1}{2}\theta \approx \frac{1}{2}\theta$:

$$F = \frac{\mu\theta v_0^2}{2\left(\frac{1}{2}\theta\right)} = \boxed{\mu v_0^2}$$

(*b*) The wave speed is the same as the speed at which the chain is moving:

$$v_0 = \boxed{\sqrt{\frac{F}{\mu}}}$$

(*c*) As seen by an observer at rest, the pulse remains at the same position because its speed along the chain is the same as the speed of the chain. With respect to a fixed point on the chain, the pulse travels through 360°.

118 •••

Picture the Problem Let Δm represent the mass of the segment of length Δx = 1 mm. We can find the wave speed from the given data for the tension in the rope and its linear density. The wavelength can be found from $v = f\lambda$. We'll use the definition of linear momentum to find the maximum transverse linear momentum of the 1-mm segment and apply Newton's 2nd law to the segment to find the maximum net force on it.

(*a*) Find the wave speed from the tension and linear density:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{10\,\text{N}}{0.1\,\text{kg/m}}} = \boxed{10.0\,\text{m/s}}$$

(*b*) Express the wavelength in terms of the speed and frequency of the wave:

$$\lambda = \frac{v}{f} = \frac{10\,\text{m/s}}{5\,\text{s}^{-1}} = \boxed{2.00\,\text{m}}$$

(*c*) Relate the maximum transverse linear momentum of the 1-mm segment to the maximum transverse speed of the wave:

$$p_{\text{max}} = \Delta m v_{\text{max}} = \mu\Delta x A\omega = 2\pi f\mu\Delta x A$$

Substitute numerical values and evaluate p_{max}:

$$p_{\text{max}} = 2\pi\left(5\,\text{s}^{-1}\right)(0.1\,\text{kg/m}) \times \left(1\times10^{-3}\,\text{m}\right)(0.04\,\text{m}) = \boxed{1.26\times10^{-4}\,\text{kg}\cdot\text{m/s}}$$

(*d*) Apply $\sum F_{\text{radial}} = m\frac{v^2}{r}$ to the 1-mm segment and simplify to obtain:

$$F_{\max} = \Delta m\frac{v^2}{A} = \mu\Delta x\frac{A^2\omega^2}{A} = \mu\Delta x A\omega^2$$
$$= \omega p_{\max} = 2\pi f p_{\max}$$

Substitute numerical values and evaluate $F_{\max}$:

$$F_{\max} = 2\pi\left(5\,\text{s}^{-1}\right)\left(1.26\times10^{-4}\,\text{kg}\cdot\text{m/s}\right)$$
$$= \boxed{3.96\,\text{mN}}$$

***119** •••

Picture the Problem We can relate the speed of the pulse to the tension in the rope and its linear density. Because the rope hangs vertically, the tension in it varies linearly with the distance from its bottom. Once we've established the result in part (*a*), we can integrate the resulting velocity equation to find the time for the pulse to travel the length of the rope and then double this time to get the round-trip time.

(*a*) Relate the speed of transverse waves to tension and linear density:

$$v = \sqrt{\frac{F}{\mu}}$$

Express the force acting on a segment of the rope of length *y*:

$$F = mg = \mu y g$$

Substitute to obtain:

$$v = \sqrt{\frac{\mu y g}{\mu}} = \boxed{\sqrt{gy}}$$

(*b*) Because the speed of the pulse varies with the distance from the bottom of the rope, express *v* as *dy*/*dt* and solve for *dt*:

$$\frac{dy}{dt} = \sqrt{gy} \text{ and } dt = \frac{1}{\sqrt{g}}\frac{dy}{\sqrt{y}}$$

Integrate the left side of the equation from 0 to *t* and the right side from 0 to 3 m:

$$\int_0^t dt' = \frac{1}{\sqrt{g}}\int_0^{3\text{m}}\frac{dy}{\sqrt{y}}$$

and

$$t = \frac{1}{\sqrt{g}}\left(2\sqrt{y}\right)\Big|_0^{3\,\text{m}} = \frac{2\sqrt{3\,\text{m}}}{\sqrt{9.81\,\text{m/s}^2}}$$
$$= 1.106\,\text{s}$$

The time for the pulse to make the round-trip is:

$$t_{\text{round-trip}} = 2t = 2\left(1.106\,\text{s}\right) = \boxed{2.21\,\text{s}}$$

120 •••

Picture the Problem We can follow the step-by-step instructions outlined above to obtain the given expressions for ΔU.

(*a*) Express the potential energy of a segment of the string:

$$\Delta U = F(\Delta\ell - \Delta x)$$

For $\Delta y/\Delta x << 1$:

$$\Delta\ell = \Delta x\left[1 + \tfrac{1}{2}(\Delta y/\Delta x)^2\right]$$

and

$$\Delta\ell - \Delta x = \Delta x\left[1 + \tfrac{1}{2}(\Delta y/\Delta x)^2\right] - \Delta x = \tfrac{1}{2}(\Delta y/\Delta x)^2\,\Delta x$$

Substitute to obtain:

$$\Delta U = F\left[\tfrac{1}{2}(\Delta y/\Delta x)^2\right] = \boxed{\tfrac{1}{2}F(\Delta y/\Delta x)^2\,\Delta x}$$

(*b*) Differentiate $y(x,t) = A\sin(kx - \omega t)$ to obtain:

$$\frac{dy}{dx} = kA\cos(kx - \omega t)$$

Approximate $\Delta y/\Delta x$ by dy/dx and substitute in our result from part (*a*):

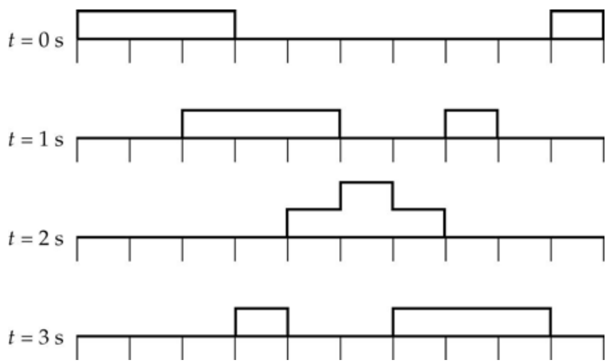
$$\Delta U = \tfrac{1}{2}F\left(kA\cos(kx - \omega t)\right)^2\Delta x = \boxed{\tfrac{1}{2}FA^2k^2\Delta x\cos^2(kx - \omega t)}$$

Chapter 16
Superposition and Standing Waves

Conceptual Problems

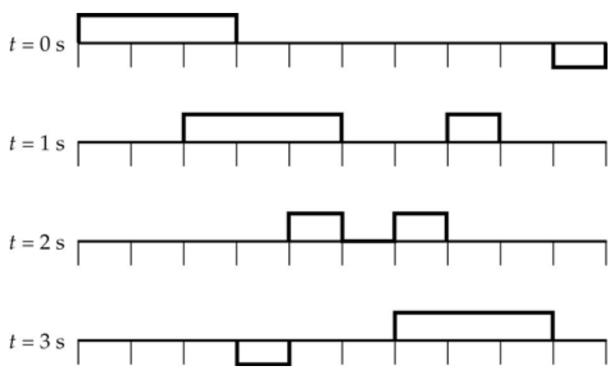
***1 ••**

Picture the Problem We can use the speeds of the pulses to determine their positions at the given times.



2 ••

Picture the Problem We can use the speeds of the pulses to determine their positions at the given times.



3 •

Determine the Concept Beats are a consequence of the alternating constructive and destructive interference of waves due to slightly different frequencies. The amplitudes of the waves play no role in producing the beats. $\boxed{(c) \text{ is correct.}}$

4 •

(*a*) True. The harmonics for a string fixed at both ends are integral multiples of the frequency of the fundamental mode (first harmonic).

(*b*) True. The harmonics for a string fixed at both ends are integral multiples of the frequency of the fundamental mode (first harmonic).

(*c*) True. If ℓ is the length of the pipe and v the speed of sound, the excited harmonics are given by $f_n = n\dfrac{v}{4\ell}$, where n = 1, 3, 5…

5 ••

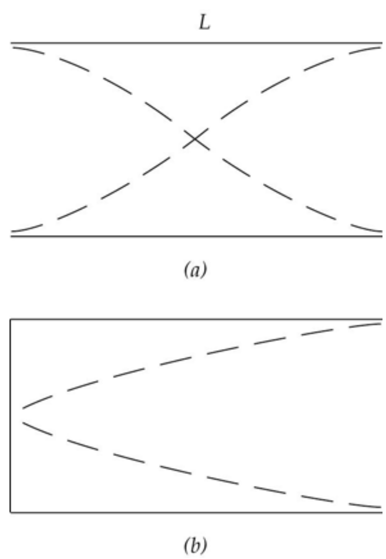
Determine the Concept Standing waves are the consequence of the constructive interference of waves that have the same amplitude and frequency but are traveling in opposite directions. $\boxed{(b)\text{ is correct.}}$

***6** •

Determine the Concept Our ears and brain find frequencies which are small-integer multiples of one another pleasing when played in combination. In particular, the ear hears frequencies related by a factor of 2 (one octave) as identical. Thus, a violin sounds much more "musical" than the sound of a drum.

7 •

Picture the Problem The first harmonic displacement-wave pattern in an organ pipe open at both ends and vibrating in its fundamental mode is represented in part (*a*) of the diagram. Part (*b*) of the diagram shows the wave pattern corresponding to the fundamental frequency for a pipe of the same length *L* that is closed at one end. Letting unprimed quantities refer to the open pipe and primed quantities refer to the closed pipe, we can relate the wavelength and, hence, the frequency of the fundamental modes using $v = f\lambda$.



Express the frequency of the first harmonic in the open pipe in terms of the speed and wavelength of the waves:

$$f_1 = \frac{v}{\lambda_1}$$

Relate the length of the open pipe to the wavelength of the fundamental mode:

$$\lambda_1 = 2L$$

Substitute to obtain:

$$f_1 = \frac{v}{2L}$$

Express the frequency of the first harmonic in the closed pipe in terms of the speed and wavelength of the waves:

$$f_1' = \frac{v}{\lambda_1'}$$

Relate the length of the closed pipe to the wavelength of the fundamental mode:

$$\lambda_1' = 4L$$

Substitute to obtain:

$$f_1' = \frac{v}{4L} = \frac{1}{2}\left(\frac{v}{2L}\right) = \frac{1}{2} f_1$$

Substitute numerical values and evaluate f_1':

$$f_1' = \frac{1}{2}(400\ \text{Hz}) = 200\ \text{Hz}$$

and $\boxed{(a)\ \text{is correct.}}$

8 ••

Picture the Problem The frequency of the fundamental mode of vibration is directly proportional to the speed of waves on the string and inversely proportional to the wavelength which, in turn, is directly proportional to the length of the string. By expressing the fundamental frequency in terms of the length L of the string and the tension F in it we can examine the various changes in lengths and tension to determine which would halve it.

Express the dependence of the frequency of the fundamental mode of vibration of the string on its wavelength:

$$f_1 = \frac{v}{\lambda_1}$$

Relate the length of the string to the wavelength of the fundamental mode:

$$\lambda_1 = 2L$$

Substitute to obtain:

$$f_1 = \frac{v}{2L}$$

Express the dependence of the speed of waves on the string on the tension

$$v = \sqrt{\frac{F}{\mu}}$$

in the string:

Substitute to obtain:

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

(*a*) Doubling the tension and the length would increase the frequency by a factor of $\sqrt{2}/2$.

(*b*) Halving the tension and keeping the length fixed would decrease the frequency by a factor of $1/\sqrt{2}$.

(*c*) Keeping the tension fixed and halving the length would double the frequency.

$\boxed{(c) \text{ is correct.}}$

9 ••

Determine the Concept We can relate the resonant frequencies of an organ pipe to the speed of sound in air and the speed of sound to the absolute temperature.

Express the dependence of the resonant frequencies on the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound to the temperature of the air:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ and R are constants, M is the molar mass of the gas (air), and T is the absolute temperature.

Substitute to obtain:

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

$\boxed{\text{Because } v \propto \sqrt{T}\text{, increasing the temperature increases the resonant frequencies.}}$

***10** •

Determine the Concept Because the two waves move independently, neither impedes the progress of the other.

11 •

Determine the Concept No; the wavelength of a wave is related to its frequency and speed of propagation ($\lambda = v/f$). The frequency of the plucked string will be the same as the wave it produces in air, but the speeds of the waves depend on the media in which they are propagating. Because the velocities of propagation differ, the wavelengths will not be the same.

12 •

Determine the Concept No; when averaged over a region in space including one or more wavelengths, the energy is unchanged.

13 •

Determine the Concept When the edges of the glass vibrate, sound waves are produced in the air in the glass. The resonance frequency of the air columns depends on the length of the air column, which depends on how much water is in the glass.

14 ••

Picture the Problem We can use $v = f\lambda$ to relate the frequency of the sound waves in the organ pipes to the speed of sound in air, nitrogen, and helium. We can use $v = \sqrt{\gamma RT/M}$ to relate the speed of sound, and hence its frequency, to the properties of the three gases.

Express the frequency of a given note as a function of its wavelength and the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound to the absolute temperature and the molar mass of the gas used in the organ:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ depends on the kind of gas, R is a constant, T is the absolute temperature, and M is the molar mass.

Substitute to obtain:

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

For air in the organ pipes we have:

$$f_{\text{air}} = \frac{1}{\lambda}\sqrt{\frac{\gamma_{\text{air}} RT}{M_{\text{air}}}} \qquad (1)$$

When nitrogen is in the organ pipes:

$$f_{N_2} = \frac{1}{\lambda}\sqrt{\frac{\gamma_{N_2} RT}{M_{N_2}}} \qquad (2)$$

Express the ratio of equation (2) to equation (1) and solve for f_{N_2}:

$$\frac{f_{N_2}}{f_{air}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{air}}\frac{M_{air}}{M_{N_2}}}$$

and

$$f_{N_2} = f_{air}\sqrt{\frac{\gamma_{N_2}}{\gamma_{air}}\frac{M_{air}}{M_{N_2}}}$$

Because $\gamma_{N_2} = \gamma_{air}$ and $M_{air} > M_{N_2}$:

$$f_{N_2} > f_{air}$$

i.e.,

f will increase for each organ pipe.

If helium were used, we'd have:

$$f_{He} = f_{air}\sqrt{\frac{\gamma_{He}}{\gamma_{air}}\frac{M_{air}}{M_{He}}}$$

Because $\gamma_{He} > \gamma_{air}$ and $M_{air} >> M_{He}$:

$$f_{He} >> f_{air}$$

i.e., the effect will be even more pronounced.

***15 ••**

Determine the Concept Increasing the tension on a piano wire increases the speed of the waves. The wavelength of these waves is determined by the length of the wire. Because the speed of the waves is the product of their wavelength and frequency, the wavelength remains the same and the frequency increases. (b) is correct.

16 ••

Determine the Concept If connected properly, the speakers will oscillate in phase and interfere constructively. If connected incorrectly, they interfere destructively. It would be difficult to detect the interference if the wavelength is short, less than the distance between the ears of the observer. Thus, one should use bass notes of low frequency and long wavelength.

17 ••

Determine the Concept The pitch is determined mostly by the resonant cavity of the mouth; the frequency of sounds he makes is directly proportional to their speed. Because $v_{He} > v_{air}$ (see Equation 15-5), the resonance frequency is higher if helium is the gas in the cavity.

***18** ••

Determine the Concept The light is being projected up from underneath the silk, so you will see light where there is a gap and darkness where two threads overlap. Because the two weaves have almost the same spatial period but not exactly identical (because the two are stretched unequally), there will be places where, for large sections of the cloth, the two weaves overlap in phase, leading to brightness, and large sections where the two overlap 90° out of phase (i.e., thread on gap and vice versa) leading to darkness. This is exactly the same idea as in the interference of two waves.

Estimation and Approximation

19 ••

Determine the Concept Pianos are tuned by ringing the tuning fork and the piano note simultaneously and tuning the piano string until the beats are far apart; i.e., the time between beats is very long. If we assume that 2 s is the maximum detectable period for the beats, then one should be able to tune the piano string to at least 0.5 Hz.

***20** •

Picture the Problem We can use $v = f_1\lambda_1$ to express the resonance frequencies in the organ pipes in terms of their wavelengths and $L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots$ to relate the length of the pipes to the resonance wavelengths.

(*a*) Relate the fundamental frequency of the pipe to its wavelength and the speed of sound:

$$f_1 = \frac{v}{\lambda_1}$$

Express the condition for constructive interference in a pipe that is open at both ends:

$$L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots \quad (1)$$

Solve for λ_1:

$$\lambda_1 = 2L$$

Substitute and evaluate f_1:

$$f_1 = \frac{v}{2L} = \frac{340\,\text{m/s}}{2(7.5\times10^{-2}\,\text{m})} = \boxed{2.27\,\text{kHz}}$$

(*b*) Relate the resonance frequencies of the pipe to their wavelengths and the speed of sound:

$$f_n = \frac{v}{\lambda_n}$$

Solve equation (2) for λ_n:

$$\lambda_n = \frac{2L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{2L} = n\frac{340\ \text{m/s}}{2(7.5\times10^{-2}\ \text{m})} = n(2.27\ \text{kHz})$$

Set f_n = 20 kHz and evaluate n:

$$n = \frac{20\ \text{kHz}}{2.27\ \text{kHz}} = 8.81$$

The eighth harmonic is within the range defined as audible. The ninth harmonic might be heard by a person with very good hearing.

21 ••

Picture the Problem Assume a pipe length of 5 m and apply the standing-wave resonance frequencies condition for a pipe that is open at both ends (the same conditions hold for a string that is fixed at both ends).

Relate the resonance frequencies for a pipe open at both ends to the length of the pipe:

$$f_n = n\frac{v}{2L},\ n = 1, 2, 3, \ldots$$

Evaluate this expression for n = 1:

$$f_1 = \frac{340\ \text{m/s}}{2(5\ \text{m})} = \boxed{34.0\ \text{Hz}}$$

Express the dependence of the speed of sound in a gas on the temperature:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ and R are constants, M is the molar mass, and T is the absolute temperature.

Because $v \propto \sqrt{T}$, the frequency will be somewhat higher in the summer.

Superposition and Interference

22 •

Picture the Problem We can use $A = 2y_0\cos\frac{1}{2}\delta$ to find the amplitude of the resultant wave.

(a) Evaluate the amplitude of the resultant wave when $\delta = \pi/6$:

$$A = 2y_0\cos\tfrac{1}{2}\delta = 2(0.02\ \text{m})\cos\frac{1}{2}\left(\frac{\pi}{6}\right) = \boxed{3.86\ \text{cm}}$$

(*b*) Proceed as in (*a*) with $\delta = \pi/3$:

$$A = 2y_0 \cos\tfrac{1}{2}\delta = 2(0.02\,\text{m})\cos\frac{1}{2}\left(\frac{\pi}{3}\right) = \boxed{3.46\,\text{cm}}$$

23 •

Picture the Problem We can use $A = 2y_0 \cos\frac{1}{2}\delta$ to find the amplitude of the resultant wave.

Evaluate the amplitude of the resultant wave when $\delta = \pi/2$:

$$A = 2y_0 \cos\tfrac{1}{2}\delta = 2(0.05\,\text{m})\cos\frac{1}{2}\left(\frac{\pi}{2}\right) = \boxed{7.07\,\text{cm}}$$

***24** •

Picture the Problem The phase shift in the waves generated by these two sources is due to their separation of $\lambda/3$. We can find the phase difference due to the path difference from $\delta = 2\pi\frac{\Delta x}{\lambda}$ and then the amplitude of the resultant wave from $A = 2y_0 \cos\frac{1}{2}\delta$.

Evaluate the phase difference δ:

$$\delta = 2\pi\frac{\Delta x}{\lambda} = 2\pi\frac{\lambda/3}{\lambda} = \frac{2}{3}\pi$$

Find the amplitude of the resultant wave:

$$A_{\text{res}} = 2y_0 \cos\tfrac{1}{2}\delta = 2A\cos\frac{1}{2}\left(\frac{2}{3}\pi\right) = 2A\cos\frac{\pi}{3} = \boxed{A}$$

25 •

Picture the Problem The phase shift in the waves generated by these two sources is due to a path difference Δx = 5.85 m – 5.00 m = 0.85 m. We can find the phase difference due to this path difference from $\delta = 2\pi\frac{\Delta x}{\lambda}$ and then the amplitude of the resultant wave from $A = 2y_0 \cos\frac{1}{2}\delta$.

(*a*) Find the phase difference due to the path difference:

$$\delta = 2\pi\frac{\Delta x}{\lambda}$$

Calculate the wavelength of the sound waves:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{100\,\text{s}^{-1}} = 3.4\,\text{m}$$

Substitute and evaluate δ:

$$\delta = 2\pi\frac{0.85\,\text{m}}{3.4\,\text{m}} = \frac{\pi}{2}\,\text{rad} = \boxed{90.0°}$$

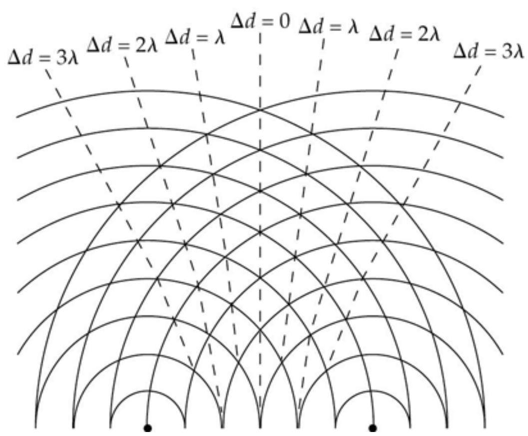
(*b*) Relate the amplitude of the resultant wave to the amplitudes of the interfering waves and the phase difference between them:

$$A = 2y_0\cos\tfrac{1}{2}\delta = 2A\cos\frac{1}{2}\left(\frac{\pi}{2}\right)$$

$$= \boxed{\sqrt{2}A}$$

***26** •

Picture the Problem The diagram is shown below. Lines of constructive interference are shown for path differences of 0, λ, 2λ, and 3λ.



27 •

Picture the Problem The intensity at the point of interest is dependent on whether the speakers are coherent and on the total phase difference in the waves arriving at the given point. We can use $\delta = 2\pi\frac{\Delta x}{\lambda}$ to determine the phase difference δ, $A = \left|2p_0\cos\tfrac{1}{2}\delta\right|$ to find the amplitude of the resultant wave, and the fact that the intensity I is proportional to the square of the amplitude to find the intensity at P for the given conditions.

(*a*) Find the phase difference δ:

$$\delta = 2\pi\frac{\tfrac{1}{2}\lambda}{\lambda} = \pi$$

Find the amplitude of the resultant wave:

$$A = \left|2p_0\cos\tfrac{1}{2}\pi\right| = 0$$

Because the intensity is proportional to A^2:

$$I = \boxed{0}$$

(*b*) The sources are incoherent and

$$I = \boxed{2I_0}$$

the intensities add:

(*c*) Express the total phase difference:

$$\delta_{\text{tot}} = \delta_{\text{sources}} + \delta_{\text{path difference}} = \pi + 2\pi\frac{\Delta x}{\lambda} = \pi + 2\pi\left(\frac{1}{2}\right) = 2\pi$$

Find the amplitude of the resultant wave:

$$A = \left|2p_0\cos\tfrac{1}{2}(2\pi)\right| = 2p_0$$

Because the intensity is proportional to A^2:

$$I = \frac{A^2}{p_0^2}I_0 = \frac{(2p_0)^2}{p_0^2}I_0 = \boxed{4I_0}$$

28 •

Picture the Problem The intensity at the point of interest is dependent on whether the speakers are coherent and on the total phase difference in the waves arriving at the given point. We can use $\delta = 2\pi\frac{\Delta x}{\lambda}$ to determine the phase difference δ, $A = \left|2p_0\cos\tfrac{1}{2}\delta\right|$ to find the amplitude of the resultant wave, and the fact that the intensity *I* is proportional to the square of the amplitude to find the intensity at *P* for the given conditions.

(*a*) Find the phase difference δ:

$$\delta = 2\pi\frac{\lambda}{\lambda} = 2\pi$$

Find the amplitude of the resultant wave:

$$A = \left|2p_0\cos\tfrac{1}{2}(2\pi)\right| = 2p_0$$

Because the intensity is proportional to A^2:

$$I = \frac{A^2}{p_0^2}I_0 = \frac{(2p_0)^2}{p_0^2}I_0 = \boxed{4I_0}$$

(*b*) The sources are incoherent and the intensities add:

$$I = \boxed{2I_0}$$

(*c*) Express the total phase difference:

$$\delta_{\text{tot}} = \delta_{\text{sources}} + \delta_{\text{path difference}} = \pi + 2\pi\frac{\Delta x}{\lambda} = \pi + 2\pi\left(\frac{\lambda}{\lambda}\right) = 3\pi$$

Find the amplitude of the resultant

$$A = \left|2p_0\cos\tfrac{1}{2}(3\pi)\right| = 0$$

wave:

Because the intensity is proportional to A^2:

$$I = \boxed{0}$$

29 •

Picture the Problem Let P be the point located a distance r_1 from speaker 1 and a distance r_2 from speaker 2. If the sound at point P is to be either a maximum or a minimum, the difference in the distances to the speakers will have to be such that this difference compensates for the 90° out-of-phase condition of the speakers.

(*a*) Express the phase shift due to the speakers in terms of a path difference:

$$\Delta r = \frac{\delta_{\text{sources}}}{360°}\lambda = \frac{90°}{360°}\lambda = \tfrac{1}{4}\lambda$$

Express the condition that $r_2 - r_1$ must satisfy in order to compensate for this path difference:

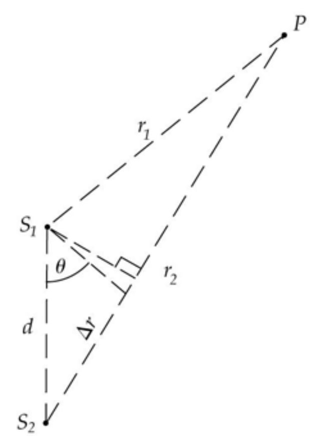
$$r_2 - r_1 = \boxed{\tfrac{1}{4}\lambda}$$

(*b*) In this case, the smallest difference in path is again $\lambda/4$, but now:

$$r_1 - r_2 = \boxed{\tfrac{1}{4}\lambda}$$

***30** ••

Picture the Problem The drawing shows a generic point P located a distance r_1 from source S_1 and a distance r_2 from source S_2. The sources are separated by a distance d and we're given that $d < \lambda/2$. Because the condition for destructive interference is that $\delta = n\pi$ where $n = 1, 2, 3,...$, we'll show that, with $d < \lambda/2$, this condition cannot be satisfied.



Relate the phase shift to the path difference and the wavelength of the sound:

$$\delta = 2\pi\frac{\Delta r}{\lambda}$$

Relate Δr to d and θ:

$$\Delta r < d\sin\theta \le d$$

Substitute to obtain:

$$\delta < 2\pi\frac{d\sin\theta}{\lambda} \le 2\pi\frac{d}{\lambda}$$

Because $d < \lambda/2$:

$$\delta < 2\pi\frac{\lambda/2}{\lambda} = \pi$$

Express the condition for destructive interference:

$$\delta = n\pi$$
where n = 1, 2, 3,…

$$\boxed{\text{Because } \delta < \pi, \text{there is no complete destructive interference in any direction.}}$$

31 ••

Picture the Problem Let the positive x direction be the direction of propagation of the wave. We can express the phase difference in terms of the separation of the two points and the wavelength of the wave and solve for λ. In part (b) we can find the phase difference by relating the time between displacements to the period of the wave. I in part (c) we can use the relationship between the speed, frequency, and wavelength of a wave to find its velocity.

(a) Relate the phase difference to the wavelength of the wave:

$$\delta = 2\pi\frac{\Delta x}{\lambda}$$

Solve for and evaluate λ:

$$\lambda = 2\pi\frac{\Delta x}{\delta} = 2\pi\frac{5\,\text{cm}}{\pi/6} = \boxed{60.0\,\text{cm}}$$

(b) Express and evaluate the period of the wave:

$$T = \frac{1}{f} = \frac{1}{40\,\text{s}^{-1}} = 25\,\text{ms}$$

Relate the time between the two displacements to the period of the wave:

$$5\,\text{ms} = \frac{1}{5}T$$

Express the phase difference corresponding to one-fifth of a period:

$$\delta = \boxed{\frac{2\pi}{5}}$$

(c) Express the wave speed in terms of its frequency and wavelength:

$$v = f\lambda = \left(40\,\text{s}^{-1}\right)\left(0.6\,\text{m}\right) = \boxed{24.0\,\text{m/s}}$$

32 ••

Picture the Problem Assume a distance of about 20 cm between your ears. When you rotate your head through 90°, you introduce a path difference of 20 cm. We can apply the equation for the phase difference due to a path difference to determine the change in phase between the sounds received by your ears as you rotate your head through 90°.

Express the phase difference due to the rotation of your head through 90°:

$$\delta = 2\pi \frac{20\,\text{cm}}{\lambda}$$

Find the wavelength of the sound waves:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{680\,\text{s}^{-1}} = 50\,\text{cm}$$

Substitute to obtain:

$$\delta = 2\pi \frac{20\,\text{cm}}{50\,\text{cm}} = \boxed{0.8\pi\,\text{rad}}$$

33 ••

Picture the Problem Because the sound intensity diminishes as the observer moves, parallel to a line through the sources, away from her initial position, we can conclude that her initial position is one at which there is constructive interference of the sound coming from the two sources. We can apply the condition for constructive interference to relate the wavelength of the sound to the path difference at her initial position and the relationship between the velocity, frequency, and wavelength of the waves to express this path difference in terms of the frequency of the sources.

Express the condition for constructive interference at (40 m, 0):

$$\Delta r = n\lambda,\ n = 1, 2, 3, \ldots \qquad (1)$$

Express the path difference Δr:

$$\Delta r = r_\text{B} - r_\text{A}$$

Using the Pythagorean theorem, find r_B:

$$r_\text{B} = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2}$$

Substitute for r_B and evaluate Δr:

$$\Delta r = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2} - 40\,\text{m} = 0.07194\,\text{m}$$

Substitute in equation (1) and solve for λ:

$$\lambda = \frac{0.07194\,\text{m}}{n}$$

Using $v = f\lambda$, express f in terms of λ and n:

$$f_n = n\frac{v}{0.07194\,\text{m}} = n\frac{340\,\text{m/s}}{0.07194\,\text{m}} = (4726\,\text{Hz})n$$

Evaluate f for n = 1 and 2:

$$f_1 = \boxed{4726\,\text{Hz}} \text{ and } f_2 = \boxed{9452\,\text{Hz}}$$

34 ••

Picture the Problem Because the sound intensity increases as the observer moves, parallel to a line through the sources, away from her initial position, we can conclude that her initial position is one at which there is destructive interference of the sound coming from the two sources. We can apply the condition for destructive interference to relate the wavelength of the sound to the path difference at her initial position and the relationship between the velocity, frequency, and wavelength of the waves to express this path difference in terms of the frequency of the sources.

Express the condition for destructive interference at (40 m, 0):

$$\Delta r = n\frac{\lambda}{2},\ n = 1, 3, 5, \ldots \qquad (1)$$

Express the path difference Δr:

$$\Delta r = r_B - r_A$$

Using the Pythagorean theorem, find r_B:

$$r_B = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2}$$

Substitute for r_B and evaluate Δr:

$$\Delta r = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2} - 40\,\text{m} = 0.07194\,\text{m}$$

Substitute in equation (1) and solve for λ:

$$\lambda = \frac{2(0.07194\,\text{m})}{n} = \frac{0.1439\,\text{m}}{n}$$

Using $v = f\lambda$, express f in terms of λ:

$$f_n = n\frac{v}{0.1439\,\text{m}} = n\frac{340\,\text{m/s}}{0.1439\,\text{m}} = (2363\,\text{Hz})n$$

Evaluate f for n = 1 and 3:

$$f_1 = \boxed{2363\,\text{Hz}}$$

and

$$f_3 = \boxed{7089\,\text{Hz}}$$

***35 ••**

Picture the Problem We can use the trigonometric identity $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ to derive the expression given in (*a*) and the speed of the envelope can be found from the second factor in this expression; i.e., from $\cos[(\Delta k/2)x - (\Delta\omega/2)t]$.

(*a*) Express the amplitude of the resultant wave function $y(x,t)$:

$$y(x,t) = A(\cos(k_1x - \omega_1t) + \cos(k_2x - \omega_2t))$$

Use the trigonometric identity $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ to obtain:

$$y(x,t) = 2A\left[\cos\frac{k_1x - \omega_1 t + k_2 x - \omega_2 t}{2}\cos\frac{k_1 x - \omega_1 t - k_2 x + \omega_2 t}{2}\right]$$
$$= 2A\left[\cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right)\cos\left(\frac{k_1 - k_2}{2}x + \frac{\omega_2 - \omega_1}{2}t\right)\right]$$

Substitute $\omega_{ave} = (\omega_1 + \omega_2)/2$, $k_{ave} = (k_1 + k_2)/2$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$ to obtain:

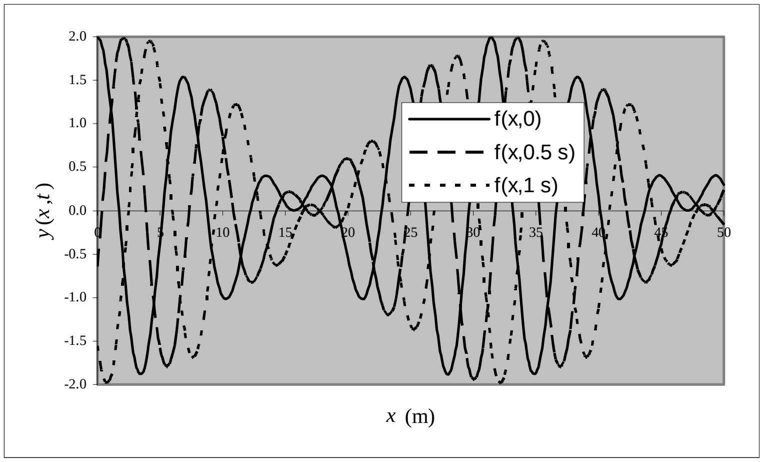
$$y(x,t) = \boxed{2A\left[\cos(k_{ave}x - \omega_{ave}t)\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right]}$$

(*b*) A spreadsheet program to calculate $y(x,t)$ between 0 m and 50 m at t = 0, 0.5 s, and 1 s follows. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
B11	B10+0.25	$x + \Delta x$
C10	COS($B$3*B10–$B$5*$C$9) + COS($B$4*B10–$B$6*$C$9)	$y(x,0)$
D10	COS($B$3*B10-$B$5*$D$9) + COS($B$4*B10–$B$6*$D$9)	$y(x,0.5\text{ s})$
E10	COS($B$3*B10–$B$5*$E$9) + COS($B$4*B10–$B$6*$E$9)	$y(x,1\text{ s})$

	A	B	C	D	E
1					
2					
3	k1=	1	m^{-1}		
4	k2=	0.8	m^{-1}		
5	w1=	1	rad/s		
6	w2=	0.9	rad/s		
7		x	y(x,0)	y(x,0.5 s)	y(x,1 s)
8		(m)			
9			0.000	2.000	4.000
10		0.00	2.000	–0.643	–1.550
11		0.25	1.949	–0.207	–1.787
12		0.50	1.799	0.241	–1.935
13		0.75	1.557	0.678	–1.984
14		1.00	1.237	1.081	–1.932
206		49.00	0.370	–0.037	0.021
207		49.25	0.397	0.003	–0.024
208		49.50	0.397	0.065	–0.075
209		49.75	0.364	0.145	–0.124
210		50.00	0.298	0.237	–0.164

The solid line is the graph of $y(x,0)$, the dashed line that of $y(x,0.5\text{ s})$, and the dotted line is the graph of $y(x,1\text{ s})$.



(*c*) Express the speed of the envelope:

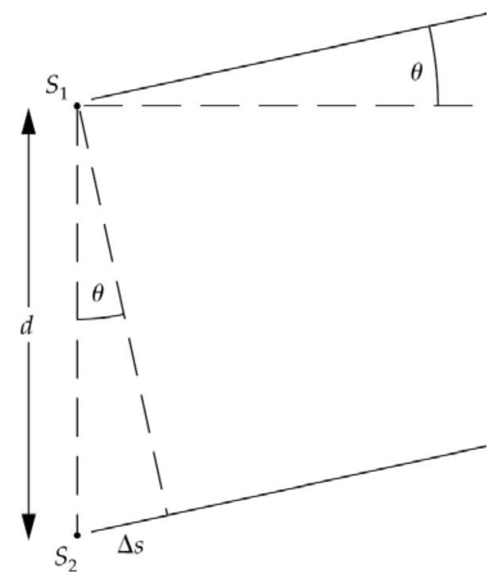
$$v_{\text{envelope}} = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2}$$

Substitute numerical values and evaluate v_{envelope}:

$$v_{\text{envelope}} = \frac{1\,\text{rad/s} - 0.9\,\text{rad/s}}{1\,\text{m}^{-1} - 0.8\,\text{m}^{-1}} = \boxed{0.500\,\text{m/s}}$$

36 ••

Picture the Problem The diagram shows the two sources separated by a distance *d* and the path difference Δs. Because the lines from the sources to the distant point are approximately parallel, the triangle shown in the diagram is approximately a right triangle and we can use trigonometry to express Δs in terms of *d* and θ. In the second part of the problem, we can apply a small-angle approximation to the larger triangle shown in Figure 16-29 to relate y_m to *D* and θ and then use the condition for constructive interference to relate y_m to *D*, λ, and *d*.



(*a*) Using the diagram, relate Δs to the separation of the sources and the angle θ:

$$\sin\theta \approx \frac{\Delta s}{d} \text{ and } \Delta s \approx \boxed{d\sin\theta}$$

(*b*) For $\theta << 1$, we can approximate

$$\Delta s \approx d\tan\theta$$

$\sin\theta$ with $\tan\theta$ to obtain:

Referring to Figure 16-29, express $\tan\theta$ in terms of y and D:

$$\tan\theta \approx \frac{y_m}{D}$$

Substitute to obtain:

$$\Delta s \approx \frac{dy_m}{D}$$

Express the condition on the phase difference for constructive interference:

$$\delta = 2\pi\frac{\Delta s}{\lambda} = 2\pi m,\ m = 1, 2, 3, \ldots$$

Substitute for Δs:

$$2\pi\frac{dy_m}{D\lambda} = 2\pi m,\ m = 1, 2, 3, \ldots$$

Simplify and solve for y_m:

$$y_m = \boxed{m\frac{D\lambda}{d}}$$

37 ••

Picture the Problem Because a maximum is heard at 0° and the sources are in phase, we can conclude that the path difference is 0. Because the next maximum is heard at 23°, the path difference to that position must be one wavelength. We can use the result of part (*a*) of Problem 36 to relate the separation of the sources to the path difference and the angle θ. We'll apply the condition for constructive interference to determine the angular locations of other points of maximum intensity in the interference pattern.

Using the result of part (*a*) of Problem 36, express the separation of the sources in terms of Δs and θ:

$$d = \frac{\Delta s}{\sin\theta}$$

Evaluate d with $\Delta s = \lambda$ and $\theta = 23°$:

$$d = \frac{\lambda}{\sin 23°} = \frac{v}{f \sin 23°}$$

$$= \frac{340\ \text{m/s}}{\left(480\ \text{s}^{-1}\right)\sin 23°} = \boxed{1.81\ \text{m}}$$

Express the condition for additional intensity maxima:

$$d\sin\theta_m = m\lambda$$

where m = 1, 2, 3, …, or

$$\theta_m = \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Evaluate this expression for $m = 2$:

$$\theta_2 = \sin^{-1}\left[\frac{2(340\,\text{m/s})}{(480\,\text{s}^{-1})(1.81\,\text{m})}\right] = \boxed{51.5°}$$

Remarks: It is easy to show that, for *m* > 2, the inverse sine function is undefined and that, therefore, there are no additional relative maxima at angles larger than 51.5°.

***38 •••**

Picture the Problem Because the speakers are driven in phase and the path difference is 0 at her initial position, the listener will hear a maximum at (*D*, 0). As she walks along a line parallel to the *y* axis she will hear a minimum wherever it is true that the path difference is an odd multiple of a half wavelength. She will hear an intensity maximum wherever the path difference is an integral multiple of a wavelength. We'll apply the condition for destructive interference in part (*a*) to determine the angular location of the first minimum and, in part (*b*), the condition for constructive interference find the angle at which she'll hear the first maximum after the one at 0°. In part (*c*), we can apply the condition for constructive interference to determine the number of maxima she can hear as keeps walking parallel to the *y* axis.

(*a*) Express the condition for destructive interference:

$$d\sin\theta_m = m\frac{\lambda}{2}$$

where m = 1, 3, 5,…, or

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$$

Evaluate this expression for $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{v}{2fd}\right) = \sin^{-1}\left[\frac{340\,\text{m/s}}{2(600\,\text{s}^{-1})(2\,\text{m})}\right]$$

$$= \boxed{8.14°}$$

(*b*) Express the condition for additional intensity maxima:

$$d\sin\theta_m = m\lambda$$

where m = 0, 1, 2, 3,…, or

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Evaluate this expression for $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{v}{fd}\right) = \sin^{-1}\left[\frac{340\,\text{m/s}}{(600\,\text{s}^{-1})(2\,\text{m})}\right]$$

$$= \boxed{16.5°}$$

(*c*) Express the limiting condition on $\sin\theta$:

$$\sin\theta_m = m\frac{\lambda}{d} \leq 1$$

Solve for *m* to obtain:

$$m \leq \frac{d}{\lambda} = \frac{fd}{v} = \frac{(600\,\text{s}^{-1})(2\,\text{m})}{340\,\text{m/s}} = 3.53$$

Because *m* must be an integer:

$$m = \boxed{3}$$

39 •••

Picture the Problem Let *d* be the separation of the two sound sources. We can express the wavelength of the sound in terms of the *d* and either of the angles at which intensity maxima are heard. We can find the frequency of the sources from its relationship to the speed of the waves and their wavelengths. Using the condition for constructive interference, we can find the angles at which intensity maxima are heard. Finally, in part (*d*), we'll use the condition for destructive interference to find the smallest angle for which the sound waves cancel.

(*a*) Express the condition for constructive interference:

$$d\sin\theta_m = m\lambda \qquad (1)$$

where *m* = 0, 1, 2, 3,…

Solve for λ:

$$\lambda = \frac{d\sin\theta_m}{m}$$

Evaluate λ for *m* = 1:

$$\lambda = (2\,\text{m})\sin(0.140\,\text{rad}) = \boxed{0.279\,\text{m}}$$

(*b*) Express the frequency of the sound in terms of its wavelength and speed:

$$f = \frac{v}{\lambda} = \frac{340\,\text{m/s}}{0.279\,\text{m}} = \boxed{1.22\,\text{kHz}}$$

(*c*) Solve equation (1) for θ_m:

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{m(0.279\,\text{m})}{2\,\text{m}}\right] = \sin^{-1}[(0.1395)m]$$

The table shows the values for θ as a function of *m*:

m	θ_m
	(rad)
3	0.432
4	0.592
5	0.772

6	0.992
7	1.354
8	undefined

(*d*) Express the condition for destructive interference:

$$d\sin\theta_m = m\frac{\lambda}{2}$$

where m = 1, 3, 5,…

Solve for θ_m:

$$\theta_m = \sin^{-1}\left(m\frac{\lambda}{2d}\right)$$

Evaluate this expression for m = 1:

$$\theta_1 = \sin^{-1}\left[\frac{0.279\,\text{m}}{2(2\,\text{m})}\right] = \boxed{0.0698\,\text{rad}}$$

40 •••

Picture the Problem The total phase shift in the waves arriving at the points of interest is the sum of the phase shift due to the difference in path lengths from the two sources to a given point and the phase shift due to the sources being out of phase by 90°. From Problem 39 we know that λ = 0.279 m. Using the conditions on the path difference Δx for constructive and destructive interference, we can find the angles at which intensity maxima are heard.

Letting the subscript ″pd ″ denote ″path difference″ and the subscript ″s″ the ″sources″, express the total phase shift δ:

$$\delta = \delta_{\text{pd}} + \delta_{\text{s}} = 2\pi\frac{\Delta x}{\lambda} + \frac{\pi}{4}$$

where Δx is the path difference between the two sources and the points at which constructive or destructive interference is heard.

Express the condition for constructive interference:

$$\delta = 2\pi\frac{\Delta x}{\lambda} + \frac{\pi}{4} = 2\pi, 4\pi, 6\pi, \ldots$$

Solve for Δx to obtain:

$$\Delta x = \frac{7}{8}\lambda, \frac{15}{8}\lambda, \frac{23}{8}\lambda, \ldots = \frac{(8m-1)}{8}\lambda$$

where m = 1, 2, 3,…

Relate Δx to d to obtain:

$$\Delta x = \frac{(8m-1)}{8}\lambda = d\sin\theta_{\text{c}}$$

where the ″c″ denotes constructive interference.

Solve for θ_c:

$$\theta_c = \sin^{-1}\left[\frac{(8m-1)\lambda}{8d}\right],\ m = 1, 2, 3,\ldots$$

The table shows the values for θ_c for m = 1 to 5:

m	θ_c
1	7.01°
2	15.2°
3	23.6°
4	35.1°
5	42.8°

Express the condition for destructive interference:

$$\delta = 2\pi\frac{\Delta x}{\lambda} + \frac{\pi}{4} = \pi, 3\pi, 5\pi, \ldots$$

Solve for Δx to obtain:

$$\Delta x = \frac{3}{8}\lambda, \frac{11}{8}\lambda, \frac{19}{8}\lambda, \ldots = \frac{(8m-5)}{8}\lambda$$

where m = 1, 2, 3,…

Letting ″d ″ denotes destructive interference, relate Δx to d to obtain:

$$\Delta x = \frac{(8m-5)}{8}\lambda = d\sin\theta_d$$

Solve for θ_d:

$$\theta_d = \sin^{-1}\left[\frac{(8m-5)\lambda}{8d}\right],\ m = 1, 2, 3,\ldots$$

The table shows the values for θ_d for m = 1 to 5:

m	θ_d
1	3.00°
2	11.1°
3	19.3°
4	28.1°
5	37.6°

41 •••

Picture the Problem We can calculate the required phase shift from the path difference and the wavelength of the radio waves using $\delta = 2\pi\frac{\Delta s}{\lambda}$.

Express the phase delay as a function of the path difference and

$$\delta = 2\pi\frac{\Delta s}{\lambda} \quad (1)$$

the wavelength of the radio waves:

Find the wavelength of the radio waves:

$$\lambda = \frac{v}{f} = \frac{3\times10^{8}\ \text{m/s}}{20\times10^{6}\ \text{s}^{-1}} = 15\,\text{m}$$

Express the path difference for the signals coming from an angle θ with the vertical:

$$\Delta s = d\sin\theta$$

Substitute numerical values and evaluate Δs:

$$\Delta s = (200\,\text{m})\sin10° = 34.73\,\text{m} = 2.315\lambda$$
$$= 2\lambda + 0.315\lambda$$

Substitute in equation (1) and evaluate δ:

$$\delta = 2\pi\frac{0.315\lambda}{\lambda} = 1.98\,\text{rad} = \boxed{113°}$$

Beats

42 •

Picture the Problem The beat frequency is the difference between the frequency of the tuning fork and the frequency of the violin string. Let f_2 = 500 Hz.

(*a*) Express the relationship between the beat frequency of the frequencies of the two tuning forks:

$$f_2 = f_1 \pm \Delta f$$
$$= 500\,\text{Hz} \pm 4\,\text{Hz}$$

Solve for f_2:

$$f_2 = \boxed{504\,\text{Hz or } 496\,\text{Hz}}$$

(*b*) $\boxed{\text{If the beat frequency is increased, then } f_2 = 504\text{ Hz; if it is diminished, } f_2 = 496\text{ Hz.}}$

43 ••

Picture the Problem The Doppler shift of the siren as heard by one of the drivers is given by the formula for source and receiver both moving and approaching each other $f_\text{r} = f_\text{s}\left[(1+u/v)/(1-u/v)\right]$, where u is the speed of the ambulance and v is the speed of sound.

(*a*) Express the beat frequency:

$$f_\text{beat} = f_\text{r} - f_\text{s}$$

where f_r is the frequency heard by either driver due to the other's siren, Express f_r:

$$f_r = f_s \frac{1+\frac{u}{v}}{1-\frac{u}{v}}$$

Substitute to obtain:

$$f_{beat} = f_s \frac{1+\frac{u}{v}}{1-\frac{u}{v}} - f_s = f_s\left(\frac{1+\frac{u}{v}}{1-\frac{u}{v}} - 1\right)$$

$$= f_s \frac{2}{\frac{v}{u}-1}$$

Substitute numerical values and evaluate f_{beat}:

$$f_{beat} = (500\,\text{Hz})\frac{2}{\frac{340\,\text{m/s}}{22.4\,\text{m/s}}-1} = \boxed{70.5\,\text{Hz}}$$

(*b*) The person on the street hears no beat frequency as the sirens of both ambulances are Doppler shifted up by the same amount (approx. 35 Hz).

Standing Waves

***44** •

Picture the Problem We can use $v = f\lambda$ to relate the second-harmonic frequency to the wavelength of the standing wave for the second harmonic.

Relate the speed of transverse waves on the string to their frequency and wavelength:

$$v = f_2\lambda_2$$

Express λ_2 in terms of the length L of the string:

$$\lambda_2 = L$$

Substitute for λ_2 and evaluate v:

$$v = f_2 L = \left(60\,\text{s}^{-1}\right)(3\,\text{m}) = \boxed{180\,\text{m/s}}$$

45 •

Picture the Problem We can find the wavelength of this standing wave from the standing-wave condition for a string fixed at both ends and its frequency from $v = f_3\lambda_3$. We can use the wave function for a standing wave on a string fixed at both ends ($y_n(x,t) = A_n \sin k_n x \cos \omega_n t$) to write the wave function for the wave described in this problem.

(*a*) Using the standing-wave condition for a string fixed at both ends, relate the length of the string to the wavelength of the harmonic mode in which it is vibrating:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for λ_3:

$$\lambda_3 = \frac{2}{3}L = \frac{2}{3}(3\,\text{m}) = \boxed{2.00\,\text{m}}$$

Express the frequency of the third harmonic in terms of the speed of transverse waves on the string and their wavelength:

$$f_3 = \frac{v}{\lambda_3} = \frac{50\,\text{m/s}}{2\,\text{m}} = \boxed{25.0\,\text{Hz}}$$

(*b*) Write the equation for a standing wave, fixed at both ends, in its third harmonic:

$$y_3(x,t) = A_3 \sin k_3 x \cos \omega_3 t$$

Evaluate k_3:

$$k_3 = \frac{2\pi}{\lambda_3} = \frac{2\pi}{2\,\text{m}} = \pi\,\text{m}^{-1}$$

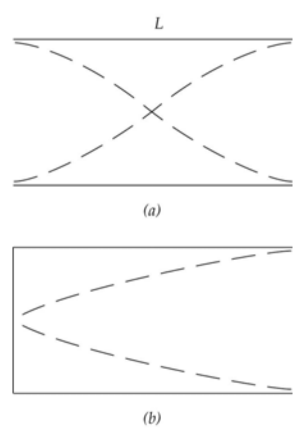
Evaluate ω_3:

$$\omega_3 = 2\pi f_3 = 2\pi(25\,\text{s}^{-1}) = 50\pi\,\text{s}^{-1}$$

Substitute to obtain: $\boxed{y_3(x,t) = (4\,\text{mm})\sin kx \cos \omega t}$ where $k = \pi\,\text{m}^{-1}$ and $\omega = 50\pi\,\text{s}^{-1}$.

46 •

Picture the Problem The first harmonic displacement-wave pattern in an organ pipe open at both ends and vibrating in its fundamental mode is represented in part (*a*) of the diagram. Part (*b*) of the diagram shows the wave pattern corresponding to the fundamental frequency for a pipe of the same length *L* that is closed at one end. We can relate the wavelength to the frequency of the fundamental modes using $v = f\lambda$.



(*a*) Express the dependence of the frequency of the fundamental mode of vibration in the open pipe on its

$$f_{1,\text{open}} = \frac{v}{\lambda_{1,\text{open}}}$$

wavelength:

Relate the length of the open pipe to the wavelength of the fundamental mode:

$$\lambda_{1,\text{open}} = 2L$$

Substitute and evaluate $f_{1,\text{open}}$:

$$f_{1,\text{open}} = \frac{v}{2L} = \frac{340\,\text{m/s}}{2(10\,\text{m})} = \boxed{17.0\,\text{Hz}}$$

(*b*) Express the dependence of the frequency of the fundamental mode of vibration in the closed pipe on its wavelength:

$$f_{1,\text{closed}} = \frac{v}{\lambda_{1,\text{closed}}}$$

Relate the length of the closed pipe to the wavelength of the fundamental mode:

$$\lambda_{1,\text{closed}} = 4L$$

Substitute to obtain:

$$f_{1,\text{closed}} = \frac{v}{4L} = \frac{340\,\text{m/s}}{4(10\,\text{m})} = \boxed{8.50\,\text{Hz}}$$

47 •

Picture the Problem We can find the speed of transverse waves on the wire using $v = \sqrt{F/\mu}$ and the wavelengths of any harmonic from $L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots$. We can use $v = f\lambda$ to find the frequency of the fundamental. For a wire fixed at both ends, the higher harmonics are integer multiples of the first harmonic (fundamental).

(*a*) Relate the speed of transverse waves on the wire to the tension in the wire and its linear density:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{968\,\text{N}}{(0.005\,\text{kg})/(1.4\,\text{m})}} = \boxed{521\,\text{m/s}}$$

(*b*) Using the standing-wave condition for a wire fixed at both ends, relate the length of the wire to the wavelength of the harmonic mode in which it is vibrating:

$$L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots$$

Solve for λ_1:

$$\lambda_1 = 2L = 2(1.4\,\text{m}) = \boxed{2.80\,\text{m}}$$

Express the frequency of the first harmonic in terms of the speed and wavelength of the waves:

$$f_1 = \frac{v}{\lambda_1} = \frac{521\,\text{m/s}}{2.80\,\text{m}} = \boxed{186\,\text{Hz}}$$

(*c*) Because, for a wire fixed at both ends, the higher harmonics are integer multiples of the first harmonic:

$$f_2 = 2f_1 = 2(186\,\text{Hz}) = \boxed{372\,\text{Hz}}$$

and

$$f_3 = 3f_1 = 3(186\,\text{Hz}) = \boxed{558\,\text{Hz}}$$

48 •

Picture the Problem We can use Equation 16-13, $f_n = n\frac{v}{4L} = nf_1, n = 1,3,5,\ldots,$ to find the resonance frequencies for a rope that is fixed at one end.

(*a*) Using the resonance-frequency condition for a rope fixed at one end, relate the resonance frequencies to the speed of the waves and the length of the rope:

$$f_n = n\frac{v}{4L} = nf_1, n = 1,3,5,\ldots$$

Solve for f_1:

$$f_1 = \frac{20\,\text{m/s}}{4(4\,\text{m})} = \boxed{1.25\,\text{Hz}}$$

(*b*) Because this rope is fixed at just one end, the system does not support a second harmonic.

(*c*) For the third harmonic, $n = 3$:

$$f_3 = 3f_1 = 3(1.25\,\text{Hz}) = \boxed{3.75\,\text{Hz}}$$

49 •

Picture the Problem We can find the fundamental frequency of the piano wire using the general expression for the resonance frequencies of a wire fixed at both ends, $f_n = n\frac{v}{2L} = nf_1, n = 1, 2, 3, \ldots,$ with $n = 1$. We can use $v = \sqrt{F/\mu}$ to express the frequencies of the fundamentals of the two wires in terms of their linear densities.

Relate the fundamental frequency of the piano wire to the speed of transverse waves on it and its linear density:

$$f_1 = \frac{v}{2L}$$

Express the dependence of the speed of transverse waves on the tension and linear density:

$$v=\sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$f_1=\frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Doubling the linear density results in a new fundamental frequency f' given by:

$$f_1'=\frac{1}{2L}\sqrt{\frac{F}{2\mu}}=\frac{1}{\sqrt{2}}\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right)=\frac{1}{\sqrt{2}}f_1$$

Substitute for f_1 to obtain:

$$f_1'=\frac{1}{\sqrt{2}}(200\,\text{Hz})=\boxed{141\,\text{Hz}}$$

***50 •**

Picture the Problem Because the frequency and wavelength of sounds waves are inversely proportional, the greatest length of the organ pipe corresponds to the lowest frequency in the normal hearing range. We can relate wavelengths to the length of the pipes using the expressions for the resonance frequencies for pipes that are open at both ends and open at one end.

Find the wavelength of a 20-Hz note:

$$\lambda_{\text{max}}=\frac{v}{f_{\text{lowest}}}=\frac{340\,\text{m/s}}{20\,\text{s}^{-1}}=17\,\text{m}$$

(*a*) Relate the length L of a closed-at-one-end organ pipe to the wavelengths of its standing waves:

$$L=n\frac{\lambda_n}{4},\ n=1,3,5,\ldots$$

Solve for and evaluate λ_1:

$$L=\frac{\lambda_{\text{max}}}{4}=\frac{17\,\text{m}}{4}=\boxed{4.25\,\text{m}}$$

(*b*) Relate the length L of an open organ pipe to the wavelengths of its standing waves:

$$L=n\frac{\lambda_n}{2},\ n=1,2,3,\ldots$$

Solve for and evaluate λ_1:

$$L=\frac{\lambda_{\text{max}}}{2}=\frac{17\,\text{m}}{2}=\boxed{8.50\,\text{m}}$$

51 ••

Picture the Problem We can find λ and f by comparing the given wave function to the general wave function for a string fixed at both ends. The speed of the waves can then be found from $v = f\lambda$. We can find the length of the string from its fourth harmonic wavelength.

(*a*) Using the wave function, relate k and λ:

$$k = \frac{2\pi}{\lambda} = 0.20\,\text{cm}^{-1}$$

Solve for λ:

$$\lambda = \frac{2\pi}{0.20\,\text{cm}^{-1}} = 10\pi\,\text{cm} = \boxed{31.4\,\text{cm}}$$

Using the wave function, relate f and ω:

$$\omega = 2\pi f = 300\,\text{s}^{-1}$$

Solve for f:

$$f = \frac{300\,\text{s}^{-1}}{2\pi} = \boxed{47.7\,\text{Hz}}$$

(*b*) Express the speed of transverse waves in terms of their frequency and wavelength:

$$v = f\lambda = (47.7\,\text{Hz})(0.314\,\text{m}) = \boxed{15.0\,\text{m/s}}$$

(*c*) Relate the length of the string to the wavelengths of its standing-wave patterns:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for L when $n = 4$:

$$L = 2\lambda_4 = 2(31.4\,\text{cm}) = \boxed{62.8\,\text{cm}}$$

52 ••

Picture the Problem We can find λ and f by comparing the given wave function to the general wave function for a string fixed at both ends. The speed of the waves can then be found from $v = f\lambda$. In a standing wave pattern, the nodes are separated by one-half wavelength.

(*a*) Express the speed of the traveling waves in terms of their frequency and wavelength:

$$v = f\lambda$$

Using the wave function, relate k and λ:

$$k = \frac{2\pi}{\lambda} = 2.5\,\text{m}^{-1}$$

Solve for λ:

$$\lambda = \frac{2\pi}{2.5\,\text{m}^{-1}} = 0.8\pi\,\text{m} = 2.51\,\text{m}$$

Using the wave function, relate ω

$$\omega = 2\pi f = 500\,\text{s}^{-1}$$

and f:

Solve for f:

$$f = \frac{500\,\text{s}^{-1}}{2\pi} = 79.6\,\text{Hz}$$

Substitute to find v:

$$v = \left(79.6\,\text{s}^{-1}\right)\left(2.51\,\text{m}\right) = \boxed{200\,\text{m/s}}$$

Express the amplitude of the standing wave in terms of the amplitude of the two traveling waves that result in the standing wave:

$$A_{\text{SW}} = 2A$$

Solve for and evaluate A:

$$A = \frac{A_{\text{SW}}}{2} = \frac{0.05\,\text{m}}{2} = \boxed{2.50\,\text{cm}}$$

(*b*) The distance between nodes is half the wavelength:

$$\frac{\lambda}{2} = \frac{2.51\,\text{m}}{2} = \boxed{1.26\,\text{m}}$$

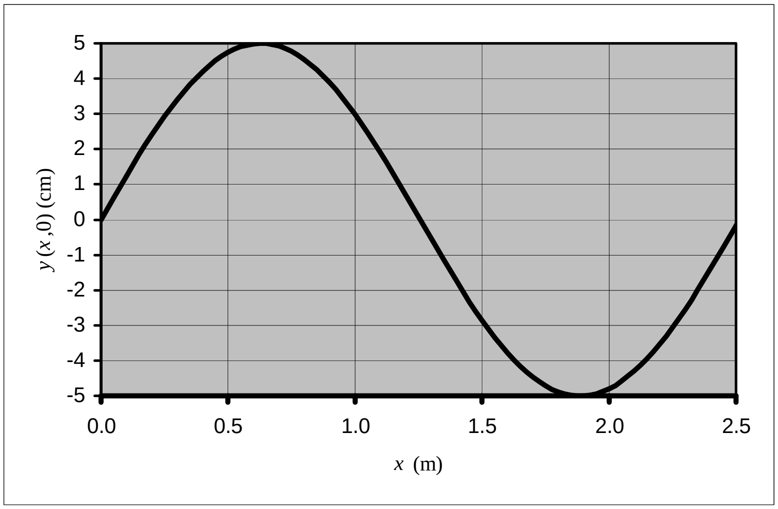
(*c*) Because there is a standing wave on the string, the shortest possible length is:

$$L_{\text{min}} = \frac{\lambda}{2} = \boxed{1.26\,\text{m}}$$

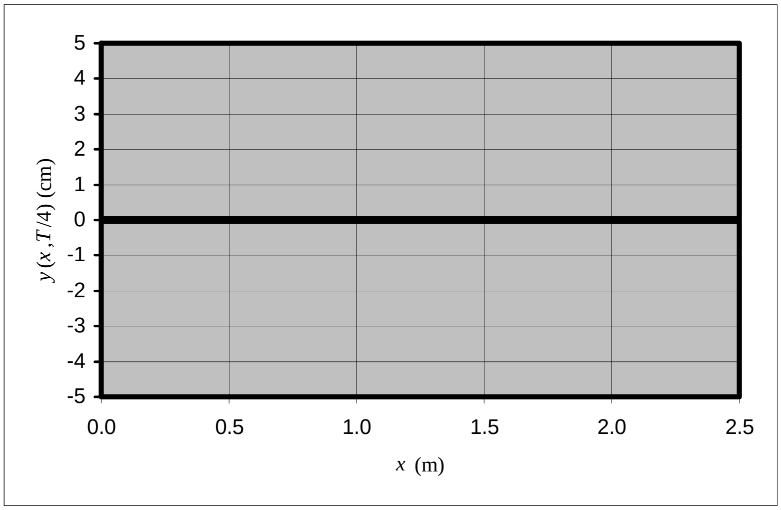
53 ••

Picture the Problem We can evaluate the wave function of Problem 52 at the given times to obtain graphs of position as a function of *x*. We can find the period of the motion from its frequency *f* and find *f* from its angular frequency ω.

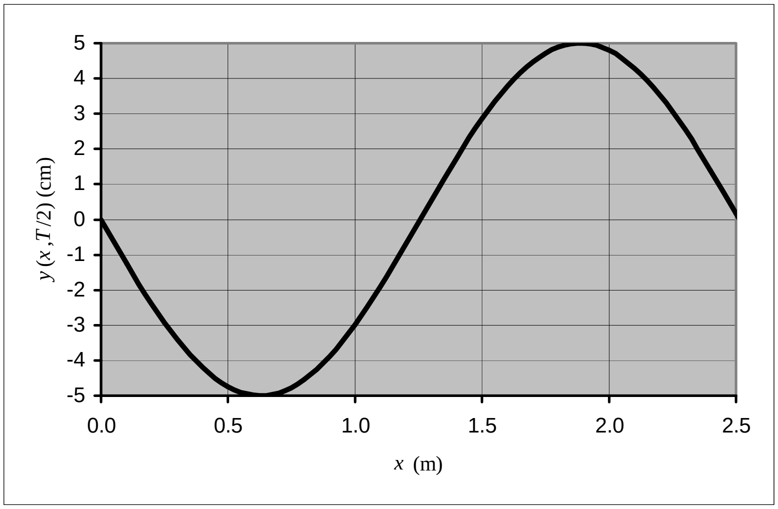
(*a*) The function $y(x,0)$ is shown below:



The functions $y(x,T/4)$ and $y(x,3T/4)$ are shown below. Because these functions are identical, only one graph is shown.



The function $y(x,T/2)$ follows:



(*b*) Express the period in terms of the frequency:

$$T=\frac{1}{f}$$

Using the wave function, relate ω and f:

$$\omega=2\pi f=500\,\text{s}^{-1}$$

Solve for f:

$$f=\frac{500\,\text{s}^{-1}}{2\pi}=79.6\,\text{Hz}$$

Substitute for f and evaluate T:

$$T=\frac{1}{79.6\,\text{s}^{-1}}=\boxed{12.6\,\text{ms}}$$

(*c*) Because the string is moving either upward or downward when $y(x)=0$ for all x, the energy of the wave is entirely kinetic energy.

***54 ••**

Picture the Problem Whether these frequencies are for a string fixed at one end only rather than for a string fixed at both ends can be decided by determining whether they are integral multiples or odd-integral multiples of a fundamental frequency. The length of the string can be found from the wave speed and the wavelength of the fundamental frequency using the standing-wave condition for a string with one end free.

(*a*) Letting the three frequencies be represented by f', f'', and f''', find the ratio of the first two frequencies:

$$\frac{f'}{f''}=\frac{75\,\text{Hz}}{125\,\text{Hz}}=\boxed{\frac{3}{5}}$$

Find the ratio of the second and third frequencies:

$$\frac{f''}{f'''} = \frac{125\,\text{Hz}}{175\,\text{Hz}} = \boxed{\frac{5}{7}}$$

(*b*) $\boxed{\text{There are no even harmonics, so the string must be fixed at one end only.}}$

(*c*) Express the resonance frequencies in terms of the fundamental frequency:

$$f_n = nf_1,\ n = 1, 3, 5, ...$$

Noting that the frequencies are multiples of 25 Hz, we can conclude that:

$$f_1 = \frac{f_3}{3} = \frac{75\,\text{Hz}}{3} = \boxed{25\,\text{Hz}}$$

(*d*) $\boxed{\begin{array}{l}\text{Because the frequencies are 3, 5, and 7 times the fundamental frequency,}\\ \text{they are the third, fifth, and seventh harmonics.}\end{array}}$

(*e*) Express the length of the string in terms of the standing-wave condition for a string fixed at one end:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, ...$$

Using $v = f_1\lambda_1$, find λ_1:

$$\lambda_1 = \frac{v}{f_1} = \frac{400\,\text{m/s}}{25\,\text{s}^{-1}} = 16\,\text{m}$$

Evaluate L for λ_1 = 16 m and n = 1:

$$L = \frac{\lambda_1}{4} = \frac{16\,\text{m}}{4} = \boxed{4.00\,\text{m}}$$

55 ••

Picture the Problem The lowest resonant frequency in this closed-at-one-end tube is its fundamental frequency. This frequency is related to its wavelength through $v = f_{min}\lambda_{max}$. We can use the relationship between the nth harmonic and the fundamental frequency, $f_n = (2n+1)f_1,\ n = 1, 2, 3, ...,$ to find the highest frequency less than or equal to 5000 Hz that will produce resonance.

(*a*) Express the length of the space above the water in terms of the standing-wave condition for a closed pipe:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, ...$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n},\ n = 1, 3, 5, ...$$

λ_{max} corresponds to n = 1:

$$\lambda_{max} = 4L = 4(1.2\,\text{m}) = 4.8\,\text{m}$$

Using $v = f_{min}\lambda_{max}$, find f_{min}:

$$f_{min} = \frac{v}{\lambda_{max}} = \frac{340\,\text{m/s}}{4.8\,\text{m}} = \boxed{70.8\,\text{Hz}}$$

(*b*) Express the nth harmonic in terms of the fundamental frequency (first harmonic):

$$f_n = (2n+1)f_1,\ n = 1, 2, 3, \ldots$$

To find the highest harmonic below 5000 Hz, let f_n = 5000 Hz:

$$5000\,\text{Hz} = (2n+1)(70.8\,\text{Hz})$$

Solve for n (an integer) to obtain:

n = 34

Evaluate f_{34}:

$$f_{34} = 69\,f_1 = 69(70.8\,\text{Hz}) = \boxed{4.89\,\text{kHz}}$$

(*c*) There are 34 harmonics higher than the fundamental frequency so the total number is:

$$\boxed{35}$$

56 ••

Picture the Problem Sound waves of frequency 460 Hz are excited in the tube, whose length L can be adjusted. Resonance occurs when the effective length of the tube $L_{eff} = L + \Delta L$ equals $\frac{1}{4}\lambda,\ \frac{3}{4}\lambda,\ \frac{5}{4}\lambda,$ and so on, where λ is the wavelength of the sound. Even though the pressure node is not exactly at the end of the tube, the wavelength can be found from the fact that the distance between water levels for successive resonances is half the wavelength. We can find the speed from $v = f\lambda$ and the end correction from the fact that, for the fundamental, $L_{eff} = \frac{1}{4}\lambda = L_1 + \Delta L,$ where L_1 is the distance from the top of the tube to the location of the first resonance.

(*a*) Relate the speed of sound in air to its wavelength and the frequency of the tuning fork:

$$v = f\lambda$$

Using the fact that nodes are separated by one-half wavelength, find the wavelength of the sound waves:

$$\lambda = 2(55.8\,\text{cm} - 18.3\,\text{cm}) = 75\,\text{cm}$$

Substitute and evaluate v:

$$v = (460\,\text{s}^{-1})(0.75\,\text{m}) = \boxed{345\,\text{m/s}}$$

(*b*) Relate the end correction ΔL to the wavelength of the sound and effective length of the tube:

$$L_{\text{eff}} = \tfrac{1}{4}\lambda = L_1 + \Delta L$$

Solve for and evaluate ΔL:

$$\Delta L = \tfrac{1}{4}\lambda - L_1 = \tfrac{1}{4}(75\,\text{cm}) - 18.3\,\text{cm} = \boxed{0.450\,\text{cm}}$$

***57** ••

Picture the Problem We can use $v = f\lambda$ to express the fundamental frequency of the organ pipe in terms of the speed of sound and $v = \sqrt{\dfrac{\gamma RT}{M}}$ to relate the speed of sound and the fundamental frequency to the absolute temperature.

Express the fundamental frequency of the organ pipe in terms of the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound to the temperature:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ and R are constants, M is the molar mass, and T is the absolute temperature.

Substitute to obtain:

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

Using primed quantities to represent the higher temperature, express the new frequency as a function of T:

$$f' = \frac{1}{\lambda'}\sqrt{\frac{\gamma RT'}{M}}$$

As we have seen, λ is proportional to the length of the pipe. For the first question, we assume the length of the pipe does not change, so $\lambda = \lambda'$. Then the ratio of f' to f is:

$$\frac{f'}{f} = \sqrt{\frac{T'}{T}}$$

Solve for and evaluate f' with $T' = 305$ K and $T = 289$ K:

$$f' = f_{305\,\text{K}} = f_{289\,\text{K}}\sqrt{\frac{305\,\text{K}}{289\,\text{K}}}$$

$$= (440.0\,\text{Hz})\sqrt{\frac{305\,\text{K}}{289\,\text{K}}}$$

$$= \boxed{452\,\text{Hz}}$$

It would be better to have the pipe expand so that v/L, where L is the length of the pipe, is independent of temperature.

58 ••

Picture the Problem We can express the wavelength of the fundamental in a pipe open at both ends in terms of the effective length of the pipe using $\lambda = 2L_{\text{eff}} = 2(L + \Delta L)$, where L is the physical length of the pipe and $\lambda = v/f$. Solving these equations simultaneously will lead us to an expression for L as a function of D.

Express the wavelength of the fundamental in a pipe open at both ends in terms of the pipe's effective length L_{eff}:

$$\lambda = 2L_{\text{eff}} = 2(L + \Delta L)$$

where L is its physical length.

Solve for L to obtain:

$$L = \frac{\lambda}{2} - \Delta L = \frac{\lambda}{2} - 0.3186D$$

Express the wavelength of middle C in terms of its frequency f and the speed of sound v:

$$\lambda = \frac{v}{f}$$

Substitute to obtain:

$$L = \frac{v}{2f} - 0.3186D$$

Substitute numerical values to express L as a function of D:

$$L = \frac{340\,\text{m/s}}{2(256\,\text{s}^{-1})} - 0.3186D$$

$$= 0.664\,\text{m} - 0.3186D$$

Evaluate L for $D = 1$ cm:

$$L = 0.664\,\text{m} - 0.3186(0.01\,\text{m})$$

$$= \boxed{66.1\,\text{cm}}$$

Evaluate L for $D = 10$ cm:

$$L = 0.664\,\text{m} - 0.3186(0.1\,\text{m})$$

$$= \boxed{63.2\,\text{cm}}$$

Evaluate L for D = 30 cm:

$$L = 0.664\,\text{m} - 0.3186(0.3\,\text{m}) = \boxed{56.8\,\text{cm}}$$

59 ••

Picture the Problem We know that, when a string is vibrating in its fundamental mode, its ends are one-half wavelength apart. We can use $v = f\lambda$ to express the fundamental frequency of the organ pipe in terms of the speed of sound and $v = \sqrt{F/\mu}$ to relate the speed of sound and the fundamental frequency to the tension in the string. We can use this relationship between f and L, the length of the string, to find the length of string when it vibrates with a frequency of 650 Hz.

(*a*) Express the wavelength of the standing wave, vibrating in its fundamental mode, to the length L of the string:

$$\lambda = 2L = 2(40\,\text{cm}) = \boxed{80\,\text{cm}}$$

(*b*) Relate the speed of the waves combining to form the standing wave to its frequency and wavelength:

$$v = f\lambda$$

Express the speed of transverse waves as a function of the tension in the string:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute and solve for F to obtain:

$$F = f^2\lambda^2\frac{m}{L}$$

where m is the mass of the string and L is its length.

Substitute numerical values and evaluate F:

$$F = \left(500\,\text{s}^{-1}\right)^2(0.8\,\text{m})^2\frac{1.2\times10^{-3}\,\text{kg}}{0.4\,\text{m}} = \boxed{480\,\text{N}}$$

(*c*) Using $v = f\lambda$ and assuming that the string is still vibrating in its fundamental mode, express its frequency in terms of its length:

$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

Solve for L:

$$L = \frac{v}{2f}$$

Letting primed quantities refer to a second length and frequency, express L' in terms of f':

$$L' = \frac{v}{2f'}$$

Express the ratio of L' to L and solve for L':

$$\frac{L'}{L} = \frac{f}{f'} \Rightarrow L' = \frac{f}{f'}L$$

Evaluate $L_{650\text{ Hz}}$:

$$L_{650\,\text{Hz}} = \frac{500\,\text{Hz}}{650\,\text{Hz}} L_{500\,\text{Hz}}$$

$$= \frac{500\,\text{Hz}}{650\,\text{Hz}}(40\,\text{cm}) = 30.77\,\text{cm}$$

You should place your finger 9.23 cm from the scroll bridge.

60 ••

Picture the Problem Let f' represent the frequencies corresponding to the A, B, C, and D notes and $x(f')$ represent the distances from the end of the string that a finger must be placed to play each of these notes. Then, the distances at which the finger must be placed are given by $x(f') = L(f_G) - L(f')$.

Express the distances at which the finger must be placed in terms of the lengths of the G string and the frequencies f' of the A, B, C, and D notes:

$$x(f') = L(f_G) - L(f') \qquad (1)$$

Assuming that it vibrates in its fundamental mode, express the frequency of the G string in terms of its length:

$$f_G = \frac{v}{\lambda_G} = \frac{v}{2L_G}$$

Solve for L_G:

$$L_G = \frac{v}{2f_G}$$

Letting primed quantities refer to the string lengths and frequencies of

$$L' = \frac{v}{2f'}$$

the A, B, C, and D notes, express L' in terms of f':

Express the ratio of L' to L and solve for L':

$$\frac{L'}{L_G} = \frac{f_G}{f'} \Rightarrow L' = \frac{f_G}{f'} L_G$$

Evaluate $L' = L(f')$ for the notes A, B, C and D to complete the table:

Note	Frequency	$L(f')$
	(Hz)	(cm)
A	220	26.73
B	247	23.81
C	262	22.44
D	294	20.00

Use equation (1) to evaluate $x(f')$ and complete the table to the right:

Note	Frequency	$L(f')$	$x(f')$
	(Hz)	(cm)	(cm)
A	220	26.73	3.27
B	247	23.81	6.19
C	262	22.44	7.56
D	294	20.00	10.0

61 ••

Picture the Problem We can use the fact that the resonance frequencies are multiples of the fundamental frequency to find both the fundamental frequency and the harmonic numbers corresponding to 375 Hz and 450 Hz. We can find the length of the string by relating it to the wavelength of the waves on it and the wavelength to the speed and frequency of the waves. The speed of the waves is, in turn, a function of the tension in the string and its linear density, both of which we are given.

(*a*) Express 375 Hz as an integer multiple of the fundamental frequency of the string:

$$nf_1 = 375\,\text{Hz} \qquad (1)$$

Express 450 Hz as an integer multiple of the fundamental frequency of the string:

$$(n+1)f_1 = 450\,\text{Hz} \qquad (2)$$

Solve equations (1) and (2) simultaneously for f_1:

$$f_1 = \boxed{75.0\,\text{Hz}}$$

(*b*) Substitute in equation (1) to obtain:

$n = 5$

$$\boxed{\text{The harmonics are the fifth and sixth.}}$$

(*c*) Express the length of the string as a function of the speed of transverse waves on it and its fundamental frequency:

$$L = \frac{\lambda}{2} = \frac{v}{2f_1}$$

Express the speed of transverse waves on the string in terms of the tension in the string and its linear density:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$L = \frac{1}{2f_1}\sqrt{\frac{F}{\mu}}$$

Substitute numerical values and evaluate *L*:

$$L = \frac{1}{2(75\,\text{s}^{-1})}\sqrt{\frac{360\,\text{N}}{4\times10^{-3}\,\text{kg/m}}} = \boxed{2.00\,\text{m}}$$

62 ••

Picture the Problem We can use the fact that the resonance frequencies are multiples of the fundamental frequency and are expressible in terms of the speed of the waves and their wavelengths to find the harmonic numbers corresponding to wavelengths of 0.54 m and 0.48 m. We can find the length of the string by using the standing-wave condition for a string fixed at both ends.

(*a*) Express the frequency of the *n*th harmonic in terms of its wavelength:

$$nf_1 = \frac{v}{\lambda_n} = \frac{v}{0.54\,\text{m}}$$

Express the frequency of the (*n* + 1)th harmonic in terms of its wavelength:

$$(n+1)f_1 = \frac{v}{\lambda_{n+1}} = \frac{v}{0.48\,\text{m}}$$

Solve these equations simultaneously for *n*:

$n = 8$

$$\boxed{\text{The harmonics are the eighth and ninth.}}$$

(*b*) Using the standing-wave condition, both ends fixed, relate the length of the string to the wavelength of its *n*th harmonic:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, ...$$

Evaluate *L* for the eighth harmonic:

$$L = 8\left(\frac{0.54\,\text{m}}{2}\right) = \boxed{2.16\,\text{m}}$$

63 ••

Picture the Problem The linear densities of the strings are related to the transverse wave speed and tension through $v = \sqrt{F/\mu}$. We can use $v = f\lambda = 2fL$ to relate the frequencies of the violin strings to their lengths and linear densities.

(*a*) Relate the speed of transverse waves on a string to the tension in the string and solve for the string's linear density:

$$v = \sqrt{\frac{F}{\mu}}$$

and

$$\mu = \frac{F}{v^2}$$

Express the dependence of the speed of the transverse waves on their frequency and wavelength:

$$v = f_E\lambda = 2f_E L$$

Substitute to obtain:

$$\mu_E = \frac{F_E}{4f_E^2L^2}$$

Substitute numerical values and evaluate μ_E:

$$\mu_E = \frac{90\,\text{N}}{4\left[1.5\left(440\,\text{s}^{-1}\right)\right]^2(0.3\,\text{m})^2} = 5.74\times10^{-4}\ \text{kg/m} = \boxed{0.574\,\text{g/m}}$$

(*b*) Evaluate μ_A:

$$\mu_A = \frac{90\,\text{N}}{4\left(440\,\text{s}^{-1}\right)^2(0.3\,\text{m})^2} = 1.29\times10^{-3}\ \text{kg/m} = \boxed{1.29\,\text{g/m}}$$

Evaluate μ_D:

$$\mu_D = \frac{90\,\text{N}}{4\left(293\,\text{s}^{-1}\right)^2\left(0.3\,\text{m}\right)^2} = 2.91\times10^{-3}\,\text{kg/m} = \boxed{2.91\,\text{g/m}}$$

Evaluate μ_G:

$$\mu_G = \frac{90\,\text{N}}{4\left(195\,\text{s}^{-1}\right)^2\left(0.3\,\text{m}\right)^2} = 6.57\times10^{-3}\,\text{kg/m} = \boxed{6.57\,\text{g/m}}$$

64 ••

Picture the Problem The spatial period is one-half the wavelength of the standing wave produced by the sound and its reflection. Hence we can solve $c = f'\lambda'$ for λ' and use $f' = f\left[1/\left(1 - v/c\right)\right]$ to derive an expression for $\lambda'/2$ in terms of c, v, and f.

(*a*) Express the wavelength of the reflected sound as a function of its frequency and the speed of sound in air:

$$\lambda' = \frac{c}{f'}$$

Use the expression for the Doppler-shift in frequency when to source is in motion to obtain:

$$f' = f\,\frac{1}{1-\dfrac{v}{c}}$$

where c is the speed of sound.

Substitute to obtain:

$$\frac{\lambda'}{2} = \frac{c}{2f'} = \frac{c}{2f\,\dfrac{1}{1-\dfrac{v}{c}}} = \frac{c}{2f}\left(1-\frac{v}{c}\right) = \frac{c-v}{2f}$$

Substitute numerical values and evaluate the spatial period of the standing wave:

$$\frac{\lambda'}{2} = \frac{340\,\text{m/s} - 22.4\,\text{m/s}}{2\left(500\,\text{s}^{-1}\right)} = \boxed{0.318\,\text{m}}$$

(*b*) | As the ambulance moves closer to the wall, the sound waves from its siren will periodically move in and out of resonance (i.e., the reflected waves will sometimes interfere constructively and sometimes partially destructively) so the intensity will periodically get louder and softer. |

65 ••

Picture the Problem Beat frequencies are heard when the strings are vibrating with slightly different frequencies. To understand the beat frequency heard when the A and E strings are bowed simultaneously, we need to consider the harmonics of both strings. In part (c) we'll relate the tension in the string to the frequency of its vibration and set up a proportion involving the frequencies corresponding to the two tensions that we can solve for the tension when the E string is perfectly tuned.

(*a*) | The two sounds produce a beat because the third harmonic of the A string equals the second harmonic of the E string, and the original frequency of the E string is slightly greater than 660 Hz. If $f_E = (660 + \Delta f)$ Hz, a beat of $2\Delta f$ will be heard. |

(*b*) Because f_{beat} increases with increasing tension, the frequency of the E string is greater than 660 Hz. Thus the frequency of the E string is:

$$f_E = 660\text{ Hz} + \tfrac{1}{2}(3\text{ Hz}) = \boxed{661.5\text{ Hz}}$$

(*c*) Express the frequency of a string as a function of its tension:

$$f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{F}{\mu}}$$

When the frequency of the E string is 660 Hz we have:

$$660\text{ Hz} = \frac{1}{\lambda}\sqrt{\frac{F_{660\text{ Hz}}}{\mu}}$$

When the frequency of the E string is 661.5 Hz we have:

$$661.5\text{ Hz} = \frac{1}{\lambda}\sqrt{\frac{80\text{ N}}{\mu}}$$

Divide the first of these equations by the second and solve for $F_{660\text{ Hz}}$ to obtain:

$$F_{660\text{ Hz}} = \left(\frac{660\text{ Hz}}{661.5\text{ Hz}}\right)^2(80\text{ N}) = \boxed{79.6\text{ N}}$$

66 ••

Picture the Problem We can use the condition for constructive interference of the waves reflected from the walls in front of and behind you to relate the path difference to the wavelength of the sound. We can find the wavelength of the sound from its frequency and the speed of sound in air.

Express the total path difference as you walk toward the far wall of the hall:

$$\Delta x = \Delta x_{\text{near wall}} + \Delta x_{\text{far wall}} \quad (1)$$

Express the condition on the path difference for constructive interference:

$$n\lambda = \Delta x \text{ where } n = 1, 2, 3, \ldots \quad (2)$$

The reduction in the distance to the nearer wall as you walk a distance *d* is:

$$\Delta x_{\text{near wall}} = 2d$$

The increase in the distance to the farther wall as you walk a distance *d* is:

$$\Delta x_{\text{far wall}} = 2d$$

Substitute in equation (1) to find the total path difference as you walk a distance *d*:

$$\Delta x = 2d + 2d = 4d$$

Relate λ to f and v:

$$\lambda = \frac{v}{f}$$

Substitute in equation (2) to obtain:

$$n\frac{v}{f} = 4d$$

Solve for and evaluate *d* for *n* = 1:

$$d = \frac{v}{4f} = \frac{340\text{ m/s}}{4\left(680\text{ s}^{-1}\right)} = \boxed{12.5\text{ cm}}$$

***67 ••**

Picture the Problem Let the wave function for the wave traveling to the right be $y_R(x,t) = A\sin(kx - \omega t - \delta)$ and the wave function for the wave traveling to the left be $y_L(x,t) = A\sin(kx + \omega t + \delta)$ and use the identity $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ to show that the sum of the wave functions can be written in the form $y(x,t) = A'\sin kx\cos(\omega t + \delta)$.

Express the sum of the traveling waves of equal amplitude moving in opposite directions:

$$y(x,t) = y_{\rm R}(x,t) + y_{\rm L}(x,t) = A\sin(kx - \omega t - \delta) \quad + A\sin(kx + \omega t + \delta)$$

Use the trigonometric identity to obtain:

$$y(x,t) = 2A\sin\left(\frac{kx - \omega t - \delta + kx + \omega t + \delta}{2}\right)\cos\left(\frac{kx - \omega t - \delta - kx - \omega t - \delta}{2}\right)$$
$$= 2A\sin kx\cos(-\omega t - \delta)$$

Because the cosine function is even; i.e., $\cos(-\theta) = \cos\theta$:

$$y(x,t) = 2A\sin kx\cos(\omega t + \delta)$$
$$= A'\sin kx\cos(\omega t + \delta)$$

where $A' = 2A$.

Thus we have:

$$y(x,t) = \boxed{A'\sin kx\cos(\omega t + \delta)}$$

provided $A' = 2A$.

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Picture the Problem We can find ω_3 and k_3 from the given information and substitute to find the wave function for the 3rd harmonic. We can use the time-derivative of this expression (the transverse speed) to express the kinetic energy of a segment of mass *dm* and length *dx* of the string. Integrating this expression will give us the maximum kinetic energy of the string in terms of its mass.

(*a*) Write the general form of the wave function for the 3rd harmonic:

$$y_3(x,t) = A_3\sin k_3x\cos\omega_3 t$$

Evaluate ω_3:

$$\omega_3 = 2\pi f_3 = 2\pi\left(100\,{\rm s}^{-1}\right) = 200\pi\,{\rm s}^{-1}$$

Using the standing-wave condition for a string fixed at one end, relate the length of the string to its 3rd harmonic wavelength:

$$L = 3\frac{\lambda_3}{4}$$

and

$$\lambda_3 = \frac{4}{3}L = \frac{4}{3}(2\,{\rm m}) = \frac{8}{3}\,{\rm m}$$

Evaluate k_3:

$$k_3 = \frac{2\pi}{\lambda_3} = \frac{2\pi}{(8/3)\,{\rm m}} = \frac{3\pi}{4}\,{\rm m}^{-1}$$

Substitute numerical values and evaluate K_{max}:

$$K_{max} = \tfrac{1}{4}m\left(200\pi\ \text{s}^{-1}\right)^2(0.03\ \text{m})^2 = \boxed{(88.8\ \text{J/kg})m}$$

Substitute to obtain:

$$y_3(x,t) = \boxed{(0.03\ \text{m})\sin\left[\left(\frac{3\pi}{4}\ \text{m}^{-1}\right)x\right]\cos\left(200\pi\ \text{s}^{-1}\right)t}$$

(*b*) Express the kinetic energy of a segment of string of mass *dm*:

$$dK = \tfrac{1}{2}dm v_y^2$$

Express the mass of the segment in terms of its length *dx* and the linear density of the string:

$$dm = \mu dx$$

Using our result in (*a*), evaluate v_y:

$$\begin{aligned} v_y &= \frac{\partial}{\partial t}\left[(0.03\ \text{m})\sin\left[\left(\frac{3\pi}{4}\ \text{m}^{-1}\right)x\right]\cos\left(200\pi\ \text{s}^{-1}\right)t\right] \\ &= -\left(200\pi\ \text{s}^{-1}\right)(0.03\ \text{m})\sin\left[\left(\frac{3\pi}{4}\ \text{m}^{-1}\right)x\right]\sin\left(200\pi\ \text{s}^{-1}\right)t \\ &= -(6\pi\ \text{m/s})\sin\left[\left(\frac{3\pi}{4}\ \text{m}^{-1}\right)x\right]\sin\left(200\pi\ \text{s}^{-1}\right)t \end{aligned}$$

Substitute to obtain:

$$dK = \boxed{\tfrac{1}{2}\left[(6\pi\ \text{m/s})\sin\left[\left(\frac{3\pi}{4}\ \text{m}^{-1}\right)x\right]\sin\left(200\pi\ \text{s}^{-1}\right)t\right]^2 \mu dx}$$

Express the condition on the time that *dK* is a maximum:

$$\sin\left(200\pi\ \text{s}^{-1}\right)t = 1$$

or

$$\left(200\pi\ \text{s}^{-1}\right)t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

Solve for and evaluate *t*:

$$t = \frac{1}{200\pi\ \text{s}^{-1}}\frac{\pi}{2}, \frac{1}{200\pi\ \text{s}^{-1}}\frac{3\pi}{2}, \ldots = \boxed{2.50\ \text{ms}, 7.50\ \text{ms}, \ldots}$$

Because the string's maximum kinetic energy occurs when $y(x,t) = 0$:

The string is a straight line.

(*c*) Integrate dK from (*b*) over the length of the string to obtain:

$$K_{max} = \int_0^L \tfrac{1}{2}[\omega A \sin kx \sin \omega t]^2 \mu dx$$
$$= \tfrac{1}{2}\mu\omega^2 A^2 \int_0^L \sin^2 kx dx$$
$$= \tfrac{1}{2}\mu\omega^2 A^2 \frac{1}{k}\left[\tfrac{1}{2}kx - \tfrac{1}{4}\sin 2kx\right]_0^L$$
$$= \tfrac{1}{4} m\omega^2 A^2$$

where m is the mass of the string.

***69 ••**

Picture the Problem We can equate the expression for the velocity of a wave on a string and the expression for the velocity of a wave in terms of its frequency and wavelength to obtain an expression for the weight that must be suspended from the end of the string in order to produce a given standing wave pattern. By using the condition on the wavelength that must be satisfied at resonance, we can express the weight on the end of the string in terms of μ, f, L, and an integer n and then evaluate this expression for $n = 1$, 2, and 3 for the first three standing wave patterns.

Express the velocity of a wave on the string in terms of the tension T in the string and its linear density μ:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$$

where mg is the weight of the object suspended from the end of the string.

Express the wave speed in terms of its wavelength λ and frequency f:

$$v = f\lambda$$

Eliminate v to obtain:

$$f\lambda = \sqrt{\frac{mg}{\mu}}$$

Solve for mg:

$$mg = \mu f^2 \lambda^2$$

Express the condition on λ that corresponds to resonance:

$$\lambda = \frac{2L}{n}, n = 1, 2, 3, \ldots$$

Substitute to obtain:

$$mg = \mu f^2 \left(\frac{2L}{n}\right)^2, n = 1, 2, 3, \ldots$$

or

$$mg = \frac{4\mu f^2 L^2}{n^2}, n = 1, 2, 3, \ldots$$

Evaluate mg for n = 1:

$$mg = \frac{4(0.415\,\text{g/m})(80\,\text{s}^{-1})^2(0.2\,\text{m})^2}{(1)^2} = \boxed{0.425\,\text{N}}$$

which corresponds, at sea level, to a mass of 43.3 g.

Evaluate mg for n = 2:

$$mg = \frac{4(0.415\,\text{g/m})(80\,\text{s}^{-1})^2(0.2\,\text{m})^2}{(2)^2} = \boxed{0.106\,\text{N}}$$

which corresponds, at sea level, to a mass of 10.8 g.

Wave Packets

70 •

Picture the Problem We can find the maximum duration of each pulse under the conditions given in the problem from the reciprocal of frequency of the pulses and the range of frequencies from the wave packet condition on $\Delta\omega$ and Δt.

(a) The maximum duration of each pulse is its period:

$$T = \frac{1}{f} = \frac{1}{10^7\,\text{s}^{-1}} = 10^{-7}\,\text{s} = \boxed{0.100\,\mu\text{s}}$$

(b) Express the wave packet condition on $\Delta\omega$ and Δt:

$$\Delta\omega\Delta t \approx 1 \text{ or } 2\pi\Delta f\Delta t \approx 1$$

Solve for Δf:

$$\Delta f \approx \frac{1}{2\pi\Delta t} = \frac{T}{2\pi}$$

Substitute numerical values and evaluate Δf:

$$\Delta f \approx \frac{10^7\,\text{s}^{-1}}{2\pi} = \boxed{1.59\,\text{MHz}}$$

71 •

Picture the Problem We can approximate the duration of the pulse from the product of the number of cycles in the interval and the period of each cycle and the wavelength from the number of complete wavelengths in Δx. We can use its definition to find the wave number k from the wavelength λ.

(a) Relate the duration of the pulse to the number of cycles in the interval and the period of each cycle:

$$\Delta t \approx NT = \boxed{\frac{N}{f_0}}$$

(*b*) There are about *N* complete wavelengths in Δx; hence:

$$\lambda \approx \boxed{\frac{\Delta x}{N}}$$

(*c*) Use its definition to express the wave number *k*:

$$k = \frac{2\pi}{\lambda} = \boxed{\frac{2\pi N}{\Delta x}}$$

(*d*) *N* is uncertain because the waveform dies out gradually rather than stopping abruptly at some time; hence, where the pulse starts and stops is not well defined.

(*e*) Using our result in part (*c*), express the uncertainty in *k*:

$$\Delta k = \frac{2\pi \Delta N}{\Delta x} = \boxed{\frac{2\pi}{\Delta x}}$$

because $\Delta N = \pm 1$.

General Problems

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Picture the Problem We can use $v = f\lambda$ and $v = \sqrt{F/\mu}$ to relate the tension in the piano wire to its fundamental frequency.

Relate the tension in the wire to the speed of transverse waves on it:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

Express the speed of the transverse in terms of their wavelength and frequency:

$$v = f\lambda$$

Equate these expressions and solve for *F* to obtain:

$$F = \frac{mf^2\lambda^2}{L}$$

Relate λ for the fundamental mode of vibration to the length of the piano wire:

$$\lambda = 2L$$

Substitute to obtain:

$$F = 4mf^2L$$

Substitute numerical values and evaluate *F*:

$$F = 4\left(7\times10^{-3}\ \text{kg}\right)\left(261.63\ \text{s}^{-1}\right)^2\left(0.8\ \text{m}\right) = \boxed{1.53\ \text{kN}}$$

73 •

Picture the Problem We can use $v = f_n\lambda_n$ to express the resonance frequencies of the ear canal in terms of their wavelengths and $L = n\frac{\lambda_n}{4}, n = 1, 3, 5, ...$ to relate the length of the ear canal to its resonance wavelengths.

(*a*) Relate the resonance frequencies to the speed of sound and the wavelength of the compressional vibrations:

$$f_n = \frac{v}{\lambda_n}$$

Express the condition for constructive interference in a pipe that is open at one end:

$$L = n\frac{\lambda_n}{4}, n = 1, 3, 5, ...$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{4L} = n\frac{340\,\text{m/s}}{4\left(2.5\times10^{-2}\,\text{m}\right)} = n(3.40\,\text{kHz})$$

Evaluate f_1, f_2, and f_3:

$$f_1 = \boxed{3.40\,\text{kHz}},$$

$$f_3 = 3\times3.40\,\text{kHz} = \boxed{10.2\,\text{kHz}},$$

and

$$f_5 = 5\times3.40\,\text{kHz} = \boxed{17.0\,\text{kHz}}$$

(*b*) Frequencies near 3400 Hz will be most readily perceived.

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Picture the Problem We can use $L = n\frac{\lambda_n}{4}, n = 1, 3, 5, ...$ to express the wavelengths of the fundamental and next two harmonics in terms of the length of the rope and $v = f_n\lambda_n$ and $v = \sqrt{\frac{F}{\mu}}$ to relate the resonance frequencies to their wavelengths.

(*a*) Express the condition for constructive interference on a rope

$$L = n\frac{\lambda_n}{4}, n = 1, 3, 5, ...$$

that is fixed at one end:

Solve for λ_n:

$$\lambda_n = \frac{4L}{n} = \frac{4(4\,\text{m})}{n} = \frac{16\,\text{m}}{n}$$

Evaluate λ_n for n = 1, 3, and 5:

$$\lambda_1 = \boxed{16.0\,\text{m}}$$

$$\lambda_3 = \frac{16\,\text{m}}{3} = \boxed{5.33\,\text{m}}$$

and

$$\lambda_5 = \frac{16\,\text{m}}{5} = \boxed{3.20\,\text{m}}$$

(*b*) Relate the resonance frequencies to the speed and wavelength of the transverse waves:

$$f_n = \frac{v}{\lambda_n}$$

Express the speed of the transverse waves as a function of the tension in the rope:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

where m and L are the mass and length of the rope.

Substitute to obtain:

$$f_n = \frac{1}{\lambda_n}\sqrt{\frac{FL}{m}} = \frac{1}{\lambda_n}\sqrt{\frac{(400\,\text{N})(4\,\text{m})}{0.16\,\text{kg}}} = \frac{100\,\text{m/s}}{\lambda_n}$$

Evaluate f_n for n = 1, 3, and 5:

$$f_1 = \frac{100\,\text{m/s}}{16\,\text{m}} = \boxed{6.25\,\text{Hz}}$$

$$f_3 = \frac{100\,\text{m/s}}{5.33\,\text{m}} = \boxed{18.8\,\text{Hz}}$$

and

$$f_5 = \frac{100\,\text{m/s}}{3.20\,\text{m}} = \boxed{31.3\,\text{Hz}}$$

75 ••

Picture the Problem The path difference at the point where the resultant wave an amplitude A is related to the phase shift between the interfering waves according to $\Delta x/\lambda = \delta/2\pi$. We can use this relationship to find the phase shift and the expression for the amplitude resulting from the superposition of two waves of the same amplitude and frequency to find the phase shift.

Express the relation between the path difference and the phase shift at the point where the resultant wave has an amplitude A:

$$\Delta x = \lambda \frac{\delta}{2\pi}$$

Express the amplitude resulting from the superposition of two waves of the same amplitude and frequency:

$$A = 2y_0 \cos \tfrac{1}{2}\delta$$

Solve for and evaluate δ:

$$\delta = 2\cos^{-1}\frac{A}{2y_0} = 2\cos^{-1}\frac{A}{2A} = \frac{2\pi}{3}$$

Substitute and simplify to obtain:

$$\Delta x = \lambda \frac{2\pi/3}{2\pi} = \boxed{\tfrac{1}{3}\lambda}$$

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Picture the Problem We can use $v = f_n\lambda_n$ to express the resonance frequencies of the string in terms of their wavelengths and $L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$ to relate the length of the string to the resonance wavelengths for a string fixed at both ends. Our strategy for part (*b*) will be the same … except that we'll use the standing-wave condition $L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$ for strings with one end free.

(*a*) Relate the frequencies of the harmonics to their wavelengths and the speed of transverse waves on the string:

$$f_n = \frac{v}{\lambda_n}$$

Express the standing-wave condition for a string with both ends fixed:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{2L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{2L}$$

Express the speed of the transverse waves as a function of the tension in the string:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$f_n = n\frac{1}{2L}\sqrt{\frac{F}{\mu}} = n\frac{1}{2(35\,\text{m})}\sqrt{\frac{18\,\text{N}}{0.0085\,\text{kg/m}}} = n(0.657\,\text{Hz})$$

Calculate the 1st four harmonics:

$$f_1 = \boxed{0.657\,\text{Hz}}$$

$$f_2 = 2(0.657\,\text{Hz}) = \boxed{1.31\,\text{Hz}}$$

$$f_3 = 3(0.657\,\text{Hz}) = \boxed{1.97\,\text{Hz}}$$

and

$$f_4 = 4(0.657\,\text{Hz}) = \boxed{2.63\,\text{Hz}}$$

(*b*) Express the standing-wave condition for a string fixed at one end:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n}$$

The resonance frequencies equation becomes:

$$f_n = n\frac{1}{4L}\sqrt{\frac{F}{\mu}} = n\frac{1}{4(35\,\text{m})}\sqrt{\frac{18\,\text{N}}{0.0085\,\text{kg/m}}} = n(0.329\,\text{Hz})$$

Calculate the 1st four harmonics:

$$f_1 = \boxed{0.329\,\text{Hz}}$$

$$f_3 = 3(0.329\,\text{Hz}) = \boxed{0.987\,\text{Hz}}$$

$$f_5 = 5(0.329\,\text{Hz}) = \boxed{1.65\,\text{Hz}}$$

and

$$f_7 = 7(0.329\,\text{Hz}) = \boxed{2.30\,\text{Hz}}$$

77 ••

Picture the Problem We'll model the shaft as a pipe of length *L* with one end open. We can relate the frequencies of the harmonics to their wavelengths and the speed of sound using $v = f_n\lambda_n$ and the depth of the mine shaft to the resonance wavelengths using the standing-wave condition for a pipe with one end open; $L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$.

Relate the frequencies of the harmonics to their wavelengths and the speed of sound:

$$f_n = \frac{v}{\lambda_n}$$

Express the standing-wave condition for a pipe with one end open:

$$L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{4L}$$

For f_n = 63.58 Hz:

$$63.58\,\text{Hz} = n\frac{v}{4L}$$

For f_{n+2} = 89.25 Hz:

$$89.25\,\text{Hz} = (n+2)\frac{v}{4L}$$

Divide either of these equations by the other and solve for n to obtain:

$$n = 4.95 \approx 5$$

Substitute in the equation for $f_n = f_5$ = 63.58 Hz:

$$f_5 = \frac{5v}{4L}$$

Solve for and evaluate L:

$$L = \frac{5v}{4f_5} = \frac{5(340\,\text{m/s})}{4(63.58\,\text{s}^{-1})} = \boxed{6.68\,\text{m}}$$

78 ••

Picture the Problem We can use the standing-wave condition for a string with one end free to find the wavelength of the 5th harmonic and the definitions of the wave number and angular frequency to calculate these quantitities. We can then substitute in the wave function for a wave in the nth harmonic to find the wave function for this standing wave.

(*a*) Express the standing-wave condition for a string with one end free:

$$L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$$

Solve for and evaluate λ_5:

$$\lambda_5 = \frac{4L}{5} = \frac{4(5\,\text{m})}{5} = \boxed{4.00\,\text{m}}$$

(*b*) Use its definition to calculate the wave number:

$$k_5 = \frac{2\pi}{\lambda_5} = \frac{2\pi}{4\,\text{m}} = \boxed{\frac{\pi}{2}\,\text{m}^{-1}}$$

(*c*) Using its definition, calculate the angular frequency:

$$\omega_5 = 2\pi f_5 = 2\pi\left(400\,\text{s}^{-1}\right) = \boxed{800\pi\,\text{s}^{-1}}$$

(*d*) Write the wave function for a standing wave in the *n*th harmonic:

$$y_n(x,t) = A\sin k_n x \cos\omega_n t$$

Substitute to obtain:

$$y_5(x,t) = A\sin(k_5 x)\cos(\omega_5 t) = \boxed{(0.03\,\text{m})\sin\left[\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\right]\cos\left(800\pi\,\text{s}^{-1}\right)t}$$

79 ••

Picture the Problem The coefficient of the factor containing the time dependence in the wave function is the maximum displacement of any point on the string. The time derivative of the wave function is the instantaneous speed of any point on the string and the coefficient of the factor containing the time dependence is the maximum speed of any point on the string.

Differentiate the wave function with respect to *t* to find the speed of any point on the string:

$$\begin{aligned} v_y &= \frac{\partial}{\partial t}\left[0.02\sin 4\pi\,x\cos 60\pi\,t\right] \\ &= -(0.02)(60\pi)\sin 4\pi\,x\sin 60\pi\,t \\ &= -1.2\pi\sin 4\pi\,x\sin 60\pi\,t \end{aligned}$$

(*a*) Referring to the wave function, express the maximum displacement of the standing wave:

$$y_{\max}(x) = (0.02\,\text{m})\sin\left[\left(4\pi\,\text{m}^{-1}\right)x\right] \quad (1)$$

Evaluate equation (1) at $x = 0.10$ m:

$$\begin{aligned} y_{\max}(0.10\,\text{m}) &= (0.02\,\text{m}) \\ &\quad\times\sin\left[\left(4\pi\,\text{m}^{-1}\right)(0.10\,\text{m})\right] \\ &= \boxed{1.90\,\text{cm}} \end{aligned}$$

Referring to the derivative of the wave function with respect to *t*, express the maximum speed of the

$$v_{y,\max}(x) = (1.2\pi\,\text{m/s})\sin\left[\left(4\pi\,\text{m}^{-1}\right)x\right] \quad (2)$$

standing wave:

Evaluate equation (2) at x = 0.10 m:

$$v_{y,\max}(0.10\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.10\,\text{m})\right] = \boxed{3.59\,\text{m/s}}$$

(b) Evaluate equation (1) at x = 0.25 m:

$$y_{\max}(0.25\,\text{m}) = (0.02\,\text{m}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.25\,\text{m})\right] = \boxed{0}$$

Evaluate equation (2) at x = 0.25 m:

$$v_{y,\max}(0.25\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.25\,\text{m})\right] = \boxed{0}$$

(c) Evaluate equation (1) at x = 0.30 m:

$$y_{\max}(0.30\,\text{m}) = (0.02\,\text{m}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.30\,\text{m})\right] = \boxed{1.18\,\text{cm}}$$

Evaluate equation (2) at x = 0.30 m:

$$v_{y,\max}(0.30\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.30\,\text{m})\right] = \boxed{2.22\,\text{m/s}}$$

(d) Evaluate equation (1) at x = 0.50 m:

$$y_{\max}(0.50\,\text{m}) = (0.02\,\text{m}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.50\,\text{m})\right] = \boxed{0}$$

Evaluate equation (2) at x = 0.50 m:

$$v_{y,\max}(0.50\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.50\,\text{m})\right] = \boxed{0}$$

80 ••

Picture the Problem In part (a) we can use the standing-wave condition for a wire fixed at both ends and the fact that nodes are separated by one-half wavelength to find the harmonic number. In part (b) we can relate the resonance frequencies to their wavelengths and the speed of transverse waves and express the speed of the transverse waves in terms of the tension in the wire and its linear density.

(*a*) Express the standing-wave condition for a wire fixed at both ends:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for *n*:

$$n = \frac{2L}{\lambda_n}$$

Solve for and evaluate λ_1:

$$\lambda_1 = 2L = 2(2.5\,\text{m}) = 5\,\text{m}$$

Relate the distance between nodes to the distance of the node closest to one end and solve for λ_n:

$$\tfrac{1}{2}\lambda_n = 0.5\,\text{m}$$

and

$$\lambda_n = 1\,\text{m}$$

Substitute and evaluate *n*:

$$n = \frac{2(2.5\,\text{m})}{1\,\text{m}} = \boxed{5}$$

(*b*) Express the resonance frequencies in terms of the their wavelengths and the speed of transverse waves on the wire:

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{\lambda_1}$$

Relate the speed of transverse waves on the wire to the tension in the wire:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute and simplify to obtain:

$$f_n = n\frac{1}{\lambda_1}\sqrt{\frac{FL}{m}} = n\frac{1}{5\,\text{m}}\sqrt{\frac{(30\,\text{N})(2.5\,\text{m})}{0.1\,\text{kg}}} = n(5.48\,\text{Hz})$$

Evaluate f_n for $n = 1, 2$, and 3:

$$f_1 = \boxed{5.48\,\text{Hz}}$$

$$f_2 = 2(5.48\,\text{Hz}) = \boxed{11.0\,\text{Hz}}$$

and

$$f_3 = 3(5.48\,\text{Hz}) = \boxed{16.4\,\text{Hz}}$$

***81 ••**

Picture the Problem We can use $v = f\lambda$ to relate the speed of sound in the gas to the distance between the piles of powder in the glass tube.

(*a*) At resonance, standing waves are set up in the tube. At a displacement antinode, the powder is moved about; at a node the powder is stationary, and so it collects at the nodes.

(*b*) Relate the speed of sound to its frequency and wavelength:

$$v = f\lambda$$

Letting *D* = distance between nodes, relate the distance between the nodes to the wavelength of the sound:

$$\lambda = 2D$$

Substitute to obtain:

$$v = \boxed{2fD}$$

(*c*) If we let the length *L* of the tube be 1.2 m and assume that v_{air} = 344 m/s (the speed of sound in air at 20°C), then the 10^{th} harmonic corresponds to *D* = 25.3 cm and a driving frequency of:

$$f_{air} = \frac{v_{air}}{2D} = \frac{344\ \text{m/s}}{2(0.253\ \text{m})} = \boxed{680\ \text{Hz}}$$

(*d*) If $f = 2$ kHz and $v_{He} = 1008$ m/s (the speed of sound in helium at 20°C), then *D* for the 10^{th} harmonic in helium would 25.3 cm and *D* for the 10^{th} harmonic in air would be 8.60 cm. Hence, neglecting end effects at the driven end, a tube whose length is the least common multiple of 8.60 cm and 25.3 cm (218 cm) would work well for the measurement of the speed of sound in either air or helium.

82 ••

Picture the Problem We can use $v = \sqrt{F/\mu}$ to express *F* as a function of *v* and $v = f\lambda$ to relate *v* to the frequency and wavelength of the string's fundamental mode. Because, for a string fixed at both ends, $f_n = nf_1$, we can extend our result in part (*a*) to part (*b*).

(*a*) Relate the speed of the transverse waves on the string to the tension in it:

$$v = \sqrt{\frac{F}{\mu}}$$

Solve for *F*:

$$F = \mu v^2 \qquad (1)$$

Relate the speed of the transverse waves on the string to their frequency and wavelength:

$$v = f_1\lambda_1$$

Express the wavelength of the fundamental mode to the length of the string:

$$\lambda_1 = 2L$$

Substitute to obtain:

$$v = 2fL$$

Substitute in equation (1) to obtain:

$$F = 4f^2L^2\mu \qquad (2)$$

Substitute numerical values and evaluate F:

$$F = 4\left(60\,\text{s}^{-1}\right)^2\left(2.5\,\text{m}\right)^2\left(8\times10^{-3}\,\text{kg/m}\right) = \boxed{720\,\text{N}}$$

(*b*) For the *n*th harmonic, equation (2) becomes:

$$F_n = f_n^2L^2\mu = n^2f_1^2L^2\mu = n^2\left(720\,\text{N}\right)$$

Evaluate this expression for $n = 2$, 3, and 4:

$$F_2 = 4\left(720\,\text{N}\right) = \boxed{2.88\,\text{kN}}$$

$$F_3 = 9\left(720\,\text{N}\right) = \boxed{6.48\,\text{kN}}$$

and

$$F_4 = 16\left(720\,\text{N}\right) = \boxed{11.5\,\text{kN}}$$

83 ••

Picture the Problem We can use the conditions $\Delta f = f_1$ and $f_n = nf_1$, where n is an integer, which must be satisfied if the pipe is open at both ends to decide whether the pipe is closed at one end or open at both ends. Once we have decided this question, we can use the condition relating Δf and the fundamental frequency to determine the latter. In part (*c*) we can use the standing-wave condition for the appropriate pipe to relate its length to its resonance wavelengths.

(*a*) Express the conditions on the frequencies for a pipe that is open at both ends:

$$\Delta f = f_1$$

and

$$f_n = nf_1$$

Evaluate $\Delta f = f_1$:

$$\Delta f = 1834\,\text{Hz} - 1310\,\text{Hz} = 524\,\text{Hz}$$

Using the 2nd condition, find n:

$$n = \frac{f_n}{f_1} = \frac{1310\,\text{Hz}}{524\,\text{Hz}} = 2.5$$

$\boxed{\text{The pipe is closed at one end.}}$

(*b*) Express the condition on the frequencies for a pipe that is open at both ends:

$$\Delta f = 2f_1$$

Solve for and evaluate f_1:

$$f_1 = \tfrac{1}{2}\Delta f = \tfrac{1}{2}(524\,\text{Hz}) = \boxed{262\,\text{Hz}}$$

(*c*) Using the standing-wave condition for a pipe open at one end, relate the length of the pipe to its resonance wavelengths:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

For $n = 1$ we have:

$$\lambda_1 = \frac{v}{f_1} \text{ and } L = \frac{\lambda_1}{4} = \frac{v}{4f_1}$$

Substitute numerical values and evaluate L:

$$L = \frac{340\,\text{m/s}}{4(262\,\text{s}^{-1})} = \boxed{32.4\,\text{cm}}$$

84 ••

Picture the Problem We can relate the speed of sound in air to the frequency of the violin string and the wavelength of the sound in the open tube that is closed at one end by water. The wavelength of the sound, in turn, is a function of the length of the air column and so we can derive an expression for the speed of sound as a function of the frequency of the transverse waves on the violin string and the length of the air column above the water. Knowing that the violin string is vibrating in its fundamental mode, we can express this frequency in terms of the tension in the string and its linear density.

Express the speed of sound in the tube in terms of its fundamental frequency and wavelength:

$$v_s = f_1\lambda_1$$

Using the standing-wave condition for a tube open at one end, relate the speed of sound to the length of the air column in the tube:

$$L_{\text{air column}} = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for λ_1:

$$\lambda_1 = 4L_{\text{air column}}$$

Substitute to obtain:

$$v_s = 4f_1L_{\text{air column}} \qquad (1)$$

Express the frequency of the transverse waves on the violin string in terms of their wavelength and the speed with which they propagate on the string:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L_{\text{string}}}$$

Relate the speed of the transverse waves on the string to the tension in it:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL_{\text{string}}}{m}}$$

Substitute to obtain:

$$f_1 = \frac{1}{2L_{\text{string}}}\sqrt{\frac{FL_{\text{string}}}{m}} = \sqrt{\frac{F}{4mL_{\text{string}}}}$$

Substitute in equation (1) to obtain:

$$v_{\text{s}} = 4L_{\text{air column}}\sqrt{\frac{F}{4mL_{\text{string}}}} = 2L_{\text{air column}}\sqrt{\frac{F}{mL_{\text{string}}}}$$

Substitute numerical values and evaluate v_{s}:

$$v_{\text{s}} = 2(0.18\,\text{m})\sqrt{\frac{(440\,\text{N})}{(10^{-3}\,\text{kg})(0.5\,\text{m})}} = \boxed{338\,\text{m/s}}$$

The method is not very accurate because it neglects end effects (see Problem 56).

85 ••

Picture the Problem We know that the superimposed traveling waves have the same wave number and angular frequency as the standing-wave function, have equal amplitudes that are half that of the standing-wave function, and travel in opposite directions. From inspection of the standing-wave function we note that $k = \frac{1}{2}\pi\,\text{m}^{-1}$ and $\omega = 40\pi\,\text{s}^{-1}$. We can express the velocity of a segment of the rope by differentiating the standing-wave function with respect to time and the acceleration by differentiating the velocity function with respect to time.

(*a*) Write the wave function for the wave traveling in the positive *x* direction:

$$y_1(x,t) = \boxed{(0.01\text{m})\sin\left[\left(\frac{\pi}{2}\text{m}^{-1}\right)x - \left(40\pi\,\text{s}^{-1}\right)t\right]}$$

Write the wave function for the wave traveling in the negative x direction:

$$y_2(x,t) = \boxed{(0.01\,\text{m})\sin\left[\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x + \left(40\pi\,\text{s}^{-1}\right)t\right]}$$

(*b*) Express the distance *d* between nodes in terms of the wavelength of the standing wave:

$$d = \tfrac{1}{2}\lambda$$

Use the wave number to find the wavelength:

$$k = \tfrac{1}{2}\pi\,\text{m}^{-1} = \frac{2\pi}{\lambda}$$

and

$$\lambda = 4\,\text{m}$$

Substitute and evaluate *d*:

$$d = \tfrac{1}{2}(4\,\text{m}) = \boxed{2.00\,\text{m}}$$

(*c*) Differentiate the given wave function with respect to *t* to express the velocity of any segment of the rope:

$$v_y(x,t) = \frac{\partial}{\partial t}\left[(0.02\,\text{m})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\cos\left(40\pi\,\text{s}^{-1}\right)t\right]$$

$$= -(0.8\pi\,\text{m/s})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\sin\left(40\pi\,\text{s}^{-1}\right)t$$

Evaluate $v_y(1\text{m},t)$:

$$v_y(1\,\text{m},t) = -(0.8\pi\,\text{m/s})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)(1\,\text{m})\sin\left(40\pi\,\text{s}^{-1}\right)t$$

$$= -(0.8\pi\,\text{m/s})\sin\left(40\pi\,\text{s}^{-1}\right)t$$

$$= \boxed{-(2.51\,\text{m/s})\sin\left(40\pi\,\text{s}^{-1}\right)t}$$

(*d*) Differentiate $v_y(x,t)$ with respect to time to obtain $a_y(x,t)$:

$$a_y(x,t) = \frac{\partial}{\partial t}\left[-(0.8\pi\,\text{m/s})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\sin\left(40\pi\,\text{s}^{-1}\right)t\right]$$

$$= -\left(32\pi^2\,\text{m/s}^2\right)\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\cos\left(40\pi\,\text{s}^{-1}\right)t$$

Evaluate $a_y(1\text{m},t)$:

$$a_y(1\text{m},t) = -\left(32\pi^2\ \text{m/s}^2\right)\sin\left(\frac{\pi}{2}\text{m}^{-1}\right)(1\text{m})\cos\left(40\pi\ \text{s}^{-1}\right)t$$
$$= -\left(32\pi^2\ \text{m/s}^2\right)\cos\left(40\pi\ \text{s}^{-1}\right)t$$
$$= \boxed{-\left(316\,\text{m/s}^2\right)\cos\left(40\pi\ \text{s}^{-1}\right)t}$$

86 ••

Picture the Problem We can use the definition of intensity to find the intensity of each speaker, the dependence of intensity on the square of the amplitude of the wave disturbance to express the amplitudes of the waves, and the dependence of the intensity on whether the speakers are coherent and their phase difference to find the intensity at the given point.

(*a*) Express the intensity as a function of the distance of a point from the source:

$$I = \frac{P}{4\pi r^2}$$

Evaluate I_1:

$$I_1 = \frac{1\,\text{mW}}{4\pi(2\,\text{m})^2} = \boxed{19.9\,\mu\text{W/m}^2}$$

Evaluate I_2:

$$I_2 = \frac{1\,\text{mW}}{4\pi(3\,\text{m})^2} = \boxed{8.84\,\mu\text{W/m}^2}$$

(*b*) Using $v = f\lambda$, find the wavelength of the sound:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{680\,\text{s}^{-1}} = 0.5\,\text{m}$$

Express the path difference in terms of λ:

$$\Delta x = 2\lambda$$

and so there is constructive interference at point *P*.

Express the intensity at point *P* due to the sound from source 1:

$$I_1 = \text{constant} \times A_1^2$$

or

$$A_1 = C\sqrt{I_1}$$

where *C* is a constant.

Express the intensity at point *P* due the sound from source 2:

$$I_2 = \text{constant} \times A_2^2$$

or

$$A_2 = C\sqrt{I_2}$$

Express the square of the resultant amplitude at point *P*:

$$A^2 = C^2\left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = C^2 I$$

Solve for and evaluate *I*:

$$I = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{19.9\,\mu\text{W/m}^2} + \sqrt{8.84\,\mu\text{W/m}^2}\right)^2 = \boxed{55.3\,\mu\text{W/m}^2}$$

(*c*) If they are driven coherently but are 180° out of phase we will have destructive interference at point *P* and the intensity is given by:

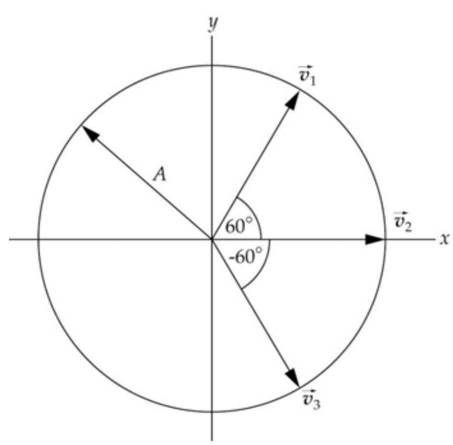
$$I = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{19.9\,\mu\text{W/m}^2} - \sqrt{8.84\,\mu\text{W/m}^2}\right)^2 = \boxed{2.21\,\mu\text{W/m}^2}$$

(*d*) Because the sources are incoherent, the intensities add arithmetically:

$$I = I_1 + I_2 = 19.9\,\mu\text{W/m}^2 + 8.84\,\mu\text{W/m}^2 = \boxed{28.7\,\mu\text{W/m}^2}$$

87 ••

Picture the Problem In Chapter 14, Section 14.1, it was shown that a harmonic function could be represented by a vector rotating at the angular frequency *ω*. The simplest way to do this problem is to use that representation. The vectors, of equal magnitude, are shown in the diagram. We can find the resultant wave function by finding the magnitude and direction of the resultant vector.



From the diagram it is evident that:

$$\sum v_y = 0$$

Find the sum of the *x* components of the vectors:

$$\sum v_x = A\cos 60° + A\cos 60° + A = 2A$$

Relate the magnitude of the resultant vector to the sum of its *x* and *y* components:

$$v = \sqrt{\left(\sum v_x\right)^2 + \left(\sum v_y\right)^2} = \sqrt{(2A)^2 + (0)^2} = 2A$$

Find the direction of the resultant vector:

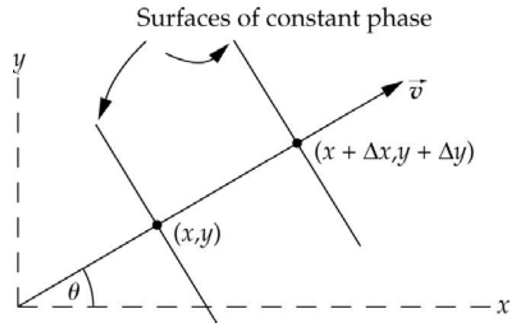
$$\theta = \tan^{-1}\left(\frac{\sum v_y}{\sum v_x}\right) = \tan^{-1}\left(\frac{0}{2A}\right) = 0$$

Express the resultant wave:

$$y_{\text{res}}(x,t) = 2A\sin(kx - \omega t)$$
$$= \boxed{0.1\sin(kx - \omega t)}$$

88 ••

Picture the Problem The diagram shows a two dimensional plane wave propagating at an angle θ with respect to the x axis. At a given point in time, the surface of constant phase for the wave is the line defined by $k_x x + k_y y = \phi$, or $y = -(k_x/k_y)x + \phi$.



The wave itself moves in a direction perpendicular to the wavefront, i.e., in a direction specified by a line with slope k_y/k_x. Choose two points (x, y) and $(x + \Delta x, y + \Delta y)$ that have a separation of 1 wavelength along such a line.

Express the phase difference ϕ between the two points that have a separation of 1 wavelength along the line $y = -(k_x/k_y)x + \phi$ in terms of the spatial separation Δr of the points:

$$\frac{\phi}{\Delta r} = \frac{2\pi}{\lambda} \text{ or } \phi = \frac{2\pi}{\lambda}\Delta r$$

where $\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

Substitute $\phi = 2\pi$ to obtain:

$$2\pi = \frac{2\pi}{\lambda}\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

or

$$\lambda = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad (1)$$

Express ϕ in terms of k_x, k_y, Δx and Δy:

$$\phi = k\Delta r = k_x\Delta x + k_y\Delta y$$

or, because $\phi = 2\pi$,

$$k_x\Delta x + k_y\Delta y = 2\pi$$

Because $\Delta y = \frac{k_y}{k_x}\Delta x$:

$$k_x\Delta x + \frac{k_y^2}{k_x}\Delta x = 2\pi$$

or

$$\Delta x = \frac{2\pi k_x}{k_x^2 + k_y^2}$$

Similarly:

$$\Delta y = \frac{2\pi k_y}{k_x^2 + k_y^2}$$

Substitute in equation (1) to obtain:

$$\lambda = \sqrt{\left(\frac{2\pi k_x}{k_x^2 + k_y^2}\right)^2 + \left(\frac{2\pi k_y}{k_x^2 + k_y^2}\right)^2}$$

$$= \frac{2\pi}{\sqrt{k_x^2 + k_y^2}}$$

Relate the wave velocity v to its angular frequency ω and wave number k:

$$v = \frac{\omega}{k} = \omega\frac{\lambda}{2\pi}$$

Substitute for λ to obtain:

$$v = \frac{\omega}{2\pi}\frac{2\pi}{\sqrt{k_x^2 + k_y^2}} = \boxed{\frac{\omega}{\sqrt{k_x^2 + k_y^2}}}$$

Express the angle between the wave velocity and the x axis:

$$\theta = \tan^{-1}\frac{\Delta y}{\Delta x} = \tan^{-1}\frac{\dfrac{2\pi k_y}{k_x^2 + k_y^2}}{\dfrac{2\pi k_x}{k_x^2 + k_y^2}}$$

$$= \boxed{\tan^{-1}\left(\frac{k_y}{k_x}\right)}$$

***89 ••**

Picture the Problem We can express the fundamental frequency of the organ pipe as a function of the air temperature and differentiate this expression with respect to the temperature to express the rate at which the frequency changes with respect to temperature. For changes in temperature that are small compared to the temperature, we can approximate the differential changes in frequency and temperature with finite changes to complete the derivation of $\Delta f/f = ½\Delta T/T$. In part (b) we'll use this relationship and the data for the frequency at 20°C to find the frequency of the fundamental at 30°C.

(*a*) Express the fundamental frequency of an organ pipe in terms of its wavelength and the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound in air to the absolute temperature:

$$v = \sqrt{\frac{\gamma RT}{M}} = C\sqrt{T}$$

where

$$C = \sqrt{\frac{\gamma R}{M}} = \text{constant}$$

Defining a new constant C', substitute to obtain:

$$f = \frac{C}{\lambda}\sqrt{T} = C'\sqrt{T}$$

because λ is constant for the fundamental frequency we ignore any change in the length of the pipe.

Differentiate this expression with respect to T:

$$\frac{df}{dT} = \frac{1}{2}C'T^{-1/2} = \frac{f}{2T}$$

Separate the variables to obtain:

$$\frac{df}{f} = \frac{1}{2}\frac{dT}{T}$$

For $\Delta T << T$, we can approximate df by Δf and dT by ΔT to obtain:

$$\boxed{\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}}$$

(*b*) Express the fundamental frequency at 30°C in terms of its frequency at 20°C:

$$f_{30} = f_{20} + \Delta f$$

Solve our result in (*a*) for Δf:

$$\Delta f = \tfrac{1}{2} f \frac{\Delta T}{T}$$

Substitute numerical values and evaluate Δf:

$$f_{30} = 200\,\text{Hz} + \tfrac{1}{2}(200\,\text{Hz})\frac{10\,\text{K}}{293\,\text{K}}$$

$$= \boxed{203\,\text{Hz}}$$

90 ••

Picture the Problem We'll use a spreadsheet program to graph the wave functions individually and their sum as functions of x at $t = 0$ and at $t = 1$ s. In (*c*) and (*d*) we can add the wave functions algebraically to find the result wave function at $t = 0$ and at $t = 1$ s.

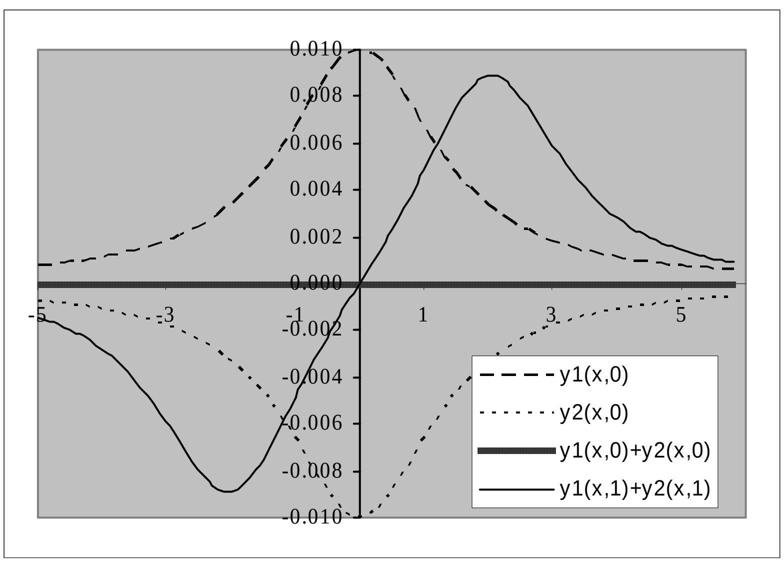
(*a*) and (*d*) A spreadsheet program to calculate values for $y_1(x,t)$ and $y_2(x,t)$ between and plot their graphs is shown below. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
A5	–5.0	x
A6	A5+0.1	$x + \Delta x$

B5	0.05/(2+(A5–2*$B$1)^2)	$y_1(x,0)$
C5	–0.05/(2+(A5+2*$B$1)^2)	$y_2(x,0)$
D5	0.05/(2+(A5–2*$B$1)^2) –0.05/(2+(A5+2*$B$1)^2)	$y_1(x,0)+y_2(x,0)$
E5	0.05/(2+(A5–2*$B$2)^2) –0.05/(2+(A5+2*$B$2)^2)	$y_1(x,1)+y_2(x,1)$

	A	B	C	D	E
1	t=	0			
2	t=	1	s		
3					
4	x	y1(x,0)	y2(x,0)	y1(x,0)+y2(x,0)	y1(x,1)+y2(x,1)
5	–5.0	0.001	–0.001	0.000	–0.001
6	–4.9	0.001	–0.001	0.000	–0.002
7	–4.8	0.001	–0.001	0.000	–0.002
8	–4.7	0.001	–0.001	0.000	–0.002
9	–4.6	0.001	–0.001	0.000	–0.002
10	–4.5	0.001	–0.001	0.000	–0.002
110	5.5	0.001	–0.001	0.000	0.001
111	5.6	0.001	–0.001	0.000	0.001
112	5.7	0.001	–0.001	0.000	0.001
113	5.8	0.001	–0.001	0.000	0.001

The four curves on the graph are identified in the legend. y_1 is traveling from left to right and y_2 from right to left. As time increases, y_1 is farther to the right and y_2 is farther to the left.



(*b*) Express the resultant wave function at t = 0:

$$y_1(x,0)+y_2(x,0)=\frac{0.02\,\text{m}^3}{2\,\text{m}^2+x^2}+\frac{-0.02\,\text{m}^3}{2\,\text{m}^2+x^2}=\boxed{0}$$

(*c*) Express the resultant wave function at t = 1 s:

$$y_1(x,1\,\text{s})+y_2(x,1\,\text{s})=\boxed{\frac{0.02\,\text{m}^3}{2\,\text{m}^2+(x-2\,\text{s})^2}+\frac{-0.02\,\text{m}^3}{2\,\text{m}^2+(x+2\,\text{s})^2}}$$

91 ••

Picture the Problem We can relate the frequency of the standing waves in the open-ended tube to its length and the speed of sound in air.

(*a*) What you hear is the fundamental mode of the tube and its overtones. A more physical explanation is that the echo of the finger snap moves back and forth along the tube with a characteristic time of $2L/c$, leading to a series of clicks from each echo. Because the clicks happen with a frequency of $c/2L$, the ear interprets this as a musical note of that frequency.

(*b*) Express the frequency of the sound in terms of the length of the tube:

$$f=\frac{v}{2L}$$

Solve for L:

$$L=\frac{v}{2f}$$

Substitute numerical values and evaluate L:

$$L=\frac{340\,\text{m/s}}{2(440\,\text{s}^{-1})}=\boxed{38.6\,\text{cm}}$$

92 ••

Picture the Problem To find the total kinetic energy of the *n*th mode of vibration, we'll need to differentiate $y_n(x,t)=A_n\sin k_nx\cos\omega_nt$ with respect to time, substitute in the expression for ΔK, and then integrate over the length of the string.

(*a*) Write the wave function for a standing wave on a string fixed at both ends:

$$y_n(x,t)=A_n\sin k_nx\cos\omega_nt$$

where $k_n=\dfrac{2\pi}{\lambda_n}$.

Using the standing-wave condition for a string with both ends fixed, relate the length of the string to the

$$L=n\frac{\lambda_n}{2},\ n=1,2,3,\ldots$$

wavelength of the nth harmonic:

Solve for λ_n:

$$\lambda_n = \frac{2L}{n}$$

Substitute in the expression for k_n to obtain:

$$k_n = n\frac{\pi}{L}$$

Differentiate this expression with respect to t:

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\left[A_n \sin k_n x \cos \omega_n t\right] = -\omega_n A_n \sin k_n x \sin \omega_n t$$

Substitute in the given expression and simplify to obtain:

$$\Delta K = \tfrac{1}{2}\mu\left(-\omega_n A_n \sin k_n x \sin \omega_n t\right)^2 \Delta x = \tfrac{1}{2}\mu\omega_n^2 A_n^2 \sin^2 k_n x \sin^2 \omega_n t \Delta x$$

Integrate this expression over the length of the string to find its total kinetic energy:

$$K = \tfrac{1}{2}\mu\omega_n^2 A_n^2 \sin^2 \omega_n t \int_0^L \sin^2\left(n\frac{\pi}{L}x\right)dx = \boxed{\tfrac{1}{4}m\omega_n^2 A_n^2 \sin^2 \omega_n t}$$

(*b*) Express the condition that $K = K_{max}$:

$$\sin^2 \omega_n t = 1 \quad (1)$$

Substitute to obtain:

$$K_{max} = \boxed{\tfrac{1}{4}m\omega_n^2 A_n^2}$$

(*c*) From equation (1), for $K = K_{max}$:

$$\sin^2 \omega_n t = 1 \text{ or } \omega_n t = \frac{\pi}{2}$$

Evaluate the wave function in (*a*) when $\omega_n t = \frac{\pi}{2}$:

$$y_n\left(x, \frac{\pi}{2\omega_n}\right) = A_n \sin k_n x \cos\frac{\pi}{2} = \boxed{0}$$

(*d*) Using the result from part (*b*), express the maximum kinetic energy:

$$K_{max} = \tfrac{1}{4}m\omega_n^2 A_n^2$$

Relate ω_n to ω_1:

$$\omega_n = n\omega_1$$

Substitute to obtain:

$$K_{max} = n^2\left(\tfrac{1}{4}m\omega_1^2 A_n^2\right)$$

or, because m and ω_1 are constants,

$$K_{max} \propto \boxed{n^2 A_n^2}$$

Remarks: Our result in part (*b*) is exactly the same result obtained in Problem 68 with ω_n and A_n replacing ω and A.

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Picture the Problem We can use $f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots$ to relate the resonant frequencies to the length of the string and the speed of transverse waves on the string and $v = \sqrt{F/\mu}$ to express the speed of the transverse waves on the string in terms of the tension in the string. Differentiating of the resulting expression with respect to *F* will lead to $\frac{df_n}{f_n} = \frac{1}{2}\frac{dF}{F}$. For changes in *f* that are small compared to *f*, we can use a differential approximation to obtain $\frac{\Delta f_n}{f_n} = \frac{1}{2}\frac{\Delta F}{F}$.

(*a*) Using the standing-wave condition for a string fixed at both ends, relate the resonant frequencies to the length of the string and the speed of transverse waves on the string:

$$f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots$$

Express the speed of transverse waves on the string in terms of the tension in the string:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} = C\sqrt{F}$$

because *n*, *L*, and μ are constants.

Differentiate f_n with respect to *F* to obtain:

$$\frac{df_n}{dF} = \frac{C}{2}\frac{1}{\sqrt{F}} = \frac{1}{2}\frac{f_n}{F}$$

Separate the variables to obtain:

$$\boxed{\frac{df_n}{f_n} = \frac{1}{2}\frac{dF}{F}}$$

Because no conditions were placed on its derivation, this expression is valid for all harmonics.

(*b*) Because Δf << *f*, one can approximate the differential quantitities in our result for part (*a*) to obtain:

$$\frac{\Delta f_n}{f_n} = \frac{1}{2}\frac{\Delta F}{F}$$

Solve for $\Delta F/F$:

$$\frac{\Delta F}{F} = 2\frac{\Delta f_n}{f_n}$$

Substitute numerical values and evaluate $\Delta F/F$:

$$\frac{\Delta F}{F} = 2\left(\frac{2\,\text{Hz}}{260\,\text{Hz}}\right) = \boxed{1.54\%}$$

94 ••

Picture the Problem Let the sources be denoted by the numerals 1 and 2. The phase difference between the two waves at point *P* is the sum of the phase difference due to the sources δ_0 and the phase difference due to the path difference δ.

(*a*) Write the wave function due to source 1:

$$f_1(x,t) = \boxed{A_0\cos(kx_1 - \omega t)}$$

Write the wave function due to source 2:

$$f_2(x,t) = \boxed{A_0\cos(k(x_1 + \Delta x) - \omega t + \delta_s)}$$

(*b*) Express the sum of the two wave functions:

$$\begin{aligned} f(x,t) &= f_1(x,t) + f_2(x,t) = A_0\cos(kx_1 - \omega t) + A_0\cos(k(x_1 + \Delta x) - \omega t + \delta_s) \\ &= A_0[\cos(kx_1 - \omega t)\cos(k(x_1 + \Delta x) - \omega t + \delta_s)] \end{aligned}$$

Use $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ to obtain:

$$f(x,t) = 2A_0\left[\cos\left(\frac{k\Delta x}{2} + \frac{\delta_s}{2}\right)\cos\left(k\left(x + \frac{\Delta x}{2}\right) - \omega t + \frac{\delta_s}{2}\right)\right]$$

Express the phase difference δ in terms of the path difference Δx and the wave number *k*:

$$\frac{\delta}{\Delta x} = \frac{2\pi}{\lambda} = k \text{ or } k\Delta x = \delta$$

Substitute to obtain:

$$f(x,t) = \boxed{2A_0\left[\cos\left(\frac{\delta+\delta_s}{2}\right)\cos\left(k\left(x+\frac{\Delta x}{2}\right)-\omega t+\frac{\delta_s}{2}\right)\right]}$$

The amplitude of the resultant wave function is the coefficient of the time-dependent factor:

$$A = \boxed{2A_0\cos\tfrac{1}{2}(\delta+\delta_s)}$$

(*c*) Express the intensity at an arbitrary point *P*:

$$I_P = C'A^2$$
$$= C'\left[2A_0\cos\tfrac{1}{2}(\delta+\delta_s)\right]^2$$
$$= C'\left[4A_0^2\cos^2\tfrac{1}{2}(\delta+\delta_s)\right]$$

Evaluate I for $\delta = 0$ and $\delta_s = Ct$:

$$I = C'\left[4A_0^2\cos^2\tfrac{1}{2}(Ct)\right]$$

Because the average value of $\cos^2\theta$ over a complete period is ½:

$$I_{ave} \propto 2A_0^2 = 2I_0$$

and

$$I \propto \boxed{4I_0\cos^2\tfrac{1}{2}(Ct)}$$

(*d*) Evaluate I for $\Delta x = \frac{1}{2}\lambda$ and $\delta_s = Ct$:

$$\Delta x = \tfrac{1}{2}\lambda \Rightarrow \delta = \pi$$
$$\therefore I = C'\left[4A_0^2\cos^2\tfrac{1}{2}(\pi+Ct)\right]$$

and at $t = 0$, $I = 0$. i.e., the waves interfere destructively.

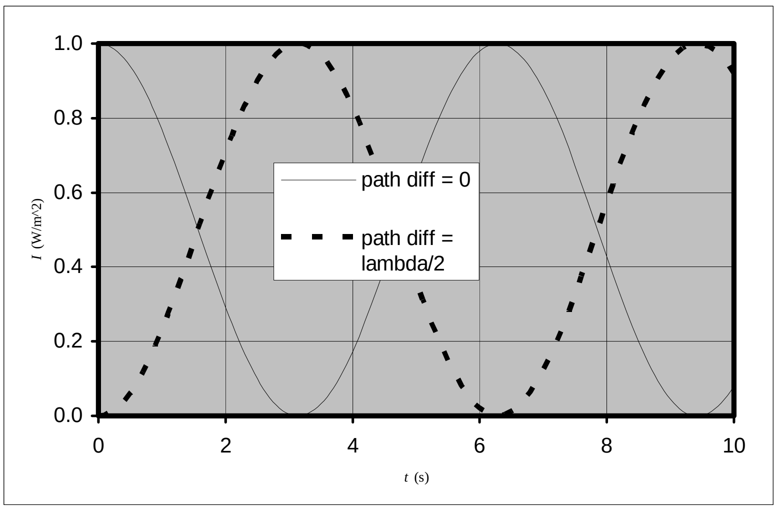
A spreadsheet program to calculate the intensity at point *P* as a function of time for a zero path difference and a path difference of λ is shown below. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
B1	1	C
B7	B6+0.1	$t+\Delta t$
C6	COS($B$6*B6/2)^2	$\cos^2\frac{1}{2}(Ct)$
D6	COS($B$6*B6/2-PI()/2)^2	$\cos^2\frac{1}{2}(\pi+Ct)$

	A	B	C	D
1	C=	1	s^{-1}	
2				
3				
4		t	I	I
5		(s)	(W/m^2)	(W/m^2)
6		0.00	1.000	0.000

7		0.10	0.998	0.002
8		0.20	0.990	0.010
9		0.30	0.978	0.022
103		9.70	0.019	0.981
104		9.80	0.035	0.965
105		9.90	0.055	0.945
106		10.00	0.080	0.920

The solid curve is the graph of $\cos^2 \frac{1}{2}(Ct)$ and the dashed curve is the graph of $\cos^2 \frac{1}{2}(\pi + Ct)$.



95 •••

Picture the Problem We can differentiate the sum of the two wave functions to find the velocity of a segment dx of the string. We can find the kinetic energy of this segment from $dK = \frac{1}{2}v_y^2 dm = \frac{1}{2}\mu v_y^2 dx$ and integrate this expression from 0 to L to find the total kinetic energy of the resultant wave.

(*a*) Express the resultant wave function:

$$y_r(x,t) = y_1(x,t) + y_2(x,t) = A_1 \cos\omega_1 t \sin k_1 x + A_2 \cos\omega_2 t \sin k_2 x$$

Differentiate this expression with respect to t to find v_y:

$$v_y(x,t) = \frac{\partial}{\partial t}\left[A_1 \cos\omega_1 t \sin k_1 x + A_2 \cos\omega_2 t \sin k_2 x\right]$$
$$= \boxed{-\omega_1 A_1 \sin\omega_1 t \sin k_1 x - \omega_2 A_2 \sin\omega_2 t \sin k_2 x}$$

(*b*) Express the kinetic energy of a segment of the string of length *dx* and mass *dm*:

$$dK = \tfrac{1}{2}v_y^2 dm = \tfrac{1}{2}\mu v_y^2 dx = \tfrac{1}{2}\mu(\omega_1 A_1 \sin\omega_1 t \sin k_1 x + \omega_2 A_2 \sin\omega_2 t \sin k_2 x)^2 dx$$

$$= \boxed{\begin{aligned}&\tfrac{1}{2}\mu\big[\omega_1^2 A_1^2 \sin^2\omega_1 t \sin^2 k_1 x + 2\omega_1\omega_2 A_1 A_2 \sin\omega_1 t \sin k_1 x \sin\omega_2 t \sin k_2 x \\ &\quad + \omega_2^2 A_2^2 \sin^2\omega_2 t \sin^2 k_2 x\big]dx\end{aligned}}$$

(*c*) Integrate *dK* from 0 to *L* to obtain:

$$\begin{aligned}
K &= \tfrac{1}{2}\mu\int_0^L \omega_1^2 A_1^2 \sin^2\omega_1 t \sin^2 k_1 x dx \\
&\quad + \tfrac{1}{2}\mu\int_0^L 2\omega_1\omega_2 A_1 A_2 \sin\omega_1 t \sin k_1 x \sin\omega_2 t \sin k_2 x dx \\
&\quad + \tfrac{1}{2}\mu\int_0^L \omega_2^2 A_2^2 \sin^2\omega_2 t \sin^2 k_2 x dx \\
&= \tfrac{1}{2}\mu\omega_1^2 A_1^2 \sin^2\omega_1 t\int_0^L \sin^2 n_1\frac{\pi}{L}x dx \\
&\quad + \mu\omega_1\omega_2 A_1 A_2 \sin\omega_1 t \sin\omega_2 t\int_0^L \sin n_1\frac{\pi}{L}x \sin n_2\frac{\pi}{L}x dx \\
&\quad + \tfrac{1}{2}\mu\omega_2^2 A_2^2 \sin^2\omega_2 t\int_0^L \sin^2 n_2\frac{\pi}{L}x dx \\
&= \left(\tfrac{1}{2}\mu\omega_1^2 A_1^2 \sin^2\omega_1 t\right)\left(\tfrac{1}{2}L\right) + \left(\mu\omega_1\omega_2 A_1 A_2 \sin\omega_1 t \sin\omega_2 t\right)(0)_{n_1\neq n_2} \\
&\qquad + \left(\tfrac{1}{2}\mu\omega_2^2 A_2^2 \sin^2\omega_2 t\right)\left(\tfrac{1}{2}L\right) \\
&= \boxed{\tfrac{1}{4}m\omega_1^2 A_1^2 \sin^2\omega_1 t + \tfrac{1}{4}m\omega_2^2 A_2^2 \sin^2\omega_2 t}
\end{aligned}$$

Note that, from Problem 92: $\tfrac{1}{4}m\omega_1^2 A_1^2 \sin^2\omega_1 t + \tfrac{1}{4}m\omega_2^2 A_2^2 \sin^2\omega_2 t = K_1 + K_2$

96 •••

Picture the Problem We can use the relationship $K_{\max} = \tfrac{1}{4}m\omega^2 A^2$ from Problem 92 to express the maximum kinetic energy of the wire and $v = f\lambda$ and $v = \sqrt{F/\mu}$ to find an expression for ω. In part (*d*) we'll use $\dfrac{\Delta U}{\Delta x} \approx \tfrac{1}{2}F\left(\dfrac{\partial y}{\partial x}\right)^2$ from Problem 15-120 to determine where the potential energy per unit length has its maximum value.

(*a*) From Problem 92 we have: $$K_{\max} = \tfrac{1}{4}m\omega^2 A^2 \qquad (1)$$

Express ω_1 in terms of f_1:

$$\omega_1 = 2\pi f_1$$

Relate f_1 to the speed of transverse waves on the wire and the wavelength of the fundamental mode:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

where L is the length of the wire.

Express the speed of the transverse waves on the wire in terms of the tension in the wire:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

Substitute and simplify to obtain:

$$f_1 = \frac{1}{2L}\sqrt{\frac{FL}{m}} = \sqrt{\frac{F}{4mL}}$$

Substitute for ω_1 and f_1 in equation (1) to obtain:

$$K_{\text{max}} = \tfrac{1}{4}m\left[2\pi\sqrt{\frac{F}{4mL}}\right]^2 A^2 = \frac{\pi^2 F}{4L}A^2$$

Substitute numerical values and evaluate K_{max}:

$$K_{\text{max}} = \frac{\pi^2(40\text{ N})}{4(2\text{ m})}\left(2\times10^{-2}\text{ m}\right)^2 = \boxed{19.7\text{ mJ}}$$

(*b*) Express the wave function for a standing wave in its first harmonic:

$$y_1(x,t) = A_1 \sin k_1 x \cos \omega_1 t \qquad (2)$$

At the instant the transverse displacement is given by (0.02 m) sin ($\pi x/2$):

$$\cos\omega_1 t = 1 \Rightarrow \omega_1 t = 0$$

and

$$K = \boxed{0}$$

(*c*) dK is a maximum where the displacement of the wire is greatest; i.e., at its midpoint:

$$x = \tfrac{1}{2}L = \tfrac{1}{2}(2\text{ m}) = \boxed{1.00\text{ m}}$$

(*d*) From Problem 15-120:

$$\frac{\Delta U}{\Delta x} \approx \tfrac{1}{2}F\left(\frac{\partial y}{\partial x}\right)^2$$

Express the condition on $\partial y/\partial x$ that maximizes $\Delta U/\Delta x$:

$$\frac{\partial y}{\partial x} = \left(\frac{\partial y}{\partial x}\right)_{\text{max}}$$

Differentiate $y_1(x,t) = A_1 \sin k_1 x \cos \omega_1 t$ with respect to x and set the derivative equal to zero for extrema:

$$\frac{\partial y_1}{\partial x} = \frac{\partial}{\partial x}\left(A_1 \sin k_1 x \cos \omega_1 t\right) = k_1 A_1 \cos k_1 x \cos \omega_1 t = 0$$

or

$$\cos k_1 x = 0$$

Solve for $k_1 x$ and then x:

$$k_1 x = \frac{\pi}{2}$$

and

$$x = \frac{\pi}{2k_1} = \frac{\pi\lambda}{2(2\pi)} = \tfrac{1}{4}\lambda = \tfrac{1}{4}(2L) = \tfrac{1}{2}(2\,\text{m}) = \boxed{1.00\,\text{m}}$$

i.e., the potential energy per unit length is a maximum at the midpoint of the wire.

Remarks: In part (*d*) we've shown that $\Delta U/\Delta x$ has an extreme value at x = 1 m. To show that $\Delta U/\Delta x$ is a *maximum* at this location, you need to examine the sign of the 2nd derivative of $y_1(x,t)$ at this point.

97 •••

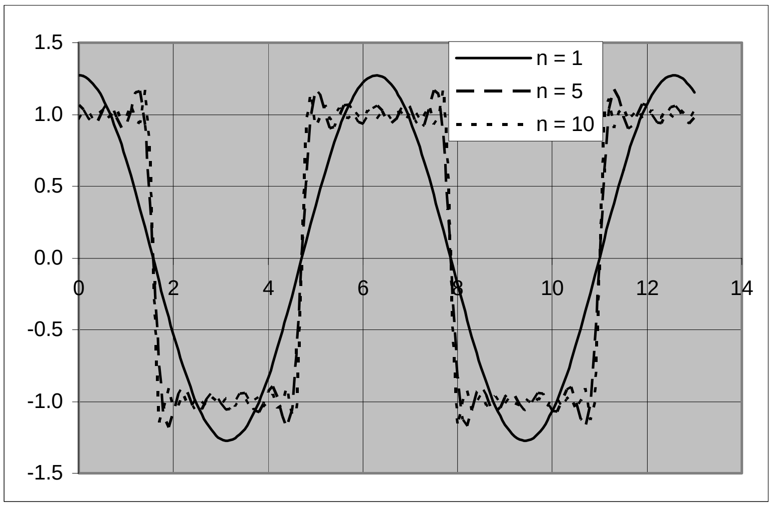
(*a*) A spreadsheet program to evaluate $f(x)$ is shown below. Typical cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
A6	A5+0.1	$x + \Delta x$
B4	2*B3+1	$2n+1$
B5	(–1)^B$3*COS(B$4*$A5) /B$4*4/PI()	$\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n \cos((2n+1)x)}{2n+1}$
C5	B5+(–1)^C$3*COS(C$4*$A5) /C$4*4/PI()	$\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n \cos((2n+1)x)}{2n+1}$

	A	B	C	D		K	L
1							
2							
3		0	1	2		9	10
4		1	3	5		19	21
5	0.0	1.2732	0.8488	1.1035		0.9682	1.0289
6	0.1	1.2669	0.8614	1.0849		1.0134	0.9828
7	0.2	1.2479	0.8976	1.0352		1.0209	0.9912
8	0.3	1.2164	0.9526	0.9706		0.9680	1.0286
9	0.4	1.1727	1.0189	0.9130		1.0057	0.9742
10	0.5	1.1174	1.0874	0.8833		1.0298	1.0010

130	12.5	1.2704	0.8544	1.0952		0.9924	1.0031
131	12.6	1.2725	0.8503	1.1013		0.9752	1.0213
132	12.7	1.2619	0.8711	1.0710		1.0287	0.9714
133	12.8	1.2386	0.9143	1.0141		1.0009	1.0126
134	12.9	1.2030	0.9740	0.9493		0.9691	1.0146
135	13.0	1.1554	1.0422	0.8990		1.0261	0.9685

The solid curve is plotted from the data in columns A and B and is the graph of *f*(*x*) for 1 term. The dashed curve is plotted from the data in columns A and F and is the graph of *f*(*x*) for 5 terms. The dotted curve is plotted from the data in columns A and K and is the graph of *f*(*x*) for 10 terms.



(*b*) Evaluate *f*(2π) to obtain:

$$f(2\pi) = \frac{4}{\pi}\left(\frac{\cos 2\pi}{1} - \frac{\cos 3(2\pi)}{3} + \frac{\cos 5(2\pi)}{5} - \ldots\right)$$

$$= \frac{4}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\right)$$

$$= 1$$

which is equivalent to the Liebnitz formula.

98 •••

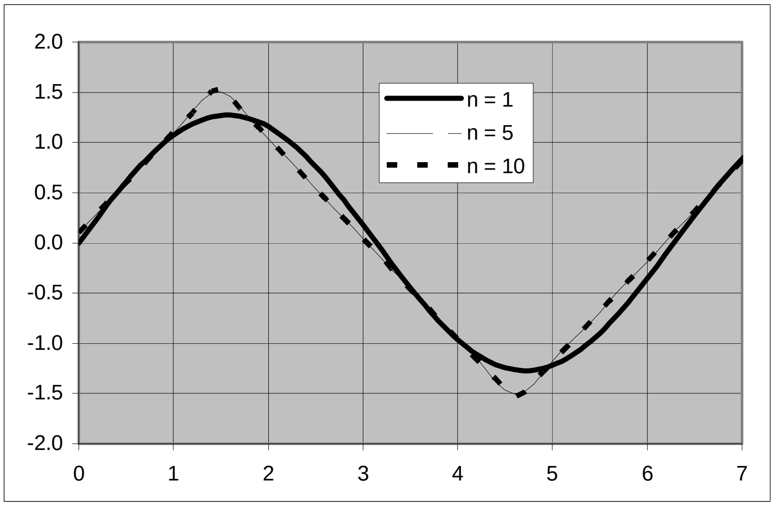
(*a*) A spreadsheet program to evaluate *f*(*x*) is shown below. Typical cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
A6	A5+0.1	$x + \Delta x$
B4	2*B3+1	$2n+1$
B5	(–1)^$B$3*sin($B$4*A5)/ ($B$4)^2*4/PI()	$\frac{4}{\pi}\sum_n \frac{(-1)^n \sin(2n+1)x}{(2n+1)^2}$

C5	B5+((−1)^$C$3*sin($C$4*A5)/ ($C$4)^2*4/PI()	$\frac{4}{\pi}\sum_{n}\frac{(-1)^{n}\sin(2n+1)x}{(2n+1)^{2}}$

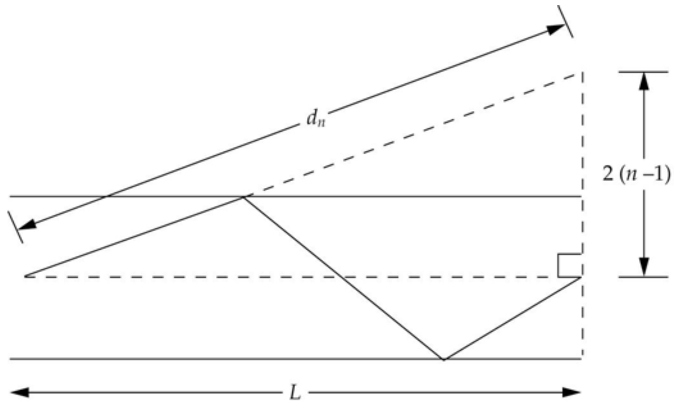
	A	B	C	D		K	L
1							
2							
3		0	1	2		9	10
4		1	3	5		19	21
5	0.0	0.0000	0.0000	0.0000		0.0000	0.0000
6	0.1	0.1271	0.0853	0.1097		0.0986	0.1011
7	0.2	0.2530	0.1731	0.2159		0.2012	0.1987
8	0.3	0.3763	0.2654	0.3163		0.3004	0.3005
9	0.4	0.4958	0.3640	0.4103		0.3983	0.4008
10	0.5	0.6104	0.4693	0.4998		0.5011	0.4985
72	6.7	0.5155	0.3812	0.4256		0.4153	0.4171
73	6.8	0.6291	0.4877	0.5146		0.5183	0.5154
74	6.9	0.7365	0.6005	0.6034		0.6171	0.6182
75	7.0	0.8365	0.7181	0.6963		0.7148	0.7166
76	7.1	0.9282	0.8380	0.7968		0.8183	0.8155

Graphs of $f(x)$ for 1, 5, and 10 terms are shown below. Note that there is little difference between the graphs for 5 terms and 10 terms of this triangular wave function.



99 •••

Picture the Problem From the diagram above, the *n*th echo will reflect $n - 1$ times going out, and the same number of times going back. If we "unfold" the ray into a straight line, we get the representation shown below. Using this figure we can express the distance d_n traveled by the *n*th echo and then use this result to express the time delay between the n_{th} and $n + 1_{\text{th}}$ echoes. The reciprocal of this time delay is the frequency corresponding to the *n*th echo.



(a) Apply the Pythagorean theorem to the right triangle whose base is L, whose height is $2(n-1)$, and whose hypotenuse is d_n to obtain:

$$d_n = 2\sqrt{4(n-1)^2 r^2 + L^2}$$

Express the time delay between the n_{th} and $n + 1_{th}$ echoes:

$$\Delta t_n = \frac{d_n}{v}$$

Substitute to obtain:

$$\Delta t_n = \frac{2}{v}\left(\sqrt{(2n)^2 r^2 + L^2} - \sqrt{[2(n-1)]^2 r^2 + L^2}\right)$$

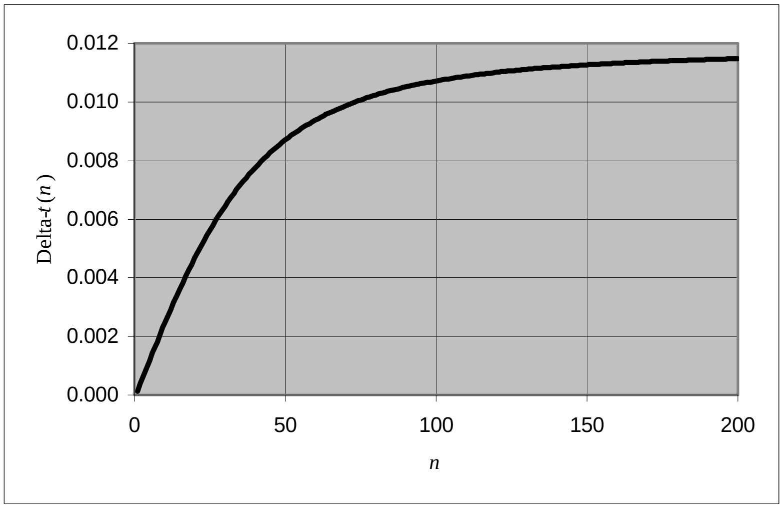
A spreadsheet program to calculate Δt_n as a function of n is shown below. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
B1	90	L
B2	1	r
B3	340	c
B8	B7+1	$n + 1$
C7	2/$B$3*((2*(B7–1) *$B$2)^2+$B$1^2)^0.5	Δt_n

	A	B	C	D
1	L=	90	m	
2	r=	1	m	
3	c=	340	m/s	
4				
5				
6		n	t(n)	delta t(n)
7		1	0.5294	0.0001
8		2	0.5295	0.0004
9		3	0.5299	0.0007
10		4	0.5306	0.0009
11		5	0.5315	0.0012

202		196	2.3544	0.0115
203		197	2.3659	0.0115
204		198	2.3773	0.0115
205		199	2.3888	0.0115
206		200	2.4003	0.0115

The graph of Δt_n as a function of n shown below was plotted using the data from columns B and D.



(*c*) The frequency heard at any time is $1/\Delta t_n$, so because Δt_n increases over time, the frequency of the culvert whistler decreases.

The highest frequency corresponds to $n = 1$ and is given by:

$$f_{\text{highest}} = \frac{1}{\Delta t_1}$$

Substitute for Δt_1 to obtain:

$$f_{\text{highest}} = \frac{1}{\Delta t_1} = \frac{v}{2\left(\sqrt{(2)^2 r^2 + L^2} - \sqrt{L^2}\right)}$$

Substitute numerical values and evaluate f_{highest}:

$$f_{\text{highest}} = \frac{340\,\text{m/s}}{2\left(\sqrt{4(1\,\text{m})^2 + (90\,\text{m})^2} - 90\,\text{m}\right)} = \boxed{7.65\,\text{kHz}}$$

The lowest frequency end can be found by examining the limit of Δt_n as $n \to \infty$:

$$\lim_{n\to\infty} \Delta t_n = \lim_{n\to\infty}\left[\frac{2}{v}\left((2n)\sqrt{r^2 + \frac{L^2}{(2n)^2}} - 2(n-1)\sqrt{r^2 + \frac{L^2}{(2(n-1))^2}}\right)\right]$$

$$= \frac{2r}{v}(2n - 2n + 2) = \frac{4r}{v}$$

Express f_{lowest} in terms of Δt_{∞}:

$$f_{\text{lowest}} = \frac{1}{\Delta t_{\infty}} = \frac{v}{4r}$$

Substitute numerical values and evaluate f_{lowest}:

$$f_{\text{lowest}} = \frac{340\,\text{m/s}}{4(1\,\text{m})} = \boxed{85.0\,\text{Hz}}$$

Chapter 17
Temperature and the Kinetic Theory of Gases

Conceptual Problems

***1 •**

(*a*) False. If two objects are in thermal equilibrium with a third, then they are in thermal equilibrium with each other.

(*b*) False. The Fahrenheit and Celsius temperature scales differ in the number of intervals between the ice-point temperature and the steam-point temperature.

(*c*) True.

(*d*) False. The result one obtains for the temperature of a given system is thermometer-dependent.

2 •

Determine the Concept Put each in thermal equilibrium with a third body; e.g., a thermometer. If each body is in thermal equilibrium with the third, then they are in thermal equilibrium with each other.

3 •

Picture the Problem We can decide which room was colder by converting 20°F to the equivalent Celsius temperature.

Using the Fahrenheit-Celsius conversion, convert 20°F to the equivalent Celsius temperature:

$$t_C = \tfrac{5}{9}(t_F - 32°) = \tfrac{5}{9}(20° - 32°) = -6.67°C$$

so $\boxed{\text{Mert's room was colder.}}$

4 ••

Picture the Problem We can apply the ideal-gas law to the two vessels to decide which of these statements is correct.

Apply the ideal-gas law to the particles in vessel 1:

$$P_1V_1 = N_1kT_1$$

Apply the ideal-gas law to the particles in vessel 2:

$$P_2V_2 = N_2kT_2$$

Divide the equation for vessel 1 by the equation for vessel 2:

$$\frac{P_1V_1}{P_2V_2} = \frac{N_1kT_1}{N_2kT_2}$$

Because the vessels are identical and are at the same temperature and pressure:

$$1 = \frac{N_1}{N_2} \text{ and } N_1 = N_2$$

$\boxed{(a) \text{ is correct.}}$

5 ••

Determine the Concept From the ideal-gas law, we have $P = nRT/V$. In the process depicted, both the temperature and the volume increase, but the temperature increases faster than does the volume. Hence, the pressure increases.

***6** ••

Determine the Concept From the ideal-gas law, we have $V = nRT/P$. In the process depicted, both the temperature and the pressure increase, but the pressure increases faster than does the temperature. Hence, the volume decreases.

7 •

True. The kinetic energy of translation K for n moles of gas is directly proportional to the absolute temperature T of the gas $\left(K = \frac{3}{2}nkT\right)$.

8 •

Determine the Concept We can use $v_{\text{rms}} = \sqrt{3RT/M}$ to relate the temperature of a gas to the rms speed of its molecules.

Express the dependence of the rms speed of the molecules of a gas on their absolute temperature:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where R is the gas constant, M is the molar mass, and T is the absolute temperature.

$\boxed{\text{Because } v_{\text{rms}} \propto \sqrt{T}\text{, the temperature must be quadrupled in order to double the rms speed of the molecules.}}$

9 •

Picture the Problem The average kinetic energy of a molecule, as a function of the temperature, is given by $K_{\text{av}} = \frac{3}{2}kT$ and the pressure, volume, and temperature of an ideal gas are related according to $PV = NkT$.

Express the average kinetic energy of a molecule in terms of its temperature:

$$K_{\text{av}} = \frac{3}{2}kT$$

From the ideal-gas law we have: $PV = NkT$

Eliminate kT between these equations to obtain: $K_{av} = \frac{3PV}{2N}$

$$\boxed{\text{If } P \text{ is doubled at constant } V,\ K_{av} \text{ increases by a factor of } 2.}$$

$$\boxed{\text{If } V \text{ is doubled at constant } P, K_{av} \text{ increases by a factor of } 2.}$$

10 ••

Picture the Problem We can express the rms speeds of the helium atoms and the methane molecules using $v_{rms} = \sqrt{3RT/M}$.

Express the rms speed of the helium atoms: $v_{rms}(\text{He}) = \sqrt{\frac{3RT}{M_{He}}}$

Express the rms speed of the methane molecules: $v_{rms}(\text{CH}_4) = \sqrt{\frac{3RT}{M_{CH_4}}}$

Divide the first of these equations by the second to obtain: $\frac{v_{rms}(\text{He})}{v_{rms}(\text{CH}_4)} = \sqrt{\frac{M_{CH_4}}{M_{He}}}$

Use Appendix C to find the molar masses of helium and methane: $\frac{v_{rms}(\text{He})}{v_{rms}(\text{CH}_4)} = \sqrt{\frac{16\,\text{g/mol}}{4\,\text{g/mol}}} = 2$

and $\boxed{(b) \text{ is correct.}}$

11 •

False. Whether the pressure changes also depends on whether and how the volume changes. In an isothermal process, the pressure can increase while the volume decreases and the temperature is constant.

12 •

Determine the Concept For the Celsius scale, the ice point (0°C) and the boiling point of water at 1 atm (100°C) are more convenient than 273 K and 373 K; temperatures in roughly this range are normally encountered. On the Fahrenheit scale, the temperature of warm-blooded animals is roughly 100°F; this may be a more convenient reference than approximately 300 K. Throughout most of the world, the Celsius scale is the standard for nonscientific purposes.

***13** •

Determine the Concept Because $10^7 >> 273$, it does not matter.

14 •

Determine the Concept The average speed of the molecules in an ideal gas depends on the square root of the kelvin temperature. Because $v_{av} \propto \sqrt{T}$, doubling the temperature while maintaining constant pressure increases the average speed by a factor of $\sqrt{2}$.

$\boxed{(d) \text{ is correct.}}$

15 •

Determine the Concept From the ideal-gas law, we have $PV = nRT$. Halving both the temperature and volume of the gas leaves the pressure unchanged. $\boxed{(b) \text{ is correct.}}$

16 •

Determine the Concept The average translational kinetic energy of the molecules of an ideal gas is given by $K = \frac{3}{2}NkT = \frac{3}{2}nRT$. The temperature of the ideal gas is related to the pressure of the gas. $\boxed{(d) \text{ is correct.}}$

17 •

Determine the Concept The only conclusion we can draw from the information that the vessel contains equal amounts, by weight, of helium and argon is that the temperatures of the helium and argon molecules are the same. $\boxed{(d) \text{ is correct.}}$

18 ••

Determine the Concept The two rooms are in thermal equilibrium and, because they are connected, the air in each is at the same pressure. Because $P = NkT/V$, and the volume of each room is identical, $N_A T_A = N_B T_B$, so the cooler room (A) has more air in it.

19 •

Determine the Concept The rms speed of an ideal gas is given by $v_{rms} = \sqrt{3RT/M}$ and its average kinetic energy by $K_{av} = \frac{3}{2}kT$. Because the gases are at the same temperature, their average kinetic energies are the same. Because $v_{rms} = \sqrt{3RT/M}$, the rms speeds are inversely proportional to the square root of the molecular masses.

20 ••

Determine the Concept The pressure is a measure of the change in momentum per second of a gas molecule on collision with the wall of the container. When the gas is heated, the average velocity, the average momentum, and pressure of the molecules increase.

***21 ••**

Determine the Concept Because the temperature remains constant, the average speed of the molecules remains constant. When the volume decreases, the molecules travel less distance between collisions, so the pressure increases because the frequency of collisions increases.

22 ••

Picture the Problem The average kinetic energies of the molecules are given by $K_{av} = \left(\frac{1}{2}mv^2\right)_{av} = \frac{3}{2}kT$. Assuming that the room's temperature distribution is uniform, we can conclude that the oxygen and nitrogen molecules have equal average kinetic energies. Because the oxygen molecules are more massive, they must be moving slower than the nitrogen molecules. $\boxed{(b)\text{ is correct.}}$

23 ••

Determine the Concept The average molecular speed of He gas at 300 K is about 1.4 km/s, so a significant fraction of He molecules have speeds in excess of earth's escape velocity (11.2 km/s). Thus, they "leak" away into space. Over time, the He content of the atmosphere decreases to almost nothing.

Estimation and Approximation

***24 ••**

Picture the Problem Assuming the steam to be an ideal gas at a temperature of 373 K, we can use the ideal-gas law to estimate the pressure inside the test tube when the water is completely boiled away.

Using the ideal-gas law, relate the pressure inside the test tube to its volume and the temperature:

$$P = \frac{NkT}{V}$$

Relate the number of particles N to the mass of water, its molar mass M, and Avogadro's number N_A:

$$\frac{m}{N} = \frac{M}{N_A}$$

Solve for N:

$$N = m\frac{N_A}{M}$$

Relate the mass of 1 mL of water to its density:

$$m = \rho V = \left(10^3\ \text{kg/m}^3\right)\left(10^{-6}\ \text{m}^3\right) = 1\,\text{g}$$

Substitute for m, N_A, and M (18 g/mol) and evaluate N:

$$N = (1\,\text{g})\frac{6.022\times10^{23}\ \text{particles/mol}}{18\,\text{g/mol}}$$

$$= 3.35\times10^{22}\ \text{particles}$$

Substitute numerical values and evaluate P:

$$P=\frac{\left(3.35\times10^{22}\text{ particles}\right)\left(1.381\times10^{-23}\text{ J/K}\right)\left(373\text{K}\right)}{10\times10^{-6}\text{ m}^3}$$

$$=172\times10^5\text{ N/m}^2\times\frac{1\text{atm}}{1.01\times10^5\text{ N/m}^2}$$

$$=\boxed{171\text{atm}}$$

25 •••

Picture the Problem We can find the escape temperatures for the earth and the moon by equating, in turn, $0.15v_e$ and v_{rms} of O_2 and H_2. We can compare these temperatures to explain the absence from the earth's upper atmosphere and from the surface of the moon.

(*a*) Express v_{rms} for O_2:

$$v_{rms}=\sqrt{\frac{3RT}{M}}$$

where R is the gas constant, T is the absolute temperature, and M is the molar mass of oxygen.

Equate $0.15v_e$ and v_{rms}:

$$0.15\sqrt{2gR_{earth}}=\sqrt{\frac{3RT}{M}}$$

Solve for T to obtain:

$$T=\frac{0.045gR_{earth}M}{3R}\qquad(1)$$

Evaluate T for O_2:

$$T=\frac{0.045\left(9.81\text{m/s}^2\right)\left(6.37\times10^6\text{ m}\right)}{3\left(8.314\text{J/mol}\cdot\text{K}\right)}\times\left(32\times10^{-3}\text{ kg/mol}\right)$$

$$=\boxed{3.61\times10^3\text{ K}}$$

(*b*) Substitute numerical values and evaluate T for H_2:

$$T=\frac{0.045\left(9.81\text{m/s}^2\right)\left(6.37\times10^6\text{ m}\right)}{3\left(8.314\text{J/mol}\cdot\text{K}\right)}\times\left(2\times10^{-3}\text{ kg/mol}\right)$$

$$=\boxed{225\text{K}}$$

(*c*) If $v_{rms}>\frac{1}{5}v_e$ or $T\geq25T_{atm}$, H_2 molecules escape. Therefore, the more energetic H_2 molecules escape from the upper atmosphere.

(*d*) Express equation (1) at the surface of the moon:

$$T=\frac{0.045g_{\text{moon}}R_{\text{moon}}M}{3R}=\frac{0.045\left(\frac{1}{6}g_{\text{earth}}\right)R_{\text{moon}}M}{3R}=\frac{0.0025g_{\text{earth}}R_{\text{moon}}M}{R}$$

Substitute numerical values and evaluate T for O_2:

$$T=\frac{0.0025\left(9.81\,\text{m/s}^2\right)\left(1.738\times10^6\,\text{m}\right)\left(32\times10^{-3}\,\text{kg/mol}\right)}{8.314\,\text{J/mol}\cdot\text{K}}=\boxed{164\,\text{K}}$$

Substitute numerical values and evaluate T for H_2:

$$T=\frac{0.0025\left(9.81\,\text{m/s}^2\right)\left(1.738\times10^6\,\text{m}\right)\left(2\times10^{-3}\,\text{kg/mol}\right)}{8.314\,\text{J/mol}\cdot\text{K}}=\boxed{10.3\,\text{K}}$$

If we assume that the temperature on the moon *with an atmosphere* would have been approximately 1000 K, then all O_2 and H_2 would have escaped during the time since the formation of the moon to the present.

26 ••

Picture the Problem We can use $v_{\text{rms}}=\sqrt{3RT/M}$ to calculate the rms speeds of H_2, O_2, and CO_2 at 273 K and then compare these speeds to 20% of the escape velocity on Mars to decide the likelihood of finding these gases in the atmosphere of Mars.

Express the rms speed of an atom as a function of the temperature:

$$v_{\text{rms}}=\sqrt{\frac{3RT}{M}}$$

(*a*) Substitute numerical values and evaluate v_{rms} for H_2:

$$v_{\text{rms},H_2}=\sqrt{\frac{3\left(8.314\,\text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right)}{2\times10^{-3}\,\text{kg/mol}}}=\boxed{1.85\,\text{km/s}}$$

(*b*) Evaluate v_{rms} for O_2:

$$v_{\text{rms},O_2}=\sqrt{\frac{3\left(8.314\,\text{J/mol}\cdot\text{K}\right)\left(273\,\text{K}\right)}{32\times10^{-3}\,\text{kg/mol}}}=\boxed{461\,\text{m/s}}$$

(*c*) Evaluate v_{rms} for CO_2:

$$v_{rms,CO_2} = \sqrt{\frac{3(8.314\,J/mol\cdot K)(273\,K)}{44\times10^{-3}\,kg/mol}} = \boxed{393\,m/s}$$

(*d*) Calculate 20% of v_{esc} for Mars:

$$v = \tfrac{1}{5}v_{esc} = \tfrac{1}{5}(5\,km/s) = 1\,km/s$$

Because v is greater than v_{rms} for CO_2 and O_2 but less than v_{rms} for H_2 O_2 and CO_2, but not H_2, should be present.

***27** ••

Picture the Problem We can use $v_{rms} = \sqrt{3RT/M}$ to calculate the rms speeds of H_2, O_2, and CO_2 at 123 K and then compare these speeds to 20% of the escape velocity on Jupiter to decide the likelihood of finding these gases in the atmosphere of Jupiter.

Express the rms speed of an atom as a function of the temperature:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

(*a*) Substitute numerical values and evaluate v_{rms} for H_2:

$$v_{rms,H_2} = \sqrt{\frac{3(8.314\,J/mol\cdot K)(123\,K)}{2\times10^{-3}\,kg/mol}} = \boxed{1.24\,km/s}$$

(*b*) Evaluate v_{rms} for O_2:

$$v_{rms,O_2} = \sqrt{\frac{3(8.314\,J/mol\cdot K)(123\,K)}{32\times10^{-3}\,kg/mol}} = \boxed{310\,m/s}$$

(*c*) Evaluate v_{rms} for CO_2:

$$v_{rms,CO_2} = \sqrt{\frac{3(8.314\,J/mol\cdot K)(123\,K)}{44\times10^{-3}\,kg/mol}} = \boxed{264\,m/s}$$

(*d*) Calculate 20% of v_{esc} for Jupiter:

$$v = \tfrac{1}{5}v_{esc} = \tfrac{1}{5}(60\,km/s) = 12\,km/s$$

Because v is greater than v_{rms} for O_2, CO_2, and H_2, O_2, CO_2, and H_2 should be found on Jupiter.

Temperature Scales

28 •

Picture the Problem We can convert both of these temperatures to the Fahrenheit scale and then express their difference to find the range of temperatures.

Solve the Fahrenheit-Celsius conversion equation for the Fahrenheit temperature:

$$t_F = \tfrac{9}{5}t_C + 32°$$

Find the Fahrenheit equivalent of −12°C:

$$t_F = \tfrac{9}{5}(-12°) + 32° = 10.4°$$

Find the Fahrenheit equivalent of −7°C:

$$t_F = \tfrac{9}{5}(-7°) + 32° = 19.4°\text{F}$$

The difference between these two temperatures is the range on the Fahrenheit scale:

$$\text{Range} = 19.4°\text{F} - 10.4°\text{F} = \boxed{9.00\text{F}°}$$

Remarks: We could take advantage of the fact that 9 F° = 5 C° to arrive at the aforementioned result in which the range of Celsius temperatures happens to be 5C°. If the temperature difference were other than 5C°, we could set up a proportion to quickly find the range on the Fahrenheit scale.

29 •

Picture the Problem We can use the Fahrenheit-Celsius conversion equation to find this temperature on the Celsius scale.

Convert 1945.4°F to the equivalent Celsius temperature:

$$t_C = \tfrac{5}{9}(t_F - 32°) = \tfrac{5}{9}(1945.4° - 32°) = \boxed{1063°\text{C}}$$

***30** •

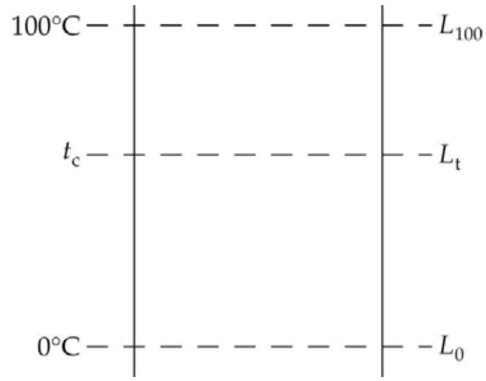
Picture the Problem We can use the Fahrenheit-Celsius conversion equation to express the temperature of the human body on the Celsius scale.

Convert 98.6°F to the equivalent Celsius temperature:

$$t_C = \tfrac{5}{9}(t_F - 32°) = \tfrac{5}{9}(98.6° - 32°) = \boxed{37.0°\text{C}}$$

31 •

Picture the Problem While we could use Equation 17-1 to relate the Celsius temperature to the length of the column of mercury in the thermometer, an alternative solution is to use the diagram to the right to set up a proportion that will relate the Celsius temperature to the calibration temperatures and to the lengths of the mercury column.



Using the diagram, set up a proportion relating the temperatures to the lengths of the column of mercury:

$$\frac{t_c - 0°\text{C}}{100°\text{C} - 0°\text{C}} = \frac{L_t - L_0}{L_{100} - L_0}$$

Solve for and evaluate L_t:

$$L_t = \frac{t_c(L_{100} - L_0)}{100°} + L_0$$
$$= \frac{t_c(24.0\,\text{cm} - 4.0\,\text{cm})}{100°} + 4.0\,\text{cm}$$
$$= \frac{t_c(20.0\,\text{cm})}{100°} + 4.0\,\text{cm}$$

(*a*) Substitute 22.0°C for t_c and evaluate L_t:

$$L_t = \frac{(22.0°\text{C})(20.0\,\text{cm})}{100°} + 4.0\,\text{cm}$$
$$= \boxed{8.40\,\text{cm}}$$

(*b*) Substitute 25.4 cm for L_t and evaluate t_c:

$$t_c = \frac{25.4\,\text{cm} - 4.0\,\text{cm}}{20\,\text{cm}} \times 100°$$
$$= \boxed{107°\text{C}}$$

32 •

Picture the Problem We can use the temperature conversion equations $t_F = \frac{9}{5}t_C + 32°$ and $t_C = T - 273.15\,\text{K}$ to convert 10^7 K to the Fahrenheit and Celsius temperatures.

Express the kelvin temperature in terms of the Celsius temperature:

$$T = t_C + 273.15\,\text{K}$$

(*a*) Solve for and evaluate t_C:

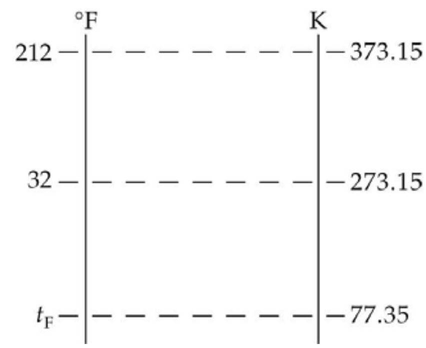
$$t_C = T - 273.15\,\text{K} = 10^7\,\text{K} - 273.15\,\text{K} \approx \boxed{10^7\,\text{K}}$$

(*b*) Use the Celsius to Fahrenheit conversion equation to evaluate t_F:

$$t_F = \tfrac{9}{5}\left(10^7\,°\text{C}\right) + 32° \approx \boxed{1.80\times10^7\,°\text{F}}$$

33 •

Picture the Problem While we could convert 77.35 K to a Celsius temperature and then convert the Celsius temperature to a Fahrenheit temperature, an alternative solution is to use the diagram to the right to set up a proportion for the direct conversion of the kelvin temperature to its Fahrenheit equivalent.



Use the diagram to set up the proportion:

$$\frac{32°\text{F} - t_F}{212°\text{F} - 32°\text{F}} = \frac{273.15\,\text{K} - 77.35\,\text{K}}{373.15\,\text{K} - 273.15\,\text{K}}$$

or

$$\frac{32°\text{F} - t_F}{180\,\text{F}°} = \frac{195}{100}$$

Solve for and evaluate t_F:

$$t_F = 32°\text{F} - \frac{195}{100}\times180\,\text{F}° = \boxed{-319\,°\text{F}}$$

34 •

Picture the Problem We can use the fact that, for a constant-volume gas thermometer, the pressure and absolute temperature are directly proportional to calibrate the given thermometer; i.e., to find the constant of proportionality relating P and T. We can then use this equation to find the temperature corresponding to a given pressure or the pressure corresponding to a given temperature.

Express the direct proportionality between the pressure and the temperature:

$$P = CT$$

where C is a constant.

Use numerical values to evaluate C:

$$C = \frac{P}{T} = \frac{0.400\,\text{atm}}{273.15\,\text{K}} = 1.464\times10^{-3}\,\text{atm/K}$$

Substitute to obtain:

$$P = \left(1.464\times10^{-3}\ \text{atm/K}\right)T \qquad (1)$$

or

$$T = \left(682.9\ \text{K/atm}\right)P \qquad (2)$$

(*a*) Use equation (2) to find the temperature:

$$T = \left(682.9\ \text{K/atm}\right)\left(0.1\text{atm}\right) = \boxed{68.3\ \text{K}}$$

(*b*) Use equation (1) to find the boiling point of sulfur:

$$P = \left(1.464\times10^{-3}\ \text{atm/K}\right)\times\left(444.6+273.15\right)\text{K} = \boxed{1.05\ \text{atm}}$$

***35 •**

Picture the Problem We can use the information that the thermometer reads 50 torr at the triple point of water to calibrate it. We can then use the direct proportionality between the absolute temperature and the pressure to either the pressure at a given temperature or the temperature for a given pressure.

Using the ideal-gas temperature scale, relate the temperature to the pressure:

$$T = \frac{273.16\ \text{K}}{P_3}P = \frac{273.16\ \text{K}}{50\ \text{torr}}P = \left(5.463\ \text{K/torr}\right)P$$

(*a*) Solve for and evaluate *P* when *T* = 300 K:

$$P = \left(0.1830\ \text{torr/K}\right)T = \left(0.1830\ \text{torr/K}\right)\left(300\ \text{K}\right) = \boxed{54.9\ \text{torr}}$$

(*b*) Find *T* when the pressure is 678 torr:

$$T = \left(5.463\ \text{K/torr}\right)\left(678\ \text{torr}\right) = \boxed{3704\ \text{K}}$$

36 •

Picture the Problem We can use the equation for the ideal-gas temperature scale to express the temperature measured by this thermometer in terms of its pressure and the given data to calibrate the thermometer.

Write the equation for the ideal-gas temperature scale:

$$T = \frac{273.16\ \text{K}}{P_3}P \qquad \text{17-4}$$

(*a*) Solve for and evaluate the thermometer's triple-point pressure:

$$P_3 = \frac{273.16\,\text{K}}{T}P = \frac{273.16\,\text{K}}{373\,\text{K}}(30\,\text{torr})$$
$$= \boxed{22.0\,\text{torr}}$$

(*b*) Substitute for P_3 in Equation 17-4:

$$T = \frac{273.16\,\text{K}}{22.0\,\text{torr}}P = \frac{273.16\,\text{K}}{22.0\,\text{torr}}(0.175\,\text{torr})$$
$$= \boxed{2.17\,\text{K}}$$

37 •

Picture the Problem We can find the temperature at which the Fahrenheit and Celsius scales give the same reading by setting $t_F = t_C$ in the temperature-conversion equation.

Set $t_F = t_C$ in $t_C = \frac{5}{9}(t_F - 32°)$:

$$t_F = \tfrac{5}{9}(t_F - 32°)$$

Solve for and evaluate t_F:

$$t_C = t_F = \boxed{-40.0°\text{C}} = \boxed{-40.0°\text{F}}$$

Remarks: If you've not already thought of doing so, you might use your graphing calculator to plot t_C versus t_F and $t_F = t_C$ (a straight line at 45°) on the same graph. Their intersection is at (–40, –40).

38 •

Picture the Problem We can use the Celsius-to-absolute conversion equation to find 371 K on the Celsius scale and the Celsius-to-Fahrenheit conversion equation to find the Fahrenheit temperature corresponding to 371 K.

Express the absolute temperature as a function of the Celsius temperature:

$$T = t_C + 273.15\,\text{K}$$

Solve for and evaluate t_C:

$$t_C = T - 273.15\,\text{K}$$
$$= 371\text{K} - 273.15\,\text{K} = \boxed{97.9°\text{C}}$$

Use the Celsius-to-Fahrenheit conversion equation to find t_F:

$$t_F = \tfrac{9}{5}t_C + 32° = \tfrac{9}{5}(97.9°) + 32°$$
$$= \boxed{208°\text{F}}$$

39 •

Picture the Problem We can use the Celsius-to-absolute conversion equation to find 90.2 K on the Celsius scale and the Celsius-to-Fahrenheit conversion equation to find the

Fahrenheit temperature corresponding to 90.2 K.

Express the absolute temperature as a function of the Celsius temperature:

$$T = t_C + 273.15\,\text{K}$$

Solve for and evaluate t_C:

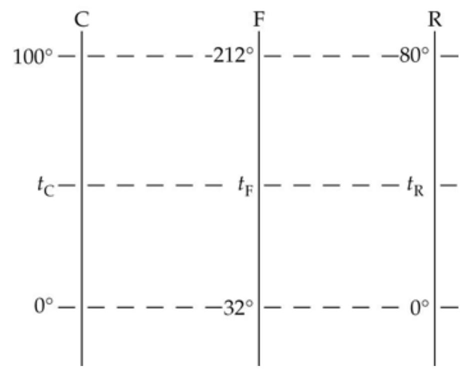
$$t_C = T - 273.15\,\text{K} = 90.2\,\text{K} - 273.15\,\text{K} = \boxed{-183°\text{C}}$$

Use the Celsius-to-Fahrenheit conversion equation to find t_F:

$$t_F = \tfrac{9}{5}t_C + 32° = \tfrac{9}{5}(-183°) + 32° = \boxed{-297°\text{F}}$$

40 ••

Picture the Problem We can use the diagram to the right to set up proportions that will allow us to convert temperatures on the Réaumur scale to Celsius and Fahrenheit temperatures.



Referring to the diagram, set up a proportion to convert temperatures on the Réaumur scale to Celsius temperatures:

$$\frac{t_C - 0°\text{C}}{100°\text{C} - 0°\text{C}} = \frac{t_R - 0°\text{R}}{80°\text{R} - 0°\text{R}}$$

Simplify to obtain:

$$\frac{t_C}{100} = \frac{t_R}{80} \text{ or } t_C = \boxed{1.25 t_R}$$

Referring to the diagram, set up a proportion to convert temperatures on the Réaumur scale to Fahrenheit temperatures:

$$\frac{t_F - 32°\text{F}}{212°\text{F} - 32°\text{F}} = \frac{t_R - 0°\text{R}}{80°\text{R} - 0°\text{R}}$$

Simplify to obtain:

$$\frac{t_F - 32}{180} = \frac{t_R}{80} \text{ or } t_F = \boxed{\tfrac{9}{4}t_R + 32}$$

***41** •••

Picture the Problem We can use the temperature dependence of the resistance of the thermistor and the given data to determine R_0 and B. Once we know these quantities, we can use the temperature-dependence equation to find the resistance at any temperature in the calibration range. Differentiation of R with respect to T will allow us to express the rate of change of resistance with temperature at both the ice point and the steam point temperatures.

(*a*) Express the resistance at the ice point as a function of temperature of the ice point:

$$7360\,\Omega = R_0 e^{B/273\,\text{K}} \qquad (1)$$

Express the resistance at the steam point as a function of temperature of the steam point:

$$153\,\Omega = R_0 e^{B/373\,\text{K}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{7360\,\Omega}{153\,\Omega} = 48.10 = e^{B/273\,\text{K} - B/373\,\text{K}}$$

Solve for B by taking the logarithm of both sides of the equation:

$$\ln 48.1 = B\left(\frac{1}{273} - \frac{1}{373}\right)\text{K}^{-1}$$

and

$$B = \frac{\ln 48.1}{\left(\frac{1}{273} - \frac{1}{373}\right)\text{K}^{-1}} = \boxed{3.94\times10^{3}\,\text{K}}$$

Solve equation (1) for R_0 and substitute for B:

$$R_0 = \frac{7360\,\Omega}{e^{B/273\,\text{K}}} = (7360\,\Omega)e^{-B/273\,\text{K}}$$

$$= (7360\,\Omega)e^{-3.94\times10^{3}\,\text{K}/273\,\text{K}}$$

$$= \boxed{3.97\times10^{-3}\,\Omega}$$

(*b*) From (*a*) we have:

$$R = \left(3.97\times10^{-3}\,\Omega\right)e^{3.94\times10^{3}\,\text{K}/T}$$

Convert 98.6°F to kelvins to obtain:

$$T = 310\,\text{K}$$

Substitute to obtain:

$$R = \left(3.97\times10^{-3}\,\Omega\right)e^{3.94\times10^{3}\,\text{K}/310\,\text{K}}$$

$$= \boxed{1.31\,\text{k}\Omega}$$

(*c*) Differentiate R with respect to T to obtain:

$$\frac{dR}{dT} = \frac{d}{dT}\left(R_0 e^{B/T}\right) = R_0 e^{B/T}\,\frac{d}{dT}\left(\frac{B}{T}\right)$$

$$= \frac{-B}{T^2}R_0 e^{B/T} = -\frac{RB}{T^2}$$

Evaluate dR/dT at the ice point:

$$\left(\frac{dR}{dT}\right)_{\text{ice point}} = -\frac{(7360\,\Omega)(3.94\times10^3\text{ K})}{(273.16\text{ K})^2} = \boxed{-389\,\Omega/\text{K}}$$

Evaluate dR/dT at the steam point:

$$\left(\frac{dR}{dT}\right)_{\text{steam point}} = -\frac{(153\,\Omega)(3.94\times10^3\text{ K})}{(373.16\text{ K})^2} = \boxed{-4.33\,\Omega/\text{K}}$$

(*d*) The thermistor is more sensitive; i.e., it has greater sensitivity at lower temperatures.

The Ideal-Gas Law

42 •

Picture the Problem Let the subscript 1 refer to the gas at 50°C and the subscript 2 to the gas at 100°C. We can apply the ideal-gas law for a fixed amount of gas to find the ratio of the final and initial volumes.

Apply the ideal-gas law for a fixed amount of gas:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

or, because $P_2 = P_1$,

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Substitute numerical values and evaluate V_2/V_1:

$$\frac{V_2}{V_1} = \frac{(273.15+100)\text{K}}{(273.15+50)\text{K}} = \boxed{1.15}$$

43 •

Picture the Problem We can use the ideal-gas law to find the number of moles of gas in the vessel and the definition of Avogadro’s number to find the number of molecules.

Apply the ideal-gas law to the gas:

$$PV = nRT$$

Solve for the number of moles of gas in the vessel:

$$n = \frac{PV}{RT}$$

Substitute numerical values and evaluate n:

$$n = \frac{(4\,\text{atm})(10\,\text{L})}{(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(273\,\text{K})} = \boxed{1.79\,\text{mol}}$$

Relate the number of molecules N in the gas in terms of the number of moles n:

$$N = nN_\text{A}$$

Substitute numerical values and evaluate N:

$$N = (1.79\,\text{mol})(6.022\times10^{23}\,\text{molecules/mol}) = \boxed{1.08\times10^{24}\,\text{molecules}}$$

44 ••

Picture the Problem We can use the ideal-gas law to relate the number of molecules in the gas to its pressure, volume, and temperature.

Solve the ideal-gas law for the number of molecules in a gas as a function of its pressure, volume, and temperature:

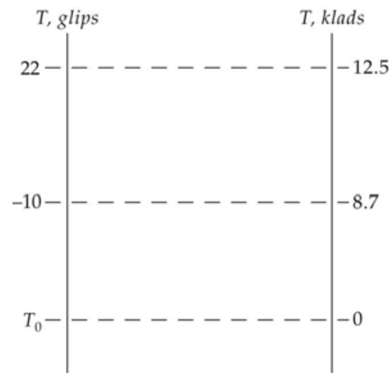
$$N = \frac{PV}{kT}$$

Substitute numerical values and evaluate N:

$$N = \frac{(10^{-8}\,\text{torr})(133.32\,\text{Pa/torr})(10^{-6}\,\text{m}^3)}{(1.381\times10^{-23}\,\text{J/K})(300\,\text{K})} = \boxed{3.22\times10^{8}}$$

45 ••

Picture the Problem The pictorial representation to the right, in which T_0 represents absolute zero, summarizes the information concerning the temperatures and pressures we are given. We know, from the ideal-gas law, that the pressure of a fixed volume of gas is proportional to its absolute temperature. We can use the diagram to set up a proportion relating the temperatures and pressures that we can solve for T_0.



Apply the ideal-gas law to obtain:

$$\frac{22\,glips - T_0}{12.5\,klads} = \frac{-10\,glips - T_0}{8.7\,klads}$$

Solve for T_0 to obtain:

$$T_0 = \boxed{-83.2\,glips}$$

Remarks: Because the gas is ideal, its pressure is directly proportional to its temperature. Hence, a graph of *P* versus *T* will be linear and the linear equation relating *P* and *T* can be solved for the temperature corresponding to zero pressure.

46 ••

Picture the Problem Let the subscript 1 refer to the tires when their pressure is 180 kPa and the subscript 2 to conditions when their pressure is 245 kPa. Assume that the air in the tires behaves as an ideal gas. Then, we can apply the ideal-gas law for a fixed amount of gas to relate the temperatures to the pressures and volumes of the tires.

(*a*) Apply the ideal-gas law for a fixed amount of gas to the air in the tires:

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1} \qquad (1)$$

Solve for T_2:

$$T_2=T_1\frac{P_2}{P_1} \text{ because } V_1 = V_2.$$

Substitute numerical values to obtain:

$$T_2=(265\,\text{K})\frac{245\,\text{kPa}}{180\,\text{kPa}}=360.7\,\text{K}$$

$$=\boxed{87.7°\text{C}}$$

(*b*) Use equation (1) with $V_2 = 1.07\,V_1$. Solve for T_2:

$$T_2=\frac{P_2V_2}{P_1V_1}T_1=1.07\left(\frac{P_2}{P_1}T_1\right)$$

Substitute numerical values and evaluate T_2:

$$T_2=1.07(360.7\,\text{K})=385.9\,\text{K}=\boxed{113°\text{C}}$$

47 ••

Picture the Problem We can apply the ideal-gas law to find the number of moles of air in the room as a function of the temperature.

(*a*) Use the ideal-gas law to relate the number of moles of air in the room to the pressure, volume, and temperature of the air:

$$n=\frac{PV}{RT} \qquad (1)$$

Substitute numerical values and evaluate *n*:

$$n=\frac{(101.3\,\text{kPa})(90\,\text{m}^3)}{(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}$$

$$=\boxed{3.66\times10^3\,\text{mol}}$$

(*b*) Letting n' represent the number of moles in the room when the temperature rises by 5 K, express the number of moles of air that leave the room:

$$\Delta n = n - n'$$

Apply the ideal-gas law to obtain:

$$n' = \frac{PV}{RT'} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{n'}{n} = \frac{T}{T'} \text{ and } n' = n\frac{T}{T'}$$

Substitute for n' to obtain:

$$\Delta n = n - n\frac{T}{T'} = n\left(1 - \frac{T}{T'}\right)$$

Substitute numerical values and evaluate Δn:

$$\Delta n = \left(3.66\times10^{3}\,\text{mol}\right)\left(1 - \frac{300\,\text{K}}{305\,\text{K}}\right) = \boxed{60.0\,\text{mol}}$$

***48 ••**

Picture the Problem Let the subscript 1 refer to helium gas at 4.2 K and the subscript 2 to the gas at 293 K. We can apply the ideal-gas law to find the volume of the gas at 4.2 K and a fixed amount of gas to find its volume at 293 K.

(*a*) Apply the ideal-gas law to the helium gas and solve for its volume:

$$V_1 = \frac{nRT_1}{P_1}$$

Substitute numerical values to obtain:

$$V_1 = n\frac{\left(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K}\right)\left(4.2\,\text{K}\right)}{1\text{atm}} = \left(0.3447\,\text{L/mol}\right)n$$

Find the number of moles in 10 g of helium:

$$n = \frac{10\,\text{g}}{4\,\text{g/mol}} = 2.5\,\text{mol}$$

Substitute for n to obtain:

$$V_1 = \left(0.3447\,\text{L/mol}\right)\left(2.5\,\text{mol}\right) = \boxed{0.862\,\text{L}}$$

(*b*) Apply the ideal-gas law for a fixed amount of gas and solve for

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

the volume of the helium gas at 293 K:

and, because $P_1 = P_2$,

$$V_2 = \frac{T_2}{T_1}V_1$$

Substitute numerical values and evaluate V_2:

$$V_2 = \frac{293\,\text{K}}{4.2\,\text{K}}(0.862\,\text{L}) = \boxed{60.1\,\text{L}}$$

49 ••

Picture the Problem Because the helium is initially in the liquid state, its temperature must be 4.2 K. Let the subscript 1 refer to helium gas at 4.2 K and the subscript 2 to the gas at 293 K. We can apply the ideal-gas law for a fixed volume of gas to relate the pressure at 293 K to the pressure at 4.2 K and use the ideal-gas law to find the pressure at 4.2 K.

Apply the ideal-gas law for a fixed amount of gas:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

Solve for its pressure at 293 K:

$$P_2 = \frac{P_1V_1T_2}{V_2T_1} = P_1\frac{T_2}{T_1} \qquad (1)$$

because $V_1 = V_2$

Apply the ideal-gas law to the helium gas at 4.2 K and solve for its pressure:

$$P_1 = \frac{nRT_1}{V_1}$$

Substitute numerical values to obtain:

$$P_1 = n\frac{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(4.2\,\text{K})}{6\,\text{L}} = (0.05744\,\text{atm/mol})n$$

Find the number of moles in 10 g of helium:

$$n = \frac{10\,\text{g}}{4\,\text{g/mol}} = 2.5\,\text{mol}$$

Substitute for n to obtain:

$$P_1 = (0.05744\,\text{atm/mol})(2.5\,\text{mol}) = 0.1436\,\text{atm}$$

Substitute in equation (1) and evaluate P_2:

$$P_2 = (0.1436\,\text{atm})\frac{293\,\text{K}}{4.2\,\text{K}} = \boxed{10.0\,\text{atm}}$$

***50** ••

Picture the Problem Let the subscript 1 refer to the tire when its temperature is 20°C and the subscript 2 to conditions when its temperature is 50°C. We can apply the ideal-gas law for a fixed amount of gas to relate the temperatures to the pressures of the air in the tire.

(*a*) Apply the ideal-gas law for a fixed amount of gas and solve for pressure at the higher temperature:

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1} \qquad (1)$$

and

$$P_2=\frac{T_2}{T_1}P_1$$

because $V_1 = V_2$

Substitute numerical values to obtain:

$$P_2=\frac{323\,\text{K}}{293\,\text{K}}(200\,\text{kPa}+101\,\text{kPa})$$
$$=332\,\text{kPa}$$

and

$$P_{2,\text{gauge}}=332\,\text{kPa}-101\,\text{kPa}$$
$$=\boxed{231\,\text{kPa}}$$

(*b*) Solve equation (1) for P_2 with $V_2 = 1.1\ V_1$ and evaluate P_2:

$$P_2=\frac{V_1T_2}{V_2T_1}P_1$$
$$=\frac{323\,\text{K}}{1.1(293\,\text{K})}(200\,\text{kPa}+101\,\text{kPa})$$
$$=302\,\text{kPa}$$

and

$$P_{2,\text{gauge}}=302\,\text{kPa}-101\,\text{kPa}=\boxed{201\,\text{kPa}}$$

51 ••

Picture the Problem Let ρ_{N_2} and ρ_{0_2} be the number densities (i.e., the number of particles per unit volume) of N_2 and O_2, respectively. We can express the density of air in terms of the densities of nitrogen and oxygen and their number densities as $\rho_{\text{air}}=m_{N_2}\rho_{N_2}+m_{0_2}\rho_{0_2}$. By applying the ideal-gas law, we can find the number density of air and, using the given composition of air, calculate the number densities of nitrogen and oxygen. Finally, we can find the masses of nitrogen and oxygen molecules from their atomic masses. Knowing ρ_{N_2}, ρ_{O_2}, m_{N_2}, and m_{O_2}, we can calculate ρ_{air}.

Express the density of air in terms of the densities of nitrogen and oxygen:

$$\rho_{\text{air}}=m_{N_2}\rho_{N_2}+m_{0_2}\rho_{0_2} \qquad (1)$$

Using the ideal-gas law, relate the number density of air N/V to its temperature and pressure:

$$PV=NkT \text{ and } \frac{N}{V}=\frac{P}{kT}$$

Substitute numerical values and evaluate the number density of air:

$$\frac{N}{V} = \frac{1.01\times10^{5}\ \text{N/m}^2}{(1.381\times10^{-23}\ \text{J/K})(297\ \text{K})} = 2.46\times10^{25}\ \text{m}^{-3}$$

Because air is approximately 74% N_2 and 26% O_2:

$$\rho_{N_2} = 0.74\frac{N}{V} = 0.74(2.46\times10^{25}\ \text{m}^{-3}) = 1.82\times10^{25}\ \text{m}^{-3}$$

and

$$\rho_{O_2} = 0.26\frac{N}{V} = 0.26(2.46\times10^{25}\ \text{m}^{-3}) = 6.40\times10^{24}\ \text{m}^{-3}$$

Calculate the masses of N_2 and O_2 molecules:

$$m_{N_2} = (28\ \text{u})(1.660\times10^{-27}\ \text{kg/u}) = 4.65\times10^{-26}\ \text{kg}$$

and

$$m_{O_2} = (32\ \text{u})(1.660\times10^{-27}\ \text{kg/u}) = 5.31\times10^{-26}\ \text{kg}$$

Substitute in equation (1) and evaluate ρ_{air}:

$$\rho_{\text{air}} = (4.65\times10^{-26}\ \text{kg})(1.82\times10^{25}\ \text{m}^{-3}) + (5.31\times10^{-26}\ \text{kg})(6.40\times10^{24}\ \text{m}^{-3}) = \boxed{1.19\ \text{kg/m}^3}$$

52 ••

Picture the Problem Let the subscript 1 refer to the conditions at the bottom of the lake and the subscript 2 to the surface of the lake and apply the ideal-gas law for a fixed amount of gas.

Apply the ideal-gas law for a fixed amount of gas:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

Solve for the volume of the bubble just before it breaks the surface:

$$V_2 = V_1\frac{T_2P_1}{T_1P_2}$$

Find the pressure at the bottom of the lake:

$$P_1 = P_{\text{atm}} + \rho gh = 101.3\ \text{kPa} + (10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(40\ \text{m}) = 493.7\ \text{kPa}$$

Substitute numerical values and evaluate V_2:

$$V_2 = \left(15\,\text{cm}^3\right)\frac{(298\,\text{K})(493.7\,\text{kPa})}{(278\,\text{K})(101.3\,\text{kPa})} = \boxed{78.4\,\text{cm}^3}$$

53 ••

Picture the Problem Assume that the volume of the balloon is not changing. Then the air inside and outside the balloon must be at the same pressure of about 1 atm. The contents of the balloon are the air molecules inside it. We can use Archimedes principle to express the buoyant force on the balloon and we can find the weight of the air molecules inside the balloon

Express the net force on the balloon and its contents:

$$F_{\text{net}} = B - w_{\text{air inside the balloon}} \qquad (1)$$

Using Archimedes principle, express the buoyant force on the balloon:

$$B = w_{\text{displaced fluid}} = m_{\text{displaced fluid}}\,g$$

or

$$B = \rho_{\text{o}} V_{\text{balloon}}\, g$$

where ρ_{o} is the density of the air outside the balloon.

Express the weight of the air inside the balloon:

$$w_{\text{air inside the balloon}} = \rho_{\text{i}} V_{\text{balloon}}\, g$$

where ρ_{i} is the density of the air inside the balloon.

Substitute in equation (1) for B and $w_{\text{air inside the balloon}}$ to obtain:

$$F_{\text{net}} = \rho_{\text{o}} V_{\text{balloon}}\, g - \rho_{\text{i}} V_{\text{balloon}}\, g = (\rho_{\text{o}} - \rho_{\text{i}}) V_{\text{balloon}}\, g \qquad (2)$$

Express the densities of the air molecules in terms of their number densities, molecular mass, and Avogadro's number:

$$\rho = \frac{M}{N_{\text{A}}}\left(\frac{N}{V}\right)$$

Using the ideal-gas law, relate the number density of air N/V to its temperature and pressure:

$$PV = NkT \text{ and } \frac{N}{V} = \frac{P}{kT}$$

Substitute to obtain:

$$\rho = \frac{M}{N_{\text{A}}}\left(\frac{P}{kT}\right)$$

Substitute in equation (2) and simplify to obtain:

$$F_{\text{net}} = \frac{MP}{N_{\text{A}}k}\left(\frac{1}{T_{\text{o}}} - \frac{1}{T_{\text{i}}}\right) V_{\text{balloon}}\, g$$

Assuming that the average molecular weight of air is 29 g/mol, substitute numerical values and evaluate F_{net}:

$$F_{net} = \frac{(29\,\text{g/mol})(1.01\times10^{5}\,\text{N/m}^2)}{(6.022\times10^{23}\,\text{particles/mol})(1.381\times10^{-23}\,\text{J/K})}\left(\frac{1}{297\,\text{K}} - \frac{1}{348\,\text{K}}\right) \times (1.5\,\text{m}^3)(9.81\,\text{m/s}^2) = \boxed{2.56\,\text{N}}$$

54 •••

Picture the Problem We can find the number of moles of helium gas in the balloon by applying the ideal-gas law to relate n to the pressure, volume, and temperature of the helium and Archimedes principle to find the volume of the helium. In part (*b*), we can apply the result of Problem 13-95 to relate atmospheric pressure to altitude and use the ideal-gas law to determine the pressure of the gas when the balloon is fully inflated. In part (c), we'll find the net force acting on the balloon at the altitude at which it is fully inflated in order to decide whether it can rise to that altitude.

(*a*) Apply the ideal-gas law to the helium in the balloon and solve for n:

$$n = \frac{PV}{RT} \quad (1)$$

Relate the net force on the balloon to its weight:

$$F_B - w_{skin} - w_{load} - w_{He} = 30\,\text{N}$$

Use Archimedes principle to express the buoyant force on the balloon in terms of the volume of the balloon:

$$F_B = w_{displaced\,air} = \rho_{air} V g$$

Substitute to obtain:

$$\rho_{air} V g - w_{skin} - w_{load} - \rho_{He} V g = 30\,\text{N}$$

Solve for the volume of the helium:

$$V = \frac{30\,\text{N} + w_{skin} + w_{load}}{(\rho_{air} - \rho_{He})g}$$

Substitute numerical values and evaluate V:

$$V = \frac{30\,\text{N} + 50\,\text{N} + 110\,\text{N}}{(1.293\,\text{kg/m}^3 - 0.179\,\text{kg/m}^3)} \times \frac{1}{(9.81\,\text{m/s}^2)} = 17.39\,\text{m}^3$$

Substitute numerical values in equation (1) and evaluate n:

$$n=\frac{(1\,\text{atm})(17.39\,\text{m}^3)\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right)}{(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(273\,\text{K})}=\boxed{776\,\text{mol}}$$

(b) Using the result of Problem 13-95, express the variation in atmospheric pressure with altitude:

$$P(h)=P_0e^{-h/h_0}$$

where h_0 = 7.93 km

Solve for h:

$$h=h_0\ln\left[\frac{P_0}{P(h)}\right] \qquad (2)$$

Neglecting changes in temperature with elevation, apply the ideal-gas law to find the pressure at which the balloon's volume will be 32 m^3:

$$P=\frac{nRT}{V}=\frac{(776\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(273\,\text{K})}{32\,\text{m}^3\times\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}}=0.543\,\text{atm}$$

Substitute in equation (2) and evaluate h:

$$h=(7.93\,\text{km})\ln\left(\frac{1\,\text{atm}}{0.543\,\text{atm}}\right)=\boxed{4.84\,\text{km}}$$

(c) Express the condition that must be satisfied if the balloon is to reach its fully inflated altitude:

$$F_{\text{net}}=F_{\text{B}}-w_{\text{tot}}\geq0 \qquad (3)$$

Express w_{tot}:

$$\begin{aligned}w_{\text{tot}}&=w_{\text{load}}+w_{\text{skin}}+w_{\text{He}}\\&=110\,N+50\,N+w_{\text{He}}\\&=160\,\text{N}+w_{\text{He}}\end{aligned}$$

Express the weight of the helium:

$$w_{\text{He}}=\rho_{\text{He}}Vg$$

Substitute for w_{He} and evaluate w_{tot}:

$$\begin{aligned}w_{\text{tot}}&=160\,\text{N}+\rho_{\text{He}}Vg\\&=160\,\text{N}+(0.179\,\text{kg/m}^3)(17.38\,\text{m}^3)\\&\quad\times(9.81\,\text{m/s}^2)\\&=190.5\,\text{N}\end{aligned}$$

Determine the buoyant force on the balloon at h = 4.84 km:

$$F_B = \rho_{\text{air},h} V g \qquad (4)$$

Express the dependence of the density of the air on atmospheric pressure:

$$\frac{P}{P_0} = \frac{\rho_{\text{air},h}}{\rho_{\text{air}}} \qquad (5)$$

or

$$\rho_{\text{air},h} = \frac{P}{P_0}\rho_{\text{air}}$$

Substitute and evaluate F_B:

$$F_B = \frac{P}{P_0}\rho_{\text{air}} V g = 0.543\left(1.293\,\text{kg/m}^3\right)\left(32\,\text{m}^3\right) \times \left(9.81\,\text{m/s}^2\right) = 219.9\,\text{N}$$

Substitute in equation (3) and evaluate F_{net}:

$$F_{\text{net}} = 219.9\,\text{N} - 190.5\,\text{N} = 29.4\,\text{N} \geq 0$$

Because $F_{\text{net}} > 0$, the balloon will rise higher than the altitude at which it is fully inflated.

(d) The balloon will rise until the net force acting on it is zero. Because the buoyant force depends on the density of the air, the balloon will rise until the density of the air has decreased sufficiently for the buoyant force to just equal the total weight of the balloon.

Substitute equation (5) in equation (2) to obtain:

$$h = h_0 \ln\frac{\rho_{\text{air}}}{\rho_{\text{air},h}}$$

Using equation (4), find the density of the air such that F_B = 190.5 N:

$$\rho_{\text{air},h} = \frac{F_B}{Vg} = \frac{190.5\,\text{N}}{\left(32\,\text{m}^3\right)\left(9.81\,\text{m/s}^2\right)} = 0.6068\,\text{kg/m}^3$$

Substitute numerical values and evaluate h:

$$h = \left(7.93\,\text{km}\right)\ln\left(\frac{1.293\,\text{kg/m}^3}{0.6068\,\text{kg/m}^3}\right) = \boxed{6.00\,\text{km}}$$

Kinetic Theory of Gases

***55** •

Picture the Problem We can express the rms speeds of argon and helium atoms by combining $PV = nRT$ and $v_{\text{rms}} = \sqrt{3RT/M}$ to obtain an expression for v_{rms} in terms of *P, V,* and *M*.

Express the rms speed of an atom as a function of the temperature:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

From the ideal-gas law we have:

$$RT = \frac{PV}{n}$$

Substitute to obtain:

$$v_{\text{rms}} = \sqrt{\frac{3PV}{nM}}$$

(*a*) Substitute numerical values and evaluate v_{rms} for an argon atom:

$$v_{\text{rms}}(\text{Ar}) = \sqrt{\frac{3(10\,\text{atm})(101.3\,\text{kPa/atm})(10^{-3}\,\text{m}^3)}{(1\,\text{mol})(40\times10^{-3}\,\text{kg/mol})}} = \boxed{276\,\text{m/s}}$$

(*b*) Substitute numerical values and evaluate v_{rms} for a helium atom:

$$v_{\text{rms}}(\text{He}) = \sqrt{\frac{3(10\,\text{atm})(101.3\,\text{kPa/atm})(10^{-3}\,\text{m}^3)}{(1\,\text{mol})(4\times10^{-3}\,\text{kg/mol})}} = \boxed{872\,\text{m/s}}$$

56 •

Picture the Problem We can express the total translational kinetic energy of the oxygen gas by combining $K = \frac{3}{2}nRT$ and the ideal-gas law to obtain an expression for *K* in terms of the pressure and volume of the gas.

Relate the total translational kinetic energy of translation to the temperature of the gas:

$$K = \tfrac{3}{2}nRT$$

Using the ideal-gas law, substitute for nRT to obtain:

$$K = \tfrac{3}{2}PV$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{3}{2}(101.3\,\text{kPa})(10^{-3}\,\text{m}^3) = \boxed{152\,\text{J}}$$

57 •

Picture the Problem Because we're given the temperature of the hydrogen atom and know its molar mass, we can find its rms speed using $v_{rms} = \sqrt{3RT/M}$ and its average kinetic energy from $K_{av} = \frac{3}{2}kT$.

Relate the rms speed of a hydrogen atom to its temperature and molar mass:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Substitute numerical values and evaluate v_{rms}:

$$v_{rms} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(10^7\,\text{K})}{10^{-3}\,\text{kg/mol}}} = \boxed{499\,\text{km/s}}$$

Express the average kinetic energy of the hydrogen atom as a function of its temperature:

$$K_{av} = \frac{3}{2}kT$$

Substitute numerical values and evaluate K_{av}:

$$K_{av} = \frac{3}{2}(1.381\times10^{-23}\,\text{J/K})(10^7\,\text{K}) = \boxed{2.07\times10^{-16}\,\text{J}}$$

***58** •

Picture the Problem Because there are 6 squared terms in the expression for the total energy of an atom in this model, we can conclude that there are 6 degrees of freedom. Because the system is in equilibrium, we can conclude that there is energy of $\frac{1}{2}kT$ per molecule or $\frac{1}{2}RT$ per mole associated with each degree of freedom.

Express the average energy per atom in the solid in terms of its temperature and the number of degrees of freedom:

$$\frac{E_{av}}{\text{atom}} = N\left(\tfrac{1}{2}kT\right) = 6\left(\tfrac{1}{2}kT\right) = \boxed{3kT}$$

Relate the total energy of one mole to its temperature and the number of degrees of freedom:

$$\frac{E_{tot}}{\text{mole}} = N\left(\tfrac{1}{2}RT\right) = 6\left(\tfrac{1}{2}RT\right) = \boxed{3RT}$$

59 •

Picture the Problem We can combine $\lambda = \dfrac{1}{\sqrt{2}n_v\pi d^2}$ and $PV = nRT$ to express the mean free path for a molecule in an ideal gas in terms of the pressure and temperature.

Express the mean free path of a molecule in an ideal gas:

$$\lambda = \frac{1}{\sqrt{2}n_v \pi d^2}$$

where

$$n_v = N/V = nN_A/V$$

Solve the ideal-gas law for the volume of the gas:

$$V = \frac{nRT}{P}$$

Substitute in our expression for n_v to obtain:

$$n_v = \frac{nN_A}{nRT}P = \frac{P}{kT}$$

Substitute in the expression for the mean free path to obtain:

$$\lambda = \boxed{\frac{kT}{\sqrt{2}P\pi d^2}}$$

60 ••

Picture the Problem We can find the collision time from the mean free path and the average (rms) speed of the helium molecules. We can use the result of Problem 43 to find the mean free path of the molecules and $v_{rms} = \sqrt{3RT/M}$ to find the average speed of the molecules.

Express the collision time in terms of the mean free path for and the average speed of a helium molecule:

$$\tau = \frac{\lambda}{v_{av}} \qquad (1)$$

Use the result of Problem 43 to express the mean free path of the gas:

$$\lambda = \frac{kT}{\sqrt{2}P\pi d^2}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{(1.381\times10^{-23}\text{ J/K})(300\text{ K})}{\sqrt{2}(7\times10^{-11}\text{ Pa})\pi(10^{-10}\text{ m})^2} = 1.332\times10^{9}\text{ m}$$

Express the average speed of the molecules:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Substitute numerical values and evaluate v_{rms}:

$$v_{rms} = \sqrt{\frac{3(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{4\times10^{-3}\text{ kg/mol}}} = 1.368\times10^{3}\text{ m/s}$$

Substitute in equation (1) and evaluate τ:

$$\tau = \frac{1.332\times10^{9}\ \text{m}}{1.368\times10^{3}\ \text{m/s}} = \boxed{9.74\times10^{5}\ \text{s}}$$

***61 ••**

Picture the Problem We can use $K = \frac{3}{2}kT$ and $\Delta U = mgh = Mgh/N_{\text{A}}$ to express the ratio of the average kinetic energy of a molecule of the gas to the change in its gravitational potential energy if it falls from the top of the container to the bottom.

Express the average kinetic energy of a molecule of the gas as a function of its temperature:

$$K = \tfrac{3}{2}kT$$

Letting h represent the height of the container, express the change in the potential energy of a molecule as it falls from the top of the container to the bottom:

$$\Delta U = mgh = \frac{Mgh}{N_{\text{A}}}$$

Express the ratio of K to ΔU and simplify to obtain:

$$\frac{K}{\Delta U} = \frac{\frac{3}{2}kT}{\frac{Mgh}{N_{\text{A}}}} = \frac{3N_{\text{A}}kT}{2Mgh}$$

Substitute numerical values and evaluate $K/\Delta U$:

$$\frac{K}{\Delta U} = \frac{3(6.022\times10^{23})(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})}{2(32\times10^{-3}\ \text{kg})(9.81\ \text{m/s}^2)(0.15\ \text{m})} = \boxed{7.95\times10^{4}}$$

The Distribution of Molecular Speeds

62 ••

Picture the Problem Equation 17-37 gives the Maxwell-Boltzmann speed distribution. Setting its derivative with respect to v equal to zero will tell us where the function's extreme values lie.

Differentiate Equation 17-37 with respect to v:

$$\frac{df}{dv} = \frac{d}{dv}\left[\frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\right]$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\left(2v - \frac{mv^3}{kT}\right)e^{-mv^2/2kT}$$

Set $df/dv = 0$ for extrema and solve for v:

$$2v - \frac{mv^3}{kT} = 0 \Rightarrow v = \boxed{\sqrt{\frac{2kT}{m}}}$$

Examination of the graph of $f(v)$ makes it clear that this extreme value is, in fact, a maximum. See Figure 17-16 and note that it is concave downward at $v = \sqrt{2kT/m}$.

Remarks: An alternative to the examination of $f(v)$ in order to conclude that $v = \sqrt{2kT/m}$ maximizes the Maxwell-Boltzmann speed distribution function is to show that $d^2f/dv^2 < 0$ at $v = \sqrt{2kT/m}$.

***63 ••**

Picture the Problem We can show that $f(v)$ is normalized by using the given integral to integrate it over all possible speeds.

Express the integral of Equation 17-37:

$$\int_0^\infty f(v)dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}dv$$

Let $a = m/2kT$ to obtain:

$$\int_0^\infty f(v)dv = \frac{4}{\sqrt{\pi}}a^{3/2}\int_0^\infty v^2 e^{-av^2}dv$$

Use the given integral to obtain:

$$\int_0^\infty f(v)dv = \frac{4}{\sqrt{\pi}}a^{3/2}\left(\frac{\sqrt{\pi}}{4}a^{-3/2}\right) = \boxed{1}$$

i.e., $f(v)$ is normalized.

64 ••

Picture the Problem In Problem 63 we showed that $f(v)$ is normalized. Hence we can evaluate v_{av} using $\int_0^\infty vf(v)dv$.

Express the average speed of the molecules in the gas:

$$v_{av} = \int_0^\infty vf(v)dv$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^\infty v^3 e^{-mv^2/2kT}dv$$

Substitute $a = m/2kT$:

$$v_{av} = \frac{4}{\sqrt{\pi}} a^{3/2} \int_0^\infty v^3 e^{-av^2} dv$$

Use the given integral to obtain:

$$v_{av} = \frac{4}{\sqrt{\pi}} a^{3/2} \left(\frac{a^{-2}}{2}\right) = \frac{2}{\sqrt{\pi}} \frac{1}{\sqrt{a}}$$

$$= \boxed{\frac{2}{\sqrt{\pi}} \sqrt{\frac{2kT}{m}}}$$

***65 ••**

Picture the Problem Choose a coordinate system in which downward is the positive direction. We can use a constant-acceleration equation to relate the fall distance to the initial velocity of the molecule, the acceleration due to gravity, the fall time, and $v_{rms} = \sqrt{3kT/m}$ to find the initial velocity of the molecules.

(*a*) Using a constant-acceleration equation, relate the fall distance to the initial velocity of a molecule, the acceleration due to gravity, and the fall time:

$$y = v_0 t + \tfrac{1}{2} g t^2 \quad (1)$$

Express the rms speed of the atom to its temperature and mass:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Substitute numerical values and evaluate v_{rms}:

$$v_{rms} = \sqrt{\frac{3(1.381\times10^{-23}\,\text{J/K})(120\,\text{nK})}{(85.47\,\text{u})(1.660\times10^{-27}\,\text{kg/u})}}$$

$$= 5.92\times10^{-3}\,\text{m/s}$$

Letting $v_{rms} = v_0$, substitute in equation (1) to obtain:

$$0.1\text{m} = (5.92\times10^{-3}\,\text{m/s})t + \tfrac{1}{2}(9.81\text{m/s}^2)t^2$$

Solve this equation to obtain:

$$t = \boxed{0.142\,\text{s}}$$

(*b*) If the atom is initially moving upward:

$$v_{rms} = v_0 = -5.92\times10^{-3}\,\text{m/s}$$

Substitute in equation (1) to obtain:

$$0.1\text{m} = (-5.92\times10^{-3}\,\text{m/s})t + \tfrac{1}{2}(9.81\text{m/s}^2)t^2$$

Solve this equation to obtain: $t = \boxed{0.143\,\text{s}}$

General Problems

66 •

Picture the Problem We can use $v_{\text{rms}} = \sqrt{3RT/M}$ to relate the temperature of the H_2 molecule to its rms speed.

Relate the rms speed of the molecule to its temperature:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Solve for the temperature:

$$T = \frac{Mv_{\text{rms}}^2}{3R}$$

Substitute numerical values and evaluate T:

$$T = \frac{(2\times10^{-3}\,\text{kg/mol})(331\,\text{m/s})^2}{3(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{8.79\,\text{K}}$$

67 ••

Picture the Problem We can use the ideal-gas law to find the initial temperature of the gas and the ideal-gas law for a fixed amount of gas to relate the volumes, pressures, and temperatures resulting from the given processes.

(*a*) Apply the ideal-gas law to express the temperature of the gas:

$$T = \frac{PV}{nR}$$

Substitute numerical values and evaluate T:

$$T = \frac{(101.3\,\text{kPa})(10\times10^{-3}\,\text{m}^3)}{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{122\,\text{K}}$$

(*b*) Use the ideal-gas law for a fixed amount of gas to relate the temperatures and volumes:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

or, because $P_1 = P_2$,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Solve for and evaluate T_2:

$$T_2 = \frac{V_2}{V_1}T_1 = 2(122\,\text{K}) = \boxed{244\,\text{K}}$$

(*c*) Use the ideal-gas law for a fixed amount of gas to relate the

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

temperatures and pressures: or, because $V_1 = V_2$,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Solve for T_2:

$$P_2 = \frac{T_2}{T_1} P_1$$

Substitute numerical values and evaluate P_2:

$$P_2 = \frac{350\,\text{K}}{244\,\text{K}}(1\,\text{atm}) = \boxed{1.43\,\text{atm}}$$

68 ••

Picture the Problem We can use the definition of pressure to express the net force on each wall of the box in terms of its area and the pressure differential between the inside and the outside of the box. We can apply the ideal-gas law for a fixed amount of gas to find the pressure inside the box.

Using the definition of pressure, express the net force on each wall of the box:

$$F = A\Delta P = A\left(P_{\text{inside}} - P_{\text{outside}}\right)$$

Use the ideal-gas law for a fixed amount of gas to relate the initial and final pressures of the gas:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

or, because $V_1 = V_2$,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Solve for and evaluate P_{inside}:

$$P_2 = P_{\text{inside}} = \frac{T_2}{T_1} P_1 = \frac{400\,\text{K}}{300\,\text{K}}(101.3\,\text{kPa}) = 135.1\,\text{kPa}$$

Substitute and evaluate F:

$$F = (0.2\,\text{m})^2(135.1\,\text{kPa} - 101.3\,\text{kPa}) = \boxed{1.35\,\text{kN}}$$

***69 ••**

Picture the Problem We can use the molar mass of water to find the number of moles in 2 L of water. Because there are two hydrogen atoms in each molecule of water, there must be as many hydrogen molecules in the gas formed by electrolysis as there were molecules of water and, because there is one oxygen atom in each molecule of water, there must be half as many oxygen molecules in the gas formed by electrolysis as there were molecules of water.

Express the electrolysis of water into H_2 and O_2:

$$n(\mathrm{H_2O}) \to n(\mathrm{H_2}) + \tfrac{1}{2}n(\mathrm{O_2})$$

Express the number of moles in 2 L of water:

$$n(\mathrm{H_2O}) = \frac{2000\,\mathrm{g}}{18\,\mathrm{g/mol}} = 111\,\mathrm{mol}$$

Because there is one hydrogen atom for each water molecule:

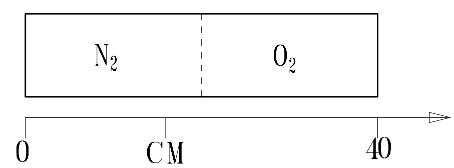
$$n(\mathrm{H_2}) = \boxed{111\,\mathrm{mol}}$$

Because there are two oxygen atoms for each water molecule:

$$n(\mathrm{O_2}) = \tfrac{1}{2}n(\mathrm{H_2O}) = \tfrac{1}{2}(111\,\mathrm{mol}) = \boxed{55.5\,\mathrm{mol}}$$

70 ••

Picture the Problem The diagram shows the cylinder before removal of the membrane. We'll assume that the gases are at the same temperature. The approximate location of the center of mass (CM) is indicated. We can find the distance the cylinder moves by finding the location of the CM after the membrane is removed.



Express the distance the cylinder will move in terms of the movement of the center of mass when the membrane is removed:

$$\Delta x = x_{\mathrm{cm,after}} - x_{\mathrm{cm,before}}$$

Apply the ideal-gas law to both collections of molecules to obtain:

$$P_{\mathrm{N_2}}V_{\mathrm{N_2}} = n(\mathrm{N_2})kT$$

and

$$P_{\mathrm{O_2}}V_{\mathrm{O_2}} = n(\mathrm{O_2})kT$$

Divide the first of these equations by the second to obtain:

$$\frac{P_{\mathrm{N_2}}}{P_{\mathrm{O_2}}} = \frac{n(\mathrm{N_2})}{n(\mathrm{O_2})}$$

or, because $P_{\mathrm{N_2}} = 2P_{\mathrm{O_2}}$,

$$\frac{2P_{\mathrm{O_2}}}{P_{\mathrm{O_2}}} = \frac{n(\mathrm{N_2})}{n(\mathrm{O_2})} \Rightarrow n(\mathrm{N_2}) = 2n(\mathrm{O_2})$$

Express the mass of O_2 in terms of its molar mass and the number of moles of oxygen:

$$m(\mathrm{O_2}) = n(\mathrm{O_2})M(\mathrm{O_2})$$

Express the mass of N_2 in terms of its molar mass and the number of moles of nitrogen:

$$m(N_2) = 2n(O_2)M(N_2).$$

Using its definition, express the center of mass before the membrane is removed:

$$x_{\mathrm{cm,before}} = \frac{\sum_i x_i m_i}{\sum_i m_i} = \frac{n(N_2)M(N_2)x_{\mathrm{cm,N_2}} + n(O_2)M(O_2)x_{\mathrm{cm,O_2}}}{n(N_2)M(N_2) + n(O_2)M(O_2)}$$
$$= \frac{2n(O_2)M(N_2)x_{\mathrm{cm,N_2}} + n(O_2)M(O_2)x_{\mathrm{cm,O_2}}}{2n(O_2)M(N_2) + n(O_2)M(O_2)}$$
$$= \frac{2M(N_2)x_{\mathrm{cm,N_2}} + M(O_2)x_{\mathrm{cm,O_2}}}{2M(N_2) + M(O_2)}$$

Substitute numerical values and evaluate $x_{\mathrm{cm,before}}$:

$$x_{\mathrm{cm,before}} = \frac{2(10\,\mathrm{cm})(28\,\mathrm{g}) + (30\,\mathrm{cm})(32\,\mathrm{g})}{2(28\,\mathrm{g}) + 32\,\mathrm{g}} = 17.27\,\mathrm{cm}$$

Locate the center of mass after the membrane is removed:

$$x_{\mathrm{cm,after}} = \frac{2(20\,\mathrm{cm})(28\,\mathrm{g}) + (20\,\mathrm{cm})(32\,\mathrm{g})}{2(28\,\mathrm{g}) + 32\,\mathrm{g}}$$
$$= 20.0\,\mathrm{cm}$$

Substitute to obtain:

$$\Delta x = 20.00\,\mathrm{cm} - 17.27\,\mathrm{cm}$$
$$= \boxed{2.73\,\mathrm{cm}}$$

Because momentum must be conserved during this process and the center of mass moved to the right, the cylinder moved 2.73 cm to the left.

71 ••

Picture the Problem We can apply the ideal-gas law to the two processes to find the number of moles of hydrogen in terms of the number of moles of nitrogen in the gas. Using the definition of molar mass, we can relate the mass of each gas to the number of moles of each gas and their molar masses.

Apply the ideal-gas law to the first case:

$$P_1 V = [2n(N_2) + n(H_2)]RT_1$$

Apply the ideal-gas law to the

$$3P_1 V = [2n(N_2) + 2n(H_2)]2RT_1$$

second case:

Divide the second of these equations by the first and simplify to express $n(H_2)$ in terms of $n(N_2)$:

$$n(H_2) = 2n(N_2) \qquad (1)$$

Relate the m_N to $n(N_2)$:

$$m_N = n(N_2)M(N_2) = n(N_2)(28\,\text{g/mol})$$

and

$$n(N_2) = \frac{m_N}{28\,\text{g/mol}}$$

Relate the m_H to $n(H_2)$:

$$m_H = n(H_2)M(H_2) = n(H_2)(2\,\text{g/mol})$$

and

$$n(H_2) = \frac{m_H}{2\,\text{g/mol}}$$

Substitute in equation (1) and solve for m_N:

$$\frac{m_H}{2\,\text{g/mol}} = \frac{2m_N}{28\,\text{g/mol}} \Rightarrow m_N = \boxed{7m_H}$$

***72 ••**

Picture the Problem Initially, we have $3P_0V = n_0RT_0$. Later, the pressures in the three vessels, each of volume V, are still equal, but the number of moles is not. The total number of moles, however, is constant and equal to the number of moles in the three vessels initially. Applying the ideal-gas law to each of the vessels will allow us to relate the number of moles in each to the final pressure and temperature. Equating this sum n_0 will leave us with an equation in P' and P_0 that we can solve for P'.

Relate the number of moles of gas in the system in the three vessels initially to the number in each vessel when the pressure is P':

$$n_0 = n_1 + n_2 + n_3$$

Relate the final pressure in the first vessel to its temperature and solve for n_1:

$$P' = \frac{n_1R(2T_0)}{V} \Rightarrow n_1 = \frac{P'V}{2RT_0}$$

Relate the final pressure in the second vessel to its temperature and solve for n_2:

$$P' = \frac{n_2R(3T_0)}{V} \Rightarrow n_2 = \frac{P'V}{3RT_0}$$

Relate the final pressure in the third vessel to its temperature and solve for n_3:

$$P' = \frac{n_3 R T_0}{V} \Rightarrow n_3 = \frac{P'V}{RT_0}$$

Substitute to obtain:

$$n_0 = \frac{P'V}{2RT_0} + \frac{P'V}{3RT_0} + \frac{P'V}{RT_0}$$
$$= \left(\frac{1}{2} + \frac{1}{3} + 1\right)\frac{P'V}{RT_0} = \frac{11}{6}\left(\frac{P'V}{RT_0}\right)$$

Express the number of moles in the three vessels initially in terms of the initial pressure and total volume:

$$n_0 = \frac{P_0(3V)}{RT_0}$$

Equate the two expressions for n_0 and solve for P' to obtain:

$$P' = \boxed{\frac{18}{11} P_0}$$

73 ••

Picture the Problem We can use the ideal-gas temperature scale to relate the temperature of the boiling substance to its pressure and the pressure at the triple point. If we assume a linear relationship between P/P_3 and P_3, we can calibrate this equation using the data from any two (or all) of the temperature measurements and then extrapolate this equation to zero gas pressure to find the ideal-gas temperature of the boiling substance.

Using the ideal-gas temperature scale, relate the temperature of the boiling substance to its pressure and the pressure at the triple point:

$$T = 273.16\,\text{K}\left(\frac{P}{P_3}\right) \qquad (1)$$

Find the temperature of the first measurement:

$$T_1 = 273.16\,\text{K}\left(\frac{734\,\text{torr}}{500\,\text{torr}}\right)$$
$$= 273.16\,\text{K}(1.4680)$$
$$= 401.00\,\text{K}$$

Find the temperature of the third measurement:

$$T_3 = 273.16\,\text{K}\left(\frac{146.65\,\text{torr}}{100\,\text{torr}}\right)$$
$$= 273.16\,\text{K}(1.4655)$$
$$= 400.59\,\text{K}$$

Assume a linear relationship between P/P_3 and P_3:

$$\frac{P}{P_3} = a + bP_3$$

where a is the pressure ratio for $P_3 = 0$.

Substitute using the data from the first measurement:

$$\frac{734\,\text{torr}}{500\,\text{torr}} = a + b(500\,\text{torr})$$

or

$$1.4680 = a + b(500\,\text{torr})$$

Substitute using the data from the third measurement:

$$\frac{146.65\,\text{torr}}{100\,\text{torr}} = a + b(100\,\text{torr})$$

or

$$1.4665 = a + b(100\,\text{torr})$$

Solve these equations simultaneously for a:

$a = 1.46613$

Substitute in equation (1) to obtain:

$$T = 273.16\,\text{K}(1.46613) = \boxed{400.49\,\text{K}}$$

***74 ••**

Picture the Problem Because the O_2 molecule resembles 2 spheres stuck together, which in cross section look something like two circles, we can estimate the radius of the molecule from the formula for the area of a circle. We can express the area, and hence the radius, of the circle in terms of the mean free path and the number density of the molecules and use the ideal-gas law to express the number density.

Express the area of two circles of diameter d that touch each other:

$$A = 2\left(\frac{\pi d^2}{4}\right) = \frac{\pi d^2}{2}$$

Solve for d to obtain:

$$d = \sqrt{\frac{2A}{\pi}} \qquad (1)$$

Relate the mean free path of the molecules to their number density and cross-sectional area:

$$\lambda = \frac{1}{n_v A}$$

Solve for A to obtain:

$$A = \frac{1}{n_v \lambda}$$

Substitute in equation (1) to obtain:

$$d = \sqrt{\frac{2}{\pi n_v \lambda}}$$

Use the ideal-gas law to relate the number density of the O_2 molecules to their temperature and pressure:

$$PV = NkT \text{ or } n_v = \frac{N}{V} = \frac{P}{kT}$$

Substitute to obtain:

$$d = \sqrt{\frac{2kT}{\pi P\lambda}}$$

Substitute numerical values and evaluate d:

$$d = \sqrt{\frac{2(1.381\times10^{-23}\text{ J/K})(300\text{ K})}{\pi(1.01\times10^{5}\text{ Pa})(7.1\times10^{-8}\text{ m})}}$$

$$= 6.06\times10^{-10}\text{ m} = \boxed{0.606\text{ nm}}$$

75 ••

Picture the Problem We can use its definition to express the mean free path of the molecules and the ideal-gas law to obtain an expression for the number density of the hydrogen gas molecules.

(*a*) Relate the mean free path of the molecules to their number density and cross-sectional area:

$$\lambda = \frac{1}{n_v A}$$

Use the ideal-gas law to relate the number density of the H_2 molecules to their temperature and pressure:

$$PV = NkT \text{ or } n_v = \frac{N}{V} = \frac{P}{kT}$$

Express the effective cross-sectional area of a H_2 molecule:

$$A = \tfrac{1}{4}\pi d^2$$

Substitute for n_v and A to obtain:

$$\lambda = \frac{4kT}{P\pi d^2}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{4(1.381\times10^{-23}\text{ J/K})(300\text{ K})}{\pi(1.01\times10^{5}\text{ N/m}^2)(1.6\times10^{-10}\text{ m})^2}$$

$$= \boxed{2.04\times10^{-6}\text{ m}}$$

(*b*) Relate the available volume per molecule to the number density n_v:

$$\frac{V}{N} = \frac{1}{n_v} = \frac{kT}{P}$$

Substitute numerical values and evaluate V/N:

$$\frac{V}{N} = \frac{(1.381\times10^{-23}\text{ J/K})(300\text{ K})}{1.01\times10^{5}\text{ N/m}^2}$$

$$= \boxed{4.10\times10^{-26}\text{ m}^3}$$

Express the volume of a spherical molecule:

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{1}{6}\pi d^3$$

Solve for d:

$$d = \sqrt[3]{\frac{6V}{\pi}}$$

Substitute numerical values and evaluate d:

$$d = \sqrt[3]{\frac{6\left(4.10\times10^{-26}\ \text{m}^3\right)}{\pi}} = \boxed{4.28\,\text{nm}}$$

The mean free path is larger by approximately a factor of 1000.

76 •••

Picture the Problem Let A be the cross-sectional area of the cylinder. We can use the ideal-gas law to find the height of the piston under equilibrium conditions. In (b), we can apply Newton's 2nd law and the ideal-gas law for a fixed amount of gas to the show that, for small displacements from its equilibrium position, the piston executes simple harmonic motion.

(a) Express the pressure inside the cylinder:

$$P_{\text{in}} = P_{\text{atm}} + \frac{Mg}{A}$$

Apply the ideal-gas law to obtain a second expression for the pressure of the gas in the cylinder:

$$P_{\text{in}} = \frac{nRT}{V} = \frac{nRT}{hA} \qquad (1)$$

Equate these two expressions:

$$P_{\text{atm}} + \frac{Mg}{A} = \frac{nRT}{hA}$$

Solve for h to obtain:

$$h = \frac{nRT}{AP_{\text{atm}} + Mg} = \frac{(2.4\,\text{m})AP_{\text{atm}}}{AP_{\text{atm}} + Mg} = \frac{2.4\,\text{m}}{1 + \dfrac{Mg}{AP_{\text{atm}}}}$$

At STP, 0.1 mol of gas occupies 2.24 L. Therefore:

$$(2.4\,\text{m})A = 2.24\times10^{-3}\ \text{m}^3$$

and

$A = 9.33\times10^{-4}\ \text{m}^2$

Substitute numerical values and evaluate h:

$$h = \frac{2.4\,\text{m}}{1 + \dfrac{(1.4\,\text{kg})(9.81\,\text{m/s}^2)}{(9.333\times10^{-4}\ \text{m}^2)(101.3\,\text{kPa})}} = \boxed{2.096\,\text{m}}$$

(*b*) Relate the frequency of vibration of the piston to its mass and a ″stiffness″ constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \qquad (2)$$

where M is the mass of the piston and k is a constant of proportionality.

Letting y be the displacement from equilibrium, apply $\sum F_y = ma_y$ to the piston in its equilibrium position:

$$P_{\text{in}}A - mg - P_{\text{atm}}A = 0$$

For a small displacement y above equilibrium:

$$P_{\text{in}}'A - mg - P_{\text{atm}}A = ma_y$$

or

$$P_{\text{in}}'A - P_{\text{in}}A = ma_y \qquad (3)$$

Using the ideal-gas law for a fixed amount of gas and constant temperature, relate P_{in}' to P_{in}:

$$P_{\text{in}}'V' = P_{\text{in}}V$$

or

$$P_{\text{in}}'(V + Ay) = P_{\text{in}}V$$

Solve for P_{in}':

$$P_{\text{in}}' = P_{\text{in}}\frac{V}{V + Ay}$$

and

$$P_{\text{in}}'A = P_{\text{in}}A\frac{Ah}{Ah + Ay} = P_{\text{in}}A\frac{1}{1 + \frac{y}{h}}$$

Substitute in equation (3) to obtain:

$$P_{\text{in}}A\left(1 + \frac{y}{h}\right)^{-1} - P_{\text{in}}A = ma_y$$

or, for $y << h$,

$$P_{\text{in}}A\left(1 - \frac{y}{h}\right) - P_{\text{in}}A \approx ma_y \qquad (4)$$

Simplify equation (4):

$$-P_{\text{in}}A\frac{y}{h} \approx ma_y$$

Substitute in equation (1) to obtain:

$$-\left(\frac{nRT}{Ah}\right)A\frac{y}{h} \approx ma_y$$

or

$$-\left(\frac{nRT}{h^2}\right)y \approx ma_y$$

Solve for a_y:

$$a_y = -\frac{nRT}{mh^2}y$$

or

$$a_y = -\frac{k}{m}y\text{, the condition for SHM}$$

where $\frac{k}{m} = \frac{nRT}{mh^2}$

Substitute in equation (2) to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{nRT}{mh^2}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{(0.1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.4\,\text{kg})(2.096\,\text{m})^2}} = \boxed{1.01\,\text{Hz}}$$

***77** •••

Picture the Problem We can show that $\int_0^V f(v)dv = I(x)$, where $f(v)$ is the Maxwell-Boltzmann distribution function, $x = mV^2/2kT$, and $I(x)$ is the integral whose values are tabulated in the problem statement. Then, we can use this table to find the value of x corresponding to the fraction of the gas molecules with speeds less than v by evaluating $I(x)$.

(a) The Maxwell-Boltzmann speed distribution $f(x)$ is given by:

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

which means that the fraction of particles with speeds between v and $v + dv$ is $f(v)dv$.

Express the fraction $F(V)$ of particles with speeds less than V = 400 m/s:

$$F(V) = \int_0^V f(v)dv$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} \int_0^V v^2 e^{-mv^2/2kT} dv$$

Change integration variables by letting $z = v\sqrt{m/2kT}$ so we can use the table of values to evaluate the integral. Then:

$$v = \sqrt{\frac{2kT}{m}}z \Rightarrow dv = \sqrt{\frac{2kT}{m}}dz$$

Substitute in the integrand of $F(V)$ to obtain:

$$v^2 e^{-mv^2/2kT}dv = z^2 \frac{2kT}{m} e^{-z^2}\left(\frac{2kT}{m}\right)^{1/2} dz = \left(\frac{2kT}{m}\right)^{3/2} z^2 e^{-z^2} dz$$

Transform the integration limits to correspond to the new integration variable $z = v\sqrt{m/2kT}$:

When $v = 0$, $z = 0$,
and
when $v = V$, $z = V\sqrt{m/2kT}$

The new lower integration limit is 0. Evaluate $z = V\sqrt{m/2kT}$ to find the upper limit:

$$z = (400\,\text{m/s})\sqrt{\frac{(32\,\text{u})(1.661\times10^{-27}\,\text{kg})}{2(1.381\times10^{-23}\,\text{J/K})(273\,\text{K})}} = 1.06$$

Evaluate $F(400$ m/s$)$ to obtain:

$$F(400\,\text{m/s}) = \int_0^{400\,\text{m/s}} f(v)dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^{400\,\text{m/s}} v^2 e^{-mv^2/2kT}dv = \frac{4}{\sqrt{\pi}}\int_0^{1.06} z^2 e^{-z^2}dz$$
$$= I(1.06)$$

where $I(x) = \frac{4}{\sqrt{\pi}}\int_0^x z^2 e^{-z^2}dz$

Letting r represent the fraction of the molecules with speeds less than 400 m/s, interpolate from the table to obtain:

$$\frac{r - 0.438}{1.06 - 1} = \frac{0.788 - 0.438}{1.5 - 1}$$
and
$$r = \boxed{48.0\%}$$

(b) Express the fraction r of the molecules with speeds between V_1 = 190 m/s and V_2 = 565 m/s:

$$r = F(V_2) - F(V_1) = I(x_2) - I(x_1)$$
where
$x_1 = V_1\sqrt{m/2kT}$ and $x_2 = V_2\sqrt{m/2kT}$

Evaluate x_1 and x_2 to obtain:

$$x_1 = (190\,\text{m/s})\sqrt{\frac{(32\,\text{u})(1.661\times10^{-27}\,\text{kg})}{2(1.381\times10^{-23}\,\text{J/K})(273\,\text{K})}} = 0.504$$

and

$$x_2 = (565\,\text{m/s})\sqrt{\frac{(32\,\text{u})(1.661\times10^{-27}\,\text{kg})}{2(1.381\times10^{-23}\,\text{J/K})(273\,\text{K})}} = 1.50$$

Substitute to obtain:

$$r = I(1.50) - I(0.504) \qquad (1)$$

Using the table, evaluate $I(1.50)$:

$$I(1.50) = 0.788$$

Letting r represent the fraction of the molecules with speeds less than 190 m/s, interpolate from the table to obtain:

$$\frac{r - 0.081}{0.504 - 0.5} = \frac{0.132 - 0.081}{0.6 - 0.5}$$

and

$$r = 0.083$$

Substitute in equation (1) to obtain:

$$r = 0.788 - 0.083 = \boxed{70.5\%}$$

Chapter 18
Heat and the First Law of Thermodynamics

Conceptual Problems

1 •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to relate the temperature changes of bodies A and B to their masses, specific heats, and the amount of heat supplied to each.

Express the change in temperature of body A in terms of its mass, specific heat, and the amount of heat supplied to it:

$$\Delta T_A = \frac{Q}{m_A c_A}$$

Express the change in temperature of body B in terms of its mass, specific heat, and the amount of heat supplied to it:

$$\Delta T_B = \frac{Q}{m_B c_B}$$

Divide the second of these equations by the first to obtain:

$$\frac{\Delta T_B}{\Delta T_A} = \frac{m_A c_A}{m_B c_B}$$

Substitute and simplify to obtain:

$$\frac{\Delta T_B}{\Delta T_A} = \frac{(2m_B)(2c_B)}{m_B c_B} = 4$$

or

$$\Delta T_B = \boxed{4\Delta T_A}$$

***2** •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to relate the temperature changes of bodies A and B to their masses, specific heats, and the amount of heat supplied to each.

Relate the temperature change of block A to its specific heat and mass:

$$\Delta T_A = \frac{Q}{M_A c_A}$$

Relate the temperature change of block B to its specific heat and mass:

$$\Delta T_B = \frac{Q}{M_B c_B}$$

Equate the temperature changes to obtain:

$$\frac{1}{M_B c_B} = \frac{1}{M_A c_A}$$

Solve for c_A:

$$c_A = \frac{M_B}{M_A} c_B$$

and $\boxed{(b) \text{ is correct.}}$

3 •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to relate the amount of energy absorbed by the aluminum and copper bodies to their masses, specific heats, and temperature changes.

Express the energy absorbed by the aluminum object:

$$Q_{Al} = m_{Al} c_{Al} \Delta T$$

Express the energy absorbed by the copper object:

$$Q_{Cu} = m_{Cu} c_{Cu} \Delta T$$

Divide the second of these equations by the first to obtain:

$$\frac{Q_{Cu}}{Q_{Al}} = \frac{m_{Cu} c_{Cu} \Delta T}{m_{Al} c_{Al} \Delta T}$$

Because the object's masses are the same and they experience the same change in temperature:

$$\frac{Q_{Cu}}{Q_{Al}} = \frac{c_{Cu}}{c_{Al}} < 1$$

or

$Q_{Cu} < Q_{Al}$ and $\boxed{(c) \text{ is correct.}}$

4 •

Determine the Concept Some examples of systems in which internal energy is converted into mechanical energy are: a steam turbine, an internal combustion engine, and a person performing mechanical work, e.g., climbing a hill.

***5** •

Determine the Concept Yes, if the heat absorbed by the system is equal to the work done by the system.

6 •

Determine the Concept According to the first law of thermodynamics, the change in the internal energy of the system is equal to the heat that enters the system plus the work done on the system. $\boxed{(b) \text{ is correct.}}$

7 •

Determine the Concept $\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$. For an ideal gas, ΔE_{int} is a function of T only. Because $W_{\text{on}} = 0$ and $Q_{\text{in}} = 0$ in a free expansion, $\Delta E_{\text{int}} = 0$ and T is constant. For a real gas, E_{int} depends on the density of the gas because the molecules exert weak attractive forces on each other. In a free expansion, these forces reduce the average kinetic energy of the molecules and, consequently, the temperature.

8 •

Determine the Concept Because the container is insulated, no energy is exchanged with the surroundings during the expansion of the gas. Neither is any work done on or by the gas during this process. Hence, the internal energy of the gas does not change and we can conclude that the equilibrium temperature will be the same as the initial temperature. Applying the ideal-gas law for a fixed amount of gas we see that the pressure at equilibrium must be half an atmosphere. $\boxed{(c) \text{ is correct.}}$

9 •

Determine the Concept The temperature of the gas increases. The average kinetic energy increases with increasing volume due to the repulsive interaction between the ions.

***10 ••**

Determine the Concept The balloon that expands isothermally is larger when it reaches the surface. The balloon that expands adiabatically will be at a lower temperature than the one that expands isothermally. Because each balloon has the same number of gas molecules and are at the same pressure, the one with the higher temperature will be bigger. An analytical argument that leads to the same conclusion is shown below.

Letting the subscript "a" denote the adiabatic process and the subscript "i" denote the isothermal process, express the equation of state for the adiabatic balloon:

$$P_0 V_0^\gamma = P_\text{f} V_{\text{f,a}}^\gamma \Rightarrow V_{\text{f,a}} = V_0 \left(\frac{P_0}{P_\text{f}}\right)^{1/\gamma}$$

For the isothermal balloon:

$$P_0 V_0 = P_\text{f} V_{\text{f,i}} \Rightarrow V_{\text{f,i}} = V_0 \left(\frac{P_0}{P_\text{f}}\right)$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{V_{\text{f,i}}}{V_{\text{f,a}}} = \frac{V_0 \left(\frac{P_0}{P_\text{f}}\right)}{V_0 \left(\frac{P_0}{P_\text{f}}\right)^{1/\gamma}} = \left(\frac{P_0}{P_\text{f}}\right)^{1-1/\lambda}$$

Because $P_0/P_f > 1$ and $\gamma > 1$: $\boxed{V_{f,i} > V_{f,a}}$

11 •

Determine the Concept The work done along each of these paths equals the area under its curve. The area is greatest for the path A→B→C and least for the path A→D→C. $\boxed{(a) \text{ is correct.}}$

12 •

Determine the Concept An adiabatic process is, by definition, one for which no heat enters or leaves the system. $\boxed{(b) \text{ is correct.}}$

13 •

(*a*) False. The heat capacity of a body is the heat needed to raise the temperature of the body by one degree.

(*b*) False. The amount of heat added to a system when it goes from one state to another is path dependent.

(*c*) False. The work done on a system when it goes from one state to another is path dependent.

(*d*) True.

(*e*) True.

(*f*) True.

(*g*) True.

***14** •

Determine the Concept For a constant-volume process, no work is done on or by the gas. Applying the first law of thermodynamics, we obtain $Q_{in} = \Delta E_{int}$. Because the temperature must change during such a process, we can conclude that $\Delta E_{int} \neq 0$ and hence $Q_{in} \neq 0$. $\boxed{(b) \text{ and } (d) \text{ are correct.}}$

15 •

Determine the Concept Because the temperature does not change during an isothermal process, the change in the internal energy of the gas is zero. Applying the first law of thermodynamics, we obtain $Q_{in} = -W_{on} = W_{\text{by the system}}$. Hence $\boxed{(d) \text{ is correct.}}$

16 ••

Determine the Concept The melting point of propane at 1 atm pressure is 83 K. Hence, at this low temperature and high pressure, C_3H_8 is a solid.

17 ••

Picture the Problem We can use the given dependence of the pressure on the volume and the ideal-gas law to show that if the volume decreases, so does the temperature.

We're given that: $P\sqrt{V} = \text{constant}$

Because the gas is an ideal gas: $PV = \left(P\sqrt{V}\right)\sqrt{V} = \text{constant}\sqrt{V} = nRT$

Solve for T: $$T = \frac{(\text{constant})\sqrt{V}}{nR}$$

Because T varies with the square root of V, if the volume decreases, the temperature decreases.

***18** ••

Determine the Concept At room temperature, most solids have a roughly constant heat capacity per *mole* of 6 cal/mol-K (Dulong-Petit law). Because 1 mole of lead is more massive than 1 mole of copper, the heat capacity of lead should be lower than the heat capacity of copper. This is, in fact, the case.

19 ••

Determine the Concept The heat capacity of a substance is proportional to the number of degrees of freedom per molecule associated with the molecule. Because there are 6 degrees of freedom per molecule in a solid and only 3 per molecule (translational) for a monatomic liquid, you would expect the solid to have the higher heat capacity.

Estimation and Approximation

***20** ••

Picture the Problem The heat capacity of lead is c = 128 J/kg·K. We'll assume that all of the work done in lifting the bag through a vertical distance of 1 m goes into raising the temperature of the lead shot and use conservation of energy to relate the number of drops of the bag and the distance through which it is dropped to the heat capacity and change in temperature of the lead shot.

(*a*) Use conservation of energy to relate the change in the potential energy of the lead shot to the change in its temperature: $Nmgh = mc\Delta T$

where N is the number of times the bag of shot is dropped.

Solve for ΔT to obtain:

$$\Delta T = \frac{Nmgh}{mc} = \frac{Ngh}{c}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{50\left(9.81\text{ m/s}^2\right)\left(1\text{ m}\right)}{128\text{ J/kg}\cdot\text{K}} = \boxed{3.83\text{ K}}$$

(*b*) It is better to use a larger mass because the rate at which heat is lost by the lead shot is proportional to its surface area while the rate at which it gains heat is proportional to its mass. The amount of heat lost varies as the surface area of the shot divided by its mass ($L^2/L^3 = L^{-1}$); which decreases as the mass increases.

21 ••

Picture the Problem Assume that the water is initially at 30°C and that the cup contains 200 g of water. We can use the definition of power to express the required time to bring the water to a boil in terms of its mass, heat capacity, change in temperature, and the rate at which energy is supplied to the water by the microwave oven.

Use the definition of power to relate the energy needed to warm the water to the elapsed time:

$$P = \frac{\Delta W}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

Solve for Δt to obtain:

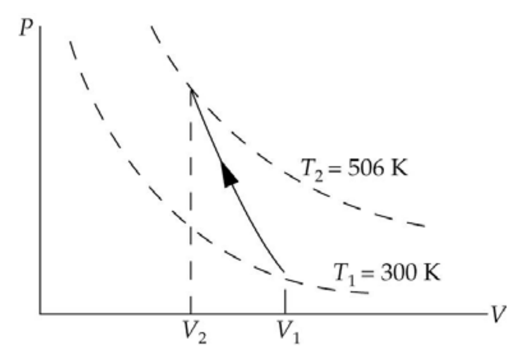
$$\Delta t = \frac{mc\Delta T}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{(0.2\text{ kg})(4.18\text{ kJ/kg}\cdot\text{K})(373\text{ K} - 303\text{ K})}{600\text{ W}} = 97.5\text{ s} = \boxed{1.63\text{ min}}$$, an elapsed time that seems to be consistent with experience.

22 •

Picture the Problem The adiabatic compression from an initial volume V_1 to a final volume V_2 between the isotherms at temperatures T_1 and T_2 is shown to the right. We'll assume a room temperature of 300 K and apply the equation for a quasi-static adiabatic process with $\gamma_{air} = 1.4$ to solve for the ratio of the initial to the final volume of the air.



Express $TV^{\gamma-1} = \text{constant}$ in terms of the initial and final values of T and V:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

Solve for V_1/V_2 to obtain:

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

Substitute numerical values and evaluate V_1/V_2:

$$\frac{V_1}{V_2} = \left(\frac{506\,\text{K}}{300\,\text{K}}\right)^{\frac{1}{1.4-1}} = \boxed{3.69}$$

23 ••

Picture the Problem We can use $Q = mc_\text{p}\Delta T$ to express the specific heat of water during heating at constant pressure in terms of the required heat and the resulting change in temperature. Further, we can use the definition of the bulk modulus to express the work done by the water as it expands. Equating the work done by the water during its expansion and the heat gained during this process will allow us to solve for c_p.

Express the heat needed to raise the temperature of a mass m of a substance whose specific heat at constant pressure is c_p by ΔT:

$$Q = mc_\text{p}\Delta T$$

Solve for c_p to obtain:

$$c_\text{p} = \frac{Q}{m\Delta T}$$

Use the definition of the bulk modulus to express the work done by the water as it expands:

$$B = \frac{\Delta P}{\Delta V/V} = \frac{V\Delta P}{\Delta V}$$

or

$$W = V\Delta P = B\Delta V$$

Assuming that the work done by the water in expanding equals the heat gained during the process, substitute to obtain:

$$c_\text{p} = \frac{B\Delta V}{m\Delta T}$$

Using the definition of the coefficient of volume expansion, express ΔV (see Chapter 20, Section 1):

$$\Delta V = \beta V\Delta T$$

Substitute to obtain:

$$c_\text{p} = \frac{B\beta V\Delta T}{m\Delta T} = \frac{B\beta V}{m}$$

Use the data given in the problem statement to find the average volume of 1 kg of water as it warms from 4°C to 100°C:

$$V = \frac{m}{\rho} = \frac{1\,\text{kg}}{\dfrac{1.0000\,\text{g/cm}^3 + 0.9584\,\text{g/cm}^3}{2}} = 1.02\times10^{-3}\,\text{m}^3$$

Substitute numerical values and evaluate c_p:

$$c_p = \frac{\left(2\times10^8\,\text{N/m}^2\right)\left(0.207\times10^{-3}\,\text{K}^{-1}\right)\left(1.02\times10^{-3}\,\text{m}^3\right)}{1\,\text{kg}} = 42.2\,\text{J/kg}\cdot\text{K}$$

Express the ratio of c_p to c_{water}:

$$\frac{c_p}{c_{water}} = \frac{42.2\,\text{J/kg}\cdot\text{K}}{4184\,\text{J/kg}\cdot\text{K}} = 1.01\times10^{-2}$$

or

$$c_p = \boxed{(1.01\%)c_{water}}$$

***24 ••**

Picture the Problem We can apply the condition for the validity of the equipartition theorem, i.e., that the spacing of the energy levels be large compared to kT, to find the critical temperature T_c:

Express the failure condition for the equipartition theorem:

$$kT_c \approx 0.15\,\text{eV}$$

Solve for T_c:

$$T_c = \frac{0.15\,\text{eV}}{k}$$

Substitute numerical values and evaluate T_c:

$$T_c = \frac{0.15\,\text{eV}\times\dfrac{1.602\times10^{-19}\,\text{J}}{\text{eV}}}{1.381\times10^{-23}\,\text{J/K}} = \boxed{1740\,\text{K}}$$

Heat Capacity; Specific Heat; Latent Heat

***25 •**

Picture the Problem We can use the conversion factor 1 cal = 4.184 J to convert 2500 kcal into joules and the definition of power to find the average output if the consumed energy is dissipated over 24 h.

(*a*) Convert 2500 kcal to joules:

$$2500\,\text{kcal} = 2500\,\text{kcal}\times\frac{4.184\,\text{J}}{\text{cal}} = \boxed{10.5\,\text{MJ}}$$

(*b*) Use the definition of average power to obtain:

$$P_{av} = \frac{\Delta E}{\Delta t} = \frac{1.05\times10^7\ \text{J}}{24\,\text{h}\times\frac{3600\,\text{s}}{\text{h}}} = \boxed{121\,\text{W}}$$

Remarks: Note that this average power output is essentially that of a widely used light bulb.

26 •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to calculate the amount of heat given off by the concrete as it cools from 25 to 20°C.

Relate the heat given off by the concrete to its mass, specific heat, and change in temperature:

$$Q = mc\Delta T$$

Substitute numerical values and evaluate Q:

$$Q = (10^5\ \text{kg})(1\,\text{kJ/kg}\cdot\text{K})(298\,\text{K} - 293\,\text{K}) = \boxed{500\,\text{MJ}}$$

27 •

Picture the Problem We can find the amount of heat that must be supplied by adding the heat required to warm the ice from –10°C to 0°C, the heat required to melt the ice, and the heat required to warm the water formed from the ice to 40°C.

Express the total heat required:

$$Q = Q_{\text{warm ice}} + Q_{\text{melt ice}} + Q_{\text{warm water}}$$

Substitute for each term to obtain:

$$Q = mc_{\text{ice}}\Delta T_{\text{ice}} + mL_{\text{f}} + mc_{\text{water}}\Delta T_{\text{water}} = m\left(c_{\text{ice}}\Delta T_{\text{ice}} + L_{\text{f}} + c_{\text{water}}\Delta T_{\text{water}}\right)$$

Substitute numerical values and evaluate Q:

$$Q = (0.06\,\text{kg})[(0.49\,\text{kcal/kg}\cdot\text{K})(273\,\text{K} - 263\,\text{K}) + 79.7\,\text{kcal/kg} + (1\,\text{kcal/kg}\cdot\text{K})(313\,\text{K} - 273\,\text{K})] = \boxed{7.48\,\text{kcal}}$$

28 ••

Picture the Problem We can find the amount of heat that must be removed by adding the heat that must be removed to cool the steam from 150°C to 100°C, the heat that must be removed to condense the steam to water, the heat that must be removed to cool the water from 100°C to 0°C, and the heat that must be removed to freeze the water.

Express the total heat that must be removed:

$$Q = Q_{\text{cool steam}} + Q_{\text{condense steam}} + Q_{\text{cool water}} + Q_{\text{freeze water}}$$

Substitute for each term to obtain:

$$\begin{aligned} Q &= mc_{\text{steam}}\Delta T_{\text{steam}} + mL_{\text{v}} + mc_{\text{water}}\Delta T_{\text{water}} + mL_{\text{f}} \\ &= m\left(c_{\text{steam}}\Delta T_{\text{steam}} + L_{\text{v}} + c_{\text{water}}\Delta T_{\text{water}} + L_{\text{f}}\right) \end{aligned}$$

Substitute numerical values and evaluate Q:

$$\begin{aligned} Q &= (0.1\,\text{kg})\left[(2.01\,\text{kJ/kg}\cdot\text{K})(423\,\text{K} - 373\,\text{K}) + 2.26\,\text{MJ/kg} \right. \\ &\quad \left. + (4.18\,\text{kJ/kg}\cdot\text{K})(373\,\text{K} - 273\,\text{K}) + 333.5\,\text{kJ/kg}\right] \\ &= 311.2\,\text{kJ} \times \frac{1\,\text{kcal}}{4.184\,\text{kJ}} \\ &= \boxed{74.4\,\text{kcal}} \end{aligned}$$

29 ••

Picture the Problem We can find the amount of nitrogen vaporized by equating the heat gained by the liquid nitrogen and the heat lost by the piece of aluminum.

Express the heat gained by the liquid nitrogen as it cools the piece of aluminum:

$$Q_{\text{N}} = m_{\text{N}} L_{\text{vN}}$$

Express the heat lost by the piece of aluminum as it cools:

$$Q_{\text{Al}} = m_{\text{Al}} c_{\text{Al}} \Delta T_{\text{Al}}$$

Equate these two expressions and solve for m_{N}:

$$m_{\text{N}} L_{\text{vN}} = m_{\text{Al}} c_{\text{Al}} \Delta T_{\text{Al}}$$

and

$$m_{\text{N}} = \frac{m_{\text{Al}} c_{\text{Al}} \Delta T_{\text{Al}}}{L_{\text{vN}}}$$

Substitute numerical values and evaluate m_{N}:

$$m_{\text{N}} = \frac{(0.05\,\text{kg})(0.90\,\text{J/kg}\cdot\text{K})(293\,\text{K} - 77\,\text{K})}{199\,\text{kJ/kg}} = 4.88\times10^{-5}\,\text{kg} = \boxed{48.8\,\text{mg}}$$

30 ••

Picture the Problem Because the heat lost by the lead as it cools is gained by the block of ice (we're assuming no heat is lost to the surroundings), we can apply the conservation of energy to determine how much ice melts.

Apply the conservation of energy to this process:

$$\Delta Q = 0$$

or

$$-m_{\text{Pb}}\left(L_{\text{f,Pb}} + c_{\text{Pb}}\Delta T_{\text{Pb}}\right) + m_{\text{w}}L_{\text{f,w}} = 0$$

Solve for m_{w}:

$$m_{\text{w}} = \frac{m_{\text{Pb}}\left(L_{\text{f,Pb}} + c_{\text{Pb}}\Delta T_{\text{Pb}}\right)}{L_{\text{f,w}}}$$

Substitute numerical values and evaluate m_{w}:

$$m_{\text{w}} = \frac{(0.5\,\text{kg})\left(24.7\,\text{kJ/kg} + (0.128\,\text{kJ/kg}\cdot\text{K})(600\,\text{K} - 273\,\text{K})\right)}{333.5\,\text{kJ/kg}} = \boxed{99.8\,\text{g}}$$

***31** ••

Picture the Problem The temperature of the bullet immediately after coming to rest in the block is the sum of its pre-collision temperature and the change in its temperature as a result of being brought to a stop in the block. We can equate the heat gained by the bullet and half its pre-collision kinetic energy to find the change in its temperature.

Express the temperature of the bullet immediately after coming to rest in terms of its initial temperature and the change in its temperature as a result of being stopped in the block:

$$T = T_{\text{i}} + \Delta T = 293\,\text{K} + \Delta T$$

Relate the heat absorbed by the bullet as it comes to rest to its kinetic energy before the collision:

$$Q = \tfrac{1}{2}K$$

Substitute for Q and K to obtain:

$$m_{\text{Pb}}c_{\text{Pb}}\Delta T = \tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{Pb}}v^2\right)$$

Solve for ΔT:

$$\Delta T = \frac{v^2}{4c_{\text{Pb}}}$$

Substitute to obtain:

$$T = 293\,\text{K} + \frac{v^2}{4c_{\text{Pb}}}$$

Substitute numerical values and evaluate T:

$$T = 293\,\text{K} + \frac{(420\,\text{m/s})^2}{4(0.128\,\text{kJ/kg}\cdot\text{K})}$$

$$= 638\,\text{K} = \boxed{365°\text{C}}$$

32 ••

Picture the Problem We can find the heat available to warm the brake drums from the initial kinetic energy of the car and the mass of steel contained in the brake drums from $Q = m_{\text{steel}}c_{\text{steel}}\Delta T$.

Express m_{steel} in terms of Q:

$$m_{\text{steel}} = \frac{Q}{c_{\text{steel}}\Delta T}$$

Find the heat available to warm the brake drums from the initial kinetic energy of the car:

$$Q = K = \tfrac{1}{2} m_{\text{car}} v^2$$

Substitute for Q to obtain:

$$m_{\text{steel}} = \frac{\tfrac{1}{2} m_{\text{car}} v^2}{c_{\text{steel}}\Delta T}$$

Substitute numerical values and evaluate m_{steel}:

$$m_{\text{steel}} = \frac{(1400\,\text{kg})\left(80\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{2\left(0.11\frac{\text{kcal}}{\text{kg}\cdot\text{K}} \times \frac{4.186\,\text{kJ}}{\text{kcal}}\right)(120\,\text{K})}$$

$$= \boxed{6.26\,\text{kg}}$$

Calorimetry

33 •

Picture the Problem Let the system consist of the piece of lead, calorimeter, and water. During this process the water will gain energy at the expense of the piece of lead. We can set the heat out of the lead equal to the heat into the water and solve for the final temperature of the lead and water.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{in}} = Q_{\text{out}}$$

Express the heat lost by the lead in terms of its specific heat and temperature change:

$$Q_{out} = m_{Pb}c_{Pb}\Delta T_{Pb}$$

Express the heat absorbed by the water in terms of its specific heat and temperature change:

$$Q_{in} = m_w c_w \Delta T_w$$

Substitute to obtain:

$$m_w c_w \Delta T_w = m_{Pb}c_{Pb}\Delta T_{Pb}$$

Substitute numerical values:

$$(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(t_f - 293\,\text{K}) = (0.2\,\text{kg})(0.128\,\text{kJ/kg}\cdot\text{K})(363\,\text{K} - t_f)$$

Solve for t_f to obtain:

$$t_f = 293.8\,\text{K} = \boxed{20.8°\text{C}}$$

***34** •

Picture the Problem During this process the water and the container will gain energy at the expense of the piece of metal. We can set the heat out of the metal equal to the heat into the water and the container and solve for the specific heat of the metal.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{gained} = Q_{lost}$$

Express the heat lost by the metal in terms of its specific heat and temperature change:

$$Q_{lost} = m_{metal}c_{metal}\Delta T_{metal}$$

Express the heat gained by the water and the container in terms of their specific heats and temperature change:

$$Q_{gained} = m_w c_w \Delta T_w + m_{container}c_{metal}\Delta T_w$$

Substitute to obtain:

$$m_w c_w \Delta T_w + m_{container}c_{metal}\Delta T_w = m_{metal}c_{metal}\Delta T_{metal}$$

Substitute numerical values:

$$\begin{aligned}(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(294.4\,\text{K} - 293\,\text{K}) + (0.2\,\text{kg})(294.4\,\text{K} - 293\,\text{K})c_{metal} \\ = (0.1\,\text{kg})(373\,\text{K} - 294.4\,\text{K})c_{metal}\end{aligned}$$

Solve for c_{metal}:

$$c_{\text{metal}} = \boxed{0.386\,\text{kJ/kg}\cdot\text{K}}$$

35 ••

Picture the Problem We can use $Q = mc\Delta T$ to express the mass m of water that can be heated through a temperature interval ΔT by an amount of heat energy Q. We can then find the amount of heat energy expended by Armstrong from the definition of power.

Express the amount of heat energy Q required to raise the temperature of a mass m of water by ΔT:

$$Q = mc\Delta T$$

Solve for m to obtain:

$$m = \frac{Q}{c\Delta T}$$

Use the definition of power to relate the heat energy expended by Armstrong to the rate at which he expended the energy:

$$P = \frac{Q}{\Delta t} \Rightarrow Q = P\Delta t$$

Substitute to obtain:

$$m = \frac{P\Delta t}{c\Delta T}$$

Substitute numerical values and evaluate m:

$$m = \frac{(400\,\text{J/s})(3600\,\text{s/h})(5\,\text{h/d})(20\,\text{d})}{(4.184\,\text{kJ/kg}\cdot\text{K})(373\,\text{K} - 297\,\text{K})} = \boxed{453\,\text{kg}}$$

36 ••

Picture the Problem During this process the ice and the water formed from the melted ice will gain energy at the expense of the glass tumbler and the water in it. We can set the heat out of the tumbler and the water that is initially at 24°C equal to the heat into the ice and ice water and solve for the final temperature of the drink.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat lost by the tumbler and the water in it in terms of their specific heats and common temperature change:

$$Q_{\text{lost}} = m_{\text{glass}}c_{\text{glass}}\Delta T + m_{\text{water}}c_{\text{water}}\Delta T$$

Express the heat gained by the ice and the melted ice in terms of their specific heats and temperature

$$Q_{\text{gained}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}}$$

changes:

Substitute to obtain:

$$m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}} = m_{\text{glass}}c_{\text{glass}}\Delta T + m_{\text{water}}c_{\text{water}}\Delta T$$

Substitute numerical values:

$$\begin{aligned}(0.03\,\text{kg})(0.49\,\text{kcal/kg}\cdot\text{K})(273\,\text{K} - 270\,\text{K}) + (0.03\,\text{kg})(79.7\,\text{kcal/kg})\\ + (0.03\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})t_{\text{f}} = (0.025\,\text{kg})(0.2\,\text{kcal/kg}\cdot\text{K})(297\,\text{K} - t_{\text{f}})\\ + (0.2\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(297\,\text{K} - t_{\text{f}})\end{aligned}$$

Solve for t_{f}:

$$t_{\text{f}} = 283.6\,\text{K} = \boxed{10.6°\text{C}}$$

37 ••

Picture the Problem Because we can not tell, without performing a couple of calculations, whether there is enough heat available in the 500 g of water to melt all of the ice, we'll need to resolve this question first.

(*a*) Determine the heat required to melt 200 g of ice:

$$\begin{aligned}Q_{\text{melt ice}} &= m_{\text{ice}}L_{\text{f}}\\ &= (0.2\,\text{kg})(79.7\,\text{kcal/kg})\\ &= 15.94\,\text{kcal}\end{aligned}$$

Determine the heat available from 500 g of water:

$$\begin{aligned}Q_{\text{water}} &= m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}}\\ &= (0.5\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})\\ &\quad\times(293\,\text{K} - 273\,\text{K})\\ &= 10\,\text{kcal}\end{aligned}$$

Because $Q_{\text{water}} < Q_{\text{melt ice}}$:

$$\boxed{\text{The final temperature is } 0°\text{C.}}$$

(*b*) Equate the energy available from the water Q_{water} to $m_{\text{ice}}L_{\text{f}}$ and solve for m_{ice}:

$$m_{\text{ice}} = \frac{Q_{\text{water}}}{L_{\text{f}}}$$

Substitute numerical values and evaluate m_{ice}:

$$m_{\text{ice}} = \frac{10\,\text{kcal}}{79.7\,\text{kcal/kg}} = \boxed{125\,\text{g}}$$

38 ••

Picture the Problem Because the bucket contains a mixture of ice and water initially, we know that its temperature must be 0°C. We can equate the heat gained by the mixture of ice and water and the heat lost by the block of copper and solve for the amount of ice initially in the bucket.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat lost by the block of copper:

$$Q_{\text{lost}} = m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}}$$

Express the heat gained by the ice and the melted ice:

$$Q_{\text{gained}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}}$$

Substitute to obtain:

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}} - m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}} = 0$$

Solve for m_{ice}:

$$m_{\text{ice}} = \frac{m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}} - m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}}}{L_{\text{f}}}$$

Substitute numerical values and evaluate m_{ice}:

$$m_{\text{ice}} = \frac{(3.5\,\text{kg})(0.0923\,\text{kcal/kg}\cdot\text{K})(353\,\text{K} - 281\,\text{K})}{79.7\,\text{kcal/kg}} - \frac{(1.2\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(281\,\text{K} - 273\,\text{K})}{79.7\,\text{kcal/kg}}$$

$$= \boxed{171\,\text{g}}$$

39 ••

Picture the Problem During this process the ice and the water formed from the melted ice will gain energy at the expense of the condensing steam and the water from the condensed steam. We can equate these quantities and solve for the final temperature of the system.

(*a*) Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat required to melt the ice and raise the temperature of the

$$Q_{\text{gained}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{water}}$$

ice water:

Express the heat available from 20 g of steam and the cooling water formed from the condensed steam:

$$Q_{\text{lost}} = m_{\text{steam}} L_{\text{v}} + m_{\text{steam}} c_{\text{water}} \Delta T_{\text{water}}$$

Substitute to obtain:

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{water}} = m_{\text{steam}} L_{\text{v}} + m_{\text{steam}} c_{\text{water}} \Delta T_{\text{water}}$$

Substitute numerical values:

$$(0.15\,\text{kg})(79.7\,\text{kcal/kg}) + (0.15\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(t_{\text{f}} - 273\,\text{K}) = (0.02\,\text{kg})(540\,\text{kcal/kg}) + (0.02\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(373\,\text{K} - t_{\text{f}})$$

Solve for t_{f}:

$$t_{\text{f}} = 277.94\,\text{K} = \boxed{4.94°\text{C}}$$

(*b*) Because the final temperture is greater than 0°C, no ice is left.

40 ••

Picture the Problem During this process the ice will gain heat and the water will lose heat. We can do a preliminary calculation to determine whether there is enough heat available to melt all of the ice and, if there is, equate the heat the heat lost by the water to the heat gained by the ice and resulting ice water as the system achieves thermal equilibrium.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Find the heat available to melt the ice:

$$Q_{\text{avail}} = m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} = (1\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K}) \times (303\,\text{K} - 273\,\text{K}) = 30\,\text{kcal}$$

Find the heat required to melt all of the ice:

$$Q_{\text{melt ice}} = m_{\text{ice}} L_{\text{f}} = (0.05\,\text{kg})(79.7\,\text{kcal/kg}) = 3.985\,\text{kcal}$$

Because $Q_{\text{avail}} > Q_{\text{melt ice}}$, we know

$$Q_{\text{lost}} = m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}}$$

that the final temperature will be greater than 273 K and we can express Q_{lost} in terms of the change in temperature of the water:

Express Q_{gained}:

$$Q_{\text{gained}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}}$$

Equate the heat gained and the heat lost to obtain:

$$m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}} = m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}}$$

Substitute numerical values to obtain:

$$(0.05\,\text{kg})(79.7\,\text{kcal/kg}) + (0.05\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(T_{\text{f}} - 273\,\text{K}) = (1\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(303\,\text{K} - T_{\text{f}})$$

Solving for T_{f} yields:

$$T_{\text{f}} = 297.8\,\text{K} = \boxed{24.8°\text{C}}$$

Find the heat required to melt 500 g of ice:

$$\begin{aligned} Q_{\text{melt ice}} &= m_{\text{ice}}L_{\text{f}} \\ &= (0.5\,\text{kg})(79.7\,\text{kcal/kg}) \\ &= 39.85\,\text{kcal} \end{aligned}$$

Because the heat required to melt 500 g of ice is greater than the heat available, the final temperature will be 0°C.

***41** ••

Picture the Problem Assume that the calorimeter is in thermal equilibrium with the water it contains. During this process the ice will gain heat in warming to 0°C and melting, as will the water formed from the melted ice. The water in the calorimeter and the calorimeter will lose heat. We can do a preliminary calculation to determine whether there is enough heat available to melt all of the ice and, if there is, equate the heat the heat lost by the water to the heat gained by the ice and resulting ice water as the system achieves thermal equilibrium.

Find the heat available to melt the ice:

$$\begin{aligned} Q_{\text{avail}} &= m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} + m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{water}} \\ &= [(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K}) + (0.2\,\text{kg})(0.9\,\text{kJ/kg}\cdot\text{K})](293\,\text{K} - 273\,\text{K}) \\ &= 45.40\,\text{kJ} \end{aligned}$$

Find the heat required to melt all of the ice:

$$\begin{aligned} Q_{\text{melt ice}} &= m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} + m_{\text{ice}} L_{\text{f}} \\ &= (0.1\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(273\,\text{K} - 253\,\text{K}) + (0.1\,\text{kg})(333.5\,\text{kJ/kg}) \\ &= 37.35\,\text{kJ} \end{aligned}$$

(*a*) Because $Q_{\text{avail}} > Q_{\text{melt ice}}$, we know that the final temperature will be greater than 0°C. Apply the conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express Q_{lost} in terms of the final temperature of the system:

$$Q_{\text{lost}} = (m_{\text{water}} c_{\text{water}} + m_{\text{cal}} c_{\text{cal}}) \Delta T_{\text{water+calorimeter}}$$

Express Q_{gained} in terms of the final temperature of the system:

$$Q_{\text{gained}} = m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} + m_{\text{ice}} L_{\text{f}}$$

Substitute to obtain:

$$m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} + m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}} = (m_{\text{water}} c_{\text{water}} + m_{\text{cal}} c_{\text{cal}}) \Delta T_{\text{water+calorimeter}}$$

Substitute numerical values:

$$\begin{aligned} &37.35\,\text{kJ} + (0.1\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(t_{\text{f}} - 273\,\text{K}) \\ &\qquad = [(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K}) + (0.2\,\text{kg})(0.9\,\text{kJ/kg}\cdot\text{K})](293\,\text{K} - t_{\text{f}}) \end{aligned}$$

Solving for t_{f} yields:

$$t_{\text{f}} = 276\,\text{K} = \boxed{2.99°\text{C}}$$

(*b*) Find the heat required to raise 200 g of ice to 0°C:

$$Q_{\text{warm ice}} = m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} = (0.2\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(273\,\text{K} - 253\,\text{K}) = 8.00\,\text{kJ}$$

Noting that there are now 600 g of water in the calorimeter, find the heat available from cooling the calorimeter and water from 3°C to 0°C:

$$\begin{aligned} Q_{\text{avail}} &= m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} + m_{\text{cal}} c_{\text{cal}} \Delta T_{\text{water}} \\ &= [(0.6\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K}) + (0.2\,\text{kg})(0.9\,\text{kJ/kg}\cdot\text{K})](293\,\text{K} - 273\,\text{K}) \\ &= 8.064\,\text{kJ} \end{aligned}$$

Express the amount of ice that will melt in terms of the difference between the heat available and the heat required to warm the ice:

$$m_{\text{melted ice}} = \frac{Q_{\text{avail}} - Q_{\text{warm ice}}}{L_{\text{f}}}$$

Substitute numerical values and evaluate $m_{\text{melted ice}}$:

$$m_{\text{melted ice}} = \frac{8.064\,\text{kJ} - 8\,\text{kJ}}{333.5\,\text{kJ/kg}} = 0.1919\,\text{g}$$

Find the ice remaining in the system:

$$m_{\text{remaining ice}} = 200\,\text{g} - 0.1919\,\text{g} = \boxed{199.8\,\text{g}}$$

(*c*) Because the initial and final conditions are the same, the answer would be the same.

42 ••

Picture the Problem Let the subscript B denote the block, w_1 the water initially in the calorimeter, and w_2 the 120 mL of water that is added to the calorimeter vessel. We can equate the heat gained by the calorimeter and its initial contents to the heat lost by the warm water and solve this equation for the specific heat of the block.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat gained by the block, the calorimeter, and the water initially in the calorimeter:

$$Q_{\text{gained}} = m_{\text{B}}c_{\text{B}}\Delta T_{\text{B}} + m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}} + m_{\text{w}_1}c_{\text{w}_1}\Delta T_{\text{w}_1} = \left(m_{\text{B}}c_{\text{B}} + m_{\text{Cu}}c_{\text{Cu}} + m_{\text{w}_1}c_{\text{w}_1}\right)\Delta T$$

because the temperature changes are the same for the block, calorimeter, and the water that is initially at 20°C.

Express the heat lost by the water that is added to the calorimeter:

$$Q_{\text{lost}} = m_{\text{w}_2}c_{\text{w}_2}\Delta T_{\text{w}_2}$$

Substitute to obtain:

$$\left(m_{\text{B}}c_{\text{B}} + m_{\text{Cu}}c_{\text{Cu}} + m_{\text{w}_1}c_{\text{w}_1}\right)\Delta T = m_{\text{w}_2}c_{\text{w}_2}\Delta T_{\text{w}_2}$$

Substitute numerical values to obtain:

$$\left[(0.1\,\text{kg})c_B + (0.025\,\text{kg})(0.386\,\text{kJ/kg}\cdot\text{K}) + (0.06\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})\right](327\,\text{K} - 293\,\text{K})$$
$$= (120\times10^{-3}\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(353\,\text{K} - 327\,\text{K})$$

Solve for c_B to obtain:

$$c_B = \boxed{1.23\,\text{kJ/kg}\cdot\text{K}}$$
$$= \boxed{0.294\,\text{cal/g}\cdot\text{K}}$$

43 ••

Picture the Problem We can find the temperature *t* by equating the heat gained by the warming water and calorimeter, and vaporization of some of the water.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat gained by the warming and vaporizing water:

$$Q_{\text{gained}} = m_{\text{w, vaporized}}L_{\text{f, w}} + m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} + m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{w}}$$

Express the heat lost by the 100-g piece of copper as it cools:

$$Q_{\text{lost}} = m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}}$$

Substitute to obtain:

$$m_{\text{w, vaporized}}L_{\text{f, w}} + m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} + m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{w}} = m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}}$$

Substitute numerical values:

$$(1.2\,\text{g})(540\,\text{cal/g}) + (200\,\text{g})(1\,\text{cal/g}\cdot\text{K})(311\,\text{K} - 289\,\text{K})$$
$$+ (150\,\text{g})(0.0923\,\text{cal/g}\cdot\text{K})(311\,\text{K} - 289\,\text{K}) = (100\,\text{g})(0.0923\,\text{cal/g}\cdot\text{K})(t - 311\,\text{K})$$

Solve for *t* to obtain:

$$t = 891\,\text{K} = \boxed{618°\text{C}}$$

44 ••

Picture the Problem We can find the final temperature of the system by equating the heat gained by the calorimeter and the water in it to the heat lost by the cooling aluminum shot. In (*b*) we'll proceed as in (*a*) but with the initial and final temperatures adjusted to minimize heat transfer between the system and its surroundings.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

(*a*) Express the heat gained by the warming water and the calorimeter:

$$Q_{\text{gained}} = m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} + m_{\text{cal}} c_{\text{Al}} \Delta T_{\text{w}}$$

Express the heat lost by the aluminum shot as it cools:

$$Q_{\text{lost}} = m_{\text{shot}} c_{\text{Al}} \Delta T_{\text{Al}}$$

Substitute to obtain:

$$\left(m_{\text{w}} c_{\text{w}} + m_{\text{cal}} c_{\text{Al}}\right)\Delta T_{\text{w}} = m_{\text{shot}} c_{\text{Al}} \Delta T_{\text{Al}}$$

Substitute numerical values to obtain:

$$\left[(500\,\text{g})(1\,\text{cal/g}\cdot\text{K}) + (200\,\text{g})(0.0923\,\text{cal/g}\cdot\text{K})\right](t_{\text{f}} - 293\,\text{K}) = (300\,\text{g})(0.215\,\text{cal/g}\cdot\text{K})(373\,\text{K} - t_{\text{f}})$$

Solve for t_{f} to obtain:

$$t_{\text{f}} = 301.9\,\text{K} = \boxed{28.9°\text{C}}$$

(*b*) Let the initial and final temperatures of the calorimeter and its contents be:

$t_{\text{i}} = 20°\text{C} - t_0$ (1)

and

$t_{\text{f}} = 20°\text{C} + t_0$

where t_{i} and t_{f} are the temperatures above and below room temperature and t_0 is the amount t_{i} and t_{f} must be below and above room temperature respectively.

Express and the heat gained by the water and calorimeter:

$$Q_{\text{in}} = m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} + m_{\text{cal}} c_{\text{Al}} \Delta T_{\text{w}} = \left(m_{\text{w}} c_{\text{w}} + m_{\text{cal}} c_{\text{Al}}\right)\Delta T_{\text{w}}$$

Express the heat lost by the aluminum shot as it cools:

$$Q_{\text{out}} = m_{\text{shot}} c_{\text{Al}} \Delta T_{\text{Al}}$$

Equate Q_{in} and Q_{out} to obtain:

$$\left(m_{\text{w}} c_{\text{w}} + m_{\text{cal}} c_{\text{Al}}\right)\Delta T_{\text{w}} = m_{\text{shot}} c_{\text{Al}} \Delta T_{\text{Al}}$$

Substitute numerical values:

$$\left[(500\,\text{g})(1\,\text{cal/g}\cdot\text{K}) + (200\,\text{g})(0.215\,\text{cal/g}\cdot\text{K})\right](293\,\text{K} + t_0 - 293\,\text{K} + t_0) = (300\,\text{g})(0.215\,\text{cal/g}\cdot\text{K})(373\,\text{K} - 293\,\text{K} - t_0)$$

Solve for and evaluate t_0:

$$t_0 = 277.49\,\text{K} = 4.49°\text{C}$$

Substitute in equation (1) to obtain:

$$t_i = 20°C - 4.49°C = \boxed{15.5°C}$$

First Law of Thermodynamics

45 •

Picture the Problem We can apply the first law of thermodynamics to find the change in internal energy of the gas during this process.

Apply the first law of thermodynamics to express the change in internal energy of the gas in terms of the heat added to the system and the work done on the gas:

$$\Delta E_{int} = Q_{in} + W_{on}$$

The work done by the gas equals the negative of the work done on the gas. Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{int} = 600\,\text{cal} \times \frac{4.184\,\text{J}}{\text{cal}} - 300\,\text{J} = \boxed{2.21\,\text{kJ}}$$

***46** •

Picture the Problem We can apply the first law of thermodynamics to find the change in internal energy of the gas during this process.

Apply the first law of thermodynamics to express the change in internal energy of the gas in terms of the heat added to the system and the work done on the gas:

$$\Delta E_{int} = Q_{in} + W_{on}$$

The work done by the gas is the negative of the work done on the gas. Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{int} = 400\,\text{kcal} \times \frac{4.184\,\text{J}}{\text{cal}} - 800\,\text{kJ} = \boxed{874\,\text{kJ}}$$

47 •

Picture the Problem We can use the first law of thermodynamics to relate the change in the bullet's internal energy to its pre-collision kinetic energy.

Using the first law of thermodynamics, relate the change in the internal energy of the bullet to the work done on it by the block of wood:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$,

$$\Delta E_{\text{int}} = W_{\text{on}} = \Delta K = -(K_{\text{f}} - K_i)$$

Substitute for ΔE_{int}, K_{f}, and K_{i} to obtain:

$$mc_{\text{Pb}}(t_{\text{f}} - t_{\text{i}}) = -\left(0 - \tfrac{1}{2}mv^2\right) = \tfrac{1}{2}mv^2$$

Solve for t_{f}:

$$t_{\text{f}} = t_{\text{i}} + \frac{v^2}{2c_{\text{Pb}}}$$

Substitute numerical values and evaluate t_{f}:

$$t_{\text{f}} = 293\,\text{K} + \frac{(200\,\text{m/s})^2}{2(0.128\,\text{kJ/kg}\cdot\text{K})} = 449\,\text{K} = \boxed{176°\text{C}}$$

48 •

Picture the Problem What is described above is clearly a limiting case because, as the water falls, it will, for example, collide with rocks and experience air drag; resulting in some of its initial potential energy being converted into internal energy. In this limiting case we can use the first law of thermodynamics to relate the change in the gravitational potential energy (take $U_{\text{g}} = 0$ at the bottom of the waterfalls) to the change in internal energy of the water and solve for the increase in temperature.

(*a*) Using the first law of thermodynamics and noting that, because the gravitational force is conservative, $W_{\text{on}} = -\Delta U$, relate the change in the internal energy of the water to the work done on it by gravity:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$,

$$\Delta E_{\text{int}} = W_{\text{on}} = -\Delta U = -(U_{\text{f}} - U_{\text{i}})$$

Substitute for ΔE_{int}, U_{f}, and U_{i} to obtain:

$$mc_{\text{w}}\Delta T = -(0 - mg\Delta h) = mg\Delta h$$

Solve for ΔT:

$$\Delta T = \frac{g\Delta h}{c_{\text{w}}}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{(9.81\,\text{m/s}^2)(50\,\text{m})}{4.18\,\text{kJ/kg}\cdot\text{K}} = \boxed{0.117\,\text{K}}$$

(*b*) Proceed as in (*a*) with Δh = 740 m:

$$\Delta T = \frac{(9.81\,\text{m/s}^2)(740\,\text{m})}{4.18\,\text{kJ/kg}\cdot\text{K}} = \boxed{1.74\,\text{K}}$$

49 •

Picture the Problem We can apply the first law of thermodynamics to find the change in internal energy of the gas during this process.

Apply the first law of thermodynamics to express the change in internal energy of the gas in terms of the heat added to the system and the work done on the gas:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

The work done by the gas is the negative of the work done on the gas. Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = 20\,\text{cal}\times\frac{4.184\,\text{J}}{\text{cal}} - 30\,\text{J} = \boxed{53.7\,\text{J}}$$

50 ••

Picture the Problem We can use the definition of kinetic energy to express the speed of the bullet upon impact in terms of its kinetic energy. The heat absorbed by the bullet is the sum of the heat required to warm to bullet from 202 K to its melting temperature of 600 K and the heat required to melt it. We can use the first law of thermodynamics to relate the impact speed of the bullet to the change in its internal energy.

Using the first law of thermodynamics, relate the change in the internal energy of the bullet to the work done on it by the target:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$,

$$\Delta E_{\text{int}} = W_{\text{on}} = \Delta K = -(K_{\text{f}} - K_i)$$

Substitute for ΔE_{int}, K_{f}, and K_{i} to obtain:

$$mc_{\text{Pb}}\Delta T_{\text{Pb}} + mL_{\text{f,Pb}} = -\left(0 - \tfrac{1}{2}mv^2\right) = \tfrac{1}{2}mv^2$$

or

$$mc_{\text{Pb}}(T_{\text{MP}} - T_{\text{i}}) + mL_{\text{f,Pb}} = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{2\left[c_{\text{Pb}}(T_{\text{MP}} - T_{\text{i}}) + L_{\text{f,Pb}}\right]}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2\{(0.128\,\text{kJ/kg}\cdot\text{K})(600\,\text{K} - 303\,\text{K}) + 24.7\,\text{kJ/kg}\}} = \boxed{354\,\text{m/s}}$$

***51 ••**

Picture the Problem We can find the rate at which heat is generated when you rub your hands together using the definition of power and the rubbing time to produce a 5°C increase in temperature from $\Delta Q = (dQ/dt)\Delta t$ and $Q = mc\Delta T$.

(*a*) Express the rate at which heat is generated as a function of the friction force and the average speed of your hands:

$$\frac{dQ}{dt} = P = f_k v = \mu F_n v$$

Substitute numerical values and evaluate *dQ*/*dt*:

$$\frac{dQ}{dt} = 0.5(35\,\text{N})(0.35\,\text{m/s}) = \boxed{6.13\,\text{W}}$$

(*b*) Relate the heat required to raise the temperature of your hands 5 K to the rate at which it is being generated:

$$\Delta Q = \frac{dQ}{dt}\Delta t = mc\Delta T$$

Solve for Δt:

$$\Delta t = \frac{mc\Delta T}{dQ/dt}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{(0.35\,\text{kg})(4\,\text{kJ/kg}\cdot\text{K})(5\,\text{K})}{6.13\,\text{W}}$$

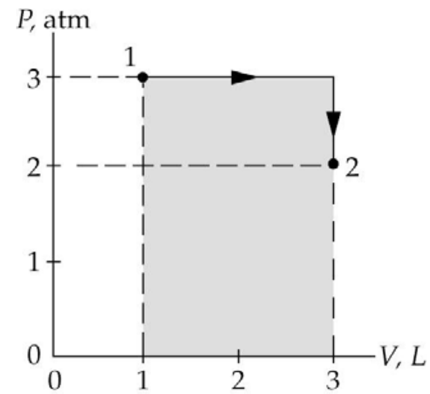
$$= 1143\,\text{s} \times \frac{1\,\text{min}}{60\,\text{s}} = \boxed{19.0\,\text{min}}$$

Work and the *PV* Diagram for a Gas

52 •

Picture the Problem We can find the work done by the gas during this process from the area under the curve. Because no work is done along the constant volume (vertical) part of the path, the work done by the gas is done during its isobaric expansion. We can then use the first law of thermodynamics to find the heat added to the system during this process.

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram.



The work done by the gas equals the area under the shaded curve:

$$W_{\text{by gas}} = P\Delta V = (3\,\text{atm})(2\,\text{L}) = \left(3\,\text{atm}\times\frac{101.3\,\text{kPa}}{\text{atm}}\right)\left(2\,\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{608\,\text{J}}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} \\ &= (E_{\text{int,2}} - E_{\text{int,1}}) - (-W_{\text{by gas}}) \\ &= (E_{\text{int,2}} - E_{\text{int,1}}) + W_{\text{by gas}} \end{aligned}$$

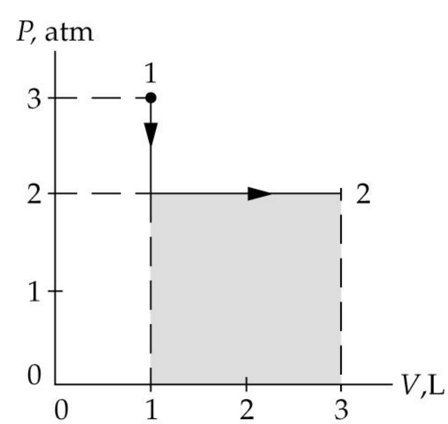
Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = (912\,\text{J} - 456\,\text{J}) + 608\,\text{J} = \boxed{1.06\,\text{kJ}}$$

53 •

Picture the Problem We can find the work done by the gas during this process from the area under the curve. Because no work is done along the constant volume (vertical) part of the path, the work done by the gas is done during its isobaric expansion. We can then use the first law of thermodynamics to find the heat added to the system during this process

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram.



The work done by the gas equals the area under the curve:

$$W_{\text{by gas}} = P\Delta V = (2\,\text{atm})(2\,\text{L}) = \left(2\,\text{atm}\times\frac{101.3\,\text{kPa}}{\text{atm}}\right)\left(2\,\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{405\,\text{J}}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} \\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) - \left(-W_{\text{by gas}}\right) \\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) + W_{\text{by gas}} \end{aligned}$$

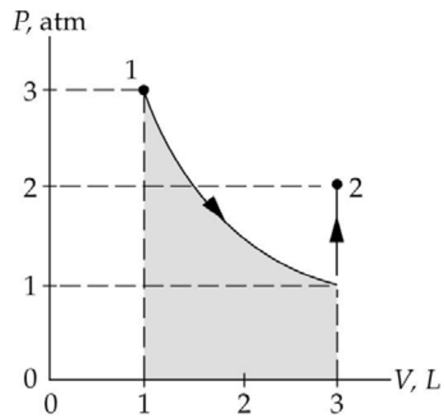
Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = (912\,\text{J} - 456\,\text{J}) + 405\,\text{J} = \boxed{861\,\text{J}}$$

***54 ••**

Picture the Problem We can find the work done by the gas during this process from the area under the curve. Because no work is done along the constant volume (vertical) part of the path, the work done by the gas is done during its isothermal expansion. We can then use the first law of thermodynamics to find the heat added to the system during this process.

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram.



The work done by the gas equals the area under the curve:

$$\begin{aligned} W_{\text{by gas}} &= \int_{V_1}^{V_2} P\,dV = nRT_1\int_{1\,\text{L}}^{3\,\text{L}}\frac{dV}{V} \\ &= P_1V_1\int_{1\,\text{L}}^{3\,\text{L}}\frac{dV}{V} = P_1V_1\left[\ln V\right]_{1\,\text{L}}^{3\,\text{L}} \\ &= P_1V\ln 3 \end{aligned}$$

Substitute numerical values and evaluate $W_{\text{by gas}}$:

$$W_{\text{by gas}} = \left(3\,\text{atm}\times\frac{101.3\,\text{kPa}}{\text{atm}}\right)\left(1\,\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\right)\ln 3 = \boxed{334\,\text{J}}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned}Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}}\\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) - \left(-W_{\text{by gas}}\right)\\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) + W_{\text{by gas}}\end{aligned}$$

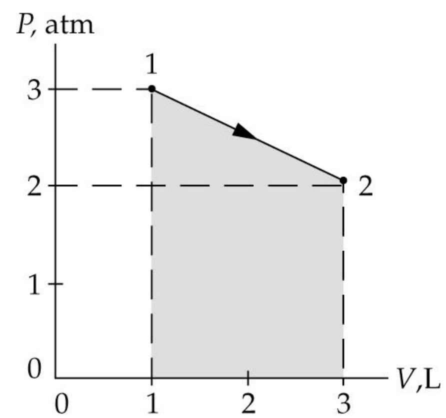
Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = \left(912\,\text{J} - 456\,\text{J}\right) + 334\,\text{J} = \boxed{790\,\text{J}}$$

55 ••

Picture the Problem We can find the work done by the gas during this process from the area under the curve. We can then use the first law of thermodynamics to find the heat added to the system during this process.

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram:



The work done by the gas equals the area under the curve:

$$\begin{aligned}W_{\text{by gas}} &= A_{\text{trapezoid}} = \tfrac{1}{2}\left(3\,\text{atm} + 2\,\text{atm}\right)\left(2\,\text{L}\right)\\ &= 5\,\text{atm}\cdot\text{L}\times\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}} = \boxed{507\,\text{J}}\end{aligned}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned}Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}}\\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) - \left(-W_{\text{by gas}}\right)\\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) + W_{\text{by gas}}\end{aligned}$$

Substitute numerical values and evaluate Q_{in}:

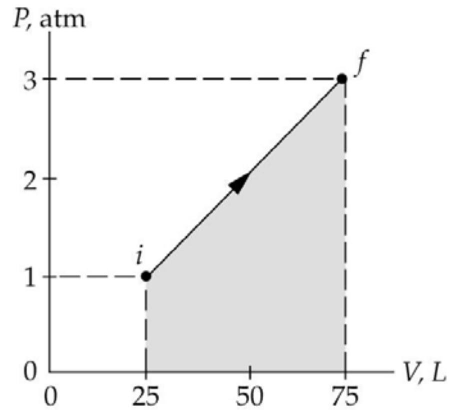
$$Q_{\text{in}} = \left(912\,\text{J} - 456\,\text{J}\right) + 507\,\text{J} = \boxed{963\,\text{J}}$$

Remarks: You could use the linearity of the path connecting the initial and final states and the coordinates of the endpoints to express *P* as a function of *V*. You could then integrate this function between 1 and 3 L to find the work done by the gas as it goes from its initial to its final state.

56 ••

Picture the Problem We can find the work done by the gas during this process from the area under the curve.

The path from the initial state *i* to the final state *f* is shown on the *PV* diagram:



The work done by the gas equals the area under the curve:

$$W_{\text{by gas}} = A_{\text{trapezoid}} = \tfrac{1}{2}(1\,\text{atm} + 3\,\text{atm})(50\,\text{L})$$
$$= 100\,\text{atm}\cdot\text{L} \times \frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}} = \boxed{10.1\text{kJ}}$$

Remarks: You could use the linearity of the path connecting the initial and final states and the coordinates of the endpoints to express *P* as a function of *V*. You could then integrate this function between 1 and 3 L to find the work done by the gas as it goes from its initial to its final state.

57 ••

Picture the Problem We can find the work done by the gas from the area under the *PV* curve provided we can find the pressure and volume coordinates of the initial and final states. We can find these coordinates by using the ideal gas law and the condition $T = AP^2$.

Apply the ideal-gas law with $n =$ 1 mol and $T = AP^2$ to obtain:

$$PV = RAP^2 \Rightarrow V = RAP \qquad (1)$$

This result tells us that the volume varies linearly with the pressure.

Solve the condition on the temperature for the pressure of the gas:

$$P_0 = \sqrt{\frac{T_0}{A}}$$

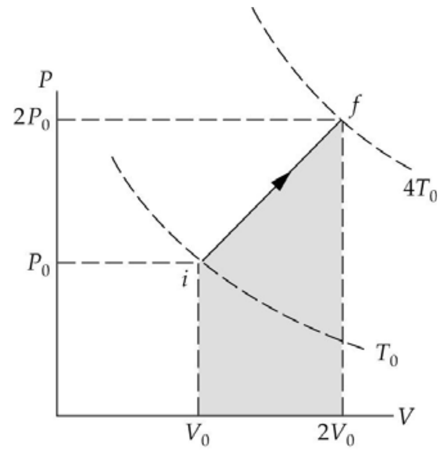
Find the pressure when the temperature is $4T_0$:

$$P = \sqrt{\frac{4T_0}{A}} = 2\sqrt{\frac{T_0}{A}} = 2P_0$$

Using equation (1), express the coordinates of the final state:

$$(2V_0, 2P_0)$$

The PV diagram for the process is shown to the right:



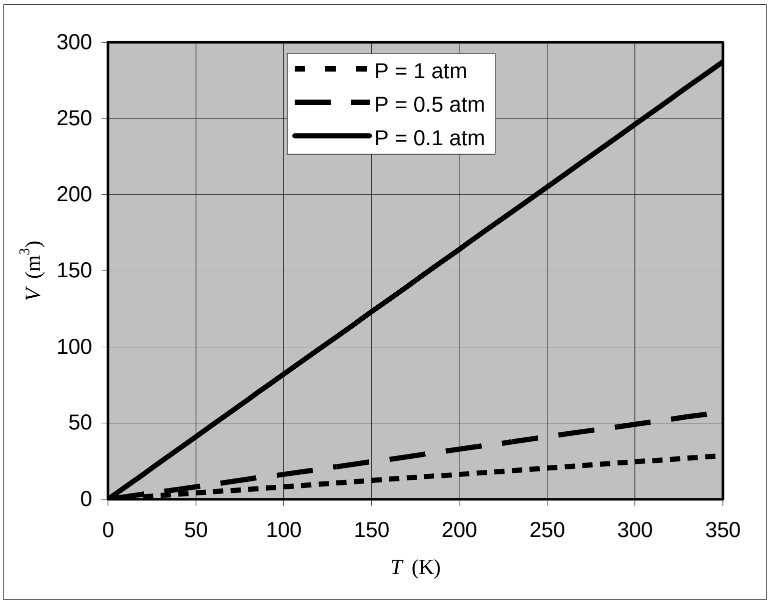
The work done by the gas equals the area under the curve:

$$W_{\text{by gas}} = A_{\text{trapezoid}} = \tfrac{1}{2}(P_0 + 3P_0)(2V_0 - V_0)$$
$$= \boxed{\tfrac{3}{2}P_0V_0}$$

***58** •

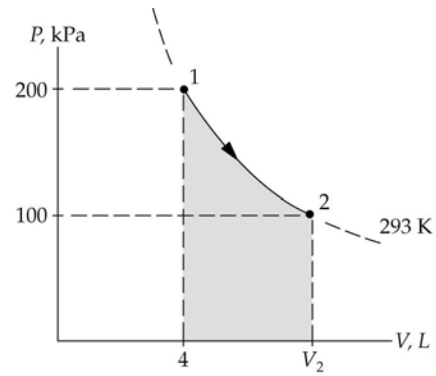
Picture the Problem From the ideal gas law, $PV = NkT$, or $V = NkT/P$. Hence, on a VT diagram, isobars will be straight lines with slope $1/P$.

A spreadsheet program was used to plot the following graph. The graph was plotted for 1 mol of gas.



59 ••

Picture the Problem The *PV* diagram shows the isothermal expansion of the ideal gas from its initial state 1 to its final state 2. We can use the ideal-gas law for a fixed amount of gas to find V_2 and then evaluate $\int PdV$ for an isothermal process to find the work done by the gas. In part (b) of the problem we can apply the first law of thermodynamics to find the heat added to the gas during the expansion.



(*a*) Express the work done by a gas during an isothermal process:

$$W_{\text{by gas}} = \int_{V_1}^{V_2} PdV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = P_1V_1\int_{V_1}^{V_2}\frac{dV}{V}$$

Apply the ideal-gas law for a fixed amount of gas undergoing an isothermal process:

$$P_1V_1 = P_2V_2 \text{ or } \frac{V_2}{V_1} = \frac{P_1}{P_1}$$

Solve for and evaluate V_2:

$$V_2 = \frac{P_1}{P_2}V_1 = \frac{200\,\text{kPa}}{100\,\text{kPa}}(4\,\text{L}) = 8\,\text{L}$$

Substitute numerical values and evaluate *W*:

$$W_{\text{by gas}} = (200\,\text{kPa})(4\,\text{L})\int_{4\text{L}}^{8\text{L}}\frac{dV}{V}$$
$$= (800\,\text{kPa}\cdot\text{L})[\ln V]_{4\text{L}}^{8\text{L}}$$
$$= (800\,\text{kPa}\cdot\text{L})\ln\left(\frac{8\,\text{L}}{4\,\text{L}}\right)$$
$$= 800\,\text{kPa}\cdot\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\ \boxed{555\,\text{J}}$$

(*b*) Apply the first law of thermodynamics to the system to obtain:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}}$$

or, because $\Delta E_{\text{int}} = 0$ for an isothermal process,

$$Q_{\text{in}} = -W_{\text{on}}$$

Because the work done by the gas is the negative of the work done on the gas:

$$Q_{\text{in}} = -(-W_{\text{by gas}}) = W_{\text{by gas}}$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{in} = \boxed{555\,\text{J}}$$

Heat Capacities of Gases and the Equipartition Theorem

60 •

Picture the Problem We can find the number of moles of the gas from its heat capacity at constant volume using $C_V = \frac{3}{2}nR$. We can find the internal energy of the gas from $E_{int} = C_V T$ and the heat capacity at constant pressure using $C_P = C_V + nR$.

(*a*) Express C_V in terms of the number of moles in the monatomic gas:

$$C_V = \tfrac{3}{2}nR$$

Solve for *n*:

$$n = \frac{2C_V}{3R}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{2(49.8\,\text{J/K})}{3(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{3.99}$$

(*b*) Relate the internal energy of the gas to its temperature:

$$E_{int} = C_V T$$

Substitute numerical values and evaluate E_{int}:

$$E_{int} = (49.8\,\text{J/K})(300\,\text{K}) = \boxed{14.9\,\text{kJ}}$$

(*c*) Relate the heat capacity at constant pressure to the heat capacity at constant volume:

$$C_P = C_V + nR = \tfrac{3}{2}nR + nR = \tfrac{5}{2}nR$$

Substitute numerical values and evaluate C_P:

$$C_P = \tfrac{5}{2}(3.99)(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{82.9\,\text{J/K}}$$

61 •

Picture the Problem The Dulong-Petit law gives the molar specific heat of a solid, c'. The specific heat is defined as $c = c'/M$ where *M* is the molar mass. Hence we can use this definition to find *M* and a periodic table to identify the element.

(*a*) Apply the Dulong-Petit law:

$$c' = 3R \text{ or } c = \frac{3R}{M}$$

Solve for M:

$$M = \frac{3R}{c}$$

Substitute numerical values and evaluate M:

$$M = \frac{24.9\,\text{J/mol}\cdot\text{K}}{0.447\,\text{kJ/kg}\cdot\text{K}} = \boxed{55.7\,\text{g/mol}}$$

Consulting the periodic table of the elements we see that the element is most likely iron.

***62 ••**

Picture the Problem The specific heats of air at constant volume and constant pressure are given by $c_V = C_V/m$ and $c_P = C_P/m$ and the heat capacities at constant volume and constant pressure are given by $C_V = \frac{5}{2}nR$ and $C_P = \frac{7}{2}nR$, respectively.

(*a*) Express the specific heats per unit mass of air at constant volume and constant pressure:

$$c_V = \frac{C_V}{m} \quad (1)$$

and

$$c_P = \frac{C_P}{m} \quad (2)$$

Express the heat capacities of a diatomic gas in terms of the gas constant R, the number of moles n, and the number of degrees of freedom:

$$C_V = \tfrac{5}{2}nR$$

and

$$C_P = \tfrac{7}{2}nR$$

Express the mass of 1 mol of air:

$$m = 0.74M_{N_2} + 0.26M_{O_2}$$

Substitute in equation (1) to obtain:

$$c_V = \frac{5nR}{2\left(0.74M_{N_2} + 0.26M_{O_2}\right)}$$

Substitute numerical values and evaluate c_V:

$$c_V = \frac{5(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}{2\left[0.74\left(28\times10^{-3}\,\text{kg}\right) + 0.26\left(32\times10^{-3}\,\text{kg}\right)\right]} = \boxed{716\,\text{J/kg}\cdot\text{K}}$$

Substitute in equation (2) to obtain:

$$c_P = \frac{7nR}{2\left(0.74M_{N_2} + 0.26M_{O_2}\right)}$$

Substitute numerical values and evaluate c_P:

$$c_P = \frac{7(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}{2\left[0.74\left(28\times10^{-3}\,\text{kg}\right)+0.26\left(32\times10^{-3}\,\text{kg}\right)\right]} = \boxed{1002\,\text{J/kg}\cdot\text{K}}$$

(*b*) Express the percent difference between the value from the *Handbook of Chemistry and Physics* and the calculated value:

$$\%\,\text{difference} = \frac{1.032\,\text{J/g}\cdot\text{K} - 1.002\,\text{J/g}\cdot\text{K}}{1.032\,\text{J/g}\cdot\text{K}} = \boxed{2.91\%}$$

63 ••

Picture the Problem We know that, during a constant-volume process, no work is done and that we can calculate the heat added during this expansion from the heat capacity at constant volume and the change in the absolute temperature. We can then use the first law of thermodynamics to find the change in the internal energy of the gas. In part (*b*), we can proceed similarly; using the heat capacity at constant pressure rather than constant volume.

(*a*) The increase in the internal energy of the ideal diatomic gas is given by:

$$\Delta E_{\text{int}} = \tfrac{5}{2}nR\Delta T$$

Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = \tfrac{5}{2}(1\,\text{mol})(8.315\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{6.24\,\text{kJ}}$$

For a constant-volume process:

$$W_{\text{on}} = \boxed{0}$$

From the 1st law of thermodynamics we have:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}}$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = 6.24\,\text{kJ} - 0 = \boxed{6.24\,\text{kJ}}$$

(*b*) Because ΔE_{int} depends only on the temperature difference:

$$\Delta E_{\text{int}} = \boxed{6.24\,\text{kJ}}$$

Relate the heat added to the gas to its heat capacity at constant pressure and the change in its temperature:

$$Q_{\text{in}} = C_P\Delta T = \left(\tfrac{5}{2}nR + nR\right)\Delta T = \tfrac{7}{2}nR\Delta T$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{in} = \tfrac{7}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{8.73\,\text{kJ}}$$

Apply the first law of thermodynamics to find W:

$$W_{on} = \Delta E_{int} - Q_{in} = 8.73\,\text{kJ} - 6.24\,\text{kJ} = \boxed{2.49\,\text{kJ}}$$

(*c*) Integrate $dW_{on} = P\,dV$ to obtain:

$$W_{on} = \int_{V_i}^{V_f} P\,dV = P(V_f - V_i) = nR(T_f - T_i)$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = (1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{2.49\,\text{kJ}}$$

64 ••

Picture the Problem Because this is a constant-volume process, we can use $Q = C_V\Delta T$ to express Q in terms of the temperature change and the ideal-gas law for a fixed amount of gas to find ΔT.

Express the amount of heat Q that must be transferred to the gas if its pressure is to triple:

$$Q = C_V\Delta T = \tfrac{5}{2}nR(T_f - T_0)$$

Using the ideal-gas law for a fixed amount of gas, relate the initial and final temperatures, pressures and volumes:

$$\frac{P_0V}{T_0} = \frac{3P_0V}{T_f}$$

Solve for T_f:

$$T_f = 3T_0$$

Substitute and simplify to obtain:

$$Q = \tfrac{5}{2}nR(2T_0) = 5(nRT_0) = \boxed{5P_0V}$$

65 ••

Picture the Problem Let the subscripts i and f refer to the initial and final states of the gas, respectively. We can use the ideal-gas law for a fixed amount of gas to express V' in terms of V and the change in temperature of the gas when 13,200 J of heat are transferred to it. We can find this change in temperature using $Q = C_p\Delta T$.

Using the ideal-gas law for a fixed amount of gas, relate the initial and

$$\frac{P_iV}{T_i} = \frac{P_fV'}{T_f}$$

final temperatures, volumes, and pressures:

Because the process is isobaric, we can solve for V' to obtain:

$$V' = V\frac{T_f}{T_i} = V\frac{T_i + \Delta T}{T_i} = V\left(1 + \frac{\Delta T}{T_i}\right)$$

Relate the heat transferred to the gas to the change in its temperature:

$$Q = C_P\Delta T = \tfrac{7}{2}nR\Delta T$$

Solve ΔT:

$$\Delta T = \frac{2Q}{7nR}$$

Substitute to obtain:

$$V' = V\left(1 + \frac{2Q}{7nRT_i}\right)$$

One mol of gas at STP occupies 22.4 L. Substitute numerical values and evaluate V':

$$V' = \left(22.4\times10^{-3}\,\text{m}^3\right)\left(1 + \frac{2(13.2\,\text{kJ})}{7(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})}\right) = \boxed{59.6\,\text{L}}$$

66 ••

Picture the Problem We can use the relationship between C_P and C_V ($C_P = C_V + nR$) to find the number of moles of this particular gas. In parts (*b*) and (*c*) we can use the number of degrees of freedom associated with monatomic and diatomic gases, respectively, to find C_P and C_V.

(*a*) Express the heat capacity of the gas at constant pressure to its heat capacity at constant volume:

$$C_P = C_V + nR$$

Solve for n:

$$n = \frac{C_P - C_V}{R}$$

Substitute numerical values and evaluate n:

$$n = \frac{29.1\,\text{J/K}}{8.314\,\text{J/mol}\cdot\text{K}} = \boxed{3.50\,\text{mol}}$$

(*b*) C_V for a monatomic gas is given by:

$$C_V = \tfrac{3}{2}nR$$

Substitute numerical values and evaluate C_V:

$$C_V = \tfrac{3}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{43.6\,\text{J/K}}$$

Express C_P for a monatomic gas:

$$C_P = \tfrac{5}{2}nR$$

Substitute numerical values and evaluate C_P:

$$C_P = \tfrac{5}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{72.7\,\text{J/K}}$$

(*c*) If the diatomic molecules rotate but do not vibrate they have 5 degrees of freedom:

$$C_V = \tfrac{5}{2}nR = \tfrac{5}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{72.7\,\text{J/K}}$$

and

$$C_P = \tfrac{7}{2}nR = \tfrac{7}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{102\,\text{J/K}}$$

***67 ••**

Picture the Problem We can find the change in the heat capacity at constant pressure as CO_2 undergoes sublimation from the energy per molecule of CO_2 in the solid and gaseous states.

Express the change in the heat capacity (at constant pressure) per mole as the CO_2 undergoes sublimation:

$$\Delta C_P = C_{P,\text{gas}} - C_{P,\text{solid}}$$

Express $C_{p,\text{gas}}$ in terms of the number of degrees of freedom per molecule:

$$C_{P,\text{gas}} = f\left(\tfrac{1}{2}Nk\right) = \tfrac{5}{2}Nk$$

because each molecule has three translational and two rotational degrees of freedom in the gaseous state.

We know, from the Dulong-Petit Law, that the molar specific heat of most solids is $3R = 3Nk$. This result is essentially a per-atom result as it was obtained for a monatomic solid with six degrees of freedom. Use this result and the fact CO_2 is triatomic to express $C_{P,\text{solid}}$:

$$C_{P,\text{solid}} = \frac{3Nk}{\text{atom}} \times 3\,\text{atoms} = 9Nk$$

Substitute to obtain:

$$\Delta C_P = \tfrac{5}{2}Nk - \tfrac{18}{2}Nk = \boxed{-\tfrac{13}{2}Nk}$$

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Picture the Problem We can find the initial internal energy of the gas from $U_i = \frac{3}{2}nRT$ and the final internal energy from the change in internal energy resulting from the addition of 500 J of heat. The work done during a constant-volume process is zero and the work done during the constant-pressure process can be found from the first law of thermodynamics.

(*a*) Express the initial internal energy of the gas in terms of its temperature:

$$E_{\text{int,i}} = \tfrac{3}{2}nRT$$

Substitute numerical values and evaluate $E_{\text{int,i}}$:

$$E_{\text{int,i}} = \tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K}) = \boxed{3.40\,\text{kJ}}$$

(*b*) Relate the final internal energy of the gas to its initial internal energy:

$$E_{\text{int,f}} = E_{\text{int,i}} + \Delta E_{\text{int}} = E_{\text{int,i}} + C_V\Delta T$$

Express the change in temperature of the gas resulting from the addition of heat:

$$\Delta T = \frac{Q_{\text{in}}}{C_P}$$

Substitute to obtain:

$$E_{\text{int,f}} = E_{\text{int,i}} + \frac{C_V}{C_P}Q_{\text{in}}$$

Substitute numerical values and evaluate $E_{\text{int,f}}$:

$$E_{\text{int,f}} = 3.40\,\text{kJ} + \frac{\frac{3}{2}nR}{\frac{5}{2}nR}(500\,\text{J}) = \boxed{3.70\,\text{kJ}}$$

(*c*) Relate the final internal energy of the gas to its initial internal energy:

$$E_{\text{int,f}} = E_{\text{int,i}} + \Delta E_{\text{int}}$$

Apply the first law of thermodynamics to the constant-volume process:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $W_{\text{on}} = 0$,

$$\Delta E_{\text{int}} = Q_{\text{in}} = 500\,\text{J}$$

Substitute numerical values and evaluate $E_{\text{int,f}}$:

$$E_{\text{int,f}} = 3.40\,\text{kJ} + 500\,\text{J} = \boxed{3.90\,\text{kJ}}$$

69 ••

Picture the Problem We can use $C_{V,\text{water}} = f\left(\frac{1}{2}Nk\right)$ to express $C_{V,\text{water}}$ and then count the number of degrees of freedom associated with a water molecule to determine *f*.

Express $C_{V,\text{water}}$ in terms of the number of degrees of freedom per molecule:

$$C_{V,\text{water}} = f\left(\tfrac{1}{2}Nk\right)$$

where *f* is the number of degrees of freedom associated with a water molecule.

There are three translational degrees of freedom and three rotational degrees of freedom. In addition, each of the hydrogen atoms can vibrate against the oxygen atom, resulting in an additional 4 degrees of freedom (2 per atom).

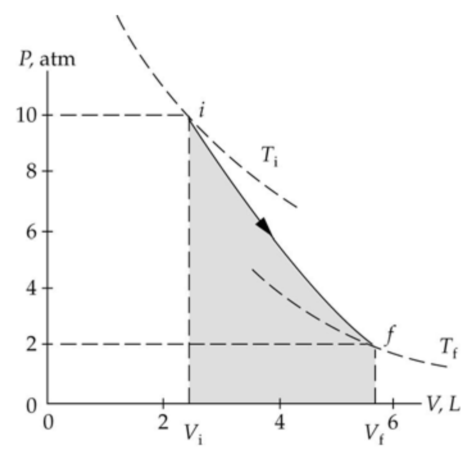
Substitute for *f* to obtain:

$$C_{V,\text{water}} = 10\left(\tfrac{1}{2}Nk\right) = \boxed{5Nk}$$

Quasi-Static Adiabatic Expansion of a Gas

***70** ••

Picture the Problem The adiabatic expansion is shown in the *PV* diagram. We can use the ideal-gas law to find the initial volume of the gas and the equation for a quasi-static adiabatic process to find the final volume of the gas. A second application of the ideal-gas law, this time at the final state, will yield the final temperature of the gas. In part (*c*) we can use the first law of thermodynamics to find the work done by the gas during this process.



(*a*) Apply the ideal-gas law to express the initial volume of the gas:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})}{10\,\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}}$$

$$= 2.24\times10^{-3}\,\text{m}^3 = \boxed{2.24\,\text{L}}$$

Use the relationship between the pressures and volumes for a quasi-static adiabatic process to express V_f:

$$P_iV_i^{\gamma}=P_fV_f^{\gamma}\Rightarrow V_f=V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma}$$

Substitute numerical values and evaluate V_f:

$$V_f=V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma}=(2.24\,\text{L})\left(\frac{10\,\text{atm}}{2\,\text{atm}}\right)^{3/5}=\boxed{5.88\,\text{L}}$$

(*b*) Apply the ideal-gas law to express the final temperature of the gas:

$$T_f=\frac{P_fV_f}{nR}$$

Substitute numerical values and evaluate T_f:

$$T_f=\frac{(2\,\text{atm})(5.88\,\text{L})}{8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K}}=\boxed{143\,\text{K}}$$

(*c*) Apply the first law of thermodynamics to express the work done on the gas:

$$W_{on}=\Delta E_{int}-Q_{in}$$

or, because the process is adiabatic,

$$W_{on}=\Delta E_{int}=C_V\Delta T=\tfrac{3}{2}nR\Delta T$$

Substitute numerical values and evaluate W_{on}:

$$W_{on}=\tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(-130\,\text{K})=-1.62\,\text{kJ}$$

Because $W_{\text{by the gas}}=-W_{on}$:

$$W_{\text{by gas}}=\boxed{1.62\,\text{kJ}}$$

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Picture the Problem We can use the temperature-volume equation for a quasi-static adiabatic process to express the final temperature of the gas in terms of its initial temperature and the ratio of its heat capacities γ. Because $C_P=C_V+nR$, we can determine γ for each of the given heat capacities at constant volume.

Express the temperature-volume relationship for a quasi-static adiabatic process:

$$T_iV_i^{\gamma-1}=T_fV_f^{\gamma-1}$$

Solve for the final temperature:

$$T_f=T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}=T_i\left(\frac{V_i}{\frac{1}{2}V_i}\right)^{\gamma-1}=T_i(2)^{\gamma-1}$$

(*a*) Evaluate γ for $C_V = \frac{3}{2}nR$:

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}nR}{\frac{3}{2}nR} = \frac{5}{3}$$

Evaluate T_f:

$$T_f = (293\,\text{K})(2)^{\frac{5}{3}-1} = \boxed{465\,\text{K}}$$

(*b*) Evaluate γ for $C_V = \frac{5}{2}nR$:

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}nR}{\frac{5}{2}nR} = \frac{7}{5}$$

Evaluate T_f:

$$T_f = (293\,\text{K})(2)^{\frac{7}{5}-1} = \boxed{387\,\text{K}}$$

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Picture the Problem We can use the temperature-volume and pressure-volume equations for a quasi-static adiabatic process to express the final temperature and pressure of the gas in terms of its initial temperature and pressure and the ratio of its heat capacities.

Express the temperature-volume relationship for a quasi-static adiabatic process:

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$

Solve for the final temperature:

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_i\left(\frac{V_i}{\frac{1}{4}V_i}\right)^{\gamma-1} = T_i(4)^{\gamma-1}$$

Using γ = 5/3 for neon, evaluate T_f:

$$T_f = (293\,\text{K})(4)^{\frac{5}{3}-1} = \boxed{738\,\text{K}}$$

Express the relationship between the pressures and volumes for a quasi-static adiabatic process:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma}$$

Solve for P_f:

$$P_f = P_i\left(\frac{V_i}{\frac{1}{4}V_i}\right)^{\gamma}$$

Substitute numerical values and evaluate P_f:

$$P_f = (1\,\text{atm})(4)^{5/3} = \boxed{10.1\,\text{atm}}$$

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Picture the Problem We can use the ideal-gas law to find the initial volume of the gas. In part (*a*) we can apply the ideal-gas law for a fixed amount of gas to find the final volume and the expression (Equation 19-16) for the work done in an isothermal process. Application of the first law of thermodynamics will allow us to find the heat absorbed by the gas during this process. In part (*b*) we can use the relationship between the pressures and volumes for a quasi-static adiabatic process to find the final volume of the gas. We can apply the ideal-gas law to find the final temperature and, as in (*a*), apply the first law of thermodynamics, this time to find the work done by the gas.

Use the ideal-gas law to express the initial volume of the gas:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{400\,\text{kPa}} = 3.12\times10^{-3}\,\text{m}^3 = 3.12\,\text{L}$$

(*a*) Because the process is isothermal:

$$T_f = T_i = \boxed{300\,\text{K}}$$

Use the ideal-gas law for a fixed amount of gas to express V_f:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because T = constant,

$$V_f = V_i\frac{P_i}{P_f}$$

Substitute numerical values and evaluate V_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right) = \boxed{7.80\,\text{L}}$$

Express the work done by the gas during the isothermal expansion:

$$W_{\text{by gas}} = nRT\ln\frac{V_f}{V_i}$$

Substitute numerical values and evaluate $W_{\text{by gas}}$:

$$W_{\text{by gas}} = (0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\times(300\,\text{K})\ln\left(\frac{7.80\,\text{L}}{3.12\,\text{L}}\right) = \boxed{1.14\,\text{kJ}}$$

Noting that the work done by the gas during the process equals the negative of the work done on the gas, apply the first law of thermodynamics to find the heat absorbed by the gas:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}} = 0 - (-1.14\,\text{kJ}) = \boxed{1.14\,\text{kJ}}$$

(*b*) Using γ = 5/3 and the relationship between the pressures and volumes for a quasi-static adiabatic process, express V_f:

$$P_iV_i^\gamma = P_fV_f^\gamma \Rightarrow V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma}$$

Substitute numerical values and evaluate V_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right)^{3/5} = \boxed{5.41\,\text{L}}$$

Apply the ideal-gas law to find the final temperature of the gas:

$$T_f = \frac{P_fV_f}{nR}$$

Substitute numerical values and evaluate T_f:

$$T_f = \frac{(160\,\text{kPa})(5.41\times10^{-3}\,\text{m}^3)}{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{208\,\text{K}}$$

For an adiabatic process:

$$Q_{in} = \boxed{0}$$

Apply the first law of thermodynamics to express the work done on the gas during the adiabatic process:

$$W_{on} = \Delta E_{int} - Q_{in} = C_V\Delta T - 0 = \tfrac{3}{2}nR\Delta T$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = \tfrac{3}{2}(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (208\,\text{K} - 300\,\text{K}) = -574\,\text{J}$$

Because the work done by the gas equals the negative of the work done on the gas:

$$W_{by\ gas} = -(-574\,\text{J}) = \boxed{574\,\text{J}}$$

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Picture the Problem We can use the ideal-gas law to find the initial volume of the gas. In part (*a*) we can apply the ideal-gas law for a fixed amount of gas to find the final volume and the expression (Equation 19-16) for the work done in an isothermal process. Application of the first law of thermodynamics will allow us to find the heat absorbed by the gas during this process. In part (*b*) we can use the relationship between the pressures and volumes for a quasi-static adiabatic process to find the final volume of the gas. We can apply the ideal-gas law to find the final temperature and, as in (*a*), apply the first law of thermodynamics, this time to find the work done by the gas.

Use the ideal-gas law to express the initial volume of the gas:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{400\,\text{kPa}} = 3.12\times10^{-3}\,\text{m}^3 = 3.12\,\text{L}$$

(*a*) Because the process is isothermal:

$$T_f = T_i = \boxed{300\,\text{K}}$$

Use the ideal-gas law for a fixed amount of gas to express V_f:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because T = constant,

$$V_f = V_i\frac{P_i}{P_f}$$

Substitute numerical values and evaluate T_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right) = \boxed{7.80\,\text{L}}$$

Express the work done by the gas during the isothermal expansion:

$$W_{\text{by gas}} = nRT\ln\frac{V_f}{V_i}$$

Substitute numerical values and evaluate $W_{\text{by gas}}$:

$$W_{\text{by gas}} = (0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\times(300\,\text{K})\ln\left(\frac{7.80\,\text{L}}{3.12\,\text{L}}\right) = \boxed{1.14\,\text{kJ}}$$

Noting that the work done by the gas during the isothermal expansion equals the negative of the work done on the gas, apply the first law of thermodynamics to find the heat absorbed by the gas:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}} = 0 - (-1.14\,\text{kJ}) = \boxed{1.14\,\text{kJ}}$$

(*b*) Using $\gamma = 1.4$ and the relationship between the pressures and volumes for a quasi-static adiabatic process, express V_f:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma} \Rightarrow V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma}$$

Substitute numerical values and evaluate V_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right)^{1/1.4} = \boxed{6.00\,\text{L}}$$

Apply the ideal-gas law to express the final temperature of the gas:

$$T_f = \frac{P_f V_f}{nR}$$

Substitute numerical values and evaluate T_f:

$$T_f = \frac{(160\,\text{kPa})(6\times10^{-3}\,\text{m}^3)}{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{231\,\text{K}}$$

For an adiabatic process:

$$Q_{in} = \boxed{0}$$

Apply the first law of thermodynamics to express the work done on the gas during the adiabatic expansion:

$$W_{on} = \Delta E_{int} - Q_{in} = C_V\Delta T - 0 = \tfrac{5}{2}nR\Delta T$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = \tfrac{5}{2}(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (231\,\text{K} - 300\,\text{K}) = -717\,\text{J}$$

Noting that the work done by the gas during the adiabatic expansion is the negative of the work done on the gas, we have:

$$W_{by\ gas} = -(-717\,\text{J}) = \boxed{717\,\text{J}}$$

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Picture the Problem We can eliminate the volumes from the equations relating the temperatures and volumes and the pressures and volumes for a quasi-static adiabatic process to obtain a relationship between the temperatures and pressures. We can find the initial volume of the gas using the ideal-gas law and the final volume using the pressure-volume relationship. In parts (*d*) and (*c*) we can find the change in the internal energy of the gas from the change in its temperature and use the first law of thermodynamics to find the work done by the gas during its expansion.

(*a*) Express the relationship between temperatures and volumes for a quasi-static adiabatic process:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

Express the relationship between pressures and volumes for a quasi-static adiabatic process:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma} \qquad (1)$$

Eliminate the volume between these two equations to obtain:

$$T_f = T_i\left(\frac{P_f}{P_i}\right)^{1-\frac{1}{\gamma}}$$

Substitute numerical values and evaluate T_f:

$$T_f = (500\,\text{K})\left(\frac{1\,\text{atm}}{5\,\text{atm}}\right)^{1-\frac{1}{5/3}} = \boxed{263\,\text{K}}$$

(*b*) Solve equation (1) for V_f:

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}}$$

Apply the ideal-gas law to express V_i:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(500\,\text{K})}{5\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}} = 4.10\,\text{L}$$

Substitute for V_i and evaluate V_f:

$$V_f = (4.10\,\text{L})\left(\frac{5\,\text{atm}}{1\text{atm}}\right)^{\frac{3}{5}} = \boxed{10.8\,\text{L}}$$

(*d*) Relate the change in the internal energy of the helium gas to the change in its temperature:

$$\Delta E_{int} = C_V\Delta T = \tfrac{3}{2}nR\Delta T$$

Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{int} = \tfrac{3}{2}(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\times(263\,\text{K}-500\,\text{K}) = \boxed{-1.48\,\text{kJ}}$$

(*c*) Use the first law of thermodynamics to express the work done on the gas:

$$W_{on} = \Delta E_{int} - Q_{in} = \Delta E_{int} - 0 = \Delta E_{int}$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = -1.48\,\text{kJ}$$

Because the work done by the gas equals the negative of W_{on}:

$$W_{\text{by gas}} = -W_{on} = -(-1.48\,\text{kJ}) = \boxed{1.48\,\text{kJ}}$$

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Picture the Problem Consider the process to be accomplished in a single compression. The initial pressure is 1 atm = 101 kPa. The final pressure is (101 + 482) kPa = 583 kPa, and the final volume is 1 L. Because air is a mixture of diatomic gases, γ_{air} = 1.4. We can find the initial volume of the air using $P_iV_i^\gamma = P_fV_f^\gamma$ and use Equation 19-39 to find the work done by the air.

Express the work done in an adiabatic process:

$$W = \frac{P_iV_i - P_fV_f}{\gamma - 1} \qquad (1)$$

Use the relationship between pressure and volume for a quasi-static adiabatic process to express the initial volume of the air:

$$P_iV_i^\gamma = P_fV_f^\gamma \Rightarrow V_i = V_f\left(\frac{P_f}{P_i}\right)^{\frac{1}{\gamma}}$$

Substitute numerical values and evaluate V_i:

$$V_i = (1\,\text{L})\left(\frac{583\,\text{kPa}}{101\,\text{kPa}}\right)^{\frac{1}{1.4}} = 3.50\,\text{L}$$

Substitute numerical values in equation (1) and evaluate W:

$$W = \frac{(101\,\text{kPa})(3.5\times10^{-3}\,\text{m}^3) - (583\,\text{kPa})(10^{-3}\,\text{m}^3)}{1.4-1} = \boxed{-574\,\text{J}}$$

where the minus sign tells us that work is done on the gas.

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Picture the Problem We can integrate PdV using the equation of state for an adiabatic process to obtain Equation 18-39.

Express the work done by the gas during this adiabatic expansion:

$$W_{\text{by gas}} = \int_{V_1}^{V_2} PdV$$

For an adiabatic process:

$$PV^\gamma = \text{constant} = C \qquad (1)$$

and

$$P = CV^{-\gamma}$$

Substitute and evaluate the integral to obtain:

$$W_{\text{by gas}} = C\int_{V_1}^{V_2} V^{-\gamma}dV = \frac{C}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$$

From equation (1) we have: $CV_2^{1-\gamma} = P_2V_2^\gamma$ and $CV_1^{1-\gamma} = P_1V_1^\gamma$

Substitute to obtain:

$$W_{\text{by gas}} = \frac{P_2V_2^\gamma - P_1V_1^\gamma}{1-\gamma} = \boxed{\frac{P_1V_1^\gamma - P_2V_2^\gamma}{\gamma - 1}},$$

which is Equation 18-39.

Cyclic Processes

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Picture the Problem To construct the *PV* diagram we'll need to determine the volume occupied by the gas at the beginning and ending points for each process. Let these points be A, B, C, and D. We can apply the ideal-gas law to the starting point (A) to find V_A. To find the volume at point B, we can use the relationship between pressure and volume for a quasi-static adiabatic process. We can use the ideal-gas law to find the volume at point C and, because they are equal, the volume at point D. We can apply the first law of thermodynamics to find the amount of heat added to or subtracted from the gas during the complete cycle.

(*a*) Using the ideal-gas law, express the volume of the gas at the starting point A of the cycle:

$$V_A = \frac{nRT_A}{P_A}$$

Substitute numerical values and evaluate V_A:

$$V_A = \frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(293\,\text{K})}{5\,\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}} = 4.81\,\text{L}$$

Use the relationship between pressure and volume for a quasi-static adiabatic process to express the volume of the gas at point B; the end point of the adiabatic expansion:

$$V_B = V_A\left(\frac{P_A}{P_B}\right)^{\frac{1}{\gamma}}$$

Substitute numerical values and evaluate V_B:

$$V_B = (4.81\,\text{L})\left(\frac{5\,\text{atm}}{1\,\text{atm}}\right)^{\frac{1}{1.4}} = 15.2\,\text{L}$$

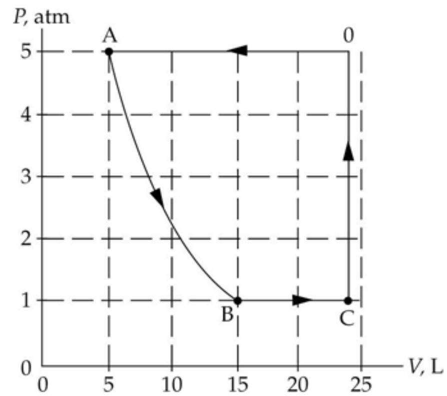
Using the ideal-gas law for a fixed amount of gas, express the volume occupied by the gas at points C and D:

$$V_C = V_D = \frac{nRT_C}{P_C}$$

Substitute numerical values and evaluate V_C:

$$V_C = \frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(293\,\text{K})}{1\,\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}} = 24.0\,\text{L}$$

The complete cycle is shown in the diagram.



(*b*) Note that for the paths A→B and B→C, $W_{\text{by gas}}$, the work done by the gas, is positive. For the path D→A, $W_{\text{by gas}}$ is negative, and greater in magnitude than $W_{A\to C}$. Therefore the total work done by the gas is negative. Find the area enclosed by the cycle by noting that each rectangle of dotted lines equals 5 atm·L and counting the rectangles:

$$W_{\text{by gas}} \approx -(13\,\text{rectangles})(5\,\text{atm}\cdot\text{L/rectangle}) = (-65\,\text{atm}\cdot\text{L})\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right) = \boxed{-6.58\,\text{kJ}}$$

(*c*) The work done on the gas equals the negative of the work done by the gas. Apply the first law of thermodynamics to find the amount of heat added to or subtracted from the gas during the complete cycle:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}} = 0 - (-6.58\,\text{kJ}) = \boxed{6.58\,\text{kJ}}$$

because $\Delta E_{\text{int}} = 0$ for the complete cycle.

(*d*) Express the work done during the complete cycle:

$$W = W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A}$$

Because A→B is an adiabatic process:

$$W_{A\to B} = \frac{P_A V_A^{\gamma} - P_B V_B^{\gamma}}{\gamma - 1}$$

Substitute numerical values and evaluate $W_{A\to B}$:

$$W_{A\to B} = \frac{(5\,\text{atm})(4.82\,\text{L}) - (1\,\text{atm})(15.2\,\text{L})}{1.4 - 1}$$
$$= (22.3\,\text{atm}\cdot\text{L})\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right)$$
$$= 2.25\,\text{kJ}$$

B→C is an isobaric process:

$$W_{B\to C} = P\Delta V$$
$$= (1\,\text{atm})(24.0\,\text{L} - 15.2\,\text{L})$$
$$= (8.80\,\text{atm}\cdot\text{L})\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right)$$
$$= 0.891\,\text{kJ}$$

C→D is a constant-volume process:

$$W_{C\to D} = 0$$

D→A is an isobaric process:

$$W_{D\to A} = P\Delta V = (5\,\text{atm})(5\,\text{L} - 24\,\text{L})$$
$$= (-95.0\,\text{atm}\cdot\text{L})\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right)$$
$$= -9.62\,\text{kJ}$$

Substitute to obtain:

$$W = 2.25\,\text{kJ} + 0.891\,\text{kJ} + 0 - 9.62\,\text{kJ}$$
$$= \boxed{-6.48\,\text{kJ}}$$

Note that our result in part (*b*) agrees with this more accurate value to within 2%.

***79 ••**

Picture the Problem The total work done as the gas is taken through this cycle is the area bounded by the two processes. Because the process from 1→2 is linear, we can use the formula for the area of a trapezoid to find the work done during this expansion. We can use $W_{\text{isothermal process}} = nRT\ln(V_f/V_i)$ to find the work done on the gas during the process 2→1. The total work is then the sum of these two terms.

Express the net work done per cycle:

$$W_{\text{total}} = W_{1\to 2} + W_{2\to 1} \qquad (1)$$

Work is done by the gas during its expansion from 1 to 2 and hence is equal to the negative of the area of the trapezoid defined by this path and the vertical lines at V_1 = 11.5 L and V_2 = 23 L. Use the formula for the area of a trapezoid to express

$$W_{1\to 2} = -A_{\text{trap}}$$
$$= -\tfrac{1}{2}(23\,\text{L} - 11.5\,\text{L})(2\,\text{atm} + 1\,\text{atm})$$
$$= -17.3\,\text{L}\cdot\text{atm}$$

$W_{1\to 2}$:

Work is done on the gas during the isothermal compression from V_2 to V_1 and hence is equal to the area under the curve representing this process. Use the expression for the work done during an isothermal process to express $W_{2\to 1}$:

$$W_{2\to 1} = nRT\ln\left(\frac{V_f}{V_i}\right)$$

Apply the ideal-gas law at point 1 to find the temperature along the isotherm 2→1:

$$T = \frac{PV}{nR} = \frac{(2\,\text{atm})(11.5\,\text{L})}{(1\,\text{mol})(8.206\times 10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 280\,\text{K}$$

Substitute numerical values and evaluate $W_{2\to 1}$:

$$W_{2\to 1} = \left|(1\,\text{mol})(8.206\times 10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(280\,\text{K})\ln\left(\frac{11.5\,\text{L}}{23\,\text{L}}\right)\right| = 15.9\,\text{L}\cdot\text{atm}$$

Substitute in equation (1) and evaluate W_{net}:

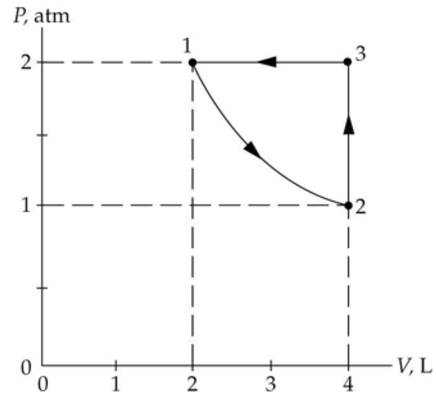
$$\begin{aligned} W_{\text{net}} &= -17.3\,\text{L}\cdot\text{atm} + 15.9\,\text{L}\cdot\text{atm} \\ &= -1.40\,\text{L}\cdot\text{atm}\times\frac{101.325\,\text{J}}{\text{L}\cdot\text{atm}} \\ &= \boxed{-142\,\text{J}} \end{aligned}$$

Remarks: The work done by the gas during each cycle is 142 J.

80 ••

Picture the Problem We can apply the ideal-gas law to find the temperatures T_1, T_2, and T_3. We can use the appropriate work and heat equations to calculate the heat added and the work done by the gas for the isothermal process (1→2), the constant-volume process (2→3), and the isobaric process (3→1).

(*a*) The cycle is shown in the diagram:



(*c*) Use the ideal-gas law to find T_1:

$$T_1 = \frac{P_1V_1}{nR} = \frac{(2\,\text{atm})(2\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{24.4\,\text{K}}$$

Because the process 1→2 is isothermal:

$$T_2 = \boxed{24.4\,\text{K}}$$

Use the ideal-gas law to find T_3:

$$T_3 = \frac{P_3V_3}{nR} = \frac{(2\,\text{atm})(4\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{48.7\,\text{K}}$$

(*b*) Because the process 1→2 is isothermal, $Q_{\text{in},1\to2} = W_{\text{by gas},1\to2}$:

$$Q_{\text{in},1\to2} = W_{\text{by gas},1\to2} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

Substitute numerical values and evaluate $Q_{\text{in},1\to2}$:

$$Q_{\text{in},1\to2} = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(24.4\,\text{K})\ln\left(\frac{4\,\text{L}}{2\,\text{L}}\right) = \boxed{281\,\text{J}}$$

Because process 2→3 takes place at constant volume:

$$W_{2\to3} = \boxed{0}$$

Because process 2→3 takes place at constant volume, $W_{\text{on},2\to3} = 0$, and:

$$Q_{\text{in},2\to3} = \Delta E_{\text{int},2\to3} = C_V\Delta T = \tfrac{3}{2}nR(T_3 - T_2)$$

Substitute numerical values and evaluate $Q_{\text{in},2\to3}$:

$$Q_{\text{in},2\to3} = \tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(48.7\,\text{K} - 24.4\,\text{K}) = \boxed{606\,\text{J}}$$

Because process 3→1 is isobaric:

$$Q_{3\to1} = C_P\Delta T = \tfrac{5}{2}nR(T_1 - T_3)$$

Substitute numerical values and evaluate $Q_{3\to1}$:

$$Q_{3\to1} = \tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(24.4\,\text{K} - 48.7\,\text{K}) = \boxed{-1.01\,\text{kJ}}$$

The work done by the gas from 3 to 1 equals the negative of the work done on the gas:

$$W_{\text{by gas},3\to1} = -P_{1,3}\Delta V = P_{1,3}(V_1 - V_3)$$

Substitute numerical values and evaluate $W_{\text{by gas},3\to2}$:

$$W_{\text{by gas},3\to1} = -(2\,\text{atm})(2\,\text{L} - 4\,\text{L})$$
$$= -(-4\,\text{atm}\cdot\text{L})\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right)$$
$$= \boxed{405\,\text{J}}$$

81 •••

Picture the Problem We can find the temperatures, pressures, and volumes at all points for this ideal monatomic gas (3 degrees of freedom) using the ideal-gas law and the work for each process by finding the areas under each curve. We can find the heat exchanged for each process from the heat capacities and the initial and final temperatures for each process.

Express the total work done by the gas per cycle:

$$W_{\text{by gas,tot}} = W_{D\to A} + W_{A\to B} + W_{B\to C} + W_{C\to D}$$

1. Use the ideal-gas law to find the volume of the gas at point D:

$$V_D = \frac{nRT_D}{P_D}$$
$$= \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K})}{(2\,\text{atm})(101.3\,\text{kPa/atm})}$$
$$= 29.5\,\text{L}$$

2. We're given that the volume of the gas at point B is three times that at point D:

$$V_B = V_C = 3V_D$$
$$= 88.6\,\text{L}$$

Use the ideal-gas law to find the pressure of the gas at point C:

$$P_C = \frac{nRT_C}{V_C} = \frac{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(360\,\text{K})}{88.6\,\text{L}} = 0.667\,\text{atm}$$

We're given that the pressure at point B is twice that at point C:

$$P_B = 2P_C = 2(0.667\,\text{atm}) = 1.33\,\text{atm}$$

3. Because path DC represents an isothermal process:

$$T_D = T_C = 360\,\text{K}$$

Use the ideal-gas law to find the temperatures at points B and A:

$$T_A = T_B = \frac{P_B V_B}{nR} = \frac{(1.333\,\text{atm})(88.6\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 720\,\text{K}$$

Because the temperature at point A is twice that at D and the volumes are the same, we can conclude that:

$$P_A = 2P_D = 4\,\text{atm}$$

The pressure, volume, and temperature at points A, B, C, and D are summarized in the table to the right.

Point	P	V	T
	(atm)	(L)	(K)
A	4	29.5	720
B	1.33	88.6	720
C	0.667	88.6	360
D	2	29.5	360

4. For the path D→A, $W_{D\to A} = 0$ and:

$$Q_{D\to A} = \Delta E_{\text{int, D}\to\text{A}} = \tfrac{3}{2} nR\Delta T_{D\to A} = \tfrac{3}{2} nR(T_A - T_D)$$

Substitute numerical values and evaluate $Q_{D\to A}$:

$$Q_{D\to A} = \tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(720\,\text{K} - 360\,\text{K}) = 8.98\,\text{kJ}$$

For the path A→B:

$$W_{A\to B} = Q_{A\to B} = nRT_{A,B}\ln\left(\frac{V_B}{V_A}\right)$$

Substitute numerical values and evaluate $W_{A\to B}$:

$$W_{A\to B} = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(720\,\text{K})\ln\left(\frac{88.6\,\text{L}}{29.5\,\text{L}}\right) = 13.2\,\text{kJ}$$

and, because process A→B is isothermal, $\Delta E_{\text{int, A}\to\text{B}} = 0$

For the path B→C, $W_{\text{B}\to\text{C}} = 0$, and:

$$Q_{\text{B}\to\text{C}} = \Delta U_{\text{B}\to\text{C}} = C_V \Delta T = \tfrac{3}{2} nR(T_C - T_B)$$

Substitute numerical values and evaluate $Q_{\text{B}\to\text{C}}$:

$$Q_{\text{B}\to\text{C}} = \tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K} - 720\,\text{K}) = -8.98\,\text{kJ}$$

For the path C→D:

$$W_{\text{C}\to\text{D}} = nRT_{\text{C,D}} \ln\left(\frac{V_D}{V_C}\right)$$

Substitute numerical values and evaluate $W_{\text{C}\to\text{D}}$:

$$W_{\text{C}\to\text{D}} = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K})\ln\left(\frac{29.5\,\text{L}}{88.6\,\text{L}}\right) = -6.58\,\text{kJ}$$

Also, because process A→B is isothermal, $\Delta E_{\text{int,A}\to\text{B}} = 0$, and

$$Q_{\text{C}\to\text{D}} = W_{\text{C}\to\text{D}} = -6.58\,\text{kJ}$$

Q_{in}, W_{on}, and ΔE_{int} are summarized for each of the processes in the table to the right.

Process	Q_{in}	W_{on}	ΔE_{int}
	(kJ)	(kJ)	(kJ)
D→A	$\boxed{8.98}$	0	8.98
A→B	$\boxed{13.2}$	–13.2	0
B→C	$\boxed{-8.98}$	0	–8.98
C→D	$\boxed{-6.58}$	6.58	0

Referring to the table, find the total work done by the gas per cycle:

$$\begin{aligned} W_{\text{tot}} &= W_{\text{D}\to\text{A}} + W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} + W_{\text{C}\to\text{D}} \\ &= 0 + 13.2\,\text{kJ} + 0 - 6.58\,\text{kJ} \\ &= \boxed{6.62\,\text{kJ}} \end{aligned}$$

Remarks: Note that, as it should be, ΔE_{int} is zero for the complete cycle.

***82** •••

Picture the Problem We can find the temperatures, pressures, and volumes at all points for this ideal diatomic gas (5 degrees of freedom) using the ideal-gas law and the work for each process by finding the areas under each curve. We can find the heat exchanged for each process from the heat capacities and the initial and final temperatures for each process.

Express the total work done by the gas per cycle:

$$W_{\text{by gas,tot}} = W_{\text{D}\to\text{A}} + W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} + W_{\text{C}\to\text{D}}$$

1. Use the ideal-gas law to find the volume of the gas at point D:

$$V_{\text{D}} = \frac{nRT_{\text{D}}}{P_{\text{D}}} = \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K})}{(2\,\text{atm})(101.3\,\text{kPa/atm})} = 29.5\,\text{L}$$

2. We're given that the volume of the gas at point B is three times that at point D:

$$V_{\text{B}} = V_{\text{C}} = 3V_{\text{D}} = 88.6\,\text{L}$$

Use the ideal-gas law to find the pressure of the gas at point C:

$$P_{\text{C}} = \frac{nRT_{\text{C}}}{V_{\text{C}}} = \frac{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(360\,\text{K})}{88.6\,\text{L}} = 0.667\,\text{atm}$$

We're given that the pressure at point B is twice that at point C:

$$P_{\text{B}} = 2P_{\text{C}} = 2(0.667\,\text{atm}) = 1.33\,\text{atm}$$

3. Because path DC represents an isothermal process:

$$T_{\text{D}} = T_{\text{C}} = 360\,\text{K}$$

Use the ideal-gas law to find the temperatures at points B and A:

$$T_{\text{A}} = T_{\text{B}} = \frac{P_{\text{B}}V_{\text{B}}}{nR} = \frac{(1.333\,\text{atm})(88.6\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 720\,\text{K}$$

Because the temperature at point A is twice that at D and the volumes are the same, we can conclude that:

$$P_{\text{A}} = 2P_{\text{D}} = 4\,\text{atm}$$

The pressure, volume, and temperature at points A, B, C, and D are summarized in the table to the right.

Point	P	V	T
	(atm)	(L)	(K)
A	4	29.5	720

B	1.33	88.6	720
C	0.667	88.6	360
D	2	29.5	360

4. For the path D→A, $W_{D\to A} = 0$ and:

$$\begin{aligned} Q_{D\to A} &= \Delta U_{D\to A} = \tfrac{5}{2} nR\Delta T_{D\to A} = \tfrac{5}{2} nR(T_A - T_D) \\ &= \tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(720\,\text{K} - 360\,\text{K}) \\ &= 15.0\,\text{kJ} \end{aligned}$$

For the path A→B:

$$W_{A\to B} = Q_{A\to B} = nRT_{A,B}\ln\left(\frac{V_B}{V_A}\right) = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(720\,\text{K})\ln\left(\frac{88.6\,\text{L}}{29.5\,\text{L}}\right)$$
$$= 13.2\,\text{kJ}$$

and, because process A→B is isothermal, $\Delta E_{\text{int},A\to B} = 0$

For the path B→C, $W_{B\to C} = 0$ and:

$$\begin{aligned} Q_{B\to C} &= \Delta U_{B\to C} = C_V\Delta T = \tfrac{5}{2} nR(T_C - T_B) = \tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K} - 720\,\text{K}) \\ &= -15.0\,\text{kJ} \end{aligned}$$

For the path C→D:

$$W_{C\to D} = nRT_{C,D}\ln\left(\frac{V_D}{V_C}\right) = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K})\ln\left(\frac{29.5\,\text{L}}{88.6\,\text{L}}\right) = -6.58\,\text{kJ}$$

Also, because process A→B is isothermal, $\Delta E_{\text{int},A\to B} = 0$ and
$Q_{C\to D} = W_{C\to D} = -6.58\,\text{kJ}$

Q_{in}, W_{on}, and ΔE_{int} are summarized for each of the processes in the table to the right.

Process	Q_{in}	W_{on}	ΔE_{int}
	(kJ)	(kJ)	(kJ)
D→A	$\boxed{15.0}$	0	15.0
A→B	$\boxed{13.2}$	–13.2	0
B→C	$\boxed{-15.0}$	0	–15.0
C→D	$\boxed{-6.58}$	6.58	0

Referring to the table and noting that the work done by the gas equals the negative of the work done on the gas, find the total work done by the gas per cycle:

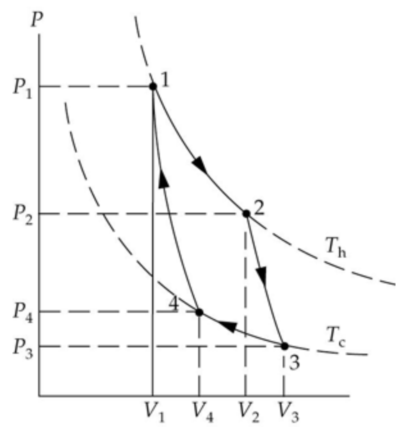
$$W_{\text{by gas,tot}} = W_{\text{D}\to\text{A}} + W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} + W_{\text{C}\to\text{D}} = 0 + 13.2\,\text{kJ} + 0 - 6.58\,\text{kJ} = \boxed{6.62\,\text{kJ}}$$

Remarks: Note that ΔE_{int} for the complete cycle is zero and that the total work done is the same for the diatomic gas of this problem and the monatomic gas of problem 81.

83 •••

Picture the Problem We can use the equations of state for adiabatic and isothermal processes to express the work done on or by the system, the heat entering or leaving the system, and the change in internal energy for each of the four processes making up the Carnot cycle. We can use the first law of thermodynamics and the definition of the efficiency of a Carnot cycle to show that the efficiency is $1 - Q_c / Q_h$.

(*a*) The cycle is shown on the *PV* diagram to the right:



(*b*) Because the process 1→2 is isothermal:

$$\Delta E_{\text{int},1\to2} = 0$$

Apply the first law of thermodynamics to obtain:

$$Q_{\text{h}} = Q_{1\to2} = W_{1\to2} = \boxed{nRT_{\text{h}}\ln\left(\frac{V_2}{V_1}\right)}$$

(*c*) Because the process 3→4 is isothermal:

$$\Delta U_{3\to4} = 0$$

Apply the first law of thermodynamics to obtain:

$$Q_c = Q_{3\to4} = W_{3\to4} = nRT_c \ln\left(\frac{V_4}{V_3}\right) = \boxed{-nRT_c \ln\left(\frac{V_3}{V_4}\right)}$$

where the minus sign tells us that heat is given off by the gas during this process.

(*d*) Apply the equation for a quasi-static adiabatic process at points 4 and 1 to obtain:

$$T_c V_4^{\gamma-1} = T_h V_1^{\gamma-1}$$

Solve for the ratio V_1/V_4:

$$\frac{V_1}{V_4} = \left(\frac{T_c}{T_h}\right)^{\frac{1}{\gamma-1}} \qquad (1)$$

Apply the equation for a quasi-static adiabatic process at points 2 and 3 to obtain:

$$T_h V_2^{\gamma-1} = T_c V_3^{\gamma-1}$$

Solve for the ratio V_2/V_3:

$$\frac{V_2}{V_3} = \left(\frac{T_c}{T_h}\right)^{\frac{1}{\gamma-1}} \qquad (2)$$

Equate equations (1) and (2) and rearrange to obtain:

$$\boxed{\frac{V_3}{V_4} = \frac{V_2}{V_1}}$$

(*e*) Express the efficiency of the Carnot cycle:

$$\varepsilon = \frac{W}{Q_h}$$

Apply the first law of thermodynamics to obtain:

$$W_{on} = \Delta E_{int,\,cycle} - Q_{in} = 0 - (Q_h - Q_c) = -(Q_h - Q_c)$$

because E_{int} is a state function and $\Delta E_{int,\,cycle} = 0$.

Substitute to obtain:

$$\varepsilon = \frac{W_{by\ the\ gas}}{Q_h} = \frac{-W_{on}}{Q_h} = \frac{Q_h - Q_c}{Q_h} = \boxed{1 - \frac{Q_c}{Q_h}}$$

(*f*) In part (*b*) we established that:

$$Q_h = nRT_h \ln\left(\frac{V_2}{V_1}\right)$$

In part (*c*) we established that the heat leaving the system along the path 3→4 is given by:

$$Q_c = nRT_c \ln\left(\frac{V_3}{V_4}\right)$$

Divide the second of these equations by the first to obtain:

$$\frac{Q_c}{Q_h} = \frac{nRT_c \ln\left(\frac{V_3}{V_4}\right)}{nRT_h \ln\left(\frac{V_2}{V_1}\right)} = \boxed{\frac{T_c}{T_h}}$$

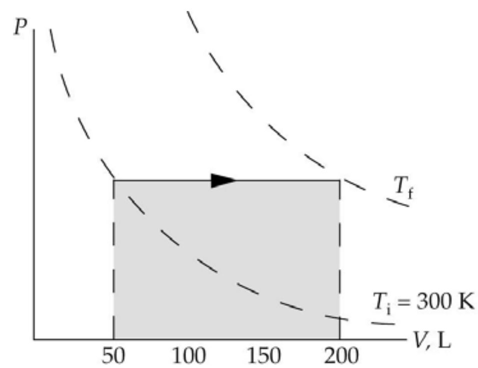
because $\frac{V_3}{V_4} = \frac{V_2}{V_1}$.

Remarks: This last result establishes that the efficiency of a Carnot cycle is also given by $\boldsymbol{\varepsilon_C = 1 - \frac{T_c}{T_h}}$ **.**

General Problems

84 •

Picture the Problem The isobaric process is shown on the *PV* diagram. We can express the heat that must be supplied to gas in terms of its heat capacity at constant pressure and the change in its temperature and then use the ideal-gas law for a fixed amount of gas to relate the final temperature to the initial temperature.



Relate Q_{in} to C_P and ΔT:

$$Q_{in} = C_P\Delta T = C_P(T_f - T_i) = \tfrac{5}{2}nR(T_f - T_i)$$

Use the ideal-gas law for a fixed amount of gas to relate the initial and final volumes, pressures, and temperatures:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because the process is isobaric,

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

Solve for T_f:

$$T_f = \frac{V_f}{V_i}T_i = \frac{200\,\text{L}}{50\,\text{L}}T_i = 4T_i$$

Substitute to obtain:

$$Q_{\text{in}} = \tfrac{5}{2} nR(4T_{\text{i}} - 3T_{\text{i}}) = \tfrac{15}{2} nRT_{\text{i}}$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = \tfrac{15}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{56.1\,\text{kJ}}$$

85 •

Picture the Problem We can use the first law of thermodynamics to relate the heat removed from the gas to the work done on the gas.

Apply the first law of thermodynamics to this process:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}} = -W_{\text{on}}$$

because $\Delta E_{\text{int}} = 0$ for an isothermal process.

Substitute numerical values to obtain:

$$Q_{\text{in}} = -180\,\text{kJ}$$

Because $Q_{\text{removed}} = -Q_{\text{in}}$:

$$Q_{\text{removed}} = \boxed{180\,\text{kJ}}$$

***86** •

Picture the Problem We can find the number of moles of the gas from the expression for the work done on or by a gas during an isothermal process.

Express the work done on the gas during the isothermal process:

$$W = nRT\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$$

Solve for n:

$$n = \frac{W}{RT\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)}$$

Substitute numerical values and evaluate n:

$$n = \frac{-180\,\text{kJ}}{(8.314\,\text{J/mol}\cdot\text{K})(293\,\text{K})\ln\left(\frac{1}{5}\right)} = \boxed{45.9\,\text{mol}}$$

87 •

Picture the Problem We can use the ideal-gas law to find the temperatures T_{A} and T_{C}. Because the process EDC is isobaric, we can find the area under this line geometrically and the first law of thermodynamics to find Q_{AEC}.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$T_A = \frac{P_A V_A}{nR} = \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{65.2\,\text{K}}$$

Using the ideal-gas law, find the temperature at point C:

$$T_C = \frac{P_C V_C}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{81.2\,\text{K}}$$

(*b*) Express the work done by the gas along the path AEC:

$$W_{AEC} = W_{AE} + W_{EC} = 0 + P_{EC}\Delta V_{EC} = (1\,\text{atm})(20\,\text{L} - 4.01\,\text{L}) = 16.0\,\text{L}\cdot\text{atm}\times\frac{101.3\,\text{J}}{\text{L}\cdot\text{atm}} = \boxed{1.62\,\text{kJ}}$$

(*c*) Apply the first law of thermodynamics to express Q_{AEC}:

$$Q_{AEC} = W_{AEC} + \Delta E_{int} = W_{AEC} + C_V\Delta T = W_{AEC} + \tfrac{3}{2}nR\Delta T = W_{AEC} + \tfrac{3}{2}nR(T_C - T_A)$$

Substitute numerical values and evaluate Q_{AEC}:

$$Q_{AEC} = 1.62\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{2.22\,\text{kJ}}$$

Remarks The difference between W_{AEC} and Q_{AEC} is the change in the internal energy $\Delta E_{int,AEC}$ during this process.

88 ••

Picture the Problem We can use the ideal-gas law to find the temperatures T_A and T_C. Because the process AB is isobaric, we can find the area under this line geometrically. We can use the expression for the work done during an isothermal expansion to find the work done between B and C and the first law of thermodynamics to find Q_{ABC}.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$T_A = \frac{P_A V_A}{nR} = \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{65.2\,\text{K}}$$

Use the ideal-gas law to find the temperature at point C:

$$T_C = \frac{P_C V_C}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{81.2\,\text{K}}$$

(*b*) Express the work done by the gas along the path ABC:

$$W_{ABC} = W_{AB} + W_{BC} = P_{AB}\Delta V_{AB} + nRT_B \ln\frac{V_C}{V_B}$$

Use the ideal-gas law to find the volume of the gas at point B:

$$V_B = \frac{nRT_B}{P_B} = \frac{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(81.2\,\text{K})}{4\,\text{atm}} = 5.00\,\text{L}$$

Substitute to obtain:

$$W_{ABC} = (4\,\text{atm})(5\,\text{L} - 4.01\,\text{L}) + (3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(81.2\,\text{K})\ln\left(\frac{20\,\text{L}}{5\,\text{L}}\right)$$

$$= 31.7\,\text{L}\cdot\text{atm}\times\frac{101.3\,\text{J}}{\text{atm}} = \boxed{3.21\,\text{kJ}}$$

(*c*) Apply the first law of thermodynamics to obtain:

$$Q_{ABC} = W_{ABC} + \Delta E_{int} = W_{ABC} + C_V\Delta T = W_{AEC} + \tfrac{3}{2}nR\Delta T = W_{AEC} + \tfrac{3}{2}nR(T_C - T_A)$$

Substitute numerical values and evaluate Q_{ABC}:

$$Q_{ABC} = 3.21\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{3.81\,\text{kJ}}$$

Remarks: The difference between W_{ABC} and Q_{ABC} is the change in the internal energy $\Delta E_{int,ABC}$ during this process.

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Picture the Problem We can use the ideal-gas law to find the temperatures T_A and T_C. Because the process DC is isobaric, we can find the area under this line geometrically. We can use the expression for the work done during an isothermal expansion to find the work done between A and D and the first law of thermodynamics to find Q_{ADC}.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$T_A = \frac{P_A V_A}{nR} = \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{65.2\,\text{K}}$$

Use the ideal-gas law to find the temperature at point C:

$$T_C = \frac{P_C V_C}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{81.2\,\text{K}}$$

(*b*) Express the work done by the gas along the path ADC:

$$W_{ADC} = W_{AD} + W_{DC} = nRT_A \ln\left(\frac{V_D}{V_A}\right) + P_{DC}\Delta V_{DC}$$

Use the ideal-gas law to find the volume of the gas at point D:

$$V_D = \frac{nRT_D}{P_D} = \frac{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(65.2\,\text{K})}{1\,\text{atm}} = 16.1\,\text{L}$$

Substitute numerical values and evaluate W_{ADC}:

$$W_{ADC} = (3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(65.2\,\text{K})\ln\left(\frac{16.1\,\text{L}}{4.01\,\text{L}}\right) + (1\,\text{atm})(20\,\text{L} - 16.1\,\text{L})$$
$$= 26.2\,\text{L}\cdot\text{atm}\times\frac{101.325\,\text{J}}{\text{L}\cdot\text{atm}} = \boxed{2.65\,\text{kJ}}$$

(*c*) Apply the first law of thermodynamics to obtain:

$$Q_{ADC} = W_{ADC} + \Delta E_{int} = W_{ADC} + C_V\Delta T = W_{ADC} + \tfrac{3}{2}nR\Delta T = W_{ADC} + \tfrac{3}{2}nR(T_C - T_A)$$

Substitute numerical values and evaluate Q_{ADC}:

$$Q_{ADC} = 2.65\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{3.25\,\text{kJ}}$$

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Picture the Problem We can use the ideal-gas law to find the temperatures T_A and T_C. Because the process AB is isobaric, we can find the area under this line geometrically. We can find the work done during the adiabatic expansion between B and C using $W_{BC} = -C_V \Delta T_{BC}$ and the first law of thermodynamics to find Q_{ABC}.

The work done by the gas along path ABC is given by:

$$W_{ABC} = W_{AB} + W_{BC} = P_{AB}\Delta V_{AB} - C_V \Delta T_{BC} = P_{AB}\Delta V_{AB} - \tfrac{3}{2} nR\Delta T_{BC}$$

because, with $Q_{in} = 0$, $W_{BC} = -\Delta E_{int,BC}$.

Use the ideal-gas law to find T_A:

$$T_A = \frac{P_A V_A}{nR} = \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 65.2\,\text{K}$$

Use the ideal-gas law to find T_B:

$$T_B = \frac{P_B V_B}{nR} = \frac{(4\,\text{atm})(8.71\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 142\,\text{K}$$

Use the ideal-gas law to find T_C:

$$T_C = \frac{P_C V_C}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 81.2\,\text{K}$$

Apply the pressure-volume relationship for a quasi-static adiabatic process to the gas at points B and C to find the volume of the gas at point B:

$$P_B V_B^{\gamma} = P_C V_C^{\gamma}$$

and

$$V_B = \left(\frac{P_C}{P_B}\right)^{\frac{1}{\gamma}} V_C = \left(\frac{1\,\text{atm}}{4\,\text{atm}}\right)^{\frac{3}{5}} (20\,\text{L})$$
$$= 8.71\,\text{L}$$

Substitute numerical values and evaluate W_{ABC}:

$$W_{ABC} = (4\,\text{atm})(8.71\,\text{L} - 4.01\,\text{L}) - \tfrac{3}{2}(3\,\text{mol})\left(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K}\right) \times (81.2\,\text{K} - 142\,\text{K})$$
$$= 41.3\,\text{L}\cdot\text{atm} \times \frac{101.325\,\text{J}}{\text{atm}} = \boxed{4.18\,\text{kJ}}$$

Apply the 1st law of thermodynamics to obtain:

$$Q_{ABC} = W_{ABC} + \Delta E_{\text{int}} = W_{ABC} + C_V \Delta T$$
$$= W_{ABC} + \tfrac{3}{2} nR\Delta T$$
$$= W_{ABC} + \tfrac{3}{2} nR(T_C - T_A)$$

Substitute numerical values and evaluate Q_{ABC}:

$$Q_{ABC} = 4.18\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{4.78\,\text{kJ}}$$

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Picture the Problem We can find *c* at *T* = 4 K by direct substitution. Because *c* is a function of *T*, we'll integrate *dQ* over the given temperature interval in order to find the heat required to heat copper from 1 to 3 K.

(*a*) Substitute for *a* and *b* to obtain:

$$c = \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)T + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)T^3$$

Evaluate *c* at *T* = 4 K:

$$c(4\,\text{K}) = \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)(4\,\text{K}) + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)(4\,\text{K})^3$$
$$= \boxed{9.20\times10^{-2}\,\text{J/kg}\cdot\text{K}}$$

(*b*) Express and evaluate the integral of *Q*:

$$Q = \int_{T_i}^{T_f} c(T)\,dT = \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)\int_{1\,\text{K}}^{3\,\text{K}} T\,dT + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)\int_{1\,\text{K}}^{3\,\text{K}} T^3\,dT$$
$$= \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)\left[\frac{T^2}{2}\right]_{1\,\text{K}}^{3\,\text{K}} + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)\left[\frac{T^4}{4}\right]_{1\,\text{K}}^{3\,\text{K}} = \boxed{0.0584\,\text{J/kg}}$$

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Picture the Problem We can use the first law of thermodynamics to relate the heat escaping from the system to the amount of work done by the gas and the change in its internal energy. We can use the expression for the work done during an isothermal process to find the temperature along the isotherm.

Apply the first law of thermodynamics to this isothermal process:

$$Q_{in} = \Delta E_{int} - W_{on}$$

For an isothermal process:

$$\Delta E_{int} = \boxed{0}$$

Substitute to obtain:

$$W_{on} = -Q_{in} = -\left(-170\,\text{cal}\times\frac{4.184\,\text{J}}{\text{cal}}\right) = 711\,\text{J}$$

Because $W_{\text{by gas}} = -W_{on}$:

$$W_{\text{by gas}} = \boxed{-711\,\text{J}}$$

Express the work done during an isothermal process:

$$W_{\text{by gas}} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

Solve for $T = T_i = T_f$:

$$T = \frac{W_{\text{by gas}}}{nR\ln\left(\frac{V_2}{V_1}\right)}$$

Substitute numerical values and evaluate T:

$$T = \frac{-711\,\text{J}}{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln\left(\frac{8\,\text{L}}{18\,\text{L}}\right)} = \boxed{52.7\,\text{K}}$$

93 ••

Picture the Problem Let the subscripts 1 and 2 refer to the initial and final values of temperature, pressure, and volume. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^\gamma = P_2V_2^\gamma$, where, for a diatomic gas, $\gamma = 1.4$. Once we've determined P_1 we can use the ideal-gas law to find T_1 and the first law of thermodynamics to find T_2. Finally, we can apply the ideal-gas law a second time to determine P_2.

Relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path:

$$W_{\text{on}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Substitute to obtain:

$$W_{\text{on}} = \frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma}V_2\right)}{\gamma - 1}$$

Solve for P_1:

$$P_1 = \frac{W(\gamma - 1)}{V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma}V_2}$$

Substitute numerical values and evaluate P_1:

$$P_1 = \frac{(-820\,\text{J})(1.4 - 1)}{18\,\text{L} - \left(\frac{18\,\text{L}}{8\,\text{L}}\right)^{1.4}(8\,\text{L})} = \boxed{47.6\,\text{kPa}}$$

Use the ideal-gas law to find T_1:

$$T_1 = \frac{P_1V_1}{nR} = \frac{(47.6\,\text{kPa})(18\,\text{L})}{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{51.5\,\text{K}}$$

Apply the first law of thermodynamics to obtain:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$ for an adiabatic process,

$$\Delta E_{\text{int}} = W_{\text{on}} = C_{\text{V}}\Delta T = \tfrac{5}{2}nR(T_2 - T_1)$$

Solve for and evaluate T_2:

$$T_2 = T_1 - \frac{W_{\text{on}}}{\frac{5}{2}nR} = 51.5\,\text{K} - \frac{-820\,\text{J}}{\frac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{71.2\,\text{K}}$$

Use the ideal-gas law to find P_2:

$$P_2 = \frac{nRT_2}{V_2} = \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(71.2\,\text{K})}{8\,\text{L}} = \boxed{148\,\text{kPa}}$$

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Picture the Problem Let the subscripts 1 and 2 refer to the initial and final state respectively. Because the gas is initially at STP, we know that V_1 = 22.4 L, P_1 = 1 atm, and T_1 = 273 K. We can use $W = -nRT\ln(V_2/V_1)$ to find the work done on the gas during an isothermal compression. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^\gamma = P_2V_2^\gamma$, where, for a diatomic gas, γ = 1.4.

(*a*) Express the work done on the gas in compressing it isothermally:

$$W_{\text{on}} = -nRT\ln\left(\frac{V_2}{V_1}\right)$$

Find the number of moles in 30 g of CO (M = 28 g/mol):

$$n = \frac{30\,\text{g}}{28\,\text{g/mol}} = 1.07\,\text{mol}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -(1.07\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})\ln\left(\frac{1}{5}\right) = \boxed{3.91\,\text{kJ}}$$

(*b*) Express the work done on the gas in compressing it adiabatically:

$$W_{\text{on}} = -\frac{P_1V_1 - P_2V_2}{\gamma - 1} = -\frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^\gamma = P_2V_2^\gamma \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

Substitute for P_2/P_1 and simplify to obtain:

$$W_{on} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right)}{\gamma - 1} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} \frac{V_1}{5}\right)}{\gamma - 1} = -\frac{P_1 V_1\left(1 - 0.2\left(\frac{V_1}{V_2}\right)^{\gamma}\right)}{\gamma - 1}$$

Substitute numerical values and evaluate W_{on}:

$$W = -\frac{(101.3\,\text{kPa})(1.07\,\text{mol})(22.4\,\text{L/mol})\left(1 - 0.2(5)^{1.4}\right)}{1.4 - 1} = \boxed{5.49\,\text{kJ}}$$

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Picture the Problem Let the subscripts 1 and 2 refer to the initial and final state respectively. Because the gas is initially at STP, we know that V_1 = 22.4 L, P_1 = 1 atm, and T_1 = 273 K. We can use $W = -nRT\ln(V_2/V_1)$ to find the work done on the gas during an isothermal compression. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^{\gamma} = P_2V_2^{\gamma}$. We can find γ using the data in Table 19-3.

(*a*) Express the work done on the gas in compressing it isothermally:

$$W_{on} = -nRT\ln\left(\frac{V_2}{V_1}\right)$$

Find the number of moles in 30 g of CO_2 (M = 44 g/mol):

$$n = \frac{30\,\text{g}}{44\,\text{g/mol}} = 0.682\,\text{mol}$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = -(0.682\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})\ln\left(\frac{1}{5}\right) = \boxed{2.49\,\text{kJ}}$$

(*b*) Express the work done on the gas in compressing it adiabatically:

$$W_{on} = -\frac{P_1V_1 - P_2V_2}{\gamma - 1} = -\frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Substitute for P_2/P_1 and simplify to obtain:

$$W_{\text{on}} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right)}{\gamma - 1} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} \frac{V_1}{5}\right)}{\gamma - 1} = -\frac{P_1V_1\left(1 - 0.2\left(\frac{V_1}{V_2}\right)^{\gamma}\right)}{\gamma - 1}$$

From Table 18-3 we have:

$$c_{\text{V}} = 3.39R$$

and

$$c_{\text{P}} = (3.39 + 1.02)R = 4.41R$$

Evaluate γ:

$$\gamma = \frac{c_{\text{P}}}{c_{\text{V}}} = \frac{4.41R}{3.39R} = 1.30$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -\frac{(101.3\,\text{kPa})(0.682\,\text{mol})(22.4\,\text{L/mol})\left(1 - 0.2(5)^{1.3}\right)}{1.3 - 1} = \boxed{3.20\,\text{kJ}}$$

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Picture the Problem Let the subscripts 1 and 2 refer to the initial and final states respectively. Because the gas is initially at STP, we know that V_1 = 22.4 L, P_1 = 1 atm, and T_1 = 273 K. We can use $W = -nRT\ln(V_2/V_1)$ to find the work done on the gas during an isothermal compression. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^{\gamma} = P_2V_2^{\gamma}$, where, for a monatomic gas, γ = 1.67.

(*a*) Express the work done on the gas in compressing it isothermally:

$$W_{\text{on}} = -nRT\ln\left(\frac{V_2}{V_1}\right)$$

Find the number of moles in 30 g of Ar (M = 40 g/mol):

$$n = \frac{30\,\text{g}}{40\,\text{g/mol}} = 0.750\,\text{mol}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -(0.75\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})\ln\left(\frac{1}{5}\right) = \boxed{2.74\,\text{kJ}}$$

(*b*) Express the work done on the gas in compressing it adiabatically:

$$W_{\text{on}} = -\frac{P_1V_1 - P_2V_2}{\gamma - 1} = -\frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Substitute for P_2/P_1 and simplify to obtain:

$$W_{\text{on}} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right)}{\gamma - 1} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} \frac{V_1}{5}\right)}{\gamma - 1} = -\frac{P_1V_1\left(1 - 0.2\left(\frac{V_1}{V_2}\right)^{\gamma}\right)}{\gamma - 1}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -\frac{(101.3\,\text{kPa})(0.75\,\text{mol})(22.4\,\text{L/mol})\left(1 - 0.2(5)^{1.67}\right)}{1.67 - 1} = \boxed{4.93\,\text{kJ}}$$

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Picture the Problem We can use conservation of energy to relate the final temperature to the heat capacities of the gas and the solid. We can apply the Dulong-Petit law to find the heat capacity of the solid at constant volume and use the fact that the gas is diatomic to find its heat capacity at constant volume.

Apply conservation of energy to this process:

$$\Delta Q = 0$$

or

$$C_{\text{V,gas}}(T_{\text{f}} - 100\,\text{K}) - C_{\text{V,solid}}(200\,\text{K} - T_{\text{f}}) = 0$$

Solve for T_{f}:

$$T_{\text{f}} = \frac{(100\,\text{K})(C_{\text{V,gas}}) + (200\,\text{K})(C_{\text{V,solid}})}{C_{\text{V,gas}} + C_{\text{V,solid}}}$$

Using the Dulong-Petit law, determine the heat capacity of the solid at constant volume:

$$C_{V,solid} = 3nR = 3(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = 49.9\,\text{J/K}$$

Determine the heat capacity of the gas at constant volume:

$$C_{V,gas} = \tfrac{5}{2}nR = \tfrac{5}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = 20.8\,\text{J/K}$$

Substitute numerical values and evaluate T_f:

$$T_f = \frac{(100\,\text{K})(20.8\,\text{J/K}) + (200\,\text{K})(49.9\,\text{J/K})}{20.8\,\text{J/K} + 49.9\,\text{J/K}} = \boxed{171\,\text{K}}$$

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Picture the Problem We can express the work done during an isobaric process as the product of the temperature and the change in volume and relate Q to ΔT through the definition of C_P. Finally, we can use the first law of thermodynamics to show that $\Delta E_{int} = C_v\Delta T$.

(*a*) For an ideal gas, the internal energy is the sum of the kinetic energies of the gas molecules, which is proportional to kT. Consequently, U is a function of T only and $\Delta E_{int} = C_V\Delta T$.

(*b*) Use the first law of thermodynamics to relate the work done on the gas, the heat entering the gas, and the change in the internal energy of the gas:

$$\Delta E_{int} = Q_{in} + W_{on}$$

At constant pressure:

$$W_{by\,gas} = P(V_f - V_i) = nR(T_f - T_i) = nR\Delta T$$

and

$$W_{on} = -W_{by\,gas} = -nR\Delta T$$

Relate Q_{in} to C_P and ΔT:

$$Q_{in} = C_P\Delta T$$

Substitute to obtain:

$$\Delta E_{int} = C_P\Delta T - nR\Delta T = (C_P - nR)\Delta T = \boxed{C_V\Delta T}$$

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Picture the Problem We can use $Q_{in} = C_V\Delta T = \frac{3}{2}nR\Delta T$ to find Q_{in} for the constant-volume process and $Q_{in} = C_P\Delta T = \frac{5}{2}nR\Delta T$ to find Q_{in} for the isobaric process. The work done by the gas is given by $W = \int_{V_i}^{V_f} PdV$. Finally, we can apply the first law of thermodynamics to find the change in the internal energy of the gas from the work done on the gas and the heat that enters the gas.

(*a*) The heat added to the gas during this process is given by:

$$Q_{in} = \tfrac{3}{2}nR\Delta T$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{in} = \tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{3.74\,\text{kJ}}$$

For a constant-volume process:

$$W_{\text{by the gas}} = \boxed{0}$$

Apply the 1st law of thermodynamics to obtain:

$$\Delta E_{int} = Q_{in} + W_{on} \qquad (1)$$

Substitute for Q_{in} and W_{on} in equation (1) and evaluate ΔE_{int}:

$$\Delta E_{int} = 3.74\,\text{kJ} + 0 = \boxed{3.74\,\text{kJ}}$$

(*b*) Relate the heat absorbed by the gas to the change in its temperature:

$$Q_{in} = C_P\Delta T = \tfrac{5}{2}nR\Delta T = \tfrac{5}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{6.24\,\text{kJ}}$$

For a constant-pressure process that begins at temperature T_i and ends at temperature T_f, the work done by the gas is given by:

$$W_{\text{by the gas}} = \int_{V_i}^{V_f} PdV = P(V_f - V_i) = nR(T_f - T_i)$$

Substitute numerical values and evaluate $W_{\text{by the gas}}$:

$$W_{\text{by the gas}} = (1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{2.49\,\text{kJ}}$$

Apply the 1st law of thermodynamics to express the change in the internal energy of the gas during this isobaric expansion:

$$\Delta E_{int} = W_{on} + Q_{in}$$

Because the $W_{\text{by gas}} = -W_{\text{on}}$:

$$\Delta E_{\text{int}} = -W_{\text{by the gas}} + Q_{\text{in}}$$

Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = -2.49\,\text{kJ} + 6.24\,\text{kJ} = \boxed{3.75\,\text{kJ}}$$

Remarks: Because ΔE_{int} depends only on the initial and final temperatures of the gas, the values for ΔE_{int} for Part (*a*) and Part (*b*) should be they same. They differ slightly due to rounding.

***100** ••

Picture the Problem We can use $Q_{\text{in}} = C_P\Delta T$ to find the change in temperature during this isobaric process and the first law of thermodynamics to relate W, Q, and ΔE_{int}. We can use $\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T$ to find the change in the internal energy of the gas during the isobaric process and the ideal-gas law for a fixed amount of gas to express the ratio of the final and initial volumes.

(*a*) Relate the change in temperature to Q_{in} and C_P and evaluate ΔT:

$$\Delta T = \frac{Q_{\text{in}}}{C_P} = \frac{Q_{\text{in}}}{\frac{7}{2}nR} = \frac{500\,\text{J}}{\frac{7}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{8.59\,\text{K}}$$

(*b*) Apply the first law of thermodynamics to relate the work done on the gas to the heat supplied and the change in its internal energy:

$$W_{\text{on}} = \Delta E_{\text{int}} - Q_{\text{in}} = C_V\Delta T - Q_{\text{in}} = \frac{5}{2}nR\Delta T - Q_{\text{in}}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = \frac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(8.59\,\text{K}) - 500\,\text{J} = -143\,\text{J}$$

Because $W_{\text{by gas}} = -W_{\text{on}}$:

$$W_{\text{by gas}} = \boxed{143\,\text{J}}$$

(*c*) Using the ideal-gas law for a fixed amount of gas, relate the initial and final pressures, volumes and temperatures:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because the process is isobaric,

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

Solve for and evaluate V_f/V_i:

$$\frac{V_f}{V_i} = \frac{T_f}{T_i} = \frac{T_i + \Delta T}{T_i} = \frac{293.15\,\text{K} + 8.59\,\text{K}}{293.15\,\text{K}} = \boxed{1.03}$$

101 ••

Picture the Problem Knowing the rate at which energy is supplied, we can obtain the data we need to plot this graph by finding the time required to warm the ice to 0°C, melt the ice, warm the water formed from the ice to 100°C, vaporize the water, and warm the water to 110°C.

Find the time required to warm the ice to 0°C:

$$\Delta t_1 = \frac{mc_{\text{ice}}\Delta T}{P} = \frac{(0.1\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(10\,\text{K})}{100\,\text{J/s}} = 20.0\,\text{s}$$

Find the time required to melt the ice:

$$\Delta t_2 = \frac{mL_f}{P} = \frac{(0.1\,\text{kg})(333.5\,\text{kJ/kg})}{100\,\text{J/s}} = 333.5\,\text{s}$$

Find the time required to heat the water to 100°C:

$$\Delta t_3 = \frac{mc_w\Delta T}{P} = \frac{(0.1\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(100\,\text{K})}{100\,\text{J/s}} = 418\,\text{s}$$

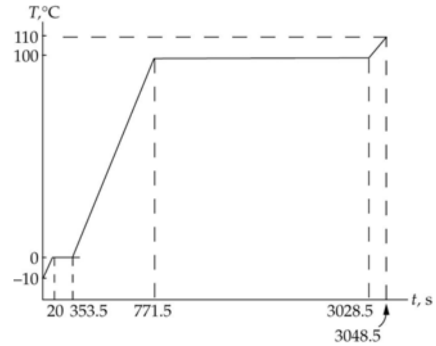
Find the time required to vaporize the water:

$$\Delta t_4 = \frac{mL_V}{P} = \frac{(0.1\,\text{kg})(2257\,\text{kJ/kg})}{100\,\text{J/s}} = 2257\,\text{s}$$

Find the time required to heat the vapor to 110°C:

$$\Delta t_5 = \frac{mc_{\text{steam}}\Delta T}{P} = \frac{(0.1\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(10\,\text{K})}{100\,\text{J/s}} = 20\,\text{s}$$

The temperature T as a function of time t is shown to the right:



***102 ••**

Picture the Problem We know that, for an adiabatic process, $Q_{in} = 0$. Hence the work done by the expanding gas equals the change in its internal energy. Because we're given the work done by the gas during the expansion, we can express the change in the temperature of the gas in terms of this work and C_V.

Express the final temperature of the gas as a result of its expansion:

$$T_f = T_i + \Delta T$$

Apply the equation for adiabatic work and solve for ΔT:

$$W_{adiabatic} = -C_V \Delta T$$

and

$$\Delta T = -\frac{W_{adiabatic}}{C_V} = -\frac{W_{adiabatic}}{\frac{5}{2}nR}$$

Substitute and evaluate T_f:

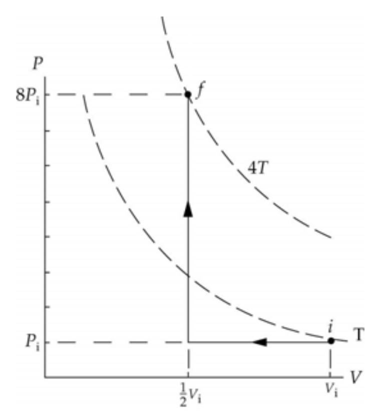
$$T_f = T_i - \frac{W_{adiabatic}}{\frac{5}{2}nR}$$

$$= 300\,K - \frac{3.5\,kJ}{\frac{5}{2}(2\,mol)(8.314\,J/mol\cdot K)}$$

$$= \boxed{216\,K}$$

103 ••

Picture the Problem Because $P_fV_f = 4P_iV_i$ and $V_f = V_i/2$, the path for which the work done by the gas is a minimum while the pressure never falls below P_i is shown on the adjacent PV diagram. We can apply the first law of thermodynamics to relate the heat transferred to the gas to its change in internal energy and the work done on the gas.



Using the first law of thermodynamics, relate the heat transferred to the gas to its change in internal energy and the work done on the gas:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

Solve for Q_{in}:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}}$$

Express the work done during this process:

$$\begin{aligned} W_{\text{on}} &= W_{\text{isobaric process}} + W_{\text{constant volume}} \\ &= P_{\text{i}}\Delta V + 0 = P_{\text{i}}\left(\tfrac{1}{2}V_{\text{i}} - V_{\text{i}}\right) \\ &= \tfrac{1}{2}P_{\text{i}}V_{\text{i}} = \tfrac{1}{2}nRT = \tfrac{1}{2}RT \end{aligned}$$

because n = 1 mol.

Express ΔE_{int} for the process:

$$\begin{aligned} \Delta E_{\text{int}} &= C_{\text{V}}\Delta T = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}nR(3T) \\ &= \tfrac{9}{2}RT \end{aligned}$$

because n = 1 mol.

Substitute to obtain:

$$Q_{\text{in}} = \tfrac{9}{2}RT - \tfrac{1}{2}RT = \boxed{4RT}$$

104 ••

Picture the Problem We can solve the ideal-gas law for the dilute solution for the increase in pressure and find the number of solute molecules dissolved in the water from their mass and molecular weight.

Solve the ideal gas law for P to obtain:

$$P = \frac{NkT}{V}$$

Express the number of solute molecules N in terms of the number of moles n and Avogadro's number and then express the number of moles in terms of the mass of the salt and its molecular mass:

$$N = nN_{\text{A}} = \frac{mN_{\text{A}}}{M_{\text{NaCl}}}$$

Substitute to obtain:

$$P = \frac{mN_{\text{A}}kT}{M_{\text{NaCl}}V}$$

Substitute numerical values and evaluate P:

$$P = \frac{(30\,\text{g})\left(6.022\times10^{23}\ \text{particles/mol}\right)\left(1.381\times10^{-23}\ \text{J/K}\right)(297\,\text{K})}{(58.4\,\text{g/mol})\left(10^{-3}\ \text{m}^3\right)} = \boxed{1.27\times10^{6}\ \text{N/m}^2}$$

105 ••

Picture the Problem Let the subscripts 1 and 2 refer to the initial and final states in this adiabatic expansion. We can use an equation describing a quasi-static adiabatic process to express the final temperature as a function of the initial temperature and the initial and final volumes.

Using the equation for a quasi-static adiabatic process, relate the initial and final volumes and temperatures:

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

Solve for and evaluate T_2:

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (300\,\text{K})(2)^{1.4-1} = \boxed{396\,\text{K}}$$

106 ••

Picture the Problem We can simplify our calculations by relating Avogadro's number N_A, Boltzmann's constant k, the number of moles n, and the number of molecules N in the gas and solving for N_Ak. We can then calculate $U_{300\,\text{K}}$ and $U_{600\,\text{K}}$ and their difference.

Express the increase in internal energy per mole resulting from the heating of diamond:

$$\Delta U = U_{600\,\text{K}} - U_{300\,\text{K}}$$

Express the relationship between Avogadro's number N_A, Boltzmann's constant k, the number of moles n, and the number of molecules N in the gas:

$$nR = Nk \Rightarrow R = \frac{N}{n}k = N_Ak$$

Substitute in the given equation to obtain:

$$U = \frac{3RT_E}{e^{T_E/T} - 1}$$

Determine $U_{300\,\text{K}}$:

$$U_{300\,\text{K}} = \frac{3(8.314\,\text{J/mol}\cdot\text{K})(1060\,\text{K})}{e^{1060\,\text{K}/300\,\text{K}} - 1} = \boxed{795\,\text{J}}$$

Determine $U_{600\,\text{K}}$:

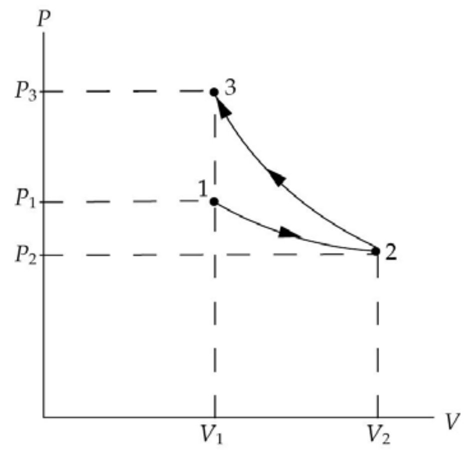
$$U_{600\,\text{K}} = \frac{3(8.314\,\text{J/mol}\cdot\text{K})(1060\,\text{K})}{e^{1060\,\text{K}/600\,\text{K}} - 1} = \boxed{5.45\,\text{kJ}}$$

Substitute to obtain:

$$\Delta U = U_{600\,\text{K}} - U_{300\,\text{K}} = 5.45\,\text{kJ} - 795\,\text{J} = \boxed{4.66\,\text{kJ}}$$

***107 •••**

Picture the Problem The isothermal expansion followed by an adiabatic compression is shown on the *PV* diagram. The path 1→2 is isothermal and the path 2→3 is adiabatic. We can apply the ideal-gas law for a fixed amount of gas and an isothermal process to find the pressure at point 2 and the pressure-volume relationship for a quasi-static adiabatic process to determine γ.



(*a*) Relate the initial and final pressures and volumes for the isothermal expansion and solve for and evaluate the final pressure:

$$P_1V_1 = P_2V_2$$

and

$$P_2 = P_1\frac{V_1}{V_2} = P_0\frac{V_1}{2V_1} = \boxed{\tfrac{1}{2}P_0}$$

(*b*) Relate the initial and final pressures and volumes for the adiabatic compression:

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

or

$$\tfrac{1}{2}P_0(2V_0)^{\gamma} = 1.32P_0V_0^{\gamma}$$

which simplifies to

$$2^{\gamma} = 2.64$$

Take the natural logarithm of both sides of this equation and solve for and evaluate γ:

$$\gamma \ln 2 = \ln 2.64$$

and

$$\gamma = \frac{\ln 2.64}{\ln 2} = 1.40$$

$$\boxed{\therefore \text{the gas is diatomic.}}$$

(*c*) In the isothermal process, T is constant, and the translational kinetic energy is unchanged.

In the adiabatic process, $T_3 = 1.32T_0$, and the translational kinetic energy increases by a factor of 1.32.

108 •••

Picture the Problem In this problem the specific heat of the combustion products depends on the temperature. Although C_P increases gradually from (9/2)R per mol to (15/2)R per mol at high temperatures, we'll assume that $C_P = 4.5R$ below $T = 2000$ K and $C_P = 7.5R$ above $T = 2000$ K. We'll also use $R = 2.0$ cal/mol·K. We can find the final temperature following combustion from the heat made available during the combustion and the final pressure by applying the ideal-gas law to the initial and final states of the gases.

(*a*) Relate the heat available in this combustion process to the change in temperature of the triatomic gases:

$$Q_{\text{available}} = nC_P\Delta T = n(7.5R)(T_f - T_i)$$

Solve for T_f to obtain:

$$T_f = \frac{Q_{\text{available}}}{7.5nR} + T_i \qquad (1)$$

Express Q available to heat the gases above 2000 K:

$$Q_{\text{available}} = Q_{\text{released}} - Q_{9\,\text{mol to }2000\,\text{K}} - Q_{\text{heat}\,CO_2} - Q_{\text{steam}} \qquad (2)$$

Express the energy released in the combustion of 1 mol of benzene:

$$Q_{\text{released}} = \tfrac{1}{2}(1516\,\text{kcal}) = 758\,\text{kcal}$$

Noting that there are 3 mol of H_2O and 6 mol of CO_2, find the heat required to form the products at 100°C:

$$\begin{aligned}Q_{\text{steam}} &= nM_w c_w\Delta T + nM_w L_V \\ &= (3\,\text{mol})(18\,\text{g/mol})\times(1\,\text{cal/g}\cdot\text{K})(373\,\text{K} - 300\,\text{K}) + (3\,\text{mol})(18\,\text{g/mol})(540\,\text{cal/g}) \\ &= 33.10\,\text{kcal}\end{aligned}$$

and

$$\begin{aligned}Q_{\text{heat}\,CO_2} &= nC_P\Delta T = 4.5nR\Delta T \\ &= 4.5(6\,\text{mol})(2\,\text{cal/mol}\cdot\text{K})\times(373\,\text{K} - 300\,\text{K}) \\ &= 3.942\,\text{kcal}\end{aligned}$$

Find Q required to heat 9 mol of gas to 2000 K:

$$\begin{aligned}Q_{9\,\text{mol to }2000\,\text{K}} &= nC_P\Delta T = 4.5nR\Delta T \\ &= 4.5(9\,\text{mol})(2\,\text{cal/mol}\cdot\text{K})\times(2000\,\text{K} - 373\,\text{K}) \\ &= 43.93\,\text{kcal}\end{aligned}$$

Substitute in equation (2) to obtain:

$$Q_{\text{available}} = 758\,\text{kcal} - 131.79\,\text{kcal} - 3.94\,\text{kcal} - 33.10\,\text{kcal} = 589.2\,\text{kcal}$$

Substitute in equation (1) and evaluate T_f:

$$T_f = \frac{589.2\,\text{kcal}}{7.5(9\,\text{mol})(2\,\text{cal/mol}\cdot\text{K})} + 2000\,\text{K} = \boxed{6364\,\text{K}}$$

Apply the ideal-gas law to express the final volume in terms of the final temperature and pressure:

$$V_f = \frac{nRT_f}{P_f} = \frac{(9\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(6364\,\text{K})}{101.3\,\text{kPa}} = \boxed{4.70\,\text{m}^3}$$

(*b*) Apply the ideal-gas law to relate the final temperature, pressure, and volume to the number of moles in the final state:

$$P_f V_f = n_f R T_f$$

Apply the ideal-gas law to relate the initial temperature, pressure, and volume to the number of moles in the initial state:

$$P_i V_i = n_i R T_i$$

Divide the first of these equations by the second and solve for P_f:

$$\frac{P_f V_f}{P_i V_i} = \frac{n_f R T_f}{n_i R T_i}$$

or, because $T_f = T_i$,

$$P_f = P_i \left(\frac{n_f}{n_i}\right)\left(\frac{V_i}{V_f}\right) \qquad (3)$$

Find the initial volume V_i occupied by 8.5 mol of gas at 300 K and 1 atm:

$$V_i = (22.4\,\text{L/mol})(8.5\,\text{mol})\left(\frac{300\,\text{K}}{273\,\text{K}}\right) = 209.2\,\text{L}$$

Substitute numerical values in equation (3) and evaluate V_f:

$$P_f = (101.3\,\text{kPa})\left(\frac{9\,\text{mol}}{8.5\,\text{mol}}\right)\left(\frac{209.2\,\text{L}}{4700\,\text{L}}\right) = 4.774\,\text{kPa}\times\frac{1\,\text{atm}}{101.325\,\text{kPa}} = \boxed{0.0471\,\text{atm}}$$

***109** •••

Picture the Problem In this problem the specific heat of the combustion products depends on the temperature. Although C_P increases gradually from (9/2)R per mol to (15/2)R per mol at high temperatures, we'll assume that $C_P = 4.5R$ below $T = 2000$ K and $C_P = 7.5R$ above $T = 2000$ K. We can find the final temperature following combustion from the heat made available during the combustion and the final pressure by applying the ideal-gas law to the initial and final states of the gases.

(*a*) Apply the ideal-gas law to find the pressure due to 3 mol at 300 K in the container prior to the reaction:

$$P_i = \frac{nRT_i}{V_i} = \frac{(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{80\,\text{L}} = \boxed{93.5\,\text{kPa}}$$

(*b*) Relate the heat available in this adiabatic process to C_V and the change in temperature of the gases:

$$\Delta E_{int} = Q_{available} = C_V(T_f - T_i)$$

Because $T > 2000$ K:

$$C_V = C_P - nR = n(7.5R) - nR = 6.5nR$$

Substitute to obtain:

$$Q_{available} = 6.5nR(T_f - T_i)$$

Solve for T_f to obtain:

$$T_f = \frac{Q_{available}}{6.5nR} + T_i \qquad (1)$$

Find Q required to raise 2 mol of CO_2 to 2000 K:

$$Q_{heat\,CO_2} = C_V\Delta T$$

For $T < 2000$ K:

$$C_V = C_P - nR = n(4.5R) - nR = 3.5nR$$

Substitute for C_V and find the heat required to warm to CO_2 to 2000 K:

$$Q_{\text{heat}\,CO_2} = 3.5nR\Delta T = 3.5(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\times(2000\,\text{K} - 300\text{K}) = 98.94\,\text{kJ}$$

Find Q available to heat 2 mol of CO_2 above 2000 K:

$$Q_{\text{available}} = 560\,\text{kJ} - 98.94\,\text{kJ} = 461.1\,\text{kJ}$$

Substitute in equation (1) and evaluate T_f:

$$T_f = \frac{461.1\,\text{kJ}}{6.5(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} + 2000\,\text{K} = \boxed{6266\,\text{K}}$$

Apply the ideal-gas law to relate the final temperature, pressure, and volume to the number of moles in the final state:

$$P_f V_f = n_f R T_f$$

Apply the ideal-gas law to relate the initial temperature, pressure, and volume to the number of moles in the initial state:

$$P_i V_i = n_i R T_i$$

Divide the first of these equations by the second and solve for P_f:

$$\frac{P_f V_f}{P_i V_i} = \frac{n_f R T_f}{n_i R T_i}$$

or, because $V_f = V_i$,

$$P_f = P_i\left(\frac{n_f}{n_i}\right)\left(\frac{T_f}{T_i}\right) \qquad (2)$$

Substitute numerical values in equation (2) and evaluate P_f:

$$P_f = (93.53\,\text{kPa})\left(\frac{2\,\text{mol}}{3\,\text{mol}}\right)\left(\frac{6266\,\text{K}}{300\,\text{K}}\right) = \boxed{1.30\,\text{MPa}}$$

(*c*) Substitute numerical values in equation (2) and evaluate P_f for $T_f = 273$ K:

$$P_f = (93.53\,\text{kPa})\left(\frac{2\,\text{mol}}{3\,\text{mol}}\right)\left(\frac{273\,\text{K}}{300\,\text{K}}\right) = \boxed{56.7\,\text{kPa}}$$

110 •••

Picture the Problem The molar heat capacity at constant volume is related to the internal energy per mole according to $c'_V = \frac{1}{n}\frac{dU}{dT}$. We can differentiate U with respect to temperature and use $nR = Nk$ or $R = N_A k$ to establish the result given in the problem statement.

From Problem 106 we have, for the internal energy per mol:

$$U = \frac{3N_A kT_E}{e^{T_E/T} - 1}$$

Relate the molar heat capacity at constant volume to the internal energy per mol:

$$c'_V = \frac{1}{n}\frac{dU}{dT}$$

Use $c'_V = \frac{1}{n}\frac{dU}{dT}$ to express c'_V:

$$c'_V = \frac{1}{n}\frac{d}{dT}\left[\frac{3N_A kT_E}{e^{T_E/T} - 1}\right] = 3RT_E\frac{d}{dT}\left[\frac{1}{e^{T_E/T} - 1}\right] = 3RT_E\left[\frac{-1}{\left(e^{T_E/T} - 1\right)^2}\right]\frac{d}{dT}\left(e^{T_E/T} - 1\right)$$

$$= 3RT_E\left[\frac{-1}{\left(e^{T_E/T} - 1\right)^2}\right]\left[e^{T_E/T}\left(-\frac{T_E}{T^2}\right)\right] = \boxed{3R\left(\frac{T_E}{T}\right)^2\frac{e^{T_E/T}}{\left(e^{T_E/T} - 1\right)^2}}$$

111 •••

Picture the Problem We can rewrite our expression for c'_V by dividing its numerator and denominator by $e^{T_E/T}$ and then using the power series for e^x to show that, for $T > T_E$, $c'_V \approx 3R$. In part (*b*), we can use the result of Problem 103 to obtain values for c'_V every 100 K between 300 K and 600 K and use this data to find ΔU numerically.

(*a*) From Problem 110 we have:

$$c'_V = 3R\left(\frac{T_E}{T}\right)^2\frac{e^{T_E/T}}{\left(e^{T_E/T} - 1\right)^2}$$

Divide the numerator and denominator by $e^{T_E/T}$ to obtain:

$$c'_V = 3R\left(\frac{T_E}{T}\right)^2\frac{1}{\frac{e^{2T_E/T} - 2e^{T_E/T} + 1}{e^{T_E/T}}}$$

$$= 3R\left(\frac{T_E}{T}\right)^2\frac{1}{e^{T_E/T} - 2 + e^{-T_E/T}}$$

Apply the power series expansion to obtain:

$$e^{T_E/T} - 2 + e^{-T_E/T} = 1 + \frac{T_E}{T} + \frac{1}{2}\left(\frac{T_E}{T}\right)^2 + \ldots - 2 + 1 - \frac{T_E}{T} + \frac{1}{2}\left(\frac{T_E}{T}\right)^2 + \ldots \approx \left(\frac{T_E}{T}\right)^2 \text{ for } T > T_E$$

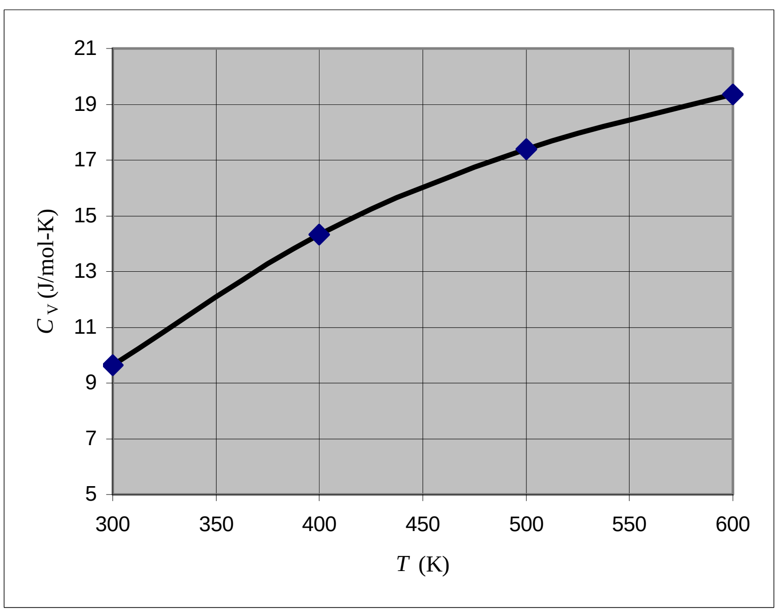
Substitute to obtain:

$$c_V' \approx 3R\left(\frac{T_E}{T}\right)^2 \frac{1}{\left(\frac{T_E}{T}\right)^2} = \boxed{3R}$$

(*b*) Use the result of Problem 110 to verify the table to the right:

T	c_V
(K)	(J/mol·K)
300	9.65
400	14.33
500	17.38
600	19.35

The following graph of specific heat as a function of temperature shown to the right was plotted using a spreadsheet program:



Integrate numerically, using the formula for the area of a trapezoid, to obtain:

$$\Delta U = \tfrac{1}{2}(100\,\text{K})(9.65+14.33)\,\text{J/mol}\cdot\text{K} + \tfrac{1}{2}(100\,\text{K})(14.33+17.38)\,\text{J/mol}\cdot\text{K}$$
$$+\tfrac{1}{2}(100\,\text{K})(17.38+19.35)\,\text{J/mol}\cdot\text{K}$$
$$= \boxed{4.62\,\text{kJ}},$$

a result in good agreement (< 1% difference) with the result of Problem 106.

112 •••

Picture the Problem In (*a*) we'll assume that $\tau = f\,(A/V, T, k, m)$ with the factors dependent on constants *a*, *b*, *c*, and *d* that we'll find using dimensional analysis. In (*b*) we'll use our result from (*a*) and assume that the diameter of the puncture is about 2 mm, that the tire volume is 0.1 m^3, and that the air temperature is 20°C.

(*a*) Express $\tau = f\,(A/V, T, k, m)$:

$$\tau = \left(\frac{A}{V}\right)^a (T)^b (k)^c (m)^d \qquad (1)$$

Rewrite this equation in terms of the dimensions of the physical quantities to obtain:

$$\text{T}^1 = (\text{L})^{-a}(\text{K})^b\left(\frac{\text{ML}^2}{\text{T}^2\text{K}}\right)^c(\text{M})^d$$

where K represents the dimension of temperature.

Simplify this dimensional equation to obtain:

$$\text{T}^1 = \text{L}^{-a}\text{K}^b\text{M}^c\text{L}^{2c}\text{K}^{-c}\text{T}^{-2c}\text{M}^d$$

or

$$\text{T}^1 = \text{L}^{2c-a}\text{K}^{b-c}\text{M}^{c+d}\text{T}^{-2c}$$

Equate exponents to obtain:

$\text{T}:\ -2c = 1$,
$\text{L}:\ 2c - a = 0$,
$\text{K}:\ b - c = 0$,
and
$\text{M}:\ c + d = 0$

Solve these equations simultaneously to obtain:

$c = -\tfrac{1}{2}$,
$a = -1$,
$b = -\tfrac{1}{2}$,
and
$d = \tfrac{1}{2}$

Substitute in equation (1):

$$\tau = \left(\frac{A}{V}\right)^{-1}(T)^{-\frac{1}{2}}(k)^{-\frac{1}{2}}(m)^{\frac{1}{2}}$$

$$= \boxed{\frac{V}{A}\sqrt{\frac{m}{kT}}}$$

(*b*) Substitute numerical values and evaluate τ:

$$\tau = \frac{0.1\,\text{m}^3}{\frac{\pi}{4}\left(2\times10^{-3}\,\text{m}\right)^2}\sqrt{\frac{\left(1.293\,\text{kg/m}^3\right)\left(0.1\,\text{m}^3\right)}{\left(8.314\,\text{J/mol}\cdot\text{K}\right)\left(293\,\text{K}\right)}} = 232\,\text{s} = \boxed{3.87\,\text{min}}$$

Chapter 19
The Second Law of Thermodynamics

Conceptual Problems

1 •

Determine the Concept Friction reduces the efficiency of the engine.

***2** •

Determine the Concept As described by the second law of thermodynamics, more heat must be transmitted to the outside world than is removed by a refrigerator or air conditioner. The heating coils on a refrigerator are inside the room–the refrigerator actually heats the room it is in. The heating coils on an air conditioner are outside, so the waste heat is vented to the outside.

3 •

Determine the Concept Increasing the temperature of the steam increases the Carnot efficiency, and generally increases the efficiency of any heat engine.

4 ••

Determine the Concept To condense, water must lose heat. Because its entropy change is given by $dS = dQ_{\text{rev}}/T$ and dQ_{rev} is negative, the entropy of the water decreases.

$\boxed{(c)\text{ is correct.}}$

***5** •

Determine the Concept

(*a*) Because the temperature changes during an adiabatic process, the internal energy of the system changes continuously during the process.

(*b*) Both the pressure and volume change during an adiabatic process and hence work is done by the system.

(*c*) $\Delta Q = 0$ during an adiabatic process. Therefore $\Delta S = 0$. $\boxed{(c)\text{ is correct.}}$

(*d*) Because the pressure and volume change during an adiabatic process, so does the temperature.

6 ••

(*a*) False. The complete conversion of mechanical energy into heat is not prohibited by either the 1st or 2nd laws of thermodynamics and is common place in energy transformations.

(*b*) True. This is the heat-engine statement of the 2nd law of thermodynamics.

(*c*) False. The efficiency of a heat engine is a function of the thermodynamic processes of its cycle.

(*d*) False. With the input of sufficient energy, a heat pump can transfer a given quantity of heat from a cold reservoir to a hot reservoir.

(*e*) False. The only restriction that the refrigerator statement of the 2nd law places on the COP is that it can not be infinite.

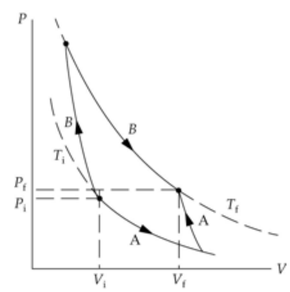
(*f*) True. The Carnot engine, as a consequence of its thermodynamic processes, is reversible.

(*g*) False. The entropy of one system can decrease at the expense of one or more other systems.

(*h*) True. This is one statement of the 2nd law of thermodynamics.

7 ••

Determine the Concept The two paths are shown on the *PV* diagram to the right. We can use the concept of a state function to choose from among the alternatives given as possible answers to the problem.



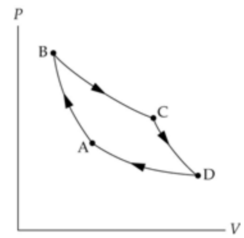
(*a*) Because E_{int} is a state function and the initial and final states are the same for the two paths and $\Delta E_{int,\,A} = \Delta E_{int,\,B}$.

(*b*) and (*c*) *S*, like E_{int}, is a state function and its change when the system moves from one state to another depends only on the system's initial and final states. It is not dependent on the process by which the change occurs and $\Delta S_A = \Delta S_B$.

(*d*) $\boxed{(d)\text{ is correct.}}$

***8 ••**

Determine the Concept The processes A→B and C→D are adiabatic; the processes B→C and D→A are isothermal. The cycle is therefore the Carnot cycle shown in the adjacent *PV* diagram.

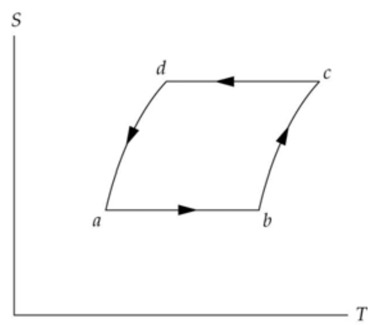


9 ••

Determine the Concept Note that A→B is an adiabatic expansion. B→C is a constant volume process in which the entropy decreases; therefore heat is released. C→D is an adiabatic compression. D→A is a constant volume process that returns the gas to its original state. The cycle is that of the Otto engine (see Figure 19-3).

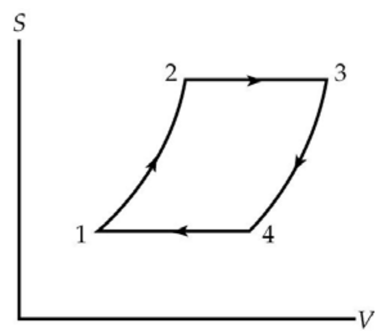
10 ••

Determine the Concept Refer to Figure 19-3. Here *a*→*b* is an adiabatic compression, so *S* is constant and *T* increases. Between *b* and *c*, heat is added to the system and both *S* and *T* increase. *c*→*d* is again isentropic, i.e., without change in entropy. *d*→*a* releases heat and both *S* and *T* decrease. The cycle on an *ST* diagram is sketched in the adjacent figure.



11 ••

Determine the Concept Referring to Figure 19-8, process 1→2 is an isothermal expansion. In this process heat is added to the system and the entropy and volume increase. Process 2→3 is adiabatic, so *S* is constant as *V* increases. Process 3→4 is an isothermal compression in which *S* decreases and *V* also decreases. Finally, process 4→1 is adiabatic, i.e., isentropic, and *S* is constant while *V* decreases. The cycle is shown in the adjacent *SV* diagram.

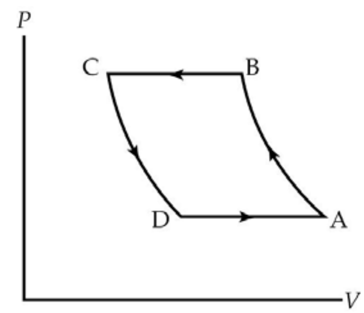


12 ••

Picture the Problem The *SV* diagram of the Otto cycle is shown in Figure 19-13. (see Problem 9)

13 ••

Determine the Concept Process A→B is at constant entropy, i.e., an adiabatic process in which the pressure increases. Process B→C is one in which *P* is constant and *S* decreases; heat is exhausted from the system and the volume decreases. Process C→D is an adiabatic compression. Process D→A returns the system to its original state at constant pressure. The cycle is shown in the adjacent *PV* diagram.



***14** •

Picture the Problem Let ΔT be the change in temperature and $\varepsilon = (T_h - T_c)/T_h$ be the initial efficiency. We can express the efficiencies of the Carnot engine resulting from the given changes in temperature and examine their ratio to decide which has the greater effect on increasing the efficiency.

If T_h is increased by ΔT, ε', the new efficiency is:

$$\varepsilon' = \frac{T_h + \Delta T - T_c}{T_h + \Delta T}$$

If T_c is reduced by ΔT, the efficiency is:

$$\varepsilon'' = \frac{T_h - T_c + \Delta T}{T_h}$$

Divide the second of these equations by the first to obtain:

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\dfrac{T_h - T_c + \Delta T}{T_h}}{\dfrac{T_h + \Delta T - T_c}{T_h + \Delta T}} = \frac{T_h + \Delta T}{T_h} > 1$$

Therefore, a reduction in the temperature of the cold reservoir by ΔT increases the efficiency more than an equal increase in the temperature of the hot reservoir.

Estimation and Approximation

15 ••

Picture the Problem The maximum efficiency of an automobile engine is given by the efficiency of a Carnot engine operating between the same two temperatures. We can use the expression for the Carnot efficiency and the equation relating *V* and *T* for a quasi-static adiabatic expansion to express the Carnot efficiency of the engine in terms of its compression ratio.

Express the Carnot efficiency of an engine operating between the temperatures T_c and T_h:

$$\varepsilon_C = 1 - \frac{T_c}{T_h}$$

Relate the temperatures T_c and T_h to the volumes V_c and V_h for a quasi-static adiabatic compression from V_c to V_h:

$$T_c V_c^{\gamma-1} = T_h V_h^{\gamma-1}$$

Solve for the ratio of T_c to T_h:

$$\frac{T_c}{T_h} = \frac{V_h^{\gamma-1}}{V_c^{\gamma-1}} = \left(\frac{V_h}{V_c}\right)^{\gamma-1}$$

Substitute to obtain:

$$\varepsilon_C = 1 - \left(\frac{V_h}{V_c}\right)^{\gamma-1}$$

Express the compression ratio *r*:

$$r = \frac{V_c}{V_h}$$

Substitute once more to obtain:

$$\varepsilon_C = 1 - \frac{1}{r^{\gamma-1}}$$

Substitute numerical values for *r* and γ (1.4 for diatomic gases) and evaluate ε_C:

$$\varepsilon_C = 1 - \frac{1}{(8)^{1.4-1}} = 0.565 = \boxed{56.5\%}$$

***16** ••

Picture the Problem If we assume that the temperature on the inside of the refrigerator is 0°C (273 K) and the room temperature to be about 30°C (303 K), then the refrigerator must be able to maintain a temperature difference of about 30 K. We can use the definition of the COP of a refrigerator and the relationship between the temperatures of the hot and cold reservoir and $|Q_h|$ and Q_c to find an upper limit on the COP of a household refrigerator. In (*b*) we can solve the definition of COP for Q_c and differentiate the resulting equation with respect to time to estimate the rate at which heat is being drawn from the refrigerator compartment.

(*a*) Using its definition, express the COP of a household refrigerator:

$$\text{COP} = \frac{Q_c}{W} \qquad (1)$$

Apply the 1st law of thermodynamics to the refrigerator to obtain:

$$W + Q_c = |Q_h|$$

Substitute for *W* and simplify to obtain:

$$\text{COP} = \frac{Q_c}{|Q_h| - Q_c} = \frac{1}{\frac{|Q_h|}{Q_c} - 1}$$

Assume, for the sake of finding the upper limit on the COP, that the refrigerator is a Carnot refrigerator and relate the temperatures of the hot and cold reservoirs to $|Q_h|$ and Q_c:

$$\frac{|Q_h|}{Q_c} = \frac{T_h}{T_c}$$

Substitute to obtain:

$$\text{COP}_{max} = \frac{1}{\frac{T_h}{T_c} - 1}$$

Substitute numerical values and evaluate COP_{max}:

$$\text{COP}_{max} = \frac{1}{\frac{303\text{K}}{273\text{K}} - 1} = \boxed{9.10}$$

(b) Solve equation (1) for Q_c:

$$Q_c = W(\text{COP}) \qquad (2)$$

Differentiate equation (2) with respect to time to obtain:

$$\frac{dQ_c}{dt} = (\text{COP})\frac{dW}{dt}$$

Substitute numerical values and evaluate dQ_c/dt:

$$\frac{dQ_c}{dt} = (9.10)(600\,\text{J/s}) = \boxed{5.46\,\text{kW}}$$

17 ••

Picture the Problem We can use the definition of intensity to find the total power of sunlight hitting the earth and the definition of the change in entropy to find the changes in the entropy of the earth and the sun resulting from the radiation from the sun.

(*a*) Using its definition, express the intensity of the sun's radiation on the earth in terms of the power delivered to the earth *P* and the earth's cross sectional area *A*:

$$I = \frac{P}{A}$$

Solve for *P* and substitute for *A* to obtain:

$$P = IA = I\pi R^2$$

where *R* is the radius of the earth.

Substitute numerical values and evaluate P:

$$P = \pi\left(1.3\,\text{kW/m}^2\right)\left(6.37\times10^6\,\text{m}\right)^2 = \boxed{1.66\times10^{17}\,\text{W}}$$

(*b*) Express dS_{earth}/dt for the earth due to the flow of solar radiation:

$$\frac{dS_{\text{earth}}}{dt} = \frac{P}{T_{\text{earth}}}$$

Substitute numerical values and evaluate dS_{earth}/dt:

$$\Delta S_{\text{earth}} = \frac{1.66\times10^{17}\,\text{W}}{290\,\text{K}} = \boxed{5.72\times10^{14}\,\text{J/K}\cdot\text{s}}$$

(*c*) Express dS_{sun}/dt for the sun due to the outflow of solar radiation *hitting the earth*:

$$\frac{dS_{\text{sun}}}{dt} = \frac{P}{T_{\text{sun}}}$$

Substitute numerical values and evaluate dS_{sun}/dt:

$$\frac{dS_{\text{sun}}}{dt} = \frac{1.66\times10^{17}\,\text{W}}{5400\,\text{K}} = \boxed{3.07\times10^{13}\,\text{J/K}\cdot\text{s}}$$

18 ••

Picture the Problem We can use the definition of intensity to find the total power radiated by the sun and the definition of the change in entropy to find the change in the entropy of the universe resulting from the radiation of 10^{11} stars in 10^{11} galaxies.

(*a*) Using its definition, express the intensity of the sun's radiation on the location of earth in terms of the total power it delivers to space P and the area of a sphere A whose radius is the distance from the sun to the earth:

$$I = \frac{P}{A}$$

Solve for P and substitute for A to obtain:

$$P = IA = 4\pi IR^2$$

where R is the distance from the sun to the earth.

Substitute numerical values and evaluate P:

$$P = 4\pi\left(1.3\,\text{kW/m}^2\right)\left(1.5\times10^{11}\,\text{m}\right)^2 = \boxed{3.68\times10^{26}\,\text{W}}$$

(*b*) Express $\Delta S_{\text{universe}}$:

$$\Delta S_{\text{universe}} = \frac{P}{T_{\text{universe}}}$$

Substitute numerical values and evaluate $\Delta S_{\text{universe}}$:

$$\Delta S_{\text{universe}} = \frac{10^{22}\left(3.68\times10^{26}\text{ W}\right)}{2.73\text{ K}} = \boxed{1.35\times10^{48}\text{ J/K}\cdot\text{s}}$$

19 ••

Picture the Problem We can use the definition of entropy change to estimate the increase in entropy of the universe as a result of the heat produced by a typical human body. The entropy change is equivalent to the entropy change if the heat from the body were added to the universe reversibly.

Express the increase in entropy of the universe as a result of the heat produced by a human body:

$$\Delta S_{\text{u}} = \frac{\Delta Q_{\text{day}}}{T_{\text{day}}} + \frac{\Delta Q_{\text{night}}}{T_{\text{night}}}$$

Using the definition of power, express the total heat produced by a human body:

$$\Delta Q = P\Delta t$$

Assume that half of the heat is produced during the day and half at night:

$$\Delta Q_{\text{day}} = \Delta Q_{\text{night}} = \tfrac{1}{2}P\Delta t$$

Substitute to obtain:

$$\Delta S_{\text{u}} = \frac{\frac{1}{2}P\Delta t}{T_{\text{day}}} + \frac{\frac{1}{2}P\Delta t}{T_{\text{night}}} = \tfrac{1}{2}P\Delta t\left(\frac{1}{T_{\text{day}}} + \frac{1}{T_{\text{night}}}\right)$$

Use $T = \frac{5}{9}\left(t_{\text{F}} - 32\right) + 273$ to obtain:

T_{day} = 294 K and T_{night} = 286 K

Substitute numerical values and evaluate ΔS_{u}:

$$\Delta S_{\text{u}} = \tfrac{1}{2}\left(100\text{ J/s}\right)\left(24\text{ h/d}\right)\left(3600\text{ s/h}\right)\left(\frac{1}{294\text{ K}} + \frac{1}{286\text{ K}}\right) = \boxed{29.8\text{ kJ/K}}$$

***20** •••

Picture the Problem If you had one molecule in a box, it would have a 50% chance of being on one side or the other. We don’t care which side the molecules are on as long as they all are on one side, so with one molecule you have a 100% chance of it being on one side or the other. With two molecules, there are four possible combinations (both on one side, both on the other, one on one side and one on the other, and the reverse), so there is a 25% (1 in 4) chance of them both being on a particular side, or a 50% chance of them both being on either side. Extending this logic, the probability of N molecules all being on one side of the box is $P = 2/2^N$, which means that, if the molecules shuffle 100 times a second, the time it would take them to cover all the combinations and all get on one side or the other is $t = \dfrac{2^N}{2(100)}$. In (e) we can apply the ideal gas law to find the number of molecules in 1 L of air at a pressure of 10^{-12} torr and an assumed temperature of 300 K.

(*a*) Evaluate t for N = 10 molecules:

$$t=\frac{2^{10}}{2(100)}=\boxed{5.12\,\text{s}}$$

(*b*) Evaluate t for N = 100 molecules:

$$t=\frac{2^{100}}{2(100)}=6.34\times10^{27}\text{ s}$$
$$=\boxed{2.01\times10^{20}\text{ y}}$$

(*c*) Evaluate t for N = 1000 molecules:

$$t=\frac{2^{1000}}{2(100)}$$

To evaluate 2^{1000} let $10^x=2^{1000}$ and take the logarithm of both sides of the equation to obtain:

$$(1000)\ln 2=x\ln 10$$

Solve for x to obtain:

$$x=301$$

Substitute to obtain:

$$t=\frac{10^{301}}{2(100)}=0.5\times10^{299}\text{ s}$$
$$=\boxed{1.58\times10^{290}\text{ y}}$$

(*d*) Evaluate t for $N=6.02\times10^{23}$ molecules:

$$t=\frac{2^{6.02\times10^{23}}}{2(100)}$$

To evaluate $2^{6.02\times10^{23}}$ let $10^x=2^{6.02\times10^{23}}$ and take the logarithm of both sides of the equation to obtain:

$$\left(6.02\times10^{23}\right)\ln 2=x\ln 10$$

Solve for x to obtain:

$$x\approx10^{23}$$

Substitute to obtain:

$$t\approx\frac{10^{10^{23}}}{2(100)}\approx\boxed{10^{10^{23}}\text{ y}}$$

(*e*) Solve the ideal gas law for the number of molecules N in the gas:

$$N=\frac{PV}{kT}$$

Assuming the gas to be at room temperature (300 K), substitute numerical values and evaluate N:

$$N=\frac{\left(10^{-12}\text{ torr}\right)\left(133.32\text{ Pa/torr}\right)\left(10^{-3}\text{ m}^3\right)}{\left(1.381\times10^{-23}\text{ J/K}\right)\left(300\text{ K}\right)}$$
$$=3.22\times10^{7}\text{ molecules}$$

Evaluate T for $N = 3.22\times10^7$ molecules:

$$t = \frac{2^{3.22\times10^7}}{2(100)}$$

To evaluate $2^{3.22\times10^7}$ let $10^x = 2^{3.22\times10^7}$ and take the logarithm of both sides of the equation to obtain:

$$(3.22\times10^7)\ln 2 = x\ln 10$$

Solve for x to obtain:

$$x \approx 10^7$$

Substitute to obtain:

$$T = \frac{10^{10^7}}{2(100)} \approx \boxed{10^{10^7}\ \text{y}}$$

Express the ratio of this waiting time to the lifetime of the universe T_{universe}:

$$\frac{T}{T_{\text{universe}}} = \frac{10^{10^7}\ \text{y}}{10^{10}\ \text{y}} \approx 10^{10^7}$$

or

$$T \approx \boxed{10^{10^7}T_{\text{universe}}}$$

Heat Engines and Refrigerators

21 •

Picture the Problem We can use the definition of the efficiency of a heat engine to relate the work done W, the heat absorbed Q_{in}, and the heat rejected each cycle Q_{out}.

(*a*) Express Q_{in} in terms of W and ε:

$$Q_{\text{in}} = \frac{W}{\varepsilon} = \frac{100\ \text{J}}{0.2} = \boxed{500\ \text{J}}$$

(*b*) Solve the definition of efficiency for and evaluate $|Q_{\text{out}}|$:

$$|Q_{\text{out}}| = Q_{\text{in}}(1-\varepsilon) = (500\ \text{J})(1-0.2) = \boxed{400\ \text{J}}$$

22 •

Picture the Problem We can use its definition to find the efficiency of a heat engine from the work done, the heat absorbed, and the heat rejected each cycle.

(*a*) Use the definition of the efficiency of a heat engine:

$$\varepsilon \equiv \frac{W}{Q_{\text{in}}} = \frac{120\ \text{J}}{400\ \text{J}} = \boxed{30\%}$$

(*b*) Solve the definition of efficiency for and evaluate $|Q_{\text{out}}|$:

$$|Q_{\text{out}}| = Q_{\text{in}}(1-\varepsilon) = (400\ \text{J})(1-0.3) = \boxed{280\ \text{J}}$$

23 •

Picture the Problem We can use its definition to find the efficiency of the engine and the definition of power to find its power output.

(*a*) Apply the definition of the efficiency of a heat engine:

$$\varepsilon = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{60\,\text{J}}{100\,\text{J}} = \boxed{40.0\%}$$

(*b*) Use the definition of power to find the power output of this engine:

$$P = \frac{\Delta W}{\Delta t} = \frac{\varepsilon Q_{in}}{\Delta t} = \frac{0.4(100\,\text{J})}{0.5\,\text{s}} = \boxed{80.0\,\text{W}}$$

***24** •

Picture the Problem We can apply their definitions to find the COP of the refrigerator and the efficiency of the heat engine.

(*a*) Using the definition of the COP, relate the heat absorbed from the cold reservoir to the work done each cycle:

$$\text{COP} = \frac{Q_c}{W}$$

Relate the work done per cycle to Q_h and Q_c:

$$W = |Q_h| - Q_c$$

Substitute to obtain:

$$\text{COP} = \frac{Q_c}{|Q_h| - Q_c}$$

Substitute numerical values and evaluate COP:

$$\text{COP} = \frac{5\,\text{kJ}}{|8\,\text{kJ}| - 5\,\text{kJ}} = \boxed{1.67}$$

(*b*) Use the definition of efficiency to relate the work done per cycle to the heat absorbed from the high-temperature reservoir:

$$\varepsilon = \frac{W}{Q_h}$$

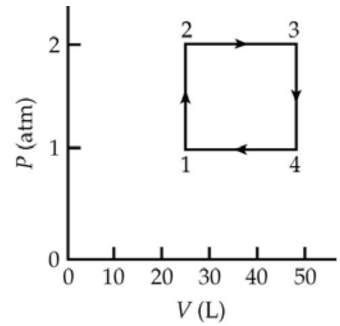
Substitute numerical values and evaluate ε:

$$\varepsilon = \frac{3\,\text{kJ}}{8\,\text{kJ}} = \boxed{37.5\%}$$

25 ••

Picture the Problem To find the heat added during each step we need to find the temperatures in states 1, 2, 3, and 4. We can then find the work done on or by the gas along each pass from the area under each straight-line segment and the heat that enters or leaves the system from $Q = C_V\Delta T$ and $Q = C_P\Delta T$. We can find the efficiency of the cycle from the work done each cycle and the heat that *enters* the system each cycle.

(*a*) The cycle is shown to the right:



Apply the ideal-gas law to state 1 to find T_1:

$$T_1 = \frac{P_1V_1}{nR} = \frac{(1\,\text{atm})(24.6\,\text{L})}{(1\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 300\,\text{K}$$

The pressure doubles while the volume remains constant between states 1 and 2. Hence:

$$T_2 = 2T_1 = 600\,K$$

The volume doubles while the pressure remains constant between states 2 and 3. Hence:

$$T_3 = 2T_2 = 1200\,K$$

The pressure is halved while the volume remains constant between states 3 and 4. Hence:

$$T_4 = \tfrac{1}{2}T_3 = 600\,K$$

For path 1→2:

$$W_{1\to2} = P\Delta V_{1\to2} = \boxed{0}$$

and

$$Q_{1\to2} = \Delta E_{\text{int},1\to2} = C_V\Delta T_{1\to2} = \tfrac{3}{2}R\Delta T_{1\to2} = \tfrac{3}{2}(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = \boxed{3.74\,\text{kJ}}$$

For path 2→3:

$$W_{2\to 3} = P\Delta V_{2\to 2} = (2\,\text{atm})(49.2\,\text{L} - 24.6\,\text{L}) = 49.20\,\text{L}\cdot\text{atm} \times \frac{101.325\,\text{J}}{\text{L}\cdot\text{atm}}$$
$$= \boxed{4.99\,\text{kJ}}$$

and

$$Q_{2\to 3} = C_\text{P}\Delta T_{2\to 3} = \tfrac{5}{2}R\Delta T_{2\to 3} = \tfrac{5}{2}(8.314\,\text{J/mol}\cdot\text{K})(1200\,\text{K} - 600\,\text{K})$$
$$= \boxed{12.5\,\text{kJ}}$$

For path 3→4:

$$W_{3\to 4} = P\Delta V_{3\to 4} = \boxed{0}$$

and

$$Q_{3\to 4} = \Delta E_{\text{int},3\to 4} = C_\text{V}\Delta T_{3\to 4} = \tfrac{3}{2}R\Delta T_{3\to 4} = \tfrac{3}{2}(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 1200\,\text{K})$$
$$= \boxed{-7.48\,\text{kJ}}$$

For path 4→1:

$$W_{4\to 1} = P\Delta V_{4\to 1} = (1\text{atm})(24.6\,\text{L} - 49.2\,\text{L}) = -24.6\,\text{L}\cdot\text{atm} \times \frac{101.3\,\text{J}}{\text{L}\cdot\text{atm}}$$
$$= \boxed{2.49\,\text{kJ}}$$

and

$$Q_{4\to 1} = C_\text{P}\Delta T_{4\to 1} = \tfrac{5}{2}R\Delta T_{4\to 1} = \tfrac{5}{2}(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 600\,\text{K})$$
$$= \boxed{-6.24\,\text{kJ}}$$

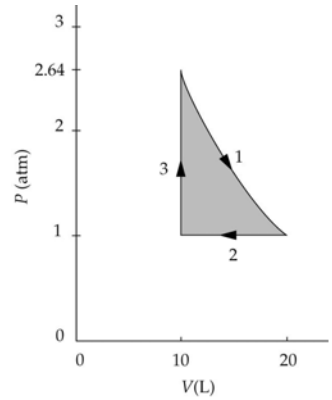
(*b*) Use its definition to find the efficiency of this cycle:

$$\varepsilon = \frac{W}{Q_\text{in}} = \frac{W_{2\to 3} + W_{4\to 1}}{Q_{1\to 2} + Q_{2\to 3}}$$
$$= \frac{4.99\,\text{kJ} - 2.49\,\text{kJ}}{3.74\,\text{kJ} + 12.5\,\text{kJ}} = \boxed{15.4\%}$$

Remarks: Note that the work done per cycle is the area bounded by the rectangular path.

26 ••

Picture the Problem The three steps in the process are shown on the *PV* diagram. We can find the efficiency of the cycle by finding the work done by the gas and the heat that enters the system per cycle.



Express the efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_{\text{in}}}$$

Find the heat entering or leaving the system during the adiabatic expansion:

$$Q_1 = 0$$

Find the heat entering or leaving the system during the isobaric compression:

$$Q_2 = C_V \Delta T_2 = \tfrac{7}{2} R\Delta T_2 = \tfrac{7}{2} P\Delta V_2$$
$$= \tfrac{7}{2}(1\,\text{atm})(10\,\text{L} - 20\,\text{L}) = -35\,\text{atm}\cdot\text{L}$$

Find the heat entering or leaving the system during the constant-volume process:

$$Q_3 = C_V \Delta T_3 = \tfrac{5}{2} R\Delta T_3 = \tfrac{5}{2} \Delta P V_3$$
$$= \tfrac{5}{2}(2.64\,\text{atm} - 1\,\text{atm})(10\,\text{L})$$
$$= 41\,\text{atm}\cdot\text{L}$$

Apply the 1st law of thermodynamics to the cycle ($\Delta E_{\text{int, cycle}} = 0$) to obtain:

$$W_{\text{on}} = \Delta E_{\text{int}} - Q_{\text{in}} = -Q_{\text{in}}$$
$$= Q_1 + Q_2 + Q_3$$
$$= 0 - 35\,\text{atm}\cdot\text{L} + 41\,\text{atm}\cdot\text{L}$$
$$= 6\,\text{atm}\cdot\text{L}$$

Substitute and evaluate ε:

$$\varepsilon = \frac{6\,\text{atm}\cdot\text{L}}{41\,\text{atm}\cdot\text{L}} = \boxed{14.6\%}$$

27 ••

Picture the Problem We can find the efficiency of the cycle by finding the work done by the gas and the heat that enters the system per cycle.

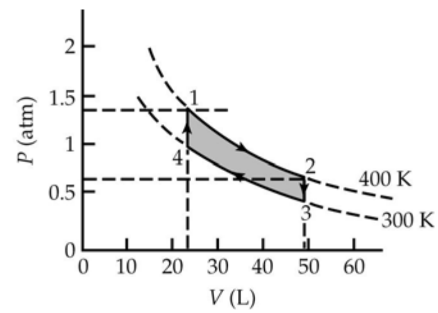
Apply the ideal-gas law to states 1, 2, 3, and 4 to find the pressures at these points:

$$P_1 = \frac{nRT_1}{V_1} = \frac{(1\,\text{mol})(8.206\times10^{-2}\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(400\,\text{K})}{24.6\,\text{K}} = 1.33\,\text{atm}$$

Proceed as above to obtain the values shown in the table:

Point	P	V	T
	(atm)	(L)	(K)
1	1.330	24.6	400
2	0.667	49.2	400
3	0.500	49.2	300
4	1.000	24.6	300

The *PV* diagram is shown to the right:



Express the efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_{\text{in}}} \quad (1)$$

Find the work done by the gas and the heat that enters the system during the isothermal expansion from 1 to 2:

$$W_{1\to2} = Q_{1\to2} = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = (1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(400\,\text{K})\ln\left(\frac{49.2\,\text{L}}{24.6\,\text{L}}\right)$$
$$= 2.305\,\text{kJ}$$

Find the work done by the gas and the heat that enters the system during the constant-volume compression from 2 to 3:

$$W_{2\to3} = 0$$

and

$$Q_{2\to3} = C_V\Delta T_{2\to3} = (21\,\text{J/K})(300\,\text{K} - 400\,\text{K}) = -2.10\,\text{kJ}$$

Find the work done by the gas and the heat that enters the system during the isothermal expansion from 3 to 4:

$$W_{3\to4} = Q_{3\to4} = nRT_3 \ln\left(\frac{V_4}{V_3}\right) = (1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})\ln\left(\frac{24.6\,\text{L}}{49.2\,\text{L}}\right)$$
$$= -1.729\,\text{kJ}$$

Find the work done by the gas and the heat that enters the system during the constant-volume process from 4 to 1:

$$W_{4\to1} = 0$$

and

$$Q_{4\to1} = C_V \Delta T_{4\to1} = (21\,\text{J/K})(400\,\text{K} - 300\,\text{K}) = 2.10\,\text{kJ}$$

Evaluate the work done each cycle:

$$W = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$$
$$= 2.305\,\text{kJ} + 0 - 1.729\,\text{kJ} + 0$$
$$= 0.5760\,\text{kJ}$$

Find the heat that enters the system each cycle:

$$Q_{\text{in}} = Q_{1\to2} + Q_{4\to1}$$
$$= 2.305\,\text{kJ} + 2.100\,\text{kJ}$$
$$= 4.405\,\text{kJ}$$

Substitute numerical values in equation (1) and evaluate ε:

$$\varepsilon = \frac{0.5760\,\text{kJ}}{4.405\,\text{kJ}} = \boxed{13.1\%}$$

***28 ••**

Picture the Problem We can use the ideal-gas law to find the temperatures of each state of the gas and the heat capacities at constant volume and constant pressure to find the heat flow for the constant-volume and isobaric processes. Because the change in internal energy is zero for the isothermal process, we can use the expression for the work done on or by a gas during an isothermal process to find the heat flow during such a process. Finally, we can find the efficiency of the cycle from its definition.

(*a*) Use the ideal-gas law to find the temperature at point 1:

$$T_1 = \frac{P_1V_1}{nR} = \frac{(100\,\text{kPa})(25\,\text{L})}{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}$$
$$= \boxed{301\,\text{K}}$$

Use the ideal-gas law to find the temperatures at points 2 and 3:

$$T_2 = T_3 = \frac{P_2V_2}{nR} = \frac{(200\,\text{kPa})(25\,\text{L})}{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{601\,\text{K}}$$

(*b*) Find the heat entering or leaving the system for the constant-volume process from 1 to 2:

$$Q_{1\to2} = C_\text{V}\Delta T_{1\to2} = \tfrac{3}{2}R\Delta T_{1\to2} = \tfrac{3}{2}(8.314\,\text{J/mol}\cdot\text{K})(601\,\text{K} - 301\,\text{K}) = \boxed{3.74\,\text{kJ}}$$

Find the heat entering or leaving the system for the isothermal process from 2 to 3:

$$Q_{2\to3} = nRT_2\ln\left(\frac{V_3}{V_2}\right) = (1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(601\,\text{K})\ln\left(\frac{50\,\text{L}}{25\,\text{L}}\right) = \boxed{3.46\,\text{kJ}}$$

Find the heat entering or leaving the system during the isobaric compression from 3 to 1:

$$Q_{3\to1} = C_\text{P}\Delta T_{3\to1} = \tfrac{5}{2}R\Delta T_{3\to1} = \tfrac{5}{2}(8.314\,\text{J/mol}\cdot\text{K})(301\,\text{K} - 601\,\text{K}) = \boxed{-6.24\,\text{kJ}}$$

(*c*) Express the efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_\text{in}} = \frac{W}{Q_{1\to2} + Q_{2\to3}} \qquad (1)$$

Apply the 1st law of thermodynamics to the cycle:

$$W = \sum Q = Q_{1\to2} + Q_{2\to3} + Q_{3\to1} = 3.74\,\text{kJ} + 3.46\,\text{kJ} - 6.24\,\text{kJ} = 0.960\,\text{kJ}$$

because, for the cycle, $\Delta U = 0$.

Substitute numerical values in equation (1) and evaluate ε:

$$\varepsilon = \frac{0.960\,\text{kJ}}{3.74\,\text{kJ} + 3.46\,\text{kJ}} = \boxed{13.3\%}$$

29 ••

Picture the Problem We can use the ideal-gas law to find the temperatures of each state of the gas. We can find the efficiency of the cycle from its definition; using the area enclosed by the cycle to find the work done per cycle and the heat entering the system between states 1 and 2 and 2 and 3 to determine Q_in.

(*a*) Use the ideal-gas law for a fixed amount of gas to find the temperature in state 2 to the temperature in state 1:

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

Solve for and evaluate T_2:

$$T_2=T_1\frac{P_2V_2}{P_1V_1}=T_1\frac{P_2}{P_1}=(200\,\text{K})\frac{(3\,\text{atm})}{(1\,\text{atm})}=\boxed{600\,\text{K}}$$

Apply the ideal-gas law for a fixed amount of gas to states 2 and 3 to obtain:

$$T_3=T_2\frac{P_3V_3}{P_2V_2}=T_2\frac{V_3}{V_2}=(600\,\text{K})\frac{(300\,\text{L})}{(100\,\text{L})}=\boxed{1800\,\text{K}}$$

Apply the ideal-gas law for a fixed amount of gas to states 3 and 4 to obtain:

$$T_4=T_3\frac{P_4V_4}{P_3V_3}=T_3\frac{P_4}{P_3}=(1800\,\text{K})\frac{(1\,\text{atm})}{(3\,\text{atm})}=\boxed{600\,\text{K}}$$

(*b*) Express the efficiency of the cycle:

$$\varepsilon=\frac{W}{Q_{\text{in}}}\qquad(1)$$

Use the area of the rectangle to find the work done each cycle:

$$W=\Delta P\Delta V=(300\,\text{L}-100\,\text{L})(3\,\text{atm}-1\,\text{atm})=400\,\text{atm}\cdot\text{L}$$

Apply the ideal-gas law to state 1 to find the product of n and R:

$$nR=\frac{P_1V_1}{T_1}=\frac{(1\,\text{atm})(100\,\text{L})}{200\,\text{K}}=0.5\,\text{L}\cdot\text{atm/K}$$

Noting that heat enters the system between states 1 and 2 and states 2 and 3, express Q_{in}:

$$\begin{aligned}Q_{\text{in}}&=Q_{1\to2}+Q_{2\to3}\\&=C_\text{V}\Delta T_{1\to2}+C_\text{P}\Delta T_{2\to3}\\&=\tfrac{5}{2}nR\Delta T_{1\to2}+\tfrac{7}{2}nR\Delta T_{2\to3}\\&=\left(\tfrac{5}{2}\Delta T_{1\to2}+\tfrac{7}{2}\Delta T_{2\to3}\right)nR\end{aligned}$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}}=\left[\tfrac{5}{2}(600\,\text{K}-200\,\text{K})+\tfrac{7}{2}(1800\,\text{K}-600\,\text{K})\right](0.5\,\text{L}\cdot\text{atm/K})=2600\,\text{atm}\cdot\text{L}$$

Substitute numerical values in equation (1) and evaluate ε:

$$\varepsilon = \frac{400\,\text{atm}\cdot\text{L}}{2600\,\text{atm}\cdot\text{L}} = \boxed{15.4\%}$$

30 •••

Picture the Problem To find the efficiency of the diesel cycle we can find the heat that enters the system and the heat that leaves the system and use the expression that gives the efficiency in terms of these quantities. Note that no heat enters or leaves the system during the adiabatic processes $a\to b$ and $c\to d$.

Express the efficiency of the cycle in terms of Q_c and Q_h:

$$\varepsilon = 1 - \frac{|Q_\text{c}|}{Q_\text{h}}$$

Express Q for the isobaric warming process $b\to c$:

$$Q_{b\to c} = |Q_\text{h}| = C_\text{P}(T_c - T_b)$$

Express Q for the constant-volume cooling process $d\to a$:

$$Q_{d\to a} = |Q_\text{c}| = C_\text{V}(T_d - T_a)$$

Substitute to obtain:

$$\varepsilon = 1 - \frac{C_\text{V}(T_d - T_a)}{C_\text{P}(T_c - T_b)} = 1 - \frac{(T_d - T_a)}{\gamma(T_c - T_b)}$$

Using the equation of state for an adiabatic process, relate the temperatures T_a and T_b:

$$T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1} \qquad (1)$$

Proceeding similarly, relate the temperatures T_c and T_d:

$$T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1} \qquad (2)$$

Use equations (1) and (2) to eliminate T_a and T_d:

$$\varepsilon = 1 - \frac{\left(T_c \frac{V_c^{\gamma-1}}{V_d^{\gamma-1}} - T_b \frac{V_b^{\gamma-1}}{V_a^{\gamma-1}}\right)}{\gamma(T_c - T_b)} = 1 - \frac{\left(\left(\frac{V_c}{V_a}\right)^{\gamma-1} - \frac{T_b}{T_c}\left(\frac{V_b}{V_a}\right)^{\gamma-1}\right)}{\gamma\left(1 - \frac{T_b}{T_c}\right)}$$

because $V_a = V_d$.

Apply the ideal-gas law for a fixed amount of gas to relate T_b and T_c:

$$\frac{T_b}{T_c}=\frac{V_b}{V_c}$$

because $P_b = P_c$.

Substitute and simplify to obtain:

$$\varepsilon=1-\frac{\left(\left(\frac{V_c}{V_a}\right)^{\gamma-1}-\frac{V_b}{V_c}\left(\frac{V_b}{V_a}\right)^{\gamma-1}\right)}{\gamma\left(1-\frac{V_b}{V_c}\right)}\cdot\frac{\frac{V_c}{V_a}}{\frac{V_c}{V_a}}=1-\frac{\left(\left(\frac{V_c}{V_a}\right)^{\gamma}-\frac{V_b}{V_a}\left(\frac{V_b}{V_a}\right)^{\gamma-1}\right)}{\gamma\left(\frac{V_c}{V_a}-\frac{V_b}{V_a}\right)}$$

$$=\boxed{1-\frac{\left(\left(\frac{V_c}{V_a}\right)^{\gamma}-\left(\frac{V_b}{V_a}\right)^{\gamma}\right)}{\gamma\left(\frac{V_c}{V_a}-\frac{V_b}{V_a}\right)}}$$

***31 ••**

Picture the Problem We can use the efficiency of a Carnot engine operating between reservoirs at body temperature and typical outdoor temperatures to find an upper limit on the efficiency of an engine operating between these temperatures.

(*a*) Express the maximum efficiency of an engine operating between body temperature and 70°F:

$$\varepsilon_{\mathrm{C}}=1-\frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}$$

Use $T=\frac{5}{9}\left(t_{\mathrm{F}}-32\right)+273$ to obtain:

$T_{\mathrm{body}} = 310$ K and $T_{\mathrm{room}} = 294$ K

Substitute numerical values and evaluate ε_{C}:

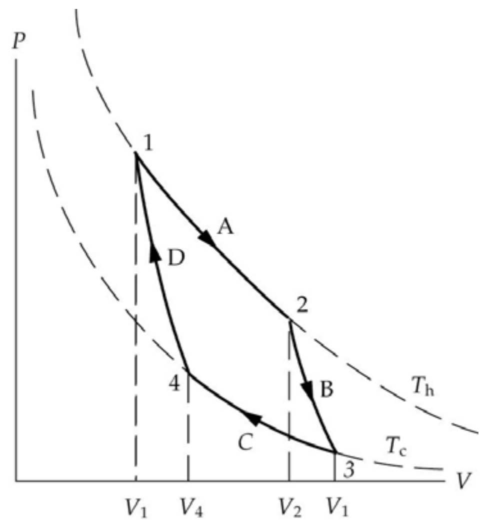
$$\varepsilon_{\mathrm{C}}=1-\frac{294\,\mathrm{K}}{310\,\mathrm{K}}=\boxed{5.16\%}$$

The fact that this efficiency is considerably less than the actual efficiency of a human body does not contradict the second law of thermodynamics. The application of the second law to chemical reactions such as the ones that supply the body with energy have not been discussed in the text but we can note that don't get our energy from heat swapping between our body and the environment. Rather, we eat food to get the energy that we need.

(*b*) Most warm – blooded animals survive under roughly the same conditions as humans. To make a heat engine work with appreciable efficiency, internal body temperatures would have to be maintained at an unreasonably high level.

32 •••

Picture the Problem The Carnot cycle's four segments (shown to the right) are: (A) an isothermal expansion at $T = T_h$ from V_1 to V_2, (B) an adiabatic expansion from V_2 to V_3, (C) an isothermal compression from V_3 to V_4 at $T = T_c$, and (D) an adiabatic compression from V_4 to V_1. We can find the Carnot efficiency for a gas described by the Clausius equation by expressing the ratio of the work done per cycle to the heat entering the system per cycle.



Express the efficiency of the Carnot cycle in terms of the work done and the heat that enters the system per cycle:

$$\varepsilon = \frac{W}{Q_h}$$

Apply the first law of thermodynamics to segment A:

$$Q_A = W_A + \Delta E_{\text{int, A}} = W_A = \int_{V_1}^{V_2} P dV$$

$$= nRT_h \int_{V_1}^{V_2} \frac{dV}{V - bn} = nRT_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right)$$

$$= Q_h$$

Follow the same procedure for segment C to obtain:

$$Q_C = W_C + \Delta E_{\text{int, C}} = W_C = \int_{V_3}^{V_4} P dV$$

$$= nRT_c \int_{V_3}^{V_4} \frac{dV}{V - bn} = nRT_c \ln\left(\frac{V_4 - bn}{V_3 - bn}\right)$$

and

$$|Q_c| = nRT_c \ln\left(\frac{V_3 - bn}{V_4 - bn}\right)$$

Apply the first law of thermodynamics to the complete cycle ($\Delta E_{\text{int, cycle}} = 0$) to express W:

$$W = Q_h - |Q_c| = nRT_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right) - nRT_c \ln\left(\frac{V_3 - bn}{V_4 - bn}\right)$$

Substitute and simplify to obtain:

$$\varepsilon = \frac{nRT_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right) - nRT_c \ln\left(\frac{V_3 - bn}{V_4 - bn}\right)}{nRT_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right)} = 1 - \frac{T_c \ln\left(\frac{V_3 - bn}{V_4 - bn}\right)}{T_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right)}$$

Apply the first law of thermodynamics to the adiabatic processes B and D:

$$dQ_B = 0 = dW_B + dE_{\text{int, B}} = PdV + C_V dT = \frac{nRT}{V - bn} dV + C_V dT$$

Separate variables and integrate to obtain:

$$\int \frac{dT}{T} = -\frac{nT}{C_V} \int \frac{dV}{V - bn} = -(\gamma - 1) \int \frac{dV}{V - bn}$$

or

$$\ln T = -(\gamma - 1)\ln(V - bn) + \text{constant} = \ln(V - bn)^{\gamma - 1} + \text{constant}$$

Simplify to obtain:

$$\ln T + \ln(V - bn)^{\gamma - 1} = \text{constant}$$

or

$$\ln T(V - bn)^{\gamma - 1} = \text{constant}$$

and

$$T(V - bn)^{\gamma - 1} = \text{constant}$$

Using this result, relate V_2 and V_3 to T_h and T_c:

$$T_h (V_2 - bn)^{\gamma - 1} = T_c (V_3 - bn)^{\gamma - 1} \quad (1)$$

Relate V_1 and V_4 to T_h and T_c:

$$T_h (V_1 - bn)^{\gamma - 1} = T_c (V_4 - bn)^{\gamma - 1} \quad (2)$$

Divide equation (1) by equation (2) and simplify to obtain:

$$\frac{T_h (V_2 - bn)^{\gamma - 1}}{T_h (V_1 - bn)^{\gamma - 1}} = \frac{T_c (V_3 - bn)^{\gamma - 1}}{T_h (V_4 - bn)^{\gamma - 1}}$$

or

$$\frac{V_2 - bn}{V_1 - bn} = \frac{V_3 - bn}{V_4 - bn}$$

Substitute in our expression for ε and simplify:

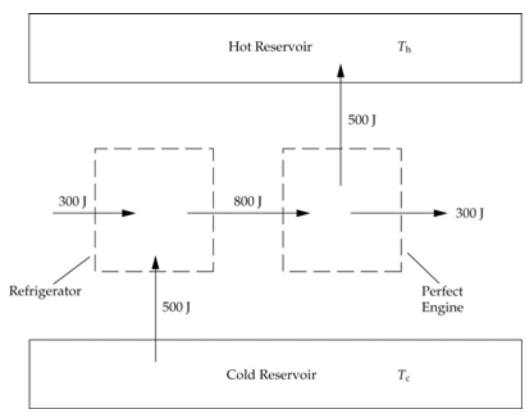
$$\varepsilon = 1 - \frac{T_c \ln\left(\frac{V_2 - bn}{V_1 - bn}\right)}{T_h \ln\left(\frac{V_2 - bn}{V_1 - bn}\right)} = \boxed{1 - \frac{T_c}{T_h}}$$

the same as for an ideal gas.

Second Law of Thermodynamics

33 ••

Determine the Concept The relationship of the perfect engine and the refrigerator to each other and to the hot and cold reservoirs is shown below. To remove 500 J from the cold reservoir and reject 800 J to the hot reservoir, 300 J of work must be done on the system. Assuming that the heat-engine statement is false, one could use the 800 J rejected to the hot reservoir to do 300 J of work. Thus, running the refrigerator connected to the ″perfect″ heat engine would have the effect of transferring 500 J of heat from the cold to the hot reservoir without any work being done, in violation of the refrigerator statement of the second law.



***34** ••

Determine the Concept The work done by the system is the area enclosed by the cycle, where we assume that we start with the isothermal expansion. It is only in this expansion that heat is extracted from a reservoir. There is no heat transfer in the adiabatic expansion or compression. Thus, we would completely convert heat to mechanical energy, without exhausting any heat to a cold reservoir, in violation of the second law.

Carnot Engines

35 •

Picture the Problem We can find the efficiency of the Carnot engine using $\varepsilon = 1 - T_c / T_h$ and the work done per cycle from $\varepsilon = W / Q_h$. We can apply conservation of energy to find the heat rejected each cycle from the heat absorbed and the work done each cycle. We can find the COP of the engine working as a refrigerator from its definition.

(*a*) Express the efficiency of the Carnot engine in terms of the temperatures of the hot and cold reservoirs:

$$\varepsilon_C = 1 - \frac{T_c}{T_h} = 1 - \frac{200\,\text{K}}{300\,\text{K}} = \boxed{33.3\%}$$

(*b*) Using the definition of efficiency, relate the work done each cycle to the heat absorbed from the hot reservoir:

$$W = \varepsilon_C Q_h = (0.333)(100\,\text{J}) = \boxed{33.3\,\text{J}}$$

(*c*) Apply conservation of energy to relate the heat given off each cycle to the heat absorbed and the work done:

$$|Q_c| = Q_h - W = 100\,\text{J} - 33.3\,\text{J} = \boxed{66.7\,\text{J}}$$

(*d*) Using its definition, express and evaluate the refrigerator's coefficient of performance:

$$\text{COP} = \frac{|Q_c|}{W} = \frac{66.7\,\text{J}}{33.3\,\text{J}} = \boxed{2.00}$$

36 •

Picture the Problem We can find the efficiency of the engine from its definition and the additional work done if the engine were reversible from $W = \varepsilon_C Q_h$, where ε_C is the Carnot efficiency.

(*a*) Express the efficiency of the engine in terms of the heat absorbed from the high-temperature reservoir and the heat exhausted to the low-temperature reservoir:

$$\varepsilon = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

$$= 1 - \frac{200\,\text{J}}{250\,\text{J}} = \boxed{20.0\%}$$

(*b*) Express the additional work done if the engine is reversible:

$$\Delta W = W_{\text{Carnot}} - W_{\text{part }a}$$

Relate the work done by a reversible engine to its Carnot efficiency:

$$W = \varepsilon_C Q_h = \left(1 - \frac{T_c}{T_h}\right) Q_h$$

$$= \left(1 - \frac{200\,\text{K}}{300\,\text{K}}\right)(250\,\text{J}) = 83.3\,\text{J}$$

Substitute and evaluate ΔW:

$$\Delta W = 83.3\,\text{J} - 50\,\text{J} = \boxed{33.3\,\text{J}}$$

37 ••

Determine the Concept Let the first engine be run as a refrigerator. Then it will remove 140 J from the cold reservoir, deliver 200 J to the hot reservoir, and require 60 J of energy to operate. Now take the second engine and run it between the same reservoirs, and let it eject 140 J into the cold reservoir, thus replacing the heat removed by the refrigerator. If ε_2, the efficiency of this engine, is greater than 30%, then Q_{h2}, the heat removed from the hot reservoir by this engine, is 140 J/$(1 - \varepsilon_2)$ > 200 J, and the work done by this engine is $W = \varepsilon_2 Q_{h2} > 60$ J. The end result of all this is that the second engine can run the refrigerator, replacing the heat taken from the cold reservoir, and do additional mechanical work. The two systems working together then convert heat into mechanical energy without rejecting any heat to a cold reservoir, in violation of the second law.

38 ••

Determine the Concept If the reversible engine is run as a refrigerator, it will require 100 J of mechanical energy to take 400 J of heat from the cold reservoir and deliver 500 J to the hot reservoir. Now let the second engine, with $\varepsilon_2 > 0.2$, operate between the same two heat reservoirs and use it to drive the refrigerator. Because $\varepsilon_2 > 0.2$, this engine will remove less than 500 J from the hot reservoir in the process of doing 100 J of work. The net result is then that no net work is done by the two systems working together, but a finite amount of heat is transferred from the cold to the hot reservoir, in violation of the refrigerator statement of the second law.

***39** ••

Picture the Problem We can use the definition of efficiency to find the efficiency of the Carnot engine operating between the two reservoirs.

(*a*) Use its definition to find the efficiency of the Carnot engine:

$$\varepsilon_C = \frac{W}{Q_h} = \frac{50\,\text{J}}{150\,\text{J}} = \boxed{33.3\%}$$

(*b*) If COP > 2, then 50 J of work will remove more than 100 J of heat from the cold reservoir and put more than 150 J of heat into the hot reservoir. So running engine (*a*) to operate the refrigerator with a COP > 2 will result in the transfer of heat from the cold to the hot reservoir without doing any net mechanical work in violation of the second law.

40 ••

Picture the Problem We can use the definitions of the efficiency of a Carnot engine and the coefficient of performance of a refrigerator to find these quantities. The work done each cycle by the Carnot engine is given by $W = \varepsilon_C Q_h$ and we can use the conservation of energy to find the heat rejected to the low-temperature reservoir.

(*a*) Use the definition of the efficiency of a Carnot engine to obtain:

$$\varepsilon_C = 1 - \frac{T_c}{T_h} = 1 - \frac{77\,\text{K}}{300\,\text{K}} = \boxed{74.3\%}$$

(*b*) Express the work done each cycle in terms of the efficiency of the engine and the heat absorbed from the high-temperature reservoir:

$$W = \varepsilon_C Q_h = (0.743)(100\,\text{J}) = \boxed{74.3\,\text{J}}$$

(*c*) Apply conservation of energy to obtain:

$$|Q_c| = Q_h - W = 100\,\text{J} - 74.3\,\text{J} = \boxed{25.7\,\text{J}}$$

(*d*) Using its definition, express and evaluate the refrigerator's coefficient of performance:

$$\text{COP} = \frac{|Q_c|}{W} = \frac{25.7\,\text{J}}{74.3\,\text{J}} = \boxed{0.346}$$

41 ••

Picture the Problem We can use the ideal-gas law for a fixed amount of gas and the equations of state for an adiabatic process to find the temperatures, volumes, and pressures at the end points of each process in the given cycle. We can use $Q = C_V \Delta T$ and $Q = C_P \Delta T$ to find the heat entering and leaving during the constant-volume and isobaric processes and the first law of thermodynamics to find the work done each cycle. Once we've calculated these quantities, we can use its definition to find the efficiency of the cycle and the definition of the Carnot efficiency to find the efficiency of a Carnot engine operating between the extreme temperatures.

(*a*) Apply the ideal-gas law for a fixed amount of gas to relate the temperature at point 3 to the temperature at point 1:

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3}$$

or, because $P_1 = P_3$,

$$T_3 = T_1 \frac{V_3}{V_1} \qquad (1)$$

Apply the ideal-gas law for a fixed amount of gas to relate the pressure at point 2 to the temperatures at points 1 and 2 and the pressure at 1:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

or, because $V_1 = V_2$,

$$P_2 = P_1\frac{T_2}{T_1} = (1\,\text{atm})\frac{423\,\text{K}}{273\,\text{K}} = 1.55\,\text{atm}$$

Apply the state equation for an adiabatic process to relate the pressures and volumes at points 2 and 3:

$$P_1V_1^{\gamma} = P_3V_3^{\gamma}$$

Noting that $V_1 = 22.4$ L, solve for and evaluate V_3:

$$V_3 = V_1\left(\frac{P_1}{P_3}\right)^{\frac{1}{\gamma}} = (22.4\,\text{L})\left(\frac{1.55\,\text{atm}}{1\,\text{atm}}\right)^{\frac{1}{1.4}} = 30.6\,\text{L}$$

Substitute in equation (1) and evaluate T_3:

$$T_3 = (273\,\text{K})\frac{30.6\,\text{L}}{22.4\,\text{L}} = 373\,\text{K}$$

and

$$t_3 = T_3 - 273 = \boxed{100°\text{C}}$$

(*b*) Process 1→2 takes place at constant volume (note that $\gamma = 1.4$ corresponds to a diatomic gas and that $C_P - C_V = R$):

$$Q_{1\to2} = C_V\Delta T_{1\to2} = \tfrac{5}{2}R\Delta T_{1\to2} = \tfrac{5}{2}(8.314\,\text{J/mol}\cdot\text{K})(423\,\text{K} - 273\,\text{K}) = \boxed{3.12\,\text{kJ}}$$

Process 2→3 takes place adiabatically:

$$Q_{2\to3} = \boxed{0}$$

Process 3→1 is isobaric (note that $C_P = C_V + R$):

$$Q_{3\to1} = C_P\Delta T_{3\to1} = \tfrac{7}{2}R\Delta T_{1\to2} = \tfrac{7}{2}(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K} - 373\,\text{K}) = \boxed{-2.91\,\text{kJ}}$$

(*c*) Use its definition to express the efficiency of this cycle:

$$\varepsilon = \frac{W}{Q_{\text{in}}}$$

Apply the first law of thermodynamics to the cycle:

$$\Delta K_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $\Delta E_{\text{int, cycle}} = 0$ (the system begins and ends in the same state) and $W_{\text{on}} = -W_{\text{by the gas}} = W$, $W = Q_{\text{in}}$.

Evaluate W:

$$W = \sum Q = Q_{1\to 2} + Q_{2\to 3} + Q_{3\to 1}$$
$$= 3.12\,\text{kJ} + 0 - 2.91\,\text{kJ}$$
$$= 0.210\,\text{kJ}$$

Substitute and evaluate ε:

$$\varepsilon = \frac{0.210\,\text{kJ}}{3.12\,\text{kJ}} = \boxed{6.73\%}$$

(*d*) Express and evaluate the efficiency of a Carnot cycle operating between 423 K and 273 K:

$$\varepsilon_\text{C} = 1 - \frac{T_\text{c}}{T_\text{h}} = 1 - \frac{273\,\text{K}}{423\,\text{K}} = \boxed{35.5\%}$$

42 ••

Picture the Problem We can find the maximum efficiency of the steam engine by calculating the Carnot efficiency of an engine operating between the given temperatures. We can apply the definition of efficiency to find the heat discharged to the engine's surroundings in 1 h.

(*a*) Find the efficiency of a Carnot engine operating between these temperatures:

$$\varepsilon_\text{max} = 1 - \frac{T_\text{c}}{T_\text{h}} = 1 - \frac{323\,\text{K}}{543\,\text{K}} = 40.5\%$$

Find the efficiency of the steam engine as a percentage of the maximum possible efficiency:

$$\varepsilon_\text{steam engine} = \frac{0.30}{0.405}\varepsilon_\text{max} = \boxed{0.741\varepsilon_\text{max}}$$

(*b*) Relate the heat discharged to the engine's surroundings to Q_h and the efficiency of the engine:

$$|Q_\text{c}| = (1-\varepsilon)Q_\text{h}$$

Using its definition, relate the efficiency of the engine to the heat intake of the engine and the work it does each cycle:

$$Q_\text{h} = \frac{W}{\varepsilon} = \frac{P\Delta t}{\varepsilon}$$

Substitute and evaluate $|Q_\text{c}|$ in 1 h:

$$|Q_\text{c}| = (1-\varepsilon)\frac{P\Delta t}{\varepsilon}$$
$$= (1-0.3)\frac{(200\,\text{kJ/s})(3600\,\text{s})}{0.3}$$
$$= \boxed{1.68\,\text{GJ}}$$

Heat Pumps

***43** •

Picture the Problem We can use the definition of the COP_{HP} and the Carnot efficiency of an engine to express the maximum efficiency of the refrigerator in terms of the reservoir temperatures. We can apply equation 19-10 and the definition of power to find the minimum power needed to run the heat pump.

(*a*) Express the COP_{HP} in terms of T_h and T_c:

$$COP_{HP} = \frac{|Q_h|}{W} = \frac{|Q_h|}{|Q_h| - Q_c} = \frac{1}{1 - \frac{Q_c}{|Q_h|}} = \frac{1}{1 - \frac{T_c}{T_h}} = \frac{T_h}{T_h - T_c}$$

Substitute numerical values and evaluate the COP_{HP}:

$$COP_{HP} = \frac{313\,K}{313\,K - 263\,K} = \boxed{6.26}$$

(*b*) Using its definition, express the power output of the engine:

$$P = \frac{W}{\Delta t}$$

Use equation 19-10 to express the work done by the heat pump:

$$W = \frac{|Q_h|}{1 + COP_{HP}}$$

Substitute and evaluate P:

$$P = \frac{|Q_h|/\Delta t}{1 + COP_{HP}} = \frac{20\,kW}{1 + 6.26} = \boxed{2.75\,kW}$$

(*c*) Find the minimum power if the COP is 60% of the efficiency of an ideal pump:

$$P_{min} = \frac{|Q_c|/\Delta t}{1 + 0.6(COP_{HP,max})} = \frac{20\,kW}{1 + 0.6(6.26)} = \boxed{4.21\,kW}$$

44 •

Picture the Problem We can use the definition of the COP to relate the heat removed from the refrigerator to its power rating and operating time. By expressing the COP in terms of T_c and T_h we can write the amount of heat removed from the refrigerator as a function of T_c, T_h, P, and Δt.

(*a*) Express the amount of heat the refrigerator can remove in a given period of time as a function of its COP:

$$Q_c = (\text{COP})W = (\text{COP})P\Delta t$$

Express the COP in terms of T_h and T_c:

$$\text{COP} = \frac{|Q_c|}{W} = \frac{|Q_c|}{\varepsilon Q_h} = \frac{Q_h - W}{\varepsilon Q_h} = \frac{1-\varepsilon}{\varepsilon} = \frac{1}{\varepsilon} - 1 = \frac{1}{1 - \frac{T_c}{T_h}} - 1 = \frac{T_c}{T_h - T_c}$$

Substitute to obtain:

$$Q_c = \left(\frac{T_c}{T_h - T_c}\right)P\Delta t$$

Substitute numerical values and evaluate Q_c:

$$Q_c = \left(\frac{273\,\text{K}}{293\,\text{K} - 273\,\text{K}}\right)(370\,\text{W})(60\,\text{s}) = \boxed{303\,\text{kJ}}$$

(*b*) Find the heat removed if the COP is 70% of the efficiency of an ideal pump:

$$Q_c' = (0.7)\left(\frac{273\,\text{K}}{293\,\text{K} - 273\,\text{K}}\right)(370\,\text{W})(60\,\text{s}) = \boxed{212\,\text{kJ}}$$

45 •

Picture the Problem We can use the definition of the COP to relate the heat removed from the refrigerator to its power rating and operating time. By expressing the COP in terms of T_c and T_h we can write the amount of heat removed from the refrigerator as a function of T_c, T_h, P, and Δt.

(*a*) Express the amount of heat the refrigerator can remove in a given period of time as a function of its COP:

$$Q_c = (\text{COP})W = (\text{COP})P\Delta t$$

Express the COP in terms of T_h and T_c:

$$\text{COP} = \frac{|Q_c|}{W} = \frac{|Q_c|}{\varepsilon Q_h} = \frac{Q_h - W}{\varepsilon Q_h} = \frac{1-\varepsilon}{\varepsilon} = \frac{1}{\varepsilon} - 1 = \frac{1}{1-\frac{T_c}{T_h}} - 1 = \frac{T_c}{T_h - T_c}$$

Substitute to obtain:

$$Q_c = \left(\frac{T_c}{T_h - T_c}\right) P\Delta t$$

Substitute numerical values and evaluate Q_c:

$$Q_c = \left(\frac{273\,\text{K}}{308\,\text{K} - 273\,\text{K}}\right)(370\,\text{W})(60\,\text{s}) = \boxed{173\,\text{kJ}}$$

(b) Find the heat removed if the COP is 70% of the efficiency of an ideal pump:

$$Q_c' = (0.7)\left(\frac{273\,\text{K}}{308\,\text{K} - 273\,\text{K}}\right)(370\,\text{W})(60\,\text{s}) = \boxed{121\,\text{kJ}}$$

Entropy Changes

46 •

Picture the Problem We can use the definition of entropy change to find the change in entropy of the water as it freezes.

Apply the definition of entropy change to obtain:

$$\Delta S = \frac{\Delta Q}{T} = \frac{-mL_f}{T}$$

Substitute numerical values and evaluate ΔS:

$$\Delta S = \frac{-(18\,\text{g})(333.5\,\text{J/g})}{273\,\text{K}} = \boxed{-22.0\,\text{J/K}}$$

***47** ••

Picture the Problem The change in the entropy of the world resulting from the freezing of this water and the cooling of the ice formed is the sum of the entropy changes of the water-ice and the freezer. Note that, while the entropy of the water decreases, the entropy of the freezer increases.

Express the change in entropy of the universe resulting from this freezing and cooling process:

$$\Delta S_u = \Delta S_{water} + \Delta S_{freezer} \qquad (1)$$

Express ΔS_{water}:

$$\Delta S_{water} = \Delta S_{freezing} + \Delta S_{cooling} \qquad (2)$$

Express $\Delta S_{freezing}$:

$$\Delta S_{freezing} = \frac{-Q_{freezing}}{T_{freezing}} \qquad (3)$$

where the minus sign is a consequence of the fact that heat is leaving the water as it freezes.

Relate $Q_{freezing}$ to the latent heat of fusion and the mass of the water:

$$Q_{freezing} = mL_f$$

Substitute in equation (3) to obtain:

$$\Delta S_{freezing} = \frac{-mL_f}{T_{freezing}}$$

Express $\Delta S_{cooling}$:

$$\Delta S_{cooling} = mC_p \ln\left(\frac{T_f}{T_i}\right)$$

Substitute in equation (2) to obtain:

$$\Delta S_{water} = \frac{-mL_f}{T_{freezing}} + mC_p \ln\left(\frac{T_f}{T_i}\right)$$

Noting that the freezer gains heat (at 263 K) from the freezing water and cooling ice, express $\Delta S_{freezer}$:

$$\Delta S_{freezer} = \frac{\Delta Q_{ice}}{T_{freezer}} + \frac{\Delta Q_{cooling\ ice}}{T_{freezer}} = \frac{mL_f}{T_{freezer}} + \frac{mC_p \Delta T}{T_{freezer}}$$

Substitute for ΔS_{water} and $\Delta S_{freezer}$ in equation (1):

$$\Delta S_u = \frac{-mL_f}{T_{freezing}} + mC_p \ln\left(\frac{T_f}{T_i}\right) + \frac{mL_f}{T_{freezer}} + \frac{mC_p \Delta T}{T_{freezer}}$$

$$= m\left[\frac{-L_f}{T_{freezing}} + C_p \ln\left(\frac{T_f}{T_i}\right) + \frac{L_f}{T_{freezer}} + \frac{C_p \Delta T}{T_{freezer}}\right]$$

Substitute numerical values and evaluate ΔS_u:

$$\Delta S_u = (0.05\,\text{kg})\left[-\frac{333.5\times10^3\ \text{J/kg}}{273\,\text{K}} + (2100\,\text{J/kg}\cdot\text{K})\ln\left(\frac{263\,\text{K}}{273\,\text{K}}\right) + \frac{333.5\times10^3\ \text{J/kg}}{263\,\text{K}} + \frac{(2100\,\text{J/kg}\cdot\text{K})(273\,\text{K}-263\,\text{K})}{263\,\text{K}}\right]$$

$$= \boxed{2.40\,\text{J/K}}$$

and, because $\Delta S_u > 0$, the entropy of the universe increases.

48 •

Picture the Problem We can use the definition of entropy change and the first law of thermodynamics to express ΔS for the ideal gas as a function of its initial and final volumes.

(*a*) Use its definition to express the entropy change of the gas:

$$\Delta S = \frac{\Delta Q}{T}$$

Apply the first law of thermodynamics to the isothermal process:

$$\Delta Q = \Delta E_{\text{int}} - W_{\text{on}} = -\left[-nRT\ln\left(\frac{V_f}{V_i}\right)\right]$$

because $\Delta E_{\text{int}} = 0$ for an isothermal process.

Substitute to obtain:

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right)$$

$$= (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln\left(\frac{80\,\text{L}}{40\,\text{L}}\right)$$

$$= \boxed{11.5\,\text{J/K}}$$

(*b*) Because the process is reversible:

$$\Delta S_u = \boxed{0}$$

Remarks: The entropy change of the environment of the gas is –11.5 J/K.

49 •

Picture the Problem We can use the definition of entropy change and the 1st law of thermodynamics to express ΔS for the ideal gas as a function of its initial and final volumes.

(*a*) Use its definition to express the entropy change of the gas:

$$\Delta S = \frac{\Delta Q}{T}$$

Apply the first law of thermodynamics to the isothermal process:

$$\Delta Q = \Delta E_{\text{int}} - W_{\text{on}} = -\left[-nRT\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)\right]$$

because $\Delta E_{\text{int}} = 0$ for an isothermal process.

Substitute to obtain:

$$\Delta S = nR\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$$

$$= (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln\left(\frac{80\,\text{L}}{40\,\text{L}}\right)$$

$$= \boxed{11.5\,\text{J/K}}$$

(*b*) Because the process is not quasi-static, it is non-reversible:

$$\Delta S_{\text{u}} > \boxed{0}$$

50 •

Picture the Problem We can use the definition of entropy change to find the change in entropy of the water as it changes to steam.

Apply the definition of entropy change to obtain:

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL_{\text{v}}}{T}$$

Substitute numerical values and evaluate ΔS:

$$\Delta S = \frac{(1\,\text{kg})(2.26\,\text{MJ/kg})}{373\,\text{K}} = \boxed{6.06\,\text{kJ/K}}$$

51 •

Picture the Problem We can use the definition of entropy change to find the change in entropy of the ice as it melts.

Apply the definition of entropy change to obtain:

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL_{\text{f}}}{T}$$

Substitute numerical values and evaluate ΔS:

$$\Delta S = \frac{(1\,\text{kg})(333.5\,\text{kJ/kg})}{273\,\text{K}} = \boxed{1.22\,\text{kJ/K}}$$

52 ••

Picture the Problem We can use the first law of thermodynamics to find the change in the internal energy of the system and the change in the entropy of the system from the change in entropy of the hot- and cold-reservoirs.

(*a*) Apply the 1st law of thermodynamics to find the change in the internal energy of the system:

$$\Delta E_{int} = Q_{in} + W_{on} = (200\,\text{J} - 100\,\text{J}) - 50\,\text{J} = \boxed{50\,\text{J}}$$

(*b*) Express the change in entropy of the system as the sum of the entropy changes of the high- and low-temperature reservoirs:

$$\Delta S = \Delta S_h - \Delta S_c = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} = \frac{200\,\text{J}}{300\,\text{K}} - \frac{100\,\text{J}}{200\,\text{K}} = \boxed{0.167\,\text{J/K}}$$

(*c*) Because the process is reversible:

$$\Delta S_u = \boxed{0}$$

(*d*) Because S_{system} is a state function:

$$\Delta E_{int} = \boxed{50\,\text{J}},\ \Delta S = \boxed{0.167\,\text{J/K}},$$

and

$$\boxed{\Delta S_u > 0}$$

***53 ••**

Picture the Problem We can use the fact that the system returns to its original state to find the entropy change for the complete cycle. Because the entropy change for the complete cycle is the sum of the entropy changes for each process, we can find the temperature *T* from the entropy changes during the 1st two processes and the heat rejected during the third.

(*a*) Because *S* is a state function of the system:

$$\Delta S_{complete\,cycle} = \boxed{0}$$

(*b*) Relate the entropy change for the complete cycle to the entropy change for each process:

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T} = 0$$

Substitute numerical values to obtain:

$$\frac{300\,\text{J}}{300\,\text{K}} + \frac{200\,\text{J}}{400\,\text{K}} + \frac{-400\,\text{J}}{T} = 0$$

Solve for *T*:

$$T = \boxed{267\,\text{K}}$$

54 ••

Picture the Problem We can use the definition of entropy change and the 1st law of thermodynamics to express ΔS for the ideal gas as a function of its initial and final volumes.

(*a*) Use its definition to express the entropy change of the gas:

$$\Delta S = \frac{\Delta Q}{T}$$

Apply the first law of thermodynamics to the isothermal process:

$$\Delta Q_{in} = \Delta E_{int} - W_{on} = -\left[-nRT\ln\left(\frac{V_f}{V_i}\right)\right]$$

because $\Delta E_{int} = 0$ for free expansion.

Substitute to obtain:

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right)$$

$$= (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln\left(\frac{80\,\text{L}}{40\,\text{L}}\right)$$

$$= \boxed{11.5\,\text{J/K}}$$

(*b*) Because the process is irreversible, $S_u > 0$ and, because no heat is exchanged:

$$\Delta S_u = \boxed{11.5\,\text{J/K}}$$

55 ••

Picture the Problem Because the ice gains heat as it melts, its entropy change is positive and can be calculated from its definition. Because the temperature of the lake is just slightly greater than 0°C and the mass of water is so much greater than that of the block of ice, the absolute value of the entropy change of the lake will be approximately equal to the entropy change of the ice as it melts.

(*a*) Use the definition of entropy change to find the entropy change of the ice:

$$\Delta S_{ice} = \frac{mL_f}{T} = \frac{(200\,\text{kg})(333.5\,\text{kJ/kg})}{273\,\text{K}}$$

$$= \boxed{244\,\text{kJ/K}}$$

(*b*) Relate the entropy change of the lake to the entropy change of the ice:

$$\Delta S_{lake} \approx -\Delta S_{ice} = \boxed{-244\,\text{kJ/K}}$$

(*c*) Because the temperature of the lake is slightly greater than that of the ice, the magnitude of the entropy change of the lake is less than 244 kJ/K and the entropy change of the universe is greater than zero. The melting of the ice is an irreversible process and $\boxed{\Delta S_u > 0}$.

56 ••

Picture the Problem We can use conservation of energy to find the equilibrium temperature of the water and apply the equations for the entropy change during a melting process and for constant-pressure processes to find the entropy change of the universe, i.e., the piece of ice and the water in the insulated container.

(*a*) Apply conservation of energy to obtain:

$$Q_{\text{lost}} = Q_{\text{gained}}$$

or

$$Q_{\text{cooling water}} = Q_{\text{melting ice}} + Q_{\text{warming water}}$$

Substitute to relate the masses of the ice and water to their temperatures, specific heats, and the final temperature of the water:

$$(100\,\text{g})(1\,\text{cal/g}\cdot\text{C}^\circ)(100^\circ\text{C} - t) = (100\,\text{g})(79.7\,\text{cal/g}) + (100\,\text{g})(1\,\text{cal/g}\cdot\text{C}^\circ)(t)$$

Solve for t to obtain:

$$t = \boxed{10.2^\circ\text{C}}$$

(*b*) Express the entropy change of the universe:

$$\Delta S_{\text{u}} = \Delta S_{\text{ice}} + \Delta S_{\text{water}}$$

Using the expression for the entropy change for a constant-pressure process, express the entropy change of the melting ice and warming ice-water:

$$\Delta S_{\text{ice}} = \Delta S_{\text{melting ice}} + \Delta S_{\text{warming water}} = \frac{mL_{\text{f}}}{T_{\text{f}}} + mc_{\text{P}}\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right)$$

Substitute numerical values to obtain:

$$\Delta S_{\text{ice}} = \frac{(0.1\,\text{kg})(333.5\,\text{kJ/kg})}{273\,\text{K}} + (0.1\,\text{kg})(4.184\,\text{kJ/kg}\cdot\text{K})\ln\left(\frac{283.2\,\text{K}}{273\,\text{K}}\right) = 138\,\text{J/K}$$

Find the entropy change of the cooling water:

$$\Delta S_{\text{water}} = (0.1\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})\ln\left(\frac{283.2\,\text{K}}{373\,\text{K}}\right) = -115\,\text{J/K}$$

Substitute for ΔS_{ice} and ΔS_{water} and evaluate the entropy change of the universe:

$$\Delta S_{\text{u}} = 138\,\text{J/K} - 115\,\text{J/K} = \boxed{23.0\,\text{J/K}}$$

Remarks: The result that $\Delta S_u > 0$ tells us that this process is irreversible.

***57** ••

Picture the Problem We can use conservation of energy to find the equilibrium temperature of the water and apply the equations for the entropy change during a constant pressure process to find the entropy changes of the copper block, the water, and the universe.

(*a*) Using the equation for the entropy change during a constant-pressure process, express the entropy change of the copper block:

$$\Delta S_{Cu} = m_{Cu} c_{Cu} \ln\left(\frac{T_f}{T_i}\right) \qquad (1)$$

Apply conservation of energy to obtain:

$$Q_{lost} = Q_{gained}$$

or

$$Q_{copper\ block} = Q_{warming\ water}$$

Substitute to relate the masses of the block and water to their temperatures, specific heats, and the final temperature T_f of the water:

$$(1\,kg)(0.386\,kJ/kg\cdot K)(373.15\,K - T_f) = (4\,L)(1\,kg/L)(4.184\,kJ/kg\cdot K)(T_f - 273.15\,K)$$

Solve for T_f:

$$T_f = 275.40\,K$$

Substitute numerical values in equation (1) and evaluate ΔS_{Cu}:

$$\Delta S_{Cu} = (1\,kg)(0.386\,kJ/kg\cdot K)\ln\left(\frac{275.40\,K}{373.15\,K}\right) = \boxed{-117\,J/K}$$

(*b*) Express the entropy change of the water:

$$\Delta S_{water} = m_{water} c_{water} \ln\left(\frac{T_f}{T_i}\right)$$

Substitute numerical values and evaluate ΔS_{water}:

$$\Delta S_{water} = (4\,kg)(4.184\,kJ/kg\cdot K)\ln\left(\frac{275.40\,K}{273.15\,K}\right) = \boxed{137\,J/K}$$

(*c*) Substitute for ΔS_{Cu} and ΔS_{water} and evaluate the entropy change of the universe:

$$\Delta S_u = \Delta S_{Cu} + \Delta S_{water} = -117\,\text{J/K} + 137\,\text{J/K} = \boxed{20.3\,\text{J/K}}$$

Remarks: The result that $\Delta S_u > 0$ tells us that this process is irreversible.

58 ••

Picture the Problem Because the mass of the water in the lake is so much greater than the mass of the piece of lead, the temperature of the lake will increase only slightly and we can reasonably assume that its final temperature is 10°C. We can apply the equation for the entropy change during a constant pressure process to find the entropy changes of the piece of lead, the water in the lake, and the universe.

Express the entropy change of the universe in terms of the entropy changes of the lead and the water in the lake:

$$\Delta S_u = \Delta S_{Pb} + \Delta S_w$$

Using the equation for the entropy change during a constant-pressure process, express and evaluate the entropy change of the lead:

$$\Delta S_{Pb} = m_{Pb}c_{Pb}\ln\left(\frac{T_f}{T_i}\right) = (2\,\text{kg})(0.128\,\text{kJ/kg}\cdot\text{K})\ln\left(\frac{283.15\,\text{K}}{373.15\,\text{K}}\right) = -70.66\,\text{J/K}$$

Find the entropy change of the water in the lake:

$$\Delta S_w = \frac{Q_w}{T_w} = \frac{Q_{Pb}}{T_w} = \frac{m_{Pb}c_{Pb}\Delta T_{Pb}}{T_w} = \frac{(2\,\text{kg})(0.128\,\text{kJ/kg}\cdot\text{K})(90\,\text{K})}{283.15\,\text{K}} = 81.37\,\text{J/K}$$

Substitute and evaluate ΔS_u:

$$\Delta S_u = -70.66\,\text{J/K} + 81.37\,\text{J/K} = \boxed{10.7\,\text{J/K}}$$

59 ••

Picture the Problem Because the air temperature will not change appreciably as a result of this crash; we can assume that the kinetic energy of the car is transformed into heat at a temperature of 20°C. We can use the definition of entropy change to find the entropy change of the universe.

Express the entropy change of the universe as a consequence of the kinetic energy of the car being transformed into heat:

$$\Delta S_u = \frac{Q}{T} = \frac{\frac{1}{2}mv^2}{T}$$

Substitute numerical values and evaluate ΔS_u:

$$\Delta S_u = \frac{\frac{1}{2}(1500\,\text{kg})\left(100\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{293.15\,\text{K}} = \boxed{1.97\,\text{kJ/K}}$$

***60 ••**

Picture the Problem The total change in entropy resulting from the mixing of these gases is the sum of the changes in their entropies.

(*a*) Express the total change in entropy resulting from the mixing of the gases:

$$\Delta S = \Delta S_A + \Delta S_B$$

Express the change in entropy of each of the gases:

$$\Delta S_A = nR\ln\left(\frac{V_{fA}}{V_{iA}}\right)$$

and

$$\Delta S_B = nR\ln\left(\frac{V_{fB}}{V_{iB}}\right)$$

Because the initial and final volumes of the gases are the same and both volumes double:

$$\Delta S = 2nR\ln\left(\frac{V_f}{V_i}\right) = 2nR\ln(2)$$

Substitute numerical values and evaluate ΔS:

$$\Delta S = 2(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln(2) = \boxed{11.5\,\text{J/K}}$$

(*b*) Because the gas molecules are indistinguishable, the entropy doesn't change. A complete description of this phonomenon has been derived using quantum mechanics.

Entropy and Work Lost

***61 ••**

Picture the Problem We can find the entropy change of the universe from the entropy changes of the high- and low-temperature reservoirs. The maximum amount of the 500 J of heat that could be converted into work can be found from the maximum efficiency of an engine operating between the two reservoirs.

(*a*) Express the entropy change of the universe:

$$\Delta S_u = \Delta S_h + \Delta S_c = -\frac{Q}{T_h} + \frac{Q}{T_c}$$
$$= -Q\left(\frac{1}{T_h} - \frac{1}{T_c}\right)$$

Substitute numerical values and evaluate ΔS_u:

$$\Delta S_u = (-500\,\text{J})\left(\frac{1}{400\,\text{K}} - \frac{1}{300\,\text{K}}\right)$$
$$= \boxed{0.417\,\text{J/K}}$$

(*b*) Express the heat that could have been converted into work in terms of the maximum efficiency of an engine operating between the two reservoirs:

$$W = \varepsilon_{\text{max}} Q_h$$

Express the maximum efficiency of an engine operating between the two reservoir temperatures:

$$\varepsilon_{\text{max}} = \varepsilon_C = 1 - \frac{T_c}{T_h}$$

Substitute and evaluate *W*:

$$W = \left(1 - \frac{T_c}{T_h}\right)Q_h = \left(1 - \frac{300\,\text{K}}{400\,\text{K}}\right)(500\,\text{J})$$
$$= \boxed{125\,\text{J}}$$

62 ••

Picture the Problem Although in the adiabatic free expansion no heat is lost by the gas, the process is irreversible and the entropy of the gas increases. In the isothermal reversible process that returns the gas to its original state, the gas releases heat to the surroundings. However, because the process is reversible, the entropy change of the universe is zero. Consequently, the net entropy change is the negative of that of the gas in the isothermal compression.

(*a*) Relate the entropy change of the universe to the entropy change of the gas during the isothermal compression:

$$\Delta S_u = -\Delta S_{\text{gas}} = -nR\ln\left(\frac{V_f}{V_i}\right)$$

Substitute numerical values and evaluate ΔS_u:

$$\Delta S_u = -(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln\left(\frac{12.3\,\text{L}}{24.6\,\text{L}}\right) = \boxed{5.76\,\text{J/K}}$$

(*b*) To the extent that the initial expansion was isothermal and reversible, no work was done and none was wasted in the cycle.

(*c*) Express the wasted work in terms of T and the entropy change of the universe:

$$W_{\text{lost}} = T\Delta S_u = (300\,\text{K})(5.76\,\text{J/K}) = \boxed{1.73\,\text{kJ}}$$

General Problems

63 •

Picture the Problem We can use the definition of power to find the work done each cycle and the definition of efficiency to find the heat that is absorbed each cycle. Application of the first law of thermodynamics will yield the heat given off each cycle.

(*a*) Use the definition of power to relate the work done in each cycle to the period of each cycle:

$$W_{\text{cycle}} = P\Delta t = (200\,\text{W})(0.1\,\text{s}) = \boxed{20.0\,\text{J}}$$

(*b*) Express the heat absorbed in each cycle in terms of the work done and the efficiency of the engine:

$$Q_{\text{h,cycle}} = \frac{W_{\text{cycle}}}{\varepsilon} = \frac{20\,\text{J}}{0.3} = \boxed{66.7\,\text{J}}$$

Apply the 1st law of thermodynamics to find the heat given off in each cycle:

$$\left|Q_{\text{c,cycle}}\right| = Q_{\text{h,cycle}} - W = 66.7\,\text{J} - 20\,\text{J} = \boxed{46.7\,\text{J}}$$

64 •

Picture the Problem We can use their definitions to find the efficiency of the engine and that of a Carnot engine operating between the same reservoirs.

(*a*) Apply the definition of efficiency:

$$\varepsilon = \frac{W}{Q_h} = 1 - \frac{|Q_c|}{Q_h} = 1 - \frac{125\,\text{J}}{150\,\text{J}} = \boxed{16.7\%}$$

(*b*) Find the efficiency of a Carnot engine operating between the same reservoirs:

$$\varepsilon_{\rm C} = 1 - \frac{T_{\rm c}}{T_{\rm h}} = 1 - \frac{293.15}{373.15} = 21.4\%$$

Express the ratio of the two efficiencies:

$$\frac{\varepsilon}{\varepsilon_{\rm C}} = \frac{16.7\%}{21.4\%} = \boxed{0.780}$$

65 •

Picture the Problem We can use the definition of efficiency to find the work done by the engine during each cycle and the first law of thermodynamics to find the heat exhausted in each cycle.

(*a*) Express the efficiency of the engine in terms of the efficiency of a Carnot engine working between the same reservoirs:

$$\varepsilon = 0.85\varepsilon_{\rm C} = 0.85\left(1 - \frac{T_{\rm c}}{T_{\rm h}}\right)$$

$$= 0.85\left(1 - \frac{200\,{\rm K}}{500\,{\rm K}}\right) = \boxed{51.0\%}$$

(*b*) Use the definition of efficiency to find the work done in each cycle:

$$W = \varepsilon Q_{\rm h} = 0.51(200\,{\rm kJ}) = \boxed{102\,{\rm kJ}}$$

(*c*) Apply the first law of thermodynamics to the cycle to obtain:

$$\left|Q_{\rm c,cycle}\right| = Q_{\rm h,cycle} - W = 200\,{\rm kJ} - 102\,{\rm kJ}$$

$$= \boxed{98.0\,{\rm kJ}}$$

***66** ••

Picture the Problem We can use the expression for the Carnot efficiency of the plant to find the highest efficiency this plant can have. We can then use this efficiency to find the power that must be supplied to the plant to generate 1 GW of power and, from this value, the power that is wasted. The rate at which heat is being delivered to the river is related to the requisite flow rate of the river by $dQ/dt = c\Delta T\rho\, dV/dt$.

(*a*) Express the Carnot efficiency of a plant operating between temperatures $T_{\rm c}$ and $T_{\rm h}$:

$$\varepsilon_{\max} = \varepsilon_{\rm C} = 1 - \frac{T_{\rm c}}{T_{\rm h}}$$

Substitute numerical values and evaluate $\varepsilon_{\rm C}$:

$$\varepsilon_{\max} = 1 - \frac{298\,{\rm K}}{500\,{\rm K}} = \boxed{0.404}$$

(*c*) Find the power that must be supplied, at 40.4% efficiency, to produce an output of 1 GW:

$$P_{\rm supplied} = \frac{P_{\rm output}}{\varepsilon_{\max}} = \frac{1\,{\rm GW}}{0.404} = \boxed{2.48\,{\rm GW}}$$

(*b*) Relate the wasted power to the power generated and the power supplied:

$$P_{\text{wasted}} = P_{\text{supplied}} - P_{\text{generated}}$$

Substitute numerical values and evaluate P_{wasted}:

$$P_{\text{wasted}} = 2.48\,\text{GW} - 1\,\text{GW} = \boxed{1.48\,\text{GW}}$$

(*d*) Express the rate at which heat is being dumped into the river:

$$\frac{dQ}{dt} = c\Delta T\frac{dm}{dt} = c\Delta T\frac{d}{dt}(\rho V) = c\Delta T\rho\frac{dV}{dt}$$

Solve for the flow rate *dV*/*dt* of the river:

$$\frac{dV}{dt} = \frac{dQ/dt}{c\Delta T\rho}$$

Substitute numerical values (see Table 19-1 for the specific heat of water) and evaluate *dV*/*dt*:

$$\frac{dV}{dt} = \frac{1.48\times10^{9}\ \text{J/s}}{(4180\,\text{J/kg})(0.5\,\text{K})(10^{3}\ \text{kg/m}^{3})} = 708\,\text{m}^{3}/\text{s} = \boxed{7.08\times10^{5}\ \text{L/s}}$$

67 •

Picture the Problem We can find the rate at which the house contributes to the increase in the entropy of the universe from the ratio of ΔS to Δt.

Using the definition of entropy change, express the rate of increase in the entropy of the universe:

$$\frac{\Delta S}{\Delta t} = \frac{\Delta Q/T}{\Delta t} = \frac{\Delta Q/\Delta t}{T}$$

Substitute numerical values and evaluate $\Delta S/\Delta t$:

$$\frac{\Delta S}{\Delta t} = \frac{30\,\text{kW}}{266\,\text{K}} = \boxed{113\,\text{W/K}}$$

68 ••

Picture the Problem Because the cycle represented in Figure 19-12 is a Carnot cycle, its efficiency is that of a Carnot engine operating between the temperatures of its isotherms.

Express the Carnot efficiency of the cycle:

$$\varepsilon_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}$$

Substitute numerical values and evaluate ε_{C}:

$$\varepsilon_{\text{C}} = 1 - \frac{300\,\text{K}}{750\,\text{K}} = \boxed{60.0\%}$$

69 ••

Picture the Problem All 500 J of mechanical energy are lost, i.e., transformed into heat in process (1). For process (2), we can find the heat that would be converted to work by a Carnot engine operating between the given temperatures and subtract amount of work from 1 kJ to find the energy that is lost. In part (*b*) we can use its definition to find the change in entropy for each process.

(*a*) For process (2):

$$W_{2,\text{max}} = W_{\text{recovered}} = \varepsilon_{\text{C}} Q_{\text{in}}$$

Find the efficiency of a Carnot engine operating between 400 K and 300 K:

$$\varepsilon_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}} = 1 - \frac{300\,K}{400\,K} = 0.25$$

Substitute to obtain:

$$W_{\text{recovered}} = 0.25(1\,\text{kJ}) = 250\,\text{J}$$

or

750 J are lost.

$$\boxed{\text{Process (1) is more wasteful of } \textit{mechanical} \text{ energy. Process (2) is more wasteful of } \textit{total} \text{ energy.}}$$

(*b*) Find the change in entropy of the universe for process (1):

$$\Delta S_1 = \frac{\Delta Q}{T} = \frac{500\,\text{J}}{300\,\text{K}} = \boxed{1.67\,\text{J/K}}$$

Express the change in entropy of the universe for process (2):

$$\Delta S_2 = \Delta S_{\text{h}} + \Delta S_{\text{c}} = -\frac{\Delta Q}{T_{\text{h}}} + \frac{\Delta Q}{T_{\text{c}}}$$

$$= \Delta Q\left(\frac{1}{T_{\text{c}}} - \frac{1}{T_{\text{h}}}\right)$$

Substitute numerical values and evaluate ΔS_2:

$$\Delta S_2 = (1\,\text{kJ})\left(\frac{1}{300\,\text{K}} - \frac{1}{400\,\text{K}}\right)$$

$$= \boxed{0.833\,\text{J/K}}$$

70 ••

Picture the Problem Denote the three states of the gas as 1, 2, and 3 with 1 being the initial state. We can use the ideal-gas law and the equation of state for an adiabatic process to find the temperatures, volumes, and pressures at points 1, 2, and 3. To find the work done during each cycle, we can use the equations for the work done during isothermal, isobaric, and adiabatic processes. Finally, we find the efficiency of the cycle from the work done each cycle and the heat that enters the system during the isothermal expansion.

(*a*) Apply the ideal-gas law to the isothermal expansion 1→2 to find P_2:

$$P_2 = P_1\frac{V_1}{V_2} = (16\,\text{atm})\frac{1\,\text{L}}{4\,\text{L}} = 4\,\text{atm}$$

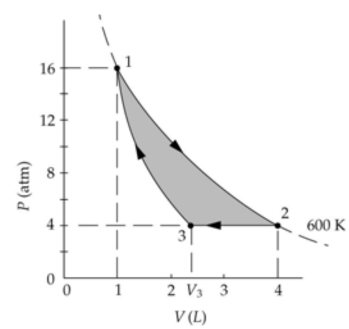
Apply the equation of state for an adiabatic process to relate the pressures and volumes at 1 and 3:

$$P_1V_1^{\gamma} = P_3V_3^{\gamma}$$

and

$$V_3 = V_1\left(\frac{P_1}{P_3}\right)^{1/\gamma} = (1\,\text{L})\left(\frac{16\,\text{atm}}{4\,\text{atm}}\right)^{1/1.67}$$
$$= 2.29\,\text{L}$$

The *PV* diagram is shown to the right:



(*b*) From (*a*) we have:

$$V_3 = \boxed{2.29\,\text{L}}$$

Apply the equation of state for an adiabatic process (γ=1.67) to relate the temperatures and volumes at 1 and 3:

$$T_3V_3^{\gamma-1} = T_1V_1^{\gamma-1}$$

and

$$T_3 = T_1\left(\frac{V_1}{V_3}\right)^{\gamma-1} = (600\,\text{K})\left(\frac{1\,\text{L}}{2.29\,\text{L}}\right)^{1.67-1}$$
$$= \boxed{344\,\text{K}}$$

(*c*) Express the work done each cycle:

$$W = W_{1\to2} + W_{2\to3} + W_{3\to1} \quad (1)$$

For the process 1→2:

$$W_{1\to2} = nRT_1\ln\left(\frac{V_2}{V_1}\right) = P_1V_1\ln\left(\frac{V_2}{V_1}\right)$$
$$= (16\,\text{atm})(1\,\text{L})\ln\left(\frac{4\,\text{L}}{1\,\text{L}}\right)$$
$$= 22.2\,\text{atm}\cdot\text{L}$$

For the process 2→3:

$$W_{2\to3} = P_2\Delta V_{2\to3}$$
$$= (4\,\text{atm})(2.29\,\text{L} - 4\,\text{L})$$
$$= -6.84\,\text{atm}\cdot\text{L}$$

For the process 3→1:

$$W_{3\to1} = -C_V \Delta T_{3\to1} = -\tfrac{3}{2} nR(T_1 - T_3)$$
$$= -\tfrac{3}{2}(P_1V_1 - P_3V_3)$$
$$= -\tfrac{3}{2}[(16\,\text{atm})(1\,\text{L}) - (4\,\text{atm})(2.29\,\text{L})]$$
$$= -10.3\,\text{atm}\cdot\text{L}$$

Substitute numerical values in equation (1) and evaluate W:

$$W = 22.2\,\text{atm}\cdot\text{L} - 6.84\,\text{atm}\cdot\text{L} - 10.3\,\text{atm}\cdot\text{L}$$
$$= \boxed{5.06\,\text{atm}\cdot\text{L}}$$

(*d*) Using its definition, express and evaluate the efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_{\text{in}}} = \frac{W}{Q_{1\to2}} = \frac{W}{W_{1\to2}}$$
$$= \frac{5.06\,\text{atm}\cdot\text{L}}{22.2\,\text{atm}\cdot\text{L}} = \boxed{22.8\%}$$

***71 ••**

Picture the Problem We can express the temperature of the cold reservoir as a function of the Carnot efficiency of an ideal engine and, given that the efficiency of the heat engine is half that of a Carnot engine, relate T_c to the work done by and the heat input to the real heat engine.

Using its definition, relate the efficiency of a Carnot engine working between the same reservoirs to the temperature of the cold reservoir:

$$\varepsilon_C = 1 - \frac{T_c}{T_h}$$

Solve for T_c:

$$T_c = T_h(1 - \varepsilon_C)$$

Relate the efficiency of the heat engine to that of a Carnot engine working between the same temperatures:

$$\varepsilon = \frac{W}{Q_{\text{in}}} = \tfrac{1}{2}\varepsilon_C \text{ or } \varepsilon_C = \frac{2W}{Q_{\text{in}}}$$

Substitute to obtain:

$$T_c = T_h\left(1 - \frac{2W}{Q_{\text{in}}}\right)$$

The work done by the gas in expanding the balloon is:

$$W = P\Delta V = (1\,\text{atm})(4\,\text{L}) = 4\,\text{atm}\cdot\text{L}$$

Substitute numerical values and evaluate T_c:

$$T_c = (393.15\,\text{K})\left(1-\frac{2\left(4\,\text{atm}\cdot\text{L}\times\frac{101.325\,\text{J}}{\text{atm}\cdot\text{L}}\right)}{4\,\text{kJ}}\right) = \boxed{313\,\text{K}}$$

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Picture the Problem We can use the definitions of the COP and ε_C to show that their relationship is COP = $T_c/(\varepsilon_C T_h)$.

Using the definition of the COP, relate the heat removed from the cold reservoir to the work done each cycle:

$$\text{COP} = \frac{Q_c}{W}$$

Apply energy conservation to relate Q_c, Q_h, and W:

$$Q_c = Q_h - W$$

Substitute to obtain:

$$\text{COP} = \frac{Q_h - W}{W}$$

Divide numerator and denominator by Q_h and simplify to obtain:

$$\text{COP} = \frac{Q_h - W}{W} = \frac{1-\frac{W}{Q_h}}{\frac{W}{Q_h}}$$

Because $\varepsilon_C = W/Q_h$:

$$\text{COP} = \frac{1-\varepsilon_C}{\varepsilon_C} = \frac{1-\left(1-\frac{T_c}{T_h}\right)}{\varepsilon_C} = \frac{\frac{T_c}{T_h}}{\varepsilon_C} = \boxed{\frac{T_c}{\varepsilon_C T_h}}$$

73 ••

Picture the Problem We can use the definition of the COP to express the work the motor must do to maintain the temperature of the freezer in terms of the rate at which heat flows into the freezer. Differentiation of this expression with respect to time will yield an expression for the power of the motor that is needed to maintain the temperature in the freezer.

Using the definition of the COP, relate the heat that must be removed from the freezer to the work done by the motor:

$$\text{COP} = \frac{Q_c}{W}$$

Solve for W:

$$W = \frac{Q_c}{\text{COP}}$$

Differentiate this expression with respect to time to express the power of the motor:

$$P = \frac{dW}{dt} = \frac{dQ_c/dt}{\text{COP}}$$

Express the maximum COP of the motor:

$$\text{COP}_{\max} = \frac{T_c}{\Delta T}$$

Substitute to obtain:

$$P = \frac{dQ_c}{dt}\frac{\Delta T}{T_c}$$

Substitute numerical values and evaluate P:

$$P = (50\,\text{W})\left(\frac{50\,\text{K}}{250\,\text{K}}\right) = \boxed{10.0\,\text{W}}$$

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Picture the Problem We can use the ideal-gas law to find the unknown temperatures, pressures, and volumes at points A, B, and C and then find the work done by the gas and the efficiency of the cycle by using the expressions for the work done on or by the gas and the heat that enters the system for the isobaric, adiabatic, and isothermal processes of the cycle.

(*a*) Apply the ideal-gas law to find the volume of the gas at A:

$$V_A = \frac{nRT_A}{P_A} = \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{5\,\text{atm}\times\dfrac{101.325\,\text{kPa}}{\text{atm}}} = \boxed{19.7\,\text{L}}$$

(*b*) We're given that:

$$V_B = 2V_A = 2(19.7\,\text{L}) = \boxed{39.4\,\text{L}}$$

Apply the ideal-gas law to this isobaric process to obtain:

$$T_B = T_A\frac{V_B}{V_A} = (600\,\text{K})\frac{2V_A}{V_A} = \boxed{1200\,\text{K}}$$

(*c*) Because the process C→A is isothermal:

$$T_C = T_A = \boxed{600\,\text{K}}$$

(*d*) Apply the equation of state for an adiabatic process ($\gamma = 1.4$) to find the volume of the gas at C:

$$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$$

and

$$V_C = V_B\left(\frac{T_B}{T_C}\right)^{\frac{1}{\gamma-1}} = (39.4\,\text{L})\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{\frac{1}{1.4-1}} = \boxed{223\,\text{L}}$$

(*e*) Express and evaluate the work done by the gas during the isobaric process AB:

$$W_{A-B} = P_A(V_B - V_A) = P_A(2V_A - V_A) = P_A V_A = (5\,\text{atm})(19.7\,\text{L}) = 98.50\,\text{atm}\cdot\text{L}\times\frac{101.325\,\text{J}}{\text{atm}\cdot\text{L}} = \boxed{9.98\,\text{kJ}}$$

Apply the first law of thermodynamics to express the work done by the gas during the adiabatic expansion BC:

$$W_{\text{on, B-C}} = \Delta E_{\text{int,B-C}} - Q_{\text{in, B-C}} = \Delta E_{\text{int, B-C}} - 0 = \Delta E_{\text{int, B-C}} = -nc_V\Delta T_{B-C} = -\tfrac{5}{2}nR\Delta T_{B-C}$$

Substitute numerical values and evaluate W_{B-C}:

$$W_{B-C} = -\tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 1200\,\text{K}) = \boxed{24.9\,\text{kJ}}$$

The work done by the gas during the isothermal compression CA is:

$$W_{C-A} = nRT_C\ln\left(\frac{V_A}{V_C}\right) = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})\ln\left(\frac{19.7\,\text{L}}{223\,\text{L}}\right) = \boxed{-24.2\,\text{kJ}}$$

(*f*) The heat absorbed during the isobaric expansion AB is:

$$Q_{A-B} = nc_P\Delta T_{A-B} = \tfrac{7}{2}nR\Delta T_{A-B} = \tfrac{7}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(1200\,\text{K} - 600\,\text{K}) = \boxed{34.9\,\text{kJ}}$$

The heat absorbed during the adiabatic expansion BC is:

$$Q_{B-C} = \boxed{0}$$

Use the first law of thermodynamics to find the heat absorbed during the isothermal compression CA:

$$Q_{C-A} = W_{C-A} + \Delta E_{int,\,C-A} = W_{C-A}$$
$$= \boxed{-24.2\,\text{kJ}}$$

because $\Delta E_{int,C-A}$ = 0 for an isothermal process.

(*g*) The thermodynamic efficiency ε is:

$$\varepsilon = \frac{W}{Q_{in}} = \frac{W_{A-B} + W_{B-C} + W_{C-A}}{Q_{A-B}}$$

Substitute numerical values and evaluate ε:

$$\varepsilon = \frac{9.98\,\text{kJ} + 24.9\,\text{kJ} - 24.2\,\text{kJ}}{34.9\,\text{kJ}}$$
$$= \boxed{30.6\%}$$

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Picture the Problem We can use the ideal-gas law to find the unknown temperatures, pressures, and volumes at points B, C, and D and then find the work done by the gas and the efficiency of the cycle by using the expressions for the work done on or by the gas and the heat that enters the system for the various thermodynamic processes of the cycle.

(*a*) Apply the ideal-gas law for a fixed amount of gas to the isothermal process AB:

$$P_B = P_A\frac{V_A}{V_B} = (5\,\text{atm})\frac{V_A}{2V_A}$$
$$= 2.50\,\text{atm} \times \frac{101.325\,\text{kPa}}{1\,\text{atm}}$$
$$= \boxed{253\,\text{kPa}}$$

(*b*) Apply the ideal-gas law for a fixed amount of gas to the adiabatic process BC:

$$T_C = T_B\frac{P_C V_C}{P_B V_B}$$

Using the ideal-gas law, find the volume at B:

$$V_B = \frac{nRT_B}{P_B}$$
$$= \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{253\,\text{kPa}}$$
$$= 39.43\,\text{L}$$

Use the equation of state for an adiabatic process and γ = 1.4 to find the volume occupied by the gas at C:

$$V_C = V_B\left(\frac{P_B}{P_C}\right)^{1/\gamma} = (39.43\,\text{L})\left(\frac{2.5\,\text{atm}}{1\,\text{atm}}\right)^{1/1.4}$$
$$= 75.87\,\text{L}$$

Substitute and evaluate T_C:

$$T_C = (600\,\text{K})\frac{(1\,\text{atm})(75.87\,\text{L})}{(2.5\,\text{atm})(39.43\,\text{L})} = \boxed{462\,\text{K}}$$

(*c*) Express the work done by the gas in one cycle:

$$W = W_{A-B} + W_{B-C} + W_{C-D} + W_{D-A}$$

The work done during the isothermal expansion AB is:

$$W_{A-B} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})\ln\left(\frac{2V_A}{V_A}\right) = 6.915\,\text{kJ}$$

The work done during the adiabatic expansion BC is:

$$W_{B-C} = -C_V\Delta T_{B-C} = -\tfrac{5}{2}nR\Delta T_{B-C} = -\tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(462\,\text{K} - 600\,\text{K})$$
$$= 5.737\,\text{kJ}$$

The work done during the isobaric compression CD is:

$$W_{C-D} = P_C(V_D - V_C) = (1\,\text{atm})(19.7\,\text{L} - 75.87\,\text{L}) = -56.17\,\text{atm}\cdot\text{L} \times \frac{101.325\,\text{J}}{\text{atm}\cdot\text{L}}$$
$$= -5.690\,\text{kJ}$$

Express and evaluate the work done during the constant-volume process DA:

$$W_{D-A} = 0$$

Substitute numerical values and evaluate W:

$$W = 6.915\,\text{kJ} + 5.737\,\text{kJ} - 5.690\,\text{kJ} + 0 = \boxed{6.96\,\text{kJ}}$$

Using its definition, express the thermodynamic efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_{in}} = \frac{W}{Q_{A-B} + Q_{D-A}} \qquad (1)$$

Express and evaluate the heat entering the system during the isothermal process AB:

$$Q_{A-B} = W_{A-B} + \Delta E_{\text{int, A-B}} = W_{A-B} = 6.915\,\text{kJ}$$

Because $\Delta E_{int} = 0$ for an isothermal process.

Express the heat entering the system

$$Q_{D-A} = C_V\Delta T_{D-A} = \tfrac{5}{2}nR\Delta T_{D-A}$$

during the constant-volume process DA:

Apply the ideal-gas law to the constant-volume process DA to obtain:

$$T_D = T_A \frac{P_D}{P_A} = (600\,\text{K})\frac{1\,\text{atm}}{5\,\text{atm}} = 120\,\text{K}$$

The heat entering the system during the process DA is:

$$Q_{D-A} = \tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 120\,\text{K}) = 20.0\,\text{kJ}$$

Substitute numerical values in equation (1) and evaluate the efficiency of the cycle:

$$\varepsilon = \frac{6.975\,\text{kJ}}{6.915\,\text{kJ} + 20.0\,\text{kJ}} = \boxed{25.9\%}$$

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Picture the Problem We can use the ideal-gas law to find the unknown temperatures, pressures, and volumes at points A, B, and C and then find the work done by the gas and the efficiency of the cycle by using the expressions for the work done on or by the gas and the heat that enters the system for the isobaric, adiabatic, and isothermal processes of the cycle.

(*a*) Apply the ideal-gas law to find the volume of the gas at A:

$$V_A = \frac{nRT_A}{P_A} = \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{5\,\text{atm}\times\frac{101.325\,\text{kPa}}{\text{atm}}} = \boxed{19.7\,\text{L}}$$

(*b*) We're given that:

$$V_B = 2V_A = 2(19.7\,\text{L}) = \boxed{39.4\,\text{L}}$$

Apply the ideal-gas law to this isobaric process to obtain:

$$T_B = T_A \frac{V_B}{V_A} = (600\,\text{K})\frac{2V_A}{V_A} = \boxed{1200\,\text{K}}$$

(*c*) Because the process CA is isothermal:

$$T_C = T_A = \boxed{600\,\text{K}}$$

(*d*) Apply the equation of state for an adiabatic process ($\gamma = 5/3$) to find the volume of the gas at C:

$$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$$

and

$$V_C = V_B\left(\frac{T_B}{T_C}\right)^{\frac{1}{\gamma-1}} = (39.4\,\text{L})\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{\frac{3}{2}}$$
$$= \boxed{111\,\text{L}}$$

(*e*) Express and evaluate the work done by the gas during the isobaric process AB:

$$W_{A-B} = P_A(V_B - V_A) = P_A(2V_A - V_A)$$
$$= P_AV_A = (5\,\text{atm})(19.7\,\text{L})$$
$$= 98.50\,\text{atm}\cdot\text{L} \times \frac{101.325\,\text{J}}{\text{atm}\cdot\text{L}}$$
$$= \boxed{9.98\,\text{kJ}}$$

Apply the first law of thermodynamics to express the work done by the gas during the adiabatic expansion BC:

$$W_{\text{on, B-C}} = \Delta E_{\text{int, B-C}} - Q_{\text{in, B-C}}$$
$$= \Delta E_{\text{int, B-C}} - 0$$
$$= \Delta E_{\text{int, B-C}} = -(nc_V\Delta T_{B-C})$$
$$= -\tfrac{3}{2}nR\Delta T_{B-C}$$

Substitute numerical values and evaluate W_{B-C}:

$$W_{\text{on, B-C}} = -\tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 1200\,\text{K}) = \boxed{14.9\,\text{kJ}}$$

The work done by the gas during the isothermal compression CA is:

$$W_{C-A} = nRT_C\ln\left(\frac{V_A}{V_C}\right) = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})\ln\left(\frac{19.7\,\text{L}}{111\,\text{L}}\right)$$
$$= \boxed{-17.2\,\text{kJ}}$$

(*f*) The heat absorbed during the isobaric expansion AB is:

$$Q_{\text{in, A-B}} = nc_P\Delta T_{A-B} = \tfrac{5}{2}nR\Delta T_{A-B} = \tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(1200\,\text{K} - 600\,\text{K})$$
$$= \boxed{24.9\,\text{kJ}}$$

Express and evaluate the heat absorbed during the adiabatic expansion BC:

$$Q_{B-C} = \boxed{0}$$

Use the first law of thermodynamics to express and evaluate the heat absorbed during the isothermal compression CA:

$$Q_{\text{C-A}} = W_{\text{C-A}} + \Delta E_{\text{int, C-A}} = W_{\text{C-A}}$$
$$= \boxed{-17.2\,\text{kJ}}$$

because $\Delta E_{\text{int}} = 0$ for an isothermal process.

(*g*) The definition of thermodynamic efficiency ε is:

$$\varepsilon = \frac{W}{Q_{\text{in}}} = \frac{W_{\text{A-B}} + W_{\text{B-C}} + W_{\text{C-A}}}{Q_{\text{A-B}}}$$

Substitute numerical values and evaluate ε:

$$\varepsilon = \frac{9.98\,\text{kJ} + 14.9\,\text{kJ} - 17.2\,\text{kJ}}{24.9\,\text{kJ}}$$
$$= \boxed{30.8\%}$$

77 ••

Picture the Problem We can use the ideal-gas law to find the unknown temperatures, pressures, and volumes at points B, C, and D and then find the work done by the gas and the efficiency of the cycle by using the expressions for the work done on or by the gas and the heat that enters the system for the various thermodynamic processes of the cycle.

(*a*) Apply the ideal-gas law for a fixed amount of gas to the isothermal process AB:

$$P_{\text{B}} = P_{\text{A}}\frac{V_{\text{A}}}{V_{\text{B}}} = (5\,\text{atm})\frac{V_{\text{A}}}{2V_{\text{A}}}$$
$$= 2.50\,\text{atm} \times \frac{101.3\,\text{kPa}}{1\,\text{atm}} = \boxed{253\,\text{kPa}}$$

(*b*) Apply the ideal-gas law for a fixed amount of gas to the adiabatic process BC:

$$T_{\text{C}} = T_{\text{B}}\frac{P_{\text{C}}V_{\text{C}}}{P_{\text{B}}V_{\text{B}}}$$

Using the ideal-gas law, find the volume at B:

$$V_{\text{B}} = \frac{nRT_{\text{B}}}{P_{\text{B}}}$$
$$= \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{253\,\text{kPa}}$$
$$= 39.43\,\text{L}$$

Use the equation of state for an adiabatic process and $\gamma = 5/3$ to find the volume occupied by the gas at C:

$$V_{\text{C}} = V_{\text{B}}\left(\frac{P_{\text{B}}}{P_{\text{C}}}\right)^{1/\gamma} = (39.43\,\text{L})\left(\frac{2.5\,\text{atm}}{1\,\text{atm}}\right)^{3/5}$$
$$= 68.33\,\text{L}$$

Substitute and evaluate T_C:

$$T_C = (600\,\text{K})\frac{(1\,\text{atm})(68.33\,\text{L})}{(2.5\,\text{atm})(39.43\,\text{L})} = \boxed{416\,\text{K}}$$

(*c*) Express the work done by the gas in one cycle:

$$W = W_{A-B} + W_{B-C} + W_{C-D} + W_{D-A} \quad (1)$$

The work done during the isothermal expansion AB is:

$$W_{A-B} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K})\ln\left(\frac{2V_A}{V_A}\right) = 6.915\,\text{kJ}$$

The work done during the adiabatic expansion BC is:

$$W_{B-C} = -C_V\Delta T_{B-C} = -\tfrac{5}{2}nR\Delta T_{B-C} = -\tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(416\,\text{K} - 600\,\text{K}) = 4.589\,\text{kJ}$$

The work done during the isobaric compression CD is:

$$W_{C-D} = P_C(V_D - V_C) = (1\,\text{atm})(19.7\,\text{L} - 68.33\,\text{L}) = -48.63\,\text{atm}\cdot\text{L}\times\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}} = -4.926\,\text{kJ}$$

The work done during the constant-volume process DA is:

$$W_{D-A} = 0$$

Substitute numerical values in equation (1) to obtain:

$$W = 6.915\,\text{kJ} + 4.589\,\text{kJ} - 4.926\,\text{kJ} + 0 = \boxed{6.58\,\text{kJ}}$$

The thermodynamic efficiency of the cycle is given by:

$$\varepsilon = \frac{W}{Q_{in}} = \frac{W}{Q_{A-B} + Q_{D-A}} \quad (2)$$

The heat entering the system during the isothermal process AB is:

$$Q_{A-B} = W_{A-B} + \Delta E_{int,\,A-B} = W_{A-B} = 6.915\,\text{kJ}$$

because $\Delta E_{int} = 0$ for an isothermal process.

The heat entering the system during the constant-volume process DA is:

$$Q_{D-A} = C_V\Delta T_{D-A} = \tfrac{3}{2}nR\Delta T_{D-A}$$

Apply the ideal-gas law to the constant-volume process DA to obtain:

$$T_D = T_A \frac{P_D}{P_A} = (600\,\text{K})\frac{1\,\text{atm}}{5\,\text{atm}} = 120\,\text{K}$$

The heat entering the system during the process DA is:

$$Q_{D-A} = \tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 120\,\text{K}) = 12.0\,\text{kJ}$$

Substitute numerical values in equation (2) and evaluate the efficiency of the cycle:

$$\varepsilon = \frac{6.58\,\text{kJ}}{6.915\,\text{kJ} + 12.0\,\text{kJ}} = \boxed{34.8\%}$$

78 ••

Picture the Problem We can express the efficiency of the Otto cycle using the result from Example 19-2. We can apply the relation $TV^{\gamma-1}$ = constant to the adiabatic processes of the Otto cycle to relate the end-point temperatures to the volumes occupied by the gas at these points and eliminate the temperatures at *c* and *d*. We can use the ideal-gas law to find the highest temperature of the gas during its cycle and use this temperature to express the efficiency of a Carnot engine. Finally, we can compare the efficiencies by examining their ratio.

The efficiency of the Otto engine is given in Example 19-2:

$$\varepsilon_O = 1 - \frac{T_d - T_a}{T_c - T_b} \qquad (1)$$

where the subscripts refer to the various points of the cycle as shown in Figure 19-3.

Apply the relation $TV^{\gamma-1}$ = constant to the adiabatic process $a\rightarrow b$ to obtain:

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1}$$

Apply the relation $TV^{\gamma-1}$ = constant to the adiabatic process $c\rightarrow d$ to obtain:

$$T_c = T_d\left(\frac{V_d}{V_c}\right)^{\gamma-1}$$

Subtract the first of these equations from the second to obtain:

$$T_c - T_b = T_d\left(\frac{V_d}{V_c}\right)^{\gamma-1} - T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1}$$

In the Otto cycle, $V_a = V_d$ and $V_c = V_b$. Substitute to obtain:

$$T_c - T_b = T_d\left(\frac{V_a}{V_b}\right)^{\gamma-1} - T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1}$$

$$= (T_d - T_a)\left(\frac{V_a}{V_b}\right)^{\gamma-1}$$

Substitute in equation (1) and simplify to obtain:

$$\varepsilon_{\mathrm{O}} = 1 - \frac{T_d - T_a}{(T_d - T_a)\left(\frac{V_a}{V_b}\right)^{\gamma-1}}$$

$$= 1 - \left(\frac{V_b}{V_a}\right)^{\gamma-1} = 1 - \frac{T_a}{T_b}$$

Note that, while T_a is the lowest temperature of the cycle, T_b is not the highest temperature.

Apply the ideal-gas law to c and b to obtain an expression for the cycle's highest temperature T_c:

$$\frac{P_c}{T_c} = \frac{P_b}{T_b} \Rightarrow T_c = T_b\frac{P_c}{P_b} > T_b$$

Express the efficiency of a Carnot engine operating between the maximum and minimum temperatures of the Otto cycle:

$$\varepsilon_{\mathrm{C}} = 1 - \frac{T_a}{T_c}$$

Express the ratio of the efficiency of a Carnot engine to the efficiency of an Otto engine operating between the same temperatures:

$$\frac{\varepsilon_{\mathrm{C}}}{\varepsilon_{\mathrm{O}}} = \frac{1 - \frac{T_a}{T_c}}{1 - \frac{T_a}{T_b}} > \boxed{1}$$ because $T_c > T_b$.

***79** •••

Picture the Problem We can use $nR = C_{\mathrm{P}} - C_{\mathrm{V}}$, $\gamma = C_{\mathrm{P}}/C_{\mathrm{V}}$, and $TV^{\gamma-1}$ = a constant to show that the entropy change for a quasi-static adiabatic expansion that proceeds from state (V_1,T_1) to state (V_2,T_2) is zero.

Express the entropy change for a general process that proceeds from state 1 to state 2:

$$\Delta S = C_{\mathrm{V}}\ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$$

For an adiabatic process:

$$\frac{T_2}{T_1}=\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Substitute and simplify to obtain:

$$\Delta S = C_V \ln\left(\frac{V_1}{V_2}\right)^{\gamma-1} + nR\ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{V_2}{V_1}\right)\left[nR + \frac{C_V \ln\left(\frac{V_1}{V_2}\right)^{\gamma-1}}{\ln\frac{V_2}{V_1}}\right]$$

$$= \ln\left(\frac{V_2}{V_1}\right)\left[nR + \frac{(\gamma-1)C_V \ln\left(\frac{V_1}{V_2}\right)}{-\ln\frac{V_1}{V_2}}\right] = \ln\left(\frac{V_2}{V_1}\right)\left[nR-(\gamma-1)C_V\right]$$

Use the relationship between C_P and C_V to obtain:

$$nR = C_P - C_V$$

Substitute for nR and γ and simplify:

$$\Delta S = \ln\left(\frac{V_2}{V_1}\right)\left[C_P - C_V - \left(\frac{C_p}{C_V}-1\right)C_V\right]$$

$$=\boxed{0}$$

80 •••

Picture the Problem

(*a*) Suppose the refrigerator statement of the second law is violated in the sense that heat Q_c is taken from the cold reservoir and an equal mount of heat is transferred to the hot reservoir and $W = 0$. The entropy change of the universe is then $\Delta S_u = Q_c/T_h - Q_c/T_c$. Because $T_h > T_c$, $S_u < 0$, i.e., the entropy of the universe would decrease.

(*b*) In this case, is heat Q_h is taken from the hot reservoir and no heat is rejected to the cold reservoir, i.e., $Q_c = 0$, then the entropy change of the universe is $\Delta S_u = -Q_h/T_h + 0$, which is negative. Again, the entropy of the universe would decrease.

(*c*) The heat-engine and refrigerator statements of the second law only state that *some* heat must be rejected to a cold reservoir and *some* work must be done to transfer heat from the cold to the hot reservoir, but these statements do not specify the minimum amount of heat rejected or work that must be done. The statement $\Delta S_u \geq 0$ is more restrictive. The heat-engine and refrigerator statements in conjunction with the Carnot efficiency are equivalent to $\Delta S_u \geq 0$.

81 •••

Picture the Problem We can express the net efficiency of the two engines in terms of W_1, W_2, and Q_h and then use $\varepsilon_1 = W_1/Q_h$ and $\varepsilon_2 = W_2/Q_m$ to eliminate W_1, W_2, Q_h, and Q_m.

Express the net efficiency of the two engines connected in series:

$$\varepsilon_{net} = \frac{W_1 + W_2}{Q_h}$$

Express the efficiencies of engines 1 and 2:

$$\varepsilon_1 = \frac{W_1}{Q_h}$$

and

$$\varepsilon_2 = \frac{W_2}{Q_m}$$

Solve for W_1 and W_2 and substitute to obtain:

$$\varepsilon_{net} = \frac{\varepsilon_1 Q_h + \varepsilon_2 Q_m}{Q_h} = \varepsilon_1 + \frac{Q_m}{Q_h}\varepsilon_2$$

Express the efficiency of engine 1 in terms of Q_m and Q_h:

$$\varepsilon_1 = 1 - \frac{Q_m}{Q_h}$$

Solve for Q_m/ Q_h:

$$\frac{Q_m}{Q_h} = 1 - \varepsilon_1$$

Substitute to obtain:

$$\varepsilon_{net} = \boxed{\varepsilon_1 + (1 - \varepsilon_1)\varepsilon_2}$$

***82** •••

Picture the Problem We can express the net efficiency of the two engines in terms of W_1, W_2, and Q_h and then use $\varepsilon_1 = W_1/Q_h$ and $\varepsilon_2 = W_2/Q_m$ to eliminate W_1, W_2, Q_h, and Q_m. Finally, we can substitute the expressions for the efficiencies of the ideal reversible engines to obtain $\varepsilon_{net} = 1 - T_c/T_h$.

Express the efficiencies of ideal reversible engines 1 and 2:

$$\varepsilon_1 = 1 - \frac{T_m}{T_h} \quad (1)$$

and

$$\varepsilon_2 = 1 - \frac{T_c}{T_m} \quad (2)$$

Express the net efficiency of the two engines connected in series:

$$\varepsilon_{net} = \frac{W_1 + W_2}{Q_h} \quad (3)$$

Express the efficiencies of engines 1 and 2:

$$\varepsilon_1 = \frac{W_1}{Q_h} \text{ and } \varepsilon_2 = \frac{W_2}{Q_m}$$

Solve for W_1 and W_2 and substitute in equation (3) to obtain:

$$\varepsilon_{net} = \frac{\varepsilon_1 Q_h + \varepsilon_2 Q_m}{Q_h} = \varepsilon_1 + \frac{Q_m}{Q_h}\varepsilon_2$$

Express the efficiency of engine 1 in terms of Q_m and Q_h:

$$\varepsilon_1 = 1 - \frac{Q_m}{Q_h}$$

Solve for Q_m / Q_h:

$$\frac{Q_m}{Q_h} = 1 - \varepsilon_1$$

Substitute to obtain:

$$\varepsilon_{net} = \varepsilon_1 + (1 - \varepsilon_1)\varepsilon_2$$

Substitute for ε_1 and ε_2 and simplify to obtain:

$$\varepsilon_{net} = 1 - \frac{T_m}{T_h} + \left(\frac{T_m}{T_h}\right)\left(1 - \frac{T_c}{T_m}\right)$$

$$= 1 - \frac{T_m}{T_h} + \frac{T_m}{T_h} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}$$

83 •••

Picture the Problem There are 26 letters and four punctuation marks (space, comma, period, and exclamation point) used in the English language, disregarding capitalization, so we have a grand total of 30 characters to choose from. This fragment is 330 characters (including spaces) long; there are then 30^{330} different possible arrangements of the character set to form a fragment this long. We can use this number of possible arrangements to express the probability that one monkey will write out this passage and then an estimate of a monkey's typing speed to approximate the time required for one million monkeys to type the passage from Shakespeare.

Assuming the monkeys type at random, express the probability P that one monkey will write out this passage:

$$P = \frac{1}{30^{330}}$$

Use the approximation $30 \approx \sqrt{1000} = 10^{1.5}$ to obtain:

$$P = \frac{1}{10^{(1.5)(330)}} = \frac{1}{10^{495}} = 10^{-495}$$

Assuming the monkeys can type at a rate of 1 character per second, it would take about 330 s to write a passage of length equal to the quotation from Shakespeare. Find the time T required for a million monkeys to type this particular passage by accident:

$$T = \frac{(330\,\text{s})(10^{495})}{10^6}$$

$$= (3.30 \times 10^{491}\,\text{s})\left(\frac{1\,\text{y}}{3.16 \times 10^7\,\text{s}}\right)$$

$$\approx \boxed{10^{484}\,\text{y}}$$

Express the ratio of T to Russell's estimate:

$$\frac{T}{T_{\text{Russell}}} = \frac{10^{484}\ \text{y}}{10^{6}\ \text{y}} = 10^{478}$$

or

$$T \approx \boxed{10^{478} T_{\text{Russell}}}$$

Chapter 20
Thermal Properties and Processes

Conceptual Problems

***1 •**

Determine the Concept The glass bulb warms and expands first, before the mercury warms and expands.

2 •

Determine the Concept The heating of the sheet causes the average separation of its molecules to increase. The consequence of this increased separation is that the area of the hole always increases. $\boxed{(b) \text{ is correct.}}$

3 •

Determine the Concept Actually, it can be hard boiled, but it does take quite a bit longer than at sea level. $\boxed{(c) \text{ is the best response.}}$

4 •

Determine the Concept Gases that cannot be liquefied by applying pressure at 20°C are those for which $T_c < 293$ K. These are He, Ar, Ne, H_2, O_2, NO.

***5 ••**

(*a*) With increasing altitude, *P* decreases; from curve OF, *T* of the liquid-gas interface diminishes, so the boiling temperature decreases. Likewise, from curve OH, the melting temperature increases with increasing altitude.

(*b*) Boiling at a lower temperature means that the cooking time will have to be increased.

6 •

Picture the Problem We can apply the Stefan-Boltzmann law to relate the rate at which an object radiates thermal energy to its environment.

Using the Stefan-Boltzmann law, relate the power radiated by a body to its temperature:

$$P_r = e\sigma AT^4$$

where *A* is the surface area of the body, σ is Stefan's constant, and *e* is the emissivity of the object.

Because *P* varies with the fourth power of *T*, tripling the temperature increases the rate at which it radiates by a factor of 3^4 and $\boxed{(d) \text{ is correct.}}$

***7** •

Determine the Concept The thermal conductivity of metal and marble is much greater than that of wood; consequently, heat transfer from the hand is more rapid.

8 •

(*a*) True

(*b*) True

(*c*) False. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature.

(*d*) False. Water contracts on heating between 0°C and 4°C.

(*e*) True

9 •

Determine the Concept Because atoms are few and far between in space, the earth can not lose heat by conduction or convection. Thermal energy is radiated through space in the form of electromagnetic waves that move at the speed of light. $\boxed{(c)\text{ is correct.}}$

10 •

Determine the Concept Because there is little, if any, molecule-to-molecule transportation of energy into a fireplace-heated room, the mechanisms are radiation and convection.

11 •

Determine the Concept In the absence of matter to support conduction and convection, radiation is the only mechanism.

12 ••

Determine the Concept Because the amount of heat lost by the house is proportional to the difference between the house temperature and that of the outside air, the rate at which the house loses heat (that must be replaced by the furnace) is greater at night when the temperature of the house is kept high than when it is allowed to cool down.

13 ••

Picture the Problem The rate at which heat is conducted through a cylinder is given by $I = dQ/dt = kA\Delta T/\Delta x$ where A is the cross-sectional area of the cylinder.

Express the rate at which heat is conducted through cylinder A:

$$I_{\text{A}} = k_{\text{A}}\pi d_{\text{A}}^2 \frac{\Delta T}{\Delta x}$$

Express the rate at which heat is conducted through cylinder B:

$$I_B = k_B \pi d_B^2 \frac{\Delta T}{\Delta x}$$

Equate these expressions to obtain:

$$k_A \pi d_A^2 \frac{\Delta T}{\Delta x} = k_B \pi d_B^2 \frac{\Delta T}{\Delta x}$$

or

$$k_A d_A^2 = k_B d_B^2$$

Because $d_A = 2d_B$:

$$k_A (2d_B)^2 = k_B d_B^2$$

and

$$4k_A = k_B \Rightarrow \boxed{(a)\text{ is correct.}}$$

14 •

Determine the Concept Most objects of everyday experience are at temperatures near the mean temperature of the earth, about 300 K. Their blackbody spectrum therefore has a peak near λ_{max} = 2.898 mm K/ 300 K $\approx$ 0.01 mm = 10 μm = 10,000 nm. These wavelengths are in the infrared region of the spectrum, so the heat which most objects radiate away can be detected most easily in the infrared, which is the spectral region where most night-vision goggles and other types of optical "heat detectors" operate. However, if the temperature of the object increases, the wavelength decreases; so the peak radiation can be found in any spectral region, not just the infrared.

***15 •**

Determine the Concept The temperature of an object is inversely proportional to the maximum wavelength at which the object radiates (Wein's displacement law). Because blue light has a shorter wavelength than red light, an object for which the wavelength of the peak of thermal emission is blue is hotter than one that is red.

Estimation and Approximation

16 •••

Picture the Problem We can express the heat current through the insulation in terms of the rate of evaporation of the liquid helium and in terms of the temperature gradient across the superinsulation. Equating these equations will allow us to solve for the thermal conductivity k of the superinsulation.

Express the heat current in terms of the rate of evaporation of the liquid helium:

$$I = L_v \frac{dm}{dt}$$

Express the heat current in terms of the temperature gradient across the superinsulation and the conductivity of the superinsulation:

$$I = kA \frac{\Delta T}{\Delta x}$$

Equate these expressions and solve for *k*:

$$k=\frac{L_{\text{v}}\Delta x\dfrac{dm}{dt}}{A\Delta T}$$

Using the definition of density, express the rate of loss of liquid helium:

$$\frac{dm}{dt}=\rho\frac{dV}{dt}$$

Substitute to obtain:

$$k=\frac{L_{\text{v}}\Delta x\rho\dfrac{dV}{dt}}{A\Delta T}$$

Express the ratio of the area of the spherical container to its volume:

$$\frac{A}{V}=\frac{4\pi r^2}{\frac{4}{3}\pi r^3}$$

Solve for *A*:

$$A=\sqrt[3]{36\pi V^2}$$

Substitute to obtain:

$$k=\frac{L_{\text{v}}\Delta x\rho\dfrac{dV}{dt}}{\sqrt[3]{36\pi V^2}\,\Delta T}$$

Substitute numerical values and evaluate *k*:

$$k=\frac{(21\,\text{kJ/kg})(7\times10^{-2}\,\text{m})(125\,\text{kg/m}^3)\left(\dfrac{0.7\times10^{-3}\,\text{m}^3}{86400\,\text{s}}\right)}{\sqrt[3]{36\pi(200\times10^{-3}\,\text{m}^3)^2}\,(288\,\text{K})}=\boxed{3.13\times10^{-6}\,\text{W/m}\cdot\text{K}}$$

17 ••

Picture the Problem We can use the thermal current equation for the thermal conductivity of the skin.

Use the thermal current equation to express the rate of conduction of thermal energy:

$$I=kA\frac{\Delta T}{\Delta x}$$

Solve for *k* to obtain:

$$k=\frac{I}{A\dfrac{\Delta T}{\Delta x}}$$

Substitute numerical values and evaluate *k*:

$$k=\frac{130\,\text{W}}{(1.8\,\text{m}^2)\dfrac{4\,\text{K}}{10^{-3}\,\text{m}}}=\boxed{18.1\,\text{mW/m}\cdot\text{K}}$$

***18 ••**

Picture the Problem The amount of heat radiated by the earth must equal the solar flux from the sun, or else the temperature on earth would continually increase. The emissivity of the earth is related to the rate at which it radiates energy into space by the Stefan-Boltzmann law $P_r = e\sigma AT^4$.

Using the Stefan-Boltzmann law, express the rate at which the earth radiates energy as a function of its emissivity e and temperature T:

$$P_r = e\sigma A'T^4$$

where A' is the surface area of the earth.

Solve for the emissivity of the earth:

$$e = \frac{P_r}{\sigma A'T^4}$$

Use its definition to express the intensity of the radiation received by the earth:

$$I = \frac{P_{absorbed}}{A}$$

where A is the cross-sectional area of the earth.

For 70% absorption of the sun's radiation incident on the earth:

$$I = \frac{0.7P_r}{A}$$

Substitute for P_r and A and simplify to obtain:

$$e = \frac{0.7AI}{\sigma AT^4} = \frac{0.7\pi R^2 I}{4\pi R^2 \sigma T^4} = \frac{0.7I}{4\sigma T^4}$$

Substitute numerical values and evaluate e:

$$e = \frac{0.7\left(1370\,\text{W/m}^2\right)}{4\left(5.670\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)(288\,\text{K})^4} = \boxed{0.615}$$

19 ••

Picture the Problem The wavelength at which maximum power is radiated by the gas falling into a black hole is related to its temperature by Wien's displacement law.

Express Wien's displacement law:

$$\lambda_{max} = \frac{2.898\,\text{mm}\cdot\text{K}}{T}$$

Substitute for T and evaluate λ_{max}:

$$\lambda_{max} = \frac{2.898\,\text{mm}\cdot\text{K}}{10^6\,\text{K}} = \boxed{2.90\,\text{nm}}$$

Thermal Expansion

20 •

Picture the Problem We can find the length of the ruler at 100°C by adding its elongation due to the increase in temperature to its length at 20°C. We can find its elongation using the definition of the coefficient of linear expansion $\alpha = (\Delta L/L)/\Delta T$.

Express the length of the ruler at 100°C in terms of its length at 20°C, its coefficient of linear expansion, and the change in its temperature:

$$L_{100°C} = L_{20°C} + \Delta L = L_{20°C} + \alpha L_{20°C}\Delta T = L_{20°C}(1+\alpha\Delta T)$$

Substitute numerical values and evaluate $L_{100°C}$:

$$L_{100°C} = (30\,\text{cm})\left[1+\left(11\times10^{-6}\,/\text{K}\right)(80\,\text{K})\right] = \boxed{30.026\,\text{cm}}$$

21 ••

Picture the Problem We can let the definition of the coefficient of linear expansion $\alpha = (\Delta L/L)/\Delta T$, with ΔA replacing ΔL and A replacing L suggest a definition of the coefficient of area expansion.

(*a*) Letting γ represent the coefficient of area expansion we have:

$$\gamma \equiv \boxed{\frac{\Delta A/A}{\Delta T}} \quad (1)$$

(*b*) For a square:

$$\Delta A = [L(1+\alpha\Delta T)]^2 - L^2 = L^2\left(1+2\alpha\Delta T+\alpha^2\Delta T^2\right) - L^2 = A\left(2\alpha\Delta T+\alpha^2\Delta T^2\right)$$

Divide both sides of the equation by A to obtain:

$$\frac{\Delta A}{A} = 2\alpha\Delta T + \alpha^2\Delta T^2$$

Substitute in equation (1) to obtain:

$$\gamma = \frac{2\alpha\Delta T+\alpha^2\Delta T^2}{\Delta T} = 2\alpha + \alpha^2\Delta T$$

Let $\Delta T\rightarrow 0$ to obtain:

$$\gamma \approx \boxed{2\alpha\Delta T}$$

For a circle:

$$\Delta A = \pi[R(1+\alpha\Delta T)]^2 - \pi R^2 = \pi R^2\left(1+2\alpha\Delta T+\alpha^2\Delta T^2\right) - \pi R^2 = A\left(2\alpha\Delta T+\alpha^2\Delta T^2\right)$$

Divide both sides of the equation by A to obtain:

$$\frac{\Delta A}{A} = 2\alpha\Delta T + \alpha^2\Delta T^2$$

Substitute in equation (1) to obtain:

$$\gamma = \frac{2\alpha\Delta T+\alpha^2\Delta T^2}{\Delta T} = 2\alpha + \alpha^2\Delta T$$

Let $\Delta T \to 0$ to obtain: $\gamma \approx \boxed{2\alpha\Delta T}$

22 ••

Picture the Problem While the mass of a sample of aluminum will remain constant with increasing temperature, its volume will increase due to thermal expansion. Consequently, its density will decrease with increasing temperature. We can use the definition of density (mass/unit volume) to express the density when its volume has increased by ΔV and the definition of the coefficient of volume expansion to relate ΔV to the increase in temperature ΔT. The relationship $\beta = 3\alpha$ will allow us to relate the coefficient of volume expansion to the coefficient of linear expansion.

Express the density of aluminum ρ' when its volume has changed by ΔV:

$$\rho' = \frac{m}{V + \Delta V} = \frac{m/V}{1 + \Delta V/V}$$

Using the definition of the coefficient of volume expansion, substitute for $\Delta V/V$ to obtain:

$$\rho' = \frac{\rho}{1 + \beta\Delta T} = \frac{\rho}{1 + 3\alpha\Delta T}$$

because $\beta = 3\alpha$.

Substitute numerical values and evaluate ρ':

$$\rho' = \frac{2.70\times10^{3}\ \text{kg/m}^{3}}{1 + 3\left(24\times10^{-6}\ /\text{K}\right)\left(200\ \text{K}\right)} = \boxed{2.66\times10^{3}\ \text{kg/m}^{3}}$$

23 ••

Picture the Problem Because the temperature of the steel shaft does not change, we need consider just the expansion of the copper collar. We can express the required temperature in terms of the initial temperature and the change in temperature that will produce the necessary increase in the diameter D of the copper collar. This increase in the diameter is related to the diameter at 20°C and the increase in temperature through the definition of the coefficient of linear expansion.

Express the temperature to which the copper collar must be raised in terms of its initial temperature and the increase in its temperature:

$$T = T_{\text{i}} + \Delta T$$

Apply the definition of the coefficient of linear expansion to express the change in temperature required for the collar to fit on the

$$\Delta T = \frac{\left(\dfrac{\Delta D}{D}\right)}{\alpha}$$

shaft:

Substitute to obtain:

$$T = T_i + \frac{\Delta D}{\alpha D}$$

Substitute numerical values and evaluate T:

$$T = 293\,\text{K} + \frac{0.02\,\text{cm}}{\left(17\times10^{-6}\,/\text{K}\right)\left(5.98\,\text{cm}\right)}$$
$$= 490\,\text{K} = \boxed{217°\text{C}}$$

***24** ••

Picture the Problem Because the temperatures of both the steel shaft and the copper collar change together, we can find the temperature change required for the collar to fit the shaft by equating their diameters for a temperature increase ΔT. These diameters are related to their diameters at 20°C and the increase in temperature through the definition of the coefficient of linear expansion.

Express the temperature to which the collar and the shaft must be raised in terms of their initial temperature and the increase in their temperature:

$$T = T_i + \Delta T \qquad (1)$$

Express the diameter of the steel shaft when its temperature has been increased by ΔT:

$$D_{\text{steel}} = D_{\text{steel,20°C}}\left(1 + \alpha_{\text{steel}}\Delta T\right)$$

Express the diameter of the copper collar when its temperature has been increased by ΔT:

$$D_{\text{Cu}} = D_{\text{Cu,20°C}}\left(1 + \alpha_{\text{Cu}}\Delta T\right)$$

If the collar is to fit over the shaft when the temperature of both has been increased by ΔT:

$$D_{\text{Cu,20°C}}\left(1 + \alpha_{\text{Cu}}\Delta T\right) = D_{\text{steel,20°C}}\left(1 + \alpha_{\text{steel}}\Delta T\right)$$

Solve for ΔT to obtain:

$$\Delta T = \frac{D_{\text{steel,20°C}} - D_{\text{Cu,20°C}}}{D_{\text{Cu,20°C}}\alpha_{\text{Cu}} - D_{\text{steel,20°C}}\alpha_{\text{steel}}}$$

Substitute in equation (1) to obtain:

$$T = T_i + \frac{D_{\text{steel,20°C}} - D_{\text{Cu,20°C}}}{D_{\text{Cu,20°C}}\alpha_{\text{Cu}} - D_{\text{steel,20°C}}\alpha_{\text{steel}}}$$

Substitute numerical values and evaluate T:

$$T = 293\,\text{K} + \frac{6.0000\,\text{cm} - 5.9800\,\text{cm}}{(5.98\,\text{cm})(17\times10^{-6}\,/\text{K}) - (6.00\,\text{cm})(11\times10^{-6}\,/\text{K})} = 854\,\text{K} = \boxed{581°\text{C}}$$

25 ••

Picture the Problem The linear expansion coefficient of the container is one-third its coefficient of volume expansion. We can relate the changes in volume of the mercury and the container to their initial volumes, temperature change, and coefficients of volume expansion, and, because we know the amount of spillage, obtain an equation that we can solve for β_c.

Relate the linear expansion coefficient of the container to its coefficient of volume expansion:

$$\alpha_c = \tfrac{1}{3}\beta_c \quad (1)$$

Express the difference in the change in the volume of the mercury and the container in terms of the spillage:

$$\Delta V_{Hg} - \Delta V_c = 7.5\,\text{mL}$$

Express ΔV_{Hg} using the definition of the coefficient of volume expansion:

$$\Delta V_{Hg} = \beta_{Hg} V_{Hg} \Delta T$$

Express ΔV_c using the definition of the coefficient of volume expansion:

$$\Delta V_c = \beta_c V_c \Delta T$$

Substitute to obtain:

$$\beta_{Hg} V_{Hg} \Delta T - \beta_c V_c \Delta T = 7.5\,\text{mL}$$

Solve for β_c:

$$\beta_c = \frac{\beta_{Hg} V_{Hg} \Delta T - 7.5\,mL}{V_c \Delta T}$$

or, because $V = V_{Hg} = V_c$,

$$\beta_c = \frac{\beta_{Hg} V \Delta T - 7.5\,mL}{V \Delta T} = \beta_{Hg} - \frac{7.5\,mL}{V \Delta T}$$

Substitute in equation (1) to obtain:

$$\alpha_c = \tfrac{1}{3}\beta_{Hg} - \frac{7.5\,mL}{3V\Delta T} = \alpha_{Hg} - \frac{7.5\,mL}{3V\Delta T}$$

Substitute numerical values and evaluate α_c:

$$\alpha_c = \tfrac{1}{3}\left(0.18\times10^{-3}/\text{K}\right) - \frac{7.5\,\text{mL}}{3(1.4\,\text{L})(40\,\text{K})} = \boxed{15.4\times10^{-6}\ \text{K}^{-1}}$$

26 ••

Picture the Problem We can use $d_{Fe,168°C} = d_{Fe,20°C}(1+\alpha_{Fe}\Delta T)$ to find the diameter of the hole in the aluminum sheet at 168°C and then $d_{Al,20°C} = d_{Al,168°C}(1-\alpha_{Al}\Delta T)$ to find the diameter of the hole when the sheet has cooled to room temperature.

Relate the diameter of the hole/steel drill bit at 168°C to its diameter at 20° C:

$$d_{Fe,168°C} = d_{Fe,20°C}(1+\alpha_{Fe}\Delta T)$$

Substitute numerical values and evaluate $d_{Fe,168°C}$:

$$d_{Fe,168°C} = (6.245\,\text{cm})\left[1+11\times10^{-6}\ \text{K}^{-1}(148\,\text{K})\right] = 6.255\,\text{cm}$$

Express the diameter of the hole in the plate at 20° C:

$$d_{Al,20°C} = d_{Al,168°C}\left(1-\alpha_{Al}\Delta T\right)$$

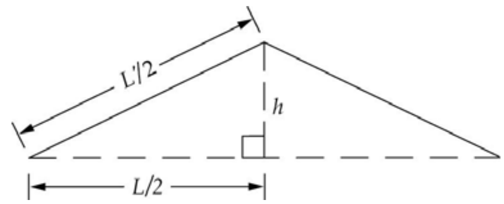
Substitute numerical values and evaluate $d_{Al,20°C}$:

$$d_{Al,20°C} = (6.255\,\text{cm})\left[1-\left(24\times10^{-6}\ \text{K}^{-1}\right)(148\,\text{K})\right] = \boxed{6.233\,\text{cm}}$$

Remarks: Note that the diameter of the hole in the plate at 20°C is less than the diameter of the drill bit at 20°C.

***27 ••**

Picture the Problem Let L be the length of the rail at 20°C and L' its length at 25°C. The diagram shows these distances and the height h of the buckle. We can use Pythagorean theorem to relate the height of the buckle to the distances L and L' and the definition of the coefficient of linear expansion to relate L and L'.



Apply the Pythagorean theorem to obtain:

$$h=\sqrt{\left(\frac{L'}{2}\right)^2-\left(\frac{L}{2}\right)^2}=\tfrac{1}{2}\sqrt{L'^2-L^2}$$

Use the definition of the coefficient of linear expansion to relate L and L':

$L'^2 = L^2(1+\alpha_{steel}\Delta T)^2$
or, because $(\alpha_{steel}\Delta T)^2 << 2\alpha_{steel}\Delta T$,
$L'^2 \approx L^2(1+2\alpha_{steel}\Delta T)$

Substitute to obtain:

$$h=\frac{1}{2}\sqrt{L^2(1+2\alpha_{steel}\Delta T)-L^2}$$
$$=\frac{L}{2}\sqrt{2\alpha_{steel}\Delta T}$$

Substitute numerical values and evaluate h:

$$h=\frac{1000\,\text{m}}{2}\sqrt{2(11\times10^{-6}\,\text{K}^{-1})(5\,\text{K})}$$
$$=\boxed{5.24\,\text{m}}$$

28 ••

Picture the Problem The amount of gas that spills is the difference between the change in the volume of the gasoline and the change in volume of the tank. We can find this difference by expressing the changes in volume of the gasoline and the tank in terms of their common volume at 10°C, their coefficients of volume expansion, and the change in the temperature.

Express the spill in terms of the change in volume of the gasoline and the change in volume of the tank:

$$V_{spill}=\Delta V_{gas}-\Delta V_{tank}$$

Relate ΔV_{gas} to the coefficient of volume expansion for gasoline:

$$\Delta V_{gas}=\beta_{gas}V\Delta T$$

Relate ΔV_{tank} to the coefficient of linear expansion for steel:

$\Delta V_{tank} = \beta_{tank} V\Delta T$
or, because $\beta_{steel} = 3\alpha_{steel}$,
$\Delta V_{tank} = 3\alpha_{steel} V\Delta T$

Substitute to obtain:

$$V_{spill} = \beta_{gas} V\Delta T - 3\alpha_{steel} V\Delta T = V\Delta T\,(\beta_{gas} - 3\alpha_{steel})$$

Substitute numerical values and evaluate V_{spill}:

$$V_{spill} = (60\,\text{L})(15\,\text{K})\left[0.9\times10^{-3}\,\text{K}^{-1} - 3\left(11\times10^{-6}\,\text{K}^{-1}\right)\right] = \boxed{0.780\,\text{L}}$$

29 ••

Picture the Problem We can relate the diameter of the capillary tube to the height the mercury rises for a 1°C increase in temperature and to the difference in the volume changes of the mercury in the bulb and the glass bulb. These volume changes can, in turn, be expressed in terms of the coefficients of volume expansion of mercury and glass.

Express the net change in volume of the mercury in the thermometer and the bulb and tube of the glass thermometer:

$\Delta V = \Delta V_{Hg} - \Delta V_{glass} = A\Delta L$
where $A = \pi d^2/4$ is the cross-sectional area of the capillary tube and d is its diameter.

Relate ΔV_{Hg} to the coefficient of linear expansion for mercury:

$\Delta V_{Hg} = \beta_{Hg} V\Delta T$

Relate ΔV_{glass} to the coefficient of linear expansion for glass:

$\Delta V_{glass} = \beta_{glass} V\Delta T$
or, because $\beta_{glass} = 3\alpha_{glass}$,
$\Delta V_{glass} = 3\alpha_{glass} V\Delta T$

Substitute to obtain:

$$\frac{\pi d^2}{4} = \beta_{Hg} V\Delta T - 3\alpha_{glass} V\Delta T = V\Delta T\left(\beta_{Hg} - 3\alpha_{glass}\right)$$

Solve for d:

$$d = \sqrt{\frac{4V\Delta T}{\pi\Delta L}\left(\beta_{Hg} - 3\alpha_{glass}\right)}$$

Substitute numerical values and evaluate d:

$$d = \sqrt{\frac{4\left(10^{-6}\,\text{m}^3\right)(1\text{K})}{\pi\left(3\times10^{-3}\,\text{m}\right)}\left(0.18\times10^{-3}\,\text{K}^{-1} - 3\left(9\times10^{-6}\,\text{K}^{-1}\right)\right)} = \boxed{0.255\,\text{mm}}$$

30 ••

Picture the Problem We can relate the volume of the thermometer bulb to the height the mercury rises for the 8 C° increase in temperature and to the difference in the volume changes of the mercury in the bulb and the glass bulb. These volume changes can, in turn, be expressed in terms of the coefficients of volume expansion of mercury and glass.

Express the net change in volume of the mercury in the thermometer and the bulb and tube of the glass thermometer:

$\Delta V = \Delta V_{Hg} - \Delta V_{glass} = A\Delta L$
where $A = \pi d^2/4$ is the cross-sectional area of the capillary tube and d is its diameter.

Relate ΔV_{Hg} to the coefficient of linear expansion for mercury:

$\Delta V_{Hg} = \beta_{Hg} V\Delta T$
or, because $\beta_{Hg} = 3\alpha_{Hg}$,
$\Delta V_{Hg} = 3\alpha_{Hg} V\Delta T$

Relate ΔV_{glass} to the coefficient of linear expansion for glass:

$\Delta V_{glass} = \beta_{glass} V\Delta T$
or, because $\beta_{glass} = 3\alpha_{glass}$,
$\Delta V_{glass} = 3\alpha_{glass} V\Delta T$

Substitute to obtain:

$$\beta_{Hg} V\Delta T - 3\alpha_{glass} V\Delta T = A\Delta L$$

Solve for V and substitute for A:

$$V = \frac{A\Delta L}{(\beta_{Hg} - 3\alpha_{glass})\Delta T} = \frac{\pi d^2 \Delta L}{4(\beta_{Hg} - 3\alpha_{glass})\Delta T}$$

Substitute numerical values and evaluate V:

$$V = \frac{\pi(0.4\times10^{-3}\text{ m})^2(7.5\times10^{-2}\text{ m})}{4[0.18\times10^{-3}\text{ K}^{-1} - 3(9\times10^{-6}\text{ K}^{-1})](8\text{ K})} = \boxed{7.70\text{ mL}}$$

31 •••

Picture the Problem We can determine whether the clock runs fast or slow from the expression for the period of a simple pendulum and the dependence of its length on the temperature. Letting T_P represent the period of the pendulum and T the temperature, we can evaluate dT_P/dT and use a differential approximation to find the time gained or lost in a 24-h period.

(*a*) Express the period of the pendulum in terms of its length:

$$T_P = 2\pi\sqrt{\frac{L}{g}}$$

Because $T_P \propto \sqrt{L}$ and L is temperature dependent, the clock runs slow.

(*b*) Because the clock runs slow at the higher temperature, we know that it will lose time. Express the loss in terms of the loss each period and the elapsed time Δt:

$$\text{Loss} = \frac{\Delta T_P}{T_P}\Delta t \qquad (1)$$

Write $\frac{dT_P}{dT}$ as the product of $\frac{dT_P}{dL}$ and $\frac{dL}{dT}$:

$$\frac{dT_P}{dT} = \frac{dT_P}{dL}\cdot\frac{dL}{dT}$$

Evaluate $\frac{dT_P}{dL}$ and simplify to obtain:

$$\frac{dT_P}{dL} = \frac{d}{dL}\left[2\pi\sqrt{\frac{L}{g}}\right] = \frac{1}{2}\left(\frac{2\pi}{g}\right)\left(\frac{L}{g}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\left(\frac{2\pi}{g}\right)\sqrt{\frac{g}{L}} = \frac{1}{2L}\left(2\pi\sqrt{\frac{L}{g}}\right)$$
$$= \frac{T_P}{2L}$$

Express the dependence of the length of the pendulum on its calibration length L_0 and the coefficient of linear expansion of brass α:

$$L = L_0(1+\alpha\Delta T)$$

Evaluate $\frac{dL}{dT}$:

$$\frac{dL}{dT} = \frac{d}{dT}\left[L_0(1+\alpha\Delta T)\right] = \alpha L_0$$

Substitute to obtain:

$$\frac{dT_P}{dT} = \left(\frac{T_P}{2L_0}\right)(\alpha L_0) = \frac{\alpha}{2}T_P$$

Use the differential approximation to obtain:

$$\frac{\Delta T_P}{\Delta T} = \frac{\alpha}{2}T_P \text{ or } \frac{\Delta T_P}{T_P} = \frac{\alpha}{2}\Delta T$$

Substitute numerical values and evaluate $\Delta T_P/T_P$:

$$\frac{\Delta T_P}{T_P} = \tfrac{1}{2}\left(19\times10^{-6}\ /\text{K}\right)(10\ \text{K})$$
$$= 9.50\times10^{-5}$$

Substitute numerical values in equation (1) to obtain:

$$\text{Loss} = \left(9.50\times10^{-5}\right)\left(24\,\text{h}\times\frac{3600\,\text{s}}{\text{h}}\right) = \boxed{8.21\,\text{s}}$$

32 •••

Picture the Problem The steel tube will fit inside the brass tube when its outside diameter equals the inside diameter of the brass tube. We can use the definition of the coefficient of linear expansion to express the diameters of the tubes when they fit in terms of the required temperature change and equate these expressions to find ΔT.

Express the temperature at which the steel tube will fit inside the brass tube in terms of their initial temperature and the change in temperature:

$$T = T_\text{i} + \Delta T = 293\,\text{K} + \Delta T \qquad (1)$$

Express the condition that the steel tube will fit inside the brass tube:

$$d_\text{steel} = d_\text{brass}$$

Relate the diameter of the steel tube to its initial diameter, coefficient of linear expansion, and the change in temperature:

$$d_\text{steel} = d_\text{0,steel}\left(1+\alpha_\text{steel}\Delta T\right)$$

Relate the diameter of the brass tube to its initial diameter, coefficient of linear expansion, and the change in temperature:

$$d_\text{brass} = d_\text{0,brass}\left(1+\alpha_\text{brass}\Delta T\right)$$

Substitute to obtain:

$$d_\text{0,steel}\left(1+\alpha_\text{steel}\Delta T\right) = d_\text{0,brass}\left(1+\alpha_\text{brass}\Delta T\right)$$

Solve for ΔT:

$$\Delta T = \frac{d_\text{0,steel} - d_\text{0,brass}}{d_\text{0,brass}\alpha_\text{brass} - d_\text{0,steel}\alpha_\text{steel}}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T == \frac{3.000\,\text{cm} - 2.997\,\text{cm}}{\left(3.000\,\text{cm}\right)\left(19\times10^{-6}\,\text{K}^{-1}\right) - \left(2.997\,\text{cm}\right)\left(11\times10^{-6}\,\text{K}^{-1}\right)} = 125\,\text{K}$$

Substitute in equation (1) to evaluate ΔT:

$$T = 293\,\text{K} + 125\,\text{K} = 418\,\text{K} = \boxed{145^\circ\text{C}}$$

***33 •••**

Picture the Problem We can use the definition of Young's modulus to express the tensile stress in the copper in terms of the strain it undergoes as its temperature returns to 20°C. We can show that $\Delta L/L$ for the circumference of the collar is the same as $\Delta d/d$ for its diameter.

Using Young's modulus, relate the stress in the collar to its strain:

$$\text{Stress} = Y \times \text{Strain} = Y\frac{\Delta L}{L_{20°\text{C}}}$$

where $L_{20°\text{C}}$ is the circumference of the collar at 20°C.

Express the circumference of the collar at the temperature at which it fits over the shaft:

$$L_T = \pi d_T$$

Express the circumference of the collar at 20°C:

$$L_{20°\text{C}} = \pi d_{20°\text{C}}$$

Substitute to obtain:

$$\text{Stress} = Y\frac{\pi d_T - \pi d_{20°\text{C}}}{\pi d_{20°\text{C}}} = Y\frac{d_T - d_{20°\text{C}}}{d_{20°\text{C}}}$$

Substitute numerical values and evaluate the stress:

$$\text{Stress} = \left(11\times10^{-10}\ \text{N/m}^2\right)\frac{0.02\,\text{cm}}{5.98\,\text{cm}} = \boxed{3.68\times10^{-12}\ \text{N/m}^2}$$

The van der Waals Equation, Liquid-Vapor Isotherms, and Phase Diagrams

34 •

Picture the Problem We can apply the ideal-gas law to find the volume of 1 mol of steam at 100°C and a pressure of 1 atm and then use the van der Waals equation to find the temperature at which the steam will this volume.

(*a*) Use the ideal-gas law to find the volume:

$$V=\frac{nRT}{P}=\frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(373\,\text{K})}{1\,\text{atm}\times\frac{101.325\,\text{kPa}}{\text{atom}}}=3.06\times10^{-2}\,\text{m}^3\times\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}=\boxed{30.6\,\text{L}}$$

(*b*) Solve van der Waals equation for *T* to obtain:

$$T=\frac{\left(P+\frac{an^2}{V^2}\right)(V-bn)}{nR}$$

Substitute numerical values and evaluate *T*:

$$T=\frac{\left(P+\frac{an^2}{V^2}\right)(V-bn)}{nR}=\left(101.3\,\text{kPa}+\frac{(0.55\,\text{Pa}\cdot\text{m}^6/\text{mol}^2)(1\,\text{mol})^2}{(3.06\times10^{-2}\,\text{m}^3)^2}\right)$$
$$\times\frac{3.06\times10^{-2}\,\text{m}^3-(30\times10^{-6}\,\text{m}^3/\text{mol})(1\,\text{mol})}{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}$$
$$=\boxed{375\,\text{K}}$$

35 ••

Picture the Problem We can find these temperatures and pressure by consulting Figure 20-3.

(*a*) At 70 kPa, water boils at: $t\approx\boxed{90°\text{C}}$

(*b*) At 0.5 atm (about 51 kPa): $t_{\text{boil}}\approx\boxed{82°\text{C}}$

(*c*) For t_{boil} = 115°C: $P\approx\boxed{170\,\text{kPa}}$

***36** ••

Picture the Problem Assume that a helium atom is spherical. Then we can find its radius from $V=\frac{4}{3}\pi r^3$ and its volume from the van der Waals equation.

Express the radius of a spherical atom in terms of its volume:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

In the van der Waals equation, b is the volume of 1 mol of molecules. For He, 1 molecule = 1 atom. Use Avogadro's number to express b in cm^3/atom:

$$b = \frac{(0.0237\,\text{L/mol})(10^3\,\text{cm}^3/\text{L})}{6.022\times10^{23}\,\text{atoms/mol}} = 3.94\times10^{-23}\,\text{cm}^3/\text{atom}$$

Substitute numerical values and evaluate r:

$$r = \sqrt[3]{\frac{3b}{4\pi}} = \sqrt[3]{\frac{3(3.94\times10^{-23}\,\text{cm}^3)}{4\pi}} = 2.11\times10^{-8}\,\text{cm} = \boxed{0.211\,\text{nm}}$$

37 •••

Picture the Problem Because, at the critical point, $dP/dV = 0$ and $d^2P/dV^2 = 0$, we can solve the van der Waals equation for P and set its first and second derivatives equal to zero to find V_c. We can then eliminate V_c between these equations to find T_c and then substitute in the van der Waals equation to express P_c. Finally, we can use their definitions to rewrite the van der Waals equation in terms of the reduced variables.

(a) Solve the van der Waals equation for P:

$$P = \frac{nRT}{V-bn} - \frac{an^2}{V^2} \qquad (1)$$

Evaluate dP/dV:

$$\frac{dP}{dV} = \frac{d}{dV}\left[\frac{nRT}{V-bn} - \frac{an^2}{V^2}\right] = -\frac{nRT}{(V-nb)^2} + \frac{2an^2}{V^3} \qquad (2)$$

$$= 0 \text{ for extrema}$$

Evaluate $\frac{d^2P}{dV^2}$:

$$\frac{d^2P}{dV^2} = \frac{d}{dV}\left[-\frac{nRT}{(V-nb)^2} + \frac{2an^2}{V^3}\right] = \frac{2nRT}{(V-nb)^3} - \frac{6an^2}{V^4} \qquad (3)$$

$$= 0 \text{ for critical points}$$

Solve equation (2) for $\frac{2an^2}{V^3}$:

$$\frac{2an^2}{V^3} = \frac{nRT}{(V-nb)^2} \qquad (4)$$

Solve equation (3) for $\frac{6an^2}{V^4}$:

$$\frac{6an^2}{V^4}=\frac{2nRT}{(V-nb)^3} \qquad (5)$$

Divide equation (4) by equation (5) and simplify to obtain:

$$\tfrac{1}{3}V=\tfrac{1}{2}(V-nb)$$

Solve for $V = V_c$:

$$V_c=3nb$$

Substitute in equation (4):

$$\frac{2an^2}{27n^3b^3}=\frac{nRT_c}{(3nb-nb)^2}$$

Simplify and solve for T_c:

$$T_c=\boxed{\frac{8a}{27Rb}}$$

Substitute for V_c and T_c in equation (1) and simplify to obtain:

$$P_c=\frac{nR\frac{8a}{27Rb}}{3bn-bn}-\frac{an^2}{(3bn)^2}=\boxed{\frac{a}{27b^2}}$$

(*b*) Using the result for V_c from (*a*), express the reduced volume V_r:

$$V_r=\frac{V}{V_c}=\frac{V}{3nb} \text{ and } V=3nbV_r$$

Using the result for T_c from (*a*), express the reduced temperature T_r:

$$T_r=\frac{T}{T_c}=\frac{27RbT}{8a}$$

and

$$T=\frac{8a}{27Rb}T_r$$

Using the result for P_c from (*a*), express the reduced pressure P_r:

$$P_r=\frac{P}{P_c}=\frac{27b^2P}{a}$$

and

$$P=\frac{a}{27b^2}P_r$$

Substitute in the van der Waals equation to obtain:

$$\left(\frac{a}{27b^2}P_r+\frac{an^2}{(3nbV_r)^2}\right)(3nbV_r-bn)=nR\frac{8a}{27Rb}T_r$$

Simplify to obtain:

$$\boxed{\left(P_\mathrm{r}+\frac{3}{V_\mathrm{r}^2}\right)(3V_\mathrm{r}-1)=8T_\mathrm{r}}$$

Heat Conduction

38 •

Picture the Problem We can use their definitions to find the thermal resistance of the bar, the thermal current in the bar, and the temperature gradient in the bar. Because the temperature varies linearly with distance along the bar, we can express the temperature in terms of the thermal gradient and evaluate this expression 25 cm from the hot end.

(*a*) Using its definition, find the thermal resistance of the bar:

$$R=\frac{\Delta x}{kA}=\frac{\Delta x}{k\pi r^2}$$

$$=\frac{2\,\mathrm{m}}{(401\,\mathrm{W/m\cdot K})\left[\pi\left(10^{-4}\,\mathrm{m}^2\right)\right]}$$

$$=\boxed{15.9\,\mathrm{K/W}}$$

(*b*) Using its definition, find the thermal current in the bar:

$$I=\frac{\Delta T}{R}=\frac{100\,\mathrm{K}}{15.9\,\mathrm{K/W}}=\boxed{6.29\,\mathrm{W}}$$

(*c*) Substitute numerical values and evaluate the temperature gradient:

$$\frac{\Delta T}{\Delta x}=\frac{100\,\mathrm{K}}{2\,\mathrm{m}}=50\,\mathrm{K/m}=\boxed{50\,\mathrm{K/m}}$$

(*d*) Express the linear dependence of the temperature in the bar on the distance from the cold end:

$$T=T_0+\frac{dT}{dx}\Delta x$$

Substitute numerical values and evaluate *T*(1.75 m):

$$T(1.75\,\mathrm{m})=273\,\mathrm{K}+(50\,\mathrm{K/m})(1.75\,\mathrm{m})$$

$$=360.5\,\mathrm{K}=\boxed{87.5°\mathrm{C}}$$

39 •

Picture the Problem We can use its definition to express the thermal current in the slab in terms of the temperature differential across it and its thermal resistance and use the definition of the R factor to express I as a function of ΔT, the cross-sectional area of the slab, and R_f.

Express the thermal current through the slab in terms of the temperature difference across it and its thermal

$$I=\frac{\Delta T}{R}$$

resistance:

Substitute to express R in terms of the insulation's R factor:

$$I = \frac{\Delta T}{R_f / A} = \frac{A\Delta T}{R_f}$$

Substitute numerical values and evaluate I:

$$I = \frac{(20\,\text{ft})(30\,\text{ft})(68°\text{F} - 30°\text{F})}{11\,\text{h}\cdot\text{ft}^2\cdot\text{F°/Btu}} = \boxed{2.07\,\text{kBtu/h}}$$

40 ••

Picture the Problem We can use $R = \Delta x/kA$ to find the thermal resistance of each cube and the fact that they are in series to find the thermal resistance of the two-cube system. We can use $I = \Delta T/R$ to find the thermal current through the cubes and the temperature at their interface.

(*a*) Using its definition, express the thermal resistance of each cube:

$$R = \frac{\Delta x}{kA}$$

Substitute numerical values and evaluate the thermal resistance of the copper cube:

$$R_{\text{Cu}} = \frac{3\,\text{cm}}{(401\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2} = \boxed{0.0831\,\text{K/W}}$$

Substitute numerical values and evaluate the thermal resistance of the aluminum cube:

$$R_{\text{Al}} = \frac{3\,\text{cm}}{(237\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2} = \boxed{0.141\,\text{K/W}}$$

(*b*) Because the cubes are in series, their thermal resistances are additive:

$$R = R_{\text{Cu}} + R_{\text{Al}} = 0.0831\,\text{K/W} + 0.141\,\text{K/W} = \boxed{0.224\,\text{K/W}}$$

(*c*) Using its definition, find the thermal current:

$$I = \frac{\Delta T}{R} = \frac{373\,\text{K} - 293\,\text{K}}{0.224\,\text{K/W}} = \boxed{357\,\text{W}}$$

(*d*) Express the temperature at the interface between the two cubes:

$$T_{\text{interface}} = 373\,\text{K} - \Delta T_{\text{Cu}}$$

Express the temperature differential across the copper cube:

$$\Delta T_{\text{Cu}} = I_{\text{Cu}} R_{\text{Cu}} = I R_{\text{Cu}}$$

Substitute numerical values and evaluate $T_{\text{interface}}$:

$$\begin{aligned} T_{\text{interface}} &= 373\,\text{K} - IR_{\text{Cu}} \\ &= 373\,\text{K} - (357\,\text{W})(0.0831\,\text{K/W}) \\ &= 343.3\,\text{K} = \boxed{70.3°\text{C}} \end{aligned}$$

41 ••

Picture the Problem We can use $I = \Delta T/R$ and $R = \Delta x/kA$ to find the thermal current in each cube. Because the currents are additive, we can find the equivalent resistance of the two-cube system from $R_{\text{eq}} = \Delta T/I$.

(*a*) Using its definition, express the thermal current through each cube:

$$I = \frac{\Delta T}{R}$$

Using its definition, express the thermal resistance of each cube:

$$R = \frac{\Delta x}{kA}$$

Substitute to obtain:

$$I = \frac{kA\Delta T}{\Delta x} \quad (1)$$

Substitute numerical values in equation (1) and evaluate the thermal current in the copper cube:

$$I_{\text{Cu}} = \frac{(401\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2(373\,\text{K} - 293\,\text{K})}{3\,\text{cm}} = \boxed{962\,\text{W}}$$

Substitute numerical values in equation (1) and evaluate the thermal current in the aluminum cube:

$$I_{\text{Al}} = \frac{(237\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2(373\,\text{K} - 293\,\text{K})}{3\,\text{cm}} = \boxed{569\,\text{W}}$$

(*b*) Because the cubes are in parallel, their total thermal currents are additive:

$$\begin{aligned} I &= I_{\text{Cu}} + I_{\text{Al}} = 962\,\text{W} + 569\,\text{W} \\ &= \boxed{1.53\,\text{kW}} \end{aligned}$$

(*c*) Use the relationship between the thermal current, temperature differential and thermal resistance to find R_{eq}:

$$\begin{aligned} R_{\text{eq}} &= \frac{\Delta T}{I} = \frac{373\,\text{K} - 293\,\text{K}}{1.53\,\text{kW}} \\ &= \boxed{0.0523\,\text{K/W}} \end{aligned}$$

42 ••

Picture the Problem The cost of operating the air conditioner is proportional to the energy used in its operation. We can use the definition of the COP to relate the rate at which the air conditioner removes heat from the house to rate at which it must do work to maintain a constant temperature differential between the interior and the exterior of the house. To obtain an expression for the minimum rate at which the air conditioner must do work, we'll assume that it is operating with the maximum efficiency possible. Doing so will allow us to derive an expression for the rate at which energy is used by the air conditioner that we can integrate to obtain the energy (and hence the cost of operation) required.

Relate the cost C of air conditioning the energy W required to operate the air conditioner:

$$C = uW \qquad (1)$$

where u is the unit cost of the energy.

Express the rate dQ/dt at which heat flows into a house provided the house is maintained at a constant temperature:

$$P = \frac{dQ}{dt} = k\Delta T$$

where ΔT is the temperature difference between the interior and exterior of the house.

Use the definition of the COP to relate the rate at which the air conditioner must remove heat dW/dt to maintain a constant temperature:

$$\text{COP} = \frac{dQ/dt}{dW/dt}$$

Solve for dW/dt:

$$dW/dt = \frac{dQ/dt}{\text{COP}}$$

Express the maximum value of the COP:

$$\text{COP}_{\text{max}} = \frac{T_c}{\Delta T}$$

where T_c is the temperature of the cold reservoir.

Letting COP = COP_{max}, substitute to obtain an expression for the minimum rate at which the air conditioner must do work in order to maintain a constant temperature:

$$\frac{dW}{dt} = \frac{dQ/dt}{T_c}\Delta T$$

Substitute for dQ/dt to obtain:

$$\frac{dW}{dt} = \frac{k\Delta T}{T_c}\Delta T = \frac{k}{T_c}(\Delta T)^2$$

Separate variables and integrate this equation to obtain:

$$W = \frac{k}{T_c}(\Delta T)^2\int_0^{\Delta t} dt' = \frac{k}{T_c}(\Delta T)^2\Delta t$$

Substitute in equation (1) to obtain:

$$C = u\frac{k}{T_c}(\Delta T)^2 \Delta t \propto \boxed{(\Delta T)^2}$$

43 •••

Picture the Problem We can follow the step-by-step instructions given in the problem statement to obtain the differential equation describing the variation of T with r. Integrating this equation will yield an equation that we can solve for the current I.

(*a*) $\boxed{\text{Conservation of energy requires that thermal current through each shell be the same.}}$

(*b*) Express the thermal current I through such a shell element in terms of the area $A = 4\pi r^2$, the thickness dr, and the temperature difference dT across the element:

$$I = -kA\frac{dT}{dr} = \boxed{-4\pi kr^2 \frac{dT}{dr}}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

(*c*) Separate the variables:

$$dT = -\frac{I}{4\pi k}\frac{dr}{r^2}$$

Integrate from $r = r_1$ to $r = r_2$:

$$\int_{T_1}^{T_2} dT = -\frac{I}{4\pi k}\int_{r_1}^{r_2}\frac{dr}{r^2}$$

and

$$T_2 - T_1 = \frac{I}{4\pi k}\left[\frac{1}{r}\right]_{r_1}^{r_2} = \frac{I}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Solve for I to obtain:

$$I = \boxed{\frac{4\pi k r_1 r_2}{r_2 - r_1}(T_2 - T_1)}$$

(*d*) When $r_2 - r_1 << r_1$:

$$r_1 \approx r_2 = r$$

Let $r_2 - r_1 = \Delta r$ and substitute to obtain:

$$I = \frac{4\pi kr^2}{\Delta r}(T_2 - T_1) = \boxed{4\pi kr^2 \frac{\Delta T}{\Delta r}}$$

which is Equation 20-7.

***44** ••

Picture the Problem We can use the expression for the thermal current to express the thickness of the walls in terms of the thermal conductivity of copper, the area of the walls, and the temperature difference between the inner and outer surfaces. Letting $\Delta A/\Delta x'$

represent the area per unit length of the pipe and L its length, we can eliminate the surface area and solve for and evaluate L.

Write the expression for the thermal current:

$$I = kA\frac{\Delta T}{\Delta x}$$

Solve for A:

$$A = \frac{I\Delta x}{k\Delta T}$$

Express the total surface area of the pipe:

$$A = \frac{\Delta A}{\Delta x'}L$$

Substitute for A and solve for L to obtain:

$$L = \frac{\dfrac{I\Delta x}{k\Delta T}}{\Delta A/\Delta x'}$$

Substitute numerical values and evaluate L:

$$L = \frac{\left[\dfrac{(3\,\text{GW})(4\times10^{-3}\,\text{m})}{(401\,\text{W/m}\cdot\text{K})(873\,\text{K} - 498\,\text{K})}\right]}{0.12\,\text{m}}$$

$$= \boxed{665\,\text{m}}$$

45 •••

Picture the Problem Consider an element with a cylindrical area of length L, radius r, and thickness dr. We can relate the heat current through this element to the conductivity of the walls of the pipe, its length and radius, and the temperature gradient across the wall. We can separate the variables in the resulting differential equation and integrate to find the rate of heat transfer.

(a) Express the heat current through the cylindrical element:

$$I = -kA\frac{dT}{dr} = -2\pi kLr\frac{dT}{dr}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

Separate the variables:

$$dT = -\frac{I}{2\pi kL}\frac{dr}{r}$$

Integrate from $r = r_1$ to $r = r_2$ and $T = T_1$ to $T = T_2$:

$$\int_{T_1}^{T_2} dT = -\frac{I}{2\pi kL}\int_{r_1}^{r_2}\frac{dr}{r}$$

and

$$T_2 - T_1 = -\frac{I}{2\pi kL}\ln r\Big]_{r_1}^{r_2}$$

$$= -\frac{I}{2\pi kL}\ln\frac{r_2}{r_1}$$

$$= \frac{I}{2\pi kL}\ln\frac{r_1}{r_2}$$

Solve for I to obtain:

$$I = \boxed{\frac{2\pi kL}{\ln(r_1/r_2)}(T_2 - T_1)}$$

Remarks: If we use the above result in Problem 44 (take 0.12 m^2 to be the outside area per unit length of the pipe), then r_1 = 1.91 cm and r_2 = 1.51 cm. Solving for L one obtains 746 m.

Radiation

46 •

Picture the Problem We can apply Wein's displacement law to find the wavelength at which the power is a maximum.

Use Wein's displacement law to relate the wavelength at which the power is a maximum to the surface temperature of the skin:

$$\lambda_{max} = \frac{2.898\,\text{mm}\cdot\text{K}}{T}$$

Substitute numerical values and evaluate λ_{max}:

$$\lambda_{max} = \frac{2.898\,\text{mm}\cdot\text{K}}{273\,\text{K} + 33\,\text{K}} = \boxed{9.47\,\mu\text{m}}$$

47 •

Picture the Problem We can apply the Stefan-Boltzmann law to find the net power radiated by the wires of its heater to the room.

Relate the net power radiated to the surface area of the heating wires, their temperature, and the room temperature:

$$P_{net} = e\sigma A\left(T^4 - T_0^4\right)$$

Solve for A:

$$A = \frac{P_{net}}{e\sigma\left(T^4 - T_0^4\right)}$$

Substitute numerical values and evaluate A:

$$A = \frac{1\,\text{kW}}{(1)\left(5.6703\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(1173\,\text{K})^4-(293\,\text{K})^4\right]} = \boxed{9.35\times10^{-3}\ \text{m}^2}$$

48 ••

Picture the Problem The rate at which the sphere absorbs radiant energy is given by $dQ/dt = mcdT/dt$ and, from the Stephan-Boltzmann law, $P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right)$, where A is the surface area of the sphere, T_0 is its temperature, and T is the temperature of the walls. We can solve the first equation for dT/dt and substitute P_{net} for dQ/dt in order to find the rate at which the temperature of the sphere changes.

Relate the rate at which the sphere absorbs radiant energy to the rate at which its temperature changes:

$$P_{\text{net}} = \frac{dQ}{dt} = mc\frac{dT}{dt}$$

Solve for dT/dt:

$$\frac{dT}{dt} = \frac{P_{\text{net}}}{mc} = \frac{P_{\text{net}}}{\rho Vc} = \frac{P_{\text{net}}}{\frac{4}{3}\pi r^3\rho c}$$

Apply the Stefan-Boltzmann law to relate the net power radiated to the sphere to the difference in temperature of the walls and the blackened copper sphere:

$$P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right) = 4\pi r^2 e\sigma\left(T^4 - T_0^4\right)$$

Substitute to obtain:

$$\frac{dT}{dt} = \frac{4\pi r^2 e\sigma\left(T^4 - T_0^4\right)}{\frac{4}{3}\pi r^3\rho c} = \frac{3e\sigma\left(T^4 - T_0^4\right)}{r\rho c}$$

Substitute numerical values and evaluate dT/dt:

$$\frac{dT}{dt} = -\frac{3(1)\left(5.6703\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(293\,\text{K})^4-(273\,\text{K})^4\right]}{\left(4\times10^{-2}\ \text{m}\right)\left(8.93\times10^3\ \text{kg/m}^3\right)\left(0.386\,\text{kJ/kg}\cdot\text{K}\right)} = \boxed{2.24\times10^{-3}\ \text{K/s}}$$

49 ••

Picture the Problem We can apply the Stephan-Boltzmann law to express the net power radiated by the incandescent lamp to its surroundings.

Express the rate at which energy is radiated to the surroundings:

$$P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right) = e\sigma A T^4\left(1 - \left(\frac{T_0}{T}\right)^4\right)$$

Evaluate $\left(T_0/T\right)^4$:

$$\left(\frac{T_0}{T}\right)^4 = \left(\frac{273\,\text{K}}{1573\,\text{K}}\right)^4 \approx 9\times10^{-4}$$

and, because this ratio is so small, we can neglect the temperature of the surroundings.

Substitute to obtain:

$$P_{\text{net}} \approx e\sigma A T^4$$

Solve for T:

$$T = \left(\frac{P_{\text{net}}}{e\sigma A}\right)^{1/4}$$

Express the temperature T' when the electric power input is doubled:

$$T' = \left(\frac{2P_{\text{net}}}{e\sigma A}\right)^{1/4}$$

Divide the second of these equations by the first:

$$\frac{T'}{T} = (2)^{1/4}$$

Solve for T':

$$T' = (2)^{1/4}T$$

Substitute numerical values and evaluate T'

$$T' = (2)^{1/4}(1573\,\text{K}) = 1871\,\text{K} = \boxed{1598°\text{C}}$$

50 ••

Picture the Problem We can differentiate $Q = mL$, where L is the latent heat of boiling for helium, with respect to time to obtain an expression for the rate at which the helium boils away.

Express the rate at which the helium boils away in terms of the rate at which its container absorbs radiant energy:

$$\frac{dm}{dt} = \frac{P_{\text{net}}}{L} = \frac{e\sigma A\left(T^4 - T_0^4\right)}{L} = \frac{e\sigma\pi d^2\left(T^4 - T_0^4\right)}{L} = \frac{e\sigma\pi d^2}{L}T^4\left(1 - \left(\frac{T_0}{T}\right)^4\right) \approx \frac{e\sigma\pi d^2}{L}T^4$$

when $T_0 << T$.

Substitute numerical values and evaluate dm/dt:

$$\frac{dm}{dt} \approx \frac{(1)\left(5.6703\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\pi(0.3\,\text{m})^2}{21\,\text{kJ/kg}}(77\,\text{K})^4 = 2.68\times10^{-5}\ \frac{\text{kg}}{\text{s}}\times\frac{3600\,\text{s}}{\text{h}}$$
$$= 9.66\times10^{-2}\ \text{kg/h} = \boxed{96.6\,\text{g/h}}$$

General Problems

***51 •**

Picture the Problem The distance by which the tape clears the ground equals the change in the radius of the circle formed by the tape placed around the earth at the equator.

Express the change in the radius of the circle defined by the steel tape:

$$\Delta R = R\alpha\Delta T$$

where R is the radius of the earth, α is the coefficient of linear expansion of steel, and ΔT is the increase in temperature.

Substitute numerical values and evaluate ΔR.

$$\Delta R = \left(6.37\times10^{6}\ \text{m}\right)\left(11\times10^{-6}\ \text{K}^{-1}\right)(30\,\text{K})$$
$$= 2.10\times10^{3}\ \text{m}$$
$$= \boxed{2.10\,\text{km}}$$

52 ••

Picture the Problem We can differentiate the definition of the density of an isotropic material with respect to T and use the definition of the coefficient of volume expansion to express the rate at which the density of the material changes with respect to temperature. Once we have an expression for $d\rho$ in terms of dT, we can apply a differential approximation to obtain $\Delta\rho$ in terms of ΔT.

Using its definition, relate the density of the material to its mass and volume:

$$\rho = \frac{m}{V}$$

Using its definition, relate the volume of the material to its coefficient of volume expansion:

$$\Delta V = \beta V\Delta T$$

Differentiate ρ with respect to T and simplify to obtain:

$$\frac{d\rho}{dT}=\frac{d\rho}{dV}\frac{dV}{dT}=-\frac{m}{V^2}\beta V$$
$$=-\frac{\rho V}{V^2}\beta V=-\rho\beta$$

or

$$d\rho=-\rho\beta dT$$

Use a differential approximation to obtain:

$$\boxed{\Delta\rho=-\rho\beta\Delta T}$$

53 ••

Picture the Problem We can apply the Stefan-Boltzmann law to express the effective temperature of the sun in terms of the total power it radiates. We can, in turn, use the intensity of the sun's radiation in the upper atmosphere of the earth to approximate the total power it radiates.

Apply the Stefan-Boltzmann law to relate the energy radiated by the sun to its temperature:

$$P_{\mathrm{r}}=e\sigma AT^4$$

Solve for T:

$$T=\sqrt[4]{\frac{P_{\mathrm{r}}}{e\sigma A}}$$

Express the area of the sun:

$$A=4\pi R_{\mathrm{S}}^2$$

Relate the intensity of the sun's radiation in the upper atmosphere to the total power radiated by the sun:

$$I=\frac{P_r}{4\pi R^2}$$

where R is the earth-sun distance.

Solve for P_{r}:

$$P_r=4\pi R^2 I$$

Substitute for P_{r} and A and simplify to obtain:

$$T=\sqrt[4]{\frac{4\pi R^2 I}{e\sigma 4\pi R_{\mathrm{S}}^2}}=\sqrt[4]{\frac{R^2 I}{e\sigma R_{\mathrm{S}}^2}}$$

Substitute numerical values and evaluate T:

$$T=\sqrt[4]{\frac{\left(1.5\times10^{11}\ \mathrm{m}\right)^2\left(1.35\,\mathrm{kW/m^2}\right)}{(1)\left(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K}\right)\left(6.96\times10^{8}\ \mathrm{m}\right)^2}}=\boxed{5767\,\mathrm{K}}$$

54 ••

Picture the Problem We can solve the thermal-current equation for the *R* factor of the material.

Use the equation for the thermal current to express *I* in terms of the temperature gradient across the insulation:

$$I = kA\frac{\Delta T}{\Delta x}$$

Rewrite this expression in terms of the *R* factor of the material:

$$I = \frac{\Delta T}{\frac{\Delta x}{kA}} = \frac{\Delta T}{\frac{R_f}{A}} = \frac{A\Delta T}{R_f}$$

Solve for the *R* factor:

$$R_f = \frac{A\Delta T}{I} = \frac{6A_{\text{one side}}\Delta T}{I}$$

Substitute numerical values and evaluate *R*:

$$R_f = \frac{6\left(12\,\text{in}\times\frac{2.54\times10^{-2}\,\text{m}}{\text{in}}\right)^2}{100\,\text{W}}(363\,\text{K} - 293\,\text{K}) = \boxed{0.390\,\frac{\text{K}\cdot\text{m}^2}{\text{W}}}$$

$$= 0.390\frac{\text{K}\cdot\text{m}^2}{\frac{\text{J}}{\text{s}}}\times\frac{9\,\text{F}^\circ}{5\,\text{K}}\times\frac{10.76\,\text{ft}^2}{\text{m}^2}\times\frac{1054\,\text{J}}{\text{Btu}}\times\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{2.21\frac{\text{F}^\circ\cdot\text{ft}^2\cdot\text{h}}{\text{Btu}}}$$

55 ••

Picture the Problem Because the temperature of the copper-aluminum interface is $(T_1 + T_2)/2$, we can conclude that the temperature differences across the two sheets must be the same. We also know, because the sheets are in series, that the heat currents through them are equal.

Express the thermal current through the aluminum sheet:

$$I_{\text{Al}} = k_{\text{Al}}A_{\text{Al}}\frac{\Delta T_{\text{Al}}}{\Delta x_{\text{Al}}}$$

Express the thermal current through the copper sheet:

$$I_{\text{Cu}} = k_{\text{Cu}}A_{\text{Cu}}\frac{\Delta T_{\text{Cu}}}{\Delta x_{\text{Cu}}}$$

Equate these currents and solve for Δx_{Al}:

$$k_{\text{Al}}A_{\text{Al}}\frac{\Delta T_{\text{Al}}}{\Delta x_{\text{Al}}} = k_{\text{Cu}}A_{\text{Cu}}\frac{\Delta T_{\text{Cu}}}{\Delta x_{\text{Cu}}}$$

and

$$\Delta x_{\text{Al}} = \Delta x_{\text{Cu}} \frac{k_{\text{Al}}}{k_{\text{Cu}}}$$

Substitute numerical values and evaluate Δx_{Al}:

$$\Delta x_{\text{Al}} = (2\,\text{cm})\frac{237\,\text{W/m}\cdot\text{K}}{401\,\text{W/m}\cdot\text{K}} = \boxed{1.18\,\text{cm}}$$

56 ••

Picture the Problem We can relate the stress in the bar to the strain due to its elongation using the definition of Young's modulus and express the strain in terms of the coefficient of linear expansion and the change in temperature of the bar.

Using the definition of Young's modulus, relate the force exerted by the bar on each wall to the strain in the bar due to the change in its length:

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

Using the definition of the coefficient of linear expansion, express the strain in the bar:

$$\frac{\Delta L}{L} = \alpha\Delta T$$

Substitute to obtain:

$$Y = \frac{F}{\alpha A \Delta T}$$

Solve for F:

$$F = \alpha A Y \Delta T$$

Substitute numerical values and evaluate F:

$$F = (11\times10^{-6}\,\text{K}^{-1})\pi(0.022\,\text{m})^2(200\,\text{GN/m}^2)(40\,\text{K}) = \boxed{1.34\times10^5\,\text{N}}$$

57 ••

Picture the Problem We can use the definition of the coefficient of volume expansion with the ideal-gas law to show that $\beta = 1/T$.

(*a*) Use the definition of the coefficient of volume expansion to express β in terms of the rate of change of the volume with temperature:

$$\beta = \frac{1}{V}\frac{dV}{dT}$$

For an ideal gas:

$$V = \frac{nRT}{P} \text{ and } \frac{dV}{dT} = \frac{nR}{P}$$

Substitute to obtain:

$$\beta = \frac{1}{V}\frac{nR}{P} = \boxed{\frac{1}{T}}$$

(*b*) Express the ratio of the experimental value to the theoretical value:

$$\frac{\beta_{\text{exp}} - \beta_{\text{th}}}{\beta_{\text{th}}} = \frac{0.003673\,\text{K}^{-1} - \frac{1}{273}\,\text{K}^{-1}}{\frac{1}{273}\,\text{K}^{-1}} < \boxed{0.3\%}$$

58 ••

Picture the Problem We can express L as the difference between L_B and L_A and express these lengths in terms of the coefficients of linear expansion brass and steel. Requiring that L be constant will lead us to the condition that $L_A/L_B = \alpha_B/\alpha_A$.

(*a*) Express the condition that L does not change when the temperature of the materials changes:

$$L = L_B - L_A = \text{constant}$$

Using the definition of the coefficient of linear expansion, substitute for L_B and L_A:

$$\begin{aligned} L &= (L_B + \alpha_B L_B \Delta T) - (L_A + \alpha_A L_A \Delta T) \\ &= (L_B - L_A) + (\alpha_B L_B - \alpha_A L_A)\Delta T \\ &= L + (\alpha_B L_B - \alpha_A L_A)\Delta T \end{aligned}$$

or

$$(\alpha_B L_B - \alpha_A L_A)\Delta T = 0$$

The condition that L remain constant when the temperature changes by ΔT is:

$$\alpha_B L_B - \alpha_A L_A = 0$$

Solve for the ratio of L_A to L_B:

$$\boxed{\frac{L_A}{L_B} = \frac{\alpha_B}{\alpha_A}}$$

(*b*) From (*a*) we have:

$$L_B = L_{steel} = L_A \frac{\alpha_A}{\alpha_B} = L_{brass}\frac{\alpha_{brass}}{\alpha_{steel}}$$

$$= (250\,\text{cm})\frac{19\times10^{-6}\,\text{K}^{-1}}{11\times10^{-6}\,\text{K}^{-1}}$$

$$= \boxed{432\,\text{cm}}$$

and

$$L = L_B - L_A = 432\,\text{cm} - 250\,\text{cm}$$

$$= \boxed{182\,\text{cm}}$$

59 ••

Picture the Problem We can apply the thermal-current equation to calculate the heat loss of the earth per second due to conduction from its core. We can also use the thermal-current equation to find the power per unit area radiated from the earth and compare this quantity to the solar constant.

Express the heat loss of the earth per unit time as a function of the thermal conductivity of the earth and its temperature gradient:

$$I = \frac{dQ}{dt} = kA\frac{\Delta T}{\Delta x} \quad (1)$$

or

$$\frac{dQ}{dt} = 4\pi R_E^2 k\frac{\Delta T}{\Delta x}$$

Substitute numerical values and evaluate dQ/dt:

$$\frac{dQ}{dt} = 4\pi(6.37\times10^6\,\text{m})^2(0.74\,\text{J/m}\cdot\text{s}\cdot\text{K})\left(\frac{1\text{C}^\circ}{30\,\text{m}}\right) = \boxed{1.26\times10^{10}\,\text{kW}}$$

Rewrite equation (1) to express the thermal current per unit area:

$$\frac{I}{A} = k\frac{\Delta T}{\Delta x}$$

Substitute numerical values and evaluate I/A:

$$\frac{I}{A} = (0.74\,\text{J/m}\cdot\text{s}\cdot\text{K})\left(\frac{1\text{C}^\circ}{30\,\text{m}}\right)$$

$$= 0.0247\,\text{W/m}^2$$

Express the ratio of I/A to the solar constant:

$$\frac{I/A}{\text{solar constant}} = \frac{0.0247\,\text{W/m}^2}{1.35\,\text{kW/m}^2}$$

$$< \boxed{0.002\%}$$

60 ••

Picture the Problem We can find the temperature of the outside of the copper bottom by finding the temperature difference between the outside of the saucepan and the boiling water. This temperature difference is related to the rate at which the water is evaporating through the thermal-current equation.

Express the temperature outside the pan in terms of the temperature inside the pan:

$$T_{out} = T_{in} + \Delta T = 373\,\text{K} + \Delta T$$

Relate the thermal current through the bottom of the saucepan to its thermal conductivity, area, and the temperature gradient between its surfaces:

$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

Solve for ΔT:

$$\Delta T = \frac{1}{kA}\frac{\Delta Q}{\Delta t}\Delta x$$

Because the water is boiling:

$$\Delta Q = mL_v$$

Substitute to obtain:

$$\Delta T = \frac{mL_v\Delta x}{kA\Delta t}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{(0.8\,\text{kg})(2.26\,\text{MJ/kg})(3\times10^{-3}\,\text{m})}{(401\,\text{W/m}\cdot\text{K})\left[\frac{\pi}{4}(0.15\,\text{m})^2\right](600\,\text{s})} = 1.28\,\text{K}$$

Substitute numerical values and evaluate T_{out}:

$$T_{out} = 373\,\text{K} + 1.28\text{K} = 374.3\,\text{K} = \boxed{101.3°\text{C}}$$

***61** ••

Picture the Problem We'll do this problem twice. First, we'll approximate the answer by disregarding the fact that the surrounding insulation is cylindrical. In the second solution, we'll obtain the exact answer by taking into account the cylindrical insulation surrounding the side of the tank. In both cases, the power required to maintain the temperature of the water in the tank is equal to the rate at which thermal energy is conducted through the insulation.

1st solution:

Using the thermal current equation, relate the rate at which energy is transmitted through the insulation to the temperature gradient, thermal conductivity of the insulation, and the area of the insulation/tank:

$$I = kA\frac{\Delta T}{\Delta x}$$

Letting d represent the inside diameter of the tank and L its inside height, express and evaluate its surface area:

$$\begin{aligned} A &= A_{\text{side}} + A_{\text{bases}} \\ &= \pi\, dL + 2\left(\frac{\pi\, d^2}{4}\right) \\ &= \pi\left(dL + \tfrac{1}{2}d^2\right) \\ &= \pi\left[(0.55\,\text{m})(1.2\,\text{m}) + \tfrac{1}{2}(0.55\,\text{m})^2\right] \\ &= 2.55\,\text{m}^2 \end{aligned}$$

Substitute numerical values and evaluate I:

$$\begin{aligned} I &= (0.035\,\text{W/m}\cdot\text{K})(2.55\,\text{m}^2)\left(\frac{74\,\text{K}}{0.05\,\text{m}}\right) \\ &= \boxed{132\,\text{W}} \end{aligned}$$

2nd solution:

Express the total heat loss as the sum of the losses through the top and bottom and the side of the hot-water tank:

$$I = I_{\text{top and bottom}} + I_{\text{side}}$$

Express I through the top and bottom surfaces:

$$\begin{aligned} I_{\text{top and bottom}} &= 2\left(kA\frac{\Delta T}{\Delta x}\right) \\ &= \tfrac{1}{2}\pi\, d^2 k\frac{\Delta T}{\Delta x} \end{aligned}$$

Substitute numerical values and evaluate $I_{\text{top and bottom}}$:

$$\begin{aligned} I_{\text{top and bottom}} &= \tfrac{1}{2}\pi(0.55\,\text{m})^2 \\ &\quad\times\frac{(0.035\,\text{W/m}\cdot\text{K})(74\,\text{K})}{0.05\,\text{m}} \\ &= 24.6\,\text{W} \end{aligned}$$

Letting r represent the inside radius of the tank, express the heat current

$$I_{\text{side}} = -kA\frac{dT}{dr} = -2\pi\, kLr\frac{dT}{dr}$$

through the cylindrical side:

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

Separate the variables:

$$dT = -\frac{I_{\text{side}}}{2\pi kL}\frac{dr}{r}$$

Integrate from $r = r_1$ to $r = r_2$ and $T = T_1$ to $T = T_2$:

$$\int_{T_1}^{T_2} dT = -\frac{I_{\text{side}}}{2\pi kL}\int_{r_1}^{r_2}\frac{dr}{r}$$

and

$$T_2 - T_1 = -\frac{I_{\text{side}}}{2\pi kL}\ln r\Big]_{r_1}^{r_2}$$

$$= -\frac{I_{\text{side}}}{2\pi kL}\ln\frac{r_2}{r_1} = \frac{I_{\text{side}}}{2\pi kL}\ln\frac{r_1}{r_2}$$

Solve for I_{side} to obtain:

$$I_{\text{side}} = \frac{2\pi kL}{\ln\frac{r_1}{r_2}}(T_2 - T_1)$$

Substitute numerical values and evaluate I_{side}:

$$I_{\text{side}} = \frac{2\pi(0.035\,\text{W/m}\cdot\text{K})(1.2\,\text{m})}{\ln\left(\frac{0.325\,\text{m}}{0.275\,\text{m}}\right)}(74\,\text{K})$$

$$= 117\,\text{W}$$

Substitute for I_{side} and evaluate I:

$$I = 24.6\,\text{W} + 117\,\text{W} = \boxed{142\,\text{W}}$$

62 •••

Picture the Problem We can use $R = \Delta T/I$ and $I = -kAdT/dt$ to express dT in terms of the linearly increasing diameter of the rod. Integrating this expression will allow us to find ΔT and, hence, R.

Express the thermal resistance of the rod in terms of the thermal current in it:

$$R = \frac{\Delta T}{I} \qquad (1)$$

Relate the thermal current in the rod to its thermal conductivity k, cross-sectional area A, and temperature gradient:

$$I = -kA\frac{dT}{dx}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

Using the dependence of the diameter of the rod on x, express the area of the rod:

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} d_0^2 (1 + ax)^2$$

Substitute to obtain:

$$I = -k\left[\frac{\pi}{4} d_0^2 (1 + ax)^2\right]\frac{dT}{dx}$$

Separate variables to obtain:

$$dT = -\frac{Idx}{k\left[\frac{\pi}{4} d_0^2 (1 + ax)^2\right]} = -\frac{4I}{\pi k d_0^2}\frac{dx}{(1 + ax)^2}$$

Integrate T from T_1 to T_2 and x from 0 to L:

$$\int_{T_1}^{T_2} dT = -\frac{4I}{\pi k d_0^2}\int_0^L \frac{dx}{(1 + ax)^2}$$

and

$$T_2 - T_1 = \Delta T = \frac{4IL}{\pi k d_0^2 (1 + aL)}$$

Substitute for ΔT and I in equation (1) and simplify to obtain:

$$R = \frac{\frac{4IL}{\pi k d_0^2 (1 + aL)}}{I} = \boxed{\frac{4L}{\pi k d_0^2 (1 + aL)}}$$

63 •••

Picture the Problem Let $\Delta T = T_2 - T_1$. We can apply Newton's 2nd law to establish the relationship between $L2$ and $L1$ and angular momentum conservation to relate ω_2 and ω_1. We can express both E_2 and E_1 in terms of their angular momenta and rotational inertias and take their ratio to establish their relationship.

Apply $\sum \tau = \frac{\Delta L}{\Delta t}$ to the spinning disk:

Because $\sum \tau = 0$, $\Delta L = 0$

and

$$\boxed{L_2 = L_1}$$

Apply conservation of angular momentum to relate the angular velocity of the disk at T_2 to the angular velocity at T_1:

$$I_2 \omega_2 = I_1 \omega_1$$

and

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$

Express I_2:

$$I_2 = mr_2^2 = mr_1^2(1+\alpha\Delta T)^2$$
$$= I_1\left(1+2\alpha\Delta T+(\alpha\Delta T)^2\right)$$
$$\approx I_1(1+2\alpha\Delta T)$$

because $(\alpha\Delta T)^2$ is small compared to $\alpha\Delta T$.

Substitute and apply the binomial expansion formula to obtain:

$$\omega_2 = \frac{I_1}{I_1(1+2\alpha\Delta T)}\omega_1$$

and, because $2\alpha\Delta T << 1$,

$$\omega_2 \approx \boxed{(1-2\alpha\Delta T)\omega_1}$$

Express E_2 in terms of L_2 and I_2:

$$E_2 = \frac{L_2^2}{2I_2} = \frac{L_1^2}{2I_2}$$

because $L_2 = L_1$.

Express E_1 in terms of L_1 and I_1:

$$E_1 = \frac{L_1^2}{2I_1}$$

Express the ratio of E_2 to E_1:

$$\frac{E_2}{E_1} = \frac{\dfrac{L_1^2}{2I_2}}{\dfrac{L_1^2}{2I_1}} = \frac{I_1}{I_2}$$

Solve for E_2 and substitute for the ratio of I_1 to I_2:

$$E_2 = E_1\frac{I_1}{I_2} = \boxed{E_1(1-2\alpha\Delta T)}$$

64 •••

Picture the Problem The amount of heat radiated by the earth must equal the solar flux from the sun, or else the temperature on earth would continually increase. The emissivity of the earth is related to the rate at which it radiates energy into space by the Stefan-Boltzmann law $P_r = e\sigma AT^4$.

Using the Stefan-Boltzmann law, express the rate at which the earth radiates energy as a function of its emissivity e and temperature T:

$$P_r = e\sigma A'T^4$$

where A' is the surface area of the earth.

Use its definition to express the intensity of the radiation P_a absorbed by the earth:

$$I = \frac{P_a}{A} \text{ or } P_a = AI$$

where A is the cross-sectional area of the earth.

For 70% absorption of the sun's radiation incident on the earth:

$$P_a = 0.7AI$$

Equate P_r and P_a and simplify:

$$0.7AI = e\sigma A'T^4$$

or

$$0.7\pi R^2 I = e\left(4\pi R^2 \sigma T^4\right)$$

Solve for T to obtain:

$$T = \sqrt[4]{\frac{0.7I}{4\sigma e}} = Ce^{-1/4} \qquad (1)$$

Substitute numerical values for I and σ and simplify to obtain:

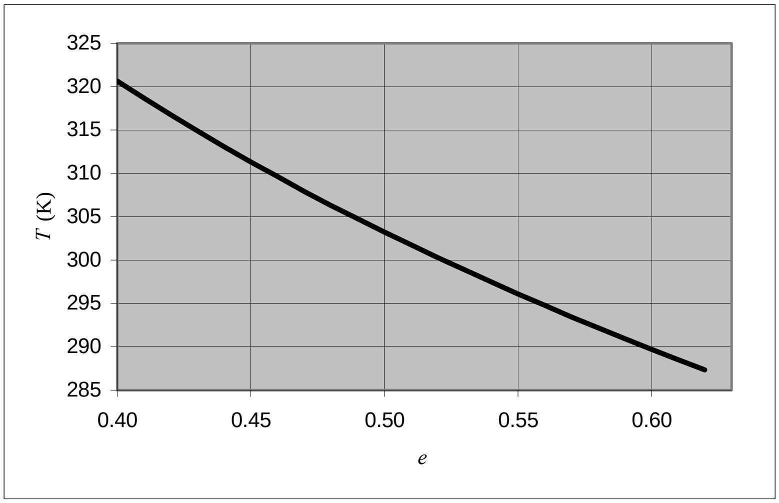
$$T = \sqrt[4]{\frac{0.7\left(1370\ \text{W/m}^2\right)}{4\left(5.670\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)e}} = \left(255\ \text{K}\right)e^{-1/4}$$

A spreadsheet program to evaluate T as a function of e is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	255	
B4	0.4	e
B5	B4+0.01	e + 0.1
C4	$B$1/(B4^0.25)	$\left(255\ \text{K}\right)e^{-1/4}$

	A	B	C	D
1	T=	255	K	
2				
3		e	T	
4		0.40	321	
5		0.41	319	
6		0.42	317	
7		0.43	315	
23		0.59	291	
24		0.60	290	
25		0.61	289	
26		0.62	287	

A graph of T as a function of e is shown below.



Treating e as a variable, differentiate equation (1) to obtain:

$$\frac{dT}{de} = -\frac{1}{4}Ce^{-5/4}de \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{dT}{T} = \frac{-\frac{1}{4}Ce^{-5/4}de}{Ce^{-1/4}} = -\frac{1}{4}\frac{de}{e}$$

Use a differential approximation to obtain:

$$\frac{\Delta T}{T} = -\frac{1}{4}\frac{\Delta e}{e}$$

Solve for Δe:

$$\Delta e = -4e\frac{\Delta T}{T}$$

Substitute numerical values ($e \approx 0.615$ for $T_{\text{earth}} = 288$ K) and evaluate Δe:

$$\Delta e = -4(0.615)\frac{1\,\text{K}}{288\,\text{K}} = \boxed{-0.00854}$$

or about a 1.39% change in e.

65 •••

Picture the Problem We can differentiate the expression for the heat that must be removed from water in order to form ice to relate dQ/dt to the rate of ice build-up dm/dt. We can apply the thermal-current equation to express the rate at which heat is removed from the water to the temperature gradient and solve this equation for dm/dt. In part (b) we can separate the variables in the differential equation relating dm/dt and ΔT and integrate to find out how long it takes for a 20-cm layer of ice to be built up.

(*a*) Relate the heat that must be removed from the water to freeze it to its mass and heat of fusion:

$$Q = mL_{\text{f}}$$

Differentiate this expression with respect to time:

$$\frac{dQ}{dt} = L_f \frac{dm}{dt}$$

Using the definition of density, relate the mass of the ice added to the bottom of the layer to its density and volume:

$$m = \rho V = \rho A x$$

Differentiate with respect to time to express the rate of build-up of the ice:

$$\frac{dm}{dt} = \rho A \frac{dx}{dt}$$

Substitute to obtain:

$$\frac{dQ}{dt} = L_f \rho A \frac{dx}{dt}$$

Apply the thermal-current equation:

$$\frac{dQ}{dt} = kA \frac{\Delta T}{x}$$

Equate these expressions and solve for dx/dt:

$$L_f \rho A \frac{dx}{dt} = kA \frac{\Delta T}{x}$$

and

$$\frac{dx}{dt} = \frac{k}{L_f \rho} \frac{\Delta T}{x} \qquad (1)$$

Substitute numerical values and evaluate dx/dt:

$$\frac{dx}{dt} = \frac{(0.592\,\text{W/m}\cdot\text{K})(10\,\text{K})}{(333.5\,\text{kJ/kg})(917\,\text{kg/m}^3)(0.01\,\text{m})} = 1.94\,\mu\text{m/s} = \boxed{0.698\,\text{cm/h}}$$

(*b*) Separate the variables in equation (1):

$$x dx = \frac{k\Delta T}{L_f \rho} dt$$

Integrate x from x_i to x_f and t' from 0 to t:

$$\int_{x_i}^{x_f} x dx = \frac{k\Delta T}{L_f \rho} \int_0^t dt'$$

and

$$\tfrac{1}{2}\left(x_f^2 - x_i^2\right) = \frac{k\Delta T}{\rho L_f} t$$

Solve for t to obtain:

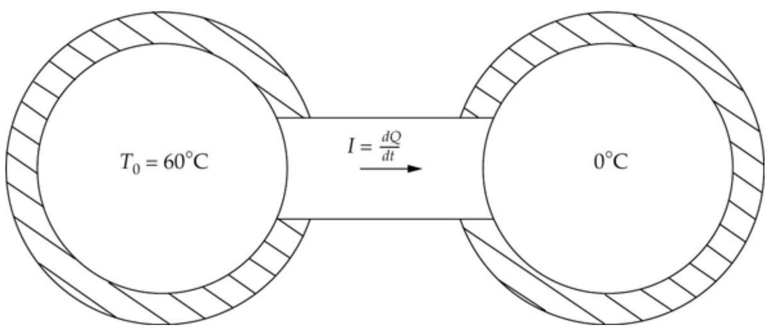
$$t = \frac{\rho L_f \left(x_f^2 - x_i^2\right)}{2k\Delta T}$$

Substitute numerical values and evaluate t:

$$t = \frac{(917\,\text{kg/m}^3)(333.5\,\text{kJ/kg})}{2(0.592\,\text{W/m}\cdot\text{K})(10\,\text{K})}\left[(0.2\,\text{m})^2 - (0.01\,\text{m})^2\right] = 1.03\times10^6\,\text{s}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1\,\text{d}}{24\,\text{h}}$$
$$= \boxed{11.9\,\text{d}}$$

***66 •••**

Picture the Problem We can use the thermal current equation and the definition of heat capacity to obtain the differential equation describing the rate at which the temperature of the water in the 200-g container is changing. Integrating this equation will yield $T = T_0e^{-t/RC}$. Substituting for dT/dt in $dQ/dt = -CdT/dt$ and integrating will lead to $Q = CT_0\left(1 - e^{-t/RC}\right)$.



(*a*) Use the thermal current equation to express the rate at which heat is conducted from the water at 60°C by the rod:

$$I = \frac{\Delta T}{R} = \frac{T}{R}$$
because the temperature of the second container is maintained at 0°C.

Using the definition of heat capacity, relate the thermal current to the rate at which the temperature of the water initially at 60°C is changing:

$$I = \frac{dQ}{dt} = -C\frac{dT}{dt} \qquad (1)$$

Equate these two expressions to obtain:

$C\dfrac{dT}{dt} = -\dfrac{1}{R}T$, the differential equation describing the rate at which the temperature of the water in the 200-g container is changing.

Separate variables to obtain:

$$\frac{dT}{T} = -\frac{1}{RC}dt$$

Integrate dT from T_0 to T and dt from 0 to t:

$$\int_{T_0}^{T}\frac{dT'}{T'} = -\frac{1}{RC}\int_0^t dt' \Rightarrow \ln\left(\frac{T}{T_0}\right) = -\frac{1}{RC}t$$

Transform from logarithmic to exponential form and solve for T to obtain:

$$\boxed{T = T_0 e^{-t/RC}} \qquad (2)$$

(*b*) Use its definition to express the thermal resistance R:

$$R = \frac{\Delta x}{kA}$$

Substitute numerical values (see Table 20-8 for the thermal conductivity of copper) and evaluate R:

$$R = \frac{0.1\,\text{m}}{(401\,\text{W/m}\cdot\text{K})(1.5\times10^{-4}\,\text{m}^2)} = \boxed{1.66\,\text{K/W}}$$

Use its definition to express the heat capacity of the water and the copper container:

$$C = m_c c_c + m_w c_w = m_c c_c + \rho_w V_w c_w$$

Substitute numerical values (see Table 18-1 for the specific heats of water and copper) and evaluate C:

$$C = (0.2\,\text{kg})(386\,\text{kJ/kg}\cdot\text{K}) + (10^3\,\text{kg/m}^3)(0.7\,\text{L})(4.18\,\text{kJ/kg}\cdot\text{K}) = \boxed{3.00\,\text{kJ/K}}$$

Evaluate the product of R and C to find the ″time constant″ τ:

$$\tau = RC = (1.66\,\text{K/W})(3.00\,\text{kJ/K}) = 4985\,\text{s} = \boxed{1.38\,\text{h}}$$

(*c*) Solve equation (1) for dQ to obtain:

$$dQ = -C\left(\frac{dT}{dt}\right)dt = -CdT$$

Integrate dQ' from $Q = 0$ to Q and dT from T_0 to T:

$$\int_0^Q dQ' = -\int_{T_0}^{T} CdT \Rightarrow Q = C(T_0 - T(t))$$

Substitute (equation (2) for $T(t)$ to obtain:

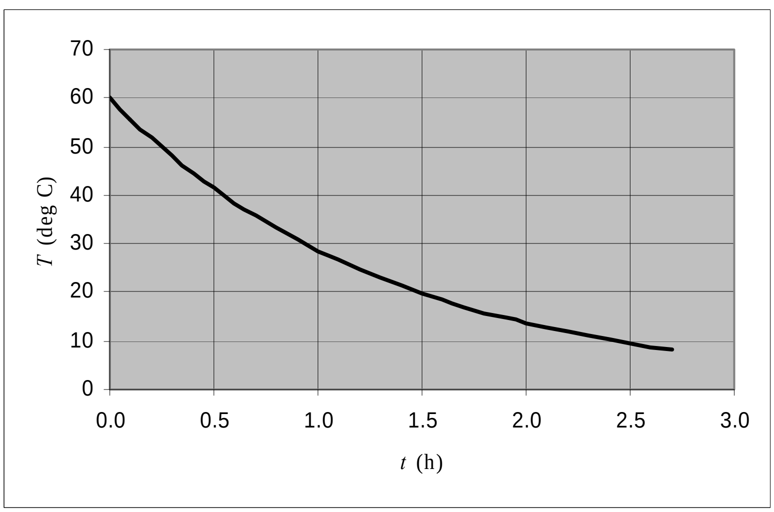
$$Q = C\left(T_0 - T_0 e^{-t/RC}\right) = \boxed{CT_0\left(1 - e^{-t/RC}\right)}$$

A spreadsheet program to evaluate Q as a function of t is shown below. The formulas used to calculate the quantities in the columns are as follows:

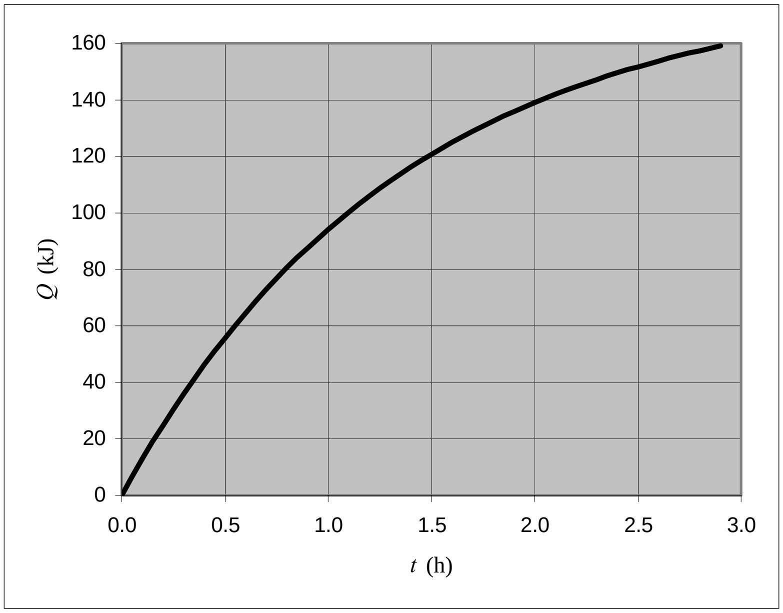
Cell	Formula/Content	Algebraic Form
D1	1.35	τ
D2	60	T_0
D3	3000	C
A6	0	t
A7	A6+0.1	$t + \Delta t$
B6	$B$2*EXP(–A6/$B$1)	$T_0 e^{-t/RC}$
C7	$B$3*$B$2*(1–EXP(–A6/$B$1))	$CT_0\left(1 - e^{-t/RC}\right)$

	A	B	C	D	E
1	tau=	1.35	h		
2	T0=	60	deg-C		
3	C=	3000	J/K		
4					
5	t (hr)	T	Q	Q/1000	
6	0.0	60.00	0.00E+00	0	
7	0.1	55.72	1.29E+04	13	
8	0.2	51.74	2.48E+04	24	
13	0.7	35.72	7.28E+04	71	
14	0.8	33.17	8.05E+04	79	
15	0.9	30.81	8.76E+04	86	
16	1.0	28.61	9.42E+04	92	
33	2.7	8.12	1.56E+05	152	
34	2.8	7.54	1.57E+05	154	
35	2.9	7.00	1.59E+05	155	

From the table we can see that the temperature of the container drops to 30°C in a little more than 0.9 h. If we wanted to know this time to the nearest hundredth of an hour, we could change the step size in the spreadsheet program to 0.01 h. A graph of T as a function of t is shown in the following graph.



A graph of Q as a function of t follows.



67 •••

Picture the Problem We can use the Stefan-Boltzmann equation and the definition of heat capacity to obtain the differential equation expressing the rate at which the temperature of the copper block decreases. We can then approximate the differential equation with a difference equation for the purpose of solving for the temperature of the block as a function of time using Euler's method.

(*a*) Express the rate at which heat is radiated away from the cube:

$$\frac{dQ}{dt} = e\sigma A\left(T^4 - T_0^4\right)$$

Using the definition of heat capacity, relate the thermal current to the rate at which the temperature of the cube is changing:

$$\frac{dQ}{dt} = -C\frac{dT}{dt}$$

Equate these expressions to obtain:

$$\boxed{\frac{dT}{dt} = -\frac{e\sigma A}{C}\left(T^4 - T_0^4\right)}$$

Approximate the differential equation by the difference equation:

$$\frac{\Delta T}{\Delta t} = -\frac{e\sigma A}{C}\left(T^4 - T_0^4\right)$$

Solve for ΔT:

$$\Delta T = -\frac{e\sigma A}{C}\left(T^4 - T_0^4\right)\Delta t$$

or

$$T_{n+1} = T_n - \frac{e\sigma A}{C}\left(T_n^4 - T_0^4\right)\Delta t \qquad (1)$$

Use the definition of heat capacity to obtain:

$$C = mc = \rho V c$$

Substitute numerical values (see Figure 13-1 for ρ_{Cu} and Table 19-1 for c_{Cu}) and evaluate C:

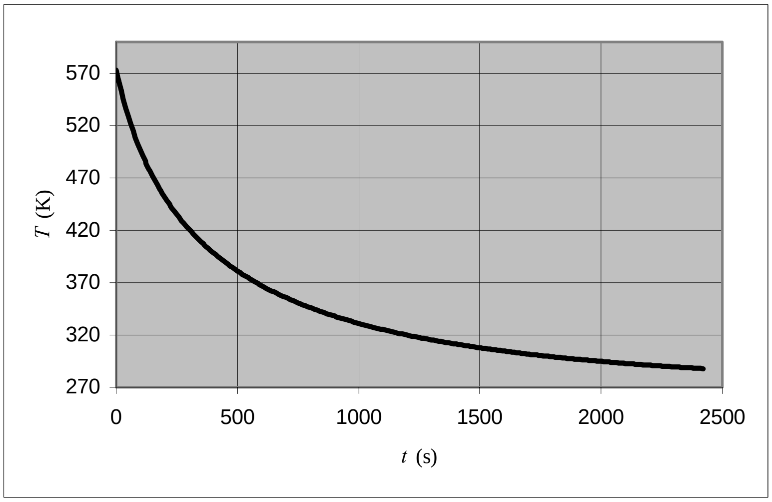
$$C = \left(8.93\times10^{3}\ \text{kg/m}^{3}\right)\left(10^{-6}\ \text{m}^{3}\right)\times\left(0.386\ \text{kJ/kg}\cdot\text{K}\right) = 3.45\ \text{J/K}$$

(*b*) A spreadsheet program to calculate T as a function of t using equation (1) is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	5.67×10^{-8}	σ
B2	6.00×10^{-4}	A
B3	3.45	C
B4	273	T_0
B5	10	Δt
A9	A8+$B$5	$t+\Delta t$
B9	B8-($B$1*$B$2/$B$3) *(B8^4–$B$4^4)*$B$5	$T_n - \frac{e\sigma A}{C}\left(T_n^4 - T_0^4\right)\Delta t$

	A	B	C
1	sigma=	5.67E–08	W/m^2·K^4
2	A=	6.00E–04	m^2
3	C=	3.45	J/K
4	T0=	273	K
5	dt=	10	s
6			
7	t (s)	T (K)	
8	0	573.00	
9	10	562.92	
10	20	553.56	
11	30	544.85	
248	2400	288.22	
249	2410	288.08	
250	2420	287.95	
251	2430	287.82	

From the spreadsheet solution, the time to cool to 15°C (288 K) is about 2420 s or $\boxed{40.5\,\text{min}.}$ A graph of *T* as a function of *t* follows.



Chapter 21
The Electric Field 1: Discrete Charge Distributions

Conceptual Problems

***1** ••

Similarities:

The force between charges and masses varies as $1/r^2$.

The force is directly proportional to the product of the charges or masses.

Differences:

There are positive and negative charges but only positive masses.

Like charges repel; like masses attract.

The gravitational constant G is many orders of magnitude smaller than the Coulomb constant k.

2 •

Determine the Concept No. In order to charge a body by induction, it must have charges that are free to move about on the body. An insulator does not have such charges.

3 ••

Determine the Concept During this sequence of events, negative charges are attracted from ground to the rectangular metal plate B. When S is opened, these charges are trapped on B and remain there when the charged body is removed. Hence B is negatively charged and $\boxed{(c)\text{ is correct.}}$

4 ••

(*a*) Connect the metal sphere to ground; bring the insulating rod near the metal sphere and disconnect the sphere from ground; then remove the insulating rod. The sphere will be negatively charged.

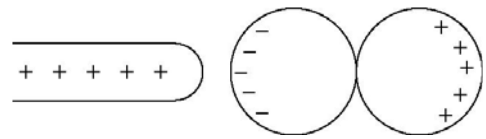
(*b*) Bring the insulating rod in contact with the metal sphere; some of the positive charge on the rod will be transferred to the metal sphere.

(*c*) Yes. First charge one metal sphere negatively by induction as in (*a*). Then use that negatively charged sphere to charge the second metal sphere positively by induction.

***5 ••**

Determine the Concept Because the spheres are conductors, there are free electrons on them that will reposition themselves when the positively charged rod is brought nearby.

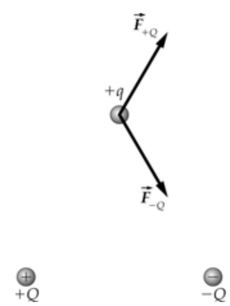
(*a*) On the sphere near the positively charged rod, the induced charge is negative and near the rod. On the other sphere, the net charge is positive and on the side far from the rod. This is shown in the diagram.



(*b*) When the spheres are separated and far apart and the rod has been removed, the induced charges are distributed uniformly over each sphere. The charge distributions are shown in the diagram.

6 •

Determine the Concept The forces acting on $+q$ are shown in the diagram. The force acting on $+q$ due to $-Q$ is along the line joining them and directed toward $-Q$. The force acting on $+q$ due to $+Q$ is along the line joining them and directed away from $+Q$.



Because charges $+Q$ and $-Q$ are equal in magnitude, the forces due to these charges are equal and their sum (the net force on $+q$) will be to the right and so $\boxed{(e)\text{ is correct.}}$ Note that the vertical components of these forces add up to zero.

***7 •**

Determine the Concept The acceleration of the positive charge is given by $\vec{a} = \frac{\vec{F}}{m} = \frac{q_0}{m}\vec{E}$. Because q_0 and m are both positive, the acceleration is in the same direction as the electric field. $\boxed{(d)\text{ is correct.}}$

***8 •**

Determine the Concept $\vec{E}$ is zero wherever the net force acting on a test charge is zero. At the center of the square the two positive charges alone would produce a net electric field of zero, and the two negative charges alone would also produce a net electric field of zero. Thus, the net force acting on a test charge at the midpoint of the square will be zero. $\boxed{(b)\text{ is correct.}}$

9 ••

(*a*) The zero net force acting on *Q* could be the consequence of equal collinear charges being equidistant from and on opposite sides of *Q*.

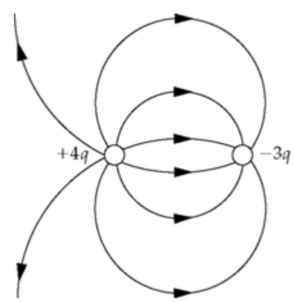
(*b*) The charges described in (*a*) could be either positive or negative and the net force on *Q* would still be zero.

(*c*) Suppose *Q* is positive. Imagine a negative charge situated to its right and a larger positive charge on the same line and the right of the negative charge. Such an arrangement of charges, with the distances properly chosen, would result in a net force of zero acting on *Q*.

(*d*) Because none of the above are correct, $\boxed{(d)\text{ is correct.}}$

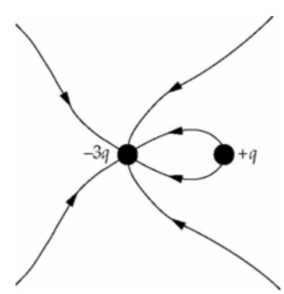
10 •

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the sketch to the right we've assigned 2 field lines to each charge *q*.



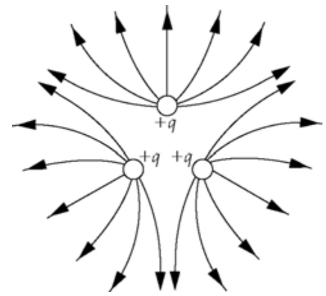
***11 •**

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch to the right we've assigned 2 field lines to each charge *q*.



***12** •

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch to the right we've assigned 7 field lines to each charge q.



13 •

Determine the Concept A positive charge will induce a charge of the opposite sign on the near surface of the nearby neutral conductor. The positive charge and the induced charge on the neutral conductor, being of opposite sign, will always attract one another. $\boxed{(a)\text{ is correct.}}$

***14** •

Determine the Concept Electric field lines around an electric dipole originate at the positive charge and terminate at the negative charge. Only the lines shown in (*d*) satisfy this requirement. $\boxed{(d)\text{ is correct.}}$

***15** ••

Determine the Concept Because $\theta \neq 0$, a dipole in a uniform electric field will experience a restoring torque whose magnitude is $pE_x \sin\theta$. Hence it will oscillate about its equilibrium orientation, $\theta = 0$. If $\theta << 1$, $\sin\theta \approx \theta$, and the motion will be simple harmonic motion. Because the field is nonuniform and is larger in the *x* direction, the force acting on the positive charge of the dipole (in the direction of increasing *x*) will be greater than the force acting on the negative charge of the dipole (in the direction of decreasing *x*) and thus there will be a net electric force on the dipole in the direction of increasing *x*. Hence, the dipole will accelerate in the *x* direction as it oscillates about $\theta = 0$.

16 ••

(*a*) False. The direction of the field is toward a negative charge.

(*b*) True.

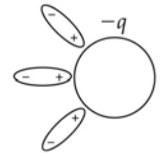
(*c*) False. Electric field lines diverge from any point in space occupied by a positive charge.

(*d*) True

(*e*) True

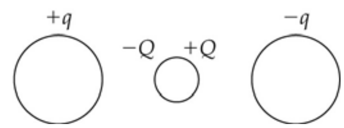
17 ••

Determine the Concept The diagram shows the metal balls before they are placed in the water. In this situation, the net electric field at the location of the sphere on the left is due only to the charge $-q$ on the sphere on the right. If the metal balls are placed in water, the water molecules around each ball tend to align themselves with the electric field. This is shown for the ball on the right with charge $-q$.



(*a*) The net electric field $\vec{E}_{\text{net}}$ that produces a force on the ball on the left is the field $\vec{E}$ due to the charge $-q$ on the ball on the right plus the field due to the layer of positive charge that surrounds the ball on the right. This layer of positive charge is due to the aligning of the water molecules in the electric field, and the amount of positive charge in the layer surrounding the ball on the left will be less than $+q$. Thus, $E_{\text{net}} < E$. Because $E_{\text{net}} < E$, the force on the ball on the left is less than it would be if the balls had not been placed in water. Hence, the force will $\boxed{\text{decrease}}$ when the balls are placed in the water.

(*b*) When a third uncharged metal ball is placed between the first two, the net electric field at the location of the sphere on the right is the field due to the charge $+q$ on the sphere on the left, plus the field due to the charge $-Q$ and $+Q$ on the sphere in the middle. This electric field is directed to the right.



The field due to the charge $-Q$ and $+Q$ on the sphere in the middle at the location of the sphere on the right is to the right. It follows that the net electric field due to the charge $+q$ on the sphere on the left, plus the field due to the charge $-Q$ and $+Q$ on the sphere in the middle is to the right and has a greater magnitude than the field due only to the charge $+q$ on the sphere on the left. Hence, the force on either sphere will $\boxed{\text{increase}}$ if a third uncharged metal ball is placed between them.

Remarks: The reduction of an electric field by the alignment of dipole moments with the field is discussed in further detail in Chapter 24.

***18 ••**

Determine the Concept Yes. A positively charged ball will induce a dipole on the metal ball, and if the two are in close proximity, the net force can be attractive.

***19 ••**

Determine the Concept Assume that the wand has a negative charge. When the charged wand is brought near the tinfoil, the side nearer the wand becomes positively charged by induction, and so it swings toward the wand. When it touches the wand, some of the negative charge is transferred to the foil, which, as a result, acquires a net negative charge and is now repelled by the wand.

Estimation and Approximation

20 ••

Picture the Problem Because it is both very small and repulsive, we can ignore the gravitational force between the spheres. It is also true that we are given no information about the masses of these spheres. We can find the largest possible value of Q by equating the electrostatic force between the charged spheres and the maximum force the cable can withstand.

Using Coulomb's law, express the electrostatic force between the two charged spheres:

$$F = \frac{kQ^2}{\ell^2}$$

Express the tensile strength S_{tensile} of steel in terms of the maximum force F_{max} in the cable and the cross-sectional area of the cable and solve for F:

$$S_{\text{tensile}} = \frac{F_{\text{max}}}{A} \Rightarrow F_{\text{max}} = AS_{\text{tensile}}$$

Equate these forces to obtain:

$$\frac{kQ^2}{\ell^2} = AS_{\text{tensile}}$$

Solve for Q:

$$Q = \ell\sqrt{\frac{AS_{\text{tensile}}}{k}}$$

Substitute numerical values and evaluate Q:

$$Q = (1\,\text{m})\sqrt{\frac{(1.5\times10^{-4}\,\text{m}^2)(5.2\times10^{8}\,\text{N/m}^2)}{8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{2.95\,\text{mC}}$$

21 ••

Picture the Problem We can use Coulomb's law to express the net force acting on the copper cube in terms of the unbalanced charge resulting from the assumed migration of half the charges to opposite sides of the cube. We can, in turn, find the unbalanced charge $Q_{\text{unbalanced}}$ from the number of copper atoms N and the number of electrons per atom.

(*a*) Using Coulomb's law, express the net force acting on the copper rod due to the imbalance in the positive and negative charges:

$$F=\frac{kQ_{\text{unbalanced}}^2}{r^2} \qquad (1)$$

Relate the number of copper atoms N to the mass m of the rod, the molar mass M of copper, and Avogadro's number N_A:

$$\frac{N}{N_A}=\frac{m}{M}=\frac{\rho_{\text{Cu}}V_{\text{rod}}}{M}$$

Solve for N to obtain:

$$N=\frac{\rho_{\text{Cu}}V_{\text{rod}}}{M}N_A$$

Substitute numerical values and evaluate N:

$$N=\frac{\left(8.93\times10^3\ \text{kg/m}^3\right)\left(0.5\times10^{-2}\ \text{m}\right)^2\left(4\times10^{-2}\ \text{m}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)}{63.54\times10^{-3}\ \text{kg/mol}}$$
$$=8.461\times10^{22}\ \text{atoms}$$

Because each atom has 29 electrons and protons, we can express $Q_{\text{unbalanced}}$ as:

$$Q_{\text{unbalanced}}=\tfrac{1}{2}(29)\left(10^{-7}\right)eN$$

Substitute numerical values and evaluate $Q_{\text{unbalanced}}$:

$$Q_{\text{unbalanced}}=\tfrac{1}{2}(29)\left(10^{-7}\right)\left(1.6\times10^{-19}\ \text{C}\right)\left(8.461\times10^{22}\right)=1.963\times10^{-2}\ \text{C}$$

Substitute for $Q_{\text{unbalanced}}$ in equation (1) to obtain:

$$F=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.963\times10^{-2}\ \text{C}\right)^2}{(0.01\,\text{m})^2}=\boxed{3.46\times10^{10}\ \text{N}}$$

(*b*) Using Coulomb's law, express the maximum force of repulsion $F_{\max}$ in terms of the maximum possible charge $Q_{\max}$:

$$F_{\max}=\frac{kQ_{\max}^2}{r^2}$$

Solve for $Q_{\max}$:

$$Q_{\max}=\sqrt{\frac{r^2F_{\max}}{k}}$$

Express $F_{\max}$ in terms of the tensile strength S_{tensile} of copper:

$$F_{\max}=S_{\text{tensile}}A$$

where A is the cross sectional area of the cube.

Substitute to obtain:

$$Q_{max} = \sqrt{\frac{r^2 S_{tensile} A}{k}}$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{max} = \sqrt{\frac{(0.01\,\text{m})^2 (2.3\times10^8\ \text{N/m}^2)(10^{-4}\ \text{m}^2)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 1.60\times10^{-5}\ \text{C}$$

Because $Q_{unbalanced} = 2Q_{max}$:

$$Q_{unbalanced} = 2(1.60\times10^{-5}\ \text{C}) = \boxed{32.0\ \mu\text{C}}$$

Remarks: A net charge of –32 μC means an excess of 2.00×10^{14} electrons, so the net imbalance as a percentage of the total number of charges is $4.06\times10^{-11} = 4\times10^{-9}$ %.

22 •••

Picture the Problem We can use the definition of electric field to express E in terms of the work done on the ionizing electrons and the distance they travel λ between collisions. We can use the ideal-gas law to relate the number density of molecules in the gas ρ and the scattering cross-section σ to the mean free path and, hence, to the electric field.

(*a*) Apply conservation of energy to relate the work done on the electrons by the electric field to the change in their kinetic energy:

$$W = \Delta K = F\Delta s$$

From the definition of electric field we have:

$$F = qE$$

Substitute for F and Δs to obtain:

$W = qE\lambda$, where the mean free path λ is the distance traveled by the electrons between ionizing collisions with nitrogen atoms.

Referring to pages 545-546 for a discussion on the mean-free path, use its definition to relate λ to the scattering cross-section σ and the number density for air molecules n:

$$\lambda = \frac{1}{\sigma n}$$

Substitute for λ and solve for E to obtain:

$$E = \frac{\sigma n W}{q}$$

Use the ideal-gas law to obtain:

$$n = \frac{N}{V} = \frac{P}{kT}$$

Substitute for n to obtain:

$$E = \frac{\sigma PW}{qkT} \qquad (1)$$

Substitute numerical values and evaluate E:

$$E = \frac{(10^{-19}\ \text{m}^2)(10^5\ \text{N/m}^2)(1\,\text{eV})(1.6\times10^{-19}\ \text{J/eV})}{(1.6\times10^{-19}\ \text{C})(1.38\times10^{-23}\ \text{J/K})(300\,\text{K})} = \boxed{2.41\times10^6\ \text{N/C}}$$

(b) From equation (1) we see that:

$$\boxed{E \propto P} \text{ and } \boxed{E \propto T^{-1}}$$

i.e., E increases linearly with pressure and varies inversely with temperature.

***23 ••**

Picture the Problem We can use Coulomb's law to express the charge on the rod in terms of the force exerted on it by the soda can and its distance from the can. We can apply Newton's 2nd law in rotational form to the can to relate its acceleration to the electric force exerted on it by the rod. Combining these equations will yield an expression for Q as a function of the mass of the can, its distance from the rod, and its acceleration.

Use Coulomb's law to relate the force on the rod to its charge Q and distance r from the soda can:

$$F = \frac{kQ^2}{r^2}$$

Solve for Q to obtain:

$$Q = \sqrt{\frac{r^2F}{k}} \qquad (1)$$

Apply $\sum \tau_{\text{center of mass}} = I\alpha$ to the can:

$$FR = I\alpha$$

Because the can rolls without slipping, we know that its linear acceleration a and angular acceleration α are related according to:

$$\alpha = \frac{a}{R}$$

where R is the radius of the soda can.

Because the empty can is a hollow cylinder:

$$I = MR^2$$

where M is the mass of the can.

Substitute for I and α and solve for F to obtain:

$$F = \frac{MR^2a}{R^2} = Ma$$

Substitute for F in equation (1):

$$Q = \sqrt{\frac{r^2Ma}{k}}$$

Substitute numerical values and evaluate Q:

$$Q=\sqrt{\frac{(0.1\,\text{m})^2(0.018\,\text{kg})(1\,\text{m/s}^2)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}=\boxed{141\,\text{nC}}$$

24 ••

Picture the Problem Because the nucleus is in equilibrium, the binding force must be equal to the electrostatic force of repulsion between the protons.

Apply $\sum\vec{F}=0$ to a proton:

$$F_{\text{binding}}-F_{\text{electrostatic}}=0$$

Solve for F_{binding}:

$$F_{\text{binding}}=F_{\text{electrostatic}}$$

Using Coulomb's law, substitute for $F_{\text{electrostatic}}$:

$$F_{\text{binding}}=\frac{kq^2}{r^2}$$

Substitute numerical values and evaluate $F_{\text{electrostatic}}$:

$$F_{\text{binding}}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(10^{-15}\ \text{m})^2}=\boxed{230\,\text{N}}$$

Electric Charge

25 •

Picture the Problem The charge acquired by the plastic rod is an integral number of electronic charges, i.e., $q=n_e(-e)$.

Relate the charge acquired by the plastic rod to the number of electrons transferred from the wool shirt:

$$q=n_e(-e)$$

Solve for and evaluate n_e:

$$n_e=\frac{q}{-e}=\frac{-0.8\,\mu\text{C}}{-1.6\times10^{-19}\ \text{C}}=\boxed{5.00\times10^{12}}$$

26 •

Picture the Problem One faraday = N_Ae**.** We can use this definition to find the number of coulombs in a faraday.

Use the definition of a faraday to calculate the number of coulombs in a faraday:

$$1\,\text{faraday}=N_Ae=(6.02\times10^{23}\ \text{electrons})(1.6\times10^{-19}\ \text{C/electron})=\boxed{9.63\times10^4\ \text{C}}$$

***27** •

Picture the Problem We can find the number of coulombs of positive charge there are in 1 kg of carbon from $Q = 6n_\text{C}e$, where n_C is the number of atoms in 1 kg of carbon and the factor of 6 is present to account for the presence of 6 protons in each atom. We can find the number of atoms in 1kg of carbon by setting up a proportion relating Avogadro's number, the mass of carbon, and the molecular mass of carbon to n_C.

Express the positive charge in terms of the electronic charge, the number of protons per atom, and the number of atoms in 1 kg of carbon:

$$Q = 6n_\text{C}e$$

Using a proportion, relate the number of atoms in 1 kg of carbon n_C, to Avogadro's number and the molecular mass M of carbon:

$$\frac{n_\text{C}}{N_\text{A}} = \frac{m_\text{C}}{M} \Rightarrow n_\text{C} = \frac{N_\text{A}m_\text{C}}{M}$$

Substitute to obtain:

$$Q = \frac{6N_\text{A}m_\text{C}e}{M}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{6\left(6.02\times10^{23}\text{ atoms/mol}\right)(1\,\text{kg})\left(1.6\times10^{-19}\text{ C}\right)}{0.012\,\text{kg/mol}} = \boxed{4.82\times10^{7}\text{ C}}$$

Coulomb's Law

28 •

Picture the Problem We can find the forces the two charges exert on each by applying Coulomb's law and Newton's 3rd law. Note that $\hat{\boldsymbol{r}}_{1,2} = \hat{\boldsymbol{i}}$ because the vector pointing from q_1 to q_2 is in the positive x direction.

(a) Use Coulomb's law to express the force that q_1 exerts on q_2:

$$\vec{\boldsymbol{F}}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{\boldsymbol{r}}_{1,2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{F}}_{1,2}$:

$$\vec{\boldsymbol{F}}_{1,2} = \frac{\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})(6\,\mu\text{C})}{(3\,\text{m})^2}\boldsymbol{i} = \boxed{(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

(*b*) Because these are action-and-reaction forces, we can apply Newton's 3rd law to obtain:

$$\vec{F}_{2,1} = -\vec{F}_{1,2} = \boxed{-(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

(*c*) If q_2 is –6.0 μC:

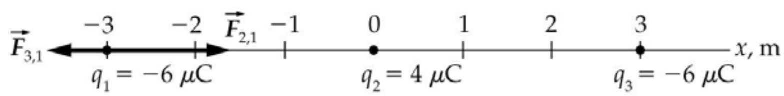
$$\vec{F}_{1,2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\,\mu\text{C})(-6\,\mu\text{C})}{(3\,\text{m})^2}\hat{\boldsymbol{i}} = \boxed{-(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

and

$$\vec{F}_{2,1} = -\vec{F}_{1,2} = \boxed{(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

29 •

Picture the Problem q_2 exerts an attractive force $\vec{F}_{2,1}$ on q_1 and q_3 a repulsive force $\vec{F}_{3,1}$. We can find the net force on q_1 by adding these forces.



Express the net force acting on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that q_2 exerts on q_1:

$$\vec{F}_{2,1} = \frac{k|q_1||q_2|}{r_{2,1}^2}\hat{\boldsymbol{i}}$$

Express the force that q_3 exerts on q_1:

$$\vec{F}_{3,1} = \frac{k|q_1||q_3|}{r_{3,1}^2}(-\hat{\boldsymbol{i}})$$

Substitute and simplify to obtain:

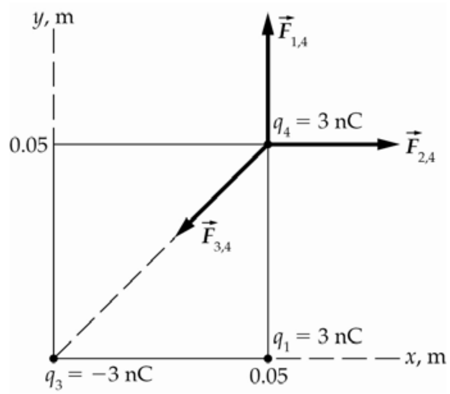
$$\vec{F}_1 = \frac{k|q_1||q_2|}{r_{2,1}^2}\hat{\boldsymbol{i}} - \frac{k|q_1||q_3|}{r_{3,1}^2}\hat{\boldsymbol{i}} = k|q_1|\left(\frac{|q_2|}{r_{2,1}^2} - \frac{|q_3|}{r_{3,1}^2}\right)\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate $\vec{F}_1$:

$$\vec{F}_1 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6\,\mu\text{C})\left(\frac{4\,\mu\text{C}}{(3\,\text{m})^2} - \frac{6\,\mu\text{C}}{(6\,\text{m})^2}\right)\hat{\boldsymbol{i}} = \boxed{(1.50\times10^{-2}\ \text{N})\hat{\boldsymbol{i}}}$$

30 ••

Picture the Problem The configuration of the charges and the forces on the fourth charge are shown in the figure … as is a coordinate system. From the figure it is evident that the net force on q_4 is along the diagonal of the square and directed away from q_3. We can apply Coulomb's law to express $\vec{F}_{1,4}$, $\vec{F}_{2,4}$ and $\vec{F}_{3,4}$ and then add them to find the net force on q_4.



Express the net force acting on q_4:

$$\vec{F}_4 = \vec{F}_{1,4} + \vec{F}_{2,4} + \vec{F}_{3,4}$$

Express the force that q_1 exerts on q_4:

$$\vec{F}_{1,4} = \frac{kq_1q_4}{r_{1,4}^2}\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{1,4}$:

$$\vec{F}_{1,4} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\text{nC})\left(\frac{3\,\text{nC}}{(0.05\,\text{m})^2}\right)\hat{j} = \left(3.24\times10^{-5}\ \text{N}\right)\hat{j}$$

Express the force that q_2 exerts on q_4:

$$\vec{F}_{2,4} = \frac{kq_2q_4}{r_{2,4}^2}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}_{2,4}$:

$$\vec{F}_{2,4} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\text{nC})\left(\frac{3\,\text{nC}}{(0.05\,\text{m})^2}\right)\hat{i} = \left(3.24\times10^{-5}\ \text{N}\right)\hat{i}$$

Express the force that q_3 exerts on q_4:

$$\vec{F}_{3,4} = \frac{kq_3q_4}{r_{3,4}^2}\hat{r}_{3,4}$$, where $\hat{r}_{3,4}$ is a unit vector pointing from q_3 to q_4.

Express $\vec{r}_{3,4}$ in terms of $\vec{r}_{3,1}$ and $\vec{r}_{1,4}$:

$$\vec{r}_{3,4} = \vec{r}_{3,1} + \vec{r}_{1,4} = (0.05\,\text{m})\hat{i} + (0.05\,\text{m})\hat{j}$$

Convert $\vec{r}_{3,4}$ to $\hat{r}_{3,4}$:

$$\hat{r}_{3,4} = \frac{\vec{r}_{3,4}}{|\vec{r}_{3,4}|} = \frac{(0.05\,\text{m})\hat{i} + (0.05\,\text{m})\hat{j}}{\sqrt{(0.05\,\text{m})^2 + (0.05\,\text{m})^2}} = 0.707\,\hat{i} + 0.707\,\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{3,4}$:

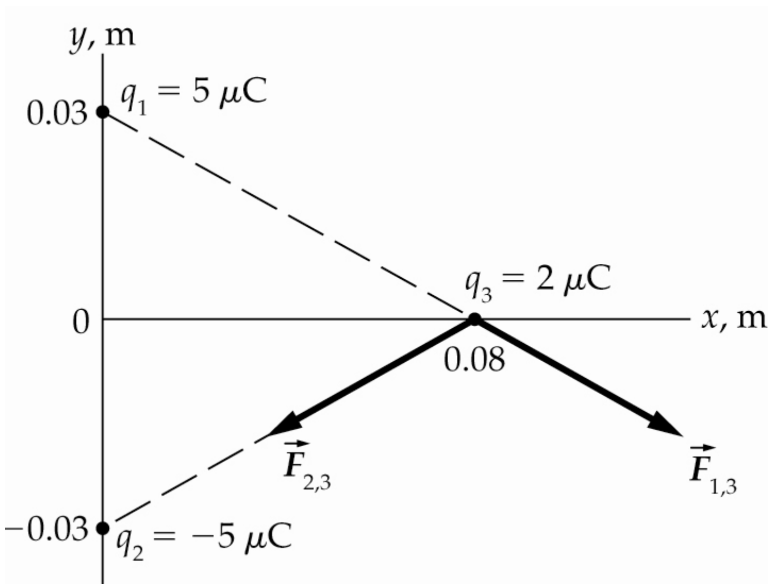
$$\vec{F}_{3,4} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3\,\text{nC})\left(\frac{3\,\text{nC}}{(0.05\sqrt{2}\,\text{m})^2}\right)(0.707\,\hat{i} + 0.707\,\hat{j})$$
$$= -(1.14\times10^{-5}\ \text{N})\hat{i} - (1.14\times10^{-5}\ \text{N})\hat{j}$$

Substitute and simplify to find $\vec{F}_4$:

$$\vec{F}_4 = (3.24\times10^{-5}\ \text{N})\hat{j} + (3.24\times10^{-5}\ \text{N})\hat{i} - (1.14\times10^{-5}\ \text{N})\hat{i} - (1.14\times10^{-5}\ \text{N})\hat{j}$$
$$= \boxed{(2.10\times10^{-5}\ \text{N})\hat{i} + (2.10\times10^{-5}\ \text{N})\hat{j}}$$

31 ••

Picture the Problem The configuration of the charges and the forces on q_3 are shown in the figure … as is a coordinate system. From the geometry of the charge distribution it is evident that the net force on the 2 μC charge is in the negative y direction. We can apply Coulomb's law to express $\vec{F}_{1,3}$ and $\vec{F}_{2,3}$ and then add them to find the net force on q_3.



The net force acting on q_3 is given by: $\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3}$

Express the force that q_1 exerts on q_3:

$$\vec{F}_{1,3} = F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j}$$

where

$$F = \frac{kq_1q_3}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\,\mu\text{C})(2\,\mu\text{C})}{(0.03\,\text{m})^2+(0.08\,\text{m})^2} = 12.3\,\text{N}$$

and

$$\theta = \tan^{-1}\left(\frac{3\,\text{cm}}{8\,\text{cm}}\right) = 20.6°$$

Express the force that q_2 exerts on q_3:

$$\vec{F}_{2,3} = -F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j}$$

Substitute for $\vec{F}_{1,3}$ and $\vec{F}_{2,3}$ and simplify to obtain:

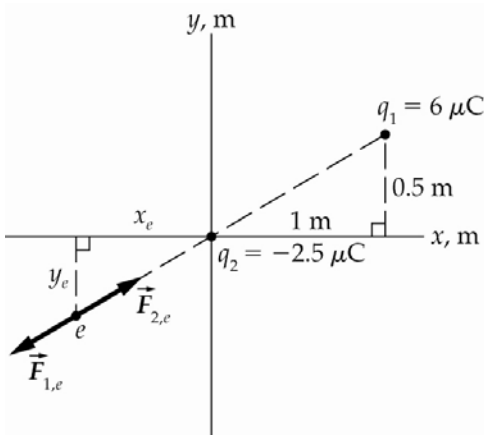
$$\vec{F}_3 = F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j} - F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j} = -2F\sin\theta\,\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_3$:

$$\vec{F}_3 = -2(12.3\,\text{N})\sin 20.6°\,\hat{j} = \boxed{-(8.66\,\text{N})\hat{j}}$$

***32 ••**

Picture the Problem The positions of the charges are shown in the diagram. It is apparent that the electron must be located along the line joining the two charges. Moreover, because it is negatively charged, it must be closer to the –2.5 μC than to the 6.0 μC charge, as is indicated in the figure. We can find the *x* and *y* coordinates of the electron's position by equating the two electrostatic forces acting on it and solving for its distance from the origin.



We can use similar triangles to express this radial distance in terms of the *x* and *y* coordinates of the electron.

Express the condition that must be

$$F_{1,e} = F_{2,e}$$

satisfied if the electron is to be in equilibrium:

Express the magnitude of the force that q_1 exerts on the electron:

$$F_{1,e} = \frac{kq_1 e}{\left(r + \sqrt{1.25\,\text{m}}\right)^2}$$

Express the magnitude of the force that q_2 exerts on the electron:

$$F_{2,e} = \frac{k|q_2|e}{r^2}$$

Substitute and simplify to obtain:

$$\frac{q_1}{\left(r + \sqrt{1.25\,\text{m}}\right)^2} = \frac{|q_2|}{r^2}$$

Substitute for q_1 and q_2 and simplify:

$$\left(-1.4\,\text{m}^{-2}\right) r^2 + \left(2.2361\,\text{m}^{-1}\right) r + 1.25\,\text{m} = 0$$

Solve for r to obtain:

$$r = 2.036\,\text{m}$$

and

$$r = -0.4386\,\text{m}$$

Because $r < 0$ is unphysical, we'll consider only the positive root.

Use the similar triangles in the diagram to establish the proportion involving the y coordinate of the electron:

$$\frac{y_e}{0.5\,\text{m}} = \frac{2.036\,\text{m}}{1.12\,\text{m}}$$

Solve for y_e:

$$y_e = 0.909\,\text{m}$$

Use the similar triangles in the diagram to establish the proportion involving the x coordinate of the electron:

$$\frac{x_e}{1\,\text{m}} = \frac{2.036\,\text{m}}{1.12\,\text{m}}$$

Solve for x_e:

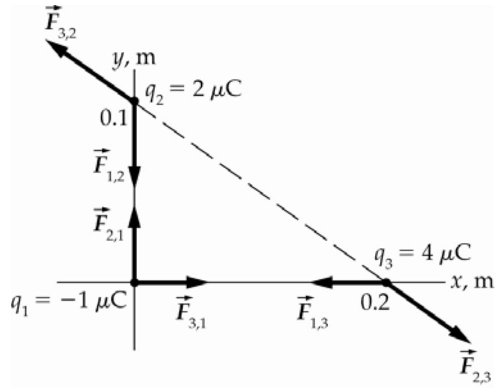
$$x_e = 1.82\,\text{m}$$

The coordinates of the electron's position are:

$$\left(x_e, y_e\right) = \boxed{\left(-1.82\,\text{m}, -0.909\,\text{m}\right)}$$

***33 ••**

Picture the Problem Let q_1 represent the charge at the origin, q_2 the charge at (0, 0.1 m), and q_3 the charge at (0.2 m, 0). The diagram shows the forces acting on each of the charges. Note the action-and-reaction pairs. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the charges.



Express the net force acting on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that q_2 exerts on q_1:

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\frac{\vec{r}_{2,1}}{r_{2,1}} = \frac{kq_2q_1}{r_{2,1}^3}\vec{r}_{2,1}$$

Substitute numerical values and evaluate $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\frac{(-1\ \mu\text{C})}{(0.1\,\text{m})^3}(-0.1\,\text{m})\hat{j} = (1.80\ \text{N})\hat{j}$$

Express the force that q_3 exerts on q_1:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3}\vec{r}_{3,1}$$

Substitute numerical values and evaluate $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\ \mu\text{C})\frac{(-1\ \mu\text{C})}{(0.2\,\text{m})^3}(-0.2\,\text{m})\hat{i} = (0.899\ \text{N})\hat{i}$$

Substitute to find $\vec{F}_1$:

$$\vec{F}_1 = \boxed{(0.899\ \text{N})\hat{i} + (1.80\ \text{N})\hat{j}}$$

Express the net force acting on q_2:

$$\vec{F}_2 = \vec{F}_{3,2} + \vec{F}_{1,2} = \vec{F}_{3,2} - \vec{F}_{2,1} = \vec{F}_{3,2} - (1.80\ \text{N})\hat{j}$$

because $\vec{F}_{1,2}$ and $\vec{F}_{2,1}$ are action-and-reaction forces.

Express the force that q_3 exerts on q_2:

$$\vec{F}_{3,2} = \frac{kq_3q_2}{r_{3,2}^3}\vec{r}_{3,2} = \frac{kq_3q_2}{r_{3,2}^3}\left[(-0.2\,\text{m})\hat{i} + (0.1\,\text{m})\hat{j}\right]$$

Substitute numerical values and evaluate $\vec{F}_{3,2}$:

$$\vec{F}_{3,2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})\frac{(2\,\mu\text{C})}{(0.224\,\text{m})^3}\left[(-0.2\,\text{m})\hat{i} + (0.1\,\text{m})\hat{j}\right]$$
$$= (-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j}$$

Find the net force acting on q_2:

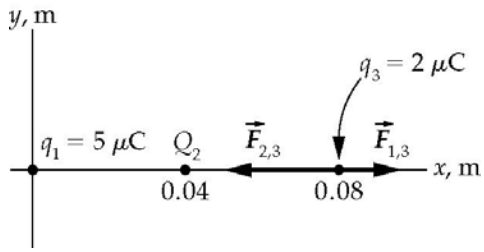
$$\vec{F}_2 = \vec{F}_{3,2} - (1.80\,\text{N})\hat{j} = (-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j} - (1.80\,\text{N})\hat{j}$$
$$= \boxed{(-1.28\,\text{N})\hat{i} - (1.16\,\text{N})\hat{j}}$$

Noting that $\vec{F}_{1,3}$ and $\vec{F}_{3,1}$ are an action-and-reaction pair, as are $\vec{F}_{2,3}$ and $\vec{F}_{3,2}$, express the net force acting on q_3:

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = -\vec{F}_{3,1} - \vec{F}_{3,2} = -(0.899\,\text{N})\hat{i} - \left[(-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j}\right]$$
$$= \boxed{(0.381\,\text{N})\hat{i} - (0.640\,\text{N})\hat{j}}$$

34 ••

Picture the Problem Let q_1 represent the charge at the origin and q_3 the charge initially at (8 cm, 0) and later at (17.75 cm, 0). The diagram shows the forces acting on q_3 at (8 cm, 0). We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the charges.



Express the net force on q_2 when it is at (8 cm, 0):

$$\vec{F}_2(8\,\text{cm},0) = \vec{F}_{1,3} + \vec{F}_{2,3} = \frac{kq_1q_3}{r_{1,3}^3}\vec{r}_{1,3} + \frac{kQ_2q_3}{r_{2,3}^3}\vec{r}_{2,3} = kq_3\left(\frac{q_1}{r_{1,3}^3}\vec{r}_{1,3} + \frac{Q_2}{r_{2,3}^3}\vec{r}_{2,3}\right)$$

Substitute numerical values to obtain:

$$(-19.7\,\mathrm{N})\hat{\boldsymbol{i}} = (8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2})(2\,\mu\mathrm{C})\left[\frac{5\,\mu\mathrm{C}}{(0.08\,\mathrm{m})^3}(0.08\,\mathrm{m})\hat{\boldsymbol{i}} + \frac{Q_2}{(0.04\,\mathrm{m})^3}(0.04\,\mathrm{m})\hat{\boldsymbol{i}}\right]$$

Solve for and evaluate Q_2:

$$Q_2 = \boxed{-3.00\,\mu\mathrm{C}}$$

Remarks: An alternative solution is to equate the electrostatic forces acting on q_2 when it is at (17.75 cm, 0).

35 ••

Picture the Problem By considering the symmetry of the array of charges we can see that the y component of the force on q is zero. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on q.

Express the net force acting on q:

$$\vec{\boldsymbol{F}}_q = \vec{\boldsymbol{F}}_{Q\text{ on }x\text{ axis},q} + 2\vec{\boldsymbol{F}}_{Q\text{ at }45°,q}$$

Express the force on q due to the charge Q on the x axis:

$$\vec{\boldsymbol{F}}_{Q\text{ on }x\text{ axis},q} = \frac{kqQ}{R^2}\hat{\boldsymbol{i}}$$

Express the net force on q due to the charges at 45°:

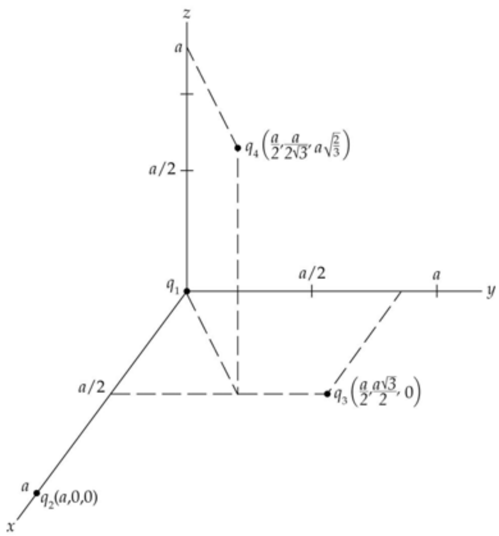
$$2\vec{\boldsymbol{F}}_{Q\text{ at }45°,q} = 2\frac{kqQ}{R^2}\cos45°\hat{\boldsymbol{i}} = \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{\boldsymbol{i}}$$

Substitute to obtain:

$$\vec{\boldsymbol{F}}_q = \frac{kqQ}{R^2}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{\boldsymbol{i}} = \boxed{\frac{kqQ}{R^2}\left(1+\frac{\sqrt{2}}{2}\right)\hat{\boldsymbol{i}}}$$

36 •••

Picture the Problem Let the H^+ ions be in the x-y plane with H_1 at (0, 0, 0), H_2 at (a, 0, 0), and H_3 at $\left(a/2, a\sqrt{3}/2, 0\right)$. The N^{-3} ion, q_4 in our notation, is then at $\left(a/2, a/2\sqrt{3}, a\sqrt{2/3}\right)$ where $a = 1.64\times10^{-10}$ m. To simplify our calculations we'll set $ke^2/a^2 = C = 8.56\times10^{-9}\,\mathrm{N}$. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each ion.



Express the net force acting on q_1: $\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$

Find $\vec{F}_{2,1}$: $\vec{F}_{2,1} = \frac{kq_1q_2}{r_{2,1}^2}\hat{r}_{2,1} = C\left(-\hat{i}\right) = -C\hat{i}$

Find $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^2}\hat{r}_{3,1}$$

$$= C\frac{\left(0 - \frac{a}{2}\right)\hat{i} + \left(0 - \frac{a\sqrt{3}}{2}\right)\hat{j}}{a}$$

$$= -C\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

Noting that the magnitude of q_4 is three times that of the other charges and that it is negative, express $\vec{F}_{4,1}$:

$$\vec{F}_{4,1} = 3C\hat{r}_{4,1} = -3C\frac{\left(0 - \frac{a}{2}\right)\hat{i} + \left(0 - \frac{a}{2\sqrt{3}}\right)\hat{j} + \left(0 - \frac{a\sqrt{2}}{\sqrt{3}}\right)\hat{k}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2\sqrt{3}}\right)^2 + \left(\frac{a\sqrt{2}}{\sqrt{3}}\right)^2}}$$

$$= 3C\frac{\left(\frac{a}{2}\right)\hat{i} + \left(\frac{a}{2\sqrt{3}}\right)\hat{j} + \left(\frac{a\sqrt{2}}{\sqrt{3}}\right)\hat{k}}{a} = 3C\left[\left(\frac{1}{2}\right)\hat{i} + \left(\frac{1}{2\sqrt{3}}\right)\hat{j} + \left(\sqrt{\frac{2}{3}}\right)\hat{k}\right]$$

Substitute to find $\vec{F}_1$:

$$\vec{F}_1 = -C\hat{i} - C\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 3C\left[\left(\frac{1}{2}\right)\hat{i} + \left(\frac{1}{2\sqrt{3}}\right)\hat{j} + \left(\sqrt{\frac{2}{3}}\right)\hat{k}\right] = \boxed{C\sqrt{6}\hat{k}}$$

From symmetry considerations:

$$\vec{F}_2 = \vec{F}_3 = \vec{F}_1 = \boxed{C\sqrt{6}\hat{k}}$$

Express the condition that molecule is in equilibrium:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Solve for and evaluate $\vec{F}_4$:

$$\vec{F}_4 = -\left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right) = -3\vec{F}_1 = \boxed{-3C\sqrt{6}\hat{k}}$$

The Electric Field

***37** •

Picture the Problem Let q represent the charge at the origin and use Coulomb's law for $\vec{E}$ due to a point charge to find the electric field at x = 6 m and –10 m.

(a) Express the electric field at a point P located a distance x from a charge q:

$$\vec{E}(x) = \frac{kq}{x^2}\hat{r}_{\mathrm{P},0}$$

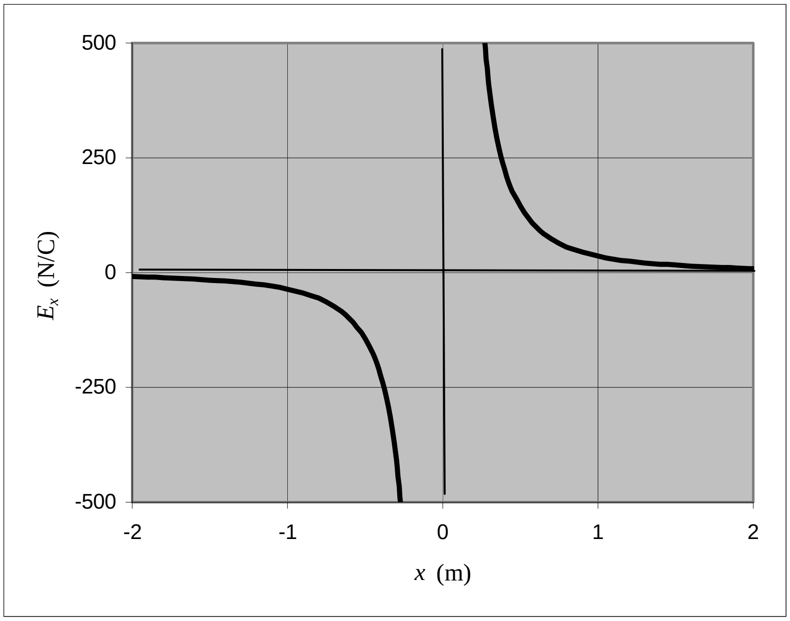
Evaluate this expression for x = 6 m:

$$\vec{E}(6\,\mathrm{m}) = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(4\,\mu\mathrm{C})}{(6\,\mathrm{m})^2}\hat{i} = \boxed{(999\,\mathrm{N/C})\hat{i}}$$

(b) Evaluate $\vec{E}$ at x = –10 m:

$$\vec{E}(-10\,\mathrm{m}) = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(4\,\mu\mathrm{C})}{(10\,\mathrm{m})^2}\left(-\hat{i}\right) = \boxed{(-360\,\mathrm{N/C})\hat{i}}$$

(c) The following graph was plotted using a spreadsheet program:



***38** •

Picture the Problem Let q represent the charges of +4 μC and use Coulomb's law for $\vec{E}$ due to a point charge and the principle of superposition for fields to find the electric field at the locations specified.

Noting that $q_1 = q_2$, use Coulomb's law and the principle of superposition to express the electric field due to the given charges at a point P a distance x from the origin:

$$\vec{E}(x) = \vec{E}_{q_1}(x) + \vec{E}_{q_2}(x) = \frac{kq_1}{x^2}\hat{r}_{q_1,\text{P}} + \frac{kq_2}{(8\,\text{m}-x)^2}\hat{r}_{q_2,\text{P}} = kq_1\left(\frac{1}{x^2}\hat{r}_{q_1,\text{P}} + \frac{1}{(8\,\text{m}-x)^2}\hat{r}_{q_2,\text{P}}\right)$$

$$= (36\,\text{kN}\cdot\text{m}^2/\text{C})\left(\frac{1}{x^2}\hat{r}_{q_1,\text{P}} + \frac{1}{(8\,\text{m}-x)^2}\hat{r}_{q_2,\text{P}}\right)$$

(a) Apply this equation to the point at $x = -2$ m:

$$\vec{\mathbf{E}}(-2\,\text{m}) = (36\,\text{kN}\cdot\text{m}^2/\text{C})\left[\frac{1}{(2\,\text{m})^2}(-\hat{\mathbf{i}}) + \frac{1}{(10\,\text{m})^2}(-\hat{\mathbf{i}})\right] = \boxed{(-9.36\,\text{kN/C})\hat{\mathbf{i}}}$$

(b) Evaluate $\vec{E}$ at $x = 2$ m:

$$\vec{E}(2\,\text{m}) = (36\,\text{kN}\cdot\text{m}^2/\text{C})\left[\frac{1}{(2\,\text{m})^2}(\hat{i}) + \frac{1}{(6\,\text{m})^2}(-\hat{i})\right] = \boxed{(8.00\,\text{kN/C})\hat{i}}$$

(*c*) Evaluate $\vec{E}$ at x = 6 m:

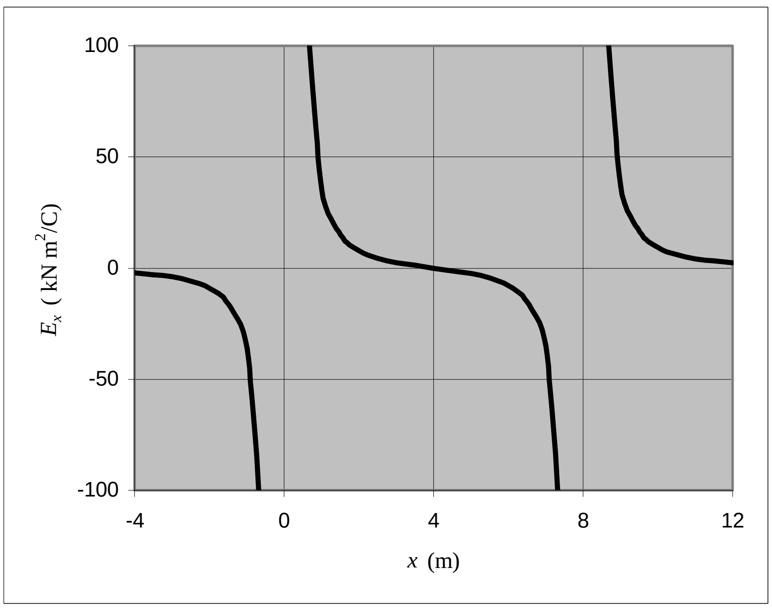
$$\vec{E}(6\,\text{m}) = (36\,\text{kN}\cdot\text{m}^2/\text{C})\left[\frac{1}{(6\,\text{m})^2}(\hat{i}) + \frac{1}{(2\,\text{m})^2}(-\hat{i})\right] = \boxed{(-8.00\,\text{kN/C})\hat{i}}$$

(*d*) Evaluate $\vec{E}$ at x = 10 m:

$$\vec{E}(10\,\text{m}) = (36\,\text{kN}\cdot\text{m}^2/\text{C})\left[\frac{1}{(10\,\text{m})^2}(\hat{i}) + \frac{1}{(2\,\text{m})^2}(\hat{i})\right] = \boxed{(9.35\,\text{kN/C})\hat{i}}$$

(*e*) From symmetry considerations: $E(4\,\text{m}) = \boxed{0}$

(*f*) The following graph was plotted using a spreadsheet program:



39 •

Picture the Problem We can find the electric field at the origin from its definition and the force on a charge placed there from $\vec{F} = q\vec{E}$. We can apply Coulomb's law to find the value of the charge placed at y = 3 cm.

(*a*) Apply the definition of electric field to obtain:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{(8\times10^{-4}\,\text{N})\hat{j}}{2\,\text{nC}} = \boxed{(400\,\text{kN/C})\hat{j}}$$

(*b*) Express and evaluate the force on a charged body in an electric field:

$$\vec{F} = q\vec{E} = (-4\,\text{nC})(400\,\text{kN/C})\hat{j} = \boxed{(-1.60\,\text{mN})\hat{j}}$$

(*c*) Apply Coulomb's law to obtain:

$$\frac{kq(-4\,\text{nC})}{(0.03\,\text{m})^2}\left(-\hat{\boldsymbol{j}}\right)=(-1.60\,\text{mN})\hat{\boldsymbol{j}}$$

Solve for and evaluate *q*:

$$q=-\frac{(1.60\,\text{mN})(0.03\,\text{m})^2}{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\text{nC})}=\boxed{-40.0\,\text{nC}}$$

40 •

Picture the Problem We can compare the electric and gravitational forces acting on an electron by expressing their ratio. We can equate these forces to find the charge that would have to be placed on a penny in order to balance the earth's gravitational force on it.

(*a*) Express the magnitude of the electric force acting on the electron:

$$F_e=eE$$

Express the magnitude of the gravitational force acting on the electron:

$$F_g=m_e g$$

Express the ratio of these forces to obtain:

$$\frac{F_e}{F_g}=\frac{eE}{mg}$$

Substitute numerical values and evaluate F_e/F_g:

$$\frac{F_e}{F_g}=\frac{\left(1.6\times10^{-19}\ \text{C}\right)(150\,\text{N/C})}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right)}=2.69\times10^{12}$$

or

$F_e=\boxed{\left(2.69\times10^{12}\right)F_g}$, i.e., the electric force is greater by a factor of 2.69×10^{12}.

(*b*) Equate the electric and gravitational forces acting on the penny and solve for *q* to obtain:

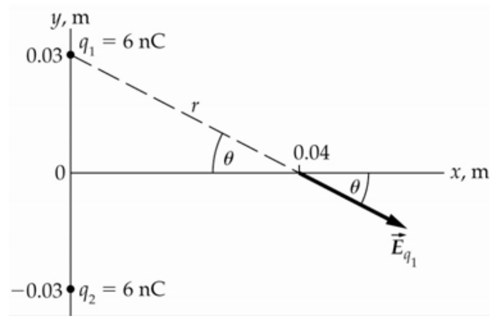
$$q=\frac{mg}{E}$$

Substitute numerical values and evaluate *q*:

$$q=\frac{\left(3\times10^{-3}\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{150\,\text{N/C}}=\boxed{1.96\times10^{-4}\ \text{C}}$$

41 ••

Picture the Problem The diagram shows the locations of the charges q_1 and q_2 and the point on the x axis at which we are to find $\vec{E}$. From symmetry considerations we can conclude that the y component of $\vec{E}$ at any point on the x axis is zero. We can use Coulomb's law for the electric field due to point charges to find the field at any point on the x axis and $\vec{F} = q\vec{E}$ to find the force on a charge q_0 placed on the x axis at $x = 4$ cm.



(*a*) Letting $q = q_1 = q_2$, express the x-component of the electric field due to one charge as a function of the distance r from either charge to the point of interest:

$$\vec{E}_x = \frac{kq}{r^2}\cos\theta\,\hat{i}$$

Express $\vec{E}_x$ for both charges:

$$\vec{E}_x = 2\frac{kq}{r^2}\cos\theta\,\hat{i}$$

Substitute for $\cos\theta$ and r, substitute numerical values, and evaluate to obtain:

$$\vec{E}_x = 2\frac{kq}{r^2}\frac{0.04\,\text{m}}{r}\hat{i} = \frac{2kq(0.04\,\text{m})}{r^3}\hat{i} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6\,\text{nC})(0.04\,\text{m})}{\left[(0.03\,\text{m})^2 + (0.04\,\text{m})^2\right]^{3/2}}\hat{i}$$

$$= \boxed{(34.5\,\text{kN/C})\hat{i}}$$

(*b*) Apply $\vec{F} = q\vec{E}$ to find the force on a charge q_0 placed on the x axis at $x = 4$ cm:

$$\vec{F} = (2\,\text{nC})(34.5\,\text{kN/C})\hat{i}$$

$$= \boxed{(69.0\,\mu\text{N})\hat{i}}$$

***42** ••

Picture the Problem If the electric field at $x = 0$ is zero, both its x and y components must be zero. The only way this condition can be satisfied with the point charges of +5.0 μC and –8.0 μC are on the x axis is if the point charge of +6.0 μC is also on the x axis. Let the subscripts 5, –8, and 6 identify the point charges and their fields. We can use Coulomb's law for $\vec{E}$ due to a point charge and the principle of superposition for fields to determine where the +6.0 μC charge should be located so that the electric field at $x = 0$ is zero.

Express the electric field at $x = 0$ in terms of the fields due to the charges of +5.0 μC, –8.0 μC, and +6.0 μC:

$$\vec{E}(0) = \vec{E}_{5\,\mu\text{C}} + \vec{E}_{-8\,\mu\text{C}} + \vec{E}_{6\,\mu\text{C}} = 0$$

Substitute for each of the fields to obtain:

$$\frac{kq_5}{r_5^2}\hat{r}_5 + \frac{kq_6}{r_6^2}\hat{r}_6 + \frac{kq_{-8}}{r_{-8}^2}\hat{r}_{-8} = 0$$

or

$$\frac{kq_5}{r_5^2}\hat{i} + \frac{kq_6}{r_6^2}\left(-\hat{i}\right) + \frac{kq_{-8}}{r_{-8}^2}\left(-\hat{i}\right) = 0$$

Divide out the unit vector $\hat{i}$ to obtain:

$$\frac{q_5}{r_5^2} - \frac{q_6}{r_6^2} - \frac{q_{-8}}{r_{-8}^2} = 0$$

Substitute numerical values to obtain:

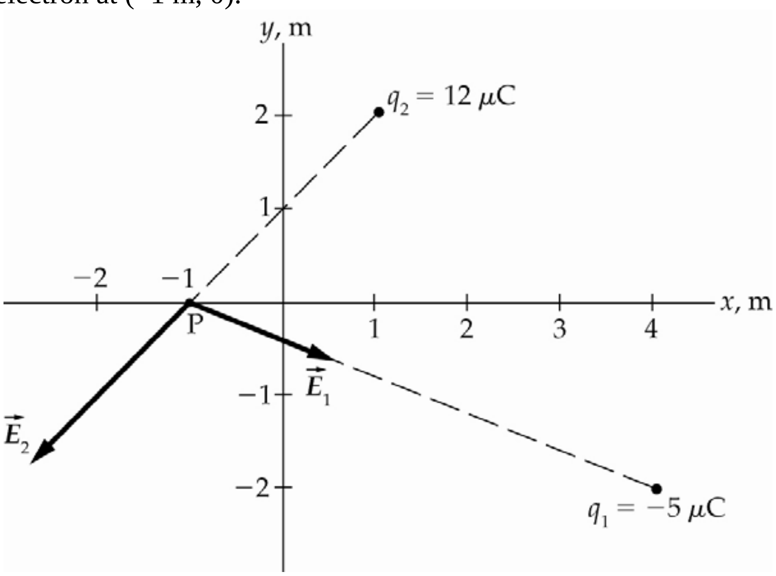
$$\frac{5}{(3\,\text{cm})^2} - \frac{6}{r_6^2} - \frac{-8}{(4\,\text{cm})^2} = 0$$

Solve for r_6:

$$r_6 = \boxed{2.38\,\text{cm}}$$

43 ••

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two charges. We can use Coulomb's law for $\vec{E}$ due to point charges and the superposition principle for electric fields to find $\vec{E}_P$. We can apply $\vec{F} = q\vec{E}$ to find the force on an electron at (–1 m, 0).



(*a*) Express the electric field at (–1 m, 0) due to the charges q_1 and q_2:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

Evaluate $\vec{E}_1$:

$$\vec{E}_1=\frac{kq_1}{r_{1,\mathrm{P}}^2}\hat{r}_{1,\mathrm{P}}=\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(-5\,\mu\mathrm{C}\right)}{\left(5\,\mathrm{m}\right)^2+\left(2\,\mathrm{m}\right)^2}\left(\frac{\left(-5\,\mathrm{m}\right)\hat{i}+\left(2\,\mathrm{m}\right)\hat{j}}{\sqrt{\left(5\,\mathrm{m}\right)^2+\left(2\,\mathrm{m}\right)^2}}\right)$$
$$=\left(-1.55\times10^3\ \mathrm{N/C}\right)\left(-0.928\,\hat{i}+0.371\,\hat{j}\right)$$
$$=\left(1.44\,\mathrm{kN/C}\right)\hat{i}+\left(-0.575\,\mathrm{kN/C}\right)\hat{j}$$

Evaluate $\vec{E}_2$:

$$\vec{E}_2=\frac{kq_2}{r_{2,\mathrm{P}}^2}\hat{r}_{2,\mathrm{P}}=\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(12\,\mu\mathrm{C}\right)}{\left(2\,\mathrm{m}\right)^2+\left(2\,\mathrm{m}\right)^2}\left(\frac{\left(-2\,\mathrm{m}\right)\hat{i}+\left(-2\,\mathrm{m}\right)\hat{j}}{\sqrt{\left(2\,\mathrm{m}\right)^2+\left(2\,\mathrm{m}\right)^2}}\right)$$
$$=\left(13.5\times10^3\ \mathrm{N/C}\right)\left(-0.707\,\hat{i}-0.707\,\hat{j}\right)$$
$$=\left(-9.54\,\mathrm{kN/C}\right)\hat{i}+\left(-9.54\,\mathrm{kN/C}\right)\hat{j}$$

Substitute for $\vec{E}_1$ and $\vec{E}_2$ and simplify to find $\vec{E}_\mathrm{P}$:

$$\vec{E}_\mathrm{P}=\left(1.44\,\mathrm{kN/C}\right)\hat{i}+\left(-0.575\,\mathrm{kN/C}\right)\hat{j}+\left(-9.54\,\mathrm{kN/C}\right)\hat{i}+\left(-9.54\,\mathrm{kN/C}\right)\hat{j}$$
$$=\left(-8.10\,\mathrm{kN/C}\right)\hat{i}+\left(-10.1\,\mathrm{kN/C}\right)\hat{j}$$

The magnitude of $\vec{E}_\mathrm{P}$ is:

$$E_\mathrm{P}=\sqrt{\left(-8.10\,\mathrm{kN/C}\right)^2+\left(-10.1\,\mathrm{kN/C}\right)^2}=\boxed{12.9\,\mathrm{kN/C}}$$

The direction of $\vec{E}_\mathrm{P}$ is:

$$\theta_E=\tan^{-1}\left(\frac{-10.1\,\mathrm{kN/C}}{-8.10\,\mathrm{kN/C}}\right)=\boxed{231^\circ}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-10.1\,\mathrm{kN/C}}{-8.10\,\mathrm{kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

(*b*) Express and evaluate the force on an electron at point P:

$$\vec{F} = q\vec{E}_P = (-1.602\times10^{-19}\text{ C})\left[(-8.10\text{ kN/C})\hat{i} + (-10.1\text{ kN/C})\hat{j}\right]$$
$$= (1.30\times10^{-15}\text{ N})\hat{i} + (1.62\times10^{-15}\text{ N})\hat{j}$$

Find the magnitude of $\vec{F}$:

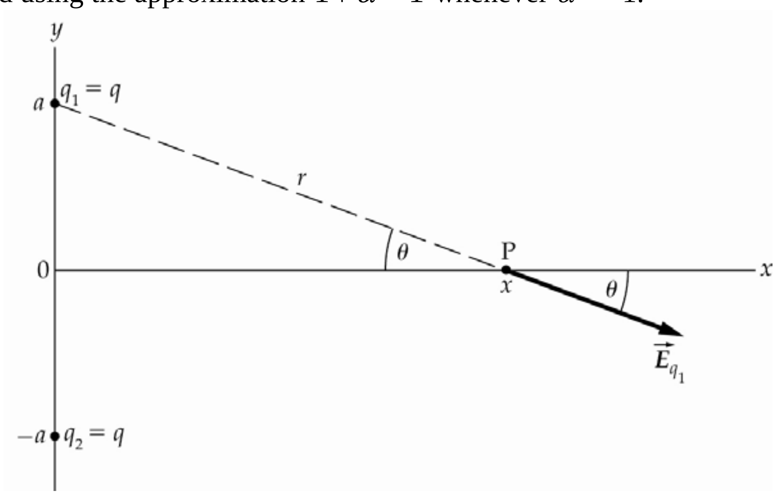
$$F = \sqrt{(1.30\times10^{-15}\text{ N})^2 + (1.62\times10^{-15}\text{ N})^2} = \boxed{2.08\times10^{-15}\text{ N}}$$

Find the direction of $\vec{F}$:

$$\theta_F = \tan^{-1}\left(\frac{1.62\times10^{-15}\text{ N}}{1.3\times10^{-15}\text{ N}}\right) = \boxed{51.3°}$$

44 ••

Picture the Problem The diagram shows the locations of the charges q_1 and q_2 and the point on the x axis at which we are to find $\vec{E}$. From symmetry considerations we can conclude that the y component of $\vec{E}$ at any point on the x axis is zero. We can use Coulomb's law for the electric field due to point charges to find the field at any point on the x axis. We can establish the results called for in parts (b) and (c) by factoring the radicand and using the approximation $1+\alpha\approx1$ whenever $\alpha << 1$.



(a) Express the x-component of the electric field due to the charges at $y = a$ and $y = -a$ as a function of the distance r from either charge to point P:

$$\vec{E}_x = 2\frac{kq}{r^2}\cos\theta\,\hat{i}$$

Substitute for $\cos\theta$ and r to obtain:

$$\vec{E}_x = 2\frac{kq}{r^2}\frac{x}{r}\hat{i} = \frac{2kqx}{r^3}\hat{i} = \frac{2kqx}{(x^2+a^2)^{3/2}}\hat{i}$$
$$= \frac{2kqx}{(x^2+a^2)^{3/2}}\hat{i}$$

and

$$E_x = \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}}}$$

(*b*) For $|x| << a, x^2 + a^2 \approx a^2$, so:

$$E_x \approx \frac{2kqx}{\left(a^2\right)^{3/2}} = \boxed{\frac{2kqx}{a^3}}$$

For $|x| >> a, x^2 + a^2 \approx x^2$, so:

$$E_x \approx \frac{2kqx}{\left(x^2\right)^{3/2}} = \boxed{\frac{2kq}{x^2}}$$

(*c*) For $x >> a$, the charges separated by a would appear to be a single charge of magnitude $2q$. Its field would be given by $E_x = \frac{2kq}{x^2}$.

Factor the radicand to obtain:

$$E_x = 2kqx\left[x^2\left(1+\frac{a^2}{x^2}\right)\right]^{-3/2}$$

For $a << x$:

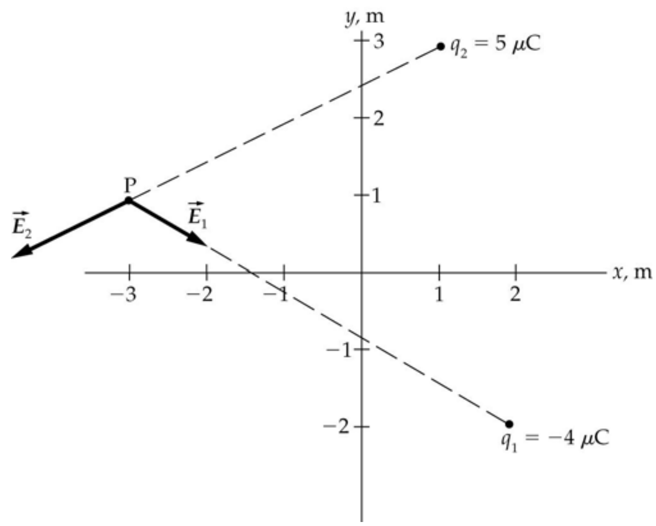
$$1+\frac{a^2}{x^2} \approx 1$$

and

$$E_x = 2kqx\left[x^2\right]^{-3/2} = \boxed{\frac{2kq}{x^2}}$$

***45 ••**

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two charges. We can use Coulomb's law for $\vec{E}$ due to point charges and the superposition principle for electric fields to find $\vec{E}_P$. We can apply $\vec{F} = q\vec{E}$ to find the force on a proton at (–3 m, 1 m).



(*a*) Express the electric field at (–3 m, 1 m) due to the charges q_1 and q_2:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

Evaluate $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4\ \mu\text{C})}{(5\,\text{m})^2 + (3\,\text{m})^2}\left(\frac{(-5\,\text{m})\hat{i} + (3\,\text{m})\hat{j}}{\sqrt{(5\,\text{m})^2 + (3\,\text{m})^2}}\right)$$
$$= (-1.06\,\text{kN/C})(-0.857\,\hat{i} + 0.514\,\hat{j}) = (0.908\,\text{kN/C})\hat{i} + (-0.544\,\text{kN/C})\hat{j}$$

Evaluate $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\ \mu\text{C})}{(4\,\text{m})^2 + (2\,\text{m})^2}\left(\frac{(-4\,\text{m})\hat{i} + (-2\,\text{m})\hat{j}}{\sqrt{(4\,\text{m})^2 + (2\,\text{m})^2}}\right)$$
$$= (2.25\,\text{kN/C})(-0.894\,\hat{i} - 0.447\,\hat{j}) = (-2.01\,\text{kN/C})\hat{i} + (-1.01\,\text{kN/C})\hat{j}$$

Substitute and simplify to find $\vec{E}_P$:

$$\vec{E}_P = (0.908\,\text{kN/C})\hat{i} + (-0.544\,\text{kN/C})\hat{j} + (-2.01\,\text{kN/C})\hat{i} + (-1.01\,\text{kN/C})\hat{j}$$
$$= (-1.10\,\text{kN/C})\hat{i} + (-1.55\,\text{kN/C})\hat{j}$$

The magnitude of $\vec{E}_P$ is:

$$E_P = \sqrt{(1.10\,\text{kN/C})^2 + (1.55\,\text{kN/C})^2} = \boxed{1.90\,\text{kN/C}}$$

The direction of $\vec{E}_P$ is:

$$\theta_E = \tan^{-1}\left(\frac{-1.55\,\text{kN/C}}{-1.10\,\text{kN/C}}\right) = \boxed{235°}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-1.55\,\text{kN/C}}{-1.10\,\text{kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

(*b*) Express and evaluate the force on a proton at point P:

$$\begin{aligned}\vec{F} &= q\vec{E}_P = \left(1.6\times10^{-19}\,\text{C}\right)\left[(-1.10\,\text{kN/C})\hat{\boldsymbol{i}} + (-1.55\,\text{kN/C})\hat{\boldsymbol{j}}\right]\\ &= \left(-1.76\times10^{-16}\,\text{N}\right)\hat{\boldsymbol{i}} + \left(-2.48\times10^{-16}\,\text{N}\right)\hat{\boldsymbol{j}}\end{aligned}$$

The magnitude of $\vec{F}$ is:

$$F = \sqrt{\left(-1.76\times10^{-16}\,\text{N}\right)^2 + \left(-2.48\times10^{-16}\,\text{N}\right)^2} = \boxed{3.04\times10^{-16}\,\text{N}}$$

The direction of $\vec{F}$ is:

$$\theta_F = \tan^{-1}\left(\frac{-2.48\times10^{-16}\,\text{N}}{-1.76\times10^{-16}\,\text{N}}\right) = \boxed{235°}$$

where, as noted above, the angle returned by your calculator for $\tan^{-1}\left(\frac{-2.48\times10^{-16}\,\text{N}}{-1.76\times10^{-16}\,\text{N}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

46 ••

Picture the Problem In Problem 44 it is shown that the electric field on the *x* axis, due to equal positive charges located at (0, *a*) and (0,–*a*), is given by $E_x = 2kqx\left(x^2 + a^2\right)^{-3/2}$. We can identify the locations at which E_x has it greatest values by setting dE_x/dx equal to zero.

(*a*) Evaluate $\frac{dE_x}{dx}$:

$$\frac{dE_x}{dx} = \frac{d}{dx}\left[2kqx\left(x^2 + a^2\right)^{-3/2}\right] = 2kq\frac{d}{dx}\left[x\left(x^2 + a^2\right)^{-3/2}\right]$$
$$= 2kq\left[x\frac{d}{dx}\left(x^2 + a^2\right)^{-3/2} + \left(x^2 + a^2\right)^{-3/2}\right]$$
$$= 2kq\left[x\left(-\frac{3}{2}\right)\left(x^2 + a^2\right)^{-5/2}(2x) + \left(x^2 + a^2\right)^{-3/2}\right]$$
$$= 2kq\left[-3x^2\left(x^2 + a^2\right)^{-5/2} + \left(x^2 + a^2\right)^{-3/2}\right]$$

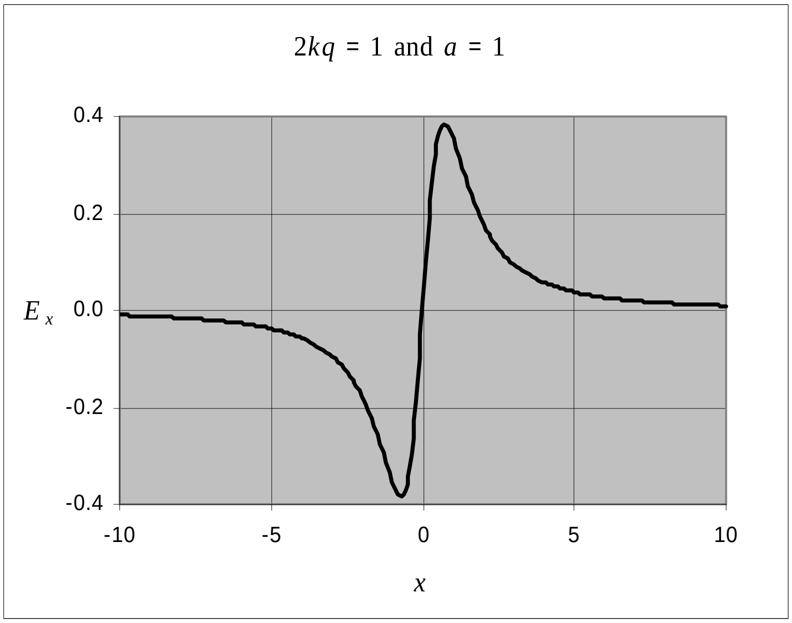
Set this derivative equal to zero:

$$-3x^2\left(x^2 + a^2\right)^{-5/2} + \left(x^2 + a^2\right)^{-3/2} = 0$$

Solve for x to obtain:

$$x = \boxed{\pm\frac{a}{\sqrt{2}}}$$

(*b*) The following graph was plotted using a spreadsheet program:



47 •••

Picture the Problem We can determine the stability of the equilibrium in Part (*a*) and Part (*b*) by considering the forces the equal charges q at $y = +a$ and $y = -a$ exert on the test charge when it is given a small displacement along either the x or y axis. The application of Coulomb's law in Part (*c*) will lead to the magnitude and sign of the charge that must be placed at the origin in order that a net force of zero is experienced by each of the three charges.

(*a*) Because E_x is in the x direction, a positive test charge that is displaced from

(0, 0) in either the $+x$ direction or the $-x$ direction will experience a force pointing away from the origin and accelerate in the direction of the force.

Consequently, the equilibrium at (0,0) is unstable for a small displacement along the x axis.

If the positive test charge is displaced in the direction of increasing y (the positive y direction), the charge at $y = +a$ will exert a greater force than the charge at $y = -a$, and the net force is then in the $-y$ direction; i.e., it is a restoring force. Similarly, if the positive test charge is displaced in the direction of decreasing y (the negative y direction), the charge at $y = -a$ will exert a greater force than the charge at $y = -a$, and the net force is then in the $-y$ direction; i.e., it is a restoring force.

Consequently, the equilibrium at (0,0) is stable for a small displacement along the y axis.

(*b*)

Following the same arguments as in Part (*a*), one finds that, for a negative test charge, the equilibrium is stable at (0,0) for displacements along the x axis and unstable for displacements along the y axis.

(*c*) Express the net force acting on the charge at $y = +a$:

$$\sum F_{q \text{ at } y=+a} = \frac{kqq_0}{a^2} + \frac{kq^2}{(2a)^2} = 0$$

Solve for q_0 to obtain:

$$q_0 = \boxed{-\tfrac{1}{4}q_0}$$

Remarks: In Part (*c*), we could just as well have expressed the net force acting on the charge at $y = -a$. Due to the symmetric distribution of the charges at $y = -a$ and $y = +a$, summing the forces acting on q_0 at the origin does not lead to a relationship between q_0 and q.

***48** •••

Picture the Problem In Problem 44 it is shown that the electric field on the x axis, due to equal positive charges located at (0, a) and (0,$-a$), is given by $E_x = 2kqx\left(x^2 + a^2\right)^{-3/2}$. We can use $T = 2\pi\sqrt{m/k'}$ to express the period of the motion in terms of the restoring constant k'.

(*a*) Express the force acting on the on the bead when its displacement from the origin is x:

$$F_x = -qE_x = -\frac{2kq^2x}{\left(x^2 + a^2\right)^{3/2}}$$

Factor a^2 from the denominator to obtain:

$$F_x = -\frac{2kq^2x}{a^2\left(\frac{x^2}{a^2}+1\right)^{3/2}}$$

For $x << a$:

$$F_x = \boxed{-\frac{2kq^2}{a^3}x}$$

i.e., the bead experiences a linear restoring force.

(*b*) Express the period of a simple harmonic oscillator:

$$T = 2\pi\sqrt{\frac{m}{k'}}$$

Obtain k' from our result in part (*a*):

$$k' = \frac{2kq^2}{a^3}$$

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{m}{\frac{2kq^2}{a^3}}} = \boxed{2\pi\sqrt{\frac{ma^3}{2kq^2}}}$$

Motion of Point Charges in Electric Fields

49 •

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the electron in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of 0.01*c* and the distance it travels while acquiring this speed.

(*a*) Use data found at the back of your text to compute *e*/*m* for an electron:

$$\frac{e}{m_e} = \frac{1.6\times10^{-19}\text{ C}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.76\times10^{11}\text{ C/kg}}$$

(*b*) Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a = \frac{F_{\text{net}}}{m_e} = \frac{eE}{m_e}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{(1.6\times10^{-19}\text{ C})(100\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = \boxed{1.76\times10^{13}\text{ m/s}^2}$$

$$\boxed{\text{The direction of the acceleration of an electron is opposite the electric field.}}$$

(*c*) Using the definition of acceleration, relate the time required for an electron to reach 0.01*c* to its acceleration:

$$\Delta t = \frac{v}{a} = \frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{0.01\left(3\times10^{8}\text{ m/s}\right)}{1.76\times10^{13}\text{ m/s}^2} = \boxed{0.170\,\mu\text{s}}$$

(*d*) Find the distance the electron travels from its average speed and the elapsed time:

$$\begin{aligned}\Delta x &= v_{\text{av}}\Delta t\\ &= \tfrac{1}{2}\left[0 + 0.01\left(3\times10^{8}\text{ m/s}\right)\right]\left(0.170\,\mu\text{s}\right)\\ &= \boxed{25.5\,\text{cm}}\end{aligned}$$

***50** •

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the proton in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of 0.01*c* and the distance it travels while acquiring this speed.

(*a*) Use data found at the back of your text to compute *e*/*m* for an electron:

$$\frac{e}{m_p} = \frac{1.6\times10^{-19}\text{ C}}{1.67\times10^{-27}\text{ kg}} = \boxed{9.58\times10^{7}\text{ C/kg}}$$

Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a = \frac{F_{\text{net}}}{m_p} = \frac{eE}{m_p}$$

Substitute numerical values and evaluate a:

$$a = \frac{\left(1.6\times10^{-19}\text{ C}\right)\left(100\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = \boxed{9.58\times10^{9}\text{ m/s}^2}$$

$$\boxed{\text{The direction of the acceleration of a proton is in the direction of the electric field.}}$$

(*b*) Using the definition of acceleration, relate the time required for an electron to reach 0.01*c* to its acceleration:

$$\Delta t = \frac{v}{a} = \frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{0.01\left(3\times10^{8}\ \text{m/s}\right)}{9.58\times10^{9}\ \text{m/s}^2} = \boxed{313\,\mu\text{s}}$$

51 •

Picture the Problem The electric force acting on the electron is opposite the direction of the electric field. We can apply Newton's 2nd law to find the electron's acceleration and use constant acceleration equations to find how long it takes the electron to travel a given distance and its deflection during this interval of time.

(*a*) Use Newton's 2nd law to relate the acceleration of the electron first to the net force acting on it and then the electric field in which it finds itself:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m_e} = \frac{-e\vec{E}}{m_e}$$

Substitute numerical values and evaluate $\vec{a}$:

$$\vec{a} = -\frac{1.6\times10^{-19}\ \text{C}}{9.11\times10^{-31}\ \text{kg}}\left(400\ \text{N/C}\right)\hat{j} = \boxed{\left(-7.03\times10^{13}\ \text{m/s}^2\right)\hat{j}}$$

(*b*) Relate the time to travel a given distance in the *x* direction to the electron's speed in the *x* direction:

$$\Delta t = \frac{\Delta x}{v_x} = \frac{0.1\text{m}}{2\times10^{6}\ \text{m/s}} = \boxed{50.0\,\text{ns}}$$

(*c*) Using a constant-acceleration equation, relate the displacement of the electron to its acceleration and the elapsed time:

$$\Delta\vec{y} = \tfrac{1}{2}\vec{a}_y\left(\Delta t\right)^2 = \tfrac{1}{2}\left(-7.03\times10^{13}\ \text{m/s}^2\right)\left(50\,\text{ns}\right)^2\hat{j} = \boxed{\left(-8.79\,\text{cm}\right)\hat{j}}$$

i.e., the electron is deflected 8.79 cm downward.

52 ••

Picture the Problem Because the electric field is uniform, the acceleration of the electron will be constant and we can apply Newton's 2nd law to find its acceleration and use a constant-acceleration equation to find its speed as it leaves the region in which there is a uniform electric field.

Using a constant-acceleration

$$v^2 = v_0^2 + 2a\Delta x$$

equation, relate the speed of the electron as it leaves the region of the electric field to its acceleration and distance of travel:

or, because $v_0 = 0$,

$$v = \sqrt{2a\Delta x}$$

Apply Newton's 2nd law to express the acceleration of the electron in terms of the electric field:

$$a = \frac{F_{\text{net}}}{m_e} = \frac{eE}{m_e}$$

Substitute to obtain:

$$v = \sqrt{\frac{2eE\Delta x}{m_e}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2\left(1.6\times10^{-19}\ \text{C}\right)\left(8\times10^{4}\ \text{N/C}\right)\left(0.05\,\text{m}\right)}{9.11\times10^{-31}\ \text{kg}}} = \boxed{3.75\times10^{7}\ \text{m/s}}$$

Remarks: Because this result is approximately 13% of the speed of light, it is only an approximation.

53 ••

Picture the Problem We can apply the work-kinetic energy theorem to relate the change in the object's kinetic energy to the net force acting on it. We can express the net force acting on the charged body in terms of its charge and the electric field.

Using the work-kinetic energy theorem, express the kinetic energy of the object in terms of the net force acting on it and its displacement:

$$W = \Delta K = F_{\text{net}}\Delta x$$

Relate the net force acting on the charged object to the electric field:

$$F_{\text{net}} = QE$$

Substitute to obtain:

$$\Delta K = K_{\text{f}} - K_{\text{i}} = QE\Delta x$$

or, because $K_{\text{i}} = 0$,

$$K_{\text{f}} = QE\Delta x$$

Solve for Q:

$$Q = \frac{K_{\text{f}}}{E\Delta x}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{0.12\,\text{J}}{(300\,\text{N/C})(0.50\,\text{m})} = \boxed{800\,\mu\text{C}}$$

***54 ••**

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of the particle in terms of the parameter t and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for y as a function of x, q, and m that we can solve for E_y.

Express the x and y coordinates of the particle as functions of time:

$$x = (v\cos\theta)t$$

and

$$y = (v\sin\theta)t - \tfrac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the particle to the net force acting on it:

$$a_y = \frac{F_{\text{net, y}}}{m} = \frac{qE_y}{m}$$

Substitute in the y-coordinate equation to obtain:

$$y = (v\sin\theta)t - \frac{qE_y}{2m}t^2$$

Eliminate the parameter t between the two equations to obtain:

$$y = (\tan\theta)x - \frac{qE_y}{2mv^2\cos^2\theta}x^2$$

Set $y = 0$ and solve for E_y:

$$E_y = \frac{mv^2\sin 2\theta}{qx}$$

Substitute the non-particle specific data to obtain:

$$E_y = \frac{m(3\times10^6\,\text{m/s})^2\sin 70°}{q(0.015\,\text{m})} = (5.64\times10^{14}\,\text{m/s}^2)\frac{m}{q}$$

(*a*) Substitute for the mass and charge of an electron and evaluate E_y:

$$E_y = (5.64\times10^{14}\,\text{m/s}^2)\frac{9.11\times10^{-31}\,\text{kg}}{1.6\times10^{-19}\,\text{C}} = \boxed{3.21\,\text{kN/C}}$$

(*b*) Substitute for the mass and charge of a proton and evaluate E_y:

$$E_y = (5.64\times10^{14}\,\text{m/s}^2)\frac{1.67\times10^{-27}\,\text{kg}}{1.6\times10^{-19}\,\text{C}} = \boxed{5.89\,\text{MN/C}}$$

55 ••

Picture the Problem We can use constant-acceleration equations to express the *x* and *y* coordinates of the electron in terms of the parameter *t* and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for *y* as a function of *x*, *q*, and *m*. We can decide whether the electron will strike the upper plate by finding the maximum value of its *y* coordinate. Should we find that it does not strike the upper plate, we can determine where it strikes the lower plate by setting $y(x) = 0$.

Express the *x* and *y* coordinates of the electron as functions of time:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the electron to the net force acting on it:

$$a_y = \frac{F_{\text{net},y}}{m_e} = \frac{eE_y}{m_e}$$

Substitute in the *y*-coordinate equation to obtain:

$$y = (v_0 \sin\theta)t - \frac{eE_y}{2m_e}t^2$$

Eliminate the parameter *t* between the two equations to obtain:

$$y(x) = (\tan\theta)x - \frac{eE_y}{2m_e v_0^2 \cos^2\theta}x^2 \qquad (1)$$

To find $y_{\max}$, set $dy/dx = 0$ for extrema:

$$\frac{dy}{dx} = \tan\theta - \frac{eE_y}{m_e v_0^2 \cos^2\theta}x' = 0\,\text{for extrema}$$

Solve for x' to obtain:

$$x' = \frac{m_e v_0^2 \sin 2\theta}{2eE_y} \quad \text{(See remark below.)}$$

Substitute x' in $y(x)$ and simplify to obtain $y_{\max}$:

$$y_{\max} = \frac{m_e v_0^2 \sin^2\theta}{2eE_y}$$

Substitute numerical values and evaluate $y_{\max}$:

$$y_{\max} = \frac{(9.11\times10^{-31}\,\text{kg})(5\times10^{6}\,\text{m/s})^2 \sin^2 45^\circ}{2(1.6\times10^{-19}\,\text{C})(3.5\times10^{3}\,\text{N/C})} = 1.02\,\text{cm}$$

and, because the plates are separated by 2 cm, the electron does not strike the upper plate.

To determine where the electron will strike the lower plate, set $y = 0$ in equation (1) and solve for x to obtain:

$$x = \frac{m_e v_0^2 \sin 2\theta}{eE_y}$$

Substitute numerical values and evaluate x:

$$x = \frac{(9.11\times10^{-31}\text{ kg})(5\times10^{6}\text{ m/s})^2 \sin 90°}{(1.6\times10^{-19}\text{ C})(3.5\times10^{3}\text{ N/C})} = \boxed{4.07\,\text{cm}}$$

Remarks: x' is an extremum, i.e., either a maximum or a minimum. To show that it is a maximum we need to show that d^2y/dx^2, evaluated at x', is negative. A simple alternative is to use your graphing calculator to show that the graph of $y(x)$ is a maximum at x'. Yet another alternative is to recognize that, because equation (1) is quadratic and the coefficient of x^2 is negative, its graph is a parabola that opens downward.

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Picture the Problem The trajectory of the electron while it is in the electric field is parabolic (its acceleration is downward and constant) and its trajectory, once it is out of the electric field is, if we ignore the small gravitational force acting on it, linear. We can use constant-acceleration equations and Newton's 2nd law to express the electron's x and y coordinates parametrically and then eliminate the parameter t to express $y(x)$. We can find the angle with the horizontal at which the electron leaves the electric field from the x and y components of its velocity and its total vertical deflection by summing its deflections over the first 4 cm and the final 12 cm of its flight.

(*a*) Using a constant-acceleration equation, express the x and y coordinates of the electron as functions of time:

$$x(t) = v_0 t$$

and

$$y(t) = v_{0,y}t + \tfrac{1}{2}a_y t^2$$

Because $v_{0,y} = 0$:

$$x(t) = v_0 t \qquad (1)$$

and

$$y(t) = \tfrac{1}{2}a_y t^2$$

Using Newton's 2nd law, relate the acceleration of the electron to the electric field:

$$a_y = \frac{F_{\text{net}}}{m_e} = \frac{-eE_y}{m_e}$$

Substitute to obtain:

$$y(t) = -\frac{eE_y}{2m_e}t^2 \qquad (2)$$

Eliminate the parameter t between equations (1) and (2) to obtain:

$$y(x) = -\frac{eE_y}{2m_e v_0^2}x^2 = -\frac{eE_y}{4K}x^2$$

Substitute numerical values and evaluate y(4 cm):

$$y(0.04\,\text{m}) = -\frac{(1.6\times10^{-19}\ \text{C})(2\times10^{4}\ \text{N/C})(0.04\,\text{m})^2}{4(2\times10^{-16}\ \text{J})} = \boxed{-6.40\,\text{mm}}$$

(b) Express the horizontal and vertical components of the electron's speed as it leaves the electric field:

$$v_x = v_0\cos\theta$$

and

$$v_y = v_0\sin\theta$$

Divide the second of these equations by the first to obtain:

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{v_y}{v_0}$$

Using a constant-acceleration equation, express v_y as a function of the electron's acceleration and its time in the electric field:

$$v_y = v_{0,y} + a_y t$$

or, because $v_{0,y} = 0$

$$v_y = a_y t = \frac{F_{\text{net},y}}{m_e}t = -\frac{eE_y}{m_e}\frac{x}{v_0}$$

Substitute to obtain:

$$\theta = \tan^{-1}\left(-\frac{eE_y x}{m_e v_0^2}\right) = \tan^{-1}\left(-\frac{eE_y x}{2K}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[-\frac{(1.6\times10^{-19}\ \text{C})(2\times10^{4}\ \text{N/C})(0.04\,\text{m})}{2(2\times10^{-16}\ \text{J})}\right] = \boxed{-17.7^\circ}$$

(c) Express the total vertical displacement of the electron:

$$y_{\text{total}} = y_{4\,\text{cm}} + y_{12\,\text{cm}}$$

Relate the horizontal and vertical distances traveled to the screen to the horizontal and vertical components of its velocity:

$$x = v_x\Delta t$$

and

$$y = v_y\Delta t$$

Eliminate Δt from these equations to obtain:

$$y=\frac{v_y}{v_x}x=(\tan\theta)x$$

Substitute numerical values and evaluate y:

$$y=[\tan(-17.7°)](0.12\,\text{m})=-3.83\text{cm}$$

Substitute for $y_{4\text{ cm}}$ and $y_{12\text{ cm}}$ and evaluate y_{total}:

$$y_{\text{total}}=-0.640\,\text{cm}-3.83\,\text{cm}$$
$$=\boxed{-4.47\,\text{cm}}$$

i.e., the electron will strike the fluorescent screen 4.47 cm below the horizontal axis.

57 •

Picture the Problem We can use its definition to find the dipole moment of this pair of charges.

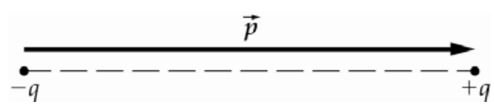
(*a*) Apply the definition of electric dipole moment to obtain:

$$\vec{\boldsymbol{p}}=q\vec{\boldsymbol{L}}$$

and

$$p=(2\,\text{pC})(4\,\mu\text{m})=\boxed{8.00\times10^{-18}\,\text{C}\cdot\text{m}}$$

(*b*) If we assume that the dipole is oriented as shown to the right, then $\vec{\boldsymbol{p}}$ is to the right; pointing from the negative charge toward the positive charge.



***58** •

Picture the Problem The torque on an electric dipole in an electric field is given by $\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{p}}\times\vec{\boldsymbol{E}}$ and the potential energy of the dipole by $U=-\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{E}}$.

Using its definition, express the torque on a dipole moment in a uniform electric field:

$$\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{p}}\times\vec{\boldsymbol{E}}$$

and

$$\tau=pE\sin\theta$$

where θ is the angle between the electric dipole moment and the electric field.

(*a*) Evaluate τ for θ = 0°:

$$\tau=pE\sin0°=\boxed{0}$$

(*b*) Evaluate τ for θ = 90°:

$$\tau = (0.5e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\sin 90° = \boxed{3.20\times10^{-24}\ \text{N}\cdot\text{m}}$$

(*c*) Evaluate τ for θ = 30°:

$$\tau = (0.5e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\sin 30° = \boxed{1.60\times10^{-24}\ \text{N}\cdot\text{m}}$$

(*d*) Using its definition, express the potential energy of a dipole in an electric field:

$$U = -\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{E}} = -pE\cos\theta$$

Evaluate U for θ = 0°:

$$U = -(0.5e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 0° = \boxed{-3.20\times10^{-24}\ \text{J}}$$

Evaluate U for θ = 90°:

$$U = -(0.5e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 90° = \boxed{0}$$

Evaluate U for θ = 30°:

$$U = -(0.5e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 30° = \boxed{-2.77\times10^{-24}\ \text{J}}$$

***59 ••**

Picture the Problem We can combine the dimension of an electric field with the dimension of an electric dipole moment to prove that, in any direction, the dimension of the far field is proportional to $1/[L]^3$ and, hence, the electric field far from the dipole falls off as $1/r^3$.

Express the dimension of an electric field:

$$[E] = \frac{[kQ]}{[L]^2}$$

Express the dimension an electric dipole moment:

$$[p] = [Q][L]$$

Write the dimension of charge in terms of the dimension of an electric dipole moment:

$$[Q] = \frac{[p]}{[L]}$$

Substitute to obtain:

$$[E] = \frac{[k][p]}{[L]^2[L]} = \boxed{\frac{[k][p]}{[L]^3}}$$

This shows that the field E due to a dipole

p falls off as $1/r^3$.

60 ••

Picture the Problem We can use its definition to find the molecule's dipole moment. From the symmetry of the system, it is evident that the x component of the dipole moment is zero.

Using its definition, express the molecule's dipole moment:

$$\vec{p} = p_x\hat{i} + p_y\hat{j}$$

From symmetry considerations we have:

$$p_x = 0$$

The y component of the molecule's dipole moment is:

$$p_y = qL = 2eL = 2\left(1.6\times10^{-19}\text{ C}\right)\left(0.058\text{ nm}\right) = 1.86\times10^{-29}\text{ C}\cdot\text{m}$$

Substitute to obtain:

$$\vec{p} = \boxed{\left(1.86\times10^{-29}\text{ C}\cdot\text{m}\right)\hat{j}}$$

61 ••

Picture the Problem We can express the net force on the dipole as the sum of the forces acting on the two charges that constitute the dipole and simplify this expression to show that $\vec{F}_{\text{net}} = Cp\hat{i}$. We can show that, under the given conditions, $\vec{F}_{\text{net}}$ is also given by $\left(dE_x/dx\right)p\hat{i}$ by differentiating the dipole's potential energy function with respect to x.

(a) Express the net force acting on the dipole:

$$\vec{F}_{\text{net}} = \vec{F}_{-q} + \vec{F}_{+q}$$

Apply Coulomb's law to express the forces on the two charges:

$$\vec{F}_{-q} = -q\vec{E} = -qC\left(x_1 - a\right)\hat{i}$$

and

$$\vec{F}_{+q} = +q\vec{E} = qC\left(x_1 + a\right)\hat{i}$$

Substitute to obtain:

$$\vec{F}_{\text{net}} = -qC\left(x_1 - a\right)\hat{i} + qC\left(x_1 + a\right)\hat{i} = 2aqC\hat{i} = \boxed{Cp\hat{i}}$$

where $p = 2aq$.

(*b*) Express the net force acting on the dipole as the spatial derivative of *U*:

$$\vec{F}_{\text{net}} = -\frac{dU}{dx}\hat{i} = -\frac{d}{dx}\left[-p_x E_x\right]\hat{i} = \boxed{p_x \frac{dE_x}{dx}\hat{i}}$$

62 •••

Picture the Problem We can express the force exerted on the dipole by the electric field as –*dU*/*dr* and the potential energy of the dipole as –*pE*. Because the field is due to a point charge, we can use Coulomb's law to express *E*. In the second part of the problem, we can use Newton's 3rd law to show that the magnitude of the electric field of the dipole along the line of the dipole a distance *r* away is approximately $2kp/r^3$.

(*a*) Express the force exerted by the electric field of the point charge on the dipole:

$$\vec{F} = -\frac{dU}{dr}\hat{r}$$

where $\hat{r}$ is a unit radial vector pointing from *Q* toward the dipole.

Express the potential energy of the dipole in the electric field:

$$U = -pE = -p\frac{kQ}{r^2}$$

Substitute to obtain:

$$\vec{F} = -\frac{d}{dr}\left[-p\frac{kQ}{r^2}\right]\hat{r} = \boxed{-\frac{2kQp}{r^3}\hat{r}}$$

(*b*) Using Newton's 3rd law, express the force that the dipole exerts on the charge *Q* at the origin:

$$\vec{F}_{\text{on}\,Q} = -\vec{F} \text{ or } F_{\text{on}\,Q}\hat{r} = -F\hat{r}$$

and

$$F_{\text{on}\,Q} = F$$

Express $F_{\text{on}\,Q}$ in terms of the field in which *Q* finds itself:

$$F_{\text{on}\,Q} = QE$$

Substitute to obtain:

$$QE = \frac{2kQp}{r^3} \Rightarrow E = \boxed{\frac{2kp}{r^3}}$$

General Problems

***63** •

Picture the Problem We can equate the gravitational force and the electric force acting on a proton to find the mass of the proton under the given condition.

(*a*) Express the condition that must be satisfied if the net force on the

$$F_{\text{g}} = F_{\text{e}}$$

proton is zero:

Use Newton's law of gravity and Coulomb's law to substitute for F_g and F_e:

$$\frac{Gm^2}{r^2}=\frac{ke^2}{r^2}$$

Solve for m to obtain:

$$m=e\sqrt{\frac{k}{G}}$$

Substitute numerical values and evaluate m:

$$m=\left(1.6\times10^{-19}\ \text{C}\right)\sqrt{\frac{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}}=\boxed{1.86\times10^{-9}\ \text{kg}}$$

(*b*) Express the ratio of F_e and Fg:

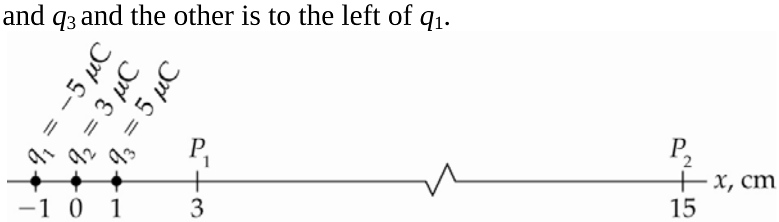
$$\frac{\dfrac{ke^2}{r^2}}{\dfrac{Gm_p^2}{r^2}}=\frac{ke^2}{Gm_p^2}$$

Substitute numerical values to obtain:

$$\frac{ke^2}{Gm_p^2}=\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.67\times10^{-27}\ \text{kg}\right)^2}=\boxed{1.24\times10^{36}}$$

64 ••

Picture the Problem The locations of the charges q_1, q_2 and q_2 and the points at which we are calculate the field are shown in the diagram. From the diagram it is evident that $\vec{E}$ along the axis has no y component. We can use Coulomb's law for $\vec{E}$ due to a point charge and the superposition principle to find $\vec{E}$ at points P_1 and P_2. Examining the distribution of the charges we can see that there are two points where E = 0. One is between q_2 and q_3 and the other is to the left of q_1.



Using Coulomb's law, express the electric field at P_1 due to the three charges:

$$\vec{E}_{P_1} = \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} = \frac{kq_1}{r_{1,P_1}^2}\hat{i} + \frac{kq_2}{r_{2,P_1}^2}\hat{i} + \frac{kq_3}{r_{3,P_1}^2}\hat{i} = k\left[\frac{q_1}{r_{1,P_1}^2} + \frac{q_2}{r_{2,P_1}^2} + \frac{q_3}{r_{3,P_1}^2}\right]\hat{i}$$

Substitute numerical values and evaluate $\vec{E}_{P_1}$:

$$\vec{E}_{P_1} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{-5\,\mu\text{C}}{(4\,\text{cm})^2} + \frac{3\,\mu\text{C}}{(3\,\text{cm})^2} + \frac{5\,\mu\text{C}}{(2\,\text{cm})^2}\right]\hat{i} = \boxed{\left(1.14\times10^8\ \text{N/C}\right)\hat{i}}$$

Express the electric field at P_2:

$$\vec{E}_{P_2} = \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} = k\left[\frac{q_1}{r_{1,P_2}^2} + \frac{q_2}{r_{2,P_2}^2} + \frac{q_3}{r_{3,P_2}^2}\right]\hat{i}$$

Substitute numerical values and evaluate $\vec{E}_{P_2}$:

$$\vec{E}_{P_2} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{-5\,\mu\text{C}}{(16\,\text{cm})^2} + \frac{3\,\mu\text{C}}{(15\,\text{cm})^2} + \frac{5\,\mu\text{C}}{(14\,\text{cm})^2}\right]\hat{i} = \boxed{\left(1.74\times10^6\ \text{N/C}\right)\hat{i}}$$

Letting x represent the x coordinate of a point where the magnitude of the electric field is zero, express the condition that $E = 0$ for the point between $x = 0$ and $x = 1$ cm:

$$E_P = k\left[\frac{q_1}{r_{1,P}^2} + \frac{q_2}{r_{2,P}^2} + \frac{q_3}{r_{3,P}^2}\right] = 0$$

or

$$\frac{-5\,\mu\text{C}}{(x+1\text{cm})^2} + \frac{3\,\mu\text{C}}{x^2} - \frac{5\,\mu\text{C}}{(1\text{cm} - x)^2} = 0$$

Solve this equation to obtain: $x = \boxed{0.417\,\text{cm}}$

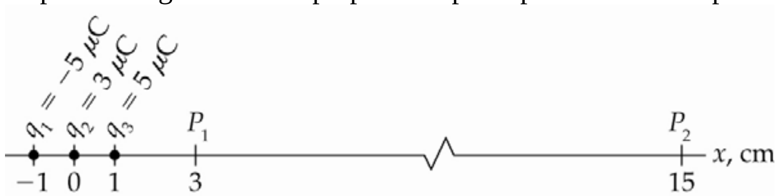
For $x < -1$ cm, let $y = -x$ to obtain:

$$\frac{5\,\mu\text{C}}{(y-1\text{cm})^2} - \frac{3\,\mu\text{C}}{y^2} - \frac{5\,\mu\text{C}}{(y+1\text{cm})^2} = 0$$

Solve this equation to obtain: $x = 6.95\,\text{cm}$ and $y = \boxed{-6.95\,\text{cm}}$

65 ••

Picture the Problem The locations of the charges q_1, q_2 and q_2 and the point P_2 at which we are calculate the field are shown in the diagram. The electric field on the *x* axis due to the dipole is given by $\vec{E}_{\text{dipole}} = 2k\vec{p}/x^3$ where $\vec{p} = 2aq_1\hat{i}$. We can use Coulomb's law for $\vec{E}$ due to a point charge and the superposition principle to find $\vec{E}$ at point P_2.



Express the electric field at P_2 as the sum of the field due to the dipole and the point charge q_2:

$$\vec{E}_{P_2} = \vec{E}_{\text{dipole}} + \vec{E}_{q_2} = \frac{2kp}{x^3}\hat{i} + \frac{kq_2}{x^2}\hat{i} = \frac{2k(2q_1a)}{x^3}\hat{i} + \frac{kq_2}{x^2}\hat{i} = \frac{k}{x^2}\left[\frac{4q_1a}{x} + q_2\right]\hat{i}$$

where a = 1 cm.

Substitute numerical values and evaluate $\vec{E}_{P_2}$:

$$\vec{E}_{P_2} = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(15\times10^{-2}\ \text{m})^2}\left[\frac{4(5\ \mu\text{C})(1\,\text{cm})}{15\,\text{cm}} + 3\mu\text{C}\right]\hat{i} = \boxed{(1.73\times10^6\ \text{N/C})\hat{i}}$$

> While the separation of the two charges of the dipole is more than 10% of the distance to the point of interest, i.e., *x* is not much greater than *a*, this result is in excellent agreement with that of Problem 64.

***66 ••**

Picture the Problem We can find the percentage of the free charge that would have to be removed by finding the ratio of the number of free electrons n_e to be removed to give the penny a charge of 15 μC to the number of free electrons in the penny. Because we're assuming the pennies to be point charges, we can use Coulomb's law to find the force of repulsion between them.

(*a*) Express the fraction *f* of the free charge to be removed as the quotient of the number of electrons to be removed and the number of free

$$f = \frac{n_e}{N}$$

electrons:

Relate N to Avogadro's number, the mass of the copper penny, and the molecular mass of copper:

$$\frac{N}{N_A}=\frac{m}{M}\Rightarrow N=N_A\frac{m}{M}$$

Relate n_e to the free charge Q to be removed from the penny:

$$Q=n_e[-e]\Rightarrow n_e=\frac{Q}{-e}$$

$$f=\frac{\frac{Q}{-e}}{N_A\frac{m}{M}}=-\frac{QM}{meN_A}$$

Substitute numerical values and evaluate f:

$$f=-\frac{(-15\,\mu\text{C})(63.5\,\text{g/mol})}{(3\,\text{g})(1.6\times10^{-19}\,\text{C})(6.02\times10^{23}\,\text{mol}^{-1})}=3.29\times10^{-9}=\boxed{3.29\times10^{-7}\%}$$

(*b*) Use Coulomb's law to express the force of repulsion between the two pennies:

$$F=\frac{kq^2}{r^2}=\frac{k(n_e e)^2}{r^2}$$

Substitute numerical values and evaluate F:

$$F=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(9.38\times10^{13})^2(1.6\times10^{-19}\,\text{C})^2}{(0.25\,\text{m})^2}=\boxed{32.4\,\text{N}}$$

67 ••

Picture the Problem Knowing the total charge of the two charges, we can use Coulomb's law to find the two combinations of charge that will satisfy the condition that both are positive and hence repel each other. If just one charge is positive, then there is just one distribution of charge that will satisfy the conditions that the force is attractive and the sum of the two charges is 6 μC.

(*a*) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F=\frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2=Q-q_1$$

Substitute to obtain:

$$F=\frac{kq_1(Q-q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$8\,\text{mN}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(6\,\mu\text{C})q_1-q_1^2\right]}{(3\,\text{m})^2}$$

Simplify to obtain:

$$q_1^2+(-6\,\mu\text{C})q_1+8.01(\mu\text{C})^2=0$$

Solve to obtain:

$$q_1=\boxed{3.99\,\mu\text{C}}\ \text{and}\ q_2=\boxed{2.01\,\mu\text{C}}$$

or

$$q_1=\boxed{2.01\,\mu\text{C}}\ \text{and}\ q_2=\boxed{3.99\,\mu\text{C}}$$

(*b*) Use Coulomb's law to express the attractive force each charge exerts on the other:

$$F=-\frac{kq_1q_2}{r_{1,2}^2}$$

Proceed as in (*a*) to obtain:

$$q_1^2+(-6\,\mu\text{C})q_1-8.01(\mu\text{C})^2=0$$

Solve to obtain:

$$q_1=\boxed{7.12\,\mu\text{C}}\ \text{and}\ q_2=\boxed{-1.12\,\mu\text{C}}$$

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Picture the Problem The electrostatic forces between the charges are responsible for the tensions in the strings. We'll assume that these are point charges and apply Coulomb's law and the principle of the superposition of forces to find the tension in each string.

Use Coulomb's law to express the net force on the charge $+q$:

$$T_1=F_{2q}+F_{4q}$$

Substitute and simplify to obtain:

$$T_1=\frac{kq(2q)}{d^2}+\frac{kq(4q)}{(2d)^2}=\boxed{\frac{3kq^2}{d^2}}$$

Use Coulomb's law to express the net force on the charge $+4q$:

$$T_2=F_q+F_{2q}$$

Substitute and simplify to obtain:

$$T_2=\frac{k(2q)(4q)}{d^2}+\frac{kq(4q)}{(2d)^2}=\boxed{\frac{9kq^2}{d^2}}$$

***69 ••**

Picture the Problem We can use Coulomb's law to express the force exerted on one charge by the other and then set the derivative of this expression equal to zero to find the distribution of the charge that maximizes this force.

Using Coulomb's law, express the force that either charge exerts on the other:

$$F = \frac{kq_1q_2}{D^2}$$

Express q_2 in terms of Q and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = \frac{kq_1(Q - q_1)}{D^2}$$

Differentiate F with respect to q_1 and set this derivative equal to zero for extreme values:

$$\frac{dF}{dq_1} = \frac{k}{D^2}\frac{d}{dq_1}[q_1(Q - q_1)]$$
$$= \frac{k}{D^2}[q_1(-1) + Q - q_1]$$
$$= 0 \text{ for extrema}$$

Solve for q_1 to obtain:

$$q_1 = \tfrac{1}{2}Q$$

and

$$q_2 = Q - q_1 = \tfrac{1}{2}Q$$

To determine whether a maximum or a minimum exists at $q_1 = \frac{1}{2}Q$, differentiate F a second time and evaluate this derivative at $q_1 = \frac{1}{2}Q$:

$$\frac{d^2F}{dq_1^2} = \frac{k}{D^2}\frac{d}{dq_1}[Q - 2q_1]$$
$$= \frac{k}{D^2}(-2)$$
$$< 0 \text{ independently of } q_1.$$

$$\boxed{\therefore q_1 = q_2 = \tfrac{1}{2}Q \text{ maximizes } F.}$$

***70 ••**

Picture the Problem We can apply Coulomb's law and the superposition of forces to relate the net force acting on the charge $q = -2\ \mu C$ to x. Because Q divides out of our equation when $F(x) = 0$, we'll substitute the data given for $x = 8.0$ cm.

Using Coulomb's law, express the net force on q as a function of x:

$$F(x) = -\frac{kqQ}{x^2} + \frac{kq(4Q)}{(12\,\text{cm} - x)^2}$$

Simplify to obtain:

$$\frac{F(x)}{kq}=\left[-\frac{1}{x^2}+\frac{4}{(12\,\text{cm}-x)^2}\right]Q$$

Solve for Q:

$$Q=\frac{F(x)}{kq\left[-\frac{1}{x^2}+\frac{4}{(12\,\text{cm}-x)^2}\right]}$$

Evaluate Q for x = 8 cm:

$$Q=\frac{126.4\,\text{N}}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\,\mu\text{C})\left[-\frac{1}{(8\,\text{cm})^2}+\frac{4}{(4\,\text{cm})^2}\right]}=\boxed{3.00\,\mu\text{C}}$$

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Picture the Problem Knowing the total charge of the two charges, we can use Coulomb's law to find the two combinations of charge that will satisfy the condition that both are positive and hence repel each other. If the spheres attract each other, then there is just one distribution of charge that will satisfy the conditions that the force is attractive and the sum of the two charges is 200 μC.

(*a*) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F=\frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2=Q-q_1$$

Substitute to obtain:

$$F=\frac{kq_1(Q-q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$80\,\text{N}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(200\,\mu\text{C})q_1-q_1^2\right]}{(0.6\,\text{m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2+(-0.2\,\text{mC})q_1+3.20\times10^{-3}(\text{mC})^2=0$$

Solve to obtain:

$$q_1=\boxed{17.5\,\mu\text{C}}\ \text{and}\ q_2=\boxed{183\,\mu\text{C}}$$

or

$q_1 = \boxed{183\,\mu\text{C}}$ and $q_2 = \boxed{17.5\,\mu\text{C}}$

(*b*) Use Coulomb's law to express the attractive force each charge exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Proceed as in (*a*) to obtain:

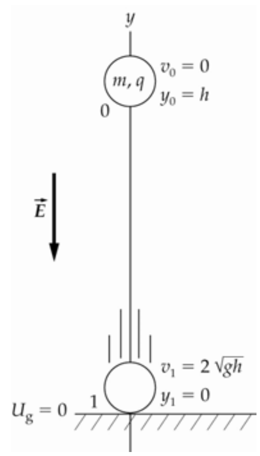
$$q_1^2 + (-0.2\,\text{mC})q_1 - 3.20\times10^{-3}(\text{mC})^2 = 0$$

Solve to obtain:

$q_1 = \boxed{-15.0\,\mu\text{C}}$ and $q_2 = \boxed{215\,\mu\text{C}}$

72 ••

Picture the Problem Choose the coordinate system shown in the diagram and let $U_g = 0$ where $y = 0$. We'll let our system include the ball and the earth. Then the work done on the ball by the electric field will change the energy of the system. The diagram summarizes what we know about the motion of the ball. We can use the work-energy theorem to our system to relate the work done by the electric field to the change in its energy.



Using the work-energy theorem, relate the work done by the electric field to the change in the energy of the system:

$$W_{\text{electric field}} = \Delta K + \Delta U_g = K_2 - K_1 + U_{g,2} - U_{g,1}$$

or, because $K_1 = U_{g,2} = 0$,

$$W_{\text{electric field}} = K_2 - U_{g,1}$$

Substitute for $W_{\text{electric field}}$, K_2, and $U_{g,0}$ and simplify:

$$qEh = \tfrac{1}{2}mv_1^2 - mgh = \tfrac{1}{2}m\left(2\sqrt{gh}\right)^2 - mgh = mgh$$

Solve for m:

$$m = \boxed{\frac{qE}{g}}$$

73 ••

Picture the Problem We can use Coulomb's law, the definition of torque, and the condition for rotational equilibrium to find the electrostatic force between the two charged bodies, the torque this force produces about an axis through the center of the meter stick, and the mass required to maintain equilibrium when it is located either 25 cm to the right or to the left of the mid-point of the rigid stick.

(*a*) Using Coulomb's law, express the electric force between the two charges:

$$F = \frac{kq_1q_2}{d^2}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\times10^{-7}\ \text{C})^2}{(0.1\,\text{m})^2} = \boxed{0.225\,\text{N}}$$

(*b*) Apply the definition of torque to obtain:

$$\tau = F\ell$$

Substitute numerical values and evaluate τ:

$$\tau = (0.225\,\text{N})(0.5\,\text{m}) = \boxed{0.113\,\text{N}\cdot\text{m, counterclockwise}}$$

(*c*) Apply $\sum\tau_{\text{center of the meter stick}} = 0$ to the meterstick:

$$\tau - mg\ell' = 0$$

Solve for *m*:

$$m = \frac{\tau}{g\ell'}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{0.113\,\text{N}}{(9.81\,\text{m/s}^2)(0.25\,\text{m})} = \boxed{0.0461\,\text{kg}}$$

(*d*) Apply $\sum\tau_{\text{center of the meter stick}} = 0$ to the meterstick:

$$-\tau + mg\ell' = 0$$

Substitute for τ:

$$-F\ell + mg\ell' = 0$$

Substitute for *F*:

$$-\frac{kq_1q_2'}{d^2} + mg\ell' = 0$$

where q' is the required charge.

Solve for q_2' to obtain:

$$q_2 = \frac{d^2mg\ell'}{kq_1\ell}$$

Substitute numerical values and evaluate q_2':

$$q_2' = \frac{(0.1\,\text{m})^2(0.0461\,\text{kg})(9.81\,\text{m/s}^2)(0.25\,\text{m})}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\times10^{-7}\ \text{C})(0.5\,\text{m})} = \boxed{5.03\times10^{-7}\ \text{C}}$$

74 ••

Picture the Problem Let the numeral 1 refer to the charge in the 1st quadrant and the numeral 2 to the charge in the 4th quadrant. We can use Coulomb's law for the electric field due to a point charge and the superposition of forces to express the field at the origin and use this equation to solve for *Q*.

Express the electric field at the origin due to the point charges *Q*:

$$\vec{E}(0,0) = \vec{E}_1 + \vec{E}_2 = \frac{kQ}{r_{1,0}^2}\hat{r}_{1,0} + \frac{kQ}{r_{2,0}^2}\hat{r}_{2,0}$$
$$= \frac{kQ}{r^3}\left[(-4\,\text{m})\hat{i} + (-2\,\text{m})\hat{j}\right] + \frac{kQ}{r^3}\left[(-4\,\text{m})\hat{i} + (2\,\text{m})\hat{j}\right] = -\frac{(8\,\text{m})kQ}{r^3}\hat{i}$$
$$= E_x\hat{i}$$

where r is the distance from each charge to the origin and $E_x = -\frac{(8\,\text{m})kQ}{r^3}$.

Express r in terms of the coordinates (x, y) of the point charges:

$$r = \sqrt{x^2 + y^2}$$

Substitute to obtain:

$$E_x = -\frac{(8\,\text{m})kQ}{\left(x^2 + y^2\right)^{3/2}}$$

Solve for Q to obtain:

$$Q = \frac{E_x\left(x^2 + y^2\right)^{3/2}}{k(8\,\text{m})}$$

Substitute numerical values and evaluate Q:

$$Q = -\frac{(4\,\text{kN/C})\left[(4\,\text{m})^2 + (2\,\text{m})^2\right]^{3/2}}{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(8\,\text{m})}$$
$$= \boxed{-4.97\,\mu\text{C}}$$

75 ••

Picture the Problem Let the numeral 1 denote one of the spheres and the numeral 2 the other. Knowing the total charge *Q* on the two spheres, we can use Coulomb's law to find the charge on each of them. A second application of Coulomb's law when the spheres carry the same charge and are 0.60 m apart will yield the force each exerts on the other.

(*a*) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = \frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$120\,\text{N} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[(200\,\mu\text{C})q_1 - q_1^2\right]}{(0.6\,\text{m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-200\,\mu\text{C})q_1 + 4810(\mu\text{C})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{28.0\,\mu\text{C}} \text{ and } q_2 = \boxed{172\,\mu\text{C}}$$

or

$$q_1 = \boxed{172\,\mu\text{C}} \text{ and } q_2 = \boxed{28.0\,\mu\text{C}}$$

(*b*) Use Coulomb's law to express the repulsive force each charge exerts on the other when $q_1 = q_2 = 100\ \mu\text{C}$:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Substitute numerical values and evaluate F:

$$F = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(100\,\mu\text{C})^2}{(0.6\,\text{m})^2} = \boxed{250\,\text{N}}$$

76 ••

Picture the Problem Let the numeral 1 denote one of the spheres and the numeral 2 the other. Knowing the total charge Q on the two spheres, we can use Coulomb's law to find the charge on each of them. A second application of Coulomb's law when the spheres carry the same charge and are 0.60 m apart will yield the force each exerts on the other.

(*a*) Use Coulomb's law to express the attractive force each charge exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = -\frac{kq_1(Q-q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$120\ \text{N} = \frac{-\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[(200\ \mu\text{C})q_1 - q_1^2\right]}{(0.6\ \text{m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-200\ \mu\text{C})q_1 - 4810(\mu\text{C})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{-21.7\ \mu\text{C}}\ \text{and}\ q_2 = \boxed{222\ \mu\text{C}}$$

or

$$q_1 = \boxed{222\ \mu\text{C}}\ \text{and}\ q_2 = \boxed{-21.7\ \mu\text{C}}$$

(*b*) Use Coulomb's law to express the repulsive force each charge exerts on the other when $q_1 = q_2 = 100\ \mu\text{C}$:

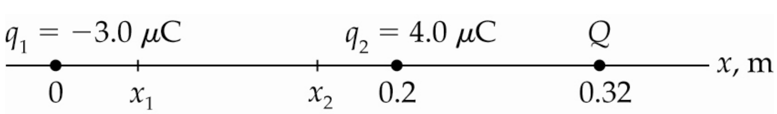
$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Substitute numerical values and evaluate *F*:

$$F = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(100\ \mu\text{C})^2}{(0.6\ \text{m})^2} = \boxed{250\ \text{N}}$$

77 ••

Picture the Problem The charge configuration is shown in the diagram as are the approximate locations, labeled x_1 and x_2, where the electric field is zero. We can determine the charge *Q* by using Coulomb's law and the superposition of forces to express the net force acting on q_2. In part (b), by inspection, the points where $E = 0$ must be between the $-3\ \mu$C and $+4\ \mu$C charges. We can use Coulomb's law for the field due to point charges and the superposition of electric fields to determine the coordinates x_1 and x_2.



(*a*) Use Coulomb's law to express the force on the 4.0-μC charge:

$$\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{Q,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{i} + \frac{kQq_2}{r_{Q,2}^2}\left(-\hat{i}\right) = kq_2\left[\frac{q_1}{r_{1,2}^2} - \frac{Q}{r_{Q,2}^2}\right]\hat{i} = F_2\hat{i}$$

Solve for Q:

$$Q = r_{Q,2}^2\left[\frac{q_1}{r_{1,2}^2} - \frac{F_2}{kq_2}\right]$$

Substitute numerical values and evaluate Q:

$$Q = (0.12\,\text{m})^2\left[\frac{-3\,\mu\text{C}}{(0.2\,\text{m})^2} - \frac{240\,\text{N}}{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})}\right] = \boxed{-97.2\,\mu\text{C}}$$

(*b*) Use Coulomb's law for electric fields and the superposition of fields to determine the coordinate x at which $E = 0$:

$$\vec{E} = -\frac{kQ}{(0.32\,\text{m} - x)^2}\hat{i} - \frac{kq_2}{(0.2\,\text{m} - x)^2}\hat{i} + \frac{kq_1}{x^2}\hat{i} = 0$$

or

$$-\frac{Q}{(0.32\,\text{m} - x)^2} - \frac{q_2}{(0.2\,\text{m} - x)^2} + \frac{q_1}{x^2} = 0$$

Substitute numerical values to obtain:

$$-\frac{-97.2\,\mu\text{C}}{(0.32\,\text{m} - x)^2} - \frac{4\,\mu\text{C}}{(0.2\,\text{m} - x)^2} + \frac{-3\,\mu\text{C}}{x^2} = 0$$

and

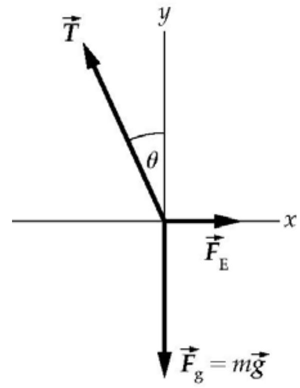
$$\frac{97.2}{(0.32\,\text{m} - x)^2} - \frac{4}{(0.2\,\text{m} - x)^2} - \frac{3}{x^2} = 0$$

Solve (preferably using a graphing calculator!) this equation to obtain:

$$x_1 = \boxed{0.0508\,\text{m}} \text{ and } x_2 = \boxed{0.169\,\text{m}}$$

***78 ••**

Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, and the electric force $\vec{F}_E$. We can use Coulomb's law to relate the electric force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*) Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_E - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg = 0$$

Eliminate *T* between these equations to obtain:

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Solve for *q*:

$$q = r\sqrt{\frac{mg\tan\theta}{k}}$$

Referring to the figure, relate the separation of the spheres *r* to the length of the pendulum *L*:

$$r = 2L\sin\theta$$

Substitute to obtain:

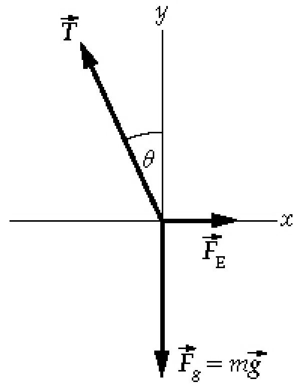
$$q = \boxed{2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}}$$

(*b*) Evaluate *q* for *m* = 10 g, *L* = 50 cm, and θ = 10°:

$$q = 2(0.5\,\text{m})\sin 10°\sqrt{\frac{(0.01\,\text{kg})(9.81\,\text{m/s}^2)\tan 10°}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{0.241\,\mu\text{C}}$$

79 ••

Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, and the electric force $\vec{F}_E$. We can use Coulomb's law to relate the electric force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*)Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_E - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg = 0$$

Eliminate *T* between these equations to obtain:

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Referring to the figure for Problem 80, relate the separation of the spheres *r* to the length of the pendulum *L*:

$$r = 2L\sin\theta$$

Substitute to obtain:

$$\tan\theta = \frac{kq^2}{4mgL^2\sin^2\theta}$$

or

$$\sin^2\theta\tan\theta = \frac{kq^2}{4mgL^2} \qquad (1)$$

Substitute numerical values and evaluate $\sin^2\theta\tan\theta$:

$$\sin^2\theta\tan\theta = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(0.75\,\mu\text{C}\right)^2}{4\left(0.01\text{kg}\right)\left(9.81\text{m/s}^2\right)\left(1.5\text{m}\right)^2} = 5.73\times10^{-3}$$

Because $\sin^2\theta\tan\theta << 1$:

$$\sin\theta \approx \tan\theta \approx \theta$$

and

$$\theta^3 \approx 5.73\times10^{-3}$$

Solve for θ to obtain:

$$\theta = 0.179\,\text{rad} = \boxed{10.3°}$$

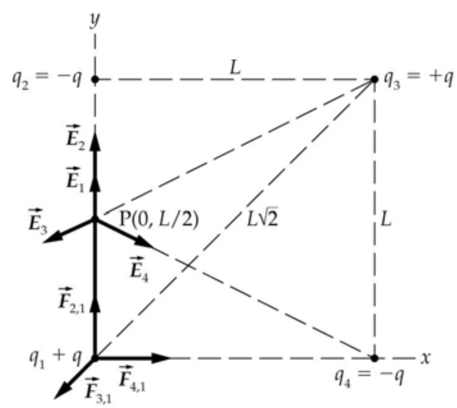
(*b*) Evaluate equation (1) with replacing q^2 with q_1q_2:

$$\sin^2\theta\tan\theta = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.5\,\mu\text{C})(1\,\mu\text{C})}{4(0.01\,\text{kg})\left(9.81\,\text{m/s}^2\right)(1.5\,\text{m})^2} = 5.09\times10^{-3} \approx \theta^3$$

Solve for θ to obtain: $\theta = 0.172\,\text{rad} = \boxed{9.86°}$

80 ••

Picture the Problem Let the origin be at the lower left-hand corner and designate the charges as shown in the diagram. We can apply Coulomb's law for point charges to find the forces exerted on q_1 by q_2, q_3, and q_4 and superimpose these forces to find the net force exerted on q_1. In part (*b*), we'll use Coulomb's law for the electric field due to a point charge and the superposition of fields to find the electric field at point $P(0, L/2)$.



(*a*) Using the superposition of forces, express the net force exerted on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

Apply Coulomb's law to express $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^3}\vec{r}_{2,1} = \frac{k(-q)q}{L^3}\left(-L\,\hat{j}\right) = \frac{kq^2}{L^2}\hat{j}$$

Apply Coulomb's law to express $\vec{F}_{4,1}$:

$$\vec{F}_{4,1} = \frac{kq_4q_1}{r_{4,1}^2}\hat{r}_{4,1} = \frac{kq_4q_1}{r_{4,1}^3}\vec{r}_{4,1} = \frac{k(-q)q}{L^3}\left(-L\,\hat{i}\right) = \frac{kq^2}{L^2}\hat{i}$$

Apply Coulomb's law to express $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^2}\hat{r}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3}\vec{r}_{3,1} = \frac{kq^2}{2^{3/2}L^3}\left(-L\,\hat{i} - L\,\hat{j}\right) = -\frac{kq^2}{2^{3/2}L^2}\left(\hat{i} + \hat{j}\right)$$

Substitute and simplify to obtain:

$$\vec{F}_1 = \frac{kq^2}{L^2}\hat{j} - \frac{kq^2}{2^{3/2}L^2}\left(\hat{i}+\hat{j}\right) + \frac{kq^2}{L^2}\hat{i}$$
$$= \frac{kq^2}{L^2}\left(\hat{i}+\hat{j}\right) - \frac{kq^2}{2^{3/2}L^2}\left(\hat{i}+\hat{j}\right)$$
$$= \boxed{\frac{kq^2}{L^2}\left(1-\frac{1}{2\sqrt{2}}\right)\left(\hat{i}+\hat{j}\right)}$$

(*b*) Using superposition of fields, express the resultant field at point *P*:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \qquad (1)$$

Use Coulomb's law to express $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} = \frac{kq}{r_{1,P}^3}\left(\frac{L}{2}\hat{j}\right) = \frac{kq}{\left(\frac{L}{2}\right)^3}\left(\frac{L}{2}\hat{j}\right) = \frac{4kq}{L^2}\hat{j}$$

Use Coulomb's law to express $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{k(-q)}{r_{2,P}^3}\left(\frac{L}{2}\hat{j}\right) = \frac{-kq}{\left(\frac{L}{2}\right)^3}\left(-\frac{L}{2}\hat{j}\right) = \frac{4kq}{L^2}\hat{j}$$

Use Coulomb's law to express $\vec{E}_3$:

$$\vec{E}_3 = \frac{kq_3}{r_{3,P}^2}\hat{r}_{3,P} = \frac{kq}{r_{3,P}^3}\left(-L\hat{i}-\frac{L}{2}\hat{j}\right) = \frac{8kq}{5^{3/2}L^2}\left(-\hat{i}-\frac{1}{2}\hat{j}\right)$$

Use Coulomb's law to express $\vec{E}_4$:

$$\vec{E}_4 = \frac{kq_4}{r_{4,P}^2}\hat{r}_{3,P} = \frac{k(-q)}{r_{4,P}^3}\left(L\hat{i}-\frac{L}{2}\hat{j}\right) = \frac{8kq}{5^{3/2}L^2}\left(\hat{i}-\frac{1}{2}\hat{j}\right)$$

Substitute in equation (1) and simplify to obtain:

$$\vec{E}_P = \frac{4kq}{L^2}\hat{j} + \frac{4kq}{L^2}\hat{j} + \frac{8kq}{5^{3/2}L^2}\left(-\hat{i}-\frac{1}{2}\hat{j}\right) + \frac{8kq}{5^{3/2}L^2}\left(\hat{i}-\frac{1}{2}\hat{j}\right) = \boxed{\frac{8kq}{L^2}\left(1+\frac{\sqrt{5}}{25}\right)\hat{j}}$$

81 ••

Picture the Problem We can apply Newton's 2nd law in rotational form to obtain the differential equation of motion of the dipole and then use the small angle approximation $\sin\theta \approx \theta$ to show that the dipole experiences a linear restoring torque and, hence, will experience simple harmonic motion.

Apply $\sum\tau = I\alpha$ to the dipole:

$$-pE\sin\theta = I\frac{d^2\theta}{dt^2}$$

where τ is negative because acts in such a direction as to decrease θ.

For small values of θ, $\sin\theta \approx \theta$ and:

$$-pE\theta = I\frac{d^2\theta}{dt^2}$$

Express the moment of inertia of the dipole:

$$I = \tfrac{1}{2}ma^2$$

Relate the dipole moment of the dipole to its charge and the charge separation:

$$p = qa$$

Substitute to obtain:

$$\tfrac{1}{2}ma^2\frac{d^2\theta}{dt^2} = -qaE\theta$$

or

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{2qE}{ma}\theta}$$

the differential equation for a simple harmonic oscillator with angular frequency $\omega = \sqrt{2qE/ma}$.

Express the period of a simple harmonic oscillator:

$$T = \frac{2\pi}{\omega}$$

Substitute to obtain:

$$T = \boxed{2\pi\sqrt{\frac{ma}{2qE}}}$$

82 ••

Picture the Problem We can apply conservation of energy and the definition of the potential energy of a dipole in an electric field to relate q to the kinetic energy of the dumbbell when it is aligned with the field.

Using conservation of energy, relate the initial potential energy of the dumbbell to its kinetic energy when it is momentarily aligned with the electric field:

$$\Delta K + \Delta U = 0$$

or, because $K_i = 0$,

$$K + \Delta U = 0$$

where K is the kinetic energy when it is aligned with the field.

Express the change in the potential energy of the dumbbell as it aligns with the electric field in terms of its dipole moment, the electric field, and the angle through which it rotates:

$$\Delta U = U_f - U_i = -pE\cos\theta_f + pE\cos\theta_f = qaE(\cos 60° - 1)$$

Substitute to obtain:

$$K + qaE(\cos 60° - 1) = 0$$

Solve for q:

$$q = \frac{K}{aE(1 - \cos 60°)}$$

Substitute numerical values and evaluate q:

$$q = \frac{5\times10^{-3}\ \text{J}}{(0.3\ \text{m})(600\ \text{N/C})(1-\cos 60°)} = \boxed{55.6\ \mu\text{C}}$$

***83** ••

Picture the Problem The forces the electron and the proton exert on each other constitute an action-and-reaction pair. Because the magnitudes of their charges are equal and their masses are the same, we find the speed of each particle by finding the speed of either one. We'll apply Coulomb's force law for point charges and Newton's 2nd law to relate v to e, m, k, and r.

Apply Newton's 2nd law to the positron:

$$\frac{ke^2}{r^2} = m\frac{v^2}{\frac{1}{2}r} \Rightarrow \frac{ke^2}{r} = 2mv^2$$

Solve for v to obtain:

$$v = \boxed{\sqrt{\frac{ke^2}{2mr}}}$$

84 ••

Picture the Problem In Problem 81 it was established that the period of an electric dipole in an electric field is given by $T = 2\pi\sqrt{ma/2qE}$. We can use this result to find the frequency of oscillation of a KBr molecule in a uniform electric field of 1000 N/C.

Express the frequency of the KBr oscillator:

$$f = \frac{1}{2\pi}\sqrt{\frac{2qE}{ma}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{2\left(1.6\times10^{-19}\ \text{C}\right)\left(1000\ \text{N/C}\right)}{\left(1.4\times10^{-25}\ \text{kg}\right)\left(0.282\ \text{nm}\right)}} = \boxed{4.53\times10^{8}\ \text{Hz}}$$

85 •••

Picture the Problem We can use Coulomb's force law for point masses and the condition for translational equilibrium to express the equilibrium position as a function of k, q, Q, m, and g. In part (b) we'll need to show that the displaced point charge experiences a linear restoring force and, hence, will exhibit simple harmonic motion.

(a) Apply the condition for translational equilibrium to the point mass:

$$\frac{kqQ}{y_0^2} - mg = 0$$

Solve for y_0 to obtain:

$$y_0 = \boxed{\sqrt{\frac{kqQ}{mg}}}$$

(b) Express the restoring force that acts on the point mass when it is displaced a distance Δy from its equilibrium position:

$$F = \frac{kqQ}{\left(y_0 + \Delta y\right)^2} - \frac{kqQ}{y_0^2} \approx \frac{kqQ}{y_0^2 + 2y_0\Delta y} - \frac{kqQ}{y_0^2}$$

because $\Delta y << y_0$.

Simplify this expression further by writing it with a common denominator:

$$F = -\frac{2y_0\Delta y kqQ}{y_0^4 + 2y_0^3\Delta y} = -\frac{2y_0\Delta y kqQ}{y_0^4\left(1 + 2\frac{\Delta y}{y_0}\right)} \approx -\frac{2\Delta y kqQ}{y_0^3}$$

again, because $\Delta y << y_0$.

From the 1st step of our solution:

$$\frac{kqQ}{y_0^2} = mg$$

Substitute to obtain:

$$F=-\frac{2mg}{y_0}\Delta y$$

Apply Newton's 2nd law to the displaced point charge to obtain:

$$m\frac{d^2\Delta y}{dt^2}=-\frac{2mg}{y_0}\Delta y$$

or

$$\boxed{\frac{d^2\Delta y}{dt^2}+\frac{2g}{y_0}\Delta y=0}$$

the differential equation of simple harmonic motion with $\boxed{\omega=\sqrt{2g/y_0}}$.

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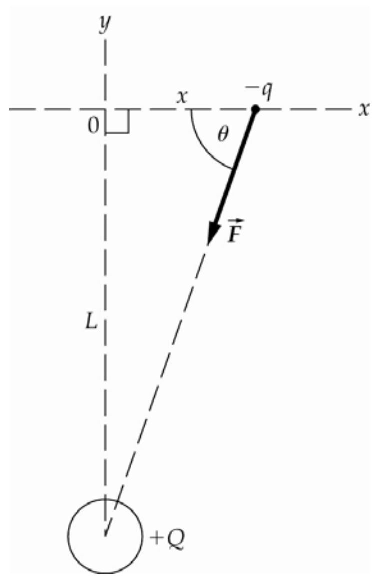
Picture the Problem The free-body diagram shows the Coulomb force the positive charge Q exerts on the bead that is constrained to move along the x axis. The x component of this force is a restoring force, i.e., it is directed toward the bead's equilibrium position. We can show that, for $x << L$, this restoring force is linear and, hence, that the bead will exhibit simple harmonic motion about its equilibrium position. Once we've obtained the differential equation of SHM we can relate the period of the motion to its angular frequency.



Using Coulomb's law for point charges, express the force F that $+Q$ exerts on $-q$:

$$F=\frac{k(-q)Q}{L^2+x^2}=-\frac{kqQ}{L^2+x^2}$$

Express the component of this force along the x axis:

$$F_x=-\frac{kqQ}{L^2+x^2}\cos\theta$$
$$=-\frac{kqQ}{L^2+x^2}\frac{x}{\sqrt{L^2+x^2}}$$
$$=-\frac{kqQ}{\left(L^2+x^2\right)^{3/2}}x$$

Factor L^2 from the denominator of this equation to obtain:

$$F_x = -\frac{kqQ}{L^3\left(1+\frac{x^2}{L^2}\right)^{3/2}}x \approx -\frac{kqQ}{L^3}x$$

because $x << L$.

Apply $\sum F_x = ma_x$ to the bead to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{L^3}x$$

or

$$\frac{d^2x}{dt^2} + \frac{kqQ}{mL^3}x = 0$$

the differential equation of simple harmonic motion with $\omega = \sqrt{kqQ/mL^3}$.

Express the period of the motion of the bead in terms of the angular frequency of the motion:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mL^3}{kqQ}} = \boxed{2\pi L\sqrt{\frac{mL}{kqQ}}}$$

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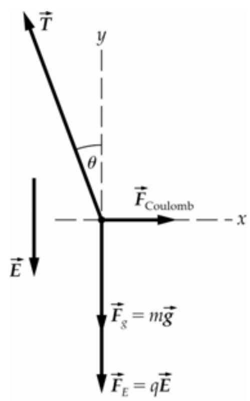
Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, $\vec{F}_{\text{Coulomb}}$ and the force $\vec{F}_E$ exerted by the electric field. We can use Coulomb's law to relate the electric force to the charges on the spheres and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*)Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_{\text{Coulomb}} - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg - qE = 0$$

Eliminate T between these equations to obtain:

$$\tan\theta = \frac{kq^2}{(mg+qE)r^2}$$

Referring to the figure for Problem 78, relate the separation of the

$$r = 2L\sin\theta$$

spheres r to the length of the pendulum L:

Substitute to obtain:

$$\tan\theta = \frac{kq^2}{4(mg+qE)L^2\sin^2\theta}$$

or

$$\sin^2\theta\tan\theta = \frac{kq^2}{4(mg+qE)L^2} \qquad (1)$$

Substitute numerical values and evaluate $\sin^2\theta\tan\theta$ to obtain:

$$\sin^2\theta\tan\theta = 3.25\times10^{-3}$$

Because $\sin^2\theta\tan\theta << 1$:

$$\sin\theta \approx \tan\theta \approx \theta$$

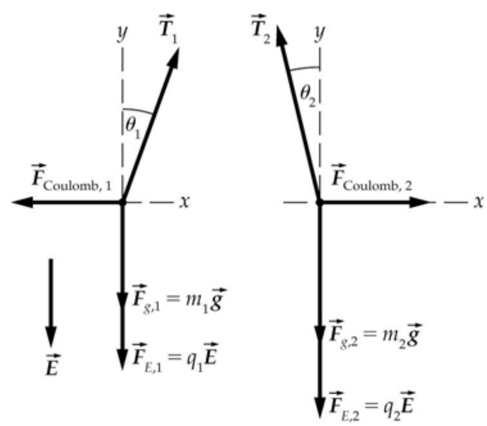
and

$$\theta^3 \approx 3.25\times10^{-3}$$

Solve for θ to obtain:

$$\theta = 0.148\,\text{rad} = \boxed{8.48°}$$

(b) The downward electrical forces acting on the two spheres are no longer equal. Let the mass of the sphere carrying the charge of 0.5 μC be m_1, and that of the sphere carrying the charge of 1.0 μC be m_2. The free-body diagrams show the tension, gravitational, and electrical forces acting on each sphere. Because we already know from part (a) that the angles are small, we can use the small-angle approximation $\sin\theta \approx \tan\theta \approx \theta$.



Apply the conditions for static equilibrium to the charged sphere whose mass is m_1:

$$\sum F_{x,1} = -\frac{kq_1q_2}{r^2} + T_1\sin\theta_1$$
$$= -\frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_1\sin\theta_1$$
$$\approx -\frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_1\theta_1$$
$$= 0$$

and

$$\sum F_{y,1} = T_{1,y} - m_1 g - q_1 E = 0$$

Apply the conditions for static equilibrium to the charged sphere whose mass is m_2:

$$\begin{aligned}\sum F_{x,2} &= \frac{kq_1q_2}{r^2} - T_2 \sin\theta_2 \\ &= \frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_2\sin\theta_2 \\ &\approx \frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_2\theta_2 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,2} = T_{2,y} - m_2 g - q_2 E = 0$$

Express θ_1 and θ_2 in terms of the components of $\vec{T}_1$ and $\vec{T}_2$:

$$\theta_1 = \frac{T_{1,x}}{T_{1,y}} \qquad (1)$$

and

$$\theta_2 = \frac{T_{2,x}}{T_{2,y}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{\dfrac{T_{1,x}}{T_{1,y}}}{\dfrac{T_{2,x}}{T_{2,y}}} = \frac{T_{2,y}}{T_{1,y}}$$

because the horizontal components of $\vec{T}_1$ and $\vec{T}_2$ are equal.

Substitute for $T_{2,y}$ and $T_{1,y}$ to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{m_2 g + q_2 E}{m_1 g + q_1 E}$$

Add equations (1) and (2) to obtain:

$$\theta_1 + \theta_2 = \frac{T_{1,x}}{T_{1,y}} + \frac{T_{2,x}}{T_{2,y}} = \frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2}\left[\frac{1}{m_1 g + q_1 E} + \frac{1}{m_2 g + q_2 E}\right]$$

Solve for $\theta_1 + \theta_2$:

$$\theta_1 + \theta_2 = \sqrt[3]{\frac{kq_1q_2}{L^2}\left[\frac{1}{m_1 g + q_1 E} + \frac{1}{m_2 g + q_2 E}\right]}$$

Substitute numerical values and evaluate $\theta_1 + \theta_2$ and θ_1/θ_2:

$$\theta_1 + \theta_2 = 0.287\,\text{rad} = 16.4°$$

and

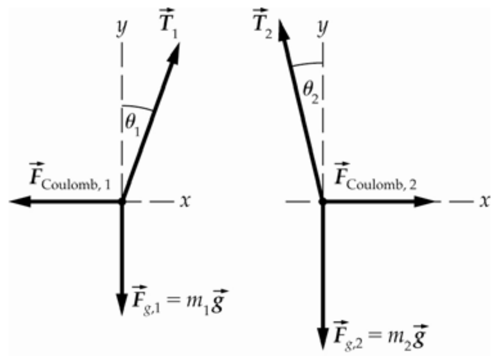
$$\frac{\theta_1}{\theta_2} = 1.34$$

Solve for θ_1 and θ_2 to obtain:

$$\theta_1 = \boxed{9.42°} \text{ and } \theta_1 = \boxed{6.98°}$$

88 •••

Picture the Problem Each sphere is in static equilibrium under the influence of a tension, gravitational and Coulomb force. Let the mass of the sphere carrying the charge of 2.0 μC be m_1 = 0.01 kg, and that of the sphere carrying the charge of 1.0 μC be m_2 = 0.02 kg. We can use Coulomb's law to relate the Coulomb force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charges on the spheres.



Apply the conditions for static equilibrium to the charged sphere whose mass is m_1:

$$\begin{aligned}\sum F_{x,1} &= -\frac{kq_1q_2}{r^2} + T_1\sin\theta_1 \\ &= -\frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_1\sin\theta_1 \\ &\approx -\frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_1\theta_1 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,1} = T_{1,y} - m_1 g = 0$$

Apply the conditions for static equilibrium to the charged sphere whose mass is m_2:

$$\begin{aligned}\sum F_{x,2} &= \frac{kq_1q_2}{r^2} - T_2\sin\theta_2 \\ &= \frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_2\sin\theta_2 \\ &\approx \frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_2\theta_2 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,2} = T_{2,y} - m_2 g = 0$$

Using the small-angle approximation $\sin\theta \approx \tan\theta \approx \theta$, express θ_1 and θ_2 in terms of the components of $\vec{T}_1$ and $\vec{T}_2$:

$$\theta_1 = \frac{T_{1,x}}{T_{1,y}} \qquad (1)$$

and

$$\theta_2 = \frac{T_{2,x}}{T_{2,y}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{\dfrac{T_{1,x}}{T_{1,y}}}{\dfrac{T_{2,x}}{T_{2,y}}} = \frac{T_{2,y}}{T_{1,y}}$$

because the horizontal components of $\vec{T}_1$ and $\vec{T}_2$ are equal.

Substitute for $T_{2,y}$ and $T_{1,y}$ to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{m_2}{m_1}$$

Add equations (1) and (2) to obtain:

$$\theta_1 + \theta_2 = \frac{T_{1,x}}{T_{1,y}} + \frac{T_{2,x}}{T_{2,y}} = \frac{kq_1q_2}{L^2(\theta_1 + \theta_2)^2}\left[\frac{1}{m_1 g} + \frac{1}{m_2 g}\right]$$

Solve for $\theta_1 + \theta_2$:

$$\theta_1 + \theta_2 = \sqrt[3]{\frac{kq_1q_2}{L^2}\left[\frac{1}{m_1 g} + \frac{1}{m_2 g}\right]}$$

Substitute numerical values and evaluate $\theta_1 + \theta_2$ and θ_1/θ_2:

$$\theta_1 + \theta_2 = 0.496\,\text{rad} = 28.4°$$

and

$$\frac{\theta_1}{\theta_2} = \frac{1}{2}$$

Solve for θ_1 and θ_2 to obtain:

$$\theta_1 = \boxed{9.47°} \text{ and } \theta_1 = \boxed{18.9°}$$

Remarks: While the small angle approximation is not as good here as it was in the preceding problems, the error introduced is less than 3%.

89 •••

Picture the Problem We can find the effective value of the gravitational field by finding the force on the bob due to $\vec{g}$ and $\vec{E}$ and equating this sum to the product of the mass of the bob and $\vec{g}'$. We can then solve this equation for $\vec{E}$ in terms of $\vec{g}$, $\vec{g}'$, q, and M and use the equation for the period of a simple pendulum to find the magnitude of $\vec{g}'$

Express the force on the bob due to $\vec{g}$ and $\vec{E}$:

$$\vec{F} = M\vec{g} + q\vec{E} = M\left(\vec{g} + \frac{q}{M}\vec{E}\right) = M\vec{g}'$$

where

$$\vec{g}' = \vec{g} + \frac{q}{M}\vec{E}$$

Solve for $\vec{E}$ to obtain:

$$\vec{E} = \frac{M}{q}\left(\vec{g}' - \vec{g}\right)$$

Using the expression for the period of a simple pendulum, find the magnitude of g':

$$T' = 2\pi\sqrt{\frac{L}{g'}}$$

and

$$g' = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1\,\text{m})}{(1.2\,\text{s})^2} = 27.4\,\text{m/s}^2$$

Substitute numerical values and evaluate $\vec{E}$:

$$\vec{E} = \frac{5\times10^{-3}\,\text{kg}}{-8.0\,\mu\text{C}}\left[\left(27.4\,\text{m/s}^2\right)\hat{j} - \left(9.81\,\text{m/s}^2\right)\hat{j}\right] = \boxed{\left(-1.10\times10^4\,\text{N/C}\right)\hat{j}}$$

***90** •••

Picture the Problem We can relate the force of attraction that each molecule exerts on the other to the potential energy function of either molecule using $F = -dU/dx$. We can relate U to the electric field at either molecule due to the presence of the other through $U = -pE$. Finally, the electric field at either molecule is given by $E = 2kp/x^3$.

Express the force of attraction between the dipoles in terms of the spatial derivative of the potential energy function of p_1:

$$F = -\frac{dU_1}{dx} \qquad (1)$$

Express the potential energy of the dipole p_1:

$$U_1 = -p_1E_1$$

where E_1 is the field at p_1 due to p_2.

Express the electric field at p_1 due to p_2:

$$E_1 = \frac{2kp_2}{x^3}$$

where x is the separation of the dipoles.

Substitute to obtain:

$$U_1 = -\frac{2kp_1p_2}{x^3}$$

Substitute in equation (1) and differentiate with respect to x:

$$F = -\frac{d}{dx}\left[-\frac{2kp_1p_2}{x^3}\right] = \frac{6kp_1p_2}{x^4}$$

Evaluate F for $p_1 = p_2 = p$ and $x = d$ to obtain:

$$F = \boxed{\frac{6kp^2}{d^4}}$$

91 •••

Picture the Problem We can use Coulomb's law for the electric field due to a point charge and superposition of fields to find the electric field at any point on the y axis. By applying Newton's 2nd law, with the charge on the ring negative, we can show that the ring experiences a linear restoring force and, therefore, will execute simple harmonic motion. We can find ω from the differential equation of motion and use $f = \omega/2\pi$ to find the frequency of the motion.

(*a*) Use Coulomb's law for the electric field due to a point charge and superposition of fields, express the field at point P on the y axis:

$$\begin{aligned}\vec{E}_P &= \vec{E}_1 + \vec{E}_2 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} + \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{kQ}{r_{1,P}^3}\vec{r}_{1,P} + \frac{kQ}{r_{2,P}^3}\vec{r}_{2,P} \\ &= \frac{kQ}{\left(a^2+y^2\right)^{3/2}}\left(\frac{L}{2}\hat{i} + y\hat{j}\right) + \frac{kQ}{\left(a^2+y^2\right)^{3/2}}\left(-\frac{L}{2}\hat{i} + y\hat{j}\right) \\ &= \boxed{\frac{2kQy}{\left(a^2+y^2\right)^{3/2}}\hat{j}}\end{aligned}$$

where $a = L/2$.

(*b*) Relate the force on the charged ring to its charge and the electric field:

$$\vec{F}_y = q\vec{E}_y = \boxed{\frac{2kqQy}{\left(a^2+y^2\right)^{3/2}}\hat{j}}$$

where q must be negative if $\vec{F}_y$ is to be a restoring force.

(*c*) Apply Newton's 2nd law to the ring to obtain:

$$m\frac{d^2y}{dt^2}=-\frac{2kqQ}{\left(a^2+y^2\right)^{3/2}}y$$

or

$$\frac{d^2y}{dt^2}=-\frac{2kqQ}{m\left(a^2+y^2\right)^{3/2}}y$$

Factor the radicand to obtain:

$$\frac{d^2y}{dt^2}=-\frac{2kqQ}{ma^3\left(1+\frac{y^2}{a^2}\right)^{3/2}}y$$

$$\approx-\frac{2kqQ}{ma^3}y=-\frac{16kqQ}{mL^3}y$$

provided $y << a = L/2$.

Thus we have:

$$\frac{d^2y}{dt^2}=-\frac{16kqQ}{mL^3}y$$

or

$$\boxed{\frac{d^2y}{dt^2}+\frac{16kqQ}{mL^3}y=0}$$

the differential equation of simple harmonic motion.

Express the frequency of the simple harmonic motion in terms of its angular frequency:

$$f=\frac{\omega}{2\pi}$$

From the differential equation describing the motion we have:

$$\omega^2=\frac{16kqQ}{mL^3}$$

and

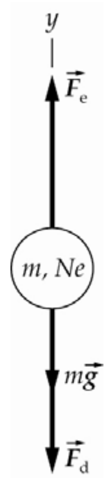
$$f=\frac{1}{2\pi}\sqrt{\frac{16kqQ}{mL^3}}$$

Substitute numerical values and evaluate *f*:

$$f=\frac{1}{2\pi}\sqrt{\frac{16\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(5\,\mu\text{C}\right)\left(2\,\mu\text{C}\right)}{\left(0.03\,\text{kg}\right)\left(0.24\,\text{m}\right)^3}}=\boxed{9.37\,\text{Hz}}$$

92 •••

Picture the Problem The free body diagram shows the forces acting on the microsphere of mass m and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force $\vec{F}_e$, its weight $m\vec{g}$, and the drag force $\vec{F}_d$. We can apply Newton's 2nd law, under terminal-speed conditions, to relate the number of excess charges N on the sphere to its mass and, using Stokes' law, find its terminal speed.



(*a*) Apply $\sum F_y = ma_y$ to the microsphere:

$$F_e - mg - F_d = ma_y$$

or, because $a_y = 0$,

$$F_e - mg - F_{d,\text{terminal}} = 0$$

Substitute for F_e, m, and $F_{d,\text{terminal}}$ to obtain:

$$qE - \rho Vg - 6\pi\eta r v_t = 0$$

or, because $q = Ne$,

$$NeE - \tfrac{4}{3}\pi r^3 \rho g - 6\pi\eta r v_t = 0$$

Solve for N to obtain:

$$N = \frac{\tfrac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_t}{eE}$$

Substitute numerical values and evaluate $\tfrac{4}{3}\pi r^3 \rho g$:

$$\tfrac{4}{3}\pi r^3 \rho g = \tfrac{4}{3}\pi\left(5.5\times10^{-7}\text{ m}\right)^3 \times\left(1.05\times10^3\text{ kg/m}^3\right)\left(9.81\text{m/s}^2\right) = 7.18\times10^{-15}\text{ N}$$

Substitute numerical values and evaluate $6\pi\eta r v_t$:

$$6\pi\eta r v_t = 6\pi\left(1.8\times10^{-5}\text{ Pa}\cdot\text{s}\right)\left(5.5\times10^{-7}\text{ m}\right) \times\left(1.16\times10^{-4}\text{ m/s}\right) = 2.16\times10^{-14}\text{ N}$$

Substitute numerical values in equation (1) and evaluate N:

$$N = \frac{7.18\times10^{-15}\text{ N} + 2.16\times10^{-14}\text{ N}}{\left(1.6\times10^{-19}\text{ C}\right)\left(6\times10^4\text{ V/m}\right)} = \boxed{3}$$

(*b*) With the field pointing upward, the electric force is downward and the application of $\sum F_y = ma_y$ to

$$F_{d,\text{terminal}} - F_e - mg = 0$$

or

$$6\pi\eta r v_t - NeE - \tfrac{4}{3}\pi r^3 \rho g - = 0$$

the bead yields:

Solve for v_t to obtain:

$$v_t = \frac{NeE + \frac{4}{3}\pi r^3 \rho g}{6\pi\eta r}$$

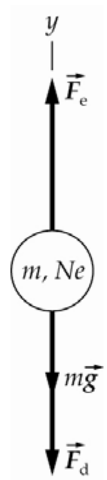
Substitute numerical values and evaluate v_t:

$$v_t = \frac{3(1.6\times10^{-19}\text{ C})(6\times10^4\text{ V/m}) + \frac{4}{3}\pi(5.5\times10^{-7}\text{ m})^3(1.05\times10^3\text{ kg/m}^3)(9.81\text{m/s}^2)}{6\pi(1.8\times10^{-5}\text{ Pa}\cdot\text{s})(5.5\times10^{-7}\text{ m})}$$

$$= \boxed{1.93\times10^{-4}\text{ m/s}}$$

***93 •••**

Picture the Problem The free body diagram shows the forces acting on the microsphere of mass *m* and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force $\vec{F}_e$, its weight $m\vec{g}$, and the drag force $\vec{F}_d$. We can apply Newton's 2nd law, under terminal-speed conditions, to relate the number of excess charges *N* on the sphere to its mass and, using Stokes' law, to its terminal speed.



(*a*) Apply $\sum F_y = ma_y$ to the microsphere when the electric field is downward:

$$F_e - mg - F_d = ma_y$$

or, because $a_y = 0$,

$$F_e - mg - F_{d,\text{terminal}} = 0$$

Substitute for F_e and $F_{d,\text{terminal}}$ to obtain:

$$qE - mg - 6\pi\eta r v_u = 0$$

or, because $q = Ne$,

$$NeE - mg - 6\pi\eta r v_u = 0$$

Solve for v_u to obtain:

$$v_u = \frac{NeE - mg}{6\pi\eta r} \quad (1)$$

With the field pointing upward, the electric force is downward and the application of $\sum F_y = ma_y$ to the microsphere yields:

$$F_{d,\text{terminal}} - F_e - mg = 0$$

or

$$6\pi\eta r v_d - NeE - mg = 0$$

Solve for v_d to obtain:

$$v_d = \frac{NeE + mg}{6\pi\eta r} \quad (2)$$

Add equations (1) and (2) to obtain:

$$v = v_{\text{u}} + v_{\text{d}} = \frac{NeE - mg}{6\pi\eta r} + \frac{NeE + mg}{6\pi\eta r} = \frac{NeE}{3\pi\eta r} = \boxed{\frac{qE}{3\pi\eta r}}$$

$$\boxed{\text{This has the advantage that you don't need to know the mass of the microsphere.}}$$

(*b*) Letting Δv represent the change in the terminal speed of the microsphere due to a gain (or loss) of one electron we have:

$$\Delta v = v_{N+1} - v_N$$

Noting that Δv will be the same whether the microsphere is moving upward or downward, express its terminal speed when it is moving upward with *N* electronic charges on it:

$$v_N = \frac{NeE - mg}{6\pi\eta r}$$

Express its terminal speed upward when it has *N* + 1 electronic charges:

$$v_{N+1} = \frac{(N+1)eE - mg}{6\pi\eta r}$$

Substitute and simplify to obtain:

$$\Delta v_{N+1} = \frac{(N+1)eE - mg}{6\pi\eta r} - \frac{NeE - mg}{6\pi\eta r} = \frac{eE}{6\pi\eta r}$$

Substitute numerical values and evaluate Δv:

$$\Delta v = \frac{(1.6\times10^{-19}\ \text{C})(6\times10^{4}\ \text{V/m})}{6\pi(1.8\times10^{-5}\ \text{Pa}\cdot\text{m})(5.5\times10^{-7}\ \text{m})} = \boxed{5.15\times10^{-5}\ \text{m/s}}$$

Chapter 22
The Electric Field 2: Continuous Charge Distributions

Conceptual Problems

***1** ••

(*a*) False. Gauss's law states that the net flux through any surface is given by $\phi_{\text{net}} = \oint_S E_n dA = 4\pi k Q_{\text{inside}}$. While it is true that Gauss's law is easiest to apply to symmetric charge distributions, it holds for *any* surface.

(*b*) True

2 ••

Determine the Concept Gauss's law states that the net flux through any surface is given by $\phi_{\text{net}} = \oint_S E_n dA = 4\pi k Q_{\text{inside}}$. To use Gauss's law the system must display some symmetry.

3 •••

Determine the Concept The electric field is that due to all the charges, inside and outside the surface. Gauss's law states that the net flux through any surface is given by $\phi_{\text{net}} = \oint_S E_n dA = 4\pi k Q_{\text{inside}}$. The lines of flux through a Gaussian surface begin on charges on one side of the surface and terminate on charges on the other side of the surface.

4 ••

Picture the Problem We can show that the charge inside a sphere of radius *r* is proportional to r^3 and that the area of a sphere is proportional to r^2. Using Gauss's law, we can show that the field must be proportional to $r^3/r^2 = r$.

Use Gauss's law to express the electric field inside a spherical charge distribution of constant volume charge density:

$$E = \frac{4\pi k Q_{\text{inside}}}{A}$$

where $A = 4\pi r^2$.

Express Q_{inside} as a function of ρ and r:

$$Q_{\text{inside}} = \rho V = \tfrac{4}{3}\pi\rho r^3$$

Substitute to obtain:

$$E = \frac{4\pi k \tfrac{4}{3}\pi\rho r^3}{4\pi r^2} = \boxed{\frac{4k\pi\rho}{3}r}$$

***5** •

(*a*) False. Consider a spherical shell, in which there is no charge, in the vicinity of an infinite sheet of charge. The electric field due to the infinite sheet would be non-zero everywhere on the spherical surface.

(*b*) True (assuming there are no charges inside the shell).

(*c*) True.

(*d*) False. Consider a spherical conducting shell. Such a surface will have equal charges on its inner and outer surfaces but, because their areas differ, so will their charge densities.

6 •

Determine the Concept Yes. The electric field on a closed surface is related to the net flux through it by Gauss's law: $\phi = \oint_S E dA = Q_{\text{inside}} / \epsilon_0$. If the net flux through the closed surface is zero, the net charge inside the surface must be zero by Gauss's law.

7 •

Determine the Concept The negative point charge at the center of the conducting shell induces a charge +*Q* on the inner surface of the shell. $\boxed{(a) \text{ is correct.}}$

8 •

Determine the Concept The negative point charge at the center of the conducting shell induces a charge +*Q* on the inner surface of the shell. Because a conductor does not have to be neutral, $\boxed{(d) \text{ is correct.}}$

***9** ••

Determine the Concept We can apply Gauss's law to determine the electric field for $r < R_1$ and $r > R_2$. We also know that the direction of an electric field at any point is determined by the direction of the electric force acting on a positively charged object located at that point.

From the application of Gauss's law we know that the electric field in both of these regions is not zero and is given by:

$$E_n = \frac{kQ}{r^2}$$

A positively charged object placed in either of these regions would experience an attractive force from the charge –*Q* located at the center of the shell. $\boxed{(b) \text{ is correct.}}$

***10 ••**

Determine the Concept We can decide what will happen when the conducting shell is grounded by thinking about the distribution of charge on the shell before it is grounded and the effect on this distribution of grounding the shell.

The negative point charge at the center of the conducting shell induces a positive charge on the inner surface of the shell and a negative charge on the outer surface.
Grounding the shell attracts positive charge from ground; resulting in the outer surface becoming electrically neutral. $\boxed{(b)\text{ is correct.}}$

11 ••

Determine the Concept We can apply Gauss's law to determine the electric field for $r < R_1$ and $r > R_2$. We also know that the direction of an electric field at any point is determined by the direction of the electric force acting on a positively charged object located at that point.

From the application of Gauss's law we know that the electric field in the region $r < R_1$ is given by $E_n = \dfrac{kQ}{r^2}$. A positively charged object placed in the region $r < R_1$ will experience an attractive force from the charge $-Q$ located at the center of the shell. With the conducting shell grounded, the net charge enclosed by a spherical Gaussian surface of radius $r > R_2$ is zero and hence the electric field in this region is zero.
$\boxed{(c)\text{ is correct.}}$

12 ••

Determine the Concept No. The electric field on a closed surface is related to the net flux through it by Gauss's law: $\phi = \oint_S EdA = Q_{\text{inside}}/\epsilon_0$. ϕ can be zero without E being zero everywhere. If the net flux through the closed surface is zero, the net charge inside the surface must be zero by Gauss's law.

13 ••

False. A physical quantity is discontinuous if its value on one side of a boundary differs from that on the other. We can show that this statement is false by citing a counterexample. Consider the field of a uniformly charged sphere. ρ is discontinuous at the surface, E is not.

Estimation and Approximation

***14 ••**

Picture the Problem We'll assume that the total charge is spread out uniformly (charge density = σ) in a thin layer at the bottom and top of the cloud and that the area of each surface of the cloud is 1 km^2. We can then use the definition of surface charge density and the expression for the electric field at the surface of a charged plane surface to estimate the total charge of the cloud.

Express the total charge Q of a thundercloud in terms of the surface area A of the cloud and the charge density σ:

$$Q = \sigma A$$

Express the electric field just outside the cloud:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for σ:

$$\sigma = \epsilon_0 E$$

Substitute for σ to obtain:

$$Q = \epsilon_0 EA$$

Substitute numerical values and evaluate Q:

$$Q = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\times10^6\ \text{V/m}\right)\left(1\,\text{km}^2\right) = \boxed{26.6\,\text{C}}$$

Remarks: This charge is in reasonably good agreement with the total charge transferred in a lightning strike of approximately 30 C.

15 ••

Picture the Problem We'll assume that the field is strong enough to produce a spark. Then we know that field must be equal to the dielectric strength of air. We can then use the relationship between the field and the charge density to estimate the latter.

Suppose the field is large enough to produce a spark. Then:

$$E \approx \boxed{3\times10^6\ \text{V/m}}$$

Because rubbing the balloon leaves it with a surface charge density of $+\sigma$ and the hair with a surface charge density of $-\sigma$, the electric field between the balloon and the hair is:

$$E = \frac{\sigma}{2\epsilon_0}$$

Solve for σ:

$$\sigma = 2\epsilon_0 E$$

Substitute numerical values and evaluate σ:

$$\sigma = 2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\times10^6\ \text{V/m}\right) = \boxed{5.31\times10^{-5}\ \text{C/m}^2}$$

16 •

Picture the Problem For $x << r$, we can model the disk as an infinite plane. For $x >> r$, we can approximate the ring charge by a point charge.

For $x << r$, express the electric field near an infinite plane of charge:

$$E_x = 2\pi k\sigma$$

(*a*) and (*b*) Because E_x is independent of x for $x << r$:

$$E_x = 2\pi\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.6\ \mu\text{C/m}^2\right) = \boxed{2.03\times10^{5}\ \text{N/C}}$$

For $x >> r$, use Coulomb's law for the electric field due to a point charge to obtain:

$$E_x(x) = \frac{kQ}{x^2} = \frac{k\pi r^2\sigma}{x^2}$$

(*c*) Evaluate E_x at x = 5 m:

$$E_x(5\,\text{m}) = \frac{\pi\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.5\,\text{cm})^2\left(3.6\ \mu\text{C/m}^2\right)}{(5\,\text{m})^2} = \boxed{2.54\,\text{N/C}}$$

(*d*) Evaluate E_x at x = 5 cm:

$$E_x(5\,\text{cm}) = \frac{\pi\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.5\,\text{cm})^2\left(3.6\ \mu\text{C/m}^2\right)}{(0.05\,\text{m})^2} = \boxed{2.54\times10^{4}\ \text{N/C}}$$

Note that this is a very poor approximation because $x = 2r$ is not much greater than r.

Calculating $\vec{E}$ From Coulomb's Law

***17** •

Picture the Problem We can use the definition of λ to find the total charge of the line of charge and the expression for the electric field on the axis of a finite line of charge to evaluate E_x at the given locations along the x axis. In part (*d*) we can apply Coulomb's law for the electric field due to a point charge to approximate the electric field at x = 250 m.

(*a*) Use the definition of linear charge density to express Q in terms of λ:

$$Q = \lambda L = (3.5\,\text{nC/m})(5\,\text{m}) = \boxed{17.5\,\text{nC}}$$

Express the electric field on the axis of a finite line charge:

$$E_x(x_0) = \frac{kQ}{x_0(x_0 - L)}$$

(*b*) Substitute numerical values and evaluate E_x at x = 6 m:

$$E_x(6\,\text{m}) = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(17.5\,\text{nC}\right)}{(6\,\text{m})(6\,\text{m}-5\,\text{m})} = \boxed{26.2\,\text{N/C}}$$

(*c*) Substitute numerical values and evaluate E_x at x = 9 m:

$$E_x(9\,\text{m}) = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(17.5\,\text{nC}\right)}{(9\,\text{m})(9\,\text{m}-5\,\text{m})} = \boxed{4.37\,\text{N/C}}$$

(*d*) Substitute numerical values and evaluate E_x at x = 250 m:

$$E_x(250\,\text{m}) = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(17.5\,\text{nC}\right)}{(250\,\text{m})(250\,\text{m}-5\,\text{m})} = \boxed{2.57\,\text{mN/C}}$$

(*e*) Use Coulomb's law for the electric field due to a point charge to obtain:

$$E_x(x) = \frac{kQ}{x^2}$$

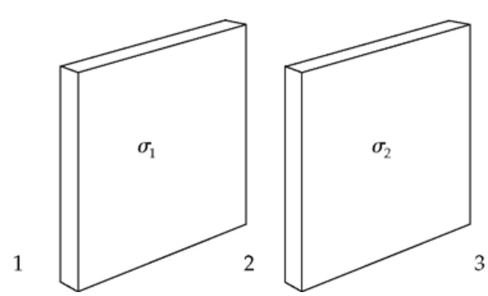
Substitute numerical values and evaluate E_x(250 m):

$$E_x(250\,\text{m}) = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(17.5\,\text{nC}\right)}{(250\,\text{m})^2} = \boxed{2.52\,\text{mN/C}}$$

Note that this result agrees to within 2% with the exact value obtained in (*d*).

18 •

Picture the Problem Let the charge densities on the two plates be σ_1 and σ_2 and denote the three regions of interest as 1, 2, and 3. Choose a coordinate system in which the positive x direction is to the right. We can apply the equation for $\vec{E}$ near an infinite plane of charge and the superposition of fields to find the field in each of the three regions.



(*a*) Use the equation for $\vec{E}$ near an infinite plane of charge to express the field in region 1 when $\sigma_1 = \sigma_2 = +3\ \mu\text{C/m}^2$:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = -2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} = -4\pi k\sigma\,\hat{i}$$

Substitute numerical values and evaluate $\vec{E}_1$:

$$\vec{E}_1 = -4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} = \boxed{-\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

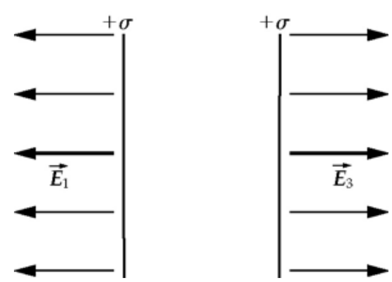
Proceed as above for region 2:

$$\vec{E}_2 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} = 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

Proceed as above for region 3:

$$\vec{E}_3 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i} = 4\pi k\sigma\hat{i} = 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} = \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

The electric field lines are shown to the right:



(*b*) Use the equation for $\vec{E}$ near an infinite plane of charge to express and evaluate the field in region 1 when $\sigma_1 = +3\ \mu\text{C/m}^2$ and $\sigma_2 = -3\ \mu\text{C/m}^2$:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} = 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

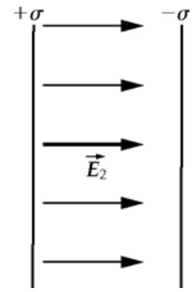
Proceed as above for region 2:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i} = 4\pi k\sigma\hat{i} = 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C}\right)\hat{i} = \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

Proceed as above for region 3:

$$\vec{E}_3 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i}$$
$$= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

The electric field lines are shown to the right:



19 •

Picture the Problem The magnitude of the electric field on the axis of a ring of charge is given by $E_x(x) = kQx\big/\left(x^2 + a^2\right)^{3/2}$ where Q is the charge on the ring and a is the radius of the ring. We can use this relationship to find the electric field on the x axis at the given distances from the ring.

Express $\vec{E}$ on the axis of a ring charge:

$$E_x(x) = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}$$

(a) Substitute numerical values and evaluate E_x for x = 1.2 cm:

$$E_x(1.2\,\text{cm}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.75\,\mu\text{C})(1.2\,\text{cm})}{\left[(1.2\,\text{cm})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{4.69\times10^5\ \text{N/C}}$$

(b) Proceed as in (a) with x = 3.6 cm:

$$E_x(3.6\,\text{cm}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.75\,\mu\text{C})(3.6\,\text{cm})}{\left[(3.6\,\text{cm})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{1.13\times10^6\ \text{N/C}}$$

(c) Proceed as in (a) with x = 4.0 m:

$$E_x(4\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.75\,\mu\text{C})(4\,\text{m})}{\left[(4\,\text{m})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{1.54\times10^3\ \text{N/C}}$$

(d) Using Coulomb's law for the electric field due to a point charge, express E_x:

$$E_x(x) = \frac{kQ}{x^2}$$

Substitute numerical values and evaluate E_x at x = 4.0 m:

$$E_x(4\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})}{(4\,\text{m})^2} = \boxed{1.55\times10^3\ \text{N/C}}$$

This result agrees to within 1% with the result obtained in Part (*c*). It is slightly larger because the point charge is nearer $x = 4$ m than is the ring.

20 •

Picture the Problem We can use $E_x(x) = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$, the expression for the electric field on the axis of a disk charge, to find E_x at x = 0.04 cm and 5 m.

Express the electric field on the axis of a disk charge:

$$E_x(x) = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

(*a*) Evaluate this expression for x = 0.04 cm:

$$E_x = 2\pi(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.6\,\mu\text{C/m}^2)\left(1 - \frac{0.04\,\text{cm}}{\sqrt{(0.04\,\text{cm})^2 + (2.5\,\text{cm})^2}}\right)$$
$$= \boxed{2.00\times10^5\ \text{N/C}}$$

This value is about 1.5% smaller than the approximate value obtained in Problem 9.

(*b*) Proceed as in (*a*) for x = 5 m:

$$E_x = 2\pi(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.6\,\mu\text{C/m}^2)\left(1 - \frac{5\,\text{m}}{\sqrt{(5\,\text{m})^2 + (2.5\,\text{cm})^2}}\right) = \boxed{2.54\ \text{N/C}}$$

Note that the exact and approximate (from Problem 16) agree to within 1%.

21 •

Picture the Problem We can use the definition of λ to find the total charge of the line of charge and the expression for the electric field on the perpendicular bisector of a finite line of charge to evaluate E_y at the given locations along the y axis. In part (*e*) we can apply Coulomb's law for the electric field due to a point charge to approximate the electric field at y = 4.5 m.

(*a*) Use the definition of linear

$$Q = \lambda L = (6\,\text{nC/m})(5\,\text{cm}) = \boxed{0.300\,\text{nC}}$$

charge density to express Q in terms of λ:

Express the electric field on the perpendicular bisector of a finite line charge:

$$E_y(y) = \frac{2k\lambda}{y} \frac{\frac{1}{2}L}{\sqrt{\left(\frac{1}{2}L\right)^2 + y^2}}$$

(*b*) Evaluate E_y at y = 4 cm:

$$E_y(4\,\text{cm}) = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.04\,\text{m}} \frac{\frac{1}{2}(6\,\text{nC/m})(0.05\,\text{m})}{\sqrt{(0.025\,\text{m})^2 + (0.04\,\text{m})^2}} = \boxed{1.43\,\text{kN/C}}$$

(*c*) Evaluate E_y at y = 12 cm:

$$E_y(12\,\text{cm}) = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.12\,\text{m}} \frac{\frac{1}{2}(6\,\text{nC/m})(0.05\,\text{m})}{\sqrt{(0.025\,\text{m})^2 + (0.12\,\text{m})^2}} = \boxed{183\,\text{N/C}}$$

(*d*) Evaluate E_y at y = 4.5 m:

$$E_y(4.5\,\text{m}) = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{4.5\,\text{m}} \frac{\frac{1}{2}(6\,\text{nC/m})(0.05\,\text{m})}{\sqrt{(0.025\,\text{m})^2 + (4.5\,\text{m})^2}} = \boxed{0.133\,\text{N/C}}$$

(*e*) Using Coulomb's law for the electric field due to a point charge, express E_y:

$$E_y(y) = \frac{kQ}{y^2}$$

Substitute numerical values and evaluate E_y at y = 4.5 m:

$$E_y(4.5\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.3\,\text{nC})}{(4.5\,\text{m})^2} = \boxed{0.133\,\text{N/C}}$$

This result agrees to three decimal places with the value calculated in Part (*d*).

22 •

Picture the Problem The electric field on the axis of a disk charge is given by $E_x = 2\pi kq\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$. We can equate this expression and $E_x = \frac{1}{2}\sigma/2\in_0$ and solve for x.

Express the electric field on the axis of a disk charge:

$$E_x = 2\pi kq\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$

We're given that: $E_x = \frac{1}{2}\sigma/2\in_0$

Equate these expressions: $$\frac{\sigma}{4\varepsilon_0} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

Simplify to obtain: $$\frac{\sigma}{4\varepsilon_0} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

or, because $k = 1/4\pi\varepsilon_0$,
$$\frac{1}{2} = 1 - \frac{x}{\sqrt{x^2+a^2}}$$

Solve for x to obtain: $$x = \boxed{\frac{a}{\sqrt{3}}}$$

23 •

Picture the Problem We can use $E_x = \frac{kQx}{\left(x^2+a^2\right)^{3/2}}$ to find the electric field at the given distances from the center of the charged ring.

(*a*) Evaluate E_x at $x = 0.2a$:
$$E_x(0.2a) = \frac{kQ(0.2a)}{\left[(0.2a)^2 + a^2\right]^{3/2}} = \boxed{0.189\frac{kQ}{a^2}}$$

(*b*) Evaluate E_x at $x = 0.5a$:
$$E_x(0.5a) = \frac{kQ(0.5a)}{\left[(0.5a)^2 + a^2\right]^{3/2}} = \boxed{0.358\frac{kQ}{a^2}}$$

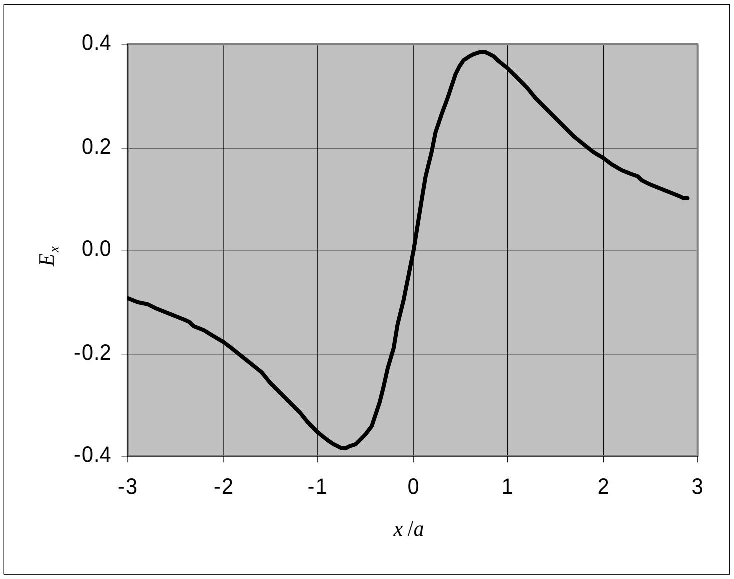
(*c*) Evaluate E_x at $x = 0.7a$:
$$E_x(0.7a) = \frac{kQ(0.7a)}{\left[(0.7a)^2 + a^2\right]^{3/2}} = \boxed{0.385\frac{kQ}{a^2}}$$

(*d*) Evaluate E_x at $x = a$:
$$E_x(a) = \frac{kQa}{\left[a^2 + a^2\right]^{3/2}} = \boxed{0.354\frac{kQ}{a^2}}$$

(*e*) Evaluate E_x at $x = 2a$:

$$E_x(2a) = \frac{2kQa}{\left[(2a)^2 + a^2\right]^{3/2}} = \boxed{0.179\frac{kQ}{a^2}}$$

The field along the *x* axis is plotted below. The *x* coordinates are in units of *x*/*a* and *E* is in units of kQ/a^2.



24 •

Picture the Problem We can use $E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$, where *R* is the radius of the disk, to find the electric field on the axis of a disk charge.

Express E_x in terms of ε_0:

$$E_x = \frac{2\pi\sigma}{4\pi \in_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

$$= \frac{\sigma}{2 \in_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

(*a*) Evaluate E_x at $x = 0.2a$:

$$E_x(0.2a) = \frac{\sigma}{2 \in_0}\left(1 - \frac{0.2a}{\sqrt{(0.2a)^2 + a^2}}\right)$$

$$= \boxed{0.804\frac{\sigma}{2 \in_0}}$$

(*b*) Evaluate E_x at $x = 0.5a$:

$$E_x(0.5a) = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{0.5a}{\sqrt{(0.5a)^2 + a^2}}\right)$$

$$= \boxed{0.553\frac{\sigma}{2\epsilon_0}}$$

(*c*) Evaluate E_x at $x = 0.7a$:

$$E_x(0.7a) = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{0.7a}{\sqrt{(0.7a)^2 + a^2}}\right)$$

$$= \boxed{0.427\frac{\sigma}{2\epsilon_0}}$$

(*d*) Evaluate E_x at $x = a$:

$$E_x(a) = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{a}{\sqrt{a^2 + a^2}}\right)$$

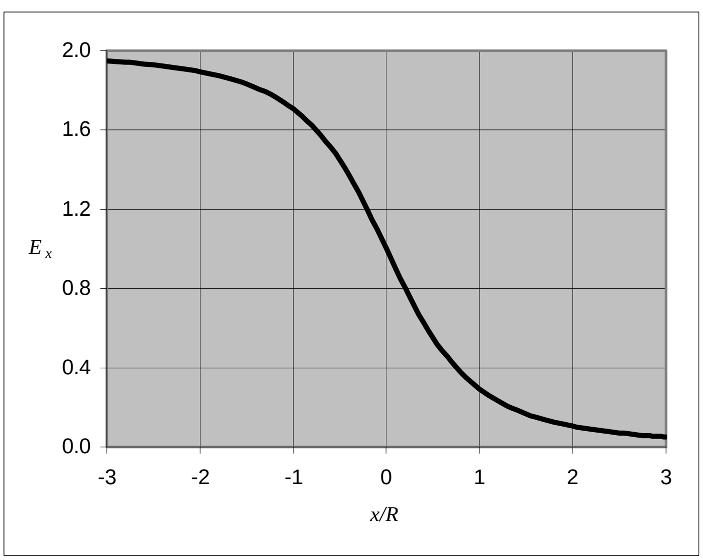
$$= \boxed{0.293\frac{\sigma}{2\epsilon_0}}$$

(*e*) Evaluate E_x at $x = 2a$:

$$E_x(2a) = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{2a}{\sqrt{(2a)^2 + a^2}}\right)$$

$$= \boxed{0.106\frac{\sigma}{2\epsilon_0}}$$

The field along the *x* axis is plotted below. The *x* coordinates are in units of *x*/*a* and *E* is in units of $\sigma/2\epsilon_0$.



***25 ••**
Picture the Problem

(*a*) The electric field on the *x* axis of a disk of radius *r* carrying a surface charge density σ is given by:

$$E_x = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+r^2}}\right)$$

(*b*) The electric field due to an infinite sheet of charge density σ is independent of the distance from the plane and is given by:

$$E_{\text{plate}} = 2\pi k\sigma$$

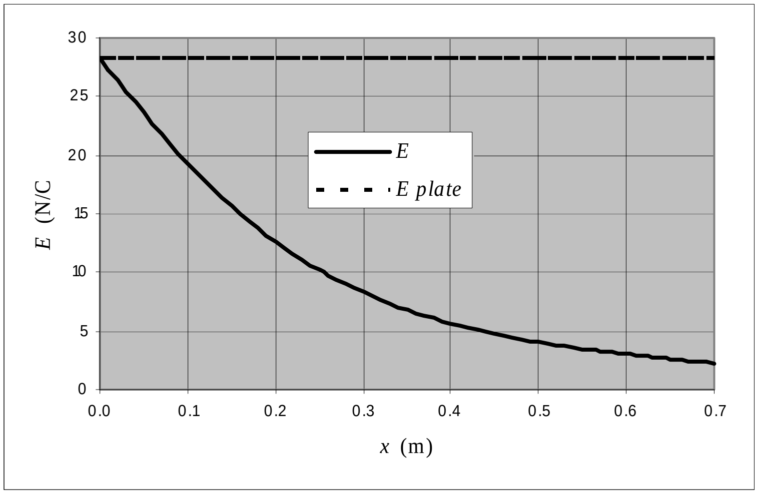
A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B3	9.00E+09	k
B4	5.00E–10	σ
B5	0.3	r
A8	0	x_0
A9	0.01	$x_0 + 0.01$
B8	2*PI()*$B$3*$B$4*(1–A8/ (A8^2+$B$5^2)^2)^0.5)	$2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+r^2}}\right)$
C8	2*PI()*$B$3*$B$4	$2\pi k\sigma$

	A	B	C
1			
2			
3	k=	9.00E+09	Nm^2/C^2
4	sigma=	5.00E-10	C/m^2
5	r=	0.3	m
6			
7	x	E(x)	E plate
8	0.00	28.27	28.3
9	0.01	27.33	28.3
10	0.02	26.39	28.3
11	0.03	25.46	28.3
12	0.04	24.54	28.3
13	0.05	23.63	28.3
14	0.06	22.73	28.3
15	0.07	21.85	28.3
73	0.65	2.60	28.3
74	0.66	2.53	28.3
75	0.67	2.47	28.3
76	0.68	2.41	28.3
77	0.69	2.34	28.3

78	0.70	2.29	28.3

The following graph shows E as a function of x. The electric field from an infinite sheet with the same charge density is shown for comparison – the magnitude of the electric fields differ by more than 10 percent for $x = 0.03$ m.



26 ••

Picture the Problem Equation 22-10 expresses the electric field on the axis of a ring charge as a function of distance along the axis from the center of the ring. We can show that it has its maximum and minimum values at $x = +a/\sqrt{2}$ and $x = -a/\sqrt{2}$ by setting its first derivative equal to zero and solving the resulting equation for x. The graph of E_x will confirm that the maximum and minimum occur at these coordinates.

Express the variation of E_x with x on the axis of a ring charge:

$$E_x = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}$$

Differentiate this expression with respect to x to obtain:

$$\frac{dE_x}{dx} = kQ\frac{d}{dx}\left[\frac{x}{\left(x^2 + a^2\right)^{3/2}}\right] = kQ\frac{\left(x^2 + a^2\right)^{3/2} - x\frac{d}{dx}\left(x^2 + a^2\right)^{3/2}}{\left(x^2 + a^2\right)^3}$$

$$= kQ\frac{\left(x^2 + a^2\right)^{3/2} - x\left(\frac{3}{2}\right)\left(x^2 + a^2\right)^{1/2}(2x)}{\left(x^2 + a^2\right)^3} = kQ\frac{\left(x^2 + a^2\right)^{3/2} - 3x^2\left(x^2 + a^2\right)^{1/2}}{\left(x^2 + a^2\right)^3}$$

Set this expression equal to zero for extrema and simplify:

$$\frac{\left(x^2 + a^2\right)^{3/2} - 3x^2\left(x^2 + a^2\right)^{1/2}}{\left(x^2 + a^2\right)^3} = 0,$$

$$\left(x^2 + a^2\right)^{3/2} - 3x^2\left(x^2 + a^2\right)^{1/2} = 0,$$

and

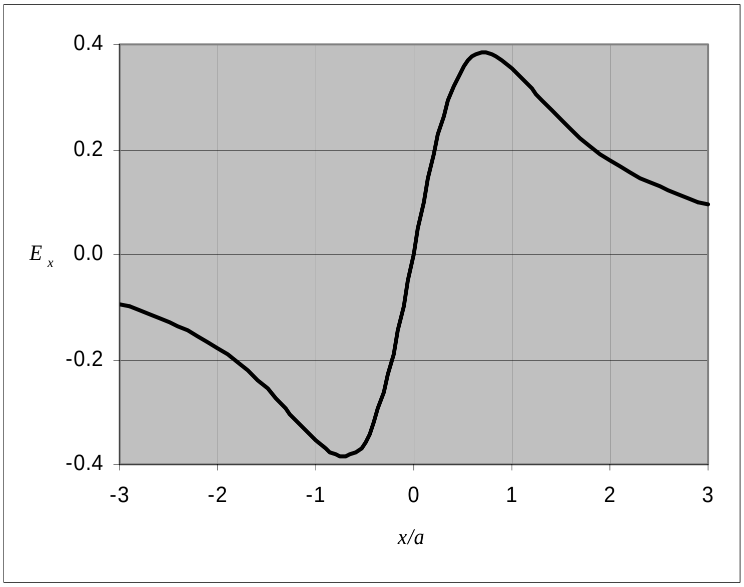
$$x^2 + a^2 - 3x^2 = 0$$

Solve for x to obtain:

$$x = \boxed{\pm \frac{a}{\sqrt{2}}}$$

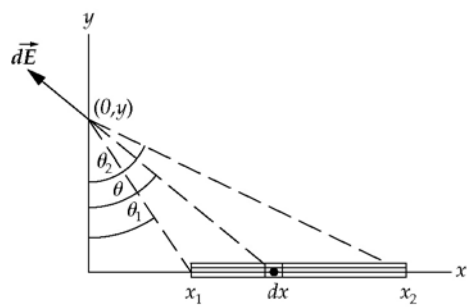
as our candidates for maxima or minima.

A plot of E, in units of kQ/a^2, versus x/a is shown to the right. This graph shows that E is a minimum at $x = -a/\sqrt{2}$ and a maximum at $x = a/\sqrt{2}$.



27 ••

Picture the Problem The line charge and point (0, y) are shown in the diagram. Also shown is a line element of length dx and the field $d\vec{E}$ its charge produces at (0, y). We can find dE_x from $d\vec{E}$ and then integrate from $x = x_1$ to $x = x_2$.



Express the x component of $d\vec{E}$:

$$dE_x = -\frac{k\lambda}{x^2 + y^2}\sin\theta dx = -\frac{k\lambda}{x^2 + y^2}\frac{x}{\sqrt{x^2 + y^2}}dx = -\frac{k\lambda x}{\left(x^2 + y^2\right)^{3/2}}dx$$

Integrate from $x = x_1$ to x_2 to obtain:

$$E_x = -k\lambda \int_{x_1}^{x_2} \frac{x}{\left(x^2+y^2\right)^{3/2}} dx$$
$$= -k\lambda \left[-\frac{1}{\sqrt{x^2+y^2}} \right]_{x_1}^{x_2}$$
$$= -k\lambda \left[-\frac{1}{\sqrt{x_2^2+y^2}} + \frac{1}{\sqrt{x_1^2+y^2}} \right]$$
$$= -\frac{k\lambda}{y} \left[-\frac{y}{\sqrt{x_2^2+y^2}} + \frac{y}{\sqrt{x_1^2+y^2}} \right]$$

From the diagram we see that:

$$\cos\theta_2 = \frac{y}{\sqrt{x_2^2+y^2}} \text{ or } \theta_2 = \tan^{-1}\left(\frac{x_2}{y}\right)$$

and

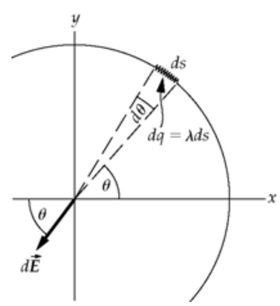
$$\cos\theta_1 = \frac{y}{\sqrt{x_1^2+y^2}} \text{ or } \theta_1 = \tan^{-1}\left(\frac{x_1}{y}\right)$$

Substitute to obtain:

$$E_x = -\frac{k\lambda}{y}\left[-\cos\theta_2 + \cos\theta_1\right]$$
$$= \boxed{\frac{k\lambda}{y}\left[\cos\theta_2 - \cos\theta_1\right]}$$

28 ••

Picture the Problem The diagram shows a segment of the ring of length ds that has a charge $dq = \lambda ds$. We can express the electric field $d\vec{E}$ at the center of the ring due to the charge dq and then integrate this expression from $\theta = 0$ to 2π to find the magnitude of the field in the center of the ring.



(*a*) and (*b*) The field $d\vec{E}$ at the center of the ring due to the charge dq is:

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y = -dE\cos\theta\,\hat{i} - dE\sin\theta\,\hat{j} \quad (1)$$

The magnitude dE of the field at the center of the ring is:

$$dE = \frac{kdq}{r^2}$$

Because $dq = \lambda ds$:

$$dE = \frac{k\lambda ds}{r^2}$$

The linear charge density varies with θ according to $\lambda(\theta) = \lambda_0 \sin\theta$:

$$dE = \frac{k\lambda_0 \sin\theta\, ds}{r^2}$$

Substitute $rd\theta$ for ds:

$$dE = \frac{k\lambda_0 \sin\theta\, r d\theta}{r^2} = \frac{k\lambda_0 \sin\theta\, d\theta}{r}$$

Substitute for dE in equation (1) to obtain:

$$d\vec{E} = -\frac{k\lambda_0 \sin\theta \cos\theta\, d\theta}{r}\hat{i} - \frac{k\lambda_0 \sin^2\theta\, d\theta}{r}\hat{j}$$

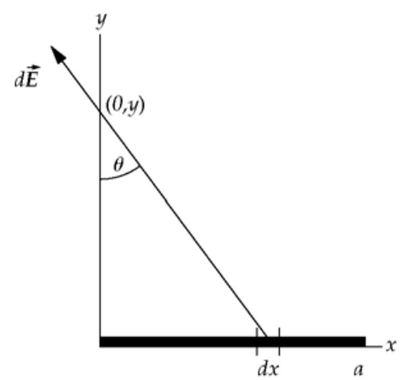
Integrate $d\vec{E}$ from $\theta = 0$ to 2π:

$$\vec{E} = -\frac{k\lambda_0}{2r}\int_0^{2\pi}\sin 2\theta\, d\theta\,\hat{i} - \frac{k\lambda_0}{r}\int_0^{2\pi}\sin^2\theta\, d\theta\,\hat{j} = 0 - \frac{\pi k\lambda_0}{r}\hat{j} = \boxed{-\frac{\pi k\lambda_0}{r}\hat{j}}$$

The field at the origin is in the negative y direction and its magnitude is $\frac{\pi k\lambda_0}{r}$.

29 ••

Picture the Problem The line charge and the point whose coordinates are (0, y) are shown in the diagram. Also shown is a segment of the line of length dx. The field that it produces at (0, y) is $d\vec{E}$. We can find dE_y from $d\vec{E}$ and then integrate from $x = 0$ to $x = a$ to find the y component of the electric field at a point on the y axis.



(a) Express the magnitude of the field $d\vec{E}$ due to charge dq of the

$$dE = \frac{kdq}{r^2}$$

element of length *dx*: where $r^2 = x^2 + y^2$

Because $dq = \lambda dx$:

$$dE = \frac{k\lambda dx}{x^2 + y^2}$$

Express the *y* component of *dE*:

$$dE_y = \frac{k\lambda}{x^2 + y^2}\cos\theta\, dx$$

Refer to the diagram to express $\cos\theta$ in terms of *x* and *y*:

$$\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

Substitute for $\cos\theta$ in the expression for dE_y to obtain:

$$dE_y = \frac{k\lambda y}{\left(x^2 + y^2\right)^{3/2}}dx$$

Integrate from $x = 0$ to $x = a$ and simplify to obtain:

$$E_y = k\lambda y\int_0^a \frac{1}{\left(x^2 + y^2\right)^{3/2}}dx$$

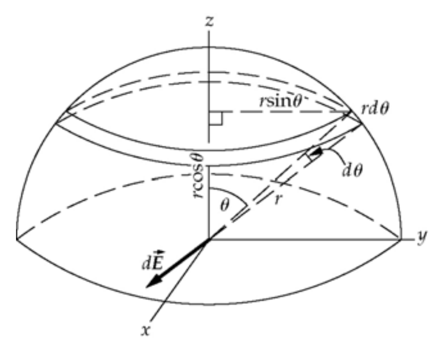
$$= k\lambda y\left[\frac{x}{y^2\sqrt{x^2 + y^2}}\right]_0^a$$

$$= k\lambda\left[\frac{a}{y\sqrt{a^2 + y^2}}\right]$$

$$= \boxed{\frac{k\lambda}{y}\frac{a}{\sqrt{a^2 + y^2}}}$$

***30** •••

Picture the Problem Consider the ring with its axis along the *z* direction shown in the diagram. Its radius is $z = r\cos\theta$ and its width is $rd\theta$. We can use the equation for the field on the axis of a ring charge and then integrate to express the field at the center of the hemispherical shell.



Express the field on the axis of the ring charge:

$$dE = \frac{kzdq}{\left(r^2\sin^2\theta + r^2\cos^2\theta\right)^{3/2}}$$

$$= \frac{kzdq}{r^3}$$

where $z = r\cos\theta$

Express the charge dq on the ring:

$$dq = \sigma dA = \sigma(2\pi r\sin\theta)rd\theta = 2\pi\sigma r^2\sin\theta d\theta$$

Substitute to obtain:

$$dE = \frac{k(r\cos\theta)2\pi\sigma r^2\sin\theta d\theta}{r^3} = 2\pi k\sigma\sin\theta\cos\theta d\theta$$

Integrate dE from $\theta = 0$ to $\pi/2$ to obtain:

$$E = 2\pi k\sigma\int_0^{\pi/2}\sin\theta\cos\theta d\theta = 2\pi k\sigma\left[\tfrac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \boxed{\pi k\sigma}$$

Gauss's Law

31 •

Picture the Problem The definition of electric flux is $\phi = \oint_S \vec{E}\cdot\hat{n}dA$. We can apply this definition to find the electric flux through the square in its two orientations.

(*a*) Apply the definition of ϕ to find the flux of the field when the square is parallel to the *yz* plane:

$$\phi = \oint_S (2\,\text{kN/C})\hat{i}\cdot\hat{i}dA = (2\,\text{kN/C})\oint_S dA = (2\,\text{kN/C})(0.1\,\text{m})^2 = \boxed{20.0\,\text{N}\cdot\text{m}^2/\text{C}}$$

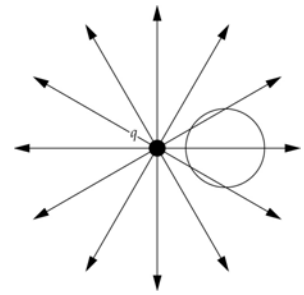
(*b*) Proceed as in (*a*) with $\hat{i}\cdot\hat{n} = \cos 30°$:

$$\phi = \oint_S (2\,\text{kN/C})\cos 30° dA = (2\,\text{kN/C})\cos 30°\oint_S dA = (2\,\text{kN/C})(0.1\,\text{m})^2\cos 30° = \boxed{17.3\,\text{N}\cdot\text{m}^2/\text{C}}$$

***32** •

Determine the Concept While the number of field lines that we choose to draw radially outward from *q* is arbitrary, we must show them originating at *q* and, in the absence of other charges, radially symmetric. The number of lines that we draw is, by agreement, in proportion to the magnitude of *q*.

(*a*) The sketch of the field lines and of the sphere is shown in the diagram to the right.



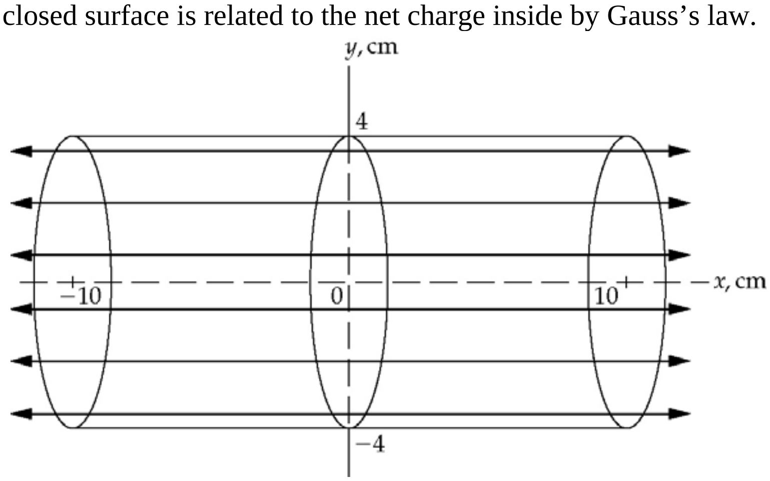
Given the number of field lines drawn from q, 3 lines enter the sphere.
Had we chosen to draw 24 field lines, 6 would have entered the spherical surface.

(*b*) The net number of lines crossing the surface is zero.

(*c*) The net flux is zero.

33 •

Picture the Problem The field at both circular faces of the cylinder is parallel to the outward vector normal to the surface, so the flux is just EA. There is no flux through the curved surface because the normal to that surface is perpendicular to $\vec{E}$. The net flux through the closed surface is related to the net charge inside by Gauss's law.



(*a*) Use Gauss's law to calculate the flux through the right circular surface:

$$\begin{aligned}\phi_{\text{right}} &= \vec{E}_{\text{right}} \cdot \hat{n}_{\text{right}} A \\ &= (300\ \text{N/C})\hat{i} \cdot \hat{i}(\pi)(0.04\ \text{m})^2 \\ &= \boxed{1.51\ \text{N}\cdot\text{m}^2/\text{C}}\end{aligned}$$

Apply Gauss's law to left circular surface:

$$\begin{aligned}\phi_{\text{left}} &= \vec{E}_{\text{left}} \cdot \hat{n}_{\text{left}} A \\ &= (-300\ \text{N/C})\hat{i} \cdot (-\hat{i})(\pi)(0.04\ \text{m})^2 \\ &= \boxed{1.51\ \text{N}\cdot\text{m}^2/\text{C}}\end{aligned}$$

(*b*) Because the field lines are parallel to the curved surface of the cylinder:

$$\phi_{\text{curved}} = \boxed{0}$$

(*c*) Express and evaluate the net flux through the entire cylindrical surface:

$$\phi_{\text{net}} = \phi_{\text{right}} + \phi_{\text{left}} + \phi_{\text{curved}}$$
$$= 1.51\ \text{N}\cdot\text{m}^2/\text{C} + 1.51\ \text{N}\cdot\text{m}^2/\text{C} + 0$$
$$= \boxed{3.02\ \text{N}\cdot\text{m}^2/\text{C}}$$

(*d*) Apply Gauss's law to obtain:

$$\phi_{\text{net}} = 4\pi k Q_{\text{inside}}$$

Solve for Q_{inside}:

$$Q_{\text{inside}} = \frac{\phi_{\text{net}}}{4\pi k}$$

Substitute numerical values and evaluate Q_{inside}:

$$Q_{\text{inside}} = \frac{3.02\ \text{N}\cdot\text{m}^2/\text{C}}{4\pi\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}$$
$$= \boxed{2.67\times10^{-11}\ \text{C}}$$

34 •

Picture the Problem We can use Gauss's law in terms of ε_0 to find the net charge inside the box.

(*a*) Apply Gauss's law in terms of ε_0 to find the net charge inside the box:

$$\phi_{\text{net}} = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$Q_{\text{inside}} = \epsilon_0\ \phi_{\text{net}}$$

Substitute numerical values and evaluate Q_{inside}:

$$Q_{\text{inside}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(6\ \text{kN}\cdot\text{m}^2/\text{C}\right)$$
$$= \boxed{5.31\times10^{-8}\ \text{C}}$$

(*b*) You can only conclude that the net charge is zero. There may be an equal number of positive and negative charges present inside the box.

35 •

Picture the Problem We can apply Gauss's law to find the flux of the electric field through the surface of the sphere.

(*a*) Use the formula for the surface area of a sphere to obtain:

$$A = 4\pi r^2 = 4\pi(0.5\ \text{m})^2 = \boxed{3.14\ \text{m}^2}$$

(*b*) Apply Coulomb's law to express and evaluate *E*:

$$E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\frac{2\ \mu\text{C}}{(0.5\,\text{m})^2} = \boxed{7.19\times10^4\ \text{N/C}}$$

(*c*) Apply Gauss's law to obtain:

$$\phi = \oint_S \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}dA = \oint_S EdA = \left(7.19\times10^4\ \text{N/C}\right)\left(3.14\,\text{m}^2\right) = \boxed{2.26\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

(*d*) No. The flux through the surface is independent of where the charge is located inside the sphere.

(*e*) Because the cube encloses the sphere, the flux through the surface of the sphere will also be the flux through the cube:

$$\phi_{\text{cube}} = \boxed{2.26\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

***36** •

Picture the Problem We'll define the flux of the gravitational field in a manner that is analogous to the definition of the flux of the electric field and then substitute for the gravitational field and evaluate the integral over the closed spherical surface.

Define the gravitational flux as:

$$\phi_g = \oint_S \vec{\boldsymbol{g}}\cdot\hat{\boldsymbol{n}}dA$$

Substitute for $\vec{\boldsymbol{g}}$ and evaluate the integral to obtain:

$$\phi_g = \oint_S\left(-\frac{Gm}{r^2}\hat{\boldsymbol{r}}\right)\cdot\hat{\boldsymbol{n}}dA = -\frac{Gm}{r^2}\oint_S dA = \left(-\frac{Gm}{r^2}\right)\left(4\pi r^2\right) = \boxed{-4\pi Gm}$$

37 ••

Picture the Problem We'll let the square be one face of a cube whose side is 40 cm. Then the charge is at the center of the cube and we can apply Gauss's law in terms of ε_0 to find the flux through the square.

Apply Gauss's law to the cube to express the net flux:

$$\phi_{\text{net}} = \frac{1}{\epsilon_0}Q_{\text{inside}}$$

Express the flux through one face of the cube:

$$\phi_{\text{square}} = \frac{1}{6\in_0} Q_{\text{inside}}$$

Substitute numerical values and evaluate ϕ_{square}:

$$\phi_{\text{square}} = \frac{2\,\mu\text{C}}{6\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{3.77\times10^{4}\ \text{N}\cdot\text{m}^2/\text{C}}$$

38 ••

Picture the Problem We can treat this portion of the earth's atmosphere as though it is a cylinder with cross-sectional area *A* and height *h*. Because the electric flux increases with altitude, we can conclude that there is charge inside the cylindrical region and use Gauss's law to find the charge and hence the charge density of the atmosphere in this region.

The definition of volume charge density is:

$$\rho = \frac{Q}{V}$$

Express the charge inside a cylinder of base area *A* and height *h* for a charge density ρ:

$$Q = \rho A h$$

Taking upward to be the positive direction, apply Gauss's law to the charge in the cylinder:

$$Q = -\left(E_h A - E_0 A\right)\in_0 = \left(E_0 A - E_h A\right)\in_0$$

where we've taken our zero at 250 m above the surface of a flat earth.

Substitute to obtain:

$$\rho = \frac{\left(E_0 A - E_h A\right)\in_0}{Ah} = \frac{\left(E_0 - E_h\right)\in_0}{h}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{\left(150\ \text{N/C} - 170\ \text{N/C}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{250\ \text{m}} = \boxed{-7.08\times10^{-13}\ \text{C/m}^3}$$

where we've been able to neglect the curvature of the earth because the maximum height of 400 m is approximately 0.006% of the radius of the earth.

Spherical Symmetry

39 •

Picture the Problem To find E_n in these three regions we can choose Gaussian surfaces of appropriate radii and apply Gauss's law. On each of these surfaces, E_r is constant and

Gauss's law relates E_r to the total charge inside the surface.

(*a*) Use Gauss's law to find the electric field in the region $r < R_1$:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

and

$$E_{r<R_1} = \frac{Q_{\text{inside}}}{\epsilon_0 A} = \boxed{0}$$

because $Q_{\text{inside}} = 0$.

Apply Gauss's law in the region $R_1 < r < R_2$:

$$E_{R_1<r<R_1} = \frac{q_1}{\epsilon_0 (4\pi r^2)} = \boxed{\frac{kq_1}{r^2}}$$

Using Gauss's law, find the electric field in the region $r > R_2$:

$$E_{r>R_2} = \frac{q_1 + q_2}{\epsilon_0 (4\pi r^2)} = \boxed{\frac{k(q_1 + q_2)}{r^2}}$$

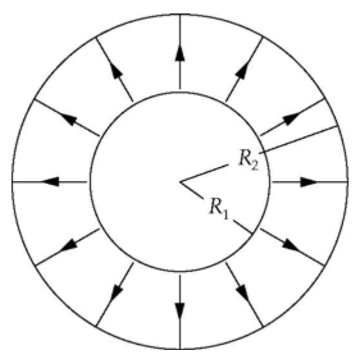
(*b*) Set $E_{r>R_2} = 0$ to obtain:

$$q_1 + q_2 = 0$$

or

$$\frac{q_1}{q_2} = \boxed{-1}$$

(*c*) The electric field lines for the situation in (*b*) with q_1 positive is shown to the right.



40 •

Picture the Problem We can use the definition of surface charge density and the formula for the area of a sphere to find the total charge on the shell. Because the charge is distributed uniformly over a spherical shell, we can choose a spherical Gaussian surface and apply Gauss's law to find the electric field as a function of the distance from the center of the spherical shell.

(*a*) Using the definition of surface charge density, relate the charge on the sphere to its area:

$$Q = \sigma A = 4\pi\sigma r^2$$
$$= 4\pi(9\,\text{nC/m}^2)(0.06\,\text{m})^2$$
$$= \boxed{0.407\,\text{nC}}$$

Apply Gauss's law to a spherical surface of radius r that is concentric the spherical shell to obtain:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{4\pi \epsilon_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

(*b*) Q_{inside} a sphere whose radius is 2 cm is zero and hence:

$$E_n(2\,\text{cm}) = \boxed{0}$$

(*c*) Q_{inside} a sphere whose radius is 5.9 cm is zero and hence:

$$E_n(5.9\,\text{cm}) = \boxed{0}$$

(*d*) Q_{inside} a sphere whose radius is 6.1 cm is 0.407 nC and hence:

$$E_n(6.1\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.061\,\text{m})^2} = \boxed{983\,\text{N/C}}$$

(*e*) Q_{inside} a sphere whose radius is 10 cm is 0.407 nC and hence:

$$E_n(10\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.1\,\text{m})^2} = \boxed{366\,\text{N/C}}$$

41 ••

Picture the Problem We can use the definition of volume charge density and the formula for the volume of a sphere to find the total charge of the sphere. Because the charge is distributed uniformly throughout the sphere, we can choose a spherical Gaussian surface and apply Gauss's law to find the electric field as a function of the distance from the center of the sphere.

(*a*) Using the definition of volume charge density, relate the charge on the sphere to its volume:

$$Q = \rho V = \tfrac{4}{3}\pi\rho r^3 = \tfrac{4}{3}\pi(450\,\text{nC/m}^3)(0.06\,\text{m})^3 = \boxed{0.407\,\text{nC}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the spherical shell to obtain:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{4\pi \in_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

Because the charge distribution is uniform, we can find the charge inside the Gaussian surface by using the definition of volume charge density to establish the proportion:

$$\frac{Q}{V} = \frac{Q_{\text{inside}}}{V'}$$

where V' is the volume of the Gaussian surface.

Solve for Q_{inside} to obtain:

$$Q_{\text{inside}} = Q\frac{V'}{V} = Q\frac{r^3}{R^3}$$

Substitute to obtain:

$$E_n(r < R) = \frac{Q_{\text{inside}}}{4\pi \in_0} \frac{1}{r^2} = \frac{kQ}{R^3} r$$

(*b*) Evaluate E_n at r = 2 cm:

$$E_n(2\,\text{cm}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.407\,\text{nC})}{(0.06\,\text{m})^3}(0.02\,\text{m}) = \boxed{339\,\text{N/C}}$$

(*c*) Evaluate E_n at r = 5.9 cm:

$$E_n(5.9\,\text{cm}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.407\,\text{nC})}{(0.06\,\text{m})^3}(0.059\,\text{m}) = \boxed{999\,\text{N/C}}$$

Apply Gauss's law to the Gaussian surface with $r > R$:

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\varepsilon_0}$$

Solve for E_n to obtain:

$$E_n = \frac{kQ_{\text{inside}}}{r^2} = \frac{kQ}{r^2}$$

(*d*) Evaluate E_n at r = 6.1 cm:

$$E_n(6.1\text{cm}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.407\,\text{nC})}{(0.061\,\text{m})^2} = \boxed{983\,\text{N/C}}$$

(*e*) Evaluate E_n at r = 10 cm:

$$E_n(10\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.1\,\text{m})^2} = \boxed{366\,\text{N/C}}$$

Note that, for $r > R$, these results are the same as those obtained for in Problem 40 for a uniform charge distribution on a spherical shell. This agreement is a consequence of the choices of σ and ρ so that the total charges on the two spheres is the same.

***42 ••**

Determine the Concept The charges on a conducting sphere, in response to the repulsive Coulomb forces each experiences, will separate until electrostatic equilibrium conditions exit. The use of a wire to connect the two spheres or to ground the outer sphere will cause additional redistribution of charge.

(*a*) Because the outer sphere is conducting, the field in the thin shell must vanish. Therefore, $-2Q$, uniformly distributed, resides on the inner surface, and $-5Q$, uniformly distributed, resides on the outer surface.

(*b*) Now there is no charge on the inner surface and $-5Q$ on the outer surface of the spherical shell. The electric field just outside the surface of the inner sphere changes from a finite value to zero.

(*c*) In this case, the $-5Q$ is drained off, leaving no charge on the outer surface and $-2Q$ on the inner surface. The total charge on the outer sphere is then $-2Q$.

43 ••

Picture the Problem By symmetry; the electric field must be radial. To find E_r inside the sphere we choose a spherical Gaussian surface of radius $r < R$. On this surface, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r\,dA = \frac{1}{\epsilon_0}Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r = \frac{Q_{\text{inside}}}{4\pi\epsilon_0}\frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

Use the definition of charge density to relate Q_{inside} to ρ and the volume defined by the Gaussian surface:

$$Q_{\text{inside}} = \rho V_{\text{Gaussian surface}} = \tfrac{4}{3}\rho\pi r^3$$

Substitute to obtain:

$$E_r(r<R)=\frac{\frac{4}{3}\rho\pi kr^3}{r^2}=\frac{4}{3}\rho\pi kr$$

Substitute numerical values and evaluate E_r at $r = 0.5R = 0.05$ m:

$$E_r(0.05\,\text{m})=\tfrac{4}{3}\pi\left(2\,\text{nC/m}^3\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.05\,\text{m})=\boxed{3.77\ \text{N/C}}$$

44 ••

Picture the Problem We can find the total charge on the sphere by expressing the charge *dq* in a spherical shell and integrating this expression between *r* = 0 and *r* = *R*. By symmetry, the electric fields must be radial. To find E_r inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find E_r outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*) Express the charge *dq* in a shell of thickness *dr* and volume $4\pi r^2\,dr$:

$$dq=4\pi r^2\rho dr=4\pi r^2(Ar)dr=4\pi Ar^3dr$$

Integrate this expression from *r* = 0 to *R* to find the total charge on the sphere:

$$Q=4\pi A\int_0^R r^3dr=\left[\pi Ar^4\right]_0^R=\boxed{\pi AR^4}$$

(*b*) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r\,dA=\frac{1}{\epsilon_0}Q_{\text{inside}}$$

or

$$4\pi r^2E_r=\frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r>R)=\frac{Q_{\text{inside}}}{4\pi\epsilon_0}\frac{1}{r^2}=\frac{kQ_{\text{inside}}}{r^2}=\frac{kA\pi R^4}{r^2}=\boxed{\frac{AR^4}{4\epsilon_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r\,dA=\frac{1}{\epsilon_0}Q_{\text{inside}}$$

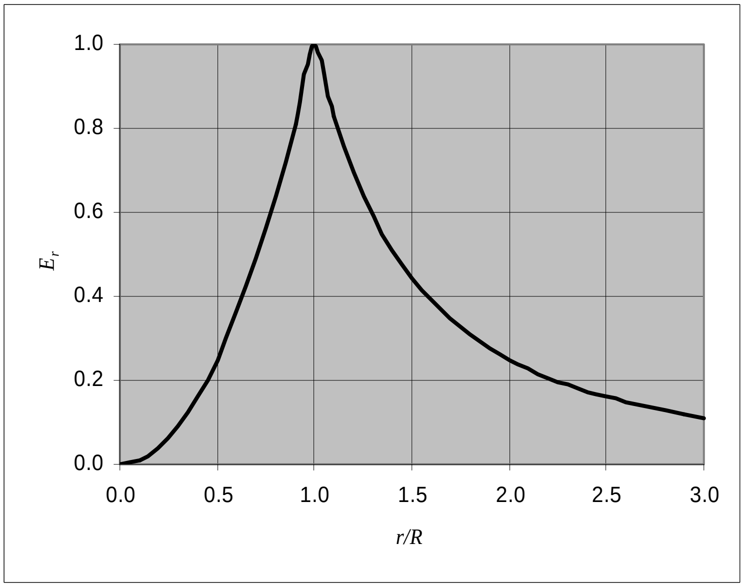
or

$$4\pi r^2E_r=\frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r<R)=\frac{Q_{\text{inside}}}{4\pi r^2\in_0}=\frac{\pi Ar^4}{4\pi r^2\in_0}=\boxed{\frac{Ar^2}{4\in_0}}$$

The graph of E_r versus r/R, with E_r in units of $A/4\in_0$, was plotted using a spreadsheet program.



Remarks: Note that the results for (*a*) and (*b*) agree at *r* = *R*.

45 ••

Picture the Problem We can find the total charge on the sphere by expressing the charge dq in a spherical shell and integrating this expression between $r = 0$ and $r = R$. By symmetry, the electric fields must be radial. To find E_r inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find E_r outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*) Express the charge dq in a shell of thickness dr and volume $4\pi r^2\,dr$:

$$dq=4\pi r^2\rho dr=4\pi r^2\frac{B}{r}dr$$
$$=4\pi Brdr$$

Integrate this expression from $r = 0$ to R to find the total charge on the sphere:

$$Q==4\pi B\int_0^R rdr=\left[2\pi Br^2\right]_0^R$$
$$=\boxed{2\pi BR^2}$$

(*b*) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r > R) = \frac{Q_{\text{inside}}}{4\pi \epsilon_0}\frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

$$= \frac{k2\pi BR^2}{r^2} = \boxed{\frac{BR^2}{2\epsilon_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

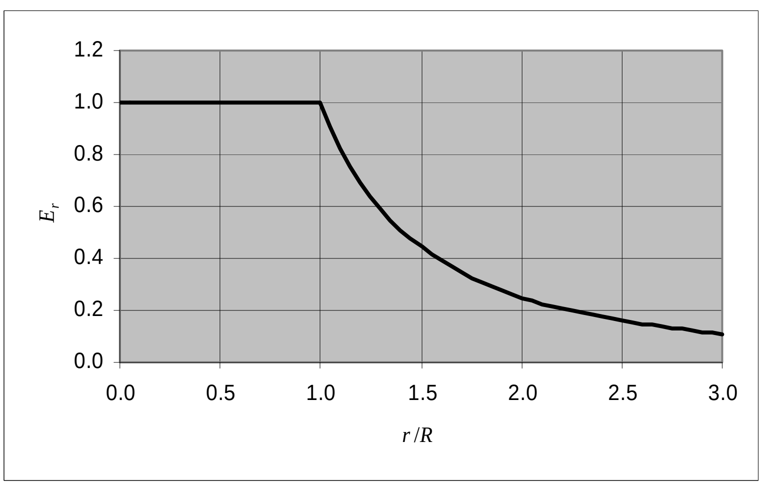
$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r < R) = \frac{Q_{\text{inside}}}{4\pi r^2 \epsilon_0} = \frac{2\pi Br^2}{4\pi r^2 \epsilon_0}$$

$$= \boxed{\frac{B}{2\epsilon_0}}$$

The graph of E_r versus r/R, with E_r in units of $B/2\epsilon_0$, was plotted using a spreadsheet program.



Remarks: Note that our results for (*a*) and (*b*) agree at *r* = *R*.

***46 ••**

Picture the Problem We can find the total charge on the sphere by expressing the charge dq in a spherical shell and integrating this expression between $r = 0$ and $r = R$. By symmetry, the electric fields must be radial. To find E_r inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find E_r outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*) Express the charge dq in a shell of thickness dr and volume $4\pi r^2\,dr$:

$$dq = 4\pi r^2 \rho dr = 4\pi r^2 \frac{C}{r^2} dr$$
$$= 4\pi C dr$$

Integrate this expression from $r = 0$ to R to find the total charge on the sphere:

$$Q = 4\pi C \int_0^R dr = \left[4\pi C r\right]_0^R$$
$$= \boxed{4\pi CR}$$

(*b*) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r > R) = \frac{Q_{\text{inside}}}{4\pi\epsilon_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$
$$= \frac{k4\pi CR}{r^2} = \boxed{\frac{CR}{\epsilon_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

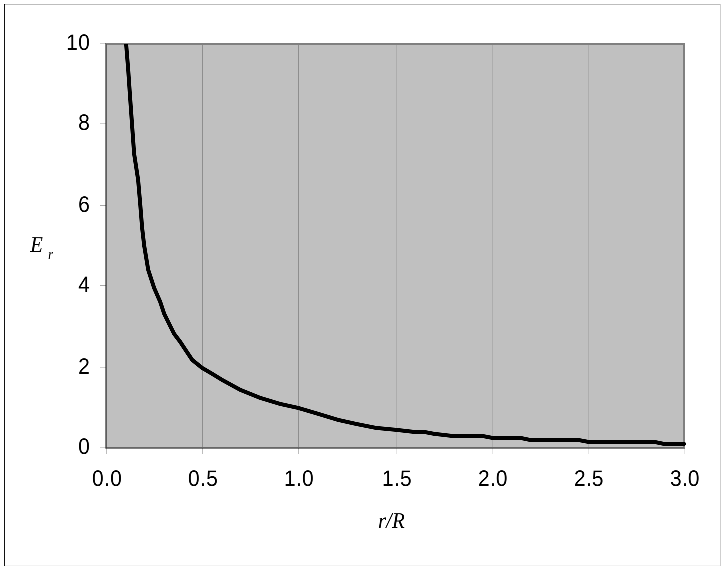
or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r < R) = \frac{Q_{\text{inside}}}{4\pi r^2 \epsilon_0} = \frac{4\pi C r}{4\pi r^2 \epsilon_0}$$
$$= \boxed{\frac{C}{\epsilon_0 r}}$$

The graph of E_r versus r/R, with E_r in units of $C/\epsilon_0 R$, was plotted using a spreadsheet program.



47 •••

Picture the Problem By symmetry, the electric fields resulting from this charge distribution must be radial. To find E_r for $r < a$ we choose a spherical Gaussian surface of radius $r < a$. To find E_r for $a < r < b$ we choose a spherical Gaussian surface of radius $a < r < b$. To find E_r for $r > b$ we choose a spherical Gaussian surface of radius $r > b$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*), (*b*) Apply Gauss's law to a spherical surface of radius *r* that is concentric with the nonconducting spherical shell to obtain:

$$\oint_S E_r dA = \frac{1}{\in_0} Q_{inside}$$

or

$$4\pi r^2 E_r = \frac{Q_{inside}}{\in_0}$$

Solve for E_r:

$$E_r(r) = \frac{Q_{inside}}{4\pi \in_0} \frac{1}{r^2} = \frac{kQ_{inside}}{r^2}$$

Evaluate $E_r(r < a)$:

$$E_r(r < a) = \frac{Q_{inside}}{4\pi \in_0} \frac{1}{r^2} = \frac{kQ_{inside}}{r^2} = \boxed{0}$$

because $\rho(r < a) = 0$ and, therefore, $Q_{inside} = 0$.

Integrate dq from $r = a$ to r to find the total charge in the spherical shell in the interval $a < r < b$:

$$Q_{\text{inside}} = 4\pi\rho\int_a^r r'^2\,dr' = \left[\frac{4\pi C r'^3}{3}\right]_a^r$$
$$= \frac{4\pi\rho}{3}\left(r^3 - a^3\right)$$

Evaluate $E_r(a < r < b)$:

$$E_r(a < r < b) = \frac{kQ_{\text{inside}}}{r^2}$$
$$= \frac{4\pi k\rho}{3r^2}\left(r^3 - a^3\right)$$
$$= \boxed{\frac{\rho}{3\in_0 r^2}\left(r^3 - a^3\right)}$$

For $r > b$:

$$Q_{\text{inside}} = \frac{4\pi\rho}{3}\left(b^3 - a^3\right)$$

and

$$E_r(r > b) = \frac{4\pi k\rho}{3r^2}\left(b^3 - a^3\right)$$
$$= \boxed{\frac{\rho}{3\in_0 r^2}\left(b^3 - a^3\right)}$$

Remarks: Note that *E* is continuous at *r* = *b*.

Cylindrical Symmetry

48 ••

Picture the Problem From symmetry, the field in the tangential direction must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shell.

Apply Gauss's law to the cylindrical surface of radius r and length L that is concentric with the infinitely long, uniformly charged cylindrical shell:

$$\oint_S E_n\,dA = \frac{1}{\in_0}Q_{\text{inside}}$$

or

$$2\pi rLE_n = \frac{Q_{\text{inside}}}{\in_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{2\pi rL\in_0} = \frac{2kQ_{\text{inside}}}{Lr}$$

For $r < R$, $Q_{\text{inside}} = 0$ and:

$$E_n(r < R) = \boxed{0}$$

For $r > R$, $Q_{\text{inside}} = \lambda L$ and:

$$E_n(r > R) = \frac{2k\lambda L}{Lr} = \frac{2k\lambda}{r} = \frac{2k(2\pi R\sigma)}{r} = \boxed{\frac{R\sigma}{\epsilon_0 r}}$$

49 ••

Picture the Problem We can use the definition of surface charge density to find the total charge on the shell. From symmetry, the electric field in the tangential direction must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the uniformly charged cylindrical shell.

(*a*) Using its definition, relate the surface charge density to the total charge on the shell:

$$Q = \sigma A = 2\pi RL\sigma$$

Substitute numerical values and evaluate Q:

$$Q = 2\pi(0.06\,\text{m})(200\,\text{m})(9\,\text{nC/m}^2) = \boxed{679\,\text{nC}}$$

(*b*) From Problem 48 we have, for $r = 2$ cm:

$$E(2\,\text{cm}) = \boxed{0}$$

(*c*) From Problem 48 we have, for $r = 5.9$ cm:

$$E(5.9\,\text{cm}) = \boxed{0}$$

(*d*) From Problem 48 we have, for $r = 6.1$ cm:

$$E_r = \frac{\sigma R}{\epsilon_0 r}$$

and

$$E(6.1\,\text{cm}) = \frac{(9\,\text{nC/m}^2)(0.06\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.061\,\text{m})} = \boxed{1.00\,\text{kN/C}}$$

(*e*) From Problem 48 we have, for $r = 10$ cm:

$$E(10\,\text{cm}) = \frac{(9\,\text{nC/m}^2)(0.06\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.1\,\text{m})} = \boxed{610\,\text{N/C}}$$

50 ••

Picture the Problem From symmetry, the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylinder.

Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{inside}$$

or

$$2\pi r L E_n = \frac{Q_{inside}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi r L \varepsilon_0} = \frac{2kQ_{inside}}{Lr}$$

Express Q_{inside} for $r < R$:

$$Q_{inside} = \rho(r)V = \rho_0\left(\pi r^2 L\right)$$

Substitute to obtain:

$$E_n(r<R) = \frac{2k\left(\pi\rho_0 L r^2\right)}{Lr} = \boxed{\frac{\rho_0}{2\epsilon_0} r}$$

or, because $\lambda = \rho\pi R^2$

$$E_n(r<R) = \boxed{\frac{\lambda}{2\pi\epsilon_0 R^2} r}$$

Express Q_{inside} for $r > R$:

$$Q_{inside} = \rho(r)V = \rho_0\left(\pi R^2 L\right)$$

Substitute to obtain:

$$E_n(r>R) = \frac{2k\left(\pi\rho_0 L R^2\right)}{Lr} = \boxed{\frac{\rho_0 R^2}{2\epsilon_0 r}}$$

or, because $\lambda = \rho\pi R^2$

$$E_n(r>R) = \boxed{\frac{\lambda}{2\pi\epsilon_0 r}}$$

51 ••

Picture the Problem We can use the definition of volume charge density to find the total charge on the cylinder. From symmetry, the electric field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the uniformly charged cylinder.

(*a*) Use the definition of volume charge density to express the total charge of the cylinder:

$$Q_{\text{tot}} = \rho V = \rho\left(\pi R^2 L\right)$$

Substitute numerical values to obtain:

$$Q_{\text{tot}} = \pi\left(300\,\text{nC/m}^3\right)\left(0.06\,\text{m}\right)^2\left(200\,\text{m}\right) = \boxed{679\,\text{nC}}$$

From Problem 50, for $r < R$, we have:

$$E_r = \frac{\rho}{2\epsilon_0}r$$

(*b*) For r = 2 cm:

$$E_r\left(2\,\text{cm}\right) = \frac{\left(300\,\text{nC/m}^3\right)\left(0.02\,\text{m}\right)}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{339\,\text{N/C}}$$

(*c*) For r = 5.9 cm:

$$E_r\left(5.9\,\text{cm}\right) = \frac{\left(300\,\text{nC/m}^3\right)\left(0.059\,\text{m}\right)}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{1.00\,\text{kN/C}}$$

From Problem 50, for $r > R$, we have:

$$E_r = \frac{\rho R^2}{2\epsilon_0 r}$$

(*d*) For r = 6.1 cm:

$$E_r\left(6.1\,\text{cm}\right) = \frac{\left(300\,\text{nC/m}^3\right)\left(0.06\,\text{m}\right)^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.061\,\text{m}\right)} = \boxed{1.00\,\text{kN/C}}$$

(*e*) For r = 10 cm:

$$E_r\left(10\,\text{cm}\right) = \frac{\left(300\,\text{nC/m}^3\right)\left(0.06\,\text{m}\right)^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.1\,\text{m}\right)} = \boxed{610\,\text{N/C}}$$

Note that, given the choice of charge densities in Problems 49 and 51, the electric fields for $r > R$ are the same.

***52 ••**

Picture the Problem From symmetry; the field tangent to the surfaces of the shells must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shells.

(*a*) Apply Gauss's law to the cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long, uniformly charged cylindrical shell:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{2kQ_{\text{inside}}}{Lr} \qquad (1)$$

For $r < R_1$, $Q_{\text{inside}} = 0$ and:

$$E_n(r < R_1) = \boxed{0}$$

Express Q_{inside} for $R_1 < r < R_2$:

$$Q_{\text{inside}} = \sigma_1 A_1 = 2\pi\sigma_1 R_1 L$$

Substitute in equation (1) to obtain:

$$E_n(R_1 < r < R_2) = \frac{2k(2\pi\sigma_1 R_1 L)}{Lr} = \boxed{\frac{\sigma_1 R_1}{\epsilon_0 r}}$$

Express Q_{inside} for $r > R_2$:

$$Q_{\text{inside}} = \sigma_1 A_1 + \sigma_2 A_2 = 2\pi\sigma_1 R_1 L + 2\pi\sigma_2 R_2 L$$

Substitute in equation (1) to obtain:

$$E_n(r > R_2) = \frac{2k(2\pi\sigma_1 R_1 L + 2\pi\sigma_2 R_2 L)}{Lr} = \boxed{\frac{\sigma_1 R_1 + \sigma_2 R_2}{\epsilon_0 r}}$$

(*b*) Set $E = 0$ for $r > R_2$ to obtain:

$$\frac{\sigma_1 R_1 + \sigma_2 R_2}{\epsilon_0 r} = 0$$

or

$$\sigma_1 R_1 + \sigma_2 R_2 = 0$$

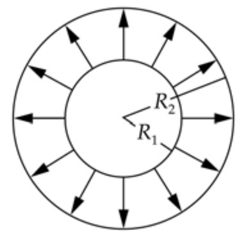
Solve for the ratio of σ_1 to σ_2:

$$\frac{\sigma_1}{\sigma_2} = \boxed{-\frac{R_2}{R_1}}$$

Because the electric field is determined by the charge inside the Gaussian surface, the field under these conditions would be as given above:

$$E_n(R_1 < r < R_2) = \boxed{\frac{\sigma_1 R_1}{\epsilon_0\, r}}$$

(*c*) Assuming that σ_1 is positive, the field lines would be directed as shown to the right.



53 ••

Picture the Problem The electric field is directed radially outward. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shell.

(*a*) Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the inner conductor:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{2kQ_{\text{inside}}}{Lr} \qquad (1)$$

For $r < 1.5$ cm, $Q_{\text{inside}} = 0$ and:

$$E_n(r < 1.5\,\text{cm}) = \boxed{0}$$

Letting $R = 1.5$ cm, express Q_{inside} for 1.5 cm < r < 4.5 cm:

$$Q_{\text{inside}} = \lambda L = 2\pi\sigma RL$$

Substitute in equation (1) to obtain:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{2k(\lambda L)}{Lr} = \frac{2k\lambda}{r}$$

Substitute numerical values and evaluate $E_n(1.5\text{ cm} < r < 4.5\text{ cm})$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = 2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6\,\text{nC/m})}{r} = \boxed{\frac{(108\,\text{N}\cdot\text{m/C})}{r}}$$

Express Q_{inside} for 4.5 cm < r < 6.5 cm:

$$Q_{\text{inside}} = 0$$

and

$$E_n(4.5\,\text{cm} < r < 6.5\,\text{cm}) = \boxed{0}$$

Letting σ_2 represent the charge density on the outer surface, express Q_{inside} for r > 6.5 cm:

$$Q_{\text{inside}} = \sigma_2 A_2 = 2\pi\sigma_2 R_2 L$$

where R_2 = 6.5 cm.

Substitute in equation (1) to obtain:

$$E_n(r > R_2) = \frac{2k(2\pi\sigma_2 R_2 L)}{Lr} = \frac{\sigma_2 R_2}{\in_0 r}$$

In (b) we show that σ_2 = 21.2 nC/m^2. Substitute numerical values to obtain:

$$E_n(r > 6.5\,\text{cm}) = \frac{(21.2\,\text{nC/m}^2)(6.5\,\text{cm})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)r} = \boxed{\frac{156\,\text{N}\cdot\text{m/C}}{r}}$$

(b) The surface charge densities on the inside and the outside surfaces of the outer conductor are given by:

$$\sigma_1 = \frac{-\lambda}{2\pi R_1} \text{ and } \sigma_2 = -\sigma_1$$

Substitute numerical values and evaluate σ_1 and σ_2:

$$\sigma_1 = \frac{-6\,\text{nC/m}}{2\pi(0.045\,\text{m})} = \boxed{-21.2\,\text{nC/m}^2}$$

and

$$\sigma_2 = \boxed{21.2\,\text{nC/m}^2}$$

54 ••

Picture the Problem From symmetry considerations, we can conclude that the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylinder.

(a) Apply Gauss's law to a cylindrical surface of radius r and length L that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\in_0}$$

where we've neglected the end areas

because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi rL \in_0} \qquad (1)$$

Express dQ_{inside} for $\rho(r) = ar$:

$$dQ_{inside} = \rho(r)dV = ar(2\pi rL)dr = 2\pi ar^2 Ldr$$

Integrate dQ_{inside} from $r = 0$ to R to obtain:

$$Q_{inside} = 2\pi aL\int_0^R r^2 dr = 2\pi aL\left[\frac{r^3}{3}\right]_0^R = \frac{2\pi aL}{3}R^3$$

Divide both sides of this equation by L to obtain an expression for the charge per unit length λ of the cylinder:

$$\lambda = \frac{Q_{inside}}{L} = \boxed{\frac{2\pi aR^3}{3}}$$

(*b*) Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r<R) = \frac{\frac{2\pi aL}{3}r^3}{2\pi \in_0 Lr} = \boxed{\frac{a}{3\in_0}r^2}$$

For $r > R$:

$$Q_{inside} = \frac{2\pi aL}{3}R^3$$

Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r>R) = \frac{\frac{2\pi aL}{3}R^3}{2\pi rL \in_0} = \boxed{\frac{aR^3}{3r\in_0}}$$

55 ••

Picture the Problem From symmetry; the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylinder.

(*a*) Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n dA = \frac{1}{\in_0}Q_{inside}$$

or

$$2\pi rLE_n = \frac{Q_{inside}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi rL\epsilon_0} \quad (1)$$

Express dQ_{inside} for $\rho(r) = br^2$:

$$dQ_{inside} = \rho(r)dV = br^2(2\pi rL)dr = 2\pi br^3 Ldr$$

Integrate dQ_{inside} from $r = 0$ to R to obtain:

$$Q_{inside} = 2\pi bL\int_0^R r^3 dr = 2\pi bL\left[\frac{r^4}{4}\right]_0^R = \frac{\pi bL}{2}R^4$$

Divide both sides of this equation by L to obtain an expression for the charge per unit length λ of the cylinder:

$$\lambda = \frac{Q_{inside}}{L} = \boxed{\frac{\pi bR^4}{2}}$$

(b) Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r < R) = \frac{\frac{\pi bL}{2}r^4}{2\pi Lr\epsilon_0} = \boxed{\frac{b}{4\epsilon_0}r^3}$$

For $r > R$:

$$Q_{inside} = \frac{\pi bL}{2}R^4$$

Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r > R) = \frac{\frac{\pi bL}{2}R^4}{2\pi rL\epsilon_0} = \boxed{\frac{bR^4}{4r\epsilon_0}}$$

56 •••

Picture the Problem From symmetry; the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylindrical shell.

Apply Gauss's law to a cylindrical surface of radius r and length L that

$$\oint_S E_n dA = \frac{1}{\epsilon_0}Q_{inside}$$

is concentric with the infinitely long nonconducting cylindrical shell:

or

$$2\pi rLE_n = \frac{Q_{inside}}{\in_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi rL \in_0}$$

For $r < a$, $Q_{inside} = 0$:

$$E_n(r < a) = \boxed{0}$$

Express Q_{inside} for $a < r < b$:

$$Q_{inside} = \rho V = \rho\pi r^2 L - \rho\pi a^2 L = \rho\pi L(r^2 - a^2)$$

Substitute for Q_{inside} to obtain:

$$E_n(a < r < b) = \frac{\rho\pi L(r^2 - a^2)}{2\pi \in_0 Lr} = \boxed{\frac{\rho(r^2 - a^2)}{2 \in_0 r}}$$

Express Q_{inside} for $r > b$:

$$Q_{inside} = \rho V = \rho\pi b^2 L - \rho\pi a^2 L = \rho\pi L(b^2 - a^2)$$

Substitute for Q_{inside} to obtain:

$$E_n(r > b) = \frac{\rho\pi L(b^2 - a^2)}{2\pi \in_0 rL} = \boxed{\frac{\rho(b^2 - a^2)}{2 \in_0 r}}$$

57 •••

Picture the Problem We can integrate the density function over the radius of the inner cylinder to find the charge on it and then calculate the linear charge density from its definition. To find the electric field for all values of *r* we can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to each region of the cable to find the electric field as a function of the distance from its centerline.

(*a*) Find the charge Q_{inner} on the inner cylinder:

$$Q_{inner} = \int_0^R \rho(r)dV = \int_0^R \frac{C}{r}2\pi rLdr = 2\pi CL\int_0^R dr = 2\pi CLR$$

Relate this charge to the linear charge density:

$$\lambda_{\text{inner}} = \frac{Q_{\text{inner}}}{L} = \frac{2\pi CLR}{L} = 2\pi CR$$

Substitute numerical values and evaluate λ_{inner}:

$$\lambda_{\text{inner}} = 2\pi(200\,\text{nC/m})(0.015\,\text{m}) = \boxed{18.8\,\text{nC/m}}$$

(*b*) Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{2\pi r L \epsilon_0}$$

Substitute to obtain, for $r < 1.5$ cm:

$$E_n(r < 1.5\,\text{cm}) = \frac{2\pi C L r}{2\pi \epsilon_0 L r} = \frac{C}{\epsilon_0}$$

Substitute numerical values and evaluate $E_n(r < 1.5\text{ cm})$:

$$E_n(r < 1.5\,\text{cm}) = \frac{200\,\text{nC/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{22.6\,\text{kN/C}}$$

Express Q_{inside} for $1.5\text{ cm} < r < 4.5\text{ cm}$:

$$Q_{\text{inside}} = 2\pi CLR$$

Substitute to obtain, for $1.5\text{ cm} < r < 4.5\text{ cm}$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{2C\pi RL}{2\pi \epsilon_0 rL} = \frac{CR}{\epsilon_0 r}$$

where $R = 1.5$ cm.

Substitute numerical values and evaluate $E_n(1.5\text{ cm} < r < 4.5\text{ cm})$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{(200\,\text{nC/m}^2)(0.015\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)r} = \boxed{\frac{339\,\text{N}\cdot\text{m/C}}{r}}$$

Because the outer cylindrical shell is a conductor:

$$E_n(4.5\,\text{cm} < r < 6.5\,\text{cm}) = \boxed{0}$$

For r > 6.5 cm, $Q_{\text{inside}} = 2\pi CLR$ and:

$$E_n(r > 6.5\,\text{cm}) = \boxed{\frac{339\,\text{N}\cdot\text{m/C}}{r}}$$

Charge and Field at Conductor Surfaces

***58** •

Picture the Problem Because the penny is in an external electric field, it will have charges of opposite signs induced on its faces. The induced charge σ is related to the electric field by $E = \sigma/\varepsilon_0$. Once we know σ, we can use the definition of surface charge density to find the total charge on one face of the penny.

(*a*) Relate the electric field to the charge density on each face of the penny:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for and evaluate σ:

$$\sigma = \epsilon_0 E = (8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(1.6\,\text{kN/C}) = \boxed{14.2\,\text{nC/m}^2}$$

(*b*) Use the definition of surface charge density to obtain:

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2}$$

Solve for and evaluate Q:

$$Q = \sigma\pi r^2 = \pi(14.2\,\text{nC/m}^2)(0.01\,\text{m})^2 = \boxed{4.45\,\text{pC}}$$

59 •

Picture the Problem Because the metal slab is in an external electric field, it will have charges of opposite signs induced on its faces. The induced charge σ is related to the electric field by $E = \sigma/\epsilon_0$.

Relate the magnitude of the electric field to the charge density on the metal slab:

$$E = \frac{\sigma}{\epsilon_0}$$

Use its definition to express σ:

$$\sigma = \frac{Q}{A} = \frac{Q}{L^2}$$

Substitute to obtain:

$$E = \frac{Q}{L^2\epsilon_0}$$

Substitute numerical values and evaluate E:

$$E = \frac{1.2\,\text{nC}}{(0.12\,\text{m})^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{9.42\,\text{kN/C}}$$

60 •

Picture the Problem We can apply its definition to find the surface charge density of the nonconducting material and calculate the electric field at either of its surfaces from $\sigma/2\epsilon_0$. When the same charge is placed on a conducting sheet, the charge will distribute itself until half the charge is on each surface.

(*a*) Use its definition to find σ:

$$\sigma = \frac{Q}{A} = \frac{6\,\text{nC}}{(0.2\,\text{m})^2} = \boxed{150\,\text{nC/m}^2}$$

(*b*) Relate the electric field on either side of the sheet to the density of charge on its surfaces:

$$E = \frac{\sigma}{2\epsilon_0} = \frac{150\,\text{nC/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{8.47\,\text{kN/C}}$$

(*c*) Because the slab is a conductor the charge will distribute uniformly on its two surfaces so that:

$$\sigma = \frac{Q}{2A} = \frac{6\,\text{nC}}{2(0.2\,\text{m})^2} = \boxed{75.0\,\text{nC/m}^2}$$

(*d*) The electric field just outside the surface of a conductor is given by:

$$E = \frac{\sigma}{\epsilon_0} = \frac{75\,\text{nC/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{8.47\,\text{kN/C}}$$

61 •

Picture the Problem We can construct a Gaussian surface in the shape of a sphere of radius r with the same center as the shell and apply Gauss's law to find the electric field as a function of the distance from this point. The inner and outer surfaces of the shell will have charges induced on them by the charge q at the center of the shell.

(*a*) Apply Gauss's law to a spherical surface of radius r that is concentric with the point charge:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0}Q_{\text{inside}}$$

or

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{4\pi r^2\epsilon_0} \qquad (1)$$

For $r < a$, $Q_{inside} = q$. Substitute in equation (1) and simplify to obtain:

$$E_n(r < a) = \frac{q}{4\pi r^2 \in_0} = \boxed{\frac{kq}{r^2}}$$

Because the spherical shell is a conductor, a charge $-q$ will be induced on its inner surface. Hence, for $a < r < b$:

$$Q_{inside} = 0$$

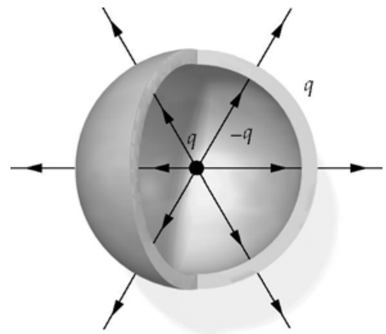
and

$$E_n(a < r < b) = \boxed{0}$$

For $r > b$, $Q_{inside} = q$. Substitute in equation (1) and simplify to obtain:

$$E_n(r > b) = \frac{q}{4\pi r^2 \in_0} = \boxed{\frac{kq}{r^2}}$$

(*b*) The electric field lines are shown in the diagram to the right:



(*c*) A charge $-q$ is induced on the inner surface. Use the definition of surface charge density to obtain:

$$\sigma_{inner} = \frac{-q}{4\pi a^2} = \boxed{-\frac{q}{4\pi a^2}}$$

A charge q is induced on the outer surface. Use the definition of surface charge density to obtain:

$$\sigma_{outer} = \boxed{\frac{q}{4\pi b^2}}$$

62 ••

Picture the Problem We can construct a spherical Gaussian surface at the surface of the earth (we'll assume the Earth is a sphere) and apply Gauss's law to relate the electric field to its total charge.

Apply Gauss's law to a spherical surface of radius R_E that is concentric with the earth:

$$\oint_S E_n dA = \frac{1}{\in_0} Q_{inside}$$

or

$$4\pi R_E^2 E_n = \frac{Q_{inside}}{\in_0}$$

Solve for $Q_{inside} = Q_{earth}$ to obtain:

$$Q_{earth} = 4\pi \in_0 R_E^2 E_n = \frac{R_E^2 E_n}{k}$$

Substitute numerical values and evaluate Q_{earth}:

$$Q_{\text{earth}} = \frac{\left(6.37\times10^{6}\ \text{m}\right)^{2}\left(150\ \text{N/C}\right)}{8.99\times10^{9}\ \text{N}\cdot\text{m}^{2}/\text{C}^{2}} = \boxed{6.77\times10^{5}\ \text{C}}$$

***63 ••**

Picture the Problem Let the inner and outer radii of the uncharged spherical conducting shell be *a* and *b* and *q* represent the positive point charge at the center of the shell. The positive point charge at the center will induce a negative charge on the inner surface of the shell and, because the shell is uncharged, an equal positive charge will be induced on its outer surface. To solve part (*b*), we can construct a Gaussian surface in the shape of a sphere of radius *r* with the same center as the shell and apply Gauss's law to find the electric field as a function of the distance from this point. In part (*c*) we can use a similar strategy with the additional charge placed on the shell.

(*a*) Express the charge density on the inner surface:

$$\sigma_{\text{inner}} = \frac{q_{\text{inner}}}{A}$$

Express the relationship between the positive point charge *q* and the charge induced on the inner surface q_{inner}:

$$q + q_{\text{inner}} = 0$$

Substitute for q_{inner} to obtain:

$$\sigma_{\text{inner}} = \frac{-q}{4\pi a^{2}}$$

Substitute numerical values and evaluate σ_{inner}:

$$\sigma_{\text{inner}} = \frac{-2.5\ \mu\text{C}}{4\pi\left(0.6\ \text{m}\right)^{2}} = \boxed{-0.553\ \mu\text{C/m}^{2}}$$

Express the charge density on the outer surface:

$$\sigma_{\text{outer}} = \frac{q_{\text{outer}}}{A}$$

Because the spherical shell is uncharged:

$$q_{\text{outer}} + q_{\text{inner}} = 0$$

Substitute for q_{outer} to obtain:

$$\sigma_{\text{outer}} = \frac{-q_{\text{inner}}}{4\pi b^{2}}$$

Substitute numerical values and evaluate σ_{outer}:

$$\sigma_{\text{outer}} = \frac{2.5\ \mu\text{C}}{4\pi\left(0.9\ \text{m}\right)^{2}} = \boxed{0.246\ \mu\text{C/m}^{2}}$$

(*b*) Apply Gauss's law to a spherical surface of radius *r* that is concentric with the point charge:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{4\pi r^2 \epsilon_0} \qquad (1)$$

For $r < a = 0.6$ m, $Q_{\text{inside}} = q$. Substitute in equation (1) and evaluate $E_n(r < 0.6\text{ m})$ to obtain:

$$E_n(r<a) = \frac{q}{4\pi r^2 \epsilon_0} = \frac{kq}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.5\,\mu\text{C})}{r^2}$$
$$= \boxed{\left(2.25\times10^4\ \text{N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

Because the spherical shell is a conductor, a charge $-q$ will be induced on its inner surface. Hence, for 0.6 m < *r* < 0.9 m:

$$Q_{\text{inside}} = 0$$

and

$$E_n(0.6\,\text{m} < r < 0.9\,\text{m}) = \boxed{0}$$

For *r* > 0.9 m, the net charge inside the Gaussian surface is *q* and:

$$E_n(r > 0.9\,\text{m}) = \frac{kq}{r^2} = \boxed{\left(2.25\times10^4\ \text{N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

(*c*) Because *E* = 0 in the conductor:

$$q_{\text{inner}} = -2.5\,\mu\text{C}$$

and

$$\sigma_{\text{inner}} = \boxed{-0.553\,\mu\text{C/m}^2}$$

as before.

Express the relationship between the charges on the inner and outer surfaces of the spherical shell:

$$q_{\text{outer}} + q_{\text{inner}} = 3.5\,\mu\text{C}$$

and

$$q_{\text{outer}} = 3.5\,\mu\text{C} - q_{\text{inner}} = 6.0\,\mu\text{C}$$

σ_{outer} is now given by:

$$\sigma_{\text{outer}} = \frac{6\,\mu\text{C}}{4\pi(0.9\,\text{m})^2} = \boxed{0.589\,\mu\text{C/m}^2}$$

For $r < a = 0.6$ m, $Q_{inside} = q$ and $E_n(r < 0.6\text{ m})$ is as it was in (*a*):

$$E_n(r < a) = \boxed{\left(2.25\times10^4\ \text{N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

Because the spherical shell is a conductor, a charge $-q$ will be induced on its inner surface. Hence, for $0.6\text{ m} < r < 0.9$ m:

$$Q_{inside} = 0$$

and

$$E_n(0.6\,\text{m} < r < 0.9\,\text{m}) = \boxed{0}$$

For $r > 0.9$ m, the net charge inside the Gaussian surface is 6 μC and:

$$E_n(r > 0.9\,\text{m}) = \frac{kq}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(6\ \mu\text{C})\frac{1}{r^2} = \boxed{\left(5.39\times10^4\ \text{N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

64 ••

Picture the Problem From Gauss's law we know that the electric field at the surface of the charged sphere is given by $E = kQ/R^2$ where Q is the charge on the sphere and R is its radius. The minimum radius for dielectric breakdown corresponds to the maximum electric field at the surface of the sphere.

Use Gauss's law to express the electric field at the surface of the charged sphere:

$$E = \frac{kQ}{R^2}$$

Express the relationship between E and R for dielectric breakdown:

$$E_{max} = \frac{kQ}{R_{min}^2}$$

Solve for R_{min}:

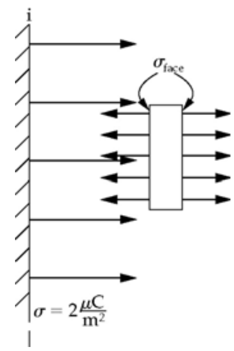
$$R_{min} = \sqrt{\frac{kQ}{E_{max}}}$$

Substitute numerical values and evaluate R_{min}:

$$R_{min} = \sqrt{\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(18\ \mu\text{C})}{3\times10^6\ \text{N/C}}} = \boxed{23.2\,\text{cm}}$$

65 ••

Picture the Problem We can use its definition to find the surface charge density just outside the face of the slab. The electric field near the surface of the slab is given by $E = \sigma_{\text{face}}/\in_0$. We can find the electric field on each side of the slab by adding the fields due to the slab and the plane of charge.



(*a*) Express the charge density per face in terms of the net charge on the slab:

$$\sigma_{\text{face}} = \frac{q}{2L^2}$$

Substitute numerical values to obtain:

$$\sigma_{\text{face}} = \frac{80\,\mu\text{C}}{2(5\,\text{m})^2} = \boxed{1.60\,\mu\text{C/m}^2}$$

Express the electric field just outside one face of the slab in terms of its surface charge density:

$$E_{\text{slab}} = \frac{\sigma_{\text{face}}}{\in_0}$$

Substitute numerical values and evaluate E_{face}:

$$E_{\text{slab}} = \frac{1.60\,\mu\text{C/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{1.81\times10^5\,\text{N/C}}$$

(*b*) Express the total field on the side of the slab closest to the infinite charged plane:

$$\vec{\boldsymbol{E}}_{\text{near}} = \vec{\boldsymbol{E}}_{\text{plane}} + \vec{\boldsymbol{E}}_{\text{slab}} = E_{\text{plane}}\hat{\boldsymbol{r}} - E_{\text{slab}}\hat{\boldsymbol{r}} = \frac{\sigma_{\text{plane}}}{2\in_0}\hat{\boldsymbol{r}} - \frac{\sigma_{\text{face}}}{\in_0}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing away from the slab.

Substitute numerical values and evaluate $\vec{\boldsymbol{E}}_{\text{near}}$:

$$\vec{\boldsymbol{E}}_{\text{near}} = \frac{2\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\boldsymbol{r}} - (1.81\times10^5\,\text{N/C})\,\hat{\boldsymbol{r}} = \boxed{(-0.680\times10^5\,\text{N/C})\,\hat{\boldsymbol{r}}}$$

Express the total field on the side of the slab away from the infinite charged plane:

$$\vec{E}_{\text{far}} = \frac{\sigma_{\text{plane}}}{2\in_0}\hat{r} + \frac{\sigma_{\text{face}}}{\in_0}\hat{r}$$

Substitute numerical values and evaluate $\vec{E}_{\text{far}}$:

$$\vec{E}_{\text{far}} = \frac{2\ \mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{r} + \left(1.81\times10^5\ \text{N/C}\right)\hat{r} = \boxed{\left(2.94\times10^5\ \text{N/C}\right)\hat{r}}$$

The charge density on the side of the slab near the plane is:

$$\sigma_{\text{near}} = \in_0 E_{\text{near}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.680\times10^5\ \text{N/C}\right) = \boxed{0.602\ \mu\text{C/m}^2}$$

The charge density on the far side of the slab is:

$$\sigma_{\text{near}} = \in_0 E_{\text{near}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.94\times10^5\ \text{N/C}\right) = \boxed{2.60\ \mu\text{C/m}^2}$$

General Problems

66 ••

Determine the Concept We can determine the direction of the electric field between spheres I and II by imagining a test charge placed between the spheres and determining the direction of the force acting on it. We can determine the amount and sign of the charge on each sphere by realizing that the charge on a given surface induces a charge of the same magnitude but opposite sign on the next surface of larger radius.

(*a*) The charge placed on sphere III has no bearing on the electric field between spheres I and II. The field in this region will be in the direction of the force exerted on a test charge placed between the spheres. Because the charge at the center is negative,

$\boxed{\text{the field will point toward the center.}}$

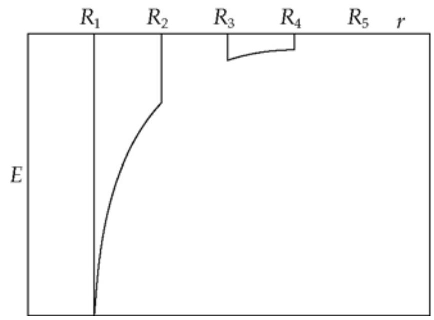
(*b*) The charge on sphere I ($-Q_0$) will induce a charge of the same magnitude but opposite sign on sphere II: $\boxed{+Q_0}$

(*c*) The induction of charge $+Q_0$ on the inner surface of sphere II will leave its outer surface with a charge of the same magnitude but opposite sign: $\boxed{-Q_0}$

(*d*) The presence of charge $-Q_0$ on the outer surface of sphere II will induce a charge of the same magnitude but opposite sign on the inner surface of sphere III: $\boxed{+Q_0}$

(*e*) The presence of charge $+Q_0$ on the inner surface of sphere III will leave the outer surface of sphere III neutral: $\boxed{0}$

(*f*) A graph of E as a function of r is shown to the right:



67 ••

Picture the Problem Because the difference between the field just to the right of the origin $E_{x,\text{right}}$ and the field just to the left of the origin $E_{x,\text{left}}$ is the field due to the nonuniform surface charge, we can express $E_{x,\text{left}}$ and the difference between $E_{x,\text{right}}$ and σ/ϵ_0.

Express the electric field just to the left of the origin in terms of $E_{x,\text{right}}$ and σ/ϵ_0:

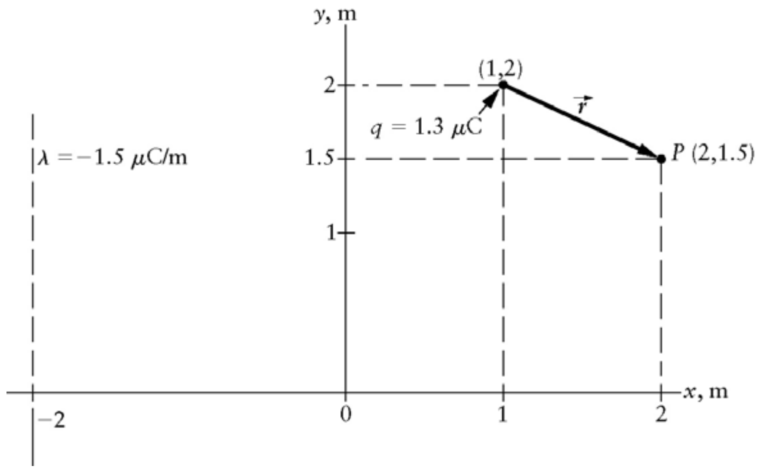
$$E_{x,\text{left}} = E_{x,\text{right}} - \frac{\sigma}{\epsilon_0}$$

Substitute numerical values and evaluate $E_{x,\text{left}}$:

$$E_{x,\text{left}} = 4.65\times10^5\ \text{N/C} - \frac{3.10\ \mu\text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{1.15\times10^5\ \text{N/C}}$$

68 ••

Picture the Problem Let *P* denote the point of interest at (2 m, 1.5 m). The electric field at *P* is the sum of the electric fields due to the infinite line charge and the point charge.



Express the resultant electric field at *P*: $\vec{E} = \vec{E}_\lambda + \vec{E}_q$

Find the field at *P* due the infinite line charge:

$$\vec{E}_\lambda = \frac{2k\lambda}{r}\hat{r} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.5\ \mu\text{C/m})}{4\,\text{m}}\hat{i} = (-6.74\,\text{kN/C})\hat{i}$$

Express the field at *P* due the point charge: $\vec{E}_q = \frac{kq}{r^2}\hat{r}$

Referring to the diagram above, determine *r* and $\hat{r}$: $r = 1.12\,\text{m}$ and $\hat{r} = 0.893\hat{i} - 0.446\hat{j}$

Substitute and evaluate $\vec{E}_q(2\,\text{m}, 1.5\,\text{m})$:

$$\begin{aligned}\vec{E}_q(2\,\text{m},1.5\,\text{m}) &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.3\,\mu\text{C})}{(1.12\,\text{m})^2}(0.893\hat{i} - 0.446\hat{j})\\ &= (9.32\,\text{kN/C})(0.893\hat{i} - 0.446\hat{j})\\ &= (8.32\,\text{kN/C})\hat{i} - (4.16\,\text{kN/C})\hat{j}\end{aligned}$$

Substitute to obtain:

$$\vec{E}(2\,\text{m},1.5\,\text{m}) = (-6.74\,\text{kN/C})\hat{i} + (8.35\,\text{kN/C})\hat{i} - (4.17\,\text{kN/C})\hat{j}$$
$$= \boxed{(1.61\,\text{kN/C})\hat{i} - (4.17\,\text{kN/C})\hat{j}}$$

***69 ••**

Picture the Problem If the patch is small enough, the field at the center of the patch comes from two contributions. We can view the field in the hole as the sum of the field from a uniform spherical shell of charge Q plus the field due to a small patch with surface charge density equal but opposite to that of the patch cut out.

(*a*) Express the magnitude of the electric field at the center of the hole:

$$E = E_{\text{spherical shell}} + E_{\text{hole}}$$

Apply Gauss's law to a spherical gaussian surface just outside the given sphere:

$$E_{\text{spherical shell}}\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\in_0} = \frac{Q}{\in_0}$$

Solve for $E_{\text{spherical shell}}$ to obtain:

$$E_{\text{spherical shell}} = \frac{Q}{4\pi \in_0 r^2}$$

The electric field due to the small hole (small enough so that we can treat it as a plane surface) is:

$$E_{\text{hole}} = \frac{-\sigma}{2\in_0}$$

Substitute and simplify to obtain:

$$E = \frac{Q}{4\pi \in_0 r^2} + \frac{-\sigma}{2\in_0}$$
$$= \frac{Q}{4\pi \in_0 r^2} - \frac{Q}{2\in_0\left(4\pi r^2\right)}$$
$$= \boxed{\frac{Q}{8\pi \in_0 r^2}}$$

(*b*) Express the force on the patch:

$$F = qE$$

where q is the charge on the patch.

Assuming that the patch has radius a, express the proportion between its charge and that of the spherical shell:

$$\frac{q}{\pi a^2} = \frac{Q}{4\pi r^2} \text{ or } q = \frac{a^2}{4r^2}Q$$

Substitute for q and E in the expression for F to obtain:

$$F = \left(\frac{a^2}{4r^2}Q\right)\left(\frac{Q}{8\pi \in_0 r^2}\right) = \boxed{\frac{Q^2 a^2}{32\pi \in_0 r^4}}$$

(*c*) The pressure is the force divided by the area of the patch:

$$P = \frac{\dfrac{Q^2 a^2}{32\pi \in_0 r^4}}{\pi a^2} = \boxed{\frac{Q^2}{32\pi^2 \in_0 r^4}}$$

70 ••

Picture the Problem The work done by the electrostatic force in expanding the soap bubble is $W=\int PdV$.

From Problem 69:

$$P=\frac{Q^2}{32\pi^2\in_0 r^4}$$

Express W in terms of dr:

$$W=\int PdV=\int P4\pi r^2dr$$

Substitute for P and simplify:

$$W=\frac{Q^2}{8\pi\in_0}\int_{0.1\,\text{m}}^{0.2\,\text{m}}\frac{dr}{r^2}$$

Evaluating the integral yields:

$$W=\frac{Q^2}{8\pi\in_0}\left[-\frac{1}{r}\right]_{0.1\,\text{m}}^{0.2\,\text{m}}=\frac{(3\,\text{nC})^2}{8\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}\left(\frac{-1}{0.2\,\text{m}}+\frac{1}{0.1\,\text{m}}\right)$$
$$=\boxed{2.02\times10^{-7}\,\text{J}}$$

71 ••

Picture the Problem We can use $E = kq/R^2$, where R is the radius of the droplet, to find the electric field at its surface. We can find R by equating the volume of the bubble at the moment it bursts to the volume of the resulting spherical droplet.

Express the field at the surface of the spherical water droplet:

$$E=\frac{kq}{R^2}\qquad(1)$$

where R is the radius of the droplet and q is its charge.

Express the volume of the bubble just before it pops:

$$V\approx 4\pi r^2t$$

where t is the thickness of the soap bubble.

Express the volume of the sphere into which the droplet collapses:

$$V=\frac{4}{3}\pi R^3$$

Because the volume of the droplet and the volume of the bubble are equal:

$$4\pi r^2t=\frac{4}{3}\pi R^3$$

Solve for R:

$$R=\sqrt[3]{3r^2t}$$

Assume a thickness t of 1 μm and evaluate R:

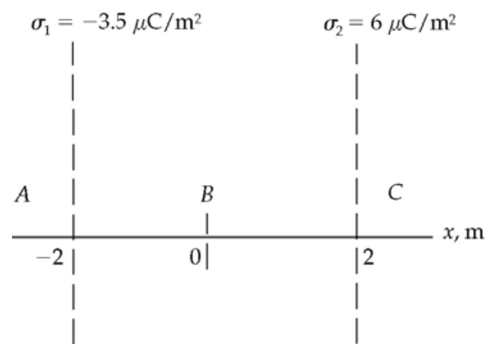
$$R=\sqrt[3]{3(0.2\,\text{m})^2(1\,\mu\text{m})}=4.93\times10^{-3}\,\text{m}$$

Substitute numerical values in equation (1) and evaluate E:

$$E=\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\text{nC})}{\left(4.93\times10^{-3}\ \text{m}\right)^2}=\boxed{1.11\times10^{6}\ \text{N/C}}$$

72 ••

Picture the Problem Let the numeral 1 refer to the infinite plane at $x = -2$ m and the numeral 2 to the plane at $x = 2$ m and let the letter A refer to the region to the left of plane 1, B to the region between the planes, and C to the region to the right of plane 2. We can use the expression for the electric field of in infinite plane of charge to express the electric field due to each plane of charge in each of the three regions. Their sum will be the resultant electric field in each region.



Express the resultant electric field as the sum of the fields due to planes 1 and 2:

$$\vec{E}=\vec{E}_1+\vec{E}_2 \qquad (1)$$

(a) Express and evaluate the field due to plane 1 in region A:

$$\vec{E}_1(A)=\frac{\sigma_1}{2\in_0}\left(-\hat{i}\right)=\frac{-3.5\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\left(-\hat{i}\right)=(198\,\text{kN/C})\hat{i}$$

Express and evaluate the field due to plane 2 in region A:

$$\vec{E}_2(A)=\frac{\sigma_2}{2\in_0}\left(-\hat{i}\right)=\frac{6\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\left(-\hat{i}\right)=(-339\,\text{kN/C})\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{E}(A)=(198\ \text{kN/C})\hat{i}+(-339\ \text{kN/C})\hat{i}=\boxed{(-141\,\text{kN/C})\hat{i}}$$

(*b*) Express and evaluate the field due to plane 1 in region *B*:

$$\vec{E}_1(B)=\frac{\sigma_1}{2\in_0}\hat{i}=\frac{-3.5\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)}\hat{i}=(-198\text{ kN/C})\hat{i}$$

Express and evaluate the field due to plane 2 in region *B*:

$$\vec{E}_2(B)=\frac{\sigma_2}{2\in_0}\left(-\hat{i}\right)=\frac{6\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)}\left(-\hat{i}\right)=(-339\text{ kN/C})\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{E}(B)=(-198\text{ kN/C})\hat{i}+(-339\text{ kN/C})\hat{i}=\boxed{(-537\text{ kN/C})\hat{i}}$$

(*c*) Express and evaluate the field due to plane 1 in region *C*:

$$\vec{E}_1(C)=\frac{\sigma_1}{2\in_0}\hat{i}=\frac{-3.5\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)}\hat{i}=(-198\text{ kN/C})\hat{i}$$

Express and evaluate the field due to plane 2 in region *C*:

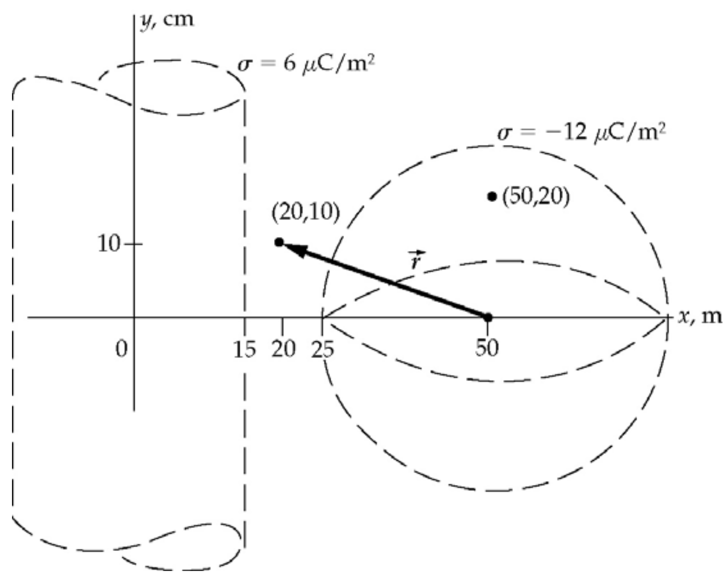
$$\vec{E}_2(C)=\frac{\sigma_2}{2\in_0}\hat{i}=\frac{6\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)}\hat{i}=(339\text{ kN/C})\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{E}(C)=(-198\text{ kN/C})\hat{i}+(339\text{ kN/C})\hat{i}=\boxed{(141\text{ kN/C})\hat{i}}$$

***73 ••**

Picture the Problem We can find the electric fields at the three points of interest by adding the electric fields due to the infinitely long cylindrical shell and the spherical shell. In Problem 42 it was established that, for an infinitely long cylindrical shell of radius *R*, $E_r(r<R)=0,$ and $E_r(r>R)=\sigma R/\in_0 r$. We know that, for a spherical shell of radius *R*, $E_r(r<R)=0,$ and $E_r(r>R)=\sigma R^2/\in_0 r^2$.



Express the resultant electric field as the sum of the fields due to the cylinder and sphere:

$$\vec{E} = \vec{E}_{\text{cyl}} + \vec{E}_{\text{sph}} \qquad (1)$$

(*a*) Express and evaluate the electric field due to the cylindrical shell at the origin:

$$\vec{E}_{\text{cyl}}(0,0) = 0$$

because the origin is inside the cylindrical shell.

Express and evaluate the electric field due to the spherical shell at the origin:

$$\vec{E}_{\text{sph}}(0,0) = \frac{\sigma R^2}{\epsilon_0\, r^2}\left(-\hat{\boldsymbol{i}}\right) = \frac{\left(-12\,\mu\text{C/m}^2\right)\left(0.25\,\text{m}\right)^2}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.5\,\text{m}\right)^2}\left(-\hat{\boldsymbol{i}}\right) = \left(339\,\text{kN/C}\right)\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(0,0) = 0 + \left(339\,\text{kN/C}\right)\hat{\boldsymbol{i}}$$

$$= \boxed{\left(339\,\text{kN/C}\right)\hat{\boldsymbol{i}}}$$

or

$$E(0,0) = \boxed{339\,\text{kN/C}}$$

and

$$\theta = \boxed{0°}$$

(*b*) Express and evaluate the electric field due to the cylindrical shell at (0.2 m, 0.1 m):

$$\vec{E}_{cyl}(0.2\,\text{m},0.1\,\text{m}) = \frac{\sigma R}{\epsilon_0\, r}\hat{i} = \frac{(6\,\mu\text{C/m}^2)(0.15\,\text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.2\,\text{m})}\hat{i} = (508\,\text{kN/C})\hat{i}$$

Express the electric field due to the charge on the spherical shell as a function of the distance from its center:

$$\vec{E}_{sph}(r) = \frac{\sigma R^2}{\epsilon_0\, r^2}\hat{r}$$

where $\hat{r}$ is a unit vector pointing from (50 cm, 0) to (20 cm, 10 cm).

Referring to the diagram shown above, find r and $\hat{r}$:

$$r = 0.316\,\text{m}$$

and

$$\vec{r} = -0.949\hat{i} + 0.316\hat{j}$$

Substitute to obtain:

$$\begin{aligned}\vec{E}_{sph}(0.2\,\text{m},0.1\,\text{m}) &= \frac{(-12\,\mu\text{C/m}^2)(0.25\,\text{m})^2}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.316\,\text{m})^2}\left(-0.949\hat{i} + 0.316\hat{j}\right)\\ &= (-849\,\text{kN/C})\left(-0.949\hat{i} + 0.316\hat{j}\right)\\ &= (806\,\text{kN/C})\hat{i} + (-268\,\text{kN/C})\hat{j}\end{aligned}$$

Substitute in equation (1) to obtain:

$$\begin{aligned}\vec{E}(0.2\,\text{m},0.1\,\text{m}) &= (508\,\text{kN/C})\hat{i} + (806\,\text{kN/C})\hat{i} + (-268\,\text{kN/C})\hat{j}\\ &= \boxed{(1310\,\text{kN/C})\hat{i} + (-268\,\text{kN/C})\hat{j}}\end{aligned}$$

or

$$E(0.2\,\text{m},0.1\,\text{m}) = \sqrt{(1310\,\text{kN/C})^2 + (-268\,\text{kN/C})^2} = \boxed{1340\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{-268\ \text{kN/C}}{1310\ \text{kN/C}}\right) = \boxed{348°}$$

(c) Express and evaluate the electric field due to the cylindrical shell at (0.5 m, 0.2 m):

$$\vec{E}_{cyl}(0.5\,\text{m},0.2\,\text{m}) = \frac{(6\,\mu\text{C/m}^2)(0.15\,\text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.5\,\text{m})}\hat{i} = (203\,\text{kN/C})\hat{i}$$

Express and evaluate the electric field due to the spherical shell at

$$\vec{E}_{sph}(0.5\,\text{m},0.2\,\text{m}) = 0$$

(0.5 m, 0.5 m): because (0.5 m, 0.2 m) is inside the spherical shell.

Substitute in equation (1) to obtain:

$$\vec{E}(0.5\,\text{m},0.2\,\text{m}) = (203\,\text{kN/C})\hat{i} + 0 = \boxed{(203\,\text{kN/C})\hat{i}}$$

or

$$E(0.5\,\text{m},0.2\,\text{m}) = \boxed{203\,\text{kN/C}}$$

and

$$\theta = \boxed{0°}$$

74 ••

Picture the Problem Let the numeral 1 refer to the plane with charge density σ_1 and the numeral 2 to the plane with charge density σ_2. We can find the electric field at the two points of interest by adding the electric fields due to the charge distributions of the two infinite planes.

Express the electric field at any point in space due to the charge distributions on the two planes:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \qquad (1)$$

(*a*) Express the electric field at (6 m, 2 m) due to plane 1:

$$\vec{E}_1(6\,\text{m},2\,\text{m}) = \frac{\sigma_1}{2\in_0}\hat{j} = \frac{65\,\text{nC/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{j} = (3.67\,\text{kN/C})\hat{j}$$

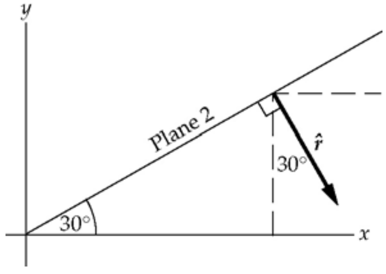
Express the electric field at (6 m, 2 m) due to plane 2:

$$\vec{E}_2(6\,\text{m},2\,\text{m}) = \frac{\sigma_2}{2\in_0}\hat{r} = \frac{45\,\text{nC/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{r} = (2.54\,\text{kN/C})\hat{r}$$

where $\hat{r}$ is a unit vector pointing from plane 2 toward the point whose coordinates are (6 m, 2 m).

Refer to the diagram below to obtain:

$$\hat{r} = \sin 30°\,\hat{i} - \cos 30°\,\hat{j}$$



Substitute to obtain:

$$\vec{E}_2(6\,\text{m},2\,\text{m}) = (2.54\,\text{kN/C})\left(\sin 30°\,\hat{i} - \cos 30°\,\hat{j}\right) = (1.27\,\text{kN/C})\hat{i} + (-2.20\,\text{kN/C})\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{E}(6\,\text{m},2\,\text{m}) = (3.67\,\text{kN/C})\hat{j} + (1.27\,\text{kN/C})\hat{i} + (-2.20\,\text{kN/C})\hat{j}$$
$$= \boxed{(1.27\,\text{kN/C})\hat{i} + (1.47\,\text{kN/C})\hat{j}}$$

(*b*) Note that $\vec{E}_1(6\,\text{m},5\,\text{m}) = \vec{E}_1(6\,\text{m},2\,\text{m})$ so that:

$$\vec{E}_1(6\,\text{m},5\,\text{m}) = \frac{\sigma}{2\in_0}\hat{j} = \frac{65\,\text{nC/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{j} = (3.67\,\text{kN/C})\hat{j}$$

Note also that $\vec{E}_2(6\,\text{m},5\,\text{m}) = -\vec{E}_2(6\,\text{m},2\,\text{m})$ so that:

$$\vec{E}_2(6\,\text{m},5\,\text{m}) = (-1.27\,\text{kN/C})\hat{i} + (2.20\,\text{kN/C})\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{E}(6\,\text{m},5\,\text{m}) = (3.67\,\text{kN/C})\hat{j} + (-1.27\,\text{kN/C})\hat{i} + (2.20\,\text{kN/C})\hat{j}$$
$$= \boxed{(-1.27\,\text{kN/C})\hat{i} + (5.87\,\text{kN/C})\hat{j}}$$

75 ••

Picture the Problem Because the atom is uncharged, we know that the integral of the electron's charge distribution over all of space must equal its charge *e*. Evaluation of this integral will lead to an expression for ρ_0. In (*b*) we can express the resultant field at any point as the sum of the fields due to the proton and the electron cloud.

(*a*) Because the atom is uncharged:

$$e = \int_0^\infty \rho(r)dV = \int_0^\infty \rho(r)4\pi r^2 dr$$

Substitute for $\rho(r)$:

$$e = \int_0^\infty \rho_0 e^{-2r/a}\,4\pi r^2 dr = 4\pi\rho_0\int_0^\infty r^2 e^{-2r/a}dr$$

Use integral tables or integration by parts to obtain:

$$\int_0^\infty r^2 e^{-2r/a}dr = \frac{a^3}{4}$$

Substitute to obtain:

$$e = 4\pi\rho_0\left(\frac{a^3}{4}\right) = \pi a^3\rho_0$$

Solve for ρ_0:

$$\rho_0 = \boxed{\frac{e}{\pi a^3}}$$

(*b*) The field will be the sum of the field due to the proton and that of the electron charge cloud:

$$E = E_{\text{p}} + E_{\text{cloud}} = \frac{kq}{r^2} + E_{\text{cloud}}$$

Express the field due to the electron cloud:

$$E_{\text{cloud}}(r) = \frac{kQ(r)}{r^2}$$

where $Q(r)$ is the net negative charge enclosed a distance r from the proton.

Substitute to obtain:

$$E(r) = \frac{ke}{r^2} + \frac{kQ(r)}{r^2}$$

As in (*a*), $Q(r)$ is given by:

$$Q(r) = \int_0^r 4\pi r' \ \rho(r')dr'$$

Integrate to find $Q(r)$ and substitute in the expression for E to obtain:

$$E(r) = \boxed{\frac{ke}{r^2}e^{-2r/a}\left(1 + \frac{2r}{a} + \frac{2r^2}{a^2}\right)}$$

***76 ••**

Picture the Problem We will assume that the radius at which they balance is large enough that only the third term in the expression matters. Apply a condition for equilibrium will yield an equation that we can solve for the distance *r*.

Apply $\sum F = 0$ to the proton:

$$\frac{2ke^2}{a^2}e^{-2r/a} - mg = 0$$

To solve for *r*, isolate the exponential factor and take the natural logarithm of both sides of the equation:

$$r = \frac{a}{2}\ln\left(\frac{2ke^2}{mga^2}\right)$$

Substitute numerical values and evaluate *r*:

$$r = \frac{0.0529\,\text{nm}}{2}\ln\left[\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.67\times10^{-27}\ \text{kg})(9.81\,\text{m/s}^2)(0.0529\,\text{nm})^2}\right] = \boxed{1.16\,\text{nm}}$$

Thus, even though the unscreened electrostatic force is 40 orders of magnitude larger than the gravitational force, screening reduces it to smaller than the gravitational force within a few nanometers.

Remarks: Note that the argument of the logarithm contains the ratio between the gravitational potential energy of a mass held a distance a_0 above the surface of the earth and the electrostatic potential energy for two unscreened charges a distance a_0 apart.

77 ••

Picture the Problem In parts (*a*) and (*b*) we can express the charges on each of the elements as the product of the linear charge density of the ring and the length of the segments. Because the lengths of the segments are the product of the angle subtended at *P* and their distances from *P*, we can express the charges in terms of their distances from *P*. By expressing the ratio of the fields due to the charges on s_1 and s_2 we can determine their dependence on r_1 and r_2 and, hence, the resultant field at *P*. We can proceed similarly in part (*c*) with *E* varying as $1/r$ rather than $1/r^2$. In part (*d*), with s_1 and s_2 representing areas, we'll use the definition of the solid angle subtended by these areas to relate their charges to their distances from point *P*.

(*a*) Express the charge q_1 on the element of length s_1:

$$q_1 = \lambda s_1 = \lambda\theta r_1$$

where θ is the angle subtended by the arcs of length s_1 and s_2.

Express the charge q_2 on the element of length s_2:

$$q_2 = \lambda s_2 = \lambda\theta r_2$$

Divide the first of these equations by the second to obtain:

$$\frac{q_1}{q_2} = \frac{\lambda\theta r_1}{\lambda\theta r_2} = \boxed{\frac{r_1}{r_2}}$$

Express the electric field at *P* due to the charge associated with the element of length s_1:

$$E_1 = \frac{kq_1}{r_1^2} = \frac{k\lambda s_1}{r_1^2} = \frac{k\lambda\theta r_1}{r_1^2} = \frac{k\lambda\theta}{r_1}$$

Express the electric field at *P* due to the charge associated with the element of length s_2:

$$E_2 = \frac{k\lambda\theta}{r_2}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_1}{E_2} = \frac{\dfrac{k\lambda\theta}{r_1}}{\dfrac{k\lambda\theta}{r_2}} = \frac{r_2}{r_1}$$

and, because $r_2 > r_1$,

$$\boxed{E_1 > E_2}$$

(*b*) The two fields point away from their segments of arc.

Because $E_1 > E_2$, the resultant field points toward s_2.

(*c*) If *E* varies as 1/*r*:

$$E_1 = \frac{kq_1}{r_1} = \frac{k\lambda s_1}{r_1} = \frac{k\lambda\theta r_1}{r_1} = k\lambda\theta$$

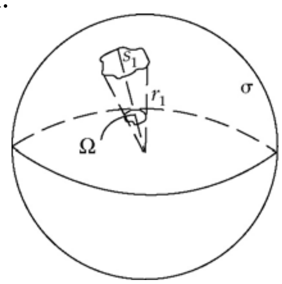
and

$$E_2 = \frac{kq_2}{r_2} = \frac{k\lambda s_2}{r_2} = \frac{k\lambda\theta r_2}{r_2} = k\lambda\theta$$

Therefore:

$$\boxed{E_1 = E_2}$$

(*d*) Use the definition of the solid angle Ω subtended by the area s_1 to obtain:



$$\frac{\Omega}{4\pi} = \frac{s_1}{4\pi r_1^2}$$

or

$$s_1 = \Omega r_1^2$$

Express the charge q_1 of the area s_1:

$$q_1 = \sigma s_1 = \sigma\Omega r_1^2$$

Similarly, for an element of area s_2:

$$s_2 = \Omega r_2^2$$

and

$$q_2 = \sigma\Omega r_2^2$$

Express the ratio of q_1 to q_2 to obtain:

$$\frac{q_1}{q_2} = \frac{\sigma\Omega r_1^2}{\sigma\Omega r_2^2} = \boxed{\frac{r_1^2}{r_2^2}}$$

Proceed as in (*a*) to obtain:

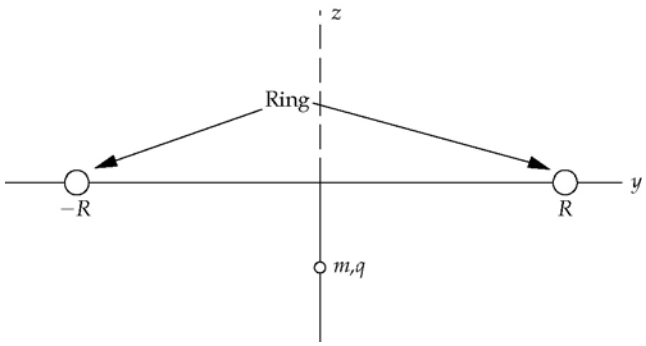
$$\frac{E_1}{E_2} = \frac{\frac{kq_1}{r_1^2}}{\frac{kq_2}{r_2^2}} = \frac{r_2^2 q_1}{r_1^2 q_2} = \frac{r_2^2 \sigma\Omega r_1^2}{r_1^2 \sigma\Omega r_2^2} = 1$$

Because the two fields are of equal magnitude and oppositely directed: $\vec{E} = 0$

If $E \propto 1/r$, then s_2 would produce the stronger field at P and $\vec{E}$ would point toward s_1.

78 ••

Picture the Problem We can apply the condition for translational equilibrium to the particle and use the expression for the electric field on the axis of a ring charge to obtain an expression for $|q|/m$. Doing so will lead us to the conclusion that $|q|/m$ will be a minimum when E_z is a maximum and so we'll use the result from Problem 26 that $z = -R/\sqrt{2}$ maximizes E_z.



(*a*) Apply $\sum F_z = 0$ to the particle: $|q|E_z - mg = 0$

Solve for $|q|/m$:

$$\frac{|q|}{m} = \frac{g}{E_z} \qquad (1)$$

Note that this result tells us that the minimum value of $|q|/m$ will be where the field due to the ring is greatest.

Express the electric field along the z axis due to the ring of charge:

$$E_z = \frac{kQz}{\left(z^2 + R^2\right)^{3/2}}$$

Differentiate this expression with respect to z to obtain:

$$\frac{dE_x}{dz} = kQ\frac{d}{dz}\left[\frac{x}{\left(z^2+R^2\right)^{3/2}}\right] = kQ\frac{\left(z^2+R^2\right)^{3/2} - z\frac{d}{dx}\left(z^2+R^2\right)^{3/2}}{\left(z^2+R^2\right)^3}$$

$$= kQ\frac{\left(z^2+R^2\right)^{3/2} - z\left(\tfrac{3}{2}\right)\left(z^2+R^2\right)^{1/2}(2z)}{\left(z^2+R^2\right)^3} = kQ\frac{\left(z^2+R^2\right)^{3/2} - 3z^2\left(z^2+R^2\right)^{1/2}}{\left(z^2+R^2\right)^3}$$

Set this expression equal to zero for extrema and simplify:

$$\frac{\left(z^2+R^2\right)^{3/2}-3z^2\left(z^2+R^2\right)^{1/2}}{\left(z^2+R^2\right)^3}=0,$$

$$\left(z^2+R^2\right)^{3/2}-3z^2\left(z^2+R^2\right)^{1/2}=0,$$

and

$$z^2+R^2-3z^2=0$$

Solve for *x* to obtain:

$$z=\pm\frac{R}{\sqrt{2}}$$

as candidates for maxima or minima.

You can either plot a graph of E_z or evaluate its second derivative at these points to show that it is a maximum at:

$$z=-\frac{R}{\sqrt{2}}$$

Substitute to obtain an expression $E_{z,\max}$:

$$E_{z,\max}=\left|\frac{kQ\left(-\frac{R}{\sqrt{2}}\right)}{\left(\left(-\frac{R}{\sqrt{2}}\right)^2+R^2\right)^{3/2}}\right|=\frac{2kQ}{\sqrt{27}R^2}$$

Substitute in equation (1) to obtain:

$$\frac{|q|}{m}=\boxed{\frac{\sqrt{27}gR^2}{2kQ}}$$

(*b*) If $|q|/m$ is twice as great as in (*a*), then the electric field should be half its value in (*a*), i.e.:

$$\frac{kQ}{\sqrt{27}R^2}=\frac{kQz}{\left(z^2+R^2\right)^{3/2}}$$

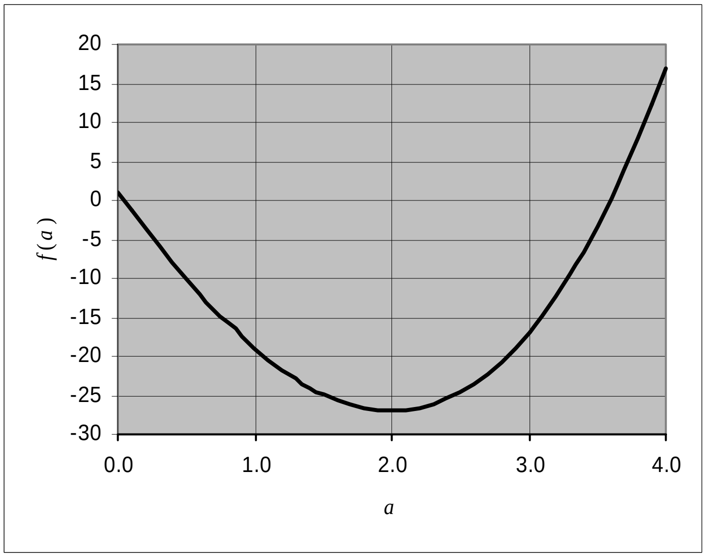
or

$$\frac{1}{27R^4}=\frac{z^2}{R^6\left(1+\frac{z^2}{R^2}\right)^3}$$

Let $a=z^2/R^2$ and simplify to obtain:

$$a^3+3a^2-24a+1=0$$

The graph of $f(a)=a^3+3a^2-24a+1$ shown below was plotted using a spreadsheet program.

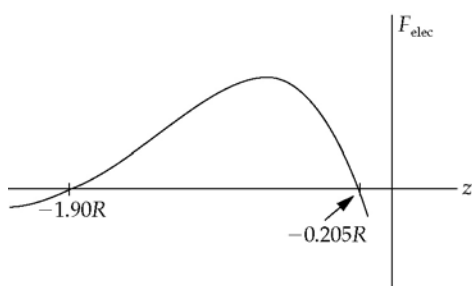


Use your calculator or trial-and-error methods to obtain: $a = 0.04188$ and $a = 3.596$

The corresponding z values are: $z = -0.205R$ and $z = -1.90R$

The condition for a stable equilibrium position is that the particle, when displaced from its equilibrium position, experiences a restoring force, i.e. a force that acts toward the equilibrium position. When the particle in this problem is just above its equilibrium position the net force on it must be downward and when it is just below the equilibrium position the net force on it must be upward. Note that the electric force is zero at the origin, so the net force there is downward and remains downward to the first equilibrium position as the weight force exceeds the electric force in this interval. The net force is upward between the first and second equilibrium positions as the electric force exceeds the weight force. The net force is downward below the second equilibrium position as the weight force exceeds the electric force. Thus, the first (higher) equilibrium position is stable and the second (lower) equilibrium position is unstable.

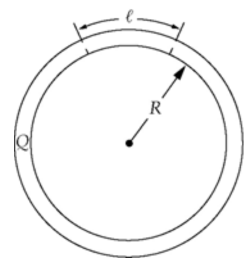
You might also find it instructive to use your graphing calculator to plot a graph of the electric force (the gravitational force is constant and only shifts the graph of the total force downward). Doing so will produce a graph similar to the one shown in the sketch to the right.



Note that the slope of the graph is negative on both sides of $-0.205R$ whereas it is positive on both sides of $-1.90R$; further evidence that $-0.205R$ is a position of stable equilibrium and $-1.90R$ a position of unstable equilibrium.

79 ••

Picture the Problem The loop with the small gap is equivalent to a closed loop and a charge of $-Q\ell/2\pi R$ at the gap. The field at the center of a closed loop of uniform line charge is zero. Thus the field is entirely due to the charge $-Q\ell/2\pi R$.



(*a*) Express the field at the center of the loop:

$$\vec{E}_{\text{center}} = \vec{E}_{\text{loop}} + \vec{E}_{\text{gap}} \qquad (1)$$

Relate the field at the center of the loop to the charge in the gap:

$$\vec{E}_{\text{gap}} = -\frac{kq}{R^2}\hat{\boldsymbol{r}}$$

Use the definition of linear charge density to relate the charge in the gap to the length of the gap:

$$\lambda = \frac{q}{\ell} = \frac{Q}{2\pi R}$$

or

$$q = \frac{Q\ell}{2\pi R}$$

Substitute to obtain:

$$\vec{E}_{\text{gap}} = -\frac{kQ\ell}{2\pi R^3}\hat{\boldsymbol{r}}$$

Substitute in equation (1) to obtain:

$$\vec{E}_{\text{center}} = 0 - \frac{kQ\ell}{2\pi R^3}\hat{\boldsymbol{r}} = -\frac{kQ\ell}{2\pi R^3}\hat{\boldsymbol{r}}$$

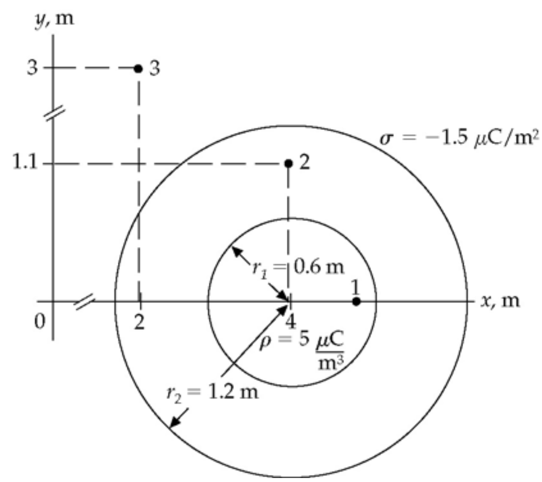
If Q is positive, the field at the origin points radially outward.

(*b*) From our result in (*a*) we see that the magnitude of $\vec{E}_{\text{center}}$ is:

$$E_{\text{center}} = \boxed{\frac{kQ\ell}{2\pi R^3}}$$

80 ••

Picture the Problem We can find the electric fields at the three points of interest, labeled 1, 2, and 3 in the diagram, by adding the electric fields due to the charge distributions on the nonconducting sphere and the spherical shell.



Express the electric field due to the nonconducting sphere and the spherical shell at any point in space:

$$\vec{E} = \vec{E}_{\text{sphere}} + \vec{E}_{\text{shell}} \qquad (1)$$

(*a*) Because (4.5 m, 0) is inside the spherical shell:

$$\vec{E}_{\text{shell}}(4.5\,\text{m},0) = 0$$

Apply Gauss's law to a spherical surface inside the nonconducting sphere to obtain:

$$\vec{E}_{\text{sphere}}(r) = \frac{4\pi}{3}k\rho r\hat{\boldsymbol{i}}$$

Evaluate $\vec{E}_{\text{sphere}}(0.5\,\text{m})$:

$$\vec{E}_{\text{sphere}}(0.5\,\text{m}) = \frac{4\pi}{3}\left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5\,\mu\text{C/m}^2\right)(0.5\,\text{m})\hat{\boldsymbol{i}} = (94.1\,\text{kN/C})\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(4.5\,\text{m},0) = (94.1\,\text{kN/C})\hat{\boldsymbol{i}} + 0 = \boxed{(94.1\,\text{kN/C})\hat{\boldsymbol{i}}}$$

Find the magnitude and direction of $\vec{E}(4.5\,\text{m},0)$:

$$E(4.5\,\text{m},0) = \boxed{94.1\,\text{kN/C}}$$

and

$$\theta = \boxed{0°}$$

(*b*) Because (4 m, 1.1m) is inside the spherical shell:

$$\vec{E}_{\text{shell}}(4\,\text{m},1.1\,\text{m}) = 0$$

Evaluate $\vec{E}_{\text{sphere}}(1.1\,\text{m})$:

$$\vec{E}_{\text{sphere}}(1.1\,\text{m}) = \frac{4\pi(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\,\mu\text{C/m}^2)(0.6\,\text{m})^3}{3(1.1\,\text{m})^2}\,\hat{\boldsymbol{j}} = (33.6\,\text{kN/C})\hat{\boldsymbol{j}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(4.5\,\text{m},0) = (33.6\,\text{kN/C})\hat{\boldsymbol{j}} + 0 = \boxed{(33.6\,\text{kN/C})\hat{\boldsymbol{j}}}$$

Find the magnitude and direction of $\vec{E}(4.5\,\text{m},1.1\,\text{m})$:

$$E(4.5\,\text{m},1.1\,\text{m}) = \boxed{33.6\,\text{kN/C}}$$

and

$$\theta = \boxed{90°}$$

(c) Because (2 m, 3 m) outside the spherical shell:

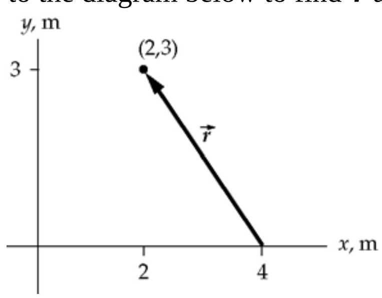
$$\vec{E}_{\text{shell}}(r) = \frac{kQ_{\text{shell}}}{r^2}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from (4 m, 0) to (2 m, 3 m).

Evaluate Q_{shell}:

$$Q_{\text{shell}} = \sigma A_{\text{shell}} = 4\pi(-1.5\,\mu\text{C/m}^2)(1.2\,\text{m})^2 = -27.1\,\mu\text{C}$$

Refer to the diagram below to find $\hat{\boldsymbol{r}}$ and r:



$r = 3.61\,\text{m}$

and

$\hat{\boldsymbol{r}} = -0.555\hat{\boldsymbol{i}} + 0.832\hat{\boldsymbol{j}}$

Substitute and evaluate $\vec{E}_{\text{shell}}(2\,\text{m},3\,\text{m})$:

$$\vec{E}_{\text{shell}}(3.61\,\text{m}) = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(-27.1\,\mu\text{C})}{(3.61\,\text{m})^2}\hat{\boldsymbol{r}}$$
$$= (-18.7\,\text{kN/C})(-0.555\hat{\boldsymbol{i}} + 0.832\hat{\boldsymbol{j}})$$
$$= (10.4\,\text{kN/C})\hat{\boldsymbol{i}} + (-15.6\,\text{kN/C})\hat{\boldsymbol{j}}$$

Express the electric field due to the charged nonconducting sphere at a distance r from its center that is greater than its radius:

$$\vec{E}_{\text{sphere}}(r) = \frac{kQ_{\text{sphere}}}{r^2}\hat{r}$$

Find the charge on the sphere:

$$Q_{\text{sphere}} = \rho V_{\text{sphere}} = \frac{4\pi}{3}\left(5\,\mu\text{C/m}^2\right)\left(0.6\,\text{m}\right)^3 = 4.52\,\mu\text{C}$$

Evaluate $\vec{E}_{\text{sphere}}(3.61\text{m})$:

$$\vec{E}_{\text{sphere}}(2\,\text{m},3\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.52\,\mu\text{C}\right)}{\left(3.61\,\text{m}\right)^2}\hat{r} = \left(3.12\,\text{kN/C}\right)\hat{r}$$
$$= \left(3.12\,\text{kN/C}\right)\left(-0.555\hat{i} + 0.832\hat{j}\right)$$
$$= \left(-1.73\,\text{kN/C}\right)\hat{i} + \left(2.59\,\text{kN/C}\right)\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{E}(2\,\text{m},3\,\text{m}) = \left(10.4\,\text{kN/C}\right)\hat{i} + \left(-15.6\,\text{kN/C}\right)\hat{j} + \left(-1.73\,\text{kN/C}\right)\hat{i} + \left(2.59\,\text{kN/C}\right)\hat{j}$$
$$= \boxed{\left(8.67\,\text{kN/C}\right)\hat{i} + \left(-13.0\,\text{kN/C}\right)\hat{j}}$$

Find the magnitude and direction of $\vec{E}(2\,\text{m},3\,\text{m})$:

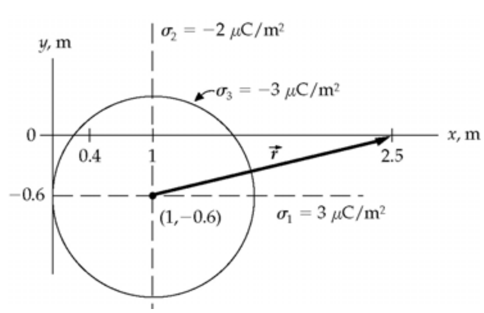
$$E(2\,\text{m},3\,\text{m}) = \sqrt{\left(8.67\,\text{kN/C}\right)^2 + \left(-13.0\,\text{kN/C}\right)^2} = \boxed{15.6\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{-13.0\,\text{kN/C}}{8.67\,\text{kN/C}}\right) = \boxed{304°}$$

81 ••

Picture the Problem Let the numeral 1 refer to the infinite plane whose charge density is σ_1 and the numeral 2 to the infinite plane whose charge density is σ_2. We can find the electric fields at the two points of interest by adding the electric fields due to the charge distributions on the infinite planes and the sphere.



Express the electric field due to the infinite planes and the sphere at any point in space:

$$\vec{E} = \vec{E}_{\text{sphere}} + \vec{E}_1 + \vec{E}_2 \qquad (1)$$

(*a*) Because (0.4 m, 0) is inside the sphere:

$$\vec{E}_{\text{sphere}}(0.4\,\text{m},0) = 0$$

Find the field at (0.4 m, 0) due to plane 1:

$$\vec{E}_1(0.4\,\text{m},0) = \frac{\sigma_1}{2\epsilon_0}\hat{\boldsymbol{j}} = \frac{3\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\boldsymbol{j}} = (169\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the field at (0.4 m, 0) due to plane 2:

$$\vec{E}_2(0.4\,\text{m},0) = \frac{\sigma_2}{2\epsilon_0}(-\hat{\boldsymbol{i}}) = \frac{-2\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}(-\hat{\boldsymbol{i}}) = (113\,\text{kN/C})\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(0.4\,\text{m},0) = 0 + (169\,\text{kN/C})\hat{\boldsymbol{j}} + (113\,\text{kN/C})\hat{\boldsymbol{i}} = (113\,\text{kN/C})\hat{\boldsymbol{i}} + (169\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the magnitude and direction of $\vec{E}(0.4\,\text{m},0)$:

$$E(0.4\,\text{m},0) = \sqrt{(113\,\text{kN/C})^2 + (169\,\text{kN/C})^2} = \boxed{203\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{169\,\text{kN/C}}{113\,\text{kN/C}}\right) = \boxed{56.2^\circ}$$

(*b*) Because (2.5 m, 0) is outside the sphere:

$$\vec{E}_{\text{sphere}}(0.4\,\text{m},0) = \frac{kQ_{\text{sphere}}}{r^2}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from (1 m, –0.6 m) to (2.5 m, 0).

Evaluate Q_{sphere}:

$$Q_{\text{sphere}} = \sigma A_{\text{sphere}} = 4\pi\sigma R^2 = 4\pi\left(-3\,\mu\text{C/m}^2\right)(1\,\text{m})^2 = -37.7\,\mu\text{C}$$

Referring to the diagram above, determine r and $\hat{\boldsymbol{r}}$:

$$r = 1.62\,\text{m}$$

and

$$\hat{\boldsymbol{r}} = 0.928\hat{\boldsymbol{i}} + 0.371\hat{\boldsymbol{j}}$$

Substitute and evaluate $\vec{\boldsymbol{E}}_{\text{sphere}}(2.5\,\text{m},0)$:

$$\vec{\boldsymbol{E}}_{\text{sphere}}(2.5\,\text{m},0) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(-37.7\,\mu C)}{(1.62\,\text{m})^2}\hat{\boldsymbol{r}} = (-129\,\text{kN/C})\left(0.928\hat{\boldsymbol{i}} + 0.371\hat{\boldsymbol{j}}\right) = (-120\,\text{kN/C})\hat{\boldsymbol{i}} + (-47.9\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the field at (2.5 m, 0) due to plane 1:

$$\vec{\boldsymbol{E}}_1(2.5\,\text{m},0) = \frac{\sigma_1}{2\in_0}\hat{\boldsymbol{j}} = \frac{3\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{\boldsymbol{j}} = (169\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the field at (2.5 m, 0) due to plane 2:

$$\vec{\boldsymbol{E}}_2(2.5\,\text{m},0) = \frac{\sigma_2}{2\in_0}\hat{\boldsymbol{i}} = \frac{-2\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{\boldsymbol{i}} = (-113\,\text{kN/C})\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{\boldsymbol{E}}(0.4\,\text{m},0) = (-120\,\text{kN/C})\hat{\boldsymbol{i}} + (-47.9\,\text{kN/C})\hat{\boldsymbol{j}} + (169\,\text{kN/C})\hat{\boldsymbol{j}} + (-113\,\text{kN/C})\hat{\boldsymbol{i}} = (-233\,\text{kN/C})\hat{\boldsymbol{i}} + (121\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the magnitude and direction of $\vec{\boldsymbol{E}}(2.5\,\text{m},0)$:

$$E(2.5\,\text{m},0)=\sqrt{(-233\,\text{kN/C})^2+(121\,\text{kN/C})^2}=\boxed{263\,\text{kN/C}}$$

and

$$\theta=\tan^{-1}\left(\frac{121\,\text{kN/C}}{-233\,\text{kN/C}}\right)=\boxed{153°}$$

82 ••

Picture the Problem Let *P* represent the point of interest at (1.5 m, 0.5 m). We can find the electric field at *P* by adding the electric fields due to the infinite plane, the infinite line, and the sphere. Once we've expressed the field at *P* in vector form, we can find its magnitude and direction.

Express the electric field at P:

$$\vec{E}=\vec{E}_{\text{plane}}+\vec{E}_{\text{line}}+\vec{E}_{\text{sphere}}$$

Find $\vec{E}_{\text{plane}}$ at *P*:

$$\vec{E}_{\text{plane}}=-\frac{\sigma}{2\in_0}\hat{i}=-\frac{2\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{i}=(-113\,\text{kN/C})\hat{i}$$

Express $\vec{E}_{\text{line}}$ at *P*:

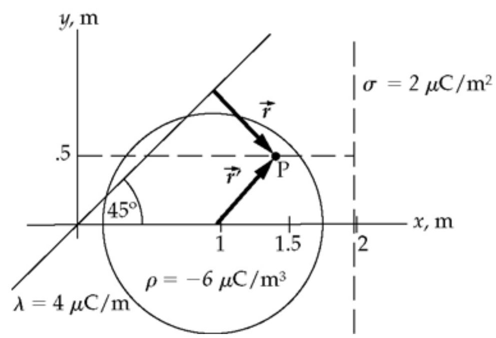
$$\vec{E}_{\text{line}}=\frac{2k\lambda}{r}\hat{r}$$

Refer to the diagram to obtain:

$$\vec{r}=(0.5\,\text{m})\hat{i}-(0.5\,\text{m})\hat{j}$$

and

$$\hat{r}=(0.707)\hat{i}-(0.707)\hat{j}$$



Substitute to obtain:

$$\vec{E}_{\text{line}}=\frac{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\,\mu\text{C/m})}{0.707\,\text{m}}\left[(0.707)\hat{i}-(0.707)\hat{j}\right]$$
$$=(102\,\text{kN/C})\left[(0.707)\hat{i}-(0.707)\hat{j}\right]=(72.1\,\text{kN/C})\hat{i}+(-72.1\,\text{kN/C})\hat{j}$$

Letting r' represent the distance from the center of the sphere to *P*,

$$\vec{E}_{\text{sphere}}=\frac{4\pi}{3}kr'\rho\hat{r}'$$

apply Gauss's law to a spherical surface of radius r' centered at (1 m, 0) to obtain an expression for $\vec{E}_{\text{sphere}}$ at P:

where $\hat{r}'$ is directed toward the center of the sphere.

Refer to the diagram used above to obtain:

$$\vec{r}' = -(0.5\,\text{m})\hat{i} - (0.5\,\text{m})\hat{j}$$

and

$$\hat{r}' = -(0.707)\hat{i} - (0.707)\hat{j}$$

Substitute to obtain:

$$\begin{aligned}\vec{E}_{\text{sphere}} &= \frac{4\pi}{3}\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.707\,\text{m})\left(-6\,\mu\text{C/m}^3\right)\left[(0.707)\hat{i} + (0.707)\hat{j}\right] \\ &= (-113\,\text{kN/C})\left(\hat{i} + \hat{j}\right) = (-113\,\text{kN/C})\hat{i} + (-113\,\text{kN/C})\hat{j}\end{aligned}$$

Substitute and evaluate $\vec{E}$:

$$\begin{aligned}\vec{E} &= (-113\,\text{kN/C})\hat{i} + (72.1\,\text{kN/C})\hat{i} + (-72.1\,\text{kN/C})\hat{j} + (-113\,\text{kN/C})\hat{i} \\ &\quad + (-113\,\text{kN/C})\hat{j} \\ &= (-154\,\text{kN/C})\hat{i} + (-185\,\text{kN/C})\hat{j}\end{aligned}$$

Finally, find the magnitude and direction of $\vec{E}$:

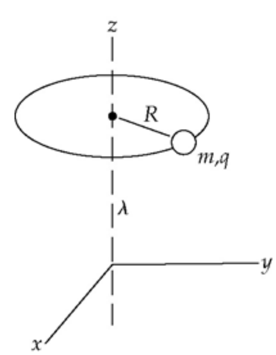
$$E = \sqrt{(-154\,\text{kN/C})^2 + (-185\,\text{kN/C})^2} = \boxed{241\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{-154\,\text{kN/C}}{-185\,\text{kN/C}}\right) = \boxed{220°}$$

83 ••

Picture the Problem We can find the period of the motion from its angular frequency and apply Newton's 2nd law to relate ω to m, q, R, and the electric field due to the infinite line charge. Because the electric field is given by $E_r = 2k\lambda/r$ we can express ω and, hence, T as a function of m, q, R, and λ.



Relate the period T of the particle to its angular frequency ω:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply Newton's 2nd law to the particle to obtain:

$$\sum F_{\text{radial}} = qE_r = mR\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{qE_r}{mR}}$$

Express the electric field at a distance R from the infinite line charge:

$$E_r = 2k\frac{\lambda}{R}$$

Substitute in the expression for ω:

$$\omega = \sqrt{\frac{2k\lambda q}{mR^2}} = \frac{1}{R}\sqrt{\frac{2k\lambda q}{m}}$$

Substitute in equation (1) to obtain:

$$T = \boxed{2\pi R\sqrt{\frac{m}{2k\lambda q}}}$$

***84 ••**

Picture the Problem Starting with the equation for the electric field on the axis of ring charge, we can factor the denominator of the expression to show that, for $x << R$, E_x is proportional to x. We can use $F_x = qE_x$ to express the force acting on the particle and apply Newton's 2nd law to show that, for small displacements from equilibrium, the particle will execute simple harmonic motion. Finally, we can find the period of the motion from its angular frequency, which we can obtain from the differential equation of motion.

(*a*) Express the electric field on the axis of the ring of charge:

$$E_x = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Factor R^2 from the denominator of E_x to obtain:

$$E_x = \frac{kQx}{\left[R^2\left(1+\frac{x^2}{R^2}\right)\right]^{3/2}} = \frac{kQx}{R^3\left(1+\frac{x^2}{R^2}\right)^{3/2}} \approx \boxed{\frac{kQ}{R^3}x}$$

provided $x << R$.

(*b*) Express the force acting on the particle as a function of its charge and the electric field:

$$F_x = qE_x = \boxed{\frac{kqQ}{R^3}x}$$

(*c*) Because the negatively charged particle experiences a linear restoring force, we know that its motion will be simple harmonic. Apply Newton's 2nd law to the negatively charged particle to obtain:

$$m\frac{d^2x}{dt^2}=-\frac{kqQ}{R^3}x$$

or

$$\boxed{\frac{d^2x}{dt^2}+\frac{kqQ}{mR^3}x=0}$$

the differential equation of simple harmonic motion.

Relate the period T of the simple harmonic motion to its angular frequency ω:

$$T=\frac{2\pi}{\omega}$$

From the differential equation we have:

$$\omega^2=\frac{kqQ}{mR^3}$$

Substitute to obtain:

$$T=\boxed{2\pi\sqrt{\frac{mR^3}{kqQ}}}$$

85 ••

Picture the Problem Starting with the equation for the electric field on the axis of a ring charge, we can factor the denominator of the expression to show that, for $x << R$, E_x is proportional to x. We can use $F_x = qE_x$ to express the force acting on the particle and apply Newton's 2nd law to show that, for small displacements from equilibrium, the particle will execute simple harmonic motion. Finally, we can find the angular frequency of the motion from the differential equation and use this expression to find its value when the radius of the ring is doubled and all other parameters remain unchanged.

Express the electric field on the axis of the ring of charge:

$$E_x=\frac{kQx}{\left(x^2+R^2\right)^{3/2}}$$

Factor R^2 from the denominator of E_x to obtain:

$$E_x=\frac{kQx}{\left[R^2\left(1+\frac{x^2}{R^2}\right)\right]^{3/2}}=\frac{kQx}{R^3\left(1+\frac{x^2}{R^2}\right)^{3/2}}\approx\frac{kQ}{R^3}x$$

provided $x << R$.

Express the force acting on the particle as a function of its charge and the electric field:

$$F_x = qE_x = \frac{kqQ}{R^3}x$$

Because the negatively charged particle experiences a linear restoring force, we know that its motion will be simple harmonic. Apply Newton's 2nd law to the negatively charged particle to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{R^3}x$$

or

$$\frac{d^2x}{dt^2} + \frac{kqQ}{mR^3}x = 0$$

the differential equation of simple harmonic motion.

The angular frequency of the simple harmonic motion of the particle is given by:

$$\omega = \sqrt{\frac{kqQ}{mR^3}} \qquad (1)$$

Express the angular frequency of the motion if the radius of the ring is doubled:

$$\omega' = \sqrt{\frac{kqQ}{m(2R)^3}} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{\omega'}{\omega} = \frac{\sqrt{\frac{kqQ}{m(2R)^3}}}{\sqrt{\frac{kqQ}{mR^3}}} = \frac{1}{\sqrt{8}}$$

Solve for and evaluate ω':

$$\omega' = \frac{\omega}{\sqrt{8}} = \frac{21\,\text{rad/s}}{\sqrt{8}} = \boxed{7.42\,\text{rad/s}}$$

86 ••

Picture the Problem Starting with the equation for the electric field on the axis of a ring charge, we can factor the denominator of the expression to show that, for $x << R$, E_x is proportional to x. We can use $F_x = qE_x$ to express the force acting on the particle and apply Newton's 2nd law to show that, for small displacements from equilibrium, the particle will execute simple harmonic motion. Finally, we can find the angular frequency of the motion from the differential equation and use this expression to find its value when the radius of the ring is doubled while keeping the linear charge density on the ring constant.

Express the electric field on the axis of the ring of charge:

$$E_x = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Factor R^2 from the denominator of E_x to obtain:

$$E_x = \frac{kQx}{\left[R^2\left(1+\frac{x^2}{R^2}\right)\right]^{3/2}} = \frac{kQx}{R^3\left(1+\frac{x^2}{R^2}\right)^{3/2}} \approx \boxed{\frac{kQ}{R^3}x}$$

provided $x << R$.

Express the force acting on the particle as a function of its charge and the electric field:

$$F_x = qE_x = \boxed{\frac{kqQ}{R^3}x}$$

Because the negatively charged particle experiences a linear restoring force, we know that its motion will be simple harmonic. Apply Newton's 2nd law to the negatively charged particle to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{R^3}x$$

or

$$\frac{d^2x}{dt^2} + \frac{kqQ}{mR^3}x = 0\,,$$

the differential equation of simple harmonic motion.

The angular frequency of the simple harmonic motion of the particle is given by:

$$\omega = \sqrt{\frac{kqQ}{mR^3}} \qquad (1)$$

Express the angular frequency of the motion if the radius of the ring is doubled while keeping the linear charge density constant (i.e., doubling Q):

$$\omega' = \sqrt{\frac{kq(2Q)}{m(2R)^3}} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{\omega'}{\omega} = \frac{\sqrt{\frac{kq(2Q)}{m(2R)^3}}}{\sqrt{\frac{kqQ}{mR^3}}} = \frac{1}{2}$$

Solve for and evaluate ω':

$$\omega' = \frac{\omega}{2} = \frac{21\,\text{rad/s}}{2} = \boxed{10.5\,\text{rad/s}}$$

87 ••

Picture the Problem We can apply Gauss's law to express $\vec{E}$ as a function of r. We can use the hint to think of the fields at points 1 and 2 as the sum of the fields due to a sphere of radius a with a uniform charge distribution ρ and a sphere of radius b, centered at $a/2$ with uniform charge distribution $-\rho$.

(*a*) The electric field at a distance r from the center of the sphere is given by:

$$\vec{E} = E\hat{r} \qquad (1)$$

where $\hat{r}$ is a unit vector pointing radially outward.

Apply Gauss's law to a spherical surface of radius r centered at the origin to obtain:

$$\oint_S E_n dA = E\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Relate Q_{enclosed} to the charge density ρ:

$$\rho = \frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi r^3} \Rightarrow Q_{\text{enclosed}} = \tfrac{4}{3}\rho\pi r^3$$

Substitute for Q_{enclosed}:

$$E\left(4\pi r^2\right) = \frac{\frac{4}{3}\rho\pi r^3}{\epsilon_0}$$

Solve for E to obtain:

$$E = \frac{\rho r}{3\epsilon_0}$$

Substitute for E in equation (1) to obtain:

$$\vec{E} = \boxed{\frac{\rho}{3\epsilon_0} r\,\hat{r}}$$

(*b*) The electric field at point 1 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_1 = \vec{E}_\rho + \vec{E}_{-\rho} = E_\rho\hat{r} + E_{-\rho}\hat{r} \qquad (2)$$

Apply Gauss's law to relate the magnitude of the field due to the positive charge distribution to the charge enclosed by the sphere:

$$E_\rho\left(4\pi a^2\right) = \frac{q_{\text{encl}}}{\epsilon_0} = \frac{\frac{4}{3}\pi a^3\rho}{\epsilon_0}$$

Solve for E_ρ:

$$E_\rho = \frac{a\rho}{3\epsilon_0} = \frac{2\rho b}{3\epsilon_0}$$

Proceed similarly for the spherical hole to obtain:

$$E_{-\rho}\left(4\pi b^2\right) = \frac{q_{\text{encl}}}{\epsilon_0} = -\frac{\frac{4}{3}\pi b^3\rho}{\epsilon_0}$$

Solve for $E_{-\rho}$:

$$E_{-\rho} = -\frac{\rho b}{3\epsilon_0}$$

Substitute in equation (2) to obtain:

$$\vec{E}_1 = \frac{2\rho b}{3\in_0}\hat{r} - \frac{\rho b}{3\in_0}\hat{r} = \boxed{\frac{\rho b}{3\in_0}\hat{r}}$$

The electric field at point 2 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_2 = \vec{E}_\rho + \vec{E}_{-\rho} = E_\rho\hat{r} + E_{-\rho}\hat{r} \qquad (3)$$

Because point 2 is at the center of the larger sphere:

$$E_\rho = 0$$

The magnitude of the field at point 2 due to the negative charge distribution is:

$$E_{-\rho} = \frac{\rho b}{3\in_0}$$

Substitute in equation (3) to obtain:

$$\vec{E}_2 = 0 + \frac{\rho b}{3\in_0}\hat{r} = \boxed{\frac{\rho b}{3\in_0}\hat{r}}$$

88 •••

Picture the Problem The electric field in the cavity is the sum of the electric field due to the uniform and positive charge distribution of the sphere whose radius is *a* and the electric field due to any charge in the spherical cavity whose radius is *b*.

The electric field at any point inside the cavity is the sum of the electric fields due to the two charge distributions:

$$\vec{E} = \vec{E}_\rho + \vec{E}_{\text{charge inside}} = E_\rho\hat{r} + E_{\text{charge inside}}\hat{r}$$

where $\hat{r}$ is a unit vector pointing radially outward.

Because there is no charge inside the cavity:

$$E_{\text{charge inside}} = 0$$

The magnitude of the field inside the cavity due to the positive charge distribution is:

$$E_\rho = \frac{\rho b}{3\in_0}$$

Substitute in the expression for $\vec{E}$ to obtain:

$$\vec{E} = 0 + \frac{\rho b}{3\in_0}\hat{r} = \boxed{\frac{\rho}{3\in_0}b\hat{r}}$$

89 ••

Picture the Problem We can use the hint given in Problem 87 to think of the fields at points 1 and 2 as the sum of the fields due to a sphere of radius *a* with a uniform charge distribution ρ and a sphere of radius *b*, centered at *a*/2 with charge *Q* spread uniformly throughout its volume.

The electric field at point 1 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_1 = \vec{E}_\rho + \vec{E}_Q = E_\rho\hat{r} + E_Q\hat{r} \qquad (1)$$

where $\hat{r}$ is a unit vector pointing radially outward.

Apply Gauss's law to relate the field due to the positive charge distribution to the charge of the sphere:

$$E_\rho\left(4\pi a^2\right)=\frac{q_{\text{encl}}}{\epsilon_0}=\frac{\frac{4}{3}\pi a^3\rho}{\epsilon_0}$$

Solve for E_ρ:

$$E_\rho=\frac{a\rho}{3\epsilon_0}=\frac{2\rho b}{3\epsilon_0}$$

Apply Gauss's law to relate the field due to the negative charge distributed uniformly throughout the volume of the cavity :

$$E_Q\left(4\pi b^2\right)=\frac{q_{\text{encl}}}{\epsilon_0}=\frac{Q}{\epsilon_0}$$

where $Q=\rho' V=\rho'\frac{4}{3}\pi b^3$

Substitute for Q to obtain:

$$E_Q\left(4\pi b^2\right)=\frac{\frac{4}{3}\pi\rho' b^3}{\epsilon_0}$$

Solve for E_Q:

$$E_Q=\frac{\rho' b}{3\epsilon_0}$$

Substitute in equation (1) to obtain:

$$\vec{E}_1=\frac{2\rho b}{3\epsilon_0}\hat{r}+\frac{\rho' b}{3\epsilon_0}\hat{r}=\boxed{\frac{(2\rho+\rho')b}{3\epsilon_0}\hat{r}}$$

The electric field at point 2 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_2=\vec{E}_\rho+\vec{E}_Q=E_\rho\hat{r}+E_Q\hat{r} \qquad (2)$$

Because point 2 is at the center of the larger sphere:

$$E_\rho=0$$

The magnitude of the field at point 2 due to the uniformly distributed charge Q was shown above to be:

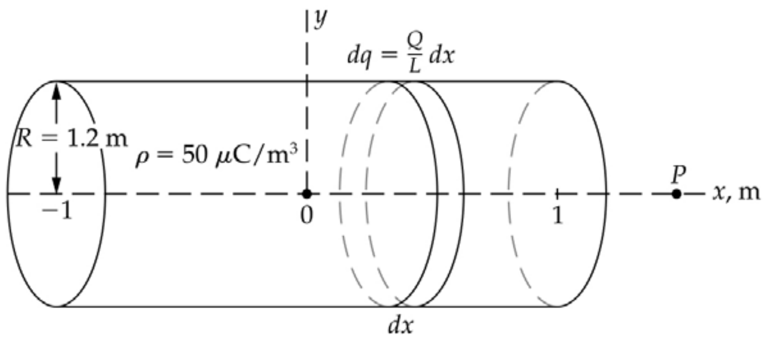
$$E_Q=\frac{\rho' b}{3\epsilon_0}$$

Substitute in equation (2) to obtain:

$$\vec{E}_2=0+\frac{\rho' b}{3\epsilon_0}\hat{r}=\boxed{\frac{\rho'}{3\epsilon_0}b\hat{r}}$$

90 ••

Picture the Problem Let the length of the cylinder be L, its radius R, and charge Q. Let P be a generic point of interest on the x axis. We can find the electric field at P by expressing the field due to an elemental disk of radius R, thickness dx, and charge dq and then integrating $E_x=2\pi k\sigma\left(1-x/\sqrt{x^2+R^2}\right)$.



Express the electric field on the x axis due to the charge carried by the disk of thickness dx:

$$dE_x = 2\pi k\rho\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)dx$$

Integrate dE_x for P beyond the end of the cylinder:

$$\begin{aligned}E_x &= 2\pi k\rho\int_{x-L/2}^{x+L/2}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)dx \\ &= 2\pi k\rho\left[L - \sqrt{\left(\frac{L}{2}+x\right)^2 + R^2} + \sqrt{\left(\frac{L}{2}-x\right)^2 + R^2}\right]\end{aligned}$$

Integrate dE_x for P inside the cylinder:

$$\begin{aligned}E_x &= 2\pi k\rho\left[\int_0^{L/2+x}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)dx - \int_0^{L/2-x}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)dx\right] \\ &= 2\pi k\rho\left[2x - \sqrt{\left(\frac{L}{2}+x\right)^2 + R^2} + \sqrt{\left(\frac{L}{2}-x\right)^2 + R^2}\right]\end{aligned}$$

The effective charge density of the disk is given by:

$$\rho = \frac{Q/L}{\pi R^2}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{50\ \mu\text{C}}{\pi(1.2\ \text{m})^2(2\ \text{m})} = 5.53\ \mu\text{C/m}^3$$

Evaluate $2\pi k\rho$:

$$2\pi k\rho = 2\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.53\ \mu\text{C/m}^3\right) = 3.12\times10^5\ \text{N/C}\cdot\text{m}$$

(*a*) Evaluate E_x(0.5 m):

$$E_x(0.5\,\text{m}) = (3.12\times10^5\ \text{N/C}\cdot\text{m})$$
$$\times\left[2(0.5\,\text{m}) - \sqrt{\left(\frac{2\,\text{m}}{2}+0.5\,\text{m}\right)^2+(1.2\,\text{m})^2}+\sqrt{\left(\frac{2\,\text{m}}{2}-0.5\,\text{m}\right)^2+(1.2\,\text{m})^2}\,\right]$$
$$=\boxed{118\,\text{kN/C}}$$

(*b*) Evaluate E_x(2 m):

$$E_x(2\,\text{m}) = (3.12\times10^5\ \text{N/C}\cdot\text{m})$$
$$\times\left[2\,\text{m} - \sqrt{\left(\frac{2\,\text{m}}{2}+2\,\text{m}\right)^2+(1.2\,\text{m})^2}+\sqrt{\left(\frac{2\,\text{m}}{2}-2\,\text{m}\right)^2+(1.2\,\text{m})^2}\,\right]$$
$$=\boxed{103\,\text{kN/C}}$$

(*c*) Evaluate E_x(20 m):

$$E_x(20\,\text{m}) = (3.12\times10^5\ \text{N/C}\cdot\text{m})$$
$$\times\left[2\,\text{m} - \sqrt{\left(\frac{2\,\text{m}}{2}+20\,\text{m}\right)^2+(1.2\,\text{m})^2}+\sqrt{\left(\frac{2\,\text{m}}{2}-20\,\text{m}\right)^2+(1.2\,\text{m})^2}\,\right]$$
$$=\boxed{1.12\,\text{kN/C}}$$

Remarks: Note that, in (*c*), the distance of 20 m is much greater than the length of the cylinder that we could have used $E_x = kQ/x^2$.

91 ••

Picture the Problem We can use $E_x = kQ/[x_0(x_0-L)]$ to express the electric fields at $x_0 = 2L$ and $x_0 = 3L$ and take the ratio of these expressions to find the field at $x_0 = 3L$.

Express the electric field along the *x* axis due to a uniform line charge on the *x* axis:

$$E_x(x_0) = \frac{kQ}{x_0(x_0-L)}$$

Evaluate E_x at $x_0 = 2L$:

$$E_x(2L) = \frac{kQ}{2L(2L-L)} = \frac{kQ}{2L^2} \qquad (1)$$

Evaluate E_x at $x_0 = 3L$:

$$E_x(3L) = \frac{kQ}{3L(3L-L)} = \frac{kQ}{6L^2} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{E_x(3L)}{E_x(2L)}=\frac{\dfrac{kQ}{6L^2}}{\dfrac{kQ}{2L^2}}=\frac{1}{3}$$

Solve for and evaluate $E_x(3L)$:

$$E_x(3L)=\frac{1}{3}E_x(2L)=\frac{1}{3}(600\ \text{N/C})$$
$$=\boxed{200\ \text{N/C}}$$

92 •••

Picture the Problem Let the coordinates of one corner of the cube be (x,y,z), and assume that the sides of the cube are Δx, Δy, and Δz and compute the flux through the faces of the cube that are parallel to the yz plane. The net flux of the electric field out of the gaussian surface is the difference between the flux out of the surface and the flux into the surface.

The net flux out of the cube is given by:

$$\phi_{\text{net}}=\phi(x+\Delta x)-\phi(x)$$

Use a Taylor series expansion to express the net flux through faces of the cube that are parallel to the yz plane:

$$\phi_{\text{net}}=\phi(x)+(\Delta x)\phi'(x)+\tfrac{1}{2}(\Delta x)^2\phi''(x)+\ldots-\phi(x)=(\Delta x)\phi'(x)+\tfrac{1}{2}(\Delta x)^2\phi''(x)+\ldots$$

Neglecting terms higher than first order we have:

$$\phi_{\text{net}}=\Delta x\phi'(x)$$

Because the electric field is in the x direction, $\phi(x)$ is:

$$\phi(x)=E_x\Delta y\Delta z$$
and
$$\phi'(x)=\frac{\partial E_x}{\partial x}\Delta y\Delta z$$

Substitute for $\phi'(x)$ to obtain:

$$\phi_{\text{net}}=\Delta x\frac{\partial E_x}{\partial x}(\Delta y\Delta z)$$
$$=\frac{\partial E_x}{\partial x}(\Delta x\Delta y\Delta z)$$
$$=\boxed{\frac{\partial E_x}{\partial x}\Delta V}$$

93 ••

Picture the Problem We can use the definition of electric flux in conjunction with the result derived in Problem 92 to show that $\nabla \cdot \vec{\boldsymbol{E}} = \rho / \in_0$.

From Gauss's law, the net flux through any surface is:

$$\phi_{\text{net}} = \frac{q_{\text{encl}}}{\in_0} = \frac{\rho}{\in_0}V$$

Generalizing our result from Problem 92 (see the remark following Problem 92):

$$\phi_{\text{net}} = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)V = \left(\nabla \cdot \vec{\boldsymbol{E}}\right)V$$

Equate these two expressions to obtain:

$$\left(\nabla \cdot \vec{\boldsymbol{E}}\right)V = \frac{\rho}{\in_0}V \text{ or } \nabla \cdot \vec{\boldsymbol{E}} = \boxed{\frac{\rho}{\in_0}}$$

***94** •••

Picture the Problem We can find the field due to the infinitely long line charge from $E = 2k\lambda/r$ and the force that acts on the dipole using $F = p\,dE/dr$.

Express the force acting on the dipole:

$$F = p\frac{dE}{dr}$$

The electric field at the location of the dipole is given by:

$$E = \frac{2k\lambda}{r}$$

Substitute to obtain:

$$F = p\frac{d}{dr}\left[\frac{2k\lambda}{r}\right] = \boxed{-\frac{2k\lambda p}{r^2}}$$

where the minus sign indicates that the dipole is attracted to the line charge.

95 ••

Picture the Problem We can find the distance from the center where the net force on either charge is zero by setting the sum of the forces acting on either point charge equal to zero. Each point charge experiences two forces; one a Coulomb force of repulsion due to the other point charge, and the second due to that fraction of the sphere's charge that is between the point charge and the center of the sphere that creates an electric field at the location of the point charge.

Apply $\sum F = 0$ to either of the point charges:

$$F_{\text{Coulomb}} - F_{\text{field}} = 0 \qquad (1)$$

Express the Coulomb force on the proton:

$$F_{\text{Coulomb}} = \frac{ke^2}{(2a)^2} = \frac{ke^2}{4a^2}$$

The force exerted by the field E is:

$$F_{\text{field}} = eE$$

Apply Gauss's law to a spherical surface of radius *a* centered at the origin:

$$E\left(4\pi a^2\right)=\frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Relate the charge density of the electron sphere to Q_{enclosed}:

$$\frac{2e}{\frac{4}{3}\pi R^3}=\frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi a^3}\Rightarrow Q_{\text{enclosed}}=\frac{2ea^3}{R^3}$$

Substitute for Q_{enclosed}:

$$E\left(4\pi a^2\right)=\frac{2ea^3}{\epsilon_0 R^3}$$

Solve for E to obtain:

$$E=\frac{ea}{2\pi\epsilon_0 R^3}\Rightarrow F_{\text{field}}=\frac{e^2a}{2\pi\epsilon_0 R^3}$$

Substitute for F_{Coulomb} and F_{field} in equation (1):

$$\frac{ke^2}{4a^2}-\frac{e^2a}{2\pi\epsilon_0 R^3}=0$$

or

$$\frac{ke^2}{4a^2}-\frac{2ke^2a}{R^3}=0$$

Solve for a to obtain:

$$a=\sqrt[3]{\frac{1}{8}}R=\boxed{0.5R}$$

96 •••

Picture the Problem We can use the result of Problem 96 to express the force acting on both point charges when they are separated by 2*a*. We can then use this expression to write the force function when the point charges are each displaced a small distance *x* from their equilibrium positions and then expand this function binomially to show that each point charge experiences a linear restoring force.

From Problem 95, the force function at the equilibrium position is:

$$F(a)=\frac{ke^2}{4a^2}-\frac{2ke^2a}{R^3}=0$$

When the charges are displaced a distance *x* symmetrically from their equilibrium positions, the force function becomes:

$$F(a+x)=\frac{ke^2}{4}(a+x)^{-2}-\frac{2ke^2}{R^3}(a+x)$$

Expand this function binomially to obtain:

$$\begin{aligned}F(a+x)&=\frac{ke^2}{4}\left(a^{-2}-2a^{-3}x+\ldots\right)-\frac{2ke^2}{R^3}a-\frac{2ke^2}{R^3}x\\&\approx\frac{ke^2}{4a^2}-\frac{ke^2}{2a^3}x-\frac{2ke^2}{R^3}a-\frac{2ke^2}{R^3}x\end{aligned}$$

Substitute for R using the result obtained in Problem 96 and simplify to obtain:

$$F_{\text{restoring}} = \boxed{-\left(\frac{3ke^2}{4a^3}\right)x}$$

Hence, we've shown that, for a small displacement from equilibrium, the point charges experience a linear restoring force.

Remarks: An alternative approach that you might find instructive is to expand the force function using the Taylor series.

97 •••

Picture the Problem Because the restoring force found in Problem 96 is linear, we can express the differential equation of the proton's motion and then identify ω^2 from this equation.

Apply $\sum F_x = ma$ to the displaced proton to obtain:

$$-\frac{3ke^2}{4r^3}x = m\frac{d^2x}{dt^2}$$

or

$$\frac{d^2x}{dt^2} = -\frac{3ke^2}{4mr^3}x = -\omega^2 x$$

where $\omega^2 = \dfrac{3ke^2}{4mr^3}$

Solve for ω:

$$\omega = \sqrt{\frac{3ke^2}{4mr^3}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{3\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{4\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.08\,\text{nm}\right)^3}} = \boxed{4.49\times10^{14}\ \text{s}^{-1}}$$

Chapter 23
Electrical Potential

Conceptual Problems

***1** •

Determine the Concept A positive charge will move in whatever direction reduces its potential energy. The positive charge will reduce its potential energy if it moves toward a region of lower electric potential.

2 ••

Picture the Problem A charged particle placed in an electric field experiences an accelerating force that does work on the particle. From the work-kinetic energy theorem we know that the work done on the particle by the net force changes its kinetic energy and that the kinetic energy K acquired by such a particle whose charge is q that is accelerated through a potential difference V is given by $K = qV$. Let the numeral 1 refer to the alpha particle and the numeral 2 to the lithium nucleus and equate their kinetic energies after being accelerated through potential differences V_1 and V_2.

Express the kinetic energy of the alpha particle when it has been accelerated through a potential difference V_1:

$$K_1 = q_1V_1 = 2eV_1$$

Express the kinetic energy of the lithium nucleus when it has been accelerated through a potential difference V_2:

$$K_2 = q_2V_2 = 3eV_2$$

Equate the kinetic energies to obtain:

$$2eV_1 = 3eV_2$$

or

$V_2 = \frac{2}{3}V_1$ and $\boxed{(b)\text{ is correct.}}$

3 •

Determine the Concept If V is constant, its gradient is zero; consequently $\vec{E} = 0$.

4 •

Determine the Concept No. E can be determined from either $E_\ell = -\dfrac{dV}{d\ell}$ provided V is known and differentiable or from $E_\ell = -\dfrac{\Delta V}{\Delta\ell}$ provided V is known at two or more points.

5 •

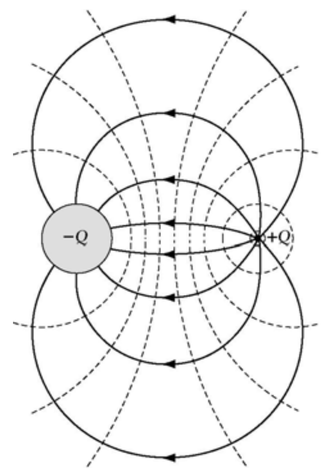
Determine the Concept Because the field lines are always perpendicular to equipotential surfaces, you move always perpendicular to the field.

6 ••

Determine the Concept V along the axis of the ring does not depend on the charge distribution. The electric field, however, does depend on the charge distribution, and the result given in Chapter 21 is valid only for a uniform distribution.

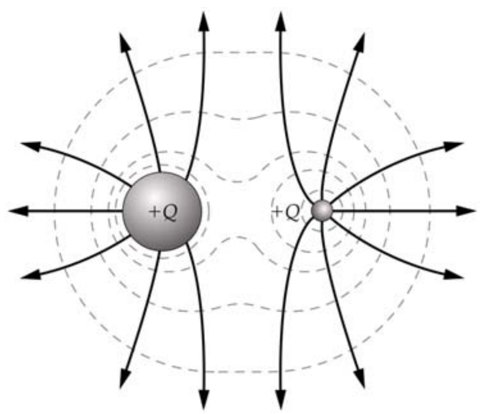
***7** ••

Picture the Problem The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge $+Q$ is the point at the right, and the metal sphere with charge $-Q$ is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere's surface.



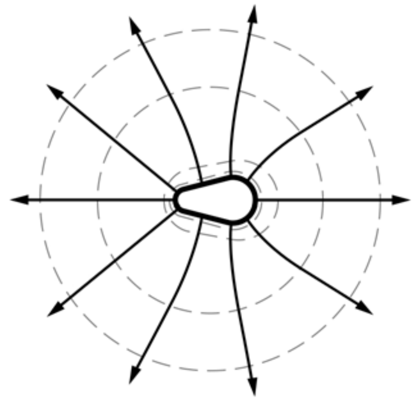
8 ••

Picture the Problem The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge $+Q$ is the point at the right, and the metal sphere with charge $+Q$ is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere's surface. Very far from both charges, the equipotential surfaces and field lines approach those of a point charge $2Q$ located at the midpoint.



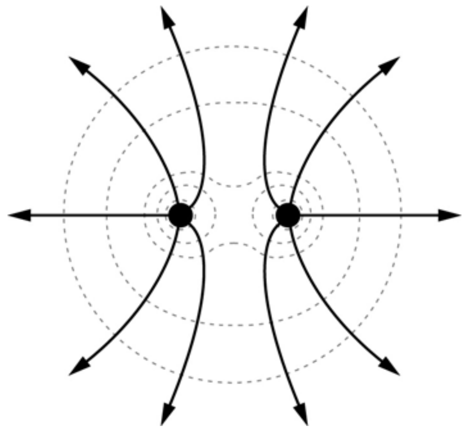
9 ••

Picture the Problem The equipotential surfaces are shown with dashed lines, the field lines are shown in solid lines. It is assumed that the conductor carries a positive charge. Near the conductor the equipotential surfaces follow the conductor's contours; far from the conductor, the equipotential surfaces are spheres centered on the conductor. The electric field lines are perpendicular to the equipotential surfaces.



10 ••

Picture the Problem The equipotential surfaces are shown with dashed lines, the electric field lines are shown with solid lines. Near each charge, the equipotential surfaces are spheres centered on each charge; far from the charges, the equipotential is a sphere centered at the midpoint between the charges. The electric field lines are perpendicular to the equipotential surfaces.



***11** •

Picture the Problem We can use Coulomb's law and the superposition of fields to find E at the origin and the definition of the electric potential due to a point charge to find V at the origin.

Apply Coulomb's law and the superposition of fields to find the electric field E at the origin:

$$\vec{E} = \vec{E}_{+Q\,\text{at}\,-a} + \vec{E}_{+Q\,\text{at}\,a} = \frac{kQ}{a^2}\hat{i} - \frac{kQ}{a^2}\hat{i} = 0$$

Express the potential V at the origin:

$$V = V_{+Q\,\text{at}\,-a} + V_{+Q\,\text{at}\,a} = \frac{kQ}{a} + \frac{kQ}{a} = \frac{2kQ}{a}$$

and $\boxed{(b)\text{ is correct.}}$

12 •

Picture the Problem We can use $\vec{E} = -\frac{\partial V}{\partial x}\hat{i}$ to find the electric field corresponding the given potential and then compare its form to those produced by the four alternatives listed.

Find the electric field corresponding to this potential function:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = -\frac{\partial}{\partial x}\left[4|x| + V_0\right]\hat{i}$$
$$= -4\frac{\partial}{\partial x}\left[|x|\right]\hat{i} = -4\begin{bmatrix} 1 \text{ if } x \geq 0 \\ -1 \text{ if } x < 0 \end{bmatrix}\hat{i}$$
$$= \begin{bmatrix} -4 \text{ if } x \geq 0 \\ 4 \text{ if } x < 0 \end{bmatrix}\hat{i}$$

Of the alternatives provided above, only a uniformly charged sheet in the *yz* plane would produce a constant electric field whose direction changes at the origin. $\boxed{(c) \text{ is correct.}}$

13 •

Picture the Problem We can use Coulomb's law and the superposition of fields to find *E* at the origin and the definition of the electric potential due to a point charge to find *V* at the origin.

Apply Coulomb's law and the superposition of fields to find the electric field *E* at the origin:

$$\vec{E} = \vec{E}_{+Q \text{ at } -a} + \vec{E}_{-Q \text{ at } a}$$
$$= \frac{kQ}{a^2}\hat{i} + \frac{kQ}{a^2}\hat{i} = \frac{2kQ}{a^2}\hat{i}$$

Express the potential *V* at the origin:

$$V = V_{+Q \text{ at } -a} + V_{-Q \text{ at } a}$$
$$= \frac{kQ}{a} + \frac{k(-Q)}{a} = 0$$

and $\boxed{(c) \text{ is correct}}$

14 ••

(*a*) False. As a counterexample, consider two equal charges at equal distances from the origin on the *x* axis. The electric field due to such an array is zero at the origin but the electric potential is not zero.

(*b*) True.

(*c*) False. As a counterexample, consider two equal-in-magnitude but opposite-in-sign charges at equal distances from the origin on the *x* axis. The electric potential due to such an array is zero at the origin but the electric field is not zero.

(*d*) True.

(*e*) True.

(*f*) True.

(*g*) False. Dielectric breakdown occurs in air at an *electric field strength* of approximately 3×10^6 V/m.

15 ••

(*a*) No. The potential at the surface of a conductor also depends on the local radius of the surface. Hence r and σ can vary in such a way that V is constant.

(*b*) Yes; yes.

***16** •

Determine the Concept When the two spheres are connected, their charges will redistribute until the two-sphere system is in electrostatic equilibrium. Consequently, the entire system must be an equipotential. $\boxed{(c)\text{ is corrent.}}$

Estimation and Approximation Problems

17 •

Picture the Problem The field of a thundercloud must be of order 3×10^6 V/m just before a lightning strike.

Express the potential difference between the cloud and the earth as a function of their separation d and electric field E between them:

$$V = Ed$$

Assuming that the thundercloud is at a distance of about 1 km above the surface of the earth, the potential difference is approximately:

$$V = \left(3\times10^6\ \text{V/m}\right)\left(10^3\ \text{m}\right) = \boxed{3.00\times10^9\ \text{V}}$$

Note that this is an upper bound, as there will be localized charge distributions on the thundercloud which raise the local electric field above the average value.

***18** •

Picture the Problem The potential difference between the electrodes of the spark plug is the product of the electric field in the gap and the separation of the electrodes. We'll assume that the separation of the electrodes is 1 mm.

Express the potential difference between the electrodes of the spark

$$V = Ed$$

plug as a function of their separation *d* and electric field *E* between them:

Substitute numerical values and evaluate *V*:

$$V = \left(2\times10^{7}\ \text{V/m}\right)\left(10^{-3}\ \text{m}\right) = \boxed{20.0\,\text{kV}}$$

19 ••

Picture the Problem We can use conservation of energy to relate the initial kinetic energy of the protons to their electrostatic potential energy when they have approached each other to the given "radius".

(*a*) Apply conservation of energy to relate the initial kinetic energy of the protons to their electrostatic potential when they are separated by a distance *r*:

$$K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$$

or, because $U_\text{i} = K_\text{f} = 0$,

$$K_\text{i} = U_\text{f}$$

Because each proton has kinetic energy *K*:

$$2K = \frac{e^2}{4\pi \in_0 r} \Rightarrow K = \frac{e^2}{8\pi \in_0 r}$$

Substitute numerical values and evaluate *K*:

$$K = \frac{\left(1.6\times10^{-19}\ \text{C}\right)^2}{8\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(10^{-15}\ \text{m}\right)} = 1.15\times10^{-13}\ \text{J}\times\frac{1\text{eV}}{1.6\times10^{-19}\ \text{J}} = \boxed{0.719\,\text{MeV}}$$

(*b*) Express and evaluate the ratio of the two energies:

$$f = \frac{K}{E_\text{rest}} = \frac{0.719\,\text{MeV}}{938\,\text{MeV}} = \boxed{0.0767\%}$$

20 ••

Picture the Problem The magnitude of the electric field for which dielectric breakdown occurs in air is about 3 MV/m. We can estimate the potential difference between you and your friend from the product of the length of the spark and the dielectric constant of air.

Express the product of the length of the spark and the dielectric constant of air:

$$V = \left(3\,\text{MV/m}\right)\left(2\,\text{mm}\right) = \boxed{6000\,\text{V}}$$

Potential Difference

21 •

Picture the Problem We can use the definition of finite potential difference to find the potential difference $V(4\text{ m}) - V(0)$ and conservation of energy to find the kinetic energy of the charge when it is at $x = 4$ m. We can also find $V(x)$ if $V(x)$ is assigned various values at various positions from the definition of finite potential difference.

(*a*) Apply the definition of finite potential difference to obtain:

$$V(4\,\text{m}) - V(0) = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_0^{4\,\text{m}} E\,d\ell$$
$$= -(2\,\text{kN/C})(4\,\text{m})$$
$$= \boxed{-8.00\,\text{kV}}$$

(*b*) By definition, ΔU is given by:

$$\Delta U = q\Delta V = (3\,\mu\text{C})(-8\,\text{kV})$$
$$= \boxed{-24.0\,\text{mJ}}$$

(*c*) Use conservation of energy to relate ΔU and ΔK:

$$\Delta K + \Delta U = 0$$
or
$$K_{4\,\text{m}} - K_0 + \Delta U = 0$$

Because $K_0 = 0$:

$$K_{4\,\text{m}} = -\Delta U = \boxed{24.0\,\text{mJ}}$$

Use the definition of finite potential difference to obtain:

$$V(x) - V(x_0) = -E_x(x - x_0)$$
$$= -(2\,\text{kV/m})(x - x_0)$$

(*d*) For $V(0) = 0$:

$$V(x) - 0 = -(2\,\text{kV/m})(x - 0)$$
or
$$V(x) = \boxed{-(2\,\text{kV/m})x}$$

(*e*) For $V(0) = 4$ kV:

$$V(x) - 4\,\text{kV} = -(2\,\text{kV/m})(x - 0)$$
or
$$V(x) = \boxed{4\,\text{kV} - (2\,\text{kV/m})x}$$

(*f*) For $V(1\text{m}) = 0$:

$$V(x) - 0 = -(2\,\text{kV/m})(x - 1)$$
or
$$V(x) = \boxed{2\,\text{kV} - (2\,\text{kV/m})x}$$

22 •

Picture the Problem Because the electric field is uniform, we can find its magnitude from $E = \Delta V/\Delta x$. We can find the work done by the electric field on the electron from the difference in potential between the plates and the charge of the electron and find the change in potential energy of the electron from the work done on it by the electric field. We can use conservation of energy to find the kinetic energy of the electron when it reaches the positive plate.

(*a*) Express the magnitude of the electric field between the plates in terms of their separation and the potential difference between them:

$$E = \frac{\Delta V}{\Delta x} = \frac{500\,\text{V}}{0.1\text{m}} = \boxed{5.00\,\text{kV/m}}$$

Because the electric force on a test charge is away from the positive plate and toward the negative plate, the positive plate is at the higher potential.

(*b*) Relate the work done by the electric field on the electron to the difference in potential between the plates and the charge of the electron:

$$W = q\Delta V = \left(1.6\times10^{-19}\ \text{C}\right)\left(500\text{V}\right) = \boxed{8.01\times10^{-17}\ \text{J}}$$

Convert 8.01×10^{-17} J to eV:

$$W = \left(8.01\times10^{-17}\ \text{J}\right)\left(\frac{1\text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = \boxed{500\,\text{eV}}$$

(*c*) Relate the change in potential energy of the electron to the work done on it as it moves from the negative plate to the positive plate:

$$\Delta U = -W = \boxed{-500\,\text{eV}}$$

Apply conservation of energy to obtain:

$$\Delta K = -\Delta U = \boxed{500\,\text{eV}}$$

23 •

Picture the Problem The Coulomb potential at a distance r from the origin relative to $V = 0$ at infinity is given by $V = kq/r$ where q is the charge at the origin. The work that must be done by an outside agent to bring a charge from infinity to a position a distance r from the origin is the product of the magnitude of the charge and the potential difference due to the charge at the origin.

(*a*) Express and evaluate the Coulomb potential of the charge:

$$V = \frac{kq}{r} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\ \mu\text{C})}{4\ \text{m}} = \boxed{4.50\ \text{kV}}$$

(*b*) Relate the work that must be done to the magnitude of the charge and the potential difference through which the charge is moved:

$$W = q\Delta V = (3\ \mu\text{C})(4.50\ \text{kV}) = \boxed{13.5\ \text{mJ}}$$

(*c*) Express the work that must be done by the outside agent in terms of the potential difference through which the 2-μC is to be moved:

$$W = q_2\Delta V_3 = \frac{kq_2q_3}{r}$$

Substitute numerical values and evaluate *W*:

$$W = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\ \mu\text{C})(3\ \mu\text{C})}{4\ \text{m}} = \boxed{13.5\ \text{mJ}}$$

24 ••

Picture the Problem In general, the work done by an external agent in separating the two ions changes both their kinetic and potential energies. Here we're assuming that they are at rest initially and that they will be at rest when they are infinitely far apart. Because their potential energy is also zero when they are infinitely far apart, the energy W_{ext} required to separate the ions to an infinite distance apart is the negative of their potential energy when they are a distance *r* apart.

Express the energy required to separate the ions in terms of the work required by an external agent to bring about this separation:

$$W_{ext} = \Delta K + \Delta U = 0 - U_i = -\frac{kq_-q_+}{r} = -\frac{k(-e)e}{r} = \frac{ke^2}{r}$$

Substitute numerical values and evaluate W_{ext}:

$$W_{ext} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{2.80\times10^{-10}\ \text{m}} = 8.24\times10^{-19}\ \text{J}$$

Convert W_{ext} to eV:

$$W = \left(8.24\times10^{-19}\ \text{J}\right)\left(\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = \boxed{5.14\,\text{eV}}$$

25 ••

Picture the Problem We can find the final speeds of the protons from the potential difference through which they are accelerated and use $E = \Delta V/\Delta x$ to find the accelerating electric field.

(*a*) Apply the work-kinetic energy theorem to the accelerated protons:

$$W = \Delta K = K_f$$

or

$$e\Delta V = \tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2\left(1.6\times10^{-19}\ \text{C}\right)\left(5\,\text{MV}\right)}{1.67\times10^{-27}\ \text{kg}}} = \boxed{3.10\times10^{7}\ \text{m/s}}$$

(*b*) Assuming the same potential change occurred *uniformly* over the distance of 2.0 m, we can use the relationship between E, ΔV, and Δx express and evaluate E:

$$E = \frac{\Delta V}{\Delta x} = \frac{5\,\text{MV}}{2\,\text{m}} = \boxed{2.50\,\text{MV/m}}$$

***26** ••

Picture the Problem The work done on the electrons by the electric field changes their kinetic energy. Hence we can use the work-kinetic energy theorem to find the kinetic energy and the speed of impact of the electrons.

Use the work-kinetic energy theorem to relate the work done by the electric field to the change in the kinetic energy of the electrons:

$$W = \Delta K = K_f$$

or

$$K_f = e\Delta V \qquad (1)$$

(*a*) Substitute numerical values and evaluate K_f:

$$K_f = (1e)(30\,\text{kV}) = \boxed{3\times10^{4}\ \text{eV}}$$

(*b*) Convert this energy to eV:

$$K_f = (3\times10^4\ \text{eV})\left(\frac{1.6\times10^{-19}\ \text{J}}{\text{eV}}\right) = \boxed{4.80\times10^{-15}\ \text{J}}$$

(*c*) From equation (1) we have:

$$\tfrac{1}{2}mv_f^2 = e\Delta V$$

Solve for v_f to obtain:

$$v_f = \sqrt{\frac{2e\Delta V}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(30\ \text{kV})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.03\times10^8\ \text{m/s}}$$

Remarks: Note that this speed is about one-third that of light.

27 ••

Picture the Problem We know that energy is conserved in the interaction between the α particle and the massive nucleus. Under the assumption that the recoil of the massive nucleus is negligible, we know that the initial kinetic energy of the α particle will be transformed into potential energy of the two-body system when the particles are at their distance of closest approach.

(*a*) Apply conservation of energy to the system consisting of the α particle and the massive nucleus:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_f = U_i = 0$ and $K_i = E$:

$$-E + U_f = 0$$

Letting r be the separation of the particles at closest approach, express U_f:

$$U_f = \frac{kq_{\text{nucleus}}q_\alpha}{r} = \frac{k(Ze)(2e)}{r} = \frac{2kZe^2}{r}$$

Substitute to obtain:

$$-E + \frac{2kZe^2}{r} = 0$$

Solve for r to obtain:

$$r = \boxed{\frac{2kZe^2}{E}}$$

(*b*) For a 5.0-MeV α particle and a gold nucleus:

$$r_5 = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(79)\left(1.6\times10^{-19}\ \text{C}\right)^2}{(5\,\text{MeV})\left(1.6\times10^{-19}\ \text{J/eV}\right)} = 4.55\times10^{-14}\ \text{m} = \boxed{45.4\,\text{fm}}$$

For a 9.0-MeV α particle and a gold nucleus:

$$r_9 = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(79)\left(1.6\times10^{-19}\ \text{C}\right)^2}{(9\,\text{MeV})\left(1.6\times10^{-19}\ \text{J/eV}\right)} = 2.53\times10^{-14}\ \text{m} = \boxed{25.3\,\text{fm}}$$

Potential Due to a System of Point Charges

28 •

Picture the Problem Let the numerals 1, 2, 3, and 4 denote the charges at the four corners of square and r the distance from each charge to the center of the square. The potential at the center of square is the algebraic sum of the potentials due to the four charges.

Express the potential at the center of the square:

$$V = \frac{kq_1}{r} + \frac{kq_2}{r} + \frac{kq_3}{r} + \frac{kq_4}{r} = \frac{k}{r}(q_1 + q_2 + q_3 + q_4) = \frac{k}{r}\sum_{i=1}^{4} q_i$$

(*a*) If the charges are positive:

$$V = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{2\sqrt{2}\ \text{m}}(4)(2\ \mu\text{C}) = \boxed{25.4\,\text{kV}}$$

(*b*) If three of the charges are positive and one is negative:

$$V = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{2\sqrt{2}\ \text{m}}(2)(2\ \mu\text{C}) = \boxed{12.7\,\text{kV}}$$

(*c*) If two are positive and two are negative:

$$V = \boxed{0}$$

29 •

Picture the Problem The potential at the point whose coordinates are (0, 3 m) is the algebraic sum of the potentials due to the charges at the three locations given.

Express the potential at the point whose coordinates are (0, 3 m):

$$V = k\sum_{i=1}^{3}\frac{q_i}{r_i} = k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right)$$

(*a*) For $q_1 = q_2 = q_3 = 2\ \mu\text{C}$:

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}+\frac{1}{3\sqrt{2}\,\text{m}}+\frac{1}{3\sqrt{5}\,\text{m}}\right) = \boxed{12.9\,\text{kV}}$$

(*b*) For $q_1 = q_2 = 2\ \mu\text{C}$ and $q_3 = -2\ \mu\text{C}$:

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}+\frac{1}{3\sqrt{2}\,\text{m}}-\frac{1}{3\sqrt{5}\,\text{m}}\right) = \boxed{7.55\,\text{kV}}$$

(*c*) For $q_1 = q_3 = 2\ \mu\text{C}$ and $q_2 = -2\ \mu\text{C}$:

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}-\frac{1}{3\sqrt{2}\,\text{m}}+\frac{1}{3\sqrt{5}\,\text{m}}\right) = \boxed{4.44\,\text{kV}}$$

30 •

Picture the Problem The potential at point C is the algebraic sum of the potentials due to the charges at points A and B and the work required to bring a charge from infinity to point C equals the change in potential energy of the system during this process.

(*a*) Express the potential at point C as the sum of the potentials due to the charges at points A and B:

$$V_C = k\left(\frac{q_A}{r_A}+\frac{q_B}{r_B}\right)$$

Substitute numerical values and evaluate V_C:

$$V_C = kq\left(\frac{1}{r_A}+\frac{1}{r_B}\right) = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}+\frac{1}{3\,\text{m}}\right) = \boxed{12.0\,\text{kV}}$$

(*b*) Express the required work in terms of the change in the potential energy of the system:

$$W = \Delta U = q_5V_C$$
$$= (5\,\mu\text{C})(12.0\,\text{kV}) = \boxed{60.0\,\text{mJ}}$$

(*c*) Proceed as in (*a*) with $q_B = -2\ \mu\text{C}$:

$$V_C = k\left(\frac{q_A}{r_A} + \frac{q_B}{r_B}\right) = \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{2\,\mu\mathrm{C}}{3\,\mathrm{m}} + \frac{-2\,\mu\mathrm{C}}{3\,\mathrm{m}}\right) = \boxed{0}$$

and $W = \Delta U = q_5 V_C = (5\,\mu\mathrm{C})(0) = \boxed{0}$

31 •

Picture the Problem The electric potential at the origin and at the north pole is the algebraic sum of the potentials at those points due to the individual charges distributed along the equator.

(*a*) Express the potential at the origin as the sum of the potentials due to the charges placed at 60° intervals along the equator of the sphere:

$$V = k\sum_{i=1}^{6}\frac{q_i}{r_i} = 6k\frac{q}{r}$$

Substitute numerical values and evaluate *V*:

$$V = 6\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{3\,\mu\mathrm{C}}{0.6\,\mathrm{m}}$$

$$= \boxed{270\,\mathrm{kV}}$$

(*b*) Using geometry, find the distance from each charge to the north pole:

$$r' = 0.6\sqrt{2}\,\mathrm{m}$$

Proceed as in (*a*) with $r' = 0.6\sqrt{2}\,\mathrm{m}$:

$$V = k\sum_{i=1}^{6}\frac{q_i}{r_i'} = 6k\frac{q}{r'}$$

$$= 6\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{3\,\mu\mathrm{C}}{0.6\sqrt{2}\,\mathrm{m}}$$

$$= \boxed{191\,\mathrm{kV}}$$

***32** •

Picture the Problem We can use the fact that the electric potential at the point of interest is the algebraic sum of the potentials at that point due to the charges *q* and *q′* to find the ratio *q/q′*.

Express the potential at the point of interest as the sum of the potentials due to the two charges:

$$\frac{kq}{a/3} + \frac{kq'}{2a/3} = 0$$

Simplify to obtain: $$q+\frac{q'}{2}=0$$

Solve for the ratio q/q': $$\frac{q}{q'}=\boxed{-\frac{1}{2}}$$

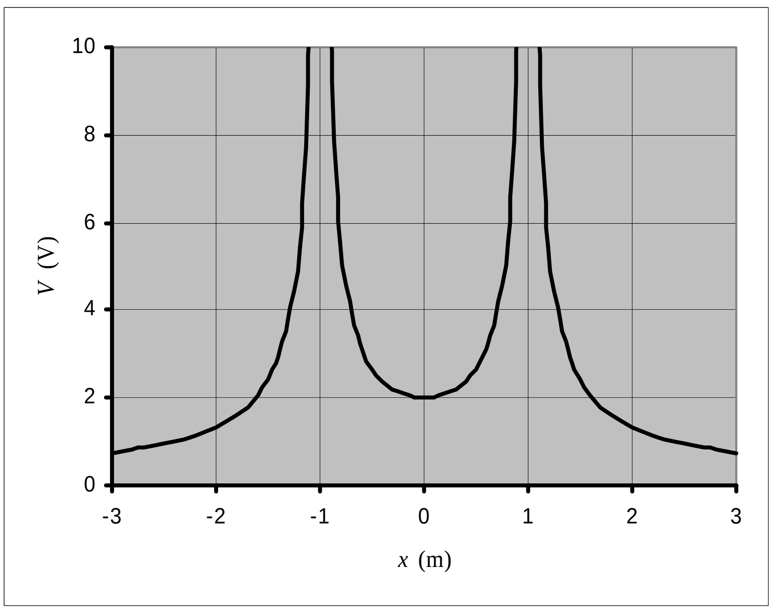
33 ••

Picture the Problem For the two charges, $r=|x-a|$ and $|x+a|$ respectively and the electric potential at x is the algebraic sum of the potentials at that point due to the charges at $x = +a$ and $x = –a$.

(*a*) Express $V(x)$ as the sum of the potentials due to the charges at $x = +a$ and $x = –a$:

$$V=\boxed{kq\left(\frac{1}{|x-a|}+\frac{1}{|x+a|}\right)}$$

(*b*) The following graph of $V(x)$ versus x for $kq = 1$ and $a = 1$ was plotted using a spreadsheet program:



(*c*) At $x = 0$: $$\frac{dV}{dx}=\boxed{0}\text{ and }E_x=-\frac{dV}{dx}=\boxed{0}$$

***34** ••

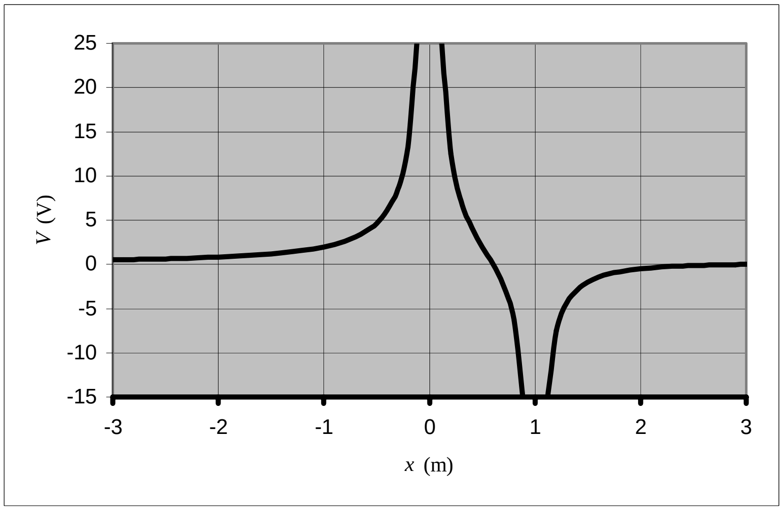
Picture the Problem For the two charges, $r=|x-a|$ and $|x|$ respectively and the electric potential at x is the algebraic sum of the potentials at that point due to the charges at $x = a$ and $x = 0$. We can use the graph and the function found in part (a) to identify the points at which $V(x) = 0$. We can find the work needed to bring a third charge $+e$ to the point

$x = \frac{1}{2}a$ on the x axis from the change in the potential energy of this third charge.

Express the potential at x:

$$V(x) = \frac{k(3e)}{|x|} + \frac{k(-2e)}{|x-a|}$$

The following graph of $V(x)$ for ke = 1 and a =1 was plotted using a spreadsheet program.



(b) From the graph we can see that $V(x)$ = 0 when:

$$x = \boxed{\pm\infty}$$

Examining the function, we see that $V(x)$ is also zero provided:

$$\frac{3}{|x|} - \frac{2}{|x-a|} = 0$$

For $x > 0$, $V(x) = 0$ when:

$$x = \boxed{3a}$$

For $0 < x < a$, $V(x) = 0$ when:

$$x = \boxed{0.6a}$$

(c) Express the work that must be done in terms of the change in potential energy of the charge:

$$W = \Delta U = qV\left(\tfrac{1}{2}a\right)$$

Evaluate the potential at $x = \frac{1}{2}a$:

$$V\left(\tfrac{1}{2}a\right) = \frac{k(3e)}{\left|\frac{1}{2}a\right|} + \frac{k(-2e)}{\left|\frac{1}{2}a - a\right|}$$

$$= \frac{6ke}{a} - \frac{4ke}{a} = \frac{2ke}{a}$$

Substitute to obtain:

$$W = e\left(\frac{2ke}{a}\right) = \boxed{\frac{2ke^2}{a}}$$

Computing the Electric Field from the Potential

35 •

Picture the Problem We can use the relationship $E_x = -(dV/dx)$ to decide the sign of $V_b - V_a$ and $E = \Delta V/\Delta x$ to find E.

(*a*) Because $E_x = -(dV/dx)$, V is greater for larger values of x. So:

$$\boxed{V_b - V_a \text{ is positive.}}$$

(*b*) Express E in terms of $V_b - V_a$ and the separation of points a and b:

$$E_x = \frac{\Delta V}{\Delta x} = \frac{V_b - V_a}{\Delta x}$$

Substitute numerical values and evaluate E_x:

$$E_x = \frac{10^5\ \text{V}}{4\,\text{m}} = \boxed{25.0\ \text{kV/m}}$$

***36** •

Picture the Problem Because $E_x = -dV/dx$, we can find the point(s) at which $E_x = 0$ by identifying the values for x for which $dV/dx = 0$.

Examination of the graph indicates that $dV/dx = 0$ at $x = 4.5$ m. Thus $E_x = 0$ at:

$$x = \boxed{4.5\,\text{m}}$$

37 •

Picture the Problem We can use $V(x) = kq/x$ to find the potential V on the x axis at $x = 3.00$ m and at $x = 3.01$ m and $E(x) = kq/r^2$ to find the electric field at $x = 3.00$ m. In part (d) we can express the off-axis potential using $V(x) = kq/r$, where $r = \sqrt{x^2 + y^2}$.

(*a*) Express the potential on the x axis as a function of x and q:

$$V(x) = \frac{kq}{x}$$

Evaluate V at $x = 3$ m:

$$V(3\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\mu\text{C})}{3\,\text{m}} = \boxed{8.99\,\text{kV}}$$

Evaluate V at x = 3.01 m:

$$V(3.01\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\,\mu\text{C})}{3.01\,\text{m}} = \boxed{8.96\,\text{kV}}$$

(b) The potential decreases as x increases and:

$$-\frac{\Delta V}{\Delta x} = -\frac{8.96\,\text{kV} - 8.99\,\text{kV}}{3.01\,\text{m} - 3.00\,\text{m}} = \boxed{3.00\,\text{kV/m}}$$

(c) Express the Coulomb field as a function of x:

$$E(x) = \frac{kq}{x^2}$$

Evaluate this expression at x = 3.00 m to obtain:

$$E(3\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\,\mu\text{C})}{(3\,\text{m})^2} = \boxed{3.00\,\text{kV/m}}$$

in agreement with our result in (b).

(d) Express the potential at (x, y) due to a point charge q at the origin:

$$V(x, y) = \frac{kq}{\sqrt{x^2 + y^2}}$$

Evaluate this expression at (3.00 m, 0.01 m):

$$V(3.00\,\text{m}, 0.01\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\,\mu\text{C})}{\sqrt{(3.00\,\text{m})^2 + (0.01\,\text{m})^2}} = \boxed{8.99\,\text{kV}}$$

For $y << x$, V is independent of y and the points (x, 0) and (x, y) are at the same potential, i.e., on an equipotential surface.

38 •

Picture the Problem We can find the potential on the x axis at x = 3.00 m by expressing it as the sum of the potentials due to the charges at the origin and at x = 6 m. We can also express the Coulomb field on the x axis as the sum of the fields due to the charges q_1 and q_2 located at the origin and at x = 6 m.

(a) Express the potential on the x axis as the sum of the potentials due to the charges q_1 and q_2 located at the origin and at x = 6 m:

$$V(x) = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Substitute numerical values and evaluate V(3 m):

$$V(x) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\times\left(\frac{3\,\mu\text{C}}{3\,\text{m}}+\frac{-3\,\mu\text{C}}{3\,\text{m}}\right) = \boxed{0}$$

(b) Express the Coulomb field on the x axis as the sum of the fields due to the charges q_1 and q_2 located at the origin and at
x = 6 m:

$$E_x = \frac{kq_1}{r_1^2}+\frac{kq_2}{r_2^2} = k\left(\frac{q_1}{r_1^2}+\frac{q_2}{r_2^2}\right)$$

Substitute numerical values and evaluate E(3 m):

$$E_x = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\times\left(\frac{3\,\mu\text{C}}{(3\,\text{m})^2}-\frac{-3\,\mu\text{C}}{(3\,\text{m})^2}\right) = \boxed{5.99\,\text{kV/m}}$$

(c) Express the potential on the x axis as the sum of the potentials due to the charges q_1 and q_2 located at the origin and at
x = 6 m:

$$V(x) = k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$$

Substitute numerical values and evaluate V(3.01 m):

$$V(3.01\,\text{m}) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\times\left(\frac{3\,\mu\text{C}}{3.01\,\text{m}}+\frac{-3\,\mu\text{C}}{2.99\,\text{m}}\right) = \boxed{-59.9\,\text{V}}$$

Compute $-\Delta V/\Delta x$:

$$-\frac{\Delta V}{\Delta x} = -\frac{-59.9\,\text{V}-0}{3.01\,\text{m}-3.00\,\text{m}} = \boxed{5.99\,\text{kV/m}} = E_x(3.00\,\text{m})$$

39 •

Picture the Problem We can use the relationship $E_y = -(dV/dy)$ to decide the sign of $V_b - V_a$ and $E = \Delta V/\Delta y$ to find E.

(a) Because $E_x = -(dV/dx)$, V is smaller for larger values of y. So:

$$\boxed{V_b - V_a \text{ is negative.}}$$

(*b*) Express E in terms of $V_b - V_a$ and the separation of points *a* and *b*:

$$E_y = \frac{\Delta V}{\Delta y} = \frac{V_b - V_a}{\Delta y}$$

Substitute numerical values and evaluate E_y:

$$E_y = \frac{2\times 10^4\ \text{V}}{4\ \text{m}} = \boxed{5.00\ \text{kV/m}}$$

40 •

Picture the Problem Given $V(x)$, we can find E_x from $-dV/dx$.

(*a*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[2000 + 3000x] = \boxed{-3.00\ \text{kV/m}}$$

(*b*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[4000 + 3000x] = \boxed{-3.00\ \text{kV/m}}$$

(*c*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[2000 - 3000x] = \boxed{3.00\ \text{kV/m}}$$

(*d*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[-2000] = \boxed{0}$$

41 ••

Picture the Problem We can express the potential at a general point on the *x* axis as the sum of the potentials due to the charges at $x = 0$ and $x = 1$ m. Setting this expression equal to zero will identify the points at which $V(x) = 0$. We can find the electric field at any point on the *x* axis from $E_x = -dV/dx$.

(*a*) Express $V(x)$ as the sum of the potentials due to the point charges at $x = 0$ and $x = 1$ m:

$$V(x) = \frac{kq}{|x|} + \frac{k(-3q)}{|x-1|} = \boxed{k\left(\frac{q}{|x|} - \frac{3q}{|x-1|}\right)}$$

(*b*) Set $V(x) = 0$:

$$k\left(\frac{q}{|x|} - \frac{3q}{|x-1|}\right) = 0$$

or

$$\frac{1}{|x|}-\frac{3}{|x-1|}=0$$

For $x < 0$:

$$\frac{1}{-x}-\frac{3}{-(x-1)}=0 \Rightarrow x=\boxed{-0.500\,\text{m}}$$

For $0 < x < 1$:

$$\frac{1}{x}-\frac{3}{-(x-1)}=0 \Rightarrow x=\boxed{0.250\,\text{m}}$$

Note also that:

$$\boxed{V(x)\to 0\,\text{as}\,x\to\pm\infty}$$

(*c*) Evaluate $V(x)$ for $0 < x < 1$:

$$V(0<x<1)=k\left(\frac{q}{x}+\frac{3q}{x-1}\right)$$

Apply $E_x = -dV/dx$ to find E_x in this region:

$$E_x(0<x<1)=-\frac{d}{dx}\left[k\left(\frac{q}{x}+\frac{3q}{x-1}\right)\right]$$
$$=kq\left[\frac{1}{x^2}+\frac{3}{(x-1)^2}\right]$$

Evaluate this expression at $x = 0.25$ m to obtain:

$$E_x(0.25\,\text{m})=kq\left[\frac{1}{(0.25\,\text{m})^2}+\frac{3}{(0.75\,\text{m})^2}\right]$$
$$=\boxed{\left(21.3\,\text{m}^{-2}\right)kq}$$

Evaluate $V(x)$ for $x < 0$:

$$V(x<0)=-kq\left(\frac{1}{x}+\frac{3}{1-x}\right)$$

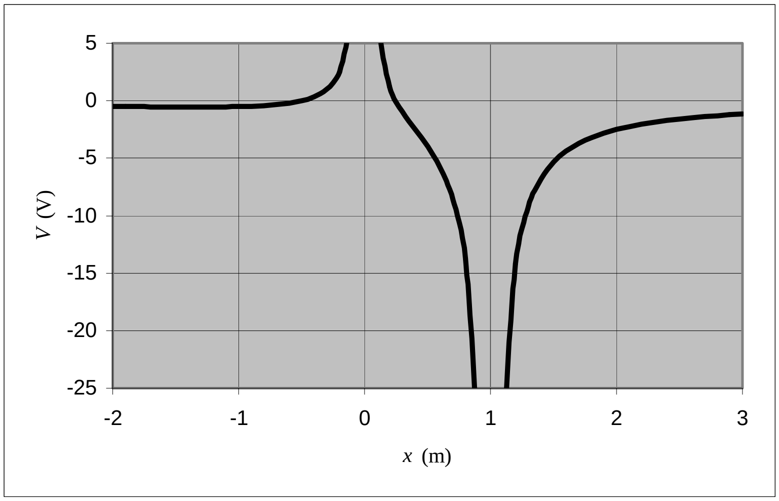
Apply $E_x = -dV/dx$ to find E_x in this region:

$$E_x(x<0)=-kq\frac{d}{dx}\left[\frac{1}{x}+\frac{3}{1-x}\right]$$
$$=kq\left[-\frac{1}{x^2}+\frac{3}{(1-x)^2}\right]$$

Evaluate this expression at $x = -0.5$ m to obtain:

$$E_x(-0.5\,\text{m})=kq\left[-\frac{1}{(-0.5\,\text{m})^2}+\frac{3}{(1.5\,\text{m})^2}\right]=\boxed{\left(-2.67\,\text{m}^{-2}\right)kq}$$

(*d*) The following graph of $V(x)$ for $kq = 1$ and $a = 1$ was plotted using a spreadsheet program:



***42 ••**

Picture the Problem Because $V(x)$ and E_x are related through $E_x = -\,dV/dx$, we can find V from E by integration.

Separate variables to obtain:

$$dV = -E_x dx = -\left(2.0x^3\ \text{kN/C}\right)dx$$

Integrate V from V_1 to V_2 and x from 1 m to 2 m:

$$\int_{V_1}^{V_2} dV = -\left(2.0\,\text{kN/C}\right)\int_{x_1}^{x_2} x^3 dx$$
$$= -\left(2.0\,\text{kN/C}\right)\left[\tfrac{1}{4}x^4\right]_{1\,\text{m}}^{2\,\text{m}}$$

Simplify to obtain:

$$V_2 - V_1 = \boxed{-7.50\,\text{kV}}$$

43 ••

Picture the Problem Let r_1 be the distance from (0, a) to (x, 0), r_2 the distance from (0, $-a$), and r_3 the distance from (a, 0) to (x, 0). We can express $V(x)$ as the sum of the potentials due to the charges at (0, a), (0, $-a$), and (a, 0) and then find E_x from $-dV/dx$.

(*a*) Express $V(x)$ as the sum of the potentials due to the charges at (0, a), (0, $-a$), and (a, 0):

$$V(x) = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

where $q_1 = q_2 = q_3 = q$

At $x = 0$, the fields due to q_1 and q_2 cancel, so $E_x(0) = -kq/a^2$; this is also obtained from (*b*) if $x = 0$.

As $x \to \infty$, i.e., for $x >> a$, the three charges appear as a point charge $3q$, so $E_x = 3kq/x^2$; this is also the result one obtains from (*b*) for $x >> a$.

Substitute for the r_i to obtain:

$$V(x) = kq\left(\frac{1}{\sqrt{x^2+a^2}} + \frac{1}{\sqrt{x^2+a^2}} + \frac{1}{|x-a|}\right) = \boxed{kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{|x-a|}\right)}$$

(*b*) For $x > a$, $x - a > 0$ and:

$$|x-a| = x-a$$

Use $E_x = -dV/dx$ to find E_x:

$$E_x(x>a) = -\frac{d}{dx}\left[kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{x-a}\right)\right] = \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}} + \frac{kq}{(x-a)^2}}$$

For $x < a$, $x - a < 0$ and:

$$|x-a| = -(x-a) = a-x$$

Use $E_x = -dV/dx$ to find E_x:

$$E_x(x<a) = -\frac{d}{dx}\left[kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{a-x}\right)\right] = \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}} - \frac{kq}{(a-x)^2}}$$

Calculations of V for Continuous Charge Distributions

44 •

Picture the Problem We can construct Gaussian surfaces just inside and just outside the spherical shell and apply Gauss's law to find the electric field at these locations. We can use the expressions for the electric potential inside and outside a spherical shell to find the potential at these locations.

(*a*) Apply Gauss's law to a spherical Gaussian surface of radius $r < 12$ cm:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

because the charge resides on the outer surface of the spherical surface. Hence

$$\vec{E}(r < 12\,\text{cm}) = \boxed{0}$$

Apply Gauss's law to a spherical Gaussian surface of radius

$$E\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$$

r > 12 cm: and

$$E(r>12\,\text{cm})=\frac{q}{4\pi r^2\,\epsilon_0}=\frac{kq}{r^2}$$

Substitute numerical values and evaluate $E(r>12\,\text{cm})$:

$$E(r>12\,\text{cm})=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{(0.12\,\text{m})^2}=\boxed{6.24\,\text{kV/m}}$$

(*b*) Express and evaluate the potential just inside the spherical shell:

$$V(r\le R)=\frac{kq}{R}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}}=\boxed{749\,\text{V}}$$

Express and evaluate the potential just outside the spherical shell:

$$V(r\ge R)=\frac{kq}{r}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}}=\boxed{749\,\text{V}}$$

(*c*) The electric potential inside a uniformly charged spherical shell is constant and given by:

$$V(r\le R)=\frac{kq}{R}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}}=\boxed{749\,\text{V}}$$

In part (*a*) we showed that: $\vec{\boldsymbol{E}}(r<12\,\text{cm})=\boxed{0}$

45 •

Picture the Problem We can use the expression for the potential due to a line charge $V=-2k\lambda\ln\frac{r}{a}$, where V = 0 at some distance r = a, to find the potential at these distances from the line.

Express the potential due to a line charge as a function of the distance from the line:

$$V=-2k\lambda\ln\frac{r}{a}$$

Because V = 0 at r = 2.5 m:

$$0=-2k\lambda\ln\frac{2.5\,\text{m}}{a},$$

$$0 = \ln\frac{2.5\,\text{m}}{a},$$

and

$$\frac{2.5\,\text{m}}{a} = \ln^{-1} 0 = 1$$

Thus we have a = 2.5 m and:

$$V = -2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.5\,\mu\text{C/m}\right)\ln\left(\frac{r}{2.5\,\text{m}}\right) = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{r}{2.5\,\text{m}}\right)$$

(*a*) Evaluate *V* at *r* = 2.0 m:

$$V = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{2\,\text{m}}{2.5\,\text{m}}\right) = \boxed{6.02\,\text{kV}}$$

(*b*) Evaluate *V* at *r* = 4.0 m:

$$V = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{4\,\text{m}}{2.5\,\text{m}}\right) = \boxed{-12.7\,\text{kV}}$$

(*c*) Evaluate *V* at *r* = 12.0 m:

$$V = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{12\,\text{m}}{2.5\,\text{m}}\right) = \boxed{-42.3\,\text{kV}}$$

46 ••

Picture the Problem The electric field on the *x* axis of a disk charge of radius *R* is given by $E_x = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$. We'll choose $V(\infty) = 0$ and integrate from $x' = \infty$ to $x' = x$ to obtain Equation 23-21.

Relate the electric potential on the axis of a disk charge to the electric field of the disk:

$$dV = -E_x dx$$

Express the electric field on the *x* axis of a disk charge:

$$E_x = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

Substitute to obtain:

$$dV = -2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx$$

Let $V(\infty) = 0$ and integrate from $x' = \infty$ to $x' = x$:

$$V = -2\pi k\sigma\int_{\infty}^{x}\left(1-\frac{x'}{\sqrt{x'^2+R^2}}\right)dx'$$

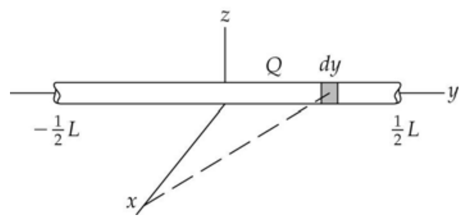
$$= 2\pi k\sigma\left(\sqrt{x^2+R^2}-x\right)$$

$$= \boxed{2\pi k\sigma|x|\left(\sqrt{1+\frac{R^2}{x^2}}-1\right)}$$

which is Equation 23-21.

***47** ••

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy. We can express the potential dV at any point on the x axis due to λdy and integrate of find $V(x, 0)$.



(*a*) Express the element of potential dV due to the line element dy:

$$dV = \frac{k\lambda}{r}dy$$

where $r = \sqrt{x^2+y^2}$

Integrate dV from $y = -L/2$ to $y = L/2$:

$$V(x,0) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{\sqrt{x^2+y^2}}$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{\sqrt{x^2+L^2/4}+L/2}{\sqrt{x^2+L^2/4}-L/2}\right)}$$

(*b*) Factor x from the numerator and denominator within the parentheses to obtain:

$$V(x,0) = \frac{kQ}{L}\ln\left(\frac{\sqrt{1+\frac{L^2}{4x^2}}+\frac{L}{2x}}{\sqrt{1+\frac{L^2}{4x^2}}-\frac{L}{2x}}\right)$$

Use $\ln\frac{a}{b} = \ln a - \ln b$ to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\ln\left(\sqrt{1+\frac{L^2}{4x^2}}+\frac{L}{2x}\right)-\ln\left(\sqrt{1+\frac{L^2}{4x^2}}-\frac{L}{2x}\right)\right\}$$

Let $\varepsilon = \dfrac{L^2}{4x^2}$ and use $(1+\varepsilon)^{1/2} = 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \ldots$ to expand $\sqrt{1+\dfrac{L^2}{4x^2}}$:

$$\left(1+\frac{L^2}{4x^2}\right)^{1/2} = 1 + \frac{1}{2}\frac{L^2}{4x^2} - \frac{1}{8}\left(\frac{L^2}{4x^2}\right)^2 + \ldots \approx 1 \text{ for } x >> L.$$

Substitute to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\ln\left(1+\frac{L}{2x}\right) - \ln\left(1-\frac{L}{2x}\right)\right\}$$

Let $\delta = \dfrac{L}{2x}$ and use $\ln(1+\delta) = \delta - \frac{1}{2}\delta^2 + \ldots$ to expand $\ln\left(1 \pm \dfrac{L}{2x}\right)$:

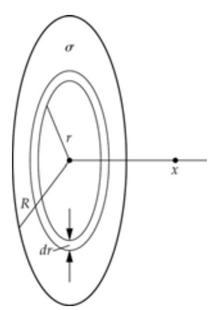
$$\ln\left(1+\frac{L}{2x}\right) \approx \frac{L}{2x} - \frac{L^2}{4x^2} \text{ and } \ln\left(1-\frac{L}{2x}\right) \approx -\frac{L}{2x} - \frac{L^2}{4x^2} \text{ for } x >> L.$$

Substitute and simplify to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\frac{L}{2x} - \frac{L^2}{4x^2} - \left(-\frac{L}{2x} - \frac{L^2}{4x^2}\right)\right\} = \boxed{\frac{kQ}{x}}$$

48 ••

Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(*a*) Express the charge dq on a ring of radius r and thickness dr:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0 \frac{R}{r}\right)dr = 2\pi\sigma_0 R dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q = 2\pi\sigma_0 R\int_0^R dr = \boxed{2\pi\sigma_0 R^2}$$

(*b*) Express the potential on the axis of the disk due to a circular element of charge $dq = 2\pi r\sigma dr$:

$$dV = \frac{kdq}{r'} = \frac{2\pi k\sigma_0 Rdr}{\sqrt{x^2 + r^2}}$$

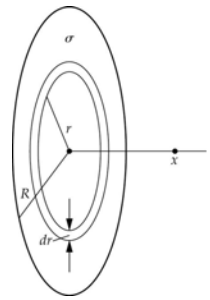
Integrate from $r = 0$ to $r = R$ to obtain:

$$V = 2\pi k\sigma_0 R\int_0^R \frac{dr}{\sqrt{x^2 + r^2}}$$

$$= \boxed{2\pi k\sigma_0 R\ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right)}$$

49 ••

Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(*a*) Express the charge dq on a ring of radius r and thickness dr:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0 \frac{r^2}{R^2}\right)dr$$

$$= \frac{2\pi\sigma_0}{R^2} r^3 dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q = \frac{2\pi\sigma_0}{R^2}\int_0^R r^3 dr = \boxed{\tfrac{1}{2}\pi\sigma_0 R^2}$$

(*b*)Express the potential on the axis of the disk due to a circular element of charge $dq = \dfrac{2\pi\sigma_0}{R^2} r^3 dr$:

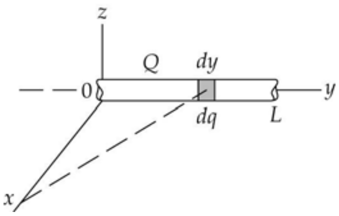
$$dV = \frac{kdq}{r'} = \frac{2\pi k\sigma_0}{R^2}\frac{r^3}{\sqrt{x^2 + r^2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$V = \frac{2\pi k\sigma_0}{R^2}\int_0^R \frac{r^3 dr}{\sqrt{x^2 + r^2}} = \boxed{\frac{2\pi k\sigma_0}{R^2}\left(\frac{R^2 - 2x^2}{3}\sqrt{x^2 + R^2} + \frac{2x^3}{3}\right)}$$

50 ••

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy. We can express the potential dV at any point on the x axis due to λdy and integrate to find $V(x, 0)$.



Express the element of potential dV due to the line element dy:

$$dV = \frac{k\lambda}{r}dy$$

where $r = \sqrt{x^2 + y^2}$

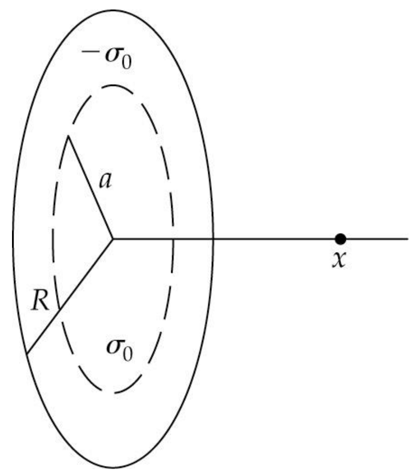
Integrate dV from $y = -L/2$ to $y = L/2$:

$$V(x,0) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{\sqrt{x^2 + y^2}}$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{\sqrt{x^2 + L^2/4} + L/2}{\sqrt{x^2 + L^2/4} - L/2}\right)}$$

***51** ••

Picture the Problem The potential at any location on the axis of the disk is the sum of the potentials due to the positive and negative charge distributions on the disk. Knowing that the total charge on the disk is zero and the charge densities are equal in magnitude will allow us to find the radius of the region that is positively charged. We can then use the expression derived in the text to find the potential due to this charge closest to the axis and integrate dV from $r = R/\sqrt{2}$ to $r = R$ to find the potential at x due to the negative charge distribution.



(*a*) Express the potential at a distance x along the axis of the disk as the sum of the potentials due to the positively and negatively charged regions of the disk:

$$V(x) = V_+(x) + V_-(x)$$

We know that the charge densities are equal in magnitude and that the

$$Q_{r<a} = Q_{r>a}$$

or

total charge carried by the disk is zero. Express this condition in terms of the charge in each of two regions of the disk:

$$\sigma_0 \pi a^2 = \sigma_0 \pi R^2 - \sigma_0 \pi a^2$$

Solve for a to obtain:

$$a = \frac{R}{\sqrt{2}}$$

Use this result and the general expression for the potential on the axis of a charged disk to express $V_+(x)$:

$$V_+(x) = 2\pi k\sigma_0\left(\sqrt{x^2 + \frac{R^2}{2}} - x\right)$$

Express the potential on the axis of the disk due to a ring of charge a distance $r > a$ from the axis of the ring:

$$dV_-(x) = -2\pi k\sigma_0 \frac{r}{r'}dr$$

where $r' = \sqrt{x^2 + r^2}$.

Integrate this expression from $r = R/\sqrt{2}$ to $r = R$ to obtain:

$$V_-(x) = -2\pi k\sigma_0 \int_{R/\sqrt{2}}^{R} \frac{r}{\sqrt{x^2 + r^2}}dr$$

$$= -2\pi k\sigma_0\left(\sqrt{x^2 + R^2} - \sqrt{x^2 + \frac{R^2}{2}}\right)$$

Substitute and simplify to obtain:

$$V(x) = 2\pi k\sigma_0\left(\sqrt{x^2 + \frac{R^2}{2}} - x\right) - 2\pi k\sigma_0\left(\sqrt{x^2 + R^2} - \sqrt{x^2 + \frac{R^2}{2}}\right)$$

$$= 2\pi k\sigma_0\left(\sqrt{x^2 + \frac{R^2}{2}} - x - \sqrt{x^2 + R^2} + \sqrt{x^2 + \frac{R^2}{2}}\right)$$

$$= \boxed{2\pi k\sigma_0\left(2\sqrt{x^2 + \frac{R^2}{2}} - \sqrt{x^2 + R^2} - x\right)}$$

(*b*) To determine V for $x >> R$, factor x from the square roots and expand using the binomial expansion:

$$\sqrt{x^2 + \frac{R^2}{2}} = x\left(1 + \frac{R^2}{2x^2}\right)^{1/2}$$

$$\approx x\left(1 + \frac{R^2}{4x^2} - \frac{R^4}{32x^4}\right)$$

and

$$\sqrt{x^2+R^2} = x\left(1+\frac{R^2}{x^2}\right)^{1/2}$$
$$\approx x\left(1+\frac{R^2}{2x^2}-\frac{R^4}{8x^4}\right)$$

Substitute to obtain:

$$V(x)\approx 2\pi k\sigma_0\left(2x\left(1+\frac{R^2}{4x^2}-\frac{R^4}{32x^4}\right)-x\left(1+\frac{R^2}{2x^2}-\frac{R^4}{8x^4}\right)-x\right)=\boxed{\frac{\pi k\sigma_0 R^4}{8x^3}}$$

52 ••

Picture the Problem Given the potential function $V(x)=2\pi k\sigma_0\left(2\sqrt{x^2+R^2/2}-\sqrt{x^2+R^2}-x\right)$ found in Problem 51(*a*), we can find E_x from $-dV/dx$. In the second part of the problem, we can find the electric field on the axis of the disk by integrating Coulomb's law for the oppositely charged regions of the disk and expressing the sum of the two fields.

Relate E_x to dV/dx:

$$E_x=-\frac{dV}{dx}$$

From Problem 51(*a*) we have:

$$V(x)=2\pi k\sigma_0\left(2\sqrt{x^2+\frac{R^2}{2}}-\sqrt{x^2+R^2}-x\right)$$

Evaluate the negative of the derivative of $V(x)$ to obtain:

$$E_x=-2\pi k\sigma_0\frac{d}{dx}\left(2\sqrt{x^2+\frac{R^2}{2}}-\sqrt{x^2+R^2}-x\right)$$
$$=\boxed{-2\pi k\sigma_0\left(\frac{2x}{\sqrt{x^2+\frac{R^2}{2}}}-\frac{x}{\sqrt{x^2+R^2}}-1\right)}$$

Express the field on the axis of the disk as the sum of the field due to the positive charge on the disk and the field due to the negative charge

$$E_x=E_{x-}+E_{x+}$$

on the disk:

The field due to the positive charge (closest to the axis) is:

$$E_{x+} = 2\pi k\sigma_0\left(1 - \frac{x}{\sqrt{x^2 + \frac{R^2}{2}}}\right)$$

To determine E_{x-} we integrate the field due to a ring charge:

$$E_{x-} = -2\pi k\sigma_0 \int_{R/\sqrt{2}}^{R} \frac{rdr}{\left(x^2 + r^2\right)^{3/2}}$$

$$= -2\pi k\sigma_0\left(\frac{x}{\sqrt{x^2 + \frac{R^2}{2}}} - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

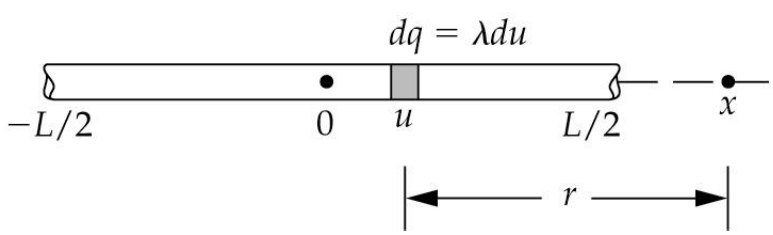
Substitute and simplify to obtain:

$$E_x = -2\pi k\sigma_0\left(\frac{x}{\sqrt{x^2 + \frac{R^2}{2}}} - \frac{x}{\sqrt{x^2 + R^2}}\right) + 2\pi k\sigma_0\left(1 - \frac{x}{\sqrt{x^2 + \frac{R^2}{2}}}\right)$$

$$= \boxed{-2\pi k\sigma_0\left(\frac{2x}{\sqrt{x^2 + \frac{R^2}{2}}} - \frac{x}{\sqrt{x^2 + R^2}} - 1\right)}$$

53 ••

Picture the Problem We can express the electric potential *dV* at *x* due to an elemental charge *dq* on the rod and then integrate over the length of the rod to find *V*(*x*). In the second part of the problem we use a binomial expansion to show that, for $x >> L/2$, our result reduces to that due to a point charge *Q*.



(*a*) Express the potential at *x* due to the element of charge *dq* located at *u*:

$$dV = \frac{kdq}{r} = \frac{k\lambda du}{x-u}$$

or, because $\lambda = Q/L$,

$$dV = \frac{kQ}{L}\frac{du}{x-u}$$

Integrate *V* from $u = -L/2$ to $L/2$ to obtain:

$$V(x) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{du}{x-u}$$

$$= \frac{kQ}{L}\ln(x-u)\Big|_{-L/2}^{L/2}$$

$$= \left[-\ln\left(x-\frac{L}{2}\right)+\ln\left(x+\frac{L}{2}\right)\right]$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right)}$$

(*b*) Divide the numerator and denominator of the argument of the logarithm by *x* to obtain:

$$\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right) = \ln\left(\frac{1+\frac{L}{2x}}{1-\frac{L}{2x}}\right) = \ln\left(\frac{1+a}{1-a}\right)$$

where $a = L/2x$.

Divide $1 + a$ by $1 - a$ to obtain:

$$\ln\left(\frac{1+a}{1-a}\right) = \ln\left(1+2a+\frac{2a^2}{1-a}\right)$$

$$= \ln\left(1+\frac{L}{x}+\frac{\frac{L^2}{x^2}}{2-\frac{L}{x}}\right)$$

$$\approx \ln\left(1+\frac{L}{x}\right)$$

provided $x >> L/2$.

Expand $\ln(1 + L/x)$ binomially to obtain:

$$\ln\left(1+\frac{L}{x}\right) \approx \frac{L}{x}$$

provided $x >> L/2$.

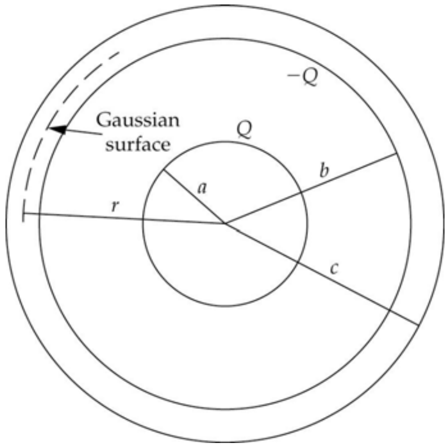
Substitute to express $V(x)$ for $x >> L/2$:

$$V(x) = \frac{kQ}{L}\frac{L}{x} = \boxed{\frac{kQ}{x}}$$, the field due to a

point charge Q.

54 ••

Picture the Problem The diagram is a cross-sectional view showing the charges on the sphere and the spherical conducting shell. A portion of the Gaussian surface over which we'll integrate E in order to find V in the region $r > b$ is also shown. For $a < r < b$, the sphere acts like point charge Q and the potential of the metal sphere is the sum of the potential due to a point charge at its center and the potential at its surface due to the charge on the inner surface of the spherical shell.



(*a*) Express $V_{r>b}$:

$$V_{r>b} = -\int E_{r>b}\,dr$$

Apply Gauss's law for $r > b$:

$$\oint_S \vec{E}_r \cdot \hat{n}\,dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0$$

and $E_{r>b} = 0$ because $Q_{\text{enclosed}} = 0$ for $r > b$.

Substitute to obtain:

$$V_{r>b} = -\int (0)\,dr = \boxed{0}$$

(*b*) Express the potential of the metal sphere:

$$V_a = V_{Q\text{ at its center}} + V_{\text{surface}}$$

Express the potential at the surface of the metal sphere:

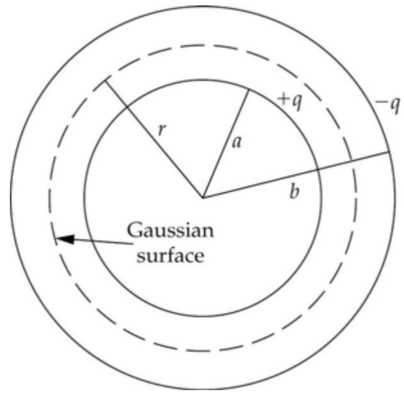
$$V_{\text{surface}} = \frac{k(-Q)}{b} = -\frac{kQ}{b}$$

Substitute and simplify to obtain:

$$V_a = \frac{kQ}{a} - \frac{kQ}{b} = \boxed{kQ\left(\frac{1}{a} - \frac{1}{b}\right)}$$

55 ••

Picture the Problem The diagram is a cross-sectional view showing the charges on the inner and outer conducting shells. A portion of the Gaussian surface over which we'll integrate *E* in order to find *V* in the region $a < r < b$ is also shown. Once we've determined how *E* varies with *r*, we can find $V_b - V_a$ from $V_b - V_a = -\int E_r dr$.



Express the potential difference $V_b - V_a$:

$$V_b - V_a = -\int E_r dr$$

Apply Gauss's law to cylindrical Gaussian surface of radius *r* and length *L*:

$$\oint_S \vec{E} \cdot \hat{n} dA = E_r (2\pi rL) = \frac{q}{\varepsilon_0}$$

Solve for E_r:

$$E_r = \frac{q}{2\pi\varepsilon_0 rL}$$

Substitute for E_r and integrate from $r = a$ to b:

$$V_b - V_a = -\frac{q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r}$$

$$= \boxed{-\frac{2kq}{L}\ln\left(\frac{b}{a}\right)}$$

56 ••

Picture the Problem Let *R* be the radius of the sphere and *Q* its charge. We can express the potential at the two locations given and solve the resulting equations simultaneously for *R* and *Q*.

Relate the potential of the sphere at its surface to its radius:

$$\frac{kQ}{R} = 450\,\text{V} \qquad (1)$$

Express the potential at a distance of 20 cm from its surface:

$$\frac{kQ}{R + 0.2\,\text{m}} = 150\,\text{V} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\frac{kQ}{R}}{\frac{kQ}{R+0.2\,\text{m}}}=\frac{450\,\text{V}}{150\,\text{V}}$$

or

$$\frac{R+0.2\,\text{m}}{R}=3$$

Solve for R to obtain:

$$R=\boxed{0.100\,\text{m}}$$

Solve equation (1) for Q:

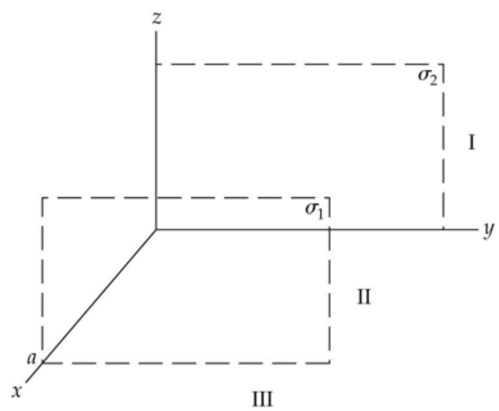
$$Q=(450\,\text{V})\frac{R}{k}$$

Substitute numerical values and evaluate Q:

$$Q=(450\,\text{V})\frac{(0.1\text{m})}{(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)}=\boxed{5.01\text{nC}}$$

57 ••

Picture the Problem Let the charge density on the infinite plane at $x = a$ be σ_1 and that on the infinite plane at $x = 0$ be σ_2. Call that region in space for which $x < 0$, region I, the region for which $0 < x < a$ region II, and the region for which $a < x$ region III. We can integrate E due to the planes of charge to find the electric potential in each of these regions.



(*a*) Express the potential in region I in terms of the electric field in that region:

$$V_\text{I}=-\int_0^x \vec{\boldsymbol{E}}_\text{I}\cdot d\vec{\boldsymbol{x}}$$

Express the electric field in region I as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{\boldsymbol{E}}_\text{I}=-\frac{\sigma_1}{2\in_0}\hat{\boldsymbol{i}}-\frac{\sigma_2}{2\in_0}\hat{\boldsymbol{i}}=-\frac{\sigma}{2\in_0}\hat{\boldsymbol{i}}-\frac{\sigma}{2\in_0}\hat{\boldsymbol{i}}=-\frac{\sigma}{\in_0}\hat{\boldsymbol{i}}$$

Substitute and evaluate V_I:

$$V_I = -\int_0^x \left(-\frac{\sigma}{\epsilon_0}\right)dx = \frac{\sigma}{\epsilon_0}x + V(0)$$
$$= \frac{\sigma}{\epsilon_0}x + 0 = \boxed{\frac{\sigma}{\epsilon_0}x}$$

Express the potential in region II in terms of the electric field in that region:

$$V_{II} = -\int \vec{E}_{II} \cdot d\vec{x} + V(0)$$

Express the electric field in region II as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{E}_{II} = -\frac{\sigma_1}{2\epsilon_0}\hat{i} + \frac{\sigma_2}{2\epsilon_0}\hat{i} = -\frac{\sigma}{2\epsilon_0}\hat{i} + \frac{\sigma}{2\epsilon_0}\hat{i}$$
$$= 0$$

Substitute and evaluate V_{II}:

$$V_{II} = -\int_0^x (0)dx = 0 + V(0) = \boxed{0}$$

Express the potential in region III in terms of the electric field in that region:

$$V_{III} = -\int_a^x \vec{E}_{III} \cdot d\vec{x}$$

Express the electric field in region III as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{E}_{III} = \frac{\sigma_1}{2\epsilon_0}\hat{i} + \frac{\sigma_2}{2\epsilon_0}\hat{i} = \frac{\sigma}{2\epsilon_0}\hat{i} + \frac{\sigma}{2\epsilon_0}\hat{i}$$
$$= \frac{\sigma}{\epsilon_0}\hat{i}$$

Substitute and evaluate V_{III}:

$$V_{III} = -\int_a^x \left(\frac{\sigma}{\epsilon_0}\right)dx = -\frac{\sigma}{\epsilon_0}x + \frac{\sigma}{\epsilon_0}a$$
$$= \boxed{\frac{\sigma}{\epsilon_0}(a - x)}$$

(*b*) Proceed as in (*a*) with $\sigma_1 = -\sigma$ and $\sigma_2 = \sigma$ to obtain:

$$V_I = \boxed{0},$$
$$V_{II} = \boxed{-\frac{\sigma}{\epsilon_0}x} \text{ and } V_{III} = \boxed{-\frac{\sigma}{\epsilon_0}a}$$

***58 ••**

Picture the Problem The potential on the axis of a disk charge of radius R and charge density σ is given by $V = 2\pi k\sigma\left[\left(x^2 + R^2\right)^{1/2} - x\right]$.

Express the potential on the axis of the disk charge:

$$V = 2\pi k\sigma\left[\left(x^2 + R^2\right)^{1/2} - x\right]$$

Factor *x* from the radical and use the binomial expansion to obtain:

$$\left(x^2 + R^2\right)^{1/2} = x\left(1 + \frac{R^2}{x^2}\right)^{1/2} = x\left[1 + \frac{R^2}{2x^2} + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\frac{R^4}{x^4} + \ldots\right]$$
$$\approx x\left[1 + \frac{R^2}{2x^2} - \frac{R^4}{8x^4}\right]$$

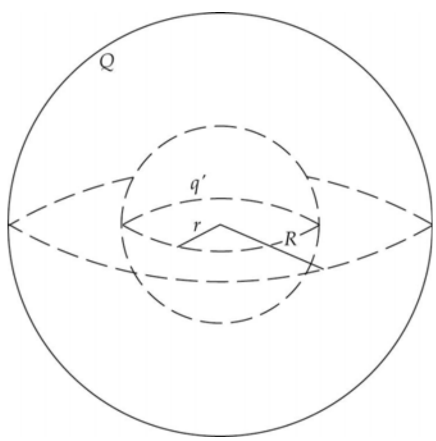
Substitute for the radical term to obtain:

$$V = 2\pi k\sigma\left\{x\left[1 + \frac{R^2}{2x^2} - \frac{R^4}{8x^4}\right] - x\right\}$$
$$= 2\pi k\sigma\left(\frac{R^2}{2x} - \frac{R^4}{8x^3}\right)$$
$$\approx 2\pi k\sigma\left(\frac{R^2}{2x}\right) = \boxed{\frac{kQ}{x}}$$

provided $x >> R$.

59 ••

Picture the Problem The diagram shows a sphere of radius *R* containing a charge *Q* uniformly distributed. We can use the definition of density to find the charge q' inside a sphere of radius *r* and the potential V_1 at *r* due to this part of the charge. We can express the potential dV_2 at *r* due to the charge in a shell of radius r' and thickness dr' at $r' > r$ using $dV_2 = kdq'/r$ and then integrate this expression from $r' = r$ to $r' = R$ to find V_2.



(*a*) Express the potential V_1 at *r* due to q':

$$V_1 = \frac{kq'}{r}$$

Use the definition of density and the fact that the charge density is uniform to relate q' to Q:

$$\rho = \frac{q'}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi R^3}$$

Solve for q':

$$q' = \frac{r^3}{R^3}Q$$

Substitute to express V_1:

$$V_1 = \frac{k}{r}\left(\frac{r^3}{R^3}Q\right) = \boxed{\frac{kQ}{R^3}r^2}$$

(*b*) Express the potential dV_2 at r due to the charge in a shell of radius r' and thickness dr' at $r' > r$:

$$dV_2 = \frac{kdq'}{r}$$

Express the charge dq' in a shell of radius r' and thickness dr' at $r' > r$:

$$dq' = 4\pi r'^2 \rho dr' = 4\pi r'^2\left(\frac{3Q}{4\pi R^3}\right)dr'$$

$$= \frac{3Q}{R^3}r'^2 dr'$$

Substitute to obtain:

$$dV_2 = \boxed{\frac{3kQ}{R^3}r'dr'}$$

(*c*) Integrate dV_2 from $r' = r$ to $r' = R$ to find V_2:

$$V_2 = \frac{3kQ}{R^3}\int_r^R r'dr' = \boxed{\frac{3kQ}{2R^3}\left(R^2 - r^2\right)}$$

(*d*) Express the potential V at r as the sum of V_1 and V_2:

$$V = V_1 + V_2$$

$$= \frac{kQ}{R^3}r^2 + \frac{3kQ}{2R^3}\left(R^2 - r^2\right)$$

$$= \boxed{\frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)}$$

60 •

Picture the Problem We can equate the expression for the electric field due to an infinite plane of charge and $-\Delta V/\Delta x$ and solve the resulting equation for the separation of the equipotential surfaces.

Express the electric field due to the infinite plane of charge:

$$E = \frac{\sigma}{2\in_0}$$

Relate the electric field to the potential:

$$E = -\frac{\Delta V}{\Delta x}$$

Equate these expressions and solve for Δx to obtain:

$$\Delta x = \frac{2\,\epsilon_0\,\Delta V}{\sigma}$$

Substitute numerical values and evaluate $|\Delta x|$:

$$|\Delta x| = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(100\ \text{V})}{3.5\,\mu\text{C/m}^2} = \boxed{0.506\ \text{mm}}$$

61 •

Picture the Problem The equipotentials are spheres centered at the origin with radii $r_i = kq/V_i$.

Evaluate r for V = 20 V:

$$r_{20\,\text{V}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{9}\times10^{-8}\ \text{C}\right)}{20\ \text{V}} = \boxed{0.499\ \text{m}}$$

Evaluate r for V = 40 V:

$$r_{40\,\text{V}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{9}\times10^{-8}\ \text{C}\right)}{40\ \text{V}} = \boxed{0.250\ \text{m}}$$

Evaluate r for V = 60 V:

$$r_{60\,\text{V}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{9}\times10^{-8}\ \text{C}\right)}{60\ \text{V}} = \boxed{0.166\ \text{m}}$$

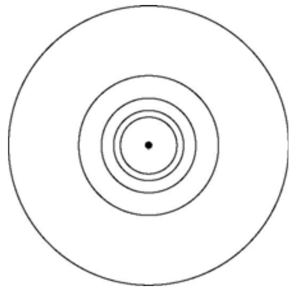
Evaluate r for V = 80 V:

$$r_{80\,\text{V}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{9}\times10^{-8}\ \text{C}\right)}{80\ \text{V}} = \boxed{0.125\ \text{m}}$$

Evaluate r for V = 100 V:

$$r_{100\,\text{V}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{9}\times10^{-8}\ \text{C}\right)}{100\ \text{V}} = \boxed{0.0999\ \text{m}}$$

The equipotential surfaces are shown in cross-section to the right:



The equipotential surfaces are not equally spaced.

62 •

Picture the Problem We can relate the dielectric strength of air (about 3 MV/m) to the maximum net charge that can be placed on a spherical conductor using the expression for the electric field at its surface. We can find the potential of the sphere when it carries its maximum charge using $V = kQ_{max}/R$.

(*a*) Express the dielectric strength of a spherical conductor in terms of the charge on the sphere:

$$E_{breakdown} = \frac{kQ_{max}}{R^2}$$

Solve for Q_{max}:

$$Q_{max} = \frac{E_{breakdown}R^2}{k}$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{max} = \frac{(3\,\text{MV/m})(0.16\,\text{m})^2}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.54\,\mu\text{C}}$$

(*b*) Because the charge carried by the sphere could be either positive or negative:

$$V_{max} = \pm\frac{kQ_{max}}{R} = \pm\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(8.54\,\mu\text{C})}{0.16\,\text{m}} = \boxed{\pm480\,\text{kV}}$$

***63** •

Picture the Problem We can solve the equation giving the electric field at the surface of a conductor for the greatest surface charge density that can exist before dielectric breakdown of the air occurs.

Relate the electric field at the surface of a conductor to the surface charge density:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for σ under dielectric breakdown of the air conditions:

$$\sigma_{\max} = \epsilon_0 E_{\text{breaddown}}$$

Substitute numerical values and evaluate $\sigma_{\max}$:

$$\sigma_{\max} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\,\text{MV/m}\right) = \boxed{26.6\,\mu\text{C/m}^2}$$

64 ••

Picture the Problem Let L and S refer to the larger and smaller spheres, respectively. We can use the fact that both spheres are at the same potential to find the electric fields near their surfaces. Knowing the electric fields, we can use $\sigma = \epsilon_0 E$ to find the surface charge density of each sphere.

Express the electric fields at the surfaces of the two spheres:

$$E_{\text{S}} = \frac{kQ_{\text{S}}}{R_{\text{S}}^2} \text{ and } E_{\text{L}} = \frac{kQ_{\text{L}}}{R_{\text{L}}^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_{\text{S}}}{E_{\text{L}}} = \frac{\dfrac{kQ_{\text{S}}}{R_{\text{S}}^2}}{\dfrac{kQ_{\text{L}}}{R_{\text{L}}^2}} = \frac{Q_{\text{S}}R_{\text{L}}^2}{Q_{\text{L}}R_{\text{S}}^2}$$

Because the potentials are equal at the surfaces of the spheres:

$$\frac{kQ_{\text{L}}}{R_{\text{L}}} = \frac{kQ_{\text{S}}}{R_{\text{S}}} \text{ and } \frac{Q_{\text{S}}}{Q_{\text{L}}} = \frac{R_{\text{S}}}{R_{\text{L}}}$$

Substitute to obtain:

$$\frac{E_{\text{S}}}{E_{\text{L}}} = \frac{R_{\text{S}}R_{\text{L}}^2}{R_{\text{L}}R_{\text{S}}^2} = \frac{R_{\text{L}}}{R_{\text{S}}}$$

Solve for E_{S}:

$$E_{\text{S}} = \frac{R_{\text{L}}}{R_{\text{S}}}E_{\text{L}} = \frac{12\,\text{cm}}{5\,\text{cm}}\left(200\,\text{kV/m}\right) = 480\,\text{kV/m}$$

Use $\sigma = \epsilon_0 E$ to find the surface charge density of each sphere:

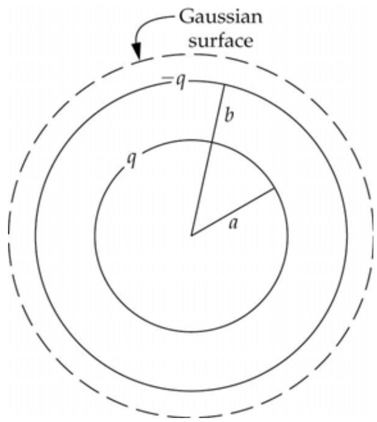
$$\sigma_{12\,\text{cm}} = \epsilon_0 E_{12\,\text{cm}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(200\,\text{kV/m}\right) = \boxed{1.77\,\mu\text{C/m}^2}$$

and

$$\sigma_{5\,\text{cm}} = \epsilon_0 E_{5\,\text{cm}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(480\,\text{kV/m}\right) = \boxed{4.25\,\mu\text{C/m}^2}$$

65 ••

Picture the Problem The diagram is a cross-sectional view showing the charges on the concentric spherical shells. The Gaussian surface over which we'll integrate E in order to find V in the region $r \geq b$ is also shown. We'll also find E in the region for which $a < r < b$. We can then use the relationship $V = -\int E dr$ to find V_a and V_b and their difference.



Express V_b:

$$V_b = -\int_{\infty}^{b} E_{r\geq a} dr$$

Apply Gauss's law for $r \geq b$:

$$\oint_S \vec{E}_r \cdot \hat{n} dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_{r\geq b} = 0$ because $Q_{\text{enclosed}} = 0$ for $r \geq b$.

Substitute to obtain:

$$V_b = -\int_{\infty}^{b} (0) dr = 0$$

Express V_a:

$$V_a = -\int_{b}^{a} E_{r\geq a} dr$$

Apply Gauss's law for $r \geq a$:

$$E_{r\geq a}\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$$

and

$$E_{r\geq a} = \frac{q}{4\pi \epsilon_0 r^2} = \frac{kq}{r^2}$$

Substitute to obtain:

$$V_a = -kq\int_{b}^{a} \frac{dr}{r^2} = \frac{kq}{a} - \frac{kq}{b}$$

The potential difference between the shells is given by:

$$V_a - V_b = V_a = \boxed{kq\left(\frac{1}{a} - \frac{1}{b}\right)}$$

***66** •••

Picture the Problem We can find the potential relative to infinity at the center of the sphere by integrating the electric field for 0 to ∞. We can apply Gauss's law to find the electric field both inside and outside the spherical shell.

The potential relative to infinity the center of the spherical shell is:

$$V = \int_0^R E_{r<R}dr + \int_R^\infty E_{r>R}dr \qquad (1)$$

Apply Gauss's law to a spherical surface of radius $r < R$ to obtain:

$$\int_S E_n dA = E_{r<R}\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Using the fact that the sphere is uniformly charged, express Q_{inside} in terms of Q:

$$\frac{Q_{\text{inside}}}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi R^3} \Rightarrow Q_{\text{inside}} = \frac{r^3}{R^3}Q$$

Substitute for Q_{inside} to obtain:

$$E_{r<R}\left(4\pi r^2\right) = \frac{r^3}{\epsilon_0 R^3}Q$$

Solve for $E_{r<R}$:

$$E_{r<R} = \frac{r}{4\pi\epsilon_0 R^3}Q = \frac{kQ}{R^3}r$$

Apply Gauss's law to a spherical surface of radius $r > R$ to obtain:

$$\int_S E_n dA = E_{r>R}\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Solve for $E_{r>R}$ to obtain:

$$E_{r>R} = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

Substitute for $E_{r<R}$ and $E_{r>R}$ in equation (1) and evaluate the resulting integral:

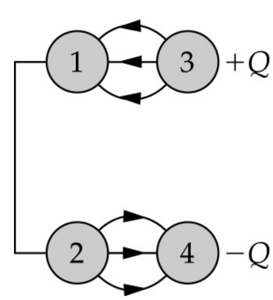
$$V = \frac{kQ}{R^3}\int_0^R r\,dr + kQ\int_R^\infty \frac{dr}{r^2}$$

$$= \frac{kQ}{R^3}\left[\frac{r^2}{2}\right]_0^R + kQ\left[-\frac{1}{r}\right]_R^\infty = \boxed{\frac{3kQ}{2R}}$$

67 ••

Picture the Problem

(*a*) The field lines are shown on the figure. The charged spheres induce charges of opposite sign on the spheres near them so that sphere 1 is negatively charged, and sphere 2 is positively charged. The total charge of the system is zero.



(*b*) $\boxed{V_1 = V_2 \text{ because the spheres are connected. From the direction of the electric field lines it follows that } V_3 > V_1.}$

(*c*) If 3 and 4 are connected, $V_3 = V_4$ and the conditions of part (*b*) can only be satisfied if all potentials are zero. Consequently the charge on each sphere is zero.

General Problems

68 •

Picture the Problem Because the charges at either end of the electric dipole are point charges, we can use the expression for the Coulomb potential to find the field at any distance from the dipole charges.

Using the expression for the potential due to a system of point charges, express the potential at the point 9.2×10^{-10} m from each of the two charges:

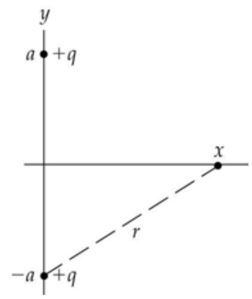
$$V = \frac{kq_+}{d} + \frac{kq_-}{d} = \frac{k}{d}(q_+ + q_-)$$

Because $q_+ = -q_-$:

$q_+ + q_- = 0$, $V = 0$ and $\boxed{(b) \text{ is correct.}}$

69 •

Picture the Problem The potential V at any point on the x axis is the sum of the Coulomb potentials due to the two point charges. Once we have found V, we can use $\vec{E} = -\overrightarrow{\text{grad}}\,V$ to find the electric field at any point on the x axis.



(*a*) Express the potential due to a system of point charges:

$$V = \sum_i \frac{kq_i}{r_i}$$

Substitute to obtain:

$$V(x) = V_{\text{charge at } +a} + V_{\text{charge at } -a} = \frac{kq}{\sqrt{x^2 + a^2}} + \frac{kq}{\sqrt{x^2 + a^2}} = \boxed{\frac{2kq}{\sqrt{x^2 + a^2}}}$$

(*b*) The electric field at any point on the *x* axis is given by:

$$\vec{E}(x) = -\overrightarrow{\text{grad}}V = -\frac{d}{dx}\left[\frac{2kq}{\sqrt{x^2+a^2}}\right]\hat{i}$$

$$= \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}}\hat{i}}$$

70 •

Picture the Problem The radius of the sphere is related to the electric field and the potential at its surface. The dielectric strength of air is about 3 MV/m.

Relate the electric field at the surface of a conducting sphere to the potential at the surface of the sphere:

$$E_r = \frac{V(r)}{r}$$

Solve for *r*:

$$r = \frac{V(r)}{E_r}$$

When *E* is a maximum, *r* is a minimum:

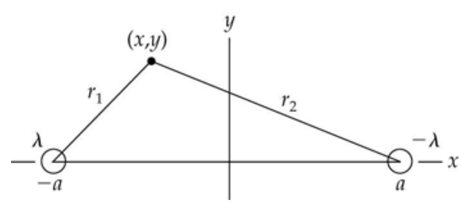
$$r_{\text{min}} = \frac{V(r)}{E_{\text{max}}}$$

Substitute numerical values and evaluate r_{min}:

$$r_{\text{min}} = \frac{10^4\ \text{V}}{3\,\text{MV/m}} = \boxed{3.33\,\text{mm}}$$

***71** ••

Picture the Problem The geometry of the wires is shown to the right. The potential at the point whose coordinates are (*x*, *y*) is the sum of the potentials due to the charge distributions on the wires.



(*a*) Express the potential at the point whose coordinates are (*x*, *y*):

$$V(x,y) = V_{\text{wire at } -a} + V_{\text{wire at } a}$$

$$= 2k\lambda\ln\left(\frac{r_{\text{ref}}}{r_1}\right) + 2k(-\lambda)\ln\left(\frac{r_{\text{ref}}}{r_2}\right)$$

$$= 2k\lambda\left[\ln\left(\frac{r_{\text{ref}}}{r_1}\right) - \ln\left(\frac{r_{\text{ref}}}{r_2}\right)\right]$$

$$= \frac{\lambda}{2\pi\in_0}\ln\left(\frac{r_2}{r_1}\right)$$

where $V(0) = 0$.

Because $r_1 = \sqrt{(x+a)^2 + y^2}$ and $r_2 = \sqrt{(x-a)^2 + y^2}$:

$$V(x, y) = \boxed{\frac{\lambda}{2\pi \in_0} \ln\left(\frac{\sqrt{(x-a)^2 + y^2}}{\sqrt{(x+a)^2 + y^2}}\right)}$$

On the y-axis, $x = 0$ and:

$$V(0, y) = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{\sqrt{a^2 + y^2}}{\sqrt{a^2 + y^2}}\right)$$

$$= \frac{\lambda}{2\pi \in_0} \ln(1) = \boxed{0}$$

(b) Evaluate the potential at $\left(\frac{1}{4}a, 0\right) = (1.25\,\text{cm}, 0)$:

$$V\left(\tfrac{1}{4}a, 0\right) = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{\sqrt{\left(\frac{1}{4}a - a\right)^2}}{\sqrt{\left(\frac{1}{4}a + a\right)^2}}\right)$$

$$= \frac{\lambda}{2\pi \in_0} \ln\left(\frac{3}{5}\right)$$

Equate $V(x,y)$ and $V\left(\frac{1}{4}a, 0\right)$:

$$\frac{3}{5} = \frac{\sqrt{(x-5)^2 + y^2}}{\sqrt{(x+5)^2 + y^2}}$$

Solve for y to obtain:

$$y = \pm\sqrt{21.25x - x^2 - 25}$$

A spreadsheet program to plot $y = \pm\sqrt{21.25x - x^2 - 25}$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

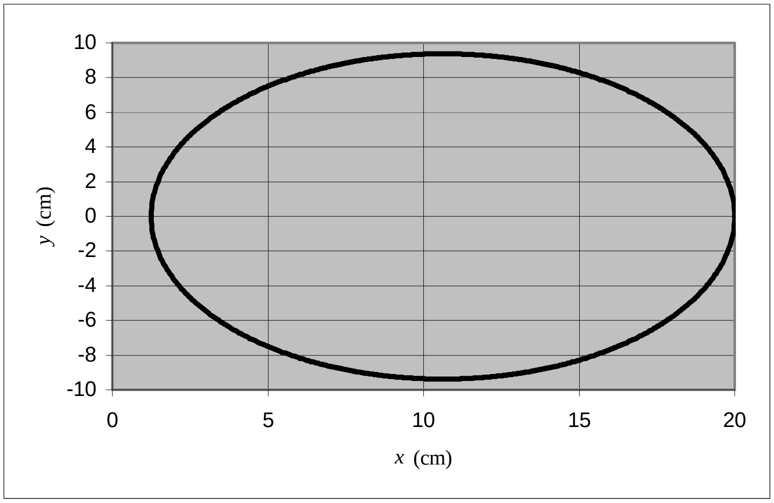
Cell	Content/Formula	Algebraic Form
A2	1.25	$\frac{1}{4}a$
A3	A2 + 0.05	$x + \Delta x$
B2	SQRT(21.25*A2 – A2^2 – 25)	$y = \sqrt{21.25x - x^2 - 25}$
B4	–SQRT(21.25*A2 – A2^2 – 25)	$y = -\sqrt{21.25x - x^2 - 25}$

	A	B	C
1	x	y_pos	y_neg
2	1.25	0.00	0.00
3	1.30	0.97	–0.97
4	1.35	1.37	–1.37
5	1.40	1.67	–1.67
6	1.45	1.93	–1.93
7	1.50	2.15	–2.15
370	19.65	2.54	–2.54
371	19.70	2.35	–2.35
372	19.75	2.15	–2.15

373	19.80	1.93	–1.93
374	19.85	1.67	–1.67
375	19.90	1.37	–1.37
376	19.95	0.97	–0.97

The following graph shows the equipotential curve in the *xy* plane for

$$V\left(\tfrac{1}{4}a,0\right)=\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{3}{5}\right).$$



72 ••

Picture the Problem We can use the expression for the potential at any point in the *xy* plane to show that the equipotential curve is a circle.

(*a*) Equipotential surfaces must satisfy the condition:

$$V=\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

Solve for r_2/r_1:

$$\frac{r_2}{r_1}=e^{\frac{2\pi\epsilon_0 V}{\lambda}}=C \text{ or } r_2=Cr_1$$

where C is a constant.

Substitute for r_1 and r_2 to obtain:

$$(x-a)^2+y^2=C^2\left[(x+a)^2+y^2\right]$$

Expand this expression, combine like terms, and simplify to obtain:

$$x^2+2a\frac{C^2+1}{C^2-1}x+y^2=-a^2$$

Complete the square by adding $\left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right]$ to both sides of the equation:

$$x^2 + 2a\frac{C^2+1}{C^2-1}x + \left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right] + y^2 = \left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right] - a^2 = \frac{4a^2C^2}{\left(C^2-1\right)^2}$$

Let $\alpha = 2a\frac{C^2+1}{C^2-1}$ and $\beta = 2a\frac{C}{C^2-1}$ to obtain:

$\boxed{(x+\alpha)^2 + y^2 = \beta^2,}$ the equation of circle in the *xy* plane with its center at $(-\alpha,0)$.

(*b*) $\boxed{\text{The three-dimensional surfaces are cylinders parallel to the wires.}}$

73 ••

Picture the Problem Expressing the charge *dq* in a spherical shell of volume $4\pi r^2 dr$ within a distance *r* of the proton and setting the integral of this expression equal to *e* will allow us to solve for the value of ρ_0 needed for charge neutrality. In part (*b*), we can use the given charge density to express the potential function due to this charge and then integrate this function to find *V* as a function of *r*.

Express the charge *dq* in a spherical shell of volume $4\pi r^2 dr$ within a distance *r* of the proton:

$$dq = \rho dV = \left(\rho_0 e^{-2r/a}\right)\left(4\pi r^2 dr\right) = 4\pi\rho_0 r^2 e^{-2r/a}dr$$

Express the condition for charge neutrality:

$$e = 4\pi\rho_0\int_0^\infty r^2 e^{-2r/a}dr$$

Integrate by parts twice to obtain:

$$e = 4\pi\rho_0\frac{a^3}{4} = \pi\rho_0 a^3$$

Solve for ρ_0:

$$\rho_0 = \boxed{\frac{e}{\pi a^3}}$$

74 •

Picture the Problem Let *Q* be the sphere's charge, *R* its radius, and *n* the number of electrons that have been removed. Then $Q = ne$, where *e* is the electronic charge. We can use the expression for the Coulomb potential of the sphere to express *Q* and then $Q = ne$ to find *n*.

Letting *n* be the number of electrons that have been removed, express the sphere's charge *Q* in terms of the electronic charge *e*:

$$Q = ne$$

Solve for *n*:

$$n = \frac{Q}{e} \quad (1)$$

Relate the potential of the sphere to its charge and radius:

$$V = \frac{kQ}{R}$$

Solve for the sphere's charge:

$$Q = \frac{VR}{k}$$

Substitute in equation (1) to obtain:

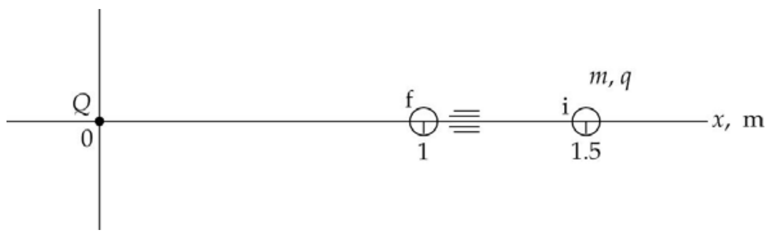
$$n = \frac{VR}{ke}$$

Substitute numerical values and evaluate n:

$$n = \frac{(400\text{ V})(0.05\text{ m})}{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})} = \boxed{1.39\times10^{10}}$$

75 •

Picture the Problem We can use conservation of energy to relate the change in the kinetic energy of the particle to the change in potential energy of the charge-and-particle system as the particle moves from x = 1.5 m to x = 1 m. The change in potential energy is, in turn, related to the change in electric potential.



Apply conservation of energy to the point charge Q and particle system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = 0$,

$$K_f + \Delta U_{if} = 0$$

Solve for K_f:

$$K_f = -\Delta U_{if}$$

Relate the difference in potential between points i and f to the change in potential energy of the system as the body whose charge is q moves from i to f:

$$\Delta U_{if} = -q\Delta V_{if} = -q(V_f - V_i) = -q\left(\frac{kQ}{x_f} - \frac{kQ}{x_i}\right) = -kqQ\left(\frac{1}{x_f} - \frac{1}{x_i}\right)$$

Substitute to obtain:

$$K_f = -kqQ\left(\frac{1}{x_f} - \frac{1}{x_i}\right)$$

Solve for Q:

$$Q = -\frac{K_f}{kq\left(\frac{1}{x_f} - \frac{1}{x_i}\right)}$$

Substitute numerical values and evaluate Q:

$$Q = -\frac{0.24\,\text{J}}{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})\left(\frac{1}{1\,\text{m}} - \frac{1}{1.5\,\text{m}}\right)} = \boxed{-20.0\,\mu\text{C}}$$

***76 ••**

Picture the Problem We can use the definition of power and the expression for the work done in moving a charge through a potential difference to find the minimum power needed to drive the moving belt.

Relate the power need to drive the moving belt to the rate at which the generator is doing work:

$$P = \frac{dW}{dt}$$

Express the work done in moving a charge q through a potential difference ΔV:

$$W = q\Delta V$$

Substitute to obtain:

$$P = \frac{d}{dt}[q\Delta V] = \Delta V\frac{dq}{dt}$$

Substitute numerical values and evaluate P:

$$P = (1.25\,\text{MV})(200\,\mu\text{C/s}) = \boxed{250\,\text{W}}$$

77 ••

Picture the Problem We can use $W_{q\rightarrow\text{final position}} = q\Delta V_{i\rightarrow f}$ to find the work required to move these charges between the given points.

(*a*) Express the required work in terms of the charge being moved and the potential due to the charge at $x = +a$:

$$\begin{aligned} W_{+Q\rightarrow+a} &= Q\Delta V_{\infty\rightarrow+a} \\ &= Q[V(a) - V(\infty)] \\ &= QV(a) = Q\left(\frac{kQ}{2a}\right) = \boxed{\frac{kQ^2}{2a}} \end{aligned}$$

(*b*) Express the required work in terms of the charge being moved and the potentials due to the charges at *x* = +*a* and *x* = –*a*:

$$\begin{aligned} W_{-Q\to 0} &= -Q\Delta V_{\infty\to 0} \\ &= -Q[V(0)-V(\infty)] \\ &= -QV(0) \\ &= -Q[V_{\text{charge at }-a}+V_{\text{charge at }+a}] \\ &= -Q\left(\frac{kQ}{a}+\frac{kQ}{a}\right)=\boxed{\frac{-2kQ^2}{a}} \end{aligned}$$

(*c*) Express the required work in terms of the charge being moved and the potentials due to the charges at *x* = +*a* and *x* = –*a*:

$$\begin{aligned} W_{-Q\to 2a} &= -Q\Delta V_{0\to 2a} \\ &= -Q[V(2a)-V(0)] \\ &= -Q[V_{\text{charge at }-a}+V_{\text{charge at }+a}-V(0)] \\ &= -Q\left(\frac{kQ}{3a}+\frac{kQ}{a}-\frac{2kQ}{a}\right) \\ &= \boxed{\frac{2kQ^2}{3a}} \end{aligned}$$

78 ••

Picture the Problem Let *q* represent the charge being moved from *x* = 50 cm to the origin, *Q* the ring charge, and *a* the radius of the ring. We can use $W_{q\to\text{final position}} = q\Delta V_{i\to f}$, where *V* is the expression for the axial field due to a ring charge, to find the work required to move *q* from *x* = 50 cm to the origin.

Express the required work in terms of the charge being moved and the potential due to the ring charge at *x* = 50 cm and *x* = 0:

$$\begin{aligned} W &= q\Delta V \\ &= q[V(0)-V(0.5\,\text{m})] \end{aligned}$$

The potential on the axis of a uniformly charged ring is:

$$V(x)=\frac{kQ}{\sqrt{x^2+a^2}}$$

Evaluate *V*(0):

$$\begin{aligned} V(0) &= \frac{kQ}{\sqrt{a^2}} \\ &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\,\text{nC})}{0.1\,\text{m}} \\ &= 180\,\text{V} \end{aligned}$$

Evaluate *V*(0.5 m):

$$\begin{aligned} V(0) &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\,\text{nC})}{\sqrt{(0.5\,\text{m})^2+(0.1\,\text{m})^2}} \\ &= 35.3\,\text{V} \end{aligned}$$

Substitute in the expression for *W* to obtain:

$$W = (1\,\text{nC})(180\,\text{V} - 35.3\,\text{V}) = \boxed{1.45\times10^{-7}\,\text{J}}$$

$$= 1.45\times10^{-7}\,\text{J}\times\frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}} = \boxed{9.06\times10^{11}\,\text{eV}}$$

79 ••

Picture the Problem We can find the speed of the proton as it strikes the negatively charged sphere from its kinetic energy and, in turn, its kinetic energy from the potential difference through which it is accelerated.

Use the definition of kinetic energy to express the speed of the proton when it strikes the negatively charged sphere:

$$v = \sqrt{\frac{2K_p}{m_p}} \qquad (1)$$

Use the work-kinetic energy theorem to relate the kinetic energy of the proton to the potential difference through which it is accelerated:

$$W = \Delta K = K_f - K_i$$

or, because $K_i = 0$ and $K_f = K_p$,

$$W = \Delta K = K_p$$

Express the work done on the proton in terms of its charge *e* and the potential difference ΔV between the spheres:

$$W = e\Delta V$$

Substitute to obtain:

$$K_p = e\Delta V$$

Substitute in equation (1) to obtain:

$$v = \sqrt{\frac{2e\Delta V}{m_p}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2(1.6\times10^{-19}\,\text{C})(100\,\text{V})}{1.67\times10^{-27}\,\text{kg}}} = \boxed{1.38\times10^{5}\,\text{m/s}}$$

80 ••

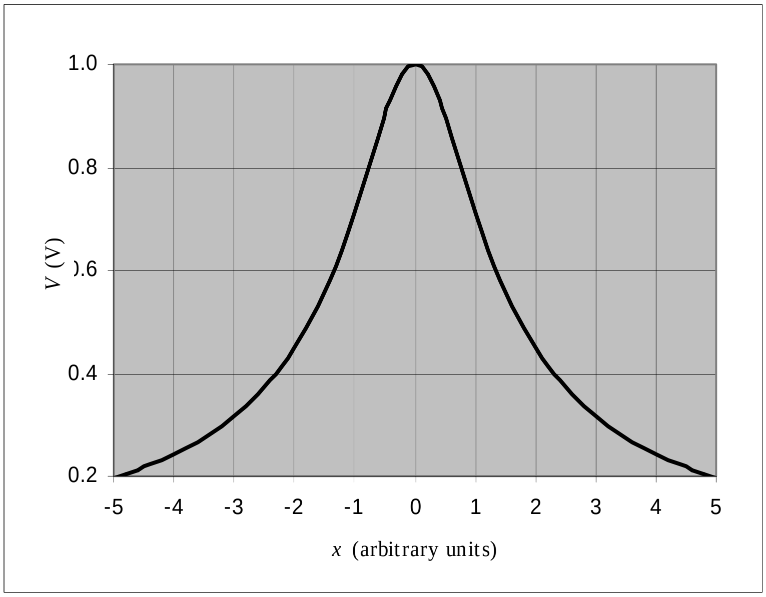
Picture the Problem Equation 23-20 is $V = kQ/\sqrt{a^2 + x^2}$.

(*a*) A spreadsheet solution is shown below for *kQ* = *a* = 1. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A4	A3 + 0.1	$x + \Delta x$
B3	1/(1+A3^2)^(1/2)	$\dfrac{kQ}{\sqrt{a^2 + x^2}}$

	A	B
1		
2	x	V(x)
3	–5.0	0.196
4	–4.8	0.204
5	–4.6	0.212
6	–4.4	0.222
7	–4.2	0.232
8	–4.0	0.243
9	–3.8	0.254
49	4.2	0.232
50	4.4	0.222
51	4.6	0.212
52	4.8	0.204
53	5.0	0.196

The following graph shows *V* as a function of *x*:



(*b*) Examining the graph we see that the maximum value of *V* occurs where:

$$x = \boxed{0}$$

Because $E = -dV/dx$, examination of the graph tells us that:

$$E(0) = \boxed{0}$$

81 ••

Picture the Problem Let R_2 be the radius of the second sphere and Q_1 and Q_2 the charges on the spheres when they have been connected by the wire. When the spheres are connected, the charge initially on the sphere of radius R_1 will redistribute until the spheres are at the same potential.

Express the common potential of the spheres when they are connected:

$$12\,\text{kV} = \frac{kQ_1}{R_1} \qquad (1)$$

and

$$12\,\text{kV} = \frac{kQ_2}{R_2} \qquad (2)$$

Express the potential of the first sphere before it is connected to the second sphere:

$$20\,\text{kV} = \frac{k(Q_1 + Q_2)}{R_1} \qquad (3)$$

Solve equation (1) for Q_1:

$$Q_1 = \frac{(12\,\text{kV})R_1}{k}$$

Solve equation (2) for Q_2:

$$Q_2 = \frac{(12\,\text{kV})R_2}{k}$$

Substitute in equation (3) to obtain:

$$20\,\text{kV} = \frac{k\left(\frac{(12\,\text{kV})R_1}{k} + \frac{(12\,\text{kV})R_2}{k}\right)}{R_1} = 12\,\text{kV} + 12\,\text{kV}\left(\frac{R_2}{R_1}\right)$$

or

$$8 = 12\left(\frac{R_2}{R_1}\right)$$

Solve for R_2:

$$R_2 = \boxed{\frac{2}{3}R_1}$$

***82 ••**

Picture the Problem We can use the definition of surface charge density to relate the radius R of the sphere to its charge Q and the potential function $V(r)=kQ/r$ to relate Q to the potential at r = 2 m.

Use its definition, relate the surface charge density σ to the charge Q on the sphere and the radius R of the sphere:

$$\sigma = \frac{Q}{4\pi R^2}$$

Solve for R to obtain:

$$R = \sqrt{\frac{Q}{4\pi\sigma}}$$

Relate the potential at r = 2.0 m to the charge on the sphere:

$$V(r) = \frac{kQ}{r}$$

Solve for Q to obtain:

$$Q = \frac{rV(r)}{k}$$

Substitute to obtain:

$$R = \sqrt{\frac{rV(r)}{4\pi k\sigma}} = \sqrt{\frac{4\pi \in_0 rV(r)}{4\pi\sigma}} = \sqrt{\frac{\in_0 rV(r)}{\sigma}}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{\frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2\,\text{m})(500\,\text{V})}{24.6\,\text{nC/m}^2}} = \boxed{0.600\,\text{m}}$$

83 ••

Picture the Problem We can use the definition of surface charge density to relate the radius R of the sphere to its charge Q and the potential function $V(r)=kQ/r$ to relate Q to the potential at r = 2 m.

Use its definition, relate the surface charge density σ to the charge Q on the disk and the radius R of the disk:

$$\sigma = \frac{Q}{\pi R^2}$$

Solve for Q to obtain:

$$Q = \pi\sigma R^2 \qquad (1)$$

Relate the potential at r to the charge on the disk:

$$V(r)=2\pi k\sigma\left(\sqrt{x^2+R^2}-x\right)$$

Substitute V(0.6 m) = 80 V:

$$80\,\text{V}=2\pi k\sigma\left(\sqrt{(0.6\,\text{m})^2+R^2}-0.6\,\text{m}\right)$$

Substitute V(1.5 m) = 40 V:

$$40\,\text{V}=2\pi k\sigma\left(\sqrt{(1.5\,\text{m})^2+R^2}-1.5\,\text{m}\right)$$

Divide the first of these equations by the second to obtain:

$$2=\frac{\sqrt{(0.6\,\text{m})^2+R^2}-0.6\,\text{m}}{\sqrt{(1.5\,\text{m})^2+R^2}-1.5\,\text{m}}$$

Solve for R to obtain:

$$R=0.800\,\text{m}$$

Express the electric field on the axis of a disk charge:

$$E_x=2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

Solve for σ to obtain:

$$\sigma=\frac{E_x}{2\pi k\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)}=\frac{2\in_0 E_x}{1-\frac{x}{\sqrt{x^2+R^2}}}$$

Evaluate σ using R = 0.8 m and E(1.5 m) = 23.5 V/m:

$$\sigma=\frac{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(23.5\,\text{V/m})}{1-\frac{1.5\,\text{m}}{\sqrt{(1.5\,\text{m})^2+(0.8\,\text{m})^2}}}=3.54\,\text{nC/m}^2$$

Substitute in equation (1) and evaluate Q:

$$Q=\pi\left(3.54\,\text{nC/m}^2\right)(0.8\,\text{m})^2=\boxed{7.12\,\text{nC}}$$

84 ••

Picture the Problem We can use $U = kq_1q_2/R$ to relate the electrostatic potential energy of the particles to their separation.

Express the electrostatic potential energy of the two particles in terms of their charge and separation:

$$U=\frac{kq_1q_2}{R}$$

Solve for R:

$$R = \frac{kq_1q_2}{U}$$

Substitute numerical values and evaluate R:

$$R = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(82)\left(1.6\times10^{-19}\ \text{C}\right)^2}{5.30\,\text{MeV}\times\dfrac{1.6\times10^{-19}\ \text{C}}{\text{eV}}} = \boxed{44.6\,\text{fm}}$$

85 ••

Picture the Problem We can use $\Delta V = E\Delta\ell$ and the expression for the electric field due to a plane of charge to find the potential difference between the two planes. The conducting slab introduced between the planes in part (*b*) will have a negative charge induced on its surface closest to the plane with the positive charge density and a positive charge induced on its other surface. We can proceed as in part (*a*) to find the potential difference between the planes with the conducting slab in place.

(*a*) Express the potential difference between the two planes:

$$\Delta V = E\Delta\ell = Ed$$

The electric field due to each plane is:

$$E = \frac{\sigma}{2\,\epsilon_0}$$

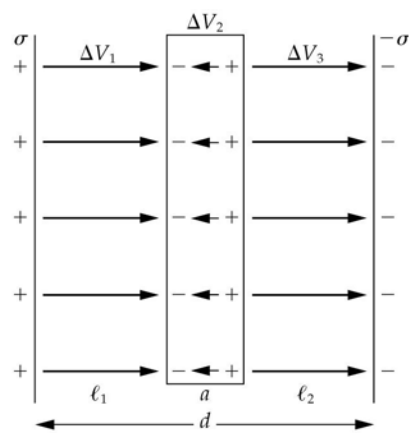
Because the charge densities are of opposite sign, the fields are additive and the resultant electric field between the planes is:

$$E = E_{\text{plane}\,1} + E_{\text{plane}\,2} = \frac{\sigma}{2\,\epsilon_0} + \frac{\sigma}{2\,\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Substitute to obtain:

$$\Delta V = \boxed{\frac{\sigma\, d}{\epsilon_0}}$$

(*b*) The diagram shows the conducting slab between the two planes and the electric field lines in the region between the original two planes.



Express the new potential difference $\Delta V'$ between the planes in terms of the potential differences ΔV_1, ΔV_2 and ΔV_3:

$$\Delta V' = \Delta V_1 + \Delta V_2 + \Delta V_3 = E_1\ell_1 + E_2 a + E_3\ell_2$$

Express the electric fields in regions 1, 2 and 3:

$$E_1 = E_3 = \frac{\sigma}{\epsilon_0} \text{ and } E_2 = 0$$

Substitute to obtain:

$$\Delta V' = \frac{\sigma}{\epsilon_0}\ell_1 + \frac{\sigma}{\epsilon_0}\ell_2 = \frac{\sigma}{\epsilon_0}(\ell_1 + \ell_2)$$

Express $\ell_1 + \ell_2$ in terms of a and d:

$$\ell_1 + \ell_2 = d - a$$

Substitute to obtain:

$$\Delta V' = \boxed{\frac{\sigma}{\epsilon_0}(d - a)}$$

86 •••

Picture the Problem We need to consider three regions, as in Example 23-5. Region I, $x > a$; region II, $0 < x < a$; and region III, $x < 0$. We can find V in each of these regions and then find E from $E = -dV/d\ell$.

(*a*) Relate E_{I} to V_{I}:

$$E_{\text{I}} = -\frac{dV_{\text{I}}}{dx}$$

In region I we have:

$$V_{\text{I}} = \frac{kq_1}{|x|} + \frac{kq_2}{|x-a|}$$

Substitute and evaluate E_{I}:

$$E_{\text{I}} = -\frac{d}{dx}\left[\frac{kq_1}{|x|} + \frac{kq_2}{|x-a|}\right]$$

Because $x > 0$:

$$|x| = x$$

For $x > a$:

$$|x-a| = x - a$$

Substitute to obtain:

$$E_{\text{I}} = -\frac{d}{dx}\left[\frac{kq_1}{x} + \frac{kq_2}{x-a}\right] = \boxed{\frac{kq_1}{x^2} + \frac{kq_2}{(x-a)^2}}$$

Proceed as above for regions II and III to obtain:

$$E_{\text{II}} = \boxed{\frac{kq_1}{x^2} - \frac{kq_2}{(x-a)^2}}$$

and

$$E_{\text{III}} = \boxed{-\frac{kq_1}{x^2} - \frac{kq_2}{(x-a)^2}}$$

(*b*) The distance between q_1 and a point on *y* axis is *y* and the distance between a point on the *y* axis and q_2 is $\sqrt{y^2 + a^2}$. Using these distances, express the potential at a point on the *y* axis:

$$V(y) = \boxed{\frac{kq_1}{|y|} + \frac{kq_2}{\sqrt{y^2 + a^2}}}$$

(*c*) To obtain the *y* component of $\vec{E}$ at a point on the *y* axis we take the derivative of *V*(*y*). For $y > 0$:

$$E_y = -\frac{d}{dy}\left[\frac{kq_1}{y} + \frac{kq_2}{\sqrt{y^2 + a^2}}\right] = \boxed{\frac{kq_1}{y^2} + \frac{kq_2 y}{(y^2 + a^2)^{3/2}}}$$

For $y < 0$:

$$E_y = -\frac{d}{dy}\left[-\frac{kq_1}{y} + \frac{kq_2}{\sqrt{y^2 + a^2}}\right] = \boxed{-\frac{kq_1}{y^2} + \frac{kq_2 y}{(y^2 + a^2)^{3/2}}}$$

These are the components of the fields due to q_1 and q_2 that one obtains using Coulomb's law.

***87** •••

Picture the Problem We can consider the relationship between the potential and the electric field to show that this arrangement is equivalent to replacing the plane by a point charge of magnitude $-q$ located a distance *d* beneath the plane. In (*b*) we can first find the field at the plane surface and then use $\sigma = \epsilon_0 E$ to find the surface charge density. In (*c*) the work needed to move the charge to a point 2*d* away from the plane is the product of the potential difference between the points at distances 2*d* and 3*d* from $-q$ multiplied by the separation Δx of these points.

(*a*) The potential anywhere on the plane is 0 in either arrangement and the electric field is perpendicular to the plane in both arrangements, so they must give the same potential everywhere in the *xy* plane. Also, because the net charge is zero, the potential at infinity is zero.

(*b*) The surface charge density is given by:

$$\sigma = \in_0 E \qquad (1)$$

At any point on the plane, the electric field points in the negative *x* direction and has magnitude:

$$E = \frac{kq}{d^2 + r^2}\cos\theta$$

where θ is the angle between the horizontal and a vector pointing from the positive charge to the point of interest on the *xz* plane and *r* is the distance along the plane from the origin (i.e., directly to the left of the charge).

Because $\cos\theta = \dfrac{d}{\sqrt{d^2 + r^2}}$:

$$E = \frac{kq}{d^2 + r^2}\frac{d}{\sqrt{d^2 + r^2}} = \frac{kqd}{\left(d^2 + r^2\right)^{3/2}} = \frac{qd}{4\pi \in_0 \left(d^2 + r^2\right)^{3/2}}$$

Substitute for *E* in equation (1) to obtain:

$$\sigma = \boxed{\frac{qd}{4\pi\left(d^2 + r^2\right)^{3/2}}}$$

88 •••

Picture the Problem We can express the potential due to the ring charges as the sum of the potentials due to each of the ring charges. To show that *V*(*x*) is a minimum at *x* = 0, we must show that the first derivative of *V*(*x*) = 0 at *x* = 0 and that the second derivative is positive. In part (*c*) we can use a Taylor expansion to show that, for *x* << *L*, the potential is of the form $V(x) = V(0) + \alpha x^2$. In part (*d*) we can obtain the potential energy function from the potential function and, noting that it is quadratic in *x*, find the ″spring″ constant and the angular frequency of oscillation of the particle provided its displacement from its equilibrium position is small.

(*a*) Express the potential due to the ring charges as the sum of the

$$V(x) = V_{\text{ring to the left}} + V_{\text{ring to the right}}$$

potentials due to each of their charges:

The potential for a ring of charge is:

$$V(x) = \frac{kQ}{\sqrt{x^2 + R^2}}$$

where R is the radius of the ring and Q is the charge of the ring.

For the ring to the left we have:

$$V_{\text{ring to the left}} = \frac{kQ}{\sqrt{(x+L)^2 + L^2}}$$

For the ring to the right we have:

$$V_{\text{ring to the right}} = \frac{kQ}{\sqrt{(x-L)^2 + L^2}}$$

Substitute to obtain:

$$V(x) = \boxed{\frac{kQ}{\sqrt{(x+L)^2 + L^2}} + \frac{kQ}{\sqrt{(x-L)^2 + L^2}}}$$

(b) Evaluate dV/dx to obtain:

$$\frac{dV}{dx} = kQ\left\{\frac{L-x}{\left[(L-x)^2 + L^2\right]^{3/2}} - \frac{L+x}{\left[(L+x)^2 + L^2\right]^{3/2}}\right\} = 0 \text{ for extrema}$$

Solve for x to obtain:

$x = 0$

Evaluate d^2V/dx^2 to obtain:

$$\frac{d^2V}{dx^2} = kQ\left\{\frac{3(L-x)^2}{\left[(L-x)^2 + L^2\right]^{5/2}} - \frac{1}{\left[(L-x)^2 + L^2\right]^{3/2}} + \frac{3(L+x)^2}{\left[(L+x)^2 + L^2\right]^{5/2}} - \frac{1}{\left[(L-x)^2 + L^2\right]^{3/2}}\right\}$$

Evaluate this expression for $x = 0$ to obtain:

$$\frac{d^2V(0)}{dx^2} = \frac{kQ}{2\sqrt{2}L^3} > 0$$

$$\boxed{\text{Hence } V(x) \text{ is a maximum at } x = 0.}$$

(*c*) The Taylor expansion of *V*(*x*) is:

$$V(x)=V(0)+V'(0)x+\tfrac{1}{2}V''(0)x^2 + \text{higher order terms}$$

For $x << L$:

$$V(x)\approx V(0)+V'(0)x+\tfrac{1}{2}V''(0)x^2$$

Substitute our results from part (*b*) to obtain:

$$V(x)=\frac{\sqrt{2}kQ}{L}+(0)x+\tfrac{1}{2}\left(\frac{kQ}{2\sqrt{2}L^3}\right)x^2 = \frac{\sqrt{2}kQ}{L}+\frac{kQ}{4\sqrt{2}L^3}x^2$$

or

$$V(x)=\boxed{V(0)+\alpha x^2}$$

where

$$V(0)=\boxed{\frac{\sqrt{2}kQ}{L}} \text{ and } \alpha=\boxed{\frac{kQ}{4\sqrt{2}L^3}}$$

(*d*) Express the angular frequency of oscillation of a simple harmonic oscillator:

$$\omega=\sqrt{\frac{k'}{m}}$$

where k' is the restoring constant.

From our result for part (*c*) and the definition of electric potential:

$$U(x)=qV(0)+\frac{1}{2}\left(\frac{kqQ}{2\sqrt{2}L^3}\right)x^2 = qV(0)+\tfrac{1}{2}k'x^2$$

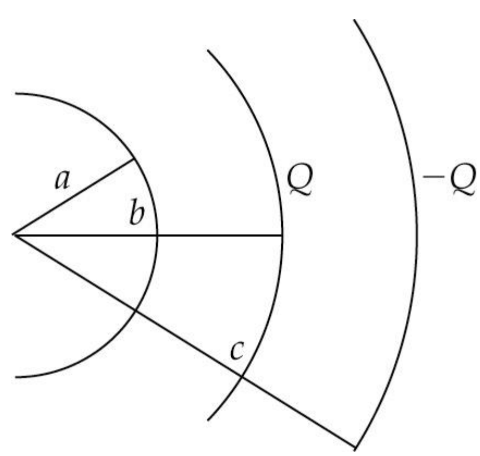
where $k'=\dfrac{kqQ}{2\sqrt{2}L^3}$

Substitute for k' in the expression for ω:

$$\omega=\boxed{\sqrt{\frac{kqQ}{2m\sqrt{2}L^3}}}$$

89 •••

Picture the Problem The diagram shows part of the shells in a cross-sectional view under the conditions of part (*a*) of the problem. We can use Gauss's law to find the electric field in the regions defined by the three surfaces and then find the electric potentials from the electric fields. In part (*b*) we can use the redistributed charges to find the charge on and potentials of the three surfaces.



(*a*) Apply Gauss's law to a spherical Gaussian surface of radius $r \geq c$ to obtain:

$$E_r\left(4\pi r^2\right)=\frac{Q_{\text{enclosed}}}{\epsilon_0}=0$$

and $E_r = 0$ because the net charge enclosed by the Gaussian surface is zero.

Because $E_r(c) = 0$:

$$V(c)=\boxed{0}$$

Apply Gauss's law to a spherical Gaussian surface of radius $b < r < c$ to obtain:

$$E_r\left(4\pi r^2\right)=\frac{Q}{\epsilon_0}$$

and

$$E_r(b<r<c)=\frac{kQ}{r^2}$$

Use $E_r(b<r<c)$ to find the potential difference between *c* and *b*:

$$V(b)-V(c)=-kQ\int_c^b\frac{dr}{r^2}$$
$$=kQ\left(\frac{1}{b}-\frac{1}{c}\right)$$

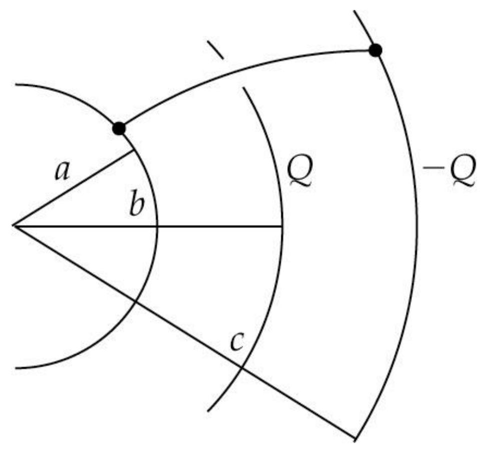
Because $V(c) = 0$:

$$V(b)=\boxed{kQ\left(\frac{1}{b}-\frac{1}{c}\right)}$$

The inner shell carries no charge, so the field between $r = a$ and $r = b$ is zero and:

$$V(a)=V(b)=\boxed{kQ\left(\frac{1}{b}-\frac{1}{c}\right)}$$

(*b*) When the inner and outer shells are connected their potentials become equal as a consequence of the redistribution of charge.



The charges on surfaces *a* and *c* are related according to:

$$Q_a+Q_c=-Q \qquad (1)$$

Q_b does not change with the connection of the inner and outer shells:

$$Q_b = \boxed{Q}$$

Express the potentials of shells *a* and *c*:

$$V(a) = V(c) = \boxed{0}$$

In the region between the $r = a$ and $r = b$, the field is kQ_a/r^2 and the potential at $r = b$ is then:

$$V(b) = kQ_a\left(\frac{1}{b} - \frac{1}{a}\right) \qquad (2)$$

The enclosed charge for $b < r < c$ is $Q_a + Q$, and by Gauss's law the field in this region is:

$$E_{b<r<c} = \frac{k(Q_a + Q)}{r^2}$$

Express the potential difference between *b* and *c*:

$$V(c) - V(b) = k(Q_a + Q)\left(\frac{1}{c} - \frac{1}{b}\right)$$
$$= -V(b)$$

because $V(c) = 0$.

Solve for $V(b)$ to obtain:

$$V(b) = k(Q_a + Q)\left(\frac{1}{b} - \frac{1}{c}\right) \qquad (3)$$

Equate equations (2) and (3) and solve for Q_a to obtain:

$$Q_a = \boxed{-Q\frac{a(c-b)}{b(c-a)}} \qquad (4)$$

Substitute equation (4) in equation (1) and solve for Q_c to obtain:

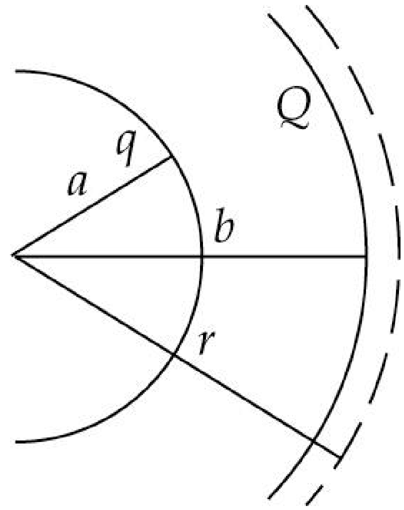
$$Q_c = \boxed{-Q\frac{c(b-a)}{b(c-a)}} \qquad (5)$$

Substitute (4) and (5) in (3) to obtain:

$$V(b) = \boxed{kQ\frac{(c-b)(b-a)}{b^2(c-a)}}$$

***90 •••**

Picture the Problem The diagram shows a cross-sectional view of a portion of the concentric spherical shells. Let the charge on the inner shell be q. The dashed line represents a spherical Gaussian surface over which we can integrate $\vec{E}\cdot\hat{n}dA$ in order to find E_r for $r \geq b$. We can find $V(b)$ from the integral of E_r between $r = \infty$ and $r = b$. We can obtain a second expression for $V(b)$ by considering the potential difference between a and b and solving the two equations simultaneously for the charge q on the inner shell.



Apply Gauss's law to a spherical surface of radius $r \geq b$:

$$E_r\left(4\pi r^2\right)=\frac{Q+q}{\varepsilon_0}$$

Solve for E_r to obtain:

$$E_r=\frac{k(Q+q)}{r^2}$$

Use E_r to find $V(b)$:

$$V(b)=-k(Q+q)\int_{\infty}^{b}\frac{dr}{r^2}$$

$$=\frac{k(Q+q)}{b}$$

We can also determine $V(b)$ by considering the potential difference between a, i.e., 0 and b:

$$V(b)=kq\left(\frac{1}{b}-\frac{1}{a}\right)$$

Equate these expressions for $V(b)$ to obtain:

$$\frac{k(Q+q)}{b}=ka\left(\frac{1}{b}-\frac{1}{a}\right)$$

Solve for q to obtain:

$$q=\boxed{-\frac{a}{b}Q}$$

91 •••

Picture the Problem We can use the hint to derive an expression for the electrostatic potential energy dU required to bring in a layer of charge of thickness dr and then integrate this expression from $r = 0$ to R to obtain an expression for the required work.

If we build up the sphere in layers, then at a given radius r the net charge on the sphere will be given by:

$$Q(r) = Q\left(\frac{r}{R}\right)^3$$

When the radius of the sphere is r, the potential relative to infinity is:

$$V(r) = \frac{Q(r)}{4\pi \in_0 r} = \frac{Q}{4\pi \in_0} \frac{r^2}{R^3}$$

Express the work dW required to bring in charge dQ from infinity to the surface of a uniformly charged sphere of radius r:

$$dW = dU = V(r)dQ$$
$$= \frac{Q}{4\pi \in_0} \frac{r^2}{R^3}\left(4\pi r^2 \frac{3Q}{4\pi R^3} dr\right)$$
$$= \frac{3Q^2}{4\pi \in_0 R^6} r^4 dr$$

Integrate dW from 0 to R to obtain:

$$W = U = \frac{3Q^2}{4\pi \in_0 R^6}\int_0^R r^4 dr$$
$$= \frac{3Q^2}{4\pi \in_0 R^6}\left[\frac{r^5}{5}\right]_0^R = \boxed{\frac{3Q^2}{20\pi \in_0 R}}$$

92 ••

Picture the Problem We can equate the rest energy of an electron and the result of Problem 91 in order to obtain an expression that we can solve for the classical electron radius.

From Problem 91 we have:

$$U = \frac{3e^2}{20\pi \in_0 R}$$

The rest mass of the electron is given by:

$$E_0 = m_0 c^2$$

Equate these energies to obtain:

$$\frac{3e^2}{20\pi \in_0 R} = m_0 c^2$$

Solve for R:

$$R = \frac{3e^2}{20\pi \in_0 m_0 c^2}$$

Substitute numerical values and evaluate R:

$$R = \frac{3(1.6\times10^{-19}\ \text{C})^2}{20\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.11\times10^5\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}$$
$$= \boxed{1.69\times10^{-15}\ \text{m}}$$

This model does not explain how the electron holds together against its own mutual repulsion.

93 ••

Picture the Problem Because the post-fission volumes of the fission products are equal, we can express the post-fission radii in terms of the radius of the pre-fission sphere.

(*a*) Relate the initial volume *V* of the uniformly charged sphere to the volumes *V*′ of the fission products:

$$V = 2V'$$

Substitute for *V* and *V*′:

$$\tfrac{4}{3}\pi R^3 = 2\left(\tfrac{4}{3}\pi R'^3\right)$$

Solve for and evaluate *R*′:

$$R' = \frac{1}{\sqrt[3]{2}}R = \boxed{0.794R}$$

(*b*) Express the difference Δ*E* in the total electrostatic energy as a result of fissioning:

$$\Delta E = E - E'$$

From Problem 91 we have:

$$E = \frac{3Q^2}{20\pi\epsilon_0 R}$$

After fissioning:

$$E' = 2\left(\frac{3Q'^2}{20\pi\epsilon_0 R'}\right) = 2\left[\frac{3\left(\frac{1}{2}Q\right)^2}{20\pi\epsilon_0 \frac{1}{\sqrt[3]{2}}R}\right]$$

$$= \frac{\sqrt[3]{2}}{2}\left(\frac{3Q^2}{20\pi\epsilon_0 R}\right) = 0.630E$$

Substitute for *E* and *E*′ to obtain:

$$\Delta E = E - 0.630E = \boxed{0.370E}$$

***94** •••

Picture the Problem We can use the definition of density to express the radius *R* of a nucleus as a function of its atomic mass *N*. We can then use the result derived in Problem 91 to express the electrostatic energies of the ^{235}U nucleus and the nuclei of the fission fragments ^{140}Xe and ^{94}Sr.

The energy released by this fission process is:

$$\Delta E = U_{^{235}\text{U}} - \left(U_{^{140}\text{Xe}} + U_{^{94}\text{Sr}}\right) \quad (1)$$

Express the mass of a nucleus in terms of its density and volume:

$$Nm = \tfrac{4}{3}\rho\pi R^3$$

where *N* is the nuclear number.

Solve for R to obtain:

$$R=\sqrt[3]{\frac{3Nm}{4\pi\rho}}$$

Substitute numerical values and evaluate R as a function of N:

$$R=\sqrt[3]{\frac{3\left(1.660\times10^{-27}\text{ kg}\right)}{4\pi\left(4\times10^{17}\text{ kg/m}^3\right)}}N^{1/3}=\left(9.97\times10^{-16}\text{ m}\right)N^{1/3}$$

The 'radius' of the ^{235}U nucleus is therefore:

$$R_U=\left(9.97\times10^{-16}\text{ m}\right)(235)^{1/3}=6.15\times10^{-15}\text{ m}$$

From Problem 91 we have:

$$U=\frac{3Q^2}{20\pi\in_0 R}$$

Substitute numerical values and evaluate the electrostatic energy of the ^{235}U nucleus:

$$U_{^{235}\text{U}}=\frac{3\left(92\times1.6\times10^{-19}\text{ C}\right)^2}{20\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.15\times10^{-15}\text{ m}\right)}$$
$$=1.91\times10^{-10}\text{ J}\times\frac{1\text{eV}}{1.6\times10^{-19}\text{ J/eV}}=1189\text{ MeV}$$

Proceed as above to find the electrostatic energy of the fission fragments ^{140}Xe and ^{94}Sr:

$$U_{^{140}\text{Xe}}=\frac{3\left(54\times1.6\times10^{-19}\text{ C}\right)^2}{20\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.15\times10^{-15}\text{ m}\right)}$$
$$=6.57\times10^{-11}\text{ J}\times\frac{1\text{eV}}{1.6\times10^{-19}\text{ J/eV}}=410\text{ MeV}$$

and

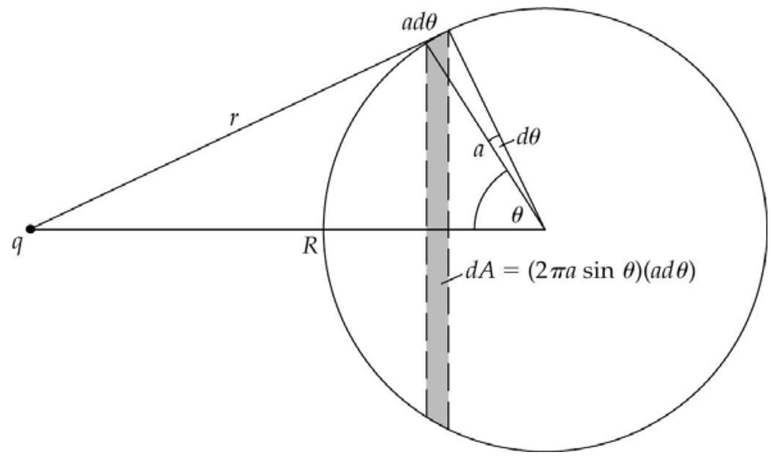
$$U_{^{94}\text{Sr}}=\frac{3\left(38\times1.6\times10^{-19}\text{ C}\right)^2}{20\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.15\times10^{-15}\text{ m}\right)}$$
$$=3.25\times10^{-11}\text{ J}\times\frac{1\text{eV}}{1.602\times10^{-19}\text{ J/eV}}=203\text{ MeV}$$

Substitute for $U_{^{235}\text{U}}$, $U_{^{140}\text{Xe}}$, and $U_{^{94}\text{Sr}}$ in equation (1) and evaluate ΔE:

$$\Delta E=1189\text{ MeV}-\left(410\text{ MeV}+203\text{ MeV}\right)=\boxed{576\text{ MeV}}$$

95 •••

Picture the Problem The geometry of the point charge and the sphere is shown below. The charge is a distance R away from the center of a spherical shell of radius a.



(*a*) The average potential over the surface of the sphere is given by:

$$V_{\text{av}} = \oint_{\text{sphere}} \frac{kdq}{r} = \oint_{\text{sphere}} \frac{k\sigma dA}{r}$$

Substitute for *k*, σ, and *dA* to obtain:

$$V_{\text{av}} = \frac{1}{4\pi \in_0} \int_0^{\pi} \frac{q(2\pi a \sin\theta)(ad\theta)}{4\pi a^2 r}$$

Apply the law of cosines to the triangle to obtain:

$$r = \sqrt{R^2 + a^2 - 2aR\cos\theta}$$

Substitute for *r* and simplify to obtain:

$$V_{\text{av}} = \frac{q}{8\pi \in_0} \int_0^{\pi} \frac{\sin\theta d\theta}{\left(R^2 + a^2 - 2aR\cos\theta\right)^{1/2}}$$

Change variables by letting $u = \cos\theta$. Then:

$$du = -\sin\theta d\theta$$

and

$$V_{\text{av}} = \frac{-q}{8\pi \in_0} \int_1^{-1} \frac{du}{\left(R^2 + a^2 - 2aRu\right)^{1/2}} \qquad (1)$$

To simplify the integrand, let:

$$\alpha = R^2 + a^2,\ \beta = 2aR \text{, and } v = \alpha - \beta u$$

Then $dv = -\beta du$ and:

$$\int_1^{-1} \frac{du}{\left(R^2 + a^2 - 2aRu\right)^{1/2}} = -\frac{1}{\beta} \int_{\ell_1}^{\ell_2} \frac{dv}{\sqrt{v}} = -\frac{2}{\beta}\sqrt{v}\Big|_{\ell_1}^{\ell_2} = -\frac{1}{aR}\sqrt{\alpha - \beta u}\Big|_1^{-1}$$

$$= -\frac{1}{aR}\left[\sqrt{\alpha + \beta} - \sqrt{\alpha - \beta}\right]$$

Substitute for α and β to obtain:

$$\int_{1}^{-1}\frac{du}{\left(R^2+a^2-2aRu\right)^{1/2}}=-\frac{1}{aR}\left[\sqrt{R^2+a^2+2aR}-\sqrt{R^2+a^2-2aR}\right]$$

$$=-\frac{1}{aR}\left[\sqrt{(R+a)^2}-\sqrt{(R-a)^2}\right]$$

$$=-\frac{1}{aR}\left[(R+a)-(R-a)\right]=-\frac{2}{R}$$

Substitute in equation (1) to obtain:

$$V_{\text{av}}=\frac{-q}{8\pi\in_0}\left(-\frac{2}{R}\right)=\boxed{\frac{q}{4\pi\in_0 R}}$$

Note that this result is the potential at the center of the sphere due to the point charge.

(*b*) The superposition principle tells us that the potential at any point is the sum of the potentials due to any charge distributions in space. Because this result is independent of any properties of the sphere, this result must hold for any sphere and any configuration of charges outside of it.

Chapter 24
Electrostatic Energy and Capacitance

Conceptual Problems

***1 •**

Determine the Concept The capacitance of a parallel-plate capacitor is a function of the surface area of its plates, the separation of these plates, and the electrical properties of the matter between them. The capacitance is, therefore, independent of the voltage across the capacitor. $\boxed{(c)\text{ is correct.}}$

2 •

Determine the Concept The capacitance of a parallel-plate capacitor is a function of the surface area of its plates, the separation of these plates, and the electrical properties of the matter between them. The capacitance is, therefore, independent of the charge of the capacitor. $\boxed{(c)\text{ is correct.}}$

3 •

Determine the Concept True. The energy density of an electrostatic field is given by $u_e = \frac{1}{2}\epsilon_0 E^2$.

4 •

Picture the Problem The energy stored in the electric field of a parallel-plate capacitor is related to the potential difference across the capacitor by $U = \frac{1}{2}QV$.

Relate the potential energy stored in the electric field of the capacitor to the potential difference across the capacitor:

$$U = \tfrac{1}{2}QV$$

$$\boxed{\text{With } Q \text{ constant, } U \text{ is directly proportional to } V. \text{ Hence, doubling } V \text{ doubles } U.}$$

***5 ••**

Picture the Problem The energy stored in a capacitor is given by $U = \frac{1}{2}QV$ and the capacitance of a parallel-plate capacitor by $C = \epsilon_0 A/d$. We can combine these relationships, using the definition of capacitance and the condition that the potential difference across the capacitor is constant, to express U as a function of d.

Express the energy stored in the capacitor:

$$U = \tfrac{1}{2}QV$$

Use the definition of capacitance to express the charge of the capacitor:

$$Q = CV$$

Substitute to obtain:

$$U = \tfrac{1}{2}CV^2$$

Express the capacitance of a parallel-plate capacitor in terms of the separation d of its plates:

$$C = \frac{\epsilon_0 A}{d}$$

where A is the area of one plate.

Substitute to obtain:

$$U = \frac{\epsilon_0 AV^2}{2d}$$

Because $U \propto \dfrac{1}{d}$, doubling the separation of the plates will reduce the energy stored in the capacitor to 1/2 its previous value:

$\boxed{(d)\text{ is correct.}}$

6 ••

Picture the Problem Let V represent the initial potential difference between the plates, U the energy stored in the capacitor initially, d the initial separation of the plates, and V', U', and d' these physical quantities when the plate separation has been doubled. We can use $U = \tfrac{1}{2}QV$ to relate the energy stored in the capacitor to the potential difference across it and $V = Ed$ to relate the potential difference to the separation of the plates.

Express the energy stored in the capacitor before the doubling of the separation of the plates:

$$U = \tfrac{1}{2}QV$$

Express the energy stored in the capacitor after the doubling of the separation of the plates:

$$U' = \tfrac{1}{2}QV'$$

because the charge on the plates does not change.

Express the ratio of U' to U:

$$\frac{U'}{U} = \frac{V'}{V}$$

Express the potential differences across the capacitor plates before and after the plate separation in terms of the electric field E between the plates:

$$V = Ed$$

and

$$V' = Ed'$$

because E depends solely on the charge on the plates and, as observed above, the charge does not change during the separation process.

Substitute to obtain:

$$\frac{U'}{U}=\frac{Ed'}{Ed}=\frac{d'}{d}$$

For $d' = 2d$:

$$\frac{U'}{U}=\frac{2d}{d}=2 \text{ and } \boxed{(b) \text{ is correct}}$$

7 •

Determine the Concept Both statements are true. The total charge stored by two capacitors in parallel is the sum of the charges on the capacitors and the equivalent capacitance is the sum of the individual capacitances. Two capacitors in series have the same charge and their equivalent capacitance is found by taking the reciprocal of the sum of the reciprocals of the individual capacitances.

8 ••

(*a*) False. Capacitors connected in series carry the same charge.

(*b*) False. The voltage across the capacitor whose capacitance is C_0 is Q/C_0 and that across the second capacitor is $Q/2C_0$.

(*c*) False. The energy stored by the capacitor whose capacitance is C_0 is $Q^2/2C_0$ and the energy stored by the second capacitor is $Q^2/4C_0$.

(*d*) $\boxed{\text{True}}$

9 •

Determine the Concept True. The capacitance of a parallel-plate capacitor filled with a dielectric of constant κ is given by $C=\dfrac{\kappa \in_0 A}{d}$ or $C \propto \kappa$.

***10** ••

Picture the Problem We can treat the configuration in (*a*) as two capacitors in parallel and the configuration in (*b*) as two capacitors in series. Finding the equivalent capacitance of each configuration and examining their ratio will allow us to decide whether (*a*) or (*b*) has the greater capacitance. In both cases, we'll let C_1 be the capacitance of the dielectric-filled capacitor and C_2 be the capacitance of the air capacitor.

In configuration (*a*) we have:

$$C_a = C_1 + C_2$$

Express C_1 and C_2:

$$C_1 = \frac{\kappa \in_0 A_1}{d_1} = \frac{\kappa \in_0 \frac{1}{2}A}{d} = \frac{\kappa \in_0 A}{2d}$$

and

$$C_2 = \frac{\in_0 A_2}{d_2} = \frac{\in_0 \frac{1}{2}A}{d} = \frac{\in_0 A}{2d}$$

Substitute for C_1 and C_2 and simplify to obtain:

$$C_a = \frac{\kappa \in_0 A}{2d} + \frac{\in_0 A}{2d} = \frac{\in_0 A}{2d}(\kappa + 1)$$

In configuration (b) we have:

$$\frac{1}{C_b} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_b = \frac{C_1 C_2}{C_1 + C_2}$$

Express C_1 and C_2:

$$C_1 = \frac{\in_0 A_1}{d_1} = \frac{\in_0 A}{\frac{1}{2}d} = \frac{2 \in_0 A}{d}$$

and

$$C_2 = \frac{\kappa \in_0 A_2}{d_2} = \frac{\kappa \in_0 A}{\frac{1}{2}d} = \frac{2\kappa \in_0 A}{d}$$

Substitute for C_1 and C_2 and simplify to obtain:

$$C_b = \frac{\left(\frac{2 \in_0 A}{d}\right)\left(\frac{2\kappa \in_0 A}{d}\right)}{\frac{2 \in_0 A}{d} + \frac{2\kappa \in_0 A}{d}} = \frac{\left(\frac{2 \in_0 A}{d}\right)\left(\frac{2\kappa \in_0 A}{d}\right)}{\frac{2 \in_0 A}{d}(\kappa + 1)} = \frac{2 \in_0 A}{d}\left(\frac{\kappa}{\kappa + 1}\right)$$

Divide C_b by C_a:

$$\frac{C_b}{C_a} = \frac{\frac{2 \in_0 A}{d}\left(\frac{\kappa}{\kappa + 1}\right)}{\frac{\in_0 A}{2d}(\kappa + 1)} = \frac{4\kappa}{(\kappa + 1)^2}$$

Because $\frac{4\kappa}{(\kappa + 1)^2} < 1$ for $\kappa > 1$:

$$\boxed{C_a > C_b}$$

11 •

(*a*) False. The capacitance of a parallel-plate capacitor is defined to be the ratio of the charge on the capacitor to the potential difference across it.

(*b*) False. The capacitance of a parallel-plate capacitor depends on the area of its plates *A*, their separation *d*, and the dielectric constant κ of the material between the plates according to $C = \kappa \in_0 A/d$.

(*c*) False. As in part (*b*), the capacitance of a parallel-plate capacitor depends on the area of its plates *A*, their separation *d*, and the dielectric constant κ of the material between the plates according to $C = \kappa \in_0 A/d$.

12 ••

Picture the Problem We can use the expression $U = \frac{1}{2}CV^2$ to express the ratio of the energy stored in the single capacitor and in the identical-capacitors-in-series combination.

Express the energy stored in capacitors when they are connected to the 100-V battery:

$$U = \tfrac{1}{2}C_{eq}V^2$$

Express the equivalent capacitance of the two identical capacitors connected in series:

$$C_{eq} = \frac{C^2}{2C} = \tfrac{1}{2}C$$

Substitute to obtain:

$$U = \tfrac{1}{2}\left(\tfrac{1}{2}C\right)V^2 = \tfrac{1}{4}CV^2$$

Express the energy stored in one capacitor when it is connected to the 100-V battery:

$$U_0 = \tfrac{1}{2}CV^2$$

Express the ratio of U to U_0:

$$\frac{U}{U_0} = \frac{\tfrac{1}{4}CV^2}{\tfrac{1}{2}CV^2} = \frac{1}{2}$$

or

$$U = \tfrac{1}{2}U_0 \text{ and } \boxed{(d) \text{ is correct}}$$

Estimation and Approximation

13 ••

Picture the Problem The outer diameter of a "typical" coaxial cable is about 5 mm, while the inner diameter is about 1 mm. From Table 24-1 we see that a reasonable range of values for κ is 3-5. We can use the expression for the capacitance of a cylindrical capacitor to estimate the capacitance per unit length of a coaxial cable.

The capacitance of a cylindrical dielectric-filled capacitor is given by:

$$C = \frac{2\pi\kappa \in_0 L}{\ln\left(\frac{R_2}{R_1}\right)}$$

where L is the length of the capacitor, R_1 is the radius of the inner conductor, and R_2 is the radius of the second (outer) conductor.

Divide both sides by L to obtain an expression for the capacitance per unit length of the cable:

$$\frac{C}{L} = \frac{2\pi\kappa \in_0}{\ln\left(\frac{R_2}{R_1}\right)} = \frac{\kappa}{2k\ln\left(\frac{R_2}{R_1}\right)}$$

If $\kappa = 3$:

$$\frac{\boldsymbol{C}}{\boldsymbol{L}} = \frac{3}{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\ln\left(\frac{2.5\,\text{mm}}{0.5\,\text{mm}}\right)} = 0.104\,\text{nF/m}$$

If $\kappa = 5$:

$$\frac{\boldsymbol{C}}{\boldsymbol{L}} = \frac{5}{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\ln\left(\frac{2.5\,\text{mm}}{0.5\,\text{mm}}\right)} = 0.173\,\text{nF/m}$$

A reasonable range of values for C/L, corresponding to $3 \le \kappa \le 5$, is:

$$\boxed{0.104\,\text{nF/m} \le \frac{\boldsymbol{C}}{\boldsymbol{L}} \le 0.173\,\text{nF/m}}$$

***14 ••**

Picture the Problem The energy stored in a capacitor is given by $U = \frac{1}{2}CV^2$.

Relate the energy stored in a capacitor to its capacitance and the potential difference across it:

$$U = \tfrac{1}{2}CV^2$$

Solve for C:

$$C = \frac{2U}{V^2}$$

The potential difference across the spark gap is related to the width of the gap d and the electric field E in the gap:

$$V = Ed$$

Substitute for V in the expression for C to obtain:

$$C = \frac{2U}{E^2 d^2}$$

Substitute numerical values and evaluate C:

$$C = \frac{2(100\,\text{J})}{(3\times10^6\,\text{V/m})^2(0.001\,\text{m})^2} = \boxed{22.2\,\mu\text{F}}$$

15 ••

Picture the Problem Because $\Delta R << R_E$ we can treat the atmosphere as a flat slab with an area equal to the surface area of the earth. Then the energy stored in the atmosphere can be estimated from $U = uV$, where u is the energy density of the atmosphere and V is its volume.

Express the electric energy stored in the atmosphere in terms of its energy density and volume:

$$U = uV$$

Because $\Delta R << R_E = 6370$ km, we can consider the volume:

$$V = A_{\text{surface area of the earth}}\Delta R = 4\pi R_E^2 \Delta R$$

Express the energy density of the Earth's atmosphere in terms of the average magnitude of its electric field:

$$u = \tfrac{1}{2}\epsilon_0 E^2$$

Substitute for V and u to obtain:

$$U = \left(\tfrac{1}{2}\epsilon_0 E^2\right)\left(4\pi R_E^2 \Delta R\right) = \frac{R_E^2 E^2 \Delta R}{2k}$$

Substitute numerical values and evaluate U:

$$U = \frac{(6370\,\text{km})^2(200\,\text{V/m})^2(1\,\text{km})}{2\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{9.03\times10^{10}\,\text{J}}$$

16 ••

Picture the Problem We'll approximate the balloon by a sphere of radius $R = 3$ m and use the expression for the capacitance of an isolated spherical conductor.

Relate the capacitance of an isolated spherical conductor to its radius:

$$C = 4\pi\epsilon_0 R = \frac{R}{k}$$

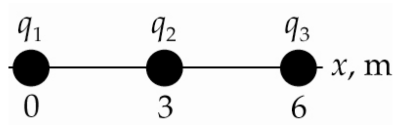
Substitute numerical values and evaluate C:

$$C = \frac{3\,\text{m}}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{0.334\,\text{nF}}$$

Electrostatic Potential Energy

17 •

The electrostatic potential energy of this system of three point charges is the work needed to bring the charges from an infinite separation to the final positions shown in the diagram.



Express the work required to assemble this system of charges:

$$U = \frac{kq_1q_2}{r_{1,2}} + \frac{kq_1q_3}{r_{1,3}} + \frac{kq_2q_3}{r_{2,3}} = k\left(\frac{q_1q_2}{r_{1,2}} + \frac{q_1q_3}{r_{1,3}} + \frac{q_2q_3}{r_{2,3}}\right)$$

Find the distances $r_{1,2}$, $r_{1,3}$, and $r_{2,3}$:

$$r_{1,2} = 3\,\text{m},\ r_{2,3} = 3\,\text{m},\ \text{and}\ r_{1,3} = 6\,\text{m}$$

(a) Evaluate U for $q_1 = q_2 = q_3 = 2\ \mu\text{C}$:

$$U = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{3\,\text{m}} + \frac{(2\,\mu\text{C})(2\,\mu\text{C})}{6\,\text{m}} + \frac{(2\,\mu\text{C})(2\,\mu\text{C})}{3\,\text{m}}\right] = \boxed{30.0\,\text{mJ}}$$

(b) Evaluate U for $q_1 = q_2 = 2\ \mu\text{C}$ and $q_3 = -2\ \mu\text{C}$:

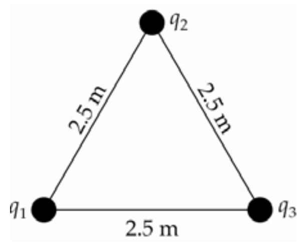
$$U = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{3\,\text{m}} + \frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{6\,\text{m}} + \frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{3\,\text{m}}\right] = \boxed{-5.99\,\text{mJ}}$$

(c) Evaluate U for $q_1 = q_3 = 2\ \mu\text{C}$ and $q_2 = -2\ \mu\text{C}$:

$$U = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{3\,\text{m}} + \frac{(2\,\mu\text{C})(2\,\mu\text{C})}{6\,\text{m}} + \frac{(-2\,\mu\text{C})(2\,\mu\text{C})}{3\,\text{m}}\right] = \boxed{-18.0\,\text{mJ}}$$

18 •

Picture the Problem The electrostatic potential energy of this system of three point charges is the work needed to bring the charges from an infinite separation to the final positions shown in the diagram.



Express the work required to assemble this system of charges:

$$U = \frac{kq_1q_2}{r_{1,2}} + \frac{kq_1q_3}{r_{1,3}} + \frac{kq_2q_3}{r_{2,3}} = k\left(\frac{q_1q_2}{r_{1,2}} + \frac{q_1q_3}{r_{1,3}} + \frac{q_2q_3}{r_{2,3}}\right)$$

Find the distances $r_{1,2}$, $r_{1,3}$, and $r_{2,3}$:

$$r_{1,2} = r_{2,3} = r_{1,3} = 2.5\,\text{m}$$

(*a*) Evaluate U for $q_1 = q_2 = q_3 = 4.2\ \mu\text{C}$:

$$U = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(4.2\,\mu\text{C})(4.2\,\mu\text{C})}{2.5\,\text{m}} + \frac{(4.2\,\mu\text{C})(4.2\,\mu\text{C})}{2.5\,\text{m}} + \frac{(4.2\,\mu\text{C})(4.2\,\mu\text{C})}{2.5\,\text{m}}\right] = \boxed{0.190\,\text{J}}$$

(*b*) Evaluate U for $q_1 = q_2 = 4.2\ \mu\text{C}$ and $q_3 = -4.2\ \mu\text{C}$:

$$U = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(4.2\,\mu\text{C})(4.2\,\mu\text{C})}{2.5\,\text{m}} + \frac{(4.2\,\mu\text{C})(-4.2\,\mu\text{C})}{2.5\,\text{m}} + \frac{(4.2\,\mu\text{C})(-4.2\,\mu\text{C})}{2.5\,\text{m}}\right] = \boxed{-63.4\,\text{mJ}}$$

(*c*) Evaluate U for $q_1 = q_2 = -4.2\ \mu\text{C}$ and $q_3 = +4.2\ \mu\text{C}$:

$$U = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(-4.2\,\mu\text{C})(-4.2\,\mu\text{C})}{2.5\,\text{m}} + \frac{(-4.2\,\mu\text{C})(4.2\,\mu\text{C})}{2.5\,\text{m}} + \frac{(-4.2\,\mu\text{C})(4.2\,\mu\text{C})}{2.5\,\text{m}}\right]$$

$$= \boxed{-63.4\,\text{mJ}}$$

***19** •

Picture the Problem The potential of an isolated spherical conductor is given by $V = kQ/r$,where Q is its charge and r its radius, and its electrostatic potential energy by $U = \frac{1}{2}QV$. We can combine these relationships to find the sphere's electrostatic potential energy.

Express the electrostatic potential energy of the isolated spherical conductor as a function of its charge Q and potential V:

$$U = \tfrac{1}{2}QV$$

Express the potential of the spherical conductor:

$$V = \frac{kQ}{r}$$

Solve for Q to obtain:

$$Q = \frac{rV}{k}$$

Substitute to obtain:

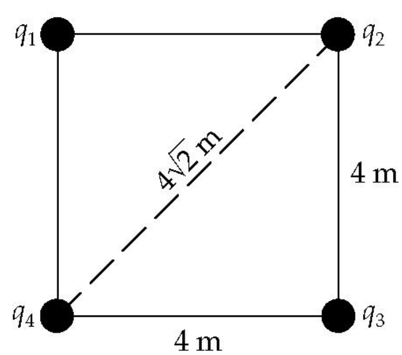
$$U = \tfrac{1}{2}\left(\frac{rV}{k}\right)V = \frac{rV^2}{2k}$$

Substitute numerical values and evaluate U:

$$U = \frac{(0.1\,\text{m})(2\,\text{kV})^2}{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{22.2\,\mu\text{J}}$$

20 ••

Picture the Problem The electrostatic potential energy of this system of four point charges is the work needed to bring the charges from an infinite separation to the final positions shown in the diagram. In part (c), depending on the configuration of the positive and negative charges, two energies are possible.



Express the work required to assemble this system of charges:

$$U=\frac{kq_1q_2}{r_{1,2}}+\frac{kq_1q_3}{r_{1,3}}+\frac{kq_1q_4}{r_{1,4}}+\frac{kq_2q_3}{r_{2,3}}+\frac{kq_2q_4}{r_{2,4}}+\frac{kq_3q_4}{r_{3,4}}$$
$$=k\left(\frac{q_1q_2}{r_{1,2}}+\frac{q_1q_3}{r_{1,3}}+\frac{q_1q_4}{r_{1,4}}+\frac{q_2q_3}{r_{2,3}}+\frac{q_2q_4}{r_{2,4}}+\frac{q_3q_4}{r_{3,4}}\right)$$

Find the distances $r_{1,2}$, $r_{1,3}$, $r_{1,4}$, $r_{2,3}$, $r_{2,4}$, and $r_{3,4}$,:

$$r_{1,2}=r_{2,3}=r_{3,4}=r_{1,4}=4\,\text{m}$$
and
$$r_{1,3}=r_{2,4}=4\sqrt{2}\,\text{m}$$

(*a*) Evaluate U for $q_1 = q_2 = q_3 = q_4 = -2\ \mu\text{C}$:

$$U=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right.$$
$$\left.+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right]$$
$$=\boxed{48.7\,\text{mJ}}$$

(*b*) Evaluate U for $q_1 = q_2 = q_3 = 2\ \mu\text{C}$ and $q_4 = -2\ \mu\text{C}$:

$$U=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{4\,\text{m}}+\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right.$$
$$\left.+\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{4\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right]$$
$$=\boxed{0}$$

(*c*) Let $q_1 = q_2 = 2\ \mu\text{C}$ and $q_3 = q_4 = -2\ \mu\text{C}$:

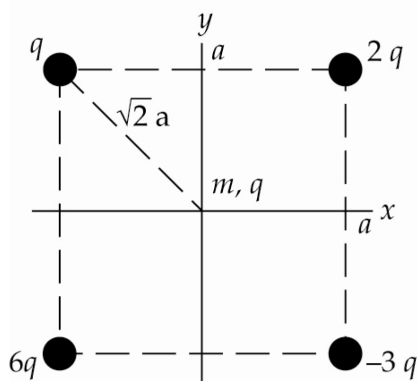
$$U=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{4\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right.$$
$$\left.+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right]$$
$$=\boxed{-12.7\,\text{mJ}}$$

Let $q_1 = q_3 = 2\ \mu\text{C}$ and $q_2 = q_4 = -2\ \mu\text{C}$:

$$U=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}+\frac{(2\,\mu\text{C})(2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right.$$
$$\left.+\frac{(-2\,\mu\text{C})(2\,\mu\text{C})}{4\,\text{m}}+\frac{(-2\,\mu\text{C})(-2\,\mu\text{C})}{4\sqrt{2}\,\text{m}}+\frac{(2\,\mu\text{C})(-2\,\mu\text{C})}{4\,\text{m}}\right]$$
$$=\boxed{-23.2\,\text{mJ}}$$

21 ••

Picture the Problem The diagram shows the four charges fixed at the corners of the square and the fifth charge that is released from rest at the origin. We can use conservation of energy to relate the initial potential energy of the fifth particle to its kinetic energy when it is at a great distance from the origin and the electrostatic potential at the origin to express U_i.



Use conservation of energy to relate the initial potential energy of the particle to its kinetic energy when it is at a great distance from the origin:

$$\Delta K+\Delta U=0$$
or, because $K_i = U_f = 0$,
$$K_f-U_i=0$$

Express the initial potential energy of the particle to its charge and the electrostatic potential at the origin:

$$U_i=qV(0)$$

Substitute for K_f and U_i to obtain:

$$\tfrac{1}{2}mv^2-qV(0)=0$$

Solve for v:

$$v=\sqrt{\frac{2qV(0)}{m}}$$

Express the electrostatic potential at the origin:

$$V(0)=\frac{kq}{\sqrt{2}a}+\frac{2kq}{\sqrt{2}a}+\frac{-3kq}{\sqrt{2}a}+\frac{6kq}{\sqrt{2}a}=\frac{6kq}{\sqrt{2}a}$$

Substitute and simplify to obtain:

$$v=\sqrt{\frac{2q}{m}\left(\frac{6kq}{\sqrt{2}a}\right)}=\boxed{q\sqrt{\frac{6\sqrt{2}k}{ma}}}$$

Capacitance

***22 •**

Picture the Problem The charge on the spherical conductor is related to its radius and potential according to $V = kQ/r$ and we can use the definition of capacitance to find the capacitance of the sphere.

(*a*) Relate the potential V of the spherical conductor to the charge on it and to its radius:

$$V = \frac{kQ}{r}$$

Solve for and evaluate Q:

$$Q = \frac{rV}{k} = \frac{(0.1\,\text{m})(2\,\text{kV})}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{22.2\,\text{nC}}$$

(*b*) Use the definition of capacitance to relate the capacitance of the sphere to its charge and potential:

$$C = \frac{Q}{V} = \frac{22.2\,\text{nC}}{2\,\text{kV}} = \boxed{11.1\text{pF}}$$

(*c*) $\boxed{\text{It doesn't. The capacitance of a sphere is a function of its radius.}}$

23 •

Picture the Problem We can use its definition to find the capacitance of this capacitor.

Use the definition of capacitance to obtain:

$$C = \frac{Q}{V} = \frac{30\,\mu\text{C}}{400\,\text{V}} = \boxed{75.0\,\text{nF}}$$

24 ••

Picture the Problem Let the separation of the spheres be *d* and their radii be *R*. Outside the two spheres the electric field is approximately the field due to point charges of $+Q$ and $-Q$, each located at the centers of spheres, separated by distance *d*. We can derive an expression for the potential at the surface of each sphere and then use the potential difference between the spheres and the definition of capacitance and to find the capacitance of the two-sphere system.

The capacitance of the two-sphere system is given by:

$$C = \frac{Q}{\Delta V}$$

where ΔV is the potential difference between the spheres.

The potential at any point outside the two spheres is:

$$V = \frac{k(+Q)}{r_1} + \frac{k(-Q)}{r_2}$$

where r_1 and r_2 are the distances from the given point to the centers of the spheres.

For a point on the surface of the sphere with charge +*Q*:

$r_1 = R$ and $r_2 = d + \delta$
where $|\delta| < R$

Substitute to obtain:

$$V_{+Q} = \frac{k(+Q)}{R} + \frac{k(-Q)}{d + \delta}$$

For $\delta << d$:

$$V_{+Q} = \frac{kQ}{R} - \frac{kQ}{d}$$

and

$$V_{-Q} = -\frac{kQ}{R} + \frac{kQ}{d}$$

The potential difference between the spheres is:

$$\Delta V = V_Q - V_{-Q} = \frac{kQ}{R} - \frac{kQ}{d} - \left(\frac{-kQ}{R} + \frac{kQ}{d}\right) = 2kQ\left(\frac{1}{R} - \frac{1}{d}\right)$$

Substitute for ΔV in the expression for *C* to obtain:

$$C = \frac{Q}{2kQ\left(\frac{1}{R} - \frac{1}{d}\right)} = \frac{2\pi \in_0}{\left(\frac{1}{R} - \frac{1}{d}\right)} = \frac{2\pi \in_0 R}{1 - \frac{R}{d}}$$

For *d* very large:

$$C = \boxed{2\pi \in_0 R}$$

The Storage of Electrical Energy

25 •

Picture the Problem Of the three equivalent expressions for the energy stored in a charged capacitor, the one that relates *U* to *C* and *V* is $U = \frac{1}{2}CV^2$.

(*a*) Express the energy stored in the capacitor as a function of *C* and *V*:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate U:

$$U = \tfrac{1}{2}(3\,\mu\text{F})(100\,\text{V})^2 = \boxed{15.0\,\text{mJ}}$$

(*b*) Express the additional energy required as the difference between the energy stored in the capacitor at 200 V and the energy stored at 100 V:

$$\begin{aligned}\Delta U &= U(200\,\text{V}) - U(100\,\text{V})\\ &= \tfrac{1}{2}(3\,\mu\text{F})(200\,\text{V})^2 - 15.0\,\text{mJ}\\ &= \boxed{45.0\,\text{mJ}}\end{aligned}$$

26 •

Picture the Problem Of the three equivalent expressions for the energy stored in a charged capacitor, the one that relates U to Q and C is $U = \dfrac{1}{2}\dfrac{Q^2}{C}$.

(*a*) Express the energy stored in the capacitor as a function of C and Q:

$$U = \frac{1}{2}\frac{Q^2}{C}$$

Substitute numerical values and evaluate U:

$$U = \frac{1}{2}\frac{(4\,\mu\text{C})^2}{10\,\mu\text{F}} = \boxed{0.800\,\mu\text{J}}$$

(*b*) Express the energy remaining when half the charge is removed:

$$U\left(\tfrac{1}{2}Q\right) = \frac{1}{2}\frac{(2\,\mu\text{C})^2}{10\,\mu\text{F}} = \boxed{0.200\,\mu\text{J}}$$

27 •

Picture the Problem Of the three equivalent expressions for the energy stored in a charged capacitor, the one that relates U to Q and C is $U = \dfrac{1}{2}\dfrac{Q^2}{C}$.

(*a*) Express the energy stored in the capacitor as a function of C and Q:

$$U = \frac{1}{2}\frac{Q^2}{C}$$

Substitute numerical values and evaluate U:

$$U(5\,\mu\text{C}) = \frac{1}{2}\frac{(5\,\mu\text{C})^2}{20\,\text{pF}} = \boxed{0.625\,\text{J}}$$

(*b*) Express the additional energy required as the difference between the energy stored in the capacitor when its charge is 5 μC and when its charge is 10 μC:

$$\begin{aligned}\Delta U &= U(10\,\mu\text{C}) - U(5\,\mu\text{C})\\ &= \frac{1}{2}\frac{(10\,\mu\text{C})^2}{20\,\text{pF}} - 0.625\,\text{J}\\ &= 2.50\,\text{J} - 0.625\,\text{J}\\ &= \boxed{1.88\,\text{J}}\end{aligned}$$

***28** •

Picture the Problem The energy per unit volume in an electric field varies with the square of the electric field according to $u = \epsilon_0 E^2/2$.

Express the energy per unit volume in an electric field:

$$u = \tfrac{1}{2}\epsilon_0 E^2$$

Substitute numerical values and evaluate *u*:

$$u = \tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\,\text{MV/m}\right)^2 = \boxed{39.8\,\text{J/m}^3}$$

29 •

Picture the Problem Knowing the potential difference between the plates, we can use $E = V/d$ to find the electric field between them. The energy per unit volume is given by $u = \tfrac{1}{2}\epsilon_0 E^2$ and we can find the capacitance of the parallel-plate capacitor using $C = \epsilon_0 A/d$.

(*a*) Express the electric field between the plates in terms of their separation and the potential difference between them:

$$E = \frac{V}{d} = \frac{100\,\text{V}}{1\text{mm}} = \boxed{100\,\text{kV/m}}$$

(*b*) Express the energy per unit volume in an electric field:

$$u = \tfrac{1}{2}\epsilon_0 E^2$$

Substitute numerical values and evaluate *u*:

$$u = \tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(100\,\text{kV/m}\right)^2 = \boxed{44.3\,\text{mJ/m}^3}$$

(*c*) The total energy is given by:

$$U = uV = uAd = \left(44.3\,\text{mJ/m}^3\right)\left(2\,\text{m}^2\right)\left(1\text{mm}\right) = \boxed{88.6\,\mu J}$$

(*d*) The capacitance of a parallel-plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2\,\text{m}^2\right)}{1\text{mm}} = \boxed{17.7\,\text{nF}}$$

(*e*) The total energy is given by:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate *U*:

$$U = \tfrac{1}{2}(17.7\,\text{nF})(100\,\text{V})^2 = \boxed{88.5\,\mu\text{J, in agreement with } (c).}$$

30 ••

Picture the Problem The total energy stored in the electric field is the product of the energy density in the space between the spheres and the volume of this space.

(*a*) The total energy *U* stored in the electric field is given by:

$$U = uV$$

where *u* is the energy density and *V* is the volume between the spheres.

The energy density of the field is:

$$u = \frac{1}{2}\in_0 E^2$$

where *E* is the field between the spheres.

The volume between the spheres is approximately:

$$V \approx 4\pi r_1^2 (r_2 - r_1)$$

Substitute for *u* and *V* to obtain:

$$U = 2\pi \in_0 E^2 r_1^2 (r_2 - r_1) \qquad (1)$$

The magnitude of the electric field between the concentric spheres is the sum of the electric fields due to each charge distribution:

$$E = E_Q + E_{-Q}$$

Because the two surfaces are so close together, the electric field between them is approximately the sum of the fields due to two plane charge distributions:

$$E = \frac{\sigma_Q}{2\in_0} + \frac{\sigma_{-Q}}{2\in_0} = \frac{\sigma_Q}{\in_0}$$

Substitute for σ_Q to obtain:

$$E \approx \frac{Q}{4\pi r_1^2 \in_0}$$

Substitute for *E* in equation (1) and simplify:

$$U = 2\pi \in_0 \left(\frac{Q}{4\pi r_1^2 \in_0}\right)^2 r_1^2 (r_2 - r_1) = \frac{Q^2}{8\pi \in_0}\frac{r_2 - r_1}{r_1^2}$$

Substitute numerical values and evaluate U:

$$U=\frac{(5\,\text{nC})^2(10.5\,\text{cm}-10.0\,\text{cm})}{8\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(10.0\,\text{cm})^2}=\boxed{56.0\,\text{nJ}}$$

(b) The capacitance of the two-sphere system is given by:

$$C=\frac{Q}{\Delta V}$$

where ΔV is the potential difference between the two spheres.

The electric potentials at the surfaces of the spheres are:

$$V_1=\frac{Q}{4\pi\in_0 r_1}\text{ and }V_2=\frac{Q}{4\pi\in_0 r_2}$$

Substitute for ΔV and simplify to obtain:

$$C=\frac{Q}{\dfrac{Q}{4\pi\in_0 r_1}-\dfrac{Q}{4\pi\in_0 r_2}}=4\pi\in_0\frac{r_1r_2}{r_2-r_1}$$

Substitute numerical values and evaluate C:

$$C=4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)\frac{(10.0\,\text{cm})(10.5\,\text{cm})}{10.5\,\text{cm}-10.0\,\text{cm}}=\boxed{0.234\,\text{nF}}$$

Use ½ Q^2/C to find the total energy stored in the electric field between the spheres:

$$U=\frac{1}{2}\left[\frac{(5\,\text{nC})^2}{0.234\,\text{nF}}\right]=\boxed{53.4\,\text{nJ}}$$

Note that our approximate result in (a) is within 5% of our exact result obtained in (b).

***31** ••

Picture the Problem We can relate the charge Q on the positive plate of the capacitor to the charge density of the plate σ using its definition. The charge density, in turn, is related to the electric field between the plates according to $\sigma=\in_0 E$ and the electric field can be found from $E = \Delta V/\Delta d$. We can use $\Delta U=\frac{1}{2}Q\Delta V$ in part (b) to find the increase in the energy stored due to the movement of the plates.

(a) Express the charge Q on the positive plate of the capacitor in terms of the plate's charge density σ and surface area A:

$$Q=\sigma A$$

Relate σ to the electric field E between the plates of the capacitor:

$$\sigma = \epsilon_0 E$$

Express E in terms of the change in V as the plates are separated a distance Δd:

$$E = \frac{\Delta V}{\Delta d}$$

Substitute for σ and E to obtain:

$$Q = \epsilon_0 EA = \epsilon_0 A \frac{\Delta V}{\Delta d}$$

Substitute numerical values and evaluate Q:

$$Q = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(500\,\text{cm}^2\right)\frac{100\,\text{V}}{0.4\,\text{cm}} = \boxed{11.1\,\text{nC}}$$

(b) Express the change in the electrostatic energy in terms of the change in the potential difference:

$$\Delta U = \tfrac{1}{2}Q\Delta V$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \tfrac{1}{2}\left(11.1\,\text{nC}\right)\left(100\,\text{V}\right) = \boxed{0.553\,\mu\text{J}}$$

32 •••

Picture the Problem By symmetry, the electric field must be radial. In part (a) we can find E_r both inside and outside the ball by choosing a spherical Gaussian surface first inside and then outside the surface of the ball and applying Gauss's law.

(a) Relate the electrostatic energy density at a distance r from the center of the ball to the electric field due to the uniformly distributed charge Q:

$$u_e = \tfrac{1}{2}\epsilon_0 E^2 \quad (1)$$

Relate the flux through the Gaussian surface to the electric field E_r on the Gaussian surface at $r < R$:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} \quad (2)$$

Using the fact that the charge is uniformly distributed, express the ratio of the charge enclosed by the Gaussian surface to the total charge of the sphere:

$$\frac{Q_{\text{inside}}}{Q} = \frac{\rho V_{\text{Gaussian surface}}}{\rho V_{\text{ball}}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3}$$

Solve for Q_{inside} to obtain:

$$Q_{\text{inside}} = Q\frac{r^3}{R^3}$$

Substitute in equation (2):

$$E_r\left(4\pi r^2\right) = \frac{Qr^3}{\in_0 R^3}$$

Solve for $E_{r<R}$:

$$E_{r<R} = \frac{Qr}{4\pi \in_0 R^3} = \frac{kQ}{R^3}r$$

Substitute in equation (1) to obtain:

$$u_e(r<R) = \tfrac{1}{2}\in_0\left(\frac{kQ}{R^3}r\right)^2 = \boxed{\frac{\in_0 k^2Q^2}{2R^6}r^2}$$

Relate the flux through the Gaussian surface to the electric field E_r on the Gaussian surface at $r > R$:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\in_0} = \frac{Q}{\in_0}$$

Solve for $E_{r>R}$:

$$E_{r>R} = \frac{Q}{4\pi r^2 \in_0} = kQr^{-2}$$

Substitute in equation (1) to obtain:

$$u_e(r>R) = \tfrac{1}{2}\in_0\left(kQr^{-2}\right)^2 = \boxed{\tfrac{1}{2}\in_0 k^2Q^2r^{-4}}$$

(*b*) Express the energy dU in a spherical shell of thickness dr and surface area $4\pi r^2$:

$$dU_{\text{shell}} = 4\pi r^2 u(r)dr$$

For $r < R$:

$$dU_{\text{shell}}(r<R) = 4\pi r^2\left(\frac{\in_0 k^2Q^2}{2R^6}r^2\right)dr = \boxed{\frac{kQ^2}{2R^6}r^4dr}$$

For $r > R$:

$$dU_{\text{shell}}(r>R) = 4\pi r^2\left(\tfrac{1}{2}\in_0 k^2Q^2r^{-4}\right)dr = \boxed{\tfrac{1}{2}kQ^2r^{-2}dr}$$

(*c*) Express the total electrostatic energy:

$$U = U(r<R) + U(r>R) \qquad (3)$$

Integrate $U_{\text{shell}}(r < R)$ from 0 to R:

$$U_{\text{shell}}(r<R)=\frac{kQ^2}{2R^6}\int_0^R r^4 dr =\frac{kQ^2}{10R}$$

Integrate $U_{\text{shell}}(r > R)$ from R to ∞:

$$U_{\text{shell}}(r>R)=\tfrac{1}{2}kQ^2\int_R^\infty r^{-2}dr=\frac{kQ^2}{2R}$$

Substitute in equation (3) to obtain:

$$U=\frac{kQ^2}{10R}+\frac{kQ^2}{2R}=\boxed{\frac{3kQ^2}{5R}}$$

The field inside the shell is zero, so the first integral vanishes. The result is greater for the sphere because it includes the field energy within the sphere.

Combinations of Capacitors

33 •

Picture the Problem We can apply the properties of capacitors connected in parallel to determine the number of 1.0-μF capacitors connected in parallel it would take to store a total charge of 1 mC with a potential difference of 10 V across each capacitor. Knowing that the capacitors are connected in parallel (parts (*a*) and (*b*)) we determine the potential difference across the combination. In part (*c*) we can use our knowledge of how potential differences add in a series circuit to find the potential difference across the combination and the definition of capacitance to find the charge on each capacitor.

(*a*) Express the number of capacitors n in terms of the charge q on each and the total charge Q:

$$n=\frac{Q}{q}$$

Relate the charge q on one capacitor to its capacitance C and the potential difference across it:

$$q=CV$$

Substitute to obtain:

$$n=\frac{Q}{CV}$$

Substitute numerical values and evaluate n:

$$n=\frac{1\,\text{mC}}{(1\,\mu\text{F})(10\,\text{V})}=\boxed{100}$$

(*b*) Because the capacitors are connected in parallel the potential difference across the combination is the same as the potential difference across each of them:

$$V_{\text{parallel combination}} = V = \boxed{10\,\text{V}}$$

(*c*) With the capacitors connected in series, the potential difference across the combination will be the sum of the potential differences across the 100 capacitors:

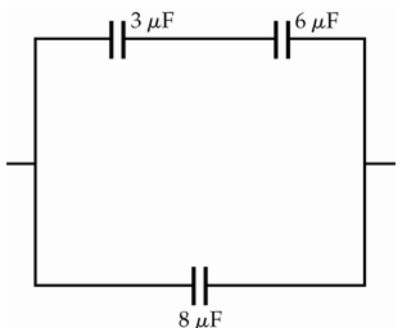
$$V_{\text{series combination}} = 100V = 100(10\,\text{V}) = \boxed{1.00\,\text{kV}}$$

Use the definition of capacitance to find the charge on each capacitor:

$$q = CV = (1\,\mu\text{F})(10\,\text{V}) = \boxed{10.0\,\mu\text{C}}$$

34 •

Picture the Problem The capacitor array is shown in the diagram. We can find the equivalent capacitance of this combination by first finding the equivalent capacitance of the 3.0-μF and 6.0-μF capacitors in series and then the equivalent capacitance of this capacitor with the 8.0-μF capacitor in parallel.



Express the equivalent capacitance for the 3.0-μF and 6.0-μF capacitors in series:

$$\frac{1}{C_{3+6}} = \frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}$$

Solve for C_{3+6}:

$$C_{3+6} = 2\,\mu\text{F}$$

Find the equivalent capacitance of a 2-μF capacitor in parallel with an 8-μF capacitor:

$$C_{2+8} = 2\,\mu\text{F} + 8\,\mu\text{F} = \boxed{10\,\mu\text{F}}$$

***35** •

Picture the Problem Because we're interested in the equivalent capacitance across terminals *a* and *c*, we need to recognize that capacitors C_1 and C_3 are in series with each other and in parallel with capacitor C_2.

Find the equivalent capacitance of C_1 and C_3 in series:

$$\frac{1}{C_{1+3}} = \frac{1}{C_1} + \frac{1}{C_3}$$

Solve for C_{1+3}:

$$C_{1+3} = \frac{C_1 C_3}{C_1 + C_3}$$

Find the equivalent capacitance of C_{1+3} and C_2 in parallel:

$$C_{eq} = C_2 + C_{1+3} = \boxed{C_2 + \frac{C_1 C_3}{C_1 + C_3}}$$

36 •

Picture the Problem Because the capacitors are connected in parallel we can add their capacitances to find the equivalent capacitance of the combination. Also, because they are in parallel, they have a common potential difference across them. We can use the definition of capacitance to find the charge on each capacitor.

(*a*) Find the equivalent capacitance of the two capacitors in parallel:

$$C_{eq} = 10.0\,\mu\text{F} + 20\,\mu\text{F} = \boxed{30\,\mu\text{F}}$$

(*b*) Because capacitors in parallel have a common potential difference across them:

$$V = V_{10} + V_{20} = \boxed{6.00\,\text{V}}$$

(*c*) Use the definition of capacitance to find the charge on each capacitor:

$$Q_{10} = C_{10}V = (10\,\mu\text{F})(6\,\text{V}) = \boxed{60.0\,\mu\text{C}}$$

and

$$Q_{20} = C_{20}V = (20\,\mu\text{F})(6\,\text{V}) = \boxed{120\,\mu\text{C}}$$

37 ••

Picture the Problem We can use the properties of capacitors in series to find the equivalent capacitance and the charge on each capacitor. We can then apply the definition of capacitance to find the potential difference across each capacitor.

(*a*) Because the capacitors are connected in series they have equal charges:

$$Q_{10} = Q_{20} = C_{eq}V$$

Express the equivalent capacitance of the two capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{10\,\mu\text{F}} + \frac{1}{20\,\mu\text{F}}$$

Solve for C_{eq} to obtain:

$$C_{eq} = \frac{(10\,\mu\text{F})(20\,\mu\text{F})}{10\,\mu\text{F} + 20\,\mu\text{F}} = 6.67\,\mu\text{F}$$

Substitute to obtain:

$$Q_{10} = Q_{20} = (6.67\,\mu\text{F})(6\,\text{V}) = \boxed{40.0\,\mu\text{C}}$$

(*b*) Apply the definition of capacitance to find the potential difference across each capacitor:

$$V_{10}=\frac{Q_{10}}{C_{10}}=\frac{40.0\ \mu\text{C}}{10\ \mu\text{F}}=\boxed{4.00\ \text{V}}$$

and

$$V_{20}=\frac{Q_{20}}{C_{20}}=\frac{40.0\ \mu\text{C}}{20\ \mu\text{F}}=\boxed{2.00\ \text{V}}$$

***38 ••**

Picture the Problem We can use the properties of capacitors connected in series and in parallel to find the equivalent capacitances for various connection combinations.

(*a*) $\boxed{\text{If their capacitance is to be a maximum, they must be connected in parallel.}}$

Find the capacitance of each capacitor:

$$C_{\text{eq}}=3C=15\ \mu\text{F}$$

and

$$C=5\ \mu\text{F}$$

(*b*) (1) Connect the three capacitors in series:

$$\frac{1}{C_{\text{eq}}}=\frac{3}{5\ \mu\text{F}}\ \text{and}\ C_{\text{eq}}=\boxed{1.67\ \mu\text{F}}$$

(2) Connect two in parallel, with the third in series with that combination:

$$C_{\text{eq, two in parallel}}=2(5\ \mu\text{F})=10\ \mu\text{F}$$

and

$$\frac{1}{C_{\text{eq}}}=\frac{1}{10\ \mu\text{F}}+\frac{1}{5\ \mu\text{F}}$$

Solve for C_{eq}:

$$C_{\text{eq}}=\frac{(10\ \mu\text{F})(5\ \mu\text{F})}{10\ \mu\text{F}+5\ \mu\text{F}}=\boxed{3.33\ \mu\text{F}}$$

(3) Connect two in series, with the third in parallel with that combination:

$$\frac{1}{C_{\text{eq, two in series}}}=\frac{2}{5\ \mu\text{F}}$$

or

$$C_{\text{eq, two in series}}=2.5\ \mu\text{F}$$

Find the capacitance equivalent to 2.5 μF and 5 μF in parallel:

$$C_{\text{eq}}=2.5\ \mu\text{F}+5\ \mu\text{F}=\boxed{7.50\ \mu\text{F}}$$

39 ••

Picture the Problem We can use the properties of capacitors connected in series and in parallel to find the equivalent capacitance between the terminals and these properties and the definition of capacitance to find the charge on each capacitor.

(*a*) Relate the equivalent capacitance of the two capacitors in series to their individual capacitances:

$$\frac{1}{C_{4+15}}=\frac{1}{4\,\mu\text{F}}+\frac{1}{15\,\mu\text{F}}$$

Solve for C_{4+15}:

$$C_{4+15}=\frac{(4\,\mu\text{F})(15\,\mu\text{F})}{4\,\mu\text{F}+15\,\mu\text{F}}=3.16\,\mu\text{F}$$

Find the equivalent capacitance of C_{4+15} in parallel with the 12-μF capacitor:

$$C_{\text{eq}}=3.16\,\mu\text{F}+12\,\mu\text{F}=\boxed{15.2\,\mu\text{F}}$$

(*b*) Using the definition of capacitance, express and evaluate the charge stored on the 12-μF capacitor:

$$\begin{aligned}Q_{12}&=C_{12}V_{12}=C_{12}V\\&=(12\,\mu\text{F})(200\,\text{V})\\&=\boxed{2.40\,\text{mC}}\end{aligned}$$

Because the capacitors in series have the same charge:

$$\begin{aligned}Q_4&=Q_{15}=C_{4+15}V\\&=(3.16\,\mu\text{F})(200\,\text{V})\\&=\boxed{0.632\,\text{mC}}\end{aligned}$$

(*c*) The total energy stored is given by:

$$U_{\text{total}}=\frac{1}{2}C_{\text{eq}}V^2$$

Substitute numerical values and evaluate U_{total}:

$$U_{\text{total}}=\frac{1}{2}(15.2\,\mu\text{F})(200\,\text{V})^2=\boxed{0.304\,\text{J}}$$

40 ••

Picture the Problem We can use the properties of capacitors in series to establish the results called for in this problem.

(*a*) Express the equivalent capacitance of two capacitors in series:

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{C_2+C_1}{C_1C_2}$$

Solve for C_{eq} by taking the reciprocal of both sides of the equation to obtain:

$$C_{\text{eq}}=\boxed{\frac{C_1C_2}{C_1+C_2}}$$

(*b*) Divide numerator and denominator of this expression by C_1 to obtain:

$$C_{eq} = \frac{C_2}{1+\frac{C_2}{C_1}} < \boxed{C_2}$$

because $1+\frac{C_2}{C_1} > 1$.

Divide numerator and denominator of this expression by C_2 to obtain:

$$C_{eq} = \frac{C_1}{1+\frac{C_1}{C_2}} < \boxed{C_1}$$

because $1+\frac{C_1}{C_2} > 1$.

Using our result from part (*a*) for two of the capacitors, add a third capacitor C_3 in series to obtain:

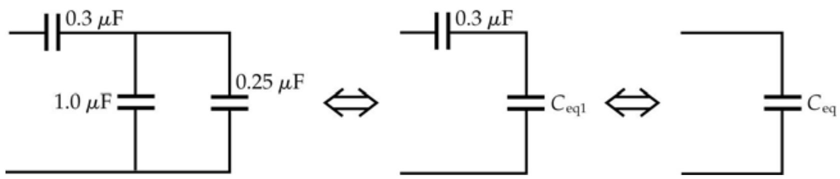
$$\frac{1}{C_{eq}} = \frac{C_1+C_2}{C_1C_2} + \frac{1}{C_3} = \frac{C_1C_3+C_2C_3+C_1C_2}{C_1C_2C_3}$$

Take the reciprocal of both sides of the equation to obtain:

$$C_{eq} = \boxed{\frac{C_1C_2C_3}{C_1C_2+C_2C_3+C_1C_3}}$$

41 ••

Picture the Problem Let C_{eq1} represent the equivalent capacitance of the parallel combination and C_{eq} the total equivalent capacitance between the terminals. We can use the equations for capacitors in parallel and then in series to find C_{eq}. Because the charge on C_{eq} is the same as on the 0.3-μF capacitor and C_{eq1}, we'll know the charge on the 0.3-μF capacitor when we have found the total charge Q_{eq} stored by the circuit. We can find the charges on the 1.0-μF and 0.25-μF capacitors by first finding the potential difference across them and then using the definition of capacitance.



(*a*) Find the equivalent capacitance for the parallel combination:

$$C_{eq1} = 1\,\mu F + 0.25\,\mu F = 1.25\,\mu F$$

The 0.30-μF capacitor is in series with C_{eq1} … find their equivalent capacitance C_{eq}:

$$\frac{1}{C_{eq}}=\frac{1}{0.3\,\mu\text{F}}+\frac{1}{1.25\,\mu\text{F}}$$

and

$$C_{eq}=\boxed{0.242\,\mu\text{F}}$$

(*b*) Express the total charge stored by the circuit Q_{eq}:

$$Q_{eq}=Q_{0.3}=Q_{1.25}=C_{eq}V=(0.242\,\mu\text{F})(10\,\text{V})=\boxed{2.42\,\mu\text{C}}$$

The 1-μF and 0.25-μF capacitors, being in parallel, have a common potential difference. Express this potential difference in terms of the 10 V across the system and the potential difference across the 0.3-μF capacitor:

$$V_{1.25}=10\,\text{V}-V_{0.3}=10\,\text{V}-\frac{Q_{0.3}}{C_{0.3}}=10\,\text{V}-\frac{2.42\,\mu\text{C}}{0.3\,\mu\text{F}}=1.93\,\text{V}$$

Using the definition of capacitance, find the charge on the 1-μF and 0.25-μF capacitors:

$$Q_1=C_1V_1=(1\,\mu\text{F})(1.93\,\text{V})=\boxed{1.93\,\mu\text{C}}$$

and

$$Q_{0.25}=C_{0.25}V_{0.25}=(0.25\,\mu\text{F})(1.93\,\text{V})=\boxed{0.483\,\mu\text{C}}$$

(*c*) The total stored energy is given by:

$$U=\tfrac{1}{2}C_{eq}V^2$$

Substitute numerical values and evaluate U:

$$U=\tfrac{1}{2}(0.242\,\mu\text{F})(10\,\text{V})^2=\boxed{12.1\,\mu\text{J}}$$

42 ••

Picture the Problem Note that there are three parallel paths between *a* and *b*. We can find the equivalent capacitance of the capacitors connected in series in the upper and lower branches and then find the equivalent capacitance of three capacitors in parallel.

(*a*) Find the equivalent capacitance of the series combination of capacitors in the upper and lower branch:

$$\frac{1}{C_{eq}}=\frac{1}{C_0}+\frac{1}{C_0}$$

or

$$\text{C}_{eq}=\frac{C_0^2}{2C_0}=\tfrac{1}{2}C_0$$

Now we have two capacitors with capacitance $C_0/2$ in parallel with a capacitor whose capacitance is C_0. Find their equivalent capacitance:

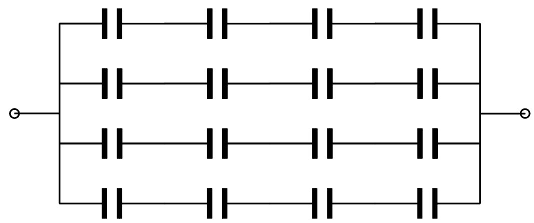
$$C'_{\text{eq}} = \tfrac{1}{2}C_0 + C_0 + \tfrac{1}{2}C_0 = \boxed{2C_0}$$

(*b*) If the central capacitance is $10C_0$, then:

$$C'_{\text{eq}} = \tfrac{1}{2}C_0 + 10C_0 + \tfrac{1}{2}C_0 = \boxed{11C_0}$$

43 ••

Picture the Problem Place four of the capacitors in series. Then the potential across each is 100 V when the potential across the combination is 400 V. The equivalent capacitance of the series is 2/4 μF = 0.5 μF. If we place four such series combinations in parallel, as shown in the circuit diagram, the total capacitance between the terminals is 2 μF.



***44** ••

Picture the Problem We can connect two capacitors in parallel, all three in parallel, two in series, three in series, two in parallel in series with the third, and two in series in parallel with the third.

Connect 2 in parallel to obtain:

$$C_{\text{eq}} = 1\,\mu\text{F} + 2\,\mu\text{F} = \boxed{3\,\mu\text{F}}$$

or

$$C_{\text{eq}} = 1\,\mu\text{F} + 4\,\mu\text{F} = \boxed{5\,\mu\text{F}}$$

or

$$C_{\text{eq}} = 2\,\mu\text{F} + 4\,\mu\text{F} = \boxed{6\,\mu\text{F}}$$

Connect all three in parallel to obtain:

$$C_{\text{eq}} = 1\,\mu\text{F} + 2\,\mu\text{F} + 4\,\mu\text{F} = \boxed{7\,\mu\text{F}}$$

Connect two in series:

$$C_{\text{eq}} = \frac{(1\,\mu\text{F})(2\,\mu\text{F})}{1\,\mu\text{F} + 2\,\mu\text{F}} = \boxed{\frac{2}{3}\,\mu\text{F}}$$

or

$$C_{\text{eq}} = \frac{(1\,\mu\text{F})(4\,\mu\text{F})}{1\,\mu\text{F} + 4\,\mu\text{F}} = \boxed{\frac{4}{5}\,\mu\text{F}}$$

or

$$C_{eq} = \frac{(2\,\mu F)(4\,\mu F)}{2\,\mu F + 4\,\mu F} = \boxed{\frac{4}{3}\,\mu F}$$

Connect all three in series:

$$C_{eq} = \frac{(1\,\mu F)(2\,\mu F)(4\,\mu F)}{(1\,\mu F)(2\,\mu F) + (2\,\mu F)(4\,\mu F) + (1\,\mu F)(4\,\mu F)} = \boxed{\frac{4}{7}\,\mu F}$$

Connect two in parallel, in series with the third:

$$C_{eq} = \frac{(4\,\mu F)(1\,\mu F + 2\,\mu F)}{1\,\mu F + 2\,\mu F + 4\,\mu F} = \boxed{\frac{12}{7}\,\mu F}$$

or

$$C_{eq} = \frac{(1\,\mu F)(4\,\mu F + 2\,\mu F)}{1\,\mu F + 2\,\mu F + 4\,\mu F} = \boxed{\frac{6}{7}\,\mu F}$$

or

$$C_{eq} = \frac{(2\,\mu F)(4\,\mu F + 1\,\mu F)}{1\,\mu F + 2\,\mu F + 4\,\mu F} = \boxed{\frac{10}{7}\,\mu F}$$

Connect two in series, in parallel with the third:

$$C_{eq} = \frac{(1\,\mu F)(2\,\mu F)}{1\,\mu F + 2\,\mu F} + 4\,\mu F = \boxed{\frac{14}{3}\,\mu F}$$

or

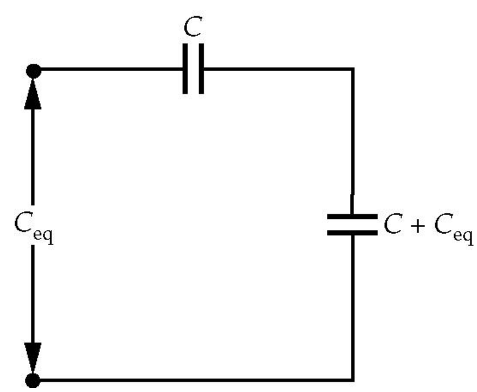
$$C_{eq} = \frac{(4\,\mu F)(2\,\mu F)}{4\,\mu F + 2\,\mu F} + 1\,\mu F = \boxed{\frac{7}{3}\,\mu F}$$

or

$$C_{eq} = \frac{(1\,\mu F)(4\,\mu F)}{1\,\mu F + 4\,\mu F} + 2\,\mu F = \boxed{\frac{14}{5}\,\mu F}$$

45 •••

Picture the Problem Let C be the capacitance of each capacitor in the ladder and let C_{eq} be the equivalent capacitance of the infinite ladder less the series capacitor in the first rung. Because the capacitance is finite and non-zero, adding one more stage to the ladder will not change the capacitance of the network. The capacitance of the two capacitor combination shown to the right is the equivalent of the infinite ladder, so it has capacitance C_{eq} also.



(*a*) The equivalent capacitance of the parallel combination of C and C_{eq} is:

$$C + C_{eq}$$

The equivalent capacitance of the series combination of C and $(C + C_{eq})$ is C_{eq}, so:

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C + C_{eq}}$$

Simply this expression to obtain a quadratic equation in C_{eq}:

$$C_{eq}^2 + CC_{eq} - C^2 = 0$$

Solve for the positive value of C_{eq} to obtain:

$$C_{eq} = \left(\frac{\sqrt{5} - 1}{2}\right)C = 0.618C$$

Because $C = 1\ \mu$F:

$$C_{eq} = \boxed{0.618\,\mu\text{F}}$$

(*b*) The capacitance C' required so that the combination has the same capacitance as the infinite ladder is:

$$C' = C + C_{eq}$$

Substitute for C_{eq} and evaluate C':

$$C' = C + 0.618C = 1.618C$$

Because $C = 1\ \mu$F:

$$C' = \boxed{1.618\,\mu\text{F}}$$

Parallel-Plate Capacitors

46 •

Picture the Problem The potential difference V across a parallel-plate capacitor, the electric field E between its plates, and the separation d of the plates are related according to $V = Ed$. We can use this relationship to find V_{max} corresponding to dielectric breakdown and the definition of capacitance to find the maximum charge on the capacitor.

(*a*) Express the potential difference V across the plates of the capacitor in terms of the electric field between the plates E and their separation d:

$$V = Ed$$

V_{max} corresponds to E_{max}:

$$V_{max} = (3\,\text{MV/m})(1.6\,\text{mm}) = \boxed{4.80\,\text{kV}}$$

(*b*) Using the definition of capacitance, find the charge Q

$$Q = CV_{max} = (2.0\,\mu\text{F})(4.80\,\text{kV}) = \boxed{9.60\,\text{mC}}$$

stored at this maximum potential difference:

47 •

Picture the Problem The potential difference V across a parallel-plate capacitor, the electric field E between its plates, and the separation d of the plates are related according to $V = Ed$. In part (*b*) we can use the definition of capacitance and the expression for the capacitance of a parallel-plate capacitor to find the required plate radius.

(*a*) Express the potential difference V across the plates of the capacitor in terms of the electric field between the plates E and their separation d:

$$V = Ed$$

Substitute numerical values and evaluate V:

$$V = \left(2\times10^4\ \text{V/m}\right)\left(2\ \text{mm}\right) = \boxed{40.0\ \text{V}}$$

(*b*) Use the definition of capacitance to relate the capacitance of the capacitor to its charge and the potential difference across it:

$$C = \frac{Q}{V}$$

Express the capacitance of a parallel-plate capacitor:

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \pi R^2}{d}$$

where R is the radius of the circular plates.

Equate these two expressions for C:

$$\frac{\epsilon_0 \pi R^2}{d} = \frac{Q}{V}$$

Solve for R to obtain:

$$R = \sqrt{\frac{Qd}{\epsilon_0 \pi V}}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{\frac{\left(10\ \mu\text{C}\right)\left(2\ \text{mm}\right)}{\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(40\ \text{V}\right)}} = \boxed{4.24\ \text{m}}$$

48 ••

Picture the Problem We can use the expression for the capacitance of a parallel-plate capacitor to find the area of each plate and the definition of capacitance to find the potential difference when the capacitor is charged to 3.2 μC. We can find the stored energy using $U = \frac{1}{2}CV^2$ and the definition of capacitance and the relationship between the potential difference across a parallel-plate capacitor and the electric field between its plates to find the charge at which dielectric breakdown occurs. Recall that $E_{\text{max, air}}$ = 3 MV/m.

(*a*) Relate the capacitance of a parallel-plate capacitor to the area *A* of its plates and their separation *d*:

$$C = \frac{\epsilon_0 A}{d}$$

Solve for *A*:

$$A = \frac{Cd}{\epsilon_0}$$

Substitute numerical values and evaluate *A*:

$$A = \frac{(0.14\,\mu\text{F})(0.5\,\text{mm})}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{7.91\,\text{m}^2}$$

(*b*) Using the definition of capacitance, express and evaluate the potential difference across the capacitor when it is charged to 3.2 μC:

$$V = \frac{Q}{C} = \frac{3.2\,\mu\text{C}}{0.14\,\mu\text{F}} = \boxed{22.9\,\text{V}}$$

(*c*) Express the stored energy as a function of the capacitor's capacitance and the potential difference across it:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate *U*:

$$U = \tfrac{1}{2}(0.14\,\mu\text{F})(22.9\,\text{V})^2 = \boxed{36.7\,\mu\text{J}}$$

(*d*) Using the definition of capacitance, relate the charge on the capacitor to breakdown potential difference:

$$Q_{\text{max}} = CV_{\text{max}}$$

Relate the maximum potential difference to the maximum electric field between the plates:

$$V_{\text{max}} = E_{\text{max}}d$$

Substitute to obtain:

$$Q_{\text{max}} = CE_{\text{max}}d$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{\text{max}} = (0.14\,\mu\text{F})(3\,\text{MV/m})(0.5\,\text{mm}) = \boxed{210\,\mu\text{C}}$$

***49 ••**

Picture the Problem The potential difference across the capacitor plates *V* is related to their separation *d* and the electric field between them according to $V = Ed$. We can use this equation with $E_{max} = 3$ MV/m to find d_{min}. In part (*b*) we can use the expression for the capacitance of a parallel-plate capacitor to find the required area of the plates.

(*a*) Use the relationship between the potential difference across the plates and the electric field between them to find the minimum separation of the plates:

$$d_{min} = \frac{V}{E_{max}} = \frac{1000\,\text{V}}{3\,\text{MV/m}} = \boxed{0.333\,\text{mm}}$$

(*b*) Use the expression for the capacitance of a parallel-plate capacitor to relate the capacitance to the area of a plate:

$$C = \frac{\epsilon_0 A}{d}$$

Solve for *A*:

$$A = \frac{Cd}{\epsilon_0}$$

Substitute numerical values and evaluate *A*:

$$A = \frac{(0.1\,\mu\text{F})(0.333\,\text{mm})}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{3.76\,\text{m}^2}$$

Cylindrical Capacitors

50 •

Picture the Problem The capacitance of a cylindrical capacitor is given by $C = 2\pi\kappa \epsilon_0 L/\ln(r_2/r_1)$ where *L* is its length and r_1 and r_2 the radii of the inner and outer conductors.

(*a*) Express the capacitance of the coaxial cylindrical shell:

$$C = \frac{2\pi\kappa \epsilon_0 L}{\ln\left(\frac{r}{R}\right)}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{2\pi(1)(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.12\,\text{m})}{\ln\left(\frac{1.5\,\text{cm}}{0.2\,\text{mm}}\right)} = \boxed{1.55\,\text{pF}}$$

(*b*) Use the definition of capacitance to express the charge per unit length:

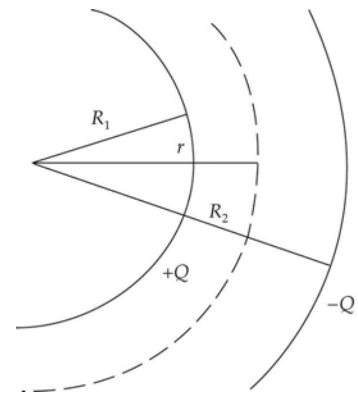
$$\lambda = \frac{Q}{L} = \frac{CV}{L}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{(1.55\,\text{pF})(1.2\,\text{kV})}{0.12\,\text{m}} = \boxed{15.5\,\text{nC/m}}$$

51 ••

Picture the Problem The diagram shows a partial cross-sectional view of the inner wire and the outer cylindrical shell. By symmetry, the electric field is radial in the space between the wire and the concentric cylindrical shell. We can apply Gauss's law to cylindrical surfaces of radii $r < R_1$, $R_1 < r < R_2$, and $r > R_2$ to find the electric field and, hence, the energy density in these regions.



(*a*) Apply Gauss's law to a cylindrical surface of radius $r < R_1$ and length L to obtain:

$$E_r(2\pi rL) = \frac{Q_{\text{inside}}}{\epsilon_0} = 0$$

and

$$E_{r<R_1} = \boxed{0}$$

Because $E = 0$ for $r < R_1$:

$$u_{r<R_1} = \boxed{0}$$

Apply Gauss's law to a cylindrical surface of radius $R_1 < r < R_2$ and length L to obtain:

$$E_r(2\pi rL) = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

where λ is the linear charge density.

Solve for E_r to obtain:

$$E_r = \frac{\lambda L}{2\pi \epsilon_0 r} = \frac{2k\lambda}{r}$$

Express the energy density in the region $R_1 < r < R_2$:

$$u = \tfrac{1}{2}\epsilon_0 E_r^2 = \tfrac{1}{2}\epsilon_0\left(\frac{2k\lambda}{r}\right)^2$$

$$= \tfrac{1}{2}\epsilon_0\left(\frac{2kQ}{rL}\right)^2 = \frac{2k^2\epsilon_0 Q^2}{r^2L^2}$$

Apply Gauss's law to a cylindrical surface of radius $r > R_2$ and length L to obtain:

$$E_r(2\pi rL) = \frac{Q_{\text{inside}}}{\epsilon_0} = 0$$

and

$$\boxed{E_{r>R_2} = 0}$$

Because $E = 0$ for $r > R_2$:

$$\boxed{u_{r>R_2} = 0}$$

(*b*) Express the energy residing in a cylindrical shell between the conductors of radius r, thickness dr, and volume $2\pi rL\,dr$:

$$dU = 2\pi rLu(r)dr = 2\pi rL\left(\frac{2k^2 \epsilon_0 Q^2}{r^2L^2}\right)dr = \frac{kQ^2}{rL}dr$$

(*c*) Integrate dU from $r = R_1$ to R_2 to obtain:

$$U = \frac{kQ^2}{L}\int_{R_1}^{R_2}\frac{dr}{r} = \boxed{\frac{kQ^2}{L}\ln\left(\frac{R_2}{R_1}\right)}$$

Use $U = \frac{1}{2}CV^2$ and the expression for the capacitance of a cylindrical capacitor to obtain:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2\left(\dfrac{2\pi \epsilon_0 L}{\ln\dfrac{R_2}{R_1}}\right)} = \boxed{\frac{kQ^2}{L}\ln\left(\frac{R_2}{R_1}\right)}$$

in agreement with the result from part (*b*).

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Picture the Problem Note that with the innermost and outermost cylinders connected together the system corresponds to two cylindrical capacitors connected in parallel. We can use $C = \dfrac{2\pi \epsilon_0 \kappa L}{\ln(R_o/R_i)}$ to express the capacitance per unit length and then calculate and add the capacitances per unit length of each of the cylindrical shell capacitors.

Relate the capacitance of a cylindrical capacitor to its length L and inner and outer radii R_i and R_o:

$$C = \frac{2\pi \epsilon_0 \kappa L}{\ln(R_o/R_i)}$$

Divide both sides of the equation by L to express the capacitance per unit

$$\frac{C}{L} = \frac{2\pi \epsilon_0 \kappa}{\ln(R_o/R_i)}$$

length:

Express the capacitance per unit length of the cylindrical system:

$$\frac{C}{L}=\left(\frac{C}{L}\right)_{\text{outer}}+\left(\frac{C}{L}\right)_{\text{inner}} \qquad (1)$$

Find the capacitance per unit length of the outer cylindrical shell combination:

$$\left(\frac{C}{L}\right)_{\text{outer}}=\frac{2\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(1)}{\ln(0.8\,\text{cm}/0.5\,\text{cm})}$$
$$=118.3\,\text{pF/m}$$

Find the capacitance per unit length of the inner cylindrical shell combination:

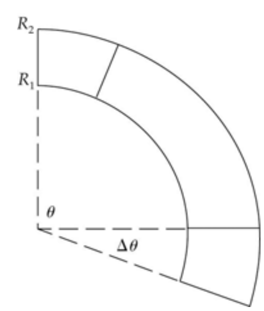
$$\left(\frac{C}{L}\right)_{\text{inner}}=\frac{2\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(1)}{\ln(0.5\,\text{cm}/0.2\,\text{cm})}$$
$$=60.7\,\text{pF/m}$$

Substitute in equation (1) to obtain:

$$\frac{C}{L}=118.3\,\text{pF/m}+60.7\,\text{pF/m}$$
$$=\boxed{179\,\text{pF/m}}$$

***53** ••

Picture the Problem We can use the expression for the capacitance of a parallel-plate capacitor of variable area and the geometry of the figure to express the capacitance of the goniometer.



The capacitance of the parallel-plate capacitor is given by:

$$C=\frac{\in_0(A-\Delta A)}{d}$$

The area of the plates is:

$$A=\pi\left(R_2^2-R_1^2\right)\frac{\theta}{2\pi}=\left(R_2^2-R_1^2\right)\frac{\theta}{2}$$

If the top plate rotates through an angle $\Delta\theta$, then the area is reduced by:

$$\Delta A=\pi\left(R_2^2-R_1^2\right)\frac{\Delta\theta}{2\pi}=\left(R_2^2-R_1^2\right)\frac{\Delta\theta}{2}$$

Substitute for A and ΔA in the expression for C to obtain:

$$C=\frac{\in_0}{d}\left[\left(R_2^2-R_1^2\right)\frac{\theta}{2}-\left(R_2^2-R_1^2\right)\frac{\Delta\theta}{2}\right]$$
$$=\boxed{\frac{\in_0\left(R_2^2-R_1^2\right)}{2d}(\theta-\Delta\theta)}$$

54 ••

Picture the Problem Let C be the capacitance of the capacitor when the pressure is P and C' be the capacitance when the pressure is $P + \Delta P$. We'll assume that (*a*) the change in the thickness of the plates is small, and (*b*) the total volume of material between the plates is conserved. We can use the expression for the capacitance of a dielectric-filled parallel-plate capacitor and the definition of Young's modulus to express the change in the capacitance ΔC of the given capacitor when the pressure on its plates is increased by ΔP.

Express the change in capacitance resulting from the decrease in separation of the capacitor plates by Δd:

$$\Delta C = C' - C = \frac{\kappa \in_0 A'}{d - \Delta d} - \frac{\kappa \in_0 A}{d}$$

Because the volume is constant:

$$A'd' = Ad$$

or

$$A' = \left(\frac{d}{d'}\right)A = \left(\frac{d}{d - \Delta d}\right)A$$

Substitute for A' in the expression for ΔC and simplify to obtain:

$$\Delta C = \frac{\kappa \in_0 A}{d - \Delta d}\left(\frac{d}{d - \Delta d}\right) - \frac{\kappa \in_0 A}{d}$$

$$= \frac{\kappa \in_0 A}{d(d - \Delta d)^2}d^2 - \frac{\kappa \in_0 A}{d}$$

$$= \frac{\kappa \in_0 A}{d}\left[\frac{d^2}{(d - \Delta d)^2} - 1\right]$$

$$= C\left[\frac{d^2}{(d - \Delta d)^2} - 1\right]$$

From the definition of Young's modulus:

$$\frac{\Delta d}{d} = -\frac{P}{Y} \Rightarrow \Delta d = -\left(\frac{P}{Y}\right)d$$

Substitute for Δd in the expression for ΔC to obtain:

$$\Delta C = \frac{\kappa \in_0 A}{d}\left[\frac{d^2}{\left\{d + \left(\frac{P}{Y}\right)d\right\}^2} - 1\right]$$

$$= C\left[\left\{1 + \left(\frac{P}{Y}\right)\right\}^{-2} - 1\right]$$

Expand $\left(1 - \frac{P}{Y}\right)^{-2}$ binomially to obtain:

$$\left(1 - \frac{P}{Y}\right)^{-2} = 1 - 2\frac{P}{Y} + 3\left(\frac{P}{Y}\right)^2 + \ldots$$

Provided $P << Y$:

$$\left(1-\frac{P}{Y}\right)^{-2} \approx 1-2\frac{P}{Y}$$

Substitute in the expression for ΔC and simplify to obtain:

$$\Delta C = C\left[1-2\frac{P}{Y}-1\right] = \boxed{-2\frac{P}{Y}C}$$

Spherical Capacitors

***55 ••**

Picture the Problem We can use the definition of capacitance and the expression for the potential difference between charged concentric spherical shells to show that $C = 4\pi \in_0 R_1R_2/(R_2 - R_1)$.

(*a*) Using its definition, relate the capacitance of the concentric spherical shells to their charge Q and the potential difference V between their surfaces:

$$C = \frac{Q}{V}$$

Express the potential difference between the conductors:

$$V = kQ\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = kQ\frac{R_2-R_1}{R_1R_2}$$

Substitute to obtain:

$$C = \frac{Q}{kQ\frac{R_2-R_1}{R_1R_2}} = \frac{R_1R_2}{k(R_2-R_1)}$$

$$= \boxed{\frac{4\pi \in_0 R_1R_2}{R_2-R_1}}$$

(*b*) Because $R_2 = R_1 + d$:

$$R_1R_2 = R_1(R_1+d)$$
$$= R_1^2 + R_1d$$
$$\approx R_1^2 = R^2$$

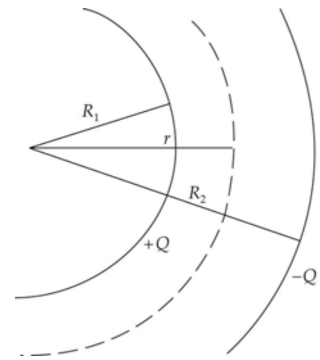
because d is small.

Substitute to obtain:

$$C \approx \frac{4\pi \in_0 R^2}{d} = \boxed{\frac{\in_0 A}{d}}$$

56 ••

Picture the Problem The diagram shows a partial cross-sectional view of the inner and outer spherical shells. By symmetry, the electric field is radial. We can apply Gauss's law to spherical surfaces of radii $r < R_1$, $R_1 < r < R_2$, and $r > R_2$ to find the electric field and, hence, the energy density in these regions.



(*a*) Apply Gauss's law to a spherical surface of radius $r < R_1$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} = 0$$

and

$$E_{r<R_1} = \boxed{0}$$

Because $E = 0$ for $r < R_1$:

$$u_{r<R_1} = \boxed{0}$$

Apply Gauss's law to a spherical surface of radius $R_1 < r < R_2$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Solve for E_r to obtain:

$$E_r = \frac{Q}{4\pi \epsilon_0 r^2} = \boxed{\frac{kQ}{r^2}}$$

Express the energy density in the region $R_1 < r < R_2$:

$$u = \tfrac{1}{2}\epsilon_0 E_r^2 = \tfrac{1}{2}\epsilon_0\left(\frac{kQ}{r^2}\right)^2$$

$$= \boxed{\frac{k^2 \epsilon_0 Q^2}{2r^4}}$$

Apply Gauss's law to a cylindrical surface of radius $r > R_2$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} = 0$$

and

$$E_{r>R_2} = \boxed{0}$$

Because $E = 0$ for $r > R_2$:

$$u_{r>R_2} = \boxed{0}$$

(*b*) Express the energy in the electrostatic field in a spherical shell of radius *r*, thickness *dr*, and volume $4\pi r^2 dr$ between the conductors:

$$dU = 4\pi r^2 u(r)dr = 4\pi r^2\left(\frac{k^2 \in_0 Q^2}{2r^4}\right)dr$$

$$= \boxed{\frac{kQ^2}{2r^2}dr}$$

(*c*) Integrate *dU* from $r = R_1$ to R_2 to obtain:

$$U = \frac{kQ^2}{2}\int_{R_1}^{R_2}\frac{dr}{r^2} = \frac{kQ^2(R_2 - R_1)}{2R_1R_2}$$

$$= \boxed{\frac{1}{2}Q^2\left(\frac{R_2 - R_1}{4\pi \in_0 R_1R_2}\right)}$$

Note that the quantity in parentheses is $1/C$, so we have $U = \frac{1}{2}Q^2/C$.

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Picture the Problem We know, from Gauss's law, that the field inside the shell is zero. Applying Gauss's law to a spherical surface of radius $R > r$ will allow us to find the energy density in this region. We can then express the energy in the electrostatic field in a spherical shell of radius *R*, thickness *dR*, and volume $4\pi R^2 dR$ outside the spherical shell and find the total energy in the electric field by integrating from *r* to ∞. If we then integrate the same expression from *r* to *R* we can find the radius *R* of the sphere such that half the total electrostatic field energy of the system is contained within that sphere.

Apply Gauss's law to a spherical shell of radius $R > r$ to obtain:

$$E_r\left(4\pi R^2\right) = \frac{Q_{\text{inside}}}{\in_0} = \frac{Q}{\in_0}$$

Solve for E_r outside the spherical shell:

$$E_r = \frac{kQ}{R^2}$$

Express the energy density in the region $R > r$:

$$u = \tfrac{1}{2}\in_0 E_R^2 = \tfrac{1}{2}\in_0\left(\frac{kQ}{R^2}\right)^2 = \frac{k^2 \in_0 Q^2}{2R^4}$$

Express the energy in the electrostatic field in a spherical shell of radius *R*, thickness *dR*, and volume $4\pi R^2 dR$ outside the spherical shell:

$$dU = 4\pi R^2 u(R)dR$$

$$= 4\pi R^2\left(\frac{k^2 \in_0 Q^2}{2R^4}\right)dR$$

$$= \frac{kQ^2}{2R^2}dR$$

Integrate *dU* from *r* to ∞ to obtain:

$$U_{\text{tot}} = \frac{kQ^2}{2}\int_r^{\infty}\frac{dR}{R^2} = \frac{kQ^2}{2r}$$

Integrate dU from r to R to obtain:

$$U = \frac{kQ^2}{2}\int_r^R \frac{dR'}{R'^2} = \frac{kQ^2}{2}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Set $U = \frac{1}{2}U_{\text{tot}}$ to obtain:

$$\frac{kQ^2}{2}\left(\frac{1}{r} - \frac{1}{R}\right) = \frac{kQ^2}{4r}$$

Solve for R:

$$R = \boxed{2r}$$

Disconnected and Reconnected Capacitors

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Picture the Problem Let C_1 represent the capacitance of the 2.0-μF capacitor and C_2 the capacitance of the 2nd capacitor. Note that when they are connected as described in the problem statement they are in parallel and, hence, share a common potential difference. We can use the equation for the equivalent capacitance of two capacitors in parallel and the definition of capacitance to relate C_2 to C_1 and to the charge stored in and the potential difference across the equivalent capacitor.

Using the definition of capacitance, find the charge on capacitor C_1:

$$Q_1 = C_1V = (2\,\mu\text{F})(12\,\text{V}) = 24\,\mu\text{C}$$

Express the equivalent capacitance of the two-capacitor system and solve for C_2:

$$C_{\text{eq}} = C_1 + C_2$$

and

$$C_2 = C_{\text{eq}} - C_1$$

Using the definition of capacitance, express C_{eq} in terms of Q_2 and V_2:

$$C_{\text{eq}} = \frac{Q_2}{V_2} = \frac{Q_1}{V_2}$$

where V_2 is the common potential difference (they are in parallel) across the two capacitors and Q_1 and Q_2 are the (equal) charges on the two capacitors.

Substitute to obtain:

$$C_2 = \frac{Q_1}{V_2} - C_1$$

Substitute numerical values and evaluate C_2:

$$C_2 = \frac{24\,\mu\text{C}}{4\,\text{V}} - 2\,\mu\text{F} = \boxed{4.00\,\mu\text{F}}$$

59 ••

Picture the Problem Because, when the capacitors are connected as described in the problem statement, they are in parallel, they will have the same potential difference across them. In part (*b*) we can find the energy lost when the connections are made by comparing the energies stored in the capacitors before and after the connections.

(*a*) Because the capacitors are in parallel:

$$V_{100} = V_{400} = \boxed{2.00\,\text{kV}}$$

(*b*) Express the energy lost when the connections are made in terms of the energy stored in the capacitors before and after their connection:

$$\Delta U = U_{\text{before}} - U_{\text{after}}$$

Express and evaluate U_{before}:

$$\begin{aligned} U_{\text{before}} &= U_{100} + U_{400} \\ &= \tfrac{1}{2}C_{100}V_{100}^2 + \tfrac{1}{2}C_{400}V_{400}^2 \\ &= \tfrac{1}{2}V^2\left(C_{100} + C_{400}\right) \\ &= \tfrac{1}{2}(2\,\text{kV})^2(500\,\text{pF}) \\ &= 1.00\,\text{mJ} \end{aligned}$$

Express and evaluate U_{after}:

$$\begin{aligned} U_{\text{after}} &= U_{100} + U_{400} \\ &= \tfrac{1}{2}C_{100}V_{100}^2 + \tfrac{1}{2}C_{400}V_{400}^2 \\ &= \tfrac{1}{2}V^2\left(C_{100} + C_{400}\right) \\ &= \tfrac{1}{2}(2\,\text{kV})^2(500\,\text{pF}) \\ &= 1.00\,\text{mJ} \end{aligned}$$

Substitute to obtain:

$$\Delta U = 1.00\,\text{mJ} - 1.00\,\text{mJ} = \boxed{0}$$

***60** ••

Picture the Problem When the capacitors are reconnected, each will have the charge it acquired while they were connected in series across the 12-V battery and we can use the definition of capacitance and their equivalent capacitance to find the common potential difference across them. In part (*b*) we can use $U = \tfrac{1}{2}CV^2$ to find the initial and final energy stored in the capacitors.

(*a*) Using the definition of capacitance, express the potential difference across each capacitor when they are reconnected:

$$V = \frac{2Q}{C_{\text{eq}}} \qquad (1)$$

where Q is the charge on each capacitor *before* they are disconnected.

Find the equivalent capacitance of the two capacitors after they are connected in parallel:

$$C_{eq} = C_1 + C_2 = 4\,\mu\text{F} + 12\,\mu\text{F} = 16\,\mu\text{F}$$

Express the charge Q on each capacitor before they are disconnected:

$$Q = C'_{eq}V$$

Express the equivalent capacitance of the two capacitors connected in series:

$$C'_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{(4\,\mu\text{F})(12\,\mu\text{F})}{4\,\mu\text{F} + 12\,\mu\text{F}} = 3\,\mu\text{F}$$

Substitute to find Q:

$$Q = (3\,\mu\text{F})(12\,\text{V}) = 36\,\mu\text{C}$$

Substitute in equation (1) and evaluate V:

$$V = \frac{2(36\,\mu\text{C})}{16\,\mu\text{F}} = \boxed{4.50\,\text{V}}$$

(*b*) Express and evaluate the energy stored in the capacitors initially:

$$U_i = \tfrac{1}{2}C'_{eq}V_i^2 = \tfrac{1}{2}(3\,\mu\text{F})(12\,\text{V})^2 = \boxed{216\,\mu\text{J}}$$

Express and evaluate the energy stored in the capacitors when they have been reconnected:

$$U_f = \tfrac{1}{2}C_{eq}V_f^2 = \tfrac{1}{2}(16\,\mu\text{F})(4.5\,\text{V})^2 = \boxed{162\,\mu\text{J}}$$

61 ••

Picture the Problem Let C_1 represent the capacitance of the 1.2-μF capacitor and C_2 the capacitance of the 2nd capacitor. Note that when they are connected as described in the problem statement they are in parallel and, hence, share a common potential difference. We can use the equation for the equivalent capacitance of two capacitors in parallel and the definition of capacitance to relate C_2 to C_1 and to the charge stored in and the potential difference across the equivalent capacitor. In part (*b*) we can use $U = \tfrac{1}{2}CV^2$ to find the energy before and after the connection was made and, hence, the energy lost when the connection was made.

(*a*) Using the definition of capacitance, find the charge on capacitor C_1:

$$Q_1 = C_1V = (1.2\,\mu\text{F})(30\,\text{V}) = 36\,\mu\text{C}$$

Express the equivalent capacitance of the two-capacitor system and solve for C_2:

$$C_{eq} = C_1 + C_2$$

and

$$C_2 = C_{eq} - C_1$$

Using the definition of capacitance, express C_{eq} in terms of Q_2 and V_2:

$$C_{eq} = \frac{Q_2}{V_2} = \frac{Q_1}{V_2}$$

where V_2 is the common potential difference (they are in parallel) across the two capacitors.

Substitute to obtain:

$$C_2 = \frac{Q_1}{V_2} - C_1$$

Substitute numerical values and evaluate C_2:

$$C_2 = \frac{36\,\mu\text{C}}{10\,\text{V}} - 1.2\,\mu\text{F} = \boxed{2.40\,\mu\text{F}}$$

(*b*) Express the energy lost when the connections are made in terms of the energy stored in the capacitors before and after their connection:

$$\begin{aligned}\Delta U &= U_{\text{before}} - U_{\text{after}} \\ &= \tfrac{1}{2}C_1V_1^2 - \tfrac{1}{2}C_{eq}V_f^2 \\ &= \tfrac{1}{2}\left(C_1V_1^2 - C_{eq}V_f^2\right)\end{aligned}$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \tfrac{1}{2}\left[(1.2\,\mu\text{F})(30\,\text{V})^2 - (3.6\,\mu\text{F})(10\,\text{V})^2\right] = \boxed{360\,\mu\text{J}}$$

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Picture the Problem Because, when the capacitors are connected as described in the problem statement, they are in parallel, they will have the same potential difference across them. In part (*b*) we can find the energy lost when the connections are made by comparing the energies stored in the capacitors before and after the connections.

(*a*) Using the definition of capacitance, express the charge Q on the capacitors when they have been reconnected:

$$\begin{aligned}Q &= Q_{400} - Q_{100} \\ &= C_{400}V_{400} - C_{100}V_{100} \\ &= (C_{400} - C_{100})V\end{aligned}$$

where V is the common potential difference to which the capacitors have been charged.

Substitute numerical values to obtain:

$$Q = (400\,\text{pF} - 100\,\text{pF})(2\,\text{kV}) = 600\,\text{nC}$$

Using the definition of capacitance, relate the equivalent capacitance, charge, and final potential difference for the parallel connection:

$$Q = (C_1 + C_2)V_f$$

Solve for and evaluate V_f:

$$V_f = \frac{Q}{C_1 + C_2} = \frac{600\,nC}{100\,pF + 400\,pF} = \boxed{1.20\,kV}$$

across both capacitors.

(*b*) Express the energy lost when the connections are made in terms of the energy stored in the capacitors before and after their connection:

$$\Delta U = U_{before} - U_{after} = \tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2 - \tfrac{1}{2}C_{eq}V_f^2 = \tfrac{1}{2}\left(C_1V_1^2 + C_2V_2^2 - C_{eq}V_f^2\right)$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \tfrac{1}{2}\left[(100\,pF)(2\,kV)^2 + (400\,pF)(2\,kV)^2 - (500\,pF)(1.2\,kV)^2\right] = \boxed{0.640\,mJ}$$

63 ••

Picture the Problem When the capacitors are reconnected, each will have a charge equal to the difference between the charges they acquired while they were connected in parallel across the 12-V battery. We can use the definition of capacitance and their equivalent capacitance to find the common potential difference across them. In part (*b*) we can use $U = \tfrac{1}{2}CV^2$ to find the initial and final energy stored in the capacitors.

(*a*) Using the definition of capacitance, express the potential difference across the capacitors when they are reconnected:

$$V_f = \frac{Q_f}{C_{eq}} = \frac{Q_f}{C_1 + C_2} \qquad (1)$$

where Q_f is the common charge on the capacitors *after* they are reconnected.

Express the final charge Q_f on each capacitor:

$$Q_f = Q_2 - Q_1$$

Use the definition of capacitance to substitute for Q_2 and Q_1:

$$Q_f = C_2V - C_1V = (C_2 - C_1)V$$

Substitute in equation (1) to obtain:

$$V_f = \frac{C_2 - C_1}{C_1 + C_2}V$$

Substitute numerical values and evaluate V_f:

$$V_f = \frac{12\,\mu F - 4\,\mu F}{12\,\mu F + 4\,\mu F}(12\,V) = \boxed{6.00\,V}$$

(*b*) Express and evaluate the energy stored in the capacitors initially:

$$U_i = \tfrac{1}{2}C_1V^2 + \tfrac{1}{2}C_2V^2 = \tfrac{1}{2}V^2(C_1 + C_2) = \tfrac{1}{2}(12\,\text{V})^2(12\,\mu\text{F} + 4\,\mu\text{F}) = \boxed{1.15\,\text{mJ}}$$

Express and evaluate the energy stored in the capacitors when they have been reconnected:

$$U_f = \tfrac{1}{2}C_1V_f^2 + \tfrac{1}{2}C_2V_f^2 = \tfrac{1}{2}V_f^2(C_1 + C_2) = \tfrac{1}{2}(6\,\text{V})^2(12\,\mu\text{F} + 4\,\mu\text{F}) = \boxed{0.288\,\text{mJ}}$$

***64 ••**

Picture the Problem Let the numeral 1 refer to the 20-pF capacitor and the numeral 2 to the 50-pF capacitor. We can use conservation of charge and the fact that the connected capacitors will have the same potential difference across them to find the charge on each capacitor. We can decide whether electrostatic potential energy is gained or lost when the two capacitors are connected by calculating the change ΔU in the electrostatic energy during this process.

(*a*) Using the fact that no charge is lost in connecting the capacitors, relate the charge Q initially on the 20-pF capacitor to the charges on the two capacitors when they have been connected:

$$Q = Q_1 + Q_2 \qquad (1)$$

Because the capacitors are in parallel, the potential difference across them is the same:

$$V_1 = V_2 \Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

Solve for Q_1 to obtain:

$$Q_1 = \frac{C_1}{C_2}Q_2$$

Substitute in equation (1) and solve for Q_2 to obtain:

$$Q_2 = \frac{Q}{1 + C_1/C_2} \qquad (2)$$

Use the definition of capacitance to find the charge Q initially on the 20-pF capacitor:

$$Q = C_1V = (20\,\text{pF})(3\,\text{kV}) = 60\,\text{nC}$$

Substitute in equation (2) and evaluate Q_2:

$$Q_2 = \frac{60\,\text{nC}}{1 + 20\,\text{pF}/50\,\text{pF}} = \boxed{42.9\,\text{nC}}$$

Substitute in equation (1) to obtain:

$$Q_1 = Q - Q_2$$
$$= 60\text{nC} - 42.9\,\text{nC} = \boxed{17.1\text{nC}}$$

(*b*) Express the change in the electrostatic potential energy of the system when the two capacitors are connected:

$$\Delta U = U_\text{f} - U_\text{i}$$
$$= \frac{Q^2}{2C_\text{eq}} - \frac{Q^2}{2C_1}$$
$$= \frac{Q^2}{2}\left(\frac{1}{C_\text{eq}} - \frac{1}{C_1}\right)$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = \frac{(60\,\text{nC})^2}{2}\left(\frac{1}{70\,\text{pF}} - \frac{1}{20\,\text{pF}}\right)$$
$$= -64.3\,\mu\text{J}$$

$\boxed{\text{Because } \Delta U < 0, \text{ electrostatic energy is lost when the two capacitors are connected.}}$

65 •••

Picture the Problem Let upper case Qs refer to the charges before S_3 is closed and lower case qs refer to the charges after this switch is closed. We can use conservation of charge to relate the charges on the capacitors before S_3 is closed to their charges when this switch is closed. We also know that the sum of the potential differences around the circuit when S_3 is closed must be zero and can use this to obtain a fourth equation relating the charges on the capacitors after the switch is closed to their capacitances. Solving these equations simultaneously will yield the charges q_1, q_2, and q_3. Knowing these charges, we can use the definition of capacitance to find the potential difference across each of the capacitors.

(*a*) With S_1 and S_2 closed, but S_3 open, the charges on and the potential differences across the capacitors do not change and:

$$V_1 = V_2 = V_3 = \boxed{200\,\text{V}}$$

(*b*) When S_3 is closed, the charges can redistribute; express the conditions on the charges that must be satisfied as a result of this

$$q_2 - q_1 = Q_2 - Q_1,$$
$$q_3 - q_2 = Q_3 - Q_2,$$

and

redistribution: $q_1 - q_3 = Q_1 - Q_3$.

Express the condition on the potential differences that must be satisfied when S_3 is closed:

$$V_1 + V_2 + V_3 = 0$$

where the subscripts refer to the three capacitors.

Use the definition of capacitance to eliminate the potential differences:

$$\frac{q_1}{C_1} + \frac{q_2}{C_2} + \frac{q_3}{C_3} = 0 \qquad (1)$$

Use the definition of capacitance to find the initial charge on each capacitor:

$$Q_1 = C_1 V = (2\,\mu\text{F})(200\,\text{V}) = 400\,\mu\text{C},$$
$$Q_2 = C_2 V = (4\,\mu\text{F})(200\,\text{V}) = 800\,\mu\text{C},$$

and

$$Q_3 = C_3 V = (6\,\mu\text{F})(200\,\text{V}) = 1200\,\mu\text{C}$$

Let $Q = Q_1$. Then: $Q_2 = 2Q$ and $Q_3 = 3Q$

Express q_2 and q_3 in terms of q_1 and Q:

$$q_2 = Q + q_1 \qquad (2)$$

and

$$q_3 = q_1 + 2Q \qquad (3)$$

Substitute in equation (1) to obtain:

$$\frac{q_1}{C_1} + \frac{Q + q_1}{C_2} + \frac{q_1 + 2Q}{C_3} = 0$$

or

$$\frac{q_1}{2\,\mu\text{F}} + \frac{Q + q_1}{4\,\mu\text{F}} + \frac{q_1 + 2Q}{6\,\mu\text{F}} = 0$$

Solve for and evaluate q_1 to obtain:

$$q_1 = -\tfrac{7}{11}Q = -\tfrac{7}{11}(400\,\mu\text{C}) = \boxed{-254\,\mu\text{C}}$$

Substitute in equation (2) to obtain:

$$q_2 = 400\,\mu\text{C} - 254\,\mu\text{C} = \boxed{146\,\mu\text{C}}$$

Substitute in equation (3) to obtain:

$$q_3 = -254\,\mu\text{C} + 2(400\,\mu\text{C}) = \boxed{546\,\mu\text{C}}$$

(*c*) Use the definition of capacitance to find the potential difference across each capacitor with S_3 closed:

$$V_1 = \frac{q_1}{C_1} = \frac{-254\,\mu\text{C}}{2\,\mu\text{F}} = \boxed{-127\,\text{V}},$$

$$V_2 = \frac{q_2}{C_2} = \frac{146\,\mu\text{C}}{4\,\mu\text{F}} = \boxed{36.5\,\text{V}},$$

and

$$V_3 = \frac{q_3}{C_3} = \frac{546\,\mu\text{C}}{6\,\mu\text{F}} = \boxed{91.0\,\text{V}}$$

***66 ••**

Picture the Problem We can use the expression for the energy stored in a capacitor to express the ratio of the energy stored in the system after the discharge of the first capacitor to the energy stored in the system prior to the discharge.

Express the energy U initially stored in the capacitor whose capacitance is C:

$$U = \frac{Q^2}{2C}$$

The energy U' stored in the two capacitors after the first capacitor has discharged is:

$$U' = \frac{\left(\frac{Q}{2}\right)^2}{2C} + \frac{\left(\frac{Q}{2}\right)^2}{2C} = \frac{Q^2}{4C}$$

Express the ratio of U' to U:

$$\frac{U'}{U} = \frac{\frac{Q^2}{4C}}{\frac{Q^2}{2C}} = \frac{1}{2} \Rightarrow U' = \boxed{\tfrac{1}{2}U}$$

Dielectrics

67 •

Picture the Problem The capacitance of a parallel-plate capacitor filled with a dielectric of constant κ is given by $C = \frac{\kappa \in_0 A}{d}$.

Relate the capacitance of the parallel-plate capacitor to the area of its plates, their separation, and the dielectric constant of the material between the plates:

$$C = \frac{\kappa \in_0 A}{d}$$

Substitute numerical values and evaluate C:

$$C = \frac{2.3\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(400\,\text{cm}^2\right)}{0.3\,\text{mm}}$$

$$= \boxed{2.71\,\text{nF}}$$

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Picture the Problem The capacitance of a cylindrical capacitor is given by $C = 2\pi\kappa \in_0 L/\ln(r_2/r_1)$, where L is its length and r_1 and r_2 the radii of the inner and outer conductors. We can use this expression, in conjunction with the definition of capacitance, to express the potential difference between the wire and the cylindrical shell in the Geiger tube. Because the electric field E in the tube is related to the linear charge density λ on the wire according to $E = 2k\lambda/\kappa r$, we can use this expression to find $2k\lambda/\kappa$ for $E = E_{max}$. In part (b) we'll use this relationship to find the charge per unit length λ on the wire.

(*a*) Use the definition of capacitance and the expression for the capacitance of a cylindrical capacitor to express the potential difference between the wire and the cylindrical shell in the tube:

$$\Delta V = \frac{Q}{C} = \frac{Q}{\dfrac{2\pi\kappa \in_0 L}{\ln(R/r)}} = \frac{2\lambda}{4\pi \in_0 \kappa}\ln\left(\frac{R}{r}\right) = \frac{2k\lambda}{\kappa}\ln\left(\frac{R}{r}\right)$$

where λ is the linear charge density, κ is the dielectric constant of the gas in the Geiger tube, r is the radius of the wire, and R the radius of the coaxial cylindrical shell of length L.

Express the electric field at a distance r greater than its radius from the center of the wire:

$$E = \frac{2k\lambda}{\kappa r}$$

Solve for $2k\lambda/\kappa$:

$$\frac{2k\lambda}{\kappa} = Er \qquad (1)$$

Noting that E is a maximum at r = 0.2 mm, evaluate $2k\lambda/\kappa$:

$$\frac{2k\lambda}{\kappa} = E_{\text{max}} r = \left(2\times10^6\ \text{V/m}\right)\left(0.2\,\text{mm}\right) = 400\,\text{V}$$

Substitute and evaluate ΔV_{max}:

$$\Delta V_{\text{max}} = \left(400\,\text{V}\right)\ln\left(\frac{1.5\,\text{cm}}{0.2\,\text{mm}}\right) = \boxed{1.73\,\text{kV}}$$

(*b*) Solve equation (1) for λ:

$$\lambda = \frac{E_{\text{max}}\kappa r}{2k}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1.8\left(2\times10^6\ \text{V/m}\right)\left(0.2\,\text{mm}\right)}{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{40.0\,\text{nC/m}}$$

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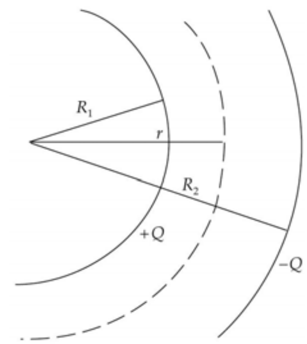
Picture the Problem The diagram shows a partial cross-sectional view of the inner and outer spherical shells. By symmetry, the electric field is radial. We can apply Gauss's law to spherical surfaces of radii $r < R_1$, $R_1 < r < R_2$, and $r > R_2$ to find the electric field and, hence, the energy density in these regions.



(*a*) Apply Gauss's law to a spherical surface of radius $r < R_1$ to obtain:

$$E_r\left(4\pi r^2\right)=\frac{Q_{\text{inside}}}{\kappa \in_0}=0$$

and

$$E_{r<R_1}=\boxed{0}$$

Because $E = 0$ for $r < R_1$:

$$u_{r<R_1}=\boxed{0}$$

Apply Gauss's law to a spherical surface of radius $R_1 < r < R_2$ to obtain:

$$E_r\left(4\pi r^2\right)=\frac{Q_{\text{inside}}}{\kappa \in_0}=\frac{Q}{\kappa \in_0}$$

Solve for E_r to obtain:

$$E_r=\frac{Q}{4\pi\kappa \in_0 r^2}=\frac{kQ}{\kappa r^2}$$

Express the energy density in the region $R_1 < r < R_2$:

$$u=\tfrac{1}{2}\kappa \in_0 E_r^2=\tfrac{1}{2}\kappa \in_0\left(\frac{kQ}{\kappa r^2}\right)^2$$

$$=\frac{k^2 \in_0 Q^2}{2\kappa r^4}$$

Apply Gauss's law to a cylindrical surface of radius $r > R_2$ to obtain:

$$E_r\left(4\pi r^2\right)=\frac{Q_{\text{inside}}}{\kappa \in_0}=0$$

and

$$E_{r>R_2}=\boxed{0}$$

Because $E = 0$ for $r > R_2$:

$$u_{r>R_2}=\boxed{0}$$

(*b*) Express the energy in the electrostatic field in a spherical shell of radius *r*, thickness *dr*, and volume $4\pi r^2 dr$ between the conductors:

$$dU = 4\pi r^2 u(r)dr = 4\pi r^2\left(\frac{k^2\kappa \in_0 Q^2}{2\kappa^2 r^4}\right)dr = \boxed{\frac{kQ^2}{2\kappa r^2}dr}$$

(*c*) Integrate *dU* from $r = R_1$ to R_2 to obtain:

$$U = \frac{kQ^2}{2\kappa}\int_{R_1}^{R_2}\frac{dr}{r^2} = \frac{kQ^2(R_2 - R_1)}{2\kappa R_1 R_2} = \boxed{\frac{1}{2}Q^2\left(\frac{R_2 - R_1}{4\pi\kappa\varepsilon_0 R_1 R_2}\right)}$$

Note that the quantity in parentheses is $1/C$, so we have $U = \frac{1}{2}Q^2/C$.

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Picture the Problem We can use the relationship between the electric field between the plates of a capacitor, their separation, and the potential difference between them to find the minimum plate separation. We can use the expression for the capacitance of a dielectric-filled parallel-plate capacitor to determine the necessary area of the plates.

(*a*) Relate the electric field of the capacitor to the potential difference across its plates:

$$E = \frac{V}{d}$$

where *d* is the plate separation.

Solve for *d*:

$$d = \frac{V}{E}$$

Noting that d_{min} corresponds to E_{max}, evaluate d_{min}:

$$d_{min} = \frac{V}{E_{max}} = \frac{2000\text{ V}}{4\times10^7\text{ V/m}} = \boxed{50.0\ \mu\text{m}}$$

(*b*) Relate the capacitance of a parallel-plate capacitor to the area of its plates:

$$C = \frac{\kappa \in_0 A}{d}$$

Solve for *A*:

$$A = \frac{Cd}{\kappa \in_0}$$

Substitute numerical values and evaluate A:

$$A=\frac{(0.1\,\mu\text{F})(50\,\mu\text{m})}{24\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}=2.35\times10^{-2}\,\text{m}^2=\boxed{235\,\text{cm}^2}$$

71 ••

Picture the Problem We can model this system as two capacitors in series, C_1 of thickness $d/4$ and C_2 of thickness $3d/4$ and use the equation for the equivalent capacitance of two capacitors connected in series.

Express the equivalent capacitance of the two capacitors connected in series:

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}$$

or

$$C_{\text{eq}}=\frac{C_1C_2}{C_1+C_2}$$

Relate the capacitance of C_1 to its dielectric constant and thickness:

$$C_1=\frac{\kappa_1\in_0 A}{\frac{1}{4}d}=\frac{4\kappa_1\in_0 A}{d}$$

Relate the capacitance of C_2 to its dielectric constant and thickness:

$$C_2=\frac{\kappa_2\in_0 A}{\frac{3}{4}d}=\frac{4\kappa_2\in_0 A}{3d}$$

Substitute and simplify to obtain:

$$C_{\text{eq}}=\frac{\left(\frac{4\kappa_1\in_0 A}{d}\right)\left(\frac{4\kappa_2\in_0 A}{3d}\right)}{\frac{4\kappa_1\in_0 A}{d}+\frac{4\kappa_2\in_0 A}{3d}}=\frac{\left(\frac{\kappa_1}{d}\right)\left(\frac{4\kappa_2}{3d}\right)}{\frac{3\kappa_1}{3d}+\frac{\kappa_2}{3d}}\in_0 A=\frac{\frac{4\kappa_1\kappa_2}{d}}{3\kappa_1+\kappa_2}\in_0 A$$

$$=\frac{4\kappa_1\kappa_2}{3\kappa_1+\kappa_2}\left(\frac{\in_0 A}{d}\right)=\boxed{\left(\frac{4\kappa_1\kappa_2}{3\kappa_1+\kappa_2}\right)C_0}$$

***72** ••

Picture the Problem Let the charge on the capacitor with the air gap be Q_1 and the charge on the capacitor with the dielectric gap be Q_2. If the capacitances of the capacitors were initially C, then the capacitance of the capacitor with the dielectric inserted is $C' = \kappa C$. We can use the conservation of charge and the equivalence of the potential difference across the capacitors to obtain two equations that we can solve simultaneously for Q_1 and Q_2.

Apply conservation of charge during

$$Q_1+Q_2=2Q \qquad (1)$$

the insertion of the dielectric to obtain:

Because the capacitors have the same potential difference across them:

$$\frac{Q_1}{C}=\frac{Q_2}{\kappa C} \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$Q_1=\boxed{\frac{2Q}{1+\kappa}} \text{ and } Q_2=\boxed{\frac{2Q\kappa}{1+\kappa}}$$

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Picture the Problem We can model this system as two capacitors in series, C_1 of thickness t and C_2 of thickness $d-t$ and use the equation for the equivalent capacitance of two capacitors connected in series.

Express the equivalent capacitance of the two capacitors connected in series:

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}$$

or

$$C_{\text{eq}}=\frac{C_1C_2}{C_1+C_2}$$

Relate the capacitance of C_1 to its dielectric constant and thickness:

$$C_1=\frac{\kappa\in_0 A}{t}$$

Relate the capacitance of C_2 to its dielectric constant and thickness:

$$C_2=\frac{\in_0 A}{d-t}$$

Substitute and simplify to obtain:

$$C_{\text{eq}}=\frac{\left(\frac{\kappa\in_0 A}{t}\right)\left(\frac{\in_0 A}{d-t}\right)}{\frac{\kappa\in_0 A}{t}+\frac{\in_0 A}{d-t}}=\frac{\left(\frac{\kappa}{t}\right)\left(\frac{1}{d-t}\right)}{\frac{\kappa}{t}+\frac{1}{d-t}}\in_0 A=\frac{\left(\frac{\kappa}{t}\right)\left(\frac{1}{d-t}\right)}{\frac{\kappa}{t}+\frac{1}{d-t}}\in_0 A$$

$$=\frac{\kappa}{\kappa(d-t)+t}\in_0 A=\boxed{\left(\frac{\kappa d}{\kappa(d-t)+t}\right)C_0}$$

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Picture the Problem Because $d \ll r$, we can model the membrane as a parallel-plate capacitor. We can use the definition of capacitance to find the charge on each side of the membrane in part (*b*) and the relationship between the potential difference across the membrane, its thickness, and the electric field in it to find the electric field called for in part (*c*).

(*a*) Express the capacitance of a parallel-plate capacitor:

$$C = \frac{\kappa \in_0 A}{d}$$

Substitute for the area of the plates:

$$C = \frac{2\pi\kappa \in_0 rL}{d} = \frac{\kappa rL}{2kd}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{3\left(10^{-5}\,\text{m}\right)\left(0.1\,\text{m}\right)}{2\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10^{-8}\,\text{m}\right)} = \boxed{16.7\,\text{nF}}$$

(*b*) Use the definition of capacitance to find the charge on each side of the membrane:

$$Q = CV = \left(16.7\,\text{nF}\right)\left(70\,\text{mV}\right) = \boxed{1.17\,\text{nC}}$$

(*c*) Express the electric field through the membrane as a function of its thickness *d* and the potential difference *V* across it:

$$E = \frac{V}{d}$$

Substitute numerical values and evaluate *E*:

$$E = \frac{70\,\text{mV}}{10^{-8}\,\text{m}} = \boxed{7.00\,\text{MV/m}}$$

***75** ••

Picture the Problem The bound charge density is related to the dielectric constant and the free charge density according to $\sigma_b = \left(1 - \frac{1}{\kappa}\right)\sigma_f$.

Solve the equation relating σ_b, σ_f, and κ for κ to obtain:

$$\kappa = \frac{1}{1 - \sigma_b/\sigma_f}$$

(*a*) Evaluate this expression for $\sigma_b/\sigma_f = 0.8$:

$$\kappa = \frac{1}{1-0.8} = \boxed{5.00}$$

(*b*) Evaluate this expression for $\sigma_b/\sigma_f = 0.2$:

$$\kappa = \frac{1}{1-0.2} = \boxed{1.25}$$

(*c*) Evaluate this expression for $\sigma_b/\sigma_f = 0.98$:

$$\kappa = \frac{1}{1-0.98} = \boxed{50.0}$$

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Picture the Problem We can use the definition of the dielectric constant to find its value. In part (*b*) we can use the expression for the electric field in the space between the charged capacitor plates to find the area of the plates and in part (*c*) we can relate the surface charge densities to the induced charges on the plates.

(*a*) Using the definition of the dielectric constant, relate the electric field without a dielectric E_0 to the field with a dielectric E:

$$E = \frac{E_0}{\kappa}$$

Solve for and evaluate κ:

$$\kappa = \frac{E_0}{E} = \frac{2.5\times10^5\ \text{V/m}}{1.2\times10^5\ \text{V/m}} = \boxed{2.08}$$

(*b*) Relate the electric field in the region between the plates to the surface charge density of the plates:

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$$

Solve for A:

$$A = \frac{Q}{E_0\ \epsilon_0}$$

Substitute numerical values and evaluate A:

$$A = \frac{10\,\text{nC}}{\left(2.5\times10^5\ \text{V/m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 4.52\times10^{-3}\ \text{m}^2 = \boxed{45.2\,\text{cm}^2}$$

(*c*) Relate the surface charge densities to the induced charges on the plates:

$$\sigma_\text{b} = \left(1-\frac{1}{\kappa}\right)\sigma_\text{f}$$

or

$$\frac{\sigma_\text{b}}{\sigma_\text{f}} = \frac{Q_\text{b}}{Q_\text{f}} = 1-\frac{1}{\kappa}$$

Solve for Q_b:

$$Q_\text{b} = \left(1-\frac{1}{\kappa}\right)Q_\text{f}$$

Substitute numerical values and evaluate Q_b:

$$Q_\text{b} = \left(1-\frac{1}{2.08}\right)(10\,\text{nC}) = \boxed{5.19\,\text{nC}}$$

***77** ••

Picture the Problem We can model this parallel-plate capacitor as a combination of two capacitors C_1 and C_2 in series with capacitor C_3 in parallel.

Express the capacitance of two series-connected capacitors in parallel with a third:

$$C = C_3 + C_s \qquad (1)$$

where

$$C_s = \frac{C_1 C_2}{C_1 + C_2} \qquad (2)$$

Express each of the capacitances C_1, C_2, and C_3 in terms of the dielectric constants, plate areas, and plate separations:

$$C_1 = \frac{\kappa_1 \in_0 \left(\tfrac{1}{2}A\right)}{\tfrac{1}{2}d} = \frac{\kappa_1 \in_0 A}{d},$$

$$C_2 = \frac{\kappa_2 \in_0 \left(\tfrac{1}{2}A\right)}{\tfrac{1}{2}d} = \frac{\kappa_2 \in_0 A}{d},$$

and

$$C_3 = \frac{\kappa_3 \in_0 \left(\tfrac{1}{2}A\right)}{d} = \frac{\kappa_3 \in_0 A}{2d}$$

Substitute in equation (2) to obtain:

$$C_s = \frac{\left(\frac{\kappa_1 \in_0 A}{d}\right)\left(\frac{\kappa_2 \in_0 A}{d}\right)}{\frac{\kappa_1 \in_0 A}{d} + \frac{\kappa_2 \in_0 A}{d}} = \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\left(\frac{\in_0 A}{d}\right)$$

Substitute in equation (1) to obtain:

$$C = \frac{\kappa_3 \in_0 A}{2d} + \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\left(\frac{\in_0 A}{d}\right) = \boxed{\left(\kappa_3 + \frac{2\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\left(\frac{\in_0 A}{2d}\right)}$$

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Picture the Problem The electric field E between the plates of a parallel-plate capacitor is related to the potential difference V between the plates and their separation d according to $V = Ed$ and the electrostatic energy U depends on the electric field according to $U = \frac{1}{2} \in_0 E_0^2 Ad$. We can use these relationships to find E, V, and U with and without the dielectric in place.

(*a*) Relate the electric field E_0 to the potential difference V between the plates and the plate separation d:

$$E_0 = \frac{V}{d} = \frac{100\,\text{V}}{4\,\text{mm}} = \boxed{25.0\,\text{kV/m}}$$

Use the definition of energy density to relate the electrostatic energy U_0 to the volume of the space between the plates:

$$U_0 = u_0 Ad$$

Express the energy density in the electric field:

$$u_0 = \tfrac{1}{2}\in_0 E_0^2$$

Substitute to obtain:

$$U_0 = \tfrac{1}{2}\in_0 E_0^2 Ad$$

Substitute numerical values and evaluate U_0:

$$U_0 = \tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(25\ \text{kV/m}\right)^2 \times\left(600\ \text{cm}^2\right)\left(4\ \text{mm}\right) = \boxed{0.664\ \mu\text{J}}$$

(*b*) With the electric field in place the electric field becomes:

$$E = \frac{E_0}{\kappa} = \frac{25\ \text{kV/m}}{4} = \boxed{6.25\ \text{kV/m}}$$

(*c*) Relate the potential difference V to the electric field E and the separation of the plates:

$$V = Ed = \left(6.25\ \text{kV/m}\right)\left(4\ \text{mm}\right) = \boxed{25.0\ \text{V}}$$

(*d*) Relate the new electrostatic energy U to the initial electrostatic energy U_0 and the dielectric constant κ:

$$U = \frac{U_0}{\kappa} = \frac{0.664\ \mu\text{J}}{4} = \boxed{0.166\ \mu\text{J}}$$

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Picture the Problem We can use the definition of capacitance and the relationship between the electric field in the capacitor and the potential difference across its plates to express C. In part (*b*) we can use $\sigma_b = \left(1 - 1/\kappa\right)\sigma_f$ and $\kappa = 1 + (3/y_0)y$ to express the ratio σ_b/σ_f and evaluate it at $y = 0$ and $y = y_0$. The application of Gauss's law in part (*c*) will yield an expression for $\rho(y)$ within the dielectric that we can integrate in part (*d*) to find the total induced bound charge.

(*a*) Using its definition, express the capacitance of the parallel-plate capacitor:

$$C = \frac{Q}{V} = \frac{\sigma A}{V} \qquad (1)$$

Express the potential difference V between the plates in terms of the electric field E between the plates:

$$dV = Edy$$

Express the electric field in the region between the plates:

$$E = \frac{E_0}{\kappa(y)} = \frac{\sigma}{\epsilon_0\,\kappa(y)}$$

Substitute to obtain:

$$dV = \frac{\sigma}{\epsilon_0\,\kappa(y)}dy = \frac{\sigma}{\epsilon_0\left\{1+\left(\frac{3}{y_0}\right)y\right\}}dy$$

Integrate from $y = 0$ to $y = y_0$:

$$V = \frac{\sigma}{\epsilon_0}\int_0^{y_0}\frac{dy}{1+(3/y_0)y}$$

$$= \frac{\sigma y_0}{3\,\epsilon_0}\ln(1+3y/y_0)\Big|_0^{y_0}$$

$$= \frac{\sigma y_0}{3\,\epsilon_0}\ln(4)$$

Substitute in equation (1) and simplify to obtain:

$$C = \frac{\sigma A}{\frac{\sigma y_0}{3\,\epsilon_0}\ln(4)} = \boxed{\frac{3\,\epsilon_0\,A}{y_0\ln(4)}}$$

(*b*) Relate σ_b to σ_f and κ:

$$\sigma_b = \left(1-\frac{1}{\kappa}\right)\sigma_f$$

and

$$\frac{\sigma_b}{\sigma_f} = 1-\frac{1}{\kappa}$$

Substitute for κ to obtain:

$$\frac{\sigma_b}{\sigma_f} = 1-\frac{1}{1+(3/y_0)y}$$

Evaluate σ_b/σ_f at $y = 0$:

$$\left.\frac{\sigma_b}{\sigma_f}\right|_{y=0} = 1-\frac{1}{1+(3/y_0)(0)} = \boxed{0}$$

Evaluate σ_b/σ_f at $y = y_0$:

$$\left.\frac{\sigma_b}{\sigma_f}\right|_{y=y_0} = 1-\frac{1}{1+(3/y_0)y_0} = \boxed{0.750}$$

(*c*) Consider a Gaussian surface of area A and width dy and recall that E into the surface is taken to be negative. Apply Gauss's law to obtain:

$$[E(y)-E(y+dy)]A = \frac{Q_{\text{inside}}}{\epsilon_0}$$

$$= \frac{Ady\rho(y)}{\epsilon_0}$$

Divide both sides of the equation by *dy*:

$$\frac{[E(y)-E(y+dy)]}{dy}=\frac{\rho(y)}{\epsilon_0}$$

or

$$-\frac{dE}{dy}=\frac{\rho(y)}{\epsilon_0}$$

Solve for $\rho(y)$ to obtain:

$$\rho(y)=-\epsilon_0\frac{dE}{dy}$$

$$=-\epsilon_0\frac{d}{dy}\left[\frac{\sigma}{\epsilon_0\,\kappa(y)}\right]$$

$$=-\sigma\frac{d}{dy}\left[\frac{1}{1+(3/y_0)y}\right]$$

$$=\boxed{\frac{3\sigma}{\left[y_0(1+3y/y_0)^2\right]}}$$

(*d*) Integrate $\rho(y)$ from $y = 0$ to $y = y_0$ to obtain:

$$\rho=3\sigma\int_0^{y_0}\frac{dy}{\left[y_0(1+3y/y_0)^2\right]}$$

$=\boxed{-\frac{3}{4}\sigma}$, the charge per unit area in the dielectric, and just cancels out the induced surface charge density.

General Problems

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Picture the Problem We can use the expression $U_0=\frac{1}{2}C_{eq}V^2$ to express the total energy stored in the combination of four capacitors in terms of their equivalent capacitance C_{eq}.

The energy stored in one capacitor when it is connected to the 100-V battery is:

$$U_0=\frac{1}{2}CV^2$$

When the four capacitors are connected to the battery in some combination, the total energy stored in them is:

$$U=\frac{1}{2}C_{eq}V^2$$

Equate U and U_0 and solve for C_{eq} to obtain:

$$C_{eq}=C$$

The equivalent capacitance C' of two capacitors of capacitance C connected in series is their product divided by their sum:

$$C' = \frac{C^2}{C+C} = \tfrac{1}{2}C$$

If we connect two of the capacitors in series in parallel with the other two capacitors connected in series, their equivalent capacitance will be:

$$C_{\mathrm{eq}} = C' + C' = \tfrac{1}{2}C + \tfrac{1}{2}C = C$$

Thus, a series combination of two of the capacitors in parallel with a series combination of the other two capacitors will result in total energy U_0 stored in all four capacitors.

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Picture the Problem We can use the equations for the equivalent capacitance of three capacitors connected in parallel and in series to find these equivalent capacitances.

(*a*) Express the equivalent capacitance of three capacitors connected in parallel:

$$C_{\mathrm{eq}} = C_1 + C_2 + C_3$$

Substitute numerical values and evaluate C_{eq}:

$$C_{\mathrm{eq}} = 2.0\,\mu\mathrm{F} + 4.0\,\mu\mathrm{F} + 8.0\,\mu\mathrm{F} = \boxed{14.0\,\mu\mathrm{F}}$$

(*b*) Express the equivalent capacitance of the three capacitors connected in series:

$$C_{\mathrm{eq}} = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_1C_3}$$

Substitute numerical values and evaluate C_{eq}:

$$C_{\mathrm{eq}} = \frac{(2\,\mu\mathrm{F})(4\,\mu\mathrm{F})(8\,\mu\mathrm{F})}{(2\,\mu\mathrm{F})(4\,\mu\mathrm{F}) + (4\,\mu\mathrm{F})(8\,\mu\mathrm{F}) + (2\,\mu\mathrm{F})(8\,\mu\mathrm{F})} = \boxed{1.14\,\mu\mathrm{F}}$$

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Picture the Problem We can first use the equation for the equivalent capacitance of two capacitors connected in parallel and then the equation for two capacitors connected in series to find the equivalent capacitance.

Find the equivalent capacitance of a 1.0-μF capacitor connected in parallel with a 2.0-μF capacitor:

$$C_{\text{eq},1} = C_1 + C_2 = 1.0\,\mu\text{F} + 2.0\,\mu\text{F} = 3.0\,\mu\text{F}$$

Find the equivalent capacitance of a 3.0-μF capacitor connected in series with a 6.0-μF capacitor:

$$\boldsymbol{C}_{\text{eq},2} = \frac{\boldsymbol{C}_{\text{eq},1}\boldsymbol{C}_6}{\boldsymbol{C}_{\text{eq},1} + \boldsymbol{C}_6} = \frac{(3.0\,\mu\text{F})(6.0\,\mu\text{F})}{3.0\,\mu\text{F} + 6.0\,\mu\text{F}} = \boxed{2.00\,\mu\text{F}}$$

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Picture the Problem The charge Q and the charge density σ are independent of the separation of the plates and do not change during the process described in the problem statement. Because the electric field E depends on σ, it too is constant. We can use $U = \frac{1}{2}CV^2$ and the relationship between V and E, together with the expression for the capacitance of a parallel-plate capacitor, to show that $U \propto d$.

Express the energy stored in the capacitor in terms of its capacitance C and the potential difference across its plates:

$$U = \tfrac{1}{2}CV^2$$

Express V in terms of E:

$$V = Ed$$

where d is the separation of the plates.

Express the capacitance of a parallel-plate capacitor:

$$C = \frac{\kappa \in_0 A}{d}$$

Substitute to obtain:

$$U = \frac{1}{2}\frac{\kappa \in_0 A}{d}(Ed)^2 = \left(\tfrac{1}{2}\kappa \in_0 AE^2\right)d$$

Because $U \propto d$, to double U one must double d. Hence:

$$d_\text{f} = 2d = 2(0.5\,\text{mm}) = \boxed{1.00\,\text{mm}}$$

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Picture the Problem We can use the equations for the equivalent capacitance of capacitors connected in parallel and in series to find the single capacitor that will store the same amount of charge as each of the networks shown above.

(*a*) Find the capacitance of the two capacitors in parallel:

$$C_{\text{eq},1} = C_0 + C_0 = 2C_0$$

Find the capacitance equivalent to $2C_0$ in series with C_0:

$$C_{eq,2} = \frac{C_{eq,1}C_0}{C_{eq,1} + C_0} = \frac{(2C_0)C_0}{2C_0 + C_0} = \boxed{\tfrac{2}{3}C_0}$$

(*b*) Find the capacitance of two capacitors of capacitance C_0 in parallel:

$$C_{eq,1} = 2C_0$$

Find the capacitance equivalent to $2C_0$ in series with $2C_0$:

$$C_{eq,2} = \frac{C_{eq,1}C_0}{C_{eq,1} + C_0} = \frac{(2C_0)(2C_0)}{2C_0 + 2C_0} = \boxed{C_0}$$

(*c*) Find the equivalent capacitance of three equal capacitors connected in parallel:

$$C_{eq} = C_0 + C_0 + C_0 = \boxed{3C_0}$$

***85** ••

Picture the Problem Note that with V applied between a and b, C_1 and C_3 are in series, and so are C_2 and C_4. Because in a series combination the potential differences across the two capacitors are inversely proportional to the capacitances, we can establish proportions involving the capacitances and potential differences for the left- and right-hand side of the network and then use the condition that $V_c = V_d$ to eliminate the potential differences and establish the relationship between the capacitances.

Letting Q represent the charge on capacitors 1 and 2, relate the potential differences across the capacitors to their common charge and capacitances:

$$V_1 = \frac{Q}{C_1}$$

and

$$V_3 = \frac{Q}{C_3}$$

Divide the first of these equations by the second to obtain:

$$\frac{V_1}{V_3} = \frac{C_3}{C_1} \qquad (1)$$

Proceed similarly to obtain:

$$\frac{V_2}{V_4} = \frac{C_4}{C_2} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{V_1V_4}{V_3V_2} = \frac{C_3C_2}{C_1C_4} \qquad (3)$$

If $V_c = V_d$ then we must have:

$$V_1 = V_2 \text{ and } V_3 = V_4$$

Substitute in equation (3) and

$$\boxed{C_2C_3 = C_1C_4}$$

rearrange to obtain:

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Picture the Problem Because the spheres are identical, each will have half the charge of the initially charged sphere when they are connected. We can find the fraction of the initial energy that is dissipated by finding the energy stored initially and the energy stored when the two spheres are connected.

Express the fraction of the initial energy that is dissipated when the two spheres are connected:

$$f=\frac{U_\text{i}-U_\text{f}}{U_\text{i}}=1-\frac{U_\text{f}}{U_\text{i}} \qquad (1)$$

Express the initial energy of the sphere whose charge is Q:

$$U_\text{i}=\frac{1}{2}\frac{Q^2}{C}$$

Relate the capacitance of a an isolated spherical conductor to its radius:

$$C=4\pi\in_0 R$$

Substitute to obtain:

$$U_\text{i}=\frac{1}{2}\frac{Q^2}{4\pi\in_0 R}=\frac{1}{2}\frac{kQ^2}{R}$$

Express the energy of the connected spheres:

$$U_\text{f}=\frac{1}{2}\frac{k(Q/2)^2}{R}+\frac{1}{2}\frac{k(Q/2)^2}{R}=\frac{1}{4}\frac{kQ^2}{R}$$

Substitute in equation (1) and simplify:

$$f=1-\frac{\frac{1}{4}\frac{kQ^2}{R}}{\frac{1}{2}\frac{kQ^2}{R}}=1-\frac{1}{2}=\boxed{\frac{1}{2}}$$

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Picture the Problem We can use the expression for the capacitance of a parallel-plate capacitor as a function of A and d to determine the effect on the capacitance of doubling the plate separation. We can use $V=Ed$ to determine the effect on the potential difference across the capacitor of doubling the plate separation. Finally, we can use $U=CV^2/2$ to determine the effect of doubling the plate separation on the energy stored in the capacitor.

(*a*) Express the capacitance of a capacitor whose plates are separated by a distance 2*d*:

$$C_\text{new}=\boxed{\frac{\in_0 A}{2d}}$$

(*b*) Express the potential difference across a parallel-plate capacitor whose plates are separated by a distance *d*:

$V = Ed$

where the electric field E depends solely on the charge on the capacitor plates.

Express the new potential difference across the plates resulting from the doubling of their separation:

$$V_{\text{new}} = E(2d) = 2(Ed) = \boxed{2V}$$

(*c*) Relate the energy stored in a parallel-plate capacitor to the separation of the plates:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2$$

When the plate separation is doubled we have:

$$U_{\text{new}} = \frac{1}{2}\frac{\epsilon_0 A}{2d}(2V)^2 = \boxed{\frac{\epsilon_0 AV^2}{d}}$$

(*d*) Relate the work required to change the plate separation from *d* to 2*d* to the change in the electrostatic potential energy of the system:

$$W = \Delta U = U_{\text{new}} - U_{\text{i}}$$
$$= \frac{\epsilon_0 AV^2}{d} - \frac{\epsilon_0 AV^2}{2d}$$
$$= \boxed{\frac{\epsilon_0 AV^2}{2d}}$$

88 ••

Picture the Problem We can use the equation for the equivalent capacitance of two capacitors in series to relate C_0 to C' and the capacitance of the dielectric-filled parallel-plate capacitor and then solve the resulting equation for C'.

Express the equivalent capacitance of the system in terms of C' and C, where C is the dielectric-filled capacitor:

$$C_0 = \frac{C'C}{C'+C}$$

Solve for C' to obtain:

$$C' = \frac{C_0 C}{C - C_0}$$

Express the capacitance of the dielectric-filled capacitor:

$$C = \frac{\kappa \epsilon_0 A}{d} = \kappa C_0$$

Substitute to obtain:

$$C' = \frac{C_0(\kappa C_0)}{\kappa C_0 - C_0} = \boxed{\frac{\kappa}{\kappa - 1}C_0}$$

89 ••

Picture the Problem Modeling the Leyden jar as a parallel-plate capacitor, we can use the equation relating the capacitance of a parallel-plate capacitor to the area A and separation d of its plates to find the jar's capacitance. To find the maximum charge the jar can carry without undergoing dielectric breakdown we can use the definition of capacitance to express Q_{max} in terms of V_{max} … and then relate V_{max} to E_{max} using $V_{max} = E_{max}d$, where d is the thickness of the glass wall of the jar.

(*a*) Treating it as a parallel-plate capacitor, express the capacitance of the Leyden jar

$$C = \frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0 (2\pi Rh)}{d} = \frac{4\pi \in_0 \kappa Rh}{2d} = \frac{\kappa Rh}{2kd}$$

where h is the height of the jar and R is its inside radius.

Substitute numerical values and evaluate C:

$$C = \frac{5(0.04\,\text{m})(0.4\,\text{m})}{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2\times10^{-3}\,\text{m})} = \boxed{2.22\,\text{nF}}$$

(*b*) Using the definition of capacitance, relate the maximum charge of the capacitor to the breakdown voltage of the dielectric:

$$Q_{max} = CV_{max}$$

Express the breakdown voltage in terms of the dielectric strength and thickness of the dielectric:

$$V_{max} = E_{max}d$$

Substitute to obtain:

$$Q_{max} = CE_{max}d$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{max} = (2.22\,\text{nF})(15\,\text{MV/m})(2\times10^{-3}\,\text{m}) = \boxed{66.6\,\mu\text{C}}$$

***90** ••

Picture the Problem The maximum voltage is related to the dielectric strength of the medium according to $V_{max} = E_{max}d$ and we can use the expression for the capacitance of a parallel-plate capacitor to determine the required area of the plates.

(*a*) Relate the maximum voltage that can be applied across this capacitor

$$V_{max} = E_{max}d$$

to the dielectric strength of silicon dioxide:

Substitute numerical values and evaluate V_{max}:

$$V_{max} = (8\times10^{6}\ \text{V/m})(5\times10^{-6}\ \text{m}) = \boxed{40.0\ \text{V}}$$

(*b*) Relate the capacitance of a parallel-plate capacitor to area *A* of its plates:

$$C = \frac{\kappa \in_0 A}{d}$$

Solve for *A* to obtain:

$$A = \frac{Cd}{\kappa \in_0}$$

Substitute numerical values and evaluate *A*:

$$A = \frac{(10\ \text{pF})(5\times10^{-6}\ \text{m})}{3.8(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.49\times10^{-6}\ \text{m}^2 = \boxed{1.49\ \text{mm}^2}$$

(*c*) Express the number of capacitors *n* in terms of the area of a square 1 cm by 1cm and the area required for each capacitor:

$$n = \frac{(1\text{cm})^2}{A} = \frac{100\ \text{mm}^2}{1.49\ \text{mm}^2} \approx \boxed{67}$$

91 ••

Picture the Problem When the battery is removed, after having initially charged both capacitors, and the separation of one of the capacitors is doubled, the charge is redistributed subject to the condition that the total charge remains constant; i.e., $Q = Q_1 + Q_2$ where *Q* is the initial charge on both capacitors and Q_2 is the charge on the capacitor whose plate separation has been doubled. We can use the conservation of charge during the plate separation process and the fact that, because the capacitors are in parallel, they share a common potential difference.

Find the equivalent capacitance of the two 2-μF parallel-plate capacitors connected in parallel:

$$C_{eq} = 2\ \mu\text{F} + 2\ \mu\text{F} = 4\ \mu\text{F}$$

Use the definition of capacitance to find the charge on the equivalent capacitor:

$$Q = C_{eq}V = (4\ \mu\text{F})(100\ \text{V}) = 400\ \mu\text{C}$$

Relate this total charge to charges distributed on capacitors 1 and 2 when the battery is removed and the separation of the plates of capacitor 2 is doubled:

$$Q = Q_1 + Q_2 \qquad (1)$$

Because the capacitors are in parallel:

$$V_1 = V_2$$

and

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2'} = \frac{Q_2}{\frac{1}{2}C_2} = \frac{2Q_2}{C_2}$$

Solve for Q_1 to obtain:

$$Q_1 = 2\left(\frac{C_1}{C_2}\right)Q_2 \qquad (2)$$

Substitute equation (2) in equation (1) and solve for Q_2 to obtain:

$$Q_2 = \frac{Q}{2(C_1/C_2)+1}$$

Substitute numerical values and evaluate Q_2:

$$Q_2 = \frac{400\,\mu\text{C}}{2(2\,\mu\text{F}/2\,\mu\text{F})+1} = \boxed{133\,\mu\text{C}}$$

Substitute in equation (1) or equation (2) and evaluate Q_1:

$$Q_1 = \boxed{267\,\mu\text{C}}$$

92 ••

Picture the Problem We can relate the electric field in the dielectric to the electric field between the capacitor's plates in the absence of a dielectric using $E = E_0/\kappa$. In part (*b*) we can express the potential difference between the plates as the sum of the potential differences across the dielectrics and then express the potential differences in terms of the electric fields in the dielectrics. In part (*c*) we can use our result from (*b*) and the definition of capacitance to express the capacitance of the dielectric-filled capacitor. In part (*d*) we can confirm the result of part (*c*) by using the addition formula for capacitors in series.

(*a*) Express the electric field E in a dielectric of constant κ in terms of the electric field E_0 in the absence of the dielectric:

$$E = \frac{E_0}{\kappa}$$

Express the electric field E_0 in the absence of the dielectrics:

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Substitute to obtain:

$$E = \frac{Q}{\kappa \in_0 A}$$

Use this relationship to express the electric fields in dielectrics whose constants are κ_1 and κ_2:

$$E_1 = \boxed{\frac{Q}{\kappa_1 \in_0 A}} \text{ and } E_2 = \boxed{\frac{Q}{\kappa_2 \in_0 A}}$$

(*b*) Express the potential difference between the plates as the sum of the potential differences across the dielectrics:

$$V = V_1 + V_2$$

Relate the potential differences to the electric fields and the thicknesses of the dielectrics:

$$V_1 = E_1 \frac{d}{2} = \frac{Qd}{2\kappa_1 \in_0 A}$$

and

$$V_2 = E_2 \frac{d}{2} = \frac{Qd}{2\kappa_2 \in_0 A}$$

Substitute and simplify to obtain:

$$V = \frac{Qd}{2\kappa_1 \in_0 A} + \frac{Qd}{2\kappa_2 \in_0 A} = \boxed{\frac{Qd}{2 \in_0 A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right)}$$

(*c*) Use the definition of capacitance to obtain:

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{2 \in_0 A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right)} = \frac{2 \in_0 A}{d\left(\frac{\kappa_1 + \kappa_2}{\kappa_1\kappa_2}\right)} = \frac{2 \in_0 A}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1 + \kappa_2}\right) = \boxed{2C_0\left(\frac{\kappa_1\kappa_2}{\kappa_1 + \kappa_2}\right)}$$

where $C_0 = \in_0 A/d$.

(*d*) Express the equivalent capacitance C of capacitors C_1 and C_2 in series:

$$C = \frac{C_1C_2}{C_1 + C_2}$$

Express C_1:

$$C_1 = \frac{\kappa_1 \in_0 A}{d/2} = \frac{2\kappa_1 \in_0 A}{d} = 2\kappa_1 C_0$$

Express C_2:

$$C_2 = \frac{\kappa_2 \in_0 A}{d/2} = \frac{2\kappa_2 \in_0 A}{d} = 2\kappa_2 C_0$$

Substitute to obtain:

$$C = \frac{(2\kappa_1 C_0)(2\kappa_2 C_0)}{2\kappa_1 C_0 + 2\kappa_2 C_0} = \boxed{2C_0\left(\frac{\kappa_1\kappa_2}{\kappa_1 + \kappa_2}\right)},$$

a result in agreement with part (*c*).

93 ••

Picture the Problem Recall that within a conductor $E = 0$. We can use the definition of capacitance to express C in terms of the charge on the capacitor Q and the potential difference across the plates V. We can then express V in terms of E and the thickness of the air gap between the plates. Finally, we can express the electric field between the plates in terms of the charge on them and their area. Substitution in our expression for C will give us C in terms of $d - t$. In part (*b*) we can use the expression for the equivalent capacitance of two capacitors connected in series to derive the same expression for C.

(*a*) Use its definition to express the capacitance of this parallel-plate capacitor:

$$C = \frac{Q}{V}$$

where Q is the charge on the capacitor.

Relate the electric potential between the plates to the electric field between the plates:

$$V = E(d - t)$$

Express the electric field E between the plates but outside the metal slab:

$$E = \frac{\sigma}{\in_0} = \frac{Q}{\in_0 A}$$

Substitute and simplify to obtain:

$$C = \frac{Q}{E(d-t)} = \frac{Q}{\frac{Q}{\in_0 A}(d-t)} = \boxed{\frac{\in_0 A}{d-t}}$$

(*b*) Express the equivalent capacitance C of two capacitors C_1 and C_2 connected in series:

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

Express the capacitances C_1 and C_2 of the plates separated by a and b, respectively:

$$C_1 = \frac{\in_0 A}{a}$$

and

$$C_2 = \frac{\epsilon_0 A}{b}$$

Substitute and simplify to obtain:

$$C = \frac{\left(\frac{\epsilon_0 A}{a}\right)\left(\frac{\epsilon_0 A}{b}\right)}{\frac{\epsilon_0 A}{a} + \frac{\epsilon_0 A}{b}} = \frac{\epsilon_0 A}{a+b}$$

Solve the constraint that $a + b + t = d$ for $a + b$ to obtain:

$$a + b = d - t$$

Substitute for $a + b$ to obtain:

$$C = \boxed{\frac{\epsilon_0 A}{d-t}}$$

***94 ••**

Picture the Problem We can express the ratio of C_{eq} to C_0 to show that the capacitance with the dielectrics in place is $(\kappa_1 + \kappa_2)/2$ times greater than that of the capacitor in the absence of the dielectrics.

(*a*) Because the capacitor plates are conductors, the potentials are the same across the entire upper and lower plates. Hence, the system is equivalent to two capacitors, each of area $A/2$, in parallel.

(*b*) Relate the capacitance C_0, in the absence of the dielectrics, to the plate area and separation:

$$C_0 = \frac{\epsilon_0 A}{d}$$

Express the equivalent capacitance of capacitors C_1 and C_2, each with plate area $A/2$, connected in parallel:

$$\begin{aligned} C_{eq} &= C_1 + C_2 \\ &= \frac{\kappa_1 \epsilon_0 \left(\frac{1}{2}A\right)}{d} + \frac{\kappa_1 \epsilon_0 \left(\frac{1}{2}A\right)}{d} \\ &= \frac{\kappa_1 \epsilon_0 A}{2d}(\kappa_1 + \kappa_2) \end{aligned}$$

Express the ratio of C_{eq} to C_0 and simplify to obtain:

$$\frac{C_{eq}}{C_0} = \frac{\frac{\kappa_1 \epsilon_0 A}{2d}(\kappa_1 + \kappa_2)}{\frac{\epsilon_0 A}{d}} = \boxed{\tfrac{1}{2}(\kappa_1 + \kappa_2)}$$

95 ••

Picture the Problem We can use $U = Q^2/2C$ and the expression for the capacitance as a function of plate separation to express U as a function of x. Differentiation of this result with respect to x will yield dU. Because the work done in increasing the plate separation a distance dx equals the change in the electrostatic potential energy of the capacitor, we can evaluate F from dU/dx. Finally, we can express F in terms of Q and E by relating E to x using $E = Vx$ and using the definition of capacitance and the expression for the capacitance of a parallel-plate capacitor.

(*a*) Relate the electrostatic energy U stored in the capacitor to its capacitance C:

$$U = \frac{1}{2}\frac{Q^2}{C}$$

Express the capacitance as a function of the plate separation:

$$C = \frac{\epsilon_0 A}{x}$$

Substitute for C to obtain:

$$U = \boxed{\frac{Q^2}{2\epsilon_0 A}x}$$

(*b*) Use the result obtained in (*a*) to evaluate dU:

$$dU = \frac{dU}{dx}dx = \frac{d}{dx}\left[\frac{Q^2}{2\epsilon_0 A}x\right]dx$$

$$= \boxed{\frac{Q^2}{2\epsilon_0 A}dx}$$

(*c*) Relate the work needed to move one plate a distance dx to the change in the electrostatic potential energy of the system:

$$W = dU = Fdx$$

Solve for and evaluate F:

$$F = \frac{dU}{dx} = \frac{d}{dx}\left[\frac{Q^2}{2\epsilon_0 A}x\right] = \boxed{\frac{Q^2}{2\epsilon_0 A}}$$

(*d*) Express the electric field between the plates in terms of their separation and their potential difference:

$$E = \frac{V}{x}$$

Use the definition of capacitance to eliminate V:

$$E = \frac{Q}{Cx}$$

Use the expression for the capacitance of a parallel-plate capacitor to eliminate C:

$$E = \frac{Q}{\frac{\epsilon_0 A}{x}x} = \frac{Q}{\epsilon_0 A}$$

Substitute in our result from part (*c*) to obtain:

$$F=\frac{Q(\epsilon_0\ AE)}{2\epsilon_0\ A}=\boxed{\tfrac{1}{2}QE}$$

The field *E* is due to the sum of the fields from charges +*Q* and –*Q* on the opposite plates of the capacitor. Each plate produces a field ½ *E* and the force is the product of charge *Q* and the field ½ *E*.

96 ••

Picture the Problem We can model this capacitor as the equivalent of two capacitors connected in parallel. Let the numeral 1 denote the capacitor with the dielectric material whose constant is κ and the numeral 2 the air-filled capacitor.

(*a*) Express the equivalent capacitance of the two capacitors in parallel:

$$C(x)=C_1+C_2 \quad (1)$$

Use the expression for the capacitance of a parallel-plate capacitor to express C_1:

$$C_1=\frac{\kappa\epsilon_0\ A_1}{d}=\frac{\kappa\epsilon_0\ bx}{d}$$

Express the capacitance C_0 of the capacitor with the dielectric removed, i.e., $x = 0$:

$$C_0=\frac{\epsilon_0\ ab}{d}$$

Divide C_1 by C_0 to obtain:

$$\frac{C_1}{C_0}=\frac{\frac{\kappa\epsilon_0\ bx}{d}}{\frac{\epsilon_0\ ab}{d}}=\frac{\kappa x}{a}$$

or

$$C_1=\frac{\kappa x}{a}C_0$$

Use the expression for the capacitance of a parallel-plate capacitor to express C_2:

$$C_2=\frac{\epsilon_0\ A_2}{d}=\frac{\epsilon_0\ b(a-x)}{d}$$

Divide C_2 by C_0 to obtain:

$$\frac{C_2}{C_0}=\frac{\frac{\epsilon_0\ b(a-x)}{d}}{\frac{\epsilon_0\ ab}{d}}=\frac{a-x}{a}$$

or

$$C_2 = \frac{a-x}{a}C_0$$

Substitute in equation (1) and simplify to obtain:

$$C(x) = \frac{\kappa x}{a}C_0 + \frac{a-x}{a}C_0 = \frac{C_0}{a}[a+(\kappa-1)x] = \boxed{\frac{\in_0 b}{d}[a+(\kappa-1)x]}$$

(*b*) Evaluate C for $x = 0$:

$$C(0) = \frac{\in_0 b}{d}[a] = \frac{\in_0 ab}{d} = \boxed{C_0}$$

as expected.

Evaluate C for $x = a$:

$$C(a) = \frac{\in_0 b}{d}[a+(\kappa-1)a] = \boxed{\frac{\kappa \in_0 ab}{d}} \text{ as expected.}$$

***97 •••**

Picture the Problem We can model this capacitor as the equivalent of two capacitors connected in parallel, one with an air gap and other filled with a dielectric of constant κ. Let the numeral 1 denote the capacitor with the dielectric material whose constant is κ and the numeral 2 the air-filled capacitor.

(*a*) Using the hint, express the energy stored in the capacitor as a function of the equivalent capacitance C_{eq}:

$$U = \frac{1}{2}\frac{Q^2}{C_{eq}}$$

The capacitances of the two capacitors are:

$$C_1 = \frac{\kappa \in_0 ax}{d} \text{ and } C_2 = \frac{\in_0 a(a-x)}{d}$$

Because the capacitors are in parallel, C_{eq} is the sum of C_1 and C_2:

$$C_{eq} = C_1 + C_2 = \frac{\kappa \in_0 ax}{d} + \frac{\in_0 a(a-x)}{d} = \frac{\in_0 a}{d}(\kappa x + a - x) = \frac{\in_0 a}{d}[(\kappa-1)x + a]$$

Substitute for C_{eq} in the expression for U and simplify to obtain:

$$U = \boxed{\frac{Q^2 d}{2\in_0 a[(\kappa-1)x+a]}}$$

(*b*) The force exerted by the electric field is given by:

$$F = -\frac{dU}{dx}$$
$$= -\frac{d}{dx}\left[\frac{1}{2\in_0 a[(\kappa-1)x+a]}\frac{Q^2 d}{}\right]$$
$$= -\frac{Q^2 d}{2\in_0 a}\frac{d}{dx}\left\{[(\kappa-1)x+a]^{-1}\right\}$$
$$= \boxed{\frac{(\kappa-1)Q^2 d}{2a\in_0 [(\kappa-1)x+a]^2}}$$

(*c*) Rewrite our result in (*b*) to obtain:

$$F = \frac{(\kappa-1)Q^2\left(\frac{a\in_0}{d}\right)}{2\left(\frac{a\in_0}{d}\right)^2[(\kappa-1)x+a]^2}$$
$$= \frac{(\kappa-1)Q^2\left(\frac{a\in_0}{d}\right)}{2C_{eq}^2}$$
$$= \boxed{\frac{(\kappa-1)a\in_0 V^2}{2d}}$$

Note that this expression is independent of x.

(*d*) This force originates from the fringing fields around the edges of the capacitor. The effect of the force is to pull the dielectric into the space between the capacitor plates.

98 ••

Picture the Problem Because capacitors connected in series have a common charge, we can find the charge on each capacitor by finding the charge on the equivalent capacitor. We can also find the total energy stored in the capacitors, with and without the dielectric inserted in one of them, by using $U = \frac{1}{2}C_{eq}V^2$. In part (*d*) we can use our knowledge of the charge on each capacitor and the definition of capacitance to the potential differences across them.

(*a*) Using the definition of capacitance, relate the charge on each capacitor to the equivalent

$$Q = Q_1 = Q_2 = C_{eq}V$$

capacitance:

Express the equivalent capacitance of two capacitors in series:

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

Substitute numerical values and evaluate C_{eq}:

$$C_{eq} = \frac{(4\,\mu\text{F})(4\,\mu\text{F})}{4\,\mu\text{F} + 4\,\mu\text{F}} = 2\,\mu\text{F}$$

Substitute numerical values and evaluate Q:

$$Q_1 = Q_2 = (2\,\mu\text{F})(24\,\text{V}) = \boxed{48.0\,\mu\text{C}}$$

(*b*) Express the energy U stored in the capacitors as a function of C_{eq} and V:

$$U = \tfrac{1}{2}C_{eq}V^2 = \tfrac{1}{2}(2\,\mu\text{F})(24\,\text{V})^2 = \boxed{576\,\mu\text{J}}$$

(*c*) Using the definition of capacitance, relate the charge on each capacitor to the new equivalent capacitance C_{eq}':

$$Q' = Q_1' = Q_2' = C_{eq}'V \qquad (1)$$

Express the new equivalent capacitance C_{eq}' when the dielectric of constant κ has been inserted between the plates of one of the capacitors:

$$C_{eq}' = \frac{C_1'C_2'}{C_1' + C_2'}$$

Letting the capacitor with the dielectric between its plates be denoted by the numeral 1, express C_1' and C_2':

$$C_1' = \kappa C_1$$
and
$$C_2' = C_2$$

Substitute to obtain:

$$C_{eq}' = \frac{\kappa C_1C_2}{\kappa C_1 + C_2}$$

Substitute numerical values and evaluate C_{eq}':

$$C_{eq}' = \frac{4.2(4\,\mu\text{F})(4\,\mu\text{F})}{4.2(4\,\mu\text{F}) + 4\,\mu\text{F}} = 3.23\,\mu\text{F}$$

Substitute in equation (1) to obtain:

$$Q_1' = Q_2' = (3.23\,\mu\text{F})(24\,\text{V}) = \boxed{77.5\,\mu\text{C}}$$

(*d*) Express the potential difference across each capacitor in terms of its

$$V_1' = \frac{Q_1'}{C_1'} = \frac{Q_1'}{\kappa C_1} = \frac{77.5\,\mu\text{C}}{4.2(4\,\mu\text{F})} = \boxed{4.61\,\text{V}}$$

charge and capacitance:

and

$$V_2' = \frac{Q_2'}{C_2'} = \frac{Q_2'}{C_2} = \frac{77.5\,\mu\text{C}}{4\,\mu\text{F}} = \boxed{19.4\,\text{V}}$$

(*e*) Express the total stored energy in terms of the equivalent capacitance:

$$U = \tfrac{1}{2}C_{\text{eq}}'V^2$$
$$= \tfrac{1}{2}(3.23\,\mu\text{F})(24\,\text{V})^2 = \boxed{930\,\mu\text{J}}$$

99 ••

Picture the Problem We can find the work required to pull the glass plate out of the capacitor by finding the change in the electrostatic energy of the system as a consequence of the removal of the dielectric plate.

Express the change in the electrostatic energy of the system resulting from the removal of the glass plate:

$$W = \Delta U = U_\text{f} - U_\text{i}$$
$$= \frac{1}{2}\frac{Q^2}{C_0} - \frac{1}{2}\frac{Q^2}{C}$$

Express the capacitance C with the dielectric plate in place in terms of the dielectric constant κ and the air-only capacitance C_0:

$$C = \kappa C_0$$

where $C_0 = \frac{\epsilon_0 A}{d}$.

Substitute and factor to obtain:

$$W = \frac{1}{2}\frac{Q^2}{C_0} - \frac{1}{2}\frac{Q^2}{\kappa C_0} = \frac{Q^2}{2C_0}\left(1 - \frac{1}{\kappa}\right)$$

Use the definition of capacitance to relate the charge on the capacitor to the potential difference across its plates:

$$Q = CV = \kappa C_0 V$$

Substitute to obtain:

$$W = \frac{\kappa^2 C_0^2 V^2}{2C_0}\left(1 - \frac{1}{\kappa}\right) = \frac{\kappa^2 C_0 V^2}{2}\left(1 - \frac{1}{\kappa}\right)$$
$$= \frac{\kappa^2 \varepsilon_0 A V^2}{2d}\left(1 - \frac{1}{\kappa}\right)$$

Substitute numerical values and evaluate W:

$$W = \frac{(5)^2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(1\text{m}^2)(12\,\text{V})^2}{2(0.5\times10^{-2}\,\text{m})}\left(1 - \frac{1}{5}\right) = \boxed{2.55\,\mu\text{J}}$$

100 ••

Picture the Problem The problem statement provides us with two conditions relating the potential between the plates of the capacitor and the charge on them. We can use the definition of capacitance to obtain simultaneous equations in *Q* and *V* and solve these equations to determine the capacitance of the capacitor and the initial and final voltages.

Using the definition of capacitance, relate the initial potential between the plates of the capacitor to the charge carried by these plates:

$$15\,\mu\text{C} = CV_\text{i}$$

Again using the definition of capacitance, express the relationship between the charge on the capacitor and the increased voltage:

$$18\,\mu\text{C} = CV_\text{f} = C(V_\text{i} + 6\,\text{V})$$

Divide the second of these equations by the first to obtain:

$$\frac{18\,\mu\text{C}}{15\,\mu\text{C}} = \frac{C(V_\text{i} + 6\,\text{V})}{CV_\text{i}}$$

or

$$6V_\text{i} = 5(V_\text{i} + 6\,\text{V})$$

Solve for *V* to obtain:

$$V_\text{i} = \boxed{30.0\,\text{V}}$$

and

$$V_\text{f} = V_\text{i} + 6\,\text{V} = \boxed{36.0\,\text{V}}$$

Substitute in either of the first two equations to obtain:

$$C = \boxed{0.500\,\mu\text{F}}$$

101 ••

Picture the Problem Let ℓ be the variable separation of the plates. We can use the definition of the work done in charging the capacitor to relate the force on the upper plate to the energy stored in the capacitor. Solving this expression for the force and substituting for the energy stored in a parallel-plate capacitor will yield an expression that we can use to decide whether the balance is stable. We can use this same expression and a condition for equilibrium to find the voltage required to balance the object whose mass is *M*.

(*a*) Express the work done in charging the capacitor (the energy stored in it) in terms of the force between the plates:

$$dW = dE = -F\,d\ell$$

or

$$F = -\frac{dE}{d\ell}$$

The energy stored in the capacitor is given by:

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\in_0 A}{\ell}\right)V^2$$

Differentiate E with respect to ℓ to obtain:

$$F = -\frac{d}{d\ell}\left[\frac{1}{2}\left(\frac{\in_0 A}{\ell}\right)V^2\right] = \left(\frac{\in_0 A}{2\ell^2}\right)V^2$$

Because $\boldsymbol{F}$ increases as ℓ decreases, a decrease in plate separation will unbalance the system and the balance is unstable.

(*b*) Apply $\sum F = 0$ to the object whose mass is M to obtain:

$$Mg - \left(\frac{\in_0 A}{2\ell^2}\right)V^2 = 0$$

Solve for V:

$$V = \boxed{\ell\sqrt{\frac{2Mg}{\in_0 A}}}$$

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Picture the Problem Recall that the dielectric strength of air is 3 MV/m. We can express the maximum energy to be stored in terms of the capacitance of the air-gap capacitor and the maximum potential difference between its plates. This maximum potential can, in turn, be expressed in terms of the maximum electric field (dielectric strength) possible in the air gap. We can solve the resulting equation for the volume of the space between the plates. In part (*b*) we can modify the equation we derive in part (*a*) to accommodate a dielectric with a constant other than 1.

(*a*) Express the energy stored in the capacitor in terms of its capacitance and the potential difference across it:

$$U_{max} = \tfrac{1}{2}CV_{max}^2$$

Express the capacitance of the air-gap parallel-plate capacitor:

$$C = \frac{\in_0 A}{d}$$

Relate the maximum potential difference across the plates to the maximum electric field between them:

$$V_{max} = E_{max}d$$

Substitute to obtain:

$$U_{max} = \tfrac{1}{2}\left(\frac{\in_0 A}{d}\right)(E_{max}d)^2 = \tfrac{1}{2}\in_0 (Ad)E_{max}^2$$

$$= \tfrac{1}{2}\in_0 \upsilon E^2$$

where $\upsilon = Ad$ is the volume between the plates.

Solve for υ:

$$\upsilon = \frac{2U_{\max}}{\epsilon_0 E_{\max}^2} \quad (1)$$

Substitute numerical values and evaluate υ:

$$\upsilon = \frac{2(100\,\text{kJ})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(3\,\text{MV/m})^2} = \boxed{2.51\times10^3\,\text{m}^3}$$

(*b*) With the dielectric in place equation (1) becomes:

$$\upsilon = \frac{2U_{\max}}{\kappa\,\epsilon_0 E_{\max}^2} \quad (2)$$

Evaluate equation (2) with $\kappa = 5$ and $E_{\max} = 3\times10^8$ V/m:

$$\upsilon = \frac{2(100\,\text{kJ})}{5(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(3\times10^8\,\text{V/m})^2} = \boxed{5.02\times10^{-2}\,\text{m}^3}$$

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Picture the Problem We can use the definition of capacitance to find the charge on each capacitor in part (*a*). In part (*b*) we can express the total energy stored as the sum of the energy stored on the two capacitors … using our result from (*a*) for the charge on each capacitor. When the dielectric is removed in part (*c*) each capacitor will carry half the charge carried by the capacitor system previously and we can proceed as in (*b*). Knowing the total charge stored by the capacitors, we can use the definition of capacitance to find the final voltage across the two capacitors in part (*d*).

(*a*) Use the definition of capacitance to express the charge on each capacitor as a function of its capacitance:

$$Q_1 = C_1 V = \boxed{(200\,\text{V})C_1}$$

and

$$Q_2 = C_2 V = \kappa C_1 V = \boxed{(200\,\text{V})\kappa C_1}$$

(*b*) Express the total stored energy of the capacitors as the sum of stored energy in each capacitor:

$$\begin{aligned} U &= U_1 + U_2 = \tfrac{1}{2}C_1V^2 + \tfrac{1}{2}C_2V^2 \\ &= \tfrac{1}{2}C_1V^2 + \tfrac{1}{2}\kappa C_1V^2 \\ &= \tfrac{1}{2}C_1V^2(1+\kappa) \\ &= \tfrac{1}{2}(200\,\text{V})^2 C_1(1+\kappa) \\ &= \boxed{(2\times10^4\,\text{V}^2)(1+\kappa)C_1} \end{aligned}$$

(*c*) With the dielectric removed, each capacitor carries charge $Q/2$. Express the final energy stored by the capacitors under this condition:

$$U_f = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2} = \frac{1}{2}\frac{Q^2}{4C_1} + \frac{1}{2}\frac{Q^2}{4C_1} = \frac{Q^2}{4C_1}$$

Using the definition of capacitance, express the total charge carried by the capacitors with the dielectric in place in C_2:

$$Q = Q_1 + Q_2 = C_1V + C_2V = C_1V + \kappa C_1V = C_1V(1+\kappa) = (200\text{ V})C_1(1+\kappa)$$

Substitute to obtain:

$$U_f = \frac{[(200\text{ V})C_1(1+\kappa)]^2}{4C_1} = \boxed{(10^4\text{ V}^2)C_1(1+\kappa)^2}$$

(*d*) Use the definition of capacitance to express the final voltage across the capacitors:

$$V_f = \frac{Q}{C_{eq}} = \frac{(200\text{ V})C_1(1+\kappa)}{2C_1} = \boxed{100(1+\kappa)\text{V}}$$

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Picture the Problem We can use the definition of capacitance and the expression for the capacitance of a cylindrical capacitor to find the potential difference between the cylinders. In part (*b*) we can apply the definition of surface charge density to find the density of the free charge σ_f on the inner and outer cylindrical surfaces. We can use the fact that that Q and Q_b are proportional to E and E_b to express Q_b at a and b and then apply the definition of surface charge density to express $\sigma_b(a)$ and $\sigma_b(b)$. In part (*d*) we can use $U = \frac{1}{2}QV$ to find the total stored electrostatic energy and in (*e*) find the mechanical work required from the change in electrostatic energy of the system resulting from the removal of the dielectric cylindrical shell.

(*a*) Using the definition of capacitance, relate the potential difference between the cylinders to their charge and capacitance:

$$V = \frac{Q}{C}$$

Express the capacitance of a cylindrical capacitor as a function of its radii a and b and length L:

$$C = \frac{2\pi \in_0 \kappa L}{\ln(b/a)}$$

Substitute to obtain:

$$V = \frac{Q\ln(b/a)}{2\pi \in_0 \kappa L} = \boxed{\frac{2kQ\ln(b/a)}{\kappa L}}$$

(*b*) Apply the definition of surface charge density to obtain:

$$\sigma_f(a) = \boxed{\frac{Q}{2\pi aL}}$$

and

$$\sigma_f(b) = \boxed{\frac{-Q}{2\pi bL}}$$

(*c*) Noting that Q and Q_b are proportional to E and E_b, express Q_b at a and b:

$$Q_b(a) = \frac{-Q(\kappa-1)}{\kappa}$$

and

$$Q_b(b) = \frac{Q(\kappa-1)}{\kappa}$$

Apply the definition of surface charge density to express $\sigma_b(a)$ and $\sigma_b(b)$:

$$\sigma_b(a) = \frac{Q_b(a)}{A} = \frac{\frac{-Q(\kappa-1)}{\kappa}}{2\pi aL} = \boxed{\frac{-Q(\kappa-1)}{2\pi aL\kappa}}$$

and

$$\sigma_b(b) = \frac{Q_b(b)}{A} = \frac{\frac{Q(\kappa-1)}{\kappa}}{2\pi bL} = \boxed{\frac{Q(\kappa-1)}{2\pi bL\kappa}}$$

(*d*) Express the total stored electrostatic energy in terms of the charge stored and the potential difference between the cylinders:

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}Q\left[\frac{2kQ\ln(b/a)}{\kappa L}\right] = \boxed{\frac{kQ^2\ln(b/a)}{\kappa L}}$$

(*e*) Express the work required to remove the dielectric cylindrical shell in terms of the change in the electrostatic potential energy of the system:

$$W = \Delta U = U' - U$$

where $U' = \kappa U$ is the electrostatic potential energy of the system with the dielectric shell in place.

Substitute for U and U' to obtain:

$$W = \kappa U - U = U(\kappa-1) = \boxed{\frac{kQ^2(\kappa-1)\ln(b/a)}{\kappa L}}$$

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Picture the Problem Let the numeral 1 denote the 35-μF capacitor and the numeral 2 the 10-μF capacitor. We can use $U = \frac{1}{2}C_{eq}V^2$ to find the energy initially stored in the system and the definition of capacitance to find the charges on the two capacitors. When the dielectric is removed from the capacitor the two capacitors will share the total charge stored equally. Finally, we can find the final stored energy from the total charge stored and the equivalent capacitance of the two equal capacitors in parallel.

(*a*) Express the stored energy of the system in terms of the equivalent capacitance and the charging potential:

$$U = \tfrac{1}{2}C_{eq}V^2$$

Express the equivalent capacitance:

$$C_{eq} = C_1 + C_2$$

Substitute to obtain:

$$U = \tfrac{1}{2}(C_1 + C_2)V^2$$

Substitute numerical values and evaluate *U*:

$$U = \tfrac{1}{2}(35\,\mu\text{F} + 10\,\mu\text{F})(100\,\text{V})^2 = \boxed{0.225\,\text{J}}$$

(*b*) Use the definition of capacitance to find the charges on the two capacitors:

$$Q_1 = C_1V = (35\,\mu\text{F})(100\,\text{V}) = \boxed{3.50\,\text{mC}}$$

and

$$Q_2 = C_2V = (10\,\mu\text{F})(100\,\text{V}) = \boxed{1.00\,\text{mC}}$$

(*c*) Because the capacitors are connected in parallel, when the dielectric is removed their charges will be equal; as will be their capacitances and:

$$Q_1 = Q_2 = \tfrac{1}{2}Q = \tfrac{1}{2}(3.5\,\text{mC} + 1\,\text{mC}) = \boxed{2.25\,\text{mC}}$$

(*d*) Express the final stored energy in terms of the total charge stored and the equivalent capacitance:

$$U_f = \frac{1}{2}\frac{Q_{tot}^2}{Q_{eq}}$$

Substitute numerical values and evaluate U_f:

$$U_f = \frac{1}{2}\frac{(4.5\,\text{mC})^2}{2(10\,\mu\text{F})} = \boxed{0.506\,\text{J}}$$

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Picture the Problem We can express the two conditions on the voltage in terms of the charges Q_1 and Q_2 and the capacitances C_1 and C_2 and solve the equations simultaneously to find Q_1 and Q_2. We can then use the definition of capacitance to find the initial voltages V_1 and V_2.

Express the condition for the series connection:

$$V_1 + V_2 = 80\,\text{V}$$

or

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = 80\,\text{V}$$

Substitute numerical values to obtain:

$$\frac{Q_1}{0.4\,\mu\text{F}} + \frac{Q_2}{1.2\,\mu\text{F}} = 80\,\text{V}$$

or

$$3Q_1 + Q_2 = 96\,\mu\text{C} \qquad (1)$$

Use the definition of capacitance to express the condition for the parallel connection:

$$\frac{Q_1 + Q_2}{C_{\text{eq}}} = 20\,\text{V}$$

Because the capacitors are now connected in parallel:

$$C_{\text{eq}} = C_1 + C_2 = 0.4\,\mu\text{F} + 1.2\,\mu\text{F} = 1.6\,\mu\text{F}$$

Substitute to obtain:

$$\frac{Q_1 + Q_2}{1.6\,\mu\text{F}} = 20\,\text{V}$$

or

$$Q_1 + Q_2 = 32\,\mu\text{C} \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$Q_1 = 32\,\mu\text{C} \text{ and } Q_2 = 0$$

Use the definition of capacitance to obtain:

$$V_1 = \frac{Q_1}{C_1} = \frac{32\,\mu\text{C}}{0.4\,\mu\text{F}} = \boxed{80.0\,\text{V}}$$

and

$$V_2 = \frac{Q_2}{C_2} = \frac{0}{0.4\,\mu\text{F}} = \boxed{0}$$

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Picture the Problem Note that, with switch S closed, C_1 and C_2 are in parallel and we can use $U_{\text{closed}} = \frac{1}{2}C_{\text{eq}}V^2$ and $C_{\text{eq}} = C_1 + C_2$ to obtain an equation we can solve for C_2.

We can use the definition of capacitance to express Q_2 in terms of V_2 and C_2 and $U_{\text{open}} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$ to obtain an equation from which we can determine V_2.

Express the energy stored in the capacitors after the switch is closed:

$$U_{\text{closed}} = \tfrac{1}{2}C_{\text{eq}}V^2$$

Express the equivalent capacitance of C_1 and C_2 in parallel:

$$C_{\text{eq}} = C_1 + C_2$$

Substitute to obtain:

$$U_{\text{closed}} = \tfrac{1}{2}(C_1 + C_2)V^2$$

Solve for C_2:

$$C_2 = \frac{2U_{\text{closed}}}{V^2} - C_1$$

Substitute numerical values and evaluate C_2:

$$C_2 = \frac{2(960\,\mu\text{J})}{(80\,\text{V})^2} - 0.2\,\mu\text{F} = \boxed{0.100\,\mu\text{F}}$$

Express the charge on C_2 when the switch is open:

$$Q_2 = C_2V_2 \qquad (1)$$

Express the energy stored in the capacitors with the switch open:

$$U_{\text{open}} = \tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2$$

Solve for V_2 to obtain:

$$V_2 = \sqrt{\frac{2U_{\text{open}} - C_1V_1^2}{C_2}}$$

Substitute in equation (1) to obtain:

$$Q_2 = C_2\sqrt{\frac{2U_{\text{open}} - C_1V_1^2}{C_2}} = \sqrt{C_2\left(2U_{\text{open}} - C_1V_1^2\right)}$$

Substitute numerical values and evaluate Q_2:

$$Q_2 = \sqrt{(0.1\,\mu\text{F})\left[2(1440\,\mu\text{J}) - (0.2\,\mu\text{F})(40\,\text{V})^2\right]} = \boxed{16.0\,\mu\text{C}}$$

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Picture the Problem We can express the electric fields in the dielectric and in the free space in terms of the charge densities and then use the fact that the electric field has the same value inside the dielectric as in the free space between the plates to establish that $\sigma_1 = 2\sigma_2$. In part (c) we can model the system as two capacitors in parallel to show that the equivalent capacitance is $3\varepsilon_0 A/2d$ and then use the definition of capacitance to show that the new potential difference is $\frac{2}{3}V$.

(*a*) The potential difference between the plates is the same for both halves (the plates are equipotential surfaces). Therefore, $E = V/d$ must be the same everywhere between the plates.

(*b*) Relate the electric field in each region to σ and κ:

$$E = \frac{\sigma}{\kappa \in_0}$$

Solve for σ:

$$\sigma = \kappa \in_0 E$$

Express σ_1 and σ_2:

$$\sigma_1 = \kappa_1 \in_0 E_1 = 2 \in_0 E_1$$

and

$$\sigma_2 = \kappa_2 \in_0 E_2 = \in_0 E_1$$

Divide the 1st of these equations by the 2nd and simplify to obtain:

$$\boxed{\sigma_1 = 2\sigma_2}$$

(*c*) Model the partially dielectric-filled capacitor as two capacitors in parallel to obtain:

$$C_{eq} = C_1 + C_2$$

where

$$C_1 = \frac{\kappa \in_0 \left(\frac{1}{2}A\right)}{d} = \frac{\kappa \in_0 A}{2d}$$

and

$$C_2 = \frac{\in_0 \left(\frac{1}{2}A\right)}{d} = \frac{\in_0 A}{2d}$$

Substitute and simplify to obtain:

$$C_{eq} = \frac{\kappa \in_0 A}{2d} + \frac{\in_0 A}{2d} = \frac{2 \in_0 A}{2d} + \frac{\in_0 A}{2d} = \boxed{\frac{3 \in_0 A}{2d}}$$

Use the definition of capacitance to relate V_f, Q_f, and C_f:

$$V_f = \frac{Q_f}{C_f}$$

Because the capacitors are in parallel:

$$Q_f = Q_i = VC_i = \frac{V \in_0 A}{d}$$

Substitute to obtain:

$$V_f = \frac{V \in_0 A}{C_f d} = \frac{V \in_0 A}{\left(\frac{3 \in_0 A}{2d}\right)d} = \boxed{\tfrac{2}{3}V}$$

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Picture the Problem Note that when the capacitors are connected in the manner described they are in parallel with each other. Let the numeral one refer to the capacitor with the air gap and the numeral 2 to the capacitor that receives the dielectric and let primes denote physical quantitities after the insertion of the dielectric. We can find the energy stored in the system from our knowledge of the charge on and capacitance of each capacitor. In part (*b*) we can find the final charges on the two capacitors by first finding the equivalent capacitance and the potential difference across the modified system of capacitors. We can use the final potential difference across the system and our knowledge of the stored charge to find the final stored energy of the system.

(*a*) Express the stored energy in the system as the sum of the energy stored in the two capacitors:

$$U = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{Q_1^2}{C_1}$$

Substitute numerical values and evaluate U:

$$U = \frac{(100\,\mu\text{C})^2}{10\,\mu\text{F}} = \boxed{1.00\,\text{mJ}}$$

(*b*) Relate the final charges Q_1' and Q_2' to the total charge stored by the capacitors:

$$Q_1' + Q_2' = 200\,\mu\text{C}$$

Express the common potential difference across the capacitors:

$$V = \frac{Q_1' + Q_2'}{C_{eq}}$$

Express the equivalent capacitance when the dielectric is inserted between the plates of capacitor 2:

$$C_{eq} = C_1 + C_2 = C_1 + \kappa C_1 = C_1(1+\kappa)$$

Substitute to obtain:

$$V = \frac{Q_1' + Q_2'}{C_1(1+\kappa)}$$

Substitute numerical values and evaluate V:

$$V = \frac{200\,\mu\text{C}}{(10\,\mu\text{F})(1+3.2)} = 4.76\,\text{V}$$

Use the definition of capacitance to find Q_1' and Q_2':

$$Q_1' = VC_1' = (4.76\,\text{V})(10\,\mu\text{F}) = \boxed{47.6\,\mu\text{C}}$$

and

$$Q_2' = VC_2' = V\kappa C_2 = (4.76\,\text{V})(3.2)(10\,\mu\text{F}) = \boxed{152\,\mu\text{C}}$$

(*c*) Express the final stored energy of the system in terms of the total charge stored and the final potential difference across the capacitors connected in parallel:

$$U_\text{f} = \tfrac{1}{2}QV = \tfrac{1}{2}(200\,\mu\text{C})(4.76\,\text{V}) = \boxed{0.476\,\text{mJ}}$$

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Picture the Problem Choose a coordinate system in which the positive *x* direction is the right and the origin is at the left edge of the capacitor. We can express an element of capacitance *dC* and then integrate this expression to find *C* for this capacitor.

Express an element of capacitance *dC* of length *b*, width *dx* and separation $d = y_0 + (y_0/a)x$:

$$dC = \frac{\in_0 b}{d}dx = \frac{\in_0 b}{y_0(1 + x/a)}dx$$

These elements are all in parallel, so the total capacitance is obtained by integration:

$$C = \frac{\in_0 b}{y_0}\int_0^{y_0}\frac{1}{1 + x/a}dx = \boxed{\frac{\in_0 ab}{y_0}\ln 2}$$

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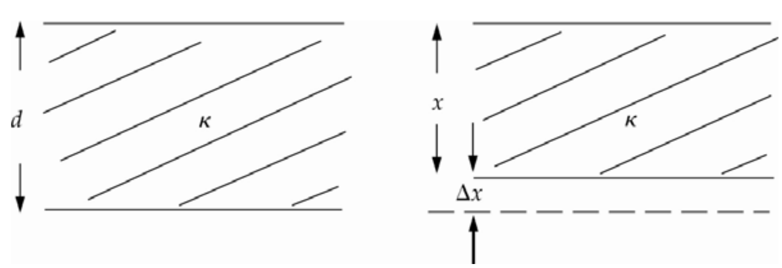
Picture the Problem The diagram to the left shows the dielectric-filled parallel-plate capacitor before compression and the diagram to the right shows the capacitor when the plate separation has been reduced to *x*. We can use the definition of capacitance and the expression for the capacitance of a parallel-plate capacitor to derive an expression for the capacitance as a function of voltage across the capacitor. We can find the maximum voltage that can be applied from the dielectric strength of the dielectric and the separation of the plates. In part (*c*) we can find the fraction of the total energy that is electrostatic field energy and the fraction that is mechanical stress energy by expressing either of these as a fraction of their sum.



(*a*) Use its definition to express the capacitance as a function of the voltage across the capacitor:

$$C(V)=\frac{Q}{V} \qquad (1)$$

The limiting value of the capacitance is:

$$C_0=\frac{\kappa \in_0 A}{d}$$

Substitute numerical values and evaluate C_0:

$$C_0=\frac{3\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)A}{0.2\,\mathrm{mm}}$$
$$=0.133A\,\mathrm{C^2/N\cdot m^3}$$

Let x be the variable separation. Because κ is independent of x:

$$C(x)=\frac{\kappa \in_0 A}{x}$$

and

$$Q(x)=C(x)V=\frac{\kappa \in_0 A}{x}V$$

Substitute in equation (1) to obtain:

$$C(V)=\frac{\frac{\kappa \in_0 A}{x}V}{V}=\frac{\kappa \in_0 A}{x} \qquad (2)$$
$$=\frac{\kappa \in_0 A}{d-\Delta x}$$

The force of attraction between the plates is given in Problem 95*c*:

$$F=\frac{Q^2(x)}{2\kappa \in_0 A}$$

Substitute to obtain:

$$F=\frac{\left(\frac{\kappa \in_0 A}{x}V\right)^2}{2\kappa \in_0 A}=-\frac{\kappa \in_0 AV^2}{2x^2}$$

where the minus sign is used to indicate that the force acts to decrease the plate separation *x*.

Apply Hooke's law to relate the stress to the strain:

$$Y=\frac{F/A}{\Delta x/x}$$

or

$$\frac{\Delta x}{x}=\frac{F}{YA}$$

Substitute for *F* to obtain:

$$\frac{\Delta x}{x}=-\frac{\kappa \in_0 V^2}{2Yx^2}$$

and

$$\Delta x = -\frac{\kappa \in_0 V^2}{2Yx} = -\frac{\kappa \in_0 V^2}{2Yd} \qquad (3)$$

provided $\Delta x << d$

The voltage across the capacitor is:

$$V = E_{\max} d = (40\,\text{kV/mm})(0.2\,\text{mm}) = 8.00\,\text{kV}$$

Substitute numerical values in equation (3) and evaluate Δx:

$$\Delta x = -\frac{3(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(8\,\text{kV})^2}{2(5\times10^6\,\text{N/m}^2)(0.2\,\text{mm})} = 8.50\times10^{-7}\,\text{m} = 8.50\times10^{-4}\,\text{mm}$$

Substitute in equation (2) to obtain:

$$C(V) = \frac{\kappa \in_0 A}{d - \frac{\kappa \in_0 V^2}{2Yd}} = \frac{\kappa \in_0 A}{d\left(1 - \frac{\kappa \in_0 V^2}{2Yd^2}\right)} = C_0\left(1 - \frac{\kappa \in_0 V^2}{2Yd^2}\right)^{-1} \approx \boxed{C_0\left(1 + \frac{\kappa \in_0 V^2}{2Yd^2}\right)}$$

provided $\Delta x << d$.

(b) Express the maximum voltage that can be applied in terms of the maximum electric field:

$$V_{\max} = E_{\max}(d - \Delta x) = (40\,\text{kV/mm})(0.2\,\text{mm} - 8.5\times10^{-4}\,\text{mm}) = \boxed{7.97\,\text{kV}}$$

(c) The fraction of the total energy of the capacitor that is mechanical stress energy is:

$$f = \frac{U_{\text{M}}}{U_{\text{M}} + U_{\text{E}}} \qquad (4)$$

Express the maximum electric field energy:

$$U_{\text{E,max}} = \tfrac{1}{2} C(V_{\max}) V_{\max}^2$$

Evaluate $C(V_{\max})$:

$$C(7.97\,\text{kV}) = (0.133A)\left[1 + (6.64\times10^{-11})(7.97\,\text{kV})^2\right]\mu\text{F/m}^2 = 0.134A\,\mu\text{F/m}^2$$

Substitute for $C(V_{\max})$ and evaluate $U_{\text{E,max}}$:

$$U_{\text{E,max}} = \tfrac{1}{2}(0.134A\,\mu\text{F/m}^2)(7.96\,\text{kV})^2 = 4.25\,\text{J/m}^2$$

The mechanical stress energy is given by:

$$U_M = \frac{1}{2}\frac{(\Delta x)^2 Y}{d}$$

Substitute numerical values and evaluate U_M:

$$U_M = \frac{1}{2}\frac{(8.5\times10^{-4}\text{ mm})^2(5\times10^6\text{ N/m}^2)}{0.2\text{ mm}} = 8.92\text{ mJ}$$

Substitute numerical values in equation (4) and evaluate f:

$$f = \frac{8.92\text{ mJ}}{8.92\text{ mJ} + 4.25\text{ J/m}^2} = \boxed{0.209\%}$$

and the fraction of the total energy that is electrostatic field energy is

$$1 - f = 1 - 0.209\% = \boxed{99.8\%}$$

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Picture the Problem Note that, due to symmetry, the electric field, wherever it exists, will be radial. We can integrate the electric flux over spherical Gaussian surfaces with radii $r < R_1$, $R_1 < r < R_2$, and $r > R_2$ to find the electric field everywhere in space. Once we know the electric field everywhere we can find the potential of the conducting sphere by using $dV = -Edr$ and integrating E in the regions $R_1 < r < R_2$ and $r > R_2$. Finally, knowing the electrostatic potential at the surface of the conducting sphere we can use $U_{\text{tot}} = \frac{1}{2}QV(R_1)$ to find the total electrostatic potential energy of the system.

(*a*) Integrate the electric flux over a spherical Gaussian surface with radius $r < R_1$ to obtain:

$$E_r(4\pi r^2) = \frac{Q_{\text{inside}}}{\epsilon_0} = 0$$

because Q_{inside} the conducting surface is zero.

Solve for $E_r(r < R_1)$ to obtain:

$$E_r(r < R_1) = \boxed{0}$$

Integrate the electric flux over a spherical Gaussian surface with radius $R_1 < r < R_2$ to obtain:

$$E_r(4\pi r^2) = \frac{Q_{\text{inside}}}{\kappa\,\epsilon_0} = \frac{Q}{\kappa\,\epsilon_0}$$

Solve for $E_r(R_1 < r < R_2)$ to obtain:

$$E_r(R_1 < r < R_2) = \frac{Q}{4\pi\,\epsilon_0\,\kappa r^2} = \boxed{\frac{kQ}{\kappa r^2}}$$

Integrate the electric flux over a spherical Gaussian surface with radius $r > R_2$ to obtain:

$$E_r(4\pi r^2) = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Solve for $E_r(r > R_2)$ to obtain:

$$E_r(r > R_2) = \frac{Q}{4\pi \epsilon_0 r^2} = \boxed{\frac{kQ}{r^2}}$$

(*b*) Express the potential at the surface of the conducting sphere in terms of the electric fields $E_r(R_1 < r < R_2)$ and $E_r(r > R_2)$:

$$V(R_1) = -\int_{\infty}^{R_1} E dr$$

$$= -kQ\int_{\infty}^{R_2} \frac{1}{r^2} dr - \frac{kQ}{\kappa}\int_{R_2}^{R_1} \frac{1}{r^2} dr$$

$$= \boxed{\frac{kQ}{\kappa}\left(\frac{R_1(\kappa - 1) + R_2}{R_1 R_2}\right)}$$

(c) Express the total electrostatic potential energy of the system in terms of $V(R_1)$ and Q:

$$U_{\text{tot}} = \tfrac{1}{2} QV(R_1)$$

$$= \tfrac{1}{2} Q\left(\frac{kQ}{\kappa}\left(\frac{R_1(\kappa - 1) + R_2}{R_1 R_2}\right)\right)$$

$$= \boxed{\frac{kQ^2}{2\kappa}\left(\frac{R_1(\kappa - 1) + R_2}{R_1 R_2}\right)}$$

Chapter 25
Electric Current and Direct-Current Circuits

Conceptual Problems

***1 •**

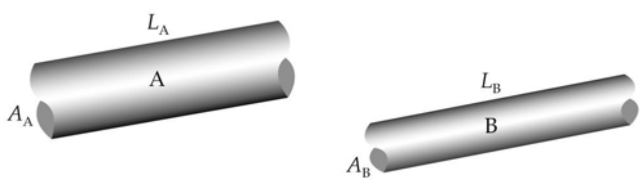
Determine the Concept When current flows, the charges are not in equilibrium. In that case, the electric field provides the force needed for the charge flow.

2 •

Determine the Concept Water, regarded as a viscous liquid flowing from a water tower through a pipe to ground is another mechanical analog of a simple circuit.

3 •

Picture the Problem The resistances of the wires are given by $R = \rho L/A$, where L is the length of the wire and A is its cross-sectional area. We can express the ratio of the resistances and use our knowledge of their lengths and diameters to find the resistance of wire A.



Express the resistance of wire A:

$$R_A = \frac{\rho L_A}{A_A}$$

where ρ is the resistivity of the wire.

Express the resistance of wire B:

$$R = \frac{\rho L_B}{A_B}$$

Divide the first of these equations by the second to obtain:

$$\frac{R_A}{R} = \frac{\frac{\rho L_A}{A_A}}{\frac{\rho L_B}{A_B}} = \frac{L_A}{L_B} \cdot \frac{A_B}{A_A}$$

or, because $L_A = L_B$,

$$R_A = \frac{A_B}{A_A} R \qquad (1)$$

Express the area of wire A in terms of its diameter:

$$A_A = \tfrac{1}{4}\pi d_A^2$$

Express the area of wire B in terms of its diameter:

$$A_B = \tfrac{1}{4}\pi d_B^2$$

Substitute in equation (1) to obtain:

$$R_A = \frac{d_B^2}{d_A^2}R$$

or, because $d_A = 2d_B$,

$$R_A = \frac{d_B^2}{(2d_B)^2}R = \tfrac{1}{4}R$$

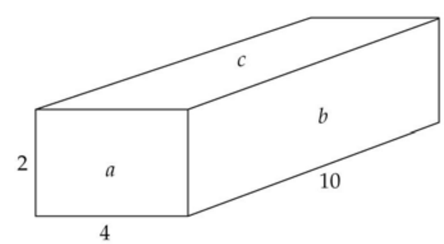
and $\boxed{(e)\text{ is correct.}}$

4 ••

Determine the Concept An emf is a source of energy that gives rise to a potential difference between two points and may result in current flow if there is a conducting path whereas a potential difference is the consequence of two points in space being at different potentials.

***5** ••

Picture the Problem The resistance of the metal bar varies directly with its length and inversely with its cross-sectional area. Hence, to minimize the resistance of the bar, we should connect to the surface for which the ratio of the length to the contact area is least.



Denoting the surfaces as *a*, *b*, and *c*, complete the table to the right:

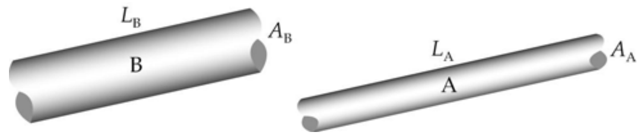
Surface	*L*	*A*	*L*/*A*
a	10	8	0.8
b	4	20	0.2
c	2	40	0.05

Because connecting to surface *c* minimizes *R*:

$\boxed{(c)\text{ is correct.}}$

6 ••

Picture the Problem The resistances of the wires are given by $R = \rho L/A$, where L is the length of the wire and A is its cross-sectional area. We can express the ratio of the resistances and use the definition of density to eliminate the cross-sectional areas of the wires in favor of the ratio of their lengths.



Express the resistance of wire A:

$$R_A = \frac{\rho L_A}{A_A}$$

where ρ is the resistivity of copper.

Express the resistance of wire B:

$$R_B = \frac{\rho L_B}{A_B}$$

Divide the first of these equations by the second to obtain:

$$\frac{R_A}{R_B} = \frac{\frac{\rho L_A}{A_A}}{\frac{\rho L_B}{A_B}} = \frac{L_A}{L_B}\frac{A_B}{A_A}$$

or, because $L_A = 2L_B$,

$$R_A = 2\frac{A_B}{A_A}R_B \qquad (1)$$

Using the definition of density, express the mass of wire A:

$$m_A = \rho' V_A = \rho' L_A A_A$$

where ρ' is the density of copper.

Express the mass of wire B

$$m_B = \rho' V_B = \rho' L_B A_B$$

Because the masses of the wires are equal:

$$\rho' L_A A_A = \rho' L_B A_B$$

or

$$\frac{A_B}{A_A} = \frac{L_A}{L_B}$$

Substitute in equation (1) to obtain:

$$R_A = 2\frac{L_A}{L_B}R_B = 2(2)R_B = 4R_B$$

and $\boxed{(b)\text{ is correct.}}$

7 •

Picture the Problem The power dissipated in the resistor is given by $P = I^2R$. We can express the power dissipated when the current is $3I$ and, assuming that the resistance does not change, express the ratio of the two rates of energy dissipation to find the power dissipated when the current is $3I$.

Express the power dissipated in the

$$P = I^2R$$

resistor when the current in it is I:

Express the power dissipated in the resistor when the current in it is $3I$:

$$P' = (3I)^2 R = 9I^2 R$$

Divide the second of these equations by the first to obtain:

$$\frac{P'}{P} = \frac{9I^2 R}{I^2 R} = 9$$

or

$P' = 9P$ and $\boxed{(d)\text{ is correct.}}$

8 •

Picture the Problem Assuming the current (which depends on the resistance) to be constant, the power dissipated in a resistor is directly proportional to the voltage drop across it.

Express the power dissipated in the resistor when the voltage drop across it is V:

$$P = \frac{V^2}{R}$$

Express the power dissipated in the resistor when the voltage drop across it is increased to $2V$:

$$P' = \frac{(2V)^2}{R} = \frac{4V^2}{R}$$

Divide the second of these equations by the first to obtain:

$$\frac{P'}{P} = \frac{\frac{4V^2}{R}}{\frac{V^2}{R}} = 4 \Rightarrow P' = 4P$$

$\boxed{(c)\text{ is correct.}}$

9 •

Determine the Concept You should decrease the resistance. Because the voltage across the resistor is constant, the heat out is given by $P = V^2/R$. Hence, decreasing the resistance will increase P.

***10** •

Picture the Problem We can find the equivalent resistance of this two-resistor combination and then apply the condition that $R_1 >> R_2$.

Express the equivalent resistance of R_1 and R_2 in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Solve for R_{eq} to obtain:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Factor R_1 from the denominator and simplify to obtain:

$$R_{eq} = \frac{R_1 R_2}{R_1\left(1 + \frac{R_2}{R_1}\right)} = \frac{R_2}{1 + \frac{R_2}{R_1}}$$

If $R_1 >> R_2$, then:

$$R_{eq} = R_{eff} \approx R_2 \text{ and } \boxed{(b) \text{ is correct}}$$

11 •

Picture the Problem We can find the equivalent resistance of this two-resistor combination and then apply the condition that $R_1 >> R_2$.

Express the equivalent resistance of R_1 and R_2 in series:

$$R_{eq} = R_1 + R_2$$

Factor R_1 to obtain:

$$R_{eq} = R_1\left(1 + \frac{R_2}{R_1}\right)$$

If $R_1 >> R_2$, then:

$$R_{eq} = R_{eff} \approx R_1 \text{ and } \boxed{(a) \text{ is correct}}$$

12 •

Picture the Problem Because the potential difference across resistors connected in parallel is the same for each resistor; we can use Ohm's law to relate the currents through the resistors to their resistances.

Using Ohm's law, express the current carried by resistor A:

$$I_A = \frac{V}{R_A} = \frac{V}{2R_B}$$

Using Ohm's law, express the current carried by resistor B:

$$I_B = \frac{V}{R_B}$$

Divide the second of these equations by the first to obtain:

$$\frac{I_B}{I_A} = \frac{\frac{V}{R_B}}{\frac{V}{2R_B}} = 2$$

and

$$I_B = 2I_A \text{ and } \boxed{(b) \text{ is correct.}}$$

***13 •**

Determine the Concept In a series circuit, because there are no alternative pathways, all resistors carry the same current. The potential difference across each resistor, keeping with Ohm's law, is given by the product of the current and the resistance and, hence, is not the same across each resistor unless the resistors are identical. $\boxed{(a)\text{ is correct.}}$

14 ••

Picture the Problem Because the potential difference across the two combinations of resistors is constant, we can use $P = V^2/R$ to relate the power delivered by the battery to the equivalent resistance of each combination of resistors.

Express the power delivered by the battery when the resistors are connected in series:

$$P_s = \frac{V^2}{R_{eq}}$$

Letting R represent the resistance of the identical resistors, express R_{eq}:

$$R_{eq} = R + R = 2R$$

Substitute to obtain:

$$P_s = \frac{V^2}{2R} \qquad (1)$$

Express the power delivered by the battery when the resistors are connected in parallel:

$$P_p = \frac{V^2}{R_{eq}}$$

Express the equivalent resistance of the identical resistors connected in parallel:

$$R_{eq} = \frac{(R)(R)}{R + R} = \tfrac{1}{2}R$$

Substitute to obtain:

$$P_p = \frac{V^2}{\frac{1}{2}R} = \frac{2V^2}{R} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{P_p}{P_s} = \frac{\frac{2V^2}{R}}{\frac{V^2}{2R}} = 4$$

Solve for and evaluate P_p:

$$P_p = 4P_s = 4(20\text{ W}) = 80\text{ W}$$

and $\boxed{(e)\text{ is correct.}}$

15 •

Determine the Concept While Kirchhoff's loop rule is a statement about potential differences around a closed loop in a circuit, recall that electric potential at a point in space is the work required to bring a charged object from infinity to the given point. Hence, the loop rule is actually a statement that energy is conserved around any closed path in an electric circuit. $\boxed{(b) \text{ is correct.}}$

16 •

Determine the Concept An ideal voltmeter would have infinite resistance. A voltmeter consists of a galvanometer movement connected in series with a large resistance. The large resistor accomplishes two purposes; 1) it protects the galvanometer movement by limiting the current drawn by it, and 2) minimizes the loading of the circuit by the voltmeter by placing a large resistance in parallel with the circuit element across which the potential difference is being measured. $\boxed{(a) \text{ is correct.}}$

***17** •

Determine the Concept An ideal ammeter would have zero resistance. An ammeter consists of a very small resistance in parallel with a galvanometer movement. The small resistance accomplishes two purposes: 1) It protects the galvanometer movement by shunting most of the current in the circuit around the galvanometer movement, and 2) It minimizes the loading of the circuit by the ammeter by minimizing the resistance of the ammeter. $\boxed{(b) \text{ is correct.}}$

18 •

Determine the Concept An ideal voltage source would have zero internal resistance. The terminal potential difference of a voltage source is given by $V = \mathcal{E} - Ir,$ where ε is the emf of the source, I is the current drawn from the source, and r is the internal resistance of the source. $\boxed{(b) \text{ is correct.}}$

19 •

Determine the Concept If we apply Kirchhoff's loop rule with the switch closed, we obtain $\varepsilon - IR - V_C = 0$. Immediately after the switch is closed, $I = 0$ and we have $\varepsilon = V_C$.
$\boxed{(b) \text{ is correct.}}$

20 ••

Determine the Concept The energy stored in the fully charged capacitor is $U = \frac{1}{2}C\mathcal{E}^2$.
During the charging process, a total charge $Q_f = \varepsilon C$ flows through the battery. The battery therefore does work $W = Q_f\varepsilon = C\varepsilon^2$. The energy dissipated in the resistor is the difference between W and U. $\boxed{(b) \text{ is correct.}}$

***21** ••

Determine the Concept Applying Kirchhoff's loop rule to the circuit, we obtain $\mathcal{E} - V_R - V_C = 0$, where V_R is the voltage drop across the resistor. Applying Ohm's law to the resistor, we obtain $V_R = IR$. Because I decreases as the capacitor is charged, V_R decreases with time. $\boxed{(e) \text{ is correct.}}$

22 ••

Picture the Problem We can express the variation of charge on the discharging capacitor as a function of time to find the time T it takes for the charge on the capacitor to drop to half its initial value. We can also express the energy remaining in the electric field of the discharging capacitor as a function of time and find the time t' for the energy to drop to half its initial value in terms of T.

Express the dependence of the charge stored on a capacitor on time:

$$Q(t) = Q_0 e^{-t/\tau}$$

where $\tau = RC$.

For $Q(t) = ½\, Q_0$:

$$\tfrac{1}{2} Q_0 = Q_0 e^{-T/\tau}$$

or

$$\tfrac{1}{2} = e^{-T/\tau}$$

Take the natural logarithm of both sides of the equation and solve for T to obtain:

$$T = \tau \ln 2$$

Express the dependence of the energy stored in a capacitor on the potential difference V_C across its terminals:

$$U(t) = \tfrac{1}{2} C V_C^2$$

Express the potential difference across a discharging capacitor as a function of time:

$$V_C = V_0 e^{-t/RC}$$

Substitute to obtain:

$$U(t) = \tfrac{1}{2} C\left(V_0 e^{-t/RC}\right)^2 = \tfrac{1}{2} C V_0^2 e^{-2t/RC} = U_0 e^{-2t/RC}$$

For $U(t) = ½\, U_0$:

$$\tfrac{1}{2} U_0 = U_0 e^{-2t'/RC}$$

or

$$\tfrac{1}{2} = e^{-2t'/RC}$$

Take the natural logarithm of both sides of the equation and solve for t' to obtain:

$$t' = \tfrac{1}{2}\tau \ln 2 = \boxed{\tfrac{1}{2}T}$$

23 •

Determine the Concept A small resistance because $P = \varepsilon^2/R$.

***24** •

Determine the Concept The potential difference across an external resistor of resistance R is given by $\dfrac{R}{r+R}V$, where r is the internal resistance and V the voltage supplied by the source. The higher R is, the higher the voltage drop across R. Put differently, the higher the resistance a voltage source sees, the less its own resistance will change the circuit.

25 •

Determine the Concept Yes. Kirchhoff's rules are statements of the conservation of energy and charge and hence apply to all circuits.

26 ••

Determine the Concept All of the current provided by the battery passes through R_1, whereas only half this current passes through R_2 and R_3. Because $P = I^2R$, the power dissipated in R_1 will be four times that dissipated in R_2 and R_3. $\boxed{(c)\text{ is correct.}}$

Estimation and Approximation

27 ••

Picture the Problem We can use Ohm's law and the definition of resistivity to find the maximum voltage that can be applied across 40 m of the 16-gauge copper wire. In part (*b*) we can find the electric field in the wire using $E = V/L$. In part (*c*) we can use $P = I^2R$ to find the power dissipated in the wire when it carries 6 A.

(*a*) Use Ohm's law to relate the potential difference across the wire to its maximum current and its resistance:

$$V_{\max} = I_{\max}R$$

Use the definition of resistivity to relate the resistance of the wire to its length and cross-sectional area:

$$R = \rho\frac{L}{A}$$

Substitute to obtain:

$$V_{\max} = I_{\max}\rho\frac{L}{A}$$

Substitute numerical values (see Tables 25-1 and 25-2) for the resistivity of copper and the cross-sectional area of 16-gauge wire:

$$V_{max} = (6\,\text{A})(1.7\times10^{-8}\,\Omega\cdot\text{m}) \times \left(\frac{40\,\text{m}}{1.309\,\text{mm}^2}\right) = \boxed{3.12\,\text{V}}$$

(*b*) Relate the electric in the wire to the potential difference between its ends and the length of the wire:

$$E = \frac{V}{L} = \frac{3.12\,\text{V}}{40\,\text{m}} = \boxed{78.0\,\text{mV/m}}$$

(*c*) Relate the power dissipated in the wire to the current in and the resistance of the wire:

$$P = I^2 R$$

Substitute for R to obtain:

$$P = I^2 \rho \frac{L}{A}$$

Substitute numerical values and evaluate P:

$$P = (6\,\text{A})^2(1.7\times10^{-8}\,\Omega\cdot\text{m})\left(\frac{40\,\text{m}}{1.309\,\text{mm}^2}\right) = \boxed{18.7\,\text{W}}$$

28 ••

Picture the Problem We can use the definition of resistivity to find the resistance of the jumper cable. In part (*b*), the application of Ohm's law will yield the potential difference across the jumper cable when it is starting a car, and, in part (*c*), we can use the expression for the power dissipated in a conductor to find the power dissipation in the jumper cable.

(*a*) Noting that a jumper cable has two leads, express the resistance of the cable in terms of the wire's resistivity and the cable's length, and cross-sectional area:

$$R = \rho \frac{L}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper) and evaluate R:

$$R = (1.7\times10^{-8}\,\Omega\cdot\text{m})\frac{6\,\text{m}}{10\,\text{mm}^2} = \boxed{0.0102\,\Omega}$$

(*b*) Apply Ohm's law to the cable to obtain:

$$V = IR = (90\,\text{A})(0.0102\,\Omega) = \boxed{0.918\,\text{V}}$$

(*c*) Use the expression for the power dissipated in a conductor to obtain:

$$P = IV = (90\,\text{A})(0.918\,\text{V}) = \boxed{82.6\,\text{W}}$$

29 ••

Picture the Problem We can combine the expression for the rate at which energy is delivered to the water to vaporize it ($P = \varepsilon^2/R$) and the expression for the resistance of a conductor ($R = \rho L/A$) to obtain an expression for the required length L of wire.

Use an expression for the power dissipated in a resistor to relate the required resistance to rate at which energy is delivered to generate the steam:

$$R = \frac{\varepsilon^2}{P}$$

Relate the resistance of the wire to its length, cross-sectional area, and resistivity:

$$R = \rho\frac{L}{A}$$

Equate these two expressions and solve for L to obtain:

$$L = \frac{\varepsilon^2 A}{\rho P}$$

Express the power required to generate the steam in terms of the rate of energy delivery:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta(mL_\text{v})}{\Delta t} = L_\text{v}\frac{\Delta m}{\Delta t}$$

Substitute to obtain:

$$L = \frac{\varepsilon^2 A}{\rho L_\text{v}\dfrac{\Delta m}{\Delta t}}$$

Substitute numerical values (see Table 25-1 for the resistivity of Nichrome and Table 18-2 for the latent heat of vaporization of water) and evaluate L:

$$L = \frac{(120\,\text{V})^2\,\dfrac{\pi}{4}(1.80\,\text{mm})^2}{(10^{-6}\,\Omega\cdot\text{m})(2257\,\text{kJ/kg})(8\,\text{g/s})} = \boxed{2.03\,\text{m}}$$

***30** ••

Picture the Problem We can find the annual savings by taking into account the costs of the two types of bulbs, the rate at which they consume energy and the cost of that energy, and their expected lifetimes.

Express the yearly savings:

$$\Delta\$ = \text{Cost}_\text{incandescent} - \text{Cost}_\text{fluorescent} \qquad (1)$$

Express the annual cost with the incandescent bulbs:

$$\text{Cost}_{\text{incandescent}} = \text{Cost}_{\text{bulbs}} + \text{Cost}_{\text{energy}}$$

Express and evaluate the annual cost of the incandescent bulbs:

$$\begin{aligned}\text{Cost}_{\text{bulbs}} &= \text{number of bulbs in use} \times \text{annual consumption of bulbs} \times \text{cost per bulb}\\ &= (6)\left(\frac{365.24\,\text{d} \times \dfrac{24\,\text{h}}{\text{d}}}{1200\,\text{h}}\right)(\$1.50) = \$65.74\end{aligned}$$

Find the cost of operating the incandescent bulbs for one year:

$$\begin{aligned}\text{Cost}_{\text{energy}} &= \text{energy consumed} \times \text{cost per unit of energy}\\ &= 6(75\,\text{W})(365.25\,\text{d})(24\,\text{h/d})(\$0.115/\text{kW}\cdot\text{h})\\ &= \$453.64\end{aligned}$$

Express the annual cost with the fluorescent bulbs:

$$\text{Cost}_{\text{fluorescent}} = \text{Cost}_{\text{bulbs}} + \text{Cost}_{\text{energy}}$$

Express and evaluate the annual cost of the fluorescent bulbs:

$$\begin{aligned}\text{Cost}_{\text{bulbs}} &= \text{number of bulbs in use} \times \text{annual consumption of bulbs} \times \text{cost per bulb}\\ &= (6)\left(\frac{365.24\,\text{d} \times \dfrac{24\,\text{h}}{\text{d}}}{8000\,\text{h}}\right)(\$6) = \$39.45\end{aligned}$$

Find the cost of operating the fluorescent bulbs for one year:

$$\begin{aligned}\text{Cost}_{\text{energy}} &= \text{energy consumed} \times \text{cost per unit of energy}\\ &= 6(20\,\text{W})\left(365.24\,\text{d} \times \frac{24\,\text{h}}{\text{d}}\right)(\$0.115/\text{kW}\cdot\text{h})\\ &= \$120.97\end{aligned}$$

Substitute in equation (1) and evaluate the cost savings $\Delta\$$:

$$\begin{aligned}\Delta\$ &= \text{Cost}_{\text{incandescent}} - \text{Cost}_{\text{fluorescent}} = (\$65.74 + \$453.64) - (\$39.45 + \$120.97)\\ &= \boxed{\$358.96}\end{aligned}$$

31 ••

Picture the Problem We can use an expression for the power dissipated in a resistor to relate the Joule heating in the wire to its resistance and the definition of resistivity to relate the resistance to the length and cross-sectional area of the wire.

Express the power the wires must dissipate in terms of the current they carry and their resistance:

$$P = I^2 R$$

Divide both sides of the equation by L to express the power dissipation per unit length:

$$\frac{P}{L} = \frac{I^2 R}{L}$$

Using the definition of resistivity, relate the resistance of the wire to its resistivity, length and cross-sectional area:

$$R = \rho\frac{L}{A} = \rho\frac{L}{\frac{\pi}{4}d^2} = \frac{4\rho L}{\pi d^2}$$

Substitute to obtain:

$$\frac{P}{L} = \frac{4\rho I^2}{\pi d^2}$$

Solve for d to obtain:

$$d = 2I\sqrt{\frac{\rho}{\pi(P/L)}}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper wire) and evaluate d:

$$d = 2(20\,\text{A})\sqrt{\frac{1.7\times10^{-8}\,\Omega\cdot\text{m}}{\pi(2\,\text{W/m})}} = \boxed{2.08\,\text{mm}}$$

***32 ••**

Picture the Problem Let r be the internal resistance of each battery and use Ohm's law to express the current in laser diode as a function of the potential difference across r. We can find the power of the laser diode from the product of the potential difference across the internal resistance of the batteries and the current delivered by them I and the time-to-discharge from the combined capacities of the two batteries and I.

(*a*) Use Ohm's law to express the current in the laser diode:

$$I = \frac{V_{\text{internal resistance}}}{2r}$$

The potential difference across the internal resistance is:

$$V_{\text{internal resistance}} = \mathcal{E} - 2.3\,\text{V}$$

Substitute to obtain:

$$I=\frac{\mathcal{E}-2.3\,\text{V}}{2r}$$

Assuming that r = 125 Ω:

$$I=\frac{2(1.55)-2.3\,\text{V}}{2(125\,\Omega)}=\boxed{3.20\,\text{mA}}$$

(*b*) The power delivered by the batteries is given by:

$$P=IV=(3.2\,\text{mA})(2.3\,\text{V})=7.36\,\text{mW}$$

The power of the laser is half this value:

$$P_{\text{laser}}=\tfrac{1}{2}P=\tfrac{1}{2}(7.36\,\text{mW})=\boxed{3.68\,\text{mW}}$$

Express the ratio of P_{laser} to P_{quoted}:

$$\frac{P_{\text{laser}}}{P_{\text{quoted}}}=\frac{3.68\,\text{mW}}{3\,\text{mW}}=1.23$$

or

$$P_{\text{laser}}=\boxed{123\%P_{\text{quoted}}}$$

(*c*) Express the time-to-discharge:

$$\Delta t=\frac{\text{Capacity}}{I}$$

Because each battery has a capacity of 20 mA·h, the series combination has a capacity of 40 mA·h and:

$$\Delta t=\frac{40\,\text{mA}\cdot\text{h}}{3.20\,\text{mA}}=\boxed{12.5\,\text{h}}$$

Current and the Motion of Charges

33 •

Picture the Problem We can relate the drift velocity of the electrons to the current density using $I=nev_{\text{d}}A$. We can find the number density of charge carriers n using $n=\rho N_{\text{A}}/M$, where ρ is the mass density, N_{A} Avogadro's number, and M the molar mass. We can find the cross-sectional area of 10-gauge wire in Table 25-2.

Use the relation between current and drift velocity to relate I and n:

$$I=nev_{\text{d}}A$$

Solve for v_{d}:

$$v_{\text{d}}=\frac{I}{neA}$$

The number density of charge carriers n is related to the mass density ρ, Avogadro's number N_A, and the molar mass M:

$$n = \frac{\rho N_A}{M}$$

For copper, $\rho = 8.93\ \text{g/cm}^3$ and $M = 63.5$ g/mol. Substitute and evaluate n:

$$n = \frac{(8.93\,\text{g/cm}^3)(6.02\times10^{23}\ \text{atoms/mol})}{63.5\,\text{g/mol}} = 8.47\times10^{28}\ \text{atoms/m}^3$$

Using Table 25-2, find the cross-sectional area A of 10-gauge wire:

$$A = 5.261\,\text{mm}^2$$

Substitute and evaluate v_d:

$$v_d = \frac{20\,\text{A}}{(8.47\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(5.261\,\text{mm}^2)} = \boxed{0.281\,\text{mm/s}}$$

34 •

Picture the Problem Note that, while the positive and negative charges flow in opposite directions, the total current is their sum.

Express the total current I in the tube as the sum of the electron current and the ion current:

$$I = I_{\text{electron}} + I_{\text{ion}}$$

The electron current is the product of the number of electrons through the cross-sectional area each second and the charge of each electron:

$$I_{\text{electron}} = ne = (2\times10^{18}\ \text{electrons/s})\times(1.60\times10^{-19}\ \text{C/electron}) = 0.320\,\text{A}$$

Proceed in the same manner to find the ion current:

$$I_{\text{ion}} = n_{\text{ion}}q_{\text{ion}} = (0.5\times10^{18}\ \text{electrons/s})\times(1.60\times10^{-19}\ \text{C/electron}) = 0.0800\,\text{A}$$

Substitute to obtain:

$$I = 0.320\,\text{A} + 0.0800\,\text{A} = \boxed{0.400\,\text{A}}$$

35 •

Picture the Problem We can solve $K = \frac{1}{2}m_e v^2$ for the velocity of an electron in the beam and use the relationship between current and drift velocity to find the beam current.

(*a*) Express the kinetic energy of the beam:

$$K = \tfrac{1}{2}m_e v^2$$

Solve for *v*:

$$v = \sqrt{\frac{2K}{m_e}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2(10\,\text{keV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{5.93\times10^{7}\ \text{m/s}}$$

(*b*) Use the relationship between current and drift velocity (here the velocity of an electron in the beam) to obtain:

$$I = nev_d A$$

Express the cross-sectional area of the beam in terms of its diameter *D*:

$$A = \tfrac{1}{4}\pi D^2$$

Substitute to obtain:

$$I = \tfrac{1}{4}\pi nev_d D^2$$

Substitute numerical values and evaluate *I*:

Substitute numerical values and evaluate *I*:

$$I = \tfrac{1}{4}\pi(5\times10^{6}\ \text{cm}^{-3})(1.60\times10^{-19}\ \text{C})(5.93\times10^{7}\ \text{m/s})(10^{-3}\ \text{m})^2 = \boxed{37.3\ \mu\text{A}}$$

36 ••

Picture the Problem We can use the definition of current, the definition of charge density, and the relationship between period and frequency to derive an expression for the current as a function of a, λ, and ω.

Use the definition of current to relate the charge ΔQ associated with a segment of the ring to the time Δt it takes the segment to pass a given point:

$$I = \frac{\Delta Q}{\Delta t}$$

Because each segment carries a charge ΔQ and the time for one

$$I = \frac{\Delta Q}{T} = \Delta Q f \qquad (1)$$

revolution is T:

Use the definition of the charge density λ to relate the charge ΔQ to the radius a of the ring:

$$\lambda = \frac{\Delta Q}{2\pi a}$$

Solve for ΔQ to obtain:

$$\Delta Q = 2\pi a\lambda$$

Substitute in equation (1) to obtain:

$$I = 2\pi a\lambda f$$

Because $\omega = 2\pi f$ we have:

$$I = \boxed{a\lambda\omega}$$

***37 ••**

Picture the Problem The current will be the same in the two wires and we can relate the drift velocity of the electrons in each wire to their current densities and the cross-sectional areas of the wires. We can find the number density of charge carriers n using $n = \rho N_A / M$, where ρ is the mass density, N_A Avogadro's number, and M the molar mass. We can find the cross-sectional area of 10- and 14-gauge wires in Table 25-2.

Relate the current density to the drift velocity of the electrons in the 10-gauge wire:

$$\frac{I_{10\,\text{gauge}}}{A_{10\,\text{gauge}}} = nev_d$$

Solve for v_d:

$$v_{d,10} = \frac{I_{10\,\text{gauge}}}{neA_{10\,\text{gauge}}}$$

The number density of charge carriers n is related to the mass density ρ, Avogadro's number N_A, and the molar mass M:

$$n = \frac{\rho N_A}{M}$$

For copper, $\rho = 8.93\ \text{g/cm}^3$ and $M = 63.5$ g/mol. Substitute and evaluate n:

$$n = \frac{(8.93\,\text{g/cm}^3)(6.02\times10^{23}\ \text{atoms/mol})}{63.5\,\text{g/mol}} = 8.47\times10^{28}\ \text{atoms/m}^3$$

Use Table 25-2 to find the cross-sectional area of 10-gauge wire:

$$A_{10} = 5.261\,\text{mm}^2$$

Substitute numerical values and evaluate $v_{d.10}$:

$$v_{\mathrm{d,10}} = \frac{15\,\mathrm{A}}{\left(8.47\times10^{28}\,\mathrm{m}^{-3}\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(5.261\,\mathrm{mm}^2\right)} = \boxed{0.210\,\mathrm{mm/s}}$$

Express the continuity of the current in the two wires:

$$I_{10\,\mathrm{gauge}} = I_{14\,\mathrm{gauge}}$$

or

$$nev_{\mathrm{d,10}}A_{10\,\mathrm{gauge}} = nev_{\mathrm{d,14}}A_{14\,\mathrm{gauge}}$$

Solve for $v_{\mathrm{d,14}}$ to obtain:

$$v_{\mathrm{d,14}} = v_{\mathrm{d,10}}\frac{A_{10\,\mathrm{gauge}}}{A_{14\,\mathrm{gauge}}}$$

Use Table 25-2 to find the cross-sectional area of 14-gauge wire:

$$A_{14} = 2.081\,\mathrm{mm}^2$$

Substitute numerical values and evaluate $v_{\mathrm{d,14}}$:

$$v_{\mathrm{d,14}} = \left(0.210\,\mathrm{mm/s}\right)\frac{5.261\,\mathrm{mm}^2}{2.081\,\mathrm{mm}^2} = \boxed{0.531\,\mathrm{mm/s}}$$

38 ••

Picture the Problem We can use $I = neAv$ to relate the number n of protons per unit volume in the beam to current I. We can find the speed of the particles in the beam from their kinetic energy. In part (*b*) we can express the number of protons N striking the target per unit time as the product of the number of protons per unit volume n in the beam and the volume of the cylinder containing those protons that will strike the target in an elapsed time Δt and solve for N. Finally, we can use the definition of current to express the charge arriving at the target as a function of time.

(*a*) Use the relation between current and drift velocity to relate I and n:

$$I = neAv$$

Solve for n to obtain:

$$n = \frac{I}{eAv}$$

Express the kinetic energy of the protons and solve for v:

$$K = \tfrac{1}{2}m_{\mathrm{p}}v^2 \Rightarrow v = \sqrt{\frac{2K}{m_{\mathrm{p}}}}$$

Relate the cross-sectional area A of the beam to its diameter D:

$$A = \tfrac{1}{4}\pi D^2$$

Substitute for v and A to obtain:

$$n=\frac{I}{\frac{1}{4}\pi eD^2\sqrt{\frac{2K}{m_p}}}=\frac{4I}{\pi eD^2}\sqrt{\frac{m_p}{2K}}$$

Substitute numerical values and evaluate n:

$$n=\frac{4(1\,\text{mA})}{\pi(1.60\times10^{-19}\,\text{C})(2\,\text{mm})^2}\sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{2(20\,\text{MeV})(1.60\times10^{-19}\,\text{J/eV})}}=\boxed{3.21\times10^{13}\,\text{mm}^{-3}}$$

(*b*) Express the number of protons N striking the target per unit time as the product of the number n of protons per unit volume in the beam and the volume of the cylinder containing those protons that will strike the target in an elapsed time Δt and solve for N:

$$\frac{N}{\Delta t}=n(vA)\Rightarrow N=nvA\Delta t$$

Substitute for v and A to obtain:

$$N=\frac{1}{4}\pi D^2 n\Delta t\sqrt{\frac{2K}{m_p}}$$

Substitute numerical values and evaluate N:

$$N=\frac{1}{4}\pi(2\,\text{mm})^2(3.21\times10^{13}\,\text{m}^{-3})(1\,\text{min})\sqrt{\frac{2(20\,\text{MeV})(1.60\times10^{-19}\,\text{J/eV})}{1.67\times10^{-27}\,\text{kg}}}=\boxed{3.75\times10^{17}}$$

(*c*) Using the definition of current, express the charge arriving at the target as a function of time:

$$Q=It=(1\,\text{mA})t=\boxed{(1\,\text{mC/s})t}$$

***39 ••**

Picture the Problem We can relate the number of protons per meter N to the number n of free charge-carrying particles per unit volume in a beam of cross-sectional area A and then use the relation between current and drift velocity to relate n to I.

(*a*) Express the number of protons per meter N in terms of the number n of free charge-carrying particles per unit volume in a beam of cross-

$$N=nA \quad (1)$$

sectional area A:

Use the relation between current and drift velocity to relate I and n:

$$I = enAv$$

Solve for n to obtain:

$$n = \frac{I}{eAv}$$

Substitute to obtain:

$$N = \frac{IA}{eAv} = \frac{I}{ev}$$

Substitute numerical values and evaluate N:

$$N = \frac{5\,\text{mA}}{(1.60\times10^{-19}\,\text{C})(3\times10^{8}\,\text{m/s})} = \boxed{1.04\times10^{8}\,\text{m}^{-1}}$$

(b) From equation (1) we have:

$$n = \frac{N}{A} = \frac{1.04\times10^{8}\,\text{m}^{-1}}{10^{-6}\,\text{m}^{2}} = \boxed{1.04\times10^{14}\,\text{m}^{-3}}$$

Resistance and Ohm's Law

40 •

Picture the Problem We can use Ohm's law to find the potential difference between the ends of the wire and $V = EL$ to find the magnitude of the electric field in the wire.

(a) Apply Ohm's law to obtain:

$$V = RI = (0.2\,\Omega)(5\,\text{A}) = \boxed{1.00\,\text{V}}$$

(b) Relate the electric field to the potential difference across the wire and the length of the wire:

$$E = \frac{V}{L} = \frac{1\,\text{V}}{10\,\text{m}} = \boxed{0.100\,\text{V/m}}$$

41 •

Picture the Problem We can apply Ohm's law to both parts of this problem, solving first for R and then for I.

(a) Apply Ohm's law to obtain:

$$R = \frac{V}{I} = \frac{100\,\text{V}}{3\,\text{A}} = \boxed{33.3\,\Omega}$$

(*b*) Apply Ohm's law a second time to obtain:

$$I=\frac{V}{R}=\frac{25\,\text{V}}{33.3\,\Omega}=\boxed{0.751\,\text{A}}$$

42 •

Picture the Problem We can use $R=\rho L/A$ to find the resistance of the block and Ohm's law to find the current in it for the given potential difference across its length.

(*a*) Relate the resistance of the block to its resistivity ρ, cross-sectional area A, and length L:

$$R=\rho\frac{L}{A}$$

Substitute numerical values (see Table 2625-1 for the resistivity of carbon) and evaluate R:

$$R=\left(3500\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{3\,\text{cm}}{(0.5\,\text{cm})^2}$$

$$=\boxed{42.0\,\text{m}\Omega}$$

(*b*) Apply Ohm's law to obtain:

$$I=\frac{V}{R}=\frac{8.4\,\text{V}}{42.0\,\text{m}\Omega}=\boxed{200\,\text{A}}$$

43 •

Picture the Problem We can solve the relation $R=\rho L/A$ for L to find the length of the carbon rod that will have a resistance of 10 Ω.

Relate the resistance of the rod to its resistivity ρ, cross-sectional area A, and length L:

$$R=\rho\frac{L}{A}$$

Solve for L to obtain:

$$L=\frac{AR}{\rho}=\frac{\pi r^2R}{\rho}$$

Substitute numerical values (see Table 2625-1 for the resistivity of carbon) and evaluate L:

$$L=\frac{\pi(0.1\,\text{mm})^2(10\,\Omega)}{3500\times10^{-8}\,\Omega\cdot\text{m}}=\boxed{8.98\,\text{mm}}$$

***44** •

Picture the Problem We can use $R=\rho L/A$ to find the resistance of the track.

(*a*) Relate the resistance of the track to its resistivity ρ, cross-sectional area A, and length L:

$$R=\rho\frac{L}{A}$$

Substitute numerical values and evaluate R:

$$R = \left(10^{-7}\,\Omega\cdot\text{m}\right)\frac{10\,\text{km}}{55\,\text{cm}^2} = \boxed{0.182\,\Omega}$$

45 •

Picture the Problem We can use Ohm's law in conjunction with $R = \rho L/A$ to find the potential difference across one wire of the extension cord.

Using Ohm's law, express the potential difference across one wire of the extension cord:

$$V = IR$$

Relate the resistance of the wire to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Substitute to obtain:

$$V = \rho\frac{LI}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 16-gauge wire) and evaluate V:

$$V = \left(1.7\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{(30\,\text{m})(5\,\text{A})}{1.309\,\text{mm}^2} = \boxed{1.95\,\text{V}}$$

46 •

Picture the Problem We can use $R = \rho L/A$ to find the length of a 14-gauge copper wire that has a resistance of 2 Ω.

(*a*) Relate the resistance of the wire to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Solve for L to obtain:

$$L = \frac{RA}{\rho}$$

Substitute numerical values (see Table 2625-1 for the resistivity of copper and Table 2625-2 for the cross-sectional area of 14-gauge wire) and evaluate L:

$$L = \frac{(2\,\Omega)\left(2.081\,\text{mm}^2\right)}{1.7\times10^{-8}\,\Omega\cdot\text{m}} = \boxed{245\,\text{m}}$$

47 ••

Picture the Problem We can use $R = \rho L/A$ to express the resistances of the glass cylinder and the copper wire. Expressing their ratio will eliminate the common cross-sectional areas and leave us with an expression we can solve for the length of the copper wire.

Relate the resistance of the glass cylinder to its resistivity, cross-sectional area, and length:

$$R_{glass} = \rho_{glass}\frac{L_{glass}}{A_{glass}}$$

Relate the resistance of the copper wire to its resistivity, cross-sectional area, and length:

$$R_{Cu} = \rho_{Cu}\frac{L_{Cu}}{A_{Cu}}$$

Divide the second of these equations by the first to obtain:

$$\frac{R_{Cu}}{R_{glass}} = \frac{\rho_{Cu}\frac{L_{Cu}}{A_{Cu}}}{\rho_{glass}\frac{L_{glass}}{A_{glass}}} = \frac{\rho_{Cu}}{\rho_{glass}}\cdot\frac{L_{Cu}}{L_{glass}}$$

or, because $A_{glass} = A_{Cu}$ and $R_{Cu} = R_{glass}$,

$$1 = \frac{\rho_{Cu}}{\rho_{glass}}\cdot\frac{L_{Cu}}{L_{glass}}$$

Solve for L_{Cu} to obtain:

$$L_{Cu} = \frac{\rho_{glass}}{\rho_{Cu}}L_{glass}$$

Substitute numerical values (see Table 25-1 for the resistivities of glass and copper) and evaluate L_{Cu}:

$$L_{Cu} = \frac{10^{12}\ \Omega\cdot\text{m}}{1.7\times10^{-8}\ \Omega\cdot\text{m}}(1\,\text{cm})$$

$$= 5.88\times10^{17}\ \text{m}\times\frac{1\,\text{c}\cdot\text{y}}{9.461\times10^{15}\ \text{m}}$$

$$= \boxed{62.2\,\text{c}\cdot\text{y}}$$

48 ••

Picture the Problem We can use Ohm's law to relate the potential differences across the two wires to their resistances and $R = \rho L/A$ to relate their resistances to their lengths, resistivities, and cross-sectional areas. Once we've found the potential differences across each wire, we can use $E = V/L$ to find the electric field in each wire.

(*b*) Apply Ohm's law to express the potential drop across each wire:

$$V_{Cu} = IR_{Cu}$$
and
$$V_{Fe} = IR_{Fe}$$

Relate the resistances of the wires to their resistivity, cross-sectional area, and length:

$$R_{Cu} = \rho_{Cu}\frac{L_{Cu}}{A_{Cu}}$$
and
$$R_{Fe} = \rho_{Fe}\frac{L_{Fe}}{A_{Fe}}$$

Substitute to obtain:

$$V_{Cu} = \frac{\rho_{Cu}L_{Cu}}{A_{Cu}}I$$
and
$$V_{Fe} = \frac{\rho_{Fe}L_{Fe}}{A_{Fe}}I$$

Substitute numerical values (see Table 2625-1 for the resistivities of copper and iron) and evaluate the potential differences:

$$V_{Cu} = \frac{(1.7\times10^{-8}\,\Omega\cdot\text{m})(80\,\text{m})}{\frac{1}{4}\pi(1\,\text{mm})^2}(2\,\text{A})$$
$$= \boxed{3.46\,\text{V}}$$
and
$$V_{Fe} = \frac{(10\times10^{-8}\,\Omega\cdot\text{m})(49\,\text{m})}{\frac{1}{4}\pi(1\,\text{mm})^2}(2\,\text{A})$$
$$= \boxed{12.5\,\text{V}}$$

(*a*) Express the electric field in each conductor in terms of its length and the potential difference across it:

$$E_{Cu} = \frac{V_{Cu}}{L_{Cu}}$$
and
$$E_{Fe} = \frac{V_{Fe}}{L_{Fe}}$$

Substitute numerical values and evaluate the electric fields:

$$E_{Cu} = \frac{3.46\,\text{V}}{80\,\text{m}} = \boxed{43.3\,\text{mV/m}}$$

and

$$E_{\text{Fe}} = \frac{12.5\,\text{V}}{49\,\text{m}} = \boxed{255\,\text{mV/m}}$$

***49 ••**

Picture the Problem We can use Ohm's law to express the ratio of the potential differences across the two wires and $R = \rho L/A$ to relate the resistances of the wires to their lengths, resistivities, and cross-sectional areas. Once we've found the ratio of the potential differences across the wires, we can use $E = V/L$ to decide which wire has the greater electric field.

(*a*) Apply Ohm's law to express the potential drop across each wire:

$$V_{\text{Cu}} = IR_{\text{Cu}}$$

and

$$V_{\text{Fe}} = IR_{\text{Fe}}$$

Divide the first of these equations by the second to express the ratio of the potential drops across the wires:

$$\frac{V_{\text{Cu}}}{V_{\text{Fe}}} = \frac{IR_{\text{Cu}}}{IR_{\text{Fe}}} = \frac{R_{\text{Cu}}}{R_{\text{Fe}}} \quad (1)$$

Relate the resistances of the wires to their resistivity, cross-sectional area, and length:

$$R_{\text{Cu}} = \rho_{\text{Cu}}\frac{L_{\text{Cu}}}{A_{\text{Cu}}}$$

and

$$R_{\text{Fe}} = \rho_{\text{Fe}}\frac{L_{\text{Fe}}}{A_{\text{Fe}}}$$

Divide the first of these equations by the second to express the ratio of the resistances of the wires:

$$\frac{R_{\text{Cu}}}{R_{\text{Fe}}} = \frac{\rho_{\text{Cu}}\dfrac{L_{\text{Cu}}}{A_{\text{Cu}}}}{\rho_{\text{Fe}}\dfrac{L_{\text{Fe}}}{A_{\text{Fe}}}} = \frac{\rho_{\text{Cu}}}{\rho_{\text{Fe}}}$$

because $L_{\text{Cu}} = L_{\text{Fe}}$ and $A_{\text{Cu}} = A_{\text{Fe}}$.

Substitute in equation (1) to obtain:

$$\frac{V_{\text{Cu}}}{V_{\text{Fe}}} = \frac{\rho_{\text{Cu}}}{\rho_{\text{Fe}}}$$

Substitute numerical values (see Table 2625-1 for the resistivities of copper and iron) and evaluate the ratio of the potential differences:

$$\frac{V_{\text{Cu}}}{V_{\text{Fe}}} = \frac{1.7\times10^{-8}\,\Omega\cdot\text{m}}{10\times10^{-8}\,\Omega\cdot\text{m}} = \boxed{0.170}$$

(*b*) Express the electric field in each conductor in terms of its length and the potential difference across it:

$$E_{\text{Cu}} = \frac{V_{\text{Cu}}}{L_{\text{Cu}}} \text{ and } E_{\text{Fe}} = \frac{V_{\text{Fe}}}{L_{\text{Fe}}}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_{\text{Cu}}}{E_{\text{Fe}}} = \frac{\dfrac{V_{\text{Cu}}}{L_{\text{Cu}}}}{\dfrac{V_{\text{Cu}}}{L_{\text{Cu}}}} = \frac{V_{\text{Cu}}}{V_{\text{Fe}}} = 0.170$$

or

$$E_{\text{Fe}} = \frac{E_{\text{Cu}}}{0.17} = 5.88E_{\text{Cu}}$$

Because $E_{\text{Fe}} = 5.88E_{\text{Cu}}$:

$$\boxed{E \text{ is greater in the iron wire.}}$$

50 ••

Picture the Problem We can use $R = \rho L/A$ to relate the resistance of the salt solution to its length, resistivity, and cross-sectional area. To find the resistance of the filled tube when it is uniformly stretched to a length of 2 m, we can use the fact that the volume of the solution is unchanged to relate the new cross-sectional area of the solution to its original cross-sectional area.

(*a*) Relate the resistance of the filled tube to the resistivity, cross-sectional area, and length of the salt solution:

$$R = \rho\frac{L}{A}$$

Substitute numerical values and evaluate *R*:

$$R = \left(10^{-3}\,\Omega\cdot\text{m}\right)\frac{1\,\text{m}}{\pi(2\,\text{mm})^2} = \boxed{79.6\,\Omega}$$

(*b*) Relate the resistance of the stretched tube to the resistivity, cross-sectional area, and length of the salt solution:

$$R' = \rho\frac{L'}{A'} = \rho\frac{2L}{A'} \qquad (1)$$

Letting *V* represent volume, express the relationship between the before-stretching volume *V* and the after-stretching volume *V*′:

$$V = V'$$

or

$$LA = L'A'$$

Solve for *A*′ to obtain:

$$A' = \frac{L}{L'}A = \tfrac{1}{2}A$$

Substitute in equation (1) to obtain:

$$R' = \rho\frac{2L}{\frac{1}{2}A} = 4\left(\rho\frac{L}{A}\right)$$
$$= 4(79.6\,\Omega) = \boxed{318\,\Omega}$$

51 ••

Picture the Problem We can use $R = \rho L/A$ to relate the resistance of the wires to their lengths, resistivities, and cross-sectional areas. To find the resistance of the stretched wire, we can use the fact that the volume of the wire does not change during the stretching process to relate the new cross-sectional area of the wire to its original cross-sectional area.

Relate the resistance of the unstretched wire to its resistivity, cross-sectional area, and length:

$$R = \rho\frac{L}{A}$$

Relate the resistance of the stretched wire to its resistivity, cross-sectional area, and length:

$$R' = \rho\frac{L'}{A'}$$

Divide the second of these equations by the first to obtain:

$$\frac{R'}{R} = \frac{\rho\frac{L'}{A'}}{\rho\frac{L}{A}} = \frac{L'}{L}\frac{A}{A'}$$

or

$$R' = 2\frac{A}{A'}R \qquad (1)$$

Letting V represent volume, express the relationship between the before-stretching volume V and the after-stretching volume V':

$$V = V'$$

or

$$LA = L'A'$$

Solve for A/A' to obtain:

$$\frac{A}{A'} = \frac{L'}{L} = 2$$

Substitute in equation (1) to obtain:

$$R' = 2(2)R = 4R = 4(0.3\,\Omega) = \boxed{1.20\,\Omega}$$

52 ••

Picture the Problem We can use $R = \rho L/A$ to find the resistance of the wire from its length, resistivity, and cross-sectional area. The electric field can be found using $E = V/L$ and Ohm's law to eliminate V. The time for an electron to travel the length of the wire can be found from $L = v_d\Delta t$, with v_d expressed in term of I using $I = neAv_d$.

(*a*) Relate the resistance of the unstretched wire to its resistivity, cross-sectional area, and length:

$$R = \rho\frac{L}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 10-gauge wire) and evaluate R:

$$R = (1.7\times10^{-8}\,\Omega\cdot\text{m})\frac{100\,\text{m}}{5.261\,\text{mm}^2} = \boxed{0.323\,\Omega}$$

(*b*) Relate the electric field in the wire to the potential difference between its ends:

$$E = \frac{V}{L}$$

Use Ohm's law to obtain:

$$E = \frac{IR}{L}$$

Substitute numerical values and evaluate E:

$$E = \frac{(30\,\text{A})(0.323\,\Omega)}{100\,\text{m}} = \boxed{96.9\,\text{mV/m}}$$

(*c*) Express the time Δt for an electron to travel a distance L in the wire in terms of its drift speed v_d:

$$\Delta t = \frac{L}{v_d}$$

Relate the current in the wire to the drift speed of the charge carriers:

$$I = neAv_d$$

Solve for v_d to obtain:

$$v_d = \frac{I}{neA}$$

Substitute to obtain:

$$\Delta t = \frac{neAL}{I}$$

Substitute numerical values (in Example 25-1 it is shown that $n = 8.47\times10^{28}\,\text{m}^{-3}$) and evaluate Δt:

$$\Delta t = \frac{(8.47\times10^{28}\ \text{m}^{-3})(1.6\times10^{-19}\ \text{C})(5.261\,\text{mm}^2)(100\,\text{m})}{30\,\text{A}} = \boxed{2.38\times10^{5}\ \text{s}}$$

53 ••

Picture the Problem We can use $R = \rho L/A$ to find express the resistance of the wire from in terms of its length, resistivity, and cross-sectional area. The fact that the volume of the copper does not change as the cube is drawn out to form the wire will allow us to eliminate either the length or the cross-sectional area of the wire and solve for its resistance.

Express the resistance of the wire in terms of its resistivity, cross-sectional area, and length:

$$R = \rho\frac{L}{A}$$

Relate the volume V of the wire to its length and cross-sectional area:

$$V = AL$$

Solve for L to obtain:

$$L = \frac{V}{A}$$

Substitute to obtain:

$$R = \rho\frac{V}{A^2}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 14-gauge wire) and evaluate R:

$$R = (1.7\times10^{-8}\,\Omega\cdot\text{m})\frac{(2\,\text{cm})^3}{(2.081\,\text{mm}^2)^2} = \boxed{0.0314\,\Omega}$$

***54** ••

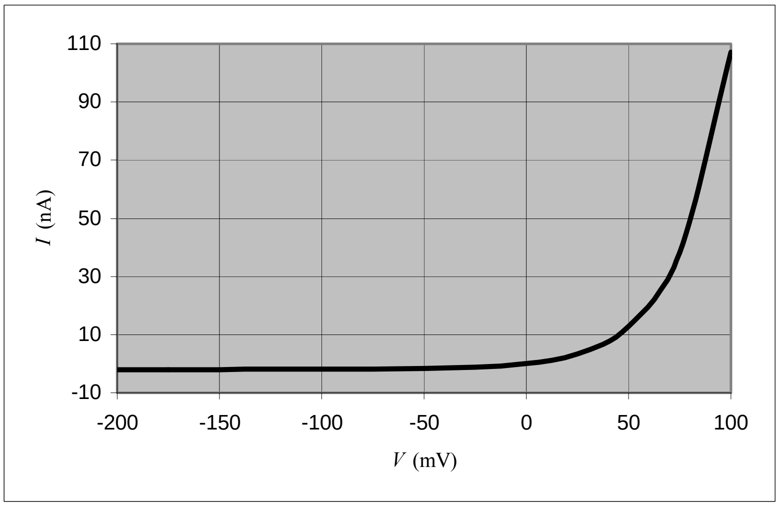
A spreadsheet program to plot I as a function of V is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	2	I_0
A5	–200	V (mV)
A6	A5 + 25	$V + \Delta V$
B5	$B$1*(EXP(A5/25) – 1)	$I_0\left(e^{V/25\,\text{mV}} - 1\right)$

	A	B	C
1	I_0=	2	nA
2			
3	V	I	

4	(mV)	(nA)	
5	–200.0	–2.00	
6	–175.0	–2.00	
7	–150.0	–2.00	
15	50.0	12.78	
16	75.0	38.17	
17	100.0	107.20	

The following graph was plotted using the data in spreadsheet table shown above.



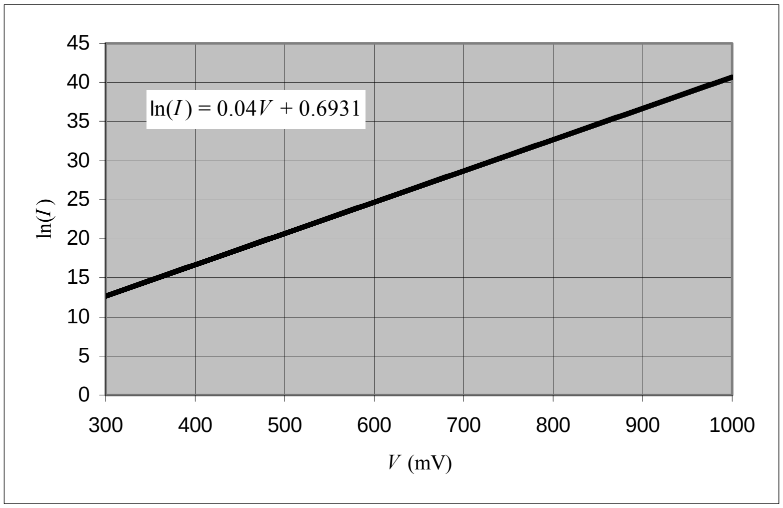
A spreadsheet program to plot ln(I) as a function of V for $V > 0.3$ V follows. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	2	2 nA
A5	300	V
A6	A5 + 10	$V + \Delta V$
B5	LN($B$1*(EXP(A5/25) – 1))	$\ln\left[I_0\left(e^{V/25\text{ mV}}-1\right)\right]$

A	B	C
I_0=	2	nA
V	ln(I)	
(mV)		
300	12.69	
310	13.09	
320	13.49	
330	13.89	
340	14.29	
350	14.69	
970	39.49	

980	39.89	
990	40.29	
1000	40.69	

A graph of ln(I) as a function of V follows. Microsoft Excel's Trendline feature was used to obtain the equation of the line.



For V >> 25 mV:

$$e^{V/25\,\text{mV}} - 1 \approx e^{V/25\,\text{mV}}$$

and

$$I \approx I_0 e^{V/25\,\text{mV}}$$

Take the natural logarithm of both sides of the equation to obtain:

$$\ln(I) = \ln\left(I_0 e^{V/25\,\text{mV}}\right) = \ln(I_0) + \frac{1}{25\,\text{mV}}V$$

which is of the form $y = mx + b$, where

$$m = \frac{1}{25\,\text{mV}} = \boxed{0.04(\text{mV})^{-1}}$$

in agreement with our graphical result.

55 ••

Picture the Problem We can use the first graph plotted in Problem 55 to conclude that, if V < 0.5 V, then the diode's resistance is effectively infinite. We can use Ohm's law to estimate the current through the diode.

(*a*) From the first graph plotted in Problem 55 we see that, if $V < 0.5$ V, then the current is negligible and the diode has essentially infinite resistance.

(*b*) Use Ohm's law to express the current flowing through the diode:

$$I = \frac{V_{\text{resistor}}}{R}$$

For a potential difference across the diode of approximately 0.5 V:

$$I = \frac{5\,\text{V} - 0.5\,\text{V}}{50\,\Omega} = \boxed{90.0\,\text{mA}}$$

56 •••

Picture the Problem We can use, as our element of resistance, a semicircular strip of width t, radius r, and thickness dr. Then $dR = (\pi r\rho/t)dr$. Because the strips are in parallel, integrating over them will give us the reciprocal of the resistance of half ring.

Integrate dR from $r = a$ to $r = b$ to obtain:

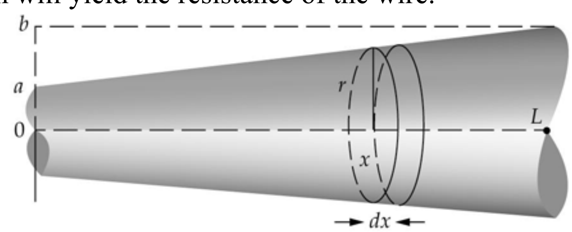
$$\frac{1}{R} = \frac{t}{\pi\rho}\int_a^b \frac{dr}{r} = \frac{t}{\pi\rho}\ln\left(\frac{b}{a}\right)$$

Take the reciprocal of both sides of the equation to obtain:

$$R = \boxed{\frac{\rho\pi}{t\ln\left(\frac{b}{a}\right)}}$$

57 •••

Picture the Problem The element of resistance we use is a segment of length dx and cross-sectional area $\pi[a + (b - a)x/L]^2$. Because these resistance elements are in series, integrating over them will yield the resistance of the wire.



Express the resistance of the chosen element of resistance:

$$dR = \rho\frac{dx}{A} = \frac{\rho}{\pi[a + (b - a)(x/L)]^2}dx$$

Integrate dR from $x = 0$ to $x = L$ and simplify to obtain:

$$R = \frac{\rho}{\pi}\int_0^L \frac{dx}{[a + (b - a)(x/L)]^2}$$

$$= \frac{\rho L}{\pi(b - a)}\left(\frac{1}{a} - \frac{1}{a + (b - a)}\right)$$

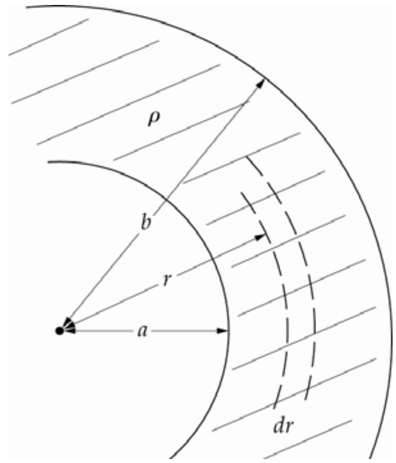
$$= \frac{\rho L}{\pi(b - a)}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{\rho L}{\pi(b - a)}\left(\frac{b - a}{ab}\right)$$

$$= \boxed{\frac{\rho L}{\pi ab}}$$

***58 •••**

Picture the Problem The diagram shows a cross-sectional view of the concentric spheres of radii a and b as well as a spherical-shell element of radius r. Using the *Hint* we can express the resistance dR of the spherical-shell element and then integrate over the volume filled with the material whose resistivity ρ is given to find the resistance between the conductors. Note that the elements of resistance are in series.



Express the element of resistance dR:

$$dR = \rho\frac{dr}{A} = \rho\frac{dr}{4\pi r^2}$$

Integrate dR from $r = a$ to $r = b$ to obtain:

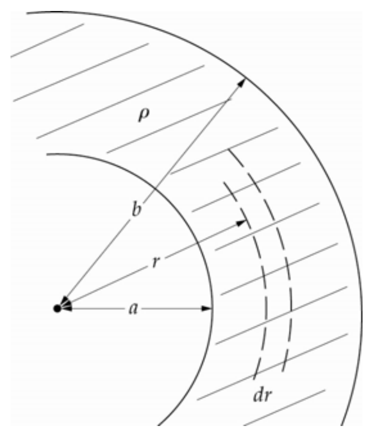
$$R = \frac{\rho}{4\pi}\int_a^b \frac{dr}{r^2} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Substitute numerical values and evaluate R:

$$R = \frac{10^9\,\Omega\cdot\text{m}}{4\pi}\left(\frac{1}{1.5\,\text{cm}} - \frac{1}{5\,\text{cm}}\right) = \boxed{3.71\times10^9\,\Omega}$$

59 •••

Picture the Problem The diagram shows a cross-sectional view of the coaxial cylinders of radii a and b as well as a cylindrical-shell element of radius r. We can express the resistance dR of the cylindrical-shell element and then integrate over the volume filled with the material whose resistivity ρ is given to find the resistance between the two cylinders. Note that the elements of resistance are in series.



Express the element of resistance dR:

$$dR = \rho\frac{dr}{A} = \rho\frac{dr}{2\pi rL}$$

Integrate dR from $r = a$ to $r = b$ to obtain:

$$R = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)}$$

(*b*) Apply Ohm's law to obtain:

$$I = \frac{V}{R} = \frac{V}{\frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)} = \frac{2\pi LV}{\rho\ln\left(\frac{b}{a}\right)}$$

Substitute numerical values and evaluate *I*:

$$I = \frac{2\pi(50\,\text{cm})(10\,\text{V})}{(30\,\Omega\cdot\text{m})\ln\left(\frac{2.5\,\text{cm}}{1.5\,\text{cm}}\right)} = \boxed{2.05\,\text{A}}$$

Temperature Dependence of Resistance

***60** •

Picture the Problem We can use $R = \rho L/A$ to find the resistance of the rod at 20°C. Ignoring the effects of thermal expansion, we can we apply the equation defining the temperature coefficient of resistivity, α, to relate the resistance at 40°C to the resistance at 20°C.

(*a*) Express the resistance of the rod at 20°C as a function of its resistivity, length, and cross-sectional area:

$$R_{20} = \rho_{20}\frac{L}{A}$$

Substitute numerical values and evaluate R_{20}:

$$R_{20} = \left(5.5\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{0.5\,\text{m}}{(1\,\text{mm})^2} = \boxed{27.5\,\text{m}\Omega}$$

(*b*) Express the resistance of the rod at 40°C as a function of its resistance at 20°C and the temperature coefficient of resistivity α:

$$\begin{aligned} R_{40} &= \rho_{40}\frac{L}{A} \\ &= \rho_{20}\left[1+\alpha\left(t_C - 20\text{C}^\circ\right)\right]\frac{L}{A} \\ &= \rho_{20}\frac{L}{A} + \rho_{20}\frac{L}{A}\alpha\left(t_C - 20\text{C}^\circ\right) \\ &= R_{20}\left[1+\alpha\left(t_C - 20\text{C}^\circ\right)\right] \end{aligned}$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of tungsten) and evaluate R_{40}:

$$R_{40} = (27.5\,\text{m}\Omega)\left[1+\left(4.5\times10^{-3}\,\text{K}^{-1}\right)(20\,\text{C}^\circ)\right] = \boxed{30.0\,\text{m}\Omega}$$

61 •

Picture the Problem The resistance of the copper wire increases with temperature according to $R_{t_C} = R_{20}[1+\alpha(t_C - 20C°)]$. We can replace R_{t_C} by $1.1R_{20}$ and solve for t_C to find the temperature at which the resistance of the wire will be 110% of its value at 20°C.

Express the resistance of the wire at $1.1R_{20}$:

$$1.1R_{20} = R_{20}[1+\alpha(t_C - 20C°)]$$

Simplify this expression to obtain:

$$1.1R_{20} = R_{20} + R_{20}\alpha(t_C - 20C°)$$

or

$$0.1 = \alpha(t_C - 20C°)$$

Solve to t_C to obtain:

$$t_C = \frac{0.1}{\alpha} + 20C°$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of copper) and evaluate t_C:

$$t_C = \frac{0.1}{3.9\times10^{-3}\,K^{-1}} + 20C° = \boxed{45.6°C}$$

62 ••

Picture the Problem Let the primed quantities denote the current and resistance at the final temperature of the heating element. We can express R' in terms of R_{20} and the final temperature of the wire t_C using $R' = R_{20}[1+\alpha(t_C - 20C°)]$ and relate I', R', I_{20}, and R_{20} using Ohm's law.

Express the resistance of the heating element at its final temperature as a function of its resistance at 20°C and the temperature coefficient of resistivity for Nichrome:

$$R' = R_{20}[1+\alpha(t_C - 20C°)] \qquad (1)$$

Apply Ohm's law to the heating element when it is first turned on:

$$V = I_{20}R_{20}$$

Apply Ohm's law to the heating element when it has reached its final temperature:

$$V = I'R'$$

Because the voltage is constant, we have:

$$I'R' = I_{20}R_{20}$$

or

$$R' = \frac{I_{20}}{I'} R_{20}$$

Substitute in equation (1) to obtain:

$$\frac{I_{20}}{I'} R_{20} = R_{20}\left[1 + \alpha\left(t_C - 20C°\right)\right]$$

or

$$\frac{I_{20}}{I'} = 1 + \alpha\left(t_C - 20C°\right)$$

Solve for t_C to obtain:

$$t_C = \frac{\frac{I_{20}}{I'} - 1}{\alpha} + 20C°$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of Nichrome) and evaluate t_C:

$$t_C = \frac{\frac{1.5\,A}{1.3\,A} - 1}{0.4 \times 10^{-3}\ \text{K}^{-1}} + 20C° = \boxed{405°\text{C}}$$

63 ••

Picture the Problem We can apply Ohm's law to find the initial current drawn by the cold heating element. The resistance of the wire at 1000°C can be found using $R_{1000} = R_{20}\left[1 + \alpha\left(t_C - 20C°\right)\right]$ and the power consumption of the heater at this temperature from $P = V^2/R_{1000}$.

(*a*) Apply Ohm's law to find the initial current I_{20} drawn by the heating element:

$$I = \frac{V}{R_{20}} = \frac{120\,\text{V}}{8\,\Omega} = \boxed{15.0\,\text{A}}$$

(*b*) Express the resistance of the heating element at 1000°C as a function of its resistance at 20°C and the temperature coefficient of resistivity for Nichrome:

$$R_{1000} = R_{20}\left[1 + \alpha\left(t_C - 20C°\right)\right]$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of Nichrome) and evaluate R_{1000}:

$$R_{1000} = (8\,\Omega)\left[1 + \left(0.4 \times 10^{-3}\ \text{K}^{-1}\right) \times \left(1000°\text{C} - 20C°\right)\right] = \boxed{11.1\,\Omega}$$

(*c*) Express and evaluate the operating wattage of this heater at 1000°C:

$$P = \frac{V^2}{R_{1000}} = \frac{(120\,\text{V})^2}{11.1\,\Omega} = \boxed{1.30\,\text{kW}}$$

64 ••

Picture the Problem We can find the resistance of the copper leads using $R_{\text{Cu}} = \rho_{\text{Cu}} L/A$ and express the percentage error in neglecting the resistance of the leads as the ratio of R_{Cu} to R_{Nichrome}. In part (*c*) we can express the change in resistance in the Nichrome wire corresponding to a change Δt_{C} in its temperature and then find Δt_{C} by substitution of the resistance of the copper wires in this equation.

(*a*) Relate the resistance of the copper leads to their resistivity, length, and cross-sectional area:

$$R_{\text{Cu}} = \rho_{\text{Cu}} \frac{L}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper) and evaluate R_{Cu}:

$$R_{\text{Cu}} = \left(1.7\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{50\,\text{cm}}{\frac{1}{4}\pi(0.6\,\text{mm})^2} = \boxed{30.1\,\text{m}\Omega}$$

(*b*) Express the percentage error as the ratio of R_{Cu} to R_{Nichrome}:

$$\%\,\text{error} = \frac{R_{\text{Cu}}}{R_{\text{Nichrome}}} = \frac{30.1\,\text{m}\Omega}{10\,\Omega} = \boxed{0.301\%}$$

(*c*) Express the change in the resistance of the Nichrome wire as its temperature changes from t_{C} to t_{C}':

$$\begin{aligned}\Delta R &= R' - R \\ &= R_{20}\left[1+\alpha\left(t_{\text{C}}' - 20\text{C}^\circ\right)\right] \\ &\quad - R_{20}\left[1+\alpha\left(t_{\text{C}} - 20\text{C}^\circ\right)\right] \\ &= R_{20}\alpha\Delta t_{\text{C}}\end{aligned}$$

Solve for Δt_{C} to obtain:

$$\Delta t_{\text{C}} = \frac{\Delta R}{R_{20}\alpha}$$

Set ΔR equal to the resistance of the copper wires (see Table 25-1 for the temperature coefficient of resistivity of Nichrome wire) and evaluate Δt_{C}:

$$\Delta t_{\text{C}} = \frac{30.1\,\text{m}\Omega}{(10\,\Omega)\left(0.4\times10^{-3}\,\text{K}^{-1}\right)} = \boxed{7.53\,\text{C}^\circ}$$

***65 •••**

Picture the Problem Expressing the total resistance of the two current-carrying (and hence warming) wires connected in series in terms of their resistivities, temperature coefficients of resistivity, lengths and temperature change will lead us to an expression in which, if $\rho_1 L_1 \alpha_1 + \rho_2 L_2 \alpha_2 = 0$, the total resistance is temperature independent. In part (b) we can apply the condition that $\rho_1 L_1 \alpha_1 + \rho_2 L_2 \alpha_2 = 0$ to find the ratio of the lengths of the carbon and copper wires.

(*a*) Express the total resistance of these two wires connected in series:

$$R = R_1 + R_2$$
$$= \rho_1 \frac{L_1}{A}(1+\alpha_1 \Delta T) + \rho_2 \frac{L_2}{A}(1+\alpha_2 \Delta T) + \frac{1}{A}\left[\rho_1 L_1 (1+\alpha_1 \Delta T) + \rho_2 L_2 (1+\alpha_2 \Delta T)\right]$$

Expand and simplify this expression to obtain:

$$R = \frac{1}{A}\left[\rho_1 L_1 + \rho_2 L_2 + (\rho_1 L_1 \alpha_1 + \rho_1 L_1 \alpha_2)\Delta T\right]$$

If $\rho_1 L_1 \alpha_1 + \rho_2 L_2 \alpha_2 = 0$, then:

$$R = \boxed{\frac{1}{A}\left[\rho_1 L_1 + \rho_2 L_2\right]}$$ independently of the temperature.

(*b*) Apply the condition for temperature independence obtained in (*a*) to the carbon and copper wires:

$$\rho_C L_C \alpha_C + \rho_{Cu} L_{Cu} \alpha_{Cu} = 0$$

Solve for the ratio of L_{Cu} to L_C:

$$\frac{L_{Cu}}{L_C} = -\frac{\rho_C \alpha_C}{\rho_{Cu} \alpha_{Cu}}$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of carbon and copper) and evaluate the ratio of L_{Cu} to L_C:

$$\frac{L_{Cu}}{L_C} = -\frac{(3500 \times 10^{-8}\ \Omega \cdot \text{m})(-0.5 \times 10^{-3}\ \text{K}^{-1})}{(1.7 \times 10^{-8}\ \Omega \cdot \text{m})(3.9 \times 10^{-3}\ \text{K}^{-1})} = \boxed{264}$$

66 •••

Picture the Problem We can use the relationship between the rate at which energy is transformed into heat and light in the filament and the resistance of and potential difference across the filament to estimate the resistance of the filament. The linear dependence of the resistivity on temperature will allow us to find the resistivity of the filament at 2500 K. We can then use the relationship between the resistance of the filament, its resistivity, and cross-sectional area to find its diameter.

(*a*) Express the wattage of the lightbulb as a function of its resistance *R* and the voltage *V* supplied by the source:

$$P = \frac{V^2}{R}$$

Solve for R to obtain:

$$R = \frac{V^2}{P}$$

Substitute numerical values and evaluate R:

$$R = \frac{(100\,\text{V})^2}{40\,\text{W}} = \boxed{250\,\Omega}$$

(b) Relate the resistance R of the filament to its resistivity ρ, radius r, and length ℓ:

$$R = \frac{\rho\ell}{\pi r^2}$$

Solve for r to obtain:

$$r = \sqrt{\frac{\rho\ell}{\pi R}}$$

and the diameter d of the filament is

$$d = 2\sqrt{\frac{\rho\ell}{\pi R}} \qquad (1)$$

Because the resistivity varies linearly with temperature, we can use a proportion to find its value at 2500 K:

$$\frac{\rho_{2500\,\text{K}} - \rho_{293\,\text{K}}}{\rho_{3500\,\text{K}} - \rho_{293\,\text{K}}} = \frac{2500\,\text{K} - 293\,\text{K}}{3500\,\text{K} - 293\,\text{K}} = \frac{2207}{3207}$$

Solve for $\rho_{2500\,\text{K}}$ to obtain:

$$\rho_{2500\,\text{K}} = \frac{2207}{3207}\left(\rho_{3500\,\text{K}} - \rho_{293\,\text{K}}\right) + \rho_{293\,\text{K}}$$

Substitute numerical values and evaluate $\rho_{2500\,\text{K}}$:

$$\rho_{2500\,\text{K}} = \frac{2207}{3207}(1.1\,\mu\Omega\cdot\text{m} - 56\,\text{n}\Omega\cdot\text{m}) + 56\,\text{n}\Omega\cdot\text{m} = 774.5\,\text{n}\Omega\cdot\text{m}$$

Substitute numerical values in equation (1) and evaluate d:

$$d = 2\sqrt{\frac{(774.5\,\text{n}\Omega\cdot\text{m})(0.5\,\text{cm})}{\pi(250\,\Omega)}} = \boxed{4.44\,\mu\text{m}}$$

67 •••

Picture the Problem We can use the relationship between the rate at which an object radiates and its temperature to find the temperature of the bulb.

(a) At a temperature T, the power emitted by a perfect blackbody is:

$$P = \sigma A T^4$$

where $\sigma = 5.67\times10^{-8}$ W/m^2·K^4 is the Stefan-Boltzmann constant.

Solve for T:

$$T = \sqrt[4]{\frac{P}{\sigma A}} = \sqrt[4]{\frac{P}{\sigma\pi d L}}$$

or, because $P = V^2/R$,

$$T = \sqrt[4]{\frac{V^2}{\sigma\pi dLR}}$$

Relate the resistance R of the filament to its resistivity ρ:

$$R = \frac{\rho L}{A} = \frac{4\rho L}{\pi d^2}$$

Substitute for R in the expression for T to obtain:

$$T = \sqrt[4]{\frac{V^2}{\sigma\pi dL\frac{4\rho L}{\pi d^2}}} = \sqrt[4]{\frac{V^2 d}{4\sigma L^2\rho}}$$

Substitute numerical values and evaluate T:

$$T = \sqrt[4]{\frac{(5\,\text{V})^2(40\times10^{-6}\,\text{m})}{4(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(0.03\,\text{m})^2(3\times10^{-5}\,\Omega\cdot\text{m})}} = \boxed{636\,\text{K}}$$

(*b*) As the filament heats up, its resistance increases, leading to more power being dissipated, leading to further heat, leading to a higher temperature, etc. This thermal runaway can burn out the filament if not controlled.

Energy in Electric Circuits

***68 •**

Picture the Problem We can use $P = V^2/R$ to find the power dissipated by the two resistors.

Express the power dissipated in a resistor as a function of its resistance and the potential difference across it:

$$P = \frac{V^2}{R}$$

(*a*) Evaluate P for V = 120 V and R = 5 Ω:

$$P = \frac{(120\,\text{V})^2}{5\,\Omega} = \boxed{2.88\,\text{kW}}$$

(*b*) Evaluate P for V = 120 V and R = 10 Ω:

$$P = \frac{(120\,\text{V})^2}{10\,\Omega} = \boxed{1.44\,\text{kW}}$$

69 •

Picture the Problem We can solve $P_{max} = I_{max}^2 R$ for the maximum current the resistor can carry and apply Ohm's law to find the maximum voltage that can be placed across the resistor.

(*a*) Express the maximum power the resistor can dissipate in terms of the current flowing through it:

$$P_{max} = I_{max}^2 R$$

Solve for I_{max} to obtain:

$$I_{max} = \sqrt{\frac{P_{max}}{R}}$$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = \sqrt{\frac{0.25\,\text{W}}{10\,\text{k}\Omega}} = \boxed{5.00\,\text{mA}}$$

(*b*) Apply Ohm's law to relate the maximum voltage across this resistor to the maximum current through it:

$$V_{max} = I_{max} R$$

Substitute numerical values and evaluate V_{max}:

$$V_{max} = (5\,\text{mA})(10\,\text{k}\Omega) = \boxed{50.0\,\text{V}}$$

70 •

Picture the Problem We can use $P = V^2/R$ to find the resistance of the heater and Ohm's law to find the current it draws.

(*a*) Express the power output of the heater in terms of its resistance and its operating voltage:

$$P = \frac{V^2}{R}$$

Solve for and evaluate R:

$$R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{1\,\text{kW}} = \boxed{57.6\,\Omega}$$

Apply Ohm's law to find the current drawn by the heater:

$$I = \frac{V}{R} = \frac{240\,\text{V}}{57.6\,\Omega} = \boxed{4.17\,\text{A}}$$

(*b*) Evaluate the power output of the heater operating at 120 V:

$$P = \frac{(120\,\text{V})^2}{57.6\,\Omega} = \boxed{250\,\text{W}}$$

71 •

Picture the Problem We can use the definition of power and the relationship between the battery's power output and its emf to find the work done by it under the given conditions.

Use the definition of power to relate the work done by the battery to the

$$P = \frac{\Delta W}{\Delta t}$$

time current is drawn from it:

Solve for the work done in time Δt:

$$\Delta W = P\Delta t$$

Express the power output of the battery as a function of the battery's emf:

$$P = \mathcal{E}I$$

Substitute to obtain:

$$\Delta W = \mathcal{E}I\Delta t$$

Substitute numerical values and evaluate ΔW:

$$\Delta W = (12\ \text{V})(3\ \text{A})(5\ \text{s}) = \boxed{180\ \text{J}}$$

72 •

Picture the Problem We can relate the terminal voltage of the battery to it emf, internal resistance, and the current delivered by it and then solve this relationship for the internal resistance.

Express the terminal potential difference of the battery in terms of its emf and internal resistance:

$$V_a - V_b = \mathcal{E} - Ir$$

Solve for r:

$$r = \frac{\mathcal{E} - (V_a - V)_b}{I}$$

Substitute numerical values and evaluate r:

$$r = \frac{12\ \text{V} - 11.4\ \text{V}}{20\ \text{A}} = \boxed{0.0300\ \Omega}$$

***73** •

Picture the Problem We can find the power delivered by the battery from the product of its emf and the current it delivers. The power delivered to the battery can be found from the product of the potential difference across the terminals of the starter (or across the battery when current is being drawn from it) and the current being delivered to it. In part (c) we can use the definition of power to relate the decrease in the chemical energy of the battery to the power it is delivering and the time during which current is drawn from it. In part (d) we can use conservation of energy to relate the energy delivered by the battery to the heat develope the battery and the energy delivered to the starter

(a) Express the power delivered by the battery as a function of its emf

$$P = \mathcal{E}I = (12\ \text{V})(20\ \text{A}) = \boxed{240\ \text{W}}$$

and the current it delivers:

(*b*) Relate the power delivered to the starter to the potential difference across its terminals:

$$P_{\text{starter}} = V_{\text{starter}} I = (11.4\ \text{V})(20\ \text{A}) = \boxed{228\ \text{W}}$$

(*c*) Use the definition of power to express the decrease in the chemical energy of the battery as it delivers current to the starter:

$$\Delta E = P\Delta t = (240\ \text{W})(3\ \text{min}) = \boxed{43.2\ \text{kJ}}$$

(*d*) Use conservation of energy to relate the energy delivered by the battery to the heat developed in the battery and the energy delivered to the starter:

$$E_{\text{delivered by battery}} = E_{\text{transformed into heat}} + E_{\text{delivered to starter}} = Q + E_{\text{delivered to starter}}$$

Express the energy delivered by the battery and the energy delivered to the starter in terms of the rate at which this energy is delivered:

$$P\Delta t = Q + P_s\Delta t$$

Solve for Q to obtain:

$$Q = (P - P_s)\Delta t$$

Substitute numerical values and evaluate Q:

$$Q = (240\ \text{W} - 228\ \text{W})(3\ \text{min}) = \boxed{2.16\ \text{kJ}}$$

74 •

Picture the Problem We can use conservation of energy to relate the emf of the battery to the potential differences across the variable resistor and the energy converted to heat within the battery. Solving this equation for I will allow us to find the current for the four values of R and we can use $P = I^2R$ to find the power delivered the battery for the four values of R.

Apply conservation of energy (Kirchhoff's loop rule) to the circuit to obtain:

$$\mathcal{E} = IR + Ir$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{R + r}$$

Express the power delivered by the battery as a function of the current drawn from it:

$$P = I^2 R$$

(*a*) For $R = 0$:

$$I = \frac{\mathcal{E}}{R + r} = \frac{6\,\text{V}}{0.3\,\Omega} = \boxed{20\,\text{A}}$$

and

$$P = (20\,\text{A})^2(0) = \boxed{0}$$

(*b*) For $R = 5\ \Omega$:

$$I = \frac{\mathcal{E}}{R + r} = \frac{6\,\text{V}}{5\,\Omega + 0.3\,\Omega} = \boxed{1.13\,\text{A}}$$

and

$$P = (1.13\,\text{A})^2(5\,\Omega) = \boxed{6.38\,\text{W}}$$

(*c*) For $R = 10\ \Omega$:

$$I = \frac{\mathcal{E}}{R + r} = \frac{6\,\text{V}}{10\,\Omega + 0.3\,\Omega} = \boxed{0.583\,\text{A}}$$

and

$$P = (0.583\text{A})^2(10\,\Omega) = \boxed{3.40\,\text{W}}$$

(*d*) For $R = \infty$:

$$I = \frac{\mathcal{E}}{R + r} = \lim_{R\to\infty}\frac{6\,\text{V}}{R + 0.3\,\Omega} = \boxed{0}$$

and

$$P = \boxed{0}$$

75 ••

Picture the Problem We can express the total stored energy ΔU in the battery in terms of its emf and the product $I\Delta t$ of the current it can deliver for a period of time Δt. We can apply the definition of power to relate the lifetime of the battery to the rate at which it is providing energy to the pair of headlights

(*a*) Express ΔU in terms of ε and the product $I\Delta t$:

$$\Delta U = \mathcal{E} I \Delta t$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (12\,\text{V})(160\,\text{A}\cdot\text{h}) = 1.92\,\text{kW}\cdot\text{h}$$
$$= 1.92\,\text{kW}\cdot\text{h} \times \frac{3.6\,\text{MJ}}{\text{kW}\cdot\text{h}}$$
$$= \boxed{6.91\,\text{MJ}}$$

(*b*) Use the definition of power to relate the lifetime of the battery to the rate at which it is providing energy to the pair of headlights:

$$\Delta t = \frac{\Delta U}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1.92\,\text{kW}\cdot\text{h}}{150\,\text{W}} = \boxed{12.8\,\text{h}}$$

76 ••

Picture the Problem We can use conservation of energy (aka Kirchhoff's loop Rule) to relate the emf at the fuse box and the voltage drop in the wires to the voltage at the outlet box (delivered to the space heater). We can find the number of 60-W li bulbs that could be supplied by the this line without blowing the fuse by dividing the current available by the current drawn by each 60-W bulb.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - V_{\text{wires}} - V_{\text{outlet}} = 0$$

or

$$\mathcal{E} - IR_{\text{wires}} - V_{\text{outlet}} = 0$$

Solve for V_{outlet} to obtain:

$$V_{\text{outlet}} = \mathcal{E} - IR_{\text{wires}}$$

Relate the resistance of the copper wires to the resistivity of copper, the length of the wires, and the cross-sectional
area of 12-gauge wire:

$$R_{\text{wires}} = \rho_{\text{Cu}}\frac{L}{A}$$

Substitute to obtain:

$$V_{\text{outlet}} = \mathcal{E} - \frac{I\rho_{\text{Cu}}L}{A}$$

Substitute numerical values (see Table 25 -1 for the resistivity of copper and Table 25-2 for the cross-sectional area of
12-gauge wire) and evaluate V_{outlet}:

$$V_{\text{outlet}} = 120\,\text{V} - \frac{(12.5\,\text{A})(1.7\times10^{-8}\,\Omega\cdot\text{m})(60\,\text{m})}{3.309\,\text{mm}^2} = \boxed{116\,\text{V}}$$

(*b*) Relate the number of bulbs N to the maximum current available and the current drawn by each 60-W bulb:

$$N = \frac{I_{\text{max}} - 12.5\,\text{A}}{I_{\text{bulb}}} \qquad (1)$$

Use Ohm's law to relate the current drawn by each bulb to the potential difference across it and it's resistance:

$$I_{\text{bulb}} = \frac{V}{R_{\text{bulb}}}$$

Express the resistance of each 60-W bulb:

$$R_{\text{bulb}} = \frac{\mathcal{E}^2}{P}$$

Substitute to obtain:

$$I_{\text{bulb}} = \frac{PV}{\mathcal{E}^2}$$

Substitute in equation (1) to obtain:

$$N = \frac{I_{\text{max}} - 12.5\,\text{A}}{\frac{PV}{\mathcal{E}^2}} = \frac{\mathcal{E}^2}{PV}\left(I_{\text{max}} - 12.5\,\text{A}\right)$$

Substitute numerical values and evaluate N:

$$N = \frac{(120\,\text{V})^2}{(60\,\text{W})(116\,\text{V})}(20\,\text{A} - 12.5\,\text{A}) = \boxed{15\,\text{bulbs}}$$

***77** ••

Picture the Problem We can use $P = fv$ to find the power the electric motor must develop to move the car at 80 km/h against a frictional force of 1200 N. We can find the total charge that can be delivered by the 10 batteries using $\Delta Q = NI\Delta t$. The total electrical energy delivered by the 10 batteries before recharging can be found using the definition of emf. We can find the distance the car can travel from the definition of work and the cost per kilometer of driving the car this distance by dividing the cost of the required energy by the distance the car has traveled.

(*a*) Express the power the electric motor must develop in terms of the speed of the car and the friction force:

$$P = fv = (1200\,\text{N})(80\,\text{km/h}) = \boxed{26.7\,\text{kW}}$$

(*b*) Use the definition of current to express the total charge that can be delivered before charging:

$$\Delta Q = NI\Delta t = 10(160\,\text{A}\cdot\text{h})\left(\frac{3600\,\text{s}}{\text{h}}\right) = \boxed{5.76\,\text{MC}}$$

where N is the number of batteries.

(*c*) Use the definition of emf to express the total electrical energy available in the batteries:

$$W = Q\mathcal{E} = (5.76\,\text{MC})(12\,\text{V}) = \boxed{69.1\,\text{MJ}}$$

(*d*) Relate the amount of work the batteries can do to the work required to overcome friction:

$$W = fd$$

Solve for and evaluate *d*:

$$d = \frac{W}{f} = \frac{69.1\,\text{MJ}}{1200\,\text{N}} = \boxed{57.6\,\text{km}}$$

(*e*) Express the cost per kilometer as the ratio of the ratio of the cost of the energy to the distance traveled before recharging:

$$\text{Cost/km} = \frac{(\$0.09/\text{kW}\cdot\text{h})\mathcal{E}It}{d} = \frac{(\$0.09/\text{kW}\cdot\text{h})(120\,\text{V})(160\,\text{A}\cdot\text{h})}{57.6\,\text{km}} = \boxed{\$0.03/\text{km}}$$

78 •••

Picture the Problem We can use the definition of power to find the current drawn by the heater and Ohm's law to find its resistance. In part (*b*) we'll use the hint to show that $\Delta P/P \approx 2\Delta V/V$ and in part (*c*) use this result to find the approximate power dissipated in the heater if the potential difference is decreased to 115 V.

(*a*) Use the definition of power to relate the current *I* drawn by the heater to its power rating *P* and the potential difference across it *V*:

$$I = \frac{P}{V}$$

Substitute numerical values and evaluate *I*:

$$I = \frac{100\,\text{W}}{120\,\text{V}} = \boxed{0.833\,\text{A}}$$

Apply Ohm's law to relate the resistance of the heater to the voltage across it and the current it draws:

$$R = \frac{V}{I} = \frac{120\,\text{V}}{0.833\,\text{A}} = \boxed{144\,\Omega}$$

(*b*) Approximate dP/dV by differentials:

$$\frac{dP}{dV} \approx \frac{\Delta P}{\Delta V} \text{ or } \Delta P \approx \frac{dP}{dV}\Delta V$$

Express the dependence of *P* on *V*:

$$P = \frac{V^2}{R}$$

Assuming R to be constant, evaluate dP/dV:

$$\frac{dP}{dV}=\frac{d}{dV}\left[\frac{V^2}{R}\right]=\frac{2V}{R}$$

Substitute to obtain:

$$\Delta P\approx\frac{2V}{R}\Delta V=2\frac{V^2}{R}\frac{\Delta V}{V}=2P\frac{\Delta V}{V}$$

Divide both sides of the equation by P to obtain:

$$\boxed{\frac{\Delta P}{P}=2\frac{\Delta V}{V}}$$

(c) Express the approximate power dissipated in the heater as the sum of its power consumption and the change in its power dissipation when the voltage is decreased by ΔV:

$$\begin{aligned}P&\approx P_0+\Delta P\\&=P_0+2P_0\frac{\Delta V}{V}\\&=P_0\left(1+2\frac{\Delta V}{V}\right)\end{aligned}$$

Substitute numerical values and evaluate P:

$$P\approx(100\text{ W})\left(1+2\left(\frac{-5\text{ V}}{120\text{ V}}\right)\right)=\boxed{91.7\text{ W}}$$

Combinations of Resistors

***79** •

Picture the Problem We can either solve this problem by using the expression for the equivalent resistance of three resistors connected in parallel and then using Ohm's law to find the current in each resistor, or we can apply Ohm's law first to find the current through each resistor and then use Ohm's law a second time to find the equivalent resistance of the parallel combination. We'll follow the first procedure.

(a) Express the equivalent resistance of the three resistors in parallel and solve for R_{eq}:

$$\frac{1}{R_{eq}}=\frac{1}{4\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}$$

and

$$\boldsymbol{R}_{eq}=\boxed{1.33\,\Omega}$$

(b) Apply Ohm's law to each of the resistors to find the current flowing through each:

$$I_4=\frac{V}{4\,\Omega}=\frac{12\text{ V}}{4\,\Omega}=\boxed{3.00\text{ A}}$$

$$I_3=\frac{V}{3\,\Omega}=\frac{12\text{ V}}{3\,\Omega}=\boxed{4.00\text{ A}}$$

and

$$I_6 = \frac{V}{6\,\Omega} = \frac{12\,\text{V}}{6\,\Omega} = \boxed{2.00\,\text{A}}$$

Remarks: You would find it instructive to use Kirchhoff's junction rule (conservation of charge) to confirm our values for the currents through the three resistors.

80 •

Picture the Problem We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We can then add that resistance and the 3-Ω resistance to find the equivalent resistance between points *a* and *b*. In part (*b*) we'll denote the currents through each resistor with subscripts corresponding to the resistance through which the current flows and apply Ohm's law to find those currents.

(*a*) Express the equivalent resistance of the two resistors in parallel and solve for $R_{\text{eq},1}$:

$$\frac{1}{R_{\text{eq},1}} = \frac{1}{R_6} + \frac{1}{R_2} = \frac{1}{6\,\Omega} + \frac{1}{2\,\Omega}$$

and

$$R_{\text{eq},1} = 1.50\,\Omega$$

Because the 3-Ω resistor is in series with $R_{\text{eq},1}$:

$$R_{\text{eq}} = R_3 + R_{\text{eq},1} = 3\,\Omega + 1.5\,\Omega = \boxed{4.50\,\Omega}$$

(*b*) Apply Ohm's law to the network to find I_3:

$$I_3 = \frac{V_{ab}}{R_{\text{eq}}} = \frac{12\,\text{V}}{4.5\,\Omega} = \boxed{2.67\,\text{A}},$$

Find the potential difference across the parallel resistors:

$$V_{6\,\&\,2} = V_{ab} - V_3 = 12\,\text{V} - (2.67\,\text{A})(3\,\Omega) = 4\,\text{V}$$

Use the common potential difference across the resistors in parallel to find the current through each of them:

$$I_6 = \frac{V_6}{R_6} = \frac{4\,\text{V}}{6\Omega} = \boxed{0.667\,\text{A}}$$

and

$$I_2 = \frac{V_{6\,\&\,2}}{R_2} = \frac{4\,\text{V}}{2\Omega} = \boxed{2.00\,\text{A}}$$

81 •

Picture the Problem Note that the resistors between *a* and *c* and between *c* and *b* are in series as are the resistors between *a* and *d* and between *d* and *b*. Hence, we have two branches in parallel, each branch consisting of two resistors *R* in series. In part (b) it will be important to note that the potential difference between point *c* and point *d* is zero.

(*a*) Express the equivalent resistance between points *a* and *b* in terms of the equivalent resistances between *acb* and *adb*:

$$\frac{1}{R_{eq}} = \frac{1}{R_{acb}} + \frac{1}{R_{adb}} = \frac{1}{2R} + \frac{1}{2R}$$

Solve for R_{eq} to obtain:

$$R_{eq} = \boxed{R}$$

(*b*) Because the potential difference between points c and d is zero, no current would flow through the resistor connected between these two points, and the addition of that resistor would not change the network.

82 ••

Picture the Problem Note that the 2-Ω resistors are in parallel with each other and with the 4-Ω resistor. We can Apply Kirchhoff's loop rule to relate the current I_3 drawn from the battery to the emf of the battery and equivalent resistance R_{eq} of the resistor network. We can find the current through the resistors connected in parallel by applying Kirchhoff's loop rule a second time. In part (*b*) we can find the power delivered by the battery from the product of its emf and the current it delivers to the circuit.

(*a*) Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E} - I_3 R_{eq} = 0$$

Solve for I_3:

$$I_3 = \frac{\mathcal{E}}{R_{eq}} \qquad (1)$$

Find the equivalent resistance of the three resistors in parallel:

$$\frac{1}{R_{eq,1}} = \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_4} = \frac{1}{2\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega}$$

and

$$R_{eq,1} = 0.8\,\Omega$$

Find the equivalent resistance of $R_{eq,1}$ and R_3 in series:

$$R_{eq} = R_3 + R_{eq,1} = 3\,\Omega + 0.8\,\Omega = 3.8\,\Omega$$

Substitute in equation (1) and evaluate I_3:

$$I_3 = \frac{6\text{ V}}{3.8\,\Omega} = \boxed{1.58\text{ A}}$$

Express the current through each of the parallel resistors in terms of their common potential difference V:

$$I_2 = \frac{V}{R_2} \text{ and } I_4 = \frac{V}{R_4}$$

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E} - I_3 R_3 - V = 0$$

Solve for V:

$$V = \mathcal{E} - I_3 R_3 = 6\,\text{V} - (1.58\,\text{A})(3\,\Omega) = 1.26\,\text{V}$$

Substitute numerical values and evaluate I_2 and I_4:

$$I_2 = \frac{1.26\,\text{V}}{2\,\Omega} = \boxed{0.630\,\text{A}}$$

and

$$I_4 = \frac{1.26\,V}{4\,\Omega} = \boxed{0.315\,\text{A}}$$

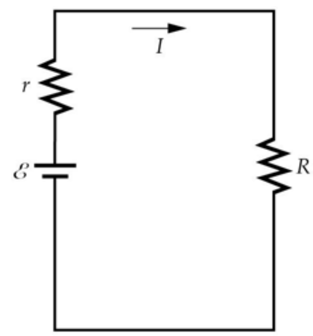
(*b*) Express P in terms of ε and I_3:

$$P = \mathcal{E} I_3 = (6\,\text{V})(1.58\,\text{A}) = \boxed{9.48\,\text{W}}$$

Remarks: Note that the currents I_3, I_2, and I_4 satisfy Kirchhoff's junction rule.

***83 ••**

Picture the Problem Let r represent the resistance of the internal resistance of the power supply, ε the emf of the power supply, R the resistance of the external resistor to be placed in series with the power supply, and I the current drawn from the power supply. We can use Ohm's law to express the potential difference across R and apply Kirchhoff's loop rule to express the current through R in terms of ε, r, and R.



Express the potential difference across the resistor whose resistance is R:

$$V_R = IR \qquad (1)$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - Ir - IR = 0$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{r + R}$$

Substitute in equation (1) to obtain:

$$V_R = \left(\frac{\mathcal{E}}{r + R}\right) R$$

Solve for *R* to obtain:

$$R = \frac{V_R r}{\mathcal{E} - V_R}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{(4.5\,\text{V})(50\,\Omega)}{5\,\text{V} - 4.5\,\text{V}} = \boxed{450\,\Omega}$$

84 ••

Picture the Problem We can apply Kirchhoff's loop rule to the two circuits described in the problem statement and solve the resulting equations simultaneously for *r* and *ε*.

(*a*) and (*b*) Apply Kirchhoff's loop rule to the two circuits to obtain:

$$\mathcal{E} - I_1 r - I_1 R_5 = 0$$

and

$$\mathcal{E} - I_2 r - I_2 R_{11} = 0$$

Substitute numerical values to obtain:

$$\mathcal{E} - (0.5\,\text{A})r - (0.5\,\text{A})(5\,\Omega) = 0$$

or

$$\mathcal{E} - (0.5\,\text{A})r = 2.5\,\text{V} \qquad (1)$$

and

$$\mathcal{E} - (0.25\,\text{A})r - (0.25\,\text{A})(11\,\Omega) = 0$$

or

$$\mathcal{E} - (0.25\,\text{A})r = 2.75\,\text{V} \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$\mathcal{E} = \boxed{3.00\,\text{V}} \text{ and } r = \boxed{1.00\,\Omega}$$

85 ••

Picture the Problem We can use the formula for the equivalent resistance for two resistors in parallel to show that $R_{eq} = R_1x/(1 + x)$.

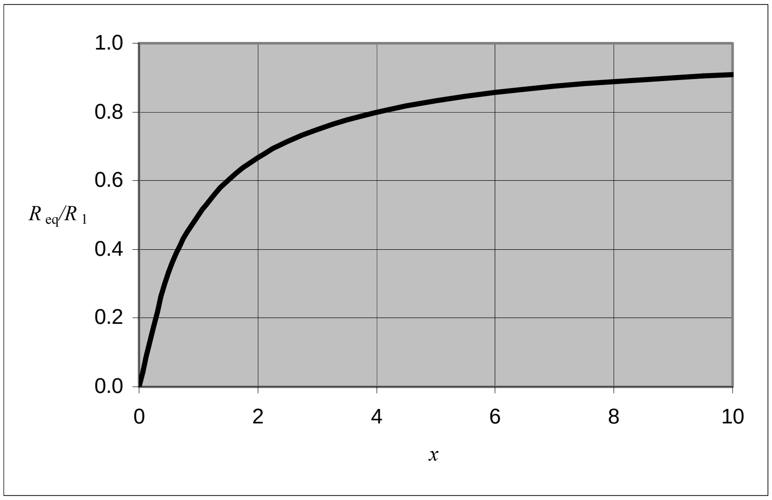
(*a*) Express the equivalent resistance of R_1 and R_2 in parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Let $x = R_2/R_1$ to obtain:

$$R_{eq} = \frac{xR_1^2}{R_1 + xR_1} = \boxed{\frac{x}{1+x}R_1}$$

(*b*) The following graph of R_{eq}/R_1 versus *x* was plotted using a spreadsheet program.



86 ••

Picture the Problem We can use Kirchhoff's loop rule to relate the required resistance to the emf of the source and the desired current. We can apply Kirchhoff's rule a second time to the circuit that includes the load resistance *r* to establish the largest value it can have if it is to change the current drawn from the source by at most 10 percent.

(*a*) Apply Kirchhoff's loop rule to the circuit that includes the source and the resistance *R* to obtain:

$$\mathcal{E} - IR = 0$$

Solve for *R* to obtain:

$$R = \frac{\mathcal{E}}{I}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{5\,\text{V}}{10\,\text{mA}} = \boxed{500\,\Omega}$$

(*b*) Letting *I*′ represent the current in the loaded circuit, express the condition that the current drops by less than 10%:

$$\frac{I - I'}{I} = 1 - \frac{I'}{I} < 0.1 \qquad (1)$$

Letting *r* represent the load resistance, apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - I'r - I'R = 0$$

Solve for *I*′ to obtain:

$$I' = \frac{\mathcal{E}}{r + R}$$

Substitute for *I* and *I*′ in equation (1):

$$1 - \frac{\frac{\mathcal{E}}{r+R}}{\frac{\mathcal{E}}{R}} < 0.1 \Rightarrow 1 - \frac{R}{r+R} < 0.1$$

Solve for r to obtain:

$$r < \frac{0.1R}{0.9}$$

Substitute the numerical value for R to obtain:

$$r < \frac{0.1(500\,\Omega)}{0.9} = \boxed{55.6\,\Omega}$$

87 ••

Picture the Problem We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We'll then have a parallel network with two resistors in series in each branch and can use the expressions for resistors in series to simplify the network to two resistors in parallel. The equivalent resistance between points *a* and *b* will be the single resistor equivalent to these two resistors. In part (*b*) we'll use the fact that the potential difference across the upper branch is the same as the potential difference across the lower branch, in conjunction with Ohm's law, to find the currents through each resistor.

(*a*) Express and evaluate the equivalent resistance of the two 6-Ω resistors in parallel and solve for $R_{\text{eq},1}$:

$$R_{\text{eq},1} = \frac{R_6 R_6}{R_6 + R_6} = \frac{(6\,\Omega)^2}{6\,\Omega + 6\,\Omega} = 3\,\Omega$$

Find the equivalent resistance of the 6-Ω resistor is in series with $R_{\text{eq},1}$:

$$R_{\text{eq},2} = R_6 + R_{\text{eq},1} = 6\,\Omega + 3\,\Omega = 9\,\Omega$$

Find the equivalent resistance of the 12-Ω resistor in series with the 6-Ω resistor:

$$R_{\text{eq},3} = R_6 + R_{12} = 6\,\Omega + 12\,\Omega = 18\,\Omega$$

Finally, find the equivalent resistance of $R_{\text{eq},3}$ in parallel with $R_{\text{eq},2}$:

$$R_{\text{eq}} = \frac{R_{\text{eq},2} R_{\text{eq},3}}{R_{\text{eq},2} + R_{\text{eq},3}} = \frac{(9\,\Omega)(18\,\Omega)}{9\,\Omega + 18\,\Omega} = \boxed{6.00\,\Omega}$$

(*b*) Apply Ohm's law to the upper branch to find the current $I_u = I_{12} = I_6$:

$$I_u = I_{12} = I_6 = \frac{V_{ab}}{R_{\text{eq},3}}, \quad = \frac{12\text{ V}}{18\,\Omega} = \boxed{0.667\text{ A}}$$

Apply Ohm's law to the lower branch to find the current $I_l = I_{6\text{-}\Omega\text{ resistor in series}}$:

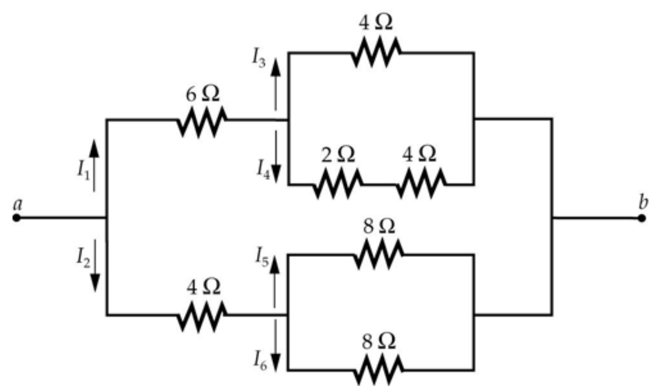
$$I_l = I_{6\text{-}\Omega\text{ resistor in series}} = \frac{V_{ab}}{R_{\text{eq},2}} = \frac{12\text{ V}}{9\,\Omega} = \boxed{1.33\text{ A}}$$

Express the current through the 6-Ω resistors in parallel:

$$I_{6\text{-}\Omega\text{ resistors in parallel}} = \tfrac{1}{2}I_l = \tfrac{1}{2}(1.33\text{ A}) = \boxed{0.667\text{ A}}$$

88 ••

Picture the Problem Assign currents in each of the resistors as shown in the diagram. We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We'll then have a parallel network with two resistors in series in each branch and can use the expressions for resistors in series to simplify the network to two resistors in parallel. The equivalent resistance between points *a* and *b* will be the single resistor equivalent to these two resistors. In part (*b*) we'll use the fact that the potential difference across the upper branch is the same as the potential difference across the lower branch, in conjunction with Ohm's law, to find the currents through each resistor.



(*a*) Express and evaluate the equivalent resistance of the resistors in parallel in the upper branch and solve for $R_{\text{eq},1}$:

$$R_{\text{eq},1} = \frac{(R_2 + R_4)R_4}{(R_2 + R_4) + R_4} = \frac{(6\,\Omega)(4\,\Omega)}{2\,\Omega + 4\,\Omega + 4\,\Omega} = 2.4\,\Omega$$

Find the equivalent resistance of the 6-Ω resistor is in series with $R_{\text{eq},1}$:

$$R_{\text{eq},2} = R_6 + R_{\text{eq},1} = 6\,\Omega + 2.4\,\Omega = 8.4\,\Omega$$

Express and evaluate the equivalent resistance of the resistors in parallel in the lower branch and solve for

$$R_{\text{eq},2} = \frac{R_8 R_8}{R_8 + R_8} = \tfrac{1}{2}R_8 = \tfrac{1}{2}(8\,\Omega) = 4\,\Omega$$

$R_{\text{eq},2}$:

Find the equivalent resistance of the 4-Ω resistor is in series with $R_{\text{eq},2}$:

$$R_{\text{eq},3} = R_4 + R_{\text{eq},2} = 4\,\Omega + 4\,\Omega = 8\,\Omega$$

Finally, find the equivalent resistance of $R_{\text{eq},2}$ in parallel with $R_{\text{eq},3}$:

$$R_{\text{eq}} = \frac{R_{\text{eq},2}R_{\text{eq},3}}{R_{\text{eq},2} + R_{\text{eq},3}} = \frac{(8.4\,\Omega)(8\,\Omega)}{8.4\,\Omega + 8\,\Omega} = \boxed{4.10\,\Omega}$$

(*b*) Apply Ohm's law to the upper branch to find the current I_1:

$$I_1 = \frac{V_{ab}}{R_{\text{eq},2}} = \frac{12\,\text{V}}{8.4\,\Omega} = \boxed{1.43\,\text{A}}$$

Find the potential difference across the 4-Ω and 6-Ω parallel combination in the upper branch:

$$V_{4\&6} = 12\,\text{V} - V_6 = 12\,\text{V} - I_{\text{u}}R_6 = 12\,\text{V} - (1.43\,\text{A})(6\,\Omega) = 3.43\,\text{V}$$

Apply Ohm's law to find the current I_4:

$$I_4 = \frac{V_6}{R_6} = \frac{3.43\,\text{V}}{6\,\Omega} = \boxed{0.572\,\text{A}}$$

Apply Ohm's law to find the current I_3:

$$I_3 = \frac{V_4}{R_4} = \frac{3.43\,\text{V}}{4\,\Omega} = \boxed{0.858\,\text{A}}$$

Apply Ohm's law to the lower branch to find the current I_2:

$$I_2 = \frac{V_{ab}}{R_{\text{eq},2}} = \frac{12\,\text{V}}{8\,\Omega} = \boxed{1.50\,\text{A}}$$

Find the potential difference across the 8-Ω and 8-Ω parallel combination in the lower branch:

$$V_{8\&8} = 12\,\text{V} - I_3R_4 = 12\,\text{V} - (1.5\,\text{A})(4\,\Omega) = 6\,\text{V}$$

Apply Ohm's law to find $I_5 = I_6$:

$$I_5 = I_6 = \frac{V_{8\&8}}{8\,\Omega} = \frac{6\,\text{V}}{8\,\Omega} = \boxed{0.750\,\text{A}}$$

***89 ••**

Picture the Problem We can use the equation for *N* identical resistors connected in parallel to relate *N* to the resistance *R* of each piece of wire and the equivalent resistance

Express the resistance of the *N* pieces connected in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{N}{R}$$

where *R* is the resistance of one of the *N* pieces.

Relate the resistance of one of the N pieces to the resistance of the wire:

$$R=\frac{R_{\text{wire}}}{N}$$

Substitute to obtain:

$$\frac{1}{R_{\text{eq}}}=\frac{N^2}{R_{\text{wire}}}$$

Solve for N:

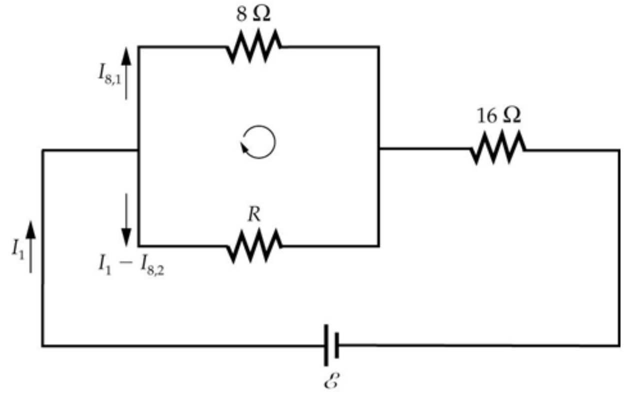
$$N=\sqrt{\frac{R_{\text{wire}}}{R_{\text{eq}}}}$$

Substitute numerical values and evaluate N:

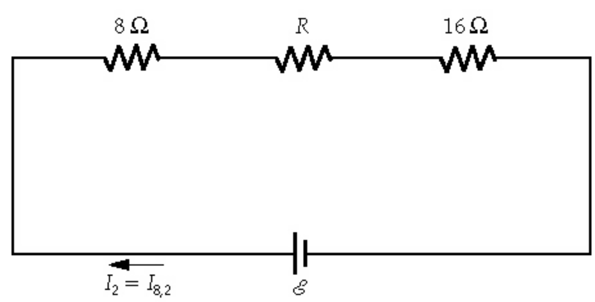
$$N=\sqrt{\frac{120\,\Omega}{1.875\,\Omega}}=\boxed{8}$$

90 ••

Picture the Problem We can assign currents as shown in the diagram of the first arrangement of resistors and apply Kirchhoff's loop rule to obtain an expression for $I_{8,1}$.



Assign currents as shown in the diagram below for the second arrangement of the resistors and apply Kirchhoff's loop rule to obtain an expression for $I_{8,2}$ that we can equate to $I_{8,1}$ and solve for R.



Apply Kirchhoff's loop rule to the first arrangement of the resistors:

$$\mathcal{E}-I_1R_{\text{eq},1}=0$$

where I_1 is the current drawn from the battery.

Solve for I_1 to obtain:

$$I_1 = \frac{\mathcal{E}}{R_{\text{eq},1}}$$

Find the equivalent resistance of the first arrangement of the resistors:

$$R_{\text{eq},1} = \frac{(8\,\Omega)R}{8\,\Omega + R} + 16\,\Omega = \frac{(24\,\Omega)R + 128\,\Omega^2}{R + 8\,\Omega}$$

Substitute to obtain:

$$I_1 = \frac{\mathcal{E}}{\dfrac{(24\,\Omega)R + 128\,\Omega^2}{R + 8\,\Omega}} = \frac{\mathcal{E}(R + 8\,\Omega)}{(24\,\Omega)R + 128\,\Omega^2}$$

Apply Kirchhoff's loop rule to the loop containing R and the 8-Ω resistor:

$$-I_8(8\,\Omega) + R(I_1 - I_8) = 0$$

Solve for $I_{8,1}$ to obtain:

$$I_{8,1} = \frac{R}{R + 8\,\Omega} I_1 = \left(\frac{R}{R + 8\,\Omega}\right)\left(\frac{\mathcal{E}(R + 8\,\Omega)}{(24\,\Omega)R + 128\,\Omega^2}\right) = \frac{\mathcal{E}R}{(24\,\Omega)R + 128\,\Omega^2}$$

Express $I_{8,2}$ in terms of I_1 and $I_{8,1}$:

$$I_{8,2} = I_1 - I_{8,1}$$

Substitute for I_1 and $I_{8,1}$ and simplify to obtain:

$$I_{8,2} = \frac{\mathcal{E}(R + 8\,\Omega)}{(24\,\Omega)R + 128\,\Omega^2} - \frac{\mathcal{E}R}{(24\,\Omega)R + 128\,\Omega^2} = \frac{(8\,\Omega)\mathcal{E}}{(24\,\Omega)R + 128\,\Omega^2}$$

Apply Kirchhoff's loop rule to the second arrangement of the resistors:

$$\mathcal{E} - I_2 R_{\text{eq},2} = 0$$

where I_2 is the current drawn from the battery.

Solve for I_2 (= $I_{8,2}$) to obtain:

$$I_2 = I_{8,2} = \frac{\mathcal{E}}{R_{\text{eq},2}}$$

Find the equivalent resistance of the second arrangement of the resistors:

$$R_{\text{eq},2} = R + 24\,\Omega$$

Substitute to obtain:

$$I_{8,2} = \frac{\mathcal{E}}{R + 24\,\Omega}$$

Equate $I_{8,1}$ and $I_{8,2}$:

$$\frac{\mathcal{E}R}{(24\,\Omega)R + 128\,\Omega^2} = \frac{(8\,\Omega)\mathcal{E}}{(24\,\Omega)R + 128\,\Omega^2}$$

Solve for R to obtain:

$$R = \boxed{8.00\,\Omega}$$

91 ••

Picture the Problem We can find the equivalent resistance R_{ab} between points *a* and *b* and then set this resistance equal, in turn, to R_1, R_3, and R_1 and solve for R_3, R_2, and R_1, respectively.

(*a*) Express the equivalent resistance between points *a* and *b*:

$$R_{ab} = \frac{R_1R_2}{R_1 + R_2} + R_3$$

Equate this expression to R_1:

$$R_1 = \frac{R_1R_2}{R_1 + R_2} + R_3$$

Solve for R_3 to obtain:

$$R_3 = \boxed{\frac{R_1^2}{R_1 + R_2}}$$

(*b*) Set R_3 equal to R_{ab}:

$$R_3 = \frac{R_1R_2}{R_1 + R_2} + R_3$$

Solve for R_2 to obtain:

$$R_2 = \boxed{0}$$

(*c*) Set R_1 equal to R_{ab}:

$$R_1 = \frac{R_1R_2}{R_1 + R_2} + R_3$$

or

$$R_1^2 - R_3R_1 - R_2R_3 = 0$$

Solve the quadratic equation for R_1 to obtain:

$$R_1 = \boxed{\frac{R_3 + \sqrt{R_3^2 + 4R_2R_3}}{2}}$$

where we've used the positive sign because resistance is a non-negative quantity.

92 ••

Picture the Problem We can substitute the given resistances in the equations derived in Problem 91 to check our results from Problem 78.

(*a*) For $R_1 = 4\ \Omega$ and $R_2 = 6\ \Omega$:

$$R_3 = \frac{R_1^2}{R_1 + R_2} = \frac{(4\,\Omega)^2}{4\,\Omega + 6\,\Omega} = \boxed{1.60\,\Omega}$$

and

$$R_{ab} = \frac{R_1R_2}{R_1 + R_2} + R_3$$

$$= \frac{(4\,\Omega)(6\,\Omega)}{4\,\Omega + 6\,\Omega} + 1.6\,\Omega = \boxed{4.00\,\Omega}$$

(*b*) For $R_1 = 4\ \Omega$ and $R_3 = 3\ \Omega$:

$$R_2 = \boxed{0}$$

and

$$R_{ab} = \frac{R_1(0)}{R_1 + 0} + R_3 = 0 + 3\,\Omega$$

$$= \boxed{3.00\,\Omega}$$

(*c*) For $R_2 = 6\ \Omega$ and $R_3 = 3\ \Omega$:

$$R_1 = \frac{3\,\Omega + \sqrt{(3\,\Omega)^2 + 4(6\,\Omega)(3\,\Omega)}}{2}$$

$$= \frac{3\,\Omega + 9\,\Omega}{2} = \boxed{6.00\,\Omega}$$

and

$$R_{ab} = \frac{R_1R_2}{R_1 + R_2} + R_3$$

$$= \frac{(6\,\Omega)(6\,\Omega)}{6\,\Omega + 6\,\Omega} + 3\,\Omega = \boxed{6.00\,\Omega}$$

Kirchhoff's Rules

***93** •

Picture the Problem We can relate the current provided by the source to the rate of Joule heating using $P = I^2R$ and use Ohm's law and Kirchhoff's rules to find the potential difference across R and the value of r.

(*a*) Relate the current I in the circuit to rate at which energy is being dissipated in the form of Joule heat:

$$P = I^2 R \text{ or } I = \sqrt{\frac{P}{R}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{8\,\text{W}}{0.5\,\Omega}} = \boxed{4.00\,\text{A}}$$

(*b*) Apply Ohm's law to find V_R:

$$V_R = IR = (4\,\text{A})(0.5\,\Omega) = \boxed{2.00\,\text{V}}$$

(*c*) Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E} - Ir - IR = 0$$

Solve for r:

$$r = \frac{\mathcal{E} - IR}{I} = \frac{\mathcal{E}}{I} - R$$

Substitute numerical values and evaluate r:

$$r = \frac{6\,\text{V}}{4\,\text{A}} - 0.5\,\Omega = \boxed{1.00\,\Omega}$$

94 •

Picture the Problem Assume that the current flows clockwise in the circuit and let $\mathcal{E}_1$ represent the 12-V source and $\mathcal{E}_2$ the 6-V source. We can apply Kirchhoff's loop rule (conservation of energy) to this series circuit to relate the current to the emfs of the sources and the resistance of the circuit. In part (*b*) we can find the power delivered or absorbed by each source using $P = \mathcal{E}I$ and in part (*c*) the rate of Joule heating using $P = I^2R$.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E}_1 - IR_2 - \mathcal{E}_2 - IR_4 = 0$$

Solve for I:

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_2 + R_4}$$

Substitute numerical values and evaluate I:

$$I = \frac{12\,\text{V} - 6\,\text{V}}{2\,\Omega + 4\,\Omega} = \boxed{1.00\,\text{A}}$$

(*b*) Express the power delivered/absorbed by each source in terms of its emf and the current drawn from or forced through it:

$$P_{12} = \mathcal{E}_{12} I = (12\,\text{V})(1\,\text{A}) = \boxed{12\,\text{W}}$$

and

$$P_6 = \mathcal{E}_6 I = (-6\,\text{V})(1\,\text{A}) = \boxed{-6\,\text{W}}$$

where the minus sign means that this source is absorbing power.

(*c*) Express the rate of Joule heating in terms of the current through and the resistance of each resistor:

$$P_2 = I^2R_2 = (1\,\text{A})^2(2\,\Omega) = \boxed{2.00\,\text{W}}$$

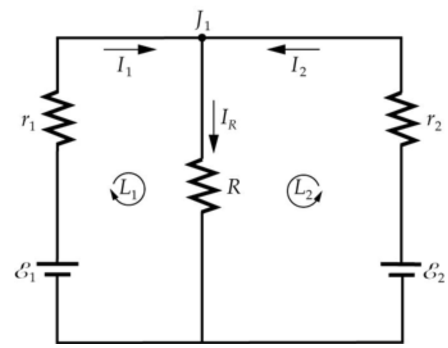
and

$$P_4 = I^2R_4 = (1\,\text{A})^2(4\,\Omega) = \boxed{4.00\,\text{W}}$$

95 ••

Picture the Problem The circuit is shown in the diagram for part (*a*). $\mathcal{E}_1$ and r_1 denote the emf of the "sick" battery and its internal resistance, $\mathcal{E}_2$ and r_2 the emf of the second battery and its internal resistance, and *R* is the load resistance. Let I_1, I_2, and I_R be the currents. We can apply Kirchhoff's rules to determine the unknown currents. In part (*c*) we can use $P = \mathcal{E}I$ to find the power delivered or absorbed by each battery and $P = I^2R$ to find the power dissipated in the internal and load resistors.

(*a*) The circuit diagram is shown to the right:



(*b*) Apply Kirchhoff's junction rule to junction 1 to obtain:

$$I_1 + I_2 = I_R \quad (1)$$

Apply Kirchhoff's loop rule to loop 1 to obtain:

$$\mathcal{E}_1 - r_1I_1 - RI_R = 0$$

or

$$11.4\,\text{V} - (0.01\,\Omega)I_1 - (2\,\Omega)I_R = 0 \quad (2)$$

Apply Kirchhoff's loop rule to loop 2 to obtain:

$$\mathcal{E}_2 - r_2I_2 - RI_R = 0$$

or

$$12.6\,\text{V} - (0.01\,\Omega)I_2 - (2\,\Omega)I_R = 0 \quad (3)$$

Solve equations (1), (2) and (3) simultaneously to obtain:

$$I_1 = \boxed{-57.0\,\text{A}},$$

$$I_2 = \boxed{63.0\,\text{A}},$$

and

$$I_R = \boxed{6.00\text{ A}}$$

where the minus sign for I_1 means that the current flows in the direction opposite to the direction we arbitrarily chose, i.e., the battery is being charged.

(*c*) Express the power delivered by the second battery in terms of its emf and the current drawn from it:

$$P_2 = \mathcal{E}_2 I_2 = (12.6\text{ V})(63.0\text{ A}) = \boxed{794\text{ W}}$$

Express the power absorbed by the second battery in terms of its emf and the current forced through it:

$$P_1 = \mathcal{E}_1 I_1 = (11.4\text{ V})(57.0\text{ A}) = \boxed{650\text{ W}}$$

Find the power dissipated in the internal resistance r_1:

$$P_{r_1} = I_1^2 r_1 = (57\text{ A})^2(0.01\,\Omega) = \boxed{32.5\text{ W}}$$

Find the power dissipated in the internal resistance r_2:

$$P_{r_2} = I_2^2 r_2 = (63\text{ A})^2(0.01\,\Omega) = \boxed{39.7\text{ W}}$$

Find the power dissipated in the load resistance R:

$$P_R = I_R^2 R = (6\text{ A})^2(2\,\Omega) = \boxed{72.0\text{ W}}$$

Remarks: Note that the sum of the power dissipated in the internal and load resistances and that absorbed by the second battery is the same as that delivered by the first battery … just as we would expect from conservation of energy.

96 ••

Picture the Problem Note that when both switches are closed the 50-Ω resistor is shorted. With both switches open, we can apply Kirchhoff's loop rule to find the current I in the 100-Ω resistor. With the switches closed, the 100-Ω resistor and R are in parallel. Hence, the potential difference across them is the same and we can express the current I_{100} in terms of the current I_{tot} flowing into the parallel branch whose resistance is R, and the resistance of the 100-Ω resistor. I_{tot}, in turn, depends on the equivalent resistance of the closed-switch circuit, so we can express $I_{100} = I$ in terms of R and solve for R.

Apply Kirchhoff's loop rule to a loop around the outside of the circuit with both switches open:

$$\mathcal{E} - (300\,\Omega)I - (100\,\Omega)I - (50\,\Omega)I = 0$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{450\,\Omega} = \frac{1.5\,\text{V}}{450\,\Omega} = 3.33\,\text{mA}$$

Relate the potential difference across the 100-Ω resistor to the potential difference across R when both switches are closed:

$$(100\,\Omega)I_{100} = RI_R$$

Apply Kirchhoff's junction rule at the junction to the left of the 100-Ω resistor and R:

$$I_{\text{tot}} = I_{100} + I_R$$

or

$$I_R = I_{\text{tot}} - I_{100}$$

where I_{tot} is the current drawn from the source when both switches are closed.

Substitute to obtain:

$$(100\,\Omega)I_{100} = R(I_{\text{tot}} - I_{100})$$

or

$$I_{100} = \frac{RI_{\text{tot}}}{R + 100\,\Omega} \qquad (1)$$

Express the current I_{tot} drawn from the source with both switches closed:

$$I_{\text{tot}} = \frac{\mathcal{E}}{R_{\text{eq}}}$$

Express the equivalent resistance when both switches are closed:

$$R_{\text{eq}} = \frac{(100\,\Omega)R}{R + 100\,\Omega} + 300\,\Omega$$

Substitute to obtain:

$$I_{\text{tot}} = \frac{1.5\,\text{V}}{\frac{(100\,\Omega)R}{R + 100\,\Omega} + 300\,\Omega}$$

Substitute in equation (1) to obtain:

$$I_{100} = \frac{R}{R + 100\,\Omega}\left(\frac{1.5\,\text{V}}{\frac{(100\,\Omega)R}{R + 100\,\Omega} + 300\,\Omega}\right)$$

$$= \frac{(1.5\,\text{V})R}{(400\,\Omega)R + 30{,}000\,\Omega^2}$$

$$= 3.33\,\text{mA}$$

Solve for and evaluate R:

$$R = \boxed{600\,\Omega}$$

Remarks: Note that we can also obtain the result in the third step by applying Kirchhoff's loop rule to the parallel branch of the circuit.

***97 ••**

Picture the Problem Let I_1 be the current delivered by the left battery, I_2 the current delivered by the right battery, and I_3 the current through the 6-Ω resistor, directed down. We can apply Kirchhoff's rules to obtain three equations that we can solve simultaneously for I_1, I_2, and I_3. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points *a* and *b* and the power delivered by both the sources.

(*a*) Apply Kirchhoff's junction rule at junction *a*:

$$I_1 + I_2 = I_3$$

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$12\text{ V} - (4\,\Omega)I_1 + (3\,\Omega)I_2 - 12\text{ V} = 0$$

or

$$-(4\,\Omega)I_1 + (3\,\Omega)I_2 = 0$$

Apply Kirchhoff's loop rule to a loop around the left-hand branch of the circuit to obtain:

$$12\text{ V} - (4\,\Omega)I_1 - (6\,\Omega)I_3 = 0$$

Solve these equations simultaneously to obtain:

$$I_1 = \boxed{0.667\text{ A}},$$

$$I_2 = \boxed{0.889\text{ A}},$$

and

$$I_3 = \boxed{1.56\text{ A}}$$

(*b*) Apply Ohm's law to find the potential difference between points *a* and *b*:

$$V_{ab} = (6\,\Omega)I_3 = (6\,\Omega)(1.56\text{ A})$$
$$= \boxed{9.36\text{ V}}$$

(*c*) Express the power delivered by the 12-V battery in the left-hand branch of the circuit:

$$P_{\text{left}} = \mathcal{E}I_1$$
$$= (12\text{ V})(0.667\text{ A}) = \boxed{8.00\text{ W}}$$

Express the power delivered by the 12-V battery in the right-hand branch of the circuit:

$$P_{\text{right}} = \mathcal{E}I_2$$
$$= (12\text{ V})(0.889\text{ A}) = \boxed{10.7\text{ W}}$$

98 ••

Picture the Problem Let I_1 be the current delivered by the 7-V battery, I_2 the current delivered by the 5-V battery, and I_3, directed up, the current through the 1-Ω resistor. We can apply Kirchhoff's rules to obtain three equations that we can solve simultaneously for I_1, I_2, and I_3. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points *a* and *b* and the power delivered by both the sources.

(*a*) Apply Kirchhoff's junction rule at junction *a*:

$$I_1 = I_2 + I_3$$

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$7\,\text{V} - (2\,\Omega)I_1 - (1\,\Omega)I_3 = 0$$

Apply Kirchhoff's loop rule to a loop around the left-hand branch of the circuit to obtain:

$$7\,\text{V} - (2\,\Omega)I_1 - (3\,\Omega)I_2 + 5\,\text{V} = 0$$

or

$$(2\,\Omega)I_1 + (3\,\Omega)I_2 = 12\,\text{V}$$

Solve these equations simultaneously to obtain:

$$I_1 = \boxed{3.00\,\text{A}},$$

$$I_2 = \boxed{2.00\,\text{A}},$$

and

$$I_3 = \boxed{1.00\,\text{A}}$$

(*b*) Apply Ohm's law to find the potential difference between points *a* and *b*:

$$V_{ab} = -5\,\text{V} + (3\,\Omega)I_2$$
$$= -5\,\text{V} + (3\,\Omega)(2\,\text{A})$$
$$= \boxed{1.00\,\text{V}}$$

(*c*) Express the power delivered by the 7-V battery:

$$P_7 = \mathcal{E}I_1 = (7\,\text{V})(3\,\text{A}) = \boxed{21.0\,\text{W}}$$

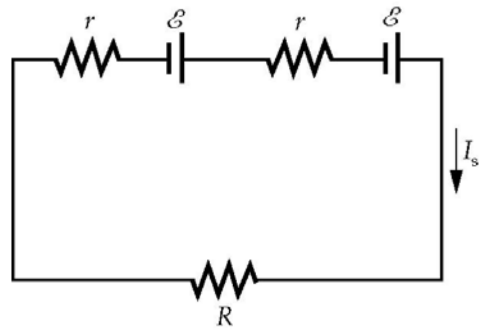
(*c*) Express the power delivered by the 5-V battery:

$$P_5 = \mathcal{E}I_2 = (5\,\text{V})(2\,\text{A}) = \boxed{10.0\,\text{W}}$$

99 ••

Picture the Problem We can apply Kirchhoff's rules to the two circuits to determine the current, and hence the power, supplied to the load resistor *R* for the two connections of the batteries. Differentiation, with respect to the load resistor, of the expressions for the power delivered to the load resistor will allow us to identify the conditions under which the power delivered is a maximum and to decide whether the power supplied to R greater when $R < r$ or when $R > r$.

The series connection of the batteries is shown to the right:



Express the power supplied to R: $P_s = I_s^2 R$

Apply Kirchhoff's loop rule to obtain: $-rI_s + \mathcal{E} - rI_s + \mathcal{E} - RI_s = 0$

Solve for I_s to obtain: $I_s = \dfrac{2\mathcal{E}}{2r + R}$

Substitute to obtain:

$$P_s = \left(\frac{2\mathcal{E}}{2r+R}\right)^2 R = \frac{4\mathcal{E}^2 R}{(2r+R)^2} \qquad (1)$$

Set the derivative, with respect to R, of equation (1) equal to zero for extrema:

$$\frac{dP_s}{dR} = \frac{d}{dR}\left[\frac{4\mathcal{E}^2 R}{(2r+R)^2}\right]$$

$$= \frac{(2r+R)^2 4\varepsilon^2 - 4\varepsilon^2 R(2)(2r+R)}{(2r+R)^4}$$

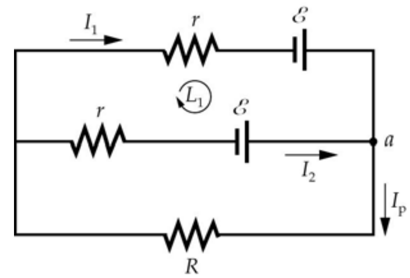
$= 0$ for extrema.

Solve for R to obtain: $R = 2r$

Examination of the second derivative of P_s at $R = 2r$ shows that $R = 2r$ corresponds to a maximum value of P_p and hence, for the series combination,

the power delivered to the load is greater if $R > r$ and is greatest when $R = 2r$.

The parallel connection of the batteries is shown to the right:



Express the power supplied to R:

$$P_p = I_p^2 R$$

Apply Kirchhoff's junction rule to point a to obtain:

$$I_p = I_1 + I_2$$

Apply Kirchhoff's loop rule to loop 1 to obtain:

$$-rI_1 + \mathcal{E} - \mathcal{E} + rI_2 = 0$$

or

$$I_1 = I_2 = \tfrac{1}{2} I_p$$

Apply Kirchhoff's loop rule to the outer loop to obtain:

$$\mathcal{E} - RI_p - rI_1 = 0$$

or

$$\mathcal{E} - RI_p - \tfrac{1}{2} rI_p = 0$$

Solve for I_p to obtain:

$$I_p = \frac{\mathcal{E}}{\frac{1}{2}r + R}$$

Substitute to obtain:

$$P_p = \left(\frac{\mathcal{E}}{\frac{1}{2}r + R}\right)^2 R = \frac{\mathcal{E}^2 R}{\left(\frac{1}{2}r + R\right)^2} \qquad (2)$$

Evaluate equation (1) when $r = R$:

$$P_s(r = R) = \frac{4\mathcal{E}^2 R}{(2R + R)^2} = \frac{4}{9}\frac{\mathcal{E}^2}{R}$$

Evaluate equation (2) when $r = R$:

$$P_p(r = R) = \frac{\mathcal{E}^2 R}{\left(\frac{1}{2}R + R\right)^2} = \frac{4}{9}\frac{\mathcal{E}^2}{R}$$

Thus, we see that if $r = R$, both arrangements provide the same power to the load.

Set the derivative, with respect to R, of equation (2) equal to zero for extrema:

$$\frac{dP_p}{dR} = \frac{d}{dR}\left[\frac{\varepsilon^2 R}{\left(\frac{1}{2}r + R\right)^2}\right] = \frac{\left(\frac{1}{2}r + R\right)^2 \varepsilon^2 - \varepsilon^2 R(2)\left(\frac{1}{2}r + R\right)}{\left(\frac{1}{2}r + R\right)^4} = 0 \text{ for extrema.}$$

Solve for R to obtain:

$$R = \tfrac{1}{2}r$$

Examination of the second derivative of P_p at $R = \frac{1}{2}r$ shows that $R = \frac{1}{2}r$ corresponds to a maximum value of P_p and hence, for the parallel combination, the power delivered to the load is greater if $R < r$ and a maximum when $R = \frac{1}{2}r$.

***100 ••**

Picture the Problem Let the current drawn from the source be I. We can use Ohm's law in conjunction with Kirchhoff's loop rule to express the output voltage as a function of V, R_1, and R_2. In (*b*) we can use the result of (*a*) to express the condition on the output voltages in terms of the effective resistance of the loaded output and the resistances R_1 and R_2.

(*a*) Use Ohm's law to express V_{out} in terms of R_2 and I:

$$V_{out} = IR_2$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$V - IR_1 - IR_2 = 0$$

Solve for I:

$$I = \frac{V}{R_1 + R_2}$$

Substitute for I in the expression for V_{out} to obtain:

$$V_{out} = \left(\frac{V}{R_1 + R_2}\right)R_2 = \boxed{V\left(\frac{R_2}{R_1 + R_2}\right)}$$

(*b*) Relate the effective resistance of the loaded circuit R_{eff} to R_2 and R_{load}:

$$\frac{1}{R_{eff}} = \frac{1}{R_2} + \frac{1}{R_{load}}$$

Solve for R_{load}:

$$R_{load} = \frac{R_2 R_{eff}}{R_2 - R_{eff}} \qquad (1)$$

Letting V'_{out} represent the output voltage under load, express the condition that V_{out} drops by less than 10 percent of its unloaded

$$\frac{V_{out} - V'_{out}}{V_{out}} = 1 - \frac{V'_{out}}{V_{out}} < 0.1 \qquad (2)$$

value:

Using the result from (*a*), express V'_{out} in terms of the effective output load R_{eff}:

$$V'_{out} = V\left(\frac{R_{eff}}{R_1 + R_{eff}}\right)$$

Substitute for V_{out} and V'_{out} in equation (2) and simplify to obtain:

$$1 - \frac{\dfrac{R_{eff}}{R_1 + R_{eff}}}{\dfrac{R_2}{R_1 + R_2}} < 0.1$$

or

$$1 - \frac{R_{eff}(R_1 + R_2)}{R_2(R_1 + R_{eff})} < 0.1$$

Solve for R_{eff}:

$$R_{eff} > \frac{0.9R_1R_2}{R_1 + 0.1R_2}$$

Substitute numerical values and evaluate R_{eff}:

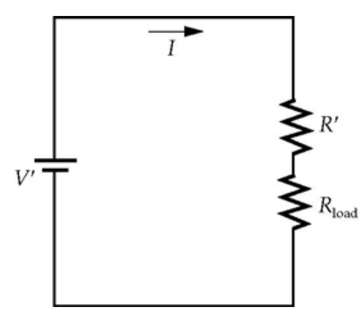
$$R_{eff} > \frac{0.9(10\,\text{k}\Omega)(10\,\text{k}\Omega)}{10\,\text{k}\Omega + 0.1(10\,\text{k}\Omega)} = 8.18\,\text{k}\Omega$$

Finally, substitute numerical values in equation (1) and evaluate R_{load}:

$$R_{load} < \frac{(10\,\text{k}\Omega)(8.18\,\text{k}\Omega)}{10\,\text{k}\Omega - 8.18\,\text{k}\Omega} = \boxed{44.9\,\text{k}\Omega}$$

101 ••

Picture the Problem In the equivalent Thevenin circuit shown to the right, R_2 is in parallel with R_{load}. We can apply Ohm's law to express V_{out} in terms of R_{eff} and I and then use Kirchhoff's loop rule to express I in terms of V, R_1, and R_{eff}. Simplification of the resulting equation will yield both of the indicated results.



(*a*) and (*b*) Use Ohm's law to express V_{out} in terms of R_{eff} and I:

$$V_{out} = IR_{eff}$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$V - IR_1 - IR_{eff} = 0$$

Solve for I:

$$I = \frac{V}{R_1 + R_{eff}}$$

Substitute for I in the expression for V_{out} to obtain:

$$V_{out} = \left(\frac{V}{R_1 + R_{eff}}\right)R_{eff} = V\left(\frac{R_{eff}}{R_1 + R_{eff}}\right)$$

Express the effective resistance R_{eff} in terms of R_{load} and R_2:

$$R_{eff} = \frac{R_2 R_{load}}{R_2 + R_{load}}$$

Substitute for R_{eff} in the expression for V_{out} to obtain:

$$V_{out} = V\left(\frac{\dfrac{R_2 R_{load}}{R_2 + R_{load}}}{R_1 + \dfrac{R_2 R_{load}}{R_2 + R_{load}}}\right)$$

Simplify to obtain:

$$\begin{aligned} V_{out} &= V\frac{R_2 R_{load}}{R_1 R_{load} + R_2 R_{load} + R_1 R_2} \\ &= \left(V\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_{load}}{R_{load} + \dfrac{R_1 R_2}{R_1 + R_2}}\right) \\ &= \left(V\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_{load}}{R_{load} + R'}\right) \\ &= V'\frac{R_{load}}{R_{load} + R'} \end{aligned}$$

where

$$R' = \boxed{\frac{R_1 R_2}{R_1 + R_2}} \text{ and } V' = \boxed{V\frac{R_2}{R_1 + R_2}}$$

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Picture the Problem Let I_1 be the current in the 1-Ω resistor, directed to the right; let I_2 be the current, directed up, in the middle branch; and let I_3 be the current in the 6-Ω resistor, directed down. We can apply Kirchhoff's rules to find these currents, the power supplied by each source, and the power dissipated in each resistor.

(*a*) Apply Kirchhoff's junction rule at the top junction to obtain:

$$I_1 + I_2 = I_3 \qquad (1)$$

Apply Kirchhoff's loop rule to the outside loop of the circuit to obtain:

$$8\,\text{V} - (1\,\Omega)I_1 + 4\,\text{V} - (2\,\Omega)I_1 - (6\,\Omega)I_3 = 0$$

or

$$(3\,\Omega)I_1 + (6\,\Omega)I_3 = 12\,\text{V} \qquad (2)$$

Apply the loop rule to the inside loop at the left-hand side of the circuit to obtain:

$$8\,\text{V} - (1\,\Omega)I_1 + 4\,\text{V} - (2\,\Omega)I_1 + (2\,\Omega)I_2 - 4\,\text{V} = 0$$

or

$$8\,\text{V} - (3\,\Omega)I_1 + (2\,\Omega)I_2 = 0 \qquad (3)$$

Solve equations (1), (2), and (3) simultaneously to obtain:

$I_1 = \boxed{2.00\ \text{A}}$,

$I_2 = \boxed{-1.00\ \text{A}}$,

and

$I_3 = \boxed{1.00\ \text{A}}$

where the minus sign indicates that the current flows downward rather than upward as we had assumed.

(*b*) Express the power delivered by the 8-V source:

$$P_8 = \mathcal{E}_8 I_1 = (8\ \text{V})(2\ \text{A}) = \boxed{16.0\ \text{W}}$$

Express the power delivered by the 4-V source:

$$P_4 = \mathcal{E}_4 I_2 = (4\ \text{V})(-1\ \text{A}) = \boxed{-4.00\ \text{W}}$$

where the minus sign indicates that this source is having current forced through it.

(*c*) Express the power dissipated in the 1-Ω resistor:

$$P_{1\Omega} = I_1^2 R_{1\Omega} = (2\ \text{A})^2(1\ \Omega) = \boxed{4.00\ \text{W}}$$

Express the power dissipated in the 2-Ω resistor in the left branch:

$$P_{2\Omega,\text{left}} = I_1^2 R_{2\Omega} = (2\ \text{A})^2(2\ \Omega) = \boxed{8.00\ \text{W}}$$

Express the power dissipated in the 2-Ω resistor in the middle branch:

$$P_{2\Omega,\text{middle}} = I_2^2 R_{2\Omega} = (1\ \text{A})^2(2\ \Omega) = \boxed{2.00\ \text{W}}$$

Express the power dissipated in the 6-Ω resistor:

$$P_{6\Omega} = I_3^2 R_{6\Omega} = (1\ \text{A})^2(6\ \Omega) = \boxed{6.00\ \text{W}}$$

103 ••

Picture the Problem Let I_1 be the current in the left branch resistor, directed up; let I_3 be the current, directed down, in the middle branch; and let I_2 be the current in the right branch, directed up. We can apply Kirchhoff's rules to find I_3 and then the potential difference between points *a* and *b*.

Relate the potential at *a* to the potential at *b*:

$$V_a - R_4 I_3 - 4\ \text{V} = V_b$$

or

$$V_a - V_b = R_4 I_3 + 4\,\text{V}$$

Apply Kirchhoff's junction rule at *a* to obtain:

$$I_1 + I_2 = I_3 \qquad (1)$$

Apply the loop rule to a loop around the outside of the circuit to obtain:

$$2\,\text{V} - (1\,\Omega)I_1 + (1\,\Omega)I_2 - 2\,\text{V} + (1\,\Omega)I_2 - (1\,\Omega)I_1 = 0$$

or

$$I_1 - I_2 = 0 \qquad (2)$$

Apply the loop rule to the left side of the circuit to obtain:

$$2\,\text{V} - (1\,\Omega)I_1 - (4\,\Omega)I_3 - 4\,\text{V} - (1\,\Omega)I_1 = 0$$

or

$$-(1\,\Omega)I_1 - (2\,\Omega)I_3 = 1\,\text{V} \qquad (3)$$

Solve equations (1), (2), and (3) simultaneously to obtain:

$I_1 = -0.200\,\text{A}$,
$I_2 = -0.200\,\text{A}$,
and
$I_3 = -0.400\,\text{A}$
where the minus signs indicate that all the directions we chose for the currents were wrong.

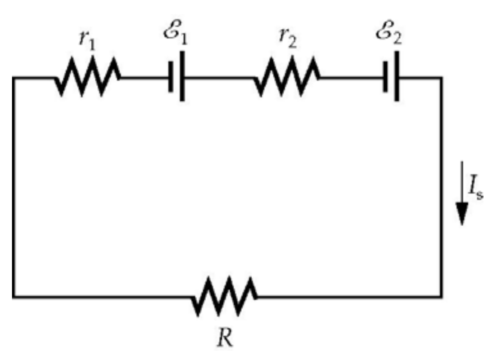
Substitute to obtain:

$$V_a - V_b = (4\,\Omega)(-0.4\,\text{A}) + 4\,\text{V} = \boxed{2.40\,\text{V}}$$

Remarks: Note that point *a* is at the higher potential.

104 ••

Picture the Problem Let ε_1 be the emf of the 9-V battery and r_1 its internal resistance of 0.8 Ω, and ε_2 be the emf of the 3-V battery and r_2 its internal resistance of 0.4 Ω. The series connection is shown to the right. We can apply Kirchhoff's rules to both connections to find the currents I_s and I_p delivered to the load resistor in the series and parallel connections.



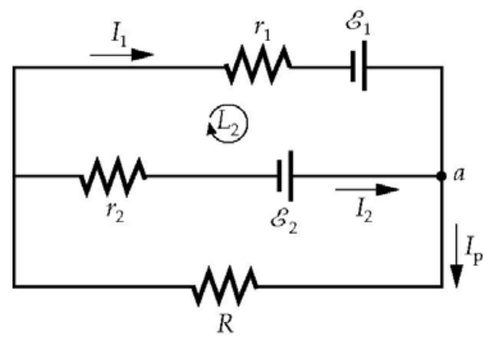
(*a*) Apply Kirchhoff's loop rule to the batteries connected in series:

$$\mathcal{E}_1 - r_2 I_s + \mathcal{E}_2 - RI_s - r_1 I_s = 0$$

Solve for I_s to obtain:

$$I_s = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R} = \boxed{\frac{12\,\text{V}}{1.2\,\Omega + R}}$$

Suppose the two batteries are connected in parallel and their terminals are then connected to R. Let I_1 be the current delivered by $\mathcal{E}_1$, I_2 be the current delivered by $\mathcal{E}_2$, and I_p the current through the load resistor R in the parallel connection.



Apply the junction rule at a to obtain:

$$I_1 + I_2 = I_p \qquad (1)$$

Apply the loop rule to a loop around the outside of the circuit:

$$\mathcal{E}_1 - RI_p - r_1 I_1 = 0$$

or

$$9\,\text{V} - RI_p - (0.8\,\Omega)I_1 = 0 \qquad (2)$$

Apply the loop rule to loop 2 to obtain:

$$\mathcal{E}_1 - \mathcal{E}_2 + r_2 I_2 - r_1 I_1 = 0$$

or

$$9\,\text{V} - 3\,\text{V} + (0.4\,\Omega)I_2 - (0.8\,\Omega)I_1 = 0$$

or

$$6\,\text{V} + (0.4\,\Omega)I_2 - (0.8\,\Omega)I_1 = 0 \qquad (3)$$

Eliminate I_2 between equations (1) and (3) to obtain:

$$I_1 = 5\,\text{A} + \tfrac{1}{3} I_p \qquad (4)$$

Substitute equation (4) in equation (2) and solve for I_p to obtain:

$$I_p = \boxed{\frac{7.5\,\text{V}}{1.5R + 0.4\,\Omega}}$$

(*b*) Evaluate I_s and I_p for $R = 0.2\ \Omega$:

$$I_s(R = 0.2\,\Omega) = \frac{12\,\text{V}}{1.2\,\Omega + 0.2\,\Omega} = \boxed{8.57\,\text{A}}$$

and

$$I_p(R=0.2\,\Omega)=\frac{7.5\text{ V}}{1.5(0.2\,\Omega)+0.4\,\Omega}=\boxed{10.7\text{ A}}$$

(*c*), (*d*), and (*e*) Proceed as in (*b*) to complete the table to the right:

	R	I_s	I_p
	(Ω)	(A)	(A)
(*c*)	0.6	6.67	5.77
(*d*)	1.0	5.45	3.95
(*e*)	1.5	4.44	2.83

Note that for $R = 0.4\,\Omega$, $I_s = I_p = 7.5$ A. When $R > 0.4\,\Omega$, the series connection gives the larger current through R.

Ammeters and Voltmeters

***105 ••**

Picture the Problem Let I be the current drawn from source and R_{eq} the resistance equivalent to R and 10 MΩ connected in parallel and apply Kirchhoff's loop rule to express the measured voltage V across R as a function of R.

The voltage measured by the voltmeter is given by:

$$V = IR_{eq} \qquad (1)$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$10\text{ V} - IR_{eq} - I(2R) = 0$$

Solve for I:

$$I = \frac{10\text{ V}}{R_{eq} + 2R}$$

Express R_{eq} in terms of R and 10-MΩ resistance in parallel with it:

$$\frac{1}{R_{eq}} = \frac{1}{10\,\text{M}\Omega} + \frac{1}{R}$$

Solve for R_{eq}:

$$R_{eq} = \frac{(10\,\text{M}\Omega)R}{R + 10\,\text{M}\Omega}$$

Substitute for I in equation (1) and simplify to obtain:

$$V = \left(\frac{10\text{ V}}{R_{eq} + 2R}\right)R_{eq} = \frac{10\text{ V}}{1 + \dfrac{2R}{R_{eq}}}$$

Substitute for R_{eq} and simplify to obtain:

$$V = \frac{(10\text{ V})(5\,\text{M}\Omega)}{R + 15\,\text{M}\Omega} \qquad (2)$$

(*a*) Evaluate equation (2) for $R = 1\ \text{k}\Omega$:

$$V = \frac{(10\,\text{V})(5\,\text{M}\Omega)}{1\,\text{k}\Omega + 15\,\text{M}\Omega} = \boxed{3.33\,\text{V}}$$

(*b*) Evaluate equation (2) for $R = 10\ \text{k}\Omega$:

$$V = \frac{(10\,\text{V})(5\,\text{M}\Omega)}{10\,\text{k}\Omega + 15\,\text{M}\Omega} = \boxed{3.33\,\text{V}}$$

(*c*) Evaluate equation (2) for $R = 1\ \text{M}\Omega$:

$$V = \frac{(10\,\text{V})(5\,\text{M}\Omega)}{1\,\text{M}\Omega + 15\,\text{M}\Omega} = \boxed{3.13\,\text{V}}$$

(*d*) Evaluate equation (2) for $R = 10\ \text{M}\Omega$:

$$V = \frac{(10\,\text{V})(5\,\text{M}\Omega)}{10\,\text{M}\Omega + 15\,\text{M}\Omega} = \boxed{2.00\,\text{V}}$$

(*e*) Evaluate equation (2) for $R = 100\ \text{M}\Omega$:

$$V = \frac{(10\,\text{V})(5\,\text{M}\Omega)}{100\,\text{M}\Omega + 15\,\text{M}\Omega} = \boxed{0.435\,\text{V}}$$

(*f*) Express the condition that the measured voltage to be within 10 percent of the *true* voltage V_{true}:

$$\frac{V_{\text{true}} - V}{V_{\text{true}}} = 1 - \frac{V}{V_{\text{true}}} < 0.1$$

Substitute for V and V_{true} to obtain:

$$1 - \frac{\dfrac{(10\,\text{V})(5\,\text{M}\Omega)}{R + 15\,\text{M}\Omega}}{IR} < 0.1$$

or, because $I = 10\ \text{V}/3R$,

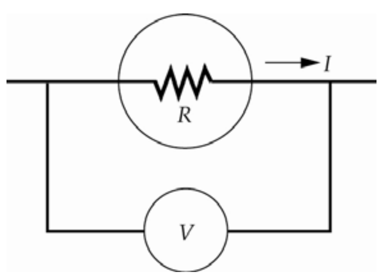
$$1 - \frac{\dfrac{(10\,\text{V})(5\,\text{M}\Omega)}{R + 15\,\text{M}\Omega}}{\dfrac{10}{3}\,\text{V}} < 0.1$$

Solve for R to obtain:

$$R < \frac{1.5\,\text{M}\Omega}{0.9} = \boxed{1.67\,\text{M}\Omega}$$

106 ••

Picture the Problem The diagram shows a voltmeter connected in parallel with a galvanometer movement whose internal resistance is R. We can apply Kirchhoff's loop rule to express R in terms of I and V.



Apply Kirchhoff's loop rule to the loop that includes the galvanometer movement and the voltmeter:

$$V - IR = 0$$

Solve for R:

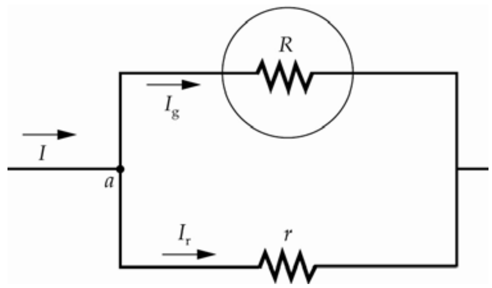
$$R = \frac{V}{I}$$

Substitute numerical values and evaluate R:

$$R = \frac{0.25\,\text{V}}{50\,\mu\text{A}} = \boxed{5.00\,\text{k}\Omega}$$

107 ••

Picture the Problem When there is a voltage drop of 0.25 V across this galvanometer, the meter reads full scale. The diagram shows the galvanometer movement with a resistor of resistance r in parallel. The purpose of this resistor is to limit the current through the movement to $I_g = 50\ \mu\text{A}$. We can apply Kirchhoff's loop rule to the circuit fragment containing the galvanometer movement and the shunt resistor to derive an expression for r.



Apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$-RI_g + rI_r = 0$$

Apply Kirchhoff's junction rule at point a to obtain:

$$I_r = I - I_g$$

Substitute for I_r in the loop equation:

$$-RI_g + r(I - I_g) = 0$$

Solve for r:

$$r = \frac{RI_g}{I - I_g}$$

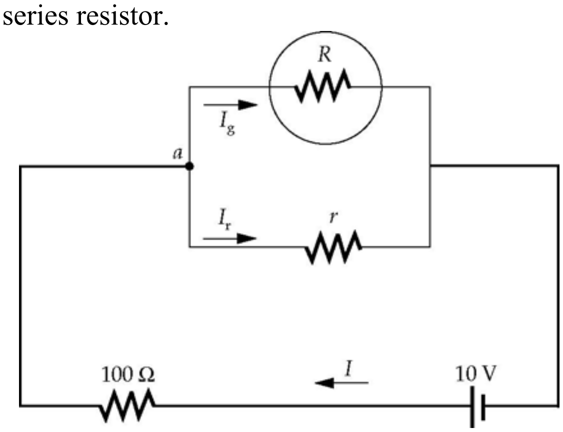
where $RI_g = 0.25$ V

Substitute numerical values and evaluate r:

$$r = \frac{0.25\,\text{V}}{100\,\text{mA} - 50\,\mu\text{A}} = \boxed{2.50\,\Omega}$$

108 ••

Picture the Problem The circuit diagram shows the ammeter connected in series with a 100-Ω resistor and a 10-V power supply. We can apply Kirchhoff's rules to obtain an expression for I as a function of r, I_g, the potential difference provided by the source, and the resistance of the series resistor.



(*a*) Apply Kirchhoff's loop rule to the inner loop of the circuit to obtain:

$$10\,\text{V} - (100\,\Omega)I - rI_r = 0$$

Apply Kirchhoff's junction rule at point *a* to obtain:

$$I_r = I - I_g$$

Substitute for I_r in the loop equation:

$$10\,\text{V} - (100\,\Omega)I - r(I - I_g) = 0$$

Solve for *I*:

$$I = \frac{10\,\text{V} + rI_g}{100\,\Omega + r} \qquad (1)$$

In Problem 107 it was established that $r = 2.50\ \Omega$. Substitute numerical values and evaluate *I*:

$$I = \frac{10\,\text{V} + (2.50\,\Omega)(50\,\mu\text{A})}{100\,\Omega + 2.50\,\Omega} = \boxed{97.6\,\text{mA}}$$

(*b*) Under these conditions, equation (1) becomes:

$$I = \frac{1\,\text{V} + rI_g}{10\,\Omega + r}$$

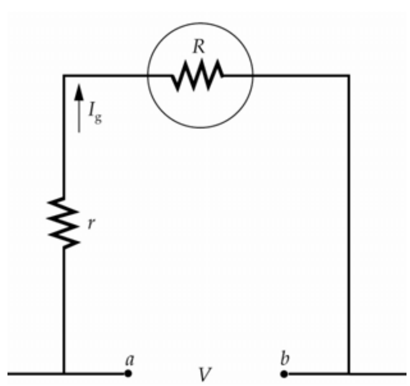
Substitute numerical values and evaluate *I*:

$$I = \frac{1\,\text{V} + (2.50\,\Omega)(50\,\mu\text{A})}{10\,\Omega + 2.50\,\Omega} = \boxed{80.0\,\text{mA}}$$

Remarks: Our result in (*b*) differs from that obtained in (*a*) by about 18 percent.

***109 ••**

Picture the Problem The circuit diagram shows a fragment of a circuit in which a resistor of resistance *r* is connected in series with the meter movement of Problem 106. The purpose of this resistor is to limit the current through the galvanometer movement to 50 μA and to produce a deflection of the galvanometer movement that is a measure of the potential difference *V*. We can apply Kirchhoff's loop rule to express *r* in terms of V_g, I_g, and *R*.



Apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$V - rI_g - RI_g = 0$$

Solve for *r*:

$$r = \frac{V - RI_g}{I_g} = \frac{V}{I_g} - R \qquad (1)$$

Use Ohm's law to relate the current I_g through the galvanometer movement to the potential difference V_g across it:

$$I_g = \frac{V_g}{R} \Rightarrow R = \frac{V_g}{I_g}$$

Use the values for V_g and I_g given in Problem 106 to evaluate R:

$$R = \frac{0.25\,\text{V}}{50\,\mu\text{A}} = 5000\,\Omega$$

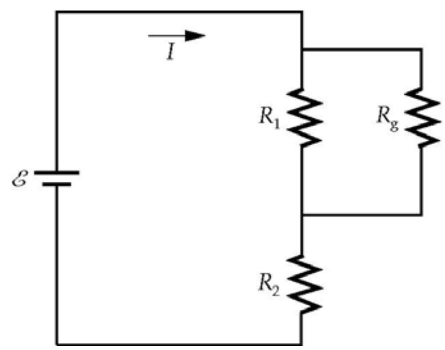
Substitute numerical values in equation (1) and evaluate r:

$$r = \frac{10\,\text{V}}{50\,\mu\text{A}} - 5000\,\Omega = \boxed{195\,\text{k}\Omega}$$

Remarks: The total series resistance is the sum of *r* and *R* or 200 kΩ.

110 ••

Picture the Problem The voltmeter shown in Figure 25-64 is equivalent to a resistor of resistance R_g = 200 kΩ as shown in the circuit diagram to the right. The voltage reading across R_1 is given by $V_1 = IR_1$. We can use Kirchhoff's loop rule and the expression for the equivalent resistance of two resistors in parallel to find I.



The voltage reading across R_1 is given by:

$$V_1 = IR_1$$

Apply Kirchhoff's loop rule to the loop including the source, R_1, and R_2:

$$\mathcal{E} - IR_{eq} - IR_2 = 0$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{R_{eq} + R_2}$$

Substitute for I in the expression for V_1:

$$V_1 = \frac{\mathcal{E}R_1}{R_{eq} + R_2} \qquad (1)$$

Express R_{eq} in terms of R_1 and R_g:

$$R_{eq} = \frac{R_1 R_g}{R_1 + R_g}$$

Substitute numerical values and evaluate R_{eq}:

$$R_{eq} = \frac{(200\,\text{k}\Omega)(200\,\text{k}\Omega)}{200\,\text{k}\Omega + 200\,\text{k}\Omega} = 100\,\text{k}\Omega$$

Substitute numerical values in equation (1) and evaluate V_1:

$$V_1 = \frac{(10\,\text{V})(200\,\text{k}\Omega)}{200\,\text{k}\Omega + 200\,\text{k}\Omega} = \boxed{5.00\,\text{V}}$$

RC Circuits

111 •

Picture the Problem We can use the definition of capacitance to find the initial charge on the capacitor and Ohm's law to find the initial current in the circuit. We can find the time constant of the circuit using its definition and the charge on the capacitor after 6 ms using $Q(t) = Q_0 e^{-t/\tau}$.

(*a*) Use the definition of capacitance to find the initial charge on the capacitor:

$$Q_0 = CV_0 = (6\,\mu\text{F})(100\,\text{V}) = \boxed{600\,\mu\text{C}}$$

(*b*) Apply Ohm's law to the resistor to obtain:

$$I_0 = \frac{V_0}{R} = \frac{100\,\text{V}}{500\,\Omega} = \boxed{0.200\,\text{A}}$$

(*c*) Use its definition to find the time constant of the circuit:

$$\tau = RC = (500\,\Omega)(6\,\mu\text{F}) = \boxed{3.00\,\text{ms}}$$

(*d*) Express the charge on the capacitor as a function of time:

$$Q(t) = Q_0 e^{-t/\tau}$$

Substitute numerical values and evaluate Q(6 ms):

$$Q(6\,\text{ms}) = (600\,\mu\text{C})e^{-6\,\text{ms}/3\,ms} = \boxed{81.2\,\mu\text{C}}$$

112 •

Picture the Problem We can use $U_0 = \frac{1}{2}CV_0^2$ to find the initial energy stored in the capacitor and $U(t) = \frac{1}{2}C(V_C(t))^2$ with $V_C(t) = V_0 e^{-t/\tau}$ to show that $U = U_0 e^{-2t/\tau}$.

(*a*) The initial energy stored in the capacitor is given by:

$$U_0 = \frac{1}{2}CV_0^2$$
$$= \frac{1}{2}(6\,\mu\text{F})(100\,\text{V})^2 = \boxed{30.0\,\text{mJ}}$$

(*b*) Express the energy stored in the discharging capacitor as a function of time:

$$U(t) = \frac{1}{2}C(V_C(t))^2$$

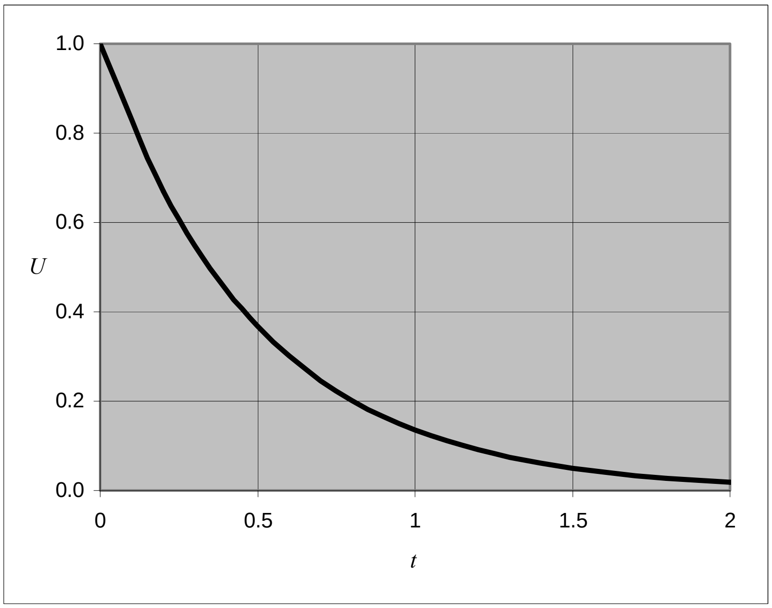
where

$$V_C(t) = V_0 e^{-t/\tau}$$

Substitute to obtain:

$$U(t) = \tfrac{1}{2}C\left(V_0 e^{-t/\tau}\right)^2 = \tfrac{1}{2}CV_0^2 e^{-2t/\tau} = \boxed{U_0 e^{-2t/\tau}}$$

(*c*) A graph of U versus t is shown below. U is in units of U_0 and t is in units of τ.



***113** ••

Picture the Problem We can find the resistance of the circuit from its time constant and use Ohm's law and the expression for the current in a charging *RC* circuit to express τ as a function of time, V_0, and $V(t)$.

Express the resistance of the resistor in terms of the time constant of the circuit:

$$R = \frac{\tau}{C} \qquad (1)$$

Using Ohm's law, express the voltage drop across the resistor as a function of time:

$$V(t) = I(t)R$$

Express the current in the circuit as a function of the elapsed time after the switch is closed:

$$I(t) = I_0 e^{-t/\tau}$$

Substitute to obtain:

$$V(t) = I_0 e^{-t/\tau}R = (I_0 R)e^{-t/\tau} = V_0 e^{-t/\tau}$$

Take the natural logarithm of both sides of the equation and solve for τ to obtain:

$$\tau = -\frac{t}{\ln\left[\frac{V(t)}{V_0}\right]}$$

Substitute in equation (1) to obtain:

$$R = -\frac{t}{C\ln\left[\frac{V(t)}{V_0}\right]}$$

Substitute numerical values and evaluate R using the data given for t = 4 s:

$$R = -\frac{4\,\text{s}}{(2\,\mu\text{F})\ln\left(\frac{20\,\text{V}}{50\,\text{V}}\right)} = \boxed{2.18\,\text{M}\Omega}$$

***114 ••**

Picture the Problem We can find the resistance of the circuit from its time constant and use the expression for the charge on a discharging capacitor as a function of time to express τ as a function of time, Q_0, and $Q(t)$.

Express the effective resistance across the capacitor in terms of the time constant of the circuit:

$$R = \frac{\tau}{C} \qquad (1)$$

Express the charge on the capacitor as a function of the elapsed time after the switch is closed:

$$Q(t) = Q_0 e^{-t/\tau}$$

Take the natural logarithm of both sides of the equation and solve for τ to obtain:

$$\tau = -\frac{t}{\ln\frac{Q(t)}{Q_0}}$$

Substitute in equation (1) to obtain:

$$R = -\frac{t}{C\ln\frac{Q(t)}{Q_0}}$$

Substitute numerical values and evaluate R:

$$R = -\frac{4\,\text{s}}{(0.12\,\mu\text{F})\ln\frac{\frac{1}{2}Q_0}{Q_0}} = \boxed{48.1\,\text{M}\Omega}$$

115 ••

Picture the Problem We can use the definition of capacitance to find the final charge on the capacitor and $Q(t) = Q_f\left(1 - e^{-t/\tau}\right)$ to express the charge on the capacitor as a function of time. In part (b) we can let $Q(t) = 0.99Q_f$ and solve for t to find the time required for the capacitor to reach 99% of its final charge.

(*a*) After a very long time has elapsed, the capacitor will be fully charged. Use the definition of capacitance to find its charge:

$$Q_f = CV = (1.6\,\mu\text{F})(5\,\text{V}) = \boxed{8.00\,\mu\text{C}}$$

(*b*) Express the charge on the capacitor as a function of time:

$$Q(t) = Q_f\left(1 - e^{-t/\tau}\right)$$

where $\tau = RC$.

When $Q = 0.99Q_f$:

$$0.99Q_f = Q_f\left(1 - e^{-t/\tau}\right)$$

or

$$0.01 = e^{-t/\tau}$$

Take the natural logarithm of both sides of the equation and solve for t to obtain:

$$t = -RC\ln(0.01)$$

Substitute numerical values and evaluate t:

$$t = -(10\,\text{k}\Omega)(1.6\,\mu\text{F})\ln(0.01) = \boxed{73.7\,\text{ms}}$$

116 ••

Picture the Problem We can use Kirchhoff's loop rule (conservation of energy) to find both the initial and steady-state currents drawn from the battery and Ohm's law to find the maximum voltage across the capacitor.

(*a*) Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$\mathcal{E} - (1.2\,\text{M}\Omega)I_0 - V_{C0} = 0$$

Because the capacitor initially is uncharged:

$$V_{C0} = 0$$

and

$$I_0 = \frac{\mathcal{E}}{1.2\,\text{M}\Omega} = \frac{120\,\text{V}}{1.2\,\text{M}\Omega} = \boxed{0.100\,\text{mA}}$$

(*b*) When a long time has passed:

$$I_{C\infty} = 0$$

Apply Kirchhoff's loop rule to a loop that includes the source and both resistors to obtain:

$$\mathcal{E} - (1.2\,\text{M}\Omega)I_\infty - (600\,\text{k}\Omega)I_\infty = 0$$

Solve for and evaluate I_∞:

$$I_\infty = \frac{\mathcal{E}}{1.2\,\text{M}\Omega + 600\,\text{k}\Omega} = \frac{120\,\text{V}}{1.2\,\text{M}\Omega + 600\,\text{k}\Omega} = \boxed{66.7\,\mu\text{A}}$$

(*c*) The maximum voltage across the capacitor equals the potential difference across the 600-kΩ under steady-state conditions. Apply Ohm's law to obtain:

$$V_{\text{C}\infty} = I_\infty R_{600\,\text{k}\Omega} = (66.7\,\mu\text{A})(600\,\text{k}\Omega) = \boxed{40.0\,\text{V}}$$

117 ••

Picture the Problem We can use $Q(t) = Q_\text{f}\left(1 - e^{-t/\tau}\right) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$ to find the charge on the capacitor at $t = \tau$ and differentiate this expression with respect to time to find the rate at which the charge is increasing (the current). The power supplied by the battery is given by $P_\tau = I_\tau \mathcal{E}$ and the power dissipated in the resistor by $P_{R,\tau} = I_\tau^2 R$. In part (f) we can differentiate $U(t) = Q^2(t)/2C$ with respect to time and evaluate the derivative at $t = \tau$ to find the rate at which the energy stored in the capacitor is increasing.

(*a*) Express the charge Q on the capacitor as a function of time:

$$Q(t) = Q_\text{f}\left(1 - e^{-t/\tau}\right) = C\mathcal{E}\left(1 - e^{-t/\tau}\right) \quad (1)$$

where $\tau = RC$.

Evaluate $Q(\tau)$ to obtain:

$$Q(\tau) = (1.5\,\mu\text{F})(6\,\text{V})\left(1 - e^{-1}\right) = \boxed{5.69\,\mu\text{C}}$$

(*b*) and (*c*) Differentiate equation (1) with respect to t to obtain:

$$\frac{dQ(t)}{dt} = I = I_0 e^{-t/\tau}$$

Apply Kirchhoff's loop rule to the circuit just after the circuit is completed to obtain:

$$\mathcal{E} - RI_0 - V_{\text{C}0} = 0$$

Because $V_{\text{C}0} = 0$ we have:

$$I_0 = \frac{\mathcal{E}}{R}$$

Substitute to obtain:

$$\frac{dQ(t)}{dt} = I(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

Substitute numerical values and evaluate $I(\tau)$:

$$I(\tau) = \frac{6\,\text{V}}{2\,\text{M}\Omega} e^{-1} = \boxed{1.10\,\mu\text{A}}$$

(*d*) Express the power supplied by the battery as the product of its emf and the current drawn from it at $t = \tau$:

$$P(\tau) = I(\tau)\mathcal{E} = (1.10\ \mu\text{C/s})(6\ \text{V}) = \boxed{6.60\ \mu\text{W}}$$

(*e*) The power dissipated in the resistor is given by:

$$P_R(\tau) = I^2(\tau)R = (1.10\ \mu\text{A})^2(2\ \text{M}\Omega) = \boxed{2.42\ \mu\text{W}}$$

(*f*) Express the energy stored in the capacitor as a function of time:

$$U(t) = \frac{Q^2(t)}{2C}$$

Differentiate this expression with respect to time to obtain:

$$\frac{dU(t)}{dt} = \frac{1}{2C}\frac{d}{dt}\left[Q^2(t)\right] = \frac{1}{2C}(2Q(t))\frac{dQ(t)}{dt} = \frac{Q(t)}{C}I(t)$$

Evaluate this expression when $t = \tau$ to obtain:

$$\frac{dU(\tau)}{dt} = \frac{Q(\tau)}{C}I(\tau) = \frac{5.69\ \mu\text{C}}{1.5\ \mu\text{F}}(1.10\ \mu\text{A}) = \boxed{4.17\ \mu\text{W}}$$

Remarks: Note that our answer for part (*f*) is the difference between the power delivered by the battery at $t = \tau$ and the rate at which energy is dissipated in the resistor at the same time.

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Picture the Problem We can apply Kirchhoff's junction rule to find the current in each branch of this circuit and then use the loop rule to obtain equations solvable for R_1, R_2, and R_3.

(*a*) Apply Kirchhoff's junction rule at the junction of the 5-μF capacitor and the 10-Ω and 50-Ω resistors under steady-state conditions:

$$I_{\text{bat}} = I_{10\,\Omega} + 5\ \text{A} \qquad (1)$$

Because the potential differences across the 5-μF capacitor and the 10-Ω resistor are the same:

$$I_{10\,\Omega} = \frac{V_{10\,\Omega}}{10\,\Omega} = \frac{V_{\text{C}}}{10\,\Omega}$$

Express the potential difference across the capacitor to its steady-state charge:

$$V_C = \frac{Q_f}{C}$$

Substitute to obtain:

$$I_{10\,\Omega} = \frac{Q_f}{(10\,\Omega)C}$$

Substitute in equation (1) to obtain:

$$I_{bat} = \frac{Q_f}{(10\,\Omega)C} + 5\,\text{A}$$

Substitute numerical values and evaluate I_{bat}:

$$I_{bat} = \frac{1000\,\mu\text{C}}{(10\,\Omega)(5\,\mu\text{F})} + 5\,\text{A} = \boxed{25.0\,\text{A}}$$

(*b*) Use Kirchhoff's junction rule to find the currents $I_{5\,\Omega}$, I_{R3}, and I_{R1}:

$$I_{5\Omega} = 10\,\text{A}\,,$$

$$I_{R3} = 15\,\text{A}\,,$$

and

$$I_{R1} = I_{bat} = 25\,\text{A}$$

Apply the loop rule to the loop that includes the battery, R_1, and the 50-Ω and 5-Ω resistors:

$$310\,\text{V} - (25\,\text{A})R_1 - (5\,\text{A})(50\,\Omega) - (10\,\text{A})(5\,\Omega) = 0$$

Solve for R_1 to obtain:

$$R_1 = \boxed{0.400\,\Omega}$$

Apply the loop rule to the loop that includes the battery, R_1, the 10-Ω resistor and R_3:

$$310\,\text{V} - (25\,\text{A})(0.4\,\Omega) - (20\,\text{A})(10\,\Omega) - (15\,\text{A})R_3 = 0$$

Solve for R_3 to obtain:

$$R_3 = \boxed{6.67\,\Omega}$$

Apply the loop rule to the loop that includes the 10-Ω and 50-Ω resistors and R_2:

$$-(20\,\text{A})(10\,\Omega) - (5\,\text{A})R_2 + (5\,\text{A})(50\,\Omega) = 0$$

Solve for R_2 to obtain:

$$R_2 = \boxed{10.0\,\Omega}$$

119 ••

Picture the Problem We can solve Equation 25-35 for dQ/dt and separate the variables in order to obtain the equation given above. Integrating this differential equation will yield Equation 25-36.

Solve Equation 25-35 for dQ/dt to obtain:

$$\frac{dQ}{dt} = \frac{\mathcal{E}C - Q}{RC}$$

Separate the variables to obtain:

$$\boxed{\frac{dQ}{\mathcal{E}C - Q} = \frac{dt}{RC}}$$

Integrate dQ' from 0 to Q and dt' from 0 to t:

$$\int_0^Q \frac{dQ'}{\mathcal{E}C - Q'} = \frac{1}{RC}\int_0^t dt'$$

and

$$\ln\left(\frac{\mathcal{E}C}{\mathcal{E}C - Q}\right) = \frac{t}{RC}$$

Transform from logarithmic to exponential form to obtain:

$$\frac{\mathcal{E}C}{\mathcal{E}C - Q} = e^{\frac{t}{RC}}$$

Solve for Q to obtain Equation 25-36:

$$Q = \mathcal{E}C\left(1 - e^{-t/RC}\right) = \boxed{Q_f\left(1 - e^{-t/RC}\right)}$$

***120 •••**

Picture the Problem We can find the time-to-discharge by expressing the voltage across the capacitor as a function of time and solving for t. We can use $U(t) = \frac{1}{2}CV^2(t)$ to find the energy released/stored in the capacitor when the lamp flashes. In part (c) we can integrate $dU_{bat} = \mathcal{E}dI(t)$ to find the energy supplied by the battery during the charging cycle.

(*a*) Express the voltage across the capacitor as a function of time:

$$V(t) = \frac{Q(t)}{C} = \frac{Q_f}{C}\left(1 - e^{-t/RC}\right) = V_f\left(1 - e^{-t/RC}\right)$$

Solve for t to obtain:

$$t = -RC\ln\left(1 - \frac{V(t)}{V_f}\right)$$

Substitute numerical values and evaluate t:

$$t = -(18\,\text{k}\Omega)(0.15\,\mu\text{F})\ln\left(1 - \frac{7\,\text{V}}{9\,\text{V}}\right) = \boxed{4.06\,\text{ms}}$$

(*b*) Express the energy stored in the capacitor as a function of time:

$$U(t) = \tfrac{1}{2}CV^2(t)$$

Substitute for $V(t)$ to obtain:

$$U(t)=\tfrac{1}{2}CV_f^2\left(1-e^{-t/RC}\right)^2$$

Substitute numerical values and evaluate $U(4.06\text{ ms})$:

$$U(4.06\,\text{ms})=\tfrac{1}{2}(0.15\,\mu\text{F})(9\,\text{V})^2\left(1-e^{-4.06\,\text{ms}/(18\,\text{k}\Omega)(0.15\,\mu\text{F})}\right)^2=\boxed{3.67\,\mu\text{J}}$$

(*c*) Relate the energy provided by the battery to its emf and the current it delivers:

$$U_{\text{bat}}(t)=\mathcal{E}\int_0^t I(t')dt'=\frac{\mathcal{E}^2}{R}\int_0^t e^{-t'/RC}dt'$$
$$=\frac{\mathcal{E}^2}{R}\left[RC\left(1-e^{-t/RC}\right)\right]$$
$$=C\mathcal{E}^2\left(1-e^{-t/RC}\right)$$

Substitute numerical values and evaluate $U_{\text{bat}}(4.06\text{ ms})$:

$$U_{\text{bat}}(4.06\,\text{ms})=(0.15\,\mu\text{F})(9\,\text{V})^2\left(1-e^{-4.06\,\text{ms}/(18\,\text{k}\Omega)(0.15\,\mu\text{F})}\right)=\boxed{9.45\,\mu\text{J}}$$

Express the fraction f of the energy supplied by the battery during the charging cycle that is dissipated in the resistor:

$$f=\frac{U_R}{U_{\text{bat}}}$$

Use conservation of energy to relate the energy supplied by the battery to the energy dissipated in the resistor and the energy released when the lamp flashes:

$$U_{\text{bat}}=U_R+U_{\text{flash}}$$
or
$$U_R=U_{\text{bat}}-U_{\text{flash}}$$

Substitute to obtain:

$$f=\frac{U_{\text{bat}}-U_{\text{flash}}}{U_{\text{bat}}}=1-\frac{U_{\text{flash}}}{U_{\text{bat}}}$$

Substitute numerical values and evaluate f:

$$f=1-\frac{3.67\,\mu\text{J}}{9.45\,\mu\text{J}}=\boxed{61.2\%}$$

***121 •••**

Picture the Problem Let $R_1 = 200\ \Omega$, $R_2 = 600\ \Omega$, I_1 and I_2 their currents, and I_3 the current into the capacitor. We can apply Kirchhoff's loop rule to find the initial battery current I_0 and the battery current I_∞ a long time after the switch is closed. In part (*c*) we can apply both the loop and junction rules to obtain equations that we can use to obtain a linear differential equation with constant coefficients describing the current in the 600-Ω resistor as a function of time. We can solve this differential equation by assuming a solution of a given form, differentiating this assumed solution and substituting it and its derivative in the differential equation. Equating coefficients, requiring the solution to hold for all values of the assumed constants, and invoking an initial condition will allow us to find the constants in the assumed solution.

(*a*) Apply Kirchhoff's loop rule to the circuit at the instant the switch is closed:

$$\mathcal{E} - (200\,\Omega)I_0 - V_{C0} = 0$$

Because the capacitor is initially uncharged:

$$V_{C0} = 0$$

Solve for and evaluate I_0:

$$I_0 = \frac{\mathcal{E}}{200\,\Omega} = \frac{50\,\text{V}}{200\,\Omega} = \boxed{0.250\,\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit after a long time has passed:

$$50\,\text{V} - (200\,\Omega)I_\infty - (600\,\Omega)I_\infty = 0$$

Solve for I_∞ to obtain:

$$I_\infty = \frac{50\,\text{V}}{800\,\Omega} = \boxed{62.5\,\text{mA}}$$

(*c*) Apply the junction rule at the junction between the 200-Ω resistor and the capacitor to obtain:

$$I_1 = I_2 + I_3 \qquad (1)$$

Apply the loop rule to the loop containing the source, the 200-Ω resistor and the capacitor to obtain:

$$\mathcal{E} - R_1 I_1 - \frac{Q}{C} = 0 \qquad (2)$$

Apply the loop rule to the loop containing the 600-Ω resistor and the capacitor to obtain:

$$\frac{Q}{C} - R_2 I_2 = 0 \qquad (3)$$

Differentiate equation (2) with respect to time to obtain:

$$\frac{d}{dt}\left[\mathcal{E} - R_1 I_1 - \frac{Q}{C}\right] = 0 - R_1\frac{dI_1}{dt} - \frac{1}{C}\frac{dQ}{dt}$$

$$= -R_1\frac{dI_1}{dt} - \frac{1}{C}I_3 = 0$$

or

$$R_1\frac{dI_1}{dt} = -\frac{1}{C}I_3 \qquad (4)$$

Differentiate equation (3) with respect to time to obtain:

$$\frac{d}{dt}\left[\frac{Q}{C}-R_2I_2\right]=\frac{1}{C}\frac{dQ}{dt}-R_2\frac{dI_2}{dt}=0$$

or

$$R_2\frac{dI_2}{dt}=\frac{1}{C}I_3 \qquad (5)$$

Using equation (1), substitute for I_3 in equation (5) to obtain:

$$\frac{dI_2}{dt}=\frac{1}{R_2C}\left(I_1-I_2\right) \qquad (6)$$

Solve equation (2) for I_1:

$$I_1=\frac{\mathcal{E}-Q/C}{R_1}=\frac{\mathcal{E}-R_2I_2}{R_1}$$

Substitute for I_1 in equation (6) and simplify to obtain the differential equation for I_2:

$$\frac{dI_2}{dt}=\frac{1}{R_2C}\left(\frac{\mathcal{E}-R_2I_2}{R_1}-I_2\right)$$

$$=\frac{\mathcal{E}}{R_1R_2C}-\left(\frac{R_1+R_2}{R_1R_2C}\right)I_2$$

To solve this linear differential equation with constant coefficients we can assume a solution of the form:

$$I_2(t)=a+be^{-t/\tau} \qquad (7)$$

Differentiate $I_2(t)$ with respect to time to obtain:

$$\frac{dI_2}{dt}=\frac{d}{dt}\left[a+be^{-t/\tau}\right]=-\frac{b}{\tau}e^{-t/\tau}$$

Substitute for I_2 and dI_2/dt to obtain:

$$-\frac{b}{\tau}e^{-t/\tau}=\frac{\mathcal{E}}{R_1R_2C}-\left(\frac{R_1+R_2}{R_1R_2C}\right)\left(a+be^{-t/\tau}\right)$$

Equate coefficients of $e^{-t/\tau}$ to obtain:

$$\tau=\frac{R_1R_2C}{R_1+R_2}$$

Requiring the equation to hold for all values of a yields:

$$a=\frac{\mathcal{E}}{R_1+R_2}$$

If I_2 is to be zero when $t=0$:

$$0=a+b$$

or

$$b=-a=-\frac{\mathcal{E}}{R_1+R_2}$$

Substitute in equation (7) to obtain:

$$I_2(t) = \frac{\mathcal{E}}{R_1 + R_2} - \frac{\mathcal{E}}{R_1 + R_2} e^{-t/\tau} = \frac{\mathcal{E}}{R_1 + R_2}\left(1 - e^{-t/\tau}\right)$$

where

$$\tau = \frac{R_1 R_2 C}{R_1 + R_2} = \frac{(200\,\Omega)(600\,\Omega)(5\,\mu\text{F})}{200\,\Omega + 600\,\Omega} = 0.750\,\text{ms}$$

Substitute numerical values and evaluate $I_2(t)$:

$$I_2(t) = \frac{50\,\text{V}}{200\,\Omega + 600\,\Omega}\left(1 - e^{-t/0.750\,\text{ms}}\right) = \boxed{(62.5\,\text{mA})\left(1 - e^{-t/0.750\,\text{ms}}\right)}$$

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Picture the Problem Let R_1 represent the 1.2-MΩ resistor and R_2 the 600-kΩ resistor. Immediately after switch S is closed, the capacitor has zero charge and so the potential difference across it (and the 600 kΩ-resistor) is zero. A long time after the switch is closed, the capacitor will be fully charged and the potential difference across it will be given by both Q/C and $I_\infty R_2$. When the switch is opened after having been closed for a long time, both the source and the 1.2-MΩ resistor will be out of the circuit and the fully charged capacitor will discharge through R_1. We can use Kirchhoff's loop to find the currents drawn from the source immediately after the switch is closed and a long time after the switch is closed, as well as the current in the *RC* circuit when the switch is again opened and the capacitor discharges through R_2.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed to obtain:

$$\mathcal{E} - I_0 R_1 - V_{C0} = 0$$

or, because $V_{C0} = 0$,

$$\mathcal{E} - I_0 R_1 = 0$$

Solve for and evaluate I_0:

$$I_0 = \frac{\mathcal{E}}{R_1} = \frac{50\,\text{V}}{1.2\,\text{M}\Omega} = \boxed{41.7\,\mu\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit a long time after the switch is closed to obtain:

$$\mathcal{E} - I_\infty R_1 - I_\infty R_2 = 0$$

Solve for and evaluate I_∞:

$$I_\infty = \frac{\mathcal{E}}{R_1 + R_2} = \frac{50\,\text{V}}{1.2\,\text{M}\Omega + 600\,\text{k}\Omega} = \boxed{27.8\,\mu\text{A}}$$

(*c*) Apply Kirchhoff's loop rule to the *RC* circuit sometime after the switch is opened and solve for $I(t)$ to obtain:

$$V_\text{C}(t) - R_2 I(t) = 0$$

or

$$I(t) = \frac{V_\text{C}(t)}{R_2}$$

Substitute for $V_\text{C}(t)$:

$$I(t) = \frac{V_{\text{C}\infty}}{R_2} e^{-t/\tau} = I_\infty e^{-t/\tau}$$

where $\tau = R_2 C$.

Substitute numerical values to obtain:

$$I(t) = (27.8\,\mu\text{A}) e^{-t/(600\,\text{k}\Omega)(2.5\,\mu\text{F})} = \boxed{(27.8\,\mu\text{A}) e^{-t/1.5\,\text{s}}}$$

123 •••

Picture the Problem In part (*a*) we can apply Kirchhoff's loop rule to the circuit immediately after the switch is closed in order to find the initial current I_0. We can find the time at which the voltage across the capacitor is 24 V by again applying Kirchhoff's loop rule to find the voltage across the resistor when this condition is satisfied and then using the expression $I(t) = I_0 e^{-t/\tau}$ for the current through the resistor as a function of time and solving for t.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

$$\mathcal{E} - 12\,\text{V} - I_0 R = 0$$

Solve for and evaluate I_0:

$$I_0 = \frac{\mathcal{E} - 12\,\text{V}}{R} = \frac{36\,\text{V} - 12\,\text{V}}{0.5\,\text{M}\Omega} = \boxed{48.0\,\mu\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit when $V_\text{C} = 24$ V and solve for V_R:

$$36\,\text{V} - 24\,\text{V} - I(t)R = 0$$

and

$$I(t)R = 12\,\text{V}$$

Express the current through the resistor as a function of I_0 and τ:

$$I(t) = I_0 e^{-t/\tau}$$

where $\tau = RC$.

Substitute to obtain:

$$RI_0e^{-t/\tau}=12\,\text{V}$$

or

$$e^{-t/\tau}=\frac{12\,\text{V}}{RI_0}$$

Take the natural logarithm of both sides of the equation to obtain:

$$-\frac{t}{\tau}=\ln\frac{12\,\text{V}}{RI_0}$$

Solve for t:

$$t=-\tau\ln\left(\frac{12\,\text{V}}{RI_0}\right)=-RC\ln\left(\frac{12\,\text{V}}{RI_0}\right)$$

Substitute numerical values and evaluate t:

$$t=-(0.5\,\text{M}\Omega)(2.5\,\mu\text{F})\ln\left[\frac{12\,\text{V}}{(0.5\,\text{M}\Omega)(48\,\mu\text{A})}\right]=\boxed{0.866\,\text{s}}$$

124 •••

Picture the Problem In part (*a*) we can apply Kirchhoff's loop rule to the circuit immediately after the switch is closed in order to find the initial current I_0. We can find the time at which the voltage across the capacitor is 24 V by again applying Kirchhoff's loop rule to find the voltage across the resistor when this condition is satisfied and then using the expression $I(t)=I_0e^{-t/\tau}$ for the current through the resistor as a function of time and solving for t.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

$$\mathcal{E}+12\,\text{V}-I_0R=0$$

Solve for and evaluate I_0:

$$I_0=\frac{\mathcal{E}+12\,\text{V}}{R}$$

$$=\frac{36\,\text{V}+12\,\text{V}}{0.5\,\text{M}\Omega}=\boxed{96.0\,\mu\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit when $V_C = 24$ V and solve for V_R:

$$36\,\text{V}-24\,\text{V}-I(t)R=0$$

and

$$I(t)R=12\,\text{V}$$

Express the current through the resistor as a function of I_0 and τ:

$$I(t)=I_0e^{-t/\tau}$$

where $\tau=RC.$

Substitute to obtain:

$$RI_0e^{-t/\tau} = 12\,\text{V}$$

or

$$e^{-t/\tau} = \frac{12\,\text{V}}{RI_0}$$

Take the natural logarithm of both sides of the equation to obtain:

$$-\frac{t}{\tau} = \ln\frac{12\,\text{V}}{RI_0}$$

Solve for t:

$$t = -\tau\ln\left(\frac{12\,\text{V}}{RI_0}\right) = -RC\ln\left(\frac{12\,\text{V}}{RI_0}\right)$$

Substitute numerical values and evaluate t:

$$t = -(0.5\,\text{M}\Omega)(2.5\,\mu\text{F})\ln\left[\frac{12\,\text{V}}{(0.5\,\text{M}\Omega)(96\,\mu\text{A})}\right] = \boxed{1.73\,\text{s}}$$

General Problems

***125 ••**

Determine the Concept Because all of the current drawn from the battery passes through R_1, we know that I_1 is greater than I_2 and I_3. Because $R_2 \neq R_3$, $I_2 \neq I_3$ and so (b) is false. Because $R_3 > R_2$, $I_3 < I_2$ and so (c) is false. $\boxed{(a)\text{ is correct.}}$

126 •• A 25-W lightbulb is connected in series with a 100-W lightbulb and a voltage V is placed across the combination. Which lightbulb is brighter? Explain.

Determine the Concept The 25-W bulb will be brighter. The brightness of a bulb is proportional to the power it dissipates. The resistance of the 25-W bulb is greater than that of the 100-W bulb, and in the series combination, the same current I flows through the bulbs. Hence, $I^2R_{25} > I^2R_{100}$.

127 •

Picture the Problem We can apply Ohm's law to find the current drawn from the battery and use Kirchhoff's loop rule to find the current in the 6-Ω resistor.

Using Ohm's law, express the current I_1 drawn from the battery:

$$I_1 = \frac{\mathcal{E}}{R_{eq}}$$

Find R_{eq}:

$$R_{eq} = 4\,\Omega + \frac{(6\,\Omega)(12\,\Omega)}{6\,\Omega + 12\,\Omega} = 8\,\Omega$$

Substitute and evaluate $I1$:

$$I_1 = \frac{24\,\text{V}}{8\,\Omega} = 3\,\text{A}$$

Apply Kirchhoff's loop rule to a loop that includes the battery and the 4-Ω and 6-Ω resistors:

$$24\,\text{V} - (4\,\Omega)(3\,\text{A}) - (6\,\Omega)I_2 = 0$$

Solve for I_2 to obtain:

$$I_2 = 2\,\text{A} \text{ and } \boxed{(b)\text{ is correct.}}$$

128 •

Picture the Problem We can use $P = I_{max}^2 R$ to find the maximum current the resistor can tolerate and Ohm's law to find the voltage across the resistor that will produce this current.

(*a*) Relate the maximum current the resistor can tolerate to its power and resistance:

$$P = I_{max}^2 R$$

Solve for and evaluate I_{max}:

$$I_{max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{5\,\text{W}}{10\,\Omega}} = \boxed{0.707\,\text{A}}$$

(*b*) Use Ohm's law to relate the voltage across the resistor to this maximum current:

$$V = I_{max}R = (0.707\,\text{A})(10\,\Omega) = \boxed{7.07\,\text{V}}$$

129 •

Picture the Problem We can use Ohm's law to find the short-circuit current drawn from the battery and the relationship between the terminal potential difference, the emf of the battery, and the current being drawn from it to find the terminal voltage when the battery is delivering a current of 20 A.

(*a*) Apply Ohm's law to the shorted battery to find the short-circuit current:

$$I_{sc} = \frac{\mathcal{E}}{r} = \frac{12\,\text{V}}{0.4\,\Omega} = \boxed{30.0\,\text{A}}$$

(*b*) Express the terminal voltage as the difference between the emf of the battery and the current being drawn from it:

$$V_{term} = \mathcal{E} - Ir$$

Substitute numerical values and evaluate V_{term}:

$$V_{term} = 12\,\text{V} - (20\,\text{A})(0.4\,\Omega) = \boxed{4.00\,\text{V}}$$

130 ••

Picture the Problem We can use Kirchhoff's loop rule to obtain two equations relating ε and r that we can solve simultaneously to find these quantities.

Use Kirchhoff's loop rule to relate the emf of the battery to the current drawn from it and the internal and external resistance:

$$\mathcal{E} - IR - Ir = 0 \qquad (1)$$

When I = 1.80 A and a 7.0-Ω resistor is connected across the battery terminals equation (1) becomes:

$$\mathcal{E} - (1.8\,\text{A})(7\,\Omega) - (1.8\,\text{A})r = 0$$

or

$$\mathcal{E} - 12.6\,\text{V} - (1.8\,\text{A})r = 0 \qquad (2)$$

When I = 2.20 A and a 12-Ω resistor is connected in parallel with the 7.0-Ω resistor:

$$\mathcal{E} - (2.2\,\text{A})R_{eq} - (2.2\,\text{A})r = 0$$

Find the equivalent resistance:

$$R_{eq} = \frac{(7\,\Omega)(12\,\Omega)}{7\,\Omega + 12\,\Omega} = 4.42\,\Omega$$

Substitute to obtain:

$$\mathcal{E} - (2.2\,\text{A})(4.42\,\Omega) - (2.2\,\text{A})r = 0$$

or

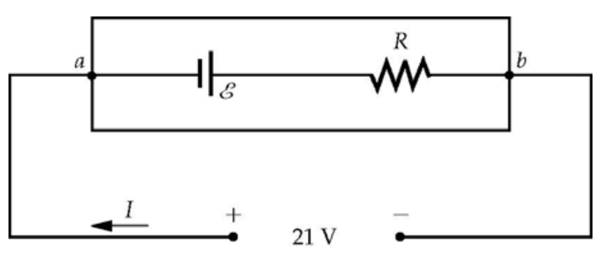
$$\mathcal{E} - 9.72\,\text{V} - (2.2\,\text{A})r = 0 \qquad (3)$$

Solve equations (2) and (3) simultaneously to obtain:

$$\mathcal{E} = \boxed{25.5\,\text{V}} \text{ and } r = \boxed{7.19\,\Omega}$$

***131** ••

Picture the Problem We can apply Kirchhoff's loop rule to the circuit that includes the box and the 21-V source to obtain two equations in the unknowns ε and R that we can solve simultaneously.



Apply Kirchhoff's loop rule to the circuit when the polarity of the 21-V source and the direction of the current are as shown in the diagram:

$$21\,\text{V} + \mathcal{E} - (1\,\text{A})R = 0$$

Apply Kirchhoff's loop rule to the circuit when the polarity of the source is reversed and the current is 2 A in the opposite direction:

$$-21\,\text{V} + \mathcal{E} + (2\,\text{A})R = 0$$

Solve these equations simultaneously to obtain:

$$R = \boxed{14.0\,\Omega} \text{ and } \mathcal{E} = \boxed{-7.00\,\text{V}}$$

132 ••

Picture the Problem When the switch is closed, the initial potential differences across the capacitors are zero (they have no charge) and the resistors in the bridge portion of the circuit are in parallel. When a long time has passed, the current through the capacitors will be zero and the resistors will be in series. In both cases, the application of Kirchhoff's loop rule to the entire circuit will yield the current in the circuit. To find the final charges on the capacitors we can use the definition of capacitance and apply Kirchhoff's loop rule to the loops containing two resistors and a capacitor to find the potential differences across the capacitors.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

$$50\,\text{V} - I_0(10\,\Omega) - I_0 R_{\text{eq}} = 0$$

Solve for I_0:

$$I_0 = \frac{50\,\text{V}}{10\,\Omega + R_{\text{eq}}}$$

Find the equivalent resistance of 15 Ω, 12 Ω, and 15 Ω in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{15\,\Omega} + \frac{1}{12\,\Omega} + \frac{1}{15\,\Omega}$$

and

$$R_{\text{eq}} = 4.62\,\Omega$$

Substitute for R_{eq} and evaluate I_0:

$$I_0 = \frac{50\,\text{V}}{10\,\Omega + 4.62\,\Omega} = \boxed{3.42\,\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit a long time after the switch is closed:

$$50\,\text{V} - I_\infty(10\,\Omega) - I_\infty R_{\text{eq}} = 0$$

Solve for I_∞:

$$I_\infty = \frac{50\,\text{V}}{10\,\Omega + R_{\text{eq}}}$$

Find the equivalent resistance of 15 Ω, 12 Ω, and 15 Ω in series:

$$R_{\text{eq}} = 15\,\Omega + 12\,\Omega + 15\,\Omega = 42\,\Omega$$

Substitute for R_{eq} and evaluate I_∞:

$$I_\infty = \frac{50\,\text{V}}{10\,\Omega + 42\,\Omega} = \boxed{0.962\,\text{A}}$$

(*c*) Using the definition of capacitance, express the charge on the capacitors in terms of their final potential differences:

$$Q_{10\,\mu\text{F}} = C_{10\,\mu\text{F}} V_{10\,\mu\text{F}} \qquad (1)$$

and

$$Q_{5\,\mu\text{F}} = C_{5\,\mu\text{F}} V_{5\,\mu\text{F}} \qquad (2)$$

Apply Kirchhoff's loop rule to the loop containing the 15-Ω and 12-Ω resistors and the 10 μF capacitor to obtain:

$$V_{10\,\mu\text{F}} - (15\,\Omega)I_\infty - (12\,\Omega)I_\infty = 0$$

Solve for $V_{10\,\mu\text{F}}$:

$$V_{10\,\mu\text{F}} = (27\,\Omega)I_\infty$$

Substitute in equation (1) and evaluate $Q_{10\,\mu\text{F}}$:

$$Q_{10\,\mu\text{F}} = C_{10\,\mu\text{F}}(27\,\Omega)I_\infty = (10\,\mu\text{F})(27\,\Omega)(0.962\,\text{A}) = \boxed{260\,\mu\text{C}}$$

Apply Kirchhoff's loop rule to the loop containing the 15-Ω and 12-Ω resistors and the 5 μF capacitor to obtain:

$$V_{5\,\mu\text{F}} - (15\,\Omega)I_\infty - (12\,\Omega)I_\infty = 0$$

Solve for $V_{5\,\mu\text{F}}$:

$$V_{5\,\mu\text{F}} = (27\,\Omega)I_\infty$$

Substitute in equation (2) and evaluate $Q_{5\,\mu\text{F}}$:

$$Q_{5\,\mu\text{F}} = C_{5\,\mu\text{F}}(27\,\Omega)I_\infty = (5\,\mu\text{F})(27\,\Omega)(0.962\,\text{A}) = \boxed{130\,\mu\text{C}}$$

***133** ••

Picture the Problem Let the current flowing through the galvanometer by I_G. By applying Kirchhoff's rules to the loops including 1) R_1, the galvanometer, and R_x, and 2) R_2, the galvanometer, and R_0, we can obtain two equations relating the unknown resistance to R_1, R_2 and R_0. Using $R = \rho L/A$ will allow us to express R_x in terms of the length of wire L_1 that corresponds to R_1 and the length of wire L_2 that corresponds to R_2.

Apply Kirchhoff's loop rule to the loop that includes R_1, the galvanometer, and R_x to obtain:

$$-R_1I_1 + R_xI_2 = 0 \quad (1)$$

Apply Kirchhoff's loop rule to the loop that includes R_2, the galvanometer, and R_0 to obtain:

$$-R_2(I_1 - I_G) + R_0(I_2 + I_G) = 0 \quad (2)$$

When the bridge is balanced, $I_G = 0$ and equations (1) and (2) become:

$$R_1I_1 = R_xI_2 \quad (3)$$

and

$$R_2I_1 = R_0I_2 \quad (4)$$

Divide equation (3) by equation (4) and solve for x to obtain:

$$R_x = R_0\frac{R_1}{R_2} \quad (5)$$

Express R_1 and R_2 in terms of their lengths, cross-sectional areas, and the resistivity of their wire:

$$R_1 = \rho\frac{L_1}{A} \text{ and } R_2 = \rho\frac{L_2}{A}$$

Substitute in equation (5) to obtain:

$$R_x = R_0\frac{L_1}{L_2}$$

(*a*) When the bridge balances at the 18-cm mark, $L_1 = 18$ cm, $L_2 = 82$ cm and:

$$R_x = (200\,\Omega)\frac{18\,\text{cm}}{82\,\text{cm}} = \boxed{43.9\,\Omega}$$

(*b*) When the bridge balances at the 60-cm mark, $L_1 = 60$ cm, $L_2 = 40$ cm and:

$$R_x = (200\,\Omega)\frac{60\,\text{cm}}{40\,\text{cm}} = \boxed{300\,\Omega}$$

(*c*) When the bridge balances at the 95-cm mark, $L_1 = 95$ cm, $L_2 = 5$ cm and:

$$R_x = (200\,\Omega)\frac{95\,\text{cm}}{5\,\text{cm}} = \boxed{3.80\,\text{k}\Omega}$$

134 ••

Picture the Problem Let the current flowing through the galvanometer by I_G. By applying Kirchhoff's rules to the loops including 1) R_1, the galvanometer, and R_x, and 2) R_2, the galvanometer, and R_0, we can obtain two equations relating the unknown resistance to R_1, R_2 and R_0. Using $R = \rho L/A$ will allow us to express R_x in terms of the length of wire L_1 that corresponds to R_1 and the length of wire L_2 that corresponds to R_2. To find the effect of an error of 2 mm in the location of the balance point we can use the relationship $\Delta R_x = (dR_x/dL)\Delta L$ to determine ΔR_x and then divide by $R_x = R_0\, L/(1-L)$ to find the fractional change (error) in R_x resulting from a given error in the determination of the balance point.

Apply Kirchhoff's loop rule to the loop that includes R_1, the galvanometer, and R_x to obtain:

$$-R_1I_1 + R_xI_2 = 0 \quad (1)$$

Apply Kirchhoff's loop rule to the loop that includes R_2, the galvanometer, and R_0 to obtain:

$$-R_2(I_1 - I_G) + R_0(I_2 + I_G) = 0 \quad (2)$$

When the bridge is balanced, $I_G = 0$ and equations (1) and (2) become:

$$R_1I_1 = R_xI_2 \quad (3)$$

and

$$R_2I_1 = R_0I_2 \quad (4)$$

Divide equation (3) by equation (4) and solve for x to obtain:

$$R_x = R_0\frac{R_1}{R_2} \quad (5)$$

Express R_1 and R_2 in terms of their lengths, cross-sectional areas, and the resistivity of their wire:

$$R_1 = \rho\frac{L_1}{A} \text{ and } R_2 = \rho\frac{L_2}{A}$$

Substitute in equation (5) to obtain:

$$R_x = R_0\frac{L_1}{L_2} \quad (6)$$

(*a*) When the bridge balances at the 98-cm mark, $L_1 = 98$ cm, $L_2 = 2$ cm and:

$$R_x = (200\,\Omega)\frac{98\,\text{cm}}{2\,\text{cm}} = \boxed{9.80\,\text{k}\Omega}$$

(*b*) Express R_x in terms of the distance to the balance point:

$$R_x = R_0\frac{L}{1-L}$$

Express the error ΔR_x in R_x resulting from an error ΔL in L:

$$\Delta R_x = \frac{dR_x}{dL}\Delta L = R_0\frac{d}{dL}\left[\frac{L}{1-L}\right]\Delta L$$

$$= R_0\frac{1}{(1-L)^2}\Delta L$$

Divide ΔR_x by R_x to obtain:

$$\frac{\Delta R_x}{R_x} = \frac{R_0 \frac{1}{(1-L)^2}\Delta L}{R_0 \frac{L}{1-L}} = \frac{1}{1-L}\frac{\Delta L}{L}$$

Evaluate $\Delta R_x/R_x$ for L = 98 cm and ΔL = 2 mm:

$$\frac{\Delta R_x}{R_x} = \frac{1\,\text{m}}{1\,\text{m} - 0.98\,\text{m}}\frac{2\,\text{mm}}{1\,\text{m}} = \boxed{10.0\%}$$

(*c*) Solve equation (6) for the ratio of L_1 to L_2:

$$\frac{L_1}{L_2} = \frac{R_x}{R_0}$$

For $L_1 = 50\,\text{cm}$, $L_2 = 50\,\text{cm}$, and $R_0 = R_x = 9.80\,\text{k}\Omega$. Hence, a resistor of approximately $10\,\text{k}\Omega$ will cause the bridge to balance near the 50 - cm mark.

135 ••

Picture the Problem Knowing the beam current and charge per proton, we can use $I = ne$ to determine the number of protons striking the target per second. The energy deposited per second is the power delivered to the target and is given by $P = IV$. We can find the elapsed time before the target temperature rises 300C° using $\Delta Q = P\Delta t = mc_{\text{Cu}}\Delta T$.

(*a*) Relate the current to the number of protons per second *n* arriving at the target:

$$I = ne$$

Solve for and evaluate *n*:

$$n = \frac{I}{e} = \frac{3.50\,\mu\text{A}}{1.60\times10^{-19}\,\text{C}} = \boxed{2.19\times10^{13}\,/\text{s}}$$

(*b*) Express the power of the beam in terms of the beam current and energy:

$$P = IV = (3.5\,\mu\text{A})(60\,\text{MeV}) = \boxed{210\,\text{J/s}}$$

(*c*) Relate the energy delivered to the target to its heat capacity and temperature change:

$$\Delta Q = P\Delta t = C_{\text{Cu}}\Delta T = mc_{\text{Cu}}\Delta T$$

Solve for Δt:

$$\Delta t = \frac{mc_{\text{Cu}}\Delta T}{P}$$

Substitute numerical values (see Table 19-1 for the specific heat of copper)and evaluate Δt:

$$\Delta t = \frac{(50\,\text{g})(0.386\,\text{kJ/kg}\cdot\text{K})(300\text{C}^\circ)}{210\,\text{J/s}} = \boxed{27.6\,\text{s}}$$

136 ••

Picture the Problem We can use the definition of current to express the current delivered by the belt in terms of the surface charge density, width, and speed of the belt. The minimum power needed to drive the belt can be found from *P* = *IV*.

(*a*) Use its definition to express the current carried by the belt:

$$I = \frac{dQ}{dt} = \sigma w \frac{dx}{dt} = \sigma w v$$

Substitute numerical values and evaluate *I*:

$$I = (5\,\text{mC/m}^2)(0.5\,\text{m})(20\,\text{m/s}) = \boxed{50.0\,\text{mA}}$$

(*b*) Express the minimum power of the motor in terms of the current delivered and the potential of the charge:

$$P = IV$$

Substitute numerical values and evaluate *P*:

$$P = (50\,\text{mA})(100\,\text{kV}) = \boxed{5.00\,\text{kW}}$$

137 ••

Picture the Problem We can differentiate the expression relating the amount of heat required to produce a given temperature change with respect to time to express the mass flow-rate required to maintain the temperature of the coils at 50°C. We can then use the definition of density to find the necessary volume flow rate.

Express the heat that must be dissipated in terms of the specific heat and mass of the water and the desired temperature change of the water:

$$Q = mc_{\text{water}}\Delta T$$

Differentiate this expression with respect to time to obtain an expression for the power dissipation:

$$P = \frac{dQ}{dt} = \frac{dm}{dt}c_{\text{water}}\Delta T$$

Solve for dm/dt:

$$\frac{dm}{dt}=\frac{P}{c_{\text{water}}\Delta T}$$

Substitute for the power dissipated to obtain:

$$\frac{dm}{dt}=\frac{IV}{c_{\text{water}}\Delta T}$$

Substitute numerical values and evaluate dm/dt:

$$\frac{dm}{dt}=\frac{(100\,\text{A})(240\,\text{V})}{(4.18\,\text{kJ/kg}\cdot\text{K})(50°\text{C}-15°\text{C})}=0.164\,\text{kg/s}$$

Using the definition of density, express the volume flow rate in terms of the mass flow rate to obtain:

$$\frac{dV}{dt}=\frac{1}{\rho}\frac{dm}{dt}=\frac{0.164\,\text{kg/s}}{10^3\,\text{kg/m}^3}=\left(0.164\times10^{-3}\,\text{m}^3/\text{s}\right)\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right)=\boxed{0.164\,\text{L/s}}$$

138 ••

Picture the Problem We can use the expressions for the capacitance of a dielectric-filled parallel-plate capacitor and the resistance of a conductor to show that $RC = \in_0\rho\kappa$.

Express the capacitance of the dielectric-filled parallel-plate capacitor:

$$C=\frac{\kappa\in_0 A}{d}$$

Express the resistance of a conductor with the same dimensions:

$$R=\frac{\rho d}{A}$$

The product of C and R is:

$$RC=\frac{\rho d}{A}\frac{\kappa\in_0 A}{d}=\boxed{\in_0\rho\kappa}$$

139 ••

Picture the Problem We can use the expressions for the capacitance of a dielectric-filled cylindrical capacitor and the resistance of a cylindrical conductor to show that $RC = \in_0\rho\kappa$.

Express the capacitance of the dielectric-filled cylindrical capacitor whose inner and outer radii are r_1 and r_2, respectively:

$$C=\frac{2\pi\ell\kappa\in_0}{\ln\left(\frac{r_2}{r_1}\right)}$$

where ℓ is the length of the capacitor.

Express the resistance of a cylindrical resistor with the same dimensions:

$$R=\frac{\rho\ln\left(\frac{r_2}{r_1}\right)}{2\pi\ell}$$

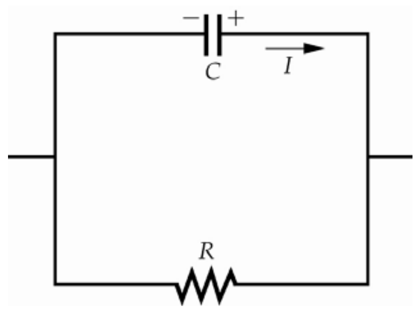
The product of C and R is:

$$RC = \frac{\rho \ln\left(\frac{r_2}{r_1}\right)}{2\pi\ell} \frac{2\pi\ell\kappa \in_0}{\ln\left(\frac{r_2}{r_1}\right)} = \boxed{\in_0 \rho\kappa}$$

This result holds independently of the geometries of the capacitor and the resistor.

***140** ••

Picture the Problem We'll assume that the capacitor is fully charged initially and apply Kirchhoff's loop rule to the circuit fragment to obtain the differential equation describing the discharge of the leaky capacitor. We'll show that the solution to this equation is the familiar expression for an exponential decay with time constant $\tau = \in_0 \rho\kappa$.



(*a*) If we think of the leaky capacitor as a resistor/capacitor combination, the voltage drop across the resistor must be the same as voltage drop across the capacitor. Hence, they must be in parallel.

(*b*) Assuming that the capacitor is initially fully charged, apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$\frac{Q}{C} - RI = 0$$

or, because $I = -\frac{dQ}{dt}$,

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0$$

Separate variables in this differential equation to obtain:

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

From Problems 138 and 139 we have:

$$RC = \in_0 \rho\kappa$$

Substitute for RC in the differential equation to obtain:

$$\frac{dQ}{Q} = -\frac{1}{\in_0 \rho\kappa}dt$$

Integrate this equation from $Q' = Q_0$ to Q to obtain:

$$Q = Q_0 e^{-t/\tau}$$

where

$$\tau = \boxed{\in_0 \rho\kappa}$$

(*c*) Because $Q/Q_0 = 0.1$:

$$e^{-t/\tau} = 0.1$$

Solve for *t* by taking the natural logarithm of both sides of the equation:

$$-\frac{t}{\tau} = \ln 0.1 \Rightarrow t = -\in_0 \rho\kappa \ln 0.1$$

Substitute numerical values and evaluate *t*:

$$t = -\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(9\times10^{13}\ \Omega\cdot\text{m}\right)(5)\ln 0.1 = 9.17\times10^3\ \text{s} = \boxed{2.55\ \text{h}}$$

141 •••

Picture the Problem We can use its definition to find the time constant of the charging circuit in part (*a*). In part (*b*) we can use the expression for the potential difference as a function of time across a charging capacitor and utilize the hint given in the problem statement to show that the voltage across the capacitor increases almost linearly over the time required to bring the potential across the switch to its critical value. In part (*c*) we can use the result derived in part (*b*) to find the value of R_1 such that *C* charges from 0.2 V to 4.2 V in 0.1 s. In part (*d*) we can use the expression for the potential difference as a function of time across a discharging capacitor to find the discharge time. Finally, in part (*e*) we can integrate I^2R_1 over the discharge time to find the rate at which energy is dissipated in R_1 during the discharge of the capacitor and use the difference in the energy stored in the capacitor initially and when the switch opens to find the rate of energy dissipation in resistance of the capacitor.

(*a*) When the capacitor is charging, the switch is open and the resistance in the charging circuit is R_1. Hence:

$$\tau = R_1C = (0.5\ \text{M}\Omega)(0.02\ \mu\text{F}) = \boxed{10.0\ \text{ms}}$$

(*b*) Express the voltage across the charging capacitor as a function of time:

$$V(t) = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$

Solve for the exponential term to obtain:

$$e^{-t/\tau} = 1 - \frac{V(t)}{\mathcal{E}} \qquad (1)$$

Noting that $V(t) << \varepsilon$, let $\eta = V(t)/\varepsilon$

$$e^{-t/\tau} = 1 - \eta$$

or

$$e^{t/\tau} = (1-\eta)^{-1} \approx 1 + \eta$$

because $\eta << 1$.

Use the power series for e^x to expand $e^{t/\tau}$:

$$e^{t/\tau} = 1 + \frac{t}{\tau} + \frac{1}{2!}\left(\frac{t}{\tau}\right)^2 + \ldots$$
$$\approx 1 + \frac{1}{\tau}t$$
provided $t/\tau << 1$.

Substitute in equation (1) to obtain:

$$1 + \frac{1}{\tau}t \approx 1 + \eta = 1 + \frac{V(t)}{\mathcal{E}}$$

Solve for t to obtain the linear relationship:

$$\boxed{V(t) = \frac{\mathcal{E}}{\tau}t} \qquad (1)$$

(c) Using the result derived in (b), relate the time Δt required to change the voltage across the capacitor by an amount ΔV to ΔV:

$$\Delta V(t) = \frac{\mathcal{E}}{\tau}\Delta t$$
or
$$\tau = R_1 C = \frac{\mathcal{E}}{\Delta V(t)}\Delta t$$

Solve for R_1:

$$R_1 = \frac{\mathcal{E}}{C\Delta V(t)}\Delta t$$

Substitute numerical values and evaluate R_1:

$$R_1 = \frac{(800\,\text{V})(0.1\,\text{s})}{(0.02\,\mu\text{F})(4.2\,\text{V} - 0.2\,\text{V})} = \boxed{1.00\,\text{G}\Omega}$$

(d) Express the potential difference across the capacitor as a function of time:

$$V_\text{C}(t) = V_\text{C0}e^{-t/\tau'}$$
where
$\tau' = R_2C$.

Solve for t to obtain:

$$t = -\tau' \ln\left(\frac{V_\text{C}(t)}{V_\text{C0}}\right) = -R_2C\ln\left(\frac{V_\text{C}(t)}{V_\text{C0}}\right)$$

Substitute numerical values and evaluate t:

$$t = -(0.001\,\Omega)(0.02\,\mu\text{F})\ln\left(\frac{0.2\,\text{V}}{4.2\,\text{V}}\right) = \boxed{60.9\,\text{ps}}$$

(e) Express the rate at which energy is dissipated in R_1 as a function of its

$$P_1 = \frac{\Delta E_1}{\Delta t} = I^2 R_1$$

resistance and the current through it:

Because the current varies with time, we need to integrate over time to find ΔE_1:

$$\Delta E_1 = \int I^2 R_1 dt = \int \left(\frac{V(t)}{R_1}\right)^2 R_1 dt$$

$$= \int \left(\frac{\mathcal{E}\, t}{\tau\, R_1}\right)^2 R_1 dt$$

$$= \left(\frac{\mathcal{E}}{\tau}\right)^2 \frac{1}{R_1} \int_{0.005\,\text{s}}^{0.105\,\text{s}} t^2 dt$$

$$= \left(\frac{800\,\text{V}}{20\,\text{s}}\right)^2 \frac{1}{1\,\text{G}\Omega} \left[\frac{t^3}{3}\right]_{0.005\,\text{s}}^{0.105\,\text{s}}$$

$$= 6.17\times 10^{-10}\ \text{J}$$

Substitute and evaluate P_1:

$$P_1 = \frac{6.17\times 10^{-10}\ \text{J}}{0.1\,\text{s}} = \boxed{6.17\,\text{nW}}$$

Express the rate at which energy is dissipated in the switch resistance:

$$P_2 = \frac{\Delta U_\text{C}}{\Delta t} = \frac{U_\text{Ci} - U_\text{Cf}}{\Delta t}$$

$$= \frac{\frac{1}{2}CV_\text{i}^2 - \frac{1}{2}CV_\text{f}^2}{\Delta t} = \frac{\frac{1}{2}C\left(V_\text{i}^2 - V_\text{f}^2\right)}{\Delta t}$$

Substitute numerical values and evaluate P_2:

$$P_2 = \frac{\frac{1}{2}(0.02\,\mu\text{F})\left[(4.2\,\text{V})^2 - (0.2\,\text{V})^2\right]}{60.9\,\text{ps}}$$

$$= \boxed{2.89\,\text{kW}}$$

142 •••

Picture the Problem We can apply both the loop and junction rules to obtain equations that we can use to obtain a linear differential equation with constant coefficients describing the current in R_2 as a function of time. We can solve this differential equation by assuming a solution of an appropriate form, differentiating this assumed solution and substituting it and its derivative in the differential equation. Equating coefficients, requiring the solution to hold for all values of the assumed constants, and invoking an initial condition will allow us to find the constants in the assumed solution. Once we know how the current varies with time in R_2, we can express the potential difference across it (as well as across C because they are in parallel). To find the voltage across the capacitor at $t = 8$ s, we can express the dependence of the voltage on time for a discharging capacitor (C is discharging after $t = 2$ s) and evaluate this function, with a time constant differing from that found in (*a*), at $t = 6$ s.

(*a*) Apply the junction rule at the junction between the two resistors to obtain:

$$I_1 = I_2 + I_3 \quad (1)$$

Apply the loop rule to the loop containing the source, R_1, and the capacitor to obtain:

$$\mathcal{E} - R_1 I_1 - \frac{Q}{C} = 0 \quad (2)$$

Apply the loop rule to the loop containing R_2 and the capacitor to obtain:

$$\frac{Q}{C} - R_2 I_2 = 0 \quad (3)$$

Differentiate equation (2) with respect to time to obtain:

$$\frac{d}{dt}\left[\mathcal{E} - R_1 I_1 - \frac{Q}{C}\right] = 0 - R_1\frac{dI_1}{dt} - \frac{1}{C}\frac{dQ}{dt}$$
$$= -R_1\frac{dI_1}{dt} - \frac{1}{C}I_3 = 0$$

or

$$R_1\frac{dI_1}{dt} = -\frac{1}{C}I_3 \quad (4)$$

Differentiate equation (3) with respect to time to obtain:

$$\frac{d}{dt}\left[\frac{Q}{C} - R_2 I_2\right] = \frac{1}{C}\frac{dQ}{dt} - R_2\frac{dI_2}{dt} = 0$$

or

$$R_2\frac{dI_2}{dt} = \frac{1}{C}I_3 \quad (5)$$

Using equation (1), substitute for I_3 in equation (5) to obtain:

$$\frac{dI_2}{dt} = \frac{1}{R_2 C}\left(I_1 - I_2\right) \quad (6)$$

Solve equation (2) for I_1:

$$I_1 = \frac{\mathcal{E} - Q/C}{R_1} = \frac{\mathcal{E} - R_2 I_2}{R_1}$$

Substitute for I_1 in equation (6) and simplify to obtain the differential equation for I_2:

$$\frac{dI_2}{dt} = \frac{1}{R_2 C}\left(\frac{\mathcal{E} - R_2 I_2}{R_1} - I_2\right)$$
$$= \frac{\mathcal{E}}{R_1 R_2 C} - \left(\frac{R_1 + R_2}{R_1 R_2 C}\right)I_2$$

To solve this linear differential equation with constant coefficients we can assume a solution of the form:

$$I_2(t) = a + be^{-t/\tau} \quad (7)$$

Differentiate $I_2(t)$ with respect to time to obtain:

$$\frac{dI_2}{dt}=\frac{d}{dt}\left[a+be^{-t/\tau}\right]=-\frac{b}{\tau}e^{-t/\tau}$$

Substitute for I_2 and dI_2/dt to obtain:

$$-\frac{b}{\tau}e^{-t/\tau}=\frac{\mathcal{E}}{R_1R_2C}-\left(\frac{R_1+R_2}{R_1R_2C}\right)\left(a+be^{-t/\tau}\right)$$

Equate coefficients of $e^{-t/\tau}$ to obtain:

$$\tau=\frac{R_1R_2C}{R_1+R_2}$$

Requiring the equation to hold for all values of a yields:

$$a=\frac{\mathcal{E}}{R_1+R_2}$$

If I_2 is to be zero when $t = 0$:

$$0=a+b$$

or

$$b=-a=-\frac{\mathcal{E}}{R_1+R_2}$$

Substitute in equation (7) to obtain:

$$I_2(t)=\frac{\mathcal{E}}{R_1+R_2}-\frac{\mathcal{E}}{R_1+R_2}e^{-t/\tau}=\frac{\mathcal{E}}{R_1+R_2}\left(1-e^{-t/\tau}\right)$$

where

$$\tau=\frac{R_1R_2C}{R_1+R_2}$$

Substitute numerical values and evaluate τ:

$$\tau=\frac{(2\,\text{M}\Omega)(5\,\text{M}\Omega)(1\,\mu\text{F})}{2\,\text{M}\Omega+5\,\text{M}\Omega}=1.43\,\text{s}$$

Substitute numerical values and evaluate $I_2(t)$:-

$$I_2(t)=\frac{10\,\text{V}}{2\,\text{M}\Omega+5\,\text{M}\Omega}\left(1-e^{-t/1.43\text{s}}\right)=(1.43\,\mu\text{A})\left(1-e^{-t/1.43\text{s}}\right)$$

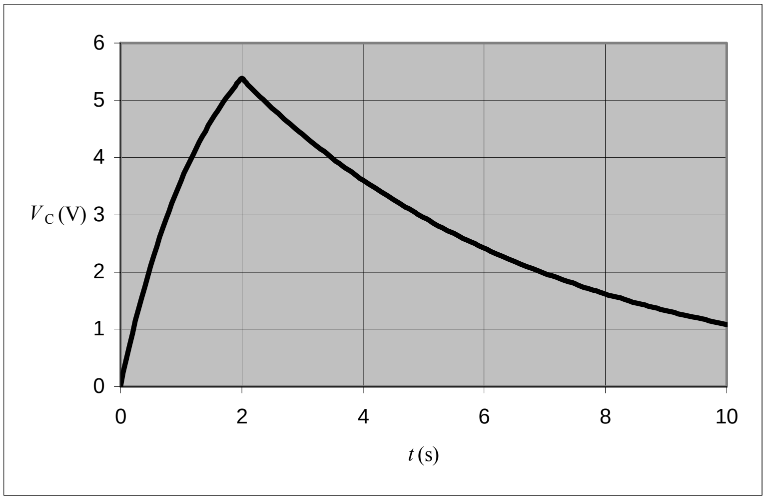
Because C and R_2 are in parallel, they have a common potential difference given by:

$$V_C(t)=V_2(t)=I_2(t)R_2=(1.43\,\mu\text{A})(5\,\text{M}\Omega)\left(1-e^{-t/1.43\text{s}}\right)=(7.15\,\text{V})\left(1-e^{-t/1.43\text{s}}\right)$$

Evaluate V_C at $t = 2$ s:

$$V_C(2\,\text{s})=(7.15\,\text{V})\left(1-e^{-2\text{s}/1.43\text{s}}\right)=5.38\,\text{V}$$

The voltage across the capacitor as a function of time is shown in the figure. The current through the 5-MΩ resistor R_2 follows the same time course, its value being $V_C/(5\times10^6)$ A.



(*b*) The value of V_C at $t = 2$ s has already been determined to be:

$$V_C(2\,\text{s}) = \boxed{5.38\,\text{V}}$$

When S is opened at $t = 2$ s, C discharges through R_2 with a time constant given by:

$$\tau' = R_2C = (5\,\text{M}\Omega)(1\,\mu\text{F}) = 5\,\text{s}$$

Express the potential difference across C as a function of time:

$$V_C(t) = V_{C0}e^{-t/\tau'} = (5.38\,\text{V})e^{-t/5\text{s}}$$

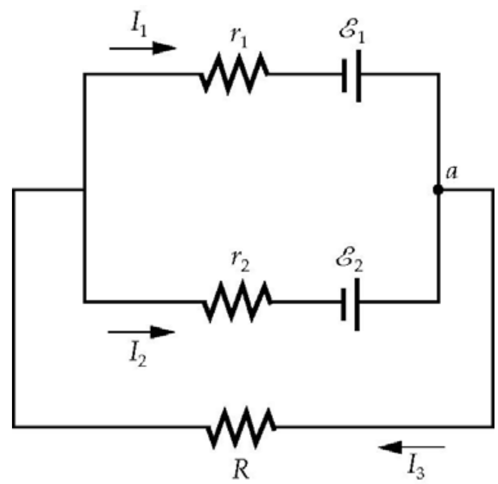
Evaluate V_C at $t = 8$ s to obtain:

$$V_C(8\,\text{s}) = (5.38\,\text{V})e^{-6\text{s}/5\text{s}} = \boxed{1.62\,\text{V}}$$

in good agreement with the graph.

143 •••

Picture the Problem Let I_1 be the current delivered by ε_1, I_2 the current delivered by ε_2, and I_3 the current through the resistor R. We can apply Kirchhoff's rules to obtain three equations in the unknowns I_1, I_2, and I_3 that we can solve simultaneously to find I_3. We can then express the power delivered by the sources to R. Setting the derivative of this expression equal to zero will allow us to solve for the value of R that maximizes the power delivered by the sources.



Apply Kirchhoff's junction rule at a to obtain:

$$I_1 + I_2 = I_3 \quad (1)$$

Apply the loop rule around the outside
of the circuit to obtain:

$$\mathcal{E}_1 - I_3R - r_1I_1 = 0 \quad (2)$$

Apply the loop rule around the inside of the circuit to obtain:

$$\mathcal{E}_2 - I_3R - r_2I_2 = 0 \quad (3)$$

Eliminate I_1 from equations (1) and (2) to obtain:

$$\mathcal{E}_1 - I_3R - r_1(I_3 - I_2) = 0 \quad (4)$$

Solve equation (3) for I_2 to obtain:

$$I_2 = \frac{\mathcal{E}_2 - I_3R}{r_2}$$

Substitute for I_2 in equation (4) to obtain:

$$\mathcal{E}_1 - I_3R - r_1\left(I_3 - \frac{\mathcal{E}_2 - I_3R}{r_2}\right) = 0$$

Solve for I_3 to obtain:

$$I_3 = \frac{\mathcal{E}_1r_2 + \mathcal{E}_2r_1}{r_1r_2 + R(r_1 + r_2)}$$

Express the power delivered to R:

$$P = I_3^2 R = \left(\frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 r_2 + R(r_1 + r_2)}\right)^2 R$$

$$= \left(\frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}\right)^2 \left[\frac{R}{(R+A)^2}\right]$$

where

$$A = \frac{r_1 r_2}{r_1 + r_2}$$

Noting that the quantity in parentheses is independent of R and that therefore we can ignore it, differentiate P with respect to R and set the derivative equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left[\frac{R}{(R+A)^2}\right]$$

$$= \frac{(R+A)^2 - R\frac{d}{dR}(R+A)^2}{(R+A)^4}$$

$$= \frac{(R+A)^2 - 2R(R+A)}{(R+A)^4}$$

$$= 0 \text{ for extrema}$$

Solve for R to obtain:

$$R = A = \frac{r_1 r_2}{r_1 + r_2}$$

To establish that this value for R corresponds to a maximum, we need to evaluate the second derivative of P with respect to R at $R = A$ and show that this quantity is negative, i.e., concave downward:

$$\frac{d^2P}{dR^2} = \frac{d}{dR}\left[\frac{(R+A)^2 - 2R(R+A)}{(R+A)^4}\right]$$

$$= \frac{2R - 4A}{(R+A)^4}$$

and

$$\left.\frac{d^2P}{dR^2}\right|_{R=A} = \frac{-2A}{(R+A)^4} < 0$$

We can conclude that:

$$R = \boxed{\frac{r_1 r_2}{r_1 + r_2}} \text{ maximizes the power}$$

delivered by the sources.

***144** •••

Picture the Problem Let Q_1 and Q_2 represent the final charges on the capacitors C_1 and C_2. Knowing that charge is conserved as it is redistributed to the two capacitors and that the final-state potential differences across the two capacitors will be the same, we can obtain two equations in the unknowns Q_1 and Q_2 that we can solve simultaneously. We can compare the initial and final energies stored in this system by expressing and simplifying their ratio. We can account for any difference between these energies by considering the role of the resistor in the circuit.

(*a*) Relate the total charge stored initially to the final charges Q_1 and Q_2 on C_1 and C_2:

$$Q = C_1V_0 = Q_1 + Q_2 \qquad (1)$$

Because, in their final state, the potential differences across the two capacitors will be the same:

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad (2)$$

Solve equation (2) for $Q2$ and substitute in equation (1) to obtain:

$$\frac{C_1}{C_2}Q_2 + Q_2 = C_1V_0$$

Solve for Q_2 to obtain:

$$Q_2 = \boxed{\frac{C_1C_2}{C_1 + C_2}V_0}$$

Substitute in either (1) or (2) and solve for Q_1 to obtain:

$$Q_1 = \boxed{\frac{C_1^2}{C_1 + C_2}V_0}$$

(*b*) Express the ratio of the initial and final energies of the system:

$$\frac{U_\text{i}}{U_\text{f}} = \frac{\frac{1}{2}C_1V_0^2}{\frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2}} = \frac{C_1V_0^2}{\frac{\left(\frac{C_1^2}{C_1 + C_2}V_0\right)^2}{C_1} + \frac{\left(\frac{C_1C_2}{C_1 + C_2}V_0\right)^2}{C_2}}$$

Simplify this expression further to obtain:

$$\frac{U_\text{i}}{U_\text{f}} = \boxed{1 + \frac{C_2}{C_1}}$$

or U_i is greater than U_f by a factor of $1 + C_2/C_1$.

(*c*) The decrease in energy equals the energy dissipated as Joule heat in the resistor connecting the two capacitors.

145 •••

Picture the Problem Let q_1 and q_2 be the time-dependent charges on the two capacitors after the switches are closed. We can use Kirchhoff's loop rule and the conservat charge to a obtain a first-order linear differential equation describing the current I_2 through R after the switches are closed. We can solve this differential equatio assuming a solution of the form $q_2(t) = a + be^{-t/\tau}$ and requiring that the solution satisfy the boundary condition that $q_2(0) = 0$ and the differential equation be satisfied for all values of t. Once we know I_2, we can find the energy dissipated in the resistor as a function of time and the total energy dissipated in the resistor.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\frac{q_1}{C_1} - IR - \frac{q_2}{C_2} = 0$$

or, because $I = dq_2/dt$,

$$\frac{q_1}{C_1} - R\frac{dq_2}{dt} - \frac{q_2}{C_2} = 0$$

Apply conservation of charge during the redistribution of charge to obtain:

$$q_1 = Q - q_2 = C_1V_0 - q_2$$

Substitute for q_1 to obtain:

$$V_0 - \frac{q_2}{C_1} - R\frac{dq_2}{dt} - \frac{q_2}{C_2} = 0$$

Rearrange to obtain the first- order differential equation:

$$R\frac{dq_2}{dt} + \left(\frac{C_1 + C_2}{C_1C_2}\right)q_2 = V_0$$

Assume a solution of the form:

$$q_2(t) = a + be^{-t/\tau} \qquad (1)$$

Differentiate the assumed solution with respect to time to obtain:

$$\frac{dq_2(t)}{dt} = \frac{d}{dt}\left[a + be^{-t/\tau}\right] = -\frac{b}{\tau}e^{-t/\tau}$$

Substitute for dq_2/dt and q_2 in the differential equation to obtain:

$$R\left(-\frac{b}{\tau}e^{-t/\tau}\right) + \left(\frac{C_1 + C_2}{C_1C_2}\right)\left(a + be^{-t/\tau}\right) = V_0$$

Rearrange to obtain:

$$\left[-\frac{R}{\tau}e^{-t/\tau}\right]b+\left(\frac{C_1+C_2}{C_1C_2}\right)a + \left[\left(\frac{C_1+C_2}{C_1C_2}\right)e^{-t/\tau}\right]b=V_0$$

If this equation is to be satisfied for all values of t:

$$a=\frac{C_1C_2}{C_1+C_2}V_0=C_{\text{eq}}V_0$$

and

$$\left[-\frac{R}{\tau}e^{-t/\tau}\right]b+\left[\left(\frac{C_1+C_2}{C_1C_2}\right)e^{-t/\tau}\right]b=0$$

or

$$-\frac{R}{\tau}+\frac{C_1+C_2}{C_1C_2}=0$$

Solve for τ to obtain:

$$\tau=R\frac{C_1C_2}{C_1+C_2}=RC_{\text{eq}}$$

Substitute the boundary condition $q_2(0) = 0$ in equation (1):

$$0=a+b$$

or

$$b=-a=-C_{\text{eq}}V_0$$

Substitute for a and b in equation (1) to obtain:

$$q_2(t)=C_{\text{eq}}V_0-C_{\text{eq}}V_0\,e^{-t/\tau} = C_{\text{eq}}V_0\left(1-e^{-t/\tau}\right)$$

Differentiate $q_2(t)$ with respect to time to find the current:

$$I(t)=\frac{dq_2(t)}{dt}=C_{\text{eq}}V_0\frac{d}{dt}\left(1-e^{-t/\tau}\right) = C_{\text{eq}}V_0\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = \frac{C_{\text{eq}}V_0}{\tau}e^{-t/\tau}=\boxed{\frac{V_0}{R}e^{-t/\tau}}$$

(b) Express the energy dissipated in the resistor as a function of time:

$$P(t)=I^2R=\left(\frac{V_0}{R}e^{-t/\tau}\right)^2R = \boxed{\frac{V_0^2}{R}e^{-2t/\tau}}$$

(c) The energy dissipated in the resistor is the integral of $P(t)$

$$E=\frac{V_0^2}{R}\int_0^\infty e^{-2t'/RC_{\text{eq}}}dt'=\boxed{\tfrac{1}{2}V_0^2C_{\text{eq}}}$$

between $t = 0$ and $t = \infty$:

> This is exactly the difference between the initial and final stored energies found in the preceding problem, which confirms the statement at the end of that problem that the difference in the stored energies equals the energy dissipated in the resistor.

146 •••

Picture the Problem We can apply Kirchhoff's loop rule to find the initial current drawn from the battery and the current drawn from the battery a long time after S_1 is closed. We can also use the loop rule to find the final voltages across the capacitors and the current in the 150-Ω resistor when S_2 is opened after having been closed for a long time.

(*a*) Apply Kirchhoff's loop rule to the loop that includes the source, the 100-Ω resistor, and the capacitor immediately after S_1 is closed to obtain:

$$12\text{ V} - I_{\text{bat}}(0)(100\,\Omega) - V_{\text{C0}} = 0$$

Because the capacitor is initially uncharged:

$$V_{\text{C0}} = 0$$

and

$$12\text{ V} - I_{\text{bat}}(0)(100\,\Omega) = 0$$

Solve for and evaluate $I_{\text{bat}}(0)$:

$$I_{\text{bat}}(0) = \frac{12\,V}{100\,\Omega} = \boxed{0.120\text{ A}}$$

(*b*) Apply Kirchhoff's loop rule to the loop that includes the source, the 100-Ω, 50-Ω, and 150-Ω resistor a long time after S_1 is closed to obtain:

$$12\text{ V} - I_\infty(100\,\Omega) - I_\infty(50\,\Omega) - I_\infty(150\,\Omega) = 0$$

Solve for and evaluate I_∞:

$$I_\infty = \frac{12\text{ V}}{100\,\Omega + 50\,\Omega + 150\,\Omega} = \boxed{40.0\text{ mA}}$$

(*c*) Apply Kirchhoff's loop rule to the loop that includes the source, the 100-Ω resistor, and C_1 a long time after both switches are closed to obtain:

$$12\text{ V} - I_{\text{bat}}(100\,\Omega) - V_{\text{C1}} = 0$$

Solve for and evaluate V_{C1}:

$$V_{C1} = 12\text{ V} - (40\text{ mA})(100\,\Omega) = \boxed{8.00\text{ V}}$$

(*d*) Apply Kirchhoff's loop rule to the loop that includes the 150-Ω resistor and C_2 a long time after both switches are closed to obtain:

$$-V_{C2} + I_{\text{bat}}(150\,\Omega) = 0$$

Solve for and evaluate V_{C2}:

$$V_{C2} = I_{\text{bat}}(150\,\Omega) = (40\text{ mA})(150\,\Omega) = \boxed{6.00\text{ V}}$$

(*e*) Apply Kirchhoff's loop rule to the loop that includes the 150-Ω resistor and C_2 after S_2 is opened to obtain:

$$V_{C_2}(t) - I(t)(150\,\Omega) = 0$$

or

$$V_{C_2}(0)e^{-t/\tau} - I(t)(150\,\Omega) = 0$$

Solve for $I(t)$ to obtain:

$$I(t) = \frac{V_{C_2}(0)}{150\,\Omega}e^{-t/\tau} = \frac{6\text{ V}}{150\,\Omega}e^{-t/(150\,\Omega)(50\,\mu\text{F})} = \boxed{(40\text{ mA})e^{-t/7.50\text{ ms}}}$$

***147 •••**

Picture the Problem We can use the definition of differential resistance and the expression for the diode current given in problem 54 to express R_d and establish the required results.

The differential resistance R_d is given by:

$$R_d = \frac{dV}{dI} = \left(\frac{dI}{dV}\right)^{-1}$$

From Problem 54, the current in the diode is given by:

$$I = I_0\left(e^{V/25\text{ mV}} - 1\right) \qquad (1)$$

Substitute for I to obtain:

$$R_d = \left\{\frac{d}{dV}\left[I_0\left(e^{V/25\text{ mV}} - 1\right)\right]\right\}^{-1} = \frac{25\text{ mV}}{I_0}e^{-V/25\text{ mV}} \qquad (2)$$

For $V > 0.6$ V, equation (1) becomes:

$$I \approx I_0 e^{V/25\text{ mV}}$$

Solve for the exponential factor to obtain:

$$e^{V/25\text{ mV}} \approx \frac{I}{I_0} \Rightarrow e^{-V/25\text{ mV}} \approx \frac{I_0}{I}$$

Substitute in equation (2) to obtain:

$$R_d \approx \frac{25\,\text{mV}}{I_0}\frac{I_0}{I} = \boxed{\frac{25\,\text{mV}}{I}}$$

Examination of equation (2) shows that, for $V < 0$, R_d increases exponentially. This result, together with that for $V > 0.6$ V, justifies the assumptions made in Problem 55.

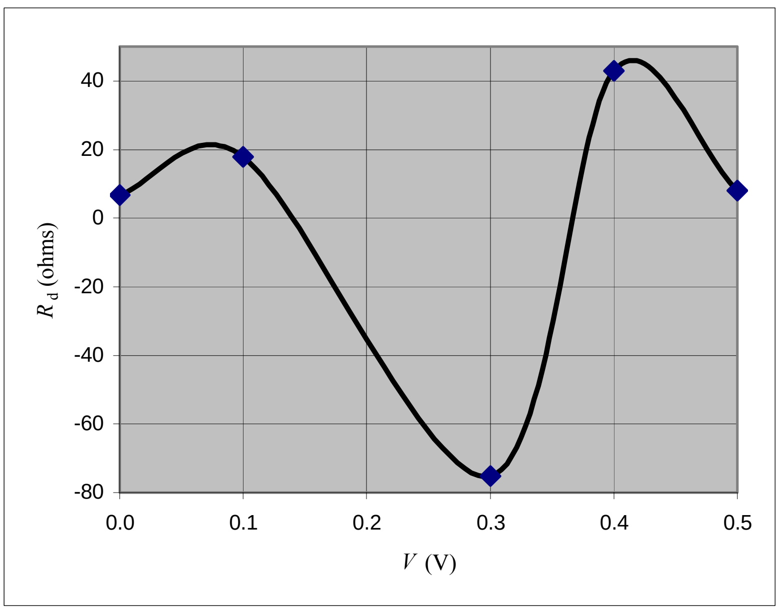
148 ••

Picture the Problem We can approximate the slope of the graph in Figure 25-77 and take its reciprocal to obtain values for R_d that we can plot as a function of V.

Use the graph in Figure 25-77 to complete the table to the right.

V (V)	R_d (Ω)
0	6.67
0.1	17.9
0.3	−75.2
0.4	42.9
0.5	8

The following graph was plotted using a spreadsheet program.



The differential resistance becomes negative at approximately 0.14 V.

149 •••

Picture the Problem We can use the definition of current to find the number of electrons accelerated in each pulse and the average current in the beam. The average and peak power of the accelerator can be found using $P_{av} = I_{av}V$ and

$P_{\text{peak}} = I_{\text{peak}} V$ and the duty factor from its definition.

(*a*) Use the definition of current to relate the number of electrons accelerated in each pulse to the duration of the pulse:

$$I_{\text{pulse}} = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t}$$

where n is the number of electrons in each pulse.

Solve for and evaluate n:

$$n = \frac{I_{\text{pulse}} \Delta t}{e} = \frac{(1.6\,\text{A})(0.1\,\mu\text{s})}{1.602 \times 10^{-19}\,\text{C}} = 9.99 \times 10^{11} \approx \boxed{10^{12}}$$

(*b*) Using the definition of current we have:

$$I_{\text{av}} = \frac{Q_{\text{pulse}}}{\Delta t_{\text{between pulses}}} = \frac{ne}{10^{-3}\,\text{s}} = \frac{10^{12}(1.60 \times 10^{-19}\,\text{C})}{10^{-3}\,\text{s}} = \boxed{0.160\,\text{mA}}$$

(*c*) Express the average power output in terms of the average current:

$$P_{\text{av}} = I_{\text{av}} V = (0.160\,\text{mA})(400\,\text{MV}) = \boxed{64.0\,\text{kW}}$$

(*d*) Express the peak power output in terms of the pulse current:

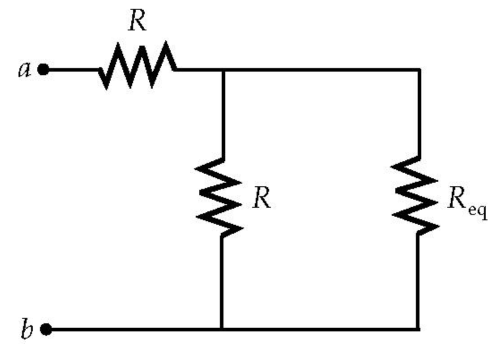
$$P_{\text{peak}} = I_{\text{pulse}} V = (1.6\,\text{A})(400\,\text{MV}) = \boxed{640\,\text{MW}}$$

(*e*) The duty factor is defined to be:

$$\text{duty factor} = \frac{\Delta t}{\text{time between pulses}} = \frac{0.1\,\mu\text{s}}{10^{-3}\,\text{s}} = \boxed{10^{-4}}$$

150 •••

Picture the Problem Let *R* be the resistance of each resistor in the ladder and let R_{eq} be the equivalent resistance of the infinite ladder. If the resistance is finite and non-zero, then adding one or more stages to the ladder will not change the resistance of the network. We can apply the rules for resistance combination to the diagram shown to the right to obtain a quadratic equation in R_{eq} that we can solve for the equivalent resistance between points *a* and *b*.



The equivalent resistance of the series combination of *R* and ($R \parallel R_{eq}$) is R_{eq}, so:

$$R_{eq} = R + R\|R_{eq} = R + \frac{RR_{eq}}{R + R_{eq}}$$

Simplify to obtain:

$$R_{eq}^2 - RR_{eq} - R^2 = 0$$

Solve for R_{eq} to obtain:

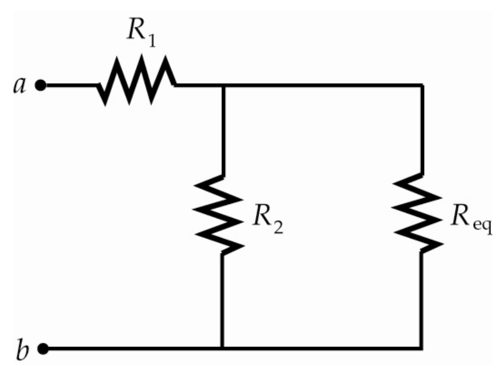
$$R_{eq} = \left(\frac{1+\sqrt{5}}{2}\right)R$$

For $R = 1\Omega$:

$$R_{eq} = \left(\frac{1+\sqrt{5}}{2}\right)(1\,\Omega) = \boxed{1.62\,\Omega}$$

***151** •••

Picture the Problem Let R_{eq} be the equivalent resistance of the infinite ladder. If the resistance is finite and non-zero, then adding one or more stages to the ladder will not change the resistance of the network. We can apply the rules for resistance combination to the diagram shown to the right to obtain a quadratic equation in R_{eq} that we can solve for the equivalent resistance between points *a* and *b*.



The equivalent resistance of the series combination of R_1 and ($R_2 \parallel R_{eq}$) is R_{eq}, so:

$$R_{eq} = R_1 + R_2\|R_{eq} = R_1 + \frac{R_2 R_{eq}}{R_2 + R_{eq}}$$

Simplify to obtain:

$$R_{eq}^2 - R_1 R_{eq} - R_1 R_2 = 0$$

Solve for the positive value of R_{eq} to obtain:

$$R_{eq} = \boxed{\frac{R_1 + \sqrt{R_1^2 + 4R_1 R_2}}{2}}$$

Chapter 26
The Magnetic Field

Conceptual Problems

***1** •

Determine the Concept Because the electrons are initially moving at 90° to the magnetic field, they will be deflected in the direction of the magnetic force acting on them. Use the right-hand rule based on the expression for the magnetic force acting on a moving charge $\vec{F} = q\vec{v} \times \vec{B}$, remembering that, for a negative charge, the force is in the direction opposite that indicated by the right-hand rule, to convince yourself that the particle will follow the path whose terminal point on the screen is 2. $\boxed{(b) \text{ is correct.}}$

2 •

Determine the Concept One cannot define the direction of the force arbitrarily. By experiment, $\vec{F}$ is perpendicular to $\vec{B}$.

3 •

Determine the Concept False. An object experiences acceleration if either its speed changes or the direction it is moving changes. The magnetic force, acting perpendicular to the direction a charged particle is moving, changes the particle's *velocity* by changing the direction it is moving and hence accelerates the particle.

4 •

Determine the Concept Yes; it will be deflected upward. Because the beam passes through undeflected when traveling from left to right, we know that the upward electric force must be balanced by a downward magnetic force. Application of the right-hand rule tells us that the magnetic field must be out of the page. When the beam is reversed, the magnetic force (as well as the electric force) acting on it is upward.

***5** •

Determine the Concept The alternating current running through the filament is changing direction every 1/60 s, so in a magnetic field the filament experiences a force which alternates in direction at that frequency.

6 •

Determine the Concept The magnitude of the torque on a current loop is given by $\tau = \mu B \sin\theta$, where θ is the angle between the magnetic field and a normal to the surface of the loop. To maximize τ, $\sin\theta = 1$ and $\theta = 90°$. Hence the normal to the plane of the loop should be perpendicular to $\vec{B}$.

7 •

(*a*) True. This is an experimental fact and is the basis for the definition of the magnetic force on a moving charged particle being expressed in terms of the cross product of $\vec{v}$ and $\vec{B}$; i.e. $\vec{F} = q\vec{v}\times\vec{B}$.

(*b*) True. This is another experimental fact. The torque on a magnet is a restoring torque, i.e., one that acts in such a direction as to align the magnet with magnetic field.

(*c*) True. We can use a right-hand rule to relate the direction of the magnetic field around the loop to the direction of the current. Doing so indicates that one side of the loop acts like a north pole and the other like a south pole.

(*d*) False. The period of a particle moving in a circular path in a magnetic field is given by $T = 2\pi\sqrt{mr/qvB}$ and, hence, is proportional to the square root of the radius of the circle.

(*e*) True. The drift velocity is related to the Hall voltage according to $v_d = V_H/Bw$ where w is the width of the Hall-effect material.

***8** • .

Determine the Concept The direction in which a particle is deflected by a magnetic field will be unchanged by any change in the definition of the direction of the magnetic field. Since we have reversed the direction of the field, we must define the direction in which particles are deflected by a "left-hand" rule instead of a "right-hand" rule.

9 •

Determine the Concept Choose a right-handed coordinate system in which east is the positive x direction and north is the positive y direction. Then the magnetic force acting on the particle is given by $\vec{F} = qv\hat{i}\times B\hat{j} = qvB\left(\hat{i}\times\hat{j}\right) = qvB\hat{k}$. Hence, the magnetic force is upward.

10 •

Determine the Concept Application of the right-hand rule tells us that this positively charged particle would have to be moving in the northwest direction with the magnetic field upward in order for the magnetic force to be toward the northeast. The situation described cannot exist. $\boxed{(e)\text{ is correct.}}$

11 •

Picture the Problem We can use Newton's 2^{nd} law for circular motion to express the radius of curvature R of each particle in terms of its charge, momentum, and the magnetic field. We can then divide the proton's radius of curvature by that of the ^{7}Li nucleus to decide which of these alternatives is correct.

Apply $\sum F_{\text{radial}} = ma_c$ to the lithium nucleus to obtain:

$$qvB = m\frac{v^2}{R}$$

Solve for *r*:

$$R = \frac{mv}{qB}$$

For the ^{7}Li nucleus this becomes:

$$R_{\text{Li}} = \frac{p_{\text{Li}}}{3eB}$$

For the proton we have:

$$R_{\text{p}} = \frac{p_{\text{p}}}{eB}$$

Divide equation (2) by equation (1) and simplify to obtain:

$$\frac{R_{\text{p}}}{R_{\text{Li}}} = \frac{\frac{p_{\text{p}}}{eB}}{\frac{p_{\text{Li}}}{3eB}} = 3\frac{p_{\text{p}}}{p_{\text{Li}}}$$

Because the momenta are equal:

$$\frac{R_{\text{p}}}{R_{\text{Li}}} = 3 \text{ and } \boxed{(a) \text{ is correct.}}$$

***12 •**

Determine the Concept Application of the right-hand rule indicates that a positively charged body would experience a downward force and, in the absence of other forces, be deflected downward. Because the direction of the magnetic force on an electron is opposite that of the force on a positively charged object, an electron will be deflected upward. $\boxed{(c) \text{ is correct.}}$

13 ••

Determine the Concept From relativity; this is equivalent to the electron moving from right to left at velocity *v* with the magnet stationary. When the electron is directly over the magnet, the field points directly up, so there is a force directed out of the page on the electron.

14 •

Similarities	Differences
Magnetic field lines are similar to electric field lines in that their density is a measure of the strength of the field; the lines point in the direction of the field; also, magnetic field lines do not cross.	They differ from electric field lines in that magnetic field lines must close on themselves (there are no isolated magnetic poles), and the force on a charge depends on the velocity of the charge and is perpendicular to the magnetic field lines.

15 •

Determine the Concept If only $\vec{F}$ and I are known, one can only conclude that the magnetic field $\vec{B}$ is in the plane perpendicular to $\vec{F}$. The specific direction of $\vec{B}$ is undetermined.

Estimation and Approximation

***16** ••

Picture the Problem If the electron enters the magnetic field in the coil with speed *v*, it will travel in a circular path under the influence of the magnetic force acting on. We can apply Newton's 2nd law to the electron in this field to obtain an expression for the magnetic field. We'll assume that the deflection of the electron is small over the distance it travels in the magnetic field, but that, once it is through the region of the magnetic field, it travels at an angle θ with respect to the direction it was originally traveling.

Apply $\sum F = ma_c$ to the electron in the magnetic field to obtain:

$$evB = m\frac{v^2}{r}$$

Solve for *B*:

$$B = \frac{mv}{er}$$

The kinetic energy of the electron is:

$$K = eV = \tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2eV}{m}}$$

Substitute for *v* in the expression for *r*:

$$B = \frac{m}{er}\sqrt{\frac{2eV}{m}} = \frac{1}{r}\sqrt{\frac{2mV}{e}}$$

Because $\theta << 1$:

$$d \approx r\sin\theta \Rightarrow r \approx \frac{d}{\sin\theta}$$

Substitute for *r* in the expression for *B* to obtain:

$$B = \frac{\sin\theta}{d}\sqrt{\frac{2mV}{e}}$$

For maximum deflection, $\theta \approx 45°$. Substitute numerical values and evaluate *B*:

$$B = \frac{\sin 45°}{0.05\,\text{m}}\sqrt{\frac{2(9.11\times10^{-31}\,\text{kg})(15\,\text{kV})}{1.60\times10^{-19}\,\text{C}}}$$

$$= \boxed{5.84\,\text{mT}}$$

17 ••

Picture the Problem Let *h* be the height of the orbit above the surface of the earth, *m* the mass of the micrometeorite, and *v* its speed. We can apply Newton's 2nd law to the orbiting micrometeorite with $F_{mag} = qvB$ to derive an expression for the charge-to-mass ratio of the micrometeorite.

(*a*) Apply $\sum F = ma_c$ to the micrometeorite orbiting under the influence of the magnetic force:

$$qvB = m\frac{v^2}{h+R_{\text{earth}}}$$

Solve for *q*/*m* to obtain:

$$\frac{q}{m} = \frac{v}{B(h+R_{\text{earth}})}$$

Substitute numerical values and evaluate *q*/*m*:

$$\frac{q}{m} = \frac{30\,\text{km/s}}{(5\times10^{-5}\,\text{T})(400\,\text{km}+6370\,\text{km})} = \boxed{88.6\,\text{C/kg}}$$

(*b*) Solve the result for *q*/*m* obtained in (*a*) for *q* to obtain:

$$q = (88.6\,\text{C/kg})m$$

Substitute numerical values and evaluate *q*:

$$q = (88.6\,\text{C/kg})(3\times10^{-10}\,\text{kg}) = \boxed{26.6\,\text{nC}}$$

Force Exerted by a Magnetic Field

18 •

Picture the Problem The magnetic force acting on a charge is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (a.k.a. ″cross″) product, and multiply by the scalar q to find $\vec{F}$.

Express the force acting on the proton:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Express $\vec{v}$:

$$\vec{v} = (4.46\,\text{Mm/s})\hat{i}$$

Express $\vec{B}$:

$$\vec{B} = (1.75\,\text{T})\hat{k}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = (1.60\times10^{-19}\,\text{C})\left[(4.46\,\text{Mm/s})\hat{i}\times(1.75\,\text{T})\hat{k}\right] = \boxed{-(1.25\,\text{pN})\hat{j}}$$

19 •

Picture the Problem The magnetic force acting on the charge is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (a.k.a. ″cross″) product, and multiply by the scalar q to find $\vec{F}$.

Express the force acting on the charge:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Substitute numerical values to obtain:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i}\times\vec{B}\right]$$

(*a*) Evaluate $\vec{F}$ for $\vec{B}$ = 0.38 T $\hat{j}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i}\times(0.38\,\text{T})\hat{j}\right] = \boxed{-(3.80\,\text{mN})\hat{k}}$$

(*b*) Evaluate $\vec{F}$ for $\vec{B}$ = 0.75 T $\hat{i}$ + 0.75 T $\hat{j}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i}\times\left\{(0.75\,\text{T})\hat{i}+(0.75\,\text{T})\hat{j}\right\}\right] = \boxed{-(7.51\text{mN})\hat{k}}$$

(*c*) Evaluate $\vec{F}$ for $\vec{B}$ = 0.65 T $\hat{i}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i}\times(0.65\,\text{T})\hat{i}\right] = \boxed{0}$$

(*d*) Evaluate $\vec{F}$ for $\vec{B}$ = 0.75 T $\hat{i}$ + 0.75 T $\hat{k}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i}\times(0.75\,\text{T})\hat{i}+(0.75\,\text{T})\hat{k}\right] = \boxed{(7.51\text{mN})\hat{j}}$$

20 •

Picture the Problem The magnetic force acting on the proton is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (a.k.a. ″cross″) product, and multiply by the scalar q to find $\vec{F}$.

Express the force acting on the proton:

$$\vec{F} = q\vec{v} \times \vec{B}$$

(*a*) Evaluate $\vec{F}$ for $\vec{v}$ = 2.7 Mm/s $\hat{i}$:

$$\vec{F} = (1.60\times10^{-19}\ \text{C})\left[(2.7\times10^6\ \text{m/s})\hat{i}\times(1.48\,\text{T})\hat{k}\right] = \boxed{-(0.640\,\text{pN})\hat{j}}$$

(*b*) Evaluate $\vec{F}$ for $\vec{v}$ = 3.7 Mm/s $\hat{j}$:

$$\vec{F} = (1.60\times10^{-19}\ \text{C})\left[(3.7\times10^6\ \text{m/s})\hat{j}\times(1.48\,\text{T})\hat{k}\right] = \boxed{(0.876\,\text{pN})\hat{i}}$$

(c) Evaluate $\vec{F}$ for $\vec{v}$ = 6.8 Mm/s $\hat{k}$:

$$\vec{F} = \left(1.60\times10^{-19}\ \text{C}\right)\left[\left(6.8\times10^{6}\ \text{m/s}\right)\hat{k}\times\left(1.48\,\text{T}\right)\hat{k}\right] = \boxed{0}$$

(d) Evaluate $\vec{F}$ for $\vec{v} = 4.0\,\text{Mm/s}\,\hat{i} + 3.0\,\text{Mm/s}\,\hat{j}$:

$$\vec{F} = \left(1.60\times10^{-19}\ \text{C}\right)\left[\left\{\left(4.0\,\text{Mm/s}\right)\hat{i} + \left(3.0\,\text{Mm/s}\right)\hat{j}\right\}\times\left(1.48\,\text{T}\right)\hat{k}\right]$$
$$= \boxed{\left(0.710\,\text{pN}\right)\hat{i} - \left(0.947\,\text{pN}\right)\hat{j}}$$

21 •

Picture the Problem The magnitude of the magnetic force acting on a segment of wire is given by $F = I\ell B\sin\theta$ where ℓ is the length of the segment of wire, B is the magnetic field, and θ is the angle between the segment of wire and the direction of the magnetic field.

Express the magnitude of the magnetic force acting on the segment of wire:

$$F = I\ell B\sin\theta$$

Substitute numerical values and evaluate F:

$$F = \left(2.6\,\text{A}\right)\left(2\,\text{m}\right)\left(0.37\,\text{T}\right)\sin 30^\circ = \boxed{0.962\,\text{N}}$$

***22** •

Picture the Problem We can use $\vec{F} = I\vec{L}\times\vec{B}$ to find the force acting on the wire segment.

Express the force acting on the wire segment:

$$\vec{F} = I\vec{L}\times\vec{B}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = \left(2.7\,\text{A}\right)\left[\left(3\,\text{cm}\right)\hat{i} + \left(4\,\text{cm}\right)\hat{j}\right]\times\left(1.3\,\text{T}\right)\hat{i} = \boxed{-\left(0.140\,\text{N}\right)\hat{k}}$$

23 •

Picture the Problem The magnetic force acting on the electron is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can form the vector product of $\vec{v}$ and $\vec{B}$ and multiply by the charge of the electron to find $\vec{F}$ and obtain its magnitude using $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$. The direction angles are given by $\theta_x = \tan^{-1}\left(F_x/F\right)$, $\theta_y = \tan^{-1}\left(F_y/F\right)$, and $\theta_z = \tan^{-1}\left(F_z/F\right)$.

Express the force acting on the proton: $\vec{F} = q\vec{v} \times \vec{B}$

Express the magnitude of $\vec{F}$ in terms of its components: $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ (1)

Substitute numerical values and evaluate $\vec{F}$:

$$\begin{aligned}\vec{F} &= \left(-1.60\times10^{-19}\ \text{C}\right)\left[\left\{(2\ \text{Mm/s})\hat{i} - (3\ \text{Mm/s})\hat{j}\right\}\times\left(0.8\hat{i} + 0.6\hat{j} - 0.4\hat{k}\right)\text{T}\right] \\ &= (-0.192\ \text{pN})\hat{k} + (-0.128\ \text{pN})\hat{j} + (-0.384\ \text{pN})\hat{k} + (-0.192\ \text{pN})\hat{i} \\ &= \boxed{-(0.192\ \text{pN})\hat{i} - (0.128\ \text{pN})\hat{j} - (0.576\ \text{pN})\hat{k}}\end{aligned}$$

Substitute in equation (1) to obtain:

$$F = \sqrt{(-0.192\ \text{pN})^2 + (-0.128\ \text{pN})^2 + (-0.576\ \text{pN})^2} = \boxed{0.621\ \text{pN}}$$

Express and evaluate the angle $\vec{F}$ makes with the x axis:

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{-0.192\ \text{pN}}{0.621\ \text{pN}}\right) = \boxed{108^\circ}$$

Express and evaluate the angle $\vec{F}$ makes with the y axis:

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{-0.128\ \text{pN}}{0.621\ \text{pN}}\right) = \boxed{102^\circ}$$

Express and evaluate the angle $\vec{F}$ makes with the z axis:

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{-0.576\ \text{pN}}{0.621\ \text{pN}}\right) = \boxed{158^\circ}$$

24 ••

Picture the Problem We can use $\vec{F} = I\vec{\ell} \times \vec{B}$ to find the force acting on the segments of the wire as well as the magnetic force acting on the wire if it were a straight segment from a to b.

Express the magnetic force acting on the wire: $\vec{F} = \vec{F}_{3\,\text{cm}} + \vec{F}_{4\,\text{cm}}$

Evaluate $\vec{F}_{3\,\text{cm}}$:

$$\vec{F}_{3\,\text{cm}} = (1.8\,\text{A})\left[(3\,\text{cm})\hat{i} \times (1.2\,\text{T})\hat{k}\right] = (0.0648\,\text{N})\left(-\hat{j}\right) = -(0.0648\,\text{N})\hat{j}$$

Evaluate $\vec{F}_{4\,\text{cm}}$:

$$\vec{F}_{4\,\text{cm}} = (1.8\,\text{A})\left[(4\,\text{cm})\hat{j} \times (1.2\,\text{T})\hat{k}\right] = (0.0864\,\text{N})\hat{i}$$

Substitute to obtain:

$$\vec{F} = -(0.0648\,\text{N})\hat{j} + (0.0864\,\text{N})\hat{i} = \boxed{(0.0864\,\text{N})\hat{i} - (0.0648\,\text{N})\hat{j}}$$

If the wire were straight from a to b:

$$\vec{\ell} = (3\,\text{cm})\hat{i} + (4\,\text{cm})\hat{j}$$

The magnetic force acting on the wire is:

$$\vec{F} = (1.8\,\text{A})\left[(3\,\text{cm})\hat{i} + (4\,\text{cm})\hat{j}\right] \times (1.2\,\text{T})\hat{k} = -(0.0648\,\text{N})\hat{j} + (0.0864\,\text{N})\hat{i} = \boxed{(0.0864\,\text{N})\hat{i} - (0.0648\,\text{N})\hat{j}}$$

in agreement with the result obtained above when we treated the two straight segments of the wire separately.

25 ••

Picture the Problem Because the magnetic field is horizontal and perpendicular to the wire, the force it exerts on the current-carrying wire will be vertical. Under equilibrium conditions, this upward magnetic force will be equal to the downward gravitational force acting on the wire.

Apply $\sum F_{\text{vertical}} = 0$ to the wire:

$$F_{\text{mag}} - w = 0$$

Express F_{mag}:

$$F_{\text{mag}} = I\ell B$$

because $\theta = 90°$.

Substitute to obtain:

$$I\ell B - mg = 0$$

Solve for I:

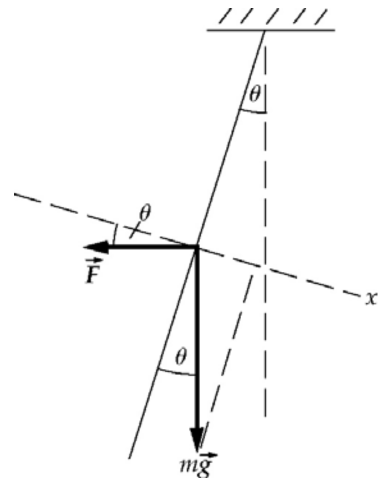
$$I = \frac{mg}{\ell B}$$

Substitute numerical values and evaluate I:

$$I = \frac{(50\,\text{g})(9.81\,\text{m/s}^2)}{(25\,\text{cm})(1.33\,\text{T})} = \boxed{1.48\,\text{A}}$$

***26 ••**

Picture the Problem The diagram shows the gaussmeter displaced from equilibrium under the influence of the gravitational and magnetic forces acting on it. We can apply the condition for translational equilibrium in the *x* direction to find the equilibrium angular displacement of the wire from the vertical. In part (b) we can solve the equation derived in part (a) for *B* and evaluate this expression for the given data to find the horizontal magnetic field sensitivity of this gaussmeter.



(*a*) Apply $\sum F_x = 0$ to the wire to obtain:

$$mg\sin\theta - F\cos\theta = 0$$

Substitute for *F* and solve for θ to obtain:

$$mg\sin\theta - I\ell B\cos\theta = 0 \qquad (1)$$

and

$$\theta = \tan^{-1}\left(\frac{I\ell B}{mg}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{(0.2\,\text{A})(0.5\,\text{m})(0.04\,\text{T})}{(0.005\,\text{kg})(9.81\,\text{m/s}^2)}\right] = \boxed{4.66°}$$

(*b*) Solve equation (1) for *B* to obtain:

$$B = \frac{mg\tan\theta}{I\ell}$$

For a displacement from vertical of 0.5 mm:

$$\tan\theta \approx \sin\theta = \frac{0.5\,\text{mm}}{0.5\,\text{m}} = 0.001$$

and

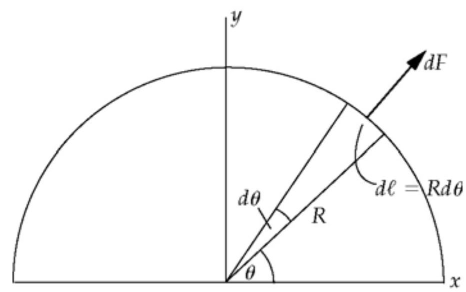
$$\theta = 0.001\,\text{rad}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{(0.005\,\text{kg})(9.81\,\text{m/s}^2)(0.001\,\text{rad})}{(20\,\text{A})(0.5\,\text{m})} = \boxed{4.91\,\mu\text{T}}$$

27 ••

Picture the Problem With the current in the direction indicated and the magnetic field in the *z* direction, pointing out of the plane of the page, the force is in the radial direction and we can integrate the element of force *dF* acting on an element of length $d\ell$ between $\theta = 0$ and π to find the force acting on the semicircular portion of the loop and use the expression for the force on a current-carrying wire in a uniform magnetic field to find the force on the straight segment of the loop.



Express the net force acting on the semicircular loop of wire:

$$F = F_{\text{semicircular loop}} + F_{\text{straight segment}} \qquad (1)$$

Express the force acting on the straight segment of the loop:

$$\vec{F}_{\text{straight segment}} = I\vec{\ell} \times \vec{B} = -2RIB$$

Express the force *dF* acting on the element of the wire of length $d\ell$:

$$dF = Id\ell B = IRBd\theta$$

Express the *x* and *y* components of *dF*:

$$dF_x = dF\cos\theta$$

and

$$dF_y = dF\sin\theta$$

Because, by symmetry, the *x* component of the force is zero, we can integrate the *y* component to find the force on the wire:

$$dF_y = IRB\sin\theta\, d\theta$$

and

$$F_y = RIB\int_0^{\pi}\sin\theta\, d\theta = 2RIB$$

Substitute in equation (1) to obtain:

$$F = 2RIB - 2RIB = \boxed{0}$$

28 ••

Picture the Problem We can use the information given in the 1st and 2nd sentences to obtain an expression containing the components of the magnetic field $\vec{B}$. We can then use the information in the 1st and 3rd sentences to obtain a second equation in these components that we can solve simultaneously for the components of $\vec{B}$.

Express the magnetic field $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (1)$$

Express $\vec{F}$ in terms of $\vec{B}$:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (4\,\text{A})[(0.1\,\text{m})\hat{k}]\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\\ &= (0.4\,\text{A}\cdot\text{m})\hat{k}\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\\ &= (0.4\,\text{A}\cdot\text{m})B_y\hat{j} - (0.4\,\text{A}\cdot\text{m})B_y\hat{i}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the second sentence of the statement of the problem to obtain:

$$(0.4\,\text{A}\cdot\text{m})B_y = 0.2\,\text{N}$$

and

$$(0.4\,\text{A}\cdot\text{m})B_x = 0.2\,\text{N}$$

Noting that B_z is undetermined, solve for B_x and B_y:

$B_x = 0.5\,\text{T}$ and $B_y = 0.5\,\text{T}$

When the wire is rotated so that the current flows in the positive x direction:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (4\,\text{A})[(0.1\,\text{m})\hat{i}]\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\\ &= (0.4\,\text{A}\cdot\text{m})\hat{i}\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\\ &= -(0.4\,\text{A}\cdot\text{m})B_z\hat{j} + (0.4\,\text{A}\cdot\text{m})B_y\hat{k}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the third sentence of the problem statement to obtain:

$$-(0.4\,\text{A}\cdot\text{m})B_z = 0$$

and

$$(0.4\,\text{A}\cdot\text{m})B_y = 0.2\,\text{N}$$

Solve for B_z and B_y to obtain:

$B_z = 0$

and, in agreement with our results above,

$B_y = 0.5\,\text{T}$

Substitute in equation (1) to obtain:

$$\vec{B} = \boxed{(0.5\,\text{T})\hat{i} + (0.5\,\text{T})\hat{j}}$$

29 ••

Picture the Problem We can use the information given in the 1st and 2nd sentences to obtain an expression containing the components of the magnetic field $\vec{B}$. We can then use the information in the 1st and 3rd sentences to obtain a second equation in these components that we can solve simultaneously for the components of $\vec{B}$.

Express the magnetic field $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (1)$$

Express $\vec{F}$ in terms of $\vec{B}$:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (2\,\text{A})\left[(0.1\,\text{m})\hat{i}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.2\,\text{A}\cdot\text{m})\hat{i}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= -(0.2\,\text{A}\cdot\text{m})B_z\hat{j} + (0.2\,\text{A}\cdot\text{m})B_y\hat{k}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the second sentence of the statement of the problem to obtain:

$$-(0.2\,\text{A}\cdot\text{m})B_z = 3\,\text{N}$$

and

$$(0.2\,\text{A}\cdot\text{m})B_y = 2\,\text{N}$$

Noting that B_x is undetermined, solve for B_z and B_y:

$$B_z = -15\,\text{T}$$

and

$$B_y = 10\,\text{T}$$

When the wire is rotated so that the current flows in the positive y direction:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (2\,\text{A})\left[(0.1\,\text{m})\hat{j}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.2\,\text{A}\cdot\text{m})\hat{j}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.2\,\text{A}\cdot\text{m})B_z\hat{i} - (0.2\,\text{A}\cdot\text{m})B_x\hat{k}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the third sentence of the problem statement to obtain:

$$(0.2\,\text{A}\cdot\text{m})B_x = -3\,\text{N}$$

and

$$-(0.2\,\text{A}\cdot\text{m})B_z = -2\,\text{N}$$

Solve for B_x and B_z to obtain:

$$B_x = -15\,\text{T}$$

and, in agreement with our results above,

$$B_z = 10\,\text{T}$$

Substitute in equation (1) to obtain: $\vec{B} = \boxed{(10\,\text{T})\hat{i} + (10\,\text{T})\hat{j} - (15\,\text{T})\hat{k}}$

30 •••

Picture the Problem We can integrate the expression for the force $d\vec{F}$ acting on an element of the wire of length $d\vec{L}$ from *a* to *b* to show that $\vec{F} = I\vec{L} \times \vec{B}$.

Express the force $d\vec{F}$ acting on the element of the wire of length $d\vec{L}$: $d\vec{F} = Id\vec{L} \times \vec{B}$

Integrate this expression to obtain:

$$\vec{F} = \int_a^b Id\vec{L} \times \vec{B}$$

Because $\vec{B}$ and *I* are constant:

$$\vec{F} = I\left(\int_a^b d\vec{L}\right) \times \vec{B} = \boxed{I\vec{L} \times \vec{B}}$$

where $\vec{L}$ is the vector from *a* to *b*.

Motion of a Point Charge in a Magnetic Field

***31** •

Picture the Problem We can apply Newton's 2nd law to the orbiting proton to relate its speed to its radius. We can then use $T = 2\pi r/v$ to find its period. In Part (*b*) we can use the relationship between *T* and *v* to determine *v*. In Part (*c*) we can use its definition to find the kinetic energy of the proton.

(*a*) Relate the period *T* of the motion of the proton to its orbital speed *v*:

$$T = \frac{2\pi r}{v} \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to the proton to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for v/r to obtain:

$$\frac{v}{r} = \frac{qB}{m}$$

Substitute to obtain:

$$T = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{2\pi(1.67 \times 10^{-27}\,\text{kg})}{(1.60 \times 10^{-19}\,\text{C})(0.75\,\text{T})} = \boxed{87.4\,\text{ns}}$$

(*b*) From equation (1) we have:

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.65\,\text{m})}{87.4\,\text{ns}}$$
$$= \boxed{4.67\times10^{7}\ \text{m/s}}$$

(*c*) Using its definition, express and evaluate the kinetic energy of the proton:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.67\times10^{-27}\ \text{kg})(4.67\times10^{7}\ \text{m/s})^2 = 1.82\times10^{-12}\ \text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}}$$
$$= \boxed{11.4\,\text{MeV}}$$

32 •

Picture the Problem We can apply Newton's 2nd law to the orbiting electron to obtain an expression for the radius of its orbit as a function of its mass *m*, charge *q*, speed *v*, and the magnetic field *B*. Using the definition of its kinetic energy will allow us to express *r* in terms of *m*, *q*, *B*, and its kinetic energy *K*. We can use $T = 2\pi r/v$ to find the period of the motion and calculate the frequency from the reciprocal of the period of the motion.

(*a*) Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the proton to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for *r*:

$$r = \frac{mv}{qB} \qquad (1)$$

Express the kinetic energy of the electron:

$$K = \tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2K}{m}} \qquad (2)$$

Substitute in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km}$$

Find the frequency from the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.110\,\text{ns}} = \boxed{9.10\,\text{GHz}}$$

Substitute numerical values and evaluate *r*:

$$r=\frac{\sqrt{2(45\,\text{keV})\left(9.11\times10^{-31}\,\text{kg}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}\right)}}{(1.6\times10^{-19}\,\text{C})(0.325\,\text{T})}=\boxed{2.20\,\text{mm}}$$

(*b*) Relate the period of the electron's motion to the radius of its orbit and its orbital speed:

$$T=\frac{2\pi r}{v}$$

Substitute equation (2) to obtain:

$$T=\frac{2\pi r}{\sqrt{\frac{2K}{m}}}=\pi r\sqrt{\frac{2m}{K}}$$

Substitute numerical values and evaluate T:

$$T=\pi(2.20\,\text{mm})\sqrt{\frac{2(9.11\times10^{-31}\,\text{kg})}{45\,\text{keV}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}}}=\boxed{0.110\,\text{ns}}$$

33 •

Picture the Problem We can apply Newton's 2nd law to the orbiting electron to obtain an expression for the radius of its orbit as a function of its mass m, charge q, speed v, and the magnetic field B.

(*a*) Apply $\sum F_{\text{radial}}=ma_{\text{c}}$ to the proton to obtain:

$$qvB=m\frac{v^2}{r}$$

Solve for r:

$$r=\frac{mv}{qB}$$

Substitute numerical values and evaluate r:

$$r=\frac{(9.11\times10^{-31}\,\text{kg})(10^{7}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(4\times10^{-7}\,\text{T})}=\boxed{142\,\text{m}}$$

(*b*) For $B=2\times10^{-5}$ T:

$$r=\frac{(9.11\times10^{-31}\,\text{kg})(10^{7}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(2\times10^{-5}\,\text{T})}=\boxed{2.84\,\text{m}}$$

34 ••

Picture the Problem We can apply Newton's 2nd law to an orbiting particle to obtain an expression for the radius of its orbit R as a function of its mass m, charge q, speed v, and the magnetic field B.

Apply $\sum F_{\text{radial}} = ma_c$ to an orbiting particle to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for r:

$$r = \frac{mv}{qB}$$

Express the kinetic energy of the particle:

$$K = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2K}{m}}$$

Substitute in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} \qquad (1)$$

Using equation (1), express the ratio R_d/R_p:

$$\frac{R_d}{R_p} = \frac{\dfrac{1}{q_d B}\sqrt{2Km_d}}{\dfrac{1}{q_p B}\sqrt{2Km_p}} = \frac{q_p}{q_d}\sqrt{\frac{m_d}{m_p}}$$

$$= \frac{e}{e}\sqrt{\frac{2m_p}{m_p}} = \boxed{\sqrt{2}}$$

Using equation (1), express the ratio R_α/R_p:

$$\frac{R_\alpha}{R_p} = \frac{\dfrac{1}{q_\alpha B}\sqrt{2Km_\alpha}}{\dfrac{1}{q_p B}\sqrt{2Km_p}} = \frac{q_p}{q_\alpha}\sqrt{\frac{m_\alpha}{m_p}}$$

$$= \frac{e}{2e}\sqrt{\frac{4m_p}{m_p}} = \boxed{1}$$

35 ••

Picture the Problem We can apply Newton's 2nd law to the orbiting particles to derive an expression for their velocities as a function of their charge, their mass, the magnetic field in which they are moving, and the radii of their orbits. We can then compare their velocities by expressing their ratio. In parts (*b*) and (*c*) we can proceed similarly starting with the definitions of kinetic energy and angular momentum.

(*a*) Apply $\sum F_{\text{radial}} = ma_c$ to an orbiting particle to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for *v*:

$$v = \frac{qBr}{m}$$

Express the velocities of the particles:

$$v_{\text{p}} = \frac{q_{\text{p}}Br}{m_{\text{p}}} \text{ and } v_\alpha = \frac{q_\alpha Br}{m_\alpha}$$

Divide the second of these equations by the first to obtain:

$$\frac{v_\alpha}{v_{\text{p}}} = \frac{\dfrac{q_\alpha Br}{m_\alpha}}{\dfrac{q_{\text{p}}Br}{m_{\text{p}}}} = \frac{q_\alpha m_{\text{p}}}{q_{\text{p}} m_\alpha} = \frac{2em_{\text{p}}}{e(4m_{\text{p}})} = \boxed{\frac{1}{2}}$$

(*b*) Express the kinetic energy of an orbiting particle:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \tfrac{1}{2}\frac{q^2B^2r^2}{m}$$

Using this relationship, express the ratio of K_α to K_{p}:

$$\frac{K_\alpha}{K_{\text{p}}} = \frac{\tfrac{1}{2}\dfrac{q_\alpha^2B^2r^2}{m_\alpha}}{\tfrac{1}{2}\dfrac{q_{\text{p}}^2B^2r^2}{m_{\text{p}}}} = \frac{q_\alpha^2 m_{\text{p}}}{q_{\text{p}}^2 m_\alpha}$$

$$= \frac{(2e)^2 m_{\text{p}}}{e^2(4m_{\text{p}})} = \boxed{1}$$

(*c*) Express the angular momenta of the particles:

$$L_\alpha = m_\alpha v_\alpha r \text{ and } L_{\text{p}} = m_{\text{p}} v_{\text{p}} r$$

Express the ratio of L_α to L_{p}:

$$\frac{L_\alpha}{L_{\text{p}}} = \frac{m_\alpha v_\alpha r}{m_{\text{p}} v_{\text{p}} r} = \frac{(4m_{\text{p}})\left(\tfrac{1}{2}v_{\text{p}}\right)}{m_{\text{p}} v_{\text{p}}} = \boxed{2}$$

36 ••

Picture the Problem We can use the definition of momentum to express *p* in terms of *v* and apply Newton's 2nd law to the orbiting particle to express *v* in terms of *q*, *B*, *R*, and *m*. In part (*b*) we can express the particle's kinetic energy in terms of its momentum and use our result from part (*a*) to show that $K = \tfrac{1}{2}B^2q^2R^2/m$.

(*a*) Express the momentum of the particle:

$$p = mv \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to the orbiting particle to obtain:

$$qvB = m\frac{v^2}{R}$$

Solve for *v*:

$$v = \frac{qBR}{m}$$

Substitute in equation (1) to obtain:

$$p = m\left(\frac{qBR}{m}\right) = \boxed{qBR}$$

(*b*) Express the kinetic energy of the orbiting particle as a function of its momentum:

$$K = \frac{p^2}{2m}$$

Substitute our result from part (*a*) to obtain:

$$K = \frac{(qBR)^2}{2m} = \boxed{\frac{q^2B^2R^2}{2m}}$$

***37 ••**

Picture the Problem The particle's velocity has a component v_1 parallel to $\vec{B}$ and a component v_2 normal to $\vec{B}$. $v_1 = v\cos\theta$ and is constant, whereas $v_2 = v\sin\theta$, being normal to $\vec{B}$, will result in a magnetic force acting on the beam of particles and circular motion perpendicular to $\vec{B}$. We can use the relationship between distance, rate, and time and Newton's 2nd law to express the distance the particle moves in the direction of the field during one period of the motion.

Express the distance moved in the direction of $\vec{B}$ by the particle during one period:

$$x = v_1 T \qquad (1)$$

Express the period of the circular motion of the particles in the beam:

$$T = \frac{2\pi r}{v_2}$$

Apply $\sum F_{\text{radial}} = ma_c$ to a particle in the beam to obtain:

$$qv_2B = m\frac{v_2^2}{r}$$

Solve for v_2:

$$v_2 = \frac{qBr}{m}$$

Substitute to obtain:

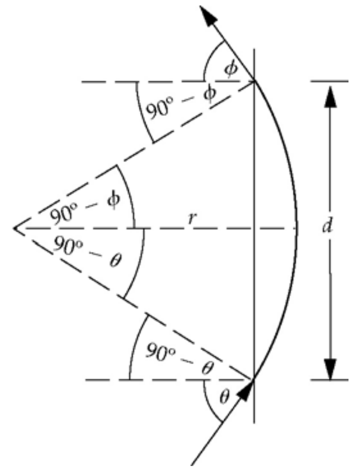
$$T = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$$

Because $v_1 = v\cos\theta$, equation (1) becomes:

$$x = (v\cos\theta)\left(\frac{2\pi m}{qB}\right) = \boxed{2\pi\left(\frac{m}{qB}\right)v\cos\theta}$$

38 ••

Picture the Problem The trajectory of the proton is shown to the right. We know that, because the proton enters the field perpendicularly to the field, its trajectory while in the field will be circular. We can use symmetry considerations to determine ϕ. The application of Newton's 2nd law to the proton while it is in the magnetic field and of trigonometry will allow us to conclude that $r = d$ and to determine their value.



From symmetry, it is evident that the angle θ in Figure 26-35 equals the angle ϕ:

$$\phi = \boxed{60.0°}$$

Use trigonometry to obtain:

$$\sin(90° - \theta) = \sin 30° = \frac{1}{2} = \frac{d/2}{r}$$

or $r = d$.

Apply $\sum F_{\text{radial}} = ma_c$ to the proton while it is in the magnetic field to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for r:

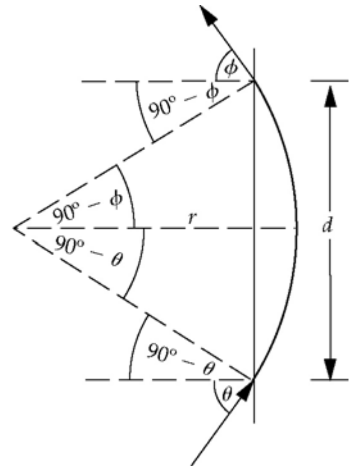
$$r = \frac{mv}{qB}$$

Substitute numerical values and evaluate $r = d$:

$$d = r = \frac{(1.67\times10^{-27}\text{ kg})(10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.8\text{T})} = \boxed{0.130\text{m}}$$

39 ••

Picture the Problem The trajectory of the proton is shown to the right. We know that, because the proton enters the field perpendicularly to the field, its trajectory while in the field will be circular. We can use symmetry considerations to determine ϕ. The application of Newton's 2nd law to the proton while it is in the magnetic field and of trigonometry will allow us to conclude that $r = d$ and to determine their value.



(*a*) From symmetry, it is evident that the angle θ in Figure 26-33 equals the angle ϕ:

$$\phi = \boxed{24.0°}$$

Use trigonometry to obtain:

$$\sin(90° - \theta) = \sin 24° = \frac{d/2}{r_p}$$

or

$$r_p = \frac{d}{2\sin 24°} = \frac{0.4\,\text{m}}{2\sin 24°} = \boxed{0.492\,\text{m}}$$

Apply $\sum F_{\text{radial}} = ma_c$ to the proton while it is in the magnetic field to obtain:

$$q_p v_p B = m_p \frac{v_p^2}{r_p}$$

Solve for and evaluate v_p:

$$v_p = \frac{q_p r_p B}{m_p}$$

Substitute numerical values and evaluate v_p:

$$v_p = \frac{(1.60\times10^{-19}\,\text{C})(0.492\,\text{m})(0.6\,\text{T})}{1.67\times10^{-27}\,\text{kg}}$$

$$= \boxed{2.83\times10^{7}\,\text{m/s}}$$

(*b*) Express v_d:

$$v_d = \frac{q_d r_d B}{m_d} = \frac{q_p r_p B}{2m_p}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{(1.60\times10^{-19}\,\text{C})(0.492\,\text{m})(0.6\,\text{T})}{2(1.67\times10^{-27}\,\text{kg})} = \boxed{1.41\times10^{7}\,\text{m/s}}$$

40 ••

Picture the Problem We can apply Newton's 2nd law of motion to express the orbital speed of the particle and then find the period of the dust particle from this orbital speed.

The period of the dust particle's motion is given by:

$$T = \frac{2\pi r}{v}$$

Apply $\sum F = ma_c$ to the particle:

$$qvB = m\frac{v^2}{r}$$

Solve for v to obtain:

$$v = \frac{qBr}{m}$$

Substitute for v in the expression for T and simplify:

$$T = \frac{2\pi rm}{qBr} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi(10\times10^{-6}\,\text{g}\times10^{-3}\,\text{kg/g})}{(0.3\,\text{nC})(10^{-9}\,\text{T})}$$

$$= 2.094\times10^{11}\,\text{s}\times\frac{1\,\text{y}}{31.56\,\text{Ms}}$$

$$= \boxed{6.64\times10^{3}\,\text{y}}$$

The Velocity Selector

***41** •

Picture the Problem Suppose that, for positively charged particles, their motion is from left to right through the velocity selector and the electric field is upward. Then the magnetic force must be downward and the magnetic field out of the page. We can apply the condition for translational equilibrium to relate v to E and B. In (*b*) and (*c*) we can use the definition of kinetic energy to find the energies of protons and electrons that pass through the velocity selector undeflected.

(*a*) Apply $\sum F_y = 0$ to the particle to obtain:

$$F_{\text{elec}} - F_{\text{mag}} = 0$$

or

$$qE - qvB = 0$$

Solve for v to obtain:

$$v = \frac{E}{B}$$

Substitute numerical values and evaluate v:

$$v = \frac{0.46\,\text{MV/m}}{0.28\,\text{T}} = \boxed{1.64\times10^{6}\ \text{m/s}}$$

(*b*) Express and evaluate the kinetic energy of protons passing through the velocity selector undeflected:

$$\begin{aligned}K_{\text{p}} &= \tfrac{1}{2}m_{\text{p}}v^{2} \\ &= \tfrac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.64\times10^{6}\ \text{m/s}\right)^{2} \\ &= 2.26\times10^{-15}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{14.0\,\text{keV}}\end{aligned}$$

(*c*) The kinetic energy of electrons passing through the velocity selector undeflected is given by:

$$\begin{aligned}K_{\text{e}} &= \tfrac{1}{2}m_{\text{e}}v^{2} \\ &= \tfrac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.64\times10^{6}\ \text{m/s}\right)^{2} \\ &= 1.23\times10^{-18}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{7.66\,\text{eV}}\end{aligned}$$

42 •

Picture the Problem Because the beam of protons is not deflected; we can conclude that the electric force acting on them is balanced by the magnetic force. Hence, we can find the magnetic force from the given data and use its definition to express the electric field.

(*a*) Use the definition of electric field to relate it to the electric force acting on the beam of protons:

$$\vec{\boldsymbol{E}}_{\text{elec}} = \frac{\vec{\boldsymbol{F}}_{\text{elec}}}{q}$$

Express the magnetic force acting on the beam of protons:

$$\vec{\boldsymbol{F}}_{\text{mag}} = qv\hat{\boldsymbol{i}}\times B\hat{\boldsymbol{j}} = qvB\hat{\boldsymbol{k}}$$

Because the electric force must be equal in magnitude but opposite in direction:

$$\vec{\boldsymbol{F}}_{\text{elec}} = -qvB\hat{\boldsymbol{k}} = -\left(1.60\times10^{-19}\ \text{C}\right)\left(12.4\,\text{km/s}\right)\left(0.85\,\text{T}\right)\hat{\boldsymbol{k}} = -\left(1.69\times10^{-15}\ \text{N}\right)\hat{\boldsymbol{k}}$$

Substitute in the equation for the electric field to obtain:

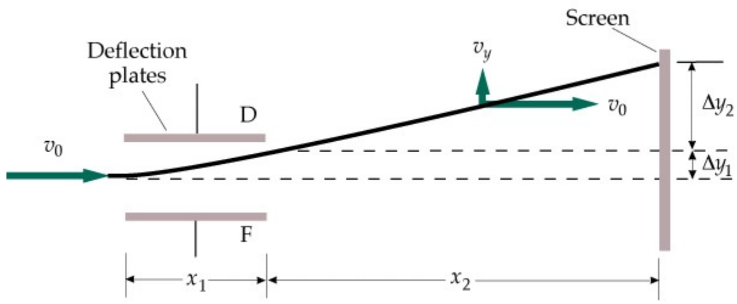
$$\begin{aligned}\vec{\boldsymbol{E}}_{\text{elec}} &= \frac{-\left(1.69\times10^{-15}\ \text{N}\right)\hat{\boldsymbol{k}}}{1.6\times10^{-19}\ \text{C}} \\ &= \boxed{-\left(10.5\,\text{kV/m}\right)\hat{\boldsymbol{k}}}\end{aligned}$$

(*b*) Because both $\vec{F}_{mag}$ and $\vec{F}_{elec}$ are reversed, electrons are not deflected.

Thomson's Measurement of *q*/*m* for Electrons and the Mass Spectrometer

***43 ••**

Picture the Problem Figure 26-18 is reproduced below. We can express the total deflection of the electron beam as the sum of the deflections while the beam is in the field between the plates and its deflection while it is in the field-free space. We can, in turn, use constant-acceleration equations to express each of these deflections. The resulting equation is in terms of v_0 and E. We can find v_0 from the kinetic energy of the beam and E from the potential difference across the plates and their separation. In part (*b*) we can equate the electric and magnetic forces acting on an electron to express B in terms of E and v_0.



(*a*) Express the total deflection Δy of the electrons:

$$\Delta y = \Delta y_1 + \Delta y_2 \qquad (1)$$

where Δy_1 is the deflection of the beam while it is in the electric field and Δy_2 is the deflection of the beam while it travels along a straight-line path outside the electric field.

Use a constant-acceleration equation to express Δy_1:

$$\Delta y_1 = \tfrac{1}{2} a_y (\Delta t)^2 \qquad (2)$$

where $\Delta t = x_1/v_0$ is the time an electron is in the electric field between the plates.

Apply Newton's 2nd law to an electron between the plates to obtain:

$$qE = ma_y$$

Solve for a_y and substitute into equation (2) to obtain:

$$a_y = \frac{qE}{m}$$

and

$$\Delta y_1 = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x_1}{v_0}\right)^2 \quad (3)$$

Express the vertical deflection Δy_2 of the electrons once they are out of the electric field:

$$\Delta y_2 = v_y \Delta t_2 \quad (4)$$

Use a constant-acceleration equation to find the vertical speed of an electron as it leaves the electric field:

$$v_y = v_{0y} + a_y \Delta t_1 = 0 + \frac{qE}{m}\left(\frac{x_1}{v_0}\right)$$

Substitute in equation (4) to obtain:

$$\Delta y_2 = \frac{qE}{m}\left(\frac{x_1}{v_0}\right)\left(\frac{x_2}{v_0}\right) = \frac{qEx_1x_2}{mv_0^2} \quad (5)$$

Substitute equations (3) and (5) in equation (1) to obtain:

$$\Delta y = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x_1}{v_0}\right)^2 + \frac{qEx_1x_2}{mv_0^2}$$

or

$$\Delta y = \frac{qEx_1}{mv_0^2}\left(\frac{x_1}{2} + x_2\right) \quad (6)$$

Use the definition of kinetic energy to find the speed of the electrons:

$$K = \tfrac{1}{2} m v_0^2$$

and

$$v_0 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.8\,\text{keV})}{9.11\times10^{-31}\,\text{kg}}} = 3.14\times10^7\ \text{m/s}$$

Express the electric field between the plates in terms of their potential difference:

$$E = \frac{V}{d}$$

Substitute numerical values and evaluate E:

$$E = \frac{V}{d} = \frac{25\,\text{V}}{1.2\,\text{cm}} = 2.08\,\text{kV/m}$$

Substitute numerical values in equation (6) and evaluate Δy:

$$\Delta y = \frac{(1.60\times10^{-19}\,\text{C})(2.08\,\text{kV/m})(6\,\text{cm})}{(9.11\times10^{-31}\,\text{kg})(31.4\,\text{Mm/s})^2}\left(\frac{6\,\text{cm}}{2} + 30\,\text{cm}\right) = \boxed{7.34\,\text{mm}}$$

(*b*) Because the electrons are deflected upward, the electric field must be downward and the magnetic field upward. Apply $\sum F_y = 0$ to an electron to obtain:

$$F_{\text{mag}} - F_{\text{elec}} = 0$$

or

$$qvB = qE$$

Solve for B:

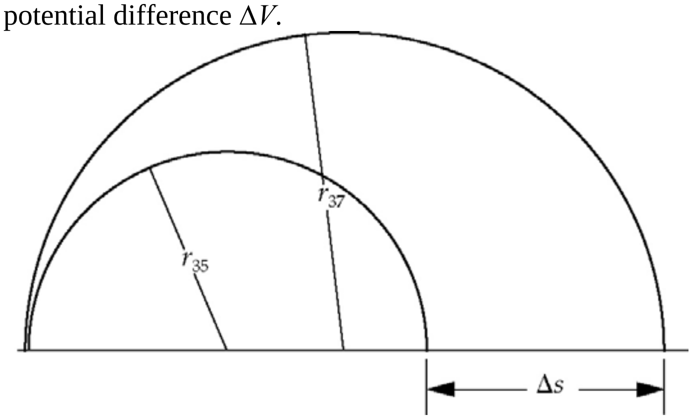
$$B = \frac{E}{v}$$

Substitute numerical values and evaluate B:

$$B = \frac{2.08\,\text{kV/m}}{3.14\times10^{7}\ \text{m/s}} = \boxed{66.2\,\mu\text{T}}$$

44 ••

Picture the Problem The diagram below represents the paths of the two ions entering the magnetic field at the left. The magnetic force acting on each causes them to travel in circular paths of differing radii due to their different masses. We can apply Newton's 2nd law to an ion in the magnetic field to obtain an expression for its radius and then express their final separation in terms of these radii that, in turn, depend on the energy with which the ions enter the field. We can connect their energy to the potential through which they are accelerated using the work-kinetic energy theorem and relate their separation Δs to the accelerating potential difference ΔV.



Express the separation Δs of the chlorine ions:

$$\Delta s = 2(r_{37} - r_{35}) \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the magnetic field of the mass spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB} \qquad (2)$$

Relate the speed of an ion as it enters the magnetic field to the potential difference through which it has been accelerated:

$$q\Delta V = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

Substitute in equation (2) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2m\Delta V}{qB^2}}$$

Use this equation to express the radii of the paths of the two chlorine isotopes to obtain:

$$r_{35} = \sqrt{\frac{2m_{35}\Delta V}{qB^2}} \text{ and } r_{37} = \sqrt{\frac{2m_{37}\Delta V}{qB^2}}$$

Substitute in equation (1) to obtain:

$$\Delta s = 2\left(\sqrt{\frac{2m_{35}\Delta V}{qB^2}} - \sqrt{\frac{2m_{35}\Delta V}{qB^2}}\right) = 2\left(\frac{1}{B}\sqrt{\frac{2\Delta V}{q}}\left(\sqrt{m_{37}} - \sqrt{m_{35}}\right)\right)$$

Solve for ΔV:

$$\Delta V = \frac{qB^2(2\Delta s)^2}{2\left(\sqrt{m_{37}} - \sqrt{m_{35}}\right)^2}$$

Substitute numerical values and evaluate ΔV:

$$\Delta V = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.2\,\text{T}\right)^2\left(\frac{1.4\,\text{cm}}{2}\right)^2}{2\left(\sqrt{37\,\text{u}} - \sqrt{35\,\text{u}}\right)^2} = \frac{5.65\times10^{-24}\,\text{C}\cdot\text{T}^2\cdot\text{m}^2}{\left(\sqrt{37} - \sqrt{35}\right)^2\left(1.66\times10^{-27}\,\text{kg}\right)} = \boxed{122\,\text{kV}}$$

45 ••

Picture the Problem We can apply Newton's 2nd law to an ion in the magnetic field to obtain an expression for r as a function of m, v, q, and B and use the work-kinetic energy theorem to express the kinetic energy in terms of the potential difference through which the ion has been accelerated. Eliminating v between these equations will allow us to express r in terms of m, q, B, and ΔV.

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the magnetic field of the mass spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB} \qquad (1)$$

Apply the work-kinetic energy theorem to relate the speed of an ion as it enters the magnetic field to the potential difference through which it has been accelerated:

$$q\Delta V = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

Substitute in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2m\Delta V}{qB^2}} \qquad (2)$$

(*a*) Substitute numerical values and evaluate equation (2) for ^{24}Mg :

$$r_{24} = \sqrt{\frac{2\left(3.983\times10^{-26}\,\text{kg}\right)\left(2.5\,\text{kV}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(557\times10^{-4}\,\text{T}\right)^2}} = \boxed{63.3\,\text{cm}}$$

(*b*) Express the difference in the radii for ^{24}Mg and ^{26}Mg:

$$\Delta r = r_{26} - r_{24}$$

Substitute numerical values and evaluate equation (2) for ^{26}Mg :

$$r_{26} = \sqrt{\frac{2\left(\frac{26}{24}\right)\left(3.983\times10^{-26}\,\text{kg}\right)\left(2.5\,\text{kV}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(557\times10^{-4}\,\text{T}\right)^2}} = 65.9\,\text{cm}$$

Substitute to obtain:

$$\Delta r = 65.9\,\text{cm} - 63.3\,\text{cm} = \boxed{2.60\,\text{cm}}$$

***46 ••**

Picture the Problem We can apply Newton's 2nd law to an ion in the magnetic field of the spectrometer to relate the diameter of its orbit to its charge, mass, velocity, and the magnetic field. If we assume that the velocity is the same for the two ions, we can then express the ratio of the two diameters as the ratio of the masses of the ions and solve for the diameter of the orbit of ^{7}Li.

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the field of the spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Express the diameter of the orbit:

$$d = \frac{2mv}{qB}$$

Express the diameters of the orbits for ^{6}Li and ^{7}Li:

$$d_6 = \frac{2m_6 v}{qB} \text{ and } d_7 = \frac{2m_7 v}{qB}$$

Assume that the velocities of the two ions are the same and divide the 2nd of these diameters by the first to obtain:

$$\frac{d_7}{d_6} = \frac{\frac{2m_7 v}{qB}}{\frac{2m_6 v}{qB}} = \frac{m_7}{m_6}$$

Solve for and evaluate d_7:

$$d_7 = \frac{m_7}{m_6} d_6 = \frac{7\,\text{u}}{6\,\text{u}}(15\,\text{cm}) = \boxed{17.5\,\text{cm}}$$

The Cyclotron

47 ••

Picture the Problem The time required for each of the ions to complete its semicircular paths is half its period. We can apply Newton's 2nd law to an ion in the magnetic field of the spectrometer to obtain an expression for r as a function of the charge and mass of the ion, its velocity, and the magnetic field.

Express the time for each ion to complete its semicircular path:

$$\Delta t = \tfrac{1}{2}T = \frac{\pi r}{v}$$

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the field of the spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Substitute to obtain:

$$\Delta t = \frac{\pi m}{qB}$$

Substitute numerical values and evaluate Δt_{58} and Δt_{60}:

$$\Delta t_{58} = \frac{58\pi\left(1.66\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.12\text{ T}\right)} = \boxed{15.8\,\mu\text{s}}$$

and

$$\Delta t_{60} = \frac{60\pi\left(1.66\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.12\text{ T}\right)} = \boxed{16.3\,\mu\text{s}}$$

48 ••

Picture the Problem We can apply a condition for equilibrium to ions passing through the velocity selector to obtain an expression relating E, B, and v that we can solve for v. We can, in turn, express E in terms of the potential difference V between the plates of the selector and their separation d. In (*b*) we can apply Newton's 2nd law to an ion in the bending field of the spectrometer to relate its diameter to its mass, charge, velocity, and the magnetic field.

(*a*) Apply $\sum F_y = 0$ to the ions in the crossed fields of the velocity selector to obtain:

$$F_{\text{elec}} - F_{\text{mag}} = 0$$

or

$$qE - qvB = 0$$

Solve for v to obtain:

$$v = \frac{E}{B}$$

Express the electric field between the plates of the velocity selector in terms of their separation and the potential difference across them:

$$E = \frac{V}{d}$$

Substitute to obtain:

$$v = \frac{V}{dB}$$

Substitute numerical values and evaluate v:

$$v = \frac{160\text{ V}}{\left(2\text{ mm}\right)\left(0.42\text{ T}\right)} = \boxed{1.90\times10^{5}\text{ m/s}}$$

(*b*) Express the difference in the diameters of the orbits of singly

$$\Delta d = d_{238} - d_{235} \qquad (1)$$

ionized ^{238}U and ^{235}U:

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to an ion in the spectrometer's magnetic field:

$$qvB = m\frac{v^2}{r}$$

Solve for the radius of the ion's orbit:

$$r = \frac{mv}{qB}$$

Express the diameter of the orbit:

$$d = \frac{2mv}{qB}$$

Express the diameters of the orbits for ^{238}U and ^{235}U:

$$d_{238} = \frac{2m_{238}v}{qB} \text{ and } d_{235} = \frac{2m_{235}v}{qB}$$

Substitute in equation (1) to obtain:

$$\Delta d = \frac{2m_{238}v}{qB} - \frac{2m_{235}v}{qB} = \frac{2v}{qB}\left(m_{238} - m_{235}\right)$$

Substitute numerical values and evaluate Δd:

$$\Delta d = \frac{2\left(1.90\times10^{5}\text{ m/s}\right)\left(238\,\text{u} - 235\,\text{u}\right)\left(\frac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.2\,\text{T}\right)} = \boxed{9.86\,\text{mm}}$$

***49 ••**

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of the protons/deuterons. By applying Newton's 2nd law, we can relate the radius of the particle's orbit to its speed and, hence, express the cyclotron frequency as a function of the particle's mass and charge and the cyclotron's magnetic field. In part (*b*) we can use the definition of kinetic energy and their maximum speed to find the maximum energy of the emerging protons.

(*a*) Express the cyclotron frequency in terms of the proton's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to a proton in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r} \qquad (1)$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Substitute to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{qB}{2\pi m} \qquad (2)$$

Substitute numerical values and evaluate f:

$$f = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(1.4\text{ T}\right)}{2\pi\left(1.67\times10^{-27}\text{ kg}\right)} = \boxed{21.3\text{ MHz}}$$

(b) Express the maximum kinetic energy of a proton:

$$K_{max} = \tfrac{1}{2}mv_{max}^2$$

Solve equation (1) for v_{max} to obtain:

$$v_{max} = \frac{qBr_{max}}{m}$$

Substitute to obtain:

$$K = \tfrac{1}{2}m\left(\frac{qBr_{max}}{m}\right)^2 = \tfrac{1}{2}\left(\frac{q^2B^2}{m}\right)r_{max}^2 \qquad (3)$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}\left(\frac{\left(1.60\times10^{-19}\text{ C}\right)^2\left(1.4\text{ T}\right)^2}{1.67\times10^{-27}\text{ kg}}\right)\left(0.7\text{ m}\right)^2$$

$$= 7.36\times10^{-12}\text{ J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{ J}}$$

$$= \boxed{46.0\text{ MeV}}$$

(c) From equation (2) we see that doubling m halves f:

$$f_{deuterons} = \tfrac{1}{2}f_{protons} = \boxed{10.7\text{ MHz}}$$

From equation (3) we see that doubling m halves K:

$$K_{deuterons} = \tfrac{1}{2}K_{protons} = \boxed{23.0\text{ MeV}}$$

50 ••

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of the protons be accelerated in the cyclotron. By applying Newton's 2nd law, we can relate the radius of the proton's orbit to its speed and, hence, express the cyclotron frequency as a function of the its mass and charge and the cyclotron's magnetic field. In part (b) we can use the definition of kinetic energy express the minimum radius required to achieve the desired emergence energy. In part (c) we can find the number of revolutions required to achieve this emergence energy from the energy acquired during each revolution.

(*a*) Express the cyclotron frequency in terms of the proton's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply $\sum F_{\text{radial}} = ma_c$ to a proton in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r}$$

Solve for *r* to obtain:

$$r = \frac{mv}{qB} \qquad (1)$$

Substitute to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{qB}{2\pi m}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.8\,\text{T}\right)}{2\pi\left(1.67\times10^{-27}\ \text{kg}\right)} = \boxed{27.4\,\text{MHz}}$$

(*b*) Using the definition of kinetic energy, relate emergence energy of the protons to their velocity:

$$K = \tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2K}{m}}$$

Substitute in equation (1) and simplify to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Substitute numerical values and evaluate r_{min}:

$$r = \frac{\sqrt{2\left(25\,\text{MeV}\right)\left(1.67\times10^{-27}\ \text{kg}\right)}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.8\,\text{T}\right)} = \boxed{0.401\,\text{m}}$$

(*c*) Express the required number of revolutions *N* in terms of the energy gained per revolution:

$$N = \frac{25\,\text{MeV}}{E_{\text{rev}}}$$

Because the beam is accelerated through a potential difference of 50

$$E_{\text{rev}} = 2q\Delta V = 100\,\text{keV}$$

kV twice during each revolution:

Substitute and evaluate N:

$$N = \frac{25\,\text{MeV}}{100\,\text{keV/rev}} = \boxed{250\,\text{rev}}$$

51 ••

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of a particle being accelerated in the cyclotron. By applying Newton's 2nd law, we can relate the radius of the particle's orbit to its speed and, hence, express the cyclotron frequency as a function of its charge-to-mass ratio and the cyclotron's magnetic field. We can then use data for the relative charges and masses of deuterons, alpha particles, and protons to establish the ratios of their cyclotron frequencies.

Express the cyclotron frequency in terms of a particle's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply $\sum F_{\text{radial}} = ma_c$ to a particle in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Substitute to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{B}{2\pi}\frac{q}{m} \qquad (1)$$

Evaluate equation (1) for deuterons:

$$f_{\text{d}} = \frac{B}{2\pi}\frac{q_{\text{d}}}{m_{\text{d}}} = \frac{B}{2\pi}\frac{e}{m_{\text{d}}}$$

Evaluate equation (1) for alpha particles:

$$f_\alpha = \frac{B}{2\pi}\frac{q_\alpha}{m_\alpha} = \frac{B}{2\pi}\frac{2e}{2m_{\text{d}}} = \frac{B}{2\pi}\frac{e}{m_{\text{d}}}$$

and

$$\boxed{f_{\text{d}} = f_\alpha}$$

Evaluate equation (1) for protons:

$$f_{\text{p}} = \frac{B}{2\pi}\frac{q_{\text{p}}}{m_{\text{p}}} = \frac{B}{2\pi}\frac{e}{\frac{1}{2}m_{\text{d}}} = 2\left(\frac{B}{2\pi}\frac{e}{m_{\text{d}}}\right)$$

$$= 2f_{\text{d}}$$

and

$$\tfrac{1}{2}f_p = \boxed{f_d = f_\alpha}$$

52 ••

Picture the Problem We can apply Newton's 2nd law to the orbiting charged particle to obtain an expression for its radius as a function of its particle's kinetic energy. Because the energy gain per revolution is constant, we can express this kinetic energy as the product of the number of orbits completed and the energy gained per revolution and, hence, show that the radius is proportional to the square root of the number of orbits completed.

Apply $\sum F_{radial} = ma_c$ to a particle in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB} \qquad (1)$$

Express the kinetic energy of the particle in terms of its speed and solve for v:

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} \qquad (2)$$

Noting that the energy gain per revolution is constant, express the kinetic energy in terms of the number of orbits N completed by the particle and energy E_{rev} gained by the particle each revolution:

$$K = NE_{rev} \qquad (3)$$

Substitute equations (2) and (3) in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2mK}$$

$$= \frac{1}{qb}\sqrt{2mNE_{rev}} = \frac{\sqrt{2mE_{rev}}}{qB}N^{1/2}$$

or $\boxed{r \propto N^{1/2}}$

Torques on Current Loops and Magnets

53 •

Picture the Problem We can use the definition of the magnetic moment of a coil to evaluate μ and the expression for the torque exerted on the coil $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$ to find the magnitude of τ.

(*a*) Using its definition, express the magnetic moment of the coil:

$$\mu = NIA = NI\pi r^2$$

Substitute numerical values and evaluate μ:

$$\mu = (20)(3\,\text{A})\pi(0.04\,\text{m})^2 = \boxed{0.302\,\text{A}\cdot\text{m}^2}$$

(*b*) Express the magnitude of the torque exerted on the coil:

$$\tau = \mu B \sin\theta$$

Substitute numerical values and evaluate τ:

$$\tau = (0.302\,\text{A}\cdot\text{m}^2)(0.5\,\text{T})\sin 60° = \boxed{0.131\,\text{N}\cdot\text{m}}$$

54 •

Picture the Problem The coil will experience the maximum torque when the plane of the coil makes an angle of 90° with the direction of $\vec{B}$. The magnitude of the maximum torque is then given by $\tau_{max} = \mu B$.

Express the maximum torque acting on the coil:

$$\tau_{max} = \mu B$$

Use its definition to express the magnetic moment of the coil:

$$\mu = NIA = NI\pi r^2$$

Substitute to obtain:

$$\tau_{max} = NI\pi r^2 B$$

Substitute numerical values and evaluate τ:

$$\tau_{max} = (400)(1.6\,\text{mA})\pi(0.75\,\text{cm})^2(0.25\,\text{T}) = \boxed{2.83\times10^{-5}\,\text{N}\cdot\text{m}}$$

***55** •

Picture the Problem We can use $\vec{\tau} = \vec{\mu}\times\vec{B}$ to find the torque on the coil in the two orientations of the magnetic field.

Express the torque acting on the coil:

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

Express the magnetic moment of the coil:

$$\vec{\mu} = \pm IA\hat{k} = \pm IL^2\hat{k}$$

(*a*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the *z* direction:

$$\vec{\tau} = \pm IL^2\hat{k}\times B\hat{k} = \pm IL^2B\left(\hat{k}\times\hat{k}\right) = \boxed{0}$$

(*b*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the *x* direction:

$$\vec{\tau} = \pm IL^2\hat{k}\times B\hat{i} = \pm IL^2B\left(\hat{k}\times\hat{i}\right) = \pm(2.5\,\text{A})(0.06\,\text{m})^2(0.3\,\text{T})\hat{j} = \boxed{\pm\left(2.70\times10^{-3}\,\text{N}\cdot\text{m}\right)\hat{j}}$$

56 •

Picture the Problem We can use $\vec{\tau} = \vec{\mu}\times\vec{B}$ to find the torque on the equilateral triangle in the two orientations of the magnetic field.

Express the torque acting on the coil:

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

Express the magnetic moment of the coil:

$$\vec{\mu} = \pm IA\hat{k}$$

Relate the area of the equilateral triangle to the length of its side:

$$A = \tfrac{1}{2}\text{base}\times\text{altitude} = \frac{1}{2}(L)\left(\frac{\sqrt{3}L}{2}\right) = \frac{\sqrt{3}}{4}L^2$$

Substitute to obtain:

$$\vec{\mu} = \pm\frac{\sqrt{3}L^2I}{4}\hat{k}$$

(*a*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the *z* direction:

$$\vec{\tau} = \pm\frac{\sqrt{3}L^2I}{4}\hat{k}\times B\hat{k} = \pm\frac{\sqrt{3}L^2IB}{4}\left(\hat{k}\times\hat{k}\right) = \boxed{0}$$

(*b*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the *x* direction:

$$\vec{\tau} = \pm\frac{\sqrt{3}L^2I}{4}\hat{k}\times B\hat{i} = \pm\frac{\sqrt{3}L^2IB}{4}\left(\hat{k}\times\hat{i}\right) = \pm\frac{\sqrt{3}(0.08\,\text{m})^2(2.5\,\text{A})(0.3\,\text{T})}{4}\hat{j} = \boxed{\pm\left(2.08\times10^{-3}\,\text{N}\cdot\text{m}\right)\hat{j}}$$

57 ••

Picture the Problem The loop will start to lift off the table when the magnetic torque equals the gravitational torque.

Express the magnetic torque acting on the loop:

$$\tau_{\text{mag}} = \mu B = I\pi R^2 B$$

Express the gravitational torque acting on the loop:

$$\tau_{\text{grav}} = mgR$$

Because the loop is in equilibrium under the influence of the two torques:

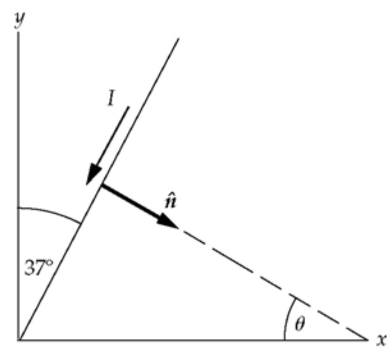
$$I\pi R^2 B = mgR$$

Solve for B to obtain:

$$B = \boxed{\frac{mg}{I\pi R}}$$

58 ••

Picture the Problem The diagram to the right shows the coil as it would appear from along the positive z axis. The right-hand rule for determining the direction of $\hat{\boldsymbol{n}}$ has been used to establish $\hat{\boldsymbol{n}}$ as shown. We can use the geometry of this figure to determine θ and to express the unit normal vector $\hat{\boldsymbol{n}}$. The magnetic moment of the coil is given by $\vec{\boldsymbol{\mu}} = NIA\hat{\boldsymbol{n}}$ and the torque exerted on the coil by $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$. Finally, we can find the potential energy of the coil in this field from $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}}$.



(*a*) Noting that θ and the angle whose measure is 37° have their right and left sides mutually perpendicular, we can conclude that:

$$\theta = \boxed{37°}$$

(*b*) Use the components of $\hat{\boldsymbol{n}}$ to express $\hat{\boldsymbol{n}}$ in terms of $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$:

$$\hat{\boldsymbol{n}} = n_x\hat{\boldsymbol{i}} + n_y\hat{\boldsymbol{j}} = \cos 37°\hat{\boldsymbol{i}} - \sin 37°\hat{\boldsymbol{j}}$$
$$= \boxed{0.799\hat{\boldsymbol{i}} - 0.602\hat{\boldsymbol{j}}}$$

(*c*) Express the magnetic moment of

$$\vec{\boldsymbol{\mu}} = NIA\hat{\boldsymbol{n}}$$

the coil:

Substitute numerical values and evaluate $\vec{\mu}$:

$$\vec{\mu} = (50)(1.75\,\text{A})(48\,\text{cm}^2)(0.799\hat{i} - 0.602\hat{j}) = \boxed{(0.336\,\text{A}\cdot\text{m}^2)\,\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}}$$

(*d*) Express the torque exerted on the coil:

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

Substitute for $\vec{\mu}$ and $\vec{B}$ to obtain:

$$\begin{aligned}\vec{\tau} &= \left\{(0.336\,\text{A}\cdot\text{m}^2)\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}\right\}\times(1.5\,\text{T})\hat{j}\\ &= (0.504\,\text{N}\cdot\text{m})(\hat{i}\times\hat{j}) - (0.380\,\text{N}\cdot\text{m})(\hat{j}\times\hat{j}) = \boxed{(0.504\,\text{N}\cdot\text{m})\hat{k}}\end{aligned}$$

(*e*) Express the potential energy of the coil in terms of its magnetic moment and the magnetic field:

$$U = -\vec{\mu}\cdot\vec{B}$$

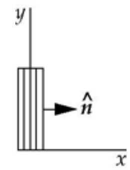
Substitute for $\vec{\mu}$ and $\vec{B}$ and evaluate *U*:

$$\begin{aligned}U &= -\left\{(0.336\,\text{A}\cdot\text{m}^2)\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}\right\}\cdot(1.5\,\text{T})\hat{j}\\ &= -(0.504\,\text{N}\cdot\text{m})(\hat{i}\cdot\hat{j}) + (0.380\,\text{N}\cdot\text{m})(\hat{j}\cdot\hat{j}) = \boxed{0.380\,\text{J}}\end{aligned}$$

59 ••

Picture the Problem We can use the right-hand rule for determining the direction of $\hat{n}$ to establish the orientation of the coil for value of $\hat{n}$ and $\vec{\tau} = \vec{\mu}\times\vec{B}$ to find the torque exerted on the coil in each orientation.

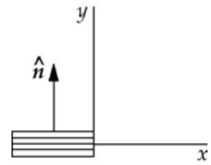
(*a*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B}$ = 2.0 T $\hat{j}$ and $\hat{n} = \hat{i}$:

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}\\ &= (50)(1.75\,\text{A})(48\,\text{cm}^2)\hat{i}\times(2\,\text{T})\hat{j}\\ &= (0.840\,\text{N}\cdot\text{m})(\hat{i}\times\hat{j})\\ &= \boxed{(0.840\,\text{N}\cdot\text{m})\hat{k}}\end{aligned}$$

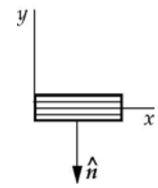
(*b*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\text{ T}\,\hat{j}$ and $\hat{n} = \hat{j}$:

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}\\ &= (50)(1.75\text{ A})(48\text{ cm}^2)\hat{j}\times(2\text{ T})\hat{j}\\ &= (0.840\text{ N}\cdot\text{m})(\hat{j}\times\hat{j})\\ &= \boxed{0}\end{aligned}$$

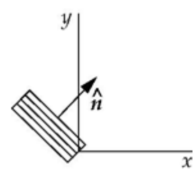
(*c*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\text{ T}\,\hat{j}$ and $\hat{n} = -\hat{j}$:

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}\\ &= -(50)(1.75\text{ A})(48\text{ cm}^2)\hat{j}\times(2\text{ T})\hat{j}\\ &= (-0.840\text{ N}\cdot\text{m})(\hat{j}\times\hat{j})\\ &= \boxed{0}\end{aligned}$$

(*d*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\text{ T}\,\hat{j}$ and $\hat{n} = (\hat{i} + \hat{j})/\sqrt{2}$:

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}\\ &= \frac{(50)(1.75\text{ A})(48\text{ cm}^2)}{\sqrt{2}}(\hat{i}+\hat{j})\times(2\text{ T})\hat{j}\\ &= (0.594\text{ N}\cdot\text{m})(\hat{i}\times\hat{j})\\ &\quad+(0.594\text{ N}\cdot\text{m})(\hat{j}\times\hat{j})\\ &= \boxed{(0.594\text{ N}\cdot\text{m})\hat{k}}\end{aligned}$$

Magnetic Moments

***60 ••**

Picture the Problem Because the small magnet can be modeled as a magnetic dipole; we can use the equation for the torque on a current loop to find its magnetic moment.

Express the magnitude of the torque

$$\tau = \mu B\sin\theta$$

acting on the magnet:

Solve for μ to obtain:

$$\mu = \frac{\tau}{B\sin\theta}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{0.10\,\text{N}\cdot\text{m}}{(0.04\,\text{T})\sin 60^\circ} = \boxed{2.89\,\text{A}\cdot\text{m}^2}$$

61 ••

Picture the Problem We can use the definition of the magnetic moment to find the magnetic moment of the given current loop and a right-hand rule to find its direction.

Using its definition, express the magnetic moment of the current loop:

$$\mu = IA$$

Express the area bounded by the loop:

$$A = \tfrac{1}{2}\left(\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right) = \frac{\pi}{2}\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$$

Substitute to obtain:

$$\mu = \frac{\pi I}{2}\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{\pi(1.5\,\text{A})}{2}\left[(0.5\,\text{m})^2 - (0.3\,\text{m})^2\right] = \boxed{0.377\,\text{A}\cdot\text{m}^2}$$

Apply the right-hand rule for determining the direction of the unit normal vector (the direction of μ) to conclude that $\boxed{\vec{\boldsymbol{\mu}}\text{ points into the page.}}$

62 ••

Picture the Problem We can use the definition of the magnetic moment of a coil to find the magnetic moment of a wire of length L that is wound into a circular coil of N loops. We can find the area of the coil from its radius R and we can find R by dividing the length of the wire by the number of turns.

Use its definition to express the magnetic moment of the coil:

$$\mu = NIA \qquad (1)$$

Express the circumference of each loop:

$$\frac{L}{N} = 2\pi R$$

where R is the radius of a loop.

Solve for R to obtain:

$$R = \frac{L}{2\pi N}$$

Express the area of the coil:

$$A = \pi R^2 = \pi\left(\frac{L}{2\pi N}\right)^2 = \frac{L^2}{4\pi N^2}$$

Substitute in equation (1) and simplify to obtain:

$$\mu = NI\left(\frac{L^2}{4\pi N^2}\right) = \boxed{\frac{IL^2}{4\pi N}}$$

63 ••

Picture the Problem We can use the definition of current and the relationship between the frequency of the motion and its period to show that $I = q\omega/2\pi$. We can use the definition of angular momentum and the moment of inertia of a point particle to show that the magnetic moment has the magnitude $\mu = \frac{1}{2}q\omega r^2$. Finally, we can express the ratio of μ to L and the fact that $\vec{\mu}$ and $\vec{L}$ are both parallel to $\vec{\omega}$ to conclude that $\vec{\mu} = (q/2m)\vec{L}$.

(*a*) Using its definition, relate the average current to the charge passing a point on the circumference of the circle in a given period of time:

$$I = \frac{\Delta q}{\Delta t} = \frac{q}{T} = qf$$

Relate the frequency of the motion to the angular frequency of the particle:

$$f = \frac{\omega}{2\pi}$$

Substitute to obtain:

$$I = \boxed{\frac{q\omega}{2\pi}}$$

From the definition of the magnetic moment we have:

$$\mu = IA = \left(\frac{q\omega}{2\pi}\right)\left(\pi r^2\right) = \boxed{\tfrac{1}{2}q\omega r^2}$$

(*b*) Express the angular momentum of the particle:

$$L = I\omega$$

The angular momentum of the particle is:

$$I = mr^2$$

Substitute to obtain: $L = (mr^2)\omega = \boxed{mr^2\omega}$

Express the ratio of μ to L and simplify to obtain:
$$\frac{\mu}{L} = \frac{\frac{1}{2}q\omega r^2}{mr^2\omega} = \frac{q}{2m} \Rightarrow \mu = \frac{q}{2m}L$$

Because $\vec{\mu}$ and $\vec{L}$ are both parallel to $\vec{\omega}$:
$$\vec{\mu} = \boxed{\frac{q}{2m}\vec{L}}$$

***64** •••

Picture the Problem We can express the magnetic moment of an element of charge *dq* in a cylinder of length *L*, radius *r*, and thickness *dr*, relate this charge to the length, radius, and thickness of the cylinder, express the current due to this rotating charge, substitute for *A* and *dI* in our expression for μ and then integrate to complete our derivation for the magnetic moment of the rotating cylinder as a function of its angular velocity.

Express the magnetic moment of an element of charge *dq* in a cylinder of length *L*, radius *r*, and thickness *dr*:
$$d\mu = AdI$$
where
$A = \pi r^2$.

Relate the charge *dq* in the cylinder to the length of the cylinder, its radius, and thickness:
$$dq = 2\pi L\rho r dr$$

Express the current due to this rotating charge:
$$dI = \frac{\omega}{2\pi}dq = \frac{\omega}{2\pi}(2\pi L\rho r dr) = L\omega\rho r dr$$

Substitute to obtain:
$$d\mu = \pi r^2(L\omega\rho r dr) = L\omega\rho\pi r^3 dr$$

Integrate *r* from R_i to R_0 to obtain:
$$\mu = L\omega\rho\pi\int_{R_i}^{R_0} r^3 dr = \boxed{\tfrac{1}{4}L\omega\rho\pi\left(R_0^4 - R_i^4\right)}$$

65 •••

Picture the Problem We can follow the step-by-step outline provided in the problem statement to establish the given results.

(*a*) Express the magnetic moment of the rotating element of charge:
$$d\mu = AdI \qquad (1)$$

The area enclosed by the rotating
$$A = \pi x^2$$

element of charge:

$$dI = \frac{dq}{\Delta t} = \frac{\lambda dx}{\Delta t}$$

where Δt is the time required for one revolution.

Express the time Δt required for one revolution:

$$\Delta t = \frac{1}{f} = \frac{2\pi}{\omega}$$

Substitute to obtain:

$$dI = \frac{\lambda\omega}{2\pi}dx$$

Substitute in equation (1) and simplify to obtain:

$$d\mu = \left(\pi x^2\right)\left(\frac{\lambda\omega}{2\pi}dx\right) = \boxed{\tfrac{1}{2}\lambda\omega x^2 dx}$$

(*b*) Integrate $d\mu$ from $x = 0$ to $x = \ell$:

$$\mu = \tfrac{1}{2}\lambda\omega\int_0^{\ell} x^2 dx = \boxed{\tfrac{1}{6}\lambda\omega\ell^3}$$

(*c*) Express the angular momentum of the rod:

$$L = I\omega$$

where L is the angular momentum of the rod and I is the moment of inertia of the rod with respect to the point about which it is rotating.

Express the moment of inertia of the rod with respect to an axis through its end:

$$I = \tfrac{1}{3}mL^2$$

where L is now the length of the rod.

Substitute to obtain:

$$L = \tfrac{1}{3}mL^2\omega$$

Divide our expression for μ by L to obtain:

$$\frac{\mu}{L} = \frac{\tfrac{1}{6}\lambda\omega L^3}{\tfrac{1}{3}mL^2\omega} = \frac{\lambda L}{2m}$$

or, because $Q = \lambda L$,

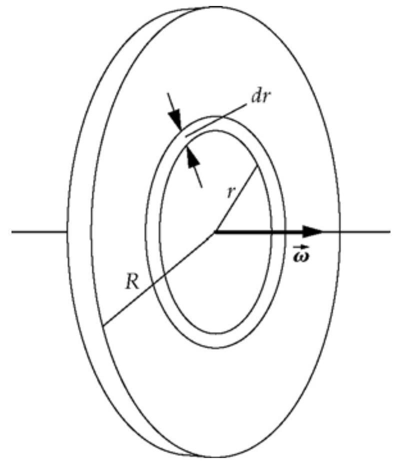
$$\mu = \frac{Q}{2m}L$$

Because $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{L}} = I\vec{\boldsymbol{\omega}}$ point in the same direction:

$$\vec{\boldsymbol{\mu}} = \boxed{\frac{Q}{2M}\vec{\boldsymbol{L}}}$$

66 •••

Picture the Problem We can express the magnetic moment of an element of current dI due to a ring of radius r, and thickness dr with charge dq. Integrating this expression from $r = 0$ to $r = R$ will give us the magnetic moment of the disk. We can integrate the charge on the ring between these same limits to find the total charge on the disk and divide μ by Q to establish the relationship between them. In part (b) we can find the angular momentum of the disk by first finding the moment of inertia of the disk by integrating r^2dm between the same limits used above.



(a) Express the magnetic moment of an element of the disk:

$$d\mu = AdI$$

The area enclosed by the rotating element of charge is:

$$A = \pi r^2$$

Express the element of current dI:

$$dI = \frac{dq}{\Delta t} = \frac{\sigma dA}{\Delta t} = f\sigma dA$$

$$= \frac{\omega}{2\pi}\left(\sigma_0 \frac{r}{R}\right)(2\pi r dr) = \frac{\sigma_0\omega}{R} r^2 dr$$

Substitute and simplify to obtain:

$$d\mu = \pi r^2 \frac{\sigma_0\omega}{R} r^2 dr = \frac{\sigma_0\pi\omega}{R} r^4 dr$$

Integrate $d\mu$ from $r = 0$ to $r = R$ to obtain:

$$\mu = \frac{\sigma_0\pi\omega}{R}\int_0^R r^4 dr = \boxed{\tfrac{1}{5}\sigma_0\pi\omega R^4} \qquad (1)$$

Express the charge dq within a distance r of the center of the disk:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0\frac{r}{R}\right)dr$$

$$= \frac{2\pi\sigma_0}{R} r^2 dr$$

Integrate dq from $r = 0$ to $r = R$ to obtain:

$$Q = \frac{2\pi\sigma_0}{R}\int_0^R r^2 dr = \tfrac{2}{3}\pi\sigma_0 R^2 \qquad (2)$$

Divide equation (1) by Q to obtain:

$$\frac{\mu}{Q}=\frac{\frac{1}{5}\sigma_0\pi\omega R^4}{\frac{2}{3}\pi\sigma_0 R^2}=\frac{3\omega R^2}{10}$$

and

$$\mu=\boxed{\tfrac{3}{10}Q\omega R^2} \qquad (3)$$

(b) Express the moment of inertia of an element of mass dm of the disk:

$$dI=r^2dm=r^2\sigma_{\mathrm{m}}dA$$
$$=r^2\left(\frac{m}{Q}\sigma\right)(2\pi r dr)$$
$$=\frac{2\pi m\left(\frac{r}{R}\sigma_0\right)}{Q}r^3dr$$
$$=\frac{2\pi m\sigma_0}{QR}r^4dr$$

Integrate dI from $r = 0$ to $r = R$ to obtain:

$$I=\frac{2\pi m\sigma_0}{QR}\int_0^R r^4dr=\frac{2\pi m\sigma_0}{5Q}R^4$$

Divide I by equation (2) and simplify to obtain:

$$\frac{I}{Q}=\frac{\frac{2\pi m\sigma_0}{5Q}R^4}{\frac{2}{3}\pi\sigma_0 R^2}=\frac{3m}{5Q}R^2$$

and

$$I=\frac{3m}{5}R^2$$

Express the angular momentum of the disk:

$$L=I\omega=\tfrac{3}{5}mR^2\omega$$

Divide equation (3) by L and simplify to obtain:

$$\frac{\mu}{L}=\frac{\frac{3}{10}Q\omega R^2}{\frac{3}{5}mR^2\omega}=\frac{Q}{2m}$$

and

$$\mu=\frac{Q}{2m}L$$

Because $\vec{\boldsymbol{\mu}}$ is in the same direction as $\vec{\boldsymbol{\omega}}$:

$$\vec{\boldsymbol{\mu}}=\boxed{\frac{Q}{2m}\vec{\boldsymbol{L}}}$$

67 •••

Picture the Problem We can use the general result from Example 26-11 and Problem 63 to express μ as a function of Q, M, and L. We can then use the definitions of surface charge density and angular momentum to substitute for Q and L to obtain the magnetic moment of the rotating sphere.

Express the magnetic moment of the spherical shell in terms of its mass, charge, and angular momentum:

$$\mu = \frac{Q}{2M}L$$

Use the definition of surface charge density to express the charge on the spherical shell is:

$$Q = \sigma A = 4\pi\sigma R^2$$

Express the angular momentum of the spherical shell:

$$L = I\omega = \tfrac{2}{3}MR^2\omega$$

Substitute to obtain:

$$\mu = \left(\frac{4\pi\sigma R^2}{2M}\right)\left(\frac{2}{3}MR^2\omega\right) = \boxed{\tfrac{4}{3}\pi\sigma R^4\omega}$$

68 •••

Picture the Problem We can use the general result from Example 26-11 and Problem 63 to express μ as a function of Q, M, and L. We can then use the definitions of volume charge density and angular momentum to substitute for Q and L to obtain the magnetic moment of the rotating sphere.

Express the magnetic moment of the solid sphere in terms of its mass, charge, and angular momentum:

$$\mu = \frac{Q}{2M}L$$

Use the definition of volume charge density to express the charge of the sphere:

$$Q = \rho V = \tfrac{4}{3}\pi\rho R^3$$

Express the angular momentum of the solid sphere:

$$L = I\omega = \tfrac{2}{5}MR^2\omega$$

Substitute to obtain:

$$\mu = \left(\frac{\tfrac{4}{3}\pi\rho R^3}{2M}\right)\left(\frac{2}{5}MR^2\omega\right) = \boxed{\tfrac{4}{15}\pi\rho R^5\omega}$$

***69** •••

Picture the Problem We can use its definition to express the torque acting on the disk and the definition of the precession frequency to find the precession frequency of the disk.

(*a*) The magnitude of the net torque acting on the disk is:

$\tau = \mu B \sin\theta$

where μ is the magnetic moment of the disk.

From example 26-11:

$$\mu = \frac{1}{4}\pi\sigma r^4\omega$$

Substitute for μ in the expression for τ to obtain:

$$\tau = \boxed{\frac{1}{4}\pi\sigma r^4\omega B\sin\theta}$$

(*b*) The precession frequency Ω is equal to the ratio of the torque divided by the spin angular momentum:

$$\Omega = \frac{\tau}{I\omega}$$

For a solid disk, the moment of inertia is given by:

$$I = \frac{1}{2}mr^2$$

Substitute for τ and I to obtain:

$$\Omega = \frac{\frac{1}{4}\pi\sigma r^4\omega B\sin\theta}{\frac{1}{2}mr^2\omega} = \boxed{\frac{\pi\sigma r^2 B}{2m}\sin\theta}$$

Remarks: It's interesting that the precession frequency is independent of ω.

The Hall Effect

70 •

Picture the Problem We can use the Hall effect equation to find the drift velocity of the electrons and the relationship between the current and the number density of charge carriers to find *n*. In (*c*) we can use a right-hand rule to decide whether *a* or *b* is at the higher potential.

(*a*) Express the Hall voltage as a function of the drift velocity of the electrons in the strip:

$$V_H = v_d Bw$$

Solve for v_d:

$$v_d = \frac{V_H}{Bw}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{4.27\,\mu\text{V}}{(2\,\text{T})(2\,\text{cm})} = \boxed{0.107\,\text{mm/s}}$$

(*b*) Express the current as a function of the number density of charge carriers:

$$I = nAqv_d$$

Solve for n:

$$n = \frac{I}{Aqv_d}$$

Substitute numerical values and evaluate n:

$$n = \frac{20\,\text{A}}{(2\,\text{cm})(0.1\,\text{cm})(1.60\times10^{-19}\,\text{C})(0.107\,\text{mm/s})} = \boxed{5.84\times10^{28}\,\text{m}^{-3}}$$

(c) Apply a right-hand rule to $I\vec{\ell}$ and $\vec{\boldsymbol{B}}$ to conclude that positive charge will accumulate at a and negative charge at b and therefore $\boxed{V_a > V_b}$.

71 ••

Picture the Problem We can use $I = nqv_dA$ to find the drift velocity and $V_H = v_dBw$ to find the Hall voltage.

(a) Express the current in the metal strip in terms of the drift velocity of the electrons:

$$I = nqv_dA$$

Solve for v_d:

$$v_d = \frac{I}{nqA}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{10\,\text{A}}{(8.47\times10^{22}\,\text{cm}^{-3})(1.60\times10^{-19}\,\text{C})(2\,\text{cm})(0.1\,\text{cm})} = \boxed{3.69\times10^{-5}\,\text{m/s}}$$

(b) Relate the Hall voltage to the drift velocity and the magnetic field:

$$V_H = v_dBw$$

Substitute numerical values and evaluate V_H:

$$V_H = (3.69\times10^{-5}\,\text{m/s})(2\,\text{T})(2\,\text{cm}) = \boxed{1.48\,\mu\text{V}}$$

***72** ••

Picture the Problem We can use $V_H = v_dBw$ to express B in terms of V_H and $I = nqv_dA$ to eliminate the drift velocity v_d and derive an expression for B in terms of V_H, n, and t.

Relate the Hall voltage to the drift velocity and the magnetic field:

$$V_H = v_dBw$$

Solve for B to obtain:

$$B = \frac{V_H}{v_d w}$$

Express the current in the metal strip in terms of the drift velocity of the electrons:

$$I = nqv_d A$$

Solve for v_d to obtain:

$$v_d = \frac{I}{nqA}$$

Substitute and simplify to obtain:

$$B = \frac{V_H}{\frac{I}{nqA}w} = \frac{nqAV_H}{Iw} = \frac{nqwtV_H}{Iw} = \frac{nqt}{I}V_H$$

Substitute numerical values and simplify to obtain:

$$B = \frac{(8.47\times10^{22}\text{ cm}^{-3})(1.60\times10^{-19}\text{ C})(0.1\text{cm})V_H}{20\text{ A}} = (6.78\times10^{5}\text{ s/m}^2)V_H$$

(a) Evaluate B for V_H = 2.00 μV:

$$B = (6.78\times10^{5}\text{ s/m}^2)(2.00\,\mu\text{V}) = \boxed{1.36\text{T}}$$

(b) Evaluate B for V_H = 5.25 μV:

$$B = (6.78\times10^{5}\text{ s/m}^2)(5.25\,\mu\text{V}) = \boxed{3.56\text{T}}$$

(c) Evaluate B for V_H = 8.00 μV:

$$B = (6.78\times10^{5}\text{ s/m}^2)(8.00\,\mu\text{V}) = \boxed{5.42\text{T}}$$

73 ••

Picture the Problem We can use $V_H = v_d Bw$ to find the Hall voltage developed across the diameter of the artery.

Relate the Hall voltage to the flow speed of the blood v_d, the diameter of the artery w, and the magnetic field B:

$$V_H = v_d Bw$$

Substitute numerical values and evaluate V_H:

$$V_H = (0.6\,\text{m/s})(0.2\,\text{T})(0.85\,\text{cm}) = \boxed{1.02\,\text{mV}}$$

74 ••

Picture the Problem Let the width of the slab be *w* and its thickness *t*. We can use the definition of the Hall electric field in the slab, the expression for the Hall voltage across it, and the definition of current density to show that the Hall coefficient is also given by 1/(*nq*).

Express the Hall coefficient:

$$R = \frac{E_y}{J_x B_z}$$

Using its definition, express the Hall electric field in the slab:

$$E_y = \frac{V_H}{w}$$

Express the current density in the slab:

$$J_x = \frac{I}{wt} = nqv_d$$

Substitute to obtain:

$$R = \frac{\frac{V_H}{w}}{nqv_d B_z} = \frac{V_H}{nqv_d w B_z}$$

Express the Hall voltage in terms of v_d, B, and w:

$$V_H = v_d B_z w$$

Substitute and simplify to obtain:

$$R = \frac{v_d B_z w}{nqv_d w B_z} = \boxed{\frac{1}{nq}}$$

***75** ••

Picture the Problem We can determine the number of conduction electrons per atom from the quotient of the number density of charge carriers and the number of charge carriers per unit volume. Let the width of a slab of aluminum be *w* and its thickness *t*. We can use the definition of the Hall electric field in the slab, the expression for the Hall voltage across it, and the definition of current density to find *n* in terms of *R* and *q* and $n_a = \rho N_A / M$, to express n_a.

Express the number of electrons per atom *N*:

$$N = \frac{n}{n_a} \qquad (1)$$

where *n* is the number density of charge carriers and n_a is the number of atoms per unit volume.

From the definition of the Hall coefficient we have:

$$R = \frac{E_y}{J_x B_z}$$

Express the Hall electric field in the slab:

$$E_y = \frac{V_H}{w}$$

Express the current density in the slab:

$$J_x = \frac{I}{wt} = nqv_d$$

Substitute to obtain:

$$R = \frac{\frac{V_H}{w}}{nqv_d B_z} = \frac{V_H}{nqv_d wB_z}$$

Express the Hall voltage in terms of v_d, B, and w:

$$V_H = v_d B_z w$$

Substitute and simplify to obtain:

$$R = \frac{v_d B_z w}{nqv_d wB_z} = \frac{1}{nq}$$

Solve for and evaluate n:

$$n = \frac{1}{Rq} \quad (2)$$

Express the number of atoms n_a per unit volume:

$$n_a = \rho \frac{N_A}{M} \quad (3)$$

Substitute equations (2) and (3) in equation (1) to obtain:

$$N = \frac{M}{qR\rho N_A}$$

Substitute numerical values and evaluate N:

$$N = \frac{27\,\text{g/mol}}{\left(-1.60\times10^{-19}\ \text{C}\right)\left(-0.3\times10^{-10}\ \text{m}^3/\text{C}\right)\left(2.7\times10^{3}\ \text{kg/m}^3\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)}$$
$$= \boxed{3.46}$$

General Problems

76 •

Picture the Problem We can use the expression for the magnetic force acting on a wire ($\vec{F} = I\vec{\ell}\times\vec{B}$) to find the force per unit length on the wire.

Express the magnetic force on the

$$\vec{F} = I\vec{\ell}\times\vec{B}$$

wire:

Substitute for $I\vec{\ell}$ and $\vec{B}$ to obtain:

$$\vec{F} = (6.5\,\text{A})\ell\hat{\boldsymbol{i}} \times (1.35\,\text{T})\hat{\boldsymbol{j}}$$

and

$$\frac{\vec{F}}{\ell} = (6.5\,\text{A})\hat{\boldsymbol{i}} \times (1.35\,\text{T})\hat{\boldsymbol{j}}$$

Simplify to obtain:

$$\frac{\vec{F}}{\ell} = (8.78\,\text{N/m})\left(\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}}\right) = \boxed{(8.78\,\text{N/m})\hat{\boldsymbol{k}}}$$

77 •

Picture the Problem We can express the period of the alpha particle's motion in terms of its orbital speed and use Newton's 2nd law to express its orbital speed in terms of known quantities. Knowing the particle's period and the radius of its motion we can find its speed and kinetic energy.

(*a*) Relate the period of the alpha particle's motion to its orbital speed:

$$T = \frac{2\pi r}{v} \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to the alpha particle to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for v to obtain:

$$v = \frac{qBr}{m}$$

Substitute and simplify to obtain:

$$T = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi\left(6.65\times10^{-27}\,\text{kg}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)(1\,\text{T})} = \boxed{0.131\,\mu\text{s}}$$

(*b*) Solve equation (1) for v:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(0.5\,\text{m})}{0.131\,\mu\text{s}} = \boxed{2.40\times10^{7}\,\text{m/s}}$$

(*c*) Express the kinetic energy of the alpha particle:

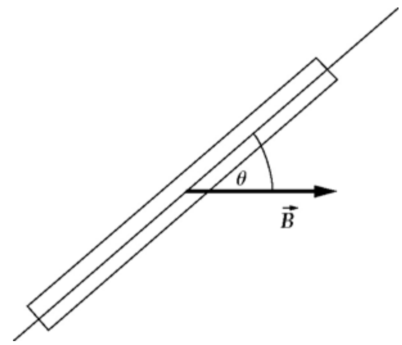
$$K = \tfrac{1}{2}mv^2$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}\left(6.65\times10^{-27}\text{ kg}\right)\left(2.41\times10^{7}\text{ m/s}\right)^2$$
$$= 1.93\times10^{-12}\text{ J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{ J}}$$
$$= \boxed{12.0\,\text{MeV}}$$

78 ••

Picture the Problem The configuration of the magnet and field are shown in the figure. We'll assume that a force $+q_m\vec{B}$ is exerted on the north pole and a force $-q_m\vec{B}$ is exerted on the south pole and show that this assumption leads to the familiar expression for the torque acting on a magnetic dipole.



Assuming that a force $+q_m\vec{B}$ is exerted on the north pole and a force $-q_m\vec{B}$ is exerted on the south pole, express the net torque acting on the bar magnet:

$$\tau = \frac{Bq_mL}{2}\sin\theta - \frac{-Bq_mL}{2}\sin\theta$$
$$= Bq_mL\sin\theta$$

Substitute for q_m to obtain:

$$\tau = B\frac{|\vec{\mu}|}{L}L\sin\theta = \mu B\sin\theta$$

or

$$\vec{\tau} = \boxed{\vec{\mu}\times\vec{B}}$$

***79** ••

Picture the Problem We can use $\vec{F} = q\vec{v}\times\vec{B}$ to show that motion of the particle in the x direction is not affected by the magnetic field. The application of Newton's 2nd law to motion of the particle in yz plane will lead us to the result that $r = mv_{0y}/qB$. By expressing the period of the motion in terms of v_{0y} we can show that the time for one complete orbit around the helix is $t = 2\pi m/qB$.

(*a*) Express the magnetic force acting on the particle:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Substitute for $\vec{v}$ and $\vec{B}$ and simplify to obtain:

$$\vec{F} = q\left(v_{0x}\hat{i} + v_{0y}\hat{j}\right)\times B\hat{i}$$
$$= qv_{0x}B\left(\hat{i}\times\hat{i}\right) + qv_{0y}B\left(\hat{j}\times\hat{i}\right)$$
$$= 0 - qv_{0y}B\hat{k} = -qv_{0y}B\hat{k}$$

i.e., the motion in the direction of the magnetic field (the x direction) is not affected by the field.

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the motion of the particle in the plane perpendicular to $\hat{i}$ (i.e., the yz plane):

$$qv_{0y}B = m\frac{v_{0y}^2}{r} \qquad (1)$$

Solve for r:

$$r = \boxed{\frac{mv_{0y}}{qB}}$$

(b) Relate the time for one orbit around the helix to the particle's orbital speed:

$$t = \frac{2\pi r}{v_{0y}}$$

Solve equation (1) for v_{0y}:

$$v_{0y} = \frac{qBr}{m}$$

Substitute and simplify to obtain:

$$t = \frac{2\pi r}{\frac{qBr}{m}} = \boxed{\frac{2\pi m}{qB}}$$

***80 ••**

Picture the Problem We can use a constant-acceleration equation to relate the velocity of the crossbar to its acceleration and Newton's 2nd law to express the acceleration of the crossbar in terms of the magnetic force acting on it. We can determine the direction of motion of the crossbar using a right-hand rule or, equivalently, by applying $\vec{F} = I\vec{\ell}\times\vec{B}$. We can find the minimum field B necessary to start the bar moving by applying a condition for static equilibrium to it.

(a) Using a constant-acceleration equation, express the velocity of the bar as a function of its acceleration and the time it has been in motion:

$$v = v_0 + at$$

or, because $v_0 = 0$,

$$v = at$$

Use Newton's 2nd law to express the acceleration of the rail:

$$a = \frac{F}{m}$$

where F is the magnitude of the magnetic force acting in the direction of the crossbar's motion.

Substitute to obtain:

$$v = \frac{F}{m}t$$

Express the magnetic force acting on the current-carrying crossbar:

$$F = ILB$$

Substitute to obtain:

$$v = \boxed{\frac{ILB}{m}t}$$

(*b*) Apply to conclude that the magnetic force is to the right and so the motion of the crossbar will also be to the right.

(*c*) Apply $\sum F_x = 0$ to the crossbar:

$$ILB_{\text{min}} - f_{\text{s,max}} = 0$$

or

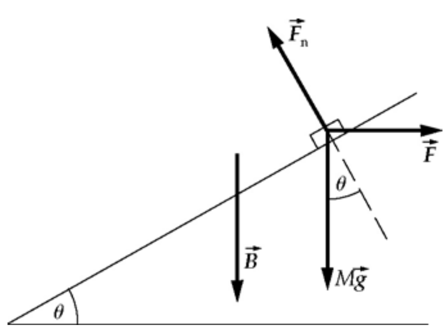
$$ILB_{\text{min}} - \mu_s mg = 0$$

Solve for B_{min} to obtain:

$$B_{\text{min}} = \boxed{\frac{\mu_s mg}{IL}}$$

81 ••

Picture the Problem Note that with the rails tilted, $\vec{F}$ still points horizontally to the right (I, and hence $\vec{\ell}$, is out of the page). Choose a coordinate system in which down the incline is the positive x direction. Then we can apply a condition for translational equilibrium to find the vertical magnetic field $\vec{B}$ is needed to keep the bar from sliding down the rails. In part (*b*) we can apply Newton's 2nd law to find the acceleration of the crossbar when B is twice its value found in (*a*).



(*a*) Apply $\sum F_x = 0$ to the crossbar

$$mg\sin\theta - I\ell B\cos\theta = 0$$

to obtain:

Solve for B:

$$B=\frac{mg}{I\ell}\tan\theta \text{ and } \vec{\boldsymbol{B}}=\boxed{-\frac{mg}{I\ell}\tan\theta\,\hat{\boldsymbol{u}}_{\text{v}}}$$

where $\hat{\boldsymbol{u}}_{\text{v}}$ is a unit vector in the vertical direction.

(*b*) Apply $\sum F_x = ma$ to the crossbar to obtain:

$$I\ell B'\cos\theta - mg\sin\theta = ma$$

Solve for a:

$$a=\frac{I\ell B'}{m}\cos\theta - g\sin\theta$$

Substitute $B' = 2B$ and simplify to obtain:

$$a=\frac{2I\ell\dfrac{mg}{I\ell}\tan\theta}{m}\cos\theta - g\sin\theta = 2g\sin\theta - g\sin\theta = \boxed{g\sin\theta}$$

Note that the direction of the acceleration is up the slope.

82 ••

Picture the Problem We're being asked to show that, for small displacements from equilibrium, the bar magnet executes simple harmonic motion. To show its motion is SHM we need to show that the bar magnet experiences a linear restoring torque when displaced from equilibrium. We can accomplish this by applying Newton's 2nd law in rotational form and using a small angle approximation to obtain the differential equation for simple harmonic motion. Once we have the DE we can identify ω and express f.

Apply $\sum \tau = I\alpha$ to the bar magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the magnet with the magnetic field and I is the moment of inertia of the magnet.

For small displacements from equilibrium, $\theta << 1$ and:

$$\sin\theta \approx \theta$$

Hence our differential equation of motion becomes:

$$I\frac{d^2\theta}{dt^2} = -\mu B\theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the bar magnet is the differential equation of simple harmonic motion. Solve this equation for $d^2\theta/dt^2$ to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{\mu B}{I}\theta = -\omega^2\theta$$

where $\omega = \sqrt{\frac{\mu B}{I}}$

Relate f to ω to obtain:

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}}$$

83 ••

Picture the Problem We can use $\vec{F} = q\vec{v}\times\vec{B}$ to find the magnitude and direction of the magnetic force experienced by an electron in the conducting wire. In (*b*) we can use a condition for translational equilibrium to relate $\vec{E}$ to $\vec{F}$. In (*c*) we can apply the definition of electric field in terms of potential difference to evaluate the difference in potential between the ends of the moving wire.

(*a*) Express the magnetic force on an electron in the conductor:

$$\vec{F} = q\vec{v}\times\vec{B} = qv\hat{i}\times B\hat{k} = qvB\left(\hat{i}\times\hat{k}\right) = -qvB\hat{j}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = -\left(-1.60\times10^{-19}\ \text{C}\right)\left(20\ \text{m/s}\right)\left(0.5\ \text{T}\right)\hat{j} = \boxed{\left(1.60\times10^{-18}\ \text{N}\right)\hat{j}}$$

(*b*) Sum the forces acting on an electron under steady-state conditions to obtain:

$$q\vec{E} + \vec{F} = 0$$

Solve for $\vec{E}$:

$$\vec{E} = -\frac{\vec{F}}{q}$$

Substitute our result in part (*a*) to obtain:

$$\vec{E} = -\frac{\left(1.60\times10^{-18}\ \text{N}\right)\hat{j}}{-1.60\times10^{-19}\ \text{C}} = \boxed{\left(10.0\ \text{V/m}\right)\hat{j}}$$

(*c*) The potential difference between the ends of the wire is:

$$\Delta V = E\Delta x = (10.0\,\text{V/m})(2\,\text{m}) = \boxed{20.0\,\text{V}}$$

84 •••

Picture the Problem We can use $T = 2\pi\sqrt{I/MgD}$ to find the period of small-displacement oscillations with no current flowing in the frame. With a current flowing, the frame will experience an additional restoring torque that will reduce its period. In part (c) we can apply the condition for rotational equilibrium to find the magnitude of the current that will put the frame in equilibrium.

(*a*) Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}} \quad (1)$$

where D is the distance from the pivot to the center of mass of the pendulum.

Express the moment of inertia of the frame:

$$I = I_{\text{hor. segment}} + 2I_{\text{vert. segment}} = m_{\text{hor. segment}}h^2 + 2\left(\tfrac{1}{3}m_{\text{ver. segment}}h^2\right)$$

where h = 10 cm.

Using the linear density of the frame, calculate $m_{\text{hor. segment}}$ and $m_{\text{ver. segment}}$:

$$m_{\text{hor. segment}} = \lambda w = (20\,\text{g/cm})(6\,\text{cm}) = 0.12\,\text{kg}$$

and

$$m_{\text{ver. segment}} = \lambda h = (20\,\text{g/cm})(10\,\text{cm}) = 0.2\,\text{kg}$$

Substitute and evaluate I:

$$I = (0.12\,\text{kg})(0.1\,\text{m})^2 + 2\left[\tfrac{1}{3}(0.2\,\text{kg})(0.1\,\text{m})^2\right] = 2.53\times10^{-3}\,\text{kg}\cdot\text{m}^2$$

Evaluate the distance D to the center of mass from the A-A axis:

$$D = \frac{2(0.05\,\text{m})(0.2\,\text{kg}) + (0.1\,\text{m})(0.12\,\text{kg})}{0.12\,\text{kg} + 0.2\,\text{kg} + 0.2\,\text{kg}} = 6.15\,\text{cm}$$

Substitute in equation (1) and evaluate T:

$$T = 2\pi\sqrt{\frac{2.53\times10^{-3}\,\text{kg}\cdot\text{m}^2}{(0.52\,\text{kg})(9.81\,\text{m/s}^2)(6.15\,\text{cm})}} = \boxed{0.564\,\text{s}}$$

(*b*) Express the restoring torque with $\vec{B}$ and *I* as shown:

$$\tau = (MgD + BIA)\theta$$

where *A* is the area of the loop and provided θ << 1 rad.

Rewrite equation (1) with this restoring torque:

$$T' = 2\pi\sqrt{\frac{I}{MgD + BIA}}$$

Evaluate *BIA*:

$$BIA = (0.2\,\text{T})(8\,\text{A})(10\,\text{cm})(6\,\text{cm}) = 9.60\times10^{-3}\,\text{N}\cdot\text{m}$$

Substitute numerical values and evaluate *T'*:

$$T = 2\pi\sqrt{\frac{2.53\times10^{-3}\,\text{kg}\cdot\text{m}^2}{0.314\,\text{N}\cdot\text{m} + 9.60\times10^{-3}\,\text{N}\cdot\text{m}}} = \boxed{0.556\,\text{s}}$$

(*c*) Apply $\sum\tau = 0$ to the frame when it is in equilibrium to obtain:

$$MgD\sin\theta - BIA\sin\theta = 0$$

Solve for *I*:

$$I = \frac{MgD}{BA}$$

Substitute numerical values and evaluate *I*:

$$I = \frac{(0.52\,\text{kg})(9.81\,\text{m/s}^2)(6.15\,\text{cm})}{(0.2\,\text{T})(10\,\text{cm})(6\,\text{cm})} = \boxed{262\,\text{A}}$$

***85 •••**

Picture the Problem We can use a constant-acceleration equation to express the height to which the wire rises in terms of its initial speed and the acceleration due to gravity. We can then use the impulse-change in momentum equation to express the initial speed of the wire in terms of the impulsive magnetic force acting on it. Finally, we can use the definition of current to relate the charge delivered by the battery to the time during which the impulsive force acts.

Using a constant-acceleration equation, relate the height *h* to the initial and final speeds and the acceleration of the wire:

$$v^2 = v_0^2 + 2a_y h$$

or, because $v = 0$ and $a_y = g$,

$$0 = v_0^2 - 2gh$$

Solve for *h*:

$$h = \frac{v_0^2}{2g} \qquad (1)$$

Use the impulse-momentum equation to relate the change in momentum of the wire to the impulsive force accelerating it:

$\Delta p = F\Delta t$ or $p_f - p_i = F\Delta t$

and, because $p_i = 0$, $mv_0 = F\Delta t$

Express the impulsive (magnetic) force acting on the wire:

$F = I\ell B$

Substitute to obtain:

$mv_0 = I\ell B\Delta t$

Solve for v_0 and substitute in equation (1):

$$h = \frac{\left(\frac{I\ell B\Delta t}{m}\right)^2}{2g} = \frac{(I\ell B\Delta t)^2}{2m^2 g}$$

Use the definition of current to relate the charge delivered by the battery to the time during which it delivers the current:

$\Delta Q = I\Delta t$

Substitute to obtain:

$$h = \frac{(\ell B\Delta Q)^2}{2m^2 g}$$

Substitute numerical values and evaluate h:

$$h = \frac{[(0.25\,\text{m})(0.4\,\text{T})(2\,\text{C})]^2}{2(0.02\,\text{kg})^2(9.81\,\text{m/s}^2)} = \boxed{5.10\,\text{m}}$$

86 •••

Picture the Problem We're being asked to show that, for small displacements from equilibrium, the circular loop executes simple harmonic motion. To show its motion is SHM we must show that the loop experiences a linear restoring torque when displaced from equilibrium. We can accomplish this by applying Newton's 2nd law in rotational form and using a small angle approximation to obtain the differential equation for simple harmonic motion. Once we have the DE we can identify ω and express the period of the motion T.

Apply $\sum \tau = I\alpha$ to the loop:

$$-IAB\sin\theta = I_{\text{inertia}}\frac{d^2\theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the loop with the magnetic field and I_{inertia} is the moment of inertia of the loop.

For small displacements from equilibrium, θ << 1 and:

$$\sin\theta \approx \theta$$

Hence, our differential equation of motion becomes:

$$I_{\text{inertia}}\frac{d^2\theta}{dt^2} = -IAB\theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the current loop is the differential equation of simple harmonic motion. Solve this equation for $d^2\theta/dt^2$ to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{IAB}{I_{\text{inertia}}}\theta$$

.

Noting that the moment of inertia of a hoop about its diameter is $\frac{1}{2}mR^2$, substitute for I_{inertia} and simplify to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{I\pi R^2 B}{\frac{1}{2}mR^2}\theta = -\frac{2I\pi B}{m}\theta = -\omega^2\theta$$

where $\omega = \sqrt{\dfrac{2\pi IB}{m}}$

Relate the period T of the motion to ω and substitute to obtain:

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{2\pi IB}}}$$

87 •••

Picture the Problem We can express $\vec{\boldsymbol{\mu}}$ in terms of its components and calculate U from $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ using $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$. Knowing U we can calculate the components of $\vec{\boldsymbol{F}}$ using $F_x = -dU/dx$ and $F_y = -dU/dy$.

Express the net force acting on the magnet in terms of its components:

$$\vec{\boldsymbol{F}} = F_x\hat{\boldsymbol{i}} + F_y\hat{\boldsymbol{j}} \qquad (1)$$

Express $\vec{\boldsymbol{\mu}}$ in terms of its components:

$$\vec{\boldsymbol{\mu}} = \mu_x\hat{\boldsymbol{i}} + \mu_y\hat{\boldsymbol{j}} + \mu_z\hat{\boldsymbol{k}}$$

Express the potential energy of the bar magnetic in the nonuniform magnetic field:

$$\begin{aligned} U &= -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} \\ &= -\left(\mu_x\hat{\boldsymbol{i}} + \mu_y\hat{\boldsymbol{j}} + \mu_z\hat{\boldsymbol{k}}\right)\cdot\left(B_x(x)\hat{\boldsymbol{i}} + B_y(y)\hat{\boldsymbol{j}}\right) \\ &= -\mu_x B_x(x) - \mu_y B_y(y) \end{aligned}$$

Because $\vec{\boldsymbol{\mu}}$ is constant but $\vec{\boldsymbol{B}}$ depends on x and y:

$$F_x = -\frac{dU}{dx} = \mu_x\left(\frac{\partial B_x}{\partial x}\right)$$

and

$$F_y = -\frac{dU}{dy} = \mu_y\left(\frac{\partial B_y}{\partial y}\right)$$

Substitute in equation (1) to obtain:

$$\boxed{\vec{F} = \mu_x \frac{\partial B_x}{\partial x}\hat{i} + \mu_y \frac{\partial B_y}{\partial y}\hat{j}}$$

***88 •••**

Picture the Problem We can apply Newton's 2nd law to the particle to derive an expression for the radius of its orbit and then express its period in terms of its orbital speed and radius.

(*a*) Because $\vec{B}$ is perpendicular to $\vec{v}$, the magnitude of force on the particle is given by:

$$F = qvB$$

Apply $\sum F = ma$ to the orbiting particle to obtain:

$$qvB = m(v)\frac{v^2}{r} = \gamma(v)m\frac{v^2}{r}$$

Solve for *r*:

$$r = \boxed{\frac{\gamma(v)mv}{qB}}$$

The period *T* of the particle's motion is related to the radius *r* of its orbit and its orbital speed *v*:

$$T = \frac{2\pi r}{v}$$

Substitute for *r* and simplify to obtain:

$$T = \boxed{\frac{2\pi\gamma(v)m}{qB}}$$

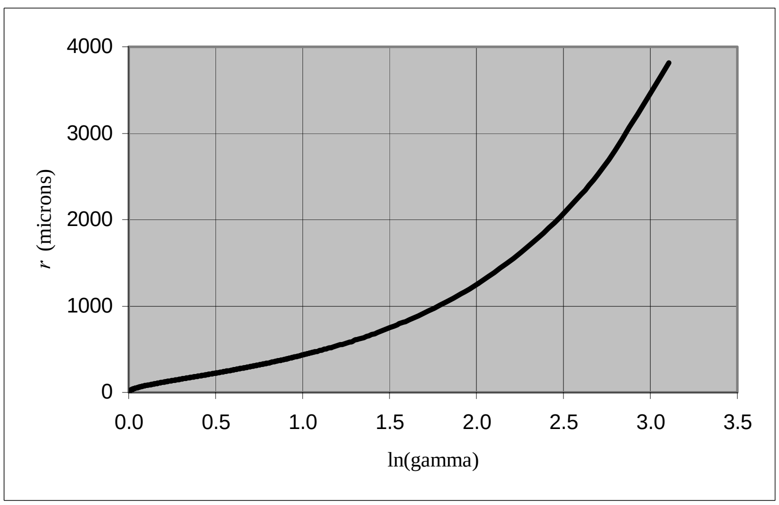
(*b*) A spreadsheet program to calculate *r* and *T* as functions of ln(γ) follows. The formulas used to calculate the quantities in the columns are given in the table.

Cell	Content/Formula	Algebraic Form
B1	9.11E−31	m
B2	1.60E−19	e
B3	10	B
B4	3.00E+08	c
A7	0.100	v/c
A8	0.101	$v/c + 0.001$
B7	1/SQRT(1 − (A7)^2)	γ
C7	LN(B7)	$\ln(\gamma)$
D7	B7*$B$1*A7*$B$4/($B$2*$B$3)	$\frac{\gamma mv}{qB}$
E7	D7*10^8	$10^6 r$

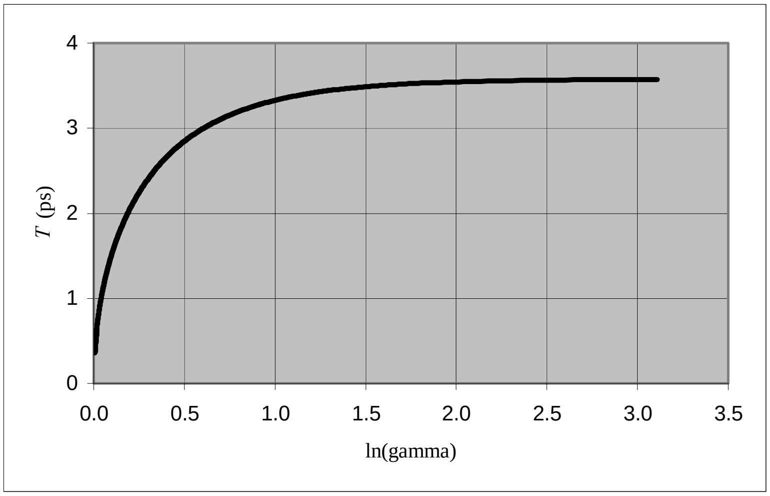
F7	(2*PI()*A7*$B$1/($B$2*$B$3))*10^12	$\frac{2\pi\gamma m}{qB}\times 10^{12}$

	A	B	C	D	E	F
1	m=	9.11E–31	kg			
2	e=	1.60E–19	C			
3	B=	10	T			
4	c=	3.00E+08	m/s			
5						
6	v/c	gamma	ln(gamma)	r	r (microns)	T (ps)
7	0.100	1.0050	0.005	1.72E-05	17.2	0.358
8	0.101	1.0051	0.005	1.73E-05	17.3	0.361
9	0.102	1.0052	0.005	1.75E-05	17.5	0.365
10	0.103	1.0053	0.005	1.77E-05	17.7	0.368
11	0.104	1.0055	0.005	1.79E-05	17.9	0.372
903	0.996	11.1915	2.415	1.90E-03	1904.0	3.563
904	0.997	12.9196	2.559	2.20E-03	2200.2	3.567
905	0.998	15.8193	2.761	2.70E-03	2696.7	3.570
906	0.999	22.3663	3.108	3.82E-03	3816.6	3.574

The following graph of r as a function of $\ln(\gamma)$ was plotted using the data in columns C and E.



The following graph of T as a function of $\ln(\gamma)$ was plotted using the data in columns C and F.


4
3
2
1
0
T (ps)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
ln(gamma)

Chapter 27
Sources of the Magnetic Field

Conceptual Problems

***1** •

Picture the Problem The electric forces are described by Coulomb's law and the laws of attraction and repulsion of charges and are independent of the fact the charges are moving. The magnetic interaction is, on the other hand, dependent on the motion of the charges. Each moving charge constitutes a current that creates a magnet field at the location of the other charge.

(*a*) The electric forces are repulsive; the magnetic forces are attractive (the two charges moving in the same direction act like two currents in the same direction).

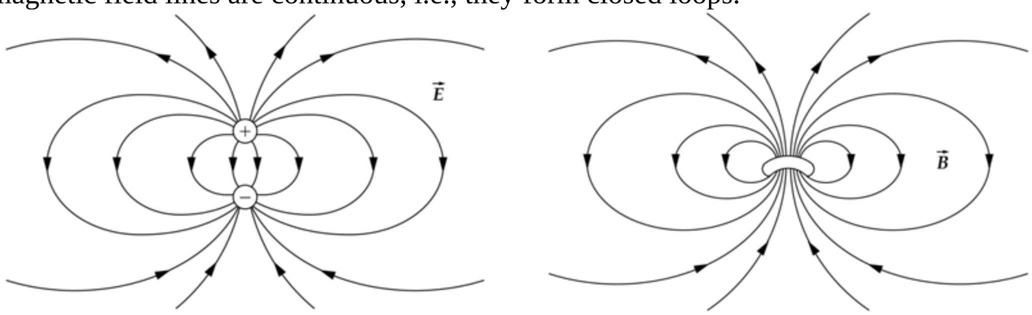
(*b*) The electric forces are again repulsive; the magnetic forces are also repulsive.

2 •

No. The magnitude of the field depends on the location within the loop.

3 •

Picture the Problem The field lines for the electric dipole are shown in the sketch to the left and the field lines for the magnetic dipole are shown in the sketch to the right. Note that, while the far fields (the fields far from the dipoles) are the same, the near fields (the fields between the two charges and inside the current loop/magnetic dipole) are not, and that, in the region between the two charges, the electric field is in the opposite direction to that of the magnetic field at the center of the magnetic dipole. It is especially important to note that while the electric field lines begin and terminate on electric charges, the magnetic field lines are continuous, i.e., they form closed loops.



4 •

Determine the Concept Applying the right-hand rule to the wire to the left we see that the magnetic field due to its current is out of the page at the midpoint. Applying the right-hand rule to the wire to the right we see that the magnetic field due to its current is out of the page at the midpoint. Hence, the sum of the magnetic fields is out of the page as well. $\boxed{(c) \text{ is correct.}}$

5 •

Determine the Concept While we could express the force wire 1 exerts on wire 2 and compare it to the force wire 2 exerts on wire 1 to show that they are the same, it is simpler to recognize that these are action and reaction forces. $\boxed{(a) \text{ is correct.}}$

***6** •

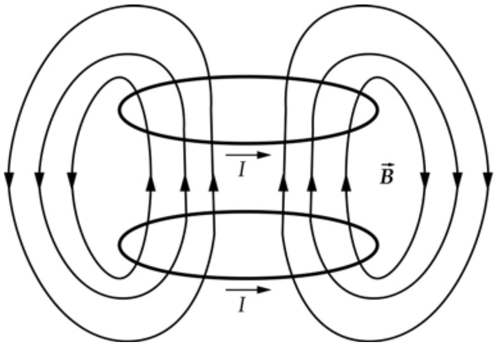
Determine the Concept Applying the right-hand rule to the wire to the left we see that the magnetic field due to the current points to west at all points north of the wire. $\boxed{(c) \text{ is correct.}}$

7 •

Determine the Concept At points to the west of the vertical wire, the magnetic field due to its current exerts a downward force on the horizontal wire and at points to the east it exerts an upward force on the horizontal wire. Hence, the net magnetic force is zero and $\boxed{(e) \text{ is correct.}}$

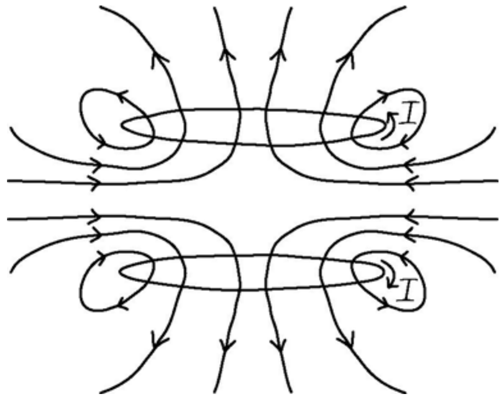
8 •

Picture the Problem The field-line sketch follows. An assumed direction for the current in the coils is shown in the diagram. Note that the field is stronger in the region between the coaxial coils and that the field lines have neither beginning nor ending points as do electric-field lines. Because there are an uncountable infinity of lines, only a representative few have been shown.



***9** •

Picture the Problem The field-line sketch is shown below. An assumed direction for the current in the coils is shown in the diagram. Note that the field lines never begin or end and that they do not touch or cross each other. Because there are an uncountable infinity of lines, only a representative few have been shown.



10 •

Determine the Concept Because all of these statements regarding Ampère's law are true, $\boxed{(e) \text{ is correct.}}$

11 •

(*a*) True

(*b*) True

***12** •

Determine the Concept The magnetic susceptibility χ_m is defined by the equation $\vec{M} = \chi_m \dfrac{\vec{B}_{app}}{\mu_0}$, where $\vec{M}$ is the magnetization vector and $\vec{B}_{app}$ is the applied magnetic field. For paramagnetic materials, χ_m is a small positive number that depends on temperature, whereas for diamagnetic materials, it is a small negative constant independent of temperature. $\boxed{(a) \text{ is correct.}}$

13 •

(*a*) False. The magnetic field due to a current element is perpendicular to the current element.

(*b*) True

(*c*) False. The magnetic field due to a long wire varies inversely with the distance from the wire.

(*d*) False. Ampère's law is easier to apply if there is a high degree of symmetry, but is valid in all situations.

(*e*) True

14 •

Determine the Concept Yes. The classical relation between magnetic moment and angular momentum is $\vec{\boldsymbol{\mu}} = \frac{q}{2m}\vec{\boldsymbol{L}}$. Thus, if its charge density is zero, a particle with angular momentum will not have a magnetic moment.

15 •

Determine the Concept No. The classical relation between magnetic moment and angular momentum is $\vec{\boldsymbol{\mu}} = \frac{q}{2m}\vec{\boldsymbol{L}}$. Thus, if the angular momentum of the particle is zero, its magnetic moment will also be zero.

16 •

Determine the Concept Yes, there is angular momentum associated with the magnetic moment. The magnitude of $\vec{\boldsymbol{L}}$ is extremely small, but very sensitive experiments have demonstrated its presence (Einstein-de Haas effect).

17 •

Determine the Concept From Ampère's law, the current enclosed by a closed path within the tube is zero, and from the cylindrical symmetry it follows that $B = 0$ everywhere within the tube.

***18** •

Determine the Concept The force per unit length experienced by each segment of the wire, due to the currents in the other segments of the wire, will be equal. These equal forces will result in the wire tending to form a circle.

19 •

Determine the Concept H_2, CO_2, and N_2 are diamagnetic ($\chi_m < 0$); O_2 is paramagnetic ($\chi_m > 0$).

Estimation and Approximation

20 ••

Picture the Problem We can use the definition of the magnetization of the earth's core to find its volume and radius.

(*a*) Express the magnetization of the earth's core in terms of the magnetic moment of the earth and the volume of the core:

$$M = \frac{\mu}{V}$$

Solve for and evaluate V:

$$V=\frac{\mu}{M}=\frac{9\times10^{22}\ \text{A}\cdot\text{m}^2}{1.5\times10^{9}\ \text{A/m}}=\boxed{6.00\times10^{13}\ \text{m}^3}$$

(b)Assuming a spherical core centered with the earth:

$$V=\tfrac{4}{3}\pi r^3$$

Solve for r:

$$r=\sqrt[3]{\frac{3V}{4\pi}}$$

Substitute numerical values and evaluate r:

$$r=\sqrt[3]{\frac{3\left(6\times10^{13}\ \text{m}^3\right)}{4\pi}}=\boxed{2.43\times10^{4}\ \text{m}}$$

***21 ••**

Picture the Problem We can model the lightning bolt as a current in a long wire and use the expression for the magnetic field due to such a current to estimate the transient magnetic field 100 m from the lightning bolt.

The magnetic field due to the current in a long, straight wire is:

$$B=\frac{\mu_0}{4\pi}\frac{2I}{r}$$

where r is the distance from the wire.

Assuming that the height of the cloud is 1 km, the charge transfer will take place in roughly 10^{-3} s and the current associated with this discharge is:

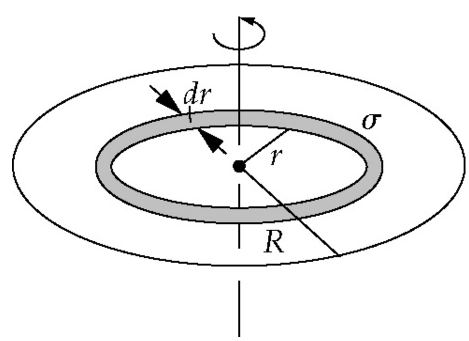
$$I=\frac{\Delta Q}{\Delta t}=\frac{30\,\text{C}}{10^{-3}\ \text{s}}=3\times10^{4}\ \text{A}$$

Substitute numerical values and evaluate B:

$$B=\frac{4\pi\times10^{-7}\ \text{N/A}^2}{4\pi}\frac{2\left(3\times10^{4}\ \text{A}\right)}{100\,\text{m}}=\boxed{60.0\ \mu\text{T}}$$

***22 ••**

Picture the Problem A rotating disk with total charge Q and surface charge density σ is shown in the diagram. We can find Q by deriving an expression for the magnetic field B at the center of the disk due to its rotation. We'll use Ampere's law to express the field dB at the center of the disk due to the element of current dI and then integrate over r to find B.



Applying Ampere's law to a circular current loop of radius *r* we obtain:

$$B=\frac{\mu_0 I}{2r}$$

The *B* field at the center of an annular ring on a rotating disk of radius *r* and thickness *dr* is:

$$dB=\frac{\mu_0}{2r}dI \qquad (1)$$

If σ represents the surface charge density, then the current in the annular ring is given by:

$$dI=\frac{\sigma(2\pi r)}{T}dr,\text{ where } \sigma=\frac{Q}{\pi R^2}$$

Because $T=\dfrac{2\pi}{\omega}$:

$$dI=\sigma\omega r dr$$

Substitute for *dI* in equation (1) to obtain:

$$dB=\frac{\mu_0}{2r}\sigma\omega r dr=\frac{\mu_0\sigma\omega}{2}dr$$

Integrate from *r* = 0 to *R* to obtain:

$$B=\frac{\mu_0\sigma\omega}{2}\int_0^R dr=\frac{\mu_0\sigma\omega R}{2}$$

Substitution for σ yields:

$$B=\frac{\mu_0\left(\dfrac{Q}{\pi R^2}\right)\omega R}{2}=\frac{\mu_0 Q\omega}{2\pi R}$$

Solve for *Q* to obtain:

$$Q=\frac{2\pi RB}{\mu_0\omega}$$

Substitute numerical values and evaluate *Q*:

$$Q=\frac{2\pi\left(10^7\text{ m}\right)(0.1\text{T})}{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(10^{-2}\text{ rad/s}\right)}=\boxed{5.00\times10^{14}\text{ C}}$$

The electric field above the sunspot is given by:

$$E=\frac{\sigma}{2\in_0}=\frac{Q}{2\pi\in_0 R^2}$$

Substitute numerical values and evaluate *E*:

$$E=\frac{5.00\times10^{14}\text{ C}}{2\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(10^7\text{ m}\right)^2}=\boxed{90.0\text{ GN/C}}$$

The Magnetic Field of Moving Point Charges

23 •

Picture the Problem We can substitute for $\vec{v}$ and q in the equation describing the magnetic field of the moving charged particle ($\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the given points of interest, and substitute to find $\vec{B}$.

Express the magnetic field of the moving charged particle:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \left(10^{-7}\text{ N/A}^2\right)\left(12\,\mu\text{C}\right)\frac{(30\text{ m/s})\hat{i}\times\hat{r}}{r^2} = \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the particle at (0, 2 m) and the point of interest at the origin:

$$\vec{r} = -(2\text{ m})\hat{j},\ r = 2\text{ m},\text{ and }\hat{r} = -\hat{j}$$

Substitute and evaluate $\vec{B}(0,0)$:

$$\vec{B}(0,0) = \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\left(-\hat{j}\right)}{(2\text{ m})^2} = \boxed{-(9.00\text{ pT})\hat{k}}$$

(*b*) Find r and $\hat{r}$ for the particle at (0, 2 m) and the point of interest at (0, 1 m):

$$\vec{r} = -(1\text{m})\hat{j},\ r = 1\text{m},\text{ and }\hat{r} = -\hat{j}$$

Substitute and evaluate $\vec{B}(0,1\text{m})$:

$$\vec{B}(0,1\text{m}) = \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\left(-\hat{j}\right)}{(1\text{m})^2} = \boxed{-(36.0\text{ pT})\hat{k}}$$

(*c*) Find r and $\hat{r}$ for the particle at (0, 2 m) and the point of interest at (0, 3 m):

$$\vec{r} = (1\text{m})\hat{j},\ r = 1\text{m},\text{ and }\hat{r} = \hat{j}$$

Substitute and evaluate $\vec{B}(0,3\text{m})$:

$$\vec{B}(0,3\text{m}) = \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\hat{j}}{(1\text{m})^2} = \boxed{(36.0\text{ pT})\hat{k}}$$

(*d*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (0, 4 m):

$$\vec{r} = (2\,\text{m})\hat{\boldsymbol{j}}\,,\; r = 2\,\text{m}\,,\text{ and }\hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Substitute and evaluate $\vec{B}(0,4\,\text{m})$:

$$\vec{B}(0,4\,\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}}{(2\,\text{m})^2} = \boxed{(9.00\,\text{pT})\hat{\boldsymbol{k}}}$$

24 •

Picture the Problem We can substitute for $\vec{v}$ and *q* in the equation describing the magnetic field of the moving charged particle ($\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{\boldsymbol{r}}}{r^2}$), evaluate *r* and $\hat{\boldsymbol{r}}$ for each of the given points of interest, and substitute to find $\vec{B}$.

The magnetic field of the moving charged particle is given by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{\boldsymbol{r}}}{r^2} = (10^{-7}\ \text{N/A}^2)(12\,\mu\text{C})\frac{(30\,\text{m/s})\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2} = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2}$$

(*a*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (1 m, 3 m):

$$\vec{r} = (1\,\text{m})\hat{\boldsymbol{i}} + (1\,\text{m})\hat{\boldsymbol{j}}\,,\; r = \sqrt{2}\,\text{m}\,,\text{ and }\hat{\boldsymbol{r}} = \frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{B}(1\text{m},3\text{m})$:

$$\vec{B}(1\text{m},3\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\times\frac{\hat{\boldsymbol{i}}\times\left(\frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}\right)}{(\sqrt{2}\,\text{m})^2} = \frac{(36.0\,\text{pT}\cdot\text{m}^2)}{\sqrt{2}}\frac{\hat{\boldsymbol{k}}}{(\sqrt{2}\,\text{m})^2} = \boxed{(12.7\,\text{pT})\hat{\boldsymbol{k}}}$$

(*b*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (2 m, 2 m):

$$\vec{r} = (2\,\text{m})\hat{\boldsymbol{i}}\,,\; r = 2\,\text{m}\,,\text{ and }\hat{\boldsymbol{r}} = \hat{\boldsymbol{i}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(2\,\text{m},2\,\text{m})$:

$$\vec{\boldsymbol{B}}(2\,\text{m},2\,\text{m})=\left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}}{(2\,\text{m})^2}=\boxed{0}$$

(*c*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (2 m, 3 m):

$\vec{\boldsymbol{r}}=(2\,\text{m})\hat{\boldsymbol{i}}+(1\,\text{m})\hat{\boldsymbol{j}}$, $r=\sqrt{5}\,\text{m}$, and

$$\hat{\boldsymbol{r}}=\frac{2}{\sqrt{5}}\hat{\boldsymbol{i}}+\frac{1}{\sqrt{5}}\hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(2\,\text{m},3\,\text{m})$:

$$\vec{\boldsymbol{B}}(2\,\text{m},3\,\text{m})=\left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{i}}+\frac{1}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{\left(\sqrt{5}\,\text{m}\right)^2}=\boxed{(3.22\,\text{pT})\hat{\boldsymbol{k}}}$$

25 •

Picture the Problem We can substitute for $\vec{\boldsymbol{v}}$ and *q* in the equation describing the magnetic field of the moving proton ($\vec{\boldsymbol{B}}=\frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}$), evaluate *r* and $\hat{\boldsymbol{r}}$ for each of the given points of interest, and substitute to find $\vec{\boldsymbol{B}}$.

The magnetic field of the moving proton is given by:

$$\vec{\boldsymbol{B}}=\frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}=\left(10^{-7}\,\text{N/A}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)\frac{\left[\left(10^4\,\text{m/s}\right)\hat{\boldsymbol{i}}+\left(2\times10^4\,\text{m/s}\right)\hat{\boldsymbol{j}}\right]\times\hat{\boldsymbol{r}}}{r^2}$$
$$=\left(1.60\times10^{-22}\,\text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{r}}}{r^2}$$

(*a*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3 m, 4 m) and the point of interest at (2 m, 2 m):

$\vec{\boldsymbol{r}}=-(1\,\text{m})\hat{\boldsymbol{i}}-(2\,\text{m})\hat{\boldsymbol{j}}$, $r=\sqrt{5}\,\text{m}$, and

$$\hat{\boldsymbol{r}}=-\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}}-\frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(1\,\text{m},3\,\text{m})$:

$$\vec{\boldsymbol{B}}(1\,\text{m},3\,\text{m})=\left(1.60\times10^{-22}\,\text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}\right)\times\left(-\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}}-\frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{r^2}$$
$$=\frac{\left(1.60\times10^{-22}\,\text{T}\cdot\text{m}^2\right)}{\sqrt{5}}\left[\frac{-2\hat{\boldsymbol{k}}+2\hat{\boldsymbol{k}}}{\left(\sqrt{5}\,\text{m}\right)^2}\right]=\boxed{0}$$

(*b*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3 m, 2 m) and the point of interest at (6 m, 4 m):

$$\vec{\boldsymbol{r}} = (3\,\text{m})\hat{\boldsymbol{i}},\ r = 3\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{i}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(6\,\text{m}, 4\,\text{m})$:

$$\vec{\boldsymbol{B}}(6\,\text{m}, 4\,\text{m}) = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{i}}}{(3\,\text{m})^2} = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\left(\frac{-2\hat{\boldsymbol{k}}}{9\,\text{m}^2}\right)$$

$$= \boxed{-\left(3.56\times10^{-23}\ \text{T}\right)\hat{\boldsymbol{k}}}$$

(*c*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3 m, 4 m) and the point of interest at the (3 m, 6 m):

$$\vec{\boldsymbol{r}} = (2\,\text{m})\hat{\boldsymbol{j}},\ r = 2\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(3\,\text{m}, 6\,\text{m})$:

$$\vec{\boldsymbol{B}}(3\,\text{m}, 6\,\text{m}) = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{j}}}{(2\,\text{m})^2} = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\left(\frac{\hat{\boldsymbol{k}}}{4\,\text{m}^2}\right)$$

$$= \boxed{\left(4.00\times10^{-23}\ \text{T}\right)\hat{\boldsymbol{k}}}$$

26 •

Picture the Problem The centripetal force acting on the orbiting electron is the Coulomb force between the electron and the proton. We can apply Newton's 2nd law to the electron to find its orbital speed and then use the expression for the magnetic field of a moving charge to find *B*.

Express the magnetic field due to the motion of the electron:

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2}$$

Apply $\sum F_{\text{radial}} = ma_c$ to the electron:

$$\frac{ke^2}{r^2} = m\frac{v^2}{r}$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{ke^2}{mr}}$$

Substitute and simplify to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{e}{r^2}\sqrt{\frac{ke^2}{mr}} = \frac{\mu_0 e^2}{4\pi r^2}\sqrt{\frac{k}{mr}}$$

Substitute numerical values and evaluate B:

$$B=\frac{\left(10^{-7}\text{ N/A}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2}{\left(5.29\times10^{-11}\text{ m}\right)^2}\sqrt{\frac{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(5.29\times10^{-11}\text{ m}\right)}}=\boxed{12.5\text{T}}$$

***27 ••**

Picture the Problem We can find the ratio of the magnitudes of the magnetic and electrostatic forces by using the expression for the magnetic field of a moving charge and Coulomb's law. Note that v and $\vec{r}$, where $\vec{r}$ is the vector from one charge to the other, are at right angles. The field $\vec{B}$ due to the charge at the origin at the location (0, b, 0) is perpendicular to v and $\vec{r}$.

Express the magnitude of the magnetic force on the moving charge at (0, b, 0):

$$F_B=qvB=\frac{\mu_0}{4\pi}\frac{q^2v^2}{b^2}$$

and, applying the right hand rule, we find that the direction of the force is toward the charge at the origin; i.e., the magnetic force between the two moving charges is attractive.

Express the magnitude of the repulsive electrostatic interaction between the two charges:

$$F_E=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{b^2}$$

Express the ratio of F_B to F_E and simplify to obtain:

$$\frac{F_B}{F_E}=\frac{\frac{\mu_0}{4\pi}\frac{q^2v^2}{b^2}}{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{b^2}}=\varepsilon_0\mu_0v^2=\boxed{\frac{v^2}{c^2}}$$

where c is the speed of light in a vacuum.

The Magnetic Field of Currents: The Biot-Savart Law

28 •

Picture the Problem We can substitute for $\vec{v}$ and q in the Biot-Savart relationship ($d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the points of interest, and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$
$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2\ \text{A})(2\ \text{mm})\hat{k}\times\hat{r}}{r^2}$$
$$= \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the point whose coordinates are (3 m, 0, 0):

$\vec{r} = (3\,\text{m})\hat{i}$, $r = 3\,\text{m}$, and $\hat{r} = \hat{i}$

Evaluate $d\vec{B}$ at (3 m, 0, 0):

$$d\vec{B}(3\,\text{m},0,0) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{i}}{(3\,\text{m})^2}$$
$$= \boxed{(44.4\,\text{pT})\hat{j}}$$

(*b*) Find r and $\hat{r}$ for the point whose coordinates are (–6 m, 0, 0):

$\vec{r} = -(6\,\text{m})\hat{i}$, $r = 6\,\text{m}$, and $\hat{r} = -\hat{i}$

Evaluate $d\vec{B}$ at (–6 m, 0, 0):

$$d\vec{B}(-6\,\text{m},0,0) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times(-\hat{i})}{(6\,\text{m})^2}$$
$$= \boxed{-(11.1\,\text{pT})\hat{j}}$$

(*c*) Find r and $\hat{r}$ for the point whose coordinates are (0, 0, 3 m):

$\vec{r} = (3\,\text{m})\hat{k}$, $r = 3\,\text{m}$, and $\hat{r} = \hat{k}$

Evaluate $d\vec{B}$ at (0, 0, 3 m):

$$d\vec{B}(0,0,3\,\text{m}) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{k}}{(3\,\text{m})^2}$$
$$= \boxed{0}$$

(*d*) Find r and $\hat{r}$ for the point whose coordinates are (0, 3 m, 0):

$\vec{r} = (3\,\text{m})\hat{j}$, $r = 3\,\text{m}$, and $\hat{r} = \hat{j}$

Evaluate $d\vec{B}$ at (0, 3 m, 0):

$$d\vec{B}(0,3\,\text{m},0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{j}}{(3\,\text{m})^2}$$
$$= \boxed{-(44.4\,\text{pT})\hat{i}}$$

29 •

Picture the Problem We can substitute for $\vec{v}$ and q in the Biot-Savart relationship ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for (0, 3 m, 4 m), and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$
$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2\,\text{A})(2\,\text{mm})\hat{k}\times\hat{r}}{r^2}$$
$$= \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

Find r and $\hat{r}$ for the point whose coordinates are (0, 3 m, 4 m):

$$\vec{r} = (3\,\text{m})\hat{j} + (4\,\text{m})\hat{k}\,,$$
$$r = 5\,\text{m}\,,$$
and
$$\hat{r} = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$$

Evaluate $d\vec{B}$ at (3 m, 0, 0):

$$d\vec{B}(3\,\text{m},0,0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\left(\frac{3}{5}\hat{j}+\frac{4}{5}\hat{k}\right)}{(5\,\text{m})^2} = \boxed{-(9.60\,\text{pT})\hat{i}}$$

***30** •

Picture the Problem We can substitute for $\vec{v}$ and q in the Biot-Savart relationship ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for the given points, and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$
$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2\,\text{A})(2\,\text{mm})\hat{k}\times\hat{r}}{r^2}$$
$$= \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

(*a*) Find *r* and $\hat{\boldsymbol{r}}$ for the point whose coordinates are (2 m, 4 m, 0):

$\vec{r} = (2\,\text{m})\hat{\boldsymbol{i}} + (4\,\text{m})\hat{\boldsymbol{j}}$,

$r = 2\sqrt{5}\,\text{m}$,

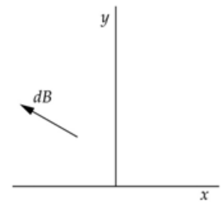
and

$$\hat{\boldsymbol{r}} = \frac{2}{2\sqrt{5}}\hat{\boldsymbol{i}} + \frac{4}{2\sqrt{5}}\hat{\boldsymbol{j}} = \frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}$$

Evaluate $d\vec{B}$ at (2 m, 4 m, 0):

$$d\vec{B}(2\,\text{m},4\,\text{m},0) = (0.400\,\text{nT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{k}}\times\left(\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{(2\sqrt{5}\,\text{m})^2} = \boxed{-(17.9\,\text{pT})\hat{\boldsymbol{i}} + (8.94\,\text{pT})\hat{\boldsymbol{j}}}$$

The diagram is shown to the right:



(*b*) Find *r* and $\hat{\boldsymbol{r}}$ for the point whose coordinates are (2 m, 0, 4 m):

$\vec{r} = (2\,\text{m})\hat{\boldsymbol{i}} + (4\,\text{m})\hat{\boldsymbol{k}}$,

$r = 2\sqrt{5}\,\text{m}$,

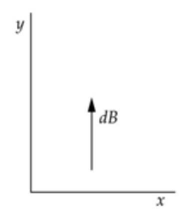
and

$$\hat{\boldsymbol{r}} = \frac{2}{2\sqrt{5}}\hat{\boldsymbol{i}} + \frac{4}{2\sqrt{5}}\hat{\boldsymbol{k}} = \frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{k}}$$

Evaluate $d\vec{B}$ at (2 m, 0, 4 m):

$$d\vec{B}(2\,\text{m},0,4\,\text{m}) = (0.400\,\text{nT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{k}}\times\left(\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{k}}\right)}{(2\sqrt{5}\,\text{m})^2} = \boxed{(8.94\,\text{pT})\hat{\boldsymbol{j}}}$$

The diagram is shown to the right:



$\vec{B}$ Due to a Current Loop

31 •

Picture the Problem We can use $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(x^2+R^2)^{3/2}}$ to find *B* on the axis of the current loop.

Express B on the axis of a current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

Substitute numerical values to obtain:

$$B_x = \left(10^{-7}\ \text{N/A}^2\right)\frac{2\pi(0.03\,\text{m})^2(2.6\,\text{A})}{\left(x^2 + (0.03\,\text{m})^2\right)^{3/2}}$$

$$= \frac{1.47\times 10^{-9}\ \text{T}\cdot\text{m}^3}{\left(x^2 + (0.03\,\text{m})^2\right)^{3/2}}$$

(a) Evaluate B at the center of the loop:

$$B(0) = \frac{1.47\times 10^{-9}\ \text{T}\cdot\text{m}^3}{\left(0 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{54.5\,\mu\text{T}}$$

(b) Evaluate B at x = 1 cm:

$$B(0.01\text{m}) = \frac{1.47\times 10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.01\,\text{m})^2 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{46.5\,\mu\text{T}}$$

(c) Evaluate B at x = 2 cm:

$$B(0.02\,\text{m}) = \frac{1.47\times 10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.02\,\text{m})^2 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{31.4\,\mu\text{T}}$$

(d) Evaluate B at x = 35 cm:

$$B(0.35\,\text{m}) = \frac{1.47\times 10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.35\,\text{m})^2 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{33.9\,\text{nT}}$$

***32** •

Picture the Problem We can solve $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$ for I with x = 0 and substitute the earth's magnetic field at the equator to find the current in the loop that would produce a magnetic field equal to that of the earth.

Express B on the axis of the current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

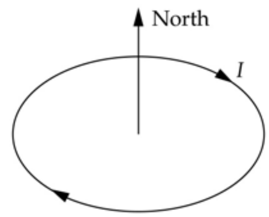
Solve for I with x = 0:

$$I = \frac{4\pi}{\mu_0}\frac{R}{2\pi}B_x$$

Substitute numerical values and evaluate I:

$$I = \frac{1}{(10^{-7}\ \text{N/A}^2)} \frac{(0.1\,\text{m})^3}{2\pi(0.1\,\text{m})^2}(0.7\,\text{G})\left(\frac{1\,\text{T}}{10^4\,\text{G}}\right) = \boxed{11.1\,\text{A}}$$

The orientation of the loop and current is shown in the sketch:



33 ••

Picture the Problem We can solve $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(x^2+R^2)^{3/2}}$ for B_0, express the ratio of B_x to B_0, and solve the resulting equation for x.

Express B on the axis of the current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(x^2+R^2)^{3/2}}$$

Evaluate B_x for $x = 0$:

$$B_0 = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}$$

Express the ratio of B_x to B_0:

$$\frac{B_x}{B_0} = \frac{\frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(x^2+R^2)^{3/2}}}{\frac{\mu_0}{4\pi}\frac{2\pi I}{R}} = \frac{R^3}{(x^2+R^2)^{3/2}}$$

Solve for x to obtain:

$$x = R\sqrt{\left(\frac{B_0}{B_x}\right)^{2/3} - 1} \qquad (1)$$

(a) Evaluate equation (1) for $B_x = 0.\ 1B_0$:

$$x = 10\,\text{cm}\sqrt{\left(\frac{B_0}{0.1B_0}\right)^{2/3} - 1} = \boxed{19.1\,\text{cm}}$$

(b) Evaluate equation (1) for $B_x = 0.\ 01B_0$:

$$x = 10\,\text{cm}\sqrt{\left(\frac{B_0}{0.01B_0}\right)^{2/3} - 1} = \boxed{45.3\,\text{cm}}$$

(*a*) Evaluate equation (1) for $B_x = 0.001B_0$:

$$x = 10\,\text{cm}\sqrt{\left(\frac{B_0}{0.001B_0}\right)^{2/3} - 1} = \boxed{99.5\,\text{cm}}$$

34 ••

Picture the Problem We can solve $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$ for I with $x = 0$ and substitute the earth's magnetic field at the equator to find the current in the loop that would produce a magnetic field equal to that of the earth.

Express B on the axis of the current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

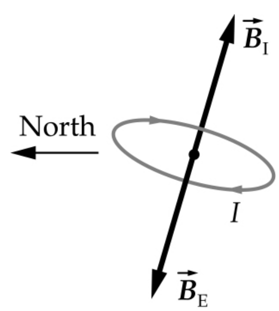
Solve for I with $x = 0$ and $B_x = B_E$:

$$I = \frac{4\pi}{\mu_0}\frac{R}{2\pi}B_E$$

Substitute numerical values and evaluate I:

$$I = \frac{1}{\left(10^{-7}\ \text{N/A}^2\right)}\frac{0.085\,\text{m}}{2\pi}(0.7\,\text{G})\left(\frac{1\,\text{T}}{10^4\,\text{G}}\right) = \boxed{9.47\,\text{A}}$$

The normal to the plane of the loop must be in the direction of the earth's field, and the current must be counterclockwise as seen from above. Here $\vec{B}_E$ denotes the earth's field and $\vec{B}_I$ the field due to the current in the coil.



35 ••

Picture the Problem We can use the expression for the magnetic field on the axis of a current loop and the expression for the electric field on the axis of ring of charge Q to plot graphs of B_x/B_0 and $E(x)/(kQ/R^2)$ as functions of x/R.

(*a*) Express B_x on the axis of a current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

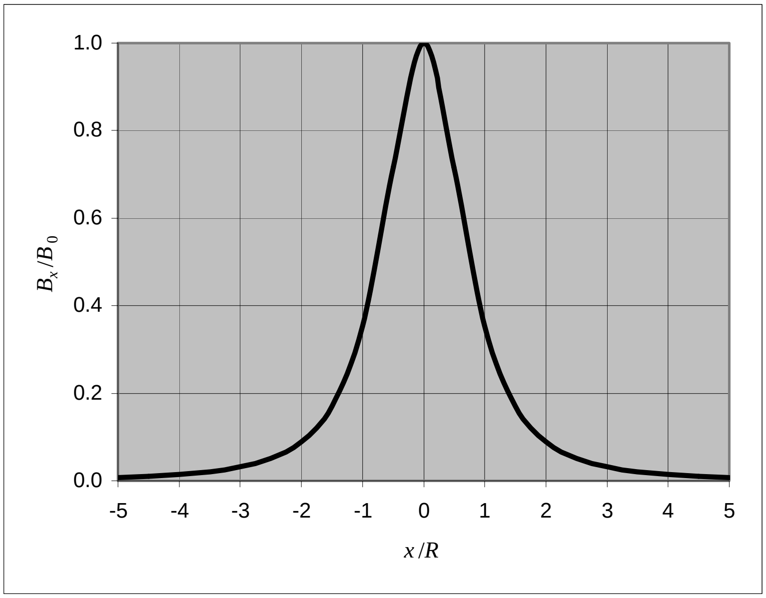
Express B_0 at the center of the loop:

$$B_0 = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(R^2\right)^{3/2}} = \frac{\mu_0 I}{2R}$$

Express the ratio of B_x to B_0 and simplify to obtain:

$$\frac{B_x}{B_0}=\frac{1}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

The graph of B_x/B_0 as a function of x/R shown below was plotted using a spreadsheet program:



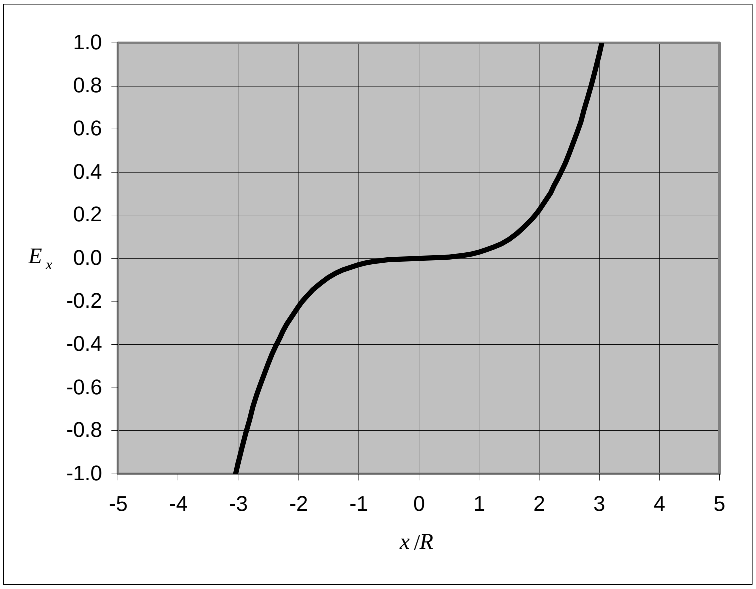
Express E_x on the axis due to a ring of radius R carrying a total charge Q:

$$E(x)=\frac{kQx}{\left(R^2+x^2\right)^{3/2}}=\frac{kQ}{R^2}\frac{\frac{x}{R}}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

Divide both sides of this equation by kQ/R^2 to obtain:

$$\frac{E(x)}{\frac{kQ}{R^2}}=\frac{\frac{x}{R}}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

The graph of E_x as a function of x/R shown below was plotted using a spreadsheet program. Here $E(x)$ is normalized, i.e., we've set $kQ/R^2 = 100$.



(*b*) Express the magnetic field on the *x* axis due to the loop centered at *x* = 0:

$$B_1(x)=\frac{\mu_0 R^2 I}{2\left(x^2+R^2\right)^{3/2}}=\frac{\mu_0 I}{2R\left(1+\dfrac{x^2}{R^2}\right)^{3/2}}$$

where N is the number of turns.

Because $B_0=\dfrac{\mu_0 I}{2R}$:

$$B_1(x)=\frac{B_0}{\left(1+\dfrac{x^2}{R^2}\right)^{3/2}}$$

or

$$\frac{B_1(x)}{B_0}=\left[1+\left(\frac{x}{R}\right)^2\right]^{-3/2}$$

Express the magnetic field on the *x* axis due to the loop centered at *x* = *R*:

$$B_2(x)=\frac{\mu_0 R^2 I}{2\left[(R-x)^2+R^2\right]^{3/2}}$$

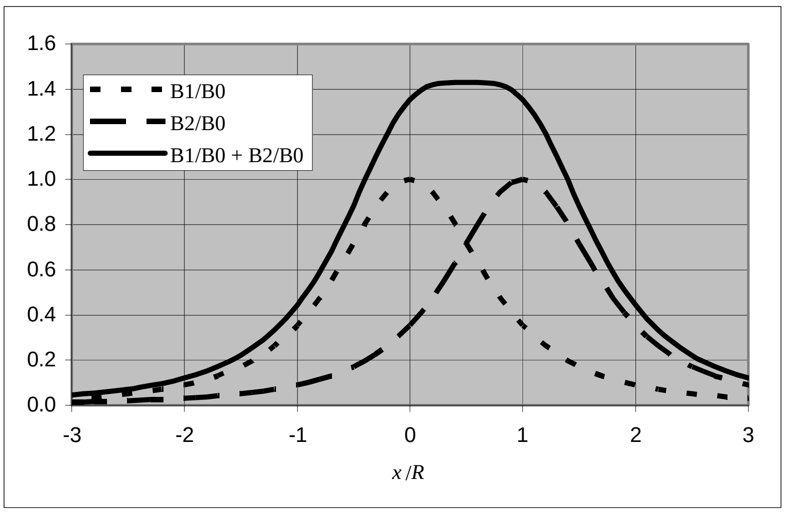
Simplify this expression to obtain:

$$B_2(x)=\frac{\mu_0 R^2 I}{2\left[(R-x)^2+R^2\right]^{3/2}}$$
$$=\frac{\mu_0 I}{2R\left[\left(1-\frac{x}{R}\right)^2+1\right]^{3/2}}$$
$$=\frac{B_0}{\left[\left(1-\frac{x}{R}\right)^2+1\right]^{3/2}}$$

or

$$\frac{B_2(x)}{B_0}=\left[\left(1-\frac{x}{R}\right)^2+1\right]^{-3/2}$$

The graphs of B_1/B_0, B_2/B_0, and $B_1/B_0 + B_2/B_0$ as functions of x/R with the second loop displaced by $d = R$ from the center of the first loop along the x axis shown below were plotted using a spreadsheet program.



Note that, midway between the two loops, $dB(x)/dx = 0$. Also, when $d = R$, $B(x)$ is nearly flat at the midpoint which shows that in the region midway between the two coils $B(x)$ is nearly constant.

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Picture the Problem Let the origin be midway between the coils so that one of them is centered at $x = -r/2$ and the other is centered at $x = r/2$. Let the numeral 1 denote the coil centered at $x = -r/2$ and the numeral 2 the coil centered at $x = r/2$. We can express the magnetic field in the region between the coils as the sum of the magnetic fields B_1 and B_2 due to the two coils.

Express the magnetic field on the x axis due to the coil centered at $x = -r/2$:

$$B_1(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2} + x\right)^2 + r^2\right]^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the x axis due to the coil centered at $x = r/2$:

$$B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2} - x\right)^2 + r^2\right]^{3/2}}$$

Add these equations to express the total magnetic field along the x axis:

$$B_x(x) = B_1(x) + B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2} + x\right)^2 + r^2\right]^{3/2}} + \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2} - x\right)^2 + r^2\right]^{3/2}}$$

$$= \frac{\mu_0 N r^2 I}{2}\left(\left[\left(\frac{r}{2} + x\right)^2 + r^2\right]^{-3/2} + \left[\left(\frac{r}{2} - x\right)^2 + r^2\right]^{-3/2}\right)$$

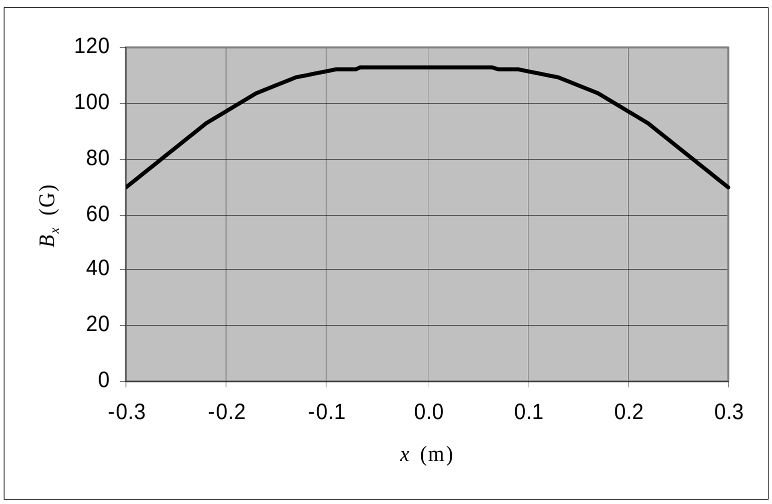
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.13×10^{-7}	μ_0
B2	0.30	r
B3	250	N
B3	15	I
B5	0.5*$B$1*$B$3*($B$2^2)*$B$4	$\text{Coeff} = \frac{\mu_0 N r^2 I}{2}$
A8	–0.30	$-r$
B8	$B$5*(($B$2/2+A8)^2+$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r}{2} + x\right)^2 + r^2\right]^{-3/2}$
C8	$B$5* (($B$2/2–A8)^2+$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r}{2} - x\right)^2 + r^2\right]^{-3/2}$
D8	10^4(B8+C8)	$B_x = 10^4(B_1 + B_2)$

	A	B	C	D
1	mu_0=	1.26E-06	N/A^2	
2	r=	0.3	m	
3	N=	250	turns	

4	I=	15	A	
5	Coeff=	2.13E–04		
6				
7	x	B_1	B_2	B(x)
8	–0.30	5.63E–03	1.34E–03	70
9	–0.29	5.86E–03	1.41E–03	73
10	–0.28	6.08E–03	1.48E–03	76
11	–0.27	6.30E–03	1.55E–03	78
12	–0.26	6.52E–03	1.62E–03	81
13	–0.25	6.72E–03	1.70E–03	84
14	–0.24	6.92E–03	1.78E–03	87
15	–0.23	7.10E–03	1.87E–03	90
61	0.23	1.87E–03	7.10E–03	90
62	0.24	1.78E–03	6.92E–03	87
63	0.25	1.70E–03	6.72E–03	84
64	0.26	1.62E–03	6.52E–03	81
65	0.27	1.55E–03	6.30E–03	78
66	0.28	1.48E–03	6.08E–03	76
67	0.29	1.41E–03	5.86E–03	73
68	0.30	1.34E–03	5.63E–03	70

The following graph of B_x as a function of x was plotted using the data in the above table.



The maximum value of B_x is 113 G. Twenty percent of this maximum value is 23 G. Referring to the table of values we see that the field is within 20 percent of 113 G in the interval $\boxed{-0.23\,\text{m} < x < 0.23\,\text{m}.}$

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Picture the Problem Let the numeral 1 denote the coil centered at the origin and the numeral 2 the coil centered at $x = R$. We can express the magnetic field in the region between the coils as the sum of the magnetic fields due to the two coils and then evaluate the derivatives at $x = R/2$.

Express the magnetic field on the x axis due to the coil centered at $x = 0$:

$$B_1(x) = \frac{\mu_0 N R^2 I}{2\left(x^2 + R^2\right)^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the x axis due to the coil centered at $x = R$:

$$B_2(x) = \frac{\mu_0 N R^2 I}{2\left[(x - R)^2 + R^2\right]^{3/2}}$$

Add these equations to express the total magnetic field along the x axis:

$$B_x(x) = B_1(x) + B_2(x) = \frac{\mu_0 N R^2 I}{2\left(x^2 + R^2\right)^{3/2}} + \frac{\mu_0 N R^2 I}{2\left[(x - R)^2 + R^2\right]^{3/2}}$$

$$= \frac{\mu_0 N R^2 I}{2}\left(\frac{1}{\left(x^2 + R^2\right)^{3/2}} + \frac{1}{\left[(x - R)^2 + R^2\right]^{3/2}}\right)$$

Evaluate x_1 and x_2 at $x = R/2$:

$$x_1\left(\tfrac{1}{2}R\right) = \sqrt{\tfrac{1}{4}R^2 + R^2} = \left(\tfrac{5}{4}R^2\right)^{1/2}$$

and

$$x_2\left(\tfrac{1}{2}R\right) = \sqrt{\left(\tfrac{1}{2}R - R\right)^2 + R^2} = \left(\tfrac{5}{4}R^2\right)^{1/2}$$

Differentiate B_x with respect to x to obtain:

$$\frac{dB_x}{dx} = \frac{\mu_0 N R^2 I}{2}\frac{d}{dx}\left(\frac{1}{x_1^3} + \frac{1}{x_2^3}\right)$$

$$= \frac{\mu_0 N R^2 I}{2}\left(\frac{x}{x_1^5} + \frac{x - R}{x_2^5}\right)$$

Evaluate dB_x/dx at $x = R/2$ to obtain:

$$\left.\frac{dB_x}{dx}\right|_{x=\frac{1}{2}R} = \frac{\mu_0 N R^2 I}{2}\left(\frac{\frac{1}{2}R}{\left(\frac{5}{4}R^2\right)^{5/2}} + \frac{-\frac{1}{2}R}{\left(\frac{5}{4}R^2\right)^{5/2}}\right) = \boxed{0}$$

Differentiate dB_x/dx with respect to x to obtain:

$$\frac{d^2B_x}{dx^2} = \frac{\mu_0 N R^2 I}{2}\frac{d}{dx}\left(\frac{x}{x_1^5} + \frac{x - R}{x_2^5}\right) = \frac{\mu_0 N R^2 I}{2}\left(\frac{1}{x_1^5} + \frac{1}{x_2^5} - \frac{5x^2}{x_1^7} - \frac{5(x - R)^2}{x_2^7}\right)$$

Evaluate d^2B_x/dx^2 at $x = R/2$ to obtain:

$$\left.\frac{d^2B_x}{dx^2}\right|_{x=\frac{1}{2}R}=\frac{\mu_0NR^2I}{2}\left(\frac{1}{\left(\frac{5}{4}R^2\right)^{5/2}}+\frac{1}{\left(\frac{5}{4}R^2\right)^{5/2}}-\frac{\frac{5}{4}R^2}{\left(\frac{5}{4}R^2\right)^{7/2}}-\frac{\frac{5}{4}R^2}{\left(\frac{5}{4}R^2\right)^{7/2}}\right)=\boxed{0}$$

Differentiate d^2B_x/dx^2 with respect to x to obtain:

$$\frac{d^3B_x}{dx^3}=\frac{\mu_0NR^2I}{2}\frac{d}{dx}\left(\frac{1}{x_1^5}+\frac{1}{x_2^5}-\frac{5x^2}{x_1^7}-\frac{5(x-R)^2}{x_2^7}\right)$$

$$=\frac{\mu_0NR^2I}{2}\left(\frac{35x^3}{x_1^9}-\frac{15x}{x_1^7}-\frac{15(x-R)}{x_2^7}-\frac{35(x-R)^3}{x_2^9}\right)$$

Evaluate d^3B_x/dx^3 at $x = R/2$ to obtain:

$$\left.\frac{d^3B_x}{dx^3}\right|_{x=\frac{1}{2}R}=\frac{\mu_0NR^2I}{2}\left(\frac{\frac{35}{8}R^3}{\left(\frac{5}{4}R^2\right)^{9/2}}-\frac{\frac{15}{2}R}{\left(\frac{5}{4}R^2\right)^{7/2}}-\frac{-\frac{15}{2}R}{\left(\frac{5}{4}R^2\right)^{7/2}}-\frac{-\frac{35}{8}R^3}{\left(\frac{5}{4}R^2\right)^{9/2}}\right)=\boxed{0}$$

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Picture the Problem Let the origin be midway between the coils so that one of them is centered at $x=-r\sqrt{3}/2$ and the other is centered at $x=r\sqrt{3}/2$. Let the numeral 1 denote the coil centered at $x=-r\sqrt{3}/2$ and the numeral 2 the coil centered at $x=r\sqrt{3}/2$. We can express the magnetic field in the region between the coils as the difference of the magnetic fields B_1 and B_2 due to the two coils.

Express the magnetic field on the x axis due to the coil centered at $x=-r\sqrt{3}/2$:

$$B_1(x)=\frac{\mu_0Nr^2I}{2\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2+r^2\right]^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the x axis due to the coil centered at $x=r\sqrt{3}/2$:

$$B_2(x)=\frac{\mu_0Nr^2I}{2\left[\left(\frac{r\sqrt{3}}{2}-x\right)^2+r^2\right]^{3/2}}$$

Subtract these equations to express the total magnetic field along the x axis:

$$B_x(x) = B_1(x) - B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2+r^2\right]^{3/2}} - \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r\sqrt{3}}{2}-x\right)^2+r^2\right]^{3/2}}$$

$$= \frac{\mu_0 N r^2 I}{2}\left(\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2+r^2\right]^{-3/2} - \left[\left(\frac{r\sqrt{3}}{2}-x\right)^2+r^2\right]^{-3/2}\right)$$

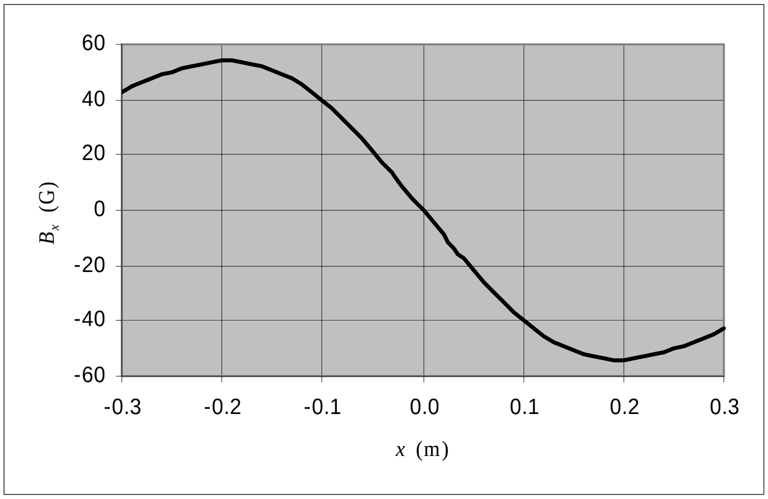
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.26×10^{-6}	μ_0
B2	0.30	r
B3	250	N
B3	15	I
B5	0.5*$B$1*$B$3*($B$2^2)*$B$4	$\text{Coeff} = \frac{\mu_0 N r^2 I}{2}$
A8	–0.30	$-r$
B8	$B$5*(($B$2*SQRT(3)/2+A8)^2 +$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2+r^2\right]^{-3/2}$
C8	$B$5* (($B$2*SQRT(3)/2–A8)^2 +$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r\sqrt{3}}{2}-x\right)^2+r^2\right]^{-3/2}$
D8	10^4*(B8–C8)	$B_x = B_1 - B_2$

	A	B	C	D
1	mu_0=	1.26E–06	N/A^2	
2	r=	0.3	m	
3	N=	250	turns	
4	I=	15	A	
5	Coeff=	2.13E–04		
6				
7	x	B_1	B_2	B(x)
8	–0.30	5.63E–03	1.34E–03	68.4
9	–0.29	5.86E–03	1.41E–03	68.9
10	–0.28	6.08E–03	1.48E–03	69.2
11	–0.27	6.30E–03	1.55E–03	69.2
12	–0.26	6.52E–03	1.62E–03	68.9
13	–0.25	6.72E–03	1.70E–03	68.4
14	–0.24	6.92E–03	1.78E–03	67.5
15	–0.23	7.10E–03	1.87E–03	66.4

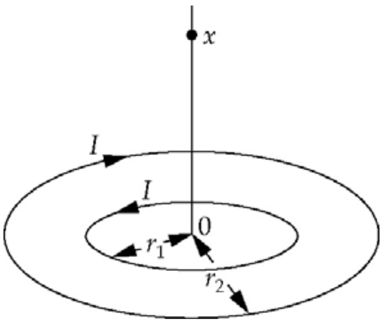
61	0.23	1.87E–03	7.10E–03	–66.4
62	0.24	1.78E–03	6.92E–03	–67.5
63	0.25	1.70E–03	6.72E–03	–68.4
64	0.26	1.62E–03	6.52E–03	–68.9
65	0.27	1.55E–03	6.30E–03	–69.2
66	0.28	1.48E–03	6.08E–03	–69.2
67	0.29	1.41E–03	5.86E–03	–68.9
68	0.30	1.34E–03	5.63E–03	–68.4

The following graph of B_x as a function of x was plotted using the data in the above table.



39 ••

Picture the Problem The diagram shows the two coils of radii r_1 and r_2 with the currents flowing in the directions given. We can use the expression for B on the axis of a current loop to express the difference of the fields due to the two loops at a distance x from their common center. We'll denote each field by the subscript identifying the radius of the current loop.



The magnitude of the field on the x axis due to the current in the inner loop is:

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi r_1^2 I}{\left(x^2 + r_1^2\right)^{3/2}} = \frac{\mu_0 r_1^2 I}{2\left(x^2 + r_1^2\right)^{3/2}}$$

The magnitude of the field on the x axis due to the current in the outer loop is:

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi r_2^2 I}{\left(x^2 + r_2^2\right)^{3/2}} = \frac{\mu_0 r_2^2 I}{2\left(x^2 + r_2^2\right)^{3/2}}$$

The resultant field at x is the difference between B_1 and B_2:

$$B_x(x) = B_1(x) - B_2(x) = \frac{\mu_0 r_1^2 I}{2(x^2 + r_1^2)^{3/2}} - \frac{\mu_0 r_2^2 I}{2(x^2 + r_2^2)^{3/2}}$$

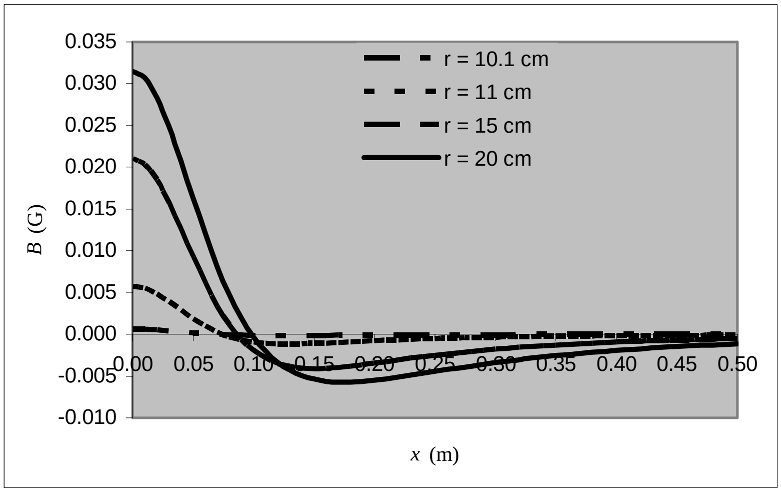
(*a*) The spreadsheet program to calculate B_x as a function of x, for r_2 = 10.1 cm, is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.26×10^{-6}	μ_0
B2	0.1	r_1
B3	1	I
B4	0.101	r_2
A7	0	x
B7	0.5*$B$1*$B$2^2*$B$3/(A7^2+$B$2^2)^(3/2)	$\frac{\mu_0 r_1^2 I}{2(x^2 + r_1^2)^{3/2}}$
C7	0.5*$B$1*$B$4^2*$B$3/(A7^2+$B$4^2)^(3/2)	$\frac{\mu_0 r_2^2 I}{2(x^2 + r_2^2)^{3/2}}$
D7	10^4*(B7–C7)	$B_x(x) = B_1(x) - B_2(x)$

The spreadsheet for B_x when r = 10.1 cm follows. The other three tables are similar.

	A	B	C	D
1	mu_0=	1.26E–06	N/A^2	
2	r_1=	0.1	m	
3	I=	1	A	
4	r_2=	0.101	m	
5	r_2=	0.11	m	
6	r_2=	0.15	m	
7	r_2=	0.2	m	
8				
9	x	B_1	B_2	B_x
10	0.00	6.30E–06	6.24E–06	6.24E–04
11	0.01	6.21E–06	6.15E–06	5.97E–04
12	0.02	5.94E–06	5.89E–06	5.21E–04
13	0.03	5.54E–06	5.49E–06	4.14E–04
14	0.04	5.04E–06	5.01E–06	2.95E–04
15	0.05	4.51E–06	4.49E–06	1.81E–04
56	0.46	6.04E–08	6.15E–08	–1.13E–05
57	0.47	5.68E–08	5.78E–08	–1.07E–05
58	0.48	5.34E–08	5.45E–08	–1.01E–05
59	0.49	5.04E–08	5.13E–08	–9.51E–06
60	0.50	4.75E–08	4.84E–08	–8.99E–06

The following graph shows $B(x)$ for r = 10.1 cm, 11 cm, 15 cm, and 20 cm.



40 •••

Picture the Problem We can approximate $B(x)$ by using the result from Problem 39 for the field due to a single coil of radius r and evaluating $B(x) \approx \frac{\partial B}{\partial r}\Delta r$ at $r = r_1$.

The magnetic field at a distance x on the axis of a coil of radius r is given by:

$$B(x) = \frac{\mu_0 I}{4\pi}\frac{2\pi r^2}{\left(x^2 + r^2\right)^{3/2}}$$

Express $B(x)$ in terms of the rate of change of B with respect to r:

$$B(x) \approx \frac{\partial B}{\partial r}\Delta r \qquad (1)$$

Evaluate the partial derivative of B with respect to r:

$$\frac{\partial}{\partial r}\left[\frac{\mu_0 I}{4\pi}\left(\frac{2\pi r^2}{\left(x^2+r^2\right)^{3/2}}\right)\right]=\frac{\mu_0 I}{4\pi}\frac{\partial}{\partial r}\left[\frac{2\pi r^2}{\left(x^2+r^2\right)^{3/2}}\right]$$

$$=\frac{\mu_0 I}{4\pi}\left[\frac{\left(x^2+r^2\right)^{3/2}\frac{\partial}{\partial r}\left(2\pi r^2\right)-2\pi r^2\frac{\partial}{\partial r}\left[\left(x^2+r^2\right)^{3/2}\right]}{\left(x^2+r_1^2\right)^3}\right]$$

$$=\frac{\mu_0 I}{4\pi}\left[\frac{\left(x^2+r^2\right)^{3/2}(4\pi r)-2\pi r^2\left[\frac{3}{2}\left(x^2+r^2\right)^{1/2}(2r)\right]}{\left(x^2+r_1^2\right)^3}\right]$$

$$=\mu_0 Ir\left[\frac{\left(x^2+r^2\right)^{3/2}-\frac{3}{2}r^2\left(x^2+r^2\right)^{1/2}}{\left(x^2+r^2\right)^3}\right]$$

$$=\mu_0 Ir\left[\frac{2\left(x^2+r^2\right)^{3/2}-3r^2\left(x^2+r^2\right)^{1/2}}{2\left(x^2+r^2\right)^3}\right]$$

$$=\mu_0 Ir\left(x^2+r^2\right)^{1/2}\left[\frac{2\left(x^2+r^2\right)-3r^2}{2\left(x^2+r^2\right)^3}\right]$$

$$=\frac{\mu_0 I}{2}\left[\frac{2x^2r-r^3}{\left(x^2+r^2\right)^{5/2}}\right]$$

Evaluate $\partial B/\partial x$ at $r = r_1$ to obtain:

$$\left.\frac{\partial B}{\partial r}\right|_{r=r_1}=\frac{\mu_0 I}{2}\left[\frac{2x^2r_1-r_1^3}{\left(x^2+r_1^2\right)^{5/2}}\right]$$

Substitute in equation (1) to obtain:

$$B(x)\approx\boxed{\left(\frac{\mu_0 I\Delta r}{2}\right)\left[\frac{2x^2r_1-r_1^3}{\left(x^2+r_1^2\right)^{5/2}}\right]}$$

Remarks: This solution shows that the field due to two coils separated by Δ*r* can be approximated by the given expression.

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Picture the Problem We can factor x from the denominator of the equation from Problem 40 to show that $B(x)\approx\left(\frac{\mu_0 I\Delta r}{2}\right)\left(\frac{2r_1}{x^3}\right)$.

From Problem 40:

$$B(x)\approx\left(\frac{\mu_0 I\Delta r}{2}\right)\left[\frac{2x^2r_1-r_1^3}{\left(x^2+r_1^2\right)^{5/2}}\right]$$

Factor x^2 from the denominator of the expression to obtain:

$$B(x) \approx \left(\frac{\mu_0 I\Delta r}{2}\right)\left[\frac{2r_1x^2 - r_1^3}{x^5\left(1+\frac{r_1^2}{x^2}\right)^{5/2}}\right]$$

For $x >> r_1$:

$$x^5\left(1+\frac{r_1^2}{x^2}\right)^{5/2} \approx x^5$$

Substitute and simplify to obtain:

$$B(x) \approx \left(\frac{\mu_0 I\Delta r}{2}\right)\left(\frac{2r_1x^2 - r_1^3}{x^5}\right)$$

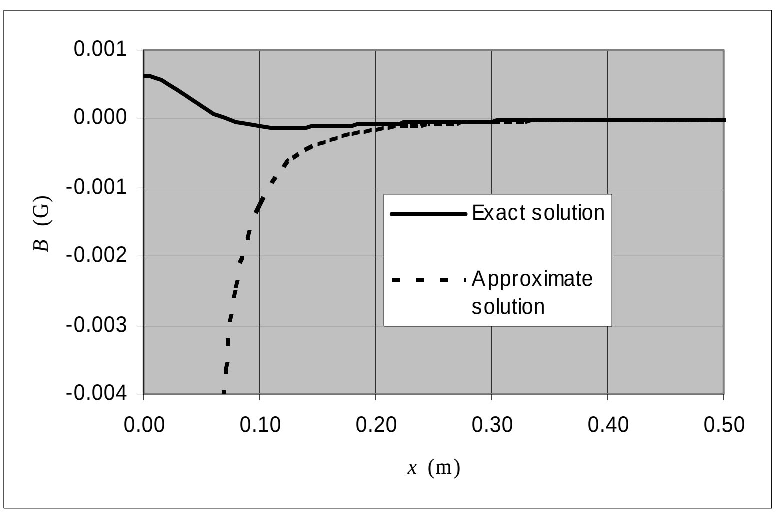
$$= \left(\frac{\mu_0 I\Delta r}{2}\right)\left(\frac{2r_1x^2}{x^5} - \frac{r_1^3}{x^5}\right)$$

$$= \left(\frac{\mu_0 I\Delta r}{2}\right)\left(\frac{2r_1}{x^3} - \frac{r_1^3}{x^5}\right)$$

For $x >> r$:

$$\boxed{B(x) \approx \left(\frac{\mu_0 I\Delta r}{2}\right)\left(\frac{2r_1}{x^3}\right)}$$

The spreadsheet-generated graph that follows provides a comparison of the exact and approximate fields. Note that the two solutions agree for large values of *x*.



Straight-Line Current Segments

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Picture the Problem The magnetic field due to the current in a long straight wire is given by $B=\frac{\mu_0}{4\pi}\frac{2I}{R}$ where I is the current in the wire and R is the distance from the wire.

Express the magnetic field due to a long straight wire:

$$B=\frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute numerical values to obtain:

$$B=\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(10\ \text{A})}{R}=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{R}$$

(*a*) Evaluate B at R = 10 cm:

$$B(10\ \text{cm})=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{0.1\text{m}}=\boxed{20.0\ \mu\text{T}}$$

(*b*) Evaluate B at R = 50 cm:

$$B(50\ \text{cm})=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{0.5\text{m}}=\boxed{4.00\ \mu\text{T}}$$

(*c*) Evaluate B at R = 2 m:

$$B(2\ \text{m})=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{2\text{m}}=\boxed{1.00\ \mu\text{T}}$$

Problems 43 to 48 refer to Figure 27-45, which shows two long straight wires in the xy plane and parallel to the x axis. One wire is at y = –6 cm and the other is at y = +6 cm. The current in each wire is 20 A.

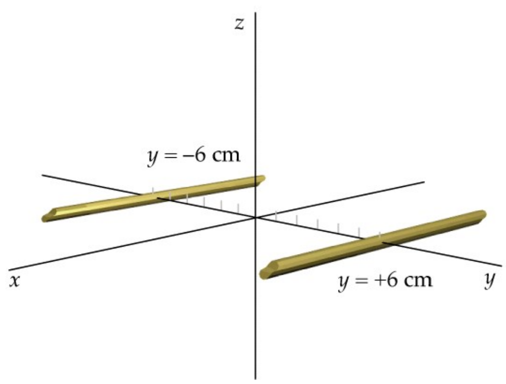


Figure 27-45 Problems 43-48

***43 •**

Picture the Problem Let + denote the wire (and current) at y = +6 cm and – the wire (and current) at y = –6 cm. We can use $B=\frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the y axis. We can apply the right-hand rule to find the direction of each of the fields and, hence, of $\vec{B}$.

(*a*) Express the resultant magnetic field at $y = -3$ cm:

$$\vec{B}(-3\,\text{cm}) = \vec{B}_+(-3\,\text{cm}) + \vec{B}_-(-3\,\text{cm})$$

Find the magnitudes of the magnetic fields at $y = -3$ cm due to each wire:

$$B_+(-3\,\text{cm}) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.09\,\text{m}} = 44.4\,\mu\text{T}$$

and

$$B_-(-3\,\text{cm}) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.03\,\text{m}} = 133\,\mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(-3\,\text{cm}) = (44.4\,\mu\text{T})\hat{\boldsymbol{k}}$$

and

$$\vec{B}_-(-3\,\text{cm}) = -(133\,\mu\text{T})\hat{\boldsymbol{k}}$$

Substitute to obtain:

$$\vec{B}(-3\,\text{cm}) = (44.4\,\mu\text{T})\hat{\boldsymbol{k}} - (133\,\mu\text{T})\hat{\boldsymbol{k}} = \boxed{-(88.6\,\mu\text{T})\hat{\boldsymbol{k}}}$$

(*b*) Express the resultant magnetic field at $y = 0$:

$$\vec{B}(0) = \vec{B}_+(0) + \vec{B}_-(0)$$

Because $\vec{B}_+(0) = -\vec{B}_-(0)$:

$$\vec{B}(0) = \boxed{0}$$

(*c*) Proceed as in (*a*) to obtain:

$$\vec{B}_+(3\,\text{cm}) = (133\,\mu\text{T})\hat{\boldsymbol{k}}\,,$$

$$\vec{B}_-(3\,\text{cm}) = -(44.4\,\mu\text{T})\hat{\boldsymbol{k}}\,,$$

and

$$\vec{B}(3\,\text{cm}) = (133\,\mu\text{T})\hat{\boldsymbol{k}} - (44.4\,\mu\text{T})\hat{\boldsymbol{k}} = \boxed{-(88.6\,\mu\text{T})\hat{\boldsymbol{k}}}$$

(*d*) Proceed as in (*a*) with $y = 9$ cm to obtain:

$$\vec{B}_+(9\,\text{cm}) = -(133\,\mu\text{T})\hat{\boldsymbol{k}}\,,$$

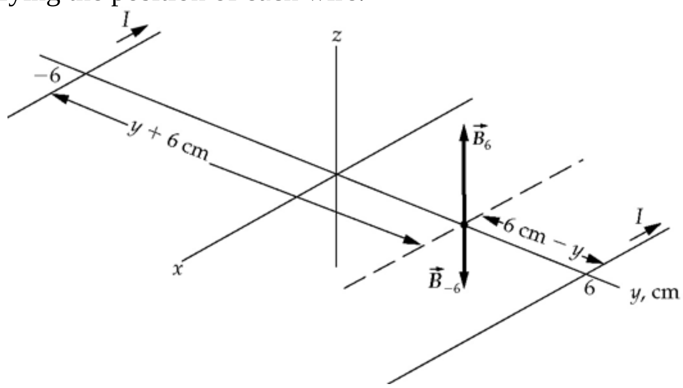
$$\vec{B}_-(9\,\text{cm}) = -(26.7\,\mu\text{T})\hat{\boldsymbol{k}}\,,$$

and

$$\vec{B}(9\,\text{cm}) = -(133\,\mu\text{T})\hat{k} - (26.7\,\mu\text{T})\hat{k}$$
$$= \boxed{-(160\,\mu\text{T})\hat{k}}$$

44 ••

Picture the Problem The diagram shows the two wires with the currents flowing in the negative *x* direction. We can use the expression for *B* due to a long, straight wire to express the difference of the fields due to the two currents. We'll denote each field by the subscript identifying the position of each wire.



The field due to the current in the wire located at $y = 6$ cm is:

$$B_6 = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} - y}$$

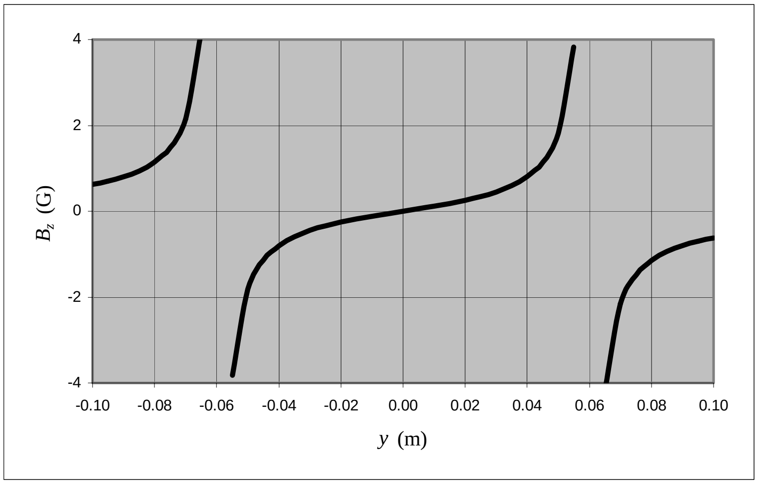
The field due to the current in the wire located at $y = -6$ cm is:

$$B_{-6} = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} + y}$$

The resultant field B_z is the difference between B_6 and B_{-6}:

$$B_z = B_6 - B_{-6} = \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} - \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{0.06\,\text{m} - y} - \frac{1}{0.06\,\text{m} + y}\right)$$

The following graph of B_z as a function of y was plotted using a spreadsheet program:



45 •

Picture the Problem Let + denote the wire (and current) at y = +6 cm and – the wire (and current) at y = –6 cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the y axis. We can apply the right-hand rule to find the direction of each of the fields and, hence, of $\vec{B}$.

(*a*) Express the resultant magnetic field at y = –3 cm:

$$\vec{B}(-3\,\text{cm}) = \vec{B}_+(-3\,\text{cm}) + \vec{B}_-(-3\,\text{cm})$$

Find the magnitudes of the magnetic fields at y = –3 cm due to each wire:

$$B_+(-3\,\text{cm}) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.09\,\text{m}} = 44.4\,\mu\text{T}$$

and

$$B_-(-3\,\text{cm}) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.03\,\text{m}} = 133\,\mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(-3\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k}$$

and

$$\vec{B}_-(-3\,\text{cm}) = -(133\,\mu\text{T})\hat{k}$$

Substitute to obtain:

$$\vec{B}(-3\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k} - (133\,\mu\text{T})\hat{k} = \boxed{-(177\,\mu\text{T})\hat{k}}$$

(*b*) Express the resultant magnetic field at $y = 0$:

$$\vec{B}(0)=\vec{B}_+(0)+\vec{B}_-(0)$$

Find the magnitudes of the magnetic fields at $y = 0$ cm due to each wire:

$$B_+(0)=\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.06\,\text{m}}$$
$$=66.7\ \mu\text{T}$$

and

$$B_-(0)=\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.06\,\text{m}}$$
$$=66.7\ \mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(0)=-(66.7\ \mu\text{T})\hat{k}$$

and

$$\vec{B}_-(0)=-(66.7\ \mu\text{T})\hat{k}$$

Substitute to obtain:

$$\vec{B}(0)=-(66.7\ \mu\text{T})\hat{k}-(66.7\ \mu\text{T})\hat{k}$$
$$=\boxed{-(133\ \mu\text{T})\hat{k}}$$

(*c*) Proceed as in (*a*) with $y = +3$ cm to obtain:

$$\vec{B}_+(3\,\text{cm})=-(133\ \mu\text{T})\hat{k}\,,$$
$$\vec{B}_-(3\,\text{cm})=-(44.4\ \mu\text{T})\hat{k}\,,$$

and

$$\vec{B}(3\,\text{cm})=-(133\ \mu\text{T})\hat{k}-(44.4\ \mu\text{T})\hat{k}$$
$$=\boxed{-(177\ \mu\text{T})\hat{k}}$$

(*d*) Proceed as in (*a*) with $y = +9$ cm to obtain:

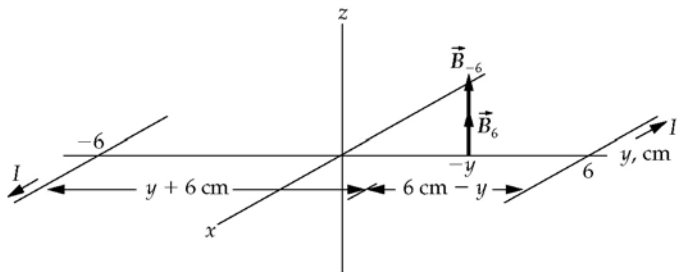
$$\vec{B}_+(9\,\text{cm})=(133\ \mu\text{T})\hat{k}\,,$$
$$\vec{B}_-(9\,\text{cm})=-(26.7\ \mu\text{T})\hat{k}\,,$$

and

$$\vec{B}(9\,\text{cm})=(133\ \mu\text{T})\hat{k}-(26.7\ \mu\text{T})\hat{k}$$
$$=\boxed{(106\ \mu\text{T})\hat{k}}$$

46 ••

Picture the Problem The diagram shows the two wires with the currents flowing in the negative *x* direction. We can use the expression for *B* due to a long, straight wire to express the difference of the fields due to the two currents. We'll denote each field by the subscript identifying the position of each wire.



The field due to the current in the wire located at y = 6 cm is:

$$B_6 = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} - y}$$

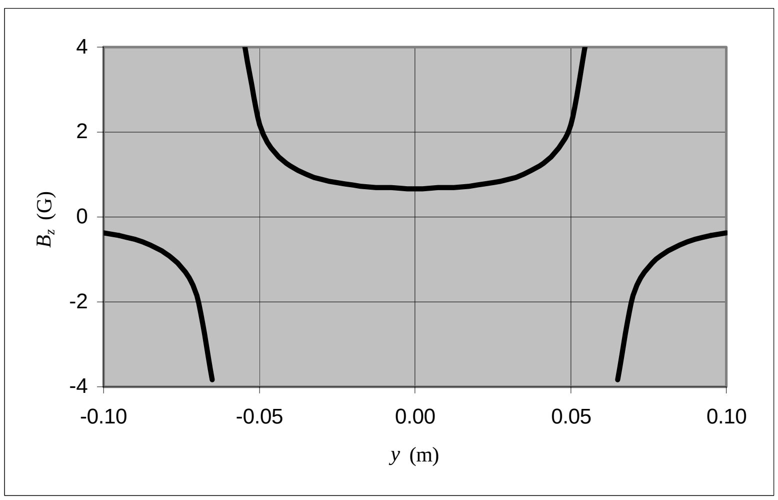
The field due to the current in the wire located at y = –6 cm is:

$$B_{-6} = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} + y}$$

The resultant field B_z is the sum of B_6 and B_{-6}:

$$B_z = B_6 - B_{-6} = \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} + \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{0.06\,\text{m} - y} + \frac{1}{0.06\,\text{m} + y}\right)$$

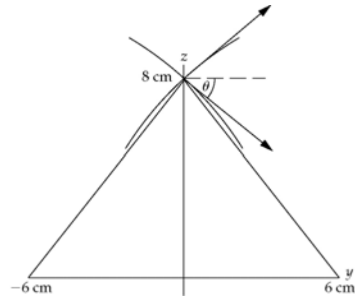
The following graph of B_z as a function of y was plotted using a spreadsheet program:



47 •

Picture the Problem Let + denote the wire (and current) at y = +6 cm and − the wire (and current) at y = –6 cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the z axis.

(*a*) Apply the right-hand rule to show that, for the currents parallel and in the negative *x* direction, the directions of the fields are as shown to the right:



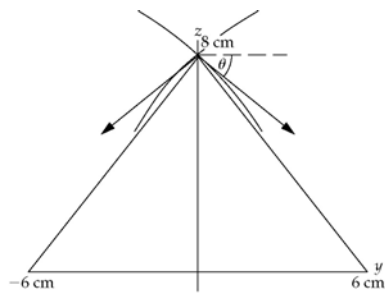
Express the magnitudes of the magnetic fields at z = +8 cm due to the current-carrying wires at y = –6 cm and y = +6 cm:

$$B_{z-} = B_{z+} = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right) \times \frac{2(20\,\text{A})}{\sqrt{(0.06\,\text{m})^2 + (0.08\,\text{m})^2}} = 40.0\,\mu\text{T}$$

Noting that the z components add to zero, express the resultant magnetic field at z = +8 cm:

$$\vec{B}(z = 8\,\text{cm}) = 2(40.0\,\mu\text{T})\sin\theta\,\hat{j} = 2(40.0\,\mu\text{T})(0.8)\,\hat{j} = \boxed{(64.0\,\mu\text{T})\,\hat{j}}$$

(*b*) Apply the right-hand rule to show that, for the currents antiparallel with the current in the wire at y = –6 cm in the negative x direction, the directions of the fields are as shown to the right:



Noting that the y components add to zero, express the resultant magnetic field at z = +8 cm:

$$\vec{B}(z = 8\,\text{cm}) = -2(40.0\,\mu\text{T})\cos\theta\,\hat{k} = -2(40.0\,\mu\text{T})(0.6)\,\hat{k} = \boxed{-(48.0\,\mu\text{T})\,\hat{k}}$$

48 •

Picture the Problem Let + denote the wire (and current) at y = +6 cm and – the wire (and current) at y = –6 cm. The forces per unit length the wires exert on each other are action and reaction forces and hence are equal in magnitude. We can use $F = I\ell B$ to express the force on either wire and $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to express the magnetic field at the location of either wire due to the current in the other.

Express the force exerted on either wire:

$$F = I\ell B$$

Express the magnetic field at either location due to the current in the wire at the other location:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute to obtain:

$$F = I\ell\left(\frac{\mu_0}{4\pi}\frac{2I}{R}\right) = \frac{2\ell\mu_0}{4\pi}\frac{I^2}{R} = \frac{\ell\mu_0}{2\pi}\frac{I^2}{R}$$

Divide both sides of the equation by ℓ to obtain:

$$\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$$

Substitute numerical values and evaluate F/ℓ:

$$\frac{F}{\ell} = \frac{2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)(20\ \text{A})^2}{0.12\ \text{m}} = \boxed{667\ \mu\text{N/m}}$$

49 •

Picture the Problem We can use $\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$ to relate the force per unit length each current-carrying wire exerts on the other to their common current.

(*a*) Because the currents repel, they are antiparallel.

(*b*)Express the force per unit length experienced by each wire:

$$\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$$

Solve for I:

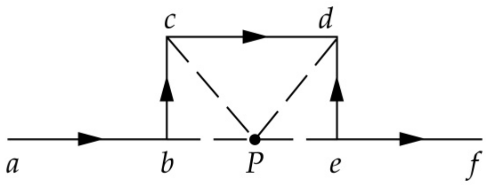
$$I = \sqrt{\frac{4\pi R}{2\mu_0}\frac{F}{\ell}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{(8.6\ \text{cm})}{2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)}(3.6\ \text{nN/m})} = \boxed{39.3\ \text{mA}}$$

50 ••

Picture the Problem Note that the current segments *a*-*b* and *e*-*f* do not contribute to the magnetic field at point *P*. The current in the segments *b*-*c*, *c*-*d*, and *d*-*e* result in a magnetic field at *P* that points into the plane of the paper. Note that the angles *bPc* and *ePd* are 45° and use the expression for *B* due to a straight wire segment to find the contributions to the field at P of segments *bc*, *cd*, and *de*.



Express the resultant magnetic field at P:

$$B = B_{bc} + B_{cd} + B_{de}$$

Express the magnetic field due to a straight line segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right) \qquad (1)$$

Use equation (1) to express B_{bc} and B_{de}:

$$B_{bc} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin 45° + \sin 0°\right) = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Use equation (1) to express B_{cd}:

$$B_{cd} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin 45° + \sin 45°\right) = 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° = 4\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute numerical values and evaluate B:

$$B = 4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{8\,\text{A}}{0.01\,\text{m}}\sin 45° = \boxed{226\,\mu\text{T}}$$

51 ••

Picture the Problem The forces acting on the wire are the upward magnetic force F_B and the downward gravitational force mg, where m is the mass of the wire. We can use a condition for translational equilibrium and the expression for the force per unit length between parallel current-carrying wires to relate the required current to the mass of the wire, its length, and the separation of the two wires.

Apply $\sum F_y = 0$ to the floating

$$F_B - mg = 0$$

wire to obtain:

Express the repulsive force acting on the upper wire:

$$F_B = 2\frac{\mu_0}{4\pi}\frac{I^2\ell}{R}$$

Substitute to obtain:

$$2\frac{\mu_0}{4\pi}\frac{I^2\ell}{R} - mg = 0$$

Solve for I:

$$I = \sqrt{\frac{4\pi mgR}{2\mu_0\ell}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{(14\times10^{-3}\text{ kg})(9.81\text{ m/s}^2)(1.5\times10^{-3}\text{ m})}{2(10^{-7}\text{ T}\cdot\text{m/A})(0.16\text{ m})}} = \boxed{80.2\text{ A}}$$

***52 ••**

Picture the Problem Note that the forces on the upper wire are away from and directed along the lines to the lower wire and that their horizontal components cancel. We can use $\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$ to find the resultant force in the upward direction (the y direction) acting on the top wire. In part (b) we can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnitude of the resultant field due to these currents.

(a) Express the force per unit length each of the lower wires exerts on the upper wire:

$$\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$$

Noting that the horizontal components add up to zero, express the net upward force per unit length on the upper wire:

$$\sum\frac{F_y}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30° + 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30° = 4\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30°$$

Substitute numerical values and evaluate $\sum \frac{F_y}{\ell}$:

$$\sum \frac{F_y}{\ell} = 4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(15\,\text{A})^2}{0.1\text{m}}\cos 30°$$

$$= \boxed{7.79\times 10^{-4}\ \text{N/m}}$$

(*b*) Noting, from the geometry of the wires, the magnetic field vectors both are at an angle of 30° with the horizontal and that their y components cancel, express the resultant magnetic field:

$$\vec{B} = 2\frac{\mu_0}{4\pi}\frac{2I}{R}\cos 30°\,\hat{i}$$

Substitute numerical values and evaluate B:

$$B = 2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(15\,\text{A})}{0.1\text{m}}\cos 30°$$

$$= \boxed{52.0\,\mu\text{T}}$$

53 ••

Picture the Problem Note that the forces on the upper wire are away from the lower left hand wire and toward the lower right hand wire and that, due to symmetry, their vertical components cancel. We can use $\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$ to find the resultant force in the x direction (to the right) acting on the top wire. In part (*b*) we can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnitude of the resultant field due to these currents.

(*a*) Express the force per unit length each of the lower wires exerts on the upper wire:

$$\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$$

Noting that the vertical components add up to zero, express the net force per unit length acting to the right on the upper wire:

$$\sum \frac{F_x}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60° + 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60°$$

$$= 4\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60°$$

Substitute numerical values and evaluate $\sum \frac{F_x}{\ell}$:

$$\sum \frac{F_x}{\ell} = 4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(15\ \text{A})^2}{0.1\,\text{m}}\cos 60° = \boxed{4.50\times10^{-4}\ \text{N/m}}$$

(*b*) Noting, from the geometry of the wires, that the magnetic field vectors both are at an angle of 30° with the horizontal and that their *x* components cancel, express the resultant magnetic field:

$$\vec{B} = -2\frac{\mu_0}{4\pi}\frac{2I}{R}\sin 30°\,\hat{j}$$

Substitute numerical values and evaluate *B*:

$$B = 2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(15\ \text{A})}{0.1\,\text{m}}\sin 30° = \boxed{30.0\ \mu\text{T}}$$

54 ••

Picture the Problem Let the numeral 1 denote the current flowing in the positive *x* direction and the magnetic field resulting from it and the numeral 2 denote the current flowing in the positive *y* direction and the magnetic field resulting from it. We can express the magnetic field anywhere in the *xy* plane using $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule and then impose the condition that $\vec{B} = 0$ to determine the set of points that satisfy this condition.

Express the resultant magnetic field due to the two current-carrying wires:

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field due to the current flowing in the positive *x* direction:

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{y}\hat{k}$$

Express the magnetic field due to the current flowing in the positive *y* direction:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2I_2}{x}\hat{k}$$

Substitute to obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{y}\hat{k} - \frac{\mu_0}{4\pi}\frac{2I}{x}\hat{k} = \left(\frac{\mu_0}{4\pi}\frac{2I}{y} - \frac{\mu_0}{4\pi}\frac{2I}{x}\right)\hat{k}$$

because $I = I_1 = I_2$.

For $\vec{B} = 0$:

$$\frac{\mu_0}{4\pi}\frac{2I}{y} - \frac{\mu_0}{4\pi}\frac{2I}{x} = 0 \Rightarrow x = y.$$

Hence, $\vec{B} = 0$ along a line that makes an angle of $45°$ with the x axis.

55 ••

Picture the Problem Let the numeral 1 denote the current flowing along the positive z axis and the magnetic field resulting from it and the numeral 2 denote the current flowing in the wire located at x = 10 cm and the magnetic field resulting from it. We can express the magnetic field anywhere in the xy plane using $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule and then impose the condition that $\vec{B} = 0$ to determine the current that satisfies this condition.

(*a*) Express the resultant magnetic field due to the two current-carrying wires:

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field at x = 2 cm due to the current flowing in the positive z direction:

$$\vec{B}_1(x = 2\,\text{cm}) = \frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}}\hat{j}$$

Express the magnetic field at x = 2 cm due to the current flowing in the wire at x = 10 cm:

$$\vec{B}_2(x = 2\,\text{cm}) = -\frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}}\hat{j}$$

Substitute to obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}}\hat{j} - \frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}}\hat{j} = \left(\frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}} - \frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}}\right)\hat{j}$$

For $\vec{B} = 0$:

$$\frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}} - \frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}} = 0$$

or

$$\frac{I_1}{2\,\text{cm}} - \frac{I_2}{8\,\text{cm}} = 0$$

Solve for and evaluate I_2:

$$I_2 = 4I_1 = 4(20\,\text{A}) = \boxed{80.0\,\text{A}}$$

(*b*) Express the magnetic field at x = 5 cm:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I_1}{5\,\text{cm}}\hat{j} - \frac{\mu_0}{4\pi}\frac{2I_2}{5\,\text{cm}}\hat{j} = \frac{2\mu_0}{4\pi(5\,\text{cm})}(I_1 - I_2)\hat{j}$$

Substitute numerical values and evaluate $\vec{B}(x = 5\,\text{cm})$:

$$\vec{B} = \frac{2(10^{-7}\,\text{T}\cdot\text{m/A})}{5\,\text{cm}}(20\,\text{A} - 80\,\text{A})\hat{j} = \boxed{-(0.240\,\text{mT})\hat{j}}$$

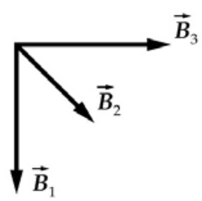
56 ••

Picture the Problem Choose a coordinate system with its origin at the lower left-hand corner of the square, the positive x axis to the right and the positive y axis upward. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnitude and direction of the magnetic field at the unoccupied corner due to each of the currents, and superimpose these fields to find the resultant field.

(*a*) Express the resultant magnetic field at the unoccupied corner:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \qquad (1)$$

When all the currents are into the paper their magnetic fields at the unoccupied corner are as shown to the right:



Express the magnetic field at the unoccupied corner due to the current I_1:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j}$$

Express the magnetic field at the unoccupied corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°(\hat{i} - \hat{j}) = \frac{\mu_0}{4\pi}\frac{2I}{2L}(\hat{i} - \hat{j})$$

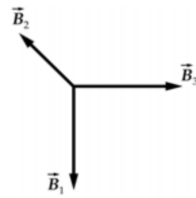
Express the magnetic field at the unoccupied corner due to the current I_3:

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i} - \hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j} + \frac{1}{2}\left(\hat{i} - \hat{j}\right) + \hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(1 + \frac{1}{2}\right)\hat{i} + \left(-1 - \frac{1}{2}\right)\hat{j}\right] = \boxed{\frac{3\mu_0 I}{4\pi L}\left[\hat{i} - \hat{j}\right]}$$

(*b*) When I_2 is out of the paper the magnetic fields at the unoccupied corner are as shown to the right:



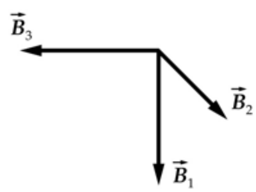
Express the magnetic field at the unoccupied corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(-\hat{i} + \hat{j}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(-\hat{i} + \hat{j}\right)$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(-\hat{i} + \hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j} + \frac{1}{2}\left(-\hat{i} + \hat{j}\right) + \hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(1 - \frac{1}{2}\right)\hat{i} + \left(-1 + \frac{1}{2}\right)\hat{j}\right] = \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j}\right] = \boxed{\frac{\mu_0 I}{4\pi L}\left[\hat{i} - \hat{j}\right]}$$

(*c*) When I_1 and I_2 are in and I_3 is out of the paper the magnetic fields at the unoccupied corner are as shown to the right:



From (*a*) or (*b*) we have:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j}$$

From (*a*) we have:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(\hat{i} - \hat{j}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i} - \hat{j}\right)$$

Express the magnetic field at the unoccupied corner due to the current I_3:

$$\vec{B}_3 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i} - \hat{j}\right) - \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j} + \frac{1}{2}\left(\hat{i} - \hat{j}\right) - \hat{i}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(-1 + \frac{1}{2}\right)\hat{i} + \left(-1 - \frac{1}{2}\right)\hat{j}\right] = \boxed{\frac{\mu_0 I}{4\pi L}\left[-\hat{i} - 3\hat{j}\right]}$$

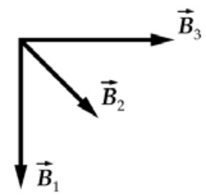
***57** ••

Picture the Problem Choose a coordinate system with its origin at the lower left-hand corner of the square, the positive *x* axis to the right and the positive *y* axis upward. Let the numeral 1 denote the wire and current in the upper left-hand corner of the square, the numeral 2 the wire and current in the lower left-hand corner (at the origin) of the square, and the numeral 3 the wire and current in the lower right-hand corner of the square. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnitude and direction of the magnetic field at, say, the upper right-hand corner due to each of the currents, superimpose these fields to find the resultant field, and then use $F = I\ell B$ to find the force per unit length on the wire.

(*a*) Express the resultant magnetic field at the upper right-hand corner:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \qquad (1)$$

When all the currents are into the paper their magnetic fields at the upper right-hand corner are as shown to the right:



Express the magnetic field due to the current I_1:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j}$$

Express the magnetic field due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}\cos 45°\left(\hat{i} - \hat{j}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(\hat{i} - \hat{j}\right)$$

Express the magnetic field due to the current I_3:

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(\hat{i}-\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left(-\hat{j}+\frac{1}{2}\left(\hat{i}-\hat{j}\right)+\hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\left(1+\frac{1}{2}\right)\hat{i}+\left(-1-\frac{1}{2}\right)\hat{j}\right] = \frac{3\mu_0 I}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

Using the expression for the magnetic force on a current-carrying wire, express the force per unit length on the wire at the upper right-hand corner:

$$\frac{F}{\ell} = BI \qquad (2)$$

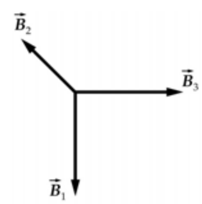
Substitute to obtain:

$$\frac{\vec{F}}{\ell} = \frac{3\mu_0 I^2}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

and

$$\frac{F}{\ell} = \sqrt{\left(\frac{3\mu_0 I^2}{4\pi a}\right)^2 + \left(\frac{3\mu_0 I^2}{4\pi a}\right)^2} = \boxed{\frac{3\sqrt{2}\mu_0 I^2}{4\pi a}}$$

(*b*) When the current in the upper right-hand corner of the square is out of the page, and the currents in the wires at adjacent corners are oppositely directed, the magnetic fields at the upper right-hand are as shown to the right:



Express the magnetic field at the upper right-hand corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}\cos 45°\left(-\hat{i}+\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(-\hat{i}+\hat{j}\right)$$

Using $\vec{B}_1$ and $\vec{B}_3$ from (*a*), substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(-\hat{i}+\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left(-\hat{j}+\frac{1}{2}\left(-\hat{i}+\hat{j}\right)+\hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\left(1-\frac{1}{2}\right)\hat{i}+\left(-1+\frac{1}{2}\right)\hat{j}\right] = \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\frac{1}{2}\hat{i}-\frac{1}{2}\hat{j}\right] = \frac{\mu_0 I}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

Substitute in equation (2) to obtain:

$$\frac{\vec{F}}{\ell}=\frac{\mu_0 I^2}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

and

$$\frac{F}{\ell}=\sqrt{\left(\frac{\mu_0 I^2}{4\pi a}\right)^2+\left(\frac{\mu_0 I^2}{4\pi a}\right)^2}$$

$$=\boxed{\frac{\sqrt{2}\mu_0 I^2}{4\pi a}}$$

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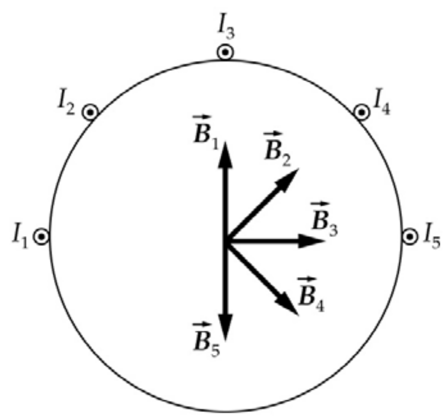
Picture the Problem The configuration is shown in the adjacent figure. Here the *z* axis points out of the plane of the paper, the *x* axis points to the right, the *y* axis points up. We can use $B=\frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnetic field due to the current in each wire and add these magnetic fields vectorially to find the resultant field.



Express the resultant magnetic field on the *z* axis:

$$\vec{B}=\vec{B}_1+\vec{B}_2+\vec{B}_3+\vec{B}_4+\vec{B}_5$$

$\vec{B}_1$ is given by: $\vec{B}_1=B\hat{j}$

$\vec{B}_2$ is given by: $\vec{B}_2=(B\cos 45°)\hat{i}+(B\sin 45°)\hat{j}$

$\vec{B}_3$ is given by: $\vec{B}_3=B\hat{i}$

$\vec{B}_4$ is given by: $\vec{B}_4=(B\cos 45°)\hat{i}-(B\sin 45°)\hat{j}$

$\vec{B}_5$ is given by: $\vec{B}_5=-B\hat{j}$

Substitute for $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, $\vec{B}_4$, and $\vec{B}_5$ and simplify to obtain:

$$\vec{B}=B\hat{j}+(B\cos 45°)\hat{i}+(B\sin 45°)\hat{j}+B\hat{i}+(B\cos 45°)\hat{i}-(B\sin 45°)\hat{j}-B\hat{j}$$
$$=(B\cos 45°)\hat{i}+B\hat{i}+(B\cos 45°)\hat{i}=(B+2B\cos 45°)\hat{i}=\left(1+\sqrt{2}\right)B\hat{i}$$

Express B due to each current at $z = 0$:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute to obtain:

$$\vec{B} = \boxed{\left(1+\sqrt{2}\right)\frac{\mu_0 I}{2\pi R}\hat{\boldsymbol{i}}}$$

$\vec{B}$ Due to a Current in a Solenoid

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Picture the Problem We can use $B_x = \frac{1}{2}\mu_0 nI\left(\frac{b}{\sqrt{b^2+R^2}}+\frac{a}{\sqrt{a^2+R^2}}\right)$ to find B at any point on the axis of the solenoid. Note that the number of turns per unit length for this solenoid is 300 turns/0.3 m = 1000 turns/m.

Express the magnetic field at any point on the axis of the solenoid:

$$B_x = \frac{1}{2}\mu_0 nI\left(\frac{b}{\sqrt{b^2+R^2}}+\frac{a}{\sqrt{a^2+R^2}}\right)$$

Substitute numerical values to obtain:

$$B_x = \tfrac{1}{2}\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(1000)(2.6\ \mathrm{A})\left(\frac{b}{\sqrt{b^2+(0.012\ \mathrm{m})^2}}+\frac{a}{\sqrt{a^2+(0.012\ \mathrm{m})^2}}\right)$$
$$= (1.63\ \mathrm{mT})\left(\frac{b}{\sqrt{b^2+(0.012\ \mathrm{m})^2}}+\frac{a}{\sqrt{a^2+(0.012\ \mathrm{m})^2}}\right)$$

(a) Evaluate B_x for $a = b = 0.15$ m:

$$B_x = (1.63\ \mathrm{mT})\left(\frac{0.15\ \mathrm{m}}{\sqrt{(0.15\ \mathrm{m})^2+(0.012\ \mathrm{m})^2}}+\frac{0.15\ \mathrm{m}}{\sqrt{(0.15\ \mathrm{m})^2+(0.012\ \mathrm{m})^2}}\right) = \boxed{3.25\ \mathrm{mT}}$$

(b) Evaluate B_x for $a = 0.1$ m and $b = 0.2$ m:

$$B_x(0.2\ \mathrm{m}) = (1.63\ \mathrm{mT})\left(\frac{0.2\ \mathrm{m}}{\sqrt{(0.2\ \mathrm{m})^2+(0.012\ \mathrm{m})^2}}+\frac{0.1\ \mathrm{m}}{\sqrt{(0.1\ \mathrm{m})^2+(0.012\ \mathrm{m})^2}}\right)$$
$$= \boxed{3.25\ \mathrm{mT}}$$

(c) Evaluate B_x (= B_{end}) for $a = 0$ and $b = 0.3$ m:

$$B_x = (1.63\,\text{mT})\left(\frac{0.3\,\text{m}}{\sqrt{(0.3\,\text{m})^2 + (0.012\,\text{m})^2}}\right) = \boxed{1.63\,\text{mT}}$$

Note that $B_{\text{end}} = \frac{1}{2} B_{\text{center}}$.

***60 •**

Picture the Problem We can use $B_x = \mu_0 nI$ to find the approximate magnetic field on the axis and inside the solenoid.

Express B_x as a function of n and I:

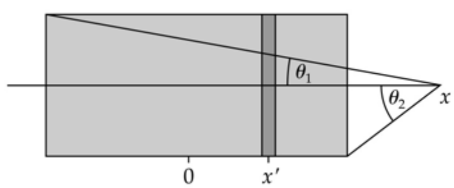
$$B_x = \mu_0 nI$$

Substitute numerical values and evaluate B_x:

$$B_x = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{600}{2.7\,\text{m}}\right)(2.5\,\text{A})$$
$$= \boxed{0.698\,\text{mT}}$$

61 •••

Picture the Problem The solenoid, extending from $x = -\ell/2$ to $x = \ell/2$, with the origin at its center, is shown in the diagram. To find the field at the point whose coordinate is x outside the solenoid we can determine the field at x due to an infinitesimal segment of the solenoid of width dx' at x', and then integrate from $x = -\ell/2$ to $x = \ell/2$. The segment may be considered as a coil ndx' carrying a current I.



Express the field dB at the axial point whose coordinate is x:

$$dB_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left[(x - x')^2 + R^2\right]^{3/2}}dx'$$

Integrate dB_x from $x = -\ell/2$ to $x = \ell/2$ to obtain:

$$B_x = \frac{\mu_0 nIR^2}{2}\int_{-\ell/2}^{\ell/2}\frac{dx'}{\left[(x - x')^2 + R^2\right]^{3/2}} = \frac{\mu_0 nI}{2}\left(\frac{x + \ell/2}{\sqrt{(x + \ell/2)^2 + R^2}} - \frac{x - \ell/2}{\sqrt{(x - \ell/2)^2 + R^2}}\right)$$

Refer to the diagram to express $\cos\theta_1$ and $\cos\theta_2$:

$$\cos\theta_1 = \frac{x + \frac{1}{2}\ell}{\left[R^2 + \left(x + \frac{1}{2}\ell\right)^2\right]^{1/2}}$$

and

$$\cos\theta_2 = \frac{x - \frac{1}{2}\ell}{\left[R^2 + \left(x - \frac{1}{2}\ell\right)^2\right]^{1/2}}$$

Substitute to obtain:

$$B = \boxed{\tfrac{1}{2}\mu_0 nI\left(\cos\theta_1 - \cos\theta_2\right)}$$

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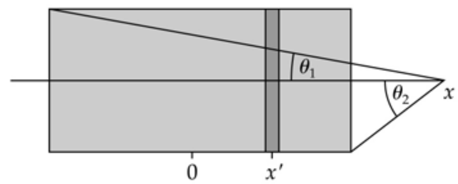
Picture the Problem We can use Equation 27-35, together with the small angle approximation for the cosine and tangent functions, to show that θ_1 and θ_2 are as given and that B is given by Equation 27-37.

(*a*) The angles θ_1 and θ_2 are shown in the diagram. Note that $\tan\theta_1 = R/(x + \ell/2)$ and $\tan\theta_2 = R/(x - \ell/2)$.



Apply the small angle approximation $\tan\theta \approx \theta$ to obtain:

$$\theta_1 \approx \boxed{\frac{R}{x + \frac{1}{2}\ell}}$$

and

$$\theta_2 \approx \boxed{\frac{R}{x - \frac{1}{2}\ell}}$$

(*b*) Express the magnetic field outside the solenoid:

$$B = \tfrac{1}{2}\mu_0 nI\left(\cos\theta_1 - \cos\theta_2\right)$$

Apply the small angle approximation for the cosine function to obtain:

$$\cos\theta_1 = 1 - \tfrac{1}{2}\left(\frac{R}{x + \frac{1}{2}\ell}\right)^2$$

and

$$\cos\theta_2 = 1 - \tfrac{1}{2}\left(\frac{R}{x - \frac{1}{2}\ell}\right)^2$$

Substitute and simplify to obtain:

$$B = \tfrac{1}{2}\mu_0 nI\left[1 - \tfrac{1}{2}\left(\frac{R}{x + \frac{1}{2}\ell}\right)^2 - 1 + \tfrac{1}{2}\left(\frac{R}{x - \frac{1}{2}\ell}\right)^2\right] = \tfrac{1}{4}\mu_0 nIR^2\left[\frac{1}{\left(x - \frac{1}{2}\ell\right)^2} - \frac{1}{\left(x + \frac{1}{2}\ell\right)^2}\right]$$

Let $r_1 = x - \frac{1}{2}\ell$ be the distance to the near end of the solenoid, $r_1 = x + \frac{1}{2}\ell$ the distance to the far

$$B = \boxed{\frac{\mu_0}{4\pi}\left(\frac{q_m}{r_1^2} - \frac{q_m}{r_2^2}\right)}$$

end, and $q_m = nI\pi R^2 = \mu/\ell$, where $\mu = nI\pi R^2$ is the magnetic moment of the solenoid to obtain:

Ampère's Law

***63** •

Picture the Problem We can apply Ampère's law to a circle centered on the axis of the cylinder and evaluate this expression for $r < R$ and $r > R$ to find B inside and outside the cylinder.

Apply Ampère's law to a circle centered on the axis of the cylinder:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$

Note that, by symmetry, the field is the same everywhere on this circle.

Evaluate this expression for $r < R$:

$$\oint_C \vec{B}_{\text{inside}} \cdot d\vec{\ell} = \mu_0(0) = 0$$

Solve for B_{inside} to obtain:

$$B_{\text{inside}} = \boxed{0}$$

Evaluate this expression for $r > R$:

$$\oint_C \vec{B}_{\text{outside}} \cdot d\vec{\ell} = B(2\pi R) = \mu_0 I$$

Solve for B_{outside} to obtain:

$$B_{\text{outside}} = \boxed{\frac{\mu_0 I}{2\pi R}}$$

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Picture the Problem We can use Ampère's law, $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$, to find the line integral $\oint_C \vec{B} \cdot d\vec{\ell}$ for each of the three paths.

(*a*) Evaluate $\oint_C \vec{B} \cdot d\vec{\ell}$ for C_1:

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = \boxed{\mu_0(8\,\text{A})}$$

Evaluate $\oint_C \vec{B} \cdot d\vec{\ell}$ for C_2:

$$\oint_{C_2} \vec{B} \cdot d\vec{\ell} = \mu_0(8\,\text{A} - 8\,\text{A}) = \boxed{0}$$

Evaluate $\oint_C \vec{B} \cdot d\vec{\ell}$ for C_3:

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = \boxed{-\mu_0(8\,\text{A})}$$ because the field is opposite the direction of integration.

(*b*) None of the paths can be used to find *B* at a general point because there the current configuration does not have cylindrical symmetry.

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Picture the Problem Let the current in the wire and outer shell be *I*. We can apply Ampère's law to a circle, concentric with the inner wire, of radius *r* to find *B* at points between the wire and the shell far from the ends ($r < R$), and outside the cable ($r > R$).

(*a*) Apply Ampère's law for $r < R$:

$$\oint_C \vec{B}_{r<R} \cdot d\vec{\ell} = B_{r<R}(2\pi r) = \mu_0 I$$

Solve for $B_{r<R}$ to obtain:

$$B_{r<R} = \boxed{\frac{\mu_0 I}{2\pi r}}$$

(*b*) Apply Ampère's law for $r > R$:

$$\oint_C \vec{B}_{r>R} \cdot d\vec{\ell} = \mu_0(0)$$

Solve for $B_{r>R}$ to obtain:

$$B_{r>R} = \boxed{0}$$

66 ••

Picture the Problem. Let the radius of the wire be *a*. We can apply Ampère's law to a circle, concentric with the center of the wire, of radius *r* to find *B* at various distances from the center of the wire.

Express Ampère's law:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$

Using the fact that the current is uniformly distributed over the cross-sectional area of the wire, relate the current enclosed by a circle of radius *r* to the total current *I* carried by the wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi a^2}$$

or

$$I_C = I\frac{r^2}{a^2}$$

Substitute and evaluate the integral to obtain:

$$B_r(2\pi r) = \frac{\mu_0 r^2}{a^2} I$$

Solve for $B_{r<a}$:

$$B_{r<a} = \frac{\mu_0 r}{2\pi a^2} I \qquad (1)$$

For $r \geq a$:

$$\oint_C \vec{B}_{r\geq a} \cdot d\vec{\ell} = B_{r\geq a}(2\pi r) = \mu_0 I$$

Solve for $B_{r\geq a}$:

$$B_{r\geq a} = \frac{\mu_0 I}{2\pi r} \qquad (2)$$

(a) Use equation (1) to evaluate B(0.1 cm):

$$B(0.1\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.001\,\text{m})}{2\pi(0.005\,\text{m})^2}(100\,\text{A}) = \boxed{8.00\times10^{-4}\ \text{T}}$$

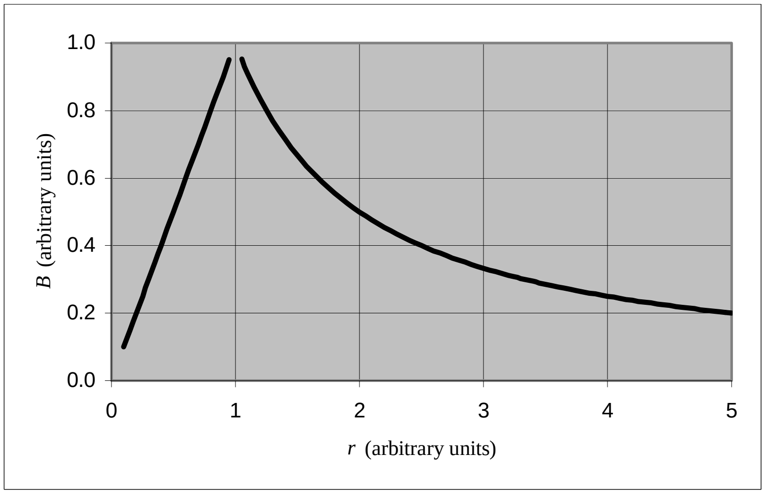
(b) Use either equation to evaluate B at the surface of the wire:

$$B(0.005\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.005\,\text{m})}{2\pi(0.005\,\text{m})^2}(100\,\text{A}) = \boxed{4.00\times10^{-3}\ \text{T}}$$

(c) Use equation (2) to evaluate B(0.7 cm):

$$B(0.007\,\text{m}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(100\,\text{A})}{2\pi(0.007\,\text{m})} = \boxed{2.86\times10^{-3}\ \text{T}}$$

(d) A graph of B as a function of r follows:



***67 ••**

Determine the Concept The contour integral consists of four portions, two horizontal portions for which $\oint_C \vec{B}\cdot d\vec{\ell} = 0$, and two vertical portions. The portion within the magnetic field gives a nonvanishing contribution, whereas the portion outside the field gives no contribution to the contour integral. Hence, the contour integral has a finite value. However, it encloses no current; thus, it appears that Ampère’s law is violated. What this demonstrates is that there must be a fringing field so that the contour integral does vanish.

68 ••

Picture the Problem Let r_1 = 1 mm, r_2 = 2 mm, and r_3 = 3 mm and apply Ampère's law in each of the three regions to obtain expressions for B in each part of the coaxial cable and outside the coaxial cable.

Apply Ampère's law to a circular path of radius $r < r_1$ to obtain:

$$B_{r<r_1}(2\pi r) = \mu_0 I_C$$

Because the current is uniformly distributed over the cross section of the inner wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi r_1^2} \Rightarrow I_C = \frac{r^2}{r_1^2} I$$

Substitute for I_C to obtain:

$$B_{r<r_1}(2\pi r) = \mu_0 \frac{r^2}{r_1^2} I$$

Solve for $B_{r<r_1}$:

$$B_{r<r_1} = \frac{2\mu_0 I}{4\pi} \frac{r}{r_1^2} \qquad (1)$$

Apply Ampère's law to a circular path of radius $r_1 < r < r_2$ to obtain:

$$B_{r_1<r<r_2}(2\pi r) = \mu_0 I$$

Solve for $B_{r_1<r<r_2}$:

$$B_{r_1<r<r_2} = \frac{2\mu_0 I}{4\pi} \frac{1}{r} \qquad (2)$$

Apply Ampère's law to a circular path of radius $r_2 < r < r_3$ to obtain:

$$B_{r_2<r<r_3}(2\pi r) = \mu_0 I_C = \mu_0 (I - I')$$

where I' is the current in the outer conductor at a distance less than r from the center of the inner conductor.

Because the current is uniformly distributed over the cross section of the outer conductor:

$$\frac{I'}{\pi r^2 - \pi r_2^2} = \frac{I}{\pi r_3^2 - \pi r_2^2}$$

Solve for I':

$$I' = \frac{r^2 - r_2^2}{r_3^2 - r_2^2} I$$

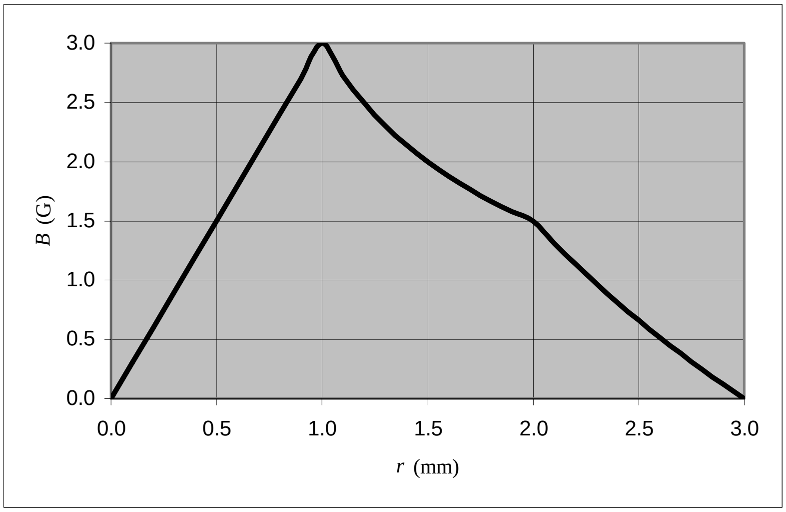
Substitute for I' to obtain:

$$B_{r_2<r<r_3}(2\pi r) = \mu_0 \left(I - \frac{r^2 - r_2^2}{r_3^2 - r_2^2} I \right)$$

Solve for $B_{r_1<r<r_2}$:

$$B_{r_2<r<r_3} = \frac{2\mu_0 I}{4\pi} \left(1 - \frac{r^2 - r_2^2}{r_3^2 - r_2^2} \right) \qquad (3)$$

A spreadsheet program was used to plot the following graph of equations (1), (2), and (3).



Apply Ampère's law to a circular path of radius $r > r_3$ to obtain:

$$B_{r>r_3}(2\pi r) = \mu_0 I_C = \mu_0(I - I) = 0$$

and $B_{r>r_3} = \boxed{0}$

69 ••

Picture the Problem We can use Ampère's law to calculate B because of the high degree of symmetry. The current through C depends on whether r is less than or the inner radius a, greater than the inner radius a but less than the outer radius b, or greater than the outer radius b.

(*a*) Apply Ampère's law to a circular path of radius $r < a$ to obtain:

$$\oint_C \vec{B}_{r<a} \cdot d\vec{\ell} = \mu_0 I_C = \mu_0(0) = 0$$

and

$$B_{r<a} = \boxed{0}$$

(*b*) Use the uniformity of the current over the cross-section of the conductor to express the current I' enclosed by a circular path whose radius satisfies the condition $a < r < b$:

$$\frac{I'}{\pi(r^2 - a^2)} = \frac{I}{\pi(b^2 - a^2)}$$

Solve for $I_C = I'$:

$$I_C = I' = I\frac{r^2 - a^2}{b^2 - a^2}$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{B}_{a<r<b} \cdot d\vec{\ell} = B_{a<r<b}(2\pi r) = \mu_0 I' = \mu_0 I \frac{r^2 - a^2}{b^2 - a^2}$$

Solve for $B_{a<r<b}$:

$$B_{a<r<b} = \boxed{\frac{\mu_0 I}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2}}$$

(*c*) Express I_C for $r > b$:

$$I_C = I$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{B}_{r>b} \cdot d\vec{\ell} = B_{r>b}(2\pi r) = \mu_0 I$$

Solve for $B_{r>b}$:

$$B_{r>b} = \boxed{\frac{\mu_0 I}{2\pi r}}$$

70 ••

Picture the Problem The number of turns enclosed within the rectangular area is *na*. Denote the corners of the rectangle, starting in the lower left-hand corner and proceeding counterclockwise, as 1, 2, 3, and 4. We can apply Ampère's law to each side of this rectangle in order to evaluate $\oint_C \vec{B} \cdot d\vec{\ell}$.

Express the integral around the closed path C as the sum of the integrals along the sides of the rectangle:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \int_{1\to2} \vec{B} \cdot d\vec{\ell} + \int_{2\to3} \vec{B} \cdot d\vec{\ell} + \int_{3\to4} \vec{B} \cdot d\vec{\ell} + \int_{4\to1} \vec{B} \cdot d\vec{\ell}$$

Evaluate $\int_{1\to2} \vec{B} \cdot d\vec{\ell}$:

$$\int_{1\to2} \vec{B} \cdot d\vec{\ell} = aB$$

For the paths $2 \to 3$ and $4 \to 1$, $\vec{B}$ is either zero (outside the solenoid) or is perpendicular to $d\vec{\ell}$ and so:

$$\int_{2\to3} \vec{B} \cdot d\vec{\ell} = \int_{4\to1} \vec{B} \cdot d\vec{\ell} = 0$$

For the path $3 \to 4$, $\vec{B}$ =0 and:

$$\int_{3\to4} \vec{B} \cdot d\vec{\ell} = 0$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = aB + 0 + 0 + 0 = aB = \mu_0 I_C = \mu_0 naI$$

Solve for B to obtain: $B = \boxed{\mu_0 nI}$

71 ••

Picture the Problem The magnetic field inside a tightly wound toroid is given by $B = \mu_0 NI/(2\pi r)$, where $a < r < b$ and a and b are the inner and outer radii of the toroid.

Express the magnetic field of a toroid:

$$B = \frac{\mu_0 NI}{2\pi r}$$

(a) Substitute numerical values and evaluate B(1.1 cm):

$$B(1.1\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(1000)(1.5\,\text{A})}{2\pi(1.1\,\text{cm})} = \boxed{27.3\,\text{mT}}$$

(b) Substitute numerical values and evaluate B(1.5 cm):

$$B(1.5\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(1000)(1.5\,\text{A})}{2\pi(1.5\,\text{cm})} = \boxed{20.0\,\text{mT}}$$

***72** ••

Picture the Problem In parts (a), (b), and (c) we can use a right-hand rule to determine the direction of the magnetic field at points above and below the infinite sheet of current. In part (d) we can evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$ around the specified path and equate it to $\mu_0 I_C$ and solve for B.

(a) $\boxed{\text{At } P \text{ the magnetic field points to the right (i.e., in the } -\hat{i} \text{ direction) since its vertical components cancel.}}$

(b) $\boxed{\text{Because the sheet is infinite, the same argument used in (a) applies; B is in the } -\hat{i} \text{ direction.}}$

(c) $\boxed{\text{Below the sheet the magnetic field points to the left, i.e., in the } \hat{i} \text{ direction. The vertical components cancel.}}$

(d) Express $\oint_C \vec{B}\cdot d\vec{\ell}$, in the counterclockwise direction, for the given path:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} + 2\int_{\perp} \vec{B}\cdot d\vec{\ell}$$

For the paths perpendicular to the sheet, $\vec{B}$ and $d\vec{\ell}$ are perpendicular to each other and:

$$\int_{\perp} \vec{B}\cdot d\vec{\ell} = 0$$

For the paths parallel to the sheet, $\vec{B}$ and $d\vec{\ell}$ are in the same direction and:

$$\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} = Bw$$

Substitute to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} = 2Bw$$
$$= \mu_0 I_C = \mu_0(\lambda w)$$

Solve for B:

$$B = \tfrac{1}{2}\mu_0\lambda \text{ and } \vec{B}_{\text{above}} = \boxed{-\tfrac{1}{2}\mu_0\lambda\hat{\boldsymbol{i}}}$$

Magnetization and Magnetic Susceptibility

73 •

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}} + \mu_0 M$ when there is an iron core with a magnetization $M = 1.2\times10^6$ A/m.

(*a*) Express the magnetic field, in the absence of a core, in the solenoid :

$$B = B_{\text{app}} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A})$$
$$= \boxed{10.1\,\text{mT}}$$

(*b*) With an iron core with a magnetization $M = 1.2\times10^6$ A/m present:

$$B_{\text{app}} = \boxed{10.1\,\text{mT}}$$

and

$$B = B_{\text{app}} + \mu_0 M = 10.1\,\text{mT} + \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(1.2\times10^6\ \text{A/m}\right) = \boxed{1.52\,\text{T}}$$

74 •

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}} + \mu_0 M$ when there is an aluminum core. We can use $M = \chi_m \frac{B_{app}}{\mu_0}$ to find the magnetization of the core with the aluminum present.

Express the magnetic field, in the absence of a core, in the solenoid :

$$B = B_{app} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A})$$

$$= \boxed{10.1\,\text{mT}}$$

Express the magnetization in the core with the aluminum present:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Use Table 27-1 to find the value of χ_m for aluminum:

$$\chi_{m,\,Al} = 2.3\times 10^{-5}$$

Substitute numerical values and evaluate M:

$$M = 2.3\times 10^{-5}\,\frac{10.1\,\text{mT}}{4\pi \times 10^{-7}\ \text{N/A}^2}$$

$$= \boxed{0.185\,\text{A/m}}$$

75 •

Picture the Problem We can use $B_{app} = \mu_0 nI$ to find B_{app} at the center of the tungsten core in the solenoid. The magnetization is related to B_{app} and χ_m according to $M = \chi_m B_{app}/\mu_0 = \chi_m nI$ and we can use $B = B_{app}(1+\chi_m)$ to find B.

Express the magnetic field, for a tungsten core, in the solenoid :

$$B_{app} = \mu_0 nI$$

Substitute numerical values and evaluate B_{app}:

$$B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A})$$

$$= \boxed{10.053\,\text{mT}}$$

Express the magnetization in the core with the aluminum present:

$$M = \chi_m \frac{B_{app}}{\mu_0} = \chi_m nI$$

Use Table 27-1 to find the value of χ_m for tungsten:

$$\chi_{m,\,tungstenl} = 6.8\times 10^{-5}$$

Substitute numerical values and evaluate M:

$$M = \left(6.8\times10^{-5}\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A}) = \boxed{0.544\,\text{A/m}}$$

Express B in terms of B_{app} and χ_m:

$$B = B_{app}\left(1+\chi_m\right)$$

Substitute numerical values and evaluate B:

$$B = (10.053\,\text{mT})\left(1+6.8\times10^{-5}\right) = \boxed{10.054\,\text{mT}}$$

76 •

Picture the Problem We can use $B = B_{app}\left(1+\chi_m\right)$ to relate B and B_{app} to the magnetic susceptibility of tungsten. Dividing both sides of this equation by B_{app} and examining the value of $\chi_{m,\,tungsten}$ will allow us to decide whether the field inside the solenoid decreases or increases when the core is removed.

Express the magnetic field inside the solenoid with the tungsten core present B in terms of B_{app} and χ_m:

$$B = B_{app}\left(1+\chi_m\right)$$

where B_{app} is the magnetic field in the absence of the tungsten core.

Express the ratio of B to B_{app}:

$$\frac{B}{B_{app}} = 1+\chi_m \qquad (1)$$

(*a*) Because $\chi_{m,\,tungsten} > 0$:

$$B > B_{app}$$

and

$\boxed{B\text{ will decrease when the tungsten core is removed.}}$

(*b*) From equation (1) the fractional change is:

$$\chi_m = 6.8\times10^{-5} = \boxed{6.8\times10^{-3}\%}$$

77 •

Picture the Problem We can use $B = B_{app}\left(1+\chi_m\right)$ to relate B and B_{app} to the magnetic susceptibility of liquid sample.

Express the magnetic field inside the solenoid with the liquid sample present B in terms of B_{app} and $\chi_{m,}$

$$B = B_{app}\left(1+\chi_{m,\,sample}\right)$$

where B_{app} is the magnetic field in the absence of the liquid sample.

sample:

The fractional change in the magnetic field in the core is:

$$\frac{\Delta B}{B_{\text{app}}} = \chi_{\text{m, sample}}$$

Substitute numerical values and evaluate $\chi_{\text{m, sample}}$:

$$\chi_{\text{m, sample}} = \frac{\Delta B}{B_{\text{app}}} = -0.004\% = \boxed{-4.00\times10^{-5}}$$

78 •

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}}(1+\chi_{\text{m}})$ when there is an aluminum or silver core.

(*a*) Express the magnetic field, in the absence of a core, in the solenoid:

$$B = B_{\text{app}} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{0.01\,\text{m}}\right)(10\,\text{A}) = \boxed{62.8\,\text{mT}}$$

(*b*) With an aluminum core:

$$B = B_{\text{app}}(1+\chi_{\text{m}})$$

Use Table 27-1 to find the value of χ_{m} for aluminum:

$$\chi_{\text{m, Al}} = 2.3\times10^{-5}$$

and

$$1+\chi_{\text{m, Al}} = 1+2.3\times10^{-5} \approx 1$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{0.01\,\text{m}}\right)(10\,\text{A}) = \boxed{62.8\,\text{mT}}$$

(*c*) With a silver core:

$$B = B_{\text{app}}(1+\chi_{\text{m}})$$

Use Table 27-1 to find the value of χ_{m} for silver:

$$\chi_{\text{m, Ag}} = -2.6\times10^{-5}$$

and

$$1+\chi_{\text{m, Ag}} = 1-2.6\times10^{-5} \approx 1$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{0.01\,\text{m}}\right)(10\ \text{A}) = \boxed{62.8\,\text{mT}}$$

***79** ••

Picture the Problem We can use the data in the table and $B_{app} = \mu_0 nI$ to plot B versus B_{app}. We can find K_m using $B = K_m B_{app}$.

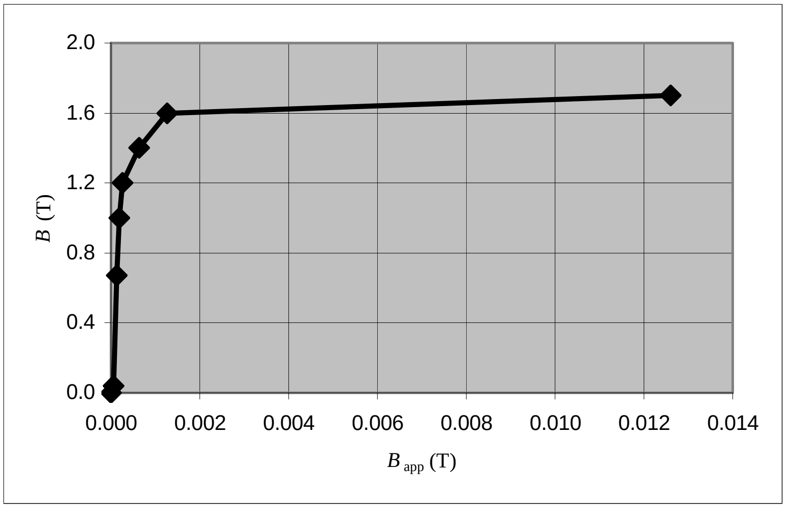
We can find the applied field B_{app} for a long solenoid using:

$$B_{app} = \mu_0 nI$$

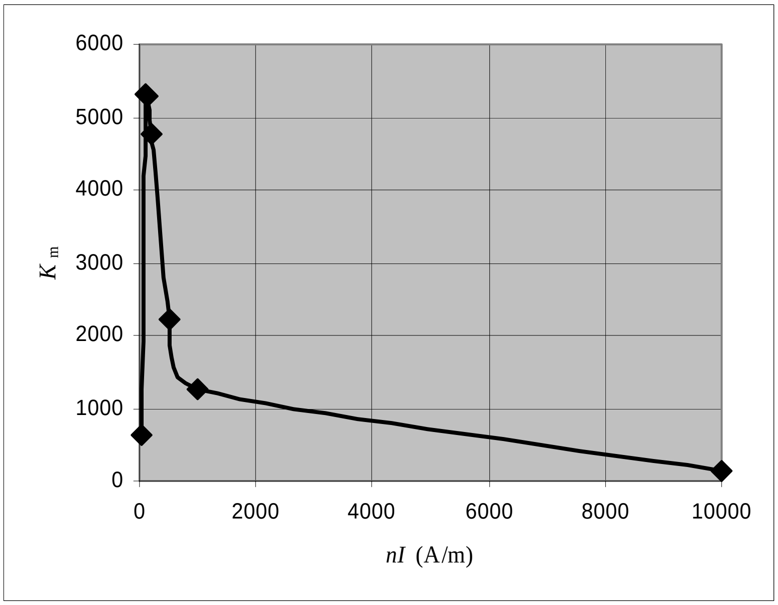
K_m can be found from B_{app} and B using:

$$K_m = \frac{B}{B_{app}}$$

The following graph was plotted using a spreadsheet program. The abscissa values for the graph were obtained by multiplying nI by μ_0. B initially rises rapidly, and then becomes nearly flat. This is characteristic of a ferromagnetic material.



The graph of K_m versus nI shown below was also plotted using a spreadsheet program. Note that K_m becomes quite large for small values of nI but then diminishes. A more revealing graph would be to plot $B/(nI)$, which would be quite large for small values of nI and then drop to nearly zero at nI = 10,000 A/m, corresponding to saturation of the magnetization.



80 ••

Picture the Problem We can use the definition of the magnetization of a sample to find M and the relationship between the Bohr magneton and the magnetic moment of the sample to find the number of electrons aligned in the sample. In part (*c*) we can express the magnetic moment of the disk in terms of the amperian surface current and solve for the latter.

(*a*) Express the magnetization of the sample in terms of its magnetic moment and volume:

$$M = \frac{\mu}{V} = \frac{\mu}{\pi r^2 d}$$

Substitute numerical values and evaluate M:

$$M = \frac{1.5\times10^{-2}\ \text{A}\cdot\text{m}^2}{\pi(1.4\ \text{cm})^2(0.3\ \text{cm})} = \boxed{8.12\times10^{3}\ \text{A/m}}$$

(*b*) Relate the magnetic moment of the sample to the Bohr magneton:

$$\mu = N\mu_B$$

Solve for and evaluate N:

$$N = \frac{\mu}{\mu_B} = \frac{1.5\times10^{-2}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{1.62\times10^{21}}$$

(*c*) Express the magnetic moment of the disk in terms of the amperian

$$\mu = AI$$

surface current:

Solve for I and substitute for μ to obtain:

$$I = \frac{\mu}{A} = \frac{MV}{A} = \frac{MAt}{A} = Mt$$

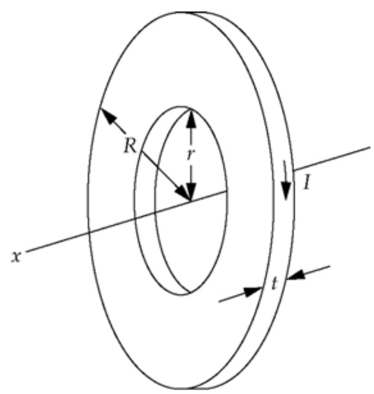
where t is the thickness of the disk.

Substitute numerical values and evaluate I:

$$I = \left(8.12\times10^{3}\ \text{A/m}\right)\left(0.3\,\text{cm}\right) = \boxed{24.4\,\text{A}}$$

81 ••

Picture the Problem We can imagine the cylinder with the hole cut out as the superposition of two uniform cylinders with radii r and R, respectively, and magnetization $-M$ and M, respectively. We can use the expression for B on the axis of a current loop to express the difference of the fields due to the two cylinders at a distance x from their common center. We'll denote each field by the subscript identifying the radius of the current loop.



From Problem 39 we have:

$$B_r = \frac{\mu_0}{4\pi}\frac{2\pi r^2 I}{\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0 r^2 I}{2\left(x^2+r^2\right)^{3/2}}$$

and

$$B_R = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 R^2 I}{2\left(x^2+R^2\right)^{3/2}}$$

The resultant field at x is the difference between B_R and B_r:

$$B_x(x) = B_R(x) - B_r(x) = \frac{\mu_0 R^2 I}{2\left(x^2+R^2\right)^{3/2}} - \frac{\mu_0 r^2 I}{2\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0 I}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}} - \frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]$$

The resultant magnetization of the disks is $M = B/\mu_0$:

$$M(x) = \frac{I}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}} - \frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]$$

The magnetization current is the product of M and the thickness of the disks:

The magnetization is related to the amperian current:

$$M = \frac{dI_{\text{amperian}}}{d\ell} \Rightarrow I_{\text{amperian}} = \int_0^t M d\ell$$

Substitute for M to obtain:

$$I_{\text{amperian}} = \int_0^t \frac{I}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}} - \frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right] d\ell = \boxed{\frac{It}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}} - \frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]}$$

Atomic Magnetic Moments

***82 ••**

Picture the Problem We can find the magnetic moment of a nickel atom μ from its relationship the saturation magnetization M_S using $M_S = n\mu$ where n is the number of molecules. n, in turn, can be found from Avogadro's number, the density of nickel, and its molar mass using $n = \frac{N_A \rho}{\text{M}}$.

Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S = n\mu$$

or

$$\mu = \frac{M_S}{n}$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n = \frac{N_A \rho}{\text{M}}$$

Substitute and simplify to obtain:

$$\mu = \frac{M_S}{\frac{N_A \rho}{\text{M}}} = \frac{\mu_0 M_S}{\frac{\mu_0 N_A \rho}{\text{M}}} = \frac{\mu_0 M_S M}{\mu_0 N_A \rho}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{(0.61\,\text{T})\left(58.7\times10^{-3}\text{ kg/mol}\right)}{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(6.02\times10^{23}\text{ atoms/mol}\right)\left(8.7\text{ g/cm}^3\right)} = 5.44\times10^{-24}\text{ A}\cdot\text{m}^2$$

Express the value of 1 Bohr magneton:

$$\mu_B = 9.27\times10^{-24}\text{ A}\cdot\text{m}^2$$

Divide μ by μ_B to obtain:

$$\frac{\mu}{\mu_B} = \frac{5.44\times10^{-24}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = 0.587$$

or

$$\mu = \boxed{0.587\mu_B}$$

83 ••

Picture the Problem We can find the magnetic moment of a cobalt atom μ from its relationship to the saturation magnetization M_S using $M_S = n\mu$, where n is the number of molecules. n, in turn, can be found from Avogadro's number, the density of cobalt, and its molar mass using $n = \frac{N_A\rho}{\text{M}}$.

Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S = n\mu$$

or

$$\mu = \frac{M_S}{n}$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n = \frac{N_A\rho}{\text{M}}$$

Substitute and simplify to obtain:

$$\mu = \frac{M_S}{\frac{N_A\rho}{\text{M}}} = \frac{\mu_0 M_S}{\frac{\mu_0 N_A\rho}{\text{M}}} = \frac{\mu_0 M_S M}{\mu_0 N_A\rho}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{(1.79\ \text{T})(58.9\times10^{-3}\ \text{kg/mol})}{(4\pi\times10^{-7}\ \text{N/A}^2)(6.02\times10^{23}\ \text{atoms/mol})(8.9\ \text{g/cm}^3)} = 1.57\times10^{-23}\ \text{A}\cdot\text{m}^2$$

Express the value of 1 Bohr magneton:

$$\mu_B = 9.27\times10^{-24}\ \text{A}\cdot\text{m}^2$$

Divide μ by μ_B to obtain:

$$\frac{\mu}{\mu_B} = \frac{1.57\times10^{-23}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = 1.69$$

or

$$\mu = \boxed{1.69\mu_B}$$

Paramagnetism

84 •

Picture the Problem We can show that $\chi_m = \mu\mu_0 M_s/3kT$ by equating Curie's law and the equation that defines χ_m ($M = \chi_m \frac{B_{app}}{\mu_0}$) and solving for χ_m.

Express Curie's law:

$$M = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

where M_S is the saturation value.

Express the magnetization of the substance in terms of its magnetic susceptibility χ_m:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Equate these expressions to obtain:

$$\chi_m \frac{B_{app}}{\mu_0} = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

or

$$\frac{\chi_m}{\mu_0} = \frac{1}{3}\frac{\mu}{kT}M_S$$

Solve for χ_m to obtain:

$$\chi_m = \boxed{\frac{\mu_0 \mu M_S}{3kT}}$$

85 ••

Picture the Problem We can use the assumption that $M = fM_S$ and Curie's law to solve these equations simultaneously for the fraction *f* of the molecules have their magnetic moments aligned with the external magnetic field.

(*a*) Assume that some fraction *f* of the molecules have their magnetic moments aligned with the external magnetic field and that the rest of the molecules are randomly oriented and so do not contribute to the magnetic field:

$$M = fM_S$$

From Curie's law we have:

$$M = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

Equate these expressions and solve for f to obtain:

$$fM_S = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

and

$$f = \boxed{\frac{\mu B}{3kT}}$$

because B given in the problem statement is the external magnetic field B_{app}.

(b) Substitute numerical values and evaluate f:

$$f = \frac{\left(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2\right)(1\,\text{T})}{3\left(1.381\times10^{-23}\ \text{J/K}\right)(300\,\text{K})} = \boxed{7.46\times10^{-4}}$$

***86** ••

Picture the Problem In (a) we can express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule and use $n = N_A\rho/\text{M}$ to express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ. We can use $\chi_m = \mu_0\mu M_S/3kT$ from Problem 84 to calculate χ_m.

(a) Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S = n\mu_B$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n = \frac{N_A\rho}{\text{M}}$$

Substitute to obtain:

$$M_S = \frac{N_A\rho}{\text{M}}\mu_B$$

Substitute numerical values and evaluate M_S:

$$M_S = \frac{\left(6.02\times10^{23}\ \text{atoms/mol}\right)\left(2.7\times10^{3}\ \text{kg/m}^3\right)\left(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2\right)}{27\,\text{g/mol}} = \boxed{5.58\times10^{5}\ \text{A/m}}$$

and

$$B_S = \mu_0 M_S = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(5.58\times10^{5}\ \text{A/m}\right) = \boxed{0.701\,\text{T}}$$

(*b*) From Problem 84 we have:

$$\chi_m = \frac{\mu_0 \mu M_S}{3kT}$$

Substitute numerical values and evaluate χ_m:

$$\chi_m = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2)(5.58\times10^5\ \text{A/m})}{3(1.381\times10^{-23}\ \text{J/K})(300\,\text{K})} = \boxed{5.23\times10^{-4}}$$

(*c*) $\boxed{\text{In calculating } \chi_m \text{ in } (b) \text{ we neglected any diamagnetic effects.}}$

87 ••

Picture the Problem We can use Equation 27-17 to express B_{app} and Equation 27-21 to express B in terms of B_{app} and M.

Express B_{app} inside a tightly wound toroid:

$$B_{app} = \boxed{\frac{\mu_0 NI}{2\pi a}} \text{ for } R - r < a < R + r$$

The resultant field B in the ring is the sum of B_{app} and $\mu_0 M$:

$$B = B_{app} + \mu_0 M = \boxed{\frac{\mu_0 NI}{2\pi a} + \mu_0 M}$$

88 ••

Picture the Problem We can find the magnetization using $M = \chi_m B_{app}/\mu_0$ and the magnetic field using $B = B_{app}(1+\chi_m)$.

(*a*) Using Equation 27-22, express the magnetization M in terms of χ_m and B_{app}:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Express B_{app} inside a tightly wound toroid:

$$B_{app} = \frac{\mu_0 NI}{2\pi r_{mean}}$$

Substitute to obtain:

$$M = \chi_m \frac{\frac{\mu_0 NI}{2\pi r_{mean}}}{\mu_0} = \chi_m \frac{NI}{2\pi r_{mean}}$$

Substitute numerical values and evaluate M:

$$M = \frac{(4\times10^{-3})(2000)(15\,\text{A})}{2\pi(0.2\,\text{m})} = \boxed{95.5\,\text{A/m}}$$

(*b*) Express B in terms of B_{app} and χ_{m}:

$$B = B_{\text{app}}(1+\chi_{\text{m}})$$

Substitute for B_{app} to obtain:

$$B = \frac{\mu_0 NI}{2\pi r_{\text{mean}}}(1+\chi_{\text{m}})$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(2000)(15\,\text{A})}{2\pi(0.2\,\text{m})}(1+4\times10^{-3}) = \boxed{30.1\,\text{mT}}$$

(*c*) Express the fractional increase in B produced by the liquid oxygen:

$$\frac{\Delta B}{B} = \frac{B-B_{\text{app}}}{B} = \frac{B_{\text{app}}(1+\chi_{\text{m}})-B_{\text{app}}}{B} = \frac{\chi_{\text{m}}B_{\text{app}}}{B} = \frac{\chi_{\text{m}}}{1+\chi_{\text{m}}} = \frac{1}{\frac{1}{\chi_{\text{m}}}+1}$$

Substitute numerical values and evaluate $\Delta B/B$:

$$\frac{\Delta B}{B} = \frac{1}{\frac{1}{4\times10^{-3}}+1} = 3.98\times10^{-3} = \boxed{0.398\%}$$

89 ••

Picture the Problem We can use $B = B_{\text{app}}(1+\chi_{\text{m}})$ and $B_{\text{app}} = \frac{\mu_0 NI}{2\pi r_{\text{mean}}} = \mu_0 nI$ to find B within the substance and $M = \chi_{\text{m}}\frac{B_{\text{app}}}{\mu_0}$ to find the magnitude of the magnetization.

(*a*) Express the magnetic field B within the substance in terms of B_{app} and χ_{m}:

$$B = B_{\text{app}}(1+\chi_{\text{m}})$$

Express B_{app} inside the toroid:

$$B_{\text{app}} = \frac{\mu_0 NI}{2\pi r_{\text{mean}}} = \mu_0 nI$$

Substitute to obtain:

$$B = \mu_0 nI(1+\chi_{\text{m}})$$

Substitute numerical values and evaluate B:

$$B = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(60\times10^2\ \text{m}^{-1}\right)(4\ \text{A})\left(1+2.9\times10^{-4}\right) = \boxed{30.2\ \text{mT}}$$

(*b*) Express the magnetization *M* in terms of χ_m and B_app:

$$M = \chi_\text{m}\frac{B_\text{app}}{\mu_0}$$

Substitute for B_app to obtain:

$$M = \chi_\text{m}\frac{\mu_0 nI}{\mu_0} = \chi_\text{m} nI$$

Substitute numerical values and evaluate *M*:

$$M = \left(2.9\times10^{-4}\right)\left(6000\ \text{m}^{-1}\right)(4\ \text{A}) = \boxed{6.96\ \text{A/m}}$$

(*c*) If there were no paramagnetic core present:

$$B = B_\text{app} = \boxed{30.2\ \text{mT}}$$

Ferromagnetism

***90** •

Picture the Problem We can use $B = K_\text{m}B_\text{app}$ to find *B* and $M = (K_\text{m}-1)B_\text{app}/\mu_0$ to find *M*.

Express *B* in terms of *M* and K_m:

$$B = K_\text{m}B_\text{app}$$

Substitute numerical values and evaluate *B*:

$$B = (5500)\left(1.57\times10^{-4}\ \text{T}\right) = \boxed{0.864\ \text{T}}$$

Relate *M* to K_m and B_app:

$$M = (K_\text{m}-1)\frac{B_\text{app}}{\mu_0} \approx \frac{K_\text{m}B_\text{app}}{\mu_0}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{(5500)\left(1.57\times10^{-4}\ \text{T}\right)}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{6.87\times10^5\ \text{A/m}}$$

91 ••

Picture the Problem We can relate the permeability μ of annealed iron to χ_m using $\mu = (1+\chi_\text{m})\mu_0$, find χ_m using Equation 27-22 ($M = \chi_\text{m}\frac{B_\text{app}}{\mu_0}$), and use its definition ($K_\text{m} = 1+\chi_\text{m}$) to evaluate K_m.

Express the permeability μ of annealed iron in terms of its magnetic susceptibility χ_m:

$$\mu = (1 + \chi_m)\mu_0 \qquad (1)$$

Using Equation 27-22, express the magnetization M in terms of χ_m and B_{app}:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Solve for and evaluate χ_m (see Table 27-2 for the product of μ_0 and M):

$$\chi_m = \frac{\mu_0 M}{B_{app}} = \frac{2.16\,\text{T}}{0.201\,\text{T}} = 10.75$$

Use its definition to express and evaluate the relative permeability K_m:

$$K_m = 1 + \chi_m = 1 + 10.75 = \boxed{11.75}$$

Substitute numerical values in equation (1) and evaluate μ:

$$\mu = (1 + 10.75)(4\pi \times 10^{-7}\ \text{N/A}^2) = \boxed{1.48 \times 10^{-5}\,\text{N/A}^2}$$

92 ••

Picture the Problem We can use the relationship between the magnetic field on the axis of a solenoid and the current in the solenoid to find the minimum current is needed in the solenoid to demagnetize the magnet.

Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

$$B_x = \mu_0 n I$$

Solve for I to obtain:

$$I = \frac{B_x}{\mu_0 n}$$

Let $B_{app} = B_x$ to obtain:

$$I = \frac{B_{app}}{\mu_0 n}$$

Substitute numerical values and evaluate I:

$$I = \frac{5.53 \times 10^{-2}\ \text{T}}{(4\pi \times 10^{-7}\ \text{N/A}^2)\left(\dfrac{600}{0.15\,\text{m}}\right)} = \boxed{11.0\,\text{A}}$$

93 ••

Picture the Problem We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find B_{app}. We can then use $B = B_{app} + \mu_0 M$ to find M and $B = K_m B_{app}$ to evaluate K_m.

(*a*) Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

$$B_x = \mu_0 n I$$

Substitute numerical values to obtain:

$$B_{app} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(50\ \text{cm}^{-1}\right)\left(2\ \text{A}\right) = \boxed{12.6\ \text{mT}}$$

(*b*) Relate M to B and B_{app}:

$$B = B_{app} + \mu_0 M$$

Solve for and evaluate M:

$$M = \frac{B - B_{app}}{\mu_0} = \frac{1.72\ \text{T} - 12.6\ \text{mT}}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{1.36\times10^6\ \text{A/m}}$$

(*c*) Express B in terms of K_m and B_{app}:

$$B = K_m B_{app}$$

Solve for and evaluate K_m:

$$K_m = \frac{B}{B_{app}} = \frac{1.72\ \text{T}}{12.6\ \text{mT}} = \boxed{137}$$

94 ••

Picture the Problem We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find B_{app}. We can then use $B = B_{app} + \mu_0 M$ to find M and $B = K_m B_{app}$ to evaluate K_m.

(*a*) Relate the magnetic field on the axis of the solenoid to the current in the solenoid:

$$B_x = \mu_0 n I$$

Substitute numerical values and evaluate B_{app}:

$$B_{app} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(50\ \text{cm}^{-1}\right)\left(0.2\ \text{A}\right) = \boxed{1.26\ \text{mT}}$$

(*b*) Relate M to B and B_{app}:

$$B = B_{app} + \mu_0 M$$

Solve for M:

$$M = \frac{B - B_{\text{app}}}{\mu_0}$$

Substitute numerical values and evaluate M:

$$M = \frac{1.58\,\text{T} - 1.26\,\text{mT}}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{1.26\times10^{6}\ \text{A/m}}$$

(c) Express B in terms of K_{m} and B_{app}:

$$B = K_{\text{m}}B_{\text{app}}$$

Solve for and evaluate K_{m}:

$$K_{\text{m}} = \frac{B}{B_{\text{app}}} = \frac{1.58\,\text{T}}{1.26\,\text{mT}} = \boxed{1.25\times10^{3}}$$

95 ••

Picture the Problem The magnetic field in the core of a hollow solenoid is related to the current in its coils according to $B_x = B_{\text{app}} = \mu_0 nI$. The presence of the iron increases the magnetic field by a factor of K_{m}. In part (b), requiring that the magnetic field be unchanged when the iron core is removed will allow us to find the current that will produce the same field within the solenoid.

(a) Relate the magnetic field on the axis of the solenoid to the current in the solenoid:

$$B_x = B_{\text{app}} = \mu_0 nI$$

Express B in terms of B_{app}:

$$B = K_{\text{m}}B_{\text{app}}$$

Substitute to obtain:

$$B = K_{\text{m}}\mu_0 nI$$

Substitute numerical values and evaluate B:

$$B = 1200\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(2000\,\text{m}^{-1}\right)\left(20\,\text{mA}\right) = \boxed{60.3\,\text{mT}}$$

(b) We require, that with the iron core removed, the magnetic field is unchanged:

$$B = K_{\text{m}}\mu_0 nI = \mu_0 nI_0$$

Solve for and evaluate I_0:

$$I_0 = K_{\text{m}}I = 1200\left(20\,\text{mA}\right) = \boxed{24.0\,\text{A}}$$

***96 ••**

Picture the Problem Because the wires carry equal currents in opposite directions, the magnetic field midway between them will be twice that due to either current alone and will be greater, by a factor of K_m, than it would be in the absence of the insulator. We can use Ampère's law to find the field, due to either current, at the midpoint of the plane of the wires and $d\vec{F} = Id\vec{\ell} \times \vec{B}$ to find the force per unit length on either wire.

(*a*) Relate the magnetic field in the insulator to the magnetic field in its absence:

$$B = K_m B_{app}$$

Apply Ampère's law to a closed circular path a distance *r* from a current-carrying wire to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_{app}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B_{app} to obtain:

$$B_{app} = \frac{\mu_0 I}{2\pi r}$$

Because there are two current carrying wires, with their currents in opposite directions, the fields are additive and:

$$B = 2K_m \frac{\mu_0 I}{2\pi r} = \frac{K_m \mu_0 I}{\pi r}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{120(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{\pi(0.02\ \text{m})} = \boxed{96.0\ \text{mT}}$$

(*b*) Express the force per unit length experienced by either wire due to the current in the other:

$$\frac{F}{\ell} = BI$$

Apply Ampère's law to obtain:

$$\int_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C = \mu_0 I$$

where *r* is the separation of the wires.

Solve for *B*:

$$B = \frac{\mu_0 I}{2\pi r} \text{ and } B_{app} = \frac{K_m \mu_0 I}{2\pi r}$$

Substitute to obtain:

$$\frac{F}{\ell} = \frac{K_m \mu_0 I^2}{2\pi r}$$

Substitute numerical values and evaluate $\frac{F}{\ell}$:

$$\frac{F}{\ell}=\frac{120\left(4\pi\times10^{-7}\text{ N/A}^2\right)(40\text{ A})^2}{2\pi(0.04\text{ m})}=\boxed{0.960\text{ N/m}}$$

97 ••

Picture the Problem We can use $B=B_{\text{app}}+\mu_0 M$ and the expression for the magnetic field inside a tightly wound toroid to find the magnetization *M*. We can find K_m from its definition, $\mu=K_m\mu_0$ to find μ, and $K_m=1+\chi_m$ to find χ_m for the iron sample.

(*a*) Relate the magnetization to *B* and B_{app}:

$$B=B_{\text{app}}+\mu_0 M$$

Solve for *M*:

$$M=\frac{B-B_{\text{app}}}{\mu_0}$$

Express the magnetic field inside a tightly wound toroid:

$$B_{\text{app}}=\frac{\mu_0 NI}{2\pi r}$$

Substitute and simplify to obtain:

$$M=\frac{B-\frac{\mu_0 NI}{2\pi r}}{\mu_0}=\frac{B}{\mu_0}-\frac{NI}{2\pi r}$$

Substitute numerical values and evaluate *M*:

$$M=\frac{1.8\text{ T}}{4\pi\times10^{-7}\text{ N/A}^2}-\frac{2000(10\text{ A})}{2\pi(0.2\text{ m})}=\boxed{1.42\times10^6\text{ A/m}}$$

(*b*) Use its definition to express K_m:

$$K_m=\frac{B}{B_{\text{app}}}=\frac{B}{\frac{\mu_0 NI}{2\pi r}}=\frac{2\pi rB}{\mu_0 NI}$$

Substitute numerical values and evaluate K_m:

$$K_m=\frac{2\pi(0.2\text{ m})(1.8\text{ T})}{\left(4\pi\times10^{-7}\text{ N/A}^2\right)(2000)(10\text{ A})}=\boxed{90.0}$$

Now that we know K_m we can find μ using:

$$\mu=K_m\mu_0=90\left(4\pi\times10^{-7}\text{ N/A}^2\right)=\boxed{1.13\times10^{-4}\text{ T}\cdot\text{m/A}}$$

Relate χ_m to K_m:

$$K_m=1+\chi_m$$

Solve for and evaluate χ_m: $\chi_m = K_m - 1 = \boxed{89.0}$

98 ••

Picture the Problem We can substitute the expression for applied magnetic field ($B_{app} = \dfrac{\mu_0 NI}{2\pi r}$) in the defining equation for K_m ($B = K_m B_{app}$) to obtain an expression for the magnetic field B in the toroid.

Relate the magnetic field in the toroid to the relative permeability of its core:

$$B = K_m B_{app}$$

Express the applied magnetic field in the toroid in terms of the current in its winding:

$$B_{app} = \frac{\mu_0 NI}{2\pi r}$$

Substitute to obtain:

$$B = \frac{K_m \mu_0 NI}{2\pi r}$$

Express the number of turns N of wire in terms of the number of turns per unit length n:

$$N = 2\pi rn$$

Substitute to obtain:

$$B = K_m \mu_0 nI$$

Substitute numerical values and evaluate B:

$$B = 500\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(60\,\text{cm}^{-1}\right)(0.2\,\text{A}) = \boxed{0.754\,\text{T}}$$

99 ••

Picture the Problem We can use Ampère's law to obtain expressions for the magnetic field inside the wire, inside the ferromagnetic material, and in the region outside the insulating ferromagnetic material.

(*a*) Apply Ampère's law to a circle of radius $r < 1$ mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C$$

Assuming that the current is distributed uniformly over the cross-sectional area of the wire (uniform current density), express I_C in terms

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi R^2}$$

or

of the total current I:

$$I_C = \frac{r^2}{R^2} I$$

Substitute to obtain:

$$B(2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

Solve for B:

$$B = \frac{\mu_0 I}{2\pi R^2} r$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{2\pi (1\ \text{mm})^2} r = \boxed{(8.00\ \text{T/m})r}$$

(*b*) Relate the magnetic field inside the ferromagnetic material to the magnetic field due to the current in the wire:

$$B = K_\text{m} B_\text{app}$$

Apply Ampère's law to a circle of radius 1 mm < r < 4 mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_\text{app}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B_app:

$$B_\text{app} = \frac{\mu_0 I}{2\pi r}$$

Substitute to obtain:

$$B = \frac{K_\text{m} \mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate B:

$$B = \frac{400(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{2\pi r} = \boxed{(3.20 \times 10^{-3}\ \text{T} \cdot \text{m})\frac{1}{r}}$$

(*c*) Apply Ampère’s law to a circle of radius r > 4 mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B:

$$B = \frac{\mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{2\pi r} = \boxed{(8.00 \times 10^{-6}\ \text{T}\cdot\text{m})\frac{1}{r}}$$

(d) Note that the field in the ferromagnetic region is that which would be produced in a nonmagnetic region by a current of 400I = 1600 A. The ampèrian current on the inside of the surface of the ferromagnetic material must therefore be (1600 – 40) A = 1560 A in the direction of I. On the outside surface there must then be an ampèrian current of 1560 A in the opposite direction.

General Problems

100 •

Picture the Problem Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P. Hence, we can use the expression for the magnetic field at the center of a current loop to find B_P.

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(15\ \text{A})}{4(0.2\ \text{m})} = \boxed{2.36 \times 10^{-5}\ \text{T}}$$

***101** •

Picture the Problem Let out of the page be the positive x direction. Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P. Hence, the resultant magnetic field at P will be the sum of the magnetic fields due to the current in the two semicircles, and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field at the center of a current loop:

$$B=\frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B=\frac{1}{2}\frac{\mu_0 I}{2R}=\frac{\mu_0 I}{4R}$$

Express $\vec{B}_1$ and $\vec{B}_2$:

$$\vec{B}_1=\frac{\mu_0 I}{4R_1}\hat{i}$$

and

$$\vec{B}_2=-\frac{\mu_0 I}{4R_2}\hat{i}$$

Substitute to obtain:

$$\vec{B}_P=\frac{\mu_0 I}{4R_1}\hat{i}-\frac{\mu_0 I}{4R_2}\hat{i}=\boxed{\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\hat{i}}$$

102 ••

Picture the Problem We can express B as a function of N, I, and R using $B=\frac{\mu_0 NI}{2R}$ and eliminate R by relating ℓ to R through $\ell=2\pi RN$.

Express the magnetic field at the center of a coil of N turns and radius R:

$$B=\frac{\mu_0 NI}{2R}$$

Relate ℓ to the number of turns N:

$$\ell=2\pi RN$$

Solve for R to obtain:

$$R=\frac{\ell}{2\pi N}$$

Substitute to obtain:

$$B=\frac{\mu_0 NI}{2\dfrac{\ell}{2\pi N}}=\boxed{\frac{\mu_0 \pi N^2 I}{\ell}}$$

103 ••

Picture the Problem The magnetic field at P (which is out of the page) is the sum of the magnetic fields due to the three parts of the wire. Let the numerals 1, 2, and 3 denote the left-hand, center (short), and right-hand wires. We can then use the expression for B due to a straight wire segment to find each of these fields and their sum.

Express the resultant magnetic field at point *P*:

$$B_P = B_1 + B_2 + B_3$$

Because $B_1 = B_3$:

$$B_P = 2B_1 + B_2$$

Express the magnetic field due to a straight wire segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

For wires 1 and 3 (the long wires), $\theta_1 = 90°$ and $\theta_2 = 45°$:

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin 90° + \sin 45°\right) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)$$

For wire 2, $\theta_1 = \theta_2 = 45°$:

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin 45° + \sin 45°\right) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right)$$

Substitute and simplify to obtain:

$$B_P = 2\left[\frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)\right] + \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{2}{\sqrt{2}}\right) = \boxed{\frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \sqrt{2}\right)}$$

***104 ••**

Picture the Problem Depending on the direction of the wire, the magnetic field due to its current (provided this field is a large enough fraction of the earth's magnetic field) will either add to or subtract from the earth's field and moving the compass over the ground in the vicinity of the wire will indicate the direction of the current.

Apply Ampère's law to a circle of radius *r* and concentric with the center of the wire:

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = B_{\text{wire}}\left(2\pi r\right) = \mu_0 I_C = \mu_0 I$$

Solve for *B* to obtain:

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate B_{wire}:

$$B_{\text{wire}} = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50\,\text{A})}{2\pi(2\,\text{m})} = 0.0500\,\text{G}$$

Express the ratio of B_{wire} to B_{earth}:

$$\frac{B_{\text{wire}}}{B_{\text{earth}}} = \frac{0.05\,\text{G}}{0.7\,\text{G}} \approx 7\%$$

Thus, the field of the current-carrying wire should be detectable with a good compass.

If the cable runs east-west, its magnetic field is in the north-south direction and thus either adds to or subtracts from the earth's field, depending on the current direction and location of the compass. Moving the compass over the region one should be able to detect the change.

If the cable runs north-south, its magnetic field is perpendicular to that of the earth, and moving the compass about one should observe a change in the direction of the compass needle.

105 ••

Picture the Problem Let I_1 and I_2 represent the currents of 20 A and 5 A, $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and $\vec{F}_4$ the forces that act on the horizontal wire at the top of the loop, and the other wires following the current in a counterclockwise direction, and $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ the magnetic fields at these wires due to I_1. Let the positive x direction be to the right and the positive y direction be upward. Note that only the components into or out of the paper of $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ contribute to the forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and $\vec{F}_4$, respectively.

(*a*) Express the forces $\vec{F}_2$ and $\vec{F}_4$ in terms of I_2 and $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{F}_2 = I_2\vec{\ell}_2\times\vec{B}_2$$
and
$$\vec{F}_4 = I_2\vec{\ell}_4\times\vec{B}_4$$

Express $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2I_1}{R_1}\hat{k}$$
and
$$\vec{B}_4 = -\frac{\mu_0}{4\pi}\frac{2I_1}{R_4}\hat{k}$$

Substitute to obtain:

$$\vec{F}_2 = -I_2\ell_2\hat{j}\times\left(-\frac{\mu_0}{4\pi}\frac{2I_1}{R_1}\hat{k}\right) = \frac{\mu_0\ell_2 I_1 I_2}{2\pi R_2}\hat{i}$$

and

$$\vec{F}_4 = I_2\ell_4\hat{j}\times\left(-\frac{\mu_0}{4\pi}\frac{2I_1}{R_4}\hat{k}\right) = -\frac{\mu_0\ell_4 I_1 I_2}{2\pi R_4}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}_2$ and $\vec{F}_4$:

$$\vec{F}_2 = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.1\,\text{m})(20\,\text{A})(5\,\text{A})}{2\pi(0.02\,\text{m})}\hat{i} = \boxed{(1.00\times10^{-4}\ \text{N})\hat{i}}$$

and

$$\vec{F}_4 = -\frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.1\,\text{m})(20\,\text{A})(5\,\text{A})}{2\pi(0.07\,\text{m})}\hat{i} = \boxed{(-0.286\times10^{-4}\ \text{N})\hat{i}}$$

(*b*) Express the net force acting on the coil:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \qquad (1)$$

Because the lengths of segments 1 and 3 are the same and the currents in these segments are in opposite directions:

$$\vec{F}_1 + \vec{F}_3 = 0$$

and

$$\vec{F}_{\text{net}} = \vec{F}_2 + \vec{F}_4$$

Substitute for $\vec{F}_2$ and $\vec{F}_4$ in equation (1) and simplify to obtain:

$$\vec{F}_{\text{net}} = (-0.250\times10^{-4}\ \text{N})\hat{j} + (1.00\times10^{-4}\ \text{N})\hat{i} + (0.250\times10^{-4}\ \text{N})\hat{j} + (-0.286\times10^{-4}\ \text{N})\hat{i} = \boxed{(0.714\times10^{-4}\ \text{N})\hat{i}}$$

106 ••

Picture the Problem Let out of the page be the positive *x* direction and the numerals 40 and 60 refer to the circular arcs whose radii are 40 cm and 60 cm. Because point *P* is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at *P*. Hence the resultant magnetic field at *P* will be the sum of the magnetic fields due to the current in the two circular arcs and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_{40} + \vec{B}_{60}$$

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of one-sixth of a current loop:

$$B = \frac{1}{6}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{12R}$$

Express $\vec{B}_{40}$ and $\vec{B}_{60}$:

$$\vec{B}_{40} = -\frac{\mu_0 I}{12R_{40}}\hat{i}$$

and

$$\vec{B}_{60} = \frac{\mu_0 I}{12R_{60}}\hat{i}$$

Substitute to obtain:

$$\vec{B}_P = -\frac{\mu_0 I}{12R_{40}}\hat{i} + \frac{\mu_0 I}{12R_{60}}\hat{i} = \frac{\mu_0 I}{12}\left(\frac{1}{R_{60}} - \frac{1}{R_{40}}\right)\hat{i}$$

Substitute numerical values and evaluate $\vec{B}_P$:

$$\vec{B}_P = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(8\,\text{A})}{12}\left(\frac{1}{0.6\,\text{m}} - \frac{1}{0.4\,\text{m}}\right)\hat{i} = \boxed{\left(-6.98\times10^{-7}\ \text{T}\right)\hat{i}}$$

107 ••

Picture the Problem Let the positive x direction be into the page and the numerals 20 and 40 refer to the circular arcs whose radii are 20 cm and 40 cm. Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P and the resultant field at P is the sum of the fields due to the two semicircular current loops.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_{20} + \vec{B}_{40}$$

Express the magnetic field at the center of a circular current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a circular current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Express $\vec{B}_{20}$ and $\vec{B}_{40}$:

$$\vec{B}_{20} = \frac{\mu_0 I}{4R_{20}}\hat{i} \text{ and } \vec{B}_{40} = \frac{\mu_0 I}{4R_{40}}\hat{i}$$

Substitute to obtain:

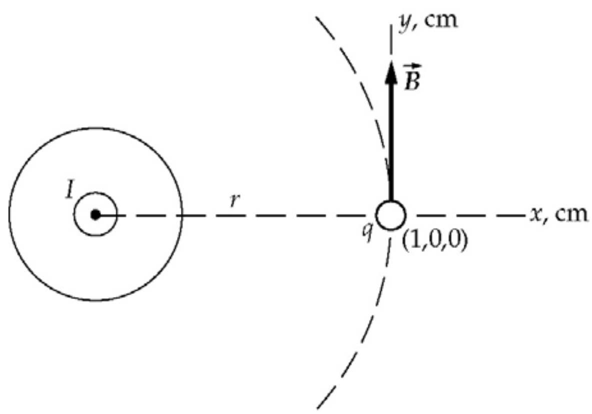
$$\vec{B}_P = \frac{\mu_0 I}{4R_{20}}\hat{i} + \frac{\mu_0 I}{4R_{40}}\hat{i} = \frac{\mu_0 I}{4}\left(\frac{1}{R_{20}} + \frac{1}{R_{40}}\right)\hat{i}$$

Substitute numerical values and evaluate B_P:

$$\vec{B}_P = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(3\,\text{A})}{4}\left(\frac{1}{0.2\,\text{m}} + \frac{1}{0.4\,\text{m}}\right)\hat{i} = \boxed{(7.07\ \mu\text{T})\hat{i}}$$

***108** ••

Picture the Problem Chose the coordinate system shown to the right. Then the current is in the positive z direction. Assume that the electron is at (1 cm, 0, 0). We can use $\vec{F} = q\vec{v}\times\vec{B}$ to relate the magnetic force on the electron to $\vec{v}$ and $\vec{B}$ and $\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j}$ to express the magnetic field at the location of the electron. We'll need to express $\vec{v}$ for each of the three situations described in the problem in order to evaluate $\vec{F} = q\vec{v}\times\vec{B}$.



Express the magnetic force acting on the electron:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Express the magnetic field due to the current in the wire as a function of distance from the wire:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j}$$

Substitute to obtain:

$$\vec{F} = q\vec{v}\times\frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j} = \frac{2q\mu_0 I}{4\pi r}\left(\vec{v}\times\hat{j}\right) \quad (1)$$

(*a*) Express the velocity of the electron when it moves directly away from the wire:

$$\vec{v} = v\hat{i}$$

Substitute to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{i}\times\hat{j}\right) = \frac{2q\mu_0 I v}{4\pi r}\hat{k}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = \frac{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(-1.6\times10^{-19}\ \text{C}\right)\left(5\times10^{6}\ \text{m/s}\right)\left(20\ \text{A}\right)\hat{k}}{4\pi\left(0.01\,\text{m}\right)} = \boxed{\left(-3.20\times10^{-16}\ \text{N}\right)\hat{k}}$$

(*b*) Express $\vec{v}$ when the electron is traveling parallel to the wire in the direction of the current:

$$\vec{v} = v\hat{k}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{k}\times\hat{j}\right) = -\frac{2q\mu_0 I v}{4\pi r}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = -\frac{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(-1.6\times10^{-19}\ \text{C}\right)\left(5\times10^{6}\ \text{m/s}\right)\left(20\ \text{A}\right)\hat{i}}{4\pi\left(0.01\,\text{m}\right)} = \boxed{\left(3.20\times10^{-16}\ \text{N}\right)\hat{i}}$$

(*c*) Express $\vec{v}$ when the electron is traveling perpendicular to the wire and tangent to a circle around the wire:

$$\vec{v} = v\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{j}\times\hat{j}\right) = \boxed{0}$$

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Picture the Problem We can apply Ampère's law to derive expressions for the magnetic field as a function of the distance from the center of the wire.

Apply Ampère's law to a closed circular path of radius $r < r_0$ to obtain:

$$B_{r<r_0}(2\pi r) = \mu_0 I_C$$

Because the current is uniformly distributed over the cross section of the wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi r_0^2} \Rightarrow I_C = \frac{r^2}{r_0^2} I$$

Substitute to obtain:

$$B_{r<r_0}(2\pi r) = \frac{\mu_0 r^2 I}{r_0^2}$$

Solve for $B_{r<r_0}$:

$$B_{r<r_0} = \frac{\mu_0 r I}{2\pi r_0^2} = \frac{\mu_0}{4\pi}\frac{2I}{r_0^2} r \qquad (1)$$

Apply Ampère's law to a closed circular path of radius $r > r_0$ to obtain:

$$B_{r>r_0}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for $B_{r>r_0}$:

$$B_{r>r_0} = \frac{\mu_0}{4\pi}\frac{2I}{r} \qquad (2)$$

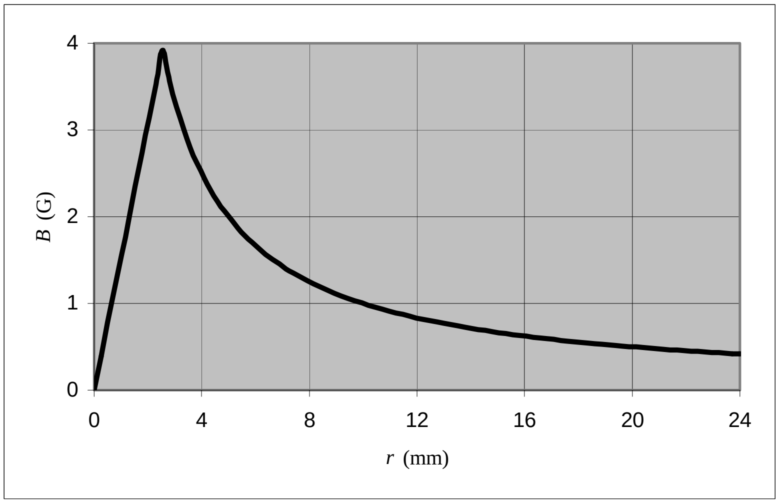
The spreadsheet program to calculate *B* as a function of *r* in the interval $0 \le r \le 10r_0$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.00E–07	$\frac{\mu_0}{4\pi}$
B2	5	I
B3	1	I
A6	2.55E–03	r (m)
B6	0.00E+00	r (mm)
C6	10^4*$B$1*2*$B$2*A6/$B$3^2	$\frac{\mu_0}{4\pi}\frac{2I}{r_0^2}r$
C17	10^4*$B$1*2*$B$2*A6/A17	$\frac{\mu_0}{4\pi}\frac{2I}{r}$

	A	B	C
1	mu/4pi=	1.00E–07	N/A^2

2	I=	5	A
3	r_0=	2.55E–03	m
4			
5	r (m)	r (mm)	B (T)
6	0.00E+00	0.00E+00	0.00E+00
7	2.55E–04	2.55E–01	3.92E–01
8	5.10E–04	5.10E–01	7.84E–01
9	7.65E–04	7.65E–01	1.18E+00
10	1.02E–03	1.02E+00	1.57E+00
102	2.45E–02	2.45E+01	4.08E–01
103	2.47E–02	2.47E+01	4.04E–01
104	2.50E–02	2.50E+01	4.00E–01
105	2.52E–02	2.52E+01	3.96E–01
106	2.55E–02	2.55E+01	3.92E–01

A graph of B as a function of r follows.



110 ••

Picture the Problem We can use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque exerted on the small coil (magnetic moment = $\vec{\mu}$) by the magnetic field $\vec{B}$ due to the current in the large coil.

Relate the torque exerted by the large coil on the small coil to the magnetic moment $\vec{\mu}$ of the small coil and the magnetic field $\vec{B}$ due to the current in the large coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

or, because the planes of the two coils are perpendicular,

$$\tau = \mu B$$

Express the magnetic moment of the small coil:

$$\mu = NIA$$

where I is the current in the coil, N is the number of turns in the coil, and A is the cross-sectional area of the coil.

Express the magnetic field at the center of the large coil:

$$B=\frac{N'\mu_0 I'}{2R}$$

where I' is the current in the large coil, N' is the number of turns in the coil, and R is its radius.

Substitute to obtain:

$$\tau=\frac{NN'II'A\mu_0}{2R}$$

Substitute numerical values and evaluate τ:

$$\tau=\frac{(50)(20)(4\,\text{A})(1\,\text{A})\pi(0.5\,\text{cm})^2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)}{2(10\,\text{cm})}=\boxed{1.97\ \mu\text{N}\cdot\text{m}}$$

***111** ••

Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for κ when there is an external field and for the period T in the absence of an external field.

Apply $\sum\tau=I\alpha$ to the bar magnet when $B\neq0$ to obtain the differential equation of motion for the magnet:

$$-\kappa\theta-\mu B\sin\theta=I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium (θ << 1):

$$-\kappa\theta-\mu B\theta\approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2}+(\kappa+\mu B)\theta=0$$

or

$$\frac{d^2\theta}{dt^2}+\left(\frac{\kappa+\mu B}{I}\right)\theta=0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2=\frac{\kappa+\mu B}{I}\qquad(1)$$

Express the moment of inertia (see Table 9-1) of the bar magnet about an axis through its center:

$$I = \tfrac{1}{12} mL^2$$

Substitute to obtain:

$$\omega^2 = \frac{\kappa + \mu B}{\tfrac{1}{12} mL^2}$$

Solve for κ to obtain:

$$\kappa = \tfrac{1}{12} mL^2\omega^2 - \mu B = \tfrac{1}{12} mL^2\left(\frac{4\pi^2}{T^2}\right) - \mu B$$

$$= \frac{\pi^2 mL^2}{3T^2} - \mu B$$

Substitute numerical values and evaluate κ:

$$\kappa = \frac{\pi^2(0.8\,\text{kg})(0.16\,\text{m})^2}{3(0.5\,\text{s})^2} - (0.12\,\text{A}\cdot\text{m}^2)(0.2\,\text{T}) = \boxed{0.246\,\text{N}\cdot\text{m/rad}}$$

Substitute $B = 0$ and $\omega = 2\pi/T$ in equation (1) to obtain:

$$\frac{4\pi^2}{T^2} = \frac{\kappa}{I}$$

Solve for T:

$$T = 2\pi\sqrt{\frac{I}{\kappa}} = 2\pi\sqrt{\frac{mL^2}{12\kappa}} = \pi L\sqrt{\frac{m}{3\kappa}}$$

Substitute numerical values and evaluate T:

$$T = \pi(0.16\,\text{m})\sqrt{\frac{0.8\,\text{kg}}{3(0.246\,\text{N}\cdot\text{m/rad})}}$$

$$= \boxed{0.523\text{s}}$$

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Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for the frequency *f* of the motion.

Apply $\sum \tau = I\alpha$ to the bar magnet to obtain the differential equation of motion for the magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium ($\theta << 1$):

$$-\mu B\theta \approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2} + \mu B\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2 = \frac{\mu B}{I}$$

Solve for ω to obtain:

$$\omega = \boxed{\sqrt{\frac{\mu B}{I}}}$$

113 ••

Picture the Problem We can use the potential energy of the displaced bar magnet to find the force acting on it to return it to its equilibrium position. While this restoring force is not, in general, linear, we can use a binomial expansion to show that for displacements that are small compared to the radius of the coil, the restoring force is linear and, hence, the motion of the bar magnet is simple harmonic motion. We can then apply Newton's 2nd law to obtain the differential equation of motion of the bar magnet and use the coefficient of the linear term to express the period of the motion.

Express the potential energy of the displaced bar magnet:

$$U = -\mu B$$

Express the magnetic field on the axis of the current loop:

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NR^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

where I is the current in the loop and R is its radius.

Substitute to obtain:

$$U = -\frac{\mu_0}{4\pi}\frac{2\pi\mu NR^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

Differentiate U with respect to x to find the restoring force acting on the bar magnet:

$$F_x = -\frac{dU}{dx}$$
$$= \tfrac{1}{2}\mu_0\mu NR^2 I\,\frac{d}{dx}\left[\left(x^2+R^2\right)^{-3/2}\right]$$
$$= -\frac{3\mu_0\mu NR^2 I}{2}\left[\frac{1}{\left(x^2+R^2\right)^{5/2}}\right]x$$

Factor R^2 from the radical to obtain:

$$F_x = -\frac{3\mu_0\mu NR^2 I}{2R^5}\left[\frac{1}{\left(1+\frac{x^2}{R^2}\right)^{5/2}}\right]x$$
$$= -\frac{3\mu_0\mu NI}{2R^3}\left(1+\frac{x^2}{R^2}\right)^{-5/2}x$$

Expand the radical factor to obtain:

$$\left(1+\frac{x^2}{R^2}\right)^{-5/2} = 1-\frac{5}{2}\frac{x^2}{R^2} + \text{higher order terms}$$

For $x << R$:

$$\left(1+\frac{x^2}{R^2}\right)^{-5/2} \approx 1$$

Substitute in F_x to obtain:

$$F_x = -\frac{3\mu_0\mu NI}{2R^3}x$$

Thus, we've shown that the bar magnet experiences a linear restoring force and, hence, its motion will be simple harmonic motion.

Apply $\sum \vec{F} = m\vec{a}$ to the bar magnet to obtain:

$$-\frac{3\mu_0\mu NI}{2R^3}x = m\frac{d^2x}{dt^2}$$

or

$$\frac{d^2x}{dt^2}+\frac{3\mu_0\mu NI}{2mR^3}x = 0$$

Because the coefficient of the linear term is the square of the angular frequency we have:

$$\omega^2 = \frac{4\pi^2}{T^2} = \frac{3\mu_0\mu NI}{2mR^3}$$

Solve for T to obtain:

$$T = 2\pi\sqrt{\frac{2mR^3}{3\mu_0\mu NI}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{2(0.1\,\text{kg})(0.1\,\text{m})^3}{3(4\pi\times10^{-7}\ \text{N/A}^2)(0.04\,\text{A}\cdot\text{m}^2)(100)(5\,\text{A})}} = \boxed{10.2\,\text{s}}$$

114 ••

Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for the frequency f of the motion.

Apply $\sum\tau = I\alpha$ to the bar magnet to obtain the differential equation of motion for the magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium ($\theta << 1$):

$$-\mu B\theta \approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2} + \mu B\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2 = 4\pi^2 f^2 = \frac{\mu B}{I}$$

where f is the frequency of oscillation.

Solve for f to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$$

or, because $\mu = 2.2N\mu_B$ where N is the number of iron atoms in the bar magnet,

$$f = \frac{1}{2\pi}\sqrt{\frac{2.2N\mu_B B}{I}}$$

From Table 9-1 we have:

$$I = \tfrac{1}{12}mL^2 = \tfrac{1}{12}\rho VL^2$$

Express the number of iron atoms in terms of Avogadro's number and the atomic weight of iron M:

$$\frac{N}{N_A} = \frac{m}{\text{M}} = \frac{\rho V}{\text{M}}$$

and

$$N = \frac{N_A \rho V}{\text{M}}$$

Substitute for I and N and simplify to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{2.2N_A \rho V \mu_B B}{\frac{1}{12}\rho VL^2 \text{M}}} = \frac{1}{\pi L}\sqrt{\frac{6.6N_A \mu_B B}{\text{M}}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{\pi(0.08\,\text{m})}\sqrt{\frac{6.6\left(6.02\times10^{23}\,/\,\text{mol}\right)\left(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2\right)\left(0.5\times10^{-4}\ \text{T}\right)}{55.85\,\text{g/mol}}}$$
$$= \boxed{0.723\,\text{Hz}}$$

115 ••

Picture the Problem We can solve the equation for the frequency f of the compass needle given in Problem 112 for magnetic dipole moment of the needle. In Parts (*b*) and (*c*) we can use their definitions to find the magnetization M and the amperian current I_{amperian}.

(*a*) In Problem 112 it is established that the frequency of the compass needle is:

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$$

where I is the moment of inertia of the needle.

Solve for μ to obtain:

$$\mu = \frac{4\pi^2 f^2 I}{B}$$

Express the moment of inertia of the needle:

$$I = \tfrac{1}{12}mL^2 = \tfrac{1}{12}\rho VL^2 = \tfrac{1}{12}\rho\pi r^2 L^3$$

Substitute to obtain:

$$\mu = \frac{\pi^3 f^2 \rho r^2 L^3}{3B}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{\pi^3\left(1.4\,\text{s}^{-1}\right)^2\left(7.96\times10^3\ \text{kg/m}^3\right)\left(0.85\times10^{-3}\ \text{m}\right)^2\left(0.03\,\text{m}\right)^3}{3\left(0.6\times10^{-4}\ \text{T}\right)} = \boxed{5.24\times10^{-2}\ \text{A}\cdot\text{m}^2}$$

(*b*) Use its definition to express the magnetization *M*:

$$M = \frac{\mu}{V}$$

Substitute to obtain:

$$M = \frac{\mu}{V} = \frac{\pi^3 f^2 \rho r^2 L^3}{3BV} = \frac{\pi^2 f^2 \rho L^2}{3B}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{\pi^2\left(1.4\,\text{s}^{-1}\right)^2\left(7.96\times10^3\ \text{kg/m}^3\right)\left(0.03\,\text{m}\right)^2}{3\left(0.6\times10^{-4}\ \text{T}\right)} = \boxed{7.70\times10^5\ \text{A/m}}$$

(*c*) Express and evaluate the amperian current on the surface of the needle:

$$I_{\text{amperian}} = ML = \left(7.70\times10^5\ \text{A/m}\right)\left(0.03\,\text{m}\right) = \boxed{2.31\times10^4\ \text{A}}$$

***116 ••**

Picture the Problem We can use the definition of angular momentum and Equation 27-27, together with the definition of the magnetization *M* of the iron bar, to derive an expression for the rotational angular velocity of the bar just after it has been demagnetized.

Assuming its angular momentum to be conserved, use the definition of *L* to express the angular momentum of the iron bar just after it has been demagnetized:

$$L = I\omega$$

Solve for the angular velocity ω:

$$\omega = \frac{L}{I}$$

Assuming that Equation 27-27 holds yields:

$$L = \frac{2m}{q}\mu = \frac{2m_e}{e}MV = \frac{2m_e}{e}M\pi r^2\ell$$

where *r* is the radius of the bar and ℓ its length.

Modeling the bar as a cylinder, express its moment of inertia with respect to its axis:

$$I=\tfrac{1}{2}mr^2=\tfrac{1}{2}\rho Vr^2=\tfrac{1}{2}\rho\pi r^4\ell$$

Substitute to obtain:

$$\omega=\frac{\frac{2m_e}{e}M\pi r^2\ell}{\frac{1}{2}\rho\pi r^4\ell}=\frac{4m_eM}{e\rho r^2}$$

Substitute numerical values (see Table 13-1 for the density of iron) and evaluate ω:

$$\omega=\frac{4\left(9.11\times10^{-31}\text{ kg}\right)\left(1.72\times10^{6}\text{ A/m}\right)}{\left(1.6\times10^{-19}\text{ C}\right)\left(7.96\times10^{3}\text{ kg/m}^3\right)\left(0.01\text{ m}\right)^2}=\boxed{4.92\times10^{-5}\text{ rad/s}}$$

117 ••

Picture the Problem The dipole moment of the bar is given by $\mu=2.219N\mu_B$, where N is the number of atoms in the bar. We can express N in terms of Avogadro's number, the density of iron, the volume of the bar, and the atomic weight of iron. We can use the definition of torque to find the torque that must be supplied to hold the iron bar perpendicular to the given magnetic field.

(*a*) Express the magnetic dipole moment of the magnetized iron bar:

$$\mu=2.219N\mu_B$$

where N is the number of iron atoms in the bar.

Express the number of iron atoms in terms of Avogadro's number and the atomic weight of iron M:

$$\frac{N}{N_A}=\frac{m}{\text{M}}=\frac{\rho V}{\text{M}}$$

and

$$N=\frac{N_A\rho V}{\text{M}}$$

Substitute to obtain:

$$\mu=\frac{2.219N_A\rho V\mu_B}{\text{M}}=\frac{2.219N_A\rho\ell A\mu_B}{\text{M}}$$

Substitute numerical values and evaluate μ:

$$\mu=\frac{2.219\left(6.02\times10^{23}\text{ mol}^{-1}\right)\left(7.96\times10^{3}\text{ kg/m}^3\right)\left(0.2\text{ m}\right)}{55.85\times10^{-3}\text{ kg/mol}}\times\left(2\times10^{-4}\text{ m}^2\right)\left(9.27\times10^{-24}\text{ A}\cdot\text{m}^2\right)=\boxed{70.6\text{ A}\cdot\text{m}^2}$$

(*b*) Express the torque required to hold the iron bar perpendicular to the magnetic field:

$$\tau = \mu B \sin\theta = \mu B \sin 90° = \mu B$$

Substitute numerical values and evaluate τ:

$$\tau = \left(70.6\,\text{A}\cdot\text{m}^2\right)\left(0.25\,\text{T}\right) = \boxed{17.7\,\text{N}\cdot\text{m}}$$

***118 ••**

Picture the Problem Note that B_e and B_c are perpendicular to each other and that the resultant magnetic field is at an angle θ with north. We can use trigonometry to relate B_c and B_e and express B_c in terms of the geometry of the coil and the current flowing in it.

Express B_c in terms of B_e:

$$B_c = B_e \tan\theta$$

where θ is the angle of the resultant field from north.

Express the field B_c due to the current in the coil:

$$B_c = \frac{N\mu_0 I}{2R}$$

where N is the number of turns.

Substitute to obtain:

$$\frac{N\mu_0 I}{2R} = B_e \tan\theta$$

Solve for I:

$$I = \boxed{\frac{2RB_e}{\mu_0 N}\tan\theta}$$

119 ••

Picture the Problem Let the positive x direction be out of the page. We can use the expressions for the magnetic fields due to an infinite straight line and a circular loop to express the net magnetic field at the center of the circular loop. We can set this net field to zero and solve for r.

Express the net magnetic field at the center of circular loop:

$$\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_{\text{loop}} + \vec{\boldsymbol{B}}_{\text{line}}$$

Letting R represent the radius of the loop, express $\vec{\boldsymbol{B}}_{\text{loop}}$:

$$\vec{\boldsymbol{B}}_{\text{loop}} = -\frac{\mu_0 I}{2R}\hat{\boldsymbol{i}}$$

Express the magnetic field due to the current in the infinite straight line:

$$\vec{\boldsymbol{B}}_{\text{line}} = \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}}$$

Substitute to obtain:

$$\vec{B} = -\frac{\mu_0 I}{2R}\hat{i} + \frac{\mu_0 I}{2\pi r}\hat{i} = \left(-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r}\right)\hat{i}$$

If $\vec{B} = 0$, then:

$$-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r} = 0$$

or

$$-\frac{1}{R} + \frac{1}{\pi r} = 0$$

Solve for r:

$$r = \frac{R}{\pi}$$

Substitute numerical values and evaluate r:

$$r = \frac{10\,\text{cm}}{\pi} = \boxed{3.18\,\text{cm}}$$

120 ••

Picture the Problem Note that only the current in the section of wire of length 2*a* contributes to the field at *P*. Hence, we can use the expression for *B* due to a straight wire segment to find the magnetic field at *P*. In Part (*b*) we can use our result from (*a*), together with the value for θ when the polygon has *N* sides, to obtain an expression for *B* at the center of a polygon of *N* sides.

Express the magnetic field at *P* due to a straight wire segment:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_1 + \sin\theta_2)$$

Because $\theta_1 = \theta_2 = \theta$:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}(2\sin\theta) = \left(\frac{\mu_0}{2\pi}\frac{I}{R}\right)\sin\theta$$

Refer to the figure to obtain:

$$\sin\theta = \frac{a}{\sqrt{a^2 + R^2}}$$

Substitute to obtain:

$$B_P = \boxed{\frac{\mu_0 a I}{2\pi R\sqrt{a^2 + R^2}}}$$

(*b*) Express θ for an *N*-sided polygon:

$$\theta = \frac{\pi}{N}$$

Because each side of the polygon contributes to *B* an amount equal to that obtained in (*a*):

$$B = \boxed{\left(\frac{N\mu_0 I}{2\pi R}\right)\sin\left(\frac{\pi}{N}\right)}$$

As $N \to \infty$:

$$\sin\left(\frac{\pi}{N}\right) \to \frac{\pi}{N}$$

and

$$B \to \left(\frac{N\mu_0 I}{2\pi R}\right)\left(\frac{\pi}{N}\right) = \boxed{\frac{\mu_0 I}{2R}}$$, the expression for the magnetic field at the center of a current-carrying circular loop.

121 ••

Picture the Problem We can use Ampère's law to derive expressions for $B(r)$ for $r < R$, $r = R$, and $r > R$ that we can evaluate for the given distances from the center of the cylindrical conductor.

Apply Ampère's law to a closed circular path a distance $r < R$ from the center of the cylindrical conductor to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(r)(2\pi r) = \mu_0 I_C = \mu_0 I(r)$$

Solve for $B(r)$ to obtain:

$$B(r) = \frac{\mu_0 I(r)}{2\pi r}$$

Substitute for $I(r)$:

$$B(r) = \frac{\mu_0 (50\text{ A/m})r}{2\pi r} = \frac{\mu_0 (50\text{ A/m})}{2\pi}$$

(*a*) and (*b*) Noting that B is independent of r, substitute numerical values and evaluate $B(5\text{ cm})$ and $B(10\text{ cm})$:

$$B(5\,\text{cm}) = B(10\,\text{cm}) = \frac{(4\pi \times 10^{-7}\text{ N/A}^2)(50\text{ A/m})}{2\pi} = \boxed{10.0\,\mu\text{T}}$$

(*c*) Apply Ampère's law to a closed circular path a distance $r > R$ from the center of the cylindrical conductor to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(r)(2\pi r) = \mu_0 I_C = \mu_0 I(R)$$

Solve for $B(r)$:

$$B(r) = \frac{\mu_0 I(R)}{2\pi r}$$

Substitute numerical values and evaluate $B(20\text{ cm})$:

$$B(20\,\text{cm}) = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50\ \text{A/m})(0.1\,\text{m})}{2\pi(0.2\,\text{m})} = \boxed{5.00\ \mu\text{T}}$$

122 ••

Picture the Problem The field $\vec{B}$ due to the 10-A current is in the *yz* plane. The net force on the wires of the square along the *y* direction cancel and do not contribute to a net torque or force. We can use $\vec{\tau} = \vec{l}\times\vec{F}$, $\vec{F} = I\vec{\ell}\times\vec{B}$, and the expression for the magnetic field due to an infinite straight wire to express the torque acting on each of the wires and hence, the net torque acting on the loop.

(*a*) Express the torque on the loop:

$$\vec{\tau} = \vec{l}\times\vec{F}$$

where $\vec{l}$ is the lever arm.

Express the magnetic force on a current element:

$$\vec{F} = I\vec{\ell}\times\vec{B}$$

Express the magnetic field at the wire at $y = 10$ cm:

$$\vec{B}_{y=10} = \frac{\mu_0}{4\pi}\frac{2I}{R}\frac{1}{\sqrt{2}}\left(-\hat{j}-\hat{k}\right)$$

where

$R = \sqrt{(0.1\,\text{m})^2 + (0.1\,\text{m})^2} = 0.141\,\text{m}$.

Substitute numerical values and evaluate $\vec{B}_{y=10}$:

$$\vec{B}_{y=10} = \frac{4\pi\times10^{-7}\ \text{N/A}^2}{4\pi\sqrt{2}}\frac{2(10\ \text{A})}{0.141\,\text{m}}\left(-\hat{j}-\hat{k}\right) = \left(1.00\times10^{-5}\ \text{T}\right)\left(-\hat{j}-\hat{k}\right)$$

Proceed similarly to obtain:

$$\vec{B}_{y=-10} = \left(1.42\times10^{-5}\ \text{T}\right)\left(-\hat{j}+\hat{k}\right)$$

Evaluate $\vec{F}_{y=10}$:

$$\begin{aligned}\vec{F}_{y=10} &= I\vec{\ell}\times\vec{B}_{y=10} = (5\ \text{A})(0.2\,\text{m})\hat{i}\times\left(1.00\times10^{-5}\ \text{T}\right)\left(-\hat{j}-\hat{k}\right)\\ &= \left(1.00\times10^{-5}\ \text{N}\right)\left[\hat{i}\times\left(-\hat{j}-\hat{k}\right)\right] = \left(1.00\times10^{-5}\ \text{N}\right)\left(-\hat{k}+\hat{j}\right)\end{aligned}$$

Evaluate $\vec{F}_{y=-10}$:

$$\begin{aligned}\vec{F}_{y=-10} &= (5\ \text{A})(-0.2\,\text{m})\hat{i}\times\left(1.00\times10^{-5}\ \text{T}\right)\left(-\hat{j}+\hat{k}\right)\\ &= \left(-1.00\times10^{-5}\ \text{N}\right)\left[\hat{i}\times\left(-\hat{j}+\hat{k}\right)\right] = \left(1.00\times10^{-5}\ \text{N}\right)\left(\hat{k}+\hat{j}\right)\end{aligned}$$

Express and evaluate the net force acting on the loop:

$$\vec{F} = \vec{F}_{y=10} + \vec{F}_{y=-10} = (1.00\times10^{-5}\ \text{N})(-\hat{k} + \hat{j}) + (1.00\times10^{-5}\ \text{N})(\hat{k} + \hat{j})$$
$$= (2.00\times10^{-5}\ \text{N})\hat{j}$$

Express and evaluate the torque about the x axis acting on the loop:

$$\tau = (0.1\,\text{m})(2.00\times10^{-5}\ \text{N}) = \boxed{2.00\times10^{-6}\ \text{N}\cdot\text{m}}$$

(*b*) The net force acting on the loop is the sum of the forces acting on the four sides (see the next to last step in (*a*)):

$$\vec{F} = \vec{F}_{y=10} + \vec{F}_{y=-10} = \boxed{(2.00\times10^{-5}\ \text{N})\hat{j}}$$

123 ••

Picture the Problem The force acting on the lower wire is given by $F_{\text{lower wire}} = I\ell B$, where I is the current in the lower wire, ℓ is the length of the wire on the balance, and B is the magnetic field at the location of the lower wire due to the current in the upper wire. We can apply Ampère's law to find B at the location of the wire on the pan of the balance.

The force experienced by the lower wire is given by:

$$F_{\text{lower wire}} = I\ell B$$

Apply Ampère's law to a closed circular path of radius r centered on the upper wire to obtain:

$$B(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B to obtain:

$$B = \frac{\mu_0 I}{2\pi r}$$

Substitute for B in the expression for the force on the lower wire:

$$F_{\text{lower wire}} = I\ell\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 \ell I^2}{2\pi r}$$

Solve for I to obtain:

$$I = \sqrt{\frac{2\pi r F_{\text{lower wire}}}{\mu_0 \ell}}$$

Noting that the force on the lower wire is the increase in the reading of the balance, substitute numerical values and evaluate I:

$$I = \sqrt{\frac{2\pi(2\,\text{cm})(5\times10^{-6}\ \text{kg})}{(4\pi\times10^{-7}\ \text{N/A}^2)(10\,\text{cm})}} = \boxed{2.24\,\text{A}}$$

124 ••

Picture the Problem We can use a proportion to relate minimum current detectible using this balance to its sensitivity and to the current and change in balance reading from Problem 123.

The minimum current I_{min} detectible is to the sensitivity of the balance as the current in Problem 123 is to the change in the balance reading in Problem 123:

$$\frac{I_{min}}{0.1\,\text{mg}} = \frac{2.24\,\text{A}}{5.0\,\text{mg}}$$

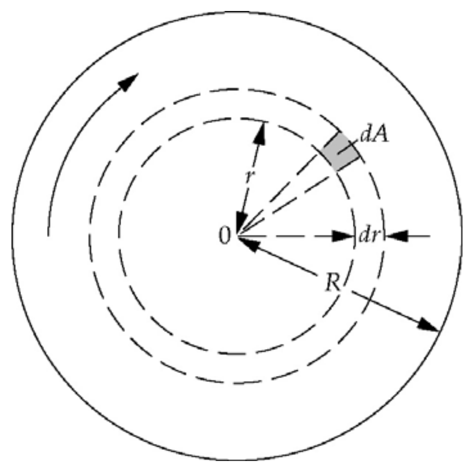
Solve for and evaluate I_{min}:

$$I_{min} = (0.1\,\text{mg})\left(\frac{2.24\,\text{A}}{5.0\,\text{mg}}\right) = \boxed{44.8\,\text{mA}}$$

The "standard" current balance can be made very sensitive by increasing the length (i.e., moment arm) of the wire balance, which one cannot do with this kind; however, this is compensated somewhat by the high sensitivity of the electronic balance.

***125** •••

Picture the Problem The diagram shows the rotating disk and the circular strip of radius *r* and width *dr* with charge *dq*. We can use the definition of surface charge density to express *dq* in terms of *r* and *dr* and the definition of current to show that $dI = \omega\sigma r\, dr$. We can then use this current and expression for the magnetic field on the axis of a current loop to obtain the results called for in (*b*) and (*c*).



(*a*) Express the total charge *dq* that passes a given point on the circular strip once each period:

$$dq = \sigma dA = 2\pi\sigma r dr$$

Using its definition, express the current in the element of width *dr*:

$$dI = \frac{dq}{dt} = \frac{2\pi\sigma r dr}{\frac{2\pi}{\omega}} = \boxed{\omega\sigma r dr}$$

(*c*) Express the magnetic field dB_x at a distance *x* along the axis of the disk due to the current loop of radius *r* and width *dr*:

$$dB_x = \frac{\mu_0}{4\pi}\frac{2\pi r^2 dI}{\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0\omega\sigma r^3}{2\left(x^2+r^2\right)^{3/2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

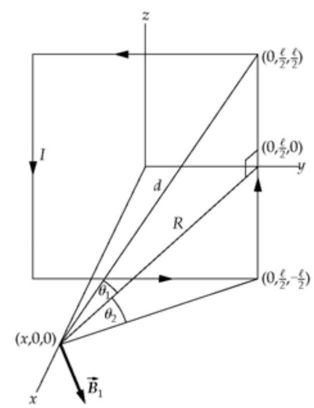
$$B_x = \frac{\mu_0\omega\sigma}{2}\int_0^R \frac{r^3}{\left(x^2+r^2\right)^{3/2}}dr = \boxed{\frac{\mu_0\omega\sigma}{2}\left(\frac{R^2+2x^2}{\sqrt{R^2+x^2}}-2x\right)}$$

(*b*) Evaluate B_x for $x = 0$:

$$B_x(0) = \frac{\mu_0\omega\sigma}{2}\left(\frac{R^2}{\sqrt{R^2}}\right) = \boxed{\tfrac{1}{2}\mu_0\sigma\omega R}$$

126 •••

Picture the Problem From the symmetry of the system it is evident that the fields due to each segment of length ℓ are the same in magnitude. We can express the magnetic field at (*x*,0,0) due to one side (segment) of the square, find its component in the *x* direction, and then multiply by four to find the resultant field.



Express *B* due to a straight wire segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1+\sin\theta_2\right)$$

where *R* is the perpendicular distance from the wire segment to the field point.

Use $\theta_1 = \theta_2$ and $R = \sqrt{x^2+\ell^2/4}$ to express *B* due to one side at (*x*,0,0):

$$B_1(x,0,0) = \frac{\mu_0}{4\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\left(2\sin\theta_1\right) = \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\left(\sin\theta_1\right)$$

Referring to the diagram, express $\sin\theta_1$:

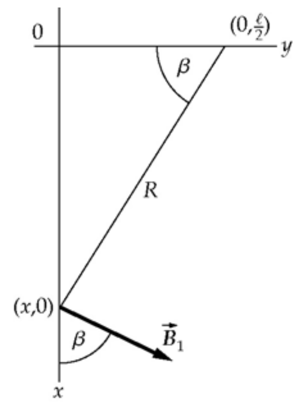
$$\sin\theta_1 = \frac{\frac{\ell}{2}}{d} = \frac{\frac{\ell}{2}}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

Substitute to obtain:

$$B_1(x,0,0) = \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2 + \frac{\ell^2}{4}}}\frac{\frac{\ell}{2}}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

$$= \frac{\mu_0 I}{4\pi\sqrt{x^2 + \frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

By symmetry, the sum of the y and z components of the fields due to the four segments must vanish, whereas the x components will add. The diagram to the right is a view of the xy plane showing the relationship between $\vec{B}_1$ and the angle β it makes with the x axis.



Express B_{1x}:

$$B_{1x} = B_1\cos\beta$$

Substitute and simplify to obtain:

$$B_{1x} = \frac{\mu_0 I}{4\pi\sqrt{x^2 + \frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2 + \frac{\ell^2}{2}}}\frac{\frac{\ell}{2}}{\sqrt{x^2 + \frac{\ell^2}{4}}}$$

$$= \frac{\mu_0 I\ell^2}{8\pi\left(x^2 + \frac{\ell^2}{4}\right)\sqrt{x^2 + \frac{\ell^2}{2}}}$$

The resultant magnetic field is the sum of the fields due to the 4 wire segments (sides of the square):

$$\vec{B} = 4B_{1x}\hat{i}$$

$$= \boxed{\frac{\mu_0 I\ell^2}{2\pi\left(x^2 + \frac{\ell^2}{4}\right)\sqrt{x^2 + \frac{\ell^2}{2}}}\hat{i}}$$

Factor x^2 from the two factors in the denominator to obtain:

$$\vec{B} = \frac{\mu_0 I\ell^2}{2\pi x^2\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{x^2\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{\boldsymbol{i}}$$

$$= \frac{\mu_0 I\ell^2}{2\pi x^3\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{\boldsymbol{i}}$$

For $x >> \ell$:

$$\vec{B} \approx \frac{\mu_0 I\ell^2}{2\pi x^3}\hat{\boldsymbol{i}} = \boxed{\frac{\mu_0 \mu}{2\pi x^3}\hat{\boldsymbol{i}}}$$

where $\mu = I\ell^2$.

Chapter 28
Magnetic Induction

Conceptual Problems

***1 •**

Determine the Concept We know that the magnetic flux (in this case the magnetic field because the area of the conducting loop is constant and its orientation is fixed) must be changing so the only issues are whether the field is increasing or decreasing and in which direction. Because the direction of the magnetic field associated with the clockwise current is into the page, the changing field that is responsible for it must be either increasing out of the page (not included in the list of possible answers) or a decreasing field directed into the page. $\boxed{(d)\text{ is correct.}}$

2 •

Determine the Concept Note that when R is constant, $\boldsymbol{B}$ in the loop to the right points out of the paper.

(*a*) If R increases, I decreases and so does B. By Lenz's law, the induced current is counterclockwise.

(*b*) If R decreases, the induced current is clockwise.

3 ••

Determine the Concept If the counterclockwise current in loop A increases, so does the magnetic flux through B. To oppose this increase in flux, the induced current in loop B will by clockwise. If the counterclockwise current in loop A decreases, so does the magnetic flux through B. To oppose this decrease in flux, the induced current in loop B will be counterclockwise. We can use $\vec{\boldsymbol{F}} = I\vec{\ell}\times\vec{\boldsymbol{B}}$ to determine the direction of the forces on each loop and, hence, whether they will attract or repel each other.

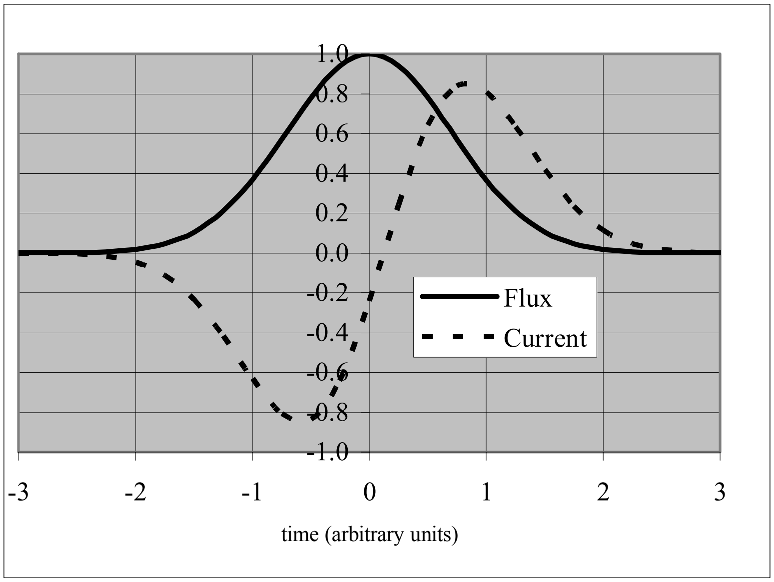
(*a*) If the current in B is clockwise the loops repel one another.

(*b*) If the current in B is counterclockwise the loops attract one another.

4 ••

Determine the Concept We know that, as the magnet moves to the right, the flux through the loop first increases until the magnet is half way through the loop and then decreases. Because the flux first increases and then decreases, the current will change directions, having its maximum values when the flux is changing most rapidly.

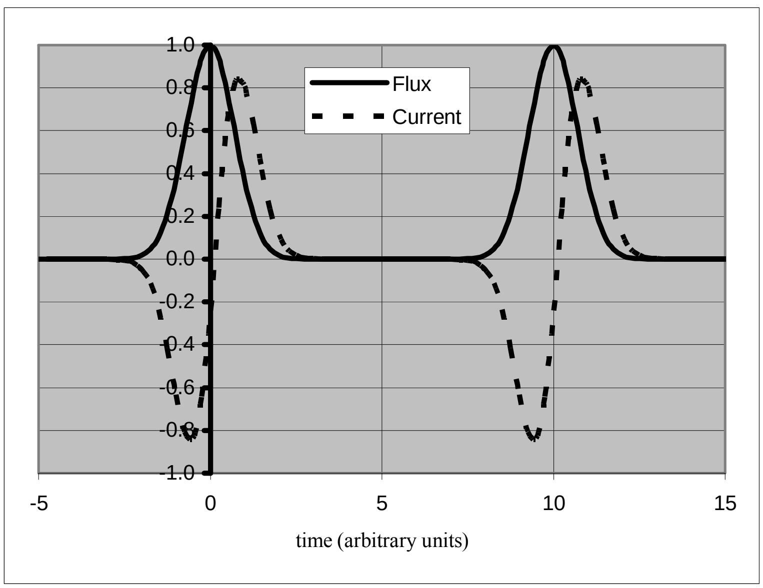
(*a*) and (*b*) The following graph shows the flux and the induced current as a function of time as the bar magnet passes through the coil. When the center of the magnet passes through the plane of the coil $d\phi_m/dt = 0$ and the current is zero.



5 ••

Determine the Concept Because the magnet moves with simple harmonic motion, the flux and the induced current will vary sinusoidally. The current will be a maximum wherever the flux is changing most rapidly and will be zero wherever the flux is momentarily constant.

(*a*), (*b*) The following graph shows the flux, ϕ_m, and the induced current (proportional to $-d\phi_m/dt$) in the loop as a function of time.



***6 •**

Determine the Concept The magnetic energy stored in an inductor is given by $U_m = \frac{1}{2}LI^2$. Doubling I quadruples U_m. $\boxed{(c)\text{ is correct.}}$

7 •

Determine the Concept The protection is needed because if the current is suddenly interrupted, the resulting emf generated across the inductor due to the large flux change can blow out the inductor. The diode allows the current to flow (in a loop) even when the switch is opened.

8 •

Determine the Concept The inductance of a coil depends on the product $n^2\ell$, where n is the number of turns per unit length and ℓ is the length of the coil. If n increases by a factor of 3, ℓ will decrease by the same factor, because the inductors are made from the same length of wire. Hence, the inductance increases by a factor of $(3)^2(1/3) = \boxed{3}$.

9 •

(*a*) False. The induced emf in a circuit is proportional to *the rate of change of* the magnetic flux through the circuit.

(*b*) True.

(*c*) True.

(*d*) False. The inductance of a solenoid is determined by its length, cross-sectional area, number of turns per unit length, and the permeability of the matter in its core.

(*e*) True.

***10 •**

Determine the Concept In the configuration shown in (*a*), energy is dissipated by eddy currents from the emf induced by the pendulum movement. In the configuration shown in (*b*), the slits inhibit the eddy currents and the braking effect is greatly reduced.

11 •

Determine the Concept The time varying magnetic field of the magnet sets up eddy currents in the metal tube. The eddy currents establish a magnetic field with a magnetic moment opposite to that of the moving magnet; thus the magnet is slowed down. If the tube is made of a nonconducting material, there are no eddy currents.

12 ••

Determine the Concept When the current is turned on, the increasing magnetic field in the coil induces a large emf in the ring. As described by Lenz's law, the direction of the current resulting from this induced emf is in such a direction that its magnetic field opposes the changing flux in the coil, i.e., the current induced in the ring will be in such a direction that the magnetic field in the coil will repel it. The demonstration will not work if a slot is cut in the ring, because the emf will not be able to induce a current in the ring.

Estimation and Approximation

***13 ••**

Picture the Problem We can use Faraday's law to relate the induced emf to the angular velocity with which the students turn the jump rope.

(*a*) It seems unlikely that the students could turn the "jump rope" wire faster than 5 revolutions per second. This corresponds to a maximum angular velocity of:

$$\omega = 5\frac{\text{rev}}{\text{s}}\times\frac{2\pi\text{ rad}}{\text{rev}} = \boxed{31.4\text{ rad/s.}}$$

(*b*) The magnetic flux ϕ_m through the rotating circular loop of wire varies sinusoidally with time according to:

$$\phi_m = BA\sin\omega t$$

and

$$\frac{d\phi_m}{dt} = BA\omega\cos\omega t$$

Because the average value of the cosine function, over one revolution, is ½, the average rate at which the flux changes through the circular loop is:

$$\left.\frac{d\phi_m}{dt}\right|_{av} = \tfrac{1}{2}BA\omega = \tfrac{1}{2}\pi r^2 B\omega$$

From Faraday's law, the magnitude of the induced emf in the loop is:

$$\mathcal{E} = \frac{d\phi_m}{dt} = \tfrac{1}{2}\pi r^2 B\omega$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = \tfrac{1}{2}\pi\left(\frac{1.5\text{ m}}{2}\right)^2\left(0.7\text{ G}\times\frac{1\text{ T}}{10^4\text{ G}}\right)(31.4\text{ rad/s}) = \boxed{1.94\text{ mV}}$$

(*c*) No. To generate an emf of 1 V, the students would have to rotate the jump rope about 500 times faster.

(*d*) The use of multiple strands of lighter wire (so that the composite wire could be rotated at the same angular speed) looped several times around would increase the induced emf.

14 •

Picture the Problem We can compare the energy density stored in the earth's electric field to that of the earth's magnetic field by finding their ratio.

The energy density in an electric field E is given by:

$$u_e = \frac{1}{2}\in_0 E^2$$

The energy density in a magnetic field B is given by:

$$u_m = \frac{B^2}{2\mu_0}$$

Express the ratio of u_m to u_e to obtain:

$$\frac{u_m}{u_e} = \frac{\frac{B^2}{2\mu_0}}{\frac{1}{2}\in_0 E^2} = \frac{B^2}{\mu_0 \in_0 E^2}$$

Substitute numerical values and evaluate u_m/u_e:

$$\frac{u_m}{u_e} == \frac{(5\times10^{-5}\text{ T})^2}{(4\pi\times10^{-7}\text{ N/A}^2)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(100\text{ V/m})^2} = 2.25\times10^4$$

or

$$u_m = \boxed{(2.25\times10^4)u_e}$$

15 ••

Picture the Problem We can apply Faraday's law to estimate the maximum emf induced by the lightning strike in the antenna.

Use Faraday's law to express the magnitude of the induced emf in antenna:

$$\mathcal{E} = \frac{d\phi_m}{dt} = \frac{d}{dt}[BA]$$

where A is the area of the antenna.

Because the lightning strike has such a short duration:

$$\mathcal{E} \approx \frac{BA}{\Delta t}$$

The magnetic field induced in the loop is given by:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

where r is the distance from the antenna to the lightning strike.

Substitute for B to obtain:

$$\mathcal{E} = \frac{\mu_0 IA}{2\pi r\Delta t}$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = \frac{(4\pi\times10^{-7}\text{ N/A}^2)\left(\frac{30\text{ C}}{1\,\mu\text{s}}\right)(0.1\text{ m}^2)}{2\pi(300\text{ m})(1\,\mu\text{s})} = \boxed{2.00\text{ kV}}$$

Magnetic Flux

16 •

Picture the Problem Because the surface is a plane with area A and $\vec{B}$ is constant in magnitude and direction over the surface and makes an angle θ with the unit normal vector, we can use $\phi_m = BA\cos\theta$ to find the magnetic flux through the coil.

Substitute for B and A to obtain:

$$\phi_m = \left(2000\,\text{G}\cdot\frac{1\,\text{T}}{10^4\,\text{G}}\right)\left(5\times10^{-2}\,\text{m}\right)^2\cos\theta = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos\theta$$

(*a*) For $\theta = 0°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 0° = 5.00\times10^{-4}\,\text{Wb} = \boxed{0.500\,\text{mWb}}$$

(*b*) For $\theta = 30°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 30° = 4.33\times10^{-4}\,\text{Wb} = \boxed{0.433\,\text{mWb}}$$

(*c*) For $\theta = 60°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 60° = 2.50\times10^{-4}\,\text{Wb} = \boxed{0.250\,\text{mWb}}$$

(*d*) For $\theta = 90°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 90° = \boxed{0}$$

***17** •

Picture the Problem Because the coil defines a plane with area A and $\vec{B}$ is constant in magnitude and direction over the surface and makes an angle θ with the unit normal vector, we can use $\phi_m = NBA\cos\theta$ to find the magnetic flux through the coil.

Substitute for N, B, and A to obtain:

$$\phi_m = NB\pi r^2\cos\theta = 25\left(0.7\,\text{G}\cdot\frac{1\,\text{T}}{10^4\,\text{G}}\right)\pi\left(5\times10^{-2}\,\text{m}\right)^2\cos\theta = \left(1.37\times10^{-5}\,\text{Wb}\right)\cos\theta$$

(*a*) When the plane of the coil is horizontal, $\theta = 90°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 90° = \boxed{0}$$

(*b*) When the plane of the coil is vertical with its axis pointing north, $\theta = 0°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 0° = \boxed{1.37\times10^{-5}\ \text{Wb}}$$

(*c*) When the plane of the coil is vertical with its axis pointing east, $\theta = 90°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 90° = \boxed{0}$$

(*d*) When the plane of the coil is vertical with its axis making an angle of 30° with north, $\theta = 30°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 30° = \boxed{1.19\times10^{-5}\ \text{Wb}}$$

18 •

Picture the Problem Because the square coil defines a plane with area A and $\vec{B}$ is constant in magnitude and direction over the surface and makes an angle θ with the unit normal vector, we can use $\phi_m = NBA\cos\theta$ to find the magnetic flux through the coil.

Substitute for N, B, and A to obtain:

$$\phi_m = NBA\cos\theta = 14(1.2\ \text{T})(5\times10^{-2}\ \text{m})^2\cos\theta = (42.0\ \text{mWb})\cos\theta$$

(*a*) For $\theta = 0°$:

$$\phi_m = (42.0\ \text{mWb})\cos 0° = \boxed{42.0\ \text{mWb}}$$

(*b*) For $\theta = 60°$:

$$\phi_m = (42.0\ \text{mWb})\cos 60° = \boxed{21.0\ \text{mWb}}$$

19 •

Picture the Problem Noting that the flux through the base must also penetrate the spherical surface, we can apply its definition to express ϕ_m.

Apply the definition of magnetic flux to obtain:

$$\phi_m = AB = \boxed{\pi R^2 B}$$

20 ••

Picture the Problem We can use $\phi_m = NBA\cos\theta$ to express the magnetic flux through the solenoid and $B = \mu_0 nI$ to relate the magnetic field in the solenoid to the current in its coils.

Express the magnetic flux through a coil with N turns:

$$\phi_m = NBA\cos\theta$$

Express the magnetic field inside a long solenoid:

$$B = \mu_0 nI$$

where n is the number of turns per unit length.

Substitute to obtain:

$$\phi_m = N\mu_0 nIA\cos\theta$$

or, because $n = N/L$ and $\theta = 0°$,

$$\phi_m = \frac{N^2\mu_0 IA}{L} = \frac{N^2\mu_0 I\pi r^2}{L}$$

Substitute numerical values and evaluate ϕ_m:

$$\phi_m = \frac{(400)^2\left(4\pi\times10^{-7}\text{ N/A}^2\right)(3\text{ A})\pi(0.01\text{ m})^2}{0.25\text{ m}} = \boxed{7.58\times10^{-4}\text{ Wb}}$$

21 ••

Picture the Problem We can use $\phi_m = NBA\cos\theta$ to express the magnetic flux through the solenoid and $B = \mu_0 nI$ to relate the magnetic field in the solenoid to the current in its coils.

Express the magnetic flux through a coil with N turns:

$$\phi_m = NBA\cos\theta$$

Express the magnetic field inside a long solenoid:

$$B = \mu_0 nI$$

where n is the number of turns per unit length.

Substitute to obtain:

$$\phi_m = N\mu_0 nIA\cos\theta$$

or, because $n = N/L$ and $\theta = 0°$,

$$\phi_m = \frac{N^2\mu_0 IA}{L} = \frac{N^2\mu_0 I\pi r^2}{L}$$

Substitute numerical values and evaluate ϕ_m:

$$\phi_m = \frac{(800)^2 (4\pi \times 10^{-7}\ \text{N/A}^2)(2\ \text{A})\pi(0.02\ \text{m})^2}{0.3\ \text{m}} = \boxed{6.74 \times 10^{-3}\ \text{Wb}}$$

22 ••

Picture the Problem We can apply the definitions of magnet flux and of the dot product to find the flux for the given unit vectors.

Apply the definition of magnetic flux to the coil to obtain:

$$\phi_m = N\int_S \vec{B} \cdot \hat{n}\,dA$$

Because $\vec{B}$ is constant:

$$\phi_m = N\vec{B} \cdot \hat{n}\int_S dA = N(\vec{B} \cdot \hat{n})A = N(\vec{B} \cdot \hat{n})\pi r^2$$

Evaluate $\vec{B}$:

$$\vec{B} = (0.4T)\hat{i}$$

Substitute numerical values and simplify to obtain:

$$\phi_m = (15)[(0.4\ \text{T})]\pi(0.04\ \text{m})^2 = (0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \hat{n}$$

(*a*) Evaluate ϕ_m for $\hat{n} = \hat{i}$:

$$\phi_m = (0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \hat{i} = \boxed{0.0302\ \text{Wb}}$$

(*b*) Evaluate ϕ_m for $\hat{n} = \hat{j}$:

$$\phi_m = (0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \hat{j} = \boxed{0}$$

(*c*) Evaluate ϕ_m for $\hat{n} = (\hat{i} + \hat{j})/\sqrt{2}$:

$$\phi_m = (0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \frac{(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{0.0302\ \text{T}\cdot\text{m}^2}{\sqrt{2}} = \boxed{0.0213\ \text{Wb}}$$

(*d*) Evaluate ϕ_m for $\hat{n} = \hat{k}$:

$$\phi_m = (0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \hat{k} = \boxed{0}$$

(*e*) Evaluate ϕ_m for $\hat{n} = 0.6\hat{i} + 0.8\hat{j}$:

$$\begin{aligned}\phi_m &= (0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot (0.6\hat{i} + 0.8\hat{j}) = 0.6(0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \hat{i} \\ &\quad + 0.8(0.0302\ \text{T}\cdot\text{m}^2)\hat{i} \cdot \hat{j} \\ &= 0.6(0.0302\ \text{T}\cdot\text{m}^2) = \boxed{0.0181\ \text{Wb}}\end{aligned}$$

23 ••

Picture the Problem The magnetic field outside the solenoid is, to a good approximation, zero. Hence, the flux through the loop is the flux in the core of the solenoid. The magnetic field inside the solenoid is uniform. Hence, the flux through this small loop is given by the same expression with R_3 replacing R_1:

(*a*) Express the flux through the large circular loop outside the solenoid:

$$\phi_m = NBA = \boxed{\mu_0 nIN\pi R_1^2}$$

(*b*) Express the flux through the small loop inside the solenoid:

$$\phi_m = NBA = \boxed{\mu_0 nIN\pi R_3^2}$$

***24 ••**

Picture the Problem We can use the hint to set up the element of area dA and express the flux $d\phi_m$ through it and then carry out the details of the integration to express ϕ_m.

(*a*) Express the flux through the strip of area dA:

$$d\phi_m = BdA$$

where $dA = bdx$.

Express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

Substitute to obtain:

$$d\phi_m = \frac{\mu_0}{2\pi}\frac{I}{x}bdx = \frac{\mu_0 Ib}{2\pi}\frac{dx}{x}$$

Integrate from $x = d$ to $x = d + a$:

$$\phi_m = \frac{\mu_0 Ib}{2\pi}\int_d^{d+a}\frac{dx}{x} = \boxed{\frac{\mu_0 Ib}{2\pi}\ln\frac{d+a}{d}}$$

(*b*) Substitute numerical values and evaluate ϕ_m:

$$\phi_m = \frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)(20\text{ A})(0.1\text{ m})}{2\pi}\ln\left(\frac{7\text{ cm}}{2\text{ cm}}\right) = \boxed{5.01\times10^{-7}\text{ Wb}}$$

25 •••

Picture the Problem Consider an element of area $dA = Ldr$ where $r \le R$. We can use its definition to express $d\phi_m$ through this area in terms of B and Ampere's law to express B as a function of I. The fact that the current is uniformly distributed over the cross-sectional area of the conductor allows us to set up a proportion from which we can obtain I as a function of r. With these substitutions in place we can integrate $d\phi_m$ to obtain ϕ_m/L.

Express the flux $d\phi_m$ through an area Ldr:

$$d\phi_m = BdA = BLdr \quad (1)$$

Apply Ampere's law to the current contained inside a cylindrical region of radius $r < R$:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\pi rB = \mu_0 I_C$$

and

$$B = \frac{\mu_0 I_C}{2\pi r}$$

Using the fact that the current I is uniformly distributed over the cross-sectional area of the conductor, express its variation with distance r from the center of the conductor:

$$\frac{I(r)}{I} = \frac{\pi r^2}{\pi R^2}$$

or

$$I(r) = I_C = I\frac{r^2}{R^2}$$

Substitute and simplify to obtain:

$$B = \frac{\mu_0 I}{2\pi r}\frac{r^2}{R^2} = \frac{\mu_0 I}{2\pi R^2} r$$

Substitute in equation (1):

$$d\phi_m = \frac{\mu_0 LI}{2\pi R^2} rdr$$

Integrate $d\phi_m$ from $r = 0$ to $r = R$ to obtain:

$$\phi_m = \frac{\mu_0 LI}{2\pi R^2}\int_0^R rdr = \frac{\mu_0 LI}{4\pi}$$

Divide both sides of this equation by L to express the magnetic flux per unit length:

$$\frac{\phi_m}{L} = \boxed{\frac{\mu_0 I}{4\pi}}$$

26 •••

Picture the Problem We can use its definition to express the flux through the rectangular region and Ampere's law to relate the magnetic field to the current in the wire and the position of the long straight wire.

(*a*) Note that for $0 \le x \le b$, B is symmetric about the wire, into the paper for the region below the wire and out of the paper for the region above the wire. Thus, for the area $2(b - x)a$:

$$\phi_{m,net} = 0$$

To find the flux through the

$$d\phi_m = BdA$$

remaining area of the rectangle, express the flux through a strip of area dA:

where $dA = adx$.

Using Ampere's law, express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

Substitute to obtain:

$$d\phi_{\rm m} = \frac{\mu_0}{2\pi}\frac{I}{x}adx = \frac{\mu_0 Ia}{2\pi}\frac{dx}{x}$$

For $0 \le x \le b$, integrate from $x' = b - x$ to $x' = x$:

$$\phi_{\rm m}(0 \le x \le b) = \frac{\mu_0 Ia}{2\pi}\int_{b-x}^{x}\frac{dx'}{x'}$$

$$= \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(\frac{x}{b-x}\right)}$$

For $x \ge b$, integrate from $x' = x$ to $x' = x + b$:

$$\phi_{\rm m}(x \ge b) = \frac{\mu_0 Ia}{2\pi}\int_{x}^{x+b}\frac{dx'}{x'}$$

$$= \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(\frac{x+b}{x}\right)}$$

(*b*) From the expressions derived in (*a*) we see that $|\phi_{\rm m}| \to \infty$ as:

$$\boxed{x \to 0}$$

The flux is a minimum ($\phi_{\rm m} = 0$) for:

$\boxed{x = \frac{1}{2}b}$ as expected from symmetry.

Induced EMF and Faraday's Law

***27 •**

Picture the Problem We can find the induced emf by applying Faraday's law to the loop. The application of Ohm's law will yield the induced current in the loop and we can find the rate of joule heating using $P = I^2R$.

(*a*) Apply Faraday's law to express the induced emf in the loop in terms of the rate of change of the magnetic field:

$$|\mathcal{E}| = \frac{d\phi_{\rm m}}{dt} = \frac{d}{dt}(AB) = A\frac{dB}{dt} = \pi R^2\frac{dB}{dt}$$

Substitute numerical values and

$$|\mathcal{E}| = \pi(0.05\,{\rm m})^2(40\,{\rm mT/s}) = \boxed{0.314\,{\rm mV}}$$

evaluate $|\mathcal{E}|$:

(*b*) Using Ohm's law, relate the induced current to the induced voltage and the resistance of the loop and evaluate *I*:

$$I = \frac{\mathcal{E}}{R} = \frac{0.314\,\text{mV}}{0.4\,\Omega} = \boxed{0.785\,\text{mA}}$$

(*c*) Express the rate at which power is dissipated in a conductor in terms of the induced current and the resistance of the loop and evaluate *P*:

$$P = I^2R = (0.785\,\text{mA})^2(0.4\,\Omega) = \boxed{0.247\,\mu\text{W}}$$

28 •

Picture the Problem Given ϕ_m as a function of time, we can use Faraday's law to express $\mathcal{E}$ as a function of time.

(*a*) Apply Faraday's law to express the induced emf in the loop in terms of the rate of change of the magnetic field:

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -\frac{d}{dt}\left[\left(t^2 - 4t\right)\times 10^{-1}\,\text{Wb}\right] = -(2t-4)\times 10^{-1}\,\text{Wb/s} = \boxed{-(0.2t-0.4)\,\text{V}}$$

(*b*) Evaluate ϕ_m at $t = 0$:

$$\phi_m(0\,\text{s}) = \left[(0)^2 - 4(0)\right]\times 10^{-1}\,\text{Wb} = \boxed{0}$$

Evaluate $\mathcal{E}$ at $t = 0$:

$$\mathcal{E}(0\,\text{s}) = -[0.2(0) - 0.4]\text{V} = \boxed{0.400\,\text{V}}$$

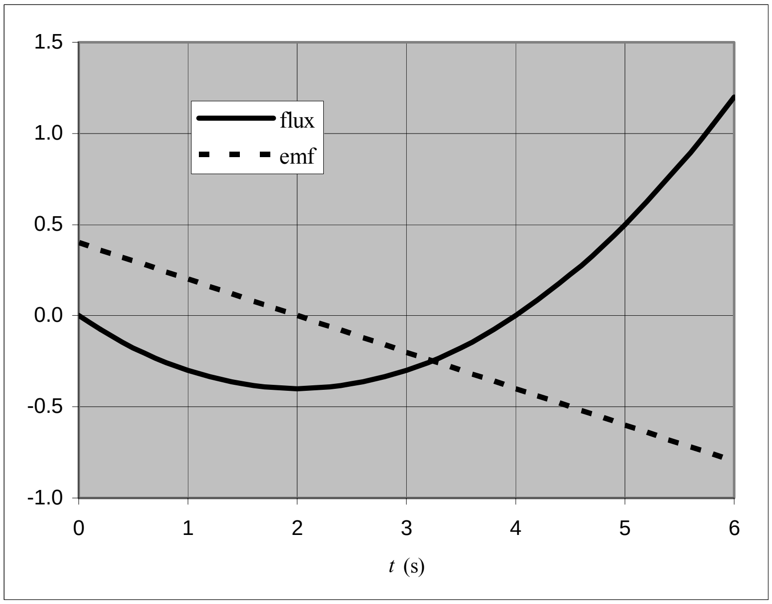
Proceed as above to complete the table to the right:

t	ϕ_m	$\mathcal{E}$
(s)	(Wb)	(V)
0	0	0
2	–0.400	0
4	0	–0.400
6	1.20	–0.800

29 •

Picture the Problem We can find the time at which the flux is a minimum by looking for the lowest point on the graph of $\mathcal{E}$ versus *t* and the emf at this time by determining the value of *V* at this time from the graph. We can interpret the graphs to find the times at which the flux is zero and the corresponding values of the emf.

(a) The flux, ϕ_m, and the induced emf, $\mathcal{E}$, are shown as functions of t in the following graph. The solid curve represents ϕ_m, the dashed curve represents $\mathcal{E}$.



(b) Referring to the graph, we see that the flux is a minimum at $t = 2$ s and that $V = 0$ at this instant.

(c) The flux is zero at $t = 0$ and $t = 4$ s. At these times, $\mathcal{E} = 0.4$ V and -0.4 V, respectively.

30 •

Picture the Problem We can use its definition to find the magnetic flux through the solenoid and Faraday's law to find the emf induced in the solenoid when the external field is reduced to zero in 1.4 s.

(a) Express the magnetic flux through the solenoid in terms of N, B, A, and θ:

$$\phi_m = NBA\cos\theta = NB\pi R^2 \cos\theta$$

Substitute numerical values and evaluate ϕ_m:

$$\phi_m = (400)(0.06\,\text{T})\pi(0.008\,\text{m})^2 \cos 50° = \boxed{3.10\,\text{mWb}}$$

(*b*) Apply Faraday's law to obtain:

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -\frac{0 - 3.10\,\text{mWb}}{1.4\,\text{s}} = \boxed{2.22\,\text{mV}}$$

***31 ••**

Picture the Problem We can use the definition of average current to express the total charge passing through the coil as a function of I_{av}. Because the induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday's law, we can express ΔQ as a function of the number of turns of the coil, the magnetic field, the resistance of the coil, and the area of the coil. Knowing the reversal time, we can find the average current from its definition and the average emf in the coil from Ohm's law.

(*a*) Express the total charge that passes through the coil in terms of the induced current:

$$\Delta Q = I_{av}\Delta t$$

Relate the induced current to the induced emf:

$$I = I_{av} = \frac{\mathcal{E}}{R}$$

Using Faraday's law, express the induced emf in terms of ϕ_m:

$$\mathcal{E} = -\frac{\Delta\phi_m}{\Delta t}$$

Substitute and simplify to obtain:

$$\Delta Q = \frac{\mathcal{E}}{R}\Delta t = \frac{-\frac{\Delta\phi_m}{\Delta t}}{R}\Delta t = -\frac{2\phi_m}{R} = -\frac{2NBA}{R} = -\frac{2NB\left(\frac{\pi}{4}d^2\right)}{R} = -\frac{NB\pi d^2}{2R}$$

where d is the diameter of the coil.

Substitute numerical values and evaluate ΔQ:

$$\Delta Q = -\frac{(100)(1\,\text{T})\pi(0.02\,\text{m})^2}{2(50\,\Omega)} = \boxed{-1.26\,\text{mC}}$$

(*b*) Apply the definition of average current to obtain:

$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{1.26\,\text{mC}}{0.1\,\text{s}} = \boxed{12.6\,\text{mA}}$$

(*c*) Using Ohm's law, relate the average emf in the coil to the average current:

$$\mathcal{E}_{av} = I_{av}R = (12.6\,\text{mA})(50\,\Omega) = \boxed{630\,\text{mV}}$$

32 ••

Picture the Problem We can use the definition of average current to express the total charge passing through the coil as a function of I_{av}. Because the induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday's law, we can express ΔQ as a function of the number of turns of the coil, the magnetic field, the resistance of the coil, and the area of the coil.

Express the total charge that passes through the coil in terms of the induced current:

$$\Delta Q = I_{av}\Delta t$$

Relate the induced current to the induced emf:

$$I = I_{av} = \frac{\mathcal{E}}{R}$$

Using Faraday's law, express the induced emf in terms of ϕ_m:

$$\mathcal{E} = -\frac{\Delta\phi_m}{\Delta t}$$

Substitute to obtain:

$$\Delta Q = \frac{\mathcal{E}}{R}\Delta t = \frac{-\frac{\Delta\phi_m}{\Delta t}}{R}\Delta t = -\frac{2\phi_m}{R} = -\frac{2NBA}{R}$$

Substitute numerical values and evaluate ΔQ:

$$\Delta Q = \left|-\frac{2(1000)(0.7\times10^{-4}\,\text{T})(300\times10^{-4}\,\text{m}^2)}{15\,\Omega}\right| = \boxed{0.280\,\text{mC}}$$

33 ••

Picture the Problem We can use Faraday's law to express the earth's magnetic field at this location in terms of the induced emf and Ohm's law to relate the induced emf to the charge that passes through the current integrator.

Using Faraday's law, express the induced emf in terms of the change in the magnetic flux as the coil is rotated through 90°:

$$\mathcal{E} = \left|-\frac{\Delta\phi_m}{\Delta t}\right| = \frac{NBA}{\Delta t} = \frac{NB\pi r^2}{\Delta t}$$

Solve for B:

$$B=\frac{\mathcal{E}\Delta t}{N\pi r^2}$$

Using Ohm's law, relate the induced emf to the induced current:

$$\mathcal{E}=IR=\frac{\Delta Q}{\Delta t}R$$

where ΔQ is the charge that passes through the current integrator.

Substitute to obtain:

$$B=\frac{\frac{\Delta Q}{\Delta t}R\Delta t}{N\pi r^2}=\frac{\Delta QR}{N\pi r^2}$$

Substitute numerical values and evaluate B:

$$B=\frac{(9.4\,\mu\text{C})(20\,\Omega)}{(300)\pi(0.05\,\text{m})^2}=\boxed{79.8\,\mu\text{T}}$$

34 ••

Picture the Problem We can use Faraday's law to express the induced emf in the coil in terms of the rate of change of the magnetic flux. We can use its definition to express the magnetic flux through the rectangular region and Ampere's law to relate the magnetic field to the current in the wire and the position of the long straight wire.

(*a*) Apply Faraday's law to relate the induced emf to the changing magnetic flux:

$$\mathcal{E}=-\frac{d\phi_\text{m}}{dt} \qquad (1)$$

Note that for $0\le x\le b$, B is symmetric about the wire, into the paper for the region below the wire and out of the paper for the region above the wire. Thus, for the area $2(b-x)a$:

$$\phi_\text{m,net}=0$$

To find the flux through the remaining area of the rectangle, express the flux through a strip of area dA:

$$d\phi_\text{m}=BdA$$

where $dA = adx$.

Using Ampere's law, express B at a distance x from a long, straight wire:

$$B=\frac{\mu_0}{4\pi}\frac{2I}{x}=\frac{\mu_0}{\pi}\frac{t}{x}$$

Substitute to obtain:

$$d\phi_{\text{m}} = \frac{\mu_0}{\pi}\frac{t}{x}a\,dx = \frac{\mu_0 ta}{\pi}\frac{dx}{x}$$

For $0 \le x \le b$, integrate from $x' = b - x$ to $x' = x$:

$$\phi_{\text{m}}(0 \le x \le b) = \frac{\mu_0 ta}{\pi}\int_{b-x}^{x}\frac{dx'}{x'}$$
$$= \frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Differentiate this expression with respect to time to obtain:

$$\frac{d\phi_{\text{m}}}{dt} = \frac{d}{dt}\left[\frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)\right]$$
$$= \frac{\mu_0 a}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Substitute in equation (1) and evaluate $\mathcal{E}$ for $x = b/4$:

$$\mathcal{E} = -\frac{\mu_0 a}{\pi}\ln\left(\frac{b/4}{b-b/4}\right) = -\frac{\mu_0 a}{\pi}\ln\left(\frac{1}{3}\right)$$
$$= \boxed{1.10\frac{\mu_0 a}{\pi}}$$

(*b*) Using Ohm's law, express and evaluate *R*:

$$R = \frac{\mathcal{E}}{I} = \frac{1.10\mu_0 a}{\pi I}$$
$$= \frac{1.10(4\pi\times10^{-7}\ \text{N/A}^2)(1.5\,\text{m})}{\pi(0.1\,\text{A})}$$
$$= \boxed{6.60\ \mu\Omega}$$

Because the magnetic flux due to *I* is increasing into the page, the induced current will be in such a direction that its magnetic field will oppose this increase; i.e, it will be out of the page. Thus the induced current is counterclockwise.

35 ••

Picture the Problem We can use Faraday's law to express the induced emf in the coil in terms of the rate of change of the magnetic flux. We can use its definition to express the magnetic flux through the rectangular region and Ampere's law to relate the magnetic field to the current in the wire and the position of the long straight wire.

(*a*) Apply Faraday's law to relate the induced emf to the changing magnetic flux:

$$\mathcal{E} = -\frac{d\phi_{\text{m}}}{dt} \qquad (1)$$

Note that for $0 \le x \le b$, B is symmetric about the wire, into the paper for the region below the wire and out of the paper for the region above the wire. Thus, for the area $2(b-x)a$:

$$\phi_{\mathrm{m,net}} = 0$$

To find the flux through the remaining area of the rectangle, express the flux through a strip of area dA:

$$d\phi_{\mathrm{m}} = B\,dA$$

where $dA = a\,dx$.

Using Ampere's law, express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{\pi}\frac{t}{x}$$

Substitute to obtain:

$$d\phi_{\mathrm{m}} = \frac{\mu_0}{\pi}\frac{t}{x}a\,dx = \frac{\mu_0 ta}{\pi}\frac{dx}{x}$$

For $0 \le x \le b$, integrate from $x' = b - x$ to $x' = x$:

$$\phi_{\mathrm{m}}(0 \le x \le b) = \frac{\mu_0 ta}{\pi}\int_{b-x}^{x}\frac{dx'}{x'} = \frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Differentiate this expression with respect to time to obtain:

$$\frac{d\phi_{\mathrm{m}}}{dt} = \frac{d}{dt}\left[\frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)\right] = \frac{\mu_0 a}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Substitute in equation (1) and evaluate $\mathcal{E}$ for $x = b/3$:

$$\mathcal{E} = -\frac{\mu_0 a}{\pi}\ln\left(\frac{b/3}{b-b/3}\right) = -\frac{\mu_0 a}{\pi}\ln\left(\frac{1}{2}\right) = \boxed{0.693\frac{\mu_0 a}{\pi}}$$

(*b*) Using Ohm's law, express and evaluate R:

$$R = \frac{\mathcal{E}}{I} = \frac{0.693\mu_0 a}{\pi I} = \frac{0.693\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)(1.5\,\mathrm{m})}{\pi(0.1\,\mathrm{A})} = \boxed{4.16\,\mu\Omega}$$

Because the magnetic flux due to I is increasing into the page, the induced current will be in such a direction that its magnetic field will oppose this increase, i.e, it will be out of the page. Thus, the induced current is counterclockwise.

Motional EMF

***36 •**

Picture the Problem We can apply the equation for the force on a charged particle moving in a magnetic field to find the magnetic force acting on an electron in the rod. We can use $\vec{E} = \vec{v} \times \vec{B}$ to find E and $V = E\ell$, where ℓ is the length of the rod, to find the potential difference between its ends.

(*a*) Relate the magnetic force on an electron in the rod to the speed of the rod, the electronic charge, and the magnetic field in which the rod is moving:

$$\vec{F} = q\vec{v} \times \vec{B}$$

and

$$F = qvB\sin\theta$$

Substitute numerical values and evaluate F:

$$F = \left(1.6\times10^{-19}\ \text{C}\right)\left(8\ \text{m/s}\right)\left(0.05\ \text{T}\right)\sin 90^\circ = \boxed{6.40\times10^{-20}\ \text{N}}$$

(*b*) Express the electrostatic field $\vec{E}$ in the rod in terms of the magnetic field $\vec{B}$:

$$\vec{E} = \vec{v} \times \vec{B}$$

and

$$E = vB\sin\theta$$

Substitute numerical values and evaluate B:

$$E = \left(8\ \text{m/s}\right)\left(0.05\ \text{T}\right)\sin 90^\circ = \boxed{0.400\ \text{V/m}}$$

(*c*) Relate the potential difference between the ends of the rod to its length ℓ and the electric field E:

$$V = E\ell$$

Substitute numerical values and evaluate V:

$$V = \left(0.4\ \text{V/m}\right)\left(0.3\ \text{m}\right) = \boxed{0.120\ \text{V}}$$

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Picture the Problem We can use $\vec{E} = \vec{v} \times \vec{B}$ to relate the speed of the rod to the electric field in the rod and magnetic field in which it is moving and $V = E\ell$ to relate the electric field in the rod to the potential difference between its ends.

Express the electrostatic field $\vec{E}$ in the rod in terms of the magnetic field $\vec{B}$ and solve for v:

$$\vec{E} = \vec{v} \times \vec{B}$$

and

$$v = \frac{E}{B \sin\theta}$$

Relate the potential difference between the ends of the rod to its length ℓ and the electric field E and solve for E:

$$V = E\ell \Rightarrow E = \frac{V}{\ell}$$

Substitute for E to obtain:

$$v = \frac{V}{B\ell \sin\theta}$$

Substitute numerical values and evaluate v:

$$v = \frac{6\,\text{V}}{(0.05\,\text{T})(0.3\,\text{m})} = \boxed{400\,\text{m/s}}$$

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Picture the Problem Because the speed of the rod is constant, an external force must act on the rod to counter the magnetic force acting on the induced current. We can use the motional-emf equation $\mathcal{E} = vB\ell$ to evaluate the induced emf, Ohm's law to find the current in the circuit, Newton's 2nd law to find the force needed to move the rod with constant velocity, and $P = Fv$ to find the power input by the force.

(*a*) Relate the induced emf in the circuit to the speed of the rod, the magnetic field, and the length of the rod:

$$\mathcal{E} = vB\ell = (10\,\text{m/s})(0.8\,\text{T})(0.2\,\text{m}) = \boxed{1.60\,\text{V}}$$

(*b*) Using Ohm's law, relate the current in the circuit to the induced emf and the resistance of the circuit:

$$I = \frac{\mathcal{E}}{R} = \frac{1.6\,\text{V}}{2\,\Omega} = \boxed{0.800\,\text{A}}$$

Note that, because the rod is moving to the right, the flux in the region defined by the rod, the rails, and the resistor is increasing. Hence, in accord with Lenz's law, the current must be counterclockwise if its magnetic field is to counter this increase in flux.

(*c*) Because the rod is moving with constant velocity, the net force acting on it must be zero. Apply

$$\sum F_x = F - F_\text{m} = 0$$

and

Newton's 2nd law to relate F to the magnetic force F_m:

$$F = F_m = BI\ell$$
$$= (0.8\,\text{T})(0.8\,\text{A})(0.2\,\text{m}) = \boxed{0.128\,\text{N}}$$

(*d*) Express the power input by the force in terms of the force and the velocity of the rod:

$$P = Fv = (0.128\,\text{N})(10\,\text{m/s}) = \boxed{1.28\,\text{W}}$$

(*e*) The rate of Joule heat production is given by:

$$P = I^2R = (0.8\,\text{A})^2(2\,\Omega) = \boxed{1.28\,\text{W}}$$

39 ••

Picture the Problem We'll need to determine how long it takes for the loop to completely enter the region in which there is a magnetic field, how long it is in the region, and how long it takes to leave the region. Once we know these times, we can use its definition to express the magnetic flux as a function of time. We can use Faraday's law to find the induced emf as a function of time.

(*a*) Find the time required for the loop to enter the region where there is a uniform magnetic field:

$$t = \frac{\ell_{\text{side of loop}}}{v} = \frac{10\,\text{cm}}{2.4\,\text{cm/s}} = 4.17\,\text{s}$$

Letting w represent the width of the loop, express and evaluate ϕ_m for $0 < t < 4.17\,\text{s}$:

$$\phi_m = NBA = NBwvt$$
$$= (1.7\,\text{T})(0.05\,\text{m})(0.024\,\text{m/s})t$$
$$= (2.04\,\text{mWb/s})t$$

Find the time during which the loop is fully in the region where there is a uniform magnetic field:

$$t = \frac{\ell_{\text{side of loop}}}{v} = \frac{10\,\text{cm}}{2.4\,\text{cm/s}} = 4.17\,\text{s}$$

i.e., the loop will begin to exit the region when t = 8.33 s.

Express ϕ_m for $4.17\,\text{s} < t < 8.33\,\text{s}$:

$$\phi_m = NBA = NB\ell w$$
$$= (1.7\,\text{T})(0.1\,\text{m})(0.05\,\text{m})$$
$$= 8.50\,\text{mWb}$$

The left-end of the loop will exit the field when t = 12.5 s. Express ϕ_m for $8.33\,\text{s} < t < 12.5\,\text{s}$:

$$\phi_m = mt + b$$

where m is the slope of the line and b is the ϕ_m-intercept.

For t = 8.33 s and ϕ_m = 8.50 mWb:

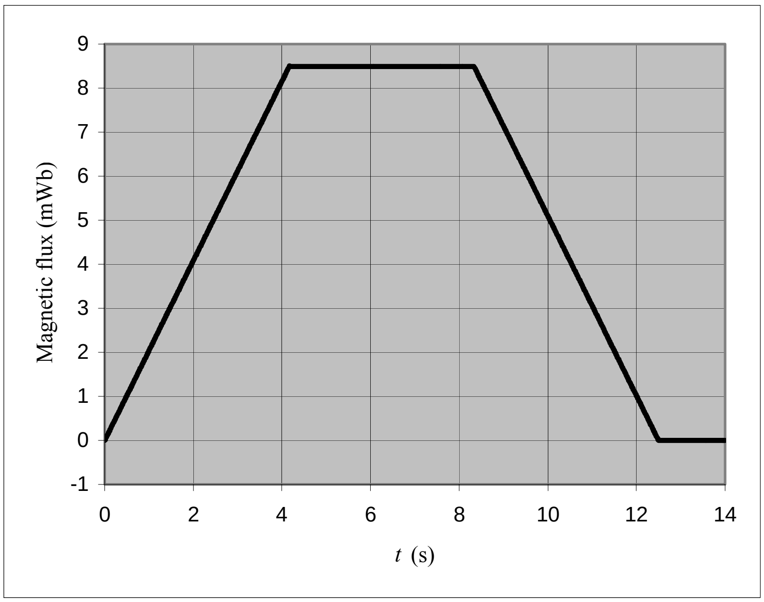
$$8.50\,\text{mWb} = m(8.33\,\text{s}) + b \qquad (1)$$

For t = 12.5 s and ϕ_m = 0: $0 = m(12.5\,\text{s}) + b$ (2)

Solve equations (1) and (2) simultaneously to obtain: $\phi_m = -(2.04\,\text{mWb/s})t + 25.5\,\text{mWb}$

The loop will be completely out of the magnetic field when $t > 12.5$ s and: $\phi_m = 0$

The following graph of $\phi_m(t)$ was plotted using a spreadsheet program.



(*b*) Using Faraday's law, relate the induced emf to the magnetic flux: $\mathcal{E} = -\frac{d\phi_m}{dt}$

During the interval $0 < t < 4.17\,\text{s}$: $\mathcal{E} = -\frac{d}{dt}[(2.04\,\text{mWb/s})t] = -2.04\,\text{mV}$

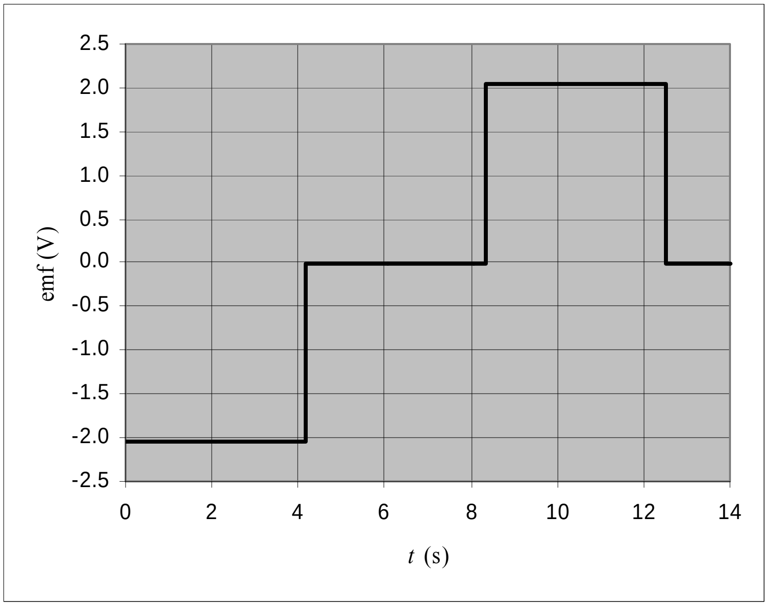
During the interval $4.17\,\text{s} < t < 8.33\,\text{s}$: $\mathcal{E} = -\frac{d}{dt}[8.50\,\text{mWb}] = 0$

During the interval $8.33\,\text{s} < t < 12.5\,\text{s}$:

$$\mathcal{E} = -\frac{d}{dt}[(-2.04\,\text{mWb/s})t + 25.5\,\text{mWb}] = 2.04\,\text{mV}$$

For $t > 12.5$ s: $\mathcal{E} = 0$

The following graph of $\varepsilon(t)$ was plotted using a spreadsheet program.



40 ••

Picture the Problem The rod is executing simple harmonic motion in the *xy* plane, i.e., in a plane perpendicular to the magnetic field. The emf induced in the rod is a consequence of its motion in this magnetic field and is given by $|\mathcal{E}| = vB\ell$. Because we're given the position of the oscillator as a function of time, we can differentiate this expression to obtain *v*.

Express the motional emf in terms of *v*, *B*, and ℓ:

$$|\mathcal{E}| = vB\ell = B\ell\frac{dx}{dt}$$

Evaluate *dx*/*dt*:

$$\frac{dx}{dt} = \frac{d}{dt}\left[(2\,\text{cm})\cos 120\pi t\right]$$
$$= -(2\,\text{cm})\left(120\,\text{s}^{-1}\right)\pi\sin 120\pi t$$
$$= -(7.54\,\text{m/s})\sin 120\pi t$$

Substitute numerical values and evaluate $|\mathcal{E}|$:

$$|\mathcal{E}| = -(1.2\,\text{T})(0.15\,\text{m})(7.54\,\text{m/s})\sin 120\pi t = \boxed{-(1.36\,\text{V})\sin 120\pi t}$$

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Picture the Problem Let *m* be the mass of the rod and *F* be the net force acting on it due to the current in it. We can obtain the equation of motion of the rod by applying

Newton's 2nd law to relate its acceleration to B, I, and ℓ. The net emf that drives I in this circuit is the emf of the battery minus the emf induced in the rod as a result of its motion.

(*a*) Letting the direction of motion of the rod be the positive x direction, apply $\sum F_x = ma_x$ to the rod:

$$BI\ell = m\frac{dv}{dt} \qquad (1)$$

where

$$I = \frac{\mathcal{E} - B\ell v}{R} \qquad (2)$$

Substitute to obtain:

$$\boxed{\frac{dv}{dt} = \frac{B\ell}{mR}(\mathcal{E} - B\ell v)}$$

(*b*) Express the condition on dv/dt when the rod has achieved its terminal speed:

$$\frac{B\ell}{mR}(\mathcal{E} - B\ell v_t) = 0$$

Solve for v_t to obtain:

$$v_t = \boxed{\frac{\mathcal{E}}{B\ell}}$$

(*c*) Substitute v_t for v in equation (2) to obtain:

$$I = \frac{\mathcal{E} - B\ell\dfrac{\mathcal{E}}{B\ell}}{R} = \boxed{0}$$

***42 ••**

Picture the Problem In Example 28-9 it is shown that the speed of the rod is given by $v = v_0 e^{-(B^2\ell^2/mR)t}$. We can use the definition of power and the expression for a motional emf to express the power dissipated in the resistance in terms of B, ℓ, v, and R. We can then separate the variables and integrate over all time to show that the total energy dissipated is equal to the initial kinetic energy of the rod.

Express the power dissipated in terms of ε and R:

$$P = \frac{\mathcal{E}^2}{R}$$

Express ε as a function of B, ℓ, and v:

$$\mathcal{E} = B\ell v$$

where

$$v = v_0 e^{-(B^2\ell^2/mR)t}$$

Substitute to obtain:

$$P = \frac{(B\ell v)^2}{R}$$

The total energy dissipated as the rod comes to rest is obtained by integrating $dE = P\,dt$:

$$E = \int_0^\infty \frac{(B\ell v)^2}{R}dt = \int_0^\infty \frac{\left(B\ell v_0 e^{-\left(B^2\ell^2/mR\right)t}\right)^2}{R}dt = \frac{B^2\ell^2 v_0^2}{R}\int_0^\infty e^{-2\left(B^2\ell^2/mR\right)t}dt$$

Evaluate the integral (by changing variables to $u = -\dfrac{2B^2\ell^2}{mR}$) to obtain:

$$E = \frac{B^2\ell^2 v_0^2}{R}\left(\frac{mR}{2B^2\ell^2}\right) = \boxed{\tfrac{1}{2}mv_0^2}$$

43 ••

Picture the Problem In Example 28-9 it is shown that the speed of the rod is given by $v = v_0 e^{-\left(B^2\ell^2/mR\right)t}$. We can write v as dx/dt, separate the variables and integrate to find the total distance traveled by the rod.

Apply the result from Example 28-9 to obtain:

$$\frac{dx}{dt} = v_0 e^{-Ct}$$

where

$$\mathrm{C} = \frac{B^2\ell^2}{mR}$$

Separate variables and integrate x' from 0 to x and t' from 0 to ∞:

$$\int_0^x dx' = v_0\int_0^\infty e^{-Ct'}dt$$

Evaluate the integrals to obtain:

$$x = \frac{v_0}{\mathrm{C}}$$

Substitute for C and simplify:

$$x = \boxed{\frac{mv_0R}{B^2\ell^2}}$$

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Picture the Problem Let *m* be the mass of the rod. The net force acting on the rod is due to the current in it. We can obtain the equation of motion for the rod by applying Newton's 2nd law to relate its acceleration to *B*, *I*, and ℓ. The net emf that drives *I* in this circuit is the emf of the capacitor minus the emf induced in the rod as a result of its motion.

(*a*) Letting the direction of motion of the rod be the positive *x* direction, apply $\sum F_x = ma_x$ to the rod:

$$BI\ell = m\frac{dv}{dt} \qquad (1)$$

where

$$I = \frac{\frac{Q}{C} - B\ell v}{R} \qquad (2)$$

Solve equation (1) for *I*:

$$I = \frac{m}{B\ell}\frac{dv}{dt}$$

or, because the capacitor is discharging,

$$-\frac{dQ}{dt} = \frac{m}{B\ell}\frac{dv}{dt}$$

Simplify to obtain:

$$dQ = -\frac{m}{B\ell}dv$$

Integrate Q' from Q_0 to Q and v' from 0 to v:

$$\int_{Q_0}^{Q} dQ' = -\frac{m}{B\ell}\int_0^v dv'$$

and

$$Q = Q_0 - \frac{m}{B\ell}v$$

Substitute in equation (2) to obtain:

$$I = \frac{\frac{Q_0 - \frac{m}{B\ell}v}{C} - B\ell v}{R} = \frac{Q_0 - \frac{m}{B\ell}v}{CR} - \frac{B\ell v}{R}$$

Substitute in equation (1) to obtain the equation of motion of the rod:

$$\frac{dv}{dt}=\frac{B\ell}{mR}\left(\frac{Q_0-\frac{m}{B\ell}v}{C}-B\ell v\right)$$

$$=\boxed{\frac{B\ell Q_0}{mRC}-\left(\frac{1}{RC}+\frac{B^2\ell^2}{mR}\right)v}$$

(*b*) When the rod has achieved its terminal speed:

$$BI\ell=m\frac{dv}{dt}=0$$

and

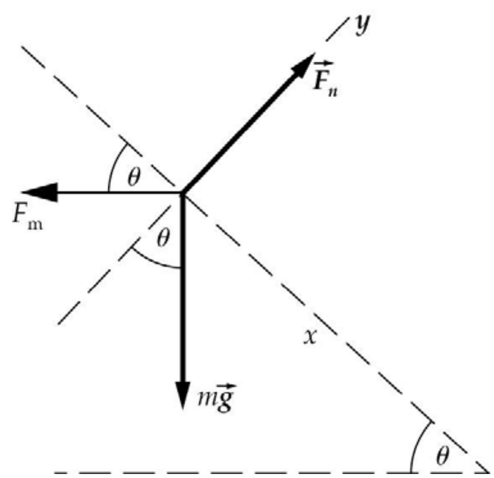
$$I=\frac{\frac{Q_f}{C}-B\ell v_t}{R}=0$$

Solve for v_t to obtain:

$$v_t=\boxed{\frac{Q_f}{CB\ell}}$$

***45 ••**

Picture the Problem The free-body diagram shows the forces acting on the rod as it slides down the inclined plane. The retarding force is the component of F_m acting up the incline, i.e., in the $-x$ direction. We can express F_m using the expression for the force acting on a conductor moving in a magnetic field. Recognizing that only the horizontal component of the rod's velocity $\vec{v}$ produces an induced emf, we can apply the expression for a motional emf in conjunction with Ohm's law to find the induced current in the rod. In part (*b*) we can apply Newton's 2nd law to obtain an expression for dv/dt and set this expression equal to zero to obtain v_t.



(*a*) Express the retarding force acting on the rod:

$$F=F_m\cos\theta \qquad (1)$$

where

$$F_m=I\ell B$$

and I is the current induced in the rod as a consequence of its motion in the magnetic field.

Express the induced emf due to the motion of the rod in the magnetic field:

$$\mathcal{E} = B\ell v\cos\theta$$

Using Ohm's law, relate the current I in the circuit to the induced emf:

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

Substitute in equation (1) to obtain:

$$F = \left(\frac{B\ell v\cos\theta}{R}\right)\ell B\cos\theta = \boxed{\frac{B^2\ell^2 v}{R}\cos^2\theta}$$

(*b*) Apply $\sum F_x = ma_x$ to the rod:

$$mg\sin\theta - \frac{B^2\ell^2 v}{R}\cos^2\theta = m\frac{dv}{dt}$$

and

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2\ell^2 v}{mR}\cos^2\theta$$

When the rod reaches its terminal velocity v_t, $dv/dt = 0$ and:

$$0 = g\sin\theta - \frac{B^2\ell^2 v_t}{mR}\cos^2\theta$$

Solve for v_t to obtain:

$$v_t = \boxed{\frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}}$$

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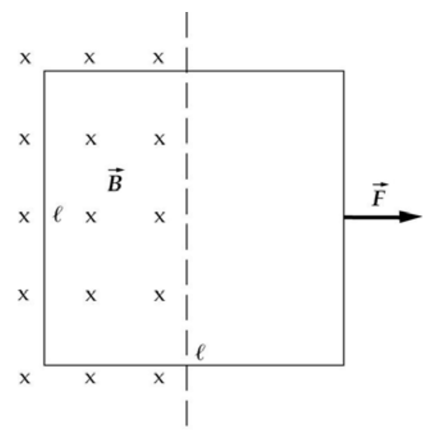
Picture the Problem The diagram shows the square loop being pulled from the magnetic field $\vec{B}$ by the constant force $\vec{F}$. The time required to pull the loop out of the magnetic field depends on the terminal speed of the loop. We can apply Newton's 2nd law and use the expressions for the magnetic force on a moving wire in a magnetic field to obtain the equation of motion for the loop and, from this equation, an expression for the terminal speed of the loop.



Apply $\sum\vec{F} = m\vec{a}$ to the square loop to obtain:

$$F - F_m = m\frac{dv}{dt} \qquad (1)$$

The magnetic force is given by:

$$F_{\text{m}} = I\ell B = \frac{\mathcal{E}\,\ell\,B}{R}$$

where R is the resistance of the loop.

Substitute for Fm in equation (1) to obtain:

$$F - \frac{\mathcal{E}\,\ell\,B}{R} = m\frac{dv}{dt} \qquad (2)$$

The induced emf ε is related to the speed of the loop:

$$\mathcal{E} = vB\ell$$

Substitute for in equation (2) to obtain the equation of motion of the loop:

$$F - \frac{\ell^2B^2}{R}v = m\frac{dv}{dt}$$

When the loop reaches its terminal speed, $dv/dt = 0$ and:

$$F - \frac{\ell^2B^2}{R}v_{\text{t}} = 0 \Rightarrow v_{\text{t}} = \frac{R}{\ell^2B^2}F$$

This result tells us that doubling F doubles the terminal speed v_{t}. Hence, doubling F will halve the time required to pull the loop from the magnetic field and $\boxed{(c)\text{ is correct.}}$

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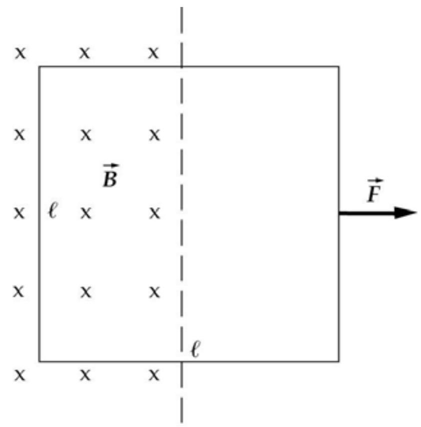
Picture the Problem The diagram shows the square loop being pulled from the magnetic field $\vec{B}$ by the constant force $\vec{F}$. The time required to pull the loop out of the magnetic field depends on the terminal speed of the loop. We can apply Newton's 2nd law and use the expressions for the magnetic force on a moving wire in a magnetic field to obtain the equation of motion for the loop and, from this equation, an expression for the terminal speed of the loop.



Apply $\sum\vec{F} = m\vec{a}$ to the square loop to obtain:

$$F - F_{\text{m}} = m\frac{dv}{dt} \qquad (1)$$

The magnetic force is given by:

$$F_{\text{m}} = I\ell B = \frac{\mathcal{E}\,\ell\,B}{R}$$

where R is the resistance of the loop.

Substitute for F_{m} in equation (1) to obtain:

$$F - \frac{\mathcal{E}\,\ell\,B}{R} = m\frac{dv}{dt} \qquad (2)$$

The induced emf ε is related to the speed of the loop:

$$\mathcal{E} = vB\ell$$

Substitute for in equation (2) to obtain the equation of motion of the loop:

$$F-\frac{\ell^2B^2}{R}v=m\frac{dv}{dt}$$

When the loop reaches its terminal speed, $dv/dt = 0$ and:

$$F-\frac{\ell^2B^2}{R}v_t=0\Rightarrow v_t=\frac{R}{\ell^2B^2}F$$

This result tells us that halving R halves the terminal speed v_t. Hence, halving R will double the time required to pull the loop from the magnetic field and $\boxed{(b)\text{ is correct.}}$

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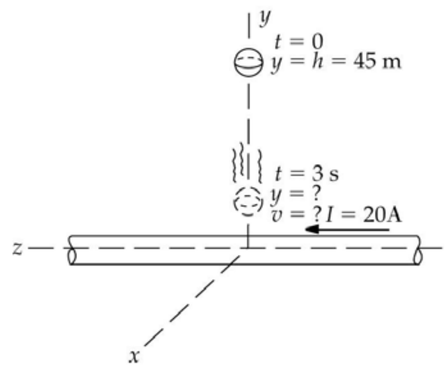
Picture the Problem The diagram shows the initial position of the sphere and its position at $t = 3$ s. We can find the velocity of the sphere and the magnetic field when $t = 3$ s and use $\vec{E}=\vec{v}\times\vec{B}$ to find $\vec{E}$. We can find the voltage across the sphere at this time from the electric field at its center and its diameter.



(*a*) Relate the electric field at the center of the sphere to the magnetic field at that location:

$$\vec{E}=\vec{v}\times\vec{B}$$

Express the magnetic field as a function of the distance y from the current-carrying wire:

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{2I}{y}\left(-\hat{i}\right)=-\frac{\mu_0}{2\pi}\frac{I}{y}\hat{i}$$

Using a constant-acceleration equation, find the position of the sphere at $t = 3$ s:

$$y=y_0+v_{0,y}\Delta t+\tfrac{1}{2}a(\Delta t)^2$$

or, because $y_0 = h$, $v_{0,y} = 0$, and $a = -g$,

$$y=h-\tfrac{1}{2}g(\Delta t)^2$$
$$=45\text{ m}-\tfrac{1}{2}\left(9.81\text{ m/s}^2\right)(3\text{ s})^2$$
$$=0.855\text{ m}$$

Substitute and evaluate $\vec{B}$:

$$\vec{B}=-\frac{\mu_0}{2\pi}\frac{I}{y}\hat{i}$$
$$=-\frac{4\pi\times10^{-7}\text{ N/A}^2}{2\pi}\frac{20\text{ A}}{0.855\text{ m}}\hat{i}$$
$$=\left(-4.68\times10^{-6}\text{ T}\right)\hat{i}$$

Using a constant-acceleration equation, find the velocity of the sphere at $t = 3$ s:

$$\vec{v} = \vec{v}_0 + \vec{a}\Delta t$$

or, because $\vec{v}_0 = 0$ and $\vec{a} = -g\,\hat{j}$

$$\vec{v} = -g\Delta t\,\hat{j} = -(9.81\,\text{m/s}^2)(3\,\text{s})\hat{j} = (-29.4\,\text{m/s})\hat{j}$$

Substitute and evaluate $\vec{E}$:

$$\vec{E} = (-29.4\,\text{m/s})\hat{j} \times (-4.68 \times 10^{-6}\,\text{T})\hat{i} = \boxed{(-0.138\,\text{mV/m})\hat{k}}$$

(b) The potential difference across the sphere depends on the electric field at the center of the sphere and the diameter of the sphere:

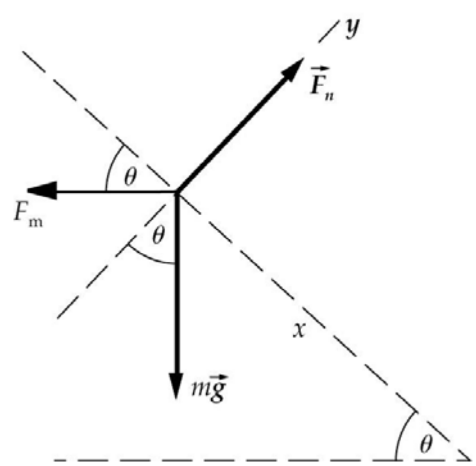
$$V = 2RE$$

Substitute numerical values and evaluate V:

$$V = 2(0.02\,\text{m})(0.138\,\text{mV/m}) = \boxed{5.52\,\mu\text{V}}$$

49 ••

Picture the Problem The free-body diagram shows the forces acting on the rod as it slides down the inclined plane. The retarding force is the component of F_m acting up the incline; i.e., in the $-x$ direction. We can express F_m using the expression for the force acting on a conductor moving in a magnetic field. We can apply the expression for a motional emf in conjunction with Ohm's law to find the induced current in the rod. In part (b) we can apply Newton's 2nd law to obtain an expression for dv/dt and set this expression equal to zero to obtain v_t.



(a) Noting that only the horizontal component of the rod's velocity $\vec{v}$ produces an induced emf, express $\mathcal{E}$ due to the motion of the rod in the magnetic field:

$$\mathcal{E} = B\ell v\cos\theta$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = (1.2\,\text{T})(15\,\text{m})v(\cos 30°) = \boxed{(15.6\,\text{T}\cdot\text{m})v}$$

(*b*) Apply Newton's 2nd law to the rod:

$$mg\sin\theta - F_{\text{m}}\cos\theta = m\frac{dv}{dt}$$

and

$$\frac{dv}{dt} = g\sin\theta - \frac{F_{\text{m}}}{m}\cos\theta \qquad (1)$$

where

$$F_{\text{m}} = I\ell B$$

and I is the current induced in the rod as a consequence of its motion in the magnetic field.

Using Ohm's law, relate the current I in the circuit to the induced emf:

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

and

$$F_{\text{m}} = \frac{B^2\ell^2 v\cos\theta}{R}$$

Substitute in equation (1) to obtain the equation of motion of the rod:

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2\ell^2 v}{mR}\cos^2\theta$$

When the rod reaches its terminal velocity v_{t}, $dv/dt = 0$:

$$0 = g\sin\theta - \frac{B^2\ell^2 v_{\text{t}}}{mR}\cos^2\theta$$

Solve for v_{t}:

$$v_{\text{t}} = \frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$$

Substitute numerical values and evaluate v_{t}:

$$v_{\text{t}} = \frac{(0.4\,\text{kg})(9.81\,\text{m/s}^2)(2\,\Omega)\sin 30°}{(1.2\,\text{T})^2(15\,\text{m})^2\cos^2 30°} = \boxed{1.61\,\text{cm/s}}$$

50 •••

Picture the Problem Let F_{f} be the friction force between the rails and cylinder, F_{m} the magnetic force on the cylinder, and I_{m} the cylinder's moment of inertia. Because the current through the rod is uniformly distributed, we can treat the current as though it were concentrated at the center of the rod. We can find the magnitude of $\vec{\boldsymbol{B}}$ by applying Newton's 2nd law to the cylinder. The application of Ohm's law to the circuit will allow us to express the net force acting on the cylinder in terms of its speed. Setting this net force equal to zero will lead us to a value for the terminal velocity of the cylinder. We can use the definition of kinetic energy (both translational and rotational) to find the kinetic energy of the cylinder when it has reached its terminal velocity.

(*a*) Apply $\sum F_x = ma_x$ to the cylinder:

$$F_{\text{m}} - F_{\text{f}} = ma_x$$

or

$$BIa - F_{\text{f}} = m\frac{dv}{dt} = mr\frac{d\omega}{dt}$$

Apply $\sum \tau = I\alpha$ to the cylinder:

$$F_{\text{f}}r = I_{\text{m}}\frac{d\omega}{dt}$$

Solve for F_{f} and substitute to obtain:

$$BIa - \frac{I_{\text{m}}}{r}\frac{d\omega}{dt} = mr\frac{d\omega}{dt}$$

Solve for $r\dfrac{d\omega}{dt}$:

$$r\frac{d\omega}{dt} = \frac{BIa}{m + \dfrac{I_{\text{m}}}{r^2}} = \frac{BIa}{m + \dfrac{\frac{1}{2}mr^2}{r^2}} = \frac{2BIa}{3m}$$

or

$$\frac{dv}{dt} = \frac{2BIa}{3m} \qquad (1)$$

Solve for B:

$$B = \frac{3m\dfrac{dv}{dt}}{2Ia}$$

Apply Ohm's law to the circuit to find I:

$$I = \frac{\mathcal{E}}{R} = \frac{12\text{V}}{6\,\Omega} = 2\text{ A}$$

Substitute numerical values and evaluate B:

$$B = \frac{3(4\text{ kg})(0.1\text{ m/s}^2)}{2(2\text{A})(0.4\text{ m})} = \boxed{0.750\text{ T}}$$

Apply $\vec{F} = I\vec{\ell} \times \vec{B}$ to determine the direction of $\vec{B}$:

$\boxed{\vec{B}\text{ is downward.}}$

(*b*) Multiply both sides of equation (1) by m to express the net force acting on the cylinder:

$$F_{\text{net}} = m\frac{dv}{dt} = \frac{2BIa}{3}$$

Use Ohm's law to express the current as a function of the emf of

$$I = \frac{\mathcal{E} - Bav}{R}$$

the battery and the induced emf in the cylinder:

Substitute to express the net force acting on the cylinder as a function of the velocity of the cylinder:

$$F_{\text{net}} = \frac{2B\left(\frac{\mathcal{E} - Bav}{R}\right)a}{3}$$
$$= \boxed{\frac{2Ba\mathcal{E}}{3R} - \frac{2B^2a^2}{3R}v}$$

(*c*) Set $F_{\text{net}} = 0$ and solve for the terminal velocity of the cylinder:

$$\frac{2Ba\mathcal{E}}{3R} - \frac{2B^2a^2}{3R}v_{\text{t}} = 0$$

and

$$v_{\text{t}} = \frac{\mathcal{E}}{Ba} = \frac{12\,\text{V}}{(0.75\,\text{T})(0.4\,\text{m})} = \boxed{40.0\,\text{m/s}}$$

(*d*) Express the total kinetic energy of the cylinder when it has reached its terminal velocity:

$$K = \tfrac{1}{2}mv_{\text{t}}^2 + \tfrac{1}{2}I_{\text{m}}\omega_{\text{t}}^2$$
$$= \tfrac{1}{2}mv_{\text{t}}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v_{\text{t}}^2}{r^2}$$
$$= \tfrac{3}{4}mv_{\text{t}}^2$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{3}{4}(4\,\text{kg})(40\,\text{m/s})^2 = \boxed{4.80\,\text{kJ}}$$

***51 •••**

Picture the Problem We can use the expression for a motional emf and Ampere's law to express the net emf induced in the moving loop. We can also use express the magnetic flux through the loop and apply Faraday's law to obtain the same result.

(*a*) Express the motional emf induced in the segments parallel to the current-carrying wire:

$$\mathcal{E} = B(x)vb$$

Using Ampere's law, express $B(d + vt)$ and $B(d + a + vt)$:

$$B(d + vt) = \frac{\mu_0 I}{2\pi(d + vt)}$$

and

$$B(d + a + vt) = \frac{\mu_0 I}{2\pi(d + a + vt)}$$

Substitute to express $\mathcal{E}_1$ for the near wire and $\mathcal{E}_2$ for the far wire:

$$\mathcal{E}_1 = \frac{\mu_0 Ivb}{2\pi(d + vt)}$$

and

$$\varepsilon_1 = \frac{\mu_0 Ivb}{2\pi(d + a + vt)}$$

Noting that the emfs both point upward and hence oppose one another, express the net emf induced in the loop:

$$\varepsilon = \varepsilon_1 - \varepsilon_2 = \frac{\mu_0 Ivb}{2\pi(d + vt)} - \frac{\mu_0 Ivb}{2\pi(d + a + vt)} = \boxed{\frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{d + vt} - \frac{1}{d + a + vt}\right)}$$

The motion of the segments perpendicular to the long wire does not change the flux through the rectangular loop. Consequently, these segments do not contribute to the the induced emf.

(*b*) From Faraday's law we have:

$$\varepsilon = -\frac{d\phi_{\mathrm{m}}}{dt}$$

Express the magnetic flux in an area of length b and width vdt:

$$d\phi_{\mathrm{m}} = B(x)dA = B(x)b\,dx$$

where, from Ampere's law,

$$B(x) = \frac{\mu_0 I}{2\pi x}$$

Substitute and integrate from $x = d + vt$ to $d + a + vt$:

$$\phi_{\mathrm{m}} = \int_{d+vt}^{d+a+vt} B(x)dx = \frac{\mu_0 Ib}{2\pi}\int_{d+vt}^{d+a+vt}\frac{dx}{x} = \frac{\mu_0 Ib}{2\pi}\ln\left[\frac{d + a + vt}{d + vt}\right]$$

Differentiate with respect to time and simplify to obtain:

$$\varepsilon = -\frac{d}{dt}\left[\frac{\mu_0 Ib}{2\pi}\ln\frac{d + a + vt}{d + vt}\right] = -\frac{\mu_0 Ib}{2\pi}\frac{d}{dt}\left[\ln\frac{d + a + vt}{d + vt}\right]$$

$$= -\frac{\mu_0 Ib}{2\pi}\left[\left(\frac{d + vt}{d + a + vt}\right)\left(\frac{(d + vt)v - (d + a + vt)v}{(d + vt)^2}\right)\right]$$

$$= -\frac{\mu_0 Ibv}{2\pi}\left[\frac{(d + vt) - (d + a + vt)}{(d + vt)(d + a + vt)}\right] = -\frac{\mu_0 Ibv}{2\pi}\left[\frac{1}{d + a + vt} - \frac{1}{d + vt}\right]$$

$$= \boxed{\frac{\mu_0 Ibv}{2\pi}\left[\frac{1}{d + vt} - \frac{1}{d + a + vt}\right]}$$

52 •••

Picture the Problem We can use $\vec{F} = q\vec{v} \times \vec{B}$ to express the magnetic force acting on the moving charged body. Expressing the emf induced in a segment of the rod of length *dr* and integrating this expression over the length of the rod will lead us to an expression for the induced emf.

(*a*) Using the equation for the magnetic force on a moving charged body, express the force acting on the charged body a distance *r* from the pivot:

$$\vec{F} = q\vec{v} \times \vec{B}$$

and

$$F = qvB\sin\theta$$

Because $\vec{v} \perp \vec{B}$ and $v = r\omega$:

$$F = \boxed{qBr\omega}$$

(*b*) Use the motional emf equation to express the emf induced in a segment of the rod of length *dr* and at a distance *r* from the pivot:

$$d\mathcal{E} = Brdv = Br\omega dr$$

Integrate this expression from $r = 0$ to $r = \ell$ to obtain:

$$\int_0^{\mathcal{E}} d\mathcal{E}' = B\omega\int_0^{\ell} rdr$$

and

$$\mathcal{E} = \boxed{\tfrac{1}{2}B\omega\ell^2}$$

(*c*) Using Faraday's law, relate the induced emf to the rate at which the flux changes:

$$|\mathcal{E}| = \frac{d\phi_{\text{m}}}{dt}$$

Express the area *dA*, for any value of θ, between *r* and $r + dr$:

$$dA = r\theta dr$$

Integrate from $r = 0$ to $r = \ell$ to obtain:

$$A = \theta\int_0^{\ell} rdr = \tfrac{1}{2}\theta\ell^2$$

Using its definition, express the magnetic flux through this area:

$$\phi_{\text{m}} = BA = \boxed{\tfrac{1}{2}B\ell^2\theta}$$

Differentiate ϕ_{m} with respect to time to obtain:

$$|\mathcal{E}| = \frac{d}{dt}\left[\tfrac{1}{2}B\ell^2\theta\right] = \tfrac{1}{2}B\ell^2\frac{d\theta}{dt} = \boxed{\tfrac{1}{2}B\ell^2\omega}$$

Inductance

53 •

Picture the Problem We can use $\phi_m = LI$ and the dependence of I on t to find the magnetic flux through the coil. We can apply Faraday's law to find the induced emf in the coil.

(*a*) Use the definition of self-inductance to express ϕ_m:

$$\phi_m = LI$$

Express I as a function of time:

$$I = 3\text{ A} + (200\text{ A/s})t$$

Substitute to obtain:

$$\phi_m = L[3\text{ A} + (200\text{ A/s})t]$$

Substitute numerical values and express ϕ_m:

$$\phi_m = (8\text{ H})[3\text{ A} + (200\text{ A/s})t] = \boxed{24\text{ Wb} + (1600\text{ H}\cdot\text{A/s})t}$$

(*b*) Use Faraday's law to relate ε, L, and dI/dt:

$$\mathcal{E} = -L\frac{dI}{dt}$$

Substitute numerical values and evaluate ε:

$$\mathcal{E} = -(8\text{ H})(200\text{ A/s}) = \boxed{-1.60\text{ kV}}$$

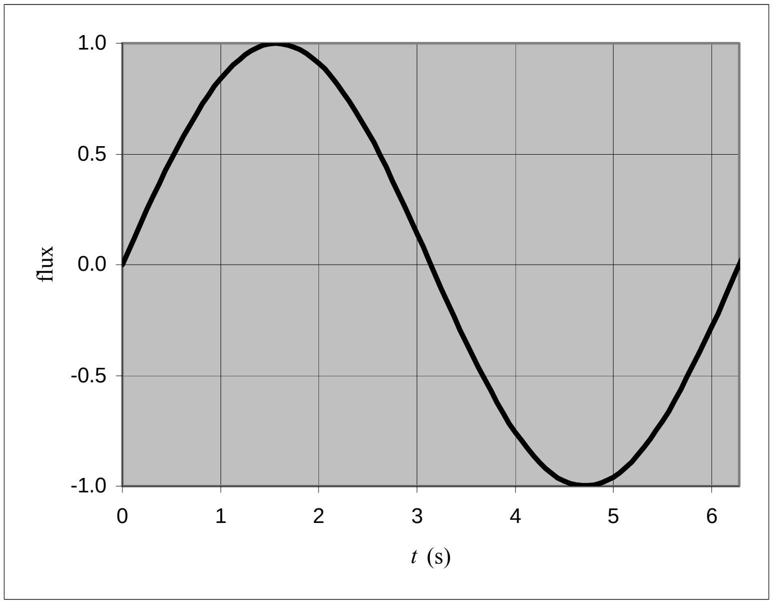
***54** •

Picture the Problem We can apply $\phi_m = LI$ to find ϕ_m and Faraday's law to find the self-induced emf as functions of time.

Use the definition of self-inductance to express ϕ_m:

$$\phi_m = LI = \boxed{LI_0 \sin 2\pi ft}$$

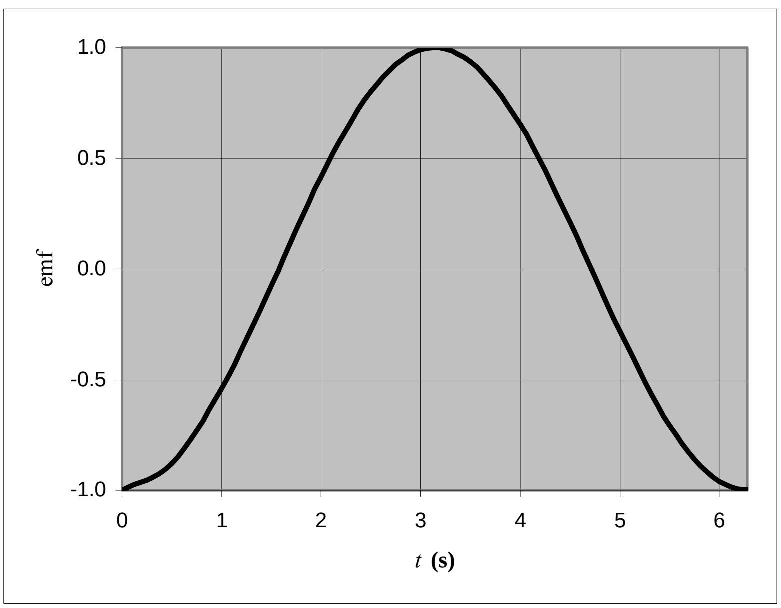
The graph of the flux ϕ_m as a function of time shown below was plotted using a spreadsheet program. The maximum value of the flux is LI_0 and we have chosen $2\pi f = 1$ rad/s.



Apply Faraday's law to relate ε, L, and dI/dt:

$$\mathcal{E} = -L\frac{dI}{dt} = -L\frac{d}{dt}\left[I_0 \sin 2\pi ft\right] = \boxed{-2\pi fLI_0 \cos 2\pi ft}$$

The graph of the emf ε as a function of time shown below was plotted using a spreadsheet program. The maximum value of the induced emf is $2\pi fLI_0$ and we have chosen $2\pi f = 1$ rad/s.



55 ••

Picture the Problem We can use $B = \mu_0 nI$ to find the magnetic field on the axis at the center of the solenoid and the definition of magnetic flux to evaluate ϕ_m. We can use the definition of magnetic flux in terms of L and I to find the self-inductance of the solenoid. Finally, we can use Faraday's law to find the induced emf in the solenoid when the current changes at 150 A/s.

(*a*) Apply the expression for B inside a long solenoid to express and evaluate B:

$$B = \mu_0 nI = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.25\,\text{m}}\right)(3\,\text{A}) = \boxed{6.03\,\text{mT}}$$

(*b*) Apply the definition of magnetic flux to obtain:

$$\phi_m = NBA = (400)(6.03\,\text{mT})\pi(0.01\,\text{m})^2 = \boxed{7.58\times10^{-4}\ \text{Wb}}$$

(*c*) Relate the self-inductance of the solenoid to the magnetic flux through it and its current:

$$L = \frac{\phi_m}{I} = \frac{7.58\times10^{-4}\ \text{Wb}}{3\,\text{A}} = \boxed{0.253\,\text{mH}}$$

(*d*) Apply Faraday's law to obtain:

$$\mathcal{E} = -L\frac{dI}{dt} = -(0.253\,\text{mH})(150\,\text{A/s}) = \boxed{-38.0\,\text{mV}}$$

56 ••

Picture the Problem We can find the mutual inductance of the two coaxial solenoids using $M_{2,1} = \frac{\phi_{m2}}{I_1} = \mu_0 n_2 n_1 \ell \pi r_1^2$.

Substitute numerical values and evaluate $M_{2,1}$:

$$M_{2,1} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{300}{0.25\,\text{m}}\right)\left(\frac{1000}{0.25\,\text{m}}\right)(0.25\,\text{m})\pi(0.02\,\text{m})^2 = \boxed{1.89\,\text{mH}}$$

***57** ••

Picture the Problem Note that the current in the two parts of the wire is in opposite directions. Consequently, the total flux in the coil is zero. We can find the resistance of the wire-wound resistor from the length of wire used and the resistance per unit length.

Because the total flux in the coil is zero:

$$L = \boxed{0}$$

Express the total resistance of the wire:

$$R = \left(18\frac{\Omega}{\text{m}}\right)L = \left(18\frac{\Omega}{\text{m}}\right)(9\,\text{m}) = \boxed{162\,\Omega}$$

58 •••

Picture the Problem We can apply Kirchhoff's loop rule to the galvanometer circuit to relate the potential difference across L_2 to the potential difference across R_2. Integration of this equation over time will yield an equation that relates the mutual inductance between the two coils to the steady-state current in circuit 1 and the charge that flows through the galvanometer.

Apply Kirchhoff's loop rule to the galvanometer circuit:

$$M\frac{dI_1}{dt} + L_2\frac{dI_2}{dt} - R_2 I_2 = 0$$

or

$$M dI_1 + L_2 dI_2 - R_2 I_2 dt = 0$$

Integrate each term from $t = 0$ to $t = \infty$:

$$M\int_0^\infty dI_1 + L_2\int_0^\infty dI_2 - R_2\int_0^\infty I_2 dt = 0$$

and

$$MI_{1\infty} + L_2 I_{2\infty} - R_2 Q = 0$$

Because $I_{2\infty} = 0$:

$$MI_{1\infty} - R_2 Q = 0$$

Solve for M:

$$M = \frac{R_2 Q}{I_{1\infty}}$$

Substitute numerical values and evaluate M:

$$M = \frac{(300\,\Omega)(2\times10^{-4}\,\text{C})}{5\,\text{A}} = \boxed{12.0\,\text{mH}}$$

59 •••

Picture the Problem We can use Ampere's law to express the magnetic field inside the rectangular toroid and the definition of magnetic flux to express ϕ_m through the toroid. We can then use the definition of self-inductance of a solenoid to express L.

Using the definition of the self-inductance of a solenoid, express L in terms of ϕ_m, N, and I:

$$L = \frac{N\phi_m}{I} \qquad (1)$$

Apply Ampere's law to a closed path or radius $a < r < b$:

$$\oint_C \vec{B}\cdot d\vec{\ell} = B2\pi r = \mu_0 I_C$$

or, because $I_C = NI$,

$$B2\pi r = \mu_0 NI$$

Solve for B to obtain:

$$B = \frac{\mu_0 NI}{2\pi r}$$

Express the flux in a strip of height H and width dr:

$$d\phi_m = BHdr$$

Substitute for B and integrate $d\phi_m$ from $r = a$ to $r = b$ to obtain:

$$\phi_m = \frac{\mu_0 NIH}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIH}{2\pi}\ln\left(\frac{b}{a}\right)$$

Substitute in equation (1) and simplify to obtain:

$$L = \boxed{\frac{\mu_0 N^2 H}{2\pi}\ln\left(\frac{b}{a}\right)}$$

Magnetic Energy

60 •

Picture the Problem The current in an LR circuit, as a function of time, is given by $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$. The energy stored in the inductor under steady-state conditions is stored in its magnetic field and is given by $U_m = \frac{1}{2}LI_f^2$.

(a) Express and evaluate I_f:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{24\text{ V}}{12\,\Omega} = \boxed{2.00\text{ A}}$$

(b) Express and evaluate the energy stored in an inductor:

$$U_m = \frac{1}{2}LI_f^2 = \frac{1}{2}(2\text{ H})(2\text{ A})^2 = \boxed{4.00\text{ J}}$$

***61** ••

Picture the Problem We can examine the ratio of u_m to u_E with $E = cB$ and $c = 1\big/\sqrt{\varepsilon_0\mu_0}$ to show that the electric and magnetic energy densities are equal.

Express the ratio of the energy density in the magnetic field to the energy density in the electric field:

$$\frac{u_m}{u_E} = \frac{\dfrac{B^2}{2\mu_0}}{\frac{1}{2}\varepsilon_0 E^2} = \frac{B^2}{\mu_0\varepsilon_0 E^2}$$

Substitute $E = cB$:

$$\frac{u_m}{u_E} = \frac{B^2}{\mu_0\varepsilon_0 c^2 B^2} = \frac{1}{\mu_0\varepsilon_0 c^2}$$

Substitute for c:

$$\frac{u_{\mathrm{m}}}{u_{\mathrm{E}}} = \frac{\mu_0 \varepsilon_0}{\mu_0 \varepsilon_0} = 1 \Rightarrow \boxed{u_{\mathrm{m}} = u_{\mathrm{E}}}$$

62 ••

Picture the Problem We can use $L = \mu_0 n^2 A\ell$ to find the inductance of the solenoid and $B = \mu_0 nI$ to find the magnetic field inside it.

(*a*) Express the magnetic energy stored in the solenoid:

$$U_{\mathrm{m}} = \tfrac{1}{2} L I^2$$

Relate the inductance of the solenoid to its dimensions and properties:

$$L = \mu_0 n^2 A\ell$$

Substitute to obtain:

$$U_{\mathrm{m}} = \tfrac{1}{2} \mu_0 n^2 A\ell I^2$$

Substitute numerical values and evaluate U_{m}:

$$U_{\mathrm{m}} = \tfrac{1}{2}\left(4\pi \times 10^{-7}\ \mathrm{N/A^2}\right)\left(\frac{2000}{0.3\,\mathrm{m}}\right)^2 \times \left(4\times 10^{-4}\ \mathrm{m^2}\right)(0.3\,\mathrm{m})(4\,\mathrm{A})^2 = \boxed{53.6\,\mathrm{mJ}}$$

(*b*) The magnetic energy per unit volume in the solenoid is:

$$\frac{U_{\mathrm{m}}}{V} = \frac{U_{\mathrm{m}}}{A\ell} = \frac{53.6\,\mathrm{mJ}}{\left(4\times 10^{-4}\ \mathrm{m^2}\right)(0.3\,\mathrm{m})} = \boxed{447\ \mathrm{J/m^3}}$$

(*c*) Express the magnetic field in the solenoid in terms of n and I:

$$B = \mu_0 nI = \mu_0 \frac{N}{\ell} I = \frac{\left(4\pi \times 10^{-7}\ \mathrm{N/A^2}\right)(2000)(4\,\mathrm{A})}{0.3\,\mathrm{m}} = \boxed{33.5\,\mathrm{mT}}$$

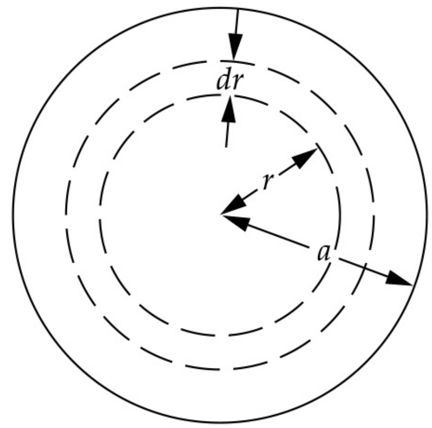
(*d*) The magnetic energy density is:

$$u_{\mathrm{m}} = \frac{B^2}{2\mu_0} = \frac{(33.5\,\mathrm{mT})^2}{2\left(4\pi \times 10^{-7}\ \mathrm{N/A^2}\right)} = \boxed{447\ \mathrm{J/m^3}}$$

in agreement with our result in Part (*b*).

63 ••

Picture the Problem Consider a cylindrical annulus of thickness dr at a radius $r < a$. We can use its definition to express the total magnetic energy dU_m inside the cylindrical annulus and divide both sides of this expression by the length of the wire to express the magnetic energy per unit length dU'_m. Integration of this expression will give us the magnetic energy per unit length within the wire.



Express the magnetic energy within the cylindrical annulus:

$$dU_m = \frac{B^2}{2\mu_0}V_{\text{annulus}} = \frac{B^2}{2\mu_0}2\pi r\ell dr = \frac{B^2}{\mu_0}\pi r\ell dr$$

Divide both sides of the equation by ℓ to express the magnetic energy per unit length dU'_m:

$$dU'_m = \frac{B^2}{\mu_0}\pi r dr \qquad (1)$$

Use Ampere's law to express the magnetic field inside the wire at a distance $r < a$ from its center:

$$2\pi r B = \mu_0 I_C$$

and

$$B = \frac{\mu_0 I_C}{2\pi r}$$

where I_C is the current inside the cylinder of radius r.

Because the current is uniformly distributed over the cross-sectional area of the wire:

$$\frac{I_C}{I} = \frac{\pi r^2}{\pi a^2} \Rightarrow I_C = \frac{r^2}{a^2}I$$

Substitute to obtain:

$$B = \frac{\mu_0 r I}{2\pi a^2}$$

Substitute for B in equation (1) to obtain:

$$dU'_m = \frac{\left(\frac{\mu_0 r I}{2\pi a^2}\right)^2}{\mu_0}\pi r dr = \frac{\mu_0 I^2}{4\pi a^4}r^3 dr$$

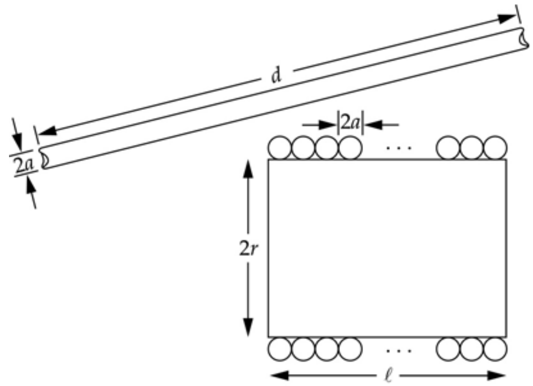
Integrate dU'_m from $r = 0$ to $r = a$:

$$U'_m = \frac{\mu_0 I^2}{4\pi a^4}\int_0^a r^3 dr = \frac{\mu_0 I^2}{4\pi a^4}\cdot\frac{a^4}{4} = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

Remarks: Note that the magnetic energy per unit length is independent of the radius of the cylinder and depends only on the total current.

***64 ••**

Picture the Problem The wire of length d and radius a is shown in the diagram, as is the inductor constructed with this wire and whose inductance L is to be found. We can use the equation for the self-inductance of a cylindrical inductor to derive an expression for L.



The self-inductance of an inductor with length ℓ, cross-sectional area A, and number of turns per unit length n is:

$$L = \mu_0 n^2 A\ell \qquad (1)$$

The number of turns N is given by:

$$N = \frac{\ell}{2a}$$

The number of turns per unit length n is:

$$n = \frac{N}{\ell} = \frac{1}{2a}$$

Assuming that $a << r$, the length of the wire d is related to n and r:

$$d = N(2\pi r) = \left(\frac{\ell}{2a}\right)2\pi r = \frac{\pi r}{a}\ell$$

Solve for ℓ to obtain:

$$\ell = \frac{ad}{\pi r}$$

Substitute for ℓ, A, and n in equation (1) to obtain:

$$L = \mu_0\left(\frac{1}{2a}\right)^2(\pi r^2)\left(\frac{ad}{\pi r}\right) = \boxed{\mu_0\left(\frac{rd}{4a}\right)}$$

65 •

Picture the Problem We can substitute numerical values in the expression derived in Problem 64 to find the self-inductance of the inductor.

From Problem 64 we have:

$$L = \frac{\mu_0 rR}{4a}$$

Substitute numerical values and evaluate L:

$$L = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(0.25\,\text{cm})(10\,\text{cm})}{4(0.5\,\text{mm})} = \boxed{0.157\ \mu\text{H}}$$

66 ••

Picture the Problem We can find the number of turns on the coil from the length of the superconducting wire and the cross-sectional radius of the coil. We can use $B = (\mu_0 NI)/(2\pi r_{\text{mean}})$ to find the magnetic field at the mean radius. We can find the energy density in the magnetic field from $u_{\text{m}} = B^2/(2\mu_0)$ and the total energy stored in the toroid by multiplying u_{m} by the volume of the toroid.

(*a*) Express the number of turns in terms of the length of the wire L and length required per turn $2\pi r$:

$$N = \frac{L}{2\pi r} = \frac{1000\,\text{m}}{2\pi(0.02\,\text{m})} = \boxed{7958}$$

(*b*) Use the equation for B inside a tightly wound toroid to find the magnetic field at the mean radius:

$$B = \frac{\mu_0 NI}{2\pi r_{\text{mean}}} = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(7958)(400\,\text{A})}{2\pi(0.25\,\text{m})} = \boxed{2.55\,\text{T}}$$

(*c*) Express and evaluate the energy density in the magnetic field:

$$u_{\text{m}} = \frac{B^2}{2\mu_0} = \frac{(2.55\,\text{T})^2}{2\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)} = \boxed{2.59 \times 10^6\ \text{J/m}^3}$$

Relate the total energy stored in the toroid to the energy density in its magnetic field and the volume of the toroid:

$$U_{\text{m}} = u_{\text{m}} V_{\text{toroid}}$$

Think of the toroid as a cylinder of radius r and height $2\pi r_{\text{mean}}$ to obtain:

$$V_{\text{toroid}} = \pi r^2 (2\pi r_{\text{mean}}) = 2\pi^2 r^2 r_{\text{mean}}$$

Substitute for V_{toroid} to obtain:

$$U_{\text{m}} = 2\pi^2 r^2 r_{\text{mean}} u_{\text{m}}$$

Substitute numerical values and evaluate U_m:

$$U_m = 2\pi^2(0.02\,\text{m})^2(0.25\,\text{m})(2.59\times10^6\ \text{J/m}^3) = \boxed{5.11\,\text{kJ}}$$

RL Circuits

67 •

Picture the Problem We can find the current using $I = I_f\left(1-e^{-t/\tau}\right)$ where $I_f = \varepsilon_0/R$ and $\tau = L/R$ and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_f\left(1-e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{100\,\text{V}}{8\,\Omega} = 12.5\,\text{A}$$

and

$$\tau = \frac{L}{R} = \frac{4\,\text{H}}{8\,\Omega} = 0.5\,\text{s}$$

Substitute to obtain:

$$I = (12.5\,\text{A})\left(1-e^{-t/0.5\,\text{s}}\right) = (12.5\,\text{A})\left(1-e^{-2t\,\text{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (12.5\,\text{A})\left(-e^{-2t\,\text{s}^{-1}}\right)\left(-2\,\text{s}^{-1}\right) = (25\,\text{A/s})e^{-2t\,\text{s}^{-1}}$$

(*a*) When $t = 0$:

$$I = (12.5\,\text{A})\left(1-e^{0}\right) = \boxed{0}$$

and

$$\frac{dI}{dt} = (25\,\text{A/s})e^{0} = \boxed{25.0\,\text{A/s}}$$

(*b*) When $t = 0.1$ s:

$$I = (12.5\,\text{A})\left(1-e^{-0.2}\right) = \boxed{2.27\,\text{A}}$$

and

$$\frac{dI}{dt} = (25\,\text{A/s})e^{-0.2} = \boxed{20.5\,\text{A/s}}$$

(*c*) When $t = 0.5$ s:

$$I = (12.5\,\text{A})\left(1-e^{-1}\right) = \boxed{7.90\,\text{A}}$$

and

$$\frac{dI}{dt} = (25\text{ A/s})e^{-1} = \boxed{9.20\text{ A/s}}$$

(*d*) When $t = 1.0$ s:

$$I = (12.5\text{ A})\left(1 - e^{-2}\right) = \boxed{10.8\text{ A}}$$

and

$$\frac{dI}{dt} = (25\text{ A/s})e^{-2} = \boxed{3.38\text{ A/s}}$$

68 •

Picture the Problem We can find the current using $I = I_0 e^{-t/\tau}$, where I_0 is the current at time $t = 0$ and $\tau = L/R$.

Express the current as a function of time:

$$I = I_0 e^{-t/\tau} = (2\text{ A})e^{-t/\tau}$$

Evaluate τ:

$$\tau = \frac{L}{R} = \frac{1\text{ mH}}{10\,\Omega} = 10^{-4}\text{ s}$$

Substitute to obtain:

$$I = (2\text{ A})e^{-10^4 t\,\text{s}^{-1}}$$

(*a*) When $t = 0.5$ ms:

$$I = (2\text{ A})e^{-10^4\left(0.5\times10^{-3}\text{ s}\right)\text{s}^{-1}} = (2\text{ A})e^{-5}$$
$$= \boxed{13.5\text{ mA}}$$

(*b*) When $t = 10$ ms:

$$I = (2\text{ A})e^{-10^4\left(10\times10^{-3}\text{ s}\right)\text{s}^{-1}} = (2\text{ A})e^{-100}$$
$$= 7.44\times10^{-44}\text{ A} \approx \boxed{0}$$

***69** ••

Picture the Problem We can find the current using $I = I_\text{f}\left(1 - e^{-t/\tau}\right)$, where $I_\text{f} = \mathcal{E}_0/R$, and $\tau = L/R$, and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_\text{f}\left(1 - e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_\text{f} = \frac{\mathcal{E}_0}{R} = \frac{12\text{ V}}{3\,\Omega} = 4\text{ A}$$

and

$$\tau = \frac{L}{R} = \frac{0.6\,\text{H}}{3\,\Omega} = 0.2\,\text{s}$$

Substitute to obtain:

$$I = (4\,\text{A})\left(1 - e^{-t/0.2\,\text{s}}\right) = (4\,\text{A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (4\,\text{A})\left(-e^{-5t\,\text{s}^{-1}}\right)\left(-5\,\text{s}^{-1}\right) = (20\,\text{A/s})e^{-5t\,\text{s}^{-1}}$$

(*a*) Find the current at $t = 0.5$ s:

$$I(0.5\,\text{s}) = (4\,\text{A})\left(1 - e^{-5(0.5\,\text{s})\text{s}^{-1}}\right) = 3.67\,\text{A}$$

The rate at which the battery supplies power at $t = 0.5$ s is:

$$P(0.5\,\text{s}) = I(0.5\,\text{s})\mathcal{E} = (3.67\,\text{A})(12\,\text{V}) = \boxed{44.0\,\text{W}}$$

(*b*) The rate of joule heating is:

$$P_\text{J}(0.5\,\text{s}) = [I(0.5\,\text{s})]^2 R = (3.67\,\text{A})^2(3\,\Omega) = \boxed{40.4\,\text{W}}$$

(*c*) Using the expression for the magnetic energy stored in an inductor, express the rate at which energy is being stored:

$$\frac{dU_\text{L}}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}$$

Substitute for *L, I,* and dI/dt to obtain:

$$\frac{dU_\text{L}}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}$$

Substitute numerical values and evaluate $\frac{dU_\text{L}}{dt}$:

$$\frac{dU_\text{L}}{dt} = (0.6\,\text{H})(4\,\text{A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)(20\,\text{A/s})e^{-5t\,\text{s}^{-1}} = (48\,\text{W})\left(1 - e^{-5t\,\text{s}^{-1}}\right)e^{-5t\,\text{s}^{-1}}$$

Evaluate this expression for $t = 0.5$ s:

$$\frac{dU_\text{L}}{dt} = (48\,\text{W})\left(1 - e^{-5(0.5\,\text{s})\text{s}^{-1}}\right)e^{-5(0.5\,\text{s})\text{s}^{-1}} = (48\,\text{W})\left(1 - e^{-2.5}\right)e^{-2.5} = \boxed{3.62\,\text{W}}$$

Remarks: Note that, to a good approximation, $dU_L/dt = P - P_J$.

70 ••

Picture the Problem We can find the current using $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$, and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_f\left(1 - e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{12\text{ V}}{3\,\Omega} = 4\text{ A}$$

and

$$\tau = \frac{L}{R} = \frac{0.6\text{ H}}{3\,\Omega} = 0.2\text{ s}$$

Substitute to obtain:

$$I = (4\text{ A})\left(1 - e^{-t/0.2\text{ s}}\right) = (4\text{ A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (4\text{ A})\left(-e^{-5t\,\text{s}^{-1}}\right)\left(-5\text{ s}^{-1}\right) = (20\text{ A/s})e^{-5t\,\text{s}^{-1}}$$

(*a*) Find the current at $t = 1$ s:

$$I(1\text{ s}) = (4\text{ A})\left(1 - e^{-5(1\text{ s})\text{s}^{-1}}\right) = 3.97\text{ A}$$

The rate at which the battery supplies power at $t = 1$ s:

$$P(1\text{ s}) = I(1\text{ s})\mathcal{E} = (3.97\text{ A})(12\text{ V}) = \boxed{47.7\text{ W}}$$

Find the current at $t = 100$ s:

$$I(100\text{ s}) = (4\text{ A})\left(1 - e^{-5(100\text{ s})\text{s}^{-1}}\right) = 4.00\text{ A}$$

The rate at which the battery supplies power at $t = 100$ s:

$$P(100\text{ s}) = I(100\text{ s})\mathcal{E} = (4\text{ A})(12\text{ V}) = \boxed{48.0\text{ W}}$$

(*b*) The rate of joule heating at $t = 1$ s is:

$$P_J(1\text{ s}) = [I(1\text{ s})]^2 R = (3.97\text{ A})^2(3\,\Omega) = \boxed{47.3\text{ W}}$$

The rate of joule heating at $t = 100$ s is:

$$P_J(100\,\text{s}) = (4\,\text{A})^2(3\Omega) = \boxed{48.0\,\text{W}}$$

Using the expression for the magnetic energy stored in an inductor, express the rate at which energy is being stored:

$$\frac{dU_L}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}$$

Substitute for L, I and dI/dt to obtain:

$$\begin{aligned}\frac{dU_L}{dt} &= \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}\\ &= (0.6\,\text{H})(4\,\text{A})\left(1-e^{-5t\,\text{s}^{-1}}\right)\\ &\quad\times(20\,\text{A/s})e^{-5t\,\text{s}^{-1}}\\ &= (48\,\text{W})\left(1-e^{-5t\,\text{s}^{-1}}\right)e^{-5t\,\text{s}^{-1}}\end{aligned}$$

Evaluate dU_L/dt for $t = 1$ s:

$$\begin{aligned}\frac{dU_L}{dt} &= (48\,\text{W})\left(1-e^{-5(1\,\text{s})\text{s}^{-1}}\right)e^{-5(1\,\text{s})\text{s}^{-1}}\\ &= (48\,\text{W})\left(1-e^{-5}\right)e^{-5}\\ &= \boxed{0.321\,\text{W}}\end{aligned}$$

Evaluate dU_L/dt for $t = 100$ s:

$$\begin{aligned}\frac{dU_L}{dt} &= (48\,\text{W})\left(1-e^{-5(100\,\text{s})\text{s}^{-1}}\right)e^{-5(100\,\text{s})\text{s}^{-1}}\\ &= (48\,\text{W})\left(1-e^{-500}\right)e^{-500}\\ &= \boxed{0}\end{aligned}$$

Remarks: Note that, to a good approximation, $dU_L/dt = P - P_J$.

71 ••

Picture the Problem If the current is initially zero in an LR circuit, its value at some later time t is given by $I = I_f\left(1-e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the time constant of the circuit from the given information and then use the definition of the time constant to find the self-inductance.

(*a*) Express the current in the circuit as a function of time:

$$I = I_f\left(1-e^{-t/\tau}\right) \text{ where } \tau = \frac{L}{R} \qquad (1)$$

Express the current when $t = 4$ s:

$$0.5I_f = I_f\left(1-e^{-4\,\text{s}/\tau}\right)$$

or

$$0.5 = 1-e^{-4\,\text{s}/\tau} \Rightarrow 0.5 = e^{-4\,\text{s}/\tau}$$

Take logarithms of both sides of this equation to obtain:

$$\ln\tfrac{1}{2} = -\frac{4\,\text{s}}{\tau}$$

Solve for and evaluate τ:

$$\tau = \frac{4\,\text{s}}{\ln 2} = \boxed{5.77\,\text{s}}$$

(*b*) Solve equation (1) for and evaluate L:

$$L = R\tau = (5\,\Omega)(5.77\,\text{s}) = \boxed{28.9\,\text{H}}$$

72 ••

Picture the Problem If the current is initially zero in an LR circuit, its value at some later time t is given by $I = I_\text{f}\left(1 - e^{-t/\tau}\right)$, where $I_\text{f} = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the number of time constants that must elapse before the current reaches any given fraction of its final value by solving this equation for t/τ.

Express the fraction of its final value to which the current has risen as a function of time:

$$\frac{I}{I_\text{f}} = 1 - e^{-t/\tau}$$

Solve for t/τ:

$$\frac{t}{\tau} = -\ln\left(1 - \frac{I}{I_\text{f}}\right)$$

(*a*) Evaluate t/τ for $I/I_\text{f} = 0.9$:

$$\left.\frac{t}{\tau}\right|_{90\%} = -\ln(1 - 0.9) = \boxed{2.30}$$

(*b*) Evaluate t/τ for $I/I_\text{f} = 0.99$:

$$\left.\frac{t}{\tau}\right|_{99\%} = -\ln(1 - 0.99) = \boxed{4.61}$$

(*c*) Evaluate t/τ for $I/I_\text{f} = 0.999$:

$$\left.\frac{t}{\tau}\right|_{99.9\%} = -\ln(1 - 0.999) = \boxed{6.91}$$

73 ••

Picture the Problem If the current is initially zero in an LR circuit, its value at some later time t is given by $I = I_\text{f}\left(1 - e^{-t/\tau}\right)$, where $I_\text{f} = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the rate of increase of the current by differentiating I with respect to time and the time for the current to reach any given fraction of its initial value by solving for t.

(*a*) Express the current in the circuit as a function of time:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)$$

Express the initial rate of increase of the current by differentiating this expression with respect to time:

$$\frac{dI}{dt} = \frac{\mathcal{E}_0}{R}\frac{d}{dt}\left(1 - e^{-t/\tau}\right)$$
$$= \frac{\mathcal{E}_0}{R}\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = \frac{\mathcal{E}_0}{\tau R}e^{-\frac{R}{L}t}$$
$$= \frac{\mathcal{E}_0}{L}e^{-\frac{R}{L}t}$$

Evaluate dI/dt at $t = 0$ to obtain:

$$\left.\frac{dI}{dt}\right|_{t=0} = \frac{\mathcal{E}_0}{L}e^0 = \frac{12\,\text{V}}{4\,\text{mH}} = \boxed{3.00\,\text{kA/s}}$$

(*b*) When $I = 0.5I_f$:

$$0.5 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 0.5$$

Evaluate dI/dt with $e^{-t/\tau} = 0.5$ to obtain:

$$\left.\frac{dI}{dt}\right|_{e^{-t/\tau}=0.5} = 0.5\frac{\mathcal{E}_0}{L} = 0.5\left(\frac{12\,\text{V}}{4\,\text{mH}}\right)$$
$$= \boxed{1.50\,\text{kA/s}}$$

(*c*) Calculate I_f from ε and R:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{12\,\text{V}}{150\,\Omega} = \boxed{80.0\,\text{mA}}$$

(*d*) When $I = 0.99I_f$:

$$0.99 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 0.01$$

Solve for and evaluate t:

$$t = -\tau\ln(0.01) = -\frac{L}{R}\ln(0.01)$$
$$= -\frac{4\,\text{mH}}{150\,\Omega}\ln(0.01) = \boxed{0.123\,\text{ms}}$$

74 ••

Picture the Problem If the current is initially zero in an *LR* circuit, its value at some later time t is given by $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the time for the current to reach any given value by solving this equation for t.

Evaluate I_f and τ:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{250\,\text{V}}{8\,\Omega} = 31.25\,\text{A}$$

and

$$\tau = \frac{L}{R} = \frac{50\,\text{H}}{8\,\Omega} = 6.25\,\text{s}$$

Solve $I = I_f\left(1 - e^{-t/\tau}\right)$ for t:

$$t = -\tau \ln\left(1 - \frac{I}{I_f}\right)$$
$$= -(6.25\,\text{s})\ln\left(1 - \frac{I}{31.25\,\text{A}}\right)$$

(*a*) Evaluate t for $I = 10$ A:

$$t\big|_{10\,\text{A}} = -(6.25\,\text{s})\ln\left(1 - \frac{10\,\text{A}}{31.25\,\text{A}}\right) = \boxed{2.41\,\text{s}}$$

(*b*) Evaluate t for $I = 30$ A:

$$t\big|_{30\,\text{A}} = -(6.25\,\text{s})\ln\left(1 - \frac{30\,\text{A}}{31.25\,\text{A}}\right) = \boxed{20.1\,\text{s}}$$

***75 •••**

Picture the Problem The self-induced emf in the inductor is proportional to the rate at which the current through it is changing. Under steady-state conditions, $dI/dt = 0$ and so the self-induced emf in the inductor is zero. We can use Kirchhoff's loop rule to obtain the current through and the voltage across the inductor as a function of time.

(*a*) Because, under steady-state conditions, the self-induced emf in the inductor is zero and because the inductor has negligible resistance, we can apply Kirchhoff's loop rule to the loop that includes the source, the 10-Ω resistor, and the inductor to find the current drawn from the battery and flowing through the inductor and the 10-Ω resistor:

$$10\,\text{V} - (10\,\Omega)I = 0$$

and

$$I = \frac{10\,\text{V}}{10\,\Omega} = \boxed{1.00\,\text{A}}$$

By applying Kirchhoff's junction rule at the junction between the resistors, we can conclude that:

$$I_{100\text{-}\Omega\text{ resistor}} = I_{\text{battery}} - I_{\text{inductor}} = \boxed{0}$$

(*b*) When the switch is closed, the current cannot immediately go to zero in the circuit because of the inductor. For a time, a current will circulate in the circuit loop between the inductor and the 100-Ω resistor. Because the current flowing through this circuit is initially 1 A, the voltage drop across the 100-Ω resistor is initially $\boxed{100\,\text{V}.}$ Conservation of energy (Kirchhoff's loop rule) requires that the voltage drop across the inductor is also $\boxed{100\,\text{V.}}$

(*c*) Apply Kirchhoff's loop rule to the *RL* circuit to obtain:

$$L\frac{dI}{dt}+IR=0$$

The solution to this differential equation is:

$$I(t)=I_0e^{-\frac{R}{L}t}=I_0e^{-\frac{t}{\tau}}$$

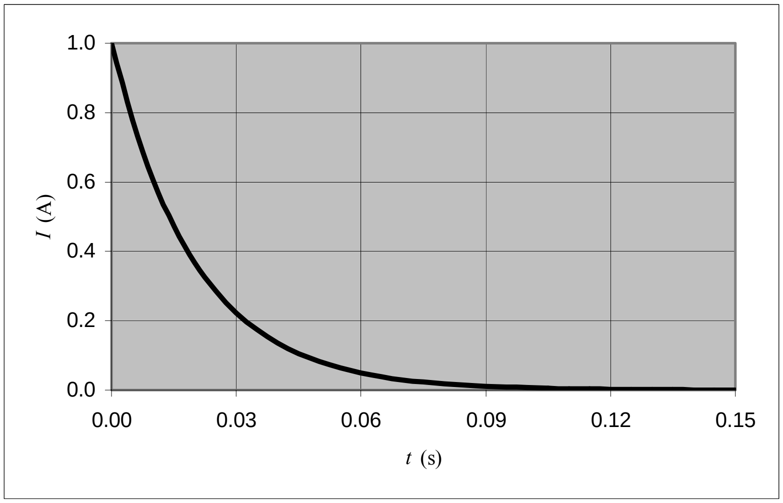
$$\text{where } \tau=\frac{L}{R}=\frac{2\,\text{H}}{100\,\Omega}=0.02\,\text{s}$$

A spreadsheet program to generate the data for graphs of the current and the voltage across the inductor as functions of time is shown below. The formulas used to calculate the quantities in the columns are as follows:

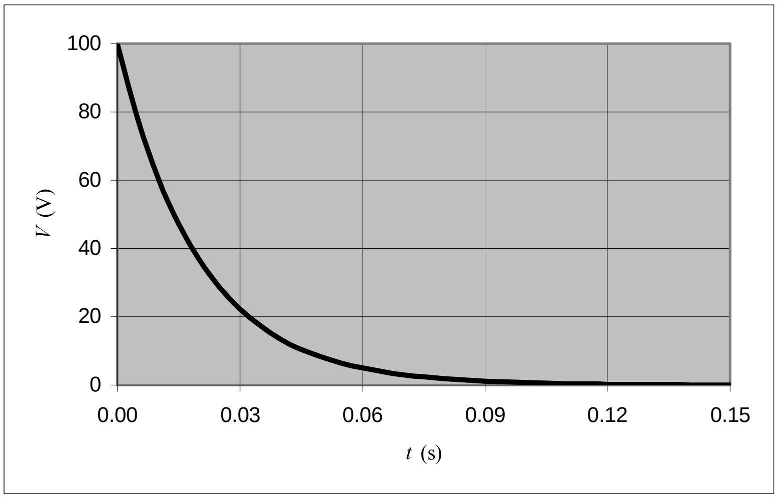
Cell	Formula/Content	Algebraic Form
B1	2	L
B2	100	R
B3	1	I_0
A6	0	t_0
B6	$B$3*EXP((−$B$2/$B$1)*A6)	$I_0e^{-\frac{R}{L}t}$

	A	B	C
1	L=	2	H
2	R=	100	ohms
3	I_0=	1	A
4			
5	t	I(t)	V(t)
6	0.000	1.00E+00	100.00
7	0.005	7.79E−01	77.88
8	0.010	6.07E−01	60.65
9	0.015	4.72E−01	47.24
10	0.020	3.68E−01	36.79
11	0.025	2.87E−01	28.65
12	0.030	2.23E−01	22.31
32	0.130	1.50E−03	0.15
33	0.135	1.17E−03	0.12
34	0.140	9.12E−04	0.09
35	0.145	7.10E−04	0.07
36	0.150	5.53E−04	0.06

The following graph of the current in the inductor as a function of time was plotted using the data in columns A and B of the spreadsheet program.



The following graph of the voltage across the inductor as a function of time was plotted using the data in columns A and C of the spreadsheet program.



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Picture the Problem We can evaluate the derivative of Equation 28-26 with respect to time at $t = 0$ to find the slope of the linear function of current as a function of time. Because the I-intercept of this equation is I_0, we can evaluate $I(t)$ at $t = \tau$ to show that the current is zero after one time constant.

Equation 28-26 describes the current in an LR circuit from which the source has been removed:

$$I = I_0 e^{-\frac{t}{\tau}}$$

Differentiate this expression with respect to t to obtain:

$$\frac{dI}{dt}=I_0\frac{d}{dt}e^{-\frac{t}{\tau}}=I_0e^{-\frac{t}{\tau}}\left(-\frac{1}{\tau}\right)$$
$$=-\frac{I_0}{\tau}e^{-\frac{t}{\tau}}$$

Evaluate dI/dt at $t = 0$:

$$\left.\frac{dI}{dt}\right|_{t=0}=-\frac{I_0}{\tau}$$

Assuming that the current decreases steadily at this rate, express I as a linear function of t to obtain:

$$I(t)=-\frac{I_0}{\tau}t+I_0$$

Evaluate this function when $t=\tau$:

$$I(\tau)=-\frac{I_0}{\tau}\tau+I_0=\boxed{0}$$

as was to have been shown.

77 ••

Picture the Problem The current in an initially energized but source-free *RL* circuit is given by $I=I_0e^{-t/\tau}$. We can find τ from this equation and then use its definition to evaluate L.

(*a*) Express the current in the *RL* circuit as a function of time:

$$I=I_0e^{-t/\tau}$$

Solve for and evaluate τ:

$$\tau=-\frac{t}{\ln\left(\frac{I}{I_0}\right)}=-\frac{45\,\text{ms}}{\ln\left(\frac{1.5\,\text{A}}{2.5\,\text{A}}\right)}=\boxed{88.1\,\text{ms}}$$

(*b*) Using the definition of the inductive time constant, relate L to R:

$$L=\tau R$$

Substitute numerical values and evaluate L:

$$L=(0.0881\,\text{s})(0.4\,\Omega)=\boxed{35.2\,\text{mH}}$$

78 •

Picture the Problem We can model this coil as a resistance-free inductor in series with an inductance-free resistor and express the potential difference across the coil as the sum of the potential differences across the inductor and the resistor. We can then use the given data to obtain two equations in the unknowns R and L and solve these equations simultaneously for the resistance and self-inductance of the coil.

Express the potential difference across the coil as the sum of the potential difference across a resistor and the potential difference across an inductor:

$$V = V_R + V_L = IR + L\frac{dI}{dt}$$

When $I = 5$ A and $dI/dt = 10$ A/s:

$$140\,\text{V} = (5\,\text{A})R + (10\,\text{A/s})L$$

When $I = 5$ A and $dI/dt = -10$ A/s:

$$60\,\text{V} = (5\,\text{A})R - (10\,\text{A/s})L$$

Add these equations to obtain:

$$200\,\text{V} = (10\,\text{A})R$$

and

$$R = \frac{200\,V}{10\,A} = \boxed{20.0\,\Omega}$$

Substitute in either of the equations to obtain:

$$L = \boxed{4.00\,\text{H}}$$

79 ••

Picture the Problem We can use the definition of inductance to express the rate at which the current changes through the inductors and the resistor and the result of Problem 88 to find the effective inductance in the circuit. We can find the final/steady-state current by applying Ohm's law.

(a) Express the rate of change of the current through the resistor:

$$\frac{dI_R}{dt} = \frac{\varepsilon}{L_{\text{eff}}}$$

Using the result given in Problem 88, find L_{eff}:

$$\frac{1}{L_{\text{eff}}} = \frac{1}{8\,\text{mH}} + \frac{1}{4\,\text{mH}}$$

and

$$L_{\text{eff}} = 2.67\,\text{mH}$$

Substitute numerical values and evaluate dI_R/dt at $t = 0$:

$$\left.\frac{dI_R}{dt}\right|_{t=0} = \frac{24\,\text{V}}{2.67\,\text{mH}} = \boxed{9.00\,\text{kA/s}}$$

Express the rate of change of the current through the 8-mH inductor:

$$\frac{dI_{8\,\text{mH}}}{dt}=\frac{\mathcal{E}}{L_{8\,\text{mH}}} \qquad (1)$$

Express the rate of change of the current through the 4-mH inductor:

$$\frac{dI_{4\,\text{mH}}}{dt}=\frac{\mathcal{E}}{L_{4\,\text{mH}}} \qquad (2)$$

Because $IR = 0$ when $t = 0$:

$$V_{L_{\text{eff}}}=V_{8\,\text{mH}}=V_{4\,\text{mH}}=24\,\text{V}$$

Substitute numerical values in equation (1) and evaluate $dI_{8\,\text{mH}}/dt$:

$$\frac{dI_{8\,\text{mH}}}{dt}=\frac{24\,\text{V}}{8\,\text{mH}}=\boxed{3.00\,\text{kA/s}}$$

Substitute numerical values in equation (2) and evaluate $dI_{4\,\text{mH}}/dt$:

$$\frac{dI_{4\,\text{mH}}}{dt}=\frac{24\,\text{V}}{4\,\text{mH}}=\boxed{6.00\,\text{kA/s}}$$

(*b*) After a long time has passed, the inductors will act as a short and the final current will be determined solely by the resistance in the circuit:

$$I_{\text{f}}=\frac{\mathcal{E}}{R}=\frac{24\,\text{V}}{15\,\Omega}=\boxed{1.60\,\text{A}}$$

***80 ••**

Picture the Problem If the current is initially zero in an *LR* circuit, its value at some later time *t* is given by $I=I_{\text{f}}\left(1-e^{-t/\tau}\right)$, where $I_{\text{f}} = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the time at which the power dissipation in the resistor equals the rate at which magnetic energy is stored in the inductor by equating expressions for these rates and using the expression for *I* and its rate of change.

Express the rate at which magnetic energy is stored in the inductor:

$$\frac{dU_L}{dt}=\frac{d}{dt}\left[\tfrac{1}{2}LI^2\right]=LI\frac{dI}{dt}$$

Express the rate at which power is dissipated in the resistor:

$$P=I^2R$$

Equate these expressions to obtain:

$$I^2R=LI\frac{dI}{dt}$$

Simplify to obtain:

$$I=\tau\frac{dI}{dt} \qquad (1)$$

Express the current and its rate of change:

$$I = I_{\mathrm{f}}\left(1 - e^{-t/\tau}\right)$$

and

$$\frac{dI}{dt} = I_{\mathrm{f}}\frac{d}{dt}\left(1 - e^{-t/\tau}\right) = -I_{\mathrm{f}}e^{-t/\tau}\left(-\frac{1}{\tau}\right)$$
$$= \frac{I_{\mathrm{f}}}{\tau}e^{-t/\tau}$$

Substitute in equation (1) to obtain:

$$I_{\mathrm{f}}\left(1 - e^{-t/\tau}\right) = \tau\left(\frac{I_{\mathrm{f}}}{\tau}e^{-t/\tau}\right)$$

or

$$1 - e^{-t/\tau} = e^{-t/\tau} \Rightarrow 1 = 2e^{-t/\tau}$$

Solve for t:

$$t = -\tau \ln \tfrac{1}{2}$$

Using $\tau = 333\ \mu\text{s}$ from Example 28-11, evaluate t to obtain:

$$t = -(333\ \mu\text{s})\ln\tfrac{1}{2} = \boxed{231\ \mu\text{s}}$$

81 •••

Picture the Problem We can integrate $dE/dt = \mathcal{E}_0 I,$ where $I = I_{\mathrm{f}}\left(1 - e^{-t/\tau}\right)$, to find the energy supplied by the battery, $dE_{\mathrm{J}}/dt = I^2R$ to find the energy dissipated in the resistor, and $U_L(\tau) = \frac{1}{2}L(I(\tau))^2$ to express the energy that has been stored in the inductor when $t = \tau$.

(*a*) Express the rate at which energy is supplied by the battery:

$$\frac{dE}{dt} = \mathcal{E}_0 I$$

Express the current in the circuit as a function of time:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)$$

Substitute to obtain:

$$\frac{dE}{dt} = \frac{\mathcal{E}_0^2}{R}\left(1 - e^{-t/\tau}\right)$$

Separate variables and integrate from $t = 0$ to $t = \tau$ to obtain:

$$E = \frac{\mathcal{E}_0^2}{R}\int_0^{\tau}\left(1 - e^{-t/\tau}\right)dt$$
$$= \frac{\mathcal{E}_0^2}{R}\left[\tau - \left(-\tau e^{-1} + \tau\right)\right]$$
$$= \frac{\mathcal{E}_0^2}{R}\frac{\tau}{e} = \frac{\mathcal{E}_0^2 L}{R^2 e}$$

Substitute numerical values and evaluate E:

$$E = \frac{(12\,\text{V})^2(0.6\,\text{H})}{(3\,\Omega)^2 e} = \boxed{3.53\,\text{J}}$$

(*b*) Express the rate at which energy is being dissipated in the resistor:

$$\frac{dE_\text{J}}{dt} = I^2 R = \left[\frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)\right]^2 R$$
$$= \frac{\mathcal{E}_0^2}{R}\left(1 - 2e^{-t/\tau} + e^{-2t/\tau}\right)$$

Separate variables and integrate from $t = 0$ to $t = \tau$ to obtain:

$$E_\text{J} = \frac{\mathcal{E}_0^2}{R}\int_0^\tau \left(1 - 2e^{-t/\tau} + e^{-2t/\tau}\right)dt$$
$$= \frac{\mathcal{E}_0^2}{R}\left(\frac{2\tau}{e} - \frac{\tau}{2} - \frac{\tau}{2e^2}\right)$$
$$= \frac{\mathcal{E}_0^2 L}{R^2}\left(\frac{2}{e} - \frac{1}{2} - \frac{1}{2e^2}\right)$$

Substitute numerical values and evaluate E_J:

$$E_\text{J} = \frac{(12\,\text{V})^2(0.6\,\text{H})}{(3\,\Omega)^2}\left(\frac{2}{e} - \frac{1}{2} - \frac{1}{2e^2}\right)$$
$$= \boxed{1.61\,\text{J}}$$

(*c*) Express the energy stored in the inductor when $t = \tau$:

$$U_L(\tau) = \tfrac{1}{2}L(I(\tau))^2$$
$$= \tfrac{1}{2}L\left(\frac{\mathcal{E}_0}{R}\left(1 - e^{-1}\right)\right)^2$$
$$= \frac{L\mathcal{E}_0^2}{2R^2}\left(1 - e^{-1}\right)^2$$

Substitute numerical values and evaluate E_L:

$$U_L(\tau) = \frac{(0.6\,\text{H})(12\,\text{V})^2}{2(3\,\Omega)^2}\left(1 - e^{-1}\right)^2$$
$$= \boxed{1.92\,\text{J}}$$

Remarks: Note that, as we would expect from energy conservation, $E = E_\text{J} + E_\text{L}$.

General Problems

82 •

Picture the Problem We can apply the definition of magnetic flux to find the flux through the coil in its two orientations with respect to the magnetic field.

(*a*) Using its definition, express the magnetic flux through the coil:

$$\phi_m = NBA\cos\theta = NB\pi r^2\cos\theta$$
$$= (6)(0.5\,\text{T})\pi(0.03\,\text{m})^2\cos 0°$$
$$= \boxed{8.48\,\text{mWb}}$$

(*b*) Proceed as in (*a*) with $\theta = 20°$:

$$\phi_m = NBA\cos\theta = NB\pi r^2\cos\theta$$
$$= (6)(0.5\,\text{T})\pi(0.03\,\text{m})^2\cos 20°$$
$$= \boxed{7.97\,\text{mWb}}$$

83 •

Picture the Problem We can apply the definition of magnetic flux to find the flux through the coil in its two orientations with respect to the magnetic field and then use Faraday's law to find the emfs induced in the coil.

Using Faraday's law, express the emf induced in the coil:

$$\mathcal{E} = -\frac{\Delta\phi_m}{\Delta t} = -\frac{\phi_{m,f} - \phi_{m,i}}{\Delta t} = \frac{\phi_{m,i}}{\Delta t}$$

because $\phi_{m,f} = 0$

(*a*) Using its definition, express the magnetic flux through the coil:

$$\phi_m = NBA\cos\theta = NB\pi r^2\cos\theta$$

Substitute to obtain:

$$\mathcal{E} = \frac{NB\pi r^2\cos\theta}{\Delta t}$$

Substitute numerical values and evaluate ε:

$$\mathcal{E} = \frac{(6)(0.5\,\text{T})\pi(0.03\,\text{m})^2\cos 0°}{1.2\,\text{s}}$$
$$= \boxed{7.07\,\text{mV}}$$

(*b*) Proceed as in (*a*) with $\theta = 20°$:

$$\mathcal{E} = \frac{(6)(0.5\,\text{T})\pi(0.03\,\text{m})^2\cos 20°}{1.2\,\text{s}}$$
$$= \boxed{6.64\,\text{mV}}$$

84 •

Picture the Problem We can apply Faraday's and Ohm's laws to obtain expressions for the induced emf that we can equate and solve for the rate at which the perpendicular magnetic field must change to induce a current of 4.0 A in the coil.

Using Faraday's law, relate the induced emf in the coil to the

$$|\mathcal{E}| = \frac{d\phi_m}{dt} = NA\frac{dB}{dt}$$

changing magnetic flux:

Using Ohm's law, relate the induced emf to the resistance of the coil and the current in it:

$$\left|\mathcal{E}\right| = IR$$

Equate these expressions and solve for dB/dt:

$$NA\frac{dB}{dt} = IR$$

and

$$\frac{dB}{dt} = \frac{IR}{NA} = \frac{IR}{N\pi r^2}$$

Substitute numerical values and evaluate dB/dt:

$$\frac{dB}{dt} = \frac{(4\,\text{A})(25\,\Omega)}{(100)\pi(0.04\,\text{m})^2} = \boxed{199\,\text{T/s}}$$

***85** ••

Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the coil (potential difference between the slip rings). In part (b) we can solve this equation for ω under the given conditions.

(*a*) Use Faraday's law to express the induced emf:

$$\mathcal{E} = -\frac{d\phi_\text{m}}{dt}$$

Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:

$$\phi_\text{m}(t) = NBA\cos\omega t$$

Substitute to obtain:

$$\mathcal{E} = -\frac{d}{dt}\left[NBA\cos\omega t\right] = -NBab\omega(-\sin\omega t) = \boxed{NBab\omega\sin\omega t}$$

(*b*) Express the condition under which $\varepsilon = \varepsilon_{max}$:

$$\sin\omega t = 1$$

Solve for and evaluate ω under this condition:

$$\omega = \frac{\varepsilon_{max}}{NBab}$$
$$= \frac{110\,\text{V}}{(1000)(2\,\text{T})(0.01\,\text{m})(0.02\,\text{m})}$$
$$= \boxed{275\,\text{rad/s}}$$

86 ••

Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the rotating coil gaussmeter.

Use Faraday's law to express the induced emf:

$$\varepsilon = -\frac{d\phi_m}{dt}$$

Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:

$$\phi_m(t) = NBA\cos\omega t$$

Substitute to obtain:

$$\varepsilon = -\frac{d}{dt}[NBA\cos\omega t]$$
$$= -NBA\omega(-\sin\omega t)$$
$$= NBA\omega\sin\omega t = \varepsilon_{max}\sin\omega t$$

where

$$\varepsilon_{max} = NBA\omega$$

Substitute numerical values and evaluate ε_{max}:

$$\varepsilon_{max} = (400)(0.45\,\text{T})(1.4\times10^{-4}\,\text{m}^2)\left(180\,\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right) = \boxed{0.475\,\text{V}}$$

The maximum induced emf occurs at the moment the plane of the coil is parallel to the magnetic field $\vec{B}$. At this instant, ϕ_m is zero, but ε is a maximum.

87 ••

Picture the Problem We can use the equality of the currents in the inductors connected in series and the additive nature of the total induced emf across the inductors to show that the inductances are additive.

Relate the total induced emf to the effective inductance L_{eff} and the rate at which the current is changing in the inductors:

$$\mathcal{E} = L_{\text{eff}} \frac{dI}{dt}$$

Because the inductors L_1 and L_2 are in series:

$$I_1 = I_2 = I$$

and

$$\frac{dI_1}{dt} = \frac{dI_2}{dt} = \frac{dI}{dt}$$

Express the total induced emf:

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$
$$= (L_1 + L_2)\frac{dI}{dt}$$

Substitute in equation (1) and simplify to obtain:

$$L_{\text{eff}} = \boxed{L_1 + L_2}$$

***88 ••**

Picture the Problem We can use the common potential difference across the parallel combination of inductors and the fact that the current into the parallel combination is the sum of the currents through each inductor to find an expression of the equivalent inductance.

Define L_{eff} by:

$$L_{\text{eff}} = \frac{\mathcal{E}}{dI/dt}$$

or

$$\frac{dI}{dt} = \mathcal{E}\frac{1}{L_{\text{eff}}} \qquad (1)$$

Relate the common potential difference across the inductors to their inductances and the rate at which the current is changing in each:

$$\mathcal{E}_1 = L_1 \frac{dI_1}{dt} \qquad (2)$$

and

$$\mathcal{E}_2 = L_2 \frac{dI_2}{dt} \qquad (3)$$

Because the current divides at the parallel junction:

$$I = I_1 + I_2$$

and

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

Solve equations (2) and (3) for dI_1/dt and dI_2/dt and substitute to obtain:

$$\frac{dI}{dt} = \frac{\mathcal{E}_1}{L_1} + \frac{\mathcal{E}_2}{L_2}$$

Express the relationship between an emf ε applied across the parallel combination of inductors and the emfs ε_1 and ε_2 across the individual inductors:

$$\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2$$

Substitute to obtain:

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2} = \mathcal{E}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

Substitute in equation (1) and solve for $1/L_{\text{eff}}$:

$$\frac{1}{L_{\text{eff}}} = \boxed{\frac{1}{L_1} + \frac{1}{L_2}}$$

***89 ••**

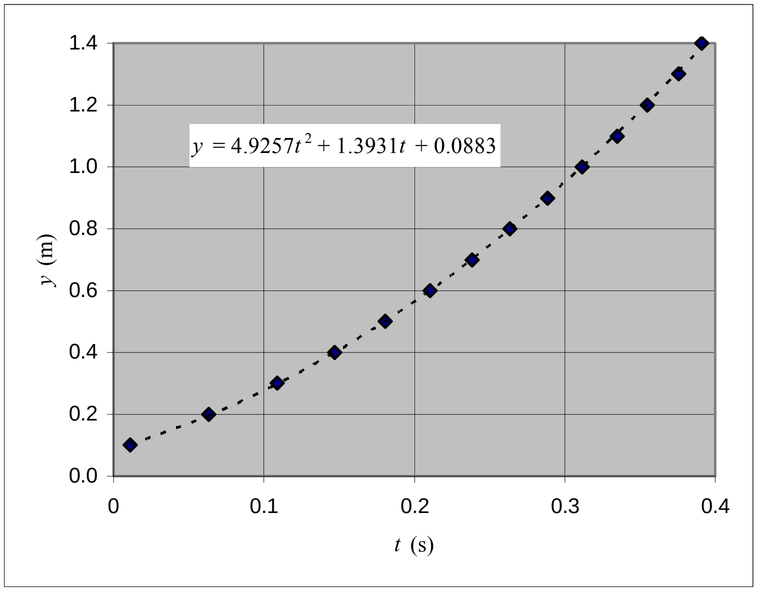
Picture the Problem

(*a*) As the magnet passes through the coil, it induces an emf because of the changing flux through the coil. This allows the coil to "sense" when the magnet is passing through it.

(*b*) One cannot use a cylinder made of conductive material because eddy currents induced in it by a falling magnet would slow the magnet.

(*c*) As the magnet approaches the loop, the flux increases, resulting in the increasing voltage signal. When the magnet is passing the coil, the flux goes from increasing to decreasing, so the induced emf becomes zero and then negative. The time at which the induced emf is zero is the time at which the magnet is at the center of the coil.

(*d*) Each time represents a point when the distance has increased by 10 cm. The following graph of distance versus time was plotted using a spreadsheet program. The regression curve, obtained using Excel's "Add Trendline" feature, is shown as a dashed line.



The coefficient of the second-degree term is $\frac{1}{2}g$. Consequently,

$$g = 2\left(4.9257\,\mathrm{m/s^2}\right) = \boxed{9.85\,\mathrm{m/s^2}}$$

90 ••

Picture the Problem The current equals the induced emf divided by the resistance. We can calculate the emf induced in the circuit as the coil moves by calculating the rate of change of the flux through the coil. The flux is proportional to the area of the coil in the magnetic field. We can find the direction of the current from Lenz's law.

(*a*) and (*c*) Express the magnitude of the induced current:

$$I = \frac{|\mathcal{E}|}{R} \qquad (1)$$

Using Faraday's law, express the magnitude of the induced emf:

$$|\mathcal{E}| = \frac{d\phi_\mathrm{m}}{dt}$$

When the coil is moving to the right (or to the left), the flux does not change (until the coil leaves the region of magnetic field). Thus:

$$|\mathcal{E}| = \frac{d\phi_\mathrm{m}}{dt} = 0$$

and

$$I = \frac{|\mathcal{E}|}{R} = \boxed{0}$$

(*b*) and (*d*) Letting *x* represent the length of the side of the rectangular coil that is in the magnetic field, express the magnetic flux through the coil:

$$\phi_\mathrm{m} = NBwx$$

Compute the rate of change of the flux when the coil is moving up or down:

$$\frac{d\phi_m}{dt} = NBw\frac{dx}{dt}$$
$$= (80)(1.4\,\text{T})(0.25\,\text{m})(2\,\text{m/s})$$
$$= 56.0\,\text{V}$$

Substitute in equation (1) to obtain:

$$I = \frac{56\,\text{V}}{24\,\Omega} = \boxed{2.33\,\text{A}}$$

(*b*) When the coil is moving upward, the outward flux increases and the induced current will be in the sense as to produce inward flux. *I* is clockwise.

(*d*) When the coil is moving downward, the outward flux decreases and the induced current will be in the sense as to produce outward flux. *I* is counterclockwise.

***91 ••**

Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the coil. We can then apply Ohm's law to find the induced current as a function of time. Note that only half of the loop is in the magnetic field.

Apply Ohm's law to relate the induced current to the induced emf:

$$I(t) = \frac{\varepsilon(t)}{R} \qquad (1)$$

Use Faraday's law to express the induced emf:

$$\varepsilon(t) = -\frac{d\phi_m(t)}{dt}$$

Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:

$$\phi_m(t) = NBA\cos\omega t$$

Substitute to obtain:

$$\varepsilon(t) = -\frac{d}{dt}[NBA\cos\omega t]$$
$$= -NBA\omega(-\sin\omega t)$$
$$= NBA\omega\sin\omega t$$

Substitute in equation (1) to obtain:

$$I(t) = \frac{NBA\omega}{R}\sin\omega t$$

Substitute numerical values and evaluate $I(t)$:

$$I(t)=\frac{(80)(1.4\,\text{T})(0.25\,\text{m})(0.15\,\text{m})(2\,\text{rad/s})}{24\Omega}\sin(2\,\text{rad/s})t$$
$$=\boxed{(0.350\,\text{A})\sin(2\,\text{rad/s})t}$$

92 ••

Picture the Problem We can use the laws of Ohm and Faraday to express the charge dQ passing through the coil in time dt and integrate this expression to show that $Q = N(\phi_{m1} - \phi_{m2})/R$.

Use Ohm's law to express the induced current in terms of the induced emf:

$$\frac{dQ}{dt}=\frac{\mathcal{E}}{R}\Rightarrow dQ=\frac{\mathcal{E}}{R}dt$$

Apply Faraday's law to obtain:

$$dQ=-\frac{N}{R}\frac{d\phi_m}{dt}dt=-\frac{N}{R}d\phi_m$$

Integrate dQ from 0 to Q and $d\phi_m = \phi_{m1}$ to ϕ_{m2} to obtain:

$$\int_0^Q dQ'=-\frac{N}{R}\int_{\phi_{m1}}^{\phi_{m2}} d\phi_m$$

and

$$Q=\boxed{\frac{N}{R}(\phi_{m1}-\phi_{m2})}$$

93 ••

Picture the Problem We can apply Faraday's law to relate the induced electric field E to the rates at which the magnetic flux is changing at distances $r < R$ and $r > R$ from the axis of the solenoid.

(*a*) Apply Faraday's law to relate the induced electric field to the magnetic flux in the solenoid within a cylindrical region of radius $r < R$:

$$\oint_C \vec{E}\cdot d\vec{\ell}=-\frac{d\phi_m}{dt}$$

or

$$E(2\pi r)=-\frac{d\phi_m}{dt} \qquad (1)$$

Express the field within the solenoid:

$$B=\mu_0 nI$$

Express the magnetic flux through an area for which $r < R$:

$$\phi_m = BA=\pi r^2\mu_0 nI$$

Substitute in equation (1) to obtain:

$$E(2\pi r) = -\frac{d}{dt}\left[\pi r^2 \mu_0 nI\right] = -\pi r^2 \mu_0 n \frac{dI}{dt}$$

Because $I = I_0 \sin \omega t$:

$$E = -\tfrac{1}{2} r\mu_0 n \frac{d}{dt}\left[I_0 \sin \omega t\right] = \boxed{-\tfrac{1}{2} r\mu_0 n I_0 \omega \cos \omega t}$$

(*b*) Proceed as in (*a*) with $r > R$ to obtain:

$$E(2\pi r) = -\frac{d}{dt}\left[\pi R^2 \mu_0 nI\right] = -\pi R^2 \mu_0 n \frac{dI}{dt} = -\pi R^2 \mu_0 n I_0 \omega \cos \omega t$$

Solve for E to obtain:

$$E = \boxed{-\frac{\mu_0 n R^2 I_0 \omega}{2r} \cos \omega t}$$

94 •••

Picture the Problem The system exhibits cylindrical symmetry, so one can use Ampère's law to determine B inside the inner cylinder, between the cylinders, and outside the outer cylinder. We can use $u_m = B^2/2\mu_0$ and the expression for B from part (a) to express the magnetic energy density in the region between the cylinders. We can integrate this expression for u_m over the volume between the cylinders to find the total magnetic energy in a volume of length ℓ . Finally, we can use our result in part (*c*) and $U_m = \tfrac{1}{2} LI^2$ to find the self-inductance of the cylinders per unit length.

(*a*) For $r < r_1$ and for $r > r_2$ the net enclosed current is zero; consequently, in these regions:

$$B = \boxed{0}$$

For $r_1 < r < r_2$:

$$2\pi r B = \mu_0 I_C \Rightarrow B = \boxed{\frac{\mu_0 I}{2\pi r}}$$

(*b*) Express the magnetic energy density in the region between the cylinders:

$$u_m = \frac{B^2}{2\mu_0}$$

Substitute for B and simplify to obtain:

$$u_{\text{m}} = \frac{\left(\frac{\mu_0 I}{2\pi r}\right)^2}{2\mu_0} = \boxed{\frac{\mu_0 I^2}{8\pi^2 r^2}}$$

(*c*) Express the magnetic energy dU_{m} in the cylindrical element of volume dV:

$$dU_{\text{m}} = u_{\text{m}} dV = \frac{\mu_0 I^2}{8\pi^2 r^2}(\ell 2\pi r dr) = \frac{\mu_0 I^2 \ell}{4\pi} \cdot \frac{dr}{r}$$

Integrate this expression from $r = r_1$ to $r = r_2$ to obtain:

$$U_{\text{m}} = \frac{\mu_0 I^2 \ell}{4\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \boxed{\frac{\mu_0}{4\pi} I^2 \ell \ln \frac{r_2}{r_1}}$$

(*d*) Express the energy in the magnetic field in terms of L and I:

$$U_{\text{m}} = \tfrac{1}{2} L I^2$$

Solve for L:

$$L = \frac{2U_{\text{m}}}{I^2}$$

From our result in (*c*):

$$\frac{U_{\text{m}}}{I^2} = \frac{\mu_0}{4\pi} \ell \ln \frac{r_2}{r_1}$$

Substitute to obtain:

$$L = 2\left(\frac{\mu_0}{4\pi} \ell \ln \frac{r_2}{r_1}\right) = \frac{\mu_0}{2\pi} \ell \ln \frac{r_2}{r_1}$$

Express the ratio L/ℓ:

$$\frac{L}{\ell} = \boxed{\frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}}$$

95 •••

Picture the Problem We can use its definition to express the magnetic flux through a rectangular element of area dA and then integrate from $r = r_1$ to $r = r_2$ to express the total flux through the region. Substituting in $L = \phi_{\text{m}}/I$ will yield the same result found in Part (*d*) of Problem 94.

Use the definition of self-inductance to relate the magnetic flux through the region of interest to the current I:

$$L = \frac{\phi_{\text{m}}}{I} \qquad (1)$$

Consider a strip of unit length ℓ and width dr at a distance r from the

$$d\phi_{\text{m}} = BdA = B\ell dr = Bdr$$

because $\ell = 1$.

axis. The flux through this area is given by:

Apply Ampere's law to express the magnetic field at a distance r from the axis:

$$2\pi rB = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Substitute to obtain:

$$d\phi_m = \frac{\mu_0 I}{2\pi}\frac{dr}{r}$$

Integrate from $r = r_1$ to $r = r_2$ to obtain:

$$\phi_m = \frac{\mu_0 I}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r}$$

and

$$\phi_m = \frac{\mu_0 I}{2\pi}\ln\frac{r_2}{r_1}$$

Substitute in equation (1) to obtain:

$$L = \boxed{\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}}$$

***96 •••**

Picture the Problem We can use $I = \varepsilon/R$ and $\varepsilon = Bv\ell$ to find the current induced in the loop and Lenz's law to determine its direction. We can apply the equation for the force on a current-carrying wire to find the net magnetic force acting on the loop and then sum the forces to find the net force on the loop. Separating the variables in the differential equation and integrating will lead us to an expression for $v(t)$ and a second integration to an expression for $y(t)$. We can solve the latter equation for $y = 1.40$ m to find the time it takes the loop to exit the magnetic field and our expression for $v(t)$ to find its exit speed. Finally, we can use a constant-acceleration equation to find its exit speed in the absence of the magnetic field.

(*a*) Relate the magnitude of the induced current to the induced emf and the resistance of the loop:

$$I = \frac{\mathcal{E}}{R}$$

Relate the induced emf to the motion of the loop:

$$\mathcal{E} = Bv\ell$$

Substitute for ε to obtain:

$$I = \boxed{\frac{B\ell}{R}v}$$

As the loop falls, the flux into the page decreases. The direction of the induced current is such that its magnetic field opposes this decrease, i.e., clockwise.

(*b*) Express the velocity-dependent force that acts on the loop in terms of the current in the loop:

$$F_v = BI\ell$$

Substitute for I to obtain:

$$F_v = B\left(\frac{B\ell}{R}\right)v\ell = \boxed{\frac{B^2\ell^2}{R}v}$$

Apply $d\vec{F} = Id\vec{\ell} \times \vec{B}$ to the horizontal portion of the loop that is in the magnetic field to conclude that the net magnetic force is upward.

Note that the magnetic force on the left side of the loop is to the left and the magnetic force on the right side of the loop is to the right.

(*c*) The net force acting on the loop is the difference between the downward gravitational force and the upward magnetic force:

$$F_{\text{net}} = mg - F_v = \boxed{mg - \frac{B^2\ell^2}{R}v}$$

(*d*) Apply Newton's 2nd law of motion to the loop to obtain its equation of motion:

$$mg - \frac{B^2\ell^2}{R}v = m\frac{dv}{dt}$$

or

$$\boxed{\frac{dv}{dt} = g - \frac{B^2\ell^2}{mR}v}$$

Factor g to obtain an alternate form of the equation of motion:

$$\frac{dv}{dt} = g\left(1 - \frac{B^2\ell^2}{mgR}v\right) = \boxed{g\left(1 - \frac{v}{v_t}\right)}$$

where $v_t = \dfrac{mgR}{B^2\ell^2}$

(*e*) Separate the variables to obtain:

$$\frac{dv}{g - \frac{B^2\ell^2}{mR}v} = dt$$

or

$$\frac{dv}{a - bv} = dt$$

where $a = g$ and $b = \dfrac{B^2\ell^2}{mR}$

Integrate v' from 0 to v and t' from 0 to t:

$$\int_0^v \frac{dv'}{a - bv'} = \int_0^t dt' \Rightarrow -\frac{1}{b}\ln\left(\frac{a-bv}{a}\right) = t$$

Transform from logarithmic to exponential form and solve for v to obtain:

$$v(t) = \frac{a}{b}\left(1 - e^{-bt}\right)$$

Noting that $v_t = \dfrac{a}{b}$, we have:

$$v(t) = \boxed{v_t\left(1 - e^{-t/\tau}\right)}$$

where $\tau = \dfrac{v_t}{a} = \dfrac{v_t}{g}$.

(*f*) Write v as dy/dt and separate variables to obtain:

$$dy = v_t\left(1 - e^{-t/\tau}\right)dt$$

Integrate y' from 0 to y and t' from 0 to t:

$$\int_0^y dy' = v_t \int_0^t \left(1 - e^{-t'/\tau}\right)dt'$$

and

$$y(t) = \boxed{v_t\left[t - \tau\left(1 - e^{-t/\tau}\right)\right]}$$

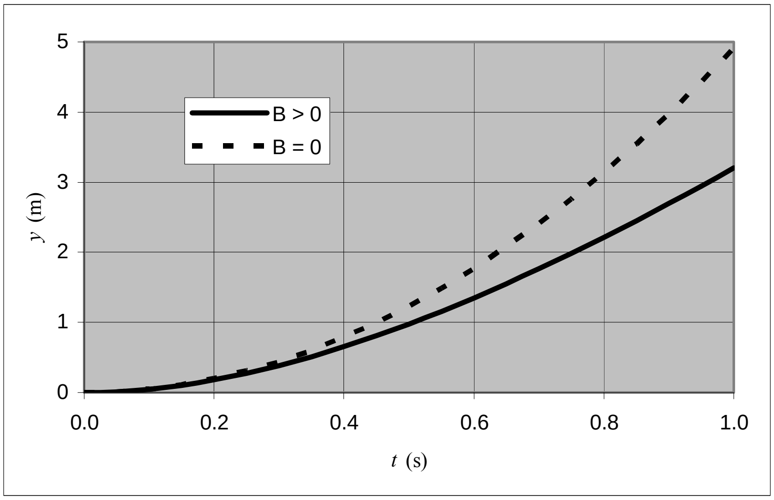
(*g*) A spreadsheet program to generate the data for graphs of position y as a function of time t is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	0.05	m
B2	0.2	R
B3	0.4	B
B4	0.3	L
B5	$B$1*$B$7*$B$2/($B$3^2*$B$4^2)	v_t
B6	$B$5/$B$7	τ
B7	9.81	g
A10	0.00	t
B10	$B$5*(A10–$B$6*(1–EXP(–A10/$B$6)))	y
C10	0.5*$B$7*A10^2	$\frac{1}{2}gt^2$

	A	B	C
1	m=	0.05	kg
2	R=	0.2	ohms
3	B=	0.4	T
4	L=	0.3	m
5	vt=	6.813	m/s
6	tau=	0.694	s
7	g=	9.81	m/s^2
8			
9	t	y	y (no B)
10	0.00	0.000	0.000
11	0.05	0.012	0.012
12	0.10	0.047	0.049
13	0.15	0.103	0.110
14	0.20	0.179	0.196
15	0.25	0.273	0.307
16	0.30	0.384	0.441
17	0.35	0.511	0.601
18	0.40	0.654	0.785
19	0.45	0.809	0.993
20	0.50	0.978	1.226
21	0.55	1.159	1.484
22	0.60	1.351	1.766
23	0.65	1.553	2.072
24	0.70	1.764	2.403
25	0.75	1.985	2.759
26	0.80	2.214	3.139
27	0.85	2.451	3.544
28	0.90	2.695	3.973
29	0.95	2.946	4.427
30	1.00	3.202	4.905

Examining the table, we see that $y = 1.4$ m when $t \approx \boxed{0.60\,\text{s}}$.

The following graph shows y as a function of t for $B \neq 0$ (solid curve) and $B = 0$ (dashed curve).



97 •••

Picture the Problem We can use the expression for the period of this spring-and-mass oscillator to find the spring constant κ. We can express the induced current in the loop by relating it to the induced emf and relating the induced emf to the velocity of the loop. Knowing that the loop is executing SHM, we can express its velocity as a sinusoidal function of time. We can use the expression for the magnetic force on a current-carrying wire in a magnetic field to express the damping force acting on the loop.

(*a*) Express the period of the mass-spring system:

$$T = 2\pi\sqrt{\frac{m}{\kappa}}$$

Solve for κ to obtain:

$$\kappa = \frac{4\pi^2 m}{T^2}$$

Substitute numerical values and evaluate κ:

$$\kappa = \frac{4\pi^2(0.5\,\text{kg})}{(0.8\,\text{s})^2} = \boxed{30.8\,\text{N/m}}$$

(*b*) Express the current in the loop in terms of its resistance and the induced emf:

$$I = \frac{\mathcal{E}}{R}$$

Relate the induced emf in the wire to the motion of the wire:

$$\mathcal{E} = Bv\ell$$

or, because $\ell = w$ (where w is the width of the loop),

$$\mathcal{E} = Bvw$$

Express the position of the mass-spring system as a function of time:

$$y = y_0 \sin \omega t$$

Differentiate this expression with respect to time to express the velocity of the system:

$$v = \frac{dy}{dt} = y_0\omega\cos\omega t$$

Substitute in our expression for I to obtain:

$$I = \boxed{\frac{By_0\omega w}{R}\cos\omega t}$$

(*c*) Express the damping force F_d acting on the loop:

$$F_d = BIw$$

Substitute for I and simplify to obtain:

$$F_d = -Bw\frac{By_0\omega w}{R}\cos\omega t = -\frac{B^2w^2}{R}y_0\omega\cos\omega t$$

Because $v = y_0\omega\cos\omega t$:

$$F_d = -\frac{B^2w^2}{R}v = \boxed{-\beta v}$$

where $\beta = \boxed{\frac{B^2w^2}{R}}$.

(*d*) Choosing the static equilibrium position of the coil as the origin, apply $\sum\vec{F} = m\vec{a}$ to the coil when it is displaced slightly from this equilibrium position to obtain:

$$-F_d - F_r = m\frac{d^2y}{dt^2}$$

where F_r is the restoring force exerted by the plastic spring.

Substituting for F_r and F_d yields the differential equation describing the motion of the coil:

$$-\beta\frac{dy}{dt} - \kappa y = m\frac{d^2y}{dt^2}$$

or

$$\frac{d^2y}{dt^2} + \frac{\beta}{m}\frac{dy}{dt} + \frac{\kappa}{m}y = 0$$

Note: compare this equation to Equation 14-35 on page 446 of Volume 1 of your textbook.

For weak damping, the solution to this differential equation is:

$$y(t) = (y_0\cos\omega t)e^{-(\beta/2m)t}$$

Note: see Equation 14-36 on page 447 of your textbook.

Differentiate $y(t)$ with respect to time to obtain the velocity of the coil:

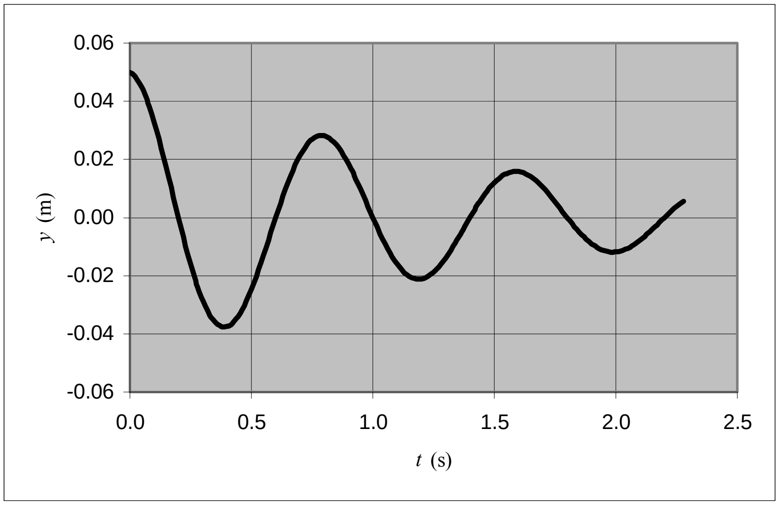
$$v(t) = -\left(\omega y_0 \sin \omega t + \frac{\beta y_0}{2m}\cos \omega t\right)e^{-\left(\frac{\beta}{2m}\right)t}$$

A spreadsheet program to generate the data for graphs of position y and velocity v as functions of time t is shown below. The formulas used to calculate the quantities in the columns are as follows:

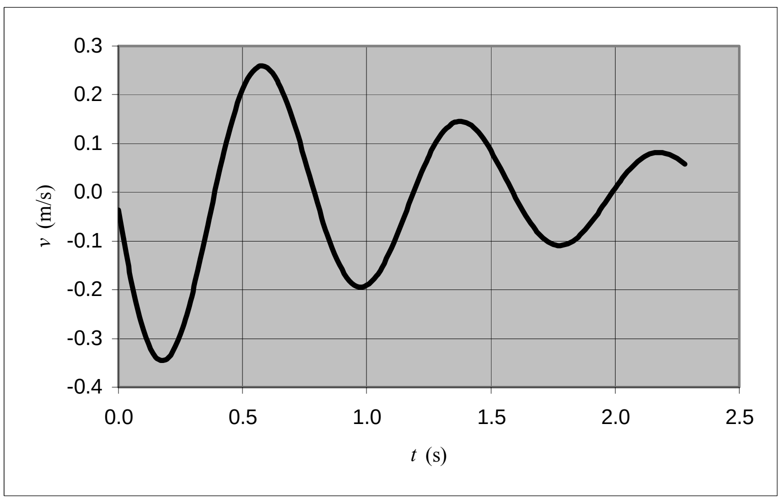
Cell	Formula/Content	Algebraic Form
B1	0.05	y_0
B2	0.8	T
B3	0.4	B
B4	0.2	R
B5	0.3	w
B6	0.05	m
B7	2*PI()/$B$2	ω
B8	$B$3^2*$B$5^2/$B$4	β
A11	0.00	t
B11	$B$1*COS($B$7*A11)* EXP((-$B$8/(2*$B$6))*A11)	$y(t)$
C11	–($B$1*$B$7*SIN($B$7*A11) +($B$8*$B$1/(2*$B$6))*COS($B$7*A11)) *EXP((–$B$8)/(2*$B$6))*A11)	$v(t)$

1	A	B	C
2	y_0=	0.05	m
3	T=	0.8	s
4	B=	0.4	T
5	R=	0.2	ohms
6	w=	0.3	m
7	m=	0.05	kg
8	omega=	7.85	s^-1
9	beta=	0.072	kg/s
10			
11	t	y	v
12	0.00	0.050	–0.036
13	0.01	0.049	–0.066
14	0.02	0.049	–0.096
15	0.03	0.048	–0.124
16	0.04	0.046	–0.151
17	0.05	0.045	–0.177
235	2.24	0.003	0.072
236	2.25	0.004	0.069
237	2.26	0.004	0.066
238	2.27	0.005	0.062
239	2.28	0.006	0.057

The graph of $y(t)$ follows:



The graph of $v(t)$ follows:



98 •••

Picture the Problem If the coil is twisted through an angle θ, a restoring torque equal to $\kappa\theta$ acts on it return it its equilibrium position. However, if it rotates back with angular speed $\omega = d\theta/dt,$ there will be an emf induced in the coil. The direction of the current resulting from this induced emf will be such that its magnetic field will oppose the change in flux resulting from the rotation of the coil. The net effect is that the motion of the coil is damped. We can apply Newton's 2nd law to relate the net restoring torque to the moment of inertia of the coil and its angular acceleration and use the laws of Faraday and Ohm to find the emf and current induced in the coil.

Apply $\sum \tau = I\alpha$ to the rotating coil to obtain:

$$\tau_{\text{restoring}} - \tau_{\text{retarding}} = I\frac{d^2\theta}{dt^2}$$

The magnitude of the retarding (damping) torque is given by:

$$\tau_{\text{retarding}} = NiBA\cos\theta$$

where i is the current induced in the coil whose cross-sectional area is A.

Substitute for $\tau_{\text{restoring}}$ and $\tau_{\text{retarding}}$ to obtain:

$$-\kappa\theta - NiBA\cos\theta = I\frac{d^2\theta}{dt^2} \qquad (1)$$

Apply Faraday's law to express the emf induced in the coil:

$$\varepsilon = -\frac{d}{dt}(NBA\sin\theta) = -(NBA\cos\theta)\frac{d\theta}{dt}$$

From Ohm's law, the magnitude of the induced current i in the coil is:

$$i = \frac{\varepsilon}{R} = \frac{NBA\cos\theta}{R}\frac{d\theta}{dt}$$

Substitute for the induced current i in equation (1) to obtain:

$$-\kappa\theta - \frac{N^2B^2A^2\cos^2\theta}{R}\frac{d\theta}{dt} = I\frac{d^2\theta}{dt^2}$$

For small displacements from equilibrium, $\cos\theta \approx 1$ and:

$$-\kappa\theta - \frac{N^2B^2A^2}{R}\frac{d\theta}{dt} \approx I\frac{d^2\theta}{dt^2}$$

Rearrange terms to obtain the differential equation of motion of the coil:

$$\frac{d^2\theta}{dt^2} + \frac{N^2B^2A^2}{RI}\frac{d\theta}{dt} + \frac{\kappa}{I}\theta \approx 0$$

Let $\beta = \dfrac{N^2B^2A^2}{RI}$ and $\omega = \sqrt{\dfrac{\kappa}{I}}$ to obtain:

$$\frac{d^2\theta}{dt^2} + \beta\frac{d\theta}{dt} + \omega^2\theta \approx 0$$

The solution to this second-order, homogeneous, linear differential equation with constant coefficients is:

$$\theta(t) = \boxed{\theta_0 e^{-(\beta/2)t}\cos\omega t}$$

Chapter 29
Alternating-Current Circuits

Conceptual Problems

***1** •

Determine the Concept Because the rms current through the resistor is given by $I_{\text{rms}} = \mathcal{E}_{\text{rms}}/R$ and both $\mathcal{E}_{\text{rms}}$ and R are independent of frequency, $\boxed{(b) \text{ is correct.}}$

2 •

Picture the Problem We can use the relationship between V and V_{peak} to decide the effect of doubling the rms voltage on the peak voltage.

Express the initial rms voltage in terms of the peak voltage:

$$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$$

Express the doubled rms voltage in terms of the new peak voltage V'_{max}:

$$2V_{\text{rms}} = \frac{V'_{\text{max}}}{\sqrt{2}}$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{2V_{\text{rms}}}{V_{\text{rms}}} = \frac{\frac{V'_{\text{max}}}{\sqrt{2}}}{\frac{V_{\text{max}}}{\sqrt{2}}} \text{ or } 2 = \frac{V'_{\text{max}}}{V_{\text{max}}}$$

Solve for V'_{max}:

$$V'_{\text{max}} = 2V_{\text{max}} \text{ and } \boxed{(a) \text{ is correct.}}$$

3 •

Determine the Concept The inductance of an inductor is determined by the details of its construction and is independent of the frequency of the circuit. The inductive reactance, on the other hand, is frequency dependent. $\boxed{(b) \text{ is correct.}}$

4 •

Determine the Concept The inductive reactance of an inductor varies with the frequency according to $X_L = \omega L$. Hence, doubling ω will double X_L. $\boxed{(a) \text{ is correct.}}$

***5 •**

Determine the Concept The capacitive reactance of an capacitor varies with the frequency according to $X_C = 1/\omega C$. Hence, doubling ω will halve X_C. $\boxed{(c)\text{ is correct.}}$

6 •

Determine the Concept Yes to both questions. While the current in the inductor is increasing, the inductor absorbs power from the generator. When the current in the inductor reverses direction, the inductor supplies power to the generator.

7 •

Determine the Concept Yes to both questions. While charge is accumulating on the capacitor, the capacitor absorbs power from the generator. When the capacitor is discharging, it supplies power to the generator.

8 •

Picture the Problem We can use the definitions of the capacitive reactance and inductive reactance to find the SI units of *LC*.

Use its definition to express the inductive reactance:

$$X_L = 2\pi fL$$

Solve for *L*:

$$L = \frac{X_L}{2\pi f}$$

Use its definition to express the capacitive reactance:

$$X_C = \frac{1}{2\pi fC}$$

Solve for *C*:

$$C = \frac{1}{2\pi f X_C}$$

Express the product of *L* and *C*:

$$LC = \frac{X_L}{2\pi f}\frac{1}{2\pi f X_C} = \frac{X_L}{4\pi^2 f^2 X_C}$$

Because the units of X_L and X_C cancel, the units of *LC* are those of $1/f^2$ or s^2.
$\boxed{(a)\text{ is correct.}}$

***9 ••**

Determine the Concept To make an *LC* circuit with a small resonance frequency requires a large inductance and large capacitance. Neither is easy to construct.

10 •

(a) True. The Q factor and the width of the resonance curve at half power are related according to $Q = \omega_0 / \Delta\omega$; i.e., they are inversely proportional to each other.

(b) True. The impedance of an *RLC* circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance $X_L = X_C$ and so $Z = R$.

(c) True. The phase angle δ is related to X_L and X_C according to $\delta = \tan^{-1} \frac{X_L - X_C}{R}$. At resonance $X_L = X_C$ and so $\delta = 0$.

11 •

Determine the Concept Yes. The power factor is defined to be $\cos\delta = R/Z$ and, because Z is frequency dependent, so is $\cos\delta$.

***12 •**

Determine the Concept Yes; the bandwidth must be wide enough to accommodate the modulation frequency.

13 •

Determine the Concept Because the power factor is defined to be $\cos\delta = R/Z$, if R = 0, then the power factor is zero.

14 •

Determine the Concept A transformer is a device used to raise or lower the voltage in a circuit without an appreciable loss of power. $\boxed{(c)\text{ is correct.}}$

15 •

True. If energy is to be conserved, the product of the current and voltage must be constant.

16 ••

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. Assuming no loss of power in the transformer, we can equate the power in the primary circuit to the power in the secondary circuit and solve for the current in the primary windings.

Assuming no loss of power in the transformer: $P_1 = P_2$

Substitute for P_1 and P_2 to obtain: $I_1V_1 = I_2V_2$

Solve for I_1:

$$I_1 = I_2\frac{V_2}{V_1} = \frac{I_2V_2}{V_1} = \frac{P_2}{V_1}$$

and $\boxed{(b)\text{ is correct}}$

17 •

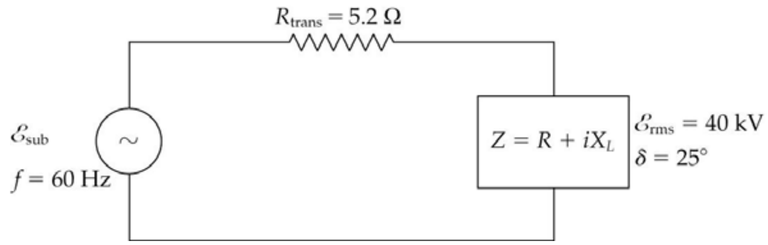
(*a*) False. The effective (rms) value of the current is not zero.

(*b*) True. The reactance of a capacitor goes to zero as *f* approaches very high frequencies.

Estimation and Approximation

***18** ••

Picture the Problem We can find the resistance and inductive reactance of the plant's total load from the impedance of the load and the phase constant. The current in the power lines can be found from the total impedance of the load the potential difference across it and the rms voltage at the substation by applying Kirchhoff's loop rule to the substation-transmission wires-load circuit. The power lost in transmission can be found from $P_{\text{trans}} = I_{\text{rms}}^2 R_{\text{trans}}$. We can find the cost savings by finding the difference in the power lost in transmission when the phase angle is reduced to 18°. Finally, we can find the capacitance that is required to reduce the phase angle to 18° by first finding the capacitive reactance using the definition of $\tan\delta$ and then applying the definition of capacitive reactance to find *C*.



(*a*) Relate the resistance and inductive reactance of the plant's total load to *Z* and δ:

$R = Z\cos\delta$
and
$X_L = Z\sin\delta$

Express *Z* in terms of the current *I* in the power lines and voltage ε_{rms} at the plant:

$$Z = \frac{\varepsilon_{\text{rms}}}{I}$$

Express the power delivered to the plant in terms of ε_{rms}, I_{rms}, and δ and

$P_{\text{av}} = \varepsilon_{\text{rms}} I_{\text{rms}}\cos\delta$
and

solve for I_{rms}:

$$I_{\text{rms}} = \frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}\cos\delta} \qquad (1)$$

Substitute to obtain:

$$Z = \frac{\varepsilon_{\text{rms}}^2\cos\delta}{P_{\text{av}}}$$

Substitute numerical values and evaluate Z:

$$Z = \frac{(40\,\text{kV})^2\cos 25°}{2.3\,\text{MW}} = 630\,\Omega$$

Substitute numerical values and evaluate R and X_L:

$$R = (630\,\Omega)\cos 25° = \boxed{571\,\Omega}$$

and

$$X_L = (630\,\Omega)\sin 25° = \boxed{266\,\Omega}$$

(b) Use equation (1) to find the current in the power lines:

$$I_{\text{rms}} = \frac{2.3\,\text{MW}}{(40\,\text{kV})\cos 25°} = \boxed{63.4\,\text{A}}$$

Apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E}_{\text{sub}} - I_{\text{rms}}R_{\text{trans}} - IZ_{\text{tot}} = 0$$

Solve for ε_{sub}:

$$\mathcal{E}_{\text{sub}} = I_{\text{rms}}(R_{\text{trans}} + Z_{\text{tot}})$$

Evaluate Z_{tot}:

$$Z_{\text{tot}} = \sqrt{R^2 + X_L^2} = \sqrt{(571\Omega)^2 + (266\,\Omega)^2} = 630\,\Omega$$

Substitute numerical values and evaluate ε_{sub}:

$$\mathcal{E}_{\text{sub}} = (63.4\,\text{A})(5.2\,\Omega + 630\,\Omega) = \boxed{40.3\,\text{kV}}$$

(c) The power lost in transmission is:

$$P_{\text{trans}} = I_{\text{rms}}^2 R_{\text{trans}} = (63.4\,\text{A})^2(5.2\,\Omega) = \boxed{20.9\,\text{kW}}$$

(d) Express the cost savings ΔC in terms of the difference in energy consumption $(P_{25°} - P_{18°})\Delta t$ and the per-unit cost u of the energy:

$$\Delta C = (P_{25°} - P_{18°})\Delta tu$$

Express the power list in

$$P_{18°} = I_{18°}^2 R_{\text{trans}}$$

transmission when $\delta = 18°$:

Find the current in the transmission lines when $\delta = 18°$:

$$I_{18°} = \frac{2.3\,\text{MW}}{(40\,\text{kV})\cos 18°} = 60.5\,\text{A}$$

Evaluate $P_{18°}$:

$$P_{18°} = (60.5\,\text{A})^2(5.2\,\Omega) = 19.0\,\text{kW}$$

Substitute numerical values and evaluate ΔC:

$$\Delta C = (20.9\,\text{kW} - 19.0\,\text{kW})(16\,\text{h/d})(30\,\text{d/month})(\$0.07\,/\,\text{kW}\cdot\text{h}) = \boxed{\$63.84}$$

Relate the new phase angle δ to the inductive reactance X_L, the reactance due to the added capacitance X_C, and the resistance of the load R:

$$\tan\delta = \frac{X_L - X_C}{R}$$

Solve for and evaluate X_C:

$$X_C = X_L - R\tan\delta$$
$$= 266\,\Omega - (571\,\Omega)\tan 18° = 80.5\,\Omega$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi(60\,\text{s}^{-1})(80.5\,\Omega)} = \boxed{33.0\,\mu\text{F}}$$

Alternating Current Generators

19 •

Picture the Problem We can use the relationship $\mathcal{E}_{\text{max}} = 2\pi NBAf$ between the maximum emf induced in the coil and its frequency to find f when ε_{max} is given and ε_{max} when f is given .

(*a*) Relate the induced emf to the angular frequency of the coil:

$$\mathcal{E} = \mathcal{E}_{\text{max}}\cos\omega t$$

where

$$\mathcal{E}_{\text{max}} = NBA\omega = 2\pi NBAf$$

Solve for f:

$$f = \frac{\mathcal{E}_{\text{max}}}{2\pi NBA}$$

Substitute numerical values and evaluate f:

$$f = \frac{10\,\text{V}}{2\pi(200)(0.5\,\text{T})(4\times10^{-4}\,\text{m}^2)}$$
$$= \boxed{39.8\,\text{Hz}}$$

(*b*) From (*a*) we have:

$$\mathcal{E}_{\max} = NBA\omega = 2\pi NBAf$$

Substitute numerical values and evaluate $\mathcal{E}_{\max}$:

$$\mathcal{E}_{\max} = 2\pi(200)(0.5\,\text{T})(4\times10^{-4}\,\text{m}^2)(60\,\text{s}^{-1}) = \boxed{15.1\,\text{V}}$$

20 •

Picture the Problem We can use the relationship $\mathcal{E}_{\max} = 2\pi NBAf$ between the maximum emf induced in the coil and the magnetic field in which it is rotating to find *B* required to generate a given emf at a given frequency.

Relate the induced emf to the magnetic field in which the coil is rotating:

$$\mathcal{E}_{\max} = NBA\omega = 2\pi NBAf$$

Solve for *B*:

$$B = \frac{\mathcal{E}_{\max}}{2\pi NfA}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{10\,\text{V}}{2\pi(200)(60\,\text{s}^{-1})(4\times10^{-4}\,\text{m}^2)} = \boxed{0.332\,\text{T}}$$

***21** •

Picture the Problem We can use the relationship $\mathcal{E}_{\max} = 2\pi NBAf$ to relate the maximum emf generated to the area of the coil, the number of turns of the coil, the magnetic field in which the coil is rotating, and the frequency at which it rotates.

(*a*) Relate the induced emf to the magnetic field in which the coil is rotating:

$$\mathcal{E}_{\max} = NBA\omega = 2\pi NBAf \qquad (1)$$

Substitute numerical values and evaluate $\mathcal{E}_{\max}$:

$$\mathcal{E}_{\max} = 2\pi(300)(0.4\,\text{T})(2\times10^{-2}\,\text{m})(1.5\times10^{-2}\,\text{m})(60\,\text{s}^{-1}) = \boxed{13.6\,\text{V}}$$

(*b*) Solve equation (1) for *f*:

$$f = \frac{\mathcal{E}_{\max}}{2\pi NBA}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{110\,\text{V}}{2\pi(300)(0.4\,\text{T})(2\times10^{-2}\,\text{m})(1.5\times10^{-2}\,\text{m})} = \boxed{486\,\text{Hz}}$$

22 •

Picture the Problem We can use the relationship $\mathcal{E}_{max} = 2\pi NBAf$ to relate the maximum emf generated to the area of the coil, the number of turns of the coil, the magnetic field in which the coil is rotating, and the frequency at which it rotates.

Relate the induced emf to the magnetic field in which the coil is rotating:

$$\mathcal{E}_{max} = NBA\omega = 2\pi NBAf$$

Solve for B:

$$B = \frac{\mathcal{E}_{max}}{2\pi NfA}$$

Substitute numerical values and evaluate B:

$$B = \frac{24\,\text{V}}{2\pi(300)(60\,\text{s}^{-1})(2\times10^{-2}\,\text{m})(1.5\times10^{-2}\,\text{m})} = \boxed{0.707\,\text{T}}$$

Alternating Current in a Resistor

***23** •

Picture the Problem We can use $P_{av} = \mathcal{E}_{rms} I_{rms}$ to find I_{rms}, $I_{max} = \sqrt{2} I_{rms}$ to find I_{max}, and $P_{max} = I_{max}\mathcal{E}_{max}$ to find P_{max}.

(*a*) Relate the average power delivered by the source to the rms voltage across the bulb and the rms current through it:

$$P_{av} = \mathcal{E}_{rms} I_{rms}$$

Solve for and evaluate I_{rms}:

$$I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms}} = \frac{100\,\text{W}}{120\,\text{V}} = \boxed{0.833\,\text{A}}$$

(*b*) Express I_{max} in terms of I_{rms}:

$$I_{max} = \sqrt{2} I_{rms}$$

Substitute for I_{rms} and evaluate I_{max}:

$$I_{max} = \sqrt{2}(0.833\,\text{A}) = \boxed{1.18\,\text{A}}$$

(*c*) Express the maximum power in terms of the maximum voltage and maximum current:

$$P_{max} = I_{max}\mathcal{E}_{max}$$

Substitute numerical values and evaluate P_{max}:

$$P_{max} = (1.18\,\text{A})\sqrt{2}(120\,\text{V}) = \boxed{200\,\text{W}}$$

24 •

Picture the Problem We can $I_{max} = \sqrt{2}I_{rms}$ to find the largest current the breaker can carry and $P_{av} = I_{rms}V_{rms}$ to find the average power supplied by this circuit.

(*a*) Express I_{max} in terms of I_{rms}:

$$I_{max} = \sqrt{2}I_{rms} = \sqrt{2}(15\,\text{A}) = \boxed{21.2\,\text{A}}$$

(*b*) Relate the average power to the rms current and voltage:

$$P_{av} = I_{rms}V_{rms} = (15\,\text{A})(120\,\text{V}) = \boxed{1.80\,\text{kW}}$$

Alternating Current in Inductors and Capacitors

25 •

Picture the Problem We can use $X_L = \omega L$ to find the reactance of the inductor at any frequency.

Express the inductive reactance as a function of *f*:

$$X_L = \omega L = 2\pi f L$$

(*a*) At $f = 60$ Hz:

$$X_L = 2\pi(60\,\text{s}^{-1})(1\,\text{mH}) = \boxed{0.377\,\Omega}$$

(*b*) At $f = 600$ Hz:

$$X_L = 2\pi(600\,\text{s}^{-1})(1\,\text{mH}) = \boxed{3.77\,\Omega}$$

(*c*) At $f = 6$ kHz:

$$X_L = 2\pi(6000\,\text{s}^{-1})(1\,\text{mH}) = \boxed{37.7\,\Omega}$$

26 •

(*a*) Relate the reactance of the inductor to its inductance:

$$X_L = \omega L = 2\pi f L$$

Solve for and evaluate *L*:

$$L = \frac{X_L}{2\pi f} = \frac{100\,\Omega}{2\pi(80\,\text{s}^{-1})} = \boxed{0.199\,\text{H}}$$

(*b*) At 160 Hz:

$$X_L = 2\pi(160\,\text{s}^{-1})(0.199\,\text{H}) = \boxed{200\,\Omega}$$

27 •

Picture the Problem We can equate the reactances of the capacitor and the inductor and then solve for the frequency.

Express the reactance of the inductor:

$$X_L = \omega L = 2\pi fL$$

Express the reactance of the capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

Equate these reactances to obtain:

$$2\pi fL = \frac{1}{2\pi fC}$$

Solve for f to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{(10\,\mu\text{F})(1\,\text{mH})}} = \boxed{1.59\,\text{kHz}}$$

28 •

Picture the Problem We can use $X_C = 1/\omega C$ to find the reactance of the capacitor at any frequency.

Express the capacitive reactance as a function of f:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

(a) At f = 60 Hz:

$$X_C = \frac{1}{2\pi(60\,\text{s}^{-1})(1\,\text{nF})} = \boxed{2.65\,\text{M}\Omega}$$

(b) At f = 6 kHz:

$$X_C = \frac{1}{2\pi(6000\,\text{s}^{-1})(1\,\text{nF})} = \boxed{26.5\,\text{k}\Omega}$$

(c) At f = 6 MHz:

$$X_C = \frac{1}{2\pi(6\times10^6\,\text{s}^{-1})(1\,\text{nF})} = \boxed{26.5\,\Omega}$$

***29** •

Picture the Problem We can use $I_{\text{max}} = \varepsilon_{\text{max}}/X_C$ and $X_C = 1/\omega C$ to express *I*max as a function of ε_{max}, f, and C. Once we've evaluate I_{max}, we can use $I_{\text{rms}} = I_{\text{max}}/\sqrt{2}$ to find I_{rms}.

Express I_{max} in terms of $\mathcal{E}_{max}$ and X_C:

$$I_{max} = \frac{\mathcal{E}_{max}}{X_C}$$

Express the capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Substitute to obtain:

$$I_{max} = 2\pi f C \mathcal{E}_{max}$$

(*a*) Substitute numerical values and evaluate I_{max}:

$$I_{max} = 2\pi\left(20\,\text{s}^{-1}\right)(20\,\mu\text{F})(10\,\text{V}) = \boxed{25.1\,\text{mA}}$$

(*b*) Express I_{rms} in terms of I_{max}:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{25.1\,\text{mA}}{\sqrt{2}} = \boxed{17.8\,\text{mA}}$$

30 •

Picture the Problem We can use $X_C = 1/\omega C = 1/2\pi f C$ to relate the reactance of the capacitor to the frequency.

Using its definition, express the reactance of a capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Solve for *f* to obtain:

$$f = \frac{1}{2\pi C X_C}$$

(*a*) Find *f* when $X_C = 1\ \Omega$:

$$f = \frac{1}{2\pi(10\,\mu\text{F})(1\Omega)} = \boxed{15.9\,\text{kHz}}$$

(*b*) Find *f* when $X_C = 100\ \Omega$:

$$f = \frac{1}{2\pi(10\,\mu\text{F})(100\,\Omega)} = \boxed{159\,\text{Hz}}$$

(*c*) Find *f* when $X_C = 0.01\ \Omega$:

$$f = \frac{1}{2\pi(10\,\mu\text{F})(0.01\Omega)} = \boxed{1.59\,\text{MHz}}$$

31 ••

Picture the Problem We can use the trigonometric identity $\cos\theta + \cos\phi = 2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$ to find the sum of the phasors V_1 and V_2 and then use this sum to express *I* as a function of time. In (*b*) we'll use a phasor diagram to obtain the same result and in (*c*) we'll use the phasor diagram appropriate to the given voltages to express the current as a function of time.

(*a*) Express the current in the resistor:

$$I=\frac{V}{R}=\frac{V_1+V_2}{R}$$

Use the trigonometric identity $\cos\theta+\cos\phi=2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$ to find V_1+V_2:

$$V_1+V_2=(5.0\,\text{V})[\cos(\omega t-\alpha)+\cos(\omega t+\alpha)]=(5\,\text{V})[2\cos\tfrac{1}{2}(2\omega t)\cos\tfrac{1}{2}(-2\alpha)]$$
$$=(10\,\text{V})\cos\frac{\pi}{6}\cos\omega t=(8.66\,\text{V})\cos\omega t$$

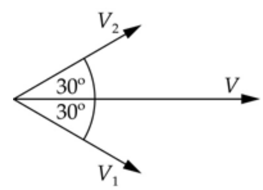
Substitute to obtain:

$$I=\frac{(8.66\,\text{V})\cos\omega t}{25\,\Omega}=\boxed{(0.346\,\text{A})\cos\omega t}$$

(*b*) Express the magnitude of the current in *R*:

$$|I|=\frac{|V|}{R}$$

The phasor diagram for the voltages is shown to the right.



Use vector addition to find $|V|$:

$$|V|=2|V_1|\cos 30°=2(5\,\text{V})\cos 30°$$
$$=8.66\,\text{V}$$

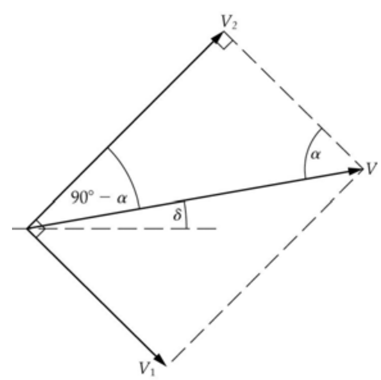
Substitute to obtain:

$$|I|=\frac{8.66\,\text{V}}{25\,\Omega}=0.346\,\text{A}$$

and

$$I=\boxed{(0.346\,\text{A})\cos\omega t}$$

(*c*) The phasor diagram is shown to the right. Note that the phase angle between V_1 and V_2 is now 90°.



Use the Pythagorean theorem to find $|V|$:

$$|V|=\sqrt{|V_1|^2+|V_2|^2}=\sqrt{(5\,\text{V})^2+(7\,\text{V})^2}$$
$$=8.60\,\text{V}$$

Express I as a function of t:

$$I = \frac{|V|}{R}\cos(\omega t + \delta)$$

where

$$\delta = 45° - (90° - \alpha) = \alpha - 45°$$
$$= \tan^{-1}\left(\frac{7\,\text{V}}{5\,\text{V}}\right) - 45° = 9.46° = 0.165\,\text{rad}$$

Substitute numerical values and evaluate I:

$$I = \frac{8.60\,\text{V}}{25\,\Omega}\cos(\omega t + 0.165\,\text{rad})$$
$$= \boxed{(0.344\,\text{A})\cos(\omega t + 0.165\,\text{rad})}$$

LC and *RLC* Circuits without a Generator

***32** •

Picture the Problem We can use $X_L = \omega L$ and $X_C = 1/\omega C$ to show the $1/\sqrt{LC}$ has the unit s^{-1}. Alternatively, we can use the dimensions of C and L to establish this result.

Substitute the units for L and C in the expression $1/\sqrt{LC}$ to obtain:

$$\frac{1}{\sqrt{\text{H}\cdot\text{F}}} = \frac{1}{\sqrt{(\Omega\cdot\text{s})\left(\dfrac{\text{s}}{\Omega}\right)}} = \frac{1}{\sqrt{\text{s}^2}} = \boxed{\text{s}^{-1}}$$

Alternatively, use the defining equation ($C = Q/V$) for capacitance to obtain the dimension of C:

$$[C] = \frac{[Q]}{[V]}$$

Solve the defining equation ($V = L\,dI/dt$) for inductance to obtain the dimension of L:

$$[L] = \frac{[V]}{\left[\dfrac{dI}{dt}\right]} = \frac{[V]}{\dfrac{[Q]}{[T]^2}} = \frac{[V][T]^2}{[Q]}$$

Express the dimension of $1/\sqrt{LC}$:

$$\left[\frac{1}{\sqrt{LC}}\right] = \left[\frac{1}{\sqrt{[L][C]}}\right] = \left[\frac{1}{\sqrt{\dfrac{[V][T]^2}{[Q]}\dfrac{[Q]}{[V]}}}\right]$$
$$= \left[\frac{1}{\sqrt{[T]^2}}\right] = \frac{1}{[T]}$$

Because the SI unit of time is the second, we've shown that $1/\sqrt{LC}$ has units of $\boxed{\text{s}^{-1}.}$

33 •

Picture the Problem We can use $T = 2\pi/\omega$ and $\omega = 1\big/\sqrt{LC}$ to relate *T* (and hence *f*) to *L* and *C*.

(*a*) Express the period of oscillation of the *LC* circuit:

$$T = \frac{2\pi}{\omega}$$

For an *LC* circuit:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute to obtain:

$$T = 2\pi\sqrt{LC} \qquad (1)$$

Substitute numerical values and evaluate *T*:

$$T = 2\pi\sqrt{(2\,\text{mH})(20\,\mu\text{F})} = \boxed{1.26\,\text{ms}}$$

(*b*) Solve equation (1) for *L* to obtain:

$$L = \frac{T^2}{4\pi^2 C} = \frac{1}{4\pi^2 f^2 C}$$

Substitute numerical values and evaluate *L*:

$$L = \frac{1}{4\pi^2\left(60\,\text{s}^{-1}\right)^2(80\,\mu\text{F})} = \boxed{88.0\,\text{mH}}$$

34 ••

Picture the Problem We can use the expression $f_0 = 1\big/2\pi\sqrt{LC}$ for the resonance frequency of an *LC* circuit to show that each circuit oscillates with the same frequency. In (*b*) we can use $I_{\text{max}} = \omega Q_0$, where Q_0 is the charge of the capacitor at time zero, and the definition of capacitance $Q_0 = CV$ to express I_{max} in terms of *ω*, *C* and *V*.

Express the resonance frequency for an *LC* circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(*a*) Express the product of *L* and *C* for each circuit:

Circuit 1 : L_1C_1,
Circuit 2 : $L_2C_2 = (2L_1)\left(\tfrac{1}{2}C_1\right) = L_1C_1$,
and
Circuit 3 : $L_3C_3 = \left(\tfrac{1}{2}L_1\right)(2C_1) = L_1C_1$

Because $L_1C_1 = L_2C_2 = L_3C_3$, the resonance frequencies of the three circuits are the same.

(*b*) Express I_{max} in terms of the

$$I_{\text{max}} = \omega Q_0$$

charge stored in the capacitor:

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substitute to obtain:

$$I_{max} = \omega CV$$

or, for ω and V constant,

$$I_{max} \propto C$$

$\boxed{\text{The circuit with } C = C_3 \text{ has the greatest } I_{max}.}$

35 ••

Picture the Problem We can use $U = \frac{1}{2}CV^2$ to find the energy stored in the electric field of the capacitor, $\omega_0 = 2\pi f_0 = 1/\sqrt{LC}$ to find f_0, and $I_{max} = \omega Q_0$ and $Q_0 = CV$ to find I_{max}.

(*a*) Express the energy stored in the system as a function of C and V:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate U:

$$U = \tfrac{1}{2}(5\,\mu\text{F})(30\text{ V})^2 = \boxed{2.25\text{ mJ}}$$

(*b*) Express the resonance frequency of the circuit in terms of L and C:

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

Solve for f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{(10\text{ mH})(5\,\mu\text{F})}} = \boxed{712\text{ Hz}}$$

(*c*) Express I_{max} in terms of the charge stored in the capacitor:

$$I_{max} = \omega Q_0$$

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substitute to obtain: $I_{max} = \omega CV$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = 2\pi\left(712\,\text{s}^{-1}\right)(5\,\mu\text{F})(30\,\text{V}) = \boxed{0.671\,\text{A}}$$

36 •

Picture the Problem We can use its definition to find the power factor of the circuit and $I_{rms} = \varepsilon/Z$ to find the rms current in the circuit. In (*c*) we can use $P_{av} = I_{rms}^2 R$ to find the average power supplied to the circuit.

(*a*) Express the power factor of the circuit:

$$\cos\delta = \frac{R}{Z}$$

Express *Z* for the circuit:

$$Z = \sqrt{R^2 + X_L^2}$$

Substitute to obtain:

$$\cos\delta = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (2\pi fL)^2}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{100\,\Omega}{\sqrt{(100\,\Omega)^2 + \left[2\pi\left(60\,\text{s}^{-1}\right)(0.4\,\text{H})\right]^2}} = \boxed{0.553}$$

(*b*) Express the rms current in terms of the rms voltage and the impedance of the circuit:

$$I_{rms} = \frac{\varepsilon}{Z} = \frac{\varepsilon}{\sqrt{R^2 + (2\pi fL)^2}}$$

Substitute numerical values and evaluate I_{rms}:

$$I_{rms} = \frac{120\,\text{V}}{\sqrt{(100\,\Omega)^2 + \left[2\pi\left(60\,\text{s}^{-1}\right)(0.4\,\text{H})\right]^2}} = \boxed{0.663\,\text{A}}$$

(*c*) Express the average power supplied to the circuit in terms of the rms current and the resistance of the inductor:

$$P_{av} = I_{rms}^2 R$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = (0.663\,\text{A})^2(100\,\Omega) = \boxed{44.0\,\text{W}}$$

***37** ••

Picture the Problem Let Q represent the instantaneous charge on the capacitor and apply Kirchhoff's loop rule to obtain the differential equation for the circuit. We can then solve this equation to obtain an expression for the charge on the capacitor as a function of time and, by differentiating this expression with respect to time, an expression for the current as a function of time. We'll use a spreadsheet program to plot the graphs.

Apply Kirchhoff's loop rule to a clockwise loop just after the switch is closed:

$$\frac{Q}{C}+L\frac{dI}{dt}=0$$

Because $I = dQ/dt$:

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0 \text{ or } \frac{d^2Q}{dt^2}+\frac{1}{LC}Q=0$$

The solution to this equation is:

$$Q(t)=Q_0\cos(\omega t-\delta)$$

where $\omega=\sqrt{\frac{1}{LC}}$

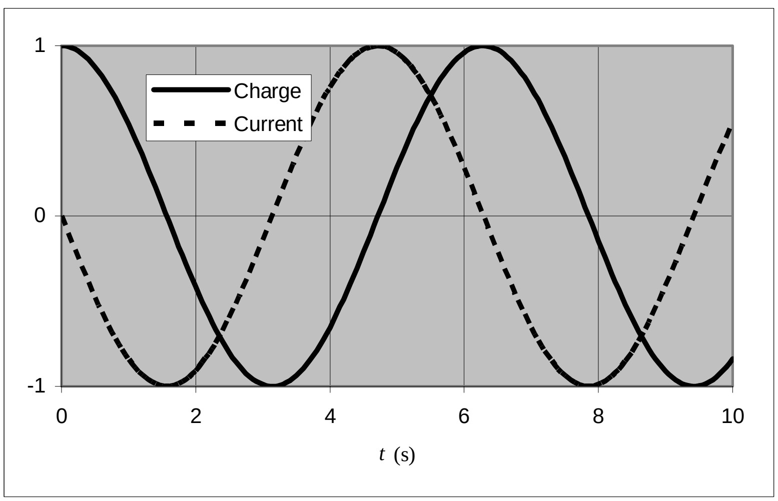
Because $Q(0) = Q_0$, $\delta = 0$ and:

$$Q(t)=Q_0\cos\omega t$$

The current in the circuit is the derivative of Q with respect to t:

$$I=\frac{dQ}{dt}=\frac{d}{dt}[Q_0\cos\omega t]=-\omega Q_0\sin\omega t$$

(a) A spreadsheet program was used to plot the following graph showing both the charge on the capacitor and the current in the circuit as functions of time. L, C, and Q_0 were all arbitrarily set equal to one to obtain these graphs. Note that the current leads the charge by one-fourth of a cycle or 90°.



(b) The equation for the current is:

$$I=-\omega Q_0\sin\omega t \qquad (1)$$

The sine and cosine functions are related through the identity:

$$-\sin\theta=\cos\left(\theta+\frac{\pi}{2}\right)$$

Use this identity to rewrite equation (1):

$$I = -\omega Q_0 \sin\omega t = \boxed{\omega Q_0 \cos\left(\omega t + \frac{\pi}{2}\right)}$$

showing that the current leads the charge by 90°.

RL Circuits with a Generator

38 ••

Picture the Problem We can express the ratio of V_R to V_L and solve this expression for the resistance R of the circuit. In (*b*) we can use the fact that, in an *LR* circuit, V_L leads V_R by 90° to find the ac input voltage.

(*a*) Express the potential differences across R and L in terms of the common current through these components:

$$V_L = IX_L = I\omega L$$

and

$$V_R = IR$$

Divide the second of these equations by the first to obtain:

$$\frac{V_R}{V_L} = \frac{IR}{I\omega L} = \frac{R}{\omega L}$$

Solve for R:

$$R = \left(\frac{V_R}{V_L}\right)\omega L$$

Substitute numerical values and evaluate R:

$$R = \left(\frac{30\,\text{V}}{40\,\text{V}}\right)2\pi\left(60\,\text{s}^{-1}\right)(1.4\,\text{H}) = \boxed{396\,\Omega}$$

(*b*) Because V_R leads V_L by 90° in an *LR* circuit:

$$V = \sqrt{V_R^2 + V_L^2}$$

Substitute numerical values and evaluate V:

$$V = \sqrt{(30\,\text{V})^2 + (40\,\text{V})^2} = \boxed{50.0\,\text{V}}$$

39 ••

Picture the Problem We can solve the expression for the impedance in an *LR* circuit for the inductive reactance and then use the definition of X_L to find L.

Express the impedance of the coil in terms of its resistance and inductive reactance:

$$Z = \sqrt{R^2 + X_L^2}$$

Solve for X_L to obtain:

$$X_L = \sqrt{Z^2 - R^2}$$

Express X_L in terms of L: $X_L = 2\pi fL$

Equate these two expressions to obtain: $2\pi fL = \sqrt{Z^2 - R^2}$

Solve for L: $$L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$

Substitute numerical values and evaluate L: $$L = \frac{\sqrt{(200\,\Omega)^2 - (80\,\Omega)^2}}{2\pi(1\,\text{kHz})} = \boxed{29.2\,\text{mH}}$$

40 ••

Picture the Problem We can express the two output voltage signals as the product of the current from each source and R = 1 kΩ. We can find the currents due to each source using the given voltage signals and the definition of the impedance for each of them.

(*a*) Express the voltage signals observed at the output side of the transmission line in terms of the potential difference across the resistor:

$V_{1,\text{out}} = I_1 R$

and

$V_{2,\text{out}} = I_2 R$

Express I_1 and I_2:

$$I_1 = \frac{V_1}{Z_1} = \frac{(10\,\text{V})\cos 100t}{\sqrt{(10^3\,\Omega)^2 + [(100\,\text{s}^{-1})(1\,\text{H})]^2}} = (9.95\,\text{mA})\cos 100t$$

and

$$I_2 = \frac{V_2}{Z_2} = \frac{(10\,\text{V})\cos 10^4 t}{\sqrt{(10^3\,\Omega)^2 + [(10^4\,\text{s}^{-1})(1\,\text{H})]^2}} = (0.995\,\text{mA})\cos 10^4 t$$

Substitute for I_1 and I_2 to obtain:

$$V_{1,\text{out}} = (10^3\,\Omega)(9.95\,\text{mA})\cos 100t = \boxed{(9.95\,\text{V})\cos 100t}$$

and

$$V_{2,\text{out}} = (10^3\,\Omega)(0.995\,\text{mA})\cos 10^4 t = \boxed{(0.995\,\text{V})\cos 10^4 t}$$

(*b*) Express the ratio of $V_{1,\text{out}}$ to $V_{2,\text{out}}$:

$$\frac{V_{1,\text{out}}}{V_{2,\text{out}}}=\frac{9.95\ \text{V}}{0.995\ \text{V}}=\boxed{10.0}$$

41 ••

Picture the Problem The average power supplied to coil is related to the power factor by $P_{\text{av}}=\mathcal{E}_{\text{rms}}I_{\text{rms}}\cos\delta$. In (*b*) we can use $P_{\text{av}}=I_{\text{rms}}^2R$ to find *R*. Because the inductance *L* is related to the resistance *R* and the phase angle δ according to $X_L=\omega L=R\tan\delta$, we can use this relationship to find the resistance of the coil. Finally, we can decide whether the current leads or lags the voltage by noting whether X_L is less than or greater than *R*.

(*a*) Express the average power supplied to the coil in terms of the power factor of the circuit:

$$P_{\text{av}}=\mathcal{E}_{\text{rms}}I_{\text{rms}}\cos\delta$$

Solve for the power factor:

$$\cos\delta=\frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}I_{\text{rms}}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta=\frac{60\ \text{W}}{(120\ \text{V})(1.5\ \text{A})}=\boxed{0.333}$$

(*b*) Express the power supplied by the source in terms of the resistance of the coil:

$$P_{\text{av}}=I_{\text{rms}}^2R$$

Solve for and evaluate *R*:

$$R=\frac{P_{\text{av}}}{I_{\text{rms}}^2}=\frac{60\ \text{W}}{(1.5\ \text{A})^2}=\boxed{26.7\ \Omega}$$

(*c*) Relate the inductive reactance to the resistance and phase angle:

$$X_L=\omega L=R\tan\delta$$

Solve for *L*:

$$L=\frac{R\tan\delta}{\omega}=\frac{R\tan\left(\cos^{-1}0.333\right)}{2\pi f}$$

Substitute numerical values and evaluate *L*:

$$L=\frac{(26.7\ \Omega)\tan 70.5^\circ}{2\pi\left(60\ \text{s}^{-1}\right)}=\boxed{0.200\ \text{H}}$$

(*d*) Evaluate X_L:

$$X_L=(26.7\ \Omega)\tan 70.5^\circ=75.4\ \Omega$$

Because $X_L>R$, the circuit is inductive and:

$$\boxed{I\text{ lags }\mathcal{E}\text{ by }70.5^\circ}.$$

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Picture the Problem We can use $I_{\max} = \mathcal{E}_{\max} \big/ \sqrt{R^2 + (\omega L)^2}$ and $V_{L,\max} = I_{\max} X_L = \omega L I_{\max}$ to find the maximum current in the circuit and the maximum voltage across the inductor. Once we've found $V_{L,\max}$ we can find $V_{L,\text{rms}}$ using $V_{L,\text{rms}} = V_{L,\max} \big/ \sqrt{2}$. We can use $P_{\text{av}} = \frac{1}{2} I_{\max}^2 R$ to find the average power dissipation, and $U_{L,\max} = \frac{1}{2} L I_{\max}^2$ to find the maximum energy stored in the magnetic field of the inductor. The average energy stored in the magnetic field of the inductor can be found from $U_{L,\text{av}} = \int P_{\text{av}} dt$.

Express the maximum current in the circuit:

$$I_{\max} = \frac{\mathcal{E}_{\max}}{Z} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (\omega L)^2}}$$

Substitute numerical values and evaluate $I_{\max}$:

$$I_{\max} = \frac{345\,\text{V}}{\sqrt{(40\,\Omega)^2 + \left[(150\pi\,\text{s}^{-1})(36\,\text{mH})\right]^2}} = \boxed{7.94\,\text{A}}$$

Relate the maximum voltage across the inductor to the current flowing through it:

$$V_{L,\max} = I_{\max} X_L = \omega L I_{\max}$$

Substitute numerical values and evaluate $V_{L,\max}$:

$$V_{L,\max} = (150\pi\,\text{s}^{-1})(36\,\text{mH})(7.94\,\text{A}) = \boxed{135\,\text{V}}$$

$V_{L,\text{rms}}$ is related to $V_{L,\max}$ according to:

$$V_{L,\text{rms}} = \frac{V_{L,\max}}{\sqrt{2}} = \frac{135\,\text{V}}{\sqrt{2}} = \boxed{95.5\,\text{V}}$$

Relate the average power dissipation to $I_{\max}$ and R:

$$P_{\text{av}} = \tfrac{1}{2} I_{\max}^2 R$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = \tfrac{1}{2}(7.94\,\text{A})^2(40\,\Omega) = \boxed{1.26\,\text{kW}}$$

The maximum energy stored in the magnetic field of the inductor is:

$$U_{L,\max} = \tfrac{1}{2} L I_{\max}^2 = \tfrac{1}{2}(36\,\text{mH})(7.94\,\text{A})^2 = \boxed{1.13\,\text{J}}$$

The definition of $U_{L,\text{av}}$ is:

$$U_{L,\text{av}} = \frac{1}{T}\int_0^T U(t)\,dt$$

$U(t)$ is given by:

$$U(t)=\frac{1}{2}L[I(t)]^2$$

Substitute for $U(t)$ to obtain:

$$U_{L,\text{av}}=\frac{L}{2T}\int_0^T[I(t)]^2dt$$

Evaluating the integral yields:

$$U_{L,\text{av}}=\frac{L}{2T}\left[\frac{1}{2}I_{\max}^2\right]T=\frac{1}{4}LI_{\max}^2$$

Substitute numerical values and evaluate $U_{L,\text{av}}$:

$$U_{L,\text{av}}=\frac{1}{4}(36\,\text{mH})(7.94\,\text{A})^2=\boxed{0.567\,\text{J}}$$

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Picture the Problem We can use the definition of the power factor to find the relationship between X_L and R when f = 60 Hz and then use the definition of X_L to relate the inductive reactance at 240 Hz to the inductive reactance at 60 Hz. We can then use the definition of the power factor to determine its value at 240 Hz.

Using the definition of the power factor, relate R and X_L:

$$\cos\delta=\frac{R}{Z}=\frac{R}{\sqrt{R^2+X_L^2}} \qquad (1)$$

Square both sides of the equation to obtain:

$$\cos^2\delta=\frac{R^2}{R^2+X_L^2}$$

Solve for $X_L^2(60\,\text{Hz})$:

$$X_L^2(60\,\text{Hz})=R^2\left(\frac{1}{\cos^2\delta}-1\right)$$

Substitute for $\cos\delta$ and simplify to obtain:

$$X_L^2(60\,\text{Hz})=R^2\left(\frac{1}{(0.866)^2}-1\right)=\tfrac{1}{3}R^2$$

Use the definition of X_L to obtain:

$$X_L^2(f)=4\pi f^2L^2$$

and

$$X_L^2(f')=4\pi f'^2L^2$$

Divide the second of these equations by the first to obtain:

$$\frac{X_L^2(f')}{X_L^2(f)}=\frac{4\pi f'^2L^2}{4\pi f^2L^2}=\frac{f'^2}{f^2}$$

$$\frac{X_L^2(f')}{X_L^2(f)}=\frac{4\pi f'^2L^2}{4\pi f^2L^2}=\frac{f'^2}{f^2}$$

or

$$X_L^2(f') = \left(\frac{f'}{f}\right)^2 X_L^2(f)$$

Substitute numerical values to obtain:

$$X_L^2(240\,\text{Hz}) = \left(\frac{240\,\text{s}^{-1}}{60\,\text{s}^{-1}}\right)^2 X_L^2(60\,\text{Hz})$$
$$= 16\left(\frac{1}{3}R^2\right) = \frac{16}{3}R^2$$

Substitute in equation (1) to obtain:

$$(\cos\delta)_{240\,\text{Hz}} = \frac{R}{\sqrt{R^2 + \frac{16}{3}R^2}}$$
$$= \sqrt{\frac{3}{19}} = \boxed{0.397}$$

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Picture the Problem We can apply Kirchhoff's loop rule to obtain expressions for I_R and I_L and then use trigonometric identities to show that $I = I_R + I_L = I_{\max}\cos(\omega t - \delta)$, where $\tan\delta = R/X_L$ and $I_{\max} = \varepsilon_{\max}/Z$ with $Z^{-2} = R^{-2} + X_L^{-2}$.

(*a*) Apply Kirchhoff's loop rule to a clockwise loop that includes the source and the resistor:

$$\varepsilon_{\max}\cos\omega t - I_R R = 0$$

Solve for I_R:

$$I_R = \boxed{\frac{\varepsilon_{\max}}{R}\cos\omega t}$$

(*b*) Apply Kirchhoff's loop rule to a clockwise loop that includes the source and the inductor:

$$\varepsilon_{\max}\cos(\omega t - 90°) - I_L X_L = 0$$

because the current lags the potential difference across the inductor by 90°.

Solve for I_L:

$$I_L = \boxed{\frac{\varepsilon_{\max}}{X_L}\cos(\omega t - 90°)}$$

(*c*) Express the current drawn from the source in terms of $I_{\max}$ and the phase constant δ:

$$I = I_R + I_L = I_{\max}\cos(\omega t - \delta)$$

Use a trigonometric identity to expand $\cos(\omega t - \delta)$:

$$I = I_{\max}(\cos\omega t\cos\delta + \sin\omega t\sin\delta)$$
$$= I_{\max}\cos\omega t\cos\delta + I_{\max}\sin\omega t\sin\delta$$

From our results in (*a*):

$$I = I_R + I_L = \frac{\mathcal{E}_{\max}}{R}\cos\omega t + \frac{\mathcal{E}_{\max}}{X_L}\cos(\omega t - 90°) = \frac{\mathcal{E}_{\max}}{R}\cos\omega t + \frac{\mathcal{E}_{\max}}{X_L}\sin\omega t$$

A useful trigonometric identity is:

$$A\cos\omega t + B\sin\omega t = \sqrt{A^2+B^2}\cos(\omega t - \delta)$$

where

$$\delta = \tan^{-1}\frac{B}{A}$$

Apply this identity to obtain:

$$I = \sqrt{\left(\frac{\mathcal{E}_{\max}}{R}\right)^2 + \left(\frac{\mathcal{E}_{\max}}{X_L}\right)^2}\cos(\omega t - \delta) \quad (1)$$

and

$$\delta = \tan^{-1}\left(\frac{\frac{\mathcal{E}_{\max}}{X_L}}{\frac{\mathcal{E}_{\max}}{R}}\right) = \tan^{-1}\left(\frac{R}{X_L}\right) \qquad (2)$$

Simplify equation (1) and rewrite equation (2) to obtain:

$$I = \sqrt{\left(\frac{\mathcal{E}_{\max}}{R}\right)^2 + \left(\frac{\mathcal{E}_{\max}}{X_L}\right)^2}\cos(\omega t - \delta) = \mathcal{E}_{\max}\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}\cos(\omega t - \delta) = \mathcal{E}_{\max}\sqrt{\left(\frac{1}{Z}\right)^2}\cos(\omega t - \delta) = \boxed{\frac{\mathcal{E}_{\max}}{Z}\cos(\omega t - \delta)}$$

where

$$\tan\delta = \boxed{\frac{R}{X_L}} \text{ and } \frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X_L^2}$$

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Picture the Problem We can use the complex numbers method to find the impedances of the parallel portion of the circuit and the total impedance of the circuit. We can then use

Kirchhoff's loop rule to obtain an expression for the current drawn form the source. Knowing the current drawn from the source, we can find the potential difference across the parallel portion of the circuit and then use this information to find the currents drawn by the load and the inductor.

(*a*) Express the rms currents in *R*, *C*, and R_L:

$$I_{R,\,\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z},\ I_{R_L,\,\text{rms}} = \frac{V_{\text{p,rms}}}{R_L},\text{and}$$

$$I_{L,\,\text{rms}} = \frac{V_{\text{p,rms}}}{X_L}$$

Express the total impedance of the circuit:

$$Z = R + Z_{\text{p}}$$

where Z_{p} is the impedance of the parallel branch of the circuit.

Use complex numbers to relate Z_{p} to R_L and X_C:

$$\frac{1}{Z_{\text{p}}} = \frac{1}{R_L} + \frac{1}{iX_L} = \frac{R_L + iX_L}{iR_L X_L}$$

or

$$Z_{\text{p}} = \frac{iR_L X_L}{R_L + iX_L}$$

Multiple the numerator and denominator of this fraction by the complex conjugate of $R_L + iX_L$ and simplify to obtain:

$$Z_{\text{p}} = \frac{iR_L X_L}{R_L + iX_L}\,\frac{R_L - iX_L}{R_L - iX_L} = \frac{R_L X_L^2}{R_L^2 + X_L^2} + i\,\frac{R_L^2 X_L}{R_L^2 + X_L^2}$$

Substitute numerical values and evaluate X_L:

$$X_L = \omega L = 2\pi f L = 2\pi\left(500\,\text{s}^{-1}\right)\left(3.2\,\text{mH}\right) = 10.1\,\Omega$$

Substitute numerical values and evaluate Z_{p}:

$$Z_{\text{p}} = \frac{(20\,\Omega)(10.1\,\Omega)^2}{(20\,\Omega)^2 + (10.1\,\Omega)^2} + i\,\frac{(20\,\Omega)^2(10.1\,\Omega)}{(20\,\Omega)^2 + (10.1\,\Omega)^2} = 4.06\,\Omega + i(8.05\,\Omega)$$

and

$$\left|Z_{\text{p}}\right| = \sqrt{(4.06\,\Omega)^2 + (8.05\,\Omega)^2} = 9.02\,\Omega$$

Substitute to evaluate *Z*:

$$Z = 4\,\Omega + 4.03\,\Omega + i(8.05\,\Omega) = 8.03\,\Omega + i(8.05\,\Omega)$$

and

$$|Z| = \sqrt{(8.03\,\Omega)^2 + (8.05\,\Omega)^2} = 11.4\,\Omega$$

Express and evaluate the power factor:

$$\cos\delta = \frac{R}{Z} = \frac{8.05\,\Omega}{11.4\,\Omega} = 0.706$$

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_{\text{rms}} - I_{R,\text{rms}}|Z| = 0$$

Solve for and evaluate $I_{R,\text{ rms}}$:

$$I_{R,\text{ rms}} = \frac{\mathcal{E}_{\text{rms}}}{|Z|} = \frac{100\,\text{V}/\sqrt{2}}{11.4\,\Omega} = \boxed{6.20\,\text{A}}$$

Express and evaluate $V_{\text{p, rms}}$:

$$V_{\text{p, rms}} = I_{R_L,\text{rms}}|Z_{\text{p}}| = (6.20\,\text{A})(9\,\Omega) = 55.8\,\text{V}$$

Substitute numerical values and evaluate $I_{R,\text{ rms}}$:

$$I_{R_L,\text{ rms}} = \frac{55.8\,\text{V}}{20\,\Omega} = \boxed{2.79\,\text{A}}$$

Substitute numerical values and evaluate $I_{L,\text{ rms}}$:

$$I_{L,\text{ rms}} = \frac{55.8\,\text{V}}{10.1\,\Omega} = \boxed{5.52\,\text{A}}$$

(*b*) Proceed as in (*a*) with $f = 2000$ Hz. Substitute numerical values and evaluate X_L:

$$X_L = \omega L = 2\pi f L = 2\pi\left(2000\,\text{s}^{-1}\right)(3.2\,\text{mH}) = 40.2\,\Omega$$

Substitute numerical values and evaluate Z_{p}:

$$Z_{\text{p}} = \frac{(20\,\Omega)(40.2\,\Omega)^2}{(20\,\Omega)^2 + (40.2\,\Omega)^2} + i\frac{(20\,\Omega)^2(40.2\,\Omega)}{(20\,\Omega)^2 + (40.2\,\Omega)^2} = 16.0\,\Omega + i(7.98\,\Omega)$$

and

$$|Z_{\text{p}}| = \sqrt{(16.0\,\Omega)^2 + (7.98\,\Omega)^2} = 17.9\,\Omega$$

Substitute to evaluate Z:

$$Z = 4\,\Omega + 16.0\,\Omega + i(7.97\,\Omega) = 20.0\,\Omega + i(7.98\,\Omega)$$

and

$$|Z| = \sqrt{(20.0\,\Omega)^2 + (7.98\,\Omega)^2} = 21.5\,\Omega$$

Find the power factor:

$$\cos\delta = \frac{R}{Z} = \frac{20.0\,\Omega}{21.5\,\Omega} = 0.930$$

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_{\text{rms}} - I_{R,\text{rms}}\left|Z\right| = 0$$

Solve for and evaluate $I_{R,\text{ rms}}$:

$$I_{R,\text{ rms}} = \frac{\mathcal{E}_{\text{rms}}}{\left|Z\right|} = \frac{100\,\text{V}/\sqrt{2}}{21.5\,\Omega} = \boxed{3.29\,\text{A}}$$

Express and evaluate $V_{\text{p, rms}}$:

$$V_{\text{p, rms}} = I_{R_L,\text{rms}}\left|Z_{\text{p}}\right| = (3.29\,\text{A})(17.9\,\Omega) = 58.9\,\text{V}$$

Substitute numerical values and evaluate $I_{R,\text{ rms}}$:

$$I_{R_L,\text{ rms}} = \frac{58.9\,\text{V}}{20\,\Omega} = \boxed{2.95\,\text{A}}$$

Substitute numerical values and evaluate $I_{L,\text{ rms}}$:

$$I_{L,\text{ rms}} = \frac{58.9\,\text{V}}{40.2\,\Omega} = \boxed{1.47\,\text{A}}$$

(*c*) Express the fraction of the power dissipated in the resistor:

$$\frac{P_{L,\text{rms}}}{P_{\text{tot}}} = \frac{I_{R_L,\text{ rm}}^2 R_L}{\mathcal{E}_{\text{rms}} I_{R,\text{ rms}}\cos\delta}$$

Evaluate this fraction for f = 500 Hz:

$$\left.\frac{P_{L,\text{rms}}}{P_{\text{tot}}}\right|_{f=500\,\text{Hz}} = \frac{(2.79\text{A})^2(20\,\Omega)}{\left(\frac{100\,\text{V}}{\sqrt{2}}\right)(6.20\,\text{A})(0.706)} = 0.503 = \boxed{50.3\%}$$

When f = 2000 Hz:

$$\left.\frac{P_{L,\text{rms}}}{P_{\text{tot}}}\right|_{f=2000\,\text{Hz}} = \frac{(2.95\,\text{A})^2(20\,\Omega)}{\left(\frac{100\,\text{V}}{\sqrt{2}}\right)(3.29\,\text{A})(0.930)} = 0.804 = \boxed{80.4\%}$$

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Picture the Problem We can treat the ac and dc components separately. For the dc component, L acts like a short circuit. For convenience we let ε_1 denote the maximum value of the ac emf. We can use $P = \mathcal{E}_1^2/R_{1,2}$ to find the power dissipated in the resistors due to the dc source. We'll apply Kirchhoff's loop rule the loop including L, R_1, and R_2 to derive an expression for the power dissipated in the resistors due to the ac source. Note that only the power dissipated in the resistor R_2 due to the ac source is frequency dependent.

(*a*) Express the total power dissipated in R_1 and R_2:

$$P = P_{dc} + P_{ac} \quad (1)$$

Express and evaluate the dc power dissipated in R_1 and R_2:

$$P_{1,dc} = \frac{\mathcal{E}_2^2}{R_1} = \frac{(16\,\text{V})^2}{10\,\Omega} = 25.6\,\text{W}$$

and

$$P_{2,dc} = \frac{\mathcal{E}_2^2}{R_2} = \frac{(16\,\text{V})^2}{8\,\Omega} = 32.0\,\text{W}$$

Express and evaluate the average ac power dissipated in R_1:

$$P_{1,ac} = \frac{1}{2}\frac{\mathcal{E}_1^2}{R_1} = \frac{1}{2}\frac{(20\,\text{V})^2}{10\,\Omega} = 20.0\,\text{W}$$

Apply Kirchhoff's loop rule to a clockwise loop that includes R_1, L, and R_2:

$$R_1 I_1 - Z_2 I_2 = 0$$

Solve for I_2:

$$I_2 = \frac{R_1}{Z_2} I_1 = \frac{R_1}{Z_2}\frac{\mathcal{E}_1}{R_1} = \frac{\mathcal{E}_1}{Z_2}$$

Express the average ac power dissipated in R_2:

$$P_{2,\,ac} = \tfrac{1}{2} I_2^2 R_2 = \tfrac{1}{2}\left(\frac{\mathcal{E}_1}{Z_2}\right)^2 R_2 = \frac{1}{2}\frac{\mathcal{E}_1^2 R_2}{Z_2^2}$$

Substitute numerical values and evaluate $P_{2,\,ac}$:

$$P_{2,\,ac} = \frac{1}{2}\left[\frac{(20\,\text{V})^2(8\,\Omega)}{(8\,\Omega)^2 + \left(2\pi\{100\,\text{s}^{-1}\}\{6\,\text{mH}\}\right)^2}\right] = 20.5\,\text{W}$$

Substitute in equation (1) to obtain:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{45.6\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 20.5\,\text{W} = \boxed{52.5\,\text{W}}$$

and

$$P = P_1 + P_2 = \boxed{98.1\,\text{W}}$$

(*b*) Proceed as in (*a*) to evaluate $P_{2,\,ac}$ with $f = 200$ Hz:

$$P_{2,\,ac} = \frac{1}{2}\left[\frac{(20\,\text{V})^2(8\,\Omega)}{(8\,\Omega)^2 + \left(2\pi\{200\,\text{s}^{-1}\}\{6\,\text{mH}\}\right)^2}\right] = 13.2\,\text{W}$$

Substitute in equation (1) to obtain:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{45.6\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 13.2\,\text{W} = \boxed{45.2\,\text{W}}$$

and

$$P = P_1 + P_2 = \boxed{90.8\,\text{W}}$$

(*c*) Proceed as in (*a*) to evaluate $P_{2,\,\text{ac}}$ with f = 800 Hz:

$$P_{2,\,\text{ac}} = \frac{1}{2}\frac{(20\,\text{V})^2(8\,\Omega)}{\left[(8\,\Omega)^2 + \left(2\pi(800\,\text{s}^{-1})(6\,\text{mH})\right)^2\right]} = 1.64\,\text{W}$$

Substitute in equation (1) to obtain:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{45.6\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 1.64\,\text{W} = \boxed{33.6\,\text{W}}$$

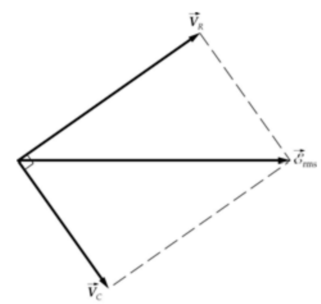
and

$$P = P_1 + P_2 = \boxed{79.2\,\text{W}}$$

47 ••

Picture the Problem We can use the phasor diagram for an *RC* circuit to find the voltage across the resistor.

Sketch the phasor diagram for the voltages in the circuit:



Use the Pythagorean theorem to express V_R:

$$V_R = \sqrt{\mathcal{E}_{\text{rms}}^2 - V_C^2}$$

Substitute numerical values and evaluate V_R:

$$V_R = \sqrt{(100\,\text{V})^2 - (80\,\text{V})^2} = \boxed{60.0\,\text{V}}$$

Filters and Rectifiers

***48** ••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. Repeating this process for the output side of the filter will yield the desired equation.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{\text{in}} - V - IR = 0$$

where V is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{\text{peak}} \cos \omega t - V - R\frac{dQ}{dt} = 0$$

Because $Q = CV$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV] = C\frac{dV}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}} \cos \omega t - V - RC\frac{dV}{dt} = 0$$

the differential equation describing the potential difference across the capacitor.

Assume a solution of the form:

$$V = V_{\text{c}} \cos \omega t + V_{\text{s}} \sin \omega t$$

Substitution of this assumed solution and its first derivative in the differential equations, followed by equating the coefficients of the sine and cosine terms, yields two coupled linear equations:

$$V_{\text{c}} + \omega RCV_{\text{s}} = V_{\text{peak}}$$

and

$$V_{\text{s}} - \omega RCV_{\text{c}} = 0$$

Solve these equations simultaneously to obtain:

$$V_{\text{c}} = \frac{1}{1+(\omega RC)^2}V_{\text{peak}}$$

and

$$V_{\text{s}} = \frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}$$

Note that the output voltage is the voltage across the resistor and that it is phase shifted relative to the input voltage:

$$V_{\text{out}} = V_{\text{H}} \cos(\omega t - \delta)$$

where V_{H} is the amplitude of the signal.

Assume that V_{H} is of the form:

$$V_{\text{H}}(t) = v_{\text{c}} \cos \omega t + v_{\text{s}} \sin \omega t$$

The input, output, and capacitor voltages are related according to:

$$V_{\text{H}}(t) = V_{\text{in}}(t) - V(t)$$

Substitute for $V_{\text{H}}(t)$, $V_{\text{peak}}(t)$, and $V(t)$ and use the previously established values for V_{c} and V_{s} to obtain:

$$v_{\text{c}} = V_{\text{peak}} - V_{\text{c}}$$

and

$$v_{\text{s}} = -V_{\text{s}}$$

Substitute for V_{c} and V_{s} to obtain:

$$v_{\text{c}} = \frac{(\omega RC)^2}{1+(\omega RC)^2}V_{\text{peak}}$$

and

$$v_{\text{s}} = -\frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}$$

V_{H}, v_{c}, and v_{s} are related according to the Pythagorean relationship:

$$V_{\text{H}} = \sqrt{v_{\text{c}}^2 + v_{\text{s}}^2}$$

Substitute for v_{c} and v_{s} to obtain:

$$V_{\text{H}} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}V_{\text{peak}} = \boxed{\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}}$$

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Picture the Problem We can use some of the intermediate results from Problem 48 to express the tangent of the phase constant.

(*a*) Because, as was shown in Problem 48, $V_{\text{H}} = \sqrt{v_{\text{c}}^2 + v_{\text{s}}^2}$:

$$\tan\delta = \frac{v_{\text{s}}}{v_{\text{c}}}$$

Also from Problem 48:

$$v_{\text{c}} = \frac{(\omega RC)^2}{1+(\omega RC)^2}V_{\text{peak}}$$

and

$$v_{\text{s}} = -\frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}$$

Substitute to obtain:

$$\tan\delta = \frac{-\frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}}{\frac{(\omega RC)^2}{1+(\omega RC)^2}V_{\text{peak}}} = \boxed{-\frac{1}{\omega RC}}$$

(*b*) Solve for δ:

$$\delta = \tan^{-1}\left[-\frac{1}{\omega RC}\right]$$

As $\omega \to 0$:

$$\delta \to \boxed{-90^\circ}$$

(*c*) As $\omega \to \infty$:

$$\delta \to \boxed{0}$$

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Picture the Problem We can use the results obtained in Problems 48 and 49 to find $f_{3\text{ dB}}$ and to plot graphs of $\log(V_{\text{out}})$ versus $\log(f)$ and δ versus $\log(f)$.

(*a*) Express the ratio V_{out}/V_{in}:

$$\frac{V_{out}}{V_{in}}=\frac{\dfrac{V_{peak}}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}}{V_{peak}}=\frac{1}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}$$

When $V_{out}=V_{in}/\sqrt{2}$:

$$\frac{1}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}=\frac{1}{\sqrt{2}}$$

Square both sides of the equation and solve for ωRC to obtain:

$$\omega RC=1\Rightarrow\ \omega=\frac{1}{RC}\Rightarrow\ f_{3\,\mathrm{dB}}=\frac{1}{2\pi RC}$$

Substitute numerical values and evaluate $f_{3\,\mathrm{dB}}$:

$$f_{3\,\mathrm{dB}}=\frac{1}{2\pi(20\,\mathrm{k\Omega})(15\,\mathrm{nF})}=\boxed{531\,\mathrm{Hz}}$$

(*b*) From Problem 48 we have:

$$V_{out}=\frac{V_{peak}}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}$$

From Problem 49 we have:

$$\delta=\tan^{-1}\left[-\frac{1}{\omega RC}\right]$$

Rewrite these expressions in terms of $f_{3\,\mathrm{dB}}$ to obtain:

$$V_{out}=\frac{V_{peak}}{\sqrt{1+\left(\dfrac{1}{2\pi fRC}\right)^2}}=\frac{V_{peak}}{\sqrt{1+\left(\dfrac{f_{3\,\mathrm{dB}}}{f}\right)^2}}$$

and

$$\delta=\tan^{-1}\left[-\frac{1}{2\pi fRC}\right]=\tan^{-1}\left[-\frac{f_{3\,\mathrm{dB}}}{f}\right]$$

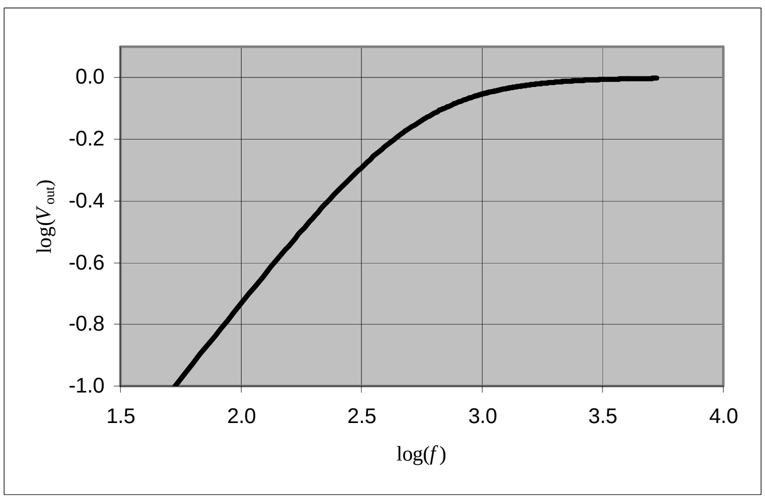
A spreadsheet program to generate the data for a graph of V_{out} versus f and δ versus f is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	1.50E–08	C
B3	1	V_{peak}
B4	531	$f_{3\,\mathrm{dB}}$
A8	53	$0.1f_{3\,\mathrm{dB}}$

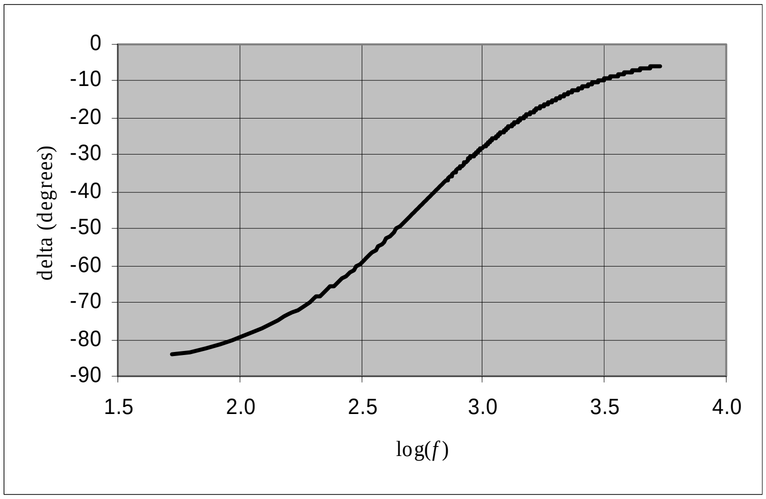
C8	\$B\$3/SQRT(1+(1(\$B\$4/A8))^2)	$\dfrac{V_{peak}}{\sqrt{1+\left(\dfrac{f_{3\,dB}}{f}\right)^2}}$
D8	LOG(C8)	$\log(V_{out})$
E8	ATAN(–\$B\$4/A8)	$\tan^{-1}\left[-\dfrac{f_{3\,dB}}{f}\right]$
F8	E8*180/PI()	δ in degrees

	A	B	C	D	E	F
1	R=	2.00E+04	ohms			
2	C=	1.50E–08	F			
3	V_peak=	1	V			
4	f_3 dB=	531	Hz			
5						
6						
7	f	log(f)	V_out	log(V_out)	delta(rad)	delta(deg)
8	53	1.72	0.099	–1.003	–1.471	–84.3
9	63	1.80	0.118	–0.928	–1.453	–83.2
10	73	1.86	0.136	–0.865	–1.434	–82.2
11	83	1.92	0.155	–0.811	–1.416	–81.1
55	523	2.72	0.702	–0.154	–0.793	–45.4
56	533	2.73	0.709	–0.150	–0.783	–44.9
57	543	2.73	0.715	–0.146	–0.774	–44.3
531	5283	3.72	0.995	–0.002	–0.100	–5.7
532	5293	3.72	0.995	–0.002	–0.100	–5.7
533	5303	3.72	0.995	–0.002	–0.100	–5.7
534	5313	3.73	0.995	–0.002	–0.100	–5.7

The following graph of $\log(V_{out})$ versus $\log(f)$ was plotted for $V_{peak} = 1$ V.



A graph of δ in degrees as a function of $\log(f)$ follows.



Referring to the spreadsheet program, we see that when $f = f_{3\,\text{dB}}$, $\delta \approx \boxed{-44.9°.}$ This result is in good agreement with its calculated value of –45.0°.

51 •••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. Because the voltage drop across the resistor is small compared to the voltage drop across the capacitor, we can express the voltage drop across the capacitor in terms of the input voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{\text{in}} - V_C - IR = 0$$

where V_C is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{\text{peak}} \cos \omega t - V_c - R\frac{dQ}{dt} = 0$$

Because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV_C] = C\frac{dV_C}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}} \cos \omega t - V_C - RC\frac{dV_C}{dt} = 0$$

the differential equation describing the potential difference across the capacitor.

Because the voltage drop across the resistor is very small compared to the voltage drop across the capacitor:

$$V_{\text{peak}} \cos \omega t - V_C \approx 0$$

and

$$V_C \approx V_{\text{peak}} \cos \omega t$$

Consequently, the potential difference across the resistor is given by:

$$V_R = RC\frac{dV_C}{dt} \approx \boxed{RC\frac{d}{dt}[V_{\text{peak}} \cos \omega t]}$$

52 ••

Picture the Problem We can use the expression for V_H from Problem 48 and the definition of β given in the problem to show that every time the frequency is halved, the output drops by 6 dB.

From Problem 48:

$$V_H = \frac{V_{peak}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

or

$$\frac{V_H}{V_{peak}} = \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

Express this ratio in terms of f and $f_{3\,dB}$:

$$\frac{V_H}{V_{peak}} = \frac{1}{\sqrt{1+\left(\frac{f_{3\,dB}}{f}\right)^2}} = \frac{f}{\sqrt{f_{3\,dB}^2\left(1+\frac{f^2}{f_{3\,dB}^2}\right)}}$$

For $f << f_{3dB}$:

$$\frac{V_H}{V_{peak}} \approx \frac{f}{\sqrt{f_{3\,dB}^2\left(1+\frac{f^2}{f_{3\,dB}^2}\right)}} = \frac{f}{f_{3\,dB}}$$

From the definition of β we have:

$$\beta = 20\log_{10}\frac{V_H}{V_{peak}}$$

Substitute for V_H/V_{peak} to obtain:

$$\beta = 20\log_{10}\frac{f}{f_{3\,dB}}$$

Doubling the frequency yields:

$$\beta' = 20\log_{10}\frac{2f}{f_{3\,dB}}$$

The change in decibel level is:

$$\Delta\beta = \beta' - \beta$$
$$= 20\log_{10}\frac{2f}{f_{3\,dB}} - 20\log_{10}\frac{f}{f_{3\,dB}}$$
$$= 20\log_{10}2 = \boxed{6.02\,dB}$$

***53 ••**

Picture the Problem We can express the instantaneous power dissipated in the resistor and then use the fact that the average value of the square of the cosine function over one cycle is ½ to establish the given result.

The instantaneous power $P(t)$ dissipated in the resistor is:

$$P(t)=\frac{V_{\text{out}}^2}{R}$$

The output voltage V_{out} is:

$$V_{\text{out}}=V_{\text{H}}\cos(\omega t-\delta)$$

From Problem 48:

$$V_{\text{H}}=\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

Substitute in the expression for $P(t)$ to obtain:

$$P(t)=\frac{V_{\text{H}}^2}{R}\cos^2(\omega t-\delta)$$

$$=\frac{V_{\text{peak}}^2}{R\left[1+\left(\frac{1}{\omega RC}\right)^2\right]}\cos^2(\omega t-\delta)$$

Because the average value of the square of the cosine function over one cycle is ½:

$$P_{\text{ave}}=\frac{V_{\text{peak}}^2}{2R\left[1+\left(\frac{1}{\omega RC}\right)^2\right]}$$

Simplify this expression to obtain:

$$\boxed{P_{\text{ave}}=\frac{V_{\text{peak}}^2}{2R}\left(\frac{(\omega RC)^2}{1+(\omega RC)^2}\right)}$$

54 ••

Picture the Problem We can solve the expression for V_{H} from Problem 48 for the required capacitance of the capacitor.

From Problem 48:

$$V_{\text{H}}=\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

We require that:

$$\frac{V_{\text{H}}}{V_{\text{peak}}}=\frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}=\frac{1}{10}$$

or

$$\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}=10$$

Solve for C to obtain:

$$C=\frac{1}{\sqrt{99}\,\omega R}=\frac{1}{2\pi\sqrt{99}\,Rf}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi\sqrt{99}(20\,\text{k}\Omega)(60\,\text{Hz})} = \boxed{13.3\,\text{nF}}$$

55 ••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution to these simultaneous equations will yield the amplitude of the output voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{\text{in}} - IR - V = 0$$

where V is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{\text{peak}} \cos\omega t - R\frac{dQ}{dt} - V = 0$$

Because $Q = CV$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV] = C\frac{dV}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}} \cos\omega t - RC\frac{dV}{dt} - V = 0$$

the differential equation describing the potential difference across the capacitor.

Assume a solution of the form:

$$V = V_{\text{c}} \cos\omega t + V_{\text{s}} \sin\omega t$$

Substitution of this assumed solution and its first derivative in the differential equation, followed by equating the coefficients of the sine and cosine terms, yields two coupled linear equations:

$$V_{\text{c}} + \omega RCV_{\text{s}} = V_{\text{peak}}$$

and

$$V_{\text{s}} - \omega RCV_{\text{c}} = 0$$

Solve these equations simultaneously to obtain:

$$V_{\text{c}} = \frac{1}{1+(\omega RC)^2}V_{\text{peak}}$$

and

$$V_{\text{s}} = \frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}$$

Note that the output voltage is the voltage across the capacitor and that it is phase shifted relative to the input voltage:

$$V_{\text{out}} = V_L \cos(\omega t - \delta)$$

where V_L is the amplitude of the signal.

V_L, V_{c}, and V_{s} are related according to the Pythagorean relationship:

$$V_L = \sqrt{V_{\text{c}}^2 + V_{\text{s}}^2}$$

Substitute for V_c and V_s to obtain:

$$V_L = \sqrt{\left[\frac{1}{1+(\omega RC)^2} V_{\text{peak}}\right]^2 + \left[\frac{\omega RC}{1+(\omega RC)^2} V_{\text{peak}}\right]^2}$$

Simplify algebraically to obtain:

$$V_L = \boxed{\frac{V_{\text{peak}}}{\sqrt{1+(\omega RC)^2}}}$$

$$\boxed{\text{As } f \to 0, V_L \to V_{\text{peak}}. \text{ As } f \to \infty, V_L \to 0.}$$

56 ••

Picture the Problem We can use some of the intermediate results from Problem 55 to express the tangent of the phase constant.

From Problem 55:

$$V_L = \sqrt{V_c^2 + V_s^2}$$

where $\tan\delta = \dfrac{V_s}{V_c}$

Also from Problem 55:

$$V_c = \frac{1}{1+(\omega RC)^2} V_{\text{peak}}$$

and

$$V_s = \frac{\omega RC}{1+(\omega RC)^2} V_{\text{peak}}$$

Substitute to obtain:

$$\tan\delta = \frac{\dfrac{\omega RC}{1+(\omega RC)^2} V_{\text{peak}}}{\dfrac{1}{1+(\omega RC)^2} V_{\text{peak}}} = \boxed{\omega RC}$$

Solve for δ:

$$\delta = \tan^{-1}(\omega RC)$$

As $\omega \to 0$:

$$\delta \to \boxed{0}$$

(c) As $\omega \to \infty$:

$$\delta \to \boxed{90°}$$

Remarks: See the spreadsheet solution in the following problem for additional evidence that our answer for Part (*c*) is correct.

***57** ••

Picture the Problem We can use the expressions for V_L and δ derived in Problem 56 to plot the graphs of V_L versus f and δ versus f for the low-pass filter of Problem 55. We'll simplify the spreadsheet program by expressing both V_L and δ as functions of $f_{3\text{ dB}}$.

From Problem 56 we have:

$$V_L = \frac{V_{\text{peak}}}{\sqrt{1+(\omega RC)^2}}$$

and

$$\delta = \tan^{-1}(\omega RC)$$

Rewrite each of these expressions in terms of $f_{3\text{ dB}}$ to obtain:

$$V_L = \frac{V_{\text{peak}}}{\sqrt{1+(2\pi fRC)^2}} = \frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{f}{f_{3\text{dB}}}\right)^2}}$$

and

$$\delta = \tan^{-1}(2\pi fRC) = \tan^{-1}\left(\frac{f}{f_{3\text{dB}}}\right)$$

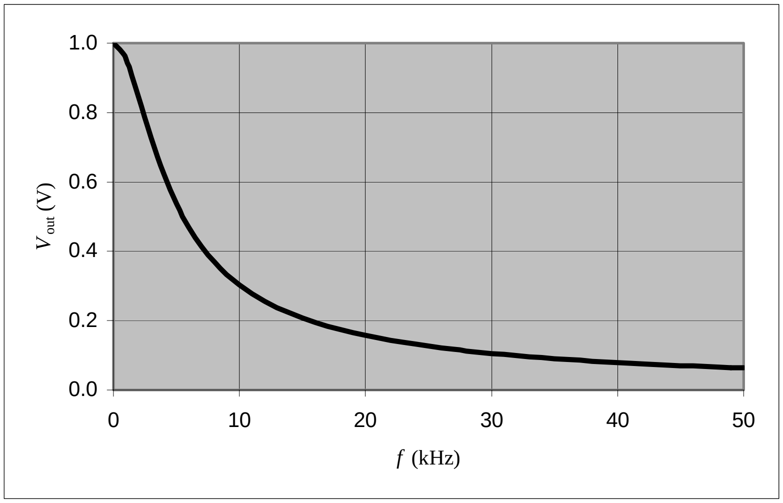
A spreadsheet program to generate the data for graphs of V_L versus f and δ versus f for the low-pass filter is shown below. Note that V_{peak} has been arbitrarily set equal to 1 V. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	5.00E–09	C
B3	1	V_{peak}
B4	(2*PI()*$B$1*$B$2)^–1	$f_{3\text{ dB}}$
B8	$B$3/SQRT(1+((A8/$B$4)^2))	$\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{f}{f_{3\text{dB}}}\right)^2}}$
C8	ATAN(A8/$B$4)	$\tan^{-1}\left(\frac{f}{f_{3\text{dB}}}\right)$
D8	C8*180/PI()	δ in degrees

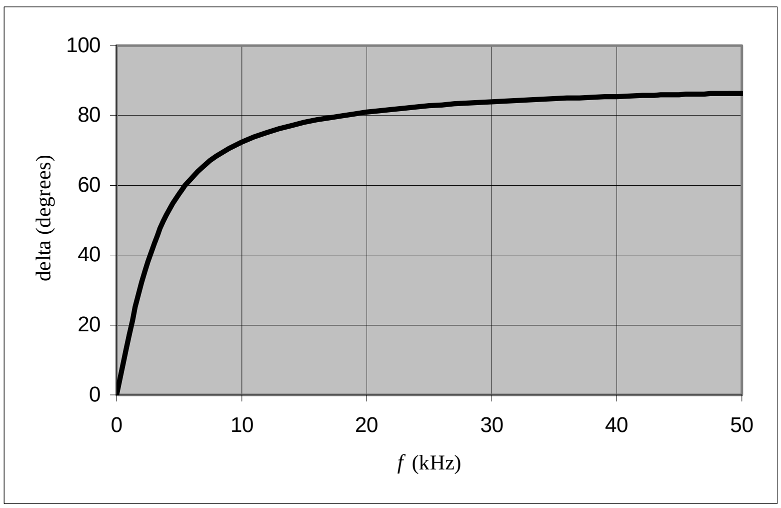
	A	B	C	D
1	R=	1.00E+04	ohms	
2	C=	5.00E–09	F	
3	V_peak=	1	V	
4	f_3 dB=	3.183	kHz	
5				
6	f(kHz)	V_out	delta(rad)	delta(deg)
7	0	1.000	0.000	0.0
8	1	0.954	0.304	17.4
9	2	0.847	0.561	32.1
10	3	0.728	0.756	43.3
54	47	0.068	1.503	86.1
55	48	0.066	1.505	86.2
56	49	0.065	1.506	86.3

57	50	0.064	1.507	86.4

A graph of V_{out} as a function of f follows:



A graph of δ as a function of f follows:



58 •••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution to these simultaneous equations will yield the amplitude of the output voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{in} - IR - V_C = 0$$

where V_C is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{peak} \cos \omega t - R\frac{dQ}{dt} - V_C = 0$$

Because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}\left[CV_C\right] = C\frac{dV_C}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}}\cos\omega t - RC\frac{dV_C}{dt} - V_C = 0$$

the differential equation describing the potential difference across the capacitor.

The output voltage is the voltage across the capacitor. Because this voltage is small :

$$V_{\text{peak}}\cos\omega t - RC\frac{dV_C}{dt} \approx 0$$

Separate the variables in this differential equation and solve for V_C:

$$V_C = \boxed{\frac{1}{RC}\int V_{\text{peak}}\cos\omega t dt}$$

***59** •••

Picture the Problem We can apply Kirchhoff's loop rule to both the input side and output side of the trap filter to obtain an expression for the impedance of the trap. Requiring that the impedance of the trap be zero will yield the frequency at which the circuit rejects signals. Defining the bandwidth as $\Delta\omega = \left|\omega - \omega_{trap}\right|$ and requiring that $\left|Z_{trap}\right| = R$ will yield an expression for the bandwidth and reveal its dependence on R.

Apply Kirchhoff's loop rule to the output of the trap circuit to obtain:

$$V_{\text{out}} - IX_L - IX_C = 0$$

Solve for V_{out}:

$$V_{\text{out}} = I\left(X_L + X_C\right) = IZ_{trap} \qquad (1)$$

where $Z_{trap} = X_L + X_C$

Apply Kirchhoff's loop rule to the input of the trap circuit to obtain:

$$V_{\text{in}} - IR - IX_L - IX_C = 0$$

Solve for I:

$$I = \frac{V_{\text{in}}}{R + X_L + X_C} = \frac{V_{\text{in}}}{R + Z_{trap}}$$

Substitute for I in equation (1) to obtain:

$$V_{\text{out}} = V_{\text{in}}\frac{Z_{trap}}{R + Z_{trap}}$$

Because $X_L = i\omega L$ and $X_C = \frac{-i}{\omega C}$:

$$Z_{trap} = i\left(\omega L - \frac{1}{\omega C}\right)$$

Note that $Z_{trap} = 0$ and $V_{\text{out}} = 0$ provided:

$$\omega = \boxed{\frac{1}{\sqrt{LC}}}$$

Let the bandwidth $\Delta\omega$ be:

$$\Delta\omega = \left|\omega - \omega_{trap}\right| \quad (2)$$

Let the frequency bandwidth to be defined by the frequency at which $\left|Z_{trap}\right| = R$. Then:

$$\omega L - \frac{1}{\omega C} = R$$

or

$$\omega^2 LC - 1 = \omega RC$$

Because $\omega_{trap} = \frac{1}{\sqrt{LC}}$:

$$\left(\frac{\omega}{\omega_{trap}}\right)^2 - 1 = \omega RC$$

For $\omega \approx \omega_{trap}$:

$$\left(\frac{\omega^2 - \omega_{trap}^2}{\omega_{trap}}\right) \approx \omega_{trap} RC$$

Solve for $\omega^2 - \omega_{trap}^2$:

$$\omega^2 - \omega_{trap}^2 = \left(\omega - \omega_{trap}\right)\left(\omega + \omega_{trap}\right)$$

Because $\omega \approx \omega_{trap}$, $\omega - \omega_{trap} \approx 2\omega_{trap}$:

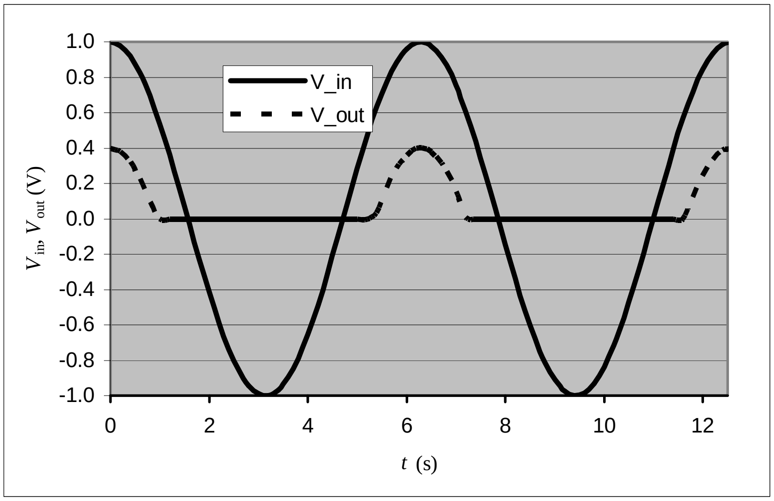
$$\omega^2 - \omega_{trap}^2 \approx 2\omega_{trap}\left(\omega - \omega_{trap}\right)$$

Substitute in equation (2) to obtain:

$$\Delta\omega = \left|\omega - \omega_{trap}\right| = \frac{RC\omega_{trap}^2}{2} = \boxed{\frac{R}{2L}}$$

60 ••

Picture the Problem For voltages greater than 0.6 V, the output voltage will mirror the input voltage minus a 0.6 V drop. But when the voltage swings below 0.6 V, the output voltage will be 0. A spreadsheet program was used to plot the following graph. The angular frequency and the peak voltage were both arbitrarily set equal to one.



61 ••

Picture the Problem We can use the decay of the potential difference across the capacitor to relate the time constant for the *RC* circuit to the frequency of the input signal. Expanding the exponential factor in the expression for V_C will allow us to find the approximate value for *C* that will limit the variation in the output voltage by less than 50 percent (or any other percentage).

The voltage across the capacitor is given by:

$$V_C = V_{\text{in}} e^{-t/RC}$$

Expand the exponential factor to obtain:

$$e^{-t/RC} \approx 1 - \frac{1}{RC}t$$

For a decay of less than 50 percent:

$$1 - \frac{1}{RC}t \le 0.5$$

Solve for *C* to obtain:

$$C \le \frac{2}{R}t$$

Because the voltage goes positive every cycle, $t = 1/60$ s and:

$$C \le \frac{2}{1\,\text{k}\Omega}\left(\frac{1}{60}\text{s}\right) = \boxed{33.3\,\mu\text{F}}$$

LC Circuits with a Generator

62 ••

Picture the Problem We know that the current leads the voltage across and capacitor and lags the voltage across an inductor. We can use $I_{L,\max} = \mathcal{E}_{\max}/X_L$ and $I_{C,\max} = \mathcal{E}_{\max}/X_C$ to find the amplitudes of these currents. The current in the generator will vanish under resonance conditions, i.e., when $|I_L| = |I_C|$. To find the currents in the inductor and capacitor at resonance, we can use the common potential difference across them and their reactances … together with our knowledge of the phase relationships mentioned above.

(*a*) Express the amplitudes of the currents through the inductor and the capacitor:

$$I_{L,\max} = \frac{\mathcal{E}_{\max}}{X_L}$$

and

$$I_{C,\max} = \frac{\mathcal{E}_{\max}}{X_C}$$

Express X_L and X_C:

$$X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}$$

Substitute to obtain:

$$I_{L,\max} = \frac{100\,\text{V}}{(4\,\text{H})\omega} = \boxed{\frac{25\,\text{V/H}}{\omega},\ \text{lagging } \mathcal{E} \text{ by } 90°}$$

and

$$I_{C,\max} = \frac{100\,\text{V}}{\dfrac{1}{(25\,\mu\text{F})\omega}} = \boxed{\left(2.5\times10^{-3}\,\text{V}\cdot\text{F}\right)\omega,\ \text{leading } \mathcal{E} \text{ by } 90°}$$

(*b*) Express the condition that $I = 0$:

$$|I_L| = |I_C|$$

or

$$\frac{\mathcal{E}}{\omega L} = \frac{\mathcal{E}}{\dfrac{1}{\omega C}} = \omega C\mathcal{E}$$

Solve for ω to obtain:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{1}{\sqrt{(4\,\text{H})(25\,\mu\text{F})}} = \boxed{100\,\text{rad/s}}$$

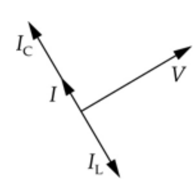
(*c*) Express the current in the inductor at $\omega = \omega_0$:

$$I_L = \left(\frac{25\,\text{V/H}}{100\,\text{s}^{-1}}\right)\cos\left[(100\,\text{rad/s})t - 90°\right] = \boxed{(0.250\,\text{A})\sin\left[\left(100\,\text{s}^{-1}\right)t\right]}$$

Express the current in the capacitor at $\omega = \omega_0$:

$$I_C = \left(2.5\times10^{-3}\,\text{V}\cdot\text{F}\right)\left(100\,\text{s}^{-1}\right)\times\cos\left[(100\,\text{rad/s})t + 90°\right] = \boxed{-(0.25\,\text{A})\sin\left[\left(100\,\text{s}^{-1}\right)t\right]}$$

(*d*) The phasor diagram is shown to the right.



63 ••

Picture the Problem We can differentiate Q with respect to time to find I as a function of time. In (b) we can find C by using $\omega = 1/\sqrt{LC}$. The energy stored in the magnetic field of the inductor is given by $U_m = \frac{1}{2}LI^2$ and the energy stored in the electric field of the capacitor by $U_e = \frac{1}{2}\frac{Q^2}{C}$.

(a) Use the definition of current to obtain:

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[(15\,\mu\text{C})\cos\left(1250\,t + \frac{\pi}{4}\right)\right]$$
$$= -(15\,\mu\text{C})(1250\,\text{s}^{-1})\sin\left(1250\,t + \frac{\pi}{4}\right)$$
$$= \boxed{-(18.75\,\text{mA})\sin\left(1250\,t + \frac{\pi}{4}\right)}$$

(b) Relate C to L and ω:

$$\omega = \frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C = \frac{1}{\omega^2 L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{(1250\,\text{s}^{-1})^2(28\,\text{mH})} = \boxed{22.86\,\mu\text{F}}$$

(c) Express and evaluate the magnetic energy U_m:

$$U_m = \frac{1}{2}LI^2 = \frac{1}{2}(28\,\text{mH})(18.75\,\text{mA})^2\sin^2\left(1250t + \frac{\pi}{4}\right)$$
$$= \boxed{(4.92\,\mu\text{J})\sin^2\left(1250t + \frac{\pi}{4}\right)}$$

Express and evaluate the electrical energy U_e:

$$U_e = \frac{1}{2}\frac{Q^2}{C}$$
$$= \frac{1}{2}\frac{(15\,\mu\text{F})^2}{22.86\,\mu\text{F}}\cos^2\left(1250\,t + \frac{\pi}{4}\right)$$
$$= \boxed{(4.92\,\mu\text{J})\cos^2\left(1250\,t + \frac{\pi}{4}\right)}$$

The total energy stored in the electric and magnetic fields is:

$$U = U_m + U_e = (4.92\,\mu\text{J})\sin^2\left(1250t + \frac{\pi}{4}\right) + (4.92\,\mu\text{J})\cos^2\left(1250t + \frac{\pi}{4}\right) = \boxed{4.92\,\mu\text{J}}$$

***64 •••**

Picture the Problem We can use the definition of the capacitance of a dielectric-filled capacitor and the expression for the resonance frequency of an *LC* circuit to derive an expression for the fractional change in the thickness of the dielectric in terms of the resonance frequency and the frequency of the circuit when the dielectric is under compression. We can then use this expression for $\Delta t/t$ to calculate the value of Young's modulus for the dielectric material.

Use its definition to express Young's modulus of the dielectric material:

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\Delta P}{\Delta t/t} \qquad (1)$$

Letting *t* be the initial thickness of the dielectric, express the initial capacitance of the capacitor:

$$C_0 = \frac{\kappa \in_0 A}{t}$$

Express the capacitance of the capacitor when it is under compression:

$$C_c = \frac{\kappa \in_0 A}{t - \Delta t}$$

Express the resonance frequency of the capacitor before the dielectric is compressed:

$$\omega_0 = \frac{1}{\sqrt{C_0 L}} = \frac{1}{\sqrt{\dfrac{\kappa \in_0 AL}{t}}}$$

When the dielectric is compressed:

$$\omega_c = \frac{1}{\sqrt{C_c L}} = \frac{1}{\sqrt{\dfrac{\kappa \in_0 AL}{t - \Delta t}}}$$

Express the ratio of ω_c to ω_0 and simplify to obtain:

$$\frac{\omega_c}{\omega_0} = \frac{\sqrt{\dfrac{\kappa \in_0 AL}{t}}}{\sqrt{\dfrac{\kappa \in_0 AL}{t - \Delta t}}} = \sqrt{1 - \frac{\Delta t}{t}}$$

Expand the radical binomially to obtain:

$$\frac{\omega_c}{\omega_0} = \left(1 - \frac{\Delta t}{t}\right)^{1/2} \approx 1 - \frac{\Delta t}{2t}$$

provided $\Delta t << t$.

Solve for $\Delta t/t$:

$$\frac{\Delta t}{t} = 2\left(1 - \frac{\omega_c}{\omega_0}\right)$$

Substitute in equation (1) to obtain:

$$Y = \frac{\Delta P}{2\left(1 - \frac{\omega_c}{\omega_0}\right)}$$

Substitute numerical values and evaluate Y:

$$Y = \frac{(800\,\text{atm})(101.325\,\text{kPa/atm})}{2\left(1 - \frac{116\,\text{MHz}}{120\,\text{MHz}}\right)} = \boxed{1.22\times10^{9}\ \text{N/m}^2}$$

65 •••

Picture the Problem We can model this capacitor as the equivalent of two capacitors connected in parallel. Let C_1 be the capacitance of the dielectric-filled capacitor and C_2 be the capacitance of the air-filled capacitor. We'll derive expressions for the capacitances of the parallel capacitors and add these expressions to obtain $C(x)$. We can then use the given resonance frequency when $x = w/2$ and the given value for L to evaluate C_0. In Part (b) we can use our result for $C(x)$ and the relationship between f, L, and $C(x)$ at resonance to express $f(x)$.

(a) Express the equivalent capacitance of the two capacitors in parallel:

$$C(x) = C_1 + C_2 = \frac{\kappa \in_0 A_1}{d} + \frac{\in_0 A_2}{d} \quad (1)$$

Express A_2 in terms of the total area of a capacitor plate A, w, and the distance x:

$$\frac{A_2}{A} = \frac{x}{w} \Rightarrow A_2 = A\frac{x}{w}$$

Express A_1 in terms of A and A_2:

$$A_1 = A - A_2 = A\left(1 - \frac{x}{w}\right)$$

Substitute in equation (1) and simplify to obtain:

$$C(x) = \frac{\kappa \in_0 A}{d}\left(1 - \frac{x}{w}\right) + \frac{\in_0 A}{d}\frac{x}{w} = \frac{\in_0 A}{d}\left[\kappa\left(1 - \frac{x}{w}\right) + \frac{x}{w}\right] = \kappa C_0\left[1 - \frac{\kappa - 1}{\kappa w}x\right]$$

where $C_0 = \dfrac{\epsilon_0 A}{d}$

Find $C(w/2)$:

$$C\left(\frac{w}{2}\right) = \kappa C_0\left[1 - \frac{\kappa - 1}{\kappa w}\frac{w}{2}\right] = \kappa C_0\left[1 - \frac{\kappa - 1}{2\kappa}\right] = C_0\frac{\kappa + 1}{2}$$

Express the resonance frequency of the circuit in terms of L and $C(x)$:

$$f(x) = \frac{1}{2\pi\sqrt{LC(x)}} \qquad (2)$$

Evaluate $f(w/2)$:

$$f\left(\frac{w}{2}\right) = \frac{1}{2\pi\sqrt{LC_0\dfrac{\kappa + 1}{2}}} = \frac{1}{2\pi}\sqrt{\frac{2}{(\kappa + 1)LC_0}}$$

Solve for C_0 to obtain:

$$C_0 = \frac{1}{2\pi^2 f^2\left(\dfrac{w}{2}\right)L(\kappa + 1)}$$

Substitute numerical values and evaluate C_0:

$$C_0 = \frac{1}{2\pi^2(90\,\text{MHz})^2(2\,\text{mH})(4.8 + 1)} = \boxed{5.39\times10^{-16}\ \text{F}}$$

(*b*) Substitute for $C(x)$ in equation (2) to obtain:

$$f(x) = \frac{1}{2\pi\sqrt{L\kappa C_0\left[1 - \dfrac{\kappa - 1}{\kappa w}x\right]}}$$

Substitute numerical values and evaluate $f(x)$:

$$f(x) = \frac{1}{2\pi\sqrt{(2\,\text{mH})(4.8)(5.39\times10^{-16}\ \text{F})\left[1 - \dfrac{4.8 - 1}{4.8(0.2\,\text{m})}x\right]}} = \boxed{\frac{70.0\,\text{MHz}}{\sqrt{1 - (3.96\,\text{m}^{-1})x}}}$$

RLC Circuits with a Generator

66 •

Picture the Problem We can use the expression for the resonance frequency of a series *RLC* circuit to obtain an expression for C as a function of f.

Express the resonance frequency as a function of L and C:

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C = \frac{1}{4\pi^2 f^2 L}$$

Substitute numerical values and evaluate the smallest value for C:

$$C_{\text{smallest}} = \frac{1}{4\pi^2(1600\,\text{kHz})^2(1\,\mu\text{H})} = 9.89\,\text{nF}$$

Substitute numerical values and evaluate the largest value for C:

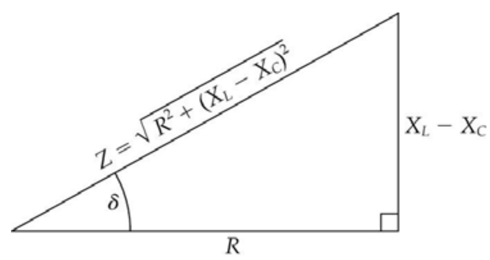
$$C_{\text{largest}} = \frac{1}{4\pi^2(500\,\text{kHz})^2(1\,\mu\text{H})} = 101\,\text{nF}$$

Therefore:

$$\boxed{9.89\,\text{nF} \le C \le 101\,\text{nF}}$$

67 •

Picture the Problem The diagram shows the relationship between δ, X_L, X_C, and R. We can use this reference triangle to express the power factor for the circuit in Example 29-5. In (*b*) we can use the reference triangle to relate ω to $\tan\delta$.



(*a*) Express the power factor for the circuit:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Evaluate X_L and X_C:

$$X_L = \omega L = (400\,\text{s}^{-1})(2\,\text{H}) = 800\,\Omega$$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{(400\,\text{s}^{-1})(2\,\mu\text{F})} = 1250\,\Omega$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{20\,\Omega}{\sqrt{(20\,\Omega)^2 + (800\,\Omega - 1250\,\Omega)^2}} = \boxed{0.0444}$$

(*b*) Express $\tan\delta$:

$$\tan\delta = \frac{X_L - X_C}{R} = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

Rewrite this equation explicitly as a quadratic equation in ω:

$$LC\omega^2 - CR\tan\delta\omega - 1 = 0$$

Substitute numerical values to obtain:

$$[(2\,\text{H})(2\,\mu\text{F})]\omega^2 - [(2\,\mu\text{F})(20\,\Omega)\tan(\cos^{-1}0.5)]\omega - 1 = 0$$

or

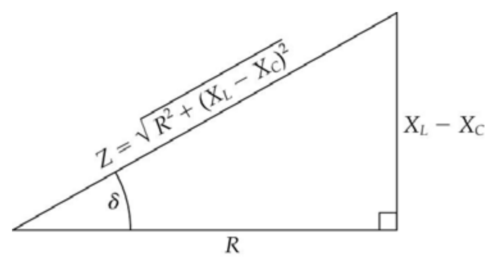
$$(4\times10^{-6}\,\text{s}^2)\omega^2 \pm (69.3\times10^{-6}\,\text{s})\omega - 1 = 0$$

Solve for ω to obtain:

$$\omega = \boxed{491\,\text{rad/s}} \text{ or } \omega = \boxed{509\,\text{rad/s}}$$

68 •

Picture the Problem The diagram shows the relationship between δ, X_L, X_C, and R. We can use this reference triangle to express the power factor for the given circuit. In (*b*) we can find the rms current from the rms potential difference and the impedance of the circuit. We can find the average power delivered by the source from the rms current and the resistance of the resistor.



(*a*) The power factor is defined to be:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

With no inductance in the circuit:

$$X_L = 0$$

and

$$\cos\delta = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{80\,\Omega}{\sqrt{(80\,\Omega)^2 + \dfrac{1}{(400\,\text{s}^{-1})^2(20\,\mu\text{F})^2}}} = \boxed{0.539}$$

(*b*) Express the rms current in the circuit:

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\dfrac{\mathcal{E}_{\max}}{\sqrt{2}}}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_{\max}}{\sqrt{2}\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$$

Insert numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{20\,\text{V}}{\sqrt{2}\sqrt{(80\,\Omega)^2 + \dfrac{1}{(400\,\text{s}^{-1})^2(20\,\mu\text{F})^2}}} = \boxed{95.3\,\text{mA}}$$

(*c*) Express and evaluate the average power delivered by the generator:

$$P_{\text{av}} = I_{\text{rms}}^2 R = (95.3\,\text{mA})^2(80\,\Omega) = \boxed{0.727\,\text{W}}$$

***69 ••**

Picture the Problem The impedance of an ac circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. We can evaluate the given expression for P_{av} first for $X_L = X_C = 0$ and then for $R = 0$.

(*a*) For $X = 0$, $Z = R$ and:

$$P_{\text{av}} = \frac{R\mathcal{E}_{\text{rms}}^2}{Z^2} = \frac{R\mathcal{E}_{\text{rms}}^2}{R^2} = \boxed{\frac{\mathcal{E}_{\text{rms}}^2}{R}}$$

(*b*), (*c*) If $R = 0$, then:

$$P_{\text{av}} = \frac{R\mathcal{E}_{\text{rms}}^2}{Z^2} = \frac{(0)\mathcal{E}_{\text{rms}}^2}{(X_L - X_C)^2} = \boxed{0}$$

Remarks: Recall that there is no energy dissipation in an ideal inductor or capacitor.

70 ••

Picture the Problem We can use $\omega_0 = 1/\sqrt{LC}$ to find the resonant frequency of the circuit, $I_{\text{rms}} = \mathcal{E}_{\text{rms}}/R$ to find the rms current at resonance, the definitions of X_C and X_L to find these reactances at ω = 8000 rad/s, the definitions of Z and I_{rms} to find the impedance and rms current at ω = 8000 rad/s, and the definition of the phase angle to find δ.

(*a*) Express the resonant frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{1}{\sqrt{(10\,\text{mH})(2\,\mu\text{F})}} = \boxed{7.07\times10^3\ \text{rad/s}}$$

(*b*) Relate the rms current at resonance to $\mathcal{E}_{rms}$ and the impedance of the circuit at resonance:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{\mathcal{E}_{max}}{\sqrt{2}R} = \frac{100\,\text{V}}{\sqrt{2}(5\,\Omega)} = \boxed{14.1\,\text{A}}$$

(*c*) Express and evaluate X_C and X_L at ω = 8000 rad/s:

$$X_C = \frac{1}{\omega C} = \frac{1}{(8000\,\text{s}^{-1})(2\,\mu\text{F})} = \boxed{62.5\,\Omega}$$

and

$$X_L = \omega L = (8000\,\text{s}^{-1})(10\,\text{mH}) = \boxed{80.0\,\Omega}$$

(*d*) Express the impedance in terms of the reactances, substitute the results from (*c*), and evaluate Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(5\,\Omega)^2 + (80\,\Omega - 62.5\,\Omega)^2} = \boxed{18.2\,\Omega}$$

Relate the rms current at ω = 8000 rad/s to $\mathcal{E}_{rms}$ and the impedance of the circuit at this frequency:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{max}}{\sqrt{2}Z} = \frac{100\,\text{V}}{\sqrt{2}(18.2\,\Omega)} = \boxed{3.89\,\text{A}}$$

(*e*) Using its definition, express and evaluate δ:

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{80\,\Omega - 62.5\,\Omega}{5\,\Omega}\right) = \boxed{74.1^\circ}$$

71 ••

Picture the Problem We can use $f_0 = 1/2\pi\sqrt{LC}$ to find the resonant frequency of the circuit, the definitions of X_C and X_L to find these reactances at f = 1000 Hz, the definitions of Z and I_{rms} to find the impedance and rms current at f= 1000 Hz, and the definition of the phase angle to find δ.

(*a*) Express the resonant frequency f_0 in terms of L and C:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{(10\,\text{mH})(2\,\mu\text{F})}} = \boxed{1.13\,\text{kHz}}$$

(*b*) Express and evaluate X_C and X_L at f = 1000 Hz:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1000\,\text{s}^{-1})(2\,\mu\text{F})} = \boxed{79.6\,\Omega}$$

and

$$X_L = 2\pi fL = 2\pi(1000\,\text{s}^{-1})(10\,\text{mH}) = \boxed{62.8\,\Omega}$$

(*c*) Express the impedance in terms of the reactances, substitute the results from (*b*), and evaluate Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(5\,\Omega)^2 + (62.8\,\Omega - 79.6\,\Omega)^2} = \boxed{17.5\,\Omega}$$

Relate the rms current at f = 1000 Hz to $\mathcal{E}_{\text{rms}}$ and the impedance of the circuit at this frequency:

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\mathcal{E}_{\text{max}}}{\sqrt{2}Z} = \frac{100\,\text{V}}{\sqrt{2}(17.5\,\Omega)} = \boxed{4.04\,\text{A}}$$

(*d*) Using its definition, express and evaluate δ:

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{62.8\,\Omega - 79.6\,\Omega}{5\,\Omega}\right) = \boxed{-73.4°}$$

72 ••

Picture the Problem Note that the reactances and, hence, the impedance of an ac circuit are frequency dependent. We can use the definitions of X_L, X_C, and Z, δ, and $\cos\delta$ to find the phase angle and the power factor of the circuit at the given frequencies.

Express the phase angle δ and the power factor $\cos\delta$ for the circuit:

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad (1)$$

and

$$\cos\delta = \frac{R}{Z} \qquad (2)$$

(*a*) Evaluate X_L, X_C, and Z at f = 900 Hz:

$$X_L = 2\pi fL = 2\pi\left(900\,\text{s}^{-1}\right)\left(10\,\text{mH}\right) = 56.5\,\Omega,$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(900\,\text{s}^{-1}\right)\left(2\,\mu\text{F}\right)} = 88.4\,\Omega,$$

and

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{\left(5\,\Omega\right)^2 + \left(56.5\,\Omega - 88.4\,\Omega\right)^2} = 32.3\,\Omega$$

Substitute in equations (1) and (2) to obtain:

$$\delta = \tan^{-1}\left(\frac{56.5\,\Omega - 88.4\,\Omega}{5\,\Omega}\right) = \boxed{-81.1^\circ}$$

and

$$\cos\delta = \frac{5\,\Omega}{32.3\,\Omega} = \boxed{0.155}$$

(*b*) Evaluate X_L, X_C, and Z at f = 1.1 kHz:

$$X_L = 2\pi fL = 2\pi\left(1100\,\text{s}^{-1}\right)\left(10\,\text{mH}\right) = 69.1\,\Omega,$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(1100\,\text{s}^{-1}\right)\left(2\,\mu\text{F}\right)} = 72.3\,\Omega,$$

and

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{\left(5\,\Omega\right)^2 + \left(69.1\,\Omega - 72.3\,\Omega\right)^2} = 5.94\,\Omega$$

Substitute in equations (1) and (2) to obtain:

$$\delta = \tan^{-1}\left(\frac{69.1\,\Omega - 72.3\,\Omega}{5\,\Omega}\right) = \boxed{-32.6^\circ}$$

and

$$\cos\delta = \frac{5\,\Omega}{5.94\,\Omega} = \boxed{0.842}$$

(*c*) Evaluate X_L, X_C, and Z at f = 1.3 kHz:

$$X_L = 2\pi fL = 2\pi(1300\,\text{s}^{-1})(10\,\text{mH}) = 81.7\,\Omega,$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1300\,\text{s}^{-1})(2\,\mu\text{F})} = 61.2\,\Omega,$$

and

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(5\,\Omega)^2 + (81.7\,\Omega - 61.2\,\Omega)^2} = 21.1\,\Omega$$

Substitute in equations (1) and (2) to obtain:

$$\delta = \tan^{-1}\left(\frac{81.7\,\Omega - 61.2\,\Omega}{5\,\Omega}\right) = \boxed{76.3°}$$

and

$$\cos\delta = \frac{5\,\Omega}{21.1\,\Omega} = \boxed{0.237}$$

73 ••

Picture the Problem The Q factor of the circuit is given by $Q = \omega_0 L/R$, the resonance width by $\Delta f = f_0/Q = \omega_0/2\pi Q$, and the power factor by $\cos\delta = R/Z$. Because Z is frequency dependent, we'll need to find X_C and X_L at ω = 8000 rad/s in order to evaluate $\cos\delta$.

Using their definitions, express the Q factor and the resonance width of the circuit:

$$Q = \frac{\omega_0 L}{R} \qquad (1)$$

and

$$\Delta f = \frac{f_0}{Q} = \frac{\omega_0}{2\pi Q} \qquad (2)$$

(*a*) Express and evaluate the resonance frequency for the circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10\,\text{mH})(2\,\mu\text{F})}} = 7.07\times10^3\ \text{rad/s}$$

Substitute numerical values in equation (1) and evaluate Q:

$$Q = \frac{(7.07\times10^3\ \text{rad/s})(10\,\text{mH})}{5\,\Omega} = \boxed{14.1}$$

(*b*) Substitute numerical values in equation (2) and evaluate Δf:

$$\Delta f = \frac{7.07\times10^3\ \text{rad/s}}{2\pi(14.1)} = \boxed{79.8\,\text{Hz}}$$

(*c*) Express the power factor of the circuit:

$$\cos\delta = \frac{R}{Z}$$

Evaluate X_L, X_C, and Z at $\omega = 8000$ rad/s:

$$X_L = \omega L = \left(8000\,\text{s}^{-1}\right)\left(10\,\text{mH}\right) = 80.0\,\Omega,$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(8000\,\text{s}^{-1}\right)\left(2\,\mu\text{F}\right)} = 62.5\,\Omega,$$

and

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{\left(5\,\Omega\right)^2 + \left(80\,\Omega - 62.5\,\Omega\right)^2} = 18.2\,\Omega$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{5\,\Omega}{18.2\,\Omega} = \boxed{0.275}$$

***74 ••**

Picture the Problem We can use its definition, $Q = f_0/\Delta f$ to find the Q factor for the circuit.

Express the Q factor for the circuit:

$$Q = \frac{f_0}{\Delta f}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{100.1\,\text{MHz}}{0.05\,\text{MHz}} = \boxed{2002}$$

75 ••

Picture the Problem We can use $I = \mathcal{E}_{\text{max}}/Z$ to find the current in the coil and the definition of the phase angle to evaluate δ. We can equate XL and XC to find the capacitance required so that the current and the voltage are in phase. Finally, we can find the voltage measured across the capacitor by using $V_C = IX_C$.

(*a*) Express the current in the coil in terms of the potential difference across it and its impedance:

$$I = \frac{\mathcal{E}_{\text{max}}}{Z}$$

Substitute numerical values and evaluate I:

$$I = \frac{100\,\text{V}}{10\,\Omega} = \boxed{10.0\,\text{A}}$$

(*b*) Express and evaluate the phase angle δ:

$$\delta = \cos^{-1}\frac{R}{Z} = \sin^{-1}\frac{X}{Z}$$
$$= \sin^{-1}\left(\frac{8\,\Omega}{10\,\Omega}\right) = \boxed{53.1^\circ}$$

(*c*) Express the condition on the reactances that must be satisfied if the current and voltage are to be in phase:

$$X_L = X_C \text{ or } X_L = \frac{1}{\omega C}$$

Solve for *C* to obtain:

$$C = \frac{1}{\omega X_L} = \frac{1}{2\pi f X_L}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(8\,\Omega\right)} = \boxed{332\,\mu\text{F}}$$

(*d*) Express the potential difference across the capacitor:

$$V_C = IX_C$$

Relate the current *I* in the circuit to the impedance of the circuit when $X_L = X_C$:

$$I = \frac{V}{R}$$

Substitute to obtain:

$$V_C = \frac{VX_C}{R} = \frac{V}{2\pi f CR}$$

Relate the impedance of the circuit to the resistance of the coil:

$$Z = \sqrt{R^2 + X^2}$$

Solve for and evaluate the resistance of the coil:

$$R = \sqrt{Z^2 - X^2} = \sqrt{\left(10\,\Omega\right)^2 - \left(8\,\Omega\right)^2}$$
$$= 6\,\Omega$$

Substitute numerical values and evaluate V_C:

$$V_C = \frac{100\,\text{V}}{2\pi\left(60\,\text{s}^{-1}\right)\left(332\,\mu\text{F}\right)\left(6\,\Omega\right)} = \boxed{133\,\text{V}}$$

76 ••

Picture the Problem We can find *C* using $V_C = I_{\text{rms}} X_C$ and I_{rms} from the potential difference across the inductor. In the absence of resistance in the circuit, the measured rms voltage across both the capacitor and inductor is $V = \left|V_L - V_C\right|$.

(*a*) Relate the capacitance *C* to the potential difference across the capacitor:

$$V_C = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{2\pi f C}$$

Solve for *C* to obtain:

$$C = \frac{I_{\text{rms}}}{2\pi f V_C}$$

Use the potential difference across the inductor to express the rms current in the circuit:

$$I_{\text{rms}} = \frac{V_L}{X_L} = \frac{V_L}{2\pi f L}$$

Substitute to obtain:

$$C = \frac{V_L}{(2\pi f)^2 L V_C}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{50\,\text{V}}{\left[2\pi\left(60\,\text{s}^{-1}\right)\right]^2 (0.25\,\text{H})(75\,\text{V})} = \boxed{18.8\,\mu\text{F}}$$

(*b*) Express the measured rms voltage V across both the capacitor and the inductor when $R = 0$:

$$V = \left|V_L - V_C\right|$$

Substitute numerical values and evaluate *V*:

$$V = \left|50\,\text{V} - 75\,\text{V}\right| = \boxed{25.0\,\text{V}}$$

77 ••

Picture the Problem We can rewrite Equation 29-51 in terms of ω, *L*, and *C* and factor *L* from the resulting expression to obtain the given equation. In (*b*) and (*c*) we can use the expansions for $\cot^{-1}x$ and $\tan^{-1}x$ to approximate δ at very low and very high frequencies.

(*a*) From Equation 29-51:

$$\tan\delta = \frac{\omega L - 1/\omega C}{R} = \frac{\omega^2 L - 1/C}{\omega R} = \frac{L\left(\omega^2 - 1/LC\right)}{\omega R} = \boxed{\frac{L\left(\omega^2 - \omega_0^2\right)}{\omega R}}$$

(*b*) Rewrite $\tan\delta$ as:

$$\tan\delta = \frac{\omega L}{R} - \frac{1}{\omega RC} \qquad (1)$$

For $\omega \ll 1$:

$$\tan\delta \approx -\frac{1}{\omega RC}$$

and

$$\cot\delta = -\omega RC \text{ or } \delta = \cot^{-1}(-\omega RC)$$

Use the expansion for $\cot^{-1}x$ to obtain:

$$\cot^{-1}x = \pm\frac{\pi}{2} - x$$

Recall that, for negative values of the argument, the angle approaches $-\pi/2$*, to obtain:

$$-\frac{\pi}{2} - \delta = -\omega RC$$

or

$$\delta = \boxed{-\frac{\pi}{2} + \omega RC}$$

(*c*) For $\omega >> 1$, equation (1) becomes:

$$\tan\delta \approx \frac{\omega L}{R} \text{ or } \delta \approx \tan^{-1}\frac{\omega L}{R}$$

Use the expansion for $\tan^{-1}x$ to obtain:

$$\tan^{-1}x = \frac{\pi}{2} - \frac{1}{x} \text{ or } \delta = \boxed{\frac{\pi}{2} - \frac{R}{\omega L}}$$

***You can easily confirm this using your graphing calculator.**

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Picture the Problem We can use the definition of the power factor to express $\cos\delta$ in the absence of an inductor and simplify the resulting equation to obtain the equation given above.

(*a*) Express the power factor for a series *RLC* circuit:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

With no inductance in the circuit, $X_L = 0$ and:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (-X_C)^2}}$$

Substitute for X_C and simplify to obtain:

$$\cos\delta = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} = \frac{R}{R\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

$$= \boxed{\frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}}$$

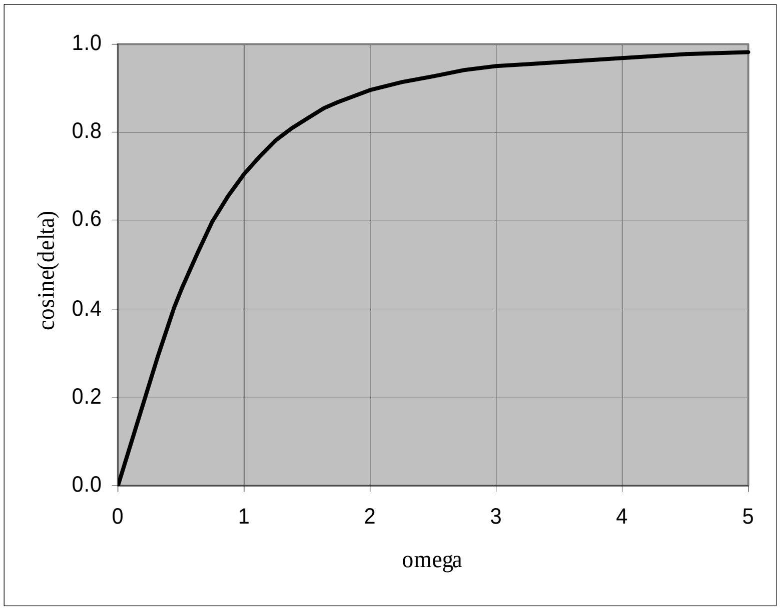
(*b*)A spreadsheet program to generate the data for a graph of $\cos\delta$ versus ωRC is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A2	0.0	ωRC
A3	A2 + 0.5	$\omega RC + 0.5$
B2	A2/(1 +A2^2)^(0.5)	$\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$

	A	B	C
1	R=	1	ohm
2	C=	1	F

3			
4	omega	cos(delta)	
5	0.0	0.000	
6	0.5	0.447	
7	1.0	0.707	
13	4.0	0.970	
14	4.5	0.976	
15	5.0	0.981	

The following graph of cos δ as a function of ω was plotted using the data in the above table. Note that both R and C were set equal to 1.



***79 ••**

Picture the Problem We can find the rms current in the circuit and then use it to find the potential differences across each of the circuit elements. We can use phasor diagrams and our knowledge the phase shifts between the voltages across the three circuit elements to find the voltage differences across their combinations.

(a) Express the potential difference between points A and B in terms of I_{rms} and X_L:

$$V_{AB} = I_{\text{rms}} X_L \quad (1)$$

Express I_{rms} in terms of ε and Z:

$$I_{\text{rms}} = \frac{\varepsilon}{Z} = \frac{\varepsilon}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Evaluate X_L and X_C to obtain:

$$X_L = 2\pi fL = 2\pi(60\,\text{s}^{-1})(137\,\text{mH}) = 51.6\,\Omega$$

and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\,\text{s}^{-1})(25\,\mu\text{F})} = 106.1\,\Omega$$

Substitute numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{115\,\text{V}}{\sqrt{(50\,\Omega)^2 + (51.6\,\Omega - 106.1\,\Omega)^2}} = 1.55\,\text{A}$$

Substitute numerical values in equation (1) and evaluate V_{AB}:

$$V_{AB} = (1.55\,\text{A})(51.6\,\Omega) = \boxed{80.0\,\text{V}}$$

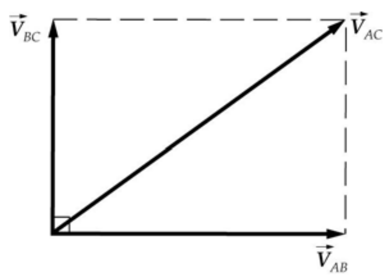
(*b*) Express the potential difference between points *B* and *C* in terms of I_{rms} and R:

$$V_{BC} = I_{\text{rms}}R = (1.55\,\text{A})(50\,\Omega) = \boxed{77.5\,\text{V}}$$

(*c*) Express the potential difference between points *C* and *D* in terms of I_{rms} and X_C:

$$V_{CD} = I_{\text{rms}}X_C = (1.55\,\text{A})(106.1\,\Omega) = \boxed{164\,\text{V}}$$

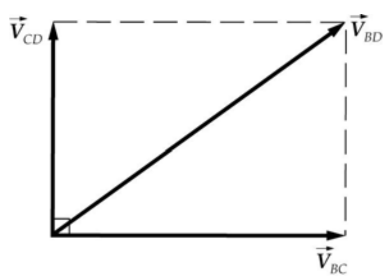
(*d*) The voltage across the inductor lags the voltage across the resistor as shown in the phasor diagram to the right:



Use the Pythagorean theorem to find V_{AC}:

$$V_{AC} = \sqrt{V_{AB}^2 + V_{BC}^2} = \sqrt{(80.0\,\text{V})^2 + (77.5\,\text{V})^2} = \boxed{111\,\text{V}}$$

(*e*) The voltage across the inductor lags the voltage across the resistor as shown in the phasor diagram to the right:



Use the Pythagorean theorem to find V_{BD}:

$$V_{BD} = \sqrt{V_{CD}^2 + V_{BC}^2} = \sqrt{(164\,\text{V})^2 + (77.5\,\text{V})^2} = \boxed{181\,\text{V}}$$

80 ••

Picture the Problem We can use $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta$ to find the power supplied to the circuit and $P_{\text{av}} = I_{\text{rms}}^2 R$ to find the resistance. In (*c*) we can relate the capacitive reactance to the impedance, inductive reactance, and resistance of the circuit and solve for the capacitance *C*. We can use the condition on X_L and X_C at resonance to find the capacitance or inductance you would need to add to the circuit to make the power factor equal to 1.

(*a*) Express the power supplied to the circuit in terms of ε_{rms}, I_{rms}, and the power factor $\cos\delta$:

$$P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = (120\,\text{V})(11\,\text{A})\cos 45° = \boxed{933\,\text{W}}$$

(*b*) Relate the power dissipated in the circuit to the resistance of the resistor:

$$P_{\text{av}} = I_{\text{rms}}^2 R \text{ or } R = \frac{P_{\text{av}}}{I_{\text{rms}}^2}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{933\,\text{W}}{(11\,\text{A})^2} = \boxed{7.71\,\Omega}$$

(*c*) Express the capacitance of the capacitor in terms of its reactance:

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} \qquad (1)$$

Relate the capacitive reactance to the impedance, inductive reactance, and resistance of the circuit:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Express the impedance of the circuit in terms of the rms emf ε and the rms current I_{rms}:

$$Z^2 = \frac{\mathcal{E}^2}{I_{\text{rms}}^2}$$

Substitute to obtain:

$$\frac{\mathcal{E}^2}{I_{\text{rms}}^2} = R^2 + (X_L - X_C)^2$$

Solve for $\left|X_L - X_C\right|$:

$$\left|X_L - X_C\right| = \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2}$$

Note that because I leads ε, the circuit is capacitive and $X_C > X_L$. Hence:

$$\left|X_L - X_C\right| = -\left(X_L - X_C\right)$$

and

$$X_C = X_L + \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2} = 2\pi f L + \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2}$$

Substitute numerical values and evaluate X_C:

$$X_C = 2\pi\left(60\,\text{s}^{-1}\right)\left(0.05\,\text{H}\right) + \sqrt{\frac{\left(120\,\text{V}\right)^2}{\left(11\text{A}\right)^2} - \left(7.71\Omega\right)^2} = 26.6\,\Omega$$

Substitute in equation (1) and evaluate C:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(26.6\,\Omega\right)} = \boxed{99.7\,\mu\text{F}}$$

(*d*) Express the relationship between X_L and X_C when $\cos\delta = 1$:

$$X_L = X_C$$

Because $X_L = 18.1\ \Omega$, we could make $X_L = X_C$ by adding 7.75 Ω of inductive reactance to the circuit. Find the *series* inductance equivalent to 7.75 Ω of inductive reactance:

$$L = \frac{X_L}{2\pi f} = \frac{7.75\,\Omega}{2\pi\left(60\,\text{s}^{-1}\right)} = \boxed{20.6\,\text{mH}}$$

Alternatively, we could make $X_L = X_C$ by reducing the capacitive reactance by 7.75 Ω. Find the capacitive reactance that you have to added in *parallel* to the existing capacitive reactance to reduce the equivalent capacitive reactance by 7.75 Ω:

$$\frac{1}{18.1\Omega} = \frac{1}{26.6\Omega} + \frac{1}{X_C}$$

and

$$X_C = 56.6\,\Omega$$

Find the capacitance corresponding to a capacitive reactance of 56.6 Ω:

$$C=\frac{1}{2\pi fX_C}=\frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(56.6\,\Omega\right)}$$
$$=\boxed{46.9\,\mu\text{F}}$$

81 ••

Picture the Problem We can find X_C using the equation relating X_C, X_L, R, and $\tan\delta$ and then solve the defining equation for X_C for C.

Express the capacitance of the circuit in terms of its capacitive reactance:

$$C=\frac{1}{\omega X_C}=\frac{1}{2\pi fX_C}$$

Express the phase angle δ in terms of X_L, X_C, and R:

$$\tan\delta=\frac{X_L-X_C}{R}$$

Solve for X_C to obtain:

$$X_C=2\pi fL-R\tan\delta$$

Substitute numerical values and evaluate X_C:

$$X_C=2\pi\left(500\,\text{s}^{-1}\right)\left(0.15\,\text{H}\right)-\left(35\,\Omega\right)\tan 75°$$
$$=341\,\Omega$$

Substitute numerical values and evaluate C:

$$C=\frac{1}{2\pi\left(500\,\text{s}^{-1}\right)\left(341\,\Omega\right)}=\boxed{0.933\,\mu\text{F}}$$

82 ••

Picture the Problem We can use the condition on X_L and X_C at resonance to find f_0. By expressing the phase angle δ in terms of X_L, X_C, and R we can obtain a quadratic equation in ω that we can solve for the frequencies corresponding to the given phase angles. We can then use these frequencies to express the ratios of f to f_0 for the given phase angles.

(*a*) Relate X_C and X_L at resonance:

$$X_L=X_C$$

or

$$2\pi f_0L=\frac{1}{2\pi f_0C}$$

Solve for f_0:

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{\left(0.35\,\text{H}\right)\left(5\,\mu\text{F}\right)}}=\boxed{120\,\text{Hz}}$$

(*b*) Express the phase angle δ in terms of X_L, X_C, and R:

$$\tan\delta = \frac{X_L - X_C}{R}$$

or

$$R\tan\delta = \omega L - \frac{1}{\omega C}$$

Rewrite this equation explicitly as a quadratic equation to obtain:

$$LC\omega^2 - (RC\tan\delta)\omega - 1 = 0$$

Substitute numerical values and simplify to obtain:

$$(1.75\times10^{-6}\ \text{F}\cdot\text{H})\omega^2 - \left[(2\times10^{-3}\ \Omega\cdot\text{H})\tan\delta\right]\omega - 1 = 0$$

or

$$(1\,\text{F}\cdot\text{H})\omega^2 - \left[(1.14\times10^{3}\ \Omega\cdot\text{H})\tan\delta\right]\omega - 5.71\times10^5 = 0$$

For $\delta = 60°$:

$$(1\,\text{F}\cdot\text{H})\omega^2 - (1.97\times10^{3}\ \Omega\cdot\text{H})\omega - 5.71\times10^5 = 0$$

Solve for the positive value of ω:

$$\omega = 2.23\times10^3\ \text{s}^{-1}$$

and

$$f = \frac{\omega}{2\pi} = \frac{2.23\times10^3\ \text{s}^{-1}}{2\pi} = 355\ \text{Hz}$$

Calculate the ratio f/f_0:

$$\frac{f}{f_0} = \frac{355\ \text{Hz}}{120\ \text{Hz}} = \boxed{2.96}$$

For $\delta = -60°$:

$$(1\,\text{F}\cdot\text{H})\omega^2 + (1.97\times10^{3}\ \Omega\cdot\text{H})\omega - 5.71\times10^5 = 0$$

Solve for the positive value of ω and then for f:

$$\omega = 256\ \text{s}^{-1}$$

and

$$f = \frac{\omega}{2\pi} = \frac{256\ \text{s}^{-1}}{2\pi} = 40.7\ \text{Hz}$$

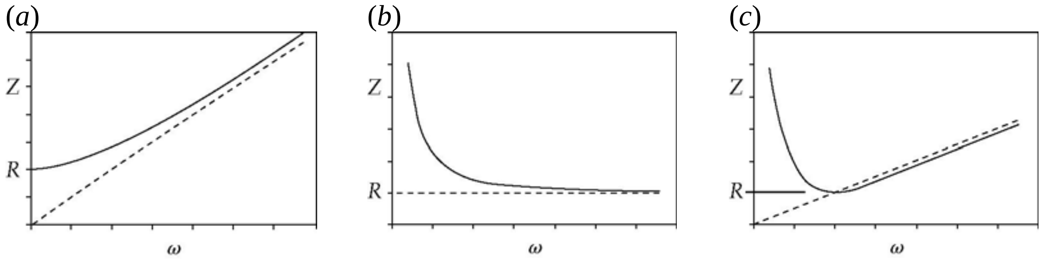
Calculate the ratio f/f_0:

$$\frac{f}{f_0} = \frac{40.7\ \text{Hz}}{120\ \text{Hz}} = \boxed{0.339}$$

Remarks: Note that these ratios are reciprocals of each other.

83 ••

Picture the Problem The impedance for the three circuits as functions of the angular frequency is shown in the three figures below. Also shown in each figure (dashed line) is the asymptotic approach for large angular frequencies.



***84** ••

Picture the Problem We can substitute for X_L and X_C in Equation 29-48 and simplify the resulting equation to obtain the given equation for I_{max}.

Equation 29-48 is:

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Substitute for X_L and X_C to obtain:

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Simplify algebraically to obtain:

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \omega^2 L^2\left(1 - \frac{1}{\omega^2 LC}\right)^2}} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \omega^2 L^2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2}} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \frac{L^2}{\omega^2}\left(\omega^2 - \omega_0^2\right)^2}}$$

$$= \frac{\mathcal{E}_{max}}{\frac{1}{\omega}\sqrt{\omega^2 R^2 + L^2\left(\omega^2 - \omega_0^2\right)^2}} = \boxed{\frac{\omega \mathcal{E}_{max}}{\sqrt{\omega^2 R^2 + L^2\left(\omega^2 - \omega_0^2\right)^2}}}$$

85 ••

Picture the Problem We can use the constraints on L and C at resonance and the given values for X_L and X_C to obtain simultaneous equations that we can solve for L and C. In (*b*) we can find Q from its definition and in (*c*) we can calculate I_{max} from ε_{max} and Z.

(*a*) Relate X_L and X_C at resonance:

$$X_L = X_C \text{ or } \omega_0 L = \frac{1}{\omega_0 C}$$

Solve for the product of L and C:

$$LC = \frac{1}{\omega_0^2} = \frac{1}{(10^4\ \text{rad/s})^2} = 10^{-8}\ \text{s}^2 \qquad (1)$$

Express X_C and X_L:

$$X_C = \frac{1}{\omega C} = 16\,\Omega$$

and

$$X_L = \omega L = 4\,\Omega$$

Eliminate ω between these equations to obtain:

$$\frac{L}{C} = 64\,\Omega^2 \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$L = \boxed{0.800\,\text{mH}} \text{ and } C = \boxed{12.5\,\mu\text{F}}$$

(*b*) Express Q in terms of R, L, and ω_0:

$$Q = \frac{\omega_0 L}{R}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{(10^4\ \text{rad/s})(0.800\,\text{mH})}{5\,\Omega} = \boxed{1.60}$$

(*c*) Relate the maximum current in the circuit to ε_{max} and Z:

$$I_{max} = \frac{\varepsilon_{max}}{Z} = \frac{\varepsilon_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = \frac{26\,\text{V}}{\sqrt{(5\,\Omega)^2 + (4\,\Omega - 16\,\Omega)^2}} = \boxed{2.00\,\text{A}}$$

86 ••

Picture the Problem We can find the maximum current in the circuit from the maximum voltage across the capacitor and the reactance of the capacitor. To find the range of inductance that is safe to use we can express Z^2 for the circuit in terms of ε_{max}^2 and I_{max}^2 and solve the resulting quadratic equation for L.

(*a*) Express the maximum current in terms of the maximum potential difference across the capacitor and its reactance:

$$I_{max} = \frac{V_{C,max}}{X_C} = \omega C V_{C,max}$$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = (2500\,\text{rad/s})(8\,\mu\text{F})(150\,\text{V}) = \boxed{3.00\,\text{A}}$$

(*b*) Relate the maximum current in the circuit to the emf of the source and the impedance of the circuit:

$$I_{max} = \frac{\mathcal{E}_{max}}{Z} \text{ or } Z^2 = \frac{\mathcal{E}_{max}^2}{I_{max}^2}$$

Express Z^2 in terms of R, X_L, and X_C:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Substitute to obtain:

$$\frac{\mathcal{E}_{max}^2}{I_{max}^2} = R^2 + (X_L - X_C)^2$$

Evaluate X_C:

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500\,\text{rad/s})(8\,\mu\text{F})} = 50\,\Omega$$

Substitute numerical values to obtain:

$$\frac{(200\,\text{V})^2}{(3\,\text{A})^2} = (60\,\Omega)^2 + ((2500\,\text{rad/s})L - 50\,\Omega)^2$$

or

$$844\,\Omega^2 = \left[(2500\,\text{s}^{-1})L - 50\,\Omega\right]^2$$

Solve for L to obtain:

$$L = \frac{50\,\Omega \pm \sqrt{844\,\Omega^2}}{2500\,\text{s}^{-1}}$$

Denoting the solutions as L_+ and L_-, find the values for the inductance:

$$L_+ = 31.6\,\text{mH} \text{ and } L_- = 8.38\,\text{mH}$$

Express the ranges for L:

$$\boxed{8.00\,\text{mH} < L < 8.38\,\text{mH}}$$

and

$$\boxed{31.6\,\text{mH} < L < 40.0\,\text{mH}}$$

87 ••

Picture the Problem We can find the impedance of the circuit from the applied emf and the current drawn by the device. In (*b*) we can use $P_{av} = I_{rms}^2 R$ to find R and the definition of the impedance of a series *RLC* circuit to find $X = X_L - X_C$.

(*a*) Express the impedance of the device in terms of the current it

$$Z = \frac{\mathcal{E}}{I}$$

draws and the emf provided by the power line:

Substitute numerical values to obtain:

$$Z=\frac{120\,\text{V}}{10\,\text{A}}=\boxed{12.0\,\Omega}$$

(*b*) Use the relationship between the average power supplied to the device and the rms current it draws to find *R*:

$$P_{\text{av}}=I_{\text{rms}}^2R$$

and

$$R=\frac{P_{\text{av}}}{I_{rms}^2}=\frac{720\,\text{W}}{(10\,\text{A})^2}=\boxed{7.20\,\Omega}$$

Express the impedance of a series *RLC* circuit:

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

or

$$Z^2=R^2+(X_L-X_C)^2$$

Solve for X_L-X_C:

$$X=X_L-X_C=\sqrt{Z^2-R^2}$$

Substitute numerical values and evaluate *X*:

$$X=\sqrt{(12\,\Omega)^2-(7.2\,\Omega)^2}=\boxed{9.60\,\Omega}$$

(*c*) $\boxed{\text{If the current leads the emf, the reactance is capacitive.}}$

***88** ••

Picture the Problem We can use the fact that when the current is a maximum, $X_L = X_C$, to find the inductance of the circuit. In (*b*), we can find *I*max from ε_{max} and the impedance of the circuit at resonance.

(*a*) Relate X_L and X_C at resonance:

$$X_L=X_C\ \text{ or }\ \omega_0L=\frac{1}{\omega_0C}$$

Solve for *L* to obtain:

$$L=\frac{1}{\omega_0^2C}$$

Substitute numerical values and evaluate *L*:

$$L=\frac{1}{(5000\,\text{s}^{-1})^2(10\,\mu\text{F})}=\boxed{4.00\,\text{mH}}$$

(*b*) Noting that, at resonance, $X = 0$, express I_{max} in terms of the applied emf and the impedance of the circuit at resonance:

$$I_{\text{max}}=\frac{\mathcal{E}_{\text{max}}}{Z}=\frac{10\,\text{V}}{100\,\Omega}=\boxed{0.100\,\text{A}}$$

89 ••

Picture the Problem We can use Ohm's law to express the current through the resistor as a function of time. Because the resistor and capacitor are in parallel they have the same potential difference across them … the emf of the source. We can relate the charge on the capacitor as a function of time to its capacitance and the potential difference across it and differentiate this expression with respect to time to express $I_C(t)$. We can then apply Kirchhoff's junction rule to express the total current drawn from the source. Using the results of (*a*) and (*b*) we can show that $I = I_R + I_C = I_{\max}\cos(\omega t + \delta)$, where $\tan\delta = R/X_C$ and $I_{\max} = \varepsilon_{\max}/Z$ with $Z^{-2} = R^{-2} + X_C^{-2}$.

(*a*) Apply Ohm's law to obtain:

$$I_R(t) = \frac{V(t)}{R} = \frac{\mathcal{E}_{\max}\cos\omega t}{R} = \boxed{\frac{\mathcal{E}_{\max}}{R}\cos\omega t}$$

(*b*) Express the potential difference across the capacitor in terms of the instantaneous charge on the capacitor:

$$V_C(t) = \frac{q(t)}{C} \text{ or } q(t) = CV_C(t)$$

Differentiate $q(t)$ to express the current to the capacitor:

$$\begin{aligned} I_C(t) &= \frac{dq(t)}{dt} = C\frac{d}{dt}\left(V_C(t)\right) \\ &= C\frac{d}{dt}\left(\mathcal{E}_{\max}\cos\omega t\right) \\ &= -\omega C\mathcal{E}_{\max}\sin\omega t \end{aligned}$$

Use the definition of X_C and the trigonometric identity $\cos(\alpha + 90°) = -\sin\alpha$ to obtain:

$$I_C(t) = \boxed{\frac{\mathcal{E}_{\max}}{X_C}\cos(\omega t + 90°)}$$

(*c*) Apply Kirchhoff's junction rule to obtain:

$$\begin{aligned} I &= I_R + I_C \\ &= \frac{\mathcal{E}_{\max}}{R}\cos\omega t + \frac{\mathcal{E}_{\max}}{X_C}\cos(\omega t + 90°) \\ &= \frac{\mathcal{E}_{\max}}{R}\cos\omega t - \frac{\mathcal{E}_{\max}}{X_C}\sin\omega t \end{aligned}$$

We know that the current is also expressible in the form:

$$I = I_{\max}\cos(\omega t + \delta)$$

Expand this expression, using the formula for the cosine of the sum of two angles, to obtain:

$$I = I_{\max}\cos\omega t\cos\delta - I_{\max}\sin\omega t\sin\delta$$

Equate these expressions and rewrite the resulting equation to obtain:

$$\left(\frac{\mathcal{E}_{\max}}{R} - I_{\max}\cos\delta\right)\cos\omega t - \left(\frac{\mathcal{E}_{\max}}{X_C} - I_{\max}\sin\delta\right) = 0$$

Express the conditions that must be satisfied if this equation is to be true for all values of t:

$$\frac{\mathcal{E}_{\max}}{R} - I_{\max}\cos\delta = 0$$

and

$$\frac{\mathcal{E}_{\max}}{X_C} - I_{\max}\sin\delta = 0$$

Rewrite these equations as:

$$I_{\max}\sin\delta = \frac{\mathcal{E}_{\max}}{X_C} \qquad (1)$$

and

$$I_{\max}\cos\delta = \frac{\mathcal{E}_{\max}}{R} \qquad (2)$$

Divide equation (1) by equation (2) and simplify to obtain:

$$\tan\delta = \boxed{\frac{R}{X_C}}$$

Square equations (1) and (2) and add to obtain:

$$I_{\max}^2\sin^2\delta + I_{\max}^2\cos^2\delta = I_{\max}^2\left(\sin^2\delta + \cos^2\delta\right) = I_{\max}^2 = \left(\frac{\mathcal{E}_{\max}}{X_C}\right)^2 + \left(\frac{\mathcal{E}_{\max}}{R}\right)^2 = \mathcal{E}_{\max}^2\left(\frac{1}{X_C^2} + \frac{1}{R^2}\right) = \frac{\mathcal{E}_{\max}^2}{Z^2}$$

or

$$I_{\max} = \boxed{\frac{\mathcal{E}_{\max}}{Z}}$$

where $Z^{-2} = \boxed{X_C^{-2} + R^{-2}}$

***90** ••

Picture the Problem Because we'll need to use it repeatedly in solving this problem, we'll begin by using complex numbers to derive an expression for the impedance Z_p of the parallel combination of C with L and R_L in series. The total impedance of the circuit is then $Z = R + Z_p$. We can apply Kirchhoff's loop rule to obtain expressions for the voltages across the load resistor with S either open or closed.

Use complex numbers to relate Z_p to R_L, X_L, and X_C:

$$\frac{1}{Z_p}=\frac{1}{-iX_C}+\frac{1}{R_L+iX_L}=\frac{R_L+i(X_L-X_C)}{X_CX_L-iR_LX_C}$$

or

$$Z_p=\frac{X_CX_L-iR_LX_C}{R_L+i(X_L-X_C)}$$

Multiple the numerator and denominator of this fraction by the complex conjugate of $R_L + i(X_L - X_C)$:

$$Z_p=\frac{X_CX_L-iR_LX_C}{R_L+i(X_L-X_C)}\,\frac{R_L-i(X_L-X_C)}{R_L-i(X_L-X_C)}$$

Simplify to obtain:

$$Z_p=\frac{R_LX_C^2}{R_L^2+(X_L-X_C)^2}-i\frac{X_C\left[R_L^2+X_L(X_L-X_C)\right]}{R_L^2+(X_L-X_C)^2}\qquad(1)$$

(*a*) **S is closed**. Because L is shorted:

$$X_L=0$$

Evaluate X_C:

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(10\,\text{s}^{-1})(8\,\mu\text{F})}=1.99\,\text{k}\Omega$$

Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p=30\,\Omega-i(0.452\,\Omega),$$

$$Z=40\,\Omega-i(0.452\,\Omega),$$

and

$$|Z|=\sqrt{(40\,\Omega)^2+(0.452\,\Omega)^2}=\boxed{40.0\,\Omega}$$

In Problem 29-77 we showed that for a parallel combination of a resistor and capacitor, the phase angle δ is given by:

$$\delta=\tan^{-1}\left(\frac{R}{X_C}\right)$$

Substitute numerical values and evaluate δ:

$$\delta = \tan^{-1}\left(\frac{40\,\Omega}{-0.452\,\Omega}\right) = \boxed{-89.4°}$$

No phasor diagram is shown because it is impossible to represent it to scale.

(*b*) **<u>S is open; i.e., the inductor is in the circuit</u>**. Find X_L:

$$X_L = \omega L = 2\pi fL = 2\pi\left(10\,\text{s}^{-1}\right)(0.15\,\text{H})$$
$$= 9.42\,\Omega$$

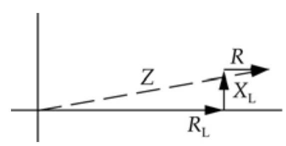
Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p = 30.3\,\Omega + i(9.01\,\Omega),$$
$$Z = 40.3\,\Omega + i(9.01\,\Omega),$$
$$|Z| = \sqrt{(40.3\,\Omega)^2 + (9.01\,\Omega)^2} = \boxed{41.3\,\Omega}$$

and

$$\delta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{9.01\,\Omega}{40.3\,\Omega}\right)$$
$$= \boxed{12.6°}$$

The phasor diagram for this case is shown to the right.



(*c*) **<u>S is closed.</u>** Apply Kirchhoff's loop rule to a loop including the source, R, and R_L:

$$\mathcal{E} - IR - V_{R_L} = 0$$

Solve for V_{R_L} :

$$V_{R_L} = \mathcal{E} - IR$$

Express the current I in the circuit:

$$I = \frac{\mathcal{E}}{Z}$$

Substitute and simplify to obtain:

$$V_{R_L} = \mathcal{E} - \frac{\mathcal{E}R}{Z} = \left(1 - \frac{R}{|Z|}\right)\mathcal{E}_{\text{max}}\cos(\omega t - \delta)$$

From (*a*) we have:

$$Z_p = 30\,\Omega - i(0.452\,\Omega),$$
$$Z = 40\,\Omega - i(0.452\,\Omega),$$
$$|Z| = 40.0\,\Omega\text{, and}$$

$$\delta = \tan^{-1}\left(\frac{-0.452}{40}\right) = -0.647° \approx 0°$$

Substitute numerical values to obtain:

$$V_{R_L} = \left(1 - \frac{10\,\Omega}{40\,\Omega}\right)(100\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t\right]$$
$$= \boxed{(75\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t\right]}$$

<u>S is open</u>. Apply Kirchhoff's loop rule to a loop including the source, *R*, *L*, and R_L when S is open:

$$\mathcal{E} - IR - IX_L - V_{R_L} = 0$$

Solve for V_{R_L}:

$$V_{R_L} = \mathcal{E} - IR - IX_L = \mathcal{E} - I(R + X_L)$$

Express the current *I* in the circuit:

$$I = \frac{\mathcal{E}}{Z}$$

Substitute to obtain:

$$V_{R_L} = \left(1 - \frac{R + X_L}{|Z|}\right)\mathcal{E}_{\max}\cos(\omega t - \delta)$$

Substitute numerical values and evaluate Z_p and Z:

$$Z_p = 30.3\,\Omega + i(9.01\,\Omega),$$
$$Z = 40.3\,\Omega + i(9.01\,\Omega),$$
$$|Z| = 41.3\,\Omega,$$

and

$$\delta = \tan^{-1}\left(\frac{X_L}{R + R_L}\right) = \tan^{-1}\left(\frac{9.42\,\Omega}{40.3\,\Omega}\right)$$
$$= 13.2°$$

Substitute numerical values and evaluate V_{R_L}:

$$V_{R_L} = \left(1 - \frac{10\,\Omega + 9.42\,\Omega}{41.3\,\Omega}\right)$$
$$\times(100\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t - 13.2°\right]$$
$$= \boxed{(53.0\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t - 13.2°\right]}$$

(*d*) Find X_L and X_C when $f = 1000$ Hz:

$$X_L = 2\pi\left(1000\,\text{s}^{-1}\right)(0.15\,\text{H}) = 942\,\Omega$$

and

$$X_C = \frac{1}{2\pi\left(1000\,\text{s}^{-1}\right)(8\,\mu\text{F})} = 19.9\,\Omega$$

S is closed. $X_L = 0$, and Z_p simplifies to:

$$Z_p = \frac{R_L X_C^2}{R_L^2 + X_C^2} - i\frac{R_L^2 X_C}{R_L^2 + X_C^2}$$

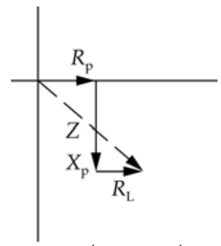
Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p = 9.17\,\Omega - i(13.8\,\Omega),$$
$$Z = 19.17\,\Omega - i(13.8\,\Omega),$$
$$|Z| = \sqrt{(19.17\,\Omega)^2 + (13.8\,\Omega)^2} = \boxed{23.6\,\Omega}$$

and

$$\delta = \tan^{-1}\left(\frac{-13.8\,\Omega}{19.17\,\Omega}\right) = \boxed{-35.7°}$$

A phasor diagram for this circuit is shown to the right,



S is open. Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p = 0.0140\,\Omega - i(20.3\,\Omega),$$
$$Z = 10.0\,\Omega - i(20.3\,\Omega),$$
$$|Z| = \sqrt{(10.0\,\Omega)^2 + (20.3\,\Omega)^2} = \boxed{22.6\,\Omega}$$

and

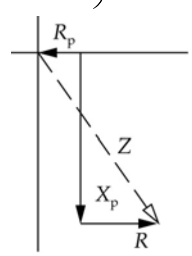
Find the total impedance, its magnitude, and phase angle for the circuit:

$$Z = 10.0\,\Omega - i(20.4\,\Omega),$$
$$Z = \sqrt{(10.0\,\Omega)^2 + (20.4\,\Omega)^2} = \boxed{22.7\,\Omega}$$

and

$$\delta = \tan^{-1}\left(\frac{-20.4\,\Omega}{10\,\Omega}\right) = \boxed{-63.9°}$$

The phasor diagram is shown to the right.



(*e*)

The load voltage at the higher frequency is much more attenuated with S open, while opening S does not reduce the low frequency load voltage significantly. Therefore, S open is the better arrangement for a low-pass filter.

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Picture the Problem We can find the resonant frequency of any parallel ac circuit by setting the imaginary part of the reciprocal of the impedance equal to zero. In (*b*) we can use complex numbers to find the impedance of each branch of the circuit and then relate the common potential difference across each branch to its impedance and the current in the resistors.

(*a*) Express the reciprocal of the impedance of the circuit:

$$\frac{1}{Z}=\frac{1}{R_1-iX_C}+\frac{1}{R_2+iX_L}$$

Rewrite this expression with a common denominator and simplify to obtain:

$$\frac{1}{Z}=\frac{(R_1+R_2)+i(X_L-X_C)}{(R_1R_2+X_CX_L)+i(R_1X_L-R_2X_C)}$$

Multiply this expression by 1 in the form of the complex conjugate of the denominator divided by itself and simplify (separate the real part of the expression from the imaginary part) to obtain:

$$\frac{1}{Z}=\frac{(R_1+R_2)(R_1R_2+X_CX_L)+(X_L-X_C)(R_1X_L-R_2X_C)}{(R_1R_2+X_CX_L)^2+(R_1X_L-R_2X_C)^2}+i\frac{(X_L-X_C)(R_1R_2+X_CX_L)-(R_1+R_2)(R_1X_L-R_2X_C)}{(R_1R_2+X_CX_L)^2+(R_1X_L-R_2X_C)^2}$$

Set the imaginary part of $1/Z$ equal to zero to obtain:

$$(X_L-X_C)(R_1R_2+X_CX_L)-(R_1+R_2)(R_1X_L-R_2X_C)=0$$

Substitute numerical values for R_1 and R_2 (suppress the units to save space and make the resulting equation more readable) to obtain:

$$\left(\omega_0L-\frac{1}{\omega_0C}\right)\left(8+\frac{L}{C}\right)-6\left(2\omega_0L-\frac{4}{\omega_0C}\right)=0$$

Simplify this equation by clearing the fractions and combining like terms to obtain:

$$\left(8LC^2+L^2C-12LC^2\right)\omega_0^2=L-16C$$

Solve for ω_0:

$$\omega_0=\sqrt{\frac{L-16C}{8LC^2+L^2C-12LC^2}}$$

Substitute numerical values for L and C and evaluate ω_0:

$$\omega_0=\sqrt{\frac{1.15\times10^{-2}}{4.28\times10^{-9}}}=\boxed{1.64\times10^{3}\ \text{rad/s}}$$

(*b*) Express the currents in each branch at resonance:

$$I_C = \frac{\mathcal{E}}{|Z_C|} \text{ and } I_L = \frac{\mathcal{E}}{|Z_L|}$$

Evaluate $Z_{C,\text{res}}$ and $|Z_{C,\text{res}}|$:

$$Z_{C,\text{res}} = 2\,\Omega - i\frac{1}{(1.64\times10^3\ \text{s}^{-1})(30\times10^{-6}\ \text{F})} = 2\,\Omega - i(20.3\,\Omega),$$

$$|Z_{C,\text{res}}| = \sqrt{(2\,\Omega)^2 + (20.3\,\Omega)^2} = 20.4\,\Omega,$$

Substitute to obtain:

$$I_{C,\text{rms}} = \frac{40\ \text{V}}{\sqrt{2}(20.4\,\Omega)} = \boxed{1.39\ \text{A}}$$

and

$$\delta_C = \tan^{-1}\left(\frac{-20.3\,\Omega}{2\,\Omega}\right) = \boxed{-84.4°}$$

Evaluate $Z_{L,\text{res}}$ and $|Z_{L,\text{res}}|$:

$$Z_{L,\text{res}} = 4\,\Omega + i(1.64\times10^3\ \text{s}^{-1})(12\times10^{-3}\ \text{H}) = 4\,\Omega + i(19.7\,\Omega),$$

$$|Z_{L,\text{res}}| = \sqrt{(4\,\Omega)^2 + (19.7\,\Omega)^2} = 20.1\,\Omega,$$

Substitute to obtain:

$$I_{L,\text{rms}} = \frac{40\ \text{V}}{\sqrt{2}(20.1\,\Omega)} = \boxed{1.41\ \text{A}}$$

and

$$\delta_L = \tan^{-1}\left(\frac{19.7\,\Omega}{4\,\Omega}\right) = \boxed{78.5°}$$

Express and evaluate the rms current supplied by the source:

$$\begin{aligned} I_{\text{rms}} &= I_{L,\text{rms}}\cos\delta_L + I_{C,\text{rms}}\cos\delta_C \\ &= (1.41\ \text{A})\cos 78.5° \\ &\quad + (1.39\ \text{A})\cos(-84.4°) \\ &= \boxed{0.417\ \text{A}} \end{aligned}$$

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Picture the Problem We can use its definition to express Q in terms of ω_0 and $\Delta\omega$. By expressing the current drawn from the source we can obtain an expression for the energy stored in the system each cycle and then use this result to establish the relationship between ω, R, L, and C when the energy stored per cycle is at half-maximum. Finally, we can solve the resulting equation for the values of ω that will allow us to determine $\Delta\omega$.

The definition of Q is:

$$Q = \frac{\omega_0}{\Delta\omega}$$

where $\Delta\omega$ is the width of the resonance at half maximum.

Express the resonance frequency of the circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitute to obtain:

$$Q = \frac{1}{\sqrt{LC}\,\Delta\omega} \qquad (1)$$

Express the current to the capacitor:

$$I_C = \frac{V}{X_C} = \omega CV$$

with I_C leading V by 90°.

Express the current in the inductor:

$$I_L = \frac{V}{X_L} = \frac{V}{\omega L}$$

with I_L lagging V by 90°.

Express the current in the resistor:

$$I_R = \frac{V}{R}$$

with I_R in phase with V.

Express the total current drawn from the source:

$$I = \frac{V}{Z} = V\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

$$= \frac{V}{R}\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

At resonance, the reactive term is zero and the total current is the current in the resistor:

$$I_0 = \frac{V}{R}$$

Substitute to obtain:

$$I = I_0\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

Express the total energy stored in the circuit per cycle:

$$U_{\text{tot}} = \frac{Q_0^2}{2C}$$

where Q_0 is the maximum charge on the capacitor.

Relate the maximum value of the current to the maximum value of the charge:

$$I_{\max} = \omega Q_0$$

Substitute to obtain:

$$U_{\text{tot}} = \frac{I_{\max}^2}{2\omega^2 C} = \frac{1}{2\omega^2 C}\frac{V^2}{R^2} = \frac{1}{2\omega^2 C}\frac{I_0^2}{1+R^2\left(\frac{1}{\omega L}-\omega C\right)^2}$$

At resonance we have:

$$U_{\text{tot,res}} = \frac{I_0^2}{2\omega^2 C}$$

At half $U_{\text{tot,res}}$:

$$\frac{1}{2}U_{\text{tot,res}} = \frac{I_0^2}{4\omega^2 C} = \frac{1}{2\omega^2 C}\frac{I_0^2}{1+R^2\left(\frac{1}{\omega L}-\omega C\right)^2}$$

or

$$\frac{1}{2} = \frac{1}{1+R^2\left(\frac{1}{\omega L}-\omega C\right)^2}$$

Solve for $R\left(\omega C - \frac{1}{\omega L}\right)$ to obtain:

$$R\left(\omega C - \frac{1}{\omega L}\right) = \pm 1 \qquad (2)$$

Rewrite equation (2) explicitly as a quadratic equation:

$$RLC\omega^2 \pm L\omega - R = 0$$

Letting + denote the roots with a positive coefficient of ω and – the roots with a negative coefficient, solve this equation for ω_+ and ω_-:

$$\omega_+ = \frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

and

$$\omega_- = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Express $\Delta\omega$:

$$\Delta\omega = \omega_+ - \omega_- = \frac{1}{RC}$$

Substitute in equation (1) to obtain:

$$Q = \frac{RC}{\sqrt{LC}} = \boxed{R\sqrt{\frac{C}{L}}}$$

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Picture the Problem We can use the expression for the resonance frequency derived by equating the capacitive and inductive reactances at resonance to express ω_0 in terms of L and C. In (b) we can use the result derived in Problem 92 to find R from Q, L, and C.

(a) Express the resonance frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f^2 L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{4\pi^2\left(4\times10^3\ \text{s}^{-1}\right)^2\left(4\,\text{mH}\right)} = \boxed{0.396\,\mu\text{F}}$$

(b) From Problem 92 we have:

$$Q = R\sqrt{\frac{C}{L}}$$

Solve for R to obtain:

$$R = Q\sqrt{\frac{L}{C}}$$

Substitute numerical values and evaluate R:

$$R = 8\sqrt{\frac{4\,\text{mH}}{0.396\,\mu\text{F}}} = \boxed{804\,\Omega}$$

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Picture the Problem We can use the expression for the resonance frequency derived by equating the capacitive and inductive reactances at resonance to express ω_0 in terms of L and C. We can use the result derived in Problem 92 to find the Q-value resulting from halving the capacitance and to find the resistance necessary to give $Q = 8$.

Express the resonance frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{\frac{1}{2}(4\,\text{mH})(0.396\,\mu\text{F})}} = \boxed{5.66\,\text{kHz}}$$

From Problem 92 we have:

$$Q = R\sqrt{\frac{C}{L}} \qquad (1)$$

Letting C' represent the halved capacitance, express Q':

$$Q' = R\sqrt{\frac{C'}{L}}$$

Divide Q' by Q and simplify to obtain:

$$\frac{Q'}{Q} = \frac{R\sqrt{\frac{C'}{L}}}{R\sqrt{\frac{C}{L}}} = \sqrt{\frac{C'}{C}}$$

Because $C' = \frac{1}{2}C$:

$$Q' = \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \boxed{5.66}$$

Solve equation (1) for R to obtain:

$$R = Q\sqrt{\frac{L}{C}}$$

Substitute numerical values and evaluate R:

$$R = 8\sqrt{\frac{4\,\text{mH}}{\frac{1}{2}(0.396\,\mu\text{F})}} = \boxed{1.14\,\text{k}\Omega}$$

95 ••

Picture the Problem We can use its definition to find the resonance frequency of this series RLC circuit and the fact that, at resonance, $Z = R$, to find the resonance current. Because, at resonance $V_L = V_C$, we can find the voltage across either element from the product of the current and its reactance. In (*c*) we can use the definition of the Q factor to find the angular frequency corresponding to $f = f_0 + \frac{1}{2}\Delta f$ and then use this result to find X_L, X_C, and Z at this frequency. Finally, we can use these values for X_L, X_C, and Z to find the rms current and the rms voltages across the inductor and capacitor.

(*a*) Express the resonance frequency f_0 in terms of L and C:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{(36\,\text{mH})(4\,\text{nF})}} = \boxed{13.26\,\text{kHz}}$$

(*b*) At resonance, $Z = R$ and:

$$I = \frac{\mathcal{E}}{R} = \frac{20\text{ V}}{100\,\Omega} = \boxed{200\,\text{mA}}$$

Express and evaluate the equal (at resonance) rms voltages across the capacitor and the inductor:

$$V_C = V_L = IX_L = \omega_0 IL = 2\pi f_0 IL$$
$$= 2\pi(13.26\,\text{kHz})(0.2\,\text{A})(36\,\text{mH})$$
$$= \boxed{600\,\text{V}}$$

(*c*) Express the rms current in the circuit and the rms voltages across the inductor and capacitor:

$$I = \frac{\mathcal{E}}{Z},\ V_L = IX_L,\ \text{and}\ V_C = IX_C$$

Express the Q factor for an RLC circuit:

$$Q = \frac{\omega_0 L}{R} \approx \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$$

Solve for Δf:

$$\Delta f = \frac{R}{\omega_0 L} f_0$$

Express f:

$$f = f_0 + \frac{R}{2\omega_0 L} f_0 = f_0\left(1 + \frac{R}{2\omega_0 L}\right)$$

Substitute numerical values and evaluate f and ω:

$$f = (13.26\,\text{kHz})$$
$$\times\left(1 + \frac{100\,\Omega}{4\pi(13.26\,\text{kHz})(36\,\text{mH})}\right)$$
$$= 13.48\,\text{kHz}$$

and

$$\omega = 2\pi f = 2\pi(13.48\,\text{kHz}) = 84.7\,\text{krad/s}$$

Calculate X_L and X_C at 84.7 krad/s:

$$X_L = \omega L = (84.7\,\text{krad/s})(36\,\text{mH})$$
$$= 3.05\,\text{k}\Omega$$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{(84.7\,\text{krad/s})(4\,\text{nF})}$$
$$= 2.95\,\text{k}\Omega$$

Now we can find Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(100\,\Omega)^2 + (3.05\,\text{k}\Omega - 2.95\,\text{k}\Omega)^2}$$
$$= 141\,\Omega$$

Substitute numerical values and evaluate I, V_L, and V_C:

$$I = \frac{20\,\text{V}}{141\,\Omega} = \boxed{142\,\text{mA}},$$

$$V_L = (142\,\text{mA})(3.05\,\text{k}\Omega) = \boxed{433\,\text{V}},$$

and

$$V_C = (142\,\text{mA})(2.95\,\text{k}\Omega) = \boxed{419\,\text{V}}$$

96 •••

Picture the Problem We can use complex numbers to find the impedance in the branches of the given circuit. We can then use Kirchhoff's loop rule to find the currents in the branches and a current phasor diagram to find the total current and its phase relative to the applied voltage.

(*a*) Use complex numbers to find Z_L, $|Z_L|$, and δ_L:

$$Z_L = R_2 + iX_L = 40\,\Omega + i(30\,\Omega),$$
$$|Z_L| = \sqrt{R_2^2 + X_L^2}$$
$$= \sqrt{(40\,\Omega)^2 + (30\,\Omega)^2} = \boxed{50.0\,\Omega}$$

and

$$\delta_L = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{30\,\Omega}{40\,\Omega}\right) = \boxed{36.9°}$$

Use complex numbers to find Z_C, $|Z_C|$, and δ_C:

$$Z_C = R_1 + iX_C = 10\,\Omega - i(10\,\Omega),$$
$$|Z_C| = \sqrt{R_1^2 + X_C^2}$$
$$= \sqrt{(10\,\Omega)^2 + (10\,\Omega)^2} = \boxed{14.1\,\Omega}$$

and

$$\delta_C = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{-10\,\Omega}{10\,\Omega}\right)$$
$$= \boxed{-45.0°}$$

(*b*) Apply Kirchhoff's loop rule to the source and the inductive branch to obtain:

$$V - I_L X_L = 0$$

or

$$I_L = \frac{V}{Z_L} = \frac{110\ \text{V}}{50\ \Omega}$$
$$= \boxed{2.20\ \text{A lagging the voltage by } 36.9°}$$

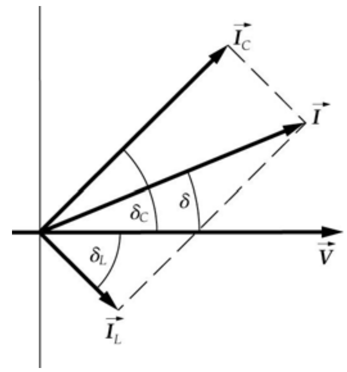
Apply Kirchhoff's loop rule to the source and the capacitive branch to obtain:

$$V - I_C X_C = 0$$

or

$$I_C = \frac{V}{Z_C} = \frac{110\ \text{V}}{14.1\Omega}$$
$$= \boxed{7.80\ \text{A leading the voltage by } 45.0°}$$

(*c*) The current phasor diagram is shown to the right.



Express the total current in terms of its horizontal and vertical components:

$$I = \sqrt{I_{\text{hor}}^2 + I_{\text{vert}}^2}$$

and

$$\delta = \tan^{-1}\left(\frac{I_{\text{vert}}}{I_{\text{hor}}}\right)$$

Find the horizontal component I_{hor} of the total current:

$$I_{\text{hor}} = I_C \cos\delta_C + I_L \cos\delta_L$$
$$= (7.80\ \text{A})\cos 45° + (2.20\ \text{A})\cos 36.9°$$
$$= 7.27\ \text{A}$$

Find the vertical component I_{vert} of the total current:

$$I_{\text{vert}} = I_C \sin\delta_C - I_L \sin\delta_L$$
$$= (7.80\ \text{A})\sin 45° - (2.20\ \text{A})\sin 36.9°$$
$$= 4.19\ \text{A}$$

Substitute numerical values and evaluate I and δ:

$$I = \sqrt{(4.19\ \text{A})^2 + (7.27\ \text{A})^2} = \boxed{8.39\ \text{A}}$$

and

$$\delta = \tan^{-1}\left(\frac{4.19\ \text{A}}{7.27\ \text{A}}\right) = \boxed{30.0°}$$

Remarks: The total current leads the applied voltage by 30.0°.

***97 •••**

Picture the Problem We can manipulate Equation 29-47 into a form that has the ratio of *L* to *R* in it and then use the definition of *Q* to eliminate *L* and *R*. In (*b*) we can approximate $\omega^2 - \omega_0^2$, near resonance, as $2\omega_0\Delta\omega$ and substitute in the result from (*a*) to obtain the desired result.

(*a*) From Equation 29-47:

$$\tan\delta = \frac{\omega L - 1/\omega C}{R} = \frac{\omega^2 L - 1/C}{\omega R} = \frac{L\left(\omega^2 - 1/LC\right)}{\omega R} = \frac{L\left(\omega^2 - \omega_0^2\right)}{\omega R}$$

Express *Q* in terms of ω_0, *L* and *R*:

$$Q = \frac{\omega_0 L}{R}$$

Solve for *L*/*R* to obtain:

$$\frac{L}{R} = \frac{Q}{\omega_0}$$

Substitute to obtain:

$$\tan\delta = \boxed{\frac{Q\left(\omega^2 - \omega_0^2\right)}{\omega\omega_0}} \qquad (1)$$

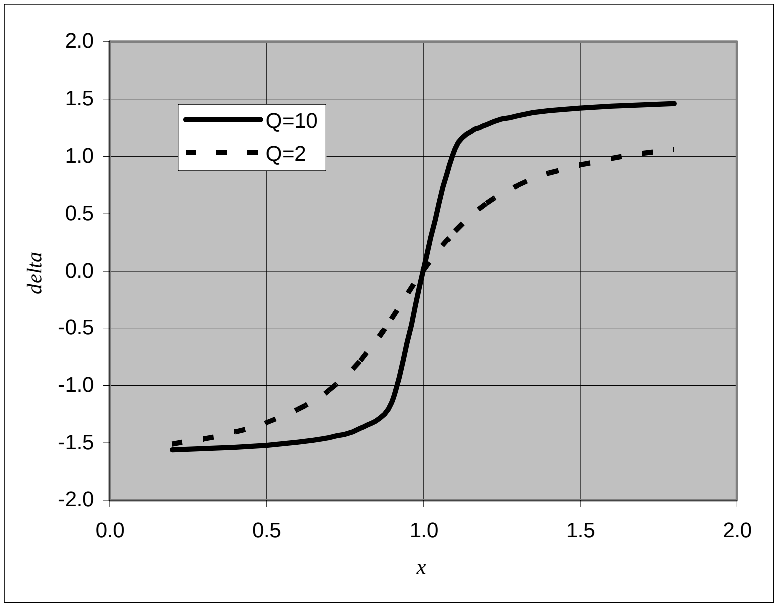
(*b*) Near resonance:

$$\omega^2 - \omega_0^2 = \left(\omega + \omega_0\right)\left(\omega - \omega_0\right) \approx 2\omega_0\Delta\omega$$

Substitute in equation (1) to obtain:

$$\tan\delta = \frac{Q\left(2\omega_0\Delta\omega\right)}{\omega\omega_0} = \boxed{\frac{2Q\left(\omega - \omega_0\right)}{\omega}}$$

(*c*) A following graph of δ as a function of $x = \omega/\omega_0$ was plotted using a spreadsheet program. The solid curve is for a high-*Q* circuit and the dashed curve is for a low-*Q* circuit.



98 •••

Picture the Problem We can rewrite Equation 29-45 in terms of the current and then differentiate Equation 29-46. Substituting for I, dI/dt, X_L, and X_C will allow us to use the trigonometric identities for the sine and cosine of the sum of two angles to rewrite the equation in such a form that we can equate the coefficients of $\sin\omega t$ and $\cos\omega t$ to obtain Equation 29-47 and an equation that is satisfied provided Z is given by Equation 29-49.

Rewrite Equation 29-45 in terms of the current:

$$L\frac{dI}{dt} + RI + \frac{1}{C}\int Idt = \mathcal{E}_{\max}\cos\omega t$$

Equation 29-46 is:

$$I = I_{\max}\cos(\omega t - \delta)$$

Differentiate Equation 29-50 with respect to time to obtain:

$$\frac{dI}{dt} = -\omega I_{\max}\sin(\omega t - \delta)$$

Evaluate $\int Idt$:

$$\int Idt = I_{\max}\int\cos(\omega t - \delta)dt = \frac{I_{\max}}{\omega}\sin(\omega t - \delta)$$

Substitute to obtain:

$$L(-\omega I_{\max}\sin(\omega t - \delta)) + RI_{\max}\cos(\omega t - \delta) + \frac{1}{C}\left(\frac{I_{\max}}{\omega}\sin(\omega t - \delta)\right) = \mathcal{E}_{\max}\cos\omega t$$

Divide through by I_{max} to obtain:

$$L(-\omega\sin(\omega t-\delta)) + R\cos(\omega t-\delta) + \frac{1}{C}\left(\frac{1}{\omega}\sin(\omega t-\delta)\right) = \frac{\mathcal{E}_{max}}{I_{max}}\cos\omega t$$

Use the definitions of X_L, X_C, and Z to obtain:

$$-X_L\sin(\omega t-\delta)+R\cos(\omega t-\delta) + X_C\sin(\omega t-\delta) = Z\cos\omega t$$

Use the trigonometric identities for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ to obtain:

$$-X_L(\sin\omega t\cos\delta-\cos\omega t\sin\delta)+R(\cos\omega t\cos\delta+\sin\omega t\sin\delta) + X_C(\sin\omega t\cos\delta-\cos\omega t\sin\delta) = Z\cos\omega t$$

Collect the terms in $\sin\omega t$ and $\cos\omega t$:

$$(-X_L\cos\delta+R\sin\delta+X_C\cos\delta)\sin\omega t + (R\cos\delta-X_C\sin\delta+X_L\sin\delta)\cos\omega t = Z\cos\omega t$$

Equate the coefficients of $\sin\omega t$ and $\cos\omega t$ to obtain:

$$-X_L\cos\delta+R\sin\delta+X_C\cos\delta=0$$

and

$$R\cos\delta-X_C\sin\delta+X_L\sin\delta=Z$$

Solve the first of these equations for $\tan\delta$:

$$\boxed{\tan\delta=\frac{X_L-X_C}{R}}$$ Equation 29-47

Rewrite the second equation as:

$$R-X_C\tan\delta+X_L\tan\delta=\frac{Z}{\cos\delta}$$

or

$$(X_L-X_C)\tan\delta+R=\frac{Z}{\cos\delta}$$

Simplify this equation to obtain Equation 29-49:

$$Z=\boxed{\sqrt{R^2+(X_L-X_C)^2}}$$

99 •••

Picture the Problem In (*a*) we can apply Kirchhoff's loop rule to obtain the 2nd order differential equation relating the charge on the capacitor to the time. In (*b*) we'll assume a solution of the form $Q=Q_{max}\cos\omega t$, differentiate it twice, and substitute for d^2Q/dt^2 and Q to show that the assumed solution satisfies the DE provided

$Q_{\max} = -\frac{\varepsilon_{\max}}{L(\omega^2 - \omega_0^2)}$. In (*c*) we'll use our results from (*a*) and (*b*) to establish the for $I_{\max}$ given in the problem statement.

(*a*) Apply Kirchhoff's loop rule to obtain:

$$\varepsilon - \frac{Q}{C} - L\frac{dI}{dt} = 0$$

Substitute for ε and rearrange the differential equation to obtain:

$$L\frac{dI}{dt} + \frac{Q}{C} = \varepsilon_{\max}\cos\omega t$$

Because $I = dQ/dt$:

$$\boxed{L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \varepsilon_{\max}\cos\omega t}$$

(*b*) Assume that the solution is:

$$Q = Q_{\max}\cos\omega t$$

Differentiate the assumed solution twice to obtain:

$$\frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$$

and

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_{\max}\cos\omega t$$

Substitute in the differential equation to obtain:

$$-\omega^2 LQ_{\max}\cos\omega t + \frac{Q_{\max}}{C}\cos\omega t = \varepsilon_{\max}\cos\omega t$$

Factor $\cos\omega t$ from the left-hand side of the equation:

$$\left(-\omega^2 LQ_{\max} + \frac{Q_{\max}}{C}\right)\cos\omega t = \varepsilon_{\max}\cos\omega t$$

If this equation is to hold for all values of *t* it must be true that:

$$-\omega^2 LQ_{\max} + \frac{Q_{\max}}{C} = \varepsilon_{\max}$$

or

$$Q_{\max} = \frac{\varepsilon_{\max}}{-\omega^2 L + \frac{1}{C}}$$

Factor *L* from the denominator and substitute for 1/*LC* to obtain:

$$Q_{\max} = \frac{\varepsilon_{\max}}{L\left(-\omega^2 + \frac{1}{LC}\right)} = \boxed{\frac{-\varepsilon_{\max}}{L(\omega^2 - \omega_0^2)}}$$

(*c*) From (*a*) and (*b*) we have:

$$I=\frac{dQ}{dt}=-\omega Q_{max}\sin\omega t$$
$$=\frac{\omega\mathcal{E}_{max}}{L\left(\omega^2-\omega_0^2\right)}\sin\omega t$$
$$=I_{max}\sin\omega t$$

where

$$I_{max}=\frac{\omega\mathcal{E}_{max}}{L\left|\omega^2-\omega_0^2\right|}=\frac{\mathcal{E}_{max}}{\frac{L}{\omega}\left|\omega^2-\omega_0^2\right|}$$
$$=\frac{\mathcal{E}_{max}}{\left|\omega L-\frac{1}{\omega C}\right|}=\frac{\mathcal{E}_{max}}{\left|X_L-X_C\right|}$$

If $\omega>\omega_0$, $X_L>X_C$ and the current lags the voltage by 90°. Therefore:

$$I=I_{max}\sin\omega t=\boxed{I_{max}\cos\left(\omega t-90°\right)}$$

If $\omega<\omega_0$, $X_L<X_C$ and the current leads the voltage by 90°. Therefore:

$$I=-I_{max}\sin\omega t=\boxed{I_{max}\cos\left(\omega t+90°\right)}$$

100 •••

Picture the Problem We can use the condition determining the half-power points to obtain a quadratic equation that we can solve for the frequencies corresponding to the half-power points. Expanding these solutions binomially will lead us to the result that $\Delta\omega=\omega_2-\omega_1\approx R/L$. We can then use the definition of Q to complete the proof that $Q\approx\omega_0/\Delta\omega$.

Equation 29-58 is:

$$P_{av}=\frac{\mathcal{E}_{rms}^2R\omega^2}{L^2\left(\omega^2-\omega_0^2\right)^2+\omega^2R^2}$$

The half-power points will occur when the denominator is twice the value near resonance, that is, when:

$$L^2(\omega^2-\omega_0^{\ 2})^2=\omega^2R^2\approx\omega_0^{\ 2}R^2$$

or

$$\left(\frac{L}{R}\right)^2\left(\omega^2-\omega_0^2\right)^2=\omega_0^2$$

Let ω_1 and ω_2 be the solutions of this equation. Then:

$$\left(\frac{L}{R}\right)^2\left(\omega_1^2-\omega_0^2\right)^2=\omega_0^2$$

and

$$\left(\frac{L}{R}\right)^2\left(\omega_2^2-\omega_0^2\right)^2=\omega_0^2$$

Solve these equations for ω_1 and ω_2 to obtain:

$$\omega_1 = \omega_0\left(1 - \frac{R}{\omega_0 L}\right)^{1/2}$$

and

$$\omega_2 = \omega_0\left(1 + \frac{R}{\omega_0 L}\right)^{1/2}$$

Expand these solutions binomially to obtain:

$$\omega_1 = \omega_0\left(1 - \frac{R}{2\omega_0 L} + \text{higher order terms}\right)$$

and

$$\omega_2 = \omega_0\left(1 + \frac{R}{2\omega_0 L} + \text{higher order terms}\right)$$

For $R << X_L$ (a condition that holds for a sharply peaked resonance):

$$\omega_1 \approx \omega_0\left(1 - \frac{R}{2\omega_0 L}\right),$$

$$\omega_2 \approx \omega_0\left(1 + \frac{R}{2\omega_0 L}\right),$$

and

$$\Delta\omega = \omega_2 - \omega_1 \approx \frac{R}{L}.$$

From the definition of Q:

$$\frac{R}{L} = \frac{\omega_0}{Q}$$

Substitute to obtain:

$$\Delta\omega \approx \frac{\omega_0}{Q}$$

Solve for Q:

$$Q \approx \boxed{\frac{\omega_0}{\Delta\omega}}$$

101 •••

Picture the Problem We'll differentiate $Q = Q_0 e^{-Rt/2L}\cos\omega' t$ twice and substitute this function and both its derivatives in the differential equation of the circuit. Rewriting the resulting equation in the form A$\cos\omega' t$ + B$\sin\omega' t$ = 0 will reveal that B vanishes. Requiring that A$\cos\omega' t$ = 0 hold for all values of t will lead to $\omega' = \sqrt{(1/LC) - (R/2L)^2}$.

Equation 29-47*b* is:

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} + R\frac{dQ}{dt} = 0$$

Assume a solution of the form: $Q = Q_0 e^{-Rt/2L} \cos\omega' t$

Differentiate $Q(t)$ twice to obtain:

$$\frac{dQ}{dt} = -Q_0 e^{-Rt/2L}\left[\omega' \sin\omega' t + \frac{R}{2L}\cos\omega' t\right]$$

and

$$\frac{d^2Q}{dt^2} = Q_0 e^{-Rt/2L}\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right]$$

Substitute these derivatives in the differential equation and simplify to obtain:

$$LQ_0 e^{-Rt/2L}\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right] + \frac{Q_0}{C} e^{-Rt/2L}\cos\omega' t$$
$$- RQ_0 e^{-Rt/2L}\left[\omega' \sin\omega' t + \frac{R}{2L}\cos\omega' t\right] = 0$$

or

$$L\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right] + \frac{1}{C}\cos\omega' t - R\left[\omega' \sin\omega' t + \frac{R}{2L}\cos\omega' t\right] = 0$$

Rewrite this equation in the form $A\cos\omega' t + B\sin\omega' t = 0$:

$$(R\omega' - R\omega')\sin\omega' t + \left[L\left(\frac{R^2}{4L^2} - \omega'^2\right) + \frac{1}{C} - \frac{R^2}{2L}\right]\cos\omega' t = 0$$

or

$$\left[\left(\frac{R^2}{4L} - L\omega'^2 + \frac{1}{C} - \frac{R^2}{2L}\right)\right]\cos\omega' t = 0$$

If this equation is to hold for all values of t, its coefficient must vanish:

$$\frac{R^2}{4L} - L\omega'^2 + \frac{1}{C} - \frac{R^2}{2L} = 0$$

Solve for ω':

$$\omega' = \boxed{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}},$$

the condition that must be satisfied if $Q = Q_0 e^{-Rt/2L}\cos\omega' t$ is the solution to Equation 29-47*b*.

***102 •••**

Picture the Problem We can use $L = \mu_0 n^2 A\ell$ to determine the inductance of the empty solenoid and the resonance condition to find the capacitance of the sample-free circuit when the resonance frequency of the circuit is 6.0000 MHz. By expressing L as a function of f_0 and then evaluating df_0/dL and approximating the derivative with $\Delta f_0/\Delta L$, we can evaluate χ from its definition.

(*a*) Express the inductance of an air-core solenoid:

$$L = \mu_0 n^2 A\ell$$

Substitute numerical values and evaluate L:

$$L = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.04\,\text{m}}\right)^2\frac{\pi}{4}(0.003\,\text{m})^2(0.04\,\text{m}) = \boxed{35.5\,\mu\text{H}}$$

(*b*) Express the condition for resonance in the LC circuit:

$$X_L = X_C$$

or

$$2\pi f_0 L = \frac{1}{2\pi f_0 C} \qquad (1)$$

Solve for C to obtain:

$$C = \frac{1}{4\pi^2 f_0^2 L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{4\pi^2(6\,\text{MHz})(35.5\,\mu\text{H})} = \boxed{119\,\mu\text{F}}$$

(*c*) Express the sample's susceptibility in terms of L and ΔL:

$$\chi = \frac{\Delta L}{L} \qquad (2)$$

Solve equation (1) for f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Differentiate f_0 with respect to L:

$$\frac{df_0}{dL} = \frac{1}{2\pi\sqrt{C}}\frac{d}{dL}L^{-1/2} = -\frac{1}{4\pi\sqrt{C}}L^{-3/2}$$

$$= -\frac{1}{4\pi L\sqrt{LC}} = -\frac{f_0}{2L}$$

Approximate df_0/dL by $\Delta f_0/\Delta L$:

$$\frac{\Delta f_0}{\Delta L} = -\frac{f_0}{2L} \text{ or } \frac{\Delta f_0}{f_0} = -\frac{\Delta L}{2L}$$

Substitute in equation (2) to obtain:

$$\chi = -2\frac{\Delta f_0}{f_0}$$

Substitute numerical values and evaluate χ:

$$\chi = -2\left(\frac{5.9989\,\text{MHz} - 6.0000\,\text{MHz}}{6.0000\,\text{MHz}_0}\right) = \boxed{3.67\times10^{-4}}$$

103 •••

Picture the Problem We can find the angular frequency ω for the circuit in Problem 91 such that the magnitudes of the reactances of the two parallel branches are equal by equating the reactances in the two branches. We can use $P = \frac{1}{2}I^2R = (\mathcal{E}/Z)^2(R/2)$, where Z is, in turn, Z_L and Z_C, to find the power dissipated in each resistor.

(*a*) When the reactances of the parallel branches are equal:

$$X_L = X_C \text{ and } \omega = \frac{1}{\sqrt{LC}}$$

Substitute numerical values to obtain:

$$\omega = \frac{1}{\sqrt{(12\,\text{mH})(30\,\mu\text{F})}} = \boxed{1.67\,\text{krad/s}}$$

(*b*) Express the power dissipation in a resistor in an ac circuit:

$$P = \tfrac{1}{2}I^2R = \tfrac{1}{2}\left(\frac{\mathcal{E}}{Z}\right)^2 R$$

Find Z_L and $|Z_L|$ at 1.67 krad/s:

$$\begin{aligned} Z_L &= R_2 + i\omega L \\ &= 4\,\Omega + i(1.67\,\text{krad/s})(12\,\text{mH}) \\ &= 4\,\Omega + i(20.0\,\Omega) \end{aligned}$$

and

$$|Z_L| = \sqrt{(4\,\Omega)^2 + (20\,\Omega)^2} = 20.4\,\Omega$$

Find Z_C and $|Z_C|$ at 1.67 krad/s:

$$\begin{aligned} Z_C &= R_1 - i\frac{1}{\omega C} \\ &= 2\,\Omega - i\frac{1}{(1.67\,\text{krad/s})(30\,\mu\text{F})} \\ &= 2\,\Omega - i(20.0\,\Omega) \end{aligned}$$

and

$$|Z_C| = \sqrt{(2\,\Omega)^2 + (20.0\,\Omega)^2} = 20.1\,\Omega$$

Evaluate the power dissipated in R_1 and R_2:

$$P_1 = \frac{1}{2}\left(\frac{\mathcal{E}}{Z_C}\right)^2 R_1 = \frac{1}{2}\left(\frac{40\text{ V}}{20.1\,\Omega}\right)^2 (2\,\Omega)$$

$$= \boxed{3.96\text{ W}}$$

and

$$P_2 = \frac{1}{2}\left(\frac{\mathcal{E}}{Z_L}\right)^2 R_2 = \frac{1}{2}\left(\frac{40\text{ V}}{20.4\,\Omega}\right)^2 (4\,\Omega)$$

$$= \boxed{7.69\text{ W}}$$

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Picture the Problem We can equate the power dissipated in the two resistors to obtain a relationship between the currents in and the resistances of the two branches. Expressing the currents in terms of the impedances of the two branches and the common potential difference across them will lead us to an equation that is quadratic in ω^2 that we can solve for ω. In (*b*) we can use complex numbers to find the reactances of each of the two parallel branches and then use these results to draw the phasor diagram of (*c*). We can use the results of (*b*) to find the impedance of the circuit in (*d*).

(*a*) Express the condition under which the power dissipation in the two resistors is the same:

$$I_1^2 R_1 = I_2^2 R_2 \text{ or } \frac{I_1^2}{I_2^2} = \frac{R_2}{R_1}$$

Express the ratio of the squares of the currents in the two resistors:

$$\frac{I_1^2}{I_1^2} = \left(\frac{\dfrac{\mathcal{E}}{Z_C}}{\dfrac{\mathcal{E}}{Z_L}}\right)^2 = \frac{Z_L^2}{Z_C^2}$$

Equate these expressions to obtain:

$$\frac{R_2}{R_1} = \frac{Z_L^2}{Z_C^2} = \frac{R_2^2 + X_L^2}{R_1^2 + X_C^2}$$

or

$$\frac{R_2}{R_1}\left(R_1^2 + X_C^2\right) = R_2^2 + X_L^2$$

Substitute for X_L, X_C, and the ratio of R_2 to R_1 to obtain:

$$2\left(4\,\Omega^2 + \frac{1}{\omega^2 C^2}\right) = 16\,\Omega^2 + \omega^2 L^2$$

or

$$8\,\Omega^2 + \frac{2}{\omega^2 C^2} = 16\,\Omega^2 + \omega^2 L^2$$

Combine like terms and clear fractions to obtain:

$$L^2C^2\omega^4 + \left(8\,\Omega^2\right)C^2\omega^2 - 2 = 0$$

Substitute numerical values to obtain:

$$\left(1.30\times10^{-13}\ \text{s}^4\right)\omega^4 + \left(7.20\times10^{-9}\ \text{s}^2\right)\omega^2 - 2 = 0$$

Use the ″solver″ capability of your calculator to solve for ω^2:

$$\omega^2 = 3.89\times10^6\ (\text{rad/s})^2$$

and

$$\omega = \boxed{1.97\times10^3\ \text{rad/s}}$$

(*b*) Express and evaluate Z_C, $\left|Z_C\right|$, and δ_C:

$$Z_C = R_1 - i\frac{1}{\omega C} = 2\,\Omega - i\frac{1}{\left(1.97\times10^3\ \text{rad/s}\right)\left(30\ \mu\text{F}\right)} = 2\,\Omega - i\left(16.9\,\Omega\right)$$

$$\left|Z_C\right| = \sqrt{\left(2\,\Omega\right)^2 + \left(16.9\,\Omega\right)^2} = \boxed{17.0\,\Omega}$$

and

$$\delta_C = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{-16.9\,\Omega}{2\,\Omega}\right) = \boxed{-83.3^\circ}$$

Express and evaluate Z_L, $\left|Z_L\right|$, and δ_L:

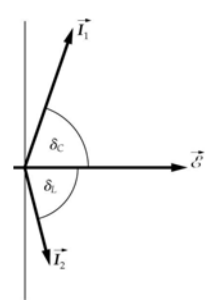
$$Z_L = R_2 + i\omega L = 4\,\Omega + i\left(1.97\times10^3\ \text{rad/s}\right)\left(12\ \text{mH}\right) = 4\,\Omega + i\left(23.6\,\Omega\right)$$

$$\left|Z_L\right| = \sqrt{\left(4\,\Omega\right)^2 + \left(23.6\,\Omega\right)^2} = \boxed{23.9\,\Omega}$$

and

$$\delta_L = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{23.9\,\Omega}{4\,\Omega}\right) = \boxed{80.5^\circ}$$

(*c*) The applied voltage and the currents in the two branches are shown on the phasor diagram to the right.



(*d*) Express the impedance of the circuit and simplify to obtain:

$$Z = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{[4\,\Omega + i(23.6\,\Omega)][2\,\Omega - i(16.9\,\Omega)]}{4\,\Omega + i(23.6\,\Omega) + 2\,\Omega - i(16.9\,\Omega)} = \frac{407\,\Omega^2 - i(20.4\,\Omega^2)}{6\,\Omega + i(6.70\,\Omega)}$$

Multiply Z by 1 in the form of the complex conjugate of $6\ \Omega + i(6.70\Omega)$ divided by itself and simplify to obtain:

$$Z = \left(\frac{407\,\Omega^2 - i(20.4\,\Omega^2)}{6\,\Omega + i(6.70\,\Omega)}\right) \times \left(\frac{6\,\Omega - i(6.70\,\Omega)}{6\,\Omega - i(6.70\,\Omega)}\right) = \frac{2.58\times10^3\,\Omega^3 - i(2.85\times10^3\,\Omega^3)}{80.9\,\Omega^2} = 31.9\,\Omega - i(35.2\,\Omega)$$

Find the magnitude of the circuit's impedance and the phase angle for the circuit:

$$|Z| = \sqrt{(31.9\,\Omega)^2 + (35.2\,\Omega)^2} = \boxed{47.5\,\Omega}$$

and

$$\delta = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{-35.2\,\Omega}{31.9\,\Omega}\right) = \boxed{-47.8^\circ}$$

The Transformer

***105** •

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can use $V_2N_1 = V_1N_2$ and $N_1I_1 = N_2I_2$ to find the turn ratio and the primary current when the transformer connections are reversed.

(*a*) Relate the number of primary and secondary turns to the primary and secondary voltages:

$$V_2N_1 = V_1N_2 \qquad (1)$$

Solve for and evaluate the ratio N_2/N_1:

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{24\,\text{V}}{120\,\text{V}} = \boxed{\frac{1}{5}}$$

(*b*) Relate the current in the primary to the current in the secondary and

$$I_1 = \frac{N_2}{N_1} I_2$$

to the turns ratio:

Express the current in the primary winding in terms of the voltage across it and its impedance:

$$I_2 = \frac{V_2}{Z_2}$$

Substitute to obtain:

$$I_1 = \frac{N_2}{N_1}\frac{V_2}{Z_2}$$

Substitute numerical values and evaluate I_1:

$$I_1 = \left(\frac{5}{1}\right)\left(\frac{120\,\text{V}}{12\,\Omega}\right) = \boxed{50.0\,\text{A}}$$

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Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can decide whether the transformer is a step-up or step-down transformer by examining the ratio of the number of turns in the secondary to the number of terms in the primary. We can relate the open-circuit voltage in the secondary to the primary voltage and the turns ratio.

(*a*) $\boxed{\text{Because there are fewer turns in the secondary than in the primary it is a step - down transformer.}}$

(*b*) Relate the open-circuit voltage V_2 in the secondary to the voltage V_1 in the primary:

$$V_2 = \frac{N_2}{N_1}V_1$$

Substitute numerical values and evaluate V_2:

$$V_2 = \frac{8}{400}(120\,\text{V}) = \boxed{2.40\,\text{V}}$$

(*c*) Because there are no losses:

$$V_1I_1 = V_2I_2$$

Solve for and evaluate I_2:

$$I_2 = \frac{V_1}{V_2}I_1 = \frac{120\,\text{V}}{2.40\,\text{V}}(0.1\text{A}) = \boxed{5.00\,\text{A}}$$

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Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can use $I_1V_1 = I_2V_2$ to find the current in the primary and $V_2N_1 = V_1N_2$ to find the number of turns in the secondary.

(*a*) Because we have 100 percent efficiency:

$$I_1V_1 = I_2V_2$$

Solve for and evaluate I_1:

$$I_1 = I_2 \frac{V_2}{V_1} = (20\,\text{A})\frac{9\,\text{V}}{120\,\text{V}} = \boxed{1.50\,\text{A}}$$

(*b*) Relate the number of primary and secondary turns to the primary and secondary voltages:

$$V_2 N_1 = V_1 N_2$$

Solve for the ratio N_2/N_1:

$$N_2 = \frac{V_2}{V_1} N_1$$

Substitute numerical values and evaluate N_2/N_1:

$$N_2 = \frac{9\,\text{V}}{120\,\text{V}}(250) = 18.8 \approx \boxed{19}$$

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Picture the Problem We can relate the input and output voltages to the number of turns in the primary and secondary using $V_2 N_1 = V_1 N_2$.

Relate the output voltages V_2 to the input voltage V_1 and the number of turns in the primary N_1 and secondary N_2:

$$V_2 = \frac{N_2}{N_1} V_1$$

Solve for N_2:

$$N_2 = N_1 \frac{V_2}{V_1}$$

Evaluate N_2 for V_2 = 2.5 V:

$$N_2 = (500)\left(\frac{2.5\,\text{V}}{120\,\text{V}}\right) = \boxed{10.4}$$

Evaluate N_2 for V_2 = 7.5 V:

$$N_2 = (500)\left(\frac{7.5\,\text{V}}{120\,\text{V}}\right) = \boxed{31.3}$$

Evaluate N_2 for V_2 = 9 V:

$$N_2 = (500)\left(\frac{9\,\text{V}}{120\,\text{V}}\right) = \boxed{37.5}$$

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Picture the Problem We can relate the input and output voltages to the number of turns in the primary and secondary using $V_2 N_1 = V_1 N_2$.

Relate the output voltages V_2 to the input voltage V_1 and the number of

$$V_2 = \frac{N_2}{N_1} V_1$$

turns in the primary N_1 and secondary N_2:

Solve for N_1:

$$N_1 = N_2 \frac{V_1}{V_2}$$

Substitute numerical values and evaluate N_1:

$$N_1 = (400)\left(\frac{2000\,\text{V}}{240\,\text{V}}\right) = \boxed{3333}$$

***110 ••**

Picture the Problem Note: In a simple circuit maximum power transfer from source to load requires that the load resistance equals the internal resistance of the source. We can use Ohm's law and the relationship between the primary and secondary currents and the primary and secondary voltages and the turns ratio of the transformer to derive an expression for the turns ratio as a function of the effective resistance of the circuit and the resistance of the speaker(s).

Express the effective loudspeaker resistance at the primary of the transformer:

$$R_{\text{eff}} = \frac{V_1}{I_1}$$

Relate V_1 to V_2, N_1, and N_2:

$$V_1 = V_2 \frac{N_1}{N_2}$$

Express I_1 in terms of I_2, N_1, and N_2:

$$I_1 = I_2 \frac{N_2}{N_1}$$

Substitute to obtain:

$$R_{\text{eff}} = \frac{V_2 \dfrac{N_1}{N_2}}{I_2 \dfrac{N_2}{N_1}} = \left(\frac{V_2}{I_2}\right)\left(\frac{N_1}{N_2}\right)^2$$

Solve for N_1/N_2:

$$\frac{N_1}{N_2} = \sqrt{\frac{I_2 R_{\text{eff}}}{V_2}} = \sqrt{\frac{R_{\text{eff}}}{R_2}} \qquad (1)$$

Evaluate N_1/N_2 for $R_{\text{eff}} = R_{\text{int}}$:

$$\frac{N_1}{N_2} = \sqrt{\frac{2000\,\Omega}{8\,\Omega}} = \boxed{15.8}$$

Express the power delivered to the two speakers connected in parallel:

$$P_{\text{sp}} = I_1^2 R_{\text{eff}} \qquad (2)$$

Find the equivalent resistance R_{sp} of the two 8-Ω speakers in parallel:

$$\frac{1}{R_{sp}} = \frac{1}{8\,\Omega} + \frac{1}{8\,\Omega} = \frac{2}{8\,\Omega} = \frac{1}{4\,\Omega}$$

and

$$R_{sp} = 4\,\Omega$$

Solve equation (1) for R_{eff} to obtain:

$$R_{eff} = R_2\left(\frac{N_1}{N_2}\right)^2$$

Substitute numerical values and evaluate R_{eff}:

$$R_{eff} = (4\,\Omega)(15.8)^2 = 999\,\Omega$$

Find the current drawn from the source:

$$I_1 = \frac{V}{R_{tot}} = \frac{12\,\text{V}}{2000\,\Omega + 999\,\Omega} = 4.00\,\text{mA}$$

Substitute numerical values in equation (2) and evaluate the power delivered to the parallel speakers:

$$P_{sp} = (4\,\text{mA})^2(999\,\Omega) = \boxed{16.0\,\text{mW}}$$

111 ••

Picture the Problem We can substitute $I_2 = V_2/Z$ in Equation 29-62 to show that $I_1 = \varepsilon/[(N_1/N_2)^2Z]$ and then use this result in $Z_{eff} = \varepsilon/I_1$ to show that $Z_{eff} = (N_1/N_2)^2Z$.

From Equation 29-62 we have:

$$I_1 = \frac{N_2}{N_1}I_2$$

or, because $I_2 = V_2/Z$,

$$I_1 = \frac{N_2}{N_1}\frac{V_2}{Z}$$

From Equation 29-61 we have:

$$V_2 = \frac{N_2}{N_1}V_1 = \frac{N_2}{N_1}\varepsilon$$

Substitute to obtain:

$$I_1 = \frac{N_2}{N_1}\frac{\frac{N_2}{N_1}\varepsilon}{Z} = \left(\frac{N_2}{N_1}\right)^2\frac{\varepsilon}{Z}$$

$$= \boxed{\frac{\varepsilon}{(N_1/N_2)^2 Z}}$$

Express the effective impedance Z_{eff} of the speaker in terms of ε and I_1:

$$Z_{eff} = \frac{\varepsilon}{I_1}$$

Substitute for I_1 to obtain:

$$Z_{\text{eff}} = \frac{\mathcal{E}}{\dfrac{\mathcal{E}}{(N_1/N_2)^2 Z}} = \boxed{(N_1/N_2)^2 Z}$$

General Problems

112 •

Picture the Problem We can use $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}$ to find the rms current and $I_{\max} = \sqrt{2} I_{\text{rms}}$ to find the maximum current drawn by the dryer.

(*a*) Express the average power delivered by the source in terms of $\mathcal{E}_{\text{rms}}$ and I_{rms}:

$$P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}$$

Solve for and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}} = \frac{5\,\text{kW}}{240\,\text{V}} = \boxed{20.8\,\text{A}}$$

(*b*) Relate the maximum current $I_{\max}$ to the rms current I_{rms}:

$$I_{\max} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(20.8\,\text{A}) = \boxed{29.5\,\text{A}}$$

(*c*) Proceed as in (*a*) and (*b*) to obtain:

$$I_{\text{rms}} = \boxed{41.6\,\text{A}} \text{ and } I_{\max} = \boxed{59.0\,\text{A}}$$

113 •

Picture the Problem We can use its definition to find the reactance of the capacitor at the given frequencies.

Express the reactance of a capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

(*a*) Evaluate X_C at f = 60 Hz:

$$X_C = \frac{1}{2\pi(60\,\text{Hz})(10\,\mu\text{F})} = \boxed{265\,\Omega}$$

(*b*) Evaluate X_C at f = 6 kHz:

$$X_C = \frac{1}{2\pi(6\,\text{kHz})(10\,\mu\text{F})} = \boxed{2.65\,\Omega}$$

(*a*) Evaluate X_C at f = 6 MHz:

$$X_C = \frac{1}{2\pi(6\,\text{MHz})(10\,\mu\text{F})} = \boxed{2.65\,\text{m}\Omega}$$

114 ••

Picture the Problem We can use its definition, $I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$ to relate the rms current to the current carried by the resistor and find $\left(I^2\right)_{\text{av}}$ by integrating I^2.

(*a*) Express the rms current in terms of the $\left(I^2\right)_{\text{av}}$:

$$I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$$

Evaluate I^2:

$$\begin{aligned} I^2 &= \left[(5\,\text{A})\sin 120\pi t + (7\,\text{A})\sin 240\pi t\right]^2 \\ &= \left(25\,\text{A}^2\right)\sin^2 120\pi t + \left(70\,\text{A}^2\right)\sin 120\pi t \sin 240\pi t + \left(49\,\text{A}^2\right)\sin^2 240\pi t \end{aligned}$$

Find $\left(I^2\right)_{\text{av}}$ by integrating I^2 from $t = 0$ to $t = T = 2\pi/\omega$ and dividing by T:

$$\begin{aligned} \left(I^2\right)_{\text{av}} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \Big\{ &\left(25\,\text{A}^2\right)\sin^2 120\pi t + \left(70\,\text{A}^2\right)\sin 120\pi t \sin 240\pi t \\ &+ \left(49\,\text{A}^2\right)\sin^2 240\pi t \Big\} dt \end{aligned}$$

Use the trigonometric identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ to simplify and evaluate the 1st and 3rd integrals and recognize that the middle term is of the form $\sin x \sin 2x$ to obtain:

$$\left(I^2\right)_{\text{av}} = 12.5\,\text{A}^2 + 0 + 24.5\,\text{A}^2 = 37.0\,\text{A}^2$$

Substitute to obtain:

$$I_{\text{rms}} = \sqrt{37.0\,\text{A}^2} = \boxed{6.08\,\text{A}}$$

(*b*) Relate the power dissipated in the resistor to its resistance and the rms current in it:

$$P = I_{\text{rms}}^2 R$$

Substitute numerical values and evaluate P:

$$P = (6.08\,\text{A})^2 (12\,\Omega) = \boxed{444\,\text{W}}$$

(*c*) Express the rms voltage across the resistor in terms of R and I_{rms}:

$$V_{\text{rms}} = I_{\text{rms}} R$$

Substitute numerical values and evaluate V_{rms}:

$$V_{\text{rms}} = (6.08\,\text{A})(12\,\Omega) = \boxed{73.0\,\text{V}}$$

***115 ••**

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average of the voltage squared, $\left(V^2\right)_{\text{av}}$ and then use the definition of the rms voltage.

(*a*) From the definition of V_{rms} we have:

$$V_{\text{rms}} = \sqrt{\left(V_0^2\right)_{\text{av}}}$$

Noting that $-V_0^2 = V_0^2$, evaluate V_{rms}:

$$V_{\text{rms}} = \sqrt{V_0^2} = V_0 = \boxed{12.0\,\text{V}}$$

(*b*) Noting that the voltage during the second half of each cycle is now zero, express the voltage during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$V = V_0$$

Express the square of the voltage during this half cycle:

$$V^2 = V_0^2$$

Calculate $\left(V^2\right)_{\text{av}}$ by integrating V^2 from t = 0 to t = $\frac{1}{2}\Delta T$ and dividing by ΔT:

$$\left(V^2\right)_{\text{av}} = \frac{V_0^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{V_0^2}{\Delta T}\left[t\right]_0^{\frac{1}{2}\Delta T} = \tfrac{1}{2}V_0^2$$

Substitute to obtain:

$$V_{\text{rms}} = \sqrt{\tfrac{1}{2}V_0^2} = \frac{V_0}{\sqrt{2}} = \frac{12\,\text{V}}{\sqrt{2}} = \boxed{8.49\,\text{V}}$$

116 ••

Picture the Problem We can use the definitions of I_{rms} and V_{rms} to find the rms value of the waveform and the average power delivered by the pulse generator.

(*a*) From the definition of I_{rms} we have:

$$I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$$

Evaluate $\left(I^2\right)_{\text{av}}$ over 1 s:

$$\left(I^2\right)_{\text{av}} = \frac{(0.1\,\text{s})I^2 + (0.9\,\text{s})(0)}{1\,\text{s}} = \frac{(0.1\,\text{s})I^2}{1\,\text{s}} = (0.1)(15\,\text{A})^2 = 22.5\,\text{A}^2$$

Substitute to obtain: $I_{\text{rms}} = \sqrt{22.5\,\text{A}^2} = \boxed{4.74\,\text{A}}$

(*b*) Express the average power delivered by the pulse generator in terms of I_{rms} and V_{rms}:

$$P_{\text{av}} = I_{\text{rms}}V_{\text{rms}}$$

From the definition of V_{rms} we have:

$$V_{\text{rms}} = \sqrt{\left(V^2\right)_{\text{av}}}$$

Evaluate $\left(V^2\right)_{\text{av}}$ over 1 s:

$$\begin{aligned}\left(V^2\right)_{\text{av}} &= \frac{(0.1\,\text{s})V^2 + (0.9\,\text{s})(0)}{1\,\text{s}} \\ &= \frac{(0.1\,\text{s})V^2}{1\,\text{s}} = (0.1)(100\,\text{V})^2 \\ &= 1000\,\text{V}^2\end{aligned}$$

Evaluate V_{rms}:

$$V_{\text{rms}} = \sqrt{1000\,\text{V}^2} = 31.6\,\text{V}$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = (4.74\,\text{A})(31.6\,\text{V}) = \boxed{150\,\text{W}}$$

117 ••

Picture the Problem We can use the definition of capacitance to find the charge on each capacitor and the definition of current to find the steady-state current in the circuit. We can find the maximum and minimum energy stored in the capacitors using $U = \frac{1}{2}C_{\text{eq}}V^2$, where V is either the maximum or the minimum potential difference across the capacitors.

(*a*) Use the definition of capacitance to express the charge on each capacitor:

$Q_1 = C_1V_1$ and $Q_2 = C_2V_2$

or, because the capacitors are in parallel,

$Q_1 = C_1V$ and $Q_2 = C_2V$

where $V = \mathcal{E} + 24\,\text{V}$

Substitute to obtain:

$$\begin{aligned}Q_1 &= C_1(\mathcal{E} + 24\,\text{V}) \\ &= (3\,\mu\text{F})[(20\,\text{V})\cos(120\pi t) + 24\,\text{V}] \\ &= \boxed{(60\,\mu\text{F})\cos(120\pi t) + 72\,\mu\text{F}}\end{aligned}$$

and

$$\begin{aligned}Q_2 &= C_2(\mathcal{E} + 24\,\text{V}) \\ &= (1.5\,\mu\text{F})[(20\,\text{V})\cos(120\pi t) + 24\,\text{V}] \\ &= \boxed{(30\,\mu\text{F})\cos(120\pi t) + 36\,\mu\text{F}}\end{aligned}$$

(*b*) Express the steady-state current as the rate at which charge is being delivered to the capacitors:

$$I = \frac{dQ}{dt} = \frac{d}{dt}(Q_1 + Q_2)$$

Substitute for Q_1 and Q_2 and evaluate I:

$$\begin{aligned} I &= \frac{d}{dt}\left[(60\ \mu\text{F})\cos(120\pi t) + 72\ \mu\text{F} \right. \\ &\quad \left. + (30\ \mu\text{F})\cos(120\pi t) + 36\ \mu\text{F}\right] \\ &= -120\pi(60\ \mu\text{F})\sin(120\pi t) \\ &\quad -120\pi(30\ \mu\text{F})\sin(120\pi t) \\ &= \boxed{-(33.9\ \text{mA})\sin(120\pi t)} \end{aligned}$$

(*c*) Express U_{max} in terms of the maximum potential difference across the capacitors:

$$U_{max} = \tfrac{1}{2}C_{eq}V_{max}^2$$

Because V_{max} = 44 V and $C_{eq} = C_1 + C_2$ = 3 μF + 1.5 μF = 4.5 μF:

$$U_{max} = \tfrac{1}{2}(4.5\ \mu\text{F})(44\ \text{V})^2 = \boxed{4.36\ \text{mJ}}$$

(*d*) Express U_{min} in terms of the minimum potential difference across the capacitors:

$$U_{min} = \tfrac{1}{2}C_{eq}V_{min}^2$$

The minimum energy stored in the capacitors occurs when $V_{min} = 24\ \text{V} - \varepsilon_{max} = 4\ \text{V}$:

$$U_{min} = \tfrac{1}{2}(4.5\ \mu\text{F})(4\ \text{V})^2 = \boxed{36.0\ \mu\text{J}}$$

118 ••

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average current I_{av}, and the average of the current squared, $(I^2)_{av}$

From the definition of I_{av} and I_{rms} we have:

$$I_{av} = \frac{1}{\Delta T}\int_0^{\Delta T} I dt \text{ and } I_{rms} = \sqrt{(I^2)_{av}}$$

(*a*) Express the current during the first half cycle of time interval ΔT:

$$I = \frac{4}{\Delta T}t$$

where I is in A when t and T are in seconds.

Evaluate I_{av}:

$$I_{av} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{4}{\Delta T}tdt = \frac{4}{(\Delta T)^2}\int_0^{\Delta T}tdt$$

$$= \frac{4}{(\Delta T)^2}\left[\frac{t^2}{2}\right]_0^{\Delta T} = \boxed{2.00\text{ A}}$$

Express the square of the current during this half cycle:

$$I^2 = \frac{16}{(\Delta T)^2}t^2$$

Noting that the average value of the squared current is the same for each time interval ΔT, calculate $\left(I^2\right)_{av}$ by integrating I^2 from $t = 0$ to $t = \Delta T$ and dividing by ΔT:

$$\left(I^2\right)_{av} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{16}{(\Delta T)^2}t^2dt$$

$$= \frac{16}{(\Delta T)^3}\left[\frac{t^3}{3}\right]_0^{\Delta T} = \frac{16}{3}$$

Substitute in the expression for I_{rms} to obtain:

$$I_{rms} = \sqrt{\frac{16}{3}\text{A}^2} = \boxed{2.31\text{A}}$$

(*b*) Noting that the current during the second half of each cycle is zero, express the current during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$I = 4\text{A}$$

Evaluate I_{av}:

$$I_{av} = \frac{4\text{A}}{\Delta T}\int_0^{\frac{1}{2}\Delta T}dt = \frac{4\text{A}}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = \boxed{2.00\text{A}}$$

Express the square of the current during this half cycle:

$$I^2 = 16\text{A}^2$$

Calculate $\left(I^2\right)_{av}$ by integrating I^2 from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by ΔT:

$$\left(I^2\right)_{av} = \frac{16\text{A}^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T}dt$$

$$= \frac{16\text{A}^2}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = 8\text{A}^2$$

Substitute in the expression for I_{rms} to obtain:

$$I_{rms} = \sqrt{8\text{A}^2} = \boxed{2.83\text{A}}$$

119 ••

Picture the Problem We can apply Kirchhoff's loop rule to express the current in the circuit in terms of the emfs of the sources and the resistance of the resistor. We can then find I_{max} and I_{min} by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find average of the current squared, $\left(I^2\right)_{av}$ and then I_{rms}.

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_1 + \mathcal{E}_2 - IR = 0$$

Solve for I:

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R}$$

Substitute numerical values to obtain:

$$I = \frac{(20\,\text{V})\cos\left(2\pi\left(180\,\text{s}^{-1}\right)t\right) + 18\,\text{V}}{36\,\Omega}$$

$$= 0.5\,\text{A} + (0.556\,\text{A})\cos\left(1131\text{s}^{-1}\right)t$$

Express the condition that must be satisfied if the current is to be a maximum:

$$\cos\left(1131\text{s}^{-1}\right)t = 1$$

Evaluate I_{max}:

$$I_{max} = 0.5\,\text{A} + 0.556\,\text{A} = \boxed{1.06\,\text{A}}$$

Express the condition that must be satisfied if the current is to be a minimum:

$$\cos\left(1131\text{s}^{-1}\right)t = -1$$

Evaluate I_{min}:

$$I_{min} = 0.5\,\text{A} - 0.556\,\text{A} = \boxed{-0.0560\,\text{A}}$$

Because the average value of $\cos\omega t = 0$:

$$I_{av} = \boxed{0.500\,\text{A}}$$

Express and evaluate the average current delivered by the source whose emf is $\mathcal{E}_2$:

$$I_2 = \frac{\mathcal{E}_2}{R} = \frac{18\,\text{V}}{36\,\Omega} = 0.5\,\text{A}$$

Because $I_1 = (0.556\,\text{A})\cos\left(1131\text{s}^{-1}\right)t$:

$$\left(I_1^2\right)_{av} = \frac{1}{5.56\,\text{ms}}\int_0^{5.56\,\text{ms}} (0.556\,\text{A})^2 \cos^2\left(1131\text{s}^{-1}\right)t\,dt$$

Use the trigonometric identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ to obtain:

$$\left(I_1^2\right)_{av} = \frac{0.309\,\text{A}^2}{2(5.56\,\text{ms})}\int_0^{5.56\,\text{ms}}\left(1+\cos 2\left(1131\,\text{s}^{-1}\right)t\right)dt$$
$$= \left(27.8\,\text{A}^2/\text{s}\right)\left[t + \frac{1}{2262\,\text{s}^{-1}}\sin\left(2262\,\text{s}^{-1}\right)t\right]_0^{5.56\,\text{ms}}$$

Evaluate $\left(I_1^2\right)_{av}$:

$$\left(I_1^2\right)_{av} = \left(27.8\,\text{A}^2/\text{s}\right)\left[5.56\,\text{ms} + \frac{1}{2262\,\text{s}^{-1}}\sin\left(2262\,\text{s}^{-1}\right)(5.56\,\text{ms})\right] = 0.1543\,\text{A}^2$$

Express $\left(I^2\right)_{av}$:

$$\left(I^2\right)_{av} = \left(I_1^2\right)_{av} + \left(I_2^2\right)_{av} = 0.1543\,\text{A}^2 + (0.5\,\text{A})^2 = 0.4043\,\text{A}^2$$

Evaluate I_{rms}:

$$I_{rms} = \sqrt{\left(I^2\right)_{av}} = \sqrt{0.4043\,\text{A}^2} = \boxed{0.636\,\text{A}}$$

***120** ••

Picture the Problem We can apply Kirchhoff's loop rule to obtain an expression for charge on the capacitor as a function of time. Differentiating this expression with respect to time will give us the current in the circuit. We can then find I_{max} and I_{min} by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. We can use the maximum value of the current to find I_{rms}.

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_1 + \mathcal{E}_2 - \frac{q(t)}{C} = 0$$

Substitute numerical values and solve for $q(t)$:

$$q(t) = (2\,\mu\text{F})(20\,\text{V})\cos\left(1131\,\text{s}^{-1}\right)t + (2\,\mu\text{F})(18\,\text{V}) = (40\,\mu\text{C})\cos\left(1131\,\text{s}^{-1}\right)t + 36\,\mu\text{C}$$

Differentiate this expression with respect to t to obtain the current as a function of time:

$$I = \frac{dq}{dt} = \frac{d}{dt}\left[(40\,\mu\text{C})\cos\left(1131\,\text{s}^{-1}\right)t + 36\,\mu\text{C}\right] = \boxed{-(45.2\,\text{mA})\sin\left(1131\,\text{s}^{-1}\right)t}$$

Express the condition that must be

$$\sin\left(1131\,\text{s}^{-1}\right)t = 1$$

satisfied if the current is to be a minimum:

and

$$I_{\min} = \boxed{-45.2\,\text{mA}}$$

Express the condition that must be satisfied if the current is to be a maximum:

$$\sin\left(1131\,\text{s}^{-1}\right)t = -1$$

and

$$I_{\max} = \boxed{45.2\,\text{mA}}$$

Because the dc source sees the capacitor as an open circuit and the average value of the sine function over a period is zero:

$$I_{\text{av}} = \boxed{0}$$

Because the peak current is 45.2 mA:

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{45.2\,\text{mA}}{\sqrt{2}} = \boxed{32.0\,\text{mA}}$$

21 ••

Picture the Problem The inductance acts as a short circuit to the constant voltage source. The current is infinite at all times. Consequently, $I_{\max} = I_{\text{rms}} = \infty$; there is no minimum current.

Chapter 30
Maxwell's Equations and Electromagnetic Waves

Conceptual Problems

***1 •**

(*a*) False. Maxwell's equations apply to both time-independent and time-dependent fields.

(*b*) True

(*c*) True

(*d*) True

(*e*) False. The magnitudes of the electric and magnetic field vectors are related according to $E = cB$.

(*f*) True

2 ••

Determine the Concept Two changes would be required. Gauss's law for magnetism would become $\oint_S B_n dA = \mu_0 q_m$ and Faraday's law would become $\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S B_n dA - \frac{I_m}{\epsilon_0}$, where I_m is the current associated with the motion of the magnetic poles.

3 •

Determine the Concept X rays have greater frequencies whereas light waves have longer wavelengths (see Table 30-1).

***4 •**

Determine the Concept The frequencies of ultraviolet radiation are greater than those of infrared radiation (see Table 30-1).

5 •

Determine the Concept Consulting Table 30-1 we see that FM radio and televisions waves have wavelengths of the order of a few meters.

6 •

Determine the Concept The dipole antenna detects the *electric* field, the loop antenna detects the *magnetic* field of the wave.

7 •

Determine the Concept The dipole antenna should be in the horizontal plane and normal to the line from the transmitter to the receiver.

***8** •

Determine the Concept A red plastic filter absorbs all the light incident on it except for the red light and a green plastic filter absorbs all the light incident on it except for the green light. If the red beam is incident on a red filter it will pass through, whereas, if it is incident on the green filter it will be absorbed. Because the green filter absorbs more energy than does the red filter, the laser beam will exert a greater force on the green filter.

Estimation and Approximation

9 ••

Picture the Problem We'll assume that the plastic bead has the same density as water. Applying a condition for translational equilibrium to the bead will allow us to relate the gravitational force acting on it to the force exerted by the laser beam. Because the force exerted by the laser beam is related to the radiation pressure and the radiation pressure to the intensity of the beam, we'll be able to find the beam's intensity. Knowing the beam's intensity, we find the total power needed to lift the bead.

Apply $\sum F_y = 0$ to the bead:

$$F_{\text{by laser beam}} - mg = 0$$

Relate the force exerted by the laser beam to the radiation pressure exerted by the beam:

$$F_{\text{by laser beam}} = P_{\text{r}}A = \frac{1}{4}\pi d^2 P_{\text{r}}$$

Substitute to obtain:

$$\frac{1}{4}\pi d^2 P_{\text{r}} - mg = 0$$

The radiation pressure P_{r} is the quotient of the intensity I and the speed of light c:

$$P_{\text{r}} = \frac{I}{c}$$

Substitute for P_{r} to obtain:

$$\frac{1}{4}\frac{\pi d^2 I}{c} - mg = 0 \quad (1)$$

Express the mass of the bead:

$$m = \rho V = \frac{1}{6}\pi\rho d^3$$

Substitute for m in equation (1) to obtain:

$$\frac{1}{4}\frac{\pi d^2 I}{c} - \frac{1}{6}\pi\rho d^3 g = 0$$

Solve for *I*:

$$I = \frac{2}{3}c\rho\, dg$$

Substitute numerical values and evaluate *I*:

$$I = \frac{2}{3}\left(3\times10^{8}\text{ m/s}\right)\left(10^{3}\text{ kg/m}^{3}\right)\left(15\,\mu m\right)\left(9.81\text{m/s}^{2}\right) = \boxed{2.94\times10^{7}\text{ W/m}^{2}}$$

The power needed is the product of the beam intensity and the cross-sectional area of the bead:

$$P = IA_{\text{bead}} = \frac{1}{4}\pi\, d^{2} I$$

Substitute numerical values and evaluate *P*:

$$P = \frac{1}{4}\pi\left(15\,\mu\text{m}\right)^{2}\left(2.94\times10^{7}\text{ W/m}^{2}\right) = \boxed{5.20\,\text{mW}}$$

10 •••

Picture the Problem The net force acting on the spacecraft is the difference between the repulsive force due to radiation pressure and the attractive gravitational force. We can apply Newton's 2nd law to the spacecraft and solve the resulting equation for the acceleration of the spacecraft. Because the acceleration turns out to be a function of *r*, we'll need to integrate *a* to find v^2. We'll assume that the sail absorbs all of the radiation incident on it.

Apply Newton's 2nd law to the spacecraft (including sail) to obtain:

$$F_{\text{r}} - F_{\text{g}} = ma$$

Solve for *a*:

$$a = \frac{F_{\text{r}} - F_{\text{g}}}{m}$$

Assuming that the sail absorbs all of the incident solar radiation:

$$F_{\text{r}} = P_{\text{r}}A = \frac{IA}{c}$$

where *A* is the area of the sail.

Because $I = \dfrac{P_{\text{s}}}{4\pi r^{2}}$:

$$F_{\text{r}} = \frac{P_{\text{s}}A}{4\pi\, r^{2}c}$$

Substitute for F_r and F_g to obtain:

$$a=\frac{\frac{P_sA}{4\pi r^2c}-\frac{GM_sm}{r^2}}{m}=\frac{P_sA}{4\pi r^2mc}-\frac{GM_s}{r^2}$$

$$=\frac{\frac{P_sA}{4\pi c}-GM_sm}{mr^2}$$

Neglecting the gravitational term:

$$a=\boxed{\frac{P_sA}{4\pi r^2mc}}$$

(*b*) Because *a* is a function of *r*, the velocity must be found by integration. Note that:

$$a=\frac{dv}{dt}=\frac{dv}{dr}\frac{dr}{dt}=v\frac{dv}{dr}\Rightarrow vdv=adr$$

Substitute for *a* and integrate v' from v_0 to v and r' from r_0 to r:

$$\int_{v_0}^{v}v'dv'=\tfrac{1}{2}\left(v^2-v_0^2\right)=\left(\frac{\frac{P_sA}{4\pi c}-GM_sm}{m}\right)\int_{r_0}^{r}\frac{dr'}{r'^2}=\left(\frac{\frac{P_sA}{4\pi c}-GM_sm}{m}\right)\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

Solve for v^2 to obtain:

$$v^2=v_0^2+2\left(\frac{\frac{P_sA}{4\pi c}-GM_sm}{m}\right)\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

Ignore the gravitational term to obtain:

$$v^2=\boxed{v_0^2+\left(\frac{P_sA}{2\pi mc}\right)\left(\frac{1}{r_0}-\frac{1}{r}\right)}$$

(*c*) This scheme is not likely to work effectively. For any reasonable mass, the surface mass density of the sail would have to be extremely small and the sail would have to be huge. Additionally, unless struts are built into the sail, it would collapse during the acceleration of the spacecraft.

11 ••

Picture the Problem We can use $I = E_{rms}B_{rms}/\mu_0$ and $B_{rms} = E_{rms}/c$ to express E_{rms} in terms of I. We can then use $B_{rms} = E_{rms}/c$ to find B_{rms}. The average power output of the sun is given by $P_{av} = 4\pi R^2 I$ where R is the earth-sun distance. The intensity and the radiation pressure at the surface of the sun can be found from the definitions of these physical quantities.

(*a*) Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{c\mu_0 I}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{(3\times10^8\text{ m/s})(4\pi\times10^{-7}\text{ N/A}^2)(1.37\text{ kW/m}^2)} = \boxed{719\text{ V/m}}$$

Use $B_{rms} = E_{rms}/c$ to evaluate B_{rms}:

$$B_{rms} = \frac{719\text{ V/m}}{3\times10^8\text{ m/s}} = \boxed{2.40\ \mu\text{T}}$$

(*b*) Express the average power output of the sun in terms of the solar constant:

$$P_{av} = 4\pi R^2 I$$

where R is the earth-sun distance.

Substitute numerical values and evaluate P_{av}:

$$P_{av} = 4\pi(1.5\times10^{11}\text{ m})^2(1.37\text{ kW/m}^2) = \boxed{3.87\times10^{26}\text{ W}}$$

(*c*) Express the intensity at the surface of the sun in terms of the sun's average power output and radius r:

$$I = \frac{P_{av}}{4\pi r^2}$$

Substitute numerical values and evaluate I at the surface of the sun:

$$I = \frac{3.87\times10^{26}\text{ W}}{4\pi(6.96\times10^8\text{ m})^2} = \boxed{6.36\times10^7\text{ W/m}^2}$$

Express the radiation pressure in terms of the intensity:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{6.36\times10^7\text{ W/m}^2}{3\times10^8\text{ m/s}} = \boxed{0.212\text{ Pa}}$$

***12 ••**

Picture the Problem We can find the radiation pressure force from the definition of pressure and the relationship between the radiation pressure and the intensity of the radiation from the sun. We can use Newton's law of gravitation to find the gravitational force the sun exerts on the earth.

The radiation pressure exerted on the earth is given by:

$$P_r = \frac{F_r}{A} \Rightarrow F_r = P_r A$$

where A is the cross-sectional area of the earth.

Express the radiation pressure in terms of the intensity of the radiation I from the sun:

$$P_r = \frac{I}{c}$$

Substituting for P_r and A yields:

$$F_r = \frac{I\pi R^2}{c}$$

Substitute numerical values and evaluate F_r:

$$F_r = \frac{\pi(1370\ \text{W/m}^2)(6370\ \text{km})^2}{3\times10^8\ \text{m/s}} = \boxed{5.82\times10^8\ \text{N}}$$

The gravitational force exerted on the earth by the sun is given by:

$$F = \frac{Gm_{\text{sun}}m_{\text{earth}}}{r^2}$$

where r is the radius of the earth's orbit.

Substitute numerical values and evaluate F:

$$F = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{(1.5\times10^{11}\ \text{m})^2} = 3.53\times10^{22}\ \text{N}$$

Express the ratio of the force due radiation pressure F_r to the gravitational force F:

$$\frac{F_r}{F} = \frac{5.82\times10^8\ \text{N}}{3.53\times10^{22}\ \text{N}} = 1.65\times10^{-14}$$

$$\boxed{\text{The gravitational force is greater by a factor of approximately } 10^{14}.}$$

***13 ••**

Picture the Problem We can find the radiation pressure force from the definition of pressure and the relationship between the radiation pressure and the intensity of the radiation from the sun. We can use Newton's law of gravitation to find the gravitational force the sun exerts on Mars.

The radiation pressure exerted on Mars is given by:

$$P_r = \frac{F_r}{A} \Rightarrow F_r = P_r A$$

where A is the cross-sectional area of Mars.

Express the radiation pressure on Mars in terms of the intensity of the radiation I_{Mars} from the sun:

$$P_r = \frac{I_{Mars}}{c}$$

Substituting for P_r and A yields:

$$F_r = \frac{I_{Mars}\pi R_{Mars}^2}{c}$$

Express the ratio of the solar constant at the earth I_{earth} to the solar constant I_{Mars} at Mars:

$$\frac{I_{Mars}}{I_{earth}} = \left(\frac{r_{earth}}{r_{Mars}}\right)^2 \Rightarrow I_{Mars} = I_{earth}\left(\frac{r_{earth}}{r_{Mars}}\right)^2$$

Substitute for I_{Mars} to obtain:

$$F_r = \frac{I_{earth}\pi R_{Mars}^2}{c}\left(\frac{r_{earth}}{r_{Mars}}\right)^2$$

Substitute numerical values and evaluate F_r:

$$F_r = \frac{\pi\left(1370\ \text{W/m}^2\right)\left(3395\ \text{km}\right)^2}{3\times10^8\ \text{m/s}}\left(\frac{1.50\times10^{11}\ \text{m}}{2.29\times10^{11}\ \text{m}}\right)^2 = \boxed{7.09\times10^7\ \text{N}}$$

The gravitational force exerted on Mars by the sun is given by:

$$F = \frac{Gm_{sun}m_{Mars}}{r^2} = \frac{Gm_{sun}\left(0.11m_{earth}\right)}{r^2}$$

where r is the radius of Mars' orbit.

Substitute numerical values and evaluate F:

$$F = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)(0.11)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(2.29\times10^{11}\ \text{m}\right)^2} = 1.66\times10^{21}\ \text{N}$$

Express the ratio of the force due radiation pressure F_r to the gravitational force F:

$$\frac{F_r}{F} = \frac{7.09\times10^7\ \text{N}}{1.66\times10^{21}\ \text{N}} = 4.27\times10^{-14}$$

Because the ratio of these forces is 1.65×10^{-14} for the earth and 4.27×10^{-14} for Mars, Mars has the larger ratio. The reason that the ratio is higher for Mars is that the dependence of the radiation pressure on the distance from the Sun is the same for both forces (r^{-2}), whereas the dependence on the radii of the planets is different. Radiation pressure varies as R^2, whereas the gravitational force varies as R^3 (assuming that the two planets have the same density, an assumption that is nearly true). Consequently, the ratio of the forces goes as $R^2/R^3 = R^{-1}$. Because Mars is smaller than earth, the ratio is larger.

***14 ••**

Picture the Problem We can use Newton's 2nd law to express the acceleration of an atom in terms of the net force acting on the atom and the relationship between radiation pressure and the intensity of the beam to find the net force. Once we know the acceleration of an atom, we can use the definition of acceleration to find the stopping time for a rubidium atom at room temperature.

(*a*) Apply $\sum F = ma$ to the atom to obtain:

$$F = ma$$

where *F* is the force exerted by the laser beam.

The radiation pressure P_r and intensity of the beam *I* are related according to:

$$P_r = \frac{F}{A} = \frac{I}{c}$$

Solve for *F* to obtain:

$$F = \frac{IA}{c} = \frac{I\lambda^2}{c}$$

Substitute for *F* in the expression of Newton's 2nd law to obtain:

$$\frac{I\lambda^2}{c} = ma$$

Solve for *a*:

$$a = \frac{I\lambda^2}{mc}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{(10\,\text{W/m}^2)(780\,\text{nm})^2}{\left(85\frac{\text{g}}{\text{mol}} \times \frac{1\,\text{mol}}{6.02\times10^{23}\,\text{particles}}\right)(3\times10^8\,\text{m/s})} = \boxed{1.44\times10^5\,\text{m/s}^2}$$

(*b*) Using the definition of acceleration, express the stopping time Δt of the atom:

$$\Delta t = \frac{v_{\text{final}} - v_{\text{initial}}}{a}$$

Because $v_{\text{final}} \approx 0$:

$$\Delta t \approx \frac{-v_{\text{initial}}}{a}$$

Using the rms speed as the initial speed of an atom, relate v_{initial} to the temperature of the gas:

$$v_{\text{initial}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Substitute in the expression for the stopping time to obtain:

$$\Delta t = -\frac{1}{a}\sqrt{\frac{3kT}{m}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = -\frac{1}{-1.44\times10^{5}\ \text{m/s}^2}\sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(300\ \text{K}\right)}{\left(85\frac{\text{g}}{\text{mol}}\times\frac{1\,\text{mol}}{6.02\times10^{23}\ \text{particles}}\right)}} = \boxed{2.06\,\text{ms}}$$

Maxwell's Displacement Current

15 •

Picture the Problem We can differentiate the expression for the electric field between the plates of a parallel-plate capacitor to find the rate of change of the electric field and the definitions of the conduction current and electric flux to compute I_d.

(*a*) Express the electric field between the plates of the parallel-plate capacitor:

$$E = \frac{Q}{\epsilon_0 A}$$

Differentiate this expression with respect to time to obtain an expression for the rate of change of the electric field:

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{Q}{\epsilon_0 A}\right] = \frac{1}{\epsilon_0 A}\frac{dQ}{dt} = \frac{I}{\epsilon_0 A}$$

Substitute numerical values and evaluate dE/dt:

$$\frac{dE}{dt} = \frac{5\,\text{A}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.023\,\text{m}\right)^2} = \boxed{3.40\times10^{14}\ \text{V/m}\cdot\text{s}}$$

(*b*) Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

Substitute for the electric flux to obtain:

$$I_d = \epsilon_0 \frac{d}{dt}\left[EA\right] = \epsilon_0 A\frac{dE}{dt}$$

Substitute numerical values and evaluate I_d:

$$I_d = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.023\,\text{m}\right)^2\left(3.40\times10^{14}\ \text{V/m}\cdot\text{s}\right) = \boxed{5.00\,\text{A}}$$

16 •

Picture the Problem We can express the displacement current in terms of the electric flux and differentiate the resulting expression to obtain I_d in terms dE/dt.

Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

Substitute for the electric flux to obtain:

$$I_d = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Because $E = (0.05\,\text{N/C})\sin 2000t$:

$$I_d = \epsilon_0 A \frac{d}{dt}[(0.05\,\text{N/C})\sin 2000t] = (2000\,\text{s}^{-1})\epsilon_0 A(0.05\,\text{N/C})\cos 2000t$$

I_d will have its maximum value when cos 2000t = 1. Hence:

$$I_{d,max} = (2000\,\text{s}^{-1})\epsilon_0 A(0.05\,\text{N/C})$$

Substitute numerical values and evaluate $I_{d,max}$:

$$I_d = (2000\,\text{s}^{-1})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(1\,\text{m}^2)(0.05\,\text{N/C}) = \boxed{8.85\times10^{-10}\,\text{A}}$$

17 ••

Picture the Problem We can use Ampere's law to a circular path of radius r between the plates and parallel to their surfaces to obtain an expression relating B to the current enclosed by the amperian loop. Assuming that the displacement current is uniformly distributed between the plates, we can relate the displacement current enclosed by the circular loop to the conduction current I.

Apply Ampere's law to a circular path of radius r between the plates and parallel to their surfaces to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\pi rB = \mu_0 I_{enclosed} = \mu_0 I$$

Assuming that the displacement current is uniformly distributed:

$$\frac{I}{\pi r^2} = \frac{I_d}{\pi R^2} \Rightarrow I = \frac{r^2}{R^2} I_d$$

where R is the radius of the circular plates.

Substitute to obtain:

$$2\pi rB = \frac{\mu_0 r^2}{R^2} I_d$$

Solve for B:

$$B = \frac{\mu_0 r}{2\pi R^2} I_{\mathrm{d}}$$

Substitute numerical values and evaluate B:

$$B(r) = \frac{(4\pi \times 10^{-7}\ \mathrm{N/A^2})(5\,\mathrm{A})}{2\pi (0.023\,\mathrm{m})^2} r = \boxed{(1.89 \times 10^{-3}\ \mathrm{T/m})r}$$

18 ••

Picture the Problem We can use the definitions of the displacement current and electric flux, together with the expression for the capacitance of an air-core-parallel-plate capacitor to show that $I_{\mathrm{d}} = C\,dV/dt$.

(*a*) Use its definition to express the displacement current I_{d}:

$$I_{\mathrm{d}} = \epsilon_0 \frac{d\phi_{\mathrm{e}}}{dt}$$

Substitute for the electric flux to obtain:

$$I_{\mathrm{d}} = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Because $E = V/d$:

$$I_{\mathrm{d}} = \epsilon_0 A \frac{d}{dt}\left[\frac{V}{d}\right] = \frac{\epsilon_0 A}{d} \frac{dV}{dt}$$

The capacitance of an air-core-parallel-plate capacitor whose plates have area A and that are separated by a distance d is given by:

$$C = \frac{\epsilon_0 A}{d}$$

Substitute to obtain:

$$I_{\mathrm{d}} = \boxed{C \frac{dV}{dt}}$$

(*b*) Substitute in the expression derived in (*a*) to obtain:

$$I_{\mathrm{d}} = (5\,\mathrm{nF})\frac{d}{dt}[(3\,\mathrm{V})\cos 500\pi t] = -(5\,\mathrm{nF})(3\,\mathrm{V})(500\pi\,\mathrm{s^{-1}})\sin 500\pi t = \boxed{-(23.6\,\mu\mathrm{A})\sin 500\pi t}$$

***19** ••

Picture the Problem We can use the conservation of charge to find I_{d}, the definitions of the displacement current and electric flux to find dE/dt, and Ampere's law to evaluate $\vec{B} \cdot d\vec{\ell}$ around the given path.

(*a*) From conservation of charge we know that:

$$I_d = I = \boxed{10.0\,\text{A}}$$

(*b*) Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Substitute for *dE*/*dt*:

$$\frac{dE}{dt} = \frac{I_d}{\epsilon_0 A}$$

Substitute numerical values and evaluate *dE*/*dt*:

$$\frac{dE}{dt} = \frac{10\,\text{A}}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.5\,\text{m}^2\right)} = \boxed{2.26\times10^{12}\,\frac{\text{V}}{\text{m}\cdot\text{s}}}$$

(*c*) Apply Ampere's law to a circular path of radius *r* between the plates and parallel to their surfaces to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}}$$

Assuming that the displacement current is uniformly distributed:

$$\frac{I_{\text{enclosed}}}{\pi r^2} = \frac{I_d}{A} \Rightarrow I_{\text{enclosed}} = \frac{\pi r^2}{A} I_d$$

where *R* is the radius of the circular plates.

Substitute for I_{enclosed} to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \frac{\mu_0 \pi r^2}{A} I_d$$

Substitute numerical values and evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \frac{\left(4\pi\times10^{-7}\,\text{N/A}^2\right)\pi(0.1\,\text{m})^2(10\,\text{A})}{0.5\,\text{m}^2} = \boxed{7.90\times10^{-7}\,\text{T}\cdot\text{m}}$$

20 •••

Picture the Problem If $Q = Q_0 e^{-t/\tau}$ is the charge on the capacitor plates, then the conduction current *I* = *dQ*/*dt*. We can use $I_d = \epsilon_0 \frac{d\phi_e}{dt}$ to find the displacement current and $I_b = \frac{dQ_b}{dt}$ to find the current due to the rate of change of the bound charges. The total current is the sum of *I*, I_d, and I_b.

(*a*) The conduction current is given by:

$$I = \frac{dQ}{dt}$$

The charge on the capacitor varies with time according to:

$$Q = Q_0 e^{-t/\tau}\text{, where } \tau = RC$$

Substitute for Q to obtain:

$$I = \frac{d}{dt}\left[Q_0 e^{-t/\tau}\right] = \boxed{\frac{Q_0}{\tau} e^{-t/\tau}}$$

This current is in the direction of the electric field, which is from the positive plate to the negative plate. By choosing the positive sign for this current we define this to be the positive direction.

(*b*) The displacement current is given by:

$$I_\mathrm{d} = \in_0 \frac{d\phi_\mathrm{e}}{dt} = \in_0 \frac{d}{dt}[EA] = \in_0 A\frac{dE}{dt}$$

Relate the electric field E to the potential difference V between the plates and the separation of the plates d:

$$E = \frac{V}{d}$$

Substitute to obtain:

$$I_\mathrm{d} = \in_0 A\frac{d}{dt}\left[\frac{V}{d}\right] = \frac{\in_0 A}{d}\frac{dV}{dt}$$

or, because $C = \dfrac{\kappa \in_0 A}{d}$,

$$I_\mathrm{d} = \frac{C}{\kappa}\frac{dV}{dt}$$

V varies with time according to:

$$V = V_0 e^{-t/\tau} = \frac{Q_0}{C} e^{-t/\tau}$$

Substituting in the expression for I_d yields:

$$I_\mathrm{d} = \frac{C}{\kappa}\frac{d}{dt}\left[\frac{Q_0}{C} e^{-t/\tau}\right] = -\frac{Q_0}{\kappa\tau} e^{-t/\tau}$$

$$= \boxed{-\frac{1}{\kappa} I}$$

(*c*) As the voltage across the dielectric decreases the magnitude of the bound charges also decreases. The current I_b due to the flow of these bound charges though a

$I_\mathrm{b} = \dfrac{dQ_\mathrm{b}}{dt}$ where Q_b is the bound charge on the surface of the dielectric next to the plate with charge Q.

stationary surface is given by:

It follows that Q and Q_b are opposite in sign and are related by Equation 24-27:

$$Q_b = -\left(1 - \frac{1}{\kappa}\right)Q$$

Substitute in the expression for I_b and carry out the differentiation to obtain:

$$I_b = \frac{d}{dt}\left[-\left(1-\frac{1}{\kappa}\right)Q\right] = -\left(1-\frac{1}{\kappa}\right)\frac{dQ}{dt}$$
$$= \boxed{-\left(1-\frac{1}{\kappa}\right)I}$$

(*d*) Add the currents found in (*a*), (*b*), and (*c*) to obtain:

$$I_{total} = I + I_d + I_b$$
$$= I - \frac{1}{\kappa}I - \left(1-\frac{1}{\kappa}\right)I$$
$$= \boxed{0}$$

Remarks: In more sophisticated treatments of electrodynamics it is conventional to refer to the sum $I_d + I_b$ as the displacement current.

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Picture the Problem We can find the conduction current as a function of time using $I = V(t)/R$ and substituting for $V(t)$. We can use $I_d = \epsilon_0 \phi_e$ to obtain an expression for the displacement current I_d as a function of time. Finally, equating the conduction and displacement currents will yield an expression for the time at which they are equal.

(*a*) Express the conduction current in terms of the potential difference between the plates of the capacitor:

$$I = \frac{V(t)}{R} = \frac{AV(t)}{\rho d}$$

Substitute for $V(t)$ to obtain:

$$I = \boxed{\frac{(0.01\,\text{V/s})A}{\rho d}t}$$

(*b*) The displacement current is given by:

$$I_d = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 \frac{d}{dt}\left(\frac{V}{d}A\right)$$
$$= \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

Substitute for *V* and simplify to obtain:

$$I_d = \frac{\epsilon_0 A}{d}\frac{d}{dt}\left[(0.01\,\text{V/s})t\right] = \boxed{\frac{(0.01\,\text{V/s})\epsilon_0 A}{d}}$$

(*c*) Set $I_d = I$ to obtain:

$$\frac{(0.01\,\text{V/s})\epsilon_0 A}{d} = \frac{A(0.01\,\text{V/s})}{\rho d}t$$

Solve for *t*:

$$t = \boxed{\epsilon_0 \rho}$$

22 ••

Picture the Problem We can use $I_d = \epsilon_0 \frac{d\phi_e}{dt}$ and the relationship between the voltage across the plates and the electric field between them to find the displacement current. The conduction current between the plates is given by $I = \frac{V}{R} = \frac{AV}{\rho d}$ where *A* is the area of the plates and *d* is their separation.

(*a*) The displacement current is given by:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Relate the electric field *E* to the potential difference *V* between the plates and the separation of the plates *d*:

$$E = \frac{V}{d}$$

Substitute to obtain:

$$I_d = \epsilon_0 A \frac{d}{dt}\left[\frac{V}{d}\right] = \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

V varies with time according to:

$$V = V_0 \cos \omega t$$

Substituting in the expression for I_d yields:

$$I_d = \frac{\epsilon_0 A}{d}\frac{d}{dt}[V_0 \cos \omega t] = -\frac{\epsilon_0 \pi r^2 V_0}{\omega d}\sin \omega t$$

Substitute numerical values and evaluate I_d:

$$I_{\mathrm{d}} = -\frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\pi\left(20\,\mathrm{cm}\right)^2\left(40\,\mathrm{V}\right)}{\left(120\pi\ \mathrm{rad/s}\right)\left(1\,\mathrm{mm}\right)}\sin\left(120\pi\ \mathrm{rad/s}\right)t$$
$$= \boxed{-\left(1.18\times10^{-10}\ \mathrm{A}\right)\sin\left(120\pi\ \mathrm{rad/s}\right)t}$$

(*b*) The conduction current between the plates is given by:

$$I = \frac{V}{R} = \frac{AV}{\rho d} = \frac{AV_0}{\rho d}\cos\omega t$$

Substitute numerical values and simplify to obtain:

$$I = \frac{\pi\left(0.2\,\mathrm{m}\right)^2\left(40\,\mathrm{V}\right)}{\left(10^4\ \Omega\cdot\mathrm{m}\right)\left(10^{-3}\ \mathrm{m}\right)}\cos\left(120\pi\ \mathrm{rad/s}\right)t$$
$$= \boxed{\left(0.503\,\mathrm{A}\right)\cos\left(120\pi\ \mathrm{rad/s}\right)t}$$

***23** •••

Picture the Problem We can follow the step-by-step instructions in the problem statement to show that Equation 30-4 gives the same result for *B* as that given in Part (*a*).

(*a*) Express the magnetic field at *P* using the expression for *B* due to a straight wire segment:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

where

$$\sin\theta_1 = \sin\theta_2 = \frac{a}{\sqrt{R^2 + a^2}}$$

Substitute for $\sin\theta_1$ and $\sin\theta_2$ to obtain:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\frac{2a}{\sqrt{R^2 + a^2}}$$
$$= \boxed{\frac{\mu_0 Ia}{2\pi R}\frac{1}{\sqrt{R^2 + a^2}}}$$

(*b*) Express the electric flux through the circular strip of radius *r* and width *dr* in the *yz* plane:

$$d\phi_{\mathrm{e}} = E_x dA = E_x\left(2\pi rdr\right)$$

The electric field due to the dipole is:

$$E_x = \frac{2kQ}{r^2 + a^2}\cos\theta_1 = \frac{2kQa}{\left(r^2 + a^2\right)^{3/2}}$$

Substitute for E_x to obtain:

$$d\phi_{\text{e}} = E_x dA = \frac{2kQa}{\left(r^2+a^2\right)^{3/2}}\left(2\pi\, rdr\right) = \frac{2Qa}{4\pi \in_0 \left(r^2+a^2\right)^{3/2}}\left(2\pi\, rdr\right) = \boxed{\frac{Qa}{\in_0 \left(r^2+a^2\right)^{3/2}}rdr}$$

(*c*) Multiply both sides of the expression for ϕ_{e} by $\in_0$:

$$\in_0 d\phi_{\text{e}} = \frac{Qa}{\left(r^2+a^2\right)^{3/2}}rdr$$

Integrate r from 0 to R to obtain:

$$\in_0 \phi_{\text{e}} = Qa\int_0^R \frac{rdr}{\left(r^2+a^2\right)^{3/2}} = Qa\left(\frac{-1}{\sqrt{R^2+a^2}}+\frac{1}{a}\right) = \boxed{Q\left(1-\frac{a}{\sqrt{R^2+a^2}}\right)}$$

(*d*) The displacement current is defined to be:

$$I_{\text{d}} = \in_0 \frac{d\phi_{\text{e}}}{dt} = \frac{d}{dt}\left[Q\left(1-\frac{a}{\sqrt{R^2+a^2}}\right)\right] = \left(1-\frac{a}{\sqrt{R^2+a^2}}\right)\frac{dQ}{dt} = -I\left(1-\frac{a}{\sqrt{R^2+a^2}}\right)$$

The total current is the sum of I and I_{d}:

$$I + I_{\text{d}} = I - I\left(1-\frac{a}{\sqrt{R^2+a^2}}\right) = \boxed{I\frac{a}{\sqrt{R^2+a^2}}}$$

(*e*) Apply Equation 30-4 (the generalized form of Ampere's law) to obtain:

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\ell} = 2\pi RB = \mu_0\left(I+I_{\text{d}}\right)$$

Solve for B:

$$B = \frac{\mu_0}{2\pi R}\left(I+I_{\text{d}}\right)$$

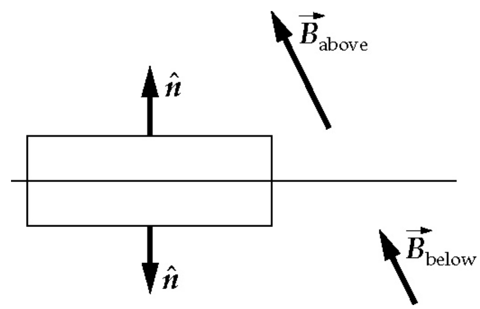
Substitute for $I + I_d$ from (*d*) to obtain:

$$B = \frac{\mu_0}{2\pi R}\left(I\frac{a}{\sqrt{R^2+a^2}}\right) = \boxed{\frac{\mu_0 Ia}{2\pi R}\frac{1}{\sqrt{R^2+a^2}}}$$

Maxwell's Equations and the Electromagnetic Spectrum

24 ••

Picture the Problem The figure shows the end view of a pillbox surrounding a small area *dA* of the surface. The normal components of the magnetic field, $\vec{B}_{\text{n, top}}$ and $\vec{B}_{\text{n, bottom}}$, are shown with different magnitudes. When performing the surface integral the normal to the surface is outward, as shown in the figure.



Apply Gauss's law for magnetism to the pillbox to obtain:

$$\oint_S \vec{B}\cdot\hat{n}dA = \int_{\text{bottom surface}} \vec{B}\cdot\hat{n}dA + \int_{\text{lateral surface}} \vec{B}\cdot\hat{n}dA + \int_{\text{top surface}} \vec{B}\cdot\hat{n}dA = 0$$

Because the horizontal component of $\vec{B}$ is zero, $\int_{\text{lateral surface}} \vec{B}\cdot\hat{n}dA = 0$, and:

$$\oint_S \vec{B}\cdot\hat{n}dA = \int_{\text{bottom surface}} \vec{B}\cdot\hat{n}dA + \int_{\text{top surface}} \vec{B}\cdot\hat{n}dA = 0 \quad (1)$$

Because $\vec{B}$ and $\hat{n}$ are oppositely directed at the bottom surface:

$$\int_{\text{bottom surface}} \vec{B}_{\text{below}}\cdot\hat{n}dA = -B_{\text{n,below}}A$$

Because $\vec{B}$ and $\hat{n}$ are parallel at the top surface:

$$\int_{\text{top surface}} \vec{B}_{\text{below}}\cdot\hat{n}dA = B_{\text{n,above}}A$$

Substitute in equation (1) to obtain:

$$-B_{\text{n, below}}A + B_{\text{n, above}}A = 0$$

Solve for $B_{\text{n,top}}$:

$\boxed{B_{\text{n, above}} = B_{\text{n, below}}}$; i.e., the normal component of $\vec{B}$ is continuous across the surface.

***25** •

Picture the Problem We can use $c = f\lambda$ to find the wavelengths corresponding to the given frequencies.

Solve $c = f\lambda$ for λ:

$$\lambda = \frac{c}{f}$$

(*a*) For f = 1000 kHz:

$$\lambda = \frac{3\times10^8\ \text{m/s}}{1000\times10^3\ \text{s}^{-1}} = \boxed{300\,\text{m}}$$

(*b*) For f = 100 MHz:

$$\lambda = \frac{3\times10^8\ \text{m/s}}{100\times10^6\ \text{s}^{-1}} = \boxed{3.00\,\text{m}}$$

***26** •

Picture the Problem We can use $c = f\lambda$ to find the frequency corresponding to the given wavelength.

Solve $c = f\lambda$ for f:

$$f = \frac{c}{\lambda}$$

Substitute numerical values and evaluate f:

$$f = \frac{3\times10^8\ \text{m/s}}{3\times10^{-2}\ \text{m}} = 10^{10}\ \text{Hz} = \boxed{10.0\,\text{GHz}}$$

27 •

Picture the Problem We can use $c = f\lambda$ to find the frequency corresponding to the given wavelength.

Solve $c = f\lambda$ for f:

$$f = \frac{c}{\lambda}$$

Substitute numerical values and evaluate f:

$$f = \frac{3\times10^8\ \text{m/s}}{0.1\times10^{-9}\ \text{m}} = \boxed{3.00\times10^{18}\ \text{Hz}}$$

Electric Dipole Radiation

28 ••

Picture the Problem We can use the intensity I_1 at a distance r = 10 m and at an angle θ = 90° to find the proportionality constant in the expression for the intensity of radiation from an electric dipole and then use the resulting equation to find the intensity at the given distances and angles.

Express the intensity of radiation as a function of r and θ:

$$I(\theta, r) = \frac{C}{r^2}\sin^2\theta \qquad (1)$$

where C is a constant of proportionality.

Express I(90°,10 m):

$$I(90°, 10\,\text{m}) = I_1 = \frac{C}{(10\,\text{m})^2}\sin^2 90° = \frac{C}{100\,\text{m}^2}$$

Solve for C:

$$C = \left(100\,\text{m}^2\right)I_1$$

Substitute in equation (1) to obtain:

$$I(\theta, r) = \frac{\left(100\,\text{m}^2\right)I_1}{r^2}\sin^2\theta \qquad (2)$$

(*a*) Evaluate equation (2) for r = 30 m and θ = 90°:

$$I(90°, 30\,\text{m}) = \frac{\left(100\,\text{m}^2\right)I_1}{(30\,\text{m})^2}\sin^2 90° = \boxed{\tfrac{1}{9}I_1}$$

(*b*) Evaluate equation (2) for r = 10 m and θ = 45°:

$$I(45°, 10\,\text{m}) = \frac{\left(100\,\text{m}^2\right)I_1}{(10\,\text{m})^2}\sin^2 45° = \boxed{\tfrac{1}{2}I_1}$$

(*c*) Evaluate equation (2) for r = 20 m and θ = 30°:

$$I(30°, 20\,\text{m}) = \frac{\left(100\,\text{m}^2\right)I_1}{(20\,\text{m})^2}\sin^2 30° = \boxed{\tfrac{1}{16}I_1}$$

29 ••

Picture the Problem We can use the intensity I_1 at a distance r = 10 m and at an angle θ = 90° to find the proportionality constant in the expression for the intensity of radiation from an electric dipole and then use the resulting equation to find the angle for a given intensity and distance and the distance corresponding to a given intensity and angle.

Express the intensity of radiation as a function of r and θ:

$$I(\theta, r) = \frac{C}{r^2}\sin^2\theta \qquad (1)$$

where C is a constant of proportionality.

Express $I(90°,10\text{ m})$:

$$I(90°,10\,\text{m}) = I_1 = \frac{C}{(10\,\text{m})^2}\sin^2 90° = \frac{C}{100\,\text{m}^2}$$

Solve for C:

$$C = (100\,\text{m}^2)I_1$$

Substitute in equation (1) to obtain:

$$I(\theta, r) = \frac{(100\,\text{m}^2)I_1}{r^2}\sin^2\theta \qquad (2)$$

(*a*) For $r = 5$ m and $I(\theta,r) = I_1$:

$$I_1 = \frac{(100\,\text{m}^2)I_1}{(5\,\text{m})^2}\sin^2\theta$$

or

$$\sin^2\theta = \tfrac{1}{4}$$

Solve for θ to obtain:

$$\theta = \sin^{-1}\tfrac{1}{2} = \boxed{30.0°}$$

(*b*) For $\theta = 45°$ and $I(\theta,r) = I_1$:

$$I_1 = \frac{(100\,\text{m}^2)I_1}{r^2}\sin^2 45°$$

or

$$r^2 = \tfrac{1}{2}(100\,\text{m}^2)$$

Solve for r to obtain:

$$r = \sqrt{\tfrac{1}{2}(100\,\text{m}^2)} = \boxed{7.07\,\text{m}}$$

30 ••

Picture the Problem We can use the intensity I at a distance $r = 4000$ m and at an angle $\theta = 90°$ to find the proportionality constant in the expression for the intensity of radiation from an electric dipole and then use the resulting equation to find the intensity at sea level and 1.5 km from the transmitter.

Express the intensity of radiation as a function of r and θ:

$$I(\theta, r) = \frac{C}{r^2}\sin^2\theta \qquad (1)$$

where C is a constant of proportionality.

Use the given data to obtain:

$$4\times10^{-12}\,\text{W/m}^2 = \frac{C}{(4\,\text{km})^2}\sin^2 90° = \frac{C}{(4\,\text{km})^2}$$

Solve for C:

$$C=(4\,\text{km})^2\left(4\times10^{-12}\ \text{W/m}^2\right)=6.40\times10^{-5}\ \text{W}$$

Substitute in equation (1) to obtain:

$$I(\theta,r)=\frac{6.40\times10^{-5}\ \text{W}}{r^2}\sin^2\theta \qquad (2)$$

For a point at sea level and 1.5 km from the transmitter:

$$\theta=\tan^{-1}\frac{2\,\text{km}}{1.5\,\text{km}}=53.1°$$

Evaluate I(53.1°,1.5 km):

$$I(53.1°,1.5\,\text{km})=\frac{6.40\times10^{-5}\ \text{W}}{(1.5\,\text{km})^2}\sin^2 53.1°=\boxed{18.2\,\text{pW/m}^2}$$

31 •••

Picture the Problem The intensity of radiation from an electric dipole is equal to $I_0(\sin^2\theta)/r^2$, where θ is the angle between the electric dipole moment and the position vector $\vec{r}$. We can integrate the intensity to express the total power radiated by the antenna and use this result to evaluate I_0. Knowing I_0 we can find the intensity at a horizontal distance of 120 km directly in front of the station.

Express the intensity of the signal as a function of r and θ:

$$I(r,\theta)=I_0\frac{\sin^2\theta}{r^2}$$

At a horizontal distance of 120 km from the station and directly in front of it:

$$I(120\,\text{km},90°)=I_0\frac{\sin^2 90°}{(120\,\text{km})^2}=\frac{I_0}{(120\,\text{km})^2} \qquad (1)$$

From the definition of intensity we have:

$$dP=IdA$$

and

$$P_{\text{tot}}=\iint I(r,\theta)dA$$

where, in polar coordinates,

$$dA=r^2\sin\theta\,d\theta\,d\phi$$

Substitute for dA to obtain:

$$P_{\text{tot}}=\int_0^{2\pi}\int_0^{\pi} I(r,\theta)r^2\sin\theta\,d\theta\,d\phi$$

Substitute for $I(r,\theta)$:

$$P_{\text{tot}} = I_0 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

From integral tables we find that:

$$\int_0^{\pi} \sin^3\theta d\theta = -\tfrac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big]_0^{\pi} = \frac{4}{3}$$

Substitute and integrate with respect to ϕ to obtain:

$$P_{\text{tot}} = \frac{4}{3} I_0 \int_0^{2\pi} d\phi = \frac{4}{3} I_0 [\phi]_0^{2\pi} = \frac{8\pi}{3} I_0$$

Solve for I_0:

$$I_0 = \frac{3}{8\pi} P_{\text{tot}}$$

Substitute for P_{tot} and evaluate I_0:

$$I_0 = \frac{3}{8\pi}(500\,\text{kW}) = 59.7\,\text{kW}$$

Substitute for I_0 in equation (1) and evaluate $I(120\text{ km},90°)$:

$$I(120\,\text{km},90°) = \frac{59.7\,\text{kW}}{(120\,\text{km})^2} = \boxed{4.15\,\mu\text{W/m}^2}$$

Express the number of photons incident on an area A in time Δt:

$$\frac{N}{A\Delta t} = \frac{N}{(P/I)\Delta t} = \frac{NI}{P\Delta t} = \frac{NI}{E} = \frac{I}{E/N} = \frac{I}{hf}$$

Substitute numerical values and evaluate I/hf:

$$\frac{I}{hf} = \frac{4.15\,\mu\text{W/m}^2}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(1.20\,\text{MHz})} = 5.21\times10^{21}\,\frac{\text{photons}}{\text{m}^2\cdot\text{s}} = \boxed{5.22\times10^{17}\,\frac{\text{photons}}{\text{cm}^2\cdot\text{s}}}$$

***32 •••**

Picture the Problem The intensity of radiation from an electric dipole is given by $I_0(\sin^2\theta)/r^2$, where θ is the angle between the electric dipole moment and the position vector $\vec{r}$. We can integrate the intensity to express the total power radiated by the antenna and use this result to evaluate I_0. Knowing I_0 we can find the total power radiated by the station.

From the definition of intensity we

$$dP = IdA$$

have:

and

$$P_{\text{tot}} = \iint I(r,\theta)\,dA$$

where, in polar coordinates,

$$dA = r^2 \sin\theta\, d\theta\, d\phi$$

Substitute for dA to obtain:

$$P_{\text{tot}} = \int_0^{2\pi}\int_0^{\pi} I(r,\theta) r^2 \sin\theta\, d\theta\, d\phi$$

Express the intensity of the signal as a function of r and θ:

$$I(r,\theta) = I_0 \frac{\sin^2\theta}{r^2} \qquad (1)$$

Substitute for $I(r,\theta)$:

$$P_{\text{tot}} = I_0 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

From integral tables we find that:

$$\int_0^{\pi} \sin^3\theta d\theta = -\tfrac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big]_0^{\pi} = \frac{4}{3}$$

Substitute and integrate with respect to ϕ to obtain:

$$P_{\text{tot}} = \frac{4}{3} I_0 \int_0^{2\pi} d\phi = \frac{4}{3} I_0 [\phi]_0^{2\pi} = \frac{8\pi}{3} I_0$$

From equation (1) we have:

$$I_0 = \frac{I(r,\theta) r^2}{\sin^2\theta}$$

Substitute to obtain:

$$P_{\text{tot}} = \frac{8\pi}{3} \frac{I(r,\theta) r^2}{\sin^2\theta}$$

or, because $\theta = 90°$,

$$P_{\text{tot}} = \frac{8\pi}{3} I(r) r^2$$

Substitute numerical values and evaluate P_{tot}:

$$P_{\text{tot}} = \frac{8\pi}{3}\left(2\times10^{-13}\ \text{W/m}^2\right)(30\,\text{km})^2$$

$$= \boxed{1.51\text{mW}}$$

33 •••

Picture the Problem The intensity of radiation from the airport's vertical dipole antenna is given by $I_0(\sin^2\theta)/r^2$, where θ is the angle between the electric dipole moment and the position vector $\vec{r}$. We can integrate the intensity to express the total power radiated by the antenna and use this result to evaluate I_0. Knowing I_0 we can find the intensity of the signal at the plane's elevation and distance from the airport.

Express the intensity of the signal as a function of r and θ:

$$I(r,\theta) = I_0 \frac{\sin^2\theta}{r^2} \qquad (1)$$

From the definition of intensity we have:

$$dP = IdA$$

and

$$P_{\text{tot}} = \iint I(r,\theta)\,dA$$

where, in polar coordinates,

$$dA = r^2 \sin\theta\, d\theta\, d\phi$$

Substitute for dA to obtain:

$$P_{\text{tot}} = \int_0^{2\pi}\int_0^{\pi} I(r,\theta) r^2 \sin\theta\, d\theta\, d\phi$$

Substitute for $I(r,\theta)$:

$$P_{\text{tot}} = I_0 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

From integral tables we find that:

$$\int_0^{\pi} \sin^3\theta d\theta = -\tfrac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big]_0^{\pi} = \frac{4}{3}$$

Substitute and integrate with respect to ϕ to obtain:

$$P_{\text{tot}} = \frac{4}{3} I_0 \int_0^{2\pi} d\phi = \frac{4}{3} I_0 [\phi]_0^{2\pi} = \frac{8\pi}{3} I_0$$

Solve for I_0:

$$I_0 = \frac{3}{8\pi} P_{\text{tot}}$$

Substitute for I_0 in equation (1):

$$I(r,\theta) = \frac{3P_{\text{tot}}}{8\pi} \frac{\sin^2\theta}{r^2}$$

At the elevation of the plane:

$$\theta = \tan^{-1}\left(\frac{2500\,\text{m}}{4000\,\text{m}}\right) = 32.0°$$

and

$$r = \sqrt{(2500\,\text{m})^2 + (4000\,\text{m})^2} = 4717\,\text{m}$$

Substitute numerical values and evaluate $I(4717\text{ m}, 32°)$:

$$I(4717\,\text{m}, 32°) = \frac{3(100\,\text{W})}{8\pi} \frac{\sin^2 32°}{(4717\,\text{m})^2} = \boxed{0.151\,\mu\text{W/m}^2}$$

Energy and Momentum in an Electromagnetic Wave

34 •

Picture the Problem We can use Pr = I/c to find the radiation pressure. The intensity of the electromagnetic wave is related to the rms values of its electric and magnetic fields according to $I = E_{rms}B_{rms}/\mu_0$, where $B_{rms} = E_{rms}/c$.

(*a*) Express the radiation pressure in terms of the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{100\,\text{W/m}^2}{3\times10^8\ \text{m/s}} = \boxed{0.333\,\mu\text{Pa}}$$

(*b*) Relate the intensity of the electromagnetic wave to E_{rms} and B_{rms}:

$$I = \frac{E_{rms}B_{rms}}{\mu_0}$$

or, because $B_{rms} = E_{rms}/c$,

$$I = \frac{E_{rms}\,E_{rms}/c}{\mu_0} = \frac{E_{rms}^2}{\mu_0 c}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{\mu_0 c I}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(3\times10^8\ \text{m/s}\right)\left(100\,\text{W/m}^2\right)} = \boxed{194\,\text{V/m}}$$

(*c*) Express B_{rms} in terms of E_{rms}:

$$B_{rms} = \frac{E_{rms}}{c}$$

Substitute numerical values and evaluate B_{rms}:

$$B_{rms} = \frac{194\,\text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{0.647\,\mu\text{T}}$$

35 •

Picture the Problem The rms values of the electric and magnetic fields are found from their amplitudes by dividing by the square root of two. The rms values of the electric and magnetic fields are related according to $B_{rms} = E_{rms}/c$. We can find the intensity of the radiation using $I = E_{rms}B_{rms}/\mu_0$ and the radiation pressure using $P_r = I/c$.

(*a*) Relate E_{rms} to E_0:

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{400\ \text{V/m}}{\sqrt{2}} = \boxed{283\,\text{V/m}}$$

(*b*) Find B_{rms} from E_{rms}:

$$B_{rms} = \frac{E_{rms}}{c} = \frac{283\,\text{V/m}}{3\times10^{8}\ \text{m/s}} = \boxed{0.943\,\mu\text{T}}$$

(*c*) The intensity of an electromagnetic wave is given by:

$$I = \frac{E_{rms}B_{rms}}{\mu_0}$$

Substitute numerical values and evaluate I:

$$I = \frac{(283\,\text{V/m})(0.943\,\mu\text{T})}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{212\,\text{W/m}^2}$$

(*d*) Express the radiation pressure in terms of the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{212\,\text{W/m}^2}{3\times10^{8}\ \text{m/s}} = \boxed{0.707\,\mu\text{Pa}}$$

36 •

Picture the Problem Given E_{rms}, we can find B_{rms} using $B_{rms} = E_{rms}/c$. The average energy density of the wave is given by $u_{av} = E_{rms}B_{rms}/\mu_0 c$ and the intensity of the wave by $I = u_{av}c$.

(*a*) Express B_{rms} in terms of E_{rms}:

$$B_{rms} = \frac{E_{rms}}{c}$$

Substitute numerical values and evaluate B_{rms}:

$$B_{rms} = \frac{400\,\text{V/m}}{3\times10^{8}\ \text{m/s}} = \boxed{1.33\,\mu\text{T}}$$

(*b*) The average energy density u_{av} is given by:

$$u_{av} = \frac{E_{rms}B_{rms}}{\mu_0 c}$$

Substitute numerical values and evaluate u_{av}:

$$u_{av} = \frac{(400\,\text{V/m})(1.33\,\mu\text{T})}{(4\pi\times10^{-7}\ \text{N/A}^2)(3\times10^{8}\ \text{m/s})} = \boxed{1.41\,\mu\text{J/m}^3}$$

(*c*) Express the intensity as the product of the average energy density and the speed of light in a vacuum:

$$I = u_{av}c$$

Substitute numerical values and evaluate I:

$$I = \left(1.41\,\mu\text{J/m}^3\right)\left(3\times10^8\text{ m/s}\right) = \boxed{423\,\text{W/m}^2}$$

37 •

Picture the Problem We can simplify the units of cB to show that this product has the same units as E.

Express the units of cB and simplify:

$$\frac{\text{m}}{\text{s}}\times\text{T} = \frac{\text{m}}{\text{s}}\times\frac{\text{N}}{\text{A}\cdot\text{m}} = \frac{\text{m}}{\text{s}}\times\frac{\text{N}}{\frac{\text{C}}{\text{s}}\cdot\text{m}} = \frac{\text{N}}{\text{C}} = \frac{\text{N}}{\text{C}}\times\frac{\text{m}}{\text{m}} = \frac{\text{J}}{\text{C}\cdot\text{m}} = \boxed{\frac{\text{V}}{\text{m}}}$$

***38** •

Picture the Problem Given B_{rms}, we can find E_{rms} using $E_{rms} = cB_{rms}$. The average energy density of the wave is given by $u_{av} = E_{rms}B_{rms}/\mu_0 c$ and the intensity of the wave by $I = u_{av}c$.

(*a*) Express E_{rms} in terms of B_{rms}:

$$E_{rms} = cB_{rms}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \left(3\times10^8\text{ m/s}\right)\left(0.245\,\mu\text{T}\right) = \boxed{73.5\,\text{V/m}}$$

(*b*) The average energy density u_{av} is given by:

$$u_{av} = \frac{E_{rms}B_{rms}}{\mu_0 c}$$

Substitute numerical values and evaluate u_{av}:

$$u_{av} = \frac{\left(73.5\,\text{V/m}\right)\left(0.245\,\mu\text{T}\right)}{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(3\times10^8\text{ m/s}\right)} = \boxed{47.8\,\text{nJ/m}^3}$$

(*c*) Express the intensity as the product of the average energy density and the speed of light in a vacuum:

$$I = u_{av}c$$

Substitute numerical values and evaluate I:

$$I = \left(47.8\,\text{nJ/m}^3\right)\left(3\times10^8\text{ m/s}\right) = \boxed{14.3\,\text{W/m}^2}$$

39 ••

Picture the Problem We can find the force exerted on the card using the definition of pressure and the relationship between radiation pressure and the intensity of the electromagnetic wave. Note that, when the card reflects all the radiation incident on it, conservation of momentum requires that the force is doubled.

(*a*) Using the definition of pressure, express the force exerted on the card by the radiation:

$$F = P_r A$$

Relate the radiation pressure to the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute to obtain:

$$F = \frac{IA}{c}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{\left(200\,\text{W/m}^2\right)\left(0.2\,\text{m}\right)\left(0.3\,\text{m}\right)}{3\times10^8\ \text{m/s}} = \boxed{40.0\,\text{nN}}$$

(*b*) If the card reflects all of the radiation incident on it, the force exerted on the card is doubled:

$$F = \boxed{80.0\,\text{nN}}$$

40 ••

Picture the Problem Only the normal component of the radiation pressure exerts a force on the card.

(*a*) Using the definition of pressure, express the force exerted on the card by the radiation:

$$F = 2P_r A\cos\theta$$

where the factor of 2 is a consequence of the fact that the card reflects the radiation incident on it.

Relate the radiation pressure to the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute to obtain:

$$F = \frac{2IA\cos\theta}{c}$$

Substitute numerical values and evaluate F:

$$F = \frac{2\left(200\ \text{W/m}^2\right)(0.2\ \text{m})(0.3\ \text{m})\cos 30°}{3\times10^8\ \text{m/s}} = \boxed{69.3\ \text{nN}}$$

***41** ••

Picture the Problem We can use $I = P_{av}/4\pi r^2$ and $I = E_{rms}B_{rms}/\mu_0$ to express E_{rms} in terms of P_{av} and the distance r from the station.

Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{P_{av}}{4\pi r^2}$$

The intensity is also given by:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0} = \frac{E_{max}^2}{2c\mu_0}$$

Equate these expressions to obtain:

$$\frac{P_{av}}{4\pi r^2} = \frac{E_{max}^2}{2c\mu_0}$$

Solve for E_{max}:

$$E_{max} = \sqrt{\frac{c\mu_0 P_{av}}{2\pi}}\left(\frac{1}{r}\right)$$

(a) Substitute numerical values and evaluate E_{max} for $r = 500$ m:

$$E_{max}(500\ \text{m}) = \sqrt{\frac{\left(3\times10^8\ \text{m/s}\right)\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50\ \text{kW})}{2\pi}}\left(\frac{1}{500\ \text{m}}\right) = \boxed{3.46\ \text{V/m}}$$

Use $B_{max} = E_{max}/c$ to evaluate B_{max}:

$$B_{max} = \frac{3.46\ \text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{11.5\ \text{nT}}$$

(b) Substitute numerical values and evaluate E_{max} for $r = 5$ km:

$$E_{max}(5\ \text{km}) = \sqrt{\frac{\left(3\times10^8\ \text{m/s}\right)\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50\ \text{kW})}{2\pi}}\left(\frac{1}{5\ \text{km}}\right) = \boxed{0.346\ \text{V/m}}$$

Use $B_{max} = E_{max}/c$ to evaluate B_{max}:

$$B_{max} = \frac{0.346\ \text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{1.15\ \text{nT}}$$

(c) Substitute numerical values and evaluate E_{max} for $r = 50$ km:

$$E_{max}(500\,\text{m}) = \sqrt{\frac{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(50\,\text{kW})}{2\pi}}\left(\frac{1}{50\,\text{km}}\right) = \boxed{0.0346\,\text{V/m}}$$

Use $B_{max} = E_{max}/c$ to evaluate B_{max}:

$$B_{max} = \frac{0.0346\,\text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{0.115\,\text{nT}}$$

42 ••

Picture the Problem We can use $I = P_{av}/A$ to express E_{rms} in terms of I. We can then use $B_{rms} = E_{rms}/c$ to find B_{rms}. The average power output of the sun is given by $P_{av} = 4\pi R^2 I$, where R is the earth-sun distance. The intensity and the radiation pressure at the surface of the sun can be found from the definitions of these physical quantities.

(*a*) From the definition of intensity we have:

$$I = \frac{P_{av}}{A} = \frac{4P_{av}}{\pi d^2}$$

Substitute numerical values and evaluate I:

$$I = \frac{4(1.5\,\text{mW})}{\pi(10^{-3}\ \text{m})^2} = \boxed{1.91\text{kW/m}^2}$$

(*b*) Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{c\mu_0 I}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(1.91\text{kW/m}^2)} = \boxed{849\,\text{V/m}}$$

Use $B_{rms} = E_{rms}/c$ to evaluate B_{rms}:

$$B_{rms} = \frac{849\,\text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{2.83\,\mu\text{T}}$$

(*d*) Express the radiation pressure in terms of the intensity:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{1.91\times10^3\ \text{W/m}^2}{3\times10^8\ \text{m/s}} = \boxed{6.37\,\mu\text{Pa}}$$

***43** ••

Picture the Problem We can use $I = E_{rms}B_{rms}/\mu_0$ and $B_{rms} = E_{rms}/c$ to express E_{rms} in terms of I. We can then use $B_{rms} = E_{rms}/c$ to find B_{rms}.

Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{c\mu_0 I}$$

Use the definition of intensity to relate the intensity of the electromagnetic wave to the power in the beam:

$$I = \frac{P}{A} = \frac{I_{trans.\,line}V}{A}$$

Substitute for I to obtain:

$$E_{rms} = \sqrt{\frac{c\mu_0 I_{trans.\,line}V}{A}}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{\frac{(3\times10^8\text{ m/s})(4\pi\times10^{-7}\text{ N/A}^2)(10^3\text{ A})(750\text{ kV})}{50\text{ m}^2}} = \boxed{75.2\text{ kV/m}}$$

Use $B_{rms} = E_{rms}/c$ to evaluate B_{rms}:

$$B_{rms} = \frac{75.2\text{ kV/m}}{3\times10^8\text{ m/s}} = \boxed{0.251\text{ mT}}$$

44 ••

Picture the Problem The spatial length L of the pulse is the product of its speed c and duration Δt. We can find the energy density within the pulse using its definition ($u = U/V$). The electric amplitude of the pulse is related to the energy density in the beam according to $u = \epsilon_0 E^2$ and we can find B from E using $B = E/c$.

(*a*) The spatial length L of the pulse is the product of its speed c and duration Δt:

$$L = c\Delta t$$

Substitute numerical values and evaluate L:

$$L = (3\times10^8\text{ m/s})(10\text{ ns}) = \boxed{3.00\text{ m}}$$

(*b*) The energy density within the pulse is the energy of the beam per unit volume:

$$u=\frac{U}{V}=\frac{U}{\pi r^2 L}$$

Substitute numerical values and evaluate *u*:

$$u=\frac{20\,\text{J}}{\pi(2\,\text{mm})^2(3.00\,\text{m})}=\boxed{531\,\text{kJ/m}^3}$$

(*c*) *E* is related to *u* according to:

$$u=\epsilon_0 E_{\text{rms}}^2=\tfrac{1}{2}\epsilon_0 E_0^2$$

Solve for E_0 to obtain:

$$E_0=\sqrt{\frac{2u}{\epsilon_0}}$$

Substitute numerical values and evaluate E_0:

$$E_0=\sqrt{\frac{2(531\text{ kJ/m}^3)}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}}=\boxed{346\,\text{MV/m}}$$

Use $B_0 = E_0/c$ to find B_0:

$$B_0=\frac{346\text{ MV/m}}{3\times10^8\text{ m/s}}=\boxed{1.15\,\text{T}}$$

***45 ••**

Picture the Problem We can determine the direction of propagation of the wave, its wavelength, and its frequency by examining the argument of the cosine function. We can find *E* from $\left|\vec{S}\right|=E^2/\mu_0 c$ and *B* from $B = E/c$. Finally, we can use the definition of the Poynting vector and the given expression for $\vec{S}$ to find $\vec{E}$ and $\vec{B}$.

(*a*) $\boxed{\text{Because the argument of the cosine function is of the form } kx-\omega t, \text{ the wave propagates in the positive } x \text{ direction.}}$

(*b*) Examining the argument of the cosine function, we note that the wave number *k* of the wave is:

$$k=\frac{2\pi}{\lambda}=10\,\text{m}^{-1}$$

Solve for and evaluate λ:

$$\lambda=\frac{2\pi}{10\,\text{m}^{-1}}=\boxed{0.628\,\text{m}}$$

Examining the argument of the cosine function, we note that the angular frequency ω of the wave is:

$$\omega=2\pi f=3\times10^9\text{ s}^{-1}$$

Solve for and evaluate f to obtain:

$$f = \frac{3\times10^9\ \text{s}^{-1}}{2\pi} = \boxed{477\,\text{MHz}}$$

(c) Express the magnitude of $\vec{S}$ in terms of E:

$$\left|\vec{S}\right| = \frac{E^2}{\mu_0 c}$$

Solve for E:

$$E = \sqrt{\mu_0 c\left|\vec{S}\right|}$$

Substitute numerical values and evaluate E:

$$E = \sqrt{\left(3\times10^8\ \text{m/s}\right)\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(100\,\text{W/m}^2\right)} = 194\,\text{V/m}$$

Because $\vec{S}(x,t) = \left(100\,\text{W/m}^2\right)\cos^2\left[10x - \left(3\times10^9\right)t\right]\hat{i}$ and $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$:

$$\boxed{\vec{E}(x,t) = (194\,\text{V/m})\cos\left[10x - \left(3\times10^9\right)t\right]\hat{j}}$$

Use $B = E/c$ to evaluate B:

$$B = \frac{194\,\text{V/m}}{3\times10^8\ \text{m/s}} = 0.647\ \mu\text{T}$$

Because $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$, the direction of $\vec{B}$ must be such that the cross product of $\vec{E}$ with $\vec{B}$ is in the positive x direction:

$$\boxed{\vec{B}(x,t) = (0.647\ \mu\text{T})\cos\left[10x - \left(3\times10^9\right)t\right]\hat{k}}$$

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Picture the Problem We can use the definition of the electric field between the plates of the parallel-plate capacitor and the definition of the displacement current to show that the displacement current in the capacitor is equal to the conduction current in the capacitor leads. In (b) we can use the definition of the Poynting vector and the directions of the electric and magnetic fields to determine the direction of the Poynting vector between the capacitor plates. In (c), we'll demonstrate that the flux of $\vec{S}$ into the region between the plates is equal to the rate of change of the energy stored in the capacitor by evaluating these quantities separately and showing that they are equal.

(a) The electric field between the plates of the capacitor is given by:

$$E = \frac{V(t)}{d} = \frac{V}{d}\left(1 - e^{-t/RC}\right)$$

The displacement current is proportional to the rate at which the

$$I_D(t) = \epsilon_0\frac{d\phi_e}{dt} = \epsilon_0\frac{d}{dt}(AE) = \epsilon_0 A\frac{dE}{dt}$$

flux is changing between the plates:

Substitute for E and carry out the details of the differentiation to obtain:

$$I_D(t) = \epsilon_0 A \frac{d}{dt}\left[\frac{V}{d}\left(1 - e^{-t/RC}\right)\right]$$
$$= \frac{\epsilon_0 AV}{d}\frac{d}{dt}\left[\left(1 - e^{-t/RC}\right)\right]$$
$$= \frac{\epsilon_0 AV}{d}\frac{d}{dt}\left[-e^{-t/RC}\right]$$
$$= \frac{\epsilon_0 AV}{dRC}e^{-t/RC}$$

Because the capacitance of an air-filled-parallel-plate capacitor is given by $C = \frac{\epsilon_0 A}{d}$:

$$I_D(t) = \frac{CV}{RC}e^{-t/RC} = \boxed{I(t)}$$

(b) Apply Ampere's law to a closed circular path of radius r (the radius of the capacitor plates) to obtain:

$$B(2\pi r) = \mu_0 I_C = \mu_0 I_D$$

Substitute for I_D from (a):

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{\pi r^2 V}{d(RC)}e^{-t/RC}$$

Solve for B to obtain:

$$B = \mu_0 \epsilon_0 \frac{rV}{2d(RC)}e^{-t/RC}$$

$$\boxed{\text{Because } \vec{E} \text{ is perpendicular to the plates of the capacitor and } \vec{B} \text{ is tangent to circles that are concentric and whose center is through the middle of the capacitor plates, } \vec{S} \text{ points radially inward toward the center of the capacitor.}}$$

(c) The magnitude of the Poynting vector is:

$$\left|\vec{S}\right| = I = \frac{BE}{\mu_0}$$

Substitute for B and E and simplify to obtain:

$$I = \boxed{\frac{\epsilon_0}{2}\frac{V^2 r}{d^2 RC}e^{-t/RC}\left(1 - e^{-t/RC}\right)}$$

The total power is:

$$P = \frac{dE}{dt} = 2\pi r d I$$

Substitute for I to obtain:

$$\frac{dE}{dt} = \in_0 \frac{V^2 \pi r^2}{dRC} e^{-t/RC}\left(1 - e^{-t/RC}\right)$$

Because the capacitance of an air-filled-parallel-plate capacitor is given by $C = \frac{\in_0 \pi r^2}{d}$:

$$\frac{dE}{dt} = \frac{V^2}{R} e^{-t/RC}\left(1 - e^{-t/RC}\right) \qquad (1)$$

The energy in the capacitor at any time is:

$$E = \frac{1}{2} C[V(t)]^2$$

Differentiate E with respect to time to obtain:

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2} C(V(t))^2\right] = CV(t)\frac{dV(t)}{dt}$$

Substitute for $V(t)$ and complete the differentiation to obtain:

$$\frac{dE}{dt} = \frac{V^2}{R} e^{-t/RC}\left(1 - e^{-t/RC}\right) \qquad (2)$$

The equivalence of equations (1) and (2) proves that the flux of $\vec{S}$ into this region is equal to the rate of change of the energy stored in the capacitor.

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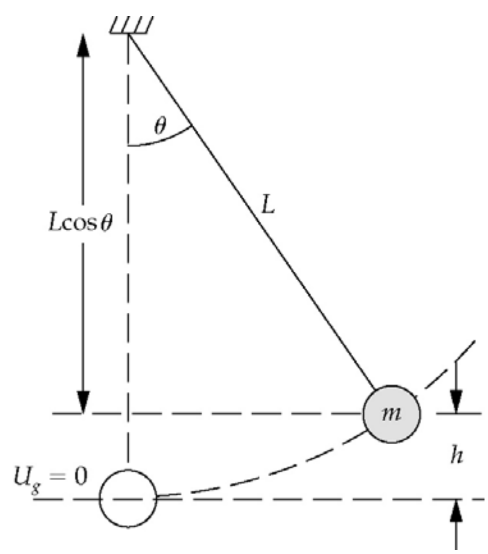
Picture the Problem The diagram shows the displacement of the pendulum bob, through an angle θ, as a consequence of the complete absorption of the radiation incident on it. We can use conservation of energy (mechanical energy is conserved *after* the collision) to relate the maximum angle of deflection of the pendulum to the initial momentum of the pendulum bob. Because the displacement of the bob during the absorption of the pulse is negligible, we can use conservation of momentum (conserved *during* the collision) to equate the momentum of the electromagnetic pulse to the initial momentum of the bob.



Apply conservation of energy to obtain:

$$K_f - K_i + U_f - U_i = 0$$

or, since $U_i = K_f = 0$ and $K_i = p_i^2/2m$,

$$-\frac{p_i^2}{2m}+U_f=0$$

U_f is given by:

$$U_f=mgh=mgL(1-\cos\theta)$$

Substitute for U_f:

$$-\frac{p_i^2}{2m}+mgL(1-\cos\theta)=0$$

Solve for θ to obtain:

$$\theta=\cos^{-1}\left(1-\frac{p_i^2}{2m^2gL}\right)$$

Use conservation of momentum to relate the momentum of the electromagnetic pulse to the initial momentum p_i of the pendulum bob:

$$p_{\text{em wave}}=\frac{U}{c}=\frac{P\Delta t}{c}=p_i$$

where Δt is the duration of the pulse.

Substitute for p_i:

$$\theta=\cos^{-1}\left(1-\frac{P^2(\Delta t)^2}{2m^2c^2gL}\right)$$

Substitute numerical values and evaluate θ:

$$\theta=\cos^{-1}\left(1-\frac{(1000\,\text{MW})^2(200\,\text{ns})^2}{2(10\,\text{mg})^2(3\times10^8\text{ m/s})^2(9.81\,\text{m/s}^2)(0.04\,\text{m})}\right)=\boxed{6.10\times10^{-3}\text{ degrees}}$$

Remarks: The solution presented here is valid only if the displacement of the bob during the absorption of the pulse is negligible. (Otherwise, the horizontal component of the momentum of the pulse-bob system is not conserved during the collision.) We can show that the displacement during the pulse-bob collision is small by solving for the speed of the bob after absorbing the pulse. Applying conservation of momentum ($mv = P(\Delta t)/c$) and solving for v gives $v = 6.67\times10^{-7}$ m/s. This speed is so slow compared to c, we can conclude that the duration of the collision is extremely close to 200 ns (the time for the pulse to travel its own length). Traveling at 6.67×10^{-7} m/s for 200 ns, the bob would travel 1.33×10^{-13} m—a distance 1000 times smaller that the diameter of a hydrogen atom. (Since 6.67×10^{-7} m/s is the maximum speed of the bob during the collision, the bob would actually travel less than 1.33×10^{-13} m during the collision.)

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Picture the Problem We can use the definitions of pressure and the relationship between radiation pressure and the intensity of the radiation to find the force due to radiation pressure on one of the mirrors.

(*a*) Because only about 0.01 percent of the energy inside the laser "leaks out", the average power of the radiation incident on one of the mirrors is:

$$P = \frac{15\,\text{W}}{10^{-4}} = \boxed{1.50\times10^{5}\ \text{W}}$$

(*b*) Use the definition of radiation pressure to obtain:

$$P_r = \frac{F}{A}$$

where *F* is the force due to radiation pressure and *A* is the area of the mirror on which the radiation is incident.

The radiation pressure is also related to the intensity of the radiation:

$$P_r = \frac{2I}{c} = \frac{2P}{Ac}$$

where *P* is the power of the laser and the factor of 2 is due to the fact that the mirror is essentially totally reflecting.

Equate the two expression for the radiation pressure and solve for *F*:

$$\frac{F}{A} = \frac{2P}{Ac} \Rightarrow F = \frac{2P}{c}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{2\left(1.50\times10^{5}\ \text{W}\right)}{3\times10^{8}\ \text{m/s}} = \boxed{1.00\,\text{mN}}$$

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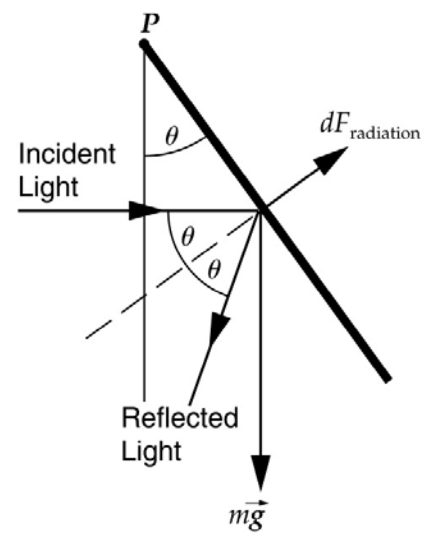
Picture the Problem The card, pivoted at point *P*, is shown in the diagram. Note that the force exerted by the radiation acts along the dashed line. Let the length of the card be ℓ, the width of the card be *w*, and the force acting on an area $dA = w\,dx$ be $dF_{\text{radiation}}$. We can find the total torque exerted on the card due to radiation pressure by integrating $d\tau_{\text{radiation}}$ over the length ℓ of the card and then relate the intensity of the light to the angle θ by applying the condition for rotational equilibrium to the card.



Express the torque, due to *F*, acting at a distance *x* from *P*:

$$d\tau_{\text{radiation}} = x\,dF_{\text{radiation}}$$

Relate $dF_{\text{radiation}}$ to the intensity of the light:

$$dF_{\text{radiation}} = \frac{2I}{c}\cos\theta\, dA$$

where the factor of 2 arises from the total reflection of the radiation incident on the mirror.

Substitute to obtain:

$$d\tau_{\text{radiation}} = \frac{2I}{c}\cos\theta\, xdA = \frac{2I}{c}\cos\theta\, xwdx$$

Integrate x from 0 to ℓ:

$$\tau_{\text{radiation}} = \frac{2Iw}{c}\cos\theta\int_0^{\ell} xdx = \frac{2Iw}{c}\cos\theta\left(\frac{\ell^2}{2}\right) = \frac{IA\ell}{c}\cos\theta$$

Apply $\sum \tau_P = 0$ to the card:

$$\frac{IA\ell}{c}\cos\theta - \left(\tfrac{1}{2}\ell\sin\theta\right)mg = 0$$

Solve for I to obtain:

$$I = \frac{mgc}{2A}\tan\theta$$

Substitute numerical values and evaluate I:

$$I = \frac{(2\,\text{g})\left(9.81\,\text{m/s}^2\right)\left(3\times10^8\ \text{m/s}\right)}{2(0.1\,\text{m})(0.15\,\text{m})}\tan 1° = \boxed{3.42\,\text{MW/m}^2}$$

The Wave Equation for Electromagnetic Waves

50 •

Picture the Problem We can show that Equation 30-17*a* is satisfied by the wave function E_y by showing that the ratio of $\partial^2E_y/\partial x^2$ to $\partial^2E_y/\partial t^2$ is $1/c^2$ where $c = \omega/k$.

Differentiate $E_y = E_0\sin(kx-\omega t)$ with respect to x:

$$\frac{\partial E_y}{\partial x} = \frac{\partial}{\partial x}\left[E_0\sin(kx-\omega t)\right] = kE_0\cos(kx-\omega t)$$

Evaluate the second partial derivative of E_y with respect to x:

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial}{\partial x}\left[kE_0\cos(kx-\omega t)\right] = -k^2E_0\sin(kx-\omega t) \qquad (1)$$

Differentiate $E_y = E_0 \sin(kx - \omega t)$ with respect to *t*:

$$\frac{\partial E_y}{\partial t} = \frac{\partial}{\partial t}\left[E_0 \sin(kx - \omega t)\right] = -\omega E_0 \cos(kx - \omega t)$$

Evaluate the second partial derivative of E_y with respect to *t*:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial}{\partial t}\left[-\omega E_0 \cos(kx - \omega t)\right] = -\omega^2 E_0 \sin(kx - \omega t) \quad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\dfrac{\partial^2 E_y}{\partial x^2}}{\dfrac{\partial^2 E_y}{\partial t^2}} = \frac{-k^2 E_0 \sin(kx - \omega t)}{-\omega^2 E_0 \sin(kx - \omega t)} = \frac{k^2}{\omega^2}$$

or

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

provided $c = \omega/k$.

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Picture the Problem Substitute numerical values and evaluate *c*:

$$c = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{3.00 \times 10^8\ \text{m/s}}$$

***52** •••

Picture the Problem We can use Figures 30-10 and 30-11and a derivation similar to that in the text to obtain the given results.

In Figure 30-11, replace B_z by E_z. For Δx small:

$$E_z(x_2) = E_z(x_1) + \frac{\partial E_z}{\partial x}\Delta x$$

Evaluate the line integral of $\vec{E}$ around the rectangular area $\Delta x \Delta z$:

$$\oint \vec{E} \cdot d\vec{\ell} \approx -\frac{\partial E_z}{\partial x}\Delta x \Delta z \quad (1)$$

Express the magnetic flux through the same area:

$$\int_S B_n\, dA = B_y \Delta x \Delta z$$

Apply Faraday's law to obtain:

$$\oint \vec{E} \cdot d\vec{\ell} \approx -\frac{\partial}{\partial t}\int_S B_n\, dA = -\frac{\partial}{\partial t}\left(B_y \Delta x \Delta z\right) = -\frac{\partial B_y}{\partial t}\Delta x \Delta z$$

Substitute in equation (1) to obtain:

$$-\frac{\partial E_z}{\partial x}\Delta x\Delta z = -\frac{\partial B_y}{\partial t}\Delta x\Delta z$$

or

$$\boxed{\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}}$$

In Figure 30-10, replace E_y by B_y and evaluate the line integral of $\vec{B}$ around the rectangular area $\Delta x\Delta z$:

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \in_0 \int_S E_n dA$$

provided there are no conduction currents.

Evaluate these integrals to obtain:

$$\boxed{\frac{\partial B_y}{\partial x} = \mu_0 \in_0 \frac{\partial E_z}{\partial t}}$$

(*b*) Using the first result obtained in (*a*), find the second partial derivative of E_z with respect to x:

$$\frac{\partial}{\partial x}\left(\frac{\partial E_z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial B_y}{\partial t}\right)$$

or

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial x}\right)$$

Use the second result obtained in (*a*) to obtain:

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial}{\partial t}\left(\mu_0\varepsilon_0 \frac{\partial E_z}{\partial t}\right) = \mu_0 \in_0 \frac{\partial^2 E_z}{\partial t^2}$$

or, because $\mu_0 \in_0 = 1/c^2$,

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}}\ .$$

Using the second result obtained in (*a*), find the second partial derivative of B_y with respect to x:

$$\frac{\partial}{\partial x}\left(\frac{\partial B_y}{\partial x}\right) = \mu_0 \in_0 \frac{\partial}{\partial x}\left(\frac{\partial E_z}{\partial t}\right)$$

or

$$\frac{\partial^2 B_y}{\partial x^2} = \mu_0 \in_0 \frac{\partial}{\partial t}\left(\frac{\partial E_z}{\partial x}\right)$$

Use the second result obtained in (*a*) to obtain:

$$\frac{\partial^2 B_y}{\partial x^2} = \mu_0 \in_0 \frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial t}\right) = \mu_0 \in_0 \frac{\partial^2 B_y}{\partial t^2}$$

or, because $\mu_0 \in_0 = 1/c^2$,

$$\boxed{\frac{\partial^2 B_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2}}\ .$$

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Picture the Problem We can show that these functions satisfy the wave equations by differentiating them twice (using the chain rule) with respect to *x* and *t* and equating the expressions for the second partial of *f* with respect to *u*.

Let $u = x - vt$. Then:

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial f}{\partial u} = \frac{\partial f}{\partial u}$$

and

$$\frac{\partial f}{\partial t} = \frac{\partial u}{\partial t}\frac{\partial f}{\partial u} = -v\frac{\partial f}{\partial u}$$

Express the second derivatives of *f* with respect to *x* and *t* to obtain:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial u^2}$$

and

$$\frac{\partial^2 f}{\partial t^2} = v^2\frac{\partial^2 f}{\partial u^2}$$

Thus, for any *f*(*u*):

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

Let $u = x + vt$. Then:

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial f}{\partial u} = \frac{\partial f}{\partial u}$$

and

$$\frac{\partial f}{\partial t} = \frac{\partial u}{\partial t}\frac{\partial f}{\partial u} = v\frac{\partial f}{\partial u}$$

Express the second derivatives of *f* with respect to *x* and *t* to obtain:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial u^2}$$

and

$$\frac{\partial^2 f}{\partial t^2} = v^2\frac{\partial^2 f}{\partial u^2}$$

Thus, for any *f*(*u*):

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

General Problems

54 •

Picture the Problem We can substitute the appropriate units and simplify to show that the units of the Poynting vector are watts per square meter and that those of radiation pressure are newtons per square meter.

(*a*) Express the units of $\vec{S}$ and simplify:

$$\frac{\frac{\text{V}}{\text{m}}\times\text{T}}{\frac{\text{N}}{\text{A}^2}}=\frac{\frac{\text{J}}{\text{C}\cdot\text{m}}\times\frac{\text{N}}{\text{C}\cdot\frac{\text{m}}{\text{s}}}}{\frac{\text{N}}{\text{A}^2}}$$

$$=\frac{\frac{\text{J}}{\text{C}}}{\frac{\text{s}}{\text{C}}}=\frac{\text{J}}{\text{s}}=\boxed{\text{W}}$$

(*b*) Express the units of P_r and simply:

$$\frac{\frac{\text{W}}{\text{m}^2}}{\frac{\text{m}}{\text{s}}}=\frac{\frac{\text{J}}{\text{s}\cdot\text{m}^2}}{\frac{\text{m}}{\text{s}}}=\frac{\frac{\text{N}\cdot\text{m}}{\text{m}^2}}{\text{m}}=\boxed{\frac{\text{N}}{\text{m}^2}}$$

55 ••

Determine the Concept The current induced in a loop antenna is proportional to the time-varying magnetic field. For maximum signal, the antenna's plane should make an angle θ = 0° with the line from the antenna to the transmitter. For any other angle, the induced current is proportional to cos θ. The intensity of the signal is therefore proportional to cos θ.

56 ••

Picture the Problem We can use $c = f\lambda$ to find the wavelength. Examination of the argument of the cosine function will reveal the direction of propagation of the wave. We can find the magnitude, wave number, and angular frequency of the electric vector from the given information and the result of (*a*) and use these results to obtain $\vec{E}(z, t)$. Finally, we can use its definition to find the Poynting vector.

(*a*) Relate the wavelength of the wave to its frequency and the speed of light:

$$\lambda=\frac{c}{f}$$

Substitute numerical values and evaluate λ:

$$\lambda=\frac{3\times10^8\ \text{m/s}}{100\text{MHz}}=\boxed{3.00\,\text{m}}$$

From the sign of the argument of the cosine function and the spatial dependence on z, we can conclude that the wave propagates in the z direction.

(*b*) Express the amplitude of $\vec{E}$:

$$E = cB = \left(3\times10^{8}\ \text{m/s}\right)\left(10^{-8}\ \text{T}\right) = 3.00\ \text{V/m}$$

Find the angular frequency and wave number of the wave:

$$\omega = 2\pi\ f = 2\pi(100\ \text{MHz}) = 6.28\times10^{8}\ \text{s}^{-1}$$

and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.00\ \text{m}} = 2.09\ \text{m}^{-1}$$

Because $\vec{S}$ is in the positive z direction, $\vec{E}$ must be in the negative y direction in order to satisfy the Poynting vector expression:

$$\boxed{\vec{E}(z,t) = -(3.00\ \text{V/m})\cos\left[\left(2.09\ \text{m}^{-1}\right)z - \left(6.28\times10^{8}\ \text{s}^{-1}\right)t\right]\hat{j}}$$

(*c*) Use its definition to express the Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{-(3.00\ \text{V/m})\left(10^{-8}\ \text{T}\right)}{4\pi\times10^{-7}\ \text{N/A}^2}\cos^2\left[\left(2.09\ \text{m}^{-1}\right)z - \left(6.28\times10^{8}\ \text{s}^{-1}\right)t\right]\left(\hat{j}\times\hat{i}\right)$$

or

$$\boxed{\vec{S} = \left(23.9\ \text{mW/m}^2\right)\cos^2\left[\left(2.09\ \text{m}^{-1}\right)z - \left(6.28\times10^{8}\ \text{s}^{-1}\right)t\right]\hat{k}}$$

The intensity of the wave is the average magnitude of the Poynting vector. The average value of the square of the cosine function is 1/2:

$$I = \left|\vec{S}\right| = \tfrac{1}{2}\left(23.9\ \text{mW/m}^2\right) = \boxed{12.0\ \text{mW/m}^2}$$

***57 ••**

Picture the Problem The maximum rms voltage induced in the loop is given by $\mathcal{E}_{\text{rms}} = A\omega B_0/\sqrt{2}$, where A is the area of the loop, B_0 is the amplitude of the magnetic field, and ω is the angular frequency of the wave. We can use the definition of density and the expression for the intensity of an electromagnetic wave to derive an expression for B_0.

The maximum induced rms emf occurs when the plane of the loop is perpendicular to $\vec{B}$:

$$\mathcal{E}_{\text{rms}} = \frac{A\omega B_0}{\sqrt{2}} = \frac{\pi R^2\omega B_0}{\sqrt{2}} \qquad (1)$$

where R is the radius of loop of wire.

From the definition of intensity we have:

$$I = \frac{P}{4\pi r^2}$$

where r is the distance from the transmitter.

The intensity is also given by:

$$I = \frac{E_0 B_0}{2\mu_0} = \frac{B_0^2 c}{2\mu_0}$$

Substitute to obtain:

$$\frac{B_0^2 c}{2\mu_0} = \frac{P}{4\pi r^2}$$

Solve for B_0:

$$B_0 = \frac{1}{r}\sqrt{\frac{\mu_0 P}{2\pi c}}$$

Substitute in equation (1) to obtain:

$$\mathcal{E}_{\text{rms}} = \frac{\pi R^2 (2\pi f)}{\sqrt{2}\, r}\sqrt{\frac{\mu_0 P}{2\pi c}} = \frac{R^2 f}{\sqrt{2}\, r}\sqrt{\frac{2\pi^3 \mu_0 P}{c}}$$

Substitute numerical values and evaluate ε_{rms}:

$$\mathcal{E}_{\text{rms}} = \frac{(0.3\,\text{m})^2 (100\,\text{MHz})}{\sqrt{2}(10^5\,\text{m})}\sqrt{\frac{2\pi^3 (4\pi \times 10^{-7}\,\text{N/A}^2)(50\,\text{kW})}{3\times 10^8\,\text{m/s}}} = \boxed{7.25\,\text{mV}}$$

58 ••

Picture the Problem The voltage induced in the piece of wire is the product of the electric field and the length of the wire. The maximum rms voltage induced in the loop is given by $\mathcal{E} = A\omega B_0$, where A is the area of the loop, B_0 is the amplitude of the magnetic field, and ω is the angular frequency of the wave.

(*a*) Because E is independent of x:

$$V = E\ell$$

where ℓ is the length of the wire.

Substitute numerical values and evaluate V:

$$V = \left[(10^{-4}\,\text{N/C})\cos 10^6 t\right](0.5\,\text{m}) = \boxed{(50.0\,\mu\text{V})\cos 10^6 t}$$

(*b*) The voltage induced in a loop is given by:

$$\mathcal{E} = \omega B_0 A$$

where A is the area of the loop and B_0 is the amplitude of the magnetic field.

Eliminate B_0 in favor of E_0 and substitute for A to obtain:

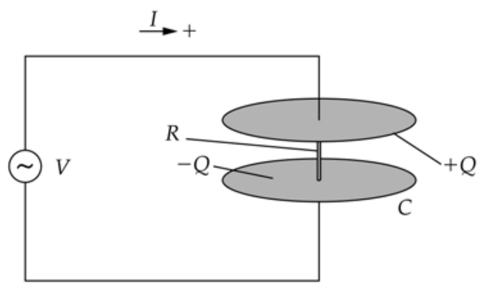
$$\mathcal{E} = \frac{\omega E_0 \pi R^2}{c}$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = \frac{\left(10^6\ \text{s}^{-1}\right)\left(10^{-4}\ \text{N/C}\right)\pi\left(0.2\ \text{m}\right)^2}{3\times10^8\ \text{m/s}} = \boxed{41.9\ \text{nV}}$$

59 ••

Picture the Problem Some of the charge entering the capacitor passes through the resistive wire while the rest of it accumulates on the upper plate. The total current is the rate at which the charge passes through the resistive wire plus the rate at which it accumulates on the upper plate. The magnetic field between the capacitor plates is due to both the current in the resistive wire and the displacement current though a surface bounded by a circle a distance r from the resistive wire. The phase difference between the supplied current and the applied voltage may be calculated using a phasor diagram.



(*a*) The current drawn by the capacitor is the sum of the conduction current through the resistance wire and dQ/dt, where Q is the charge on the upper plate of the capacitor:

$$I = I_c + \frac{dQ}{dt} \qquad (1)$$

Express the conduction current I_c in terms of the potential difference between the plates and the resistance of the wire:

$$I_c = \frac{V}{R} = \frac{V_0}{R}\sin\omega t$$

Express the displacement current between the capacitor plates. Let C be the capacitance of the capacitor:

$$Q = CV$$

so

$$\frac{dQ}{dt} = C\frac{dV}{dt} = \omega C V_0 \cos\omega t$$

Substitute in equation (1):

$$I = \frac{V_0}{R}\sin\omega t + \omega C V_0 \cos\omega t \qquad (2)$$

Using Equation 24-10 for the capacitance of a parallel-plate capacitor with plate area *A* and plate separation *d* we have:

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \pi a^2}{d}$$

Substituting for *C* equation 2 gives:

$$\boxed{I = V_0\left(\frac{1}{R}\sin\omega t + \frac{\omega \epsilon_0 \pi a^2}{d}\cos\omega t\right)}$$

(*b*) Apply the generalized form of Ampere's law to a circular path of radius *r* centered within the plates of the capacitor, where I'_{d} is the displacement current through the flat surface *S* bounded by the path and I_{c} is the conduction current through the same surface:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0\left(I_{\text{c}} + I'_{\text{d}}\right)$$

By symmetry the line integral is *B* times the circumference of the circle of radius *r*:

$$B(2\pi r) = \mu_0\left(I_{\text{c}} + I'_{\text{d}}\right) \qquad (3)$$

In the region between the capacitor plates there is a uniform electric field due to the surface charges +*Q* and –*Q*. The associated displacement current through *S* is:

$$I'_{\text{d}} = \epsilon_0 \frac{d\phi_{\text{e}}}{dt} = \epsilon_0 \frac{d}{dt}\left(A'E\right)$$

$$= \epsilon_0 A' \frac{dE}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt}$$

provided $(r \le a)$

To evaluate the displacement current we first must evaluate *E* everywhere on *S*. Near the surface of a conductor $E = \sigma/\epsilon_0$ (Equation 22-25), where σ is the surface charge density:

$E = \sigma/\epsilon_0$, where $\sigma = Q/A = Q/\left(\pi a^2\right)$

so

$$E = \frac{Q}{\epsilon_0 \pi a^2}$$

Substituting for E in the equation for I'_{d} gives:

$$I'_{\text{d}} = \epsilon_0 \pi r^2 \frac{dE}{dt} = \epsilon_0 \pi r^2 \frac{d}{dt}\left(\frac{Q}{\epsilon_0 \pi a^2}\right)$$
$$= \frac{r^2}{d^2}\frac{dQ}{dt} = \frac{r^2}{d^2}\frac{d}{dt}\left(V_0 \sin \omega t\right)$$
$$= \omega \frac{r^2}{d^2} V_0 \cos \omega t$$

Substituting for I_{c} and I'_{d} in equation (3) and solving for B gives:

$$B(r) = \frac{\mu_0\left(I_{\text{c}} + I'_{\text{d}}\right)}{2\pi r}$$
$$= \frac{\mu_0}{2\pi r}\left(\frac{V_0}{R}\sin \omega t + \omega \frac{r^2}{a^2} V_0 \cos \omega t\right)$$
$$= \boxed{\frac{\mu_0 V_0}{2\pi r}\left(\frac{1}{R}\sin \omega t + \omega \frac{r^2}{a^2} \cos \omega t\right)}$$

(c) Both the charge Q and the conduction current I_{c} are in phase with V. However, dQ/dt, which is equal to the displacement current I_{d} through S for $r \geq a$, lags V by 90°. (Mathematically, cos ωt lags behind sin ωt by 90°.) The voltage V leads the current $I = I_{\text{c}} + I_{\text{d}}$ by phase angle δ. The current relation is expressed in terms of the current amplitudes:

$$I = I_{\text{c}} + I_{\text{d}}$$

or

$$I_{\text{max}} \sin(\omega t + \delta) = I_{\text{c,max}} \sin \omega t + I_{\text{d,max}} \cos \omega t$$

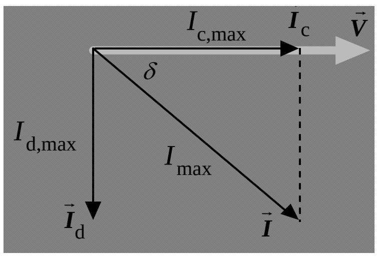
The values of the conduction and displacement current amplitudes are obtained by comparison with the answer to part (a):

$$I_{\text{c,max}} = \frac{V_0}{R}$$

and

$$I_{\text{d,max}} = \frac{\omega \epsilon_0 \pi a^2 V_0}{d}$$

A phasor diagram for adding the currents I_{c} and I_{d} is shown to the right. The conduction current I_{c} is in phase with the voltage V across the resistor and I_{d} lags behind it by 90°:



From the phasor diagram we have:

$$\tan\delta = \frac{I_{\text{d,max}}}{I_{\text{c,max}}} = \frac{V_0 \dfrac{\omega \in_0 \pi a^2}{d}}{V_0/R}$$

$$= \frac{R\omega \in_0 \pi a^2}{d}$$

so

$$\boxed{\delta = \tan^{-1}\left(\frac{R\omega \in_0 \pi a^2}{d}\right)}$$

Remarks: The capacitor and the resistive wire are connected in parallel. The potential difference across each of them is the applied voltage $V_0 \sin \omega t$.

60 ••

Picture the Problem The total force on the surface is the sum of the force due to the reflected radiation and the force due to the absorbed radiation. From the conservation of momentum, the force due to the 10 kW that are reflected is twice the force due to the 10 kW that are absorbed.

Express the total force on the surface:

$$F_{\text{tot}} = F_{\text{r}} + F_a$$

Substitute for F_{r} and F_{a} to obtain:

$$F_{\text{tot}} = \frac{2\left(\frac{1}{2}P\right)}{c} + \frac{\frac{1}{2}P}{c} = \frac{3P}{2c}$$

Substitute numerical values and evaluate F_{tot}:

$$F_{\text{tot}} = \frac{3(20\,\text{kW})}{2(3\times10^8\ \text{m/s})} = \boxed{0.100\,\text{mN}}$$

***61** ••

Picture the Problem We can use the definition of the Poynting vector and the relationship between $\vec{B}$ and $\vec{E}$ to find the instantaneous Poynting vectors for each of the resultant wave motions and the fact that the time average of the cross product term is zero for $\omega_1 \neq \omega_2$, and ½ for the square of cosine function to find the time-averaged Poynting vectors.

(*a*) Because $\vec{E}_1$ and $\vec{E}_2$ propagate in the *x* direction:

$$\vec{E}\times\vec{B} = \mu_0 S\hat{\boldsymbol{i}} \Rightarrow \vec{B} = B\hat{\boldsymbol{k}}$$

Express *B* in terms of E_1 and E_2:

$$B = \frac{1}{c}(E_1 + E_2)$$

Substitute for E_1 and E_2 to obtain:

$$\vec{B} = \frac{1}{c}\left[E_{1,0}\cos(k_1x - \omega_1 t) + E_{2,0}\cos(k_2x - \omega_2 t + \delta)\right]\hat{k}$$

Express the instantaneous Poynting vector for the resultant wave motion:

$$\begin{aligned}\vec{S} &= \frac{1}{\mu_0}\left(E_{1,0}\cos(k_1x - \omega_1 t) + E_{2,0}\cos(k_2x - \omega_2 t + \delta)\right)\hat{j} \\ &\quad \times \frac{1}{c}\left(E_{1,0}\cos(k_1x - \omega_1 t) + E_{2,0}\cos(k_2x - \omega_2 t + \delta)\right)\hat{k} \\ &= \frac{1}{\mu_0 c}\left(E_{1,0}\cos(k_1x - \omega_1 t) + E_{2,0}\cos(k_2x - \omega_2 t + \delta)\right)^2\left(\hat{j}\times\hat{k}\right) \\ &= \boxed{\begin{aligned}\frac{1}{\mu_0 c}\Big[&E_{1,0}^2\cos^2(k_1x - \omega_1 t) + 2E_{1,0}E_{2,0}\cos(k_1x - \omega_1 t) \\ &\times\cos(k_2x - \omega_2 t + \delta) + E_{2,0}^2\cos^2(k_2x - \omega_2 t + \delta)\Big]\,\hat{i}\end{aligned}}\end{aligned}$$

(*b*) The time average of the cross product term is zero for $\omega_1 \neq \omega_2$, and the time average of the square of the cosine terms is ½:

$$\boxed{\vec{S}_{\text{av}} = \frac{1}{2\mu_0 c}\left[E_{1,0}^2 + E_{2,0}^2\right]\hat{i}}$$

(*c*) In this case $\vec{B}_2 = -B\hat{k}$ because the wave with $k = k_2$ propagates in the $-\hat{i}$ direction. The magnetic field is then:

$$\vec{B} = \frac{1}{c}\left[E_{1,0}\cos(k_1x - \omega_1 t) - E_{2,0}\cos(k_2x + \omega_2 t + \delta)\right]\hat{k}$$

Express the instantaneous Poynting vector for the resultant wave motion:

$$\begin{aligned}\vec{S} &= \frac{1}{\mu_0}\left(E_{1,0}\cos(k_1x - \omega_1 t) + E_{2,0}\cos(k_2x - \omega_2 t + \delta)\right)\hat{j} \\ &\quad \times \frac{1}{c}\left(E_{1,0}\cos(k_1x - \omega_1 t) - E_{2,0}\cos(k_2x + \omega_2 t + \delta)\right)\hat{k} \\ &= \boxed{\frac{1}{\mu_0 c}\left[E_{1,0}^2\cos^2(k_1x - \omega_1 t) - E_{2,0}^2\cos^2(k_2x + \omega_2 t + \delta)\right]\hat{i}}\end{aligned}$$

(*d*) The time average of the square of the cosine terms is ½:

$$\boxed{\vec{S}_{\text{av}} = \frac{1}{2\mu_0 c}\left[E_{1,0}^2 - E_{2,0}^2\right]\hat{i}}$$

***62** ••

Picture the Problem We can use the definitions of power and intensity to express the area of the surface as a function of *P*, *I*, and the efficiency ε.

Use the definition of power to relate the required surface area to the intensity of the solar radiation:

$$P = \frac{E}{t}\varepsilon = IA\varepsilon$$

where ε is the efficiency of the system.

Solve for A to obtain:

$$A = \frac{P}{I\varepsilon}$$

Substitute numerical values and evaluate A:

$$A = \frac{25\,\text{kW}}{0.3\left(0.75\,\text{kW/m}^2\right)} = \boxed{111\,\text{m}^2}$$

63 ••

Picture the Problem We can use the relationship between the average value of the Poynting vector (the intensity), E_0, and B_0 to find B_0. The application of Faraday's law will allow us to find the emf induced in the antenna. The emf induced in a 2-m wire oriented in the direction of the electric field can be found using $\mathcal{E} = E\ell$ and the relationship between E and B.

(*a*) The intensity of the signal is related the amplitude of the magnetic field in the wave:

$$S_{\text{av}} = I = \frac{E_0 B_0}{2\mu_0} = \frac{cB_0^2}{2\mu_0}$$

Solve for B_0:

$$B_0 = \sqrt{\frac{2\mu_0 I}{c}}$$

Substitute numerical values and evaluate B_0:

$$B_0 = \sqrt{\frac{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(10^{-14}\ \text{W/m}^2\right)}{3\times10^8\ \text{m/s}}} = \boxed{9.15\times10^{-15}\ \text{T}}$$

(*b*) Apply Faraday's law to the antenna coil to obtain:

$$|\mathcal{E}| = \frac{d}{dt}(BA) = A\frac{d}{dt}\left(NK_\text{m}B_0\sin\omega t\right) = NK_\text{m}AB_0\omega\cos\omega t$$

Substitute numerical values and evaluate $|\mathcal{E}|$:

$$|\mathcal{E}| = 2000(200)\pi(0.01\,\text{m})^2\left(9.15\times10^{-15}\ \text{T}\right)[2\pi(140\,\text{kHz})]\cos[2\pi(140\,\text{kHz})]t = \boxed{(1.01\,\mu\text{V})\cos\left(8.80\times10^5\ \text{s}^{-1}\right)t}$$

(*c*) The voltage induced in the wire

$$\mathcal{E} = E\ell$$

is the product of its length ℓ and the amplitude of electric field E_0:

Relate E to B:

$$E = cB = cB_0 \sin \omega t$$

Substitute for E to obtain:

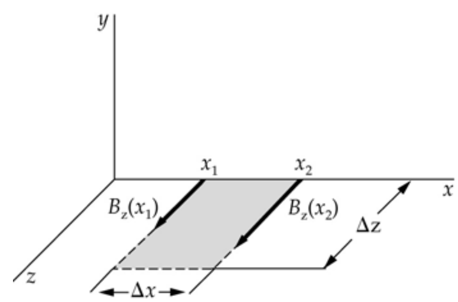
$$\mathcal{E} = c\ell B_0 \sin \omega t$$

Substitute numerical values and evaluate $|\mathcal{E}|$:

$$\mathcal{E} = (3\times10^8\ \text{m/s})(2\ \text{m})(9.15\times10^{-15}\ \text{T})\sin[2\pi(140\ \text{kHz})]t$$
$$= \boxed{(5.49\ \mu\text{V})\sin(8.80\times10^5\ \text{s}^{-1})t}$$

64 ••

Picture the Problem We'll choose the curve with sides Δx and Δz in the xy plane shown in the diagram and apply Equation 30-6d to show that $\dfrac{\partial B_z}{\partial x} = -\mu_0 \in_0 \dfrac{\partial E_y}{\partial t}$.



Because Δx is very small, we can approximate the difference in B_z at the points x_1 and x_2 by:

$$B_z(x_2) - B_z(x_1) = \Delta B \approx \frac{\partial B_z}{\partial x}\Delta x$$

Then:

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\ell} \approx \mu_0 \in_0 \frac{\partial E_y}{\partial t}\Delta x \Delta z$$

The flux of the electric field through this curve is approximately:

$$\int_S E_{\text{n}}\,dA = E_y \Delta x \Delta y$$

Apply Faraday's law to obtain:

$$\frac{\partial B_z}{\partial x}\Delta x\Delta z = -\mu_0 \in_0 \frac{\partial E_y}{\partial t}\Delta x\Delta z$$

or

$$\boxed{\frac{\partial B_z}{\partial x} = -\mu_0 \in_0 \frac{\partial E_y}{\partial t}}$$

***65** •••

Picture the Problem We can use Ohm's law to relate the electric field E in the conductor to I, ρ, and a and Ampere's law to find the magnetic field B just outside the conductor.

Knowing $\vec{E}$ and $\vec{B}$ we can find $\vec{S}$ and, using its normal component, show that the rate of energy flow into the conductor equals I^2R, where R is the resistance.

(*a*) Apply Ohm's law to the cylindrical conductor to obtain:

$$V = IR = \frac{I\rho L}{A} = \frac{I\rho L}{\pi a^2} = EL$$

Solve for E:

$$E = \boxed{\frac{I\rho}{\pi a^2}}$$

(*b*) Apply Ampere's law to a circular path of radius a at the surface of the cylindrical conductor:

$$\int_C \vec{B}\cdot d\vec{\ell} = B(2\pi a) = \mu_0 I_{\text{enclosed}} = \mu_0 I$$

Solve for B to obtain:

$$B = \boxed{\frac{\mu_0 I}{2\pi a}}$$

(*c*) The electric field at the surface of the conductor is in the direction of the current and the magnetic field at the surface is tangent to the surface. Use the results of (*a*) and (*b*) and the right-hand rule to evaluate $\vec{S}$:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\left(\frac{I\rho}{\pi a^2}\right)\hat{\boldsymbol{u}}_{\text{parallel}} \times \left(\frac{\mu_0 I}{2\pi a}\right)\hat{\boldsymbol{u}}_{\text{tangent}} = \boxed{-\frac{I^2\rho}{2\pi^2 a^3}\hat{\boldsymbol{r}}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector directed radially outward from the cylindrical conductor.

(*d*) The flux through the surface of the conductor into the conductor is:

$$\oint S_n dA = S(2\pi aL)$$

Substitute for S_n, the *inward* component of $\vec{S}$, and simplify to obtain:

$$\oint S_n dA = \frac{I^2\rho}{2\pi^2 a^3}(2\pi aL) = \frac{I^2\rho L}{\pi a^2}$$

Since $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi a^2}$:

$$\oint S_n dA = \boxed{I^2 R}$$

66 •••

Picture the Problem We can use Faraday's law to express the induced electric field at a distance $r < R$ from the solenoid axis in terms of the rate of change of magnetic flux and $B = n\mu_0 at$ to express B in terms of the current in the windings of the solenoid. We can use the results of (*a*) to find the magnitude and direction of the Poynting vector $\vec{S}$ at the cylindrical surface $r = R$ just inside the solenoid windings. In part (c) we'll use the definition of flux and the expression for the magnetic energy in a given region to show that the flux of $\vec{S}$ into the solenoid equals the rate of increase of the magnetic energy inside the solenoid.

(*a*) Apply Faraday's law to a circular path of radius $r < R$:

$$\oint_C E \cdot d\ell = E(2\pi r) = -\frac{d\phi_m}{dt}$$

Solve for E to obtain:

$$E = -\frac{1}{2\pi r}\frac{d\phi_m}{dt} \qquad (1)$$

Express the magnetic field inside a long solenoid:

$$B = n\mu_0 I = n\mu_0 at$$

The magnetic flux through a circle of radius r is:

$$\phi_m = BA = n\mu_0 at\pi r^2$$

Substitute in equation (1) to obtain:

$$E = -\frac{1}{2\pi r}\frac{d}{dt}\left[n\mu_0 at\pi r^2\right] = \boxed{-\frac{n\mu_0 a r}{2}}$$

(*b*) Express the magnitude of $\vec{S}$ at $r = R$:

$$S = \frac{EB}{\mu_0}$$

At the cylindrical surface just inside the windings:

$$B = n\mu_0 at$$

Substitute to obtain:

$$S = \frac{\left(\frac{n\mu_0 a R}{2}\right)(n\mu_0 at)}{\mu_0} = \frac{n^2\mu_0 a^2 Rt}{2}$$

Because the field $\vec{E}$ is tangential and directed so as to give an induced current that opposes the increase in $\vec{B}$, $\vec{E}\times\vec{B}$ is a vector that points toward the axis of the solenoid. Hence:

$$\vec{S} = \boxed{-\frac{n^2\mu_0 a^2 Rt}{2}\hat{r}}$$

where $\hat{r}$ is a unit vector that points radially outward.

(*c*) Consider a cylindrical surface of length L and radius R. Because $\vec{S}$ points inward, the energy flowing

$$\oint S_n dA = 2\pi RLS = 2\pi RL\left(\frac{n^2\mu_0 a^2 Rt}{2}\right)$$
$$= n^2\pi\mu_0 R^2 La^2 t$$

into the solenoid per unit time is:

Express the magnetic energy in the solenoid:

$$U_B = u_m V = \frac{B^2}{2\mu_0}\left(\pi R^2 L\right) = \frac{(\mu_0 nat)^2}{2\mu_0}\left(\pi R^2 L\right) = \frac{n^2\pi\,\mu_0 R^2 L a^2 t^2}{2}$$

Evaluate dU_B/dt:

$$\frac{dU_B}{dt} = \frac{d}{dt}\left[\frac{n^2\pi\,\mu_0 R^2 L a^2 t^2}{2}\right] = \boxed{n^2\pi\,\mu_0 R^2 L a^2 t} = \oint S_n dA$$

***67 •••**

Picture the Problem We can use a condition for translational equilibrium to obtain an expression relating the forces due to gravity and radiation pressure that act on the particles. We can express the force due to radiation pressure in terms of the radiation pressure and the effective cross sectional area of the particles and the radiation pressure in terms of the intensity of the solar radiation. We can solve the resulting equation for *r*.

Apply the condition for translational equilibrium to the particle:

$$F_r - F_g = 0$$

or, since $F_r = P_r A$ and $F_g = mg$,

$$P_r A - \frac{GM_s m}{R^2} = 0 \qquad (1)$$

The radiation pressure P_r depends on the intensity of the radiation I:

$$P_r = \frac{I}{c}$$

The intensity of the solar radiation at a distance R is:

$$I = \frac{P}{4\pi R^2}$$

Substitute to obtain:

$$P_r = \frac{P}{4\pi R^2 c}$$

Substitute for P_r, A, and m in equation (1):

$$\frac{P}{4\pi R^2 c}\left(\pi r^2\right) - \frac{\frac{4}{3}\pi r^3 \rho G M_s}{R^2} = 0$$

Solve for *R* to obtain:

$$r=\frac{3P}{16\pi\,\rho c GM_{\text{s}}}$$

Substitute numerical values and evaluate *r*:

$$r=\frac{3\left(3.83\times10^{26}\text{ W}\right)}{16\pi\left(1\text{ g/cm}^3\right)\left(3\times10^{8}\text{ m/s}\right)\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)}$$
$$=\boxed{0.574\,\mu\text{m}}$$

68 •••

Picture the Problem

(*a*) At a perfectly conducting surface $\vec{E}=0$. Therefore, the sum of the electric fields of the incident and reflected wave must add to zero, and so $\vec{E}_{\text{i}}=-\vec{E}_{\text{r}}$.

(*b*) Let the incident and reflected waves be described by:

$$E_{\text{i}}=E_{0y}\cos(\omega t-kx)$$
and
$$E_{\text{r}}=-E_{0y}\cos(\omega t+kx)$$

Use the trigonometric identity cos($\alpha+\beta$) = cosαcosβ – sinαsinβ to obtain:

$$\begin{aligned}E_{\text{i}}+E_{\text{r}}&=E_{0y}\cos(\omega t-kx)-E_{0y}\cos(\omega t+kx)=E_{0y}\left[\cos(\omega t-kx)-\cos(\omega t+kx)\right]\\&=E_{0y}\left[\cos\omega t\cos(-kx)-\sin\omega t\sin(-kx)-\cos\omega t\cos kx+\sin\omega t\sin(kx)\right]\\&=E_{0y}\left[\cos\omega t\cos kx+\sin\omega t\sin kx-\cos\omega t\cos kx+\sin\omega t\sin kx\right]\\&=\boxed{2E_{0y}\sin\omega t\sin kx}\text{, the equation of a standing wave.}\end{aligned}$$

(*c*) Because $\vec{E}\times\vec{B}=\mu_0\vec{S}$ and $\vec{S}$ is in the direction of propagation of the wave, we see that for the incident wave $B_{\text{i}}=B_z\cos(\omega t-kx)$. Since both $\vec{S}$ and E_y are reversed for the reflected wave, $B_r=B_z\cos(\omega t+kx)$. So the magnetic field vectors are in the direction at the reflecting surface and add at that surface. Hence $\vec{B}=2\vec{B}_r$.

***69** •••

Picture the Problem Let the point source be a distance *a* above the plane. Consider a ring of radius *r* and thickness *dr* in the plane and centered at the point directly below the light source. Express the force of force on this ring and integrate the resulting expression to obtain *F*.

The intensity anywhere along this infinitesimal ring is $P/4\pi(r^2 + a^2)$ and the element of force dF on this ring of area $2\pi rdr$ is given by:

$$dF = \frac{P\,rdr}{c(r^2+a^2)}\frac{a}{\sqrt{r^2+a^2}} = \frac{Pardr}{c(r^2+a^2)^{3/2}}$$

where we have taken into account that only the normal component of the incident radiation contributes to the force on the plane, and that the plane is a perfectly reflecting plane.

Integrate dF from $r = 0$ to $r = \infty$:

$$F = \frac{Pa}{c}\int_0^\infty \frac{rdr}{(r^2+a^2)^{3/2}}$$

From integral tables:

$$\int_0^\infty \frac{rdr}{(r^2+a^2)^{3/2}} = \left.\frac{-1}{\sqrt{r^2+a^2}}\right]_0^\infty = \frac{1}{a}$$

Substitute to obtain:

$$F = \frac{Pa}{c}\left(\frac{1}{a}\right) = \frac{P}{c}$$

Substitute numerical values and evaluate F:

$$F = \frac{1\,\text{MW}}{3\times10^8\ \text{m/s}} = \boxed{3.33\,\text{mN}}$$

Chapter 31
Properties of Light

Conceptual Problems

1 •

Determine the Concept The population inversion between the state $E_{2,\text{Ne}}$ and the state 1.96 eV below it (see Figure 31-9) is achieved by inelastic collisions between neon atoms and helium atoms excited to the state $E_{2,\text{He}}$.

2 ••

Determine the Concept Although the excited atoms emit the light of the same frequency on returning to the ground state, the light is emitted in a random direction, not exclusively in the direction of the incident beam. Consequently, the beam intensity is greatly diminished.

3 •

Determine the Concept The layer of water greatly reduces the light reflected back from the car's headlights, but increases the light reflected by the road of light from the headlights of oncoming cars.

4 •

Determine the Concept When light passes from air into water its wavelength changes ($\lambda_{\text{water}} = \lambda_{\text{air}}/n_{\text{water}}$), its speed changes ($v_{\text{water}} = c/n_{\text{water}}$), and the direction of its propagation changes in accordance with Snell's law. $\boxed{(c)\text{ is correct.}}$

***5** ••

Determine the Concept The change in atmospheric density results in refraction of the light from the sun, bending it toward the earth. Consequently, the sun can be seen even after it is just below the horizon. Also, the light from the lower portion of the sun is refracted more than that from the upper portion, so the lower part appears to be slightly higher in the sky. The effect is an apparent flattening of the disk into an ellipse.

6 •

Determine the Concept (*a*) Yes. (*b*) Her procedure is based on Fermat's principle in that, since the ball presumably travels at constant speed, the path that requires the least time of travel corresponds to the shortest distance of travel.

7 •

Determine the Concept Because she can run faster than she can swim, she should choose the path that will maximize her running distance. Path *LES* is the path that satisfies this criterion.

8 •

Picture the Problem The intensity of the light transmitted by the second polarizer is given by $I_{\text{trans}} = I_0 \cos^2\theta$, where $I_0 = \frac{1}{2}I$. Therefore, $I_{\text{trans}} = \frac{1}{2}I\cos^2\theta$ and

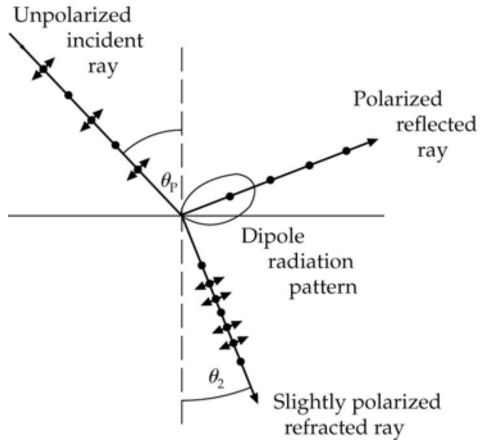
$\boxed{(b) \text{ is correct.}}$

9 •

Picture the Problem Polarized light can be produced from unpolarized light by absorption, reflection, birefringence, and scattering. Therefore, $\boxed{(d) \text{ is correct.}}$

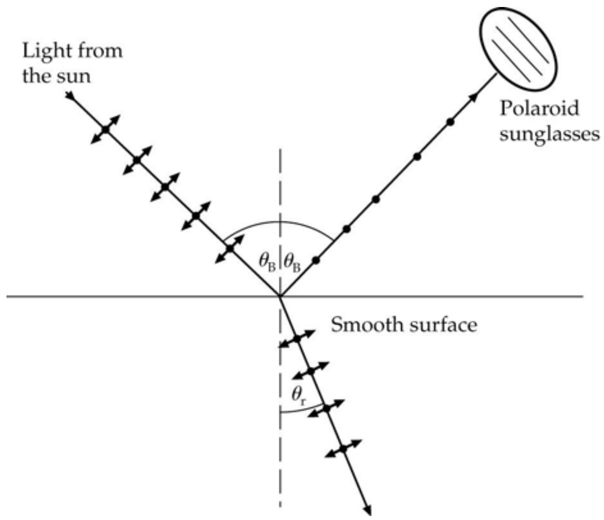
***10** ••

Determine the Concept The diagram shows that the radiated intensity for a dipole is zero in the direction of the dipole moment. Because the dipole axis is in the same direction as the polarization, for light polarized parallel to plane of incidence, the dipole axis will point in the same direction as the reflected wave, i.e., in the direction described by Brewster's law. As the diagram indicates, there is zero field in the direction of the refracted ray. On the other hand, if the incoming wave is polarized perpendicular to the plane of incidence, the dipole axis will never point along the direction of propagation for the reflected or refracted wave.



11 ••

Determine the Concept The diagram shows unpolarized light from the sun incident on the smooth surface at the polarizing angle for that particular surface. The reflected light is polarized perpendicular to the plane of incidence, i.e., in the horizontal direction. The sunglasses are shown in the correct orientation to pass vertically polarized light and block the reflected sunlight.



12 •

(*a*) True.

(*b*) False. Most of the light incident normally on an air–glass interface is transmitted.

(*c*) False. The relationship between the angles of incidence and refraction depends on the indices of refraction on both sides of the interface.

(*d*) False. The index of refraction of water is a function of the wavelength of light.

(*e*) True.

13 ••

Picture the Problem Because the speed of light in a given medium is inversely proportional to the index of refraction of the medium, we can decide which of the statements are true by referring to Figure 31-26.

(*a*) The graphs of *n* vs. λ are not horizontal lines and so the speed of light is a function of its wavelength.

(*b*) Because the index of refraction decreases with wavelength, violet light has the lowest speed and red light the highest speed.

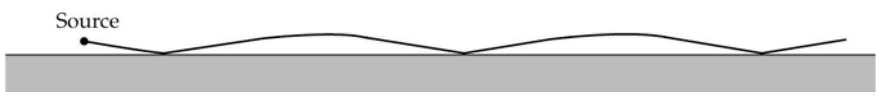
(*c*) Because the index of refraction decreases with wavelength, violet light has the lowest speed and red light the highest speed. $\boxed{(c)\text{ is correct.}}$

(*d*) Examination of Figure 31-26 tells us that this statement is false.

(*e*) Examination of Figure 31-26 tells us that this statement is false.

***14 ••**

Picture the Problem The sound is reflected specularly from the surface of the water (we assume it is calm). It is then refracted back toward the water in the region above the water because the speed of sound depends on the temperature of the air and is greater at the higher temperature. The pattern of the sound wave is shown schematically below.



15 •

Determine the Concept In resonance absorption, the molecules respond to the frequency of the light through the Einstein photon relation $E = hf$. Thus, the color appears to be the same in spite of the fact that the wavelength has changed.

Estimation and Approximation

16 •

Picture the Problem We can use the distance, rate, and time relationship to estimate the time required to travel 6 km (see Problem 31-14).

Express the distance D to light traveled in terms of its speed c and the elapsed time Δt:

$$D = c\Delta t$$

Solve for Δt:

$$\Delta t = \frac{D}{c}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{6\,\text{km}}{2.998\times10^{8}\ \text{m/s}} = \boxed{20.0\,\mu\text{s}}$$

17 •

Picture the Problem We can use the period of Io's motion and the position of the earth at *B* to find the number of eclipses of Io during the earth's movement and then use this information to find the number of days before a night-time eclipse. During the 42.5 h between eclipses of Jupiter's moon, the earth moves from *A* to *B*, increasing the distance from Jupiter by approximately the distance from the earth to the Sun, making the path for the light longer and introducing a delay in the onset of the eclipse.

(*a*) Find the time it takes the earth to travel from point *A* to point *B*:

$$t_{A\to B}=\frac{T_{\text{earth}}}{4}=\frac{365.24\,\text{d}}{4}\times\frac{24\,\text{h}}{\text{d}}=2191\,\text{h}$$

Because there are 42.5 h between eclipses of Io, the number of eclipses *N* occurring in the time it takes for the earth to move from *A* to *B* is:

$$N=\frac{t_{A\to B}}{T_{\text{Io}}}=\frac{2191\,\text{h}}{42.5\,\text{h}}=51.55$$

Hence, in one-fourth of a year, there will be 51.55 eclipses. Because we want to find the next occurrence that happens in the evening hours, we'll use 52 as the number of eclipses. We'll also assume that Jupiter is visible so that the eclipse of Io can be observed at the time we determine.

Relate the time $t(N)$ at which the Nth eclipse occurs to *N* and the period T_{Io} of Io:

$$t(N)=NT_{\text{Io}}$$

Evaluate $t(52)$ to obtain:

$$t(52)=(52)\left(42.5\,\text{h}\times\frac{1\,\text{d}}{24\,\text{h}}\right)=92.083\,\text{d}$$

Subtract the number of whole days to find the clock time *t*:

$$t=t(52)-92\,\text{d}=92.083\,\text{d}-92\,\text{d}=0.083\,\text{d}\times\frac{24\,\text{h}}{\text{d}}=1.992\,\text{h}\approx\boxed{2\,\text{am}}$$

Because June, July, and August have 30, 31, and 31 d, respectively, the date is:

$$\boxed{\text{September 1}}$$

(*b*) Express the time delay Δt in the arrival of light from Io due to the earth's location at *B*:

$$\Delta t=\frac{r_{\text{earth-sun}}}{c}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1.5\times10^{11}\ \text{m}}{2.998\times10^{8}\ \text{m/s}} = 500\,\text{s}$$
$$= 8.33\,\text{min}$$

$$\boxed{\text{Hence, the eclipse will actually occur at } 2:08\,\text{pm.}}$$

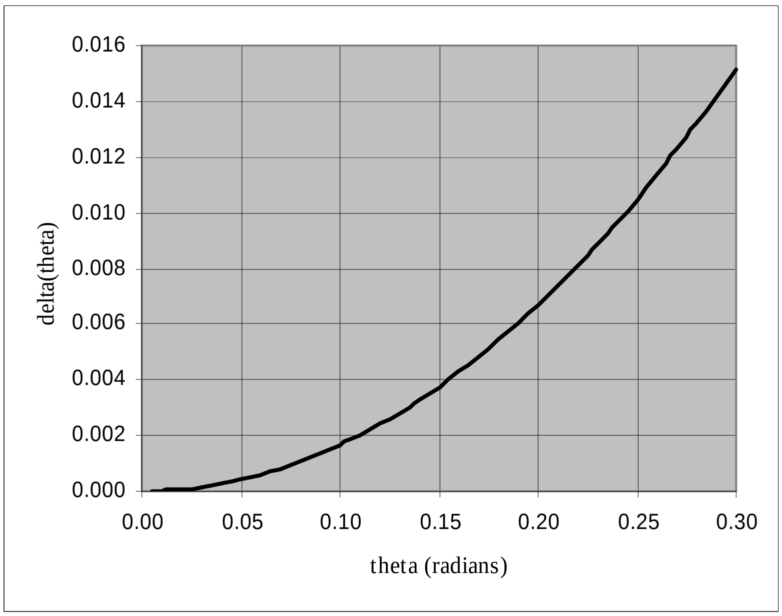
18 ••

Picture the Problem We can express the relative error in using the small angle approximation and then either 1) use trial-and-error methods, 2) use a spreadsheet program, or 3) use the Solver capability of a scientific calculator to solve the transcendental equation the results from setting the error function equal to 0.01.

Express the relative error δ in using the small angle approximation:

$$\delta(\theta) = \frac{\theta - \sin\theta}{\sin\theta} = \frac{\theta}{\sin\theta} - 1$$

A spreadsheet program was used to plot the following graph of $\delta(\theta)$.



From the graph, we can see that $\delta(\theta)$ < 1% for $\theta \le 0.24$ radians. In degree measure, $\theta \le \boxed{14°}$

Remarks: Using the Solver program on a TI-85 gave θ = 0.244 radians.

Sources of Light

19 •

Picture the Problem We can use the definition of power to find the total energy of the pulse. The ratio of the total energy to the energy per photon will yield the number of photons emitted in the pulse.

(*a*) Use the definition of power to obtain:

$$E = P\Delta t$$

Substitute numerical values and evaluate *E*:

$$E = (10\,\text{MW})(1.5\,\text{ns}) = \boxed{15.0\,\text{mJ}}$$

(*b*) Relate the number of photons *N* to the total energy in the pulse and the energy of a single photon E_{photon}:

$$N = \frac{E}{E_{\text{photon}}}$$

The energy of a photon is given by:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Substitute for E_{photon} to obtain:

$$N = \frac{\lambda E}{hc}$$

Substitute numerical values (the wavelength of light emitted by a ruby laser is 694.3 nm) and evaluate *N*:

$$N = \frac{(694.3\,\text{nm})(15.0\,\text{mJ})}{1240\,\text{eV}\cdot\text{nm}} \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{5.25\times10^{16}}$$

20 •

Picture the Problem We can express the number of photons emitted per second as the ratio of the power output of the laser and energy of a single photon.

Relate the number of photons per second *n* to the power output of the pulse and the energy of a single photon E_{photon}:

$$n = \frac{P}{E_{\text{photon}}}$$

The energy of a photon is given by:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Substitute for E_{photon} to obtain:

$$n = \frac{\lambda P}{hc}$$

Substitute numerical values and evaluate n:

$$n = \frac{(632.8\,\text{nm})(4\,\text{mW})}{1240\,\text{eV}\cdot\text{nm}}\,\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{1.28\times10^{16}\ \text{photons/s}}$$

21 •

Picture the Problem We can use the Einstein equation for photon energy to find the wavelength of the radiation for resonance absorption. We can use the same relationship, with $E_{\text{Raman}} = E_{\text{inc}} - \Delta E$ where ΔE is the energy for resonance absorption, to find the wavelength of the Raman scattered light.

(*a*) Use the Einstein equation for photon energy to relate the wavelength of the radiation to energy of the first excited state:

$$\lambda = \frac{hc}{E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{2.85\,\text{eV}} = \boxed{435\,\text{nm}}$$

(*b*) The wavelength of the Raman scattered light is given by:

$$\lambda_{\text{Raman}} = \frac{1240\,\text{eV}\cdot\text{nm}}{E_{\text{Raman}}}$$

Relate the energy of the Raman scattered light E_{Raman} to the energy of the incident light E_{inc}:

$$E_{\text{Raman}} = E_{\text{inc}} - \Delta E = \frac{1240\,\text{eV}\cdot\text{nm}}{320\,\text{nm}} - 2.85\,\text{eV} = 1.025\,\text{eV}$$

Substitute numerical values and evaluate λ_{Raman}:

$$\lambda_{\text{Raman}} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.025\,\text{eV}} = \boxed{1210\,\text{nm}}$$

22 ••

Picture the Problem The incident radiation will excite atoms of the gas to higher energy states. The scattered light that is observed is a consequence of these atoms returning to their ground state. The energy difference between the ground state and the atomic state excited by the irradiation is given by $\Delta E = hf = \dfrac{hc}{\lambda}$.

The energy difference between the ground state and the atomic state excited by the irradiation is given by:

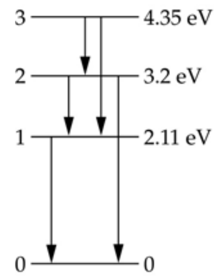
$$\Delta E = hf = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{fm}}{\lambda}$$

Substitute 368 nm for λ and evaluate ΔE:

$$\Delta E = \frac{1240\,\text{eV}\cdot\text{fm}}{368\,\text{nm}} = \boxed{3.37\,\text{eV}}$$

23 ••

Picture the Problem The ground state and the three excited energy levels are shown in the diagram to the right. Because the wavelength is related to the energy of a photon by $\lambda = hc/\Delta E$, longer wavelengths correspond to smaller energy differences.



(*a*) The maximum wavelength of radiation that will result in resonance fluorescence corresponds to an excitation to the 3.2 eV level followed by decays to the 2.11 eV level and the ground state:

$$\lambda_{max} = \frac{1240\,\text{eV}\cdot\text{fm}}{3.2\,\text{eV}} = \boxed{387.5\,\text{nm}}$$

The fluorescence wavelengths are:

$$\lambda_{21} = \frac{1240\,\text{eV}\cdot\text{fm}}{3.2\,\text{eV} - 2.11\,\text{eV}} = \boxed{1138\,\text{nm}}$$

and

$$\lambda_{10} = \frac{1240\,\text{eV}\cdot\text{fm}}{2.11\,\text{eV} - 0} = \boxed{587.7\,\text{nm}}$$

(*b*) For excitation:

$$\lambda_{03} = \frac{1240\,\text{eV}\cdot\text{fm}}{4.35\,\text{eV}} = \boxed{285.1\,\text{nm}}$$

The fluorescence wavelengths corresponding to the possible transitions are:

$$\lambda_{32} = \frac{1240\,\text{eV}\cdot\text{fm}}{4.35\,\text{eV} - 3.2\,\text{eV}} = \boxed{1078\,\text{nm}}$$

$$\lambda_{21} = \frac{1240\,\text{eV}\cdot\text{fm}}{3.2\,\text{eV} - 2.11\,\text{eV}} = \boxed{1138\,\text{nm}}$$

$$\lambda_{10} = \frac{1240\,\text{eV}\cdot\text{fm}}{2.11\,\text{eV} - 0} = \boxed{587.7\,\text{nm}}$$

$$\lambda_{31} = \frac{1240\,\text{eV}\cdot\text{fm}}{4.35\,\text{eV} - 2.11\,\text{eV}} = \boxed{553.6\,\text{nm}}$$

and

$$\lambda_{20} = \frac{1240\,\text{eV}\cdot\text{fm}}{3.2\,\text{eV} - 0} = \boxed{387.5\,\text{nm}}$$

***24 ••**

Determine the Concept The energy difference between the ground state and the first excited state is $3E_0$ = 40.8 eV, corresponding to a wavelength of 30.4 nm. This is in the far ultraviolet, well outside the visible range of wavelengths. There will be no dark lines in the transmitted radiation.

The Speed of Light

25 •

Picture the Problem We can use the distance, rate, and time relationship to find the distance to the spaceship.

Relate the distance D to the spaceship to the speed of electromagnetic radiation in a vacuum and to the time for the message to reach the astronauts:

$$D = c\Delta t$$

Noting that the time for the message to reach the astronauts is half the time for Mission Control to hear their response, substitute numerical values and evaluate D:

$$D = \left(2.998\times10^{8}\ \text{m/s}\right)\left(2.5\,\text{s}\right) = 7.50\times10^{8}\ \text{m}$$

and

$\boxed{(a)\ \text{is correct.}}$

26 •

Picture the Problem We can use the conversion factor, found in EP-3, to convert a distance in km into $c{\cdot}y$:

Convert $D = 2\times10^{19}$ km into light-years:

$$D = 2\times10^{19}\ \text{km}\times\frac{1c\cdot y}{9.46\times10^{15}\ \text{m}} = \boxed{2.11\times10^{6}\ c\cdot y}$$

27 •

Picture the Problem We can use the distance, rate, and time relationship to find the time delay between sending the signal from the earth and receiving it on Mars.

Relate the distance D to Mars to the

$$D = c\Delta t$$

speed of electromagnetic radiation in a vacuum and to the travel time for the signal:

Solve for Δt:

$$\Delta t = \frac{D}{c}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{9.7\times 10^{10}\text{ m}}{2.998\times 10^{8}\text{ m/s}} = 324\,\text{s}$$

$$= \boxed{5\,\text{min}\,23\,\text{s}}$$

28 •

Picture the Problem We can use the given information that the uncertainty in the measured distance Δx is related to the uncertainty in the time Δt by $\Delta x = c\Delta t$ to evaluate Δx.

The uncertainty in the distance is:

$$\Delta x = \pm c\Delta t$$

Substitute numerical values and evaluate Δt:

$$\Delta x = \pm\left(2.998\times 10^{8}\text{ m/s}\right)\left(1.0\,\text{ns}\right)$$

$$= \boxed{\pm 30.0\,\text{cm}}$$

***29** ••

Picture the Problem We can use the distance, rate, and time relationship to find the time difference Galileo would need to be able to measure the speed of light successfully.

(*a*) Relate the distance separating Galileo and his assistant to the speed of light and the time required for it travel to the assistant and back to Galileo:

$$D = c\Delta t$$

Solve for Δt:

$$\Delta t = \frac{D}{c}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2\left(3\,\text{km}\right)}{3\times 10^{8}\text{ m/s}} = \boxed{20.0\,\mu\text{s}}$$

(*b*) Express the ratio of the human reaction time to the transit time for the light:

$$\frac{\Delta t_{\text{reaction}}}{\Delta t} = \frac{0.2\,\text{s}}{20\,\mu\text{s}} = 10^{4}$$

or

$$\Delta t_{\text{reaction}} = \boxed{10^4 \Delta t}$$

Reflection and Refraction

30 •

Picture the Problem Let the subscript 1 refer to air and the subscript 2 to water and use the equation relating the intensity of reflected light at normal incidence to the intensity of the incident light and the indices of refraction of the media on either side of the interface.

Express the intensity I of the light reflected from an air-water interface at normal incidence in terms of the indices of refraction and the intensity I_0 of the incident light:

$$I = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 I_0$$

Solve for the ratio I/I_0:

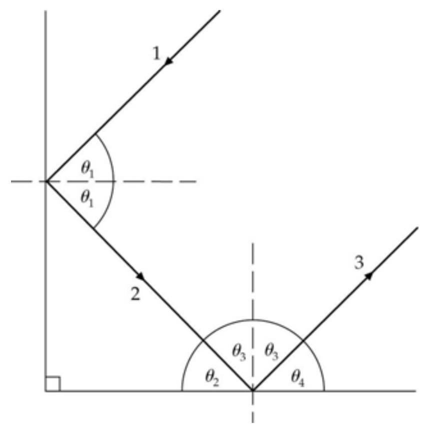
$$\frac{I}{I_0} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Substitute numerical values and evaluate I/I_0:

$$\frac{I}{I_0} = \left(\frac{1 - 1.33}{1 + 1.33}\right)^2 = 0.0201 = \boxed{2.01\%}$$

***31** ••

Picture the Problem The diagram shows ray 1 incident on the vertical surface at an angle θ_1, reflected as ray 2, and incident on the horizontal surface at an angle of incidence θ_3. We'll prove that rays 1 and 3 are parallel by showing that $\theta_1 = \theta_4$, i.e., by showing that they make equal angles with the horizontal. Note that the law of reflection has been used in identifying equal angles of incidence and reflection.



We know that the angles of the right triangle formed by ray 2 and the two mirror surfaces add up to 180°:

$$\theta_2 + 90° + 90° - \theta_1 = 180°$$

or

$$\theta_1 = \theta_2$$

The sum of θ_2 and θ_3 is 90°:

$$\theta_3 = 90° - \theta_2$$

Because $\theta_1 = \theta_2$:

$$\theta_3 = 90° - \theta_1$$

The sum of θ_4 and θ_3 is 90°:

$$\theta_3 + \theta_4 = 90°$$

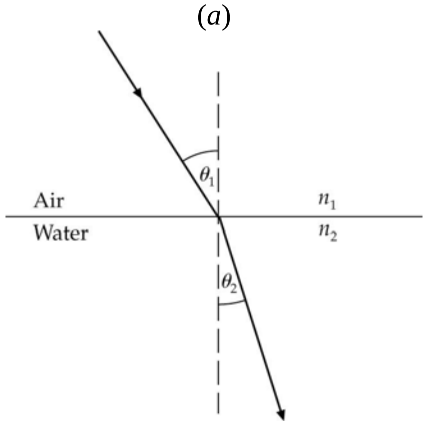
Substitute for θ_3 to obtain:

$$90° - \theta_1 + \theta_4 = 90° \Rightarrow \theta_1 = \boxed{\theta_4}$$

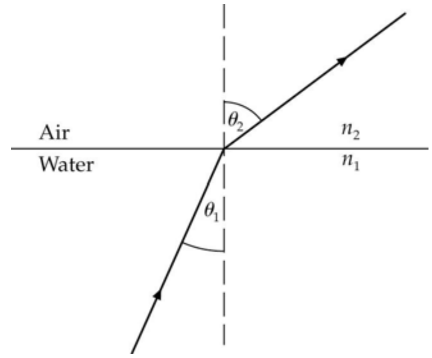
32 ••

Picture the Problem Diagrams showing the light rays for the two cases are shown below. In (a) the light travels from air into water and in (*b*) it travels from water into air.

(*a*)



(*b*)



(*a*) Apply Snell's law to the air-water interface to obtain:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where the angles of incidence and refraction are θ_1 and θ_2, respectively.

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

A spreadsheet program to graph θ_2 as a function of θ_1 is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	1	n_1
B2	1.33333	n_2
A6	0	θ_1 (deg)
A7	A6 + 5	$\theta_1 + \Delta\theta$
B6	A6*PI()/180	$\theta_1 \times \frac{\pi}{180}$
C6	ASIN(($B$1/$B$2)*SIN(B6))	$\sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$
D6	C6*180/PI()	$\theta_2 \times \frac{180}{\pi}$

	A	B	C	D
1	n1=	1		
2	n2=	1.33333		
3				
4	theta1	theta1	theta2	theta2
5	(deg)	(rad)	(rad)	(deg)
6	0	0.00	0.000	0.00
7	1	0.02	0.013	0.75
8	2	0.03	0.026	1.50
9	3	0.05	0.039	2.25
21	87	1.52	0.847	48.50
22	88	1.54	0.847	48.55
23	89	1.55	0.848	48.58
24	90	1.57	0.848	48.59

A graph of θ_2 as a function of θ_1 follows:

(*b*) Change the contents of cell B1 to 1.33333 and the contents of cell B2 to 1 to obtain the following graph:

Note that as the angle of incidence approaches the critical angle for a water-air interface (48.6°), the angle of refraction approaches 90°. No light will be refracted into the air if the angle of incidence is greater than 48.6°.

33 •

Picture the Problem We can use the definition of the index of refraction to find the speed of light in water and in glass.

The definition of the index of refraction is:

$$n = \frac{c}{v}$$

Solve for v to obtain:

$$v = \frac{c}{n}$$

Substitute numerical values and evaluate v_{water}:

$$v_{\text{water}} = \frac{3\times10^8\ \text{m/s}}{1.33} = \boxed{2.25\times10^8\ \text{m/s}}$$

Substitute numerical values and evaluate v_{glass}:

$$v_{\text{glass}} = \frac{3\times10^8\ \text{m/s}}{1.5} = \boxed{2.00\times10^8\ \text{m/s}}$$

34 •

Picture the Problem Let the subscript 1 refer to the air and the subscript 2 to the silicate glass and apply Snell's law to the air-glass interface.

Apply Snell's law to the air-glass interface to obtain:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

Substitute numerical values for the light of wavelength 400 nm and evaluate $\theta_{2,\,400\text{ nm}}$:

$$\theta_{2,\,400\text{ nm}} = \sin^{-1}\left(\frac{1}{1.66}\sin 45°\right) = \boxed{25.2°}$$

Substitute numerical values for the light of wavelength 700 nm and evaluate $\theta_{2,\,700\text{ nm}}$:

$$\theta_{2,\,700\text{ nm}} = \sin^{-1}\left(\frac{1}{1.61}\sin 45°\right) = \boxed{26.1°}$$

35 ••

Picture the Problem Let the subscript 1 refer to the water and the subscript 2 to the glass and apply Snell's law to the water-glass interface.

Apply Snell's law to the water-glass interface to obtain:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

(*a*) Evaluate θ_2 for $\theta_1 = 60°$:

$$\theta_2 = \sin^{-1}\left(\frac{1.33}{1.5}\sin 60°\right) = \boxed{50.2°}$$

(*b*) Evaluate θ_2 for $\theta_1 = 45°$:

$$\theta_2 = \sin^{-1}\left(\frac{1.33}{1.5}\sin 45°\right) = \boxed{38.8°}$$

(*c*) Evaluate θ_2 for $\theta_1 = 30°$:

$$\theta_2 = \sin^{-1}\left(\frac{1.33}{1.5}\sin 30°\right) = \boxed{26.3°}$$

36 ••

Picture the Problem Let the subscript 1 refer to the glass and the subscript 2 to the water and apply Snell's law to the glass-water interface.

Apply Snell's law to the water-glass interface to obtain:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

(*a*) Evaluate θ_2 for θ_1 = 60°:

$$\theta_2 = \sin^{-1}\left(\frac{1.5}{1.33}\sin 60°\right) = \boxed{77.6°}$$

(*b*) Evaluate θ_2 for θ_1 = 45°:

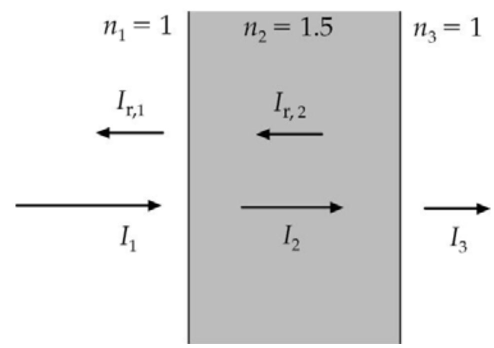
$$\theta_2 = \sin^{-1}\left(\frac{1.5}{1.33}\sin 45°\right) = \boxed{52.9°}$$

(*c*) Evaluate θ_2 for θ_1 = 30°:

$$\theta_2 = \sin^{-1}\left(\frac{1.5}{1.33}\sin 30°\right) = \boxed{34.3°}$$

***37 ••**

Picture the Problem Let the subscript 1 refer to the medium to the left (air) of the first interface, the subscript 2 to glass, and the subscript 3 to the medium (air) to the right of the second interface. Apply the equation relating the intensity of reflected light at normal incidence to the intensity of the incident light and the indices of refraction of the media on either side of the interface to both interfaces. We'll neglect multiple reflections at glass-air interfaces.



Express the intensity of the transmitted light in the second medium:

$$I_2 = I_1 - I_{r,1} = I_1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 I_1$$
$$= I_1\left[1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2\right]$$

Express the intensity of the transmitted light in the third medium:

$$I_3 = I_2 - I_{r,2} = I_2 - \left(\frac{n_2 - n_3}{n_2 + n_3}\right)^2 I_2$$
$$= I_2\left[1 - \left(\frac{n_2 - n_3}{n_2 + n_3}\right)^2\right]$$

Substitute for I_2 to obtain:

$$I_3 = I_1\left[1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2\right]\left[1 - \left(\frac{n_2 - n_3}{n_2 + n_3}\right)^2\right]$$

Solve for the ratio I_3/I_1:

$$\frac{I_3}{I_1} = \left[1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2\right]\left[1 - \left(\frac{n_2 - n_3}{n_2 + n_3}\right)^2\right]$$

Substitute numerical values and evaluate I_3/I_1:

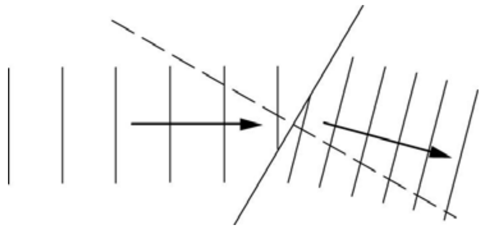
$$\frac{I_3}{I_1} = \left[1 - \left(\frac{1-1.5}{1+1.5}\right)^2\right]\left[1 - \left(\frac{1.5-1}{1.5+1}\right)^2\right]$$

$$= 0.922 = \boxed{92.2\%}$$

38 ••

Picture the Problem As the line enters the muddy field, its speed is reduced by half and the direction of the forward motion of the line is changed. In this case, the forward motion in the muddy field makes an angle of 14.5° with respect to the normal of the boundary line. Note that the separation between successive lines in the muddy field is half that in the dry field.

Picture the Problem As the line enters the muddy field, its speed is reduced by half and the direction of the forward motion of the line is changed. In this case, the forward motion in the muddy field makes an angle of 14.5° with respect to the normal of the boundary line. Note that the separation between successive lines in the muddy field is half that in the dry field.



39 ••

Picture the Problem We can apply Snell's law consecutively, first to the n_1-n_2 interface and then to the n_2-n_3 interface.

Apply Snell's law to the n_1-n_2 interface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

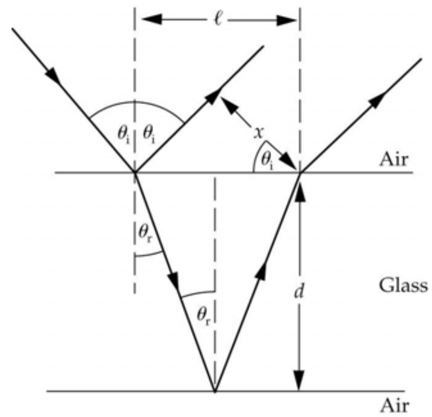
Apply Snell's law to the n_2-n_3 interface:

$$n_2 \sin\theta_2 = n_3 \sin\theta_3$$

Equate the two expressions for $n_2 \sin\theta_2$ to obtain:

$$\boxed{n_1 \sin\theta_1 = n_3 \sin\theta_3}$$

***40 •••**



Picture the Problem Let x be the perpendicular separation between the two rays and let ℓ be the separation between the points of emergence of the two rays on the glass surface. We can use the geometry of the refracted and reflected rays to express x as a function of ℓ, d, θ_r, and θ_i. Setting the derivative of the resulting equation equal to zero will yield the value of θ_i that maximizes x.

(*a*) Express ℓ in terms of d and the angle of refraction θ_r:

$$\ell = 2d\tan\theta_r$$

Express x as a function of ℓ, d, θ_r, and θ_i:

$$x = 2d\tan\theta_r\cos\theta_i$$

Differentiate x with respect to θ_i:

$$\frac{dx}{d\theta_i} = 2d\frac{d}{d\theta_i}\left(\tan\theta_r\cos\theta_i\right) = 2d\left(-\tan\theta_r\sin\theta_i + \sec^2\theta_r\cos\theta_i\frac{d\theta_r}{d\theta_i}\right) \quad (1)$$

Apply Snell's law to the air-glass interface:

$$n_1\sin\theta_i = n_2\sin\theta_r \quad (2)$$

or, since $n_1 = 1$ and $n_2 = n$,

$$\sin\theta_i = n\sin\theta_r$$

Differentiate implicitly with respect to θ_i to obtain:

$$\cos\theta_i\,d\theta_i = n\cos\theta_r\,d\theta_r$$

or

$$\frac{d\theta_r}{d\theta_i} = \frac{1}{n}\frac{\cos\theta_i}{\cos\theta_r}$$

Substitute in equation (1) to obtain:

$$\frac{dx}{d\theta_i} = 2d\left(-\frac{\sin\theta_r}{\cos\theta_r}\sin\theta_i + \frac{1}{n}\frac{\cos\theta_i}{\cos^2\theta_r}\frac{\cos\theta_i}{\cos\theta_r}\right) = 2d\left(\frac{1}{n}\frac{\cos^2\theta_i}{\cos^3\theta_r} - \frac{\sin\theta_r\sin\theta_i}{\cos\theta_r}\right)$$

Substitute $1-\sin^2\theta_i$ for $\cos^2\theta_i$ and $\frac{1}{n}\sin\theta_i$ for $\sin\theta_r$ to obtain:

$$\frac{dx}{d\theta_i}=2d\left(\frac{1-\sin^2\theta_i}{n\cos^3\theta_r}-\frac{\sin^2\theta_i}{n\cos\theta_r}\right)$$

Multiply the second term in parentheses by $\cos^2\theta_r/\cos^2\theta_r$ and simplify to obtain:

$$\frac{dx}{d\theta_i}=2d\left(\frac{1-\sin^2\theta_i}{n\cos^3\theta_r}-\frac{\sin^2\theta_i\cos^2\theta_r}{n\cos^3\theta_r}\right)=\frac{2d}{n\cos^3\theta_r}\left(1-\sin^2\theta_i-\sin^2\theta_i\cos^2\theta_r\right)$$

Substitute $1-\sin^2\theta_r$ for $\cos^2\theta_r$:

$$\frac{dx}{d\theta_i}=\frac{2d}{n\cos^3\theta_r}\left[1-\sin^2\theta_i-\sin^2\theta_i\left(1-\sin^2\theta_r\right)\right]$$

Substitute $\frac{1}{n}\sin\theta_i$ for $\sin\theta_r$ to obtain:

$$\frac{dx}{d\theta_i}=\frac{2d}{n\cos^3\theta_r}\left[1-\sin^2\theta_i-\sin^2\theta_i\left(1-\frac{1}{n^2}\sin^2\theta_i\right)\right]$$

Factor out $1/n^2$, simplify, and set equal to zero to obtain:

$$\frac{dx}{d\theta_i}=\frac{2d}{n^3\cos^3\theta_r}\left[\sin^4\theta_i-2n^2\sin^2\theta_i+n^2\right]=0 \text{ for extrema}$$

If $dx/d\theta_1=0$, then it must be true that:

$$\sin^4\theta_i-2n^2\sin^2\theta_i+n^2=0$$

Solve this quartic equation for θ_i to obtain:

$$\theta_i=\boxed{\sin^{-1}\left(n\sqrt{1-\sqrt{1-\frac{1}{n^2}}}\right)}$$

(*b*) Evaluate θ_1 for $n = 1.60$:

$$\theta_i=\sin^{-1}\left(1.6\sqrt{1-\sqrt{1-\frac{1}{(1.6)^2}}}\right)=\boxed{48.5°}$$

In (*a*) we showed that:

$$x=2d\tan\theta_r\cos\theta_i$$

Solve equation (2) for θ_r:

$$\theta_r = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i\right)$$

Substitute numerical values and evaluate θ_r:

$$\theta_r = \sin^{-1}\left(\frac{1}{1.6}\sin 48.5°\right) = 27.9°$$

Substitute numerical values and evaluate x:

$$x = 2(4\,\text{cm})\tan 27.9°\cos 48.5° = \boxed{2.81\,\text{cm}}$$

Total Internal Reflection

41 •

Picture the Problem Let the subscript 1 refer to the glass and the subscript 2 to the water and use Snell's law under total internal reflection conditions.

Use Snell's law to obtain:

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

When there is total internal reflection:

$$\theta_1 = \theta_c \text{ and } \theta_2 = 90°$$

Substitute to obtain:

$$n_1\sin\theta_c = n_2\sin 90° = n_2$$

Solve for θ_c:

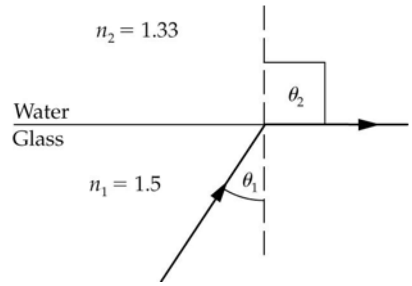
$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$

Substitute numerical values and evaluate θ_c:

$$\theta_c = \sin^{-1}\frac{1.33}{1.5} = \boxed{62.5°}$$

42 ••

Picture the Problem Let the index of refraction of glass be represented by n_1 and the index of refraction of water by n_2 and apply Snell's law to the glass-water interface under total internal reflection conditions.



Apply Snell's law to the glass-water interface:

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

At the critical angle, $\theta_1 = \theta_c$ and

$$n_1\sin\theta_c = n_2\sin 90°$$

$\theta_2 = 90°$:

Solve for θ_c:

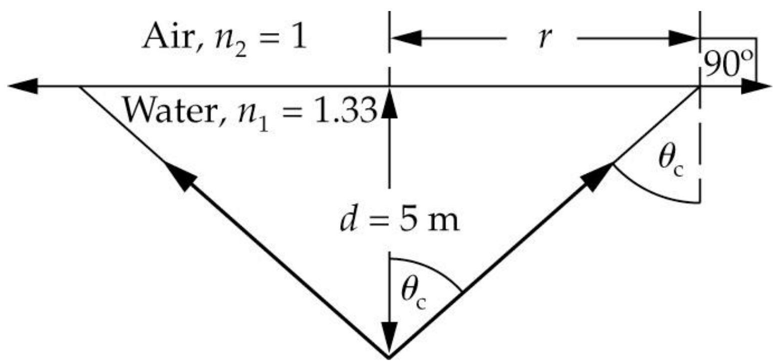
$$\theta_c = \sin^{-1}\left[\frac{n_2}{n_1}\sin 90°\right]$$

Substitute numerical values and evaluate θ_c:

$$\theta_c = \sin^{-1}\left[\frac{1.33}{1.5}\sin 90°\right] = \boxed{62.5°}$$

43 ••

Picture the Problem We can apply Snell's law to the water-air interface to express the critical angle θ_c in terms of the indices of refraction of water (n_1) and air (n_2) and then relate the radius of the circle to the depth d of the point source and θ_c.



Express the area of the circle whose radius is r:

$$A = \pi r^2$$

Relate the radius of the circle to the depth d of the point source and the critical angle θ_c:

$$r = d\tan\theta_c$$

Apply Snell's law to the water-air interface to obtain:

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2$$

Solve for θ_c:

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$

Substitute for r and θ_c to obtain:

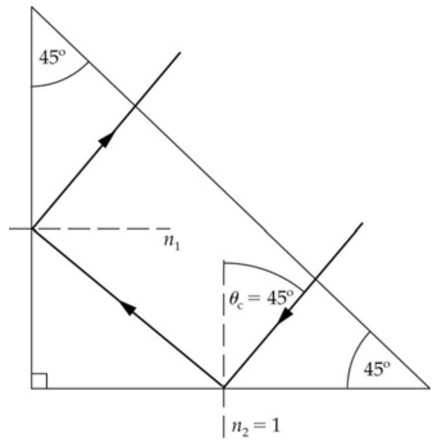
$$A = \pi\left[d\tan\theta_c\right]^2 = \pi\left[d\tan\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]^2$$

Substitute numerical values and evaluate A:

$$A = \pi\left[(5\,\text{m})\tan\left(\sin^{-1}\frac{1}{1.33}\right)\right]^2 = \boxed{102\,\text{m}^2}$$

44 ••

Picture the Problem We can use the definition of the index of refraction to express the speed of light in the prism in terms of the index of refraction n_1 of the prism. The application of Snell's law at the glass-air interface will allow us to relate the index of refraction of the prism to the critical angle for total internal reflection. Finally, we can use the geometry of the isosceles-right-triangle prism to conclude that $\theta_c = 45°$.



Express the speed of light v in the prism in terms of its index of refraction n_1:

$$v = \frac{c}{n_1}$$

Apply Snell's law to the glass-air interface to obtain:

$$n_1 \sin\theta_c = n_2 \sin 90° = 1$$

Solve for n_1:

$$n_1 = \frac{1}{\sin\theta_c}$$

Substitute to obtain:

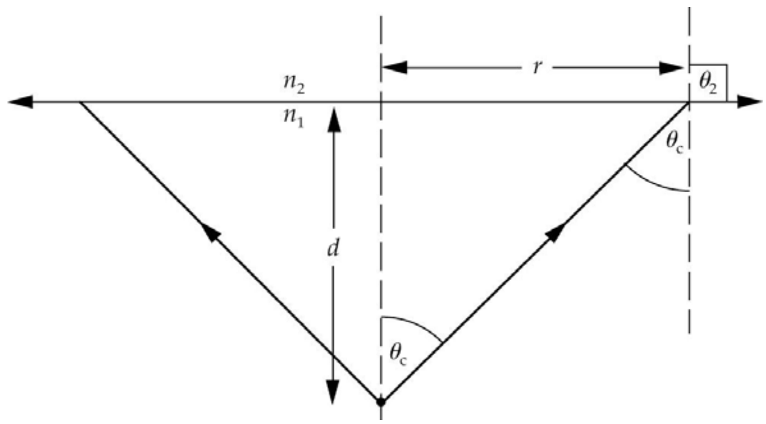
$$v = c\sin\theta_c$$

Substitute numerical values and evaluate v:

$$v = (2.998\times10^8\ \text{m/s})\sin 45° = \boxed{2.12\times10^8\ \text{m/s}}$$

45 ••

Picture the Problem The observer above the surface of the fluid will not see any light until the angle of incidence of the light at the fluid-air interface is less than or equal to the critical angle for the two media. We can use Snell's law to express the index of refraction of the fluid in terms of the critical angle and use the geometry of card and light source to express the critical angle.



Apply Snell's law to the fluid-air interface to obtain:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Light is seen by the observer when $\theta_1 = \theta_c$ and $\theta_2 = 90°$:

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2$$

Because the medium above the interface is air, $n_2 = 1$. Solve for n_1 to obtain:

$$n_1 = \frac{1}{\sin\theta_c}$$

From the geometry of the diagram:

$$\tan\theta_c = \frac{r}{d} \Rightarrow \theta_c = \tan^{-1}\frac{r}{d}$$

Substitute to obtain:

$$n_1 = \frac{1}{\sin\left[\tan^{-1}\frac{r}{d}\right]}$$

Substitute numerical values and evaluate n_1:

$$n_1 = \frac{1}{\sin\left[\tan^{-1}\frac{6\,\text{cm}}{5\,\text{cm}}\right]} = \boxed{1.30}$$

***46 ••**

Picture the Problem We can use the geometry of the figure, the law of refraction at the air-n_1 interface, and the condition for total internal reflection at the n_1-n_2 interface to show that the numerical aperture is given by $\sqrt{n_2^2 - n_3^2}$.

Referring to the figure, note that:

$$\sin\theta_c = \frac{n_3}{n_2} = \frac{a}{c}$$

and

$$\sin\theta_2 = \frac{b}{c}$$

Apply the Pythagorean theorem to the right triangle to obtain:

$$a^2 + b^2 = c^2$$

or

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

Solve for $\frac{b}{c}$:

$$\frac{b}{c} = \sqrt{1 - \frac{a^2}{c^2}}$$

Substitute for $\frac{a}{c}$ and $\frac{b}{c}$ to obtain:

$$\sin\theta_2 = \sqrt{1 - \frac{n_3^2}{n_2^2}}$$

Use the law of refraction to relate θ_1 and θ_2:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Substitute for $\sin\theta_2$ and let $n_1 = 1$ (air) to obtain:

$$\sin\theta_1 = n_2\sqrt{1 - \frac{n_3^2}{n_2^2}} = \boxed{\sqrt{n_2^2 - n_3^2}}$$

47 •

Picture the Problem We can use the result of Problem 46 to find the maximum angle of incidence under the given conditions.

From Problem 46:

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

Solve for θ_0:

$$\theta_0 = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \sin^{-1}\left(\sqrt{(1.492)^2 - (1.489)^2}\right) = \boxed{5.43°}$$

48 ••

Picture the Problem Examination of the figure reveals that, if the length of the tube is L, the distance traveled by the pulse that enters at an angle θ_0 is the ratio of a to b multiplied by L. Let the subscripts 1 and 2 denote the pulses entering the tube normally and at an angle θ_0, respectively.

Express the difference in time Δt needed for the two pulses to travel a distance L:

$$\Delta t = t_2 - t_1 = \frac{L\frac{b}{a}}{\frac{c}{n_1}} - \frac{L}{\frac{c}{n_1}}$$

Substitute for t_2 and t_1 and simplify to obtain:

$$\Delta t = \frac{L\frac{b}{a}}{\frac{c}{n_1}} - \frac{L}{\frac{c}{n_1}} = \frac{n_1 L}{c}\left(\frac{b}{a} - 1\right) \qquad (1)$$

Referring to the figure, note that:

$$\sin\theta_c = \frac{b}{a}$$

From Snell's law, the sine of the critical angle is also given by:

$$\sin\theta_c = \frac{n_2}{n_1} \Rightarrow \frac{b}{a} = \frac{n_2}{n_1}$$

Substitute for b/a in equation (1) and simplify to obtain:

$$\Delta t = \frac{n_1 L}{c}\left(\frac{n_2}{n_1} - 1\right) = \frac{L}{c}(n_2 - n_1)$$

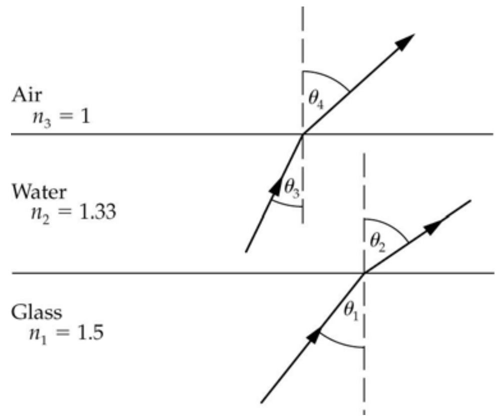
Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{15\,\text{km}}{3\times10^8\ \text{m/s}}(1.492 - 1.489)$$

$$= \boxed{150\,\text{ns}}$$

49 •••

Picture the Problem Let the index of refraction of glass be represented by n_1, the index of refraction of water by n_2, and the index of refraction of air by n_3. We can apply Snell's law to the glass-water interface under total internal reflection conditions to find the critical angle for total internal reflection. The application of Snell's law to glass-air and glass-water interfaces will allow us to decide whether there are angles of incidence greater than θ_c for glass-to-air refraction for which light rays will leave the glass and the water and pass into the air.



(*a*) Apply Snell's law to the glass-water interface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

At the critical angle, $\theta_1 = \theta_c$ and $\theta_2 = 90°$:

$$n_1 \sin\theta_c = n_2 \sin 90°$$

Solve for θ_c:

$$\theta_c = \sin^{-1}\left[\frac{n_2}{n_1}\sin 90°\right]$$

Substitute numerical values and evaluate θ_c:

$$\theta_c = \sin^{-1}\left[\frac{1.33}{1.5}\sin 90°\right] = \boxed{62.5°}$$

(*b*) Apply Snell's law to a glass-air interface:

$$n_1 \sin\theta_c = n_3 \sin 90°$$

or

$$1.5\sin\theta_c = \sin 90° = 1$$

Solve for θ_c:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

Apply Snell's law to a ray incident at the critical angle for a glass-water interface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

or

$$1.5\sin 41.8° = 1.33\sin\theta_2$$

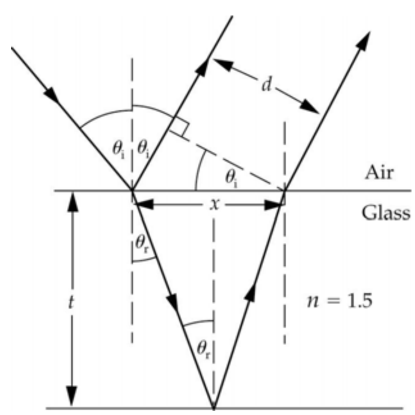
Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{1.5\sin 41.8°}{1.33}\right) = 48.7°$$

Note that θ_2 equals the critical angle for a water-air interface. Therefore, the ray will not leave the water for $\theta_1 \geq 41.8°$.

50 •••

Picture the Problem The situation is shown in the adjacent figure. We can use the geometry of the diagram and trigonometric relationships to derive an expression for *d* in terms of the angles of incidence and refraction. Applying Snell's law will yield θ_r.



Express the distance *x* in terms of *t* and θ_r:

$$x = 2t\tan\theta_r$$

The separation of the reflected rays is:

$$d = x\cos\theta_i$$

Substitute to obtain:

$$d = 2t\tan\theta_r\cos\theta_i \qquad (1)$$

Apply Snell's law at the air-glass interface to obtain:

$$\sin\theta_i = n\sin\theta_r$$

Solve for θ_r:

$$\theta_r = \sin^{-1}\left(\frac{\sin\theta_i}{n}\right)$$

Substitute in equation (1) to obtain:

$$d = 2t\tan\left[\sin^{-1}\left(\frac{\sin\theta_i}{n}\right)\right]\cos\theta_i$$

Substitute numerical values and evaluate *d*:

$$d = 2(3\,\text{cm})\tan\left[\sin^{-1}\left(\frac{\sin 40°}{1.5}\right)\right]\cos 40° = \boxed{2.18\,\text{cm}}$$

Dispersion

***51 ••**

Picture the Problem We can apply Snell's law of refraction to express the angles of refraction for red and violet light in silicate flint glass.

Express the difference between the angle of refraction for violet light and for red light:

$$\Delta\theta = \theta_{r,\text{red}} - \theta_{r,\text{violet}} \qquad (1)$$

Apply Snell's law of refraction to the interface to obtain:

$$\sin 45° = n\sin\theta_r$$

Solve for θ_r:

$$\theta_r = \sin^{-1}\left(\frac{1}{\sqrt{2}n}\right)$$

Substitute in equation (1):

$$\Delta\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}n_{\text{red}}}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}n_{\text{violet}}}\right)$$

Substitute numerical values and evaluate $\Delta\theta$:

$$\Delta\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}(1.60)}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}(1.66)}\right) = 26.23° - 25.21° = \boxed{1.02°}$$

52 ••

Picture the Problem The transit times will be different because the speed with which light of various wavelengths propagates in silicate crown glass is dependent on the index of refraction. We can use Table 31-26 to estimate the indices of refraction for pulses of wavelengths 500 and 700 nm.

Express the difference in time needed for two short pulses of light to travel a distance L in the fiber:

$$\Delta t = \frac{L}{v_{500}} - \frac{L}{v_{700}}$$

Substitute for L, v_{500}, and v_{700} and simplify to obtain:

$$\Delta t = \frac{n_{500}L}{c} - \frac{n_{700}L}{c} = \frac{L}{c}\left(n_{500} - n_{700}\right)$$

Use Table 31-26 to find the indices of refraction of silicate crown glass for the two wavelengths:

$$n_{500} \approx 1.55$$

and

$$n_{700} \approx 1.50$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{15\,\text{km}}{2.998\times10^{8}\text{ m/s}}(1.55 - 1.50) = \boxed{2.50\,\mu\text{s}}$$

Polarization

53 •

Picture the Problem The polarizing angle is given by Brewster's law: $\tan\theta_{\text{p}} = n_2/n_1$ where n_1 and n_2 are the indices of refraction on the near and far sides of the interface, respectively.

Use Brewster's law to obtain:

$$\theta_{\text{p}} = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

(*a*) For $n_1 = 1$ and $n_2 = 1.33$:

$$\theta_{\text{p}} = \tan^{-1}\left(\frac{1.33}{1}\right) = \boxed{53.1°}$$

(*b*) For $n_1 = 1$ and $n_2 = 1.50$:

$$\theta_{\text{p}} = \tan^{-1}\left(\frac{1.50}{1}\right) = \boxed{56.3°}$$

54 •

Picture the Problem The intensity of the transmitted light I is related to the intensity of the incident light I_0 and the angle the transmission axis makes with the horizontal θ

according to $I = I_0 \cos^2 \theta$.

Express the intensity of the transmitted light in terms of the intensity of the incident light and the angle the transmission axis makes with the horizontal:

$$I = I_0 \cos^2 \theta$$

Solve for θ:

$$\theta = \cos^{-1} \sqrt{\frac{I}{I_0}}$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1} \sqrt{0.15} = 67.2°$$

and $\boxed{(d) \text{ is correct.}}$

55 •

Picture the Problem Let I_n be the intensity after the *n*th polarizing sheet and use $I = I_0 \cos^2 \theta$ to find the intensity of the light transmitted through all three sheets for θ = 45° and θ = 30°.

(*a*) Express the intensity of the light between the first and second sheets:

$$I_1 = \tfrac{1}{2} I_0$$

Express the intensity of the light between the second and third sheets:

$$I_2 = I_1 \cos^2 \theta_{1,2} = \tfrac{1}{2} I_0 \cos^2 45° = \tfrac{1}{4} I_0$$

Express the intensity of the light that has passed through the third sheet:

$$I_3 = I_2 \cos^2 \theta_{2,3} = \tfrac{1}{4} I_0 \cos^2 45° = \boxed{\tfrac{1}{8} I_0}$$

(*b*) Express the intensity of the light between the first and second sheets:

$$I_1 = \tfrac{1}{2} I_0$$

Express the intensity of the light between the second and third sheets:

$$I_2 = I_1 \cos^2 \theta_{1,2} = \tfrac{1}{2} I_0 \cos^2 30° = \tfrac{3}{8} I_0$$

Express the intensity of the light that has passed through the third sheet:

$$I_3 = I_2 \cos^2 \theta_{2,3} = \tfrac{3}{8} I_0 \cos^2 60° = \boxed{\tfrac{3}{32} I_0}$$

56 ••

Picture the Problem Because the light is polarized in the vertical direction and the first polarizer is also vertically polarized, no loss of intensity results from the first transmission. We can use Malus's law to find the intensity of the light after it has passed through the second polarizer.

The intensity of the beam is the ratio of its power to cross-sectional area:

$$I = \frac{P}{A}$$

Express the intensity of the light between the first and second polarizers:

$$I_1 = I_0 \text{ and } P_1 = P_0$$

Express Malus's law in terms of the power of the beam:

$$\frac{P}{A} = \frac{P_0}{A}\cos^2\theta \Rightarrow P = P_0\cos^2\theta$$

Express the power of the beam after the second transmission:

$$P_2 = P_1\cos^2\theta_{1,2} = P_0\cos^2\theta_{12}$$

Substitute numerical values and evaluate I_2:

$$P_2 = (5\,\text{mW})\cos^2 27° = \boxed{3.97\,\text{mW}}$$

57 ••

Picture the Problem Assume that light is incident in air (n_1 = 1). We can use the relationship between the polarizing angle and the angle of refraction to determine the latter and Brewster's law to find the index of refraction of the substance.

(*a*) At the polarizing angle, the sum of the angles of polarization and refraction is 90°:

$$\theta_\text{p} + \theta_\text{r} = 90°$$

Solve for θ_r:

$$\theta_\text{r} = 90° - \theta_\text{p}$$

Substitute for θ_p to obtain:

$$\theta_\text{r} = 90° - 60° = \boxed{30.0°}$$

(*b*) From Brewster's law we have:

$$\tan\theta_\text{p} = \frac{n_2}{n_1}$$

or, because n_1 = 1,

$$n_2 = \tan\theta_\text{p}$$

Substitute for θ_p and evaluate n_2:

$$n_2 = \tan 60° = \boxed{1.73}$$

58 ••

Picture the Problem Let I_n be the intensity after the *n*th polarizing sheet and use $I = I_0 \cos^2\theta$ to find the intensity of the light transmitted through the three sheets.

Express the intensity of the light between the first and second sheets:

$$I_1 = \tfrac{1}{2} I_0$$

Express the intensity of the light between the second and third sheets:

$$I_2 = I_1 \cos^2\theta_{1,2} = \tfrac{1}{2} I_0 \cos^2\theta$$

Express the intensity of the light that has passed through the third sheet and simplify to obtain:

$$\begin{aligned} I_3 &= I_2 \cos^2\theta_{2,3} \\ &= \tfrac{1}{2} I_0 \cos^2\theta \cos^2(90° - \theta) \\ &= \tfrac{1}{2} I_0 \cos^2\theta \sin^2\theta \\ &= \tfrac{1}{8} I_0 (2\cos\theta \sin\theta)^2 \\ &= \boxed{\tfrac{1}{8} I_0 \sin^2 2\theta} \end{aligned}$$

Because the sine function is a maximum when its argument is 90°, the maximum value of I_3 occurs when:

$$\boxed{\theta = 45.0°}$$

59 ••

Picture the Problem Let I_n be the intensity after the *n*th polarizing sheet, use $I = I_0 \cos^2\theta$ to find the intensity of the light transmitted through each sheet, and replace θ with ωt.

Express the intensity of the light between the first and second sheets:

$$I_1 = \tfrac{1}{2} I_0$$

Express the intensity of the light between the second and third sheets:

$$I_2 = I_1 \cos^2\theta_{1,2} = \tfrac{1}{2} I_0 \cos^2\omega t$$

Express the intensity of the light that has passed through the third sheet and simplify to obtain:

$$\begin{aligned} I_3 &= I_2 \cos^2\theta_{2,3} \\ &= \tfrac{1}{2} I_0 \cos^2\omega t \cos^2(90° - \omega t) \\ &= \tfrac{1}{2} I_0 \cos^2\omega t \sin^2\omega t \\ &= \tfrac{1}{8} I_0 (2\cos\omega t \sin\omega t)^2 \\ &= \boxed{\tfrac{1}{8} I_0 \sin^2 2\omega t} \end{aligned}$$

***60 ••**

Picture the Problem Let I_n be the intensity after the nth polarizing sheet and use $I = I_0 \cos^2\theta$ to find the ratio of I_{n+1} to I_n.

(*a*) Find the ratio of I_{n+1} to I_n:

$$\frac{I_{n+1}}{I_n} = \cos^2\frac{\pi}{2N}$$

Because there are N such reductions of intensity:

$$\frac{I_{N+1}}{I_1} = \frac{I_{N+1}}{I_0} = \cos^{2N}\left(\frac{\pi}{2N}\right)$$

and

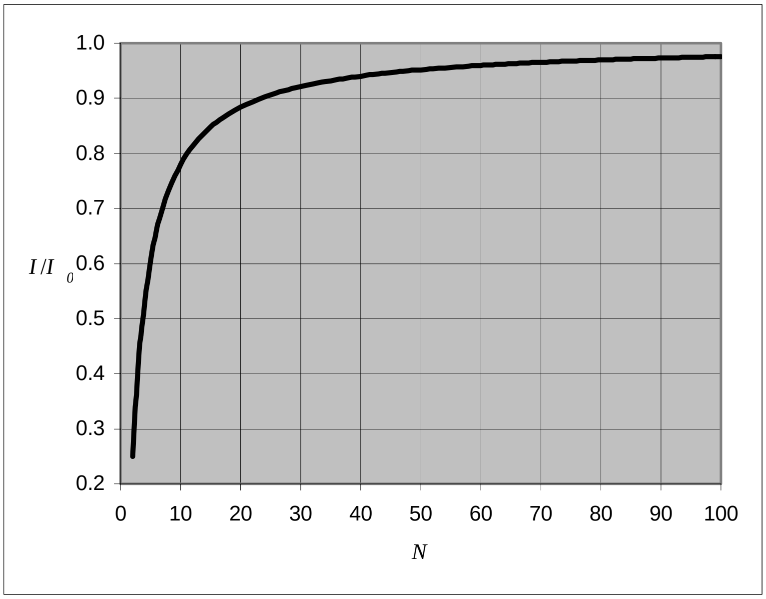
$$I_{N+1} = \boxed{I_0 \cos^{2N}\left(\frac{\pi}{2N}\right)}$$

(*b*) A spreadsheet program to graph I_{N+1}/I_0 as a function of N is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A2	2	N
A3	A2 + 1	$N + 1$
B2	(cos(PI()/(2*A2))^(2*A2)	$\cos^{2N}\left(\frac{\pi}{2N}\right)$

	A	B
1	N	I/I_0
2	2	0.250
3	3	0.422
4	4	0.531
5	5	0.605
95	95	0.974
96	96	0.975
97	97	0.975
98	98	0.975
99	99	0.975
100	100	0.976

A graph of I/I_0 as a function of N follows.



(*c*) In each case, the polarization of the transmitted beam is perpendicular to that of the incident beam.

61 ••

Picture the Problem Let I_n be the intensity after the *n*th polarizing sheet and use $I = I_0 \cos^2\theta$ to find the ratio of I_{n+1} to I_n. Because each sheet introduces a 2% loss of intensity, the net transmission after *N* sheets $(0.98)^N$.

Find the ratio of I_{n+1} to I_n:

$$\frac{I_{n+1}}{I_n} = (0.98)\cos^2\frac{\pi}{2N}$$

Because there are *N* such reductions of intensity:

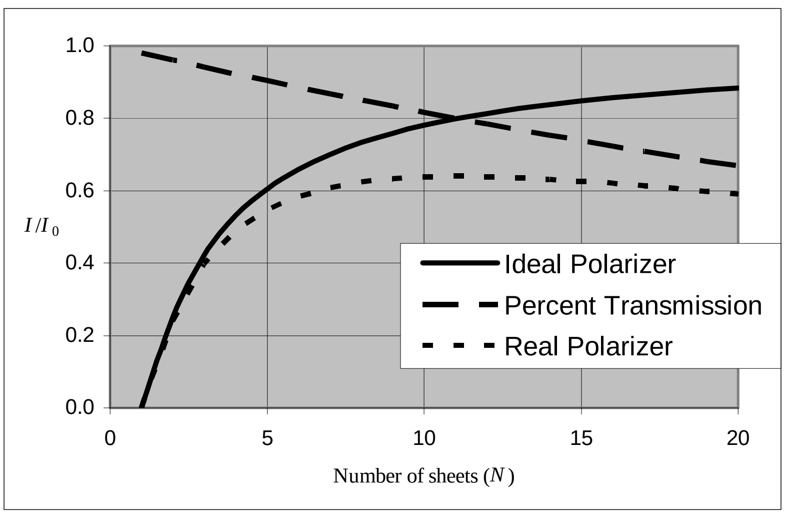
$$\frac{I_{N+1}}{I_0} = (0.98)^N \cos^{2N}\left(\frac{\pi}{2N}\right)$$

(*b*) A spreadsheet program to graph I_{N+1}/I_0 for an ideal polarizer as a function of *N*, the percent transmission, and I_{N+1}/I_0 for a real polarizer as a function of *N* is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A3	1	N
B2	(cos(PI()/(2*A2))^(2*A2)	$\cos^{2N}\left(\frac{\pi}{2N}\right)$
C3	(0.98)^A3	$(0.98)^N$
D4	B3*C3	$(0.98)^N \cos^{2N}\left(\frac{\pi}{2N}\right)$

	A	B	C	D
1		Ideal	Percent	Real
2	N	Polarizer	Transmission	Polarizer
3	1	0.000	0.980	0.000
4	2	0.250	0.960	0.240
5	3	0.422	0.941	0.397
6	4	0.531	0.922	0.490
7	5	0.605	0.904	0.547
8	6	0.660	0.886	0.584
9	7	0.701	0.868	0.608
10	8	0.733	0.851	0.624
11	9	0.759	0.834	0.633
12	10	0.781	0.817	0.638
13	11	0.798	0.801	0.639
14	12	0.814	0.785	0.638
15	13	0.827	0.769	0.636
16	14	0.838	0.754	0.632
17	15	0.848	0.739	0.626
18	16	0.857	0.724	0.620
19	17	0.865	0.709	0.613
20	18	0.872	0.695	0.606
21	19	0.878	0.681	0.598
22	20	0.884	0.668	0.590

A graph of I/I_0 as a function of N for the quantities described above follows:



Inspection of the table, as well as of the graph, tells us that the optimum number of sheets is 13.

***62** ••

Picture the Problem A circularly polarized wave is said to be *right circularly polarized* if the electric and magnetic fields rotate clockwise when viewed along the direction of propagation and *left circularly polarized* if the fields rotate counterclockwise.

For a circularly polarized wave, the x and y components of the electric field are given by:

$E_x = E_0 \cos \omega t$

and

$E_y = E_0 \sin \omega t$ or $E_y = -E_0 \sin \omega t$

for left and right circular polarization, respectively.

For a wave polarized along the x axis:

$$\vec{E}_{\text{right}} + \vec{E}_{\text{left}} = E_0 \cos \omega t \,\hat{i} + E_0 \cos \omega t \,\hat{i} = \boxed{2E_0 \cos \omega t \,\hat{i}}$$

63 ••

Picture the Problem Let I_n be the intensity after the nth polarizing sheet and use $I = I_0 \cos^2 \theta$ to find the intensity of the light transmitted by the four sheets.

(a) Express the intensity of the light between the first and second sheets:

$I_1 = \frac{1}{2} I_0$

Express the intensity of the light between the second and third sheets:

$I_2 = I_1 \cos^2 \theta_{1,2} = \frac{1}{2} I_0 \cos^2 30° = \frac{3}{8} I_0$

Express the intensity of the light between the third and fourth sheets:

$I_3 = I_2 \cos^2 \theta_{2,3} = \frac{3}{8} I_0 \cos^2 30° = \frac{9}{32} I_0$

Express the intensity of the light to the right of the fourth sheet:

$$I_4 = I_3 \cos^2 \theta_{3,4} = \frac{9}{32} I_0 \cos^2 30° = \frac{27}{128} I_0 = \boxed{0.211 I_0}$$

Note that, for the single sheet between the two end sheets at $\theta = 45^{\circ}$, $I = 0.125 I_0$. Using two sheets at relative angles of 30° increases the transmitted intensity.

Remarks: We could also apply the result obtained in Problem 60(*a*) to solve this problem.

***64** ••

Picture the Problem We can use the components of $\vec{E}$ to show that $\vec{E}$ is constant in time and rotates with angular frequency ω.

Express the magnitude of $\vec{E}$ in terms of its components:

$E = \sqrt{E_x^2 + E_y^2}$

Substitute for E_x and E_y to obtain:

$$E = \sqrt{\left[E_0 \sin(kx - \omega t)\right]^2 + \left[E_0 \cos(kx - \omega t)\right]^2} = \sqrt{E_0^2\left[\sin^2(kx - \omega t) + \cos^2(kx - \omega t)\right]}$$
$$= E_0$$

and the $\vec{E}$ vector rotates in the yz plane with angular frequency ω.

65 ••

Picture the Problem We can apply the given definitions of right and left circular polarization to the electric field and magnetic fields of the wave.

The electric field of the wave in Problem 64 is:

$$\vec{E} = E_0 \sin(kx - \omega t)\hat{j} + E_0 \cos(kx - \omega t)\hat{k}$$

The corresponding magnetic field is:

$$\vec{B} = B_0 \sin(kx - \omega t)\hat{k} - B_0 \cos(kx - \omega t)\hat{j}$$

Because these fields rotate clockwise when viewed along the direction of propagation, the wave is $\boxed{\text{right circularly polarized.}}$

For a left circularly polarized wave traveling in the opposite direction:

$$\vec{E} = \boxed{E_0 \sin(kx + \omega t)\hat{j} - E_0 \cos(kx + \omega t)\hat{k}}$$

General Problems

66 •

Picture the Problem We can use $v = f\lambda$ and the definition of the index of refraction to relate the wavelength of light in a medium whose index of refraction is n to the wavelength of light in air.

(a) The wavelength λ_n of light in a medium whose index of refraction is n is given by:

$$\lambda_n = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda_0}{n}$$

Substitute numerical values and evaluate λ_{water}:

$$\lambda_{\text{water}} = \frac{700\,\text{nm}}{n_{\text{water}}} = \frac{700\,\text{nm}}{1.33} = \boxed{526\,\text{nm}}$$

(*b*) Because the color observed depends on the frequency of the light, a swimmer observes the same color in air and in water.

67 ••

Picture the Problem We can use Snell's law, under critical angle and polarization conditions, to relate the polarizing angle of the substance to the critical angle for internal reflection.

Apply Snell's law, under critical angle conditions, to the interface:

$$n_1 \sin\theta_c = n_2 \qquad (1)$$

Apply Snell's law, under polarization conditions, to the interface:

$$n_1 \sin\theta_p = n_2 \sin(90° - \theta_p) = n_2 \cos\theta_p$$

or

$$\tan\theta_p = \frac{n_2}{n_1}$$

Solve for θ_p:

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) \qquad (2)$$

Solve equation (1) for the ratio of n_2 to n_1:

$$\frac{n_2}{n_1} = \sin\theta_c$$

Substitute for n_2/n_1 in equation (2) to obtain:

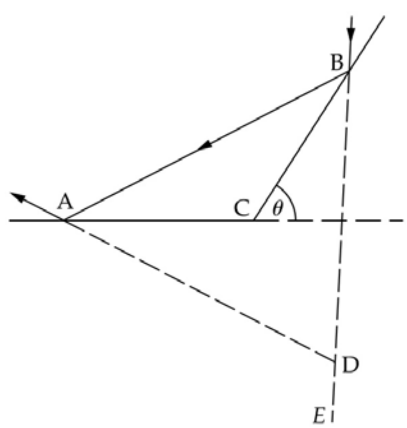
$$\theta_p = \tan^{-1}(\sin\theta_c)$$

Substitute numerical values and evaluate θ_p:

$$\theta_p = \tan^{-1}(\sin 45°) = \boxed{35.3°}$$

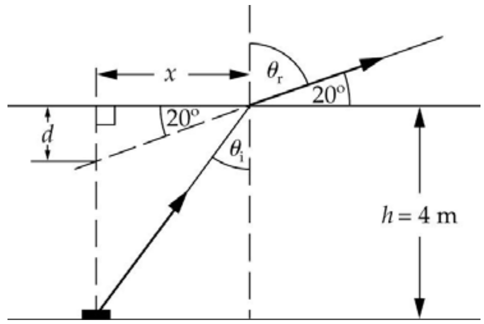
***68** ••

Picture the Problem Angle *ADE* is the angle between the direction of the incoming ray and that reflected by the two mirror surfaces. Note that triangle *ABC* is isosceles and that angles *CAB* and *ABC* are equal and their sum equals θ. Also from the law of reflection, angles *CAD* and *CBD* equal angle *ABC*. Because angle *BAD* is twice *BAC* and angle *DBA* is twice *CBA*, angle *ADE* is twice the angle θ.



69 ••

Picture the Problem The sketch shows the ray from the coin passing through the water to the eye of the observer. We can use trigonometry to express the apparent depth *d* in terms of the depth *h* of the water, the 20° angle, and the angle of incidence θ_i. The application of Snell's law at the interface will yield an expression for θ_i.



Express the apparent depth *d* in terms of the distance *x*:

$$d = x\tan 20° \qquad (1)$$

Relate the distance *x* to the depth of the water and the angle θ_i:

$$x = h\tan\theta_i$$

Substitute for *x* in equation (1) to obtain:

$$d = h\tan\theta_i\tan 20° \qquad (2)$$

Apply Snell's law to the water-air interface:

$$n_1\sin\theta_i = n_2\sin\theta_r$$

Solve for θ_i:

$$\theta_i = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_r\right)$$

Substitute for θ_i in equation (2) to obtain:

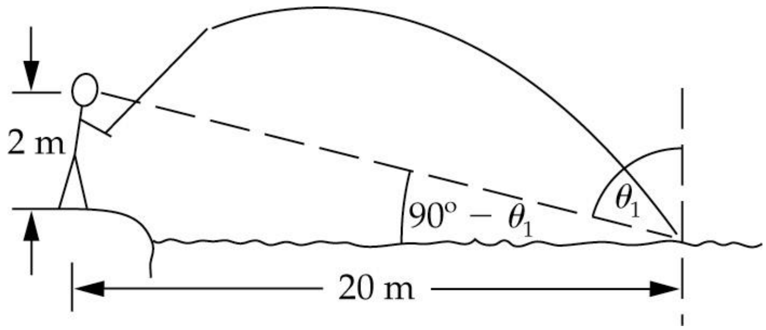
$$d = h\tan\left[\sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_r\right)\right]\tan 20°$$

Substitute numerical values and evaluate *d*:

$$d = (4\,\text{m})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\sin 70°\right)\right]\tan 20° = \boxed{1.45\,\text{m}}$$

70 ••

Picture the Problem Assume that the sound source is the voice of the fisherman and that the fisherman's mouth is 2 m from the surface of the water as shown below. We can apply Snell's law at the air-water interface to find θ_c and use trigonometry to find θ_1. If we can show that $\theta_1 > \theta_c$, then we can conclude that the noise on shore cannot possibly be sensed by fish 20 m from shore.



Apply Snell's law at the air-water interface 20 m from the shore:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

For $\theta_1 = \theta_c$:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{v_1}{v_2}\right)$$

Substitute numerical values and evaluate θ_c:

$$\theta_c = \sin^{-1}\left(\frac{330\,\text{m/s}}{1450\,\text{m/s}}\right) = 13.2°$$

Relate θ_1 to the distance from the shore and the distance from the surface of the water to the fisherman's mouth:

$$\tan(90° - \theta_1) = \frac{2\,\text{m}}{20\,\text{m}}$$

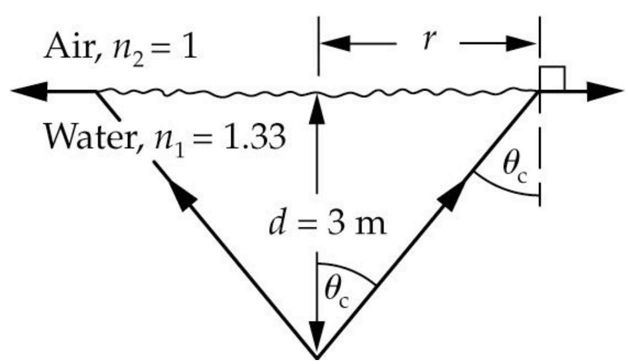
Solve for and evaluate θ_1:

$$\theta_1 = 90° - \tan^{-1}(0.1) = 84.3°$$

Because $\theta_1 > \theta_c$, all the sound is reflected at air - water interface.

***71 ••**

Picture the Problem We can apply Snell's law to the water-air interface to express the critical angle θ_c in terms of the indices of refraction of water (n_1) and air (n_2) and then relate the radius of the circle to the depth d of the swimmer and θ_c.



Relate the radius of the circle to the depth *d* of the point source and the critical angle θ_c:

$$r = d\tan\theta_c$$

Apply Snell's law to the water-air interface to obtain:

$$n_1\sin\theta_c = n_2\sin 90° = n_2$$

Solve for θ_c:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Substitute for θ_c to obtain:

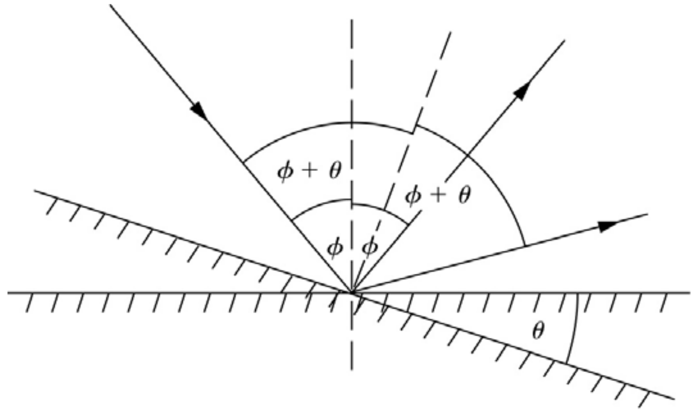
$$r = d\tan\left[\sin^{-1}\left(\frac{n_2}{n_1}\right)\right]$$

Substitute numerical values and evaluate *r*:

$$r = (3\,\text{m})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = \boxed{3.42\,\text{m}}$$

72 ••

Picture the Problem Let ϕ be the initial angle of incidence. Since the angle of reflection with the normal to the mirror is also ϕ, the angle between incident and reflected rays is 2ϕ. If the mirror is now rotated by a further angle θ, the angle of incidence is increased by θ to $\phi + \theta$, and so is the angle of reflection. Consequently, the reflected beam is rotated by 2θ relative to the incident beam.



73 ••

Picture the Problem We can apply Snell's law at the glass-air interface to express θ_c in terms of the index of refraction of the glass and use Figure 31-25 to find the index of refraction of the glass for the given wavelengths of light.

Apply Snell's law at the glass-air interface:

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

If $\theta_1 = \theta_c$ and $n_2 = 1$:

$$n_1 \sin\theta_c = \sin 90° = 1$$

and

$$\theta_c = \sin^{-1}\left(\frac{1}{n_1}\right)$$

(a) For violet light of wavelength 400 nm, $n_2 = 1.67$:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.67}\right) = \boxed{36.8°}$$

(b) For red light of wavelength 700 nm, $n_2 = 1.60$:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.60}\right) = \boxed{38.7°}$$

74 ••

Picture the Problem We'll neglect multiple reflections at the glass-air interfaces. We can use the expression (Equation 31-11) for the reflected intensity at an interface to express the intensity of the light in the glass slab as the difference between the intensity of the incident beam and the reflected beam. Repeating this analysis at the glass-air interface will lead to the desired result.

Express the intensity of the light transmitted into the glass:

$$I_{\text{glass}} = I_0 - I_{\text{R},1}$$

where $I_{\text{R},1}$ is the intensity of the light reflected at the air-glass interface.

The intensity of the light reflected at the air-glass interface is:

$$I_{\text{R},1} = \left(\frac{1-n}{1+n}\right)^2 I_0$$

Substitute and simplify to obtain:

$$\begin{aligned} I_{\text{glass}} &= I_0 - \left(\frac{1-n}{1+n}\right)^2 I_0 \\ &= I_0\left[1 - \left(\frac{1-n}{1+n}\right)^2\right] \\ &= I_0\left[\frac{4n}{(1+n)^2}\right] \end{aligned}$$

Express the intensity of the light transmitted at the glass-air interface:

$$I_{\text{T}} = I_{\text{glass}} - I_{\text{R},2}$$

where $I_{\text{R},2}$ is the intensity of the light reflected at the glass-air interface.

The intensity of the light reflected at the glass-air interface is:

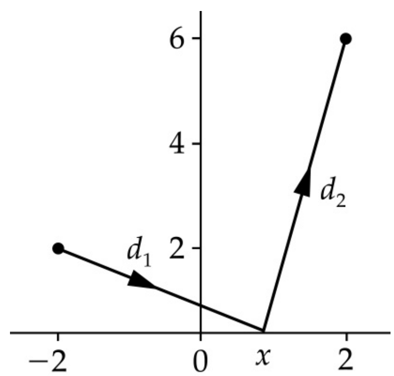
$$I_{R,2}=\left(\frac{1-n}{1+n}\right)^2 I_{glass}$$
$$=\left(\frac{1-n}{1+n}\right)^2\left[\frac{4n}{(1+n)^2}\right]I_0$$

Substitute and simplify to obtain:

$$I_T=I_0\left[\frac{4n}{(1+n)^2}\right]-\left(\frac{1-n}{1+n}\right)^2\left[\frac{4n}{(1+n)^2}\right]I_0$$
$$=I_0\left[1-\left(\frac{1-n}{1+n}\right)^2\right]\left[\frac{4n}{(1+n)^2}\right]$$
$$=I_0\left[\frac{4n}{(1+n)^2}\right]\left[\frac{4n}{(1+n)^2}\right]$$
$$=\boxed{I_0\left[\frac{4n}{(1+n)^2}\right]^2}$$

75 ••

Picture the Problem We can write an expression for the total distance traveled by the light as a function of *x* and set the derivative of this expression equal to zero to find the value of *x* that minimizes the distance traveled by the light. The adjacent figure shows the two points and the reflecting surface. The *x* and *y* coordinates are in meters.



(*a*) Express the total distance *D* traveled by the light:

$$D=d_1+d_2$$
$$=\sqrt{(x+2)^2+4}+\sqrt{(2-x)^2+36}$$

Differentiate *D* with respect to *x*:

$$\frac{dD}{dx}=\frac{d}{dx}\left[\sqrt{(x+2)^2+4}+\sqrt{(2-x)^2+36}\right]$$
$$=\tfrac{1}{2}\left[(x+2)^2+4\right]^{-\frac{1}{2}}2(x+2)+\tfrac{1}{2}\left[(2-x)^2+36\right]^{-\frac{1}{2}}2(2-x)(-1)=0\text{ for extrema}$$

Simplify this expression to obtain:

$$\frac{x+2}{\sqrt{(x+2)^2+4}} - \frac{2-x}{\sqrt{(2-x)^2+36}} = 0$$

Solve for x to obtain:

$$x = \boxed{-1.00\,\text{m}}$$

(*b*) With x = –1 m:

$$\theta_\text{i} = \tan^{-1}\left[\frac{-2-(-1)}{0-2}\right] = \tan^{-1}\left(\frac{1}{2}\right) = \boxed{26.6°}$$

and

$$\theta_\text{r} = \tan^{-1}\left[\frac{-1-(2)}{0-6}\right] = \tan^{-1}\left(\frac{3}{6}\right) = \boxed{26.6°}$$

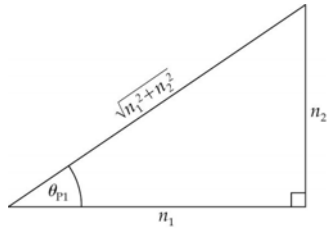
***76 ••**

Picture the Problem Let the angle of refraction at the first interface by θ_1 and the angle of refraction at the second interface be θ_2. We can apply Snell's law at each interface and eliminate θ_1 and n_2 to show that $\theta_2 = \theta_{P2}$.

Apply Snell's Brewster's law at the n_1-n_2 interface:

$$\tan\theta_{P1} = \frac{n_2}{n_1}$$

Draw a reference triangle consistent with Brewster's law:



Apply Snell's law at the n_1-n_2 interface:

$$n_1\sin\theta_{P1} = n_2\sin\theta_1$$

Solve for θ_1 to obtain:

$$\theta_1 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_{P1}\right)$$

Referring to the reference triangle we note that:

$$\theta_1 = \sin^{-1}\left(\frac{n_1}{n_2}\frac{n_2}{\sqrt{n_1^2+n_2^2}}\right) = \sin^{-1}\left(\frac{n_1}{\sqrt{n_1^2+n_2^2}}\right)$$

i.e., θ_1 is the complement of θ_{p1}.

Apply Snell's law at the n_2-n_1 interface:

$$n_2 \sin\theta_1 = n_1 \sin\theta_2$$

Solve for θ_2 to obtain:

$$\theta_2 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_1\right)$$

Refer to the reference triangle again to obtain:

$$\theta_2 = \sin^{-1}\left(\frac{n_2}{n_1}\frac{n_1}{\sqrt{n_1^2 + n_2^2}}\right)$$

$$= \sin^{-1}\left(\frac{n_2}{\sqrt{n_1^2 + n_2^2}}\right) = \boxed{\theta_{P2}}$$

Equate these expressions for $n_2 \sin\theta_1$ to obtain:

$$n_1 \sin\theta_P = n_1 \sin\theta_2 \Rightarrow \theta_2 = \boxed{\theta_P}$$

77 ••

Picture the Problem We can use Brewster's law in conjunction with index of refraction data from Figure 31-29 to calculate the polarization angles for the air-glass interface.

From Brewster's law we have:

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

or, for $n_1 = 1$,

$$\theta_p = \tan^{-1} n_2$$

For silicate flint glass, $n_2 \approx 1.62$ and:

$$\theta_p = \tan^{-1}(1.62) = \boxed{58.3°}$$

For borate flint glass, $n_2 \approx 1.57$ and:

$$\theta_p = \tan^{-1}(1.57) = \boxed{57.5°}$$

For quartz glass, $n_2 \approx 1.54$ and:

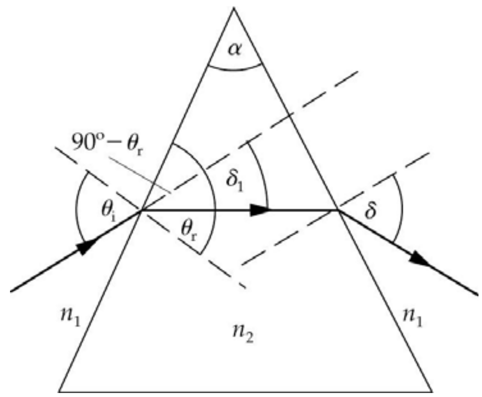
$$\theta_p = \tan^{-1}(1.54) = \boxed{57.0°}$$

For silicate crown glass, $n_2 \approx 1.51$ and:

$$\theta_p = \tan^{-1}(1.51) = \boxed{56.5°}$$

78 •••

Picture the Problem The diagram to the right shows the angles of incidence, refraction, and deviation at the first interface. We can use the geometry of this symmetric passage of the light to express θ_r in terms of α and δ_1 in terms of θ_r and α. We can then use a symmetry argument to express the deviation at the second interface and the total deviation δ. Finally, we can apply Snell's law at the first interface to complete the derivation of the given expression.



(*a*) With respect to the normal to the left face of the prism, let the angle of incidence be θ_i and the angle of refraction be θ_r. From the geometry of the figure, it is evident that:

$$\theta_r = \tfrac{1}{2}\alpha$$

Express the angle of deviation at the refracting surface:

$$\delta_1 = \theta_i - \theta_r = \theta_i - \tfrac{1}{2}\alpha$$

By symmetry, the angle of deviation at the second refracting surface is also of this magnitude. Thus:

$$\delta = 2\delta_1 = 2\theta_i - \alpha$$

Solve for θ_i:

$$\theta_i = \tfrac{1}{2}(\alpha + \delta)$$

Apply Snell's law, with $n_1 = 1$ and $n_2 = n$, to the first interface:

$$\sin\theta_i = n\sin\tfrac{1}{2}\alpha$$

Substitute for θ_i to obtain:

$$\boxed{\sin\frac{\alpha+\delta}{2} = n\sin\frac{\alpha}{2}} \qquad (1)$$

(*b*) The angular separation is:

$$\Delta\delta = \delta_{violet} - \delta_{red}$$

Solve equation (1) for δ:

$$\delta = 2\sin^{-1}\left[n\sin\frac{\alpha}{2}\right] - \alpha$$

Substitute to obtain:

$$\Delta\delta = 2\sin^{-1}\left[n_{\text{violet}}\sin\frac{\alpha}{2}\right] - \alpha - \left\{2\sin^{-1}\left[n_{\text{red}}\sin\frac{\alpha}{2}\right] - \alpha\right\}$$
$$= 2\sin^{-1}\left[n_{\text{violet}}\sin\frac{\alpha}{2}\right] - 2\sin^{-1}\left[n_{\text{red}}\sin\frac{\alpha}{2}\right]$$

Substitute numerical values and evaluate $\Delta\delta$:

$$\Delta\delta = 2\sin^{-1}\left[1.52\sin\frac{60°}{2}\right] - 2\sin^{-1}\left[1.48\sin\frac{60°}{2}\right] = \boxed{3.47°}$$

***79 ••**

Picture the Problem We can apply Snell's law at the critical angle and the polarizing angle to show that tan θ_p = sin θ_c.

(*a*) Apply Snell's law at the medium-vacuum interface:

$$n_1\sin\theta_1 = n_2\sin\theta_r$$

For $\theta_1 = \theta_c$, $n_1 = n$, and $n_2 = 1$:

$$n\sin\theta_c = \sin 90° = 1$$

For $\theta_1 = \theta_p$, $n_1 = n$, and $n_2 = 1$:

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1}{n} \Rightarrow n\tan\theta_p = 1$$

Because both expressions equal one:

$$\boxed{\tan\theta_p = \sin\theta_c}$$

(*b*) For any value of θ:

$$\tan\theta > \sin\theta \Rightarrow \boxed{\theta_p > \theta_c}$$

80 ••

Picture the Problem Let the numeral 1 refer to the side of the interface from which the light is incident and the numeral 2 to the refraction side of the interface. We can apply Snell's law, under the conditions described in the problem statement, at the interface to derive an expression for *n* as a function of the angle of incidence (also the polarizing angle).

(*a*) Apply Snell's law at the air-medium interface:

$$\sin\theta_1 = n\sin\theta_2$$

Because the reflected and refracted rays are mutually perpendicular:

$$\theta_1 + \theta_2 = 90° \Rightarrow \theta_2 = 90° - \theta_1$$

Substitute for θ_2 to obtain:

$$\sin\theta_1 = n\sin(90° - \theta_1) = n\cos\theta_1$$

or

$$n = \tan\theta_1 = \tan\theta_p$$

Substitute for θ_p and evaluate n:

$$n = \tan 58° = \boxed{1.60}$$

(*b*) Apply Snell's law at the interface under conditions of total internal reflection:

$$n_2 \sin\theta_c = n_1 \sin 90° = n_1$$

Because $n_1 = 1$:

$$\theta_c = \sin^{-1}\left(\frac{1}{n_2}\right) = \sin^{-1}\left(\frac{1}{n}\right)$$

Substitute for n and evaluate θ_c:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.6}\right) = \boxed{38.7°}$$

81 ••

Picture the Problem We can apply Snell's law at the glass–liquid and liquid–air interfaces to find the refractive index of the unknown liquid, the angle of incidence (glass-air interface) for total internal reflection, and the angle of refraction of a ray into the liquid film.

(*a*) Apply Snell's law, under critical-angle conditions, at the glass–liquid interface:

$$\sin\theta_c = \frac{n_{\text{liquid}}}{n_{\text{glass}}}$$

Solve for n_{liquid}:

$$n_{\text{liquid}} = n_{\text{glass}} \sin\theta_c$$

Substitute numerical values and evaluate n_{liquid}:

$$n_{\text{liquid}} = (1.655)\sin 53.7° = \boxed{1.33}$$

(*b*) With the liquid removed:

$$\theta_c = \sin^{-1}\left(\frac{1}{n_{\text{glass}}}\right)$$

Substitute numerical values and evaluate θ_c:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.655}\right) = \boxed{37.2°}$$

(*c*) Apply Snell's law at the glass–liquid interface:

$$n_{\text{glass}} \sin\theta_1 = n_{\text{liquid}} \sin\theta_2$$

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left[\frac{n_{\text{glass}}}{n_{\text{liquid}}}\sin\theta_1\right]$$

Substitute numerical values and evaluate θ_2:

$$\theta_2 = \sin^{-1}\left[\frac{1.655}{1.33}\sin 37.2°\right] = \boxed{48.8°}$$

Because θ_2 is also the angle of incidence at the liquid – air interface and because it is larger than the critical angle for total internal reflection at this interface, no light will emerge.

82 ••

Picture the Problem We can use Equation 31-18 and the result of Problem 86 to find the angular separation of these colors in the primary rainbow.

Express the angular separation $\Delta\phi$ of the colors:

$$\Delta\phi = \phi_{\text{d,blue}} - \phi_{\text{d,red}} \qquad (1)$$

From Equation 31-18, with $n_{\text{air}} = 1$ and $n_{\text{water}} = n$:

$$\phi_{\text{d}} = \pi + 2\theta_1 - 4\sin^{-1}\left(\frac{\sin\theta_1}{n}\right)$$

From Problem 86:

$$\cos\theta_{1\text{m}} = \sqrt{\frac{n^2-1}{3}}$$

or

$$\theta_{1\text{m}} = \cos^{-1}\left[\sqrt{\frac{n^2-1}{3}}\right]$$

Substitute to obtain:

$$\phi_{\text{d}} = \pi + 2\cos^{-1}\left[\sqrt{\frac{n^2-1}{3}}\right] - 4\sin^{-1}\left(\frac{\sin\left\{\cos^{-1}\left[\sqrt{\frac{n^2-1}{3}}\right]\right\}}{n}\right)$$

Evaluate ϕ_{d} for blue light in water:

$$\phi_{\text{d,blue}} = \pi + 2\cos^{-1}\left[\sqrt{\frac{(1.3435)^2 - 1}{3}}\right] - 4\sin^{-1}\left(\frac{\sin\left\{\cos^{-1}\left[\sqrt{\frac{(1.3435)^2 - 1}{3}}\right]\right\}}{1.3435}\right)$$

$$= 139.42°$$

Evaluate ϕ_{d} for red light in water:

$$\phi_{\text{d,red}} = \pi + 2\cos^{-1}\left[\sqrt{\frac{(1.3318)^2 - 1}{3}}\right] - 4\sin^{-1}\left(\frac{\sin\left\{\cos^{-1}\left[\sqrt{\frac{(1.3318)^2 - 1}{3}}\right]\right\}}{1.3318}\right)$$

$$= 137.75°$$

Substitute in equation (1) and evaluate $\Delta\phi$:

$$\Delta\phi = 139.42 - 137.75° = \boxed{1.67°}$$

83 ••

Picture the Problem We can use the result, obtained in Problem 74, that each slab reduces the intensity of the transmitted light by $\left[\frac{4n}{(n+1)^2}\right]^2$, to find the ratio of the transmitted intensity to the incident intensity through N parallel slabs of glass for light of normal incidence.

(*a*) From Problem 74, each slab reduces the intensity by the factor:

$$\left[\frac{4n}{(n+1)^2}\right]^2$$

For N slabs:

$$I_{\text{t}} = I_0\left[\frac{4n}{(n+1)^2}\right]^{2N}$$

and

$$\frac{I_{\text{t}}}{I_0} = \boxed{\left[\frac{4n}{(n+1)^2}\right]^{2N}} \qquad (1)$$

(*b*) Evaluate equation (1) with $N = 3$ and $n = 1.5$:

$$\frac{I_{\text{t}}}{I_0} = \left[\frac{4(1.5)}{(1.5+1)^2}\right]^{2(3)} = \boxed{0.783}$$

(*c*) Begin the solution of equation (1) for *N* by taking the logarithm (arbitrarily to base 10) of both sides of the equation:

$$\log\left(\frac{I_t}{I_0}\right)=\log\left[\frac{4n}{(n+1)^2}\right]^{2N}=2N\log\left[\frac{4n}{(n+1)^2}\right]$$

Solve for *N*:

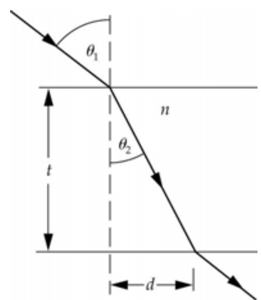
$$N=\frac{\log\left(\frac{I_t}{I_0}\right)}{2\log\left[\frac{4n}{(n+1)^2}\right]}$$

Substitute numerical values and evaluate *N*:

$$N=\frac{\log(0.1)}{2\log\left[\frac{4(1.5)}{(1.5+1)^2}\right]}=28.2\approx\boxed{28}$$

84 ••

Picture the Problem We can apply Snell's law at the air-slab interface to express the index of refraction *n* in terms of θ_1 and θ_2 and then use the geometry of the figure to relate θ_2 to *t* and *d*.



Apply Snell's law to the first interface:

$$\sin\theta_1=n\sin\theta_2$$

Solve for *n*:

$$n=\frac{\sin\theta_1}{\sin\theta_2}$$

From the diagram:

$$d=t\tan\theta_2\Rightarrow\theta_2=\tan^{-1}\left(\frac{d}{t}\right)$$

Substitute to obtain:

$$n=\boxed{\frac{\sin\theta_1}{\sin\left[\tan^{-1}\left(\frac{d}{t}\right)\right]}}$$

***85** ••

Picture the Problem The angle that the rain appears to make with the vertical, according to the marathoner, is the angle whose tangent is the ratio of v_{runner} to v_{rain}. The circular motion of the star is analogous to the circular motion of the cloud with $v_{\text{runner}} = v_{\text{earth}}$ and $v_{\text{rain}} = c$.

(*a*) The angle that the rain appears to make with the vertical to the marathoner is given by:

$$\theta = \tan^{-1}\left(\frac{v_{\text{runner}}}{v_{\text{rain}}}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{4\text{ m/s}}{9\text{ m/s}}\right) = \boxed{24.0^\circ}$$

(*b*) The cloud moves in a circle whose radius is given by:

$$R = H\tan\theta$$

Substitute numerical values and evaluate R:

$$R = (10\text{ km})\tan 24^\circ = \boxed{4.45\text{ km}}$$

(*c*) Here $v_{\text{runner}} = v_{\text{earth}}$ and $v_{\text{rain}} = c$:

$$\theta = \boxed{\tan^{-1}\left(\frac{v_{\text{earth}}}{c}\right)} \qquad (1)$$

where $\theta = \frac{1}{2}(\text{angular diameter})$

(*d*) From equation (1):

$$c = \frac{v_{\text{earth}}}{\tan\theta} = \frac{2\pi R_{\text{earth-sun}}}{T_{\text{earth}}\tan\theta}$$

Convert 20.6″ to degrees:

$$20.6'' = 20.6'' \times \frac{1'}{60''} \times \frac{1^\circ}{60'} = 5.722 \times 10^{-3\circ}$$

Substitute numerical values and evaluate c:

$$c = \frac{2\pi(1.5\times10^{11}\text{ m})}{(1\text{ y})(3.156\times10^{7}\text{ s/y})\tan(20.6'')} = \boxed{2.99\times10^{8}\text{ m/s}}$$

Substitute numerical values and evaluate c:

$$c = \frac{2\pi(1.5\times10^{11}\text{ m})}{(1\text{ y})(3.156\times10^{7}\text{ s/y})\tan(5.722\times10^{-3\circ})} = \boxed{2.99\times10^{8}\text{ m/s}}$$

86 •••

Picture the Problem We can follow the directions given in the problem statement and use the hint to establish the given result.

(*a*) Equation 31-18 is:

$$\phi_d = \pi + 2\theta_1 - 4\sin^{-1}\left(\frac{n_{air}\sin\theta_1}{n_{water}}\right)$$

For $n_{air} = 1$ and $n_{water} = n$:

$$\phi_d = \pi + 2\theta_1 - 4\sin^{-1}\left(\frac{\sin\theta_1}{n}\right)$$

Use the hint to differentiate ϕ_d with respect to θ_1:

$$\frac{d\phi_d}{d\theta_1} = \frac{d}{d\theta_1}\left[\pi + 2\theta_1 - 4\sin^{-1}\left(\frac{\sin\theta_1}{n}\right)\right] = \boxed{2 - \frac{4\cos\theta_1}{\sqrt{n^2 - \sin^2\theta_1}}}$$

(*b*) Set $d\phi_d/d\theta_1 = 0$:

$$2 - \frac{4\cos\theta_1}{\sqrt{n^2 - \sin^2\theta_1}} = 0 \text{ for extrema}$$

Simplify to obtain:

$$16\cos^2\theta_1 = 4\left(n^2 - \sin^2\theta_1\right)$$

Replace $\sin^2\theta_1$ with $1 - \cos^2\theta_1$ and simplify:

$$12\cos^2\theta_1 = 4n^2 - 4$$

Solve for $\cos\theta_1 = \cos\theta_{1m}$:

$$\cos\theta_{1m} = \boxed{\sqrt{\frac{n^2 - 1}{3}}}$$

and

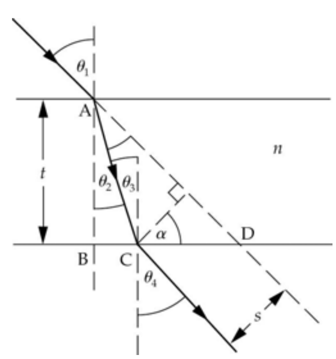
$$\theta_{1m} = \cos^{-1}\left[\sqrt{\frac{n^2 - 1}{3}}\right]$$

Evaluate θ_{1m} for $n = 1.33$:

$$\theta_{1m} = \cos^{-1}\left[\sqrt{\frac{(1.33)^2 - 1}{3}}\right] = \boxed{59.6°}$$

87 •••

Picture the Problem Let the thickness of the slab be t and the separation of the incident and emerging rays be ds. We can apply Snell's law at both interfaces and use the geometry of the diagram and trigonometric relationships to show that the emerging ray and incident ray are parallel and to derive an expression for d.



Apply Snell's law at the two interfaces to obtain:

$$\sin\theta_1 = n\sin\theta_2 \quad (1)$$

and

$$n\sin\theta_3 = \sin\theta_4$$

Because θ_2 and θ_3 are equal (they are alternate interior angles formed by parallel lines and a transversal):

$$\sin\theta_1 = n\sin\theta_3$$

and

$$n\sin\theta_3 = \sin\theta_4$$

Substitute for $n\sin\theta_3$ in the first of these equations to obtain:

$\sin\theta_1 = \sin\theta_4 \Rightarrow \theta_1 = \theta_4$ and

the emerging ray and incident ray are parallel.

Express the distance d_{BD} in terms of t and θ_1:

$$d_{BD} = t\tan\theta_1$$

The distance d_{BC} is:

$$d_{BC} = t\tan\theta_2$$

Use the distances d_{BD} and d_{BC} to express the distance d_{CD}:

$$d_{CD} = d_{BD} - d_{BC} = t(\tan\theta_1 - \tan\theta_2)$$

Because α and θ_1 have their right and left sides mutually perpendicular, they are equal and:

$$s = t(\tan\theta_1 - \tan\theta_2)\cos\alpha = t(\tan\theta_1 - \tan\theta_2)\cos\theta_1 \quad (2)$$

Substitute for $\tan\theta_1$ and $\tan\theta_2$ and simplify to obtain:Solve equation (1) for θ_2:

$$s = t\left(\frac{\sin\theta_1}{\cos\theta_1} - \frac{\sin\theta_2}{\cos\theta_2}\right)\cos\theta_1$$

$$= t\left(\sin\theta_1 - \frac{\sin\theta_2\cos\theta_1}{\cos\theta_2}\right)$$

$$= \frac{t(\sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1)}{\cos\theta_2}$$

$$= \boxed{\frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}}$$

Remarks: One can also derive this expression using the law of sines.

88 ••

Picture the Problem We can use Snell's law to determine θ_2 and then apply the result of Problem 87 to find s.

From Problem 87 we have:

$$s = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$$

Apply Snell's law to the first interface to obtain:

$$\sin\theta_1 = n\sin\theta_2$$

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right)$$

Substitute numerical values and evaluate θ_2:

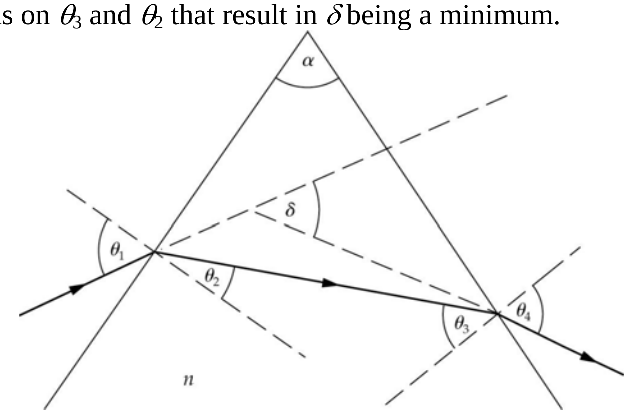
$$\theta_2 = \sin^{-1}\left(\frac{\sin 30°}{1.5}\right) = 19.47°$$

Substitute numerical values and evaluate s:

$$s = \frac{(15\,\text{mm})\sin(30° - 19.47°)}{\cos(19.47°)} = \boxed{2.91\,\text{mm}}$$

89 •••

Picture the Problem The figure below shows the prism and the path of the ray through it. The dashed lines are the normals to the prism faces. The triangle formed by the interior ray and the prism faces has interior angles of α, 90° – θ_2, and 90° – θ_3. Consequently, $\theta_2 + \theta_3 = \alpha$. We can apply Snell's law at both interfaces to express the angle of deviation δ as a function of θ_3 and then set the derivative of this function equal to zero to find the conditions on θ_3 and θ_2 that result in δ being a minimum.



Express the angle of deviation:

$$\delta = \theta_1 + \theta_2 - \alpha \quad (1)$$

Apply Snell's law to relate θ_1 to θ_2 and θ_3 to θ_4:

$$\sin\theta_1 = n\sin\theta_2 \quad (2)$$

and

$$n\sin\theta_3 = \sin\theta_4 \quad (3)$$

Solve equation (2) for θ_1 and

$$\theta_1 = \sin^{-1}(n\sin\theta_2)$$

equation (3) for θ_4: and

$$\theta_4 = \sin^{-1}(n \sin\theta_3)$$

Substitute in equation (1) to obtain:

$$\delta = \sin^{-1}(n\sin\theta_2) + \sin^{-1}(n\sin\theta_3) - \alpha = \sin^{-1}[n\sin(\theta_3 - \alpha)] + \sin^{-1}(n\sin\theta_3) - \alpha$$

Note that the only variable in this expression is θ_3. To determine the condition that minimizes δ, take the derivative of δ with respect to θ_3 and set it equal to zero.

$$\begin{aligned}\frac{d\delta}{d\theta_3} &= \frac{d}{d\theta_3}\left\{\sin^{-1}[n\sin(\theta_3-\alpha)] + \sin^{-1}(n\sin\theta_3) - \alpha\right\} \\ &= -\frac{n\cos(\alpha-\theta_3)}{\sqrt{1-[n\sin(\alpha-\theta_3)]^2}} + \frac{n\cos\theta_3}{\sqrt{1-(n\sin\theta_3)^2}} = 0 \text{ for extrema}\end{aligned}$$

This equation is satisfied provided: $\alpha - \theta_3 = \theta_3 \Rightarrow \theta_3 = \frac{1}{2}\alpha$

Because $\theta_2 = \alpha - \theta_3$: $\theta_2 = \alpha - \frac{1}{2}\alpha = \frac{1}{2}\alpha$

Because $\theta_2 = \theta_3$, we can conclude that the deviation angle is a minimum if the ray passes through the prism symmetrically.

Remarks: Setting $d\delta/d\theta_3 = 0$ establishes the condition on θ_3 that δ is either a maximum or a minimum. To establish that δ is indeed a minimum when $\theta_3 = \theta_2 = \frac{1}{2}\alpha$, we can either show that $d^2\delta/d\theta_3^2$, evaluated at $\theta_3 = \frac{1}{2}\alpha$, is positive or, alternatively, plot a graph of $\delta(\theta_3)$ to show that it is concave upward at $\theta_3 = \frac{1}{2}\alpha$.

Chapter 32
Optical Images

Conceptual Problems

1 •

Determine the Concept Yes. Note that a virtual image is ″seen″ because the eye focuses the diverging rays to form a real image on the retina. Similarly, the camera lens can focus the diverging rays onto the film.

2 •

Determine the Concept Yes; the mirror image is a left-handed coordinate system.

3 ••

(*a*) False. The virtual image formed by a concave mirror when the object is between the focal point and the vertex of the mirror depends on the distance of the object from the vertex.

(*b*) False. When the object is outside the focal point, the image is real.

(*c*) True.

(*d*) False. When the object is between the center of curvature and the focal point, the image is enlarged and real.

***4 ••**

Determine the Concept Let *s* be the object distance and *f* the focal length of the mirror.

(*a*) If $s < f$, the image is virtual, upright, and larger than the object.

(*b*) If $s < f$, the image is virtual, upright, and larger than the object.

(*c*) If $s > 2f$, the image is real, inverted, and smaller than the object.

(*d*) If $f < s < 2f$, the image is real, inverted, and larger than the object.

5 ••

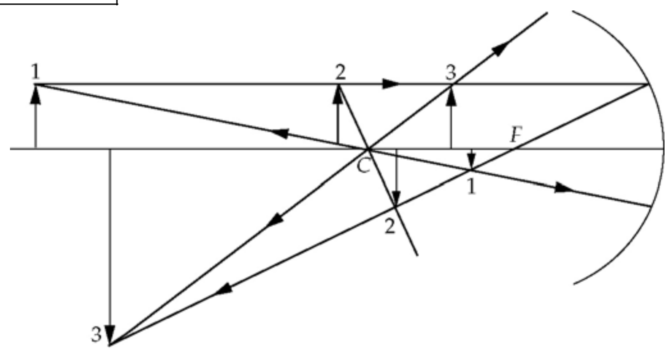
Determine the Concept A convex mirror always produces a virtual, upright image that is s
than the object. It never produces an enlarged image.

6 ••

Determine the Concept They appear more distant because the images are smaller than they would be in a flat mirror.

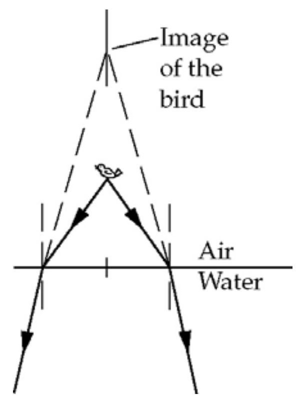
7 ••

Picture the Problem The ray diagram shows three object positions 1, 2, and 3 as the object is moved from a great distance toward the focal point *F* of a concave mirror. The real images corresponding to each of these object positions are labeled with the same numeral. (*b*) is correct.



8 •

Picture the Problem The diagram shows two rays (from the bundle of rays) of light refracted at the air-water interface. Because the index of refraction of water is greater than that of air, the rays are bent toward the normal. The diver will, therefore, think that the rays are diverging from a point above the bird and so the bird appears to be farther from the surface than it actually is.



***9** •

Determine the Concept

(*a*) The lens will be positive if its index of refraction is greater than that of the surrounding medium and the lens is thicker in the middle than at the edges. Conversely, if the index of refraction of the lens is less than that of the surrounding medium, the lens will be positive if it is thinner at its center than at the edges.

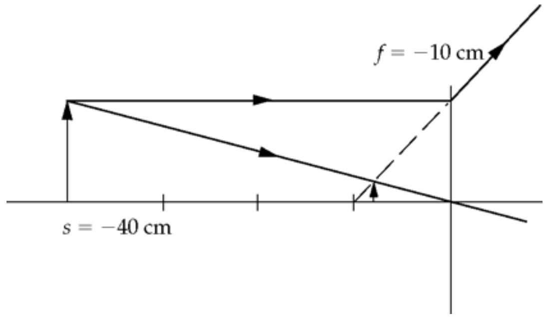
(*b*) The lens will be negative if its index of refraction is greater than that of the surrounding medium and the lens is thinner at the center than at the edges. Conversely, if the index of refraction of the lens is less than that of the surrounding medium, the lens will be negative if it is thicker at the center than at the edges.

10 •

Determine the Concept The focal length depends on the index of refraction, and *n* is a function of wavelength.

11 ••

Picture the Problem We can use a ray diagram to determine the general features of the image. In the diagram shown, the parallel ray and central ray have been used to locate the image.

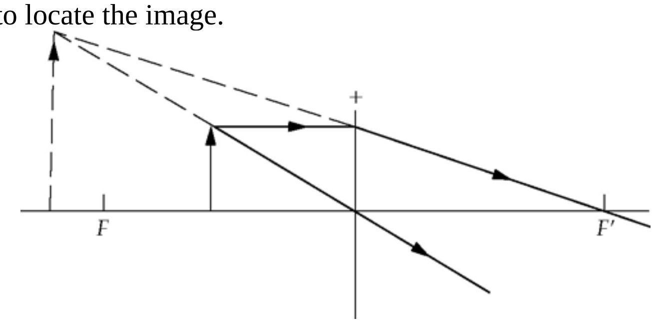


From the diagram, we see that the image is virtual (only one of the rays from the head of the object actually pass through the head of the image), upright, and diminished.

$\boxed{(d)\text{ is correct.}}$

12 ••

Picture the Problem We can use a ray diagram to determine the general features of the image. In the diagram shown, the ray parallel to the principle axis and the central ray have been used to locate the image.



From the diagram, we see that the image is virtual (neither ray from the head of the object passes through the head of the image), upright, and enlarged. $\boxed{(c)\text{ is correct.}}$

13 •

Determine the Concept The muscles in the eye change the thickness of the lens and thereby change the focal length of the lens to accommodate objects at different distances. A camera, on the other hand, has a fixed focal length so that focusing is accomplished by varying the distance between the lens and the film.

***14 •**

Determine the Concept The eye muscles of a farsighted person lack the ability to shorten the focal length of the lens in the eye sufficiently to form an image on the retina of the eye. A convex lens (a lens that is thicker in the middle than at the circumference) will bring the image forward onto the retina. $\boxed{(a)\text{ is correct.}}$

15 ••

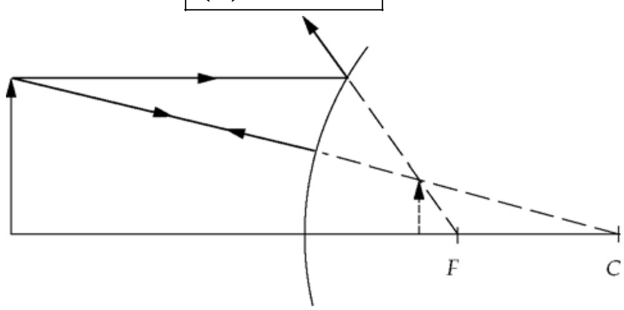
Determine the Concept Refraction of light at the water-cornea interface is less than at the air-cornea interface and so an image that would normally (that is, without a corrective lens) be in front of the retina, is formed on the retina. $\boxed{(b)\text{ is correct.}}$

16 ••

Determine the Concept A nearsighted person's lenses form sharp images (unless the person is also astigmatic) of nearby object's on the retinas of her eyes. The corrective lenses (convex) give a reduced image of the object and, therefore, should be removed.
$\boxed{(b)\text{ is correct.}}$

***17 •**

Determine the Concept Referring to the ray diagram show below we note that the image is always virtual and diminished. $\boxed{(d)\text{ is correct.}}$



18 •

Determine the Concept Converging lenses can form real or virtual images that can be enlarged or reduced. $\boxed{(c)\text{ is correct.}}$

19 •

Picture the Problem We can apply the lens maker's equation to the air-glass lens and to the water-glass lens to find the ratio of their focal lengths.

Apply the lens maker's equation to the air-glass interface:

$$\frac{1}{f_{\text{air}}} = (1.6 - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Apply the lens maker's equation to the water-glass interface:

$$\frac{1}{f_{\text{water}}} = (1.6 - 1.33)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Divide the first of these equations by the second to obtain:

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{(1.6-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{(1.6-1.33)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = 2.22$$

or $f_{\text{water}} = 2.22 f_{\text{air}}$ and $\boxed{(a)\text{ is correct.}}$

20 ••

(*a*) True.

(*b*) True.

(*c*) False. Where the rays intersect the axis of a spherical mirror depends on how far from the axis they are reflected from the mirror.

(*d*) True.

(*e*) False. The image distance for a virtual image is negative.

***21** •

Determine the Concept Microscopes ordinarily produce images (either the intermediate one produced by the objective or the one viewed through the eyepiece) that are larger than the object being viewed. A telescope, on the other hand, ordinarily produces images that are much reduced compared to the object. The object is normally viewed from a great distance and the telescope magnifies the angle subtended by the object.

Estimation and Approximation

22 ••

Picture the Problem We can use the lens-maker's equation to obtain a relationship between the two radii of curvature of the lenses we are to design.

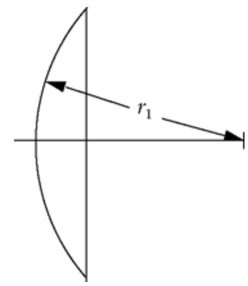
For a thin lens of focal length 27 cm and index of refraction of 1.6:

$$\frac{1}{27\,\text{cm}} = (1.6-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

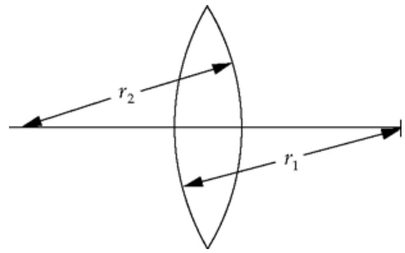
or

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{16.2\,\text{cm}}$$

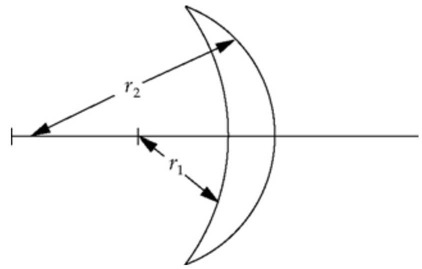
One solution is a plano-convex lens (one with a flat surface and a convex surface). Let $r_2 = \infty$. Then $r_1 = \boxed{16.2\,\text{cm}}$ and $r_2 = \boxed{\infty}$.



Another design is a double convex lens (one with both surfaces convex and radii of curvature that are equal in magnitude) obtained by letting $r_2 = -r_1$. Then $r_1 = \boxed{32.4\,\text{cm}}$ and $r_2 = \boxed{-32.4\,\text{cm}}$.



A third possibility is a double convex lens with unequal curvature, e.g., let $r_2 = 12$ cm. Then $r_1 = \boxed{6.89\,\text{cm}}$ and $r_2 = \boxed{12.0\,\text{cm}}$.



23 ••

Picture the Problem We can use the lens-maker's equation to obtain a relationship between the two radii of curvature of the lenses we are to design.

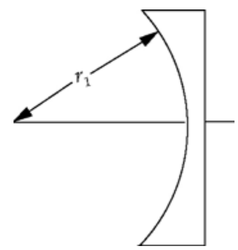
For a thin lens of focal length –27 cm and index of refraction of 1.6:

$$\frac{1}{-27\,\text{cm}} = (1.6-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

or

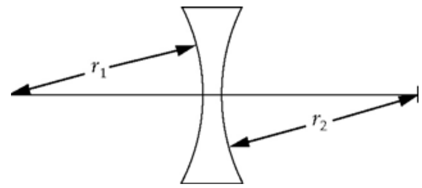
$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{-16.2\,\text{cm}}$$

One solution is a plano-concave lens (one with a flat surface and a concave surface), Let $r_2 = \infty$. Then $r_1 = \boxed{-16.2\,\text{cm}}$ and $r_2 = \boxed{\infty}$.

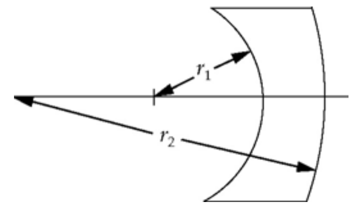


Another design is a biconcave lens

(one with both surfaces concave) by letting $r_2 = -r_1$. Then $r_1 = \boxed{-32.4\,\text{cm}}$ and $r_2 = \boxed{32.4\,\text{cm}}$.



A third possibility is a lens with r_2 = 8.1 cm. Then $r_1 = \boxed{5.40\,\text{cm}}$ and $r_2 = \boxed{16.2\,\text{cm}}$.



***24 ••**

Picture the Problem Because the focal length of a spherical lens depends on its radii of curvature and the magnification depends on the focal length, there is a practical upper limit to the magnification.

Use equation 32-20 to relate the magnification M of a simple magnifier to its focal length f:

$$M = \frac{x_{np}}{f}$$

Use the lens-maker's equation to relate the focal length of a lens to its radii of curvature and the index of refraction of the material from which it is constructed:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

For a plano-convex lens, $r_2 = \infty$. Hence:

$$\frac{1}{f} = \frac{n-1}{r_1} \Rightarrow f = \frac{r_1}{n-1}$$

Substitute in the expression for M and simplify to obtain:

$$M = \frac{(n-1)x_{np}}{r_1}$$

Note that the smallest reasonable value for r_1 will maximize M.

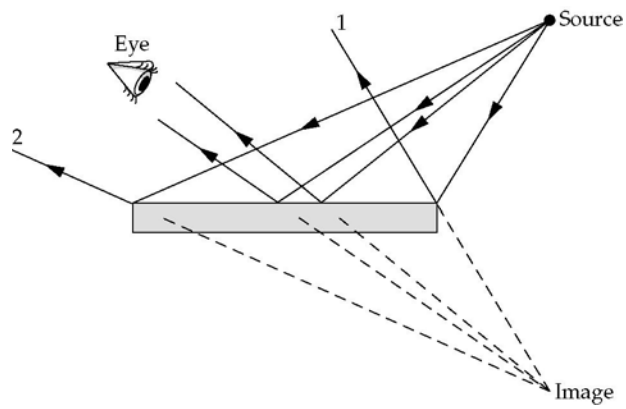
A reasonable smallest value for the radius of a magnifier is 1 cm. Use this value and n = 1.5 to estimate M_{max}:

$$M_{max} = \frac{(1.5-1)(25\,\text{cm})}{1\,\text{cm}} = \boxed{12.5}$$

Plane Mirrors

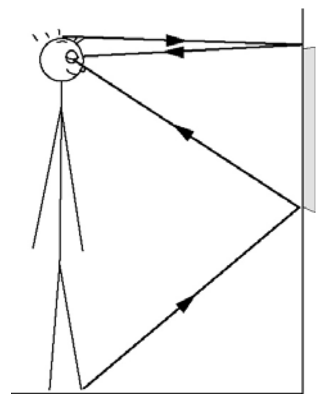
25 •

Determine the Concept Rays from the source and reflected by the mirror are shown. The reflected rays appear to diverge from the image. The eye can see the image if it is in the region between rays 1 and 2.



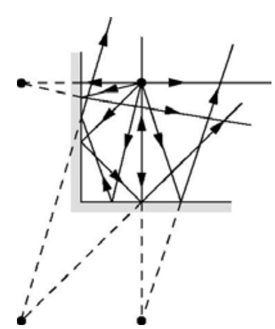
26 •

Determine the Concept The mirror must be half the height of the person, i.e., 81 cm. The top of the mirror must be 7.5 cm below the top of the head, or 154.5 cm above the floor. The bottom of the mirror must be 73.5 cm above the floor. A ray diagram showing rays from the person's feet and the top of her head reaching her eyes is shown to the right.



***27** •

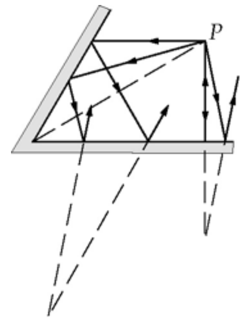
Determine the Concept Draw rays of light from the object that satisfy the law of reflection at the two mirror surfaces. Three virtual images are formed, as shown in the adjacent figure. The eye should be to the right and above the mirrors in order to see these images.



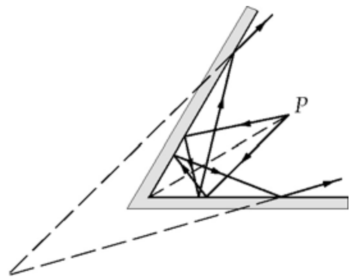
28 •

Determine the Concept Draw rays of light from the object that satisfy the law of reflection at the two mirror surfaces. The images are located at the intersection of the dashed lines (extensions of the reflected rays).

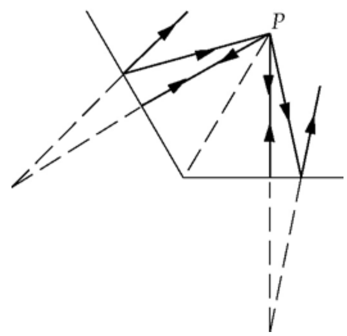
(*a*) The diagram to the right shows selected rays emanating from a point object (*P*) that form the two virtual images directly below the horizontal mirror:



The diagram to the right shows selected rays emanating from the point object (*P*) that form the image that lies on the bisector of the angle. There are two additional virtual images to the left of the mirror that is at 60° with the horizontal. Hence, the total number of images formed when a point object is on the bisector of the 60° angle is five.



(*b*) The diagram to the right shows selected rays emanating from a point object (*P*) that form the two virtual images at the intersection of the dashed lines (extensions of the reflected rays):



29 ••

Determine the Concept

(*a*) The first image in the mirror on the left is 10 cm behind the mirror. The mirror on the right forms an image 20 cm behind that mirror or 50 cm from the left mirror. This image will result in a second image 50 cm behind the left mirror. The first image in the left mirror is 40 cm from the right mirror and forms an image 40 cm behind the right mirror or 70 cm from the left mirror. That image gives an image 70 cm behind the left mirror. The fourth image behind the left mirror is 110 cm behind that mirror.

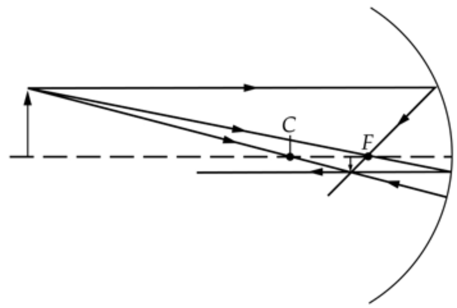
(*b*) Proceeding as in Part (*a*) for the mirror on the right, one finds the location of the images to be 20 cm, 40 cm, 80 cm, and 100 cm behind the right-hand mirror.

Spherical Mirrors

***30 ••**

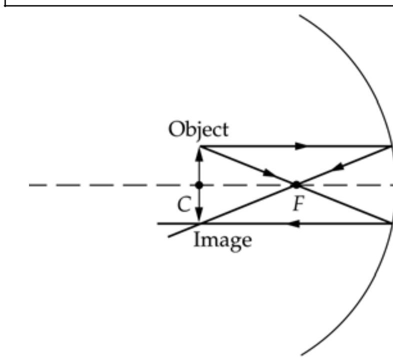
Picture the Problem The easiest rays to use in locating the image are 1) the ray parallel to the principal axis and passes through the focal point of the mirror, the ray that passes through the center of curvature of the spherical mirror and is reflected back on itself, and 2) the ray that passes through the focal point of the spherical mirror and is reflected parallel to the principal axis. We can use any two of these rays emanating from the top of the object to locate the image of the object.

(*a*) The ray diagram is shown to the right. The image is real, inverted, and reduced.



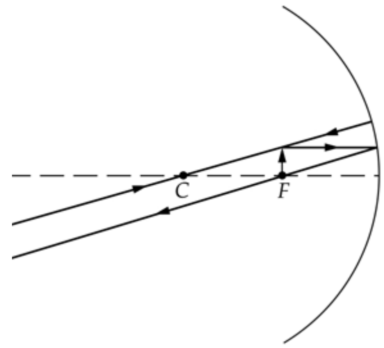
The image is real, inverted, and reduced.

(*b*) The ray diagram is shown to the right.



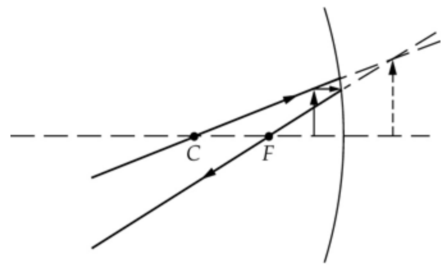
The image is real, inverted, and the same size as the object.

(*c*) The ray diagram is shown to the right. The object is at the focal point of the mirror.



The emerging rays are parallel and do not form an image.

(*d*) The ray diagram is shown to the right.



The image is virtual, erect, and enlarged.

31 •

Picture the Problem In describing the images, we must indicate where they are located, how large they are in relationship to the object, whether they are real or virtual, and whether they are upright or inverted. The object distance *s*, the image distance *s*′, and the focal length of a mirror are related according to $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, where $f=\frac{1}{2}r$ and *r* is the radius of curvature of the mirror. In this problem, *f* = 20 cm because *r* is positive for a concave mirror.

Solve the mirror equation for *s*′:

$$s'=\frac{fs}{s-f}$$

(*a*) When *s* = 50 cm:

$$s'=\frac{(12\,\text{cm})(50\,\text{cm})}{50\,\text{cm}-12\,\text{cm}}=\boxed{15.8\,\text{cm}}$$

The lateral magnification of the image is:

$$m=-\frac{s'}{s}=-\frac{15.8\,\text{cm}}{50\,\text{cm}}=-0.316$$

Because the image distance is positive and the lateral magnification is less than one and negative, we can conclude that the image real, inverted, and reduced.

(*b*) When s = 24 cm:

$$s' = \frac{(12\,\text{cm})(24\,\text{cm})}{24\,\text{cm} - 12\,\text{cm}} = \boxed{24.0\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{24\,\text{cm}}{24\,\text{cm}} = -1$$

Because the image distance is positive and the lateral magnification is one and negative, we can conclude that the image real, inverted, and the same size as the object.

(*c*) When s = 12 cm:

$$s' = \frac{(12\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - 12\,\text{cm}} = \boxed{\infty}$$

and $\boxed{\text{there is no image.}}$

(*d*) When s = 8 cm:

$$s' = \frac{(12\,\text{cm})(8\,\text{cm})}{8\,\text{cm} - 12\,\text{cm}} = \boxed{-24.0\,\text{cm}}$$

The lateral magnification of the image is:

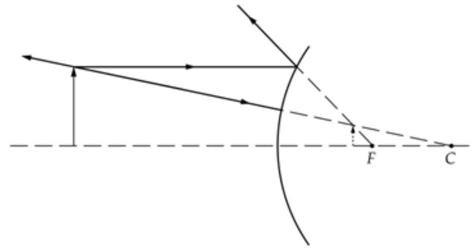
$$m = -\frac{s'}{s} = -\frac{-24\,\text{cm}}{8\,\text{cm}} = 3$$

Because the image distance is negative and the lateral magnification is three and positive, we can conclude that the image virtual, erect, and three times the size of the object.

32 ••

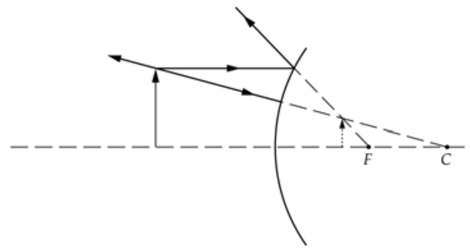
Picture the Problem The easiest rays to use in locating the image are 1) the ray parallel to the principal axis and passes through the focal point of the mirror, the ray that passes through the center of curvature of the spherical mirror and is reflected back on itself, and 2) the ray that passes through the focal point of the spherical mirror and is reflected parallel to the principal axis. We can use any two of these rays emanating from the top of the object to locate the image of the object.

(*a*) The ray diagram is shown to the right.



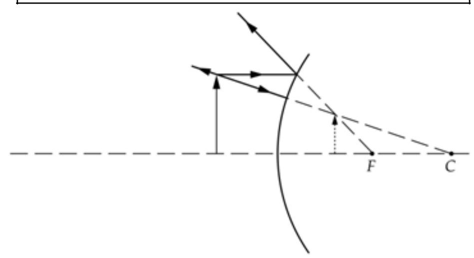
The image is virtual, upright, and reduced.

(*b*) The ray diagram is shown to the right.



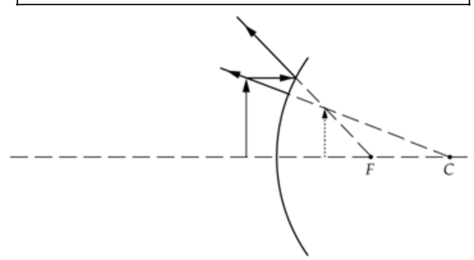
The image is virtual, upright, and reduced.

(*c*) The ray diagram is shown to the right.



The image is virtual, upright, and reduced.

(*d*) The ray diagram is shown to the right.



The image is virtual, upright, and reduced.

33 •

Picture the Problem In describing the images, we must indicate where they are located, how large they are in relationship to the object, whether they are real or virtual, and whether they are upright or inverted. The object distance *s*, the image distance *s*′, and the focal length of a mirror are related according to $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, where $f=\frac{1}{2}r$ and *r* is the radius of curvature of the mirror. In this problem, *f* = –20 cm because *r* is negative for a convex mirror.

Solve the mirror equation for s':

$$s' = \frac{fs}{s - f}$$

(*a*) When s = 55 cm:

$$s' = \frac{(-12\,\text{cm})(55\,\text{cm})}{55\,\text{cm} - (-12\,\text{cm})} = \boxed{-9.85\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-9.85\,\text{cm}}{55\,\text{cm}} = 0.179$$

Because the image distance is negative and the lateral magnification is less than one in magnitude and positive, we can conclude that the image is virtual, upright, and reduced.

(*b*) When s = 24 cm:

$$s' = \frac{(-12\,\text{cm})(24\,\text{cm})}{24\,\text{cm} - (-12\,\text{cm})} = \boxed{-8.00\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-8\,\text{cm}}{24\,\text{cm}} = 0.333$$

Because the image distance is negative and the lateral magnification is less than one in magnitude and positive, we can conclude that the image is virtual, upright, and reduced.

(*c*) When s = 12 cm:

$$s' = \frac{(-12\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - (-12\,\text{cm})} = \boxed{-6.00\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-6\,\text{cm}}{12\,\text{cm}} = \frac{1}{2}$$

Because the image distance is negative and the lateral magnification is one – half in magnitude and positive, we can conclude that the image virtual, upright, and half the size of the object.

(*d*) When s = 8 cm:

$$s' = \frac{(-12\,\text{cm})(8\,\text{cm})}{8\,\text{cm} - (-12\,\text{cm})} = \boxed{-4.80\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-4.80\,\text{cm}}{8\,\text{cm}} = 0.600$$

Because the image distance is negative and the lateral magnification is less than one and positive, we can conclude that the image is virtual, upright, and reduced.

34 •

Picture the Problem We can solve the mirror equation for 1/*s*′ and then examine the implications of *f* < 0 and *s* > 0.

Solve the mirror equation for 1/*s*′:

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$$

For a convex mirror:

$$f < 0$$

With *s* > 0, the numerator is positive and the denominator negative. Consequently:

$$\frac{1}{s'} < 0 \Rightarrow \boxed{s' < 0}$$

***35** •

Picture the Problem We can use the mirror equation and the definition of the lateral magnification to find the radius of curvature of the mirror.

(*a*) Express the mirror equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = \frac{2}{r}$$

Solve for *r*:

$$r = \frac{2ss'}{s'+s} \qquad (1)$$

The lateral magnification of the mirror is given by:

$$m = -\frac{s'}{s}$$

Solve for *s*′:

$$s' = -ms$$

Substitute for *s*′ in equation (1) to obtain:

$$r = \frac{-2ms}{1-m}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{-2(5.5)(2.1\,\text{cm})}{1-5.5} = \boxed{5.13\,\text{cm}}$$

(*b*) The mirror must be concave. A convex mirror always produces a diminished virtual image.

36 ••

Picture the Problem We can use the mirror equation and the relationship between the focal length of a mirror and its radius of curvature to find the location of the image. We can then use the definition of the lateral magnification of the mirror to find the height of the image formed in the mirror.

(*a*) and (*b*) Solve the mirror equation for for s':

$$s' = \frac{fs}{s - f}$$

Relate the focal length of the mirror to its radius of curvature:

$$f = \tfrac{1}{2}r$$

Substitute to obtain:

$$s' = \frac{\tfrac{1}{2}rs}{s - \tfrac{1}{2}r} = \frac{rs}{2s - r}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(-1.2\,\text{m})(10\,\text{m})}{2(10\,\text{m}) - (-1.2\,\text{m})} = \boxed{-0.566\,\text{m}}$$

and

the image is 56.6 cm behind the mirror.

(*c*) Express the lateral magnification of the mirror:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

Substitute numerical values and evaluate y':

$$y' = -\frac{-0.566\,\text{m}}{10\,\text{m}}(2\,\text{m}) = \boxed{11.3\,\text{cm}}$$

37 ••

Picture the Problem We can use the mirror equation to locate the image formed in this mirror and the expression for the lateral magnification of the mirror to find the diameter of the image.

Solve the mirror equation for the location of the image of the moon:

$$s' = \frac{fs}{f - s}$$

Because $f = \frac{1}{2}r$:

$$s' = \frac{\frac{1}{2}rs}{\frac{1}{2}r - s} = \frac{rs}{r - 2s}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(8\,\text{m})(3.8\times10^8\ \text{m})}{8\,\text{m} - 2(3.8\times10^8\ \text{m})} = \boxed{-4.00\,\text{m}}$$

Express the lateral magnification of the mirror:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

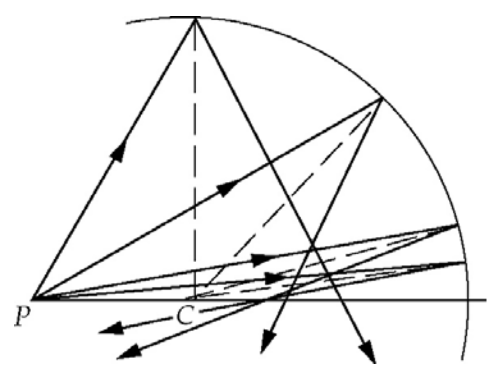
$$y' = -\frac{s'}{s}y$$

Substitute numerical values and evaluate y':

$$y' = -\frac{-4\,\text{m}}{3.8\times10^8\ \text{m}}(3.5\times10^6\ \text{m}) = \boxed{3.68\,\text{cm}}$$

38 ••

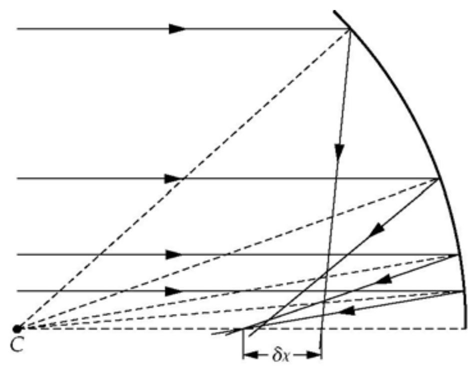
Picture the Problem The rays from the point object are shown in the diagram to the right. Note that the rays that reflect from the mirror far from the axis do not converge at the same point as those that reflect from the mirror close to the mirror axis. For the small-angle rays, the point of convergence is 4.5 cm from the mirror. The 60° ray crosses the axis at 3 cm from the mirror. Consequently, the image extends from 4.5 cm to 3.0 cm, or about 1.5 cm along the axis.



***39** ••

Picture the Problem

(a) The figure to the right shows the mirror and the four rays drawn to scale. Using a calibrated ruler, the spread of the crossing points is $\delta x \approx 1.0$ cm. Note that the triangles formed by the center of curvature, the point of reflection on the mirror, and the point of intersection of the reflected ray and the mirror axis are isosceles triangles.



Express the equal angles of the isosceles triangles:

$$\theta_r = \sin^{-1}\left(\frac{y}{R}\right)$$

where y is the distance of the incoming ray from the mirror axis and R is the radius of curvature of the mirror.

Using the law of cosines, the distance between the point of intersection and the mirror is given by:

$$d = R\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{y}{R}\right)\right)\right]^{-1}\right\}$$

Evaluate d for $y/R = 2/3$:

$$d = (6\,\text{cm})\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right)\right]^{-1}\right\} = 1.975\,\text{cm}$$

Evaluate d for $y/R = 1/12$:

$$d = (6\,\text{cm})\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{1}{12}\right)\right)\right]^{-1}\right\} = 2.990\,\text{cm}$$

Express the spread δx:

$$\delta x = 2.990\,\text{cm} - 1.975\,\text{cm} = \boxed{1.01\,\text{cm}}$$

in good agreement with the result obtained above.

(*b*) Evaluate *d* for *y*/*R* = 1/3:

$$d = (6\,\text{cm})\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)\right]^{-1}\right\}$$
$$= 2.818\,\text{cm}$$

Express the new spread $\delta x'$:

$$\delta x' = 2.990\,\text{cm} - 2.818\,\text{cm} = 0.172\,\text{cm}$$

Express the ratio of $\delta x'$ to δx:

$$\frac{\delta x'}{\delta x} = \frac{0.172\,\text{cm}}{1.01\,\text{cm}} = 17.0\%$$

By blocking off the edges of the mirror so that only paraxial rays within 2 cm of the mirror axis are reflected, the spread is reduced by 83.0%.

40 ••

Picture the Problem We can use the mirror equation to find the focal length of the mirror and then apply it a second time to find the object position after the mirror has been moved.

Solve the mirror equation for *f*:

$$f = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{(100\,\text{cm})(75\,\text{cm})}{75\,\text{cm} + 100\,\text{cm}} = 42.86\,\text{cm}$$

Solve the mirror equation for *s*:

$$s = \frac{fs'}{s' - f}$$

Find *s* for *f* = –42.86 cm and *s*′ = – 35 cm:

$$s = \frac{(-42.86\,\text{cm})(-35\,\text{cm})}{-35\,\text{cm} - (-42.86\,\text{cm})} = 190.9\,\text{cm}$$

The distance *d* the mirror moved is:

$$d = 190.9\,\text{cm} - 100\,\text{cm} = \boxed{90.9\,\text{cm}}$$

41 ••

Picture the Problem We can use the mirror equation, with $s = \infty$, to find the image distance in the large mirror. Because this image serves as a virtual object for the small mirror, we can use the mirror equation a second time to find the focal length and, hence, the radius of curvature of the small mirror.

(*a*) Express the mirror equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$$

Because $s = \infty$:

$$\frac{1}{s'} = \frac{2}{r} \text{ and } s' = \tfrac{1}{2}r$$

Substitute numerical values and evaluate s':

$$s' = \tfrac{1}{2}(5\,\text{m}) = 2.5\,\text{m}$$

This image serves as a virtual object for the small mirror at $s = -0.5$ m. Solve the mirror equation for the focal length of the small mirror:

$$f_{\text{small}} = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate f_{small}:

$$f_{\text{small}} = \frac{(-0.5\,\text{m})(2\,\text{m})}{2\,\text{m} + (-0.5\,\text{m})} = -0.667\,\text{m}$$

The radius of curvature is twice the focal length:

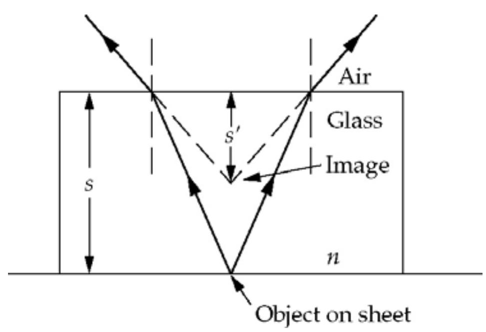
$$r_{\text{small}} = 2f_{\text{small}} = 2(-0.667\,\text{m}) = \boxed{-1.33\,\text{m}}$$

(*b*) $\boxed{\text{Because } f_{\text{small}} < 0, \text{ the small mirror is convex.}}$

Images Formed by Refraction

42 •

Picture the Problem The diagram shows two rays (from the bundle of rays) of light refracted at the glass-air interface. Because the index of refraction of air is less than that of water, the rays are bent away from the normal. The writing on the paper will, therefore, appear to be closer than it actually is. We can use the equation for refraction at a single surface to find the distance s'.



Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

Here we have $n_1 = n$, $n_2 = 1$, and $r = \infty$. Therefore:

$$\frac{n}{s} + \frac{1}{s'} = 0$$

Solve for s':

$$s' = -\frac{s}{n}$$

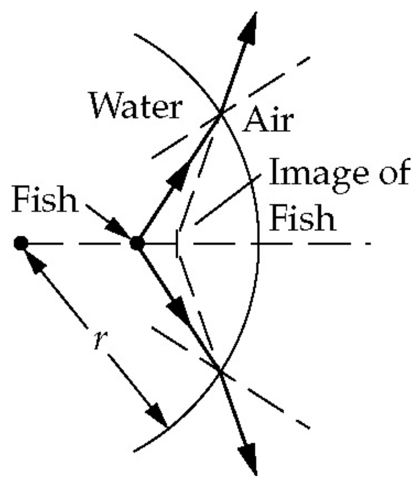
Substitute numerical values and evaluate s':

$$s' = -\frac{2\,\text{cm}}{1.5} = \boxed{-1.33\,\text{cm}}$$

where the minus sign tells us that the image is 1.33 cm below the glass surface.

43 •

Picture the Problem The diagram shows two rays (from the bundle of rays) of light refracted at the water-air interface. Because the index of refraction of air is less than that of water, the rays are bent away from the normal. The fish will, therefore, appear to be closer than it actually is. We can use the equation for refraction at a single surface to find the distance s'. We'll assume that the glass bowl is thin enough that we can ignore the refraction of the light passing through it.



(a) Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

Here we have $n_1 = n$ and $n_2 = 1$. Therefore:

$$\frac{n}{s} + \frac{1}{s'} = \frac{1-n}{r}$$

Solve for s':

$$s' = \frac{rs}{s(1-n) - nr}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(-20\,\text{cm})(10\,\text{cm})}{(10\,\text{cm})(1-1.33) - (1.33)(-20\,\text{cm})} = \boxed{-8.54\,\text{cm}}$$

where the minus sign tells us that the image is 8.54 cm from the front surface of the bowl.

(*b*) Repeat (*a*) with *s* = 30 cm:

$$s' = \frac{(-20\,\text{cm})(30\,\text{cm})}{(30\,\text{cm})(1-1.33)-(1.33)(-20\,\text{cm})} = \boxed{-35.9\,\text{cm}}$$

where the minus sign tells us that the image is 35.9 cm from the front surface of the bowl.

***44 ••**

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \qquad (1)$$

Here we have n_1 = 1 and n_2 = *n* = 1.5. Therefore:

$$\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{r}$$

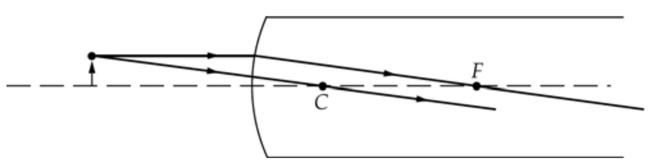
Solve for *s′*:

$$s'=\frac{nrs}{s(n-1)-r}$$

(*a*) Substitute numerical values (*s* = 35 cm and *r* = 7.2 cm) and evaluate *s′*:

$$s' = \frac{(1.5)(7.2\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5-1)-(7.2\,\text{cm})} = \boxed{36.7\,\text{cm}}$$

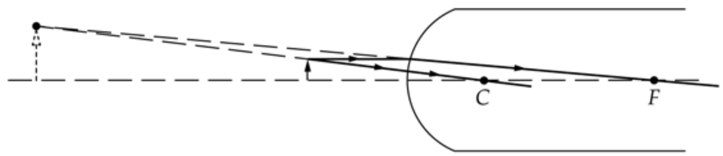
where the positive distance tells us that the image is 36.7 cm in back of the surface and is $\boxed{\text{real.}}$



(*b*) Substitute numerical values (*s* = 6.5 cm and *r* = 7.2 cm) and evaluate *s′*:

$$s' = \frac{(1.5)(7.2\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1)-(7.2\,\text{cm})} = \boxed{-17.8\,\text{cm}}$$

where the minus sign tells us that the image is 17.8 cm in front of the surface and is virtual.



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n}{s'} = \frac{n-1}{r}$$

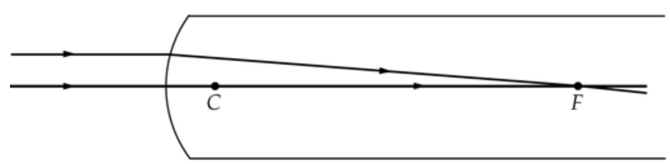
Solve for s':

$$s' = \frac{nr}{n-1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(1.5)(7.2\,\text{cm})}{1.5-1} = \boxed{21.6\,\text{cm}}$$

i.e., the image is at the focal point, is real, and of zero size.



45 ••

Picture the Problem We can use the equation for refraction at a single surface to find the image distance that corresponds to parallel light rays in the rod.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r} \qquad (1)$$

Parallel rays imply that $s' = \infty$. Therefore:

$$\frac{1}{s} = \frac{n-1}{r}$$

Solve for s:

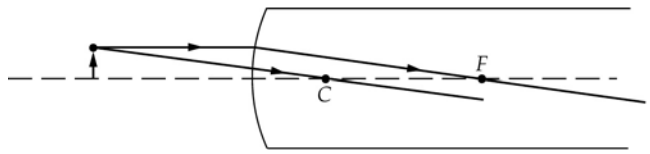
$$s = \frac{r}{n-1}$$

Substitute numerical values and

$$s = \frac{7.2\,\text{cm}}{1.5-1} = \boxed{14.4\,\text{cm}}$$

evaluate s:

The ray diagram is shown below:



46 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \quad (1)$$

Here we have $n_1 = 1$ and $n_2 = n = 1.5$. Therefore:

$$\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{r}$$

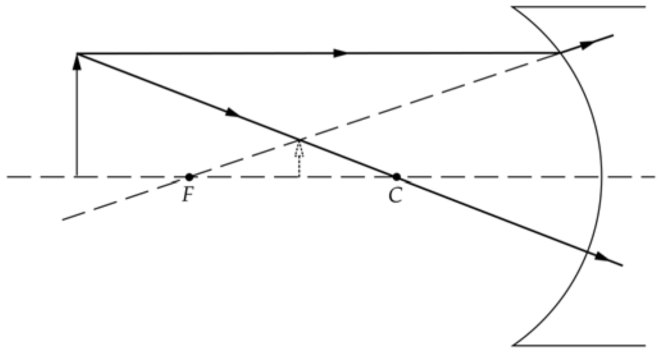
Solve for s':

$$s'=\frac{nrs}{s(n-1)-r}$$

(a) Substitute numerical values (s = 35 cm and r = –7.2 cm) and evaluate s':

$$s'=\frac{(1.5)(-7.2\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5-1)-(-7.2\,\text{cm})}=\boxed{-15.3\,\text{cm}}$$

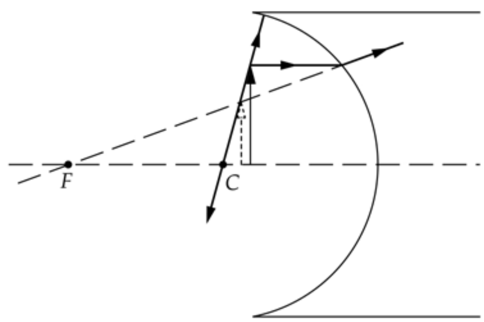
where the minus sign tells us that the image is 15.3 cm in front of the surface of the rod and is $\boxed{\text{virtual.}}$



(*b*) Substitute numerical values (s = 6.5 cm and r = –7.2 cm) and evaluate s':

$$s' = \frac{(1.5)(-7.2\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1)-(-7.2\,\text{cm})} = \boxed{-6.72\,\text{cm}}$$

where the minus sign tells us that the image is 6.72 cm in front of the surface of the rod (located at the object) and is $\boxed{\text{virtual.}}$



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n}{s'} = \frac{n-1}{r}$$

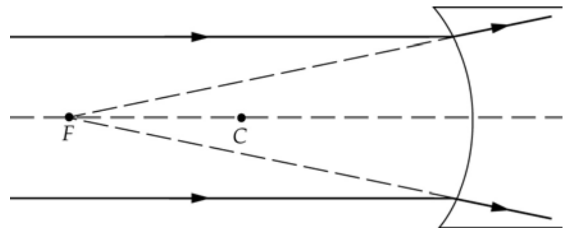
Solve for s':

$$s' = \frac{nr}{n-1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(1.5)\,(-7.2\,\text{cm})}{1.5-1} = \boxed{-21.6\,\text{cm}}$$

where the minus sign tells us that the image is 21.6 cm in front of the surface of the rod and is $\boxed{\text{virtual.}}$



47 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \quad (1)$$

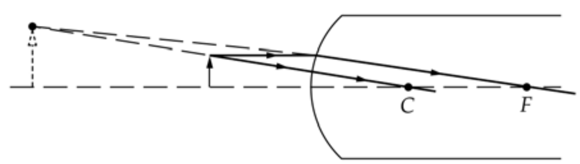
Solve for s':

$$s'=\frac{n_2 rs}{s(n_2-n_1)-n_1 r}$$

(*a*) Substitute numerical values (s = 35 cm, n_1 = 1.33, n_2 = 1.5, and r = 7.2 cm) and evaluate s':

$$s'=\frac{(1.5)(7.2\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5-1.33)-(1.33)(7.2\,\text{cm})}=\boxed{-104\,\text{cm}}$$

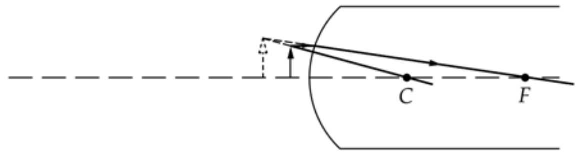
where the negative distance tells us that the image is 104 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*b*) Substitute numerical values (s = 6.5 cm) and evaluate s':

$$s'=\frac{(1.5)(7.2\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1.33)-(1.33)(7.2\,\text{cm})}=\boxed{-8.29\,\text{cm}}$$

where the minus sign tells us that the image is 8.29 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

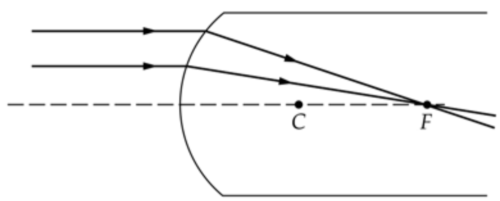
Solve for s':

$$s' = -\frac{n_2}{n_2 - n_1} r$$

Substitute numerical values and evaluate s':

$$s' = \frac{1.5}{1.5 - 1.33}(7.2\,\text{cm}) = \boxed{63.5\,\text{cm}}$$

i.e., the image is 63.5 cm to the right of the surface (at the focal point) and is $\boxed{\text{real.}}$



48 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r} \qquad (1)$$

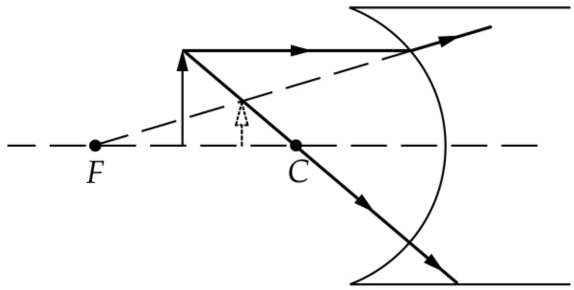
Solve for s':

$$s' = \frac{n_2 r s}{s(n_2 - n_1) - n_1 r}$$

(*a*) Substitute numerical values (s = 35 cm) and evaluate s':

$$s' = \frac{(1.5)(-7.5\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5 - 1.33) - (1.33)(-7.5\,\text{cm})}$$
$$= \boxed{-24.7\,\text{cm}}$$

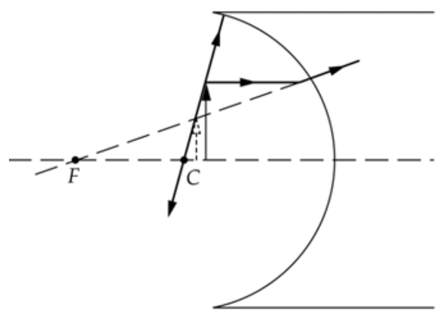
where the minus sign tells us that the image is 24.7 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*b*) Substitute numerical values (s = 6.5 cm) and evaluate s':

$$s' = \frac{(1.5)(-7.5\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1.33)-(1.33)(-7.5\,\text{cm})} = \boxed{-6.60\,\text{cm}}$$

where the minus sign tells us that the image is 6.60 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

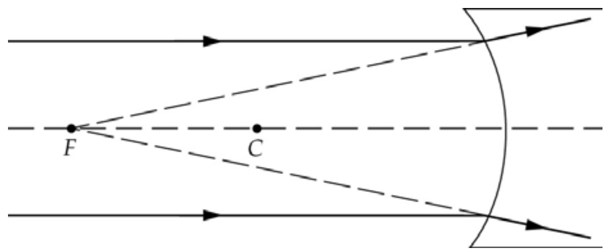
Solve for s':

$$s' = \frac{n_2 r}{n_2 - n_1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(1.5)(-7.5\,\text{cm})}{1.5-1.33} = \boxed{-66.2\,\text{cm}}$$

i.e., the image is at the focal point, is $\boxed{\text{virtual,}}$ and of zero size.



***49** ••

Picture the Problem We can use the equation for refraction at a single surface to find the images due to refraction at the ends of the glass rod. The image formed by the refraction at the first surface will serve as the object for the second surface. The sign of the final image distance will tell us whether the image is real or virtual.

(*a*) Use the equation for refraction at a single surface to relate the image and object distances at the first surface:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \qquad (1)$$

Solve for s':

$$s'=\frac{n_2 rs}{s(n_2-n_1)-n_1 r}$$

Substitute numerical values and evaluate s':

$$s'=\frac{(1.6)(8\,\text{cm})(20\,\text{cm})}{(20\,\text{cm})(1.6-1)-(8\,\text{cm})}=\boxed{64.0\,\text{cm}}$$

(*b*) The object for the second surface is 96 cm – 64 cm = 32 cm from the surface whose radius is 16 cm. Substitute numerical values and evaluate s':

$$s'=\frac{(1)(-16\,\text{cm})(32\,\text{cm})}{(32\,\text{cm})(1-1.6)-(1.6)(-16\,\text{cm})}=\boxed{-80.0\,\text{cm}}$$

(*c*) $\boxed{\text{The final image is 96 cm} - \text{80 cm} = \text{16 cm from the surface whose radius is 8 cm and is virtual.}}$

50 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images due to refraction at the ends of the glass rod. The image formed by the refraction at the first surface will serve as the object for the second surface. The sign of the final image distance will tell us whether the image is real or virtual.

(*a*) Use the equation for refraction at a single surface to relate the image and object distances at the first surface:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \qquad (1)$$

Solve for *s*′:

$$s'=\frac{n_2 rs}{s(n_2-n_1)-n_1 r}$$

Substitute numerical values and evaluate *s*′:

$$s'=\frac{(1.6)(16\,\text{cm})(20\,\text{cm})}{(20\,\text{cm})(1.6-1)-(16\,\text{cm})}=\boxed{-128\,\text{cm}}$$

(*b*) The object for the second surface is 96 cm + 128 cm = 224 cm from the surface whose radius is 8 cm. Substitute numerical values and evaluate *s*′:

$$s'=\frac{(1)(-8\,\text{cm})(224\,\text{cm})}{(224\,\text{cm})(1-1.6)-(1.6)(-8\,\text{cm})}=\boxed{14.7\,\text{cm}}$$

(*c*) $\boxed{\text{The final image is 14.7 cm from the far end of the rod and is real.}}$

Thin Lenses

51 •

Picture the Problem We can use the lens-maker's equation to find the focal length of each of the lenses.

The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

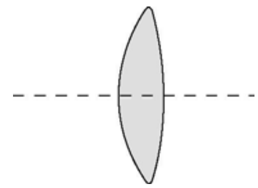
(*a*) For r_1 = 15 cm and r_2 = –26 cm:

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{15\,\text{cm}}-\frac{1}{-26\,\text{cm}}\right)$$

and

$$f=\boxed{19.0\,\text{cm}}$$

A double convex lens is shown to the right:



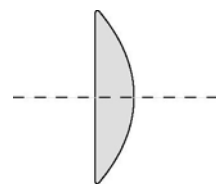
(*b*) For $r_1 = \infty$ and $r_2 = -15$ cm:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{-15\,\text{cm}}\right)$$

and

$$f = \boxed{30.0\,\text{cm}}$$

A plano-convex lens is shown to the right:



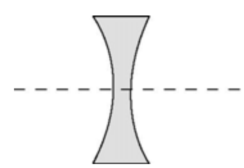
(*c*) For $r_1 = -15$ cm and $r_2 = +15$ cm:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{-15\,\text{cm}} - \frac{1}{15\,\text{cm}}\right)$$

and

$$f = \boxed{-15.0\,\text{cm}}$$

A double concave lens is shown to the right:



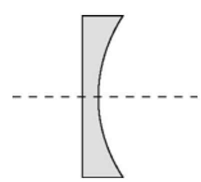
(*d*) For $r_1 = \infty$ and $r_2 = +26$ cm:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{26\,\text{cm}}\right)$$

and

$$f = \boxed{-52.0\,\text{cm}}$$

A plano-concave lens is shown to the right:



52 •

Picture the Problem We can use the lens-maker's equation to find the focal length of the lens.

The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

Substitute numerical values to obtain:

$$\frac{1}{f}=(1.62-1)\left(\frac{1}{-100\,\text{cm}}-\frac{1}{-40\,\text{cm}}\right)$$

Solve for *f*:

$$f=\boxed{108\,\text{cm}}$$

***53** •

Picture the Problem We can use the lens-maker's equation to find the focal length of the lens and the thin-lens equation to locate the image. We can use $m=-\frac{s'}{s}$ to find the lateral magnification of the image.

(*a*) The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

Substitute numerical values to obtain:

$$\frac{1}{f}=(1.45-1)\left(\frac{1}{-30\,\text{cm}}-\frac{1}{25\,\text{cm}}\right)$$

Solve for *f*:

$$f=\boxed{-30.3\,\text{cm}}$$

(*b*) Use the thin-lens equation to relate the image and object distances:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Solve for *s*′:

$$s'=\frac{fs}{s-f}$$

Substitute numerical values and evaluate *s*′:

$$s'=\frac{(-30.3\,\text{cm})(80\,\text{cm})}{80\,\text{cm}-(-30.3\,\text{cm})}=\boxed{-22.0\,\text{cm}}$$

(c) The lateral magnification of the image is given by:

$$m = -\frac{s'}{s}$$

Substitute numerical values and evaluate m:

$$m = -\frac{-22\,\text{cm}}{80\,\text{cm}} = \boxed{0.275}$$

(d) Because $s' < 0$ and $m > 0$, the image is $\boxed{\text{virtual and upright.}}$

54 •

Picture the Problem We can use the lens-maker's equation to find the focal length of each of the lenses described in the problem statement.

The lens-maker's equation is:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

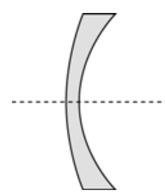
(a) For r_1 = 20 cm, r_2 = 10 cm:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{20\,\text{cm}} - \frac{1}{10\,\text{cm}}\right)$$

and

$$f = \boxed{-33.3\,\text{cm}}$$

A sketch of the lens is shown to the right:



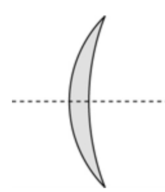
(b) For r_1 = 10 cm, r_2 = 20 cm:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{10\,\text{cm}} - \frac{1}{20\,\text{cm}}\right)$$

and

$$f = \boxed{33.3\,\text{cm}}$$

A sketch of the lens is shown to the right:



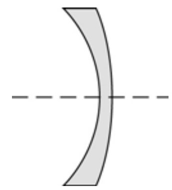
(c) For $r_1 = -10$ cm, $r_2 = -20$ cm:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{-10\,\text{cm}} - \frac{1}{-20\,\text{cm}}\right)$$

and

$$f = \boxed{-33.3\,\text{cm}}$$

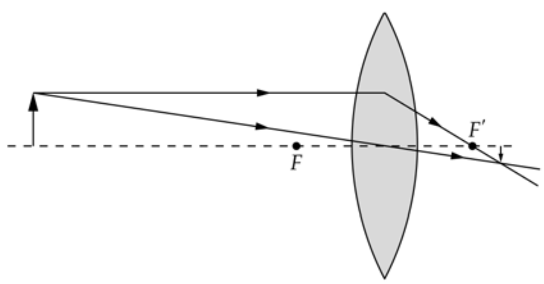
A sketch of the lens is shown to the right:



Remarks: Note that the lenses that are thicker on their axis than on their circumferences are positive (converging) lenses and those that are thinner on their axis are negative (diverging) lenses.

***55** •

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = \frac{y'}{y} = -\frac{s'}{s}$.



The image is real, inverted, and diminished.

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s-f}$$

Use the definition of the power of the lens to find its focal length:

$$f = \frac{1}{P} = \frac{1}{10\,\text{m}^{-1}} = 0.1\,\text{m} = 10\,\text{cm}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(10\,\text{cm})(25\,\text{cm})}{25\,\text{cm} - 10\,\text{cm}} = \boxed{16.7\,\text{cm}}$$

Use the lateral magnification equation to relate the height of the image y' to the height y of the object and the image and object distances:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

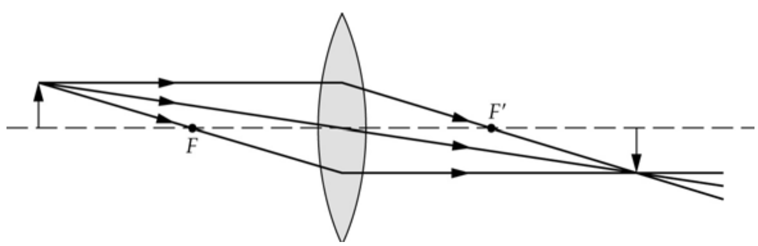
Substitute numerical values and evaluate y':

$$y' = -\frac{16.7\,\text{cm}}{25\,\text{cm}}(3\,\text{cm}) = \boxed{-2.00\,\text{cm}}$$

Because $s' > 0$ and $y' = -2.00\,\text{cm}$, the image is real, inverted, and diminished in agreement with the ray diagram.

56 •

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = \frac{y'}{y} = -\frac{s'}{s}$.



The image is real and inverted and appears to be the same size as the object.

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s-f}$$

Use the definition of the power of the lens to find its focal length:

$$f = \frac{1}{P} = \frac{1}{10\,\text{m}^{-1}} = 0.1\,\text{m} = 10\,\text{cm}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - 10\,\text{cm}} = \boxed{20.0\,\text{cm}}$$

Use the lateral magnification equation to relate the height of the image y' to the height y of the object and the image and object distances:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

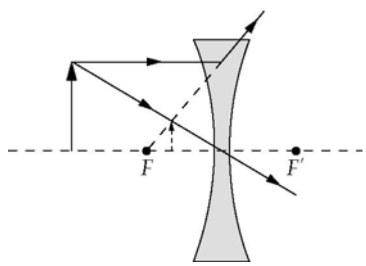
Substitute numerical values and evaluate y':

$$y' = -\frac{20\,\text{cm}}{20\,\text{cm}}(1\,\text{cm}) = \boxed{-1.00\,\text{cm}}$$

Because $s' > 0$ and $y' = -1\,\text{cm}$, the image is real, inverted, and the same size as the object in agreement with the ray diagram.

57 •

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = \frac{y'}{y} = -\frac{s'}{s}$.



The image is virtual, upright, and diminished.

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s - f}$$

Use the definition of the power of the lens to find its focal length:

$$f = \frac{1}{P} = \frac{1}{-10\,\text{m}^{-1}} = -0.1\,\text{m} = -10\,\text{cm}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(-10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - (-10\,\text{cm})} = \boxed{-6.67\,\text{cm}}$$

Use the lateral magnification equation to relate the height of the image y' to the height y of the object and the image and object distances:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

Substitute numerical values and evaluate y':

$$y' = -\frac{-6.67\,\text{cm}}{20\,\text{cm}}(1.5\,\text{cm}) = \boxed{0.500\,\text{cm}}$$

$$\boxed{\begin{array}{l}\text{Because } s' < 0 \text{ and } y' = 0.500\text{ cm, the image is virtual, erect, and}\\ \text{about one-third the size of the object in agreement with the ray diagram.}\end{array}}$$

58 ••

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = -\frac{s'}{s}$.

(*a*) A negative object distance implies that the object is a virtual object, i.e., that light rays converge on the object rather than diverge from the object. A virtual object can occur in a two-lens system when the first lens forms an image that is at a distance $-|s|$ from the second lens.

(*b*) The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s - f}$$

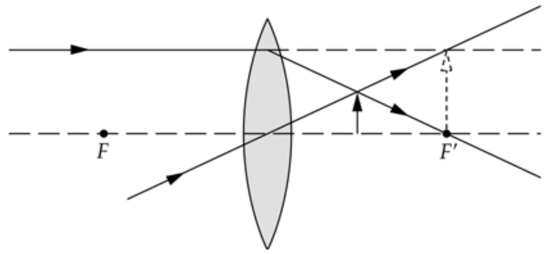
Substitute numerical values and evaluate s':

$$s' = \frac{(20\,\text{cm})(-20\,\text{cm})}{-20\,\text{cm} - (20\,\text{cm})} = \boxed{10.0\,\text{cm}}$$

The lateral magnification is:

$$m = -\frac{s'}{s} = -\frac{10\,\text{cm}}{-20\,\text{cm}} = \boxed{0.500}$$

The parallel and central rays were used to locate the image in the ray diagram shown below:



Because $s' > 0$ and $m > 0$, the image is real, erect, and one-half the size of the virtual object.

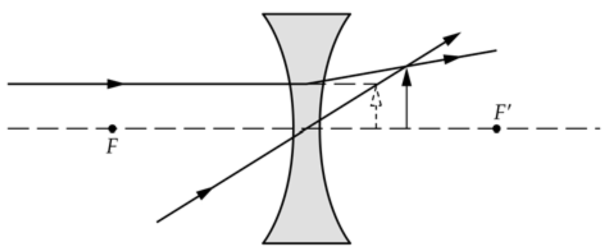
(*c*) Proceed as in (*b*) with $s = -10$ cm and $f = -30$ cm:

$$s' = \frac{(-30\,\text{cm})(-10\,\text{cm})}{-10\,\text{cm} - (-30\,\text{cm})} = \boxed{15.0\,\text{cm}}$$

and

$$m = -\frac{s'}{s} = -\frac{15\,\text{cm}}{-10\,\text{cm}} = \boxed{1.500}$$

The parallel and central rays were used to locate the image in the ray diagram shown below:

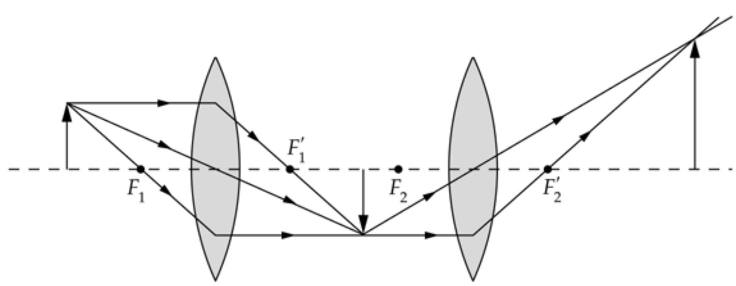


Because $s' > 0$ and $m = 1.5$, the image is real, erect, and one and one-half times the size of the virtual object.

***59 ••**

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

(*a*) The parallel, central, and focal rays were used to locate the image formed by the first lens and the parallel and central rays to locate the image formed by the second lens.



Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - 10\,\text{cm}} = 20\,\text{cm}$$

Find the lateral magnification of the first image:

$$m_1 = -\frac{s_1'}{s} = -\frac{20\,\text{cm}}{20\,\text{cm}} = -1$$

Because the lenses are separated by 35 cm, the object distance for the second lens is 35 cm – 20 cm = 15 cm. Equation (1) applied to the second lens is:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(10\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 10\,\text{cm}} = 30\,\text{cm}$$

and the final image is $\boxed{85.0\,\text{cm}}$ from the object.

Find the lateral magnification of the second image:

$$m_2 = -\frac{s_2'}{s} = -\frac{30\,\text{cm}}{15\,\text{cm}} = -2$$

$\boxed{\text{Because } s'_2 > 0 \text{ and } m = m_1 m_2 = 2, \text{ the image is real, erect, and twice the size of the object.}}$

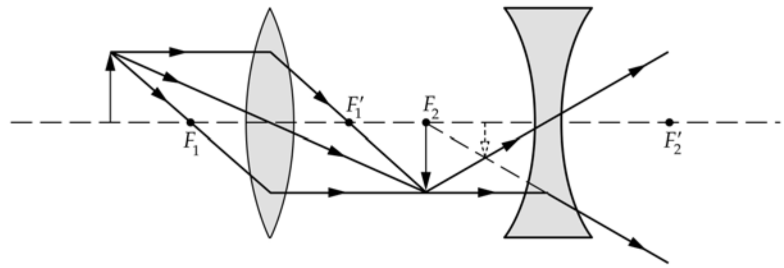
The overall lateral magnification of the image is the product of the magnifications of each image:

$$m = m_1 m_2 = (-1)(-2) = \boxed{2.00}$$

60 ••

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

(*a*) The parallel, central, and focal rays were used to locate the image formed by the first lens and the parallel and central rays to locate the image formed by the second lens.



Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - 10\,\text{cm}} = 20\,\text{cm}$$

Find the lateral magnification of the first image:

$$m_1 = -\frac{s'_1}{s} = -\frac{20\,\text{cm}}{20\,\text{cm}} = -1$$

Because the lenses are separated by 35 cm, the object distance for the second lens is
35 cm – 20 cm = 15 cm. Equation (1) applied to the second lens is:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(-15\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - (-15\,\text{cm})} = -7.5\,\text{cm}$$

and the final image is $\boxed{47.5\,\text{cm}}$ from the object.

Find the lateral magnification of the second image:

$$m_2 = -\frac{s_2'}{s} = -\frac{-7.5\,\text{cm}}{15\,\text{cm}} = 0.5$$

Because $s'_2 < 0$ and $m = m_1 m_2 = -0.5$, the image is virtual, inverted, and half as large as the object.

The overall lateral magnification of the image is the product of the magnifications of each image:

$$m = m_1 m_2 = (-1)(0.5) = \boxed{-0.500}$$

61 ••

Picture the Problem We can use the thin-lens equation and the definition of the lateral magnification to show that $s = (m - 1)f/m$.

(*a*) Express the thin-lens equation:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Express the lateral magnification of the image and solve for *s*′:

$$m=-\frac{s'}{s}\Rightarrow s'=-ms$$

Substitute to obtain:

$$\frac{1}{s}+\frac{1}{-ms}=\frac{1}{f}$$

Solve for *s*:

$$s=\boxed{\frac{(m-1)f}{m}}$$

(*b*) The magnification *m* is:

$$m=-\frac{y'}{y}=-\frac{24\,\text{mm}}{1.75\,\text{m}}=-0.0137$$

Substitute numerical values and evaluate *s*:

$$s=\frac{(-0.0137-1)(50\,\text{mm})}{-0.0137}=\boxed{3.70\,\text{m}}$$

62 ••

Picture the Problem We can plot the first graph by solving the thin-lens equation for the image distance *s*′ and the second graph by using the definition of the magnification of the image.

(*a*) and (*b*) Solve the thin-lens equation for *s*′ to obtain:

$$s'=\frac{fs}{s-f}$$

The magnification of the image is given by:

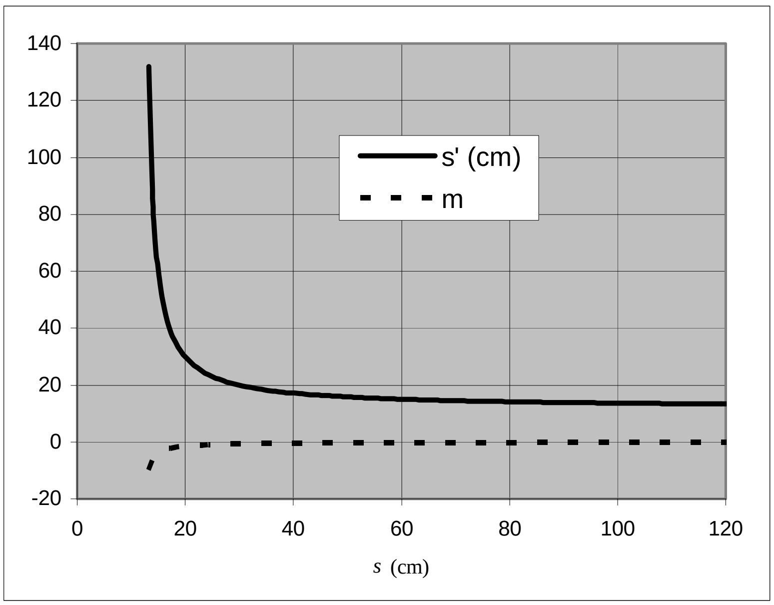
$$m=-\frac{s'}{s}$$

A spreadsheet program to calculate *s*′as a function of *s* is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	12	f
A4	13.2	s
A5	A4 + 1	$s+\Delta s$
B4	$B$1*A4/(A4 – $B$1)	$\frac{fs}{s-f}$
C5	–B4/A4	$-\frac{s'}{s}$

	A	B	C
1	f=	12	cm
2			
3	s	s'	m
4	13.2	132.00	–10.00
5	14.2	77.45	–5.45
6	15.2	57.00	–3.75
7	16.2	46.29	–2.86
8	17.2	39.69	–2.31
9	18.2	35.23	–1.94
108	117.2	13.37	–0.11
109	118.2	13.36	–0.11
110	119.2	13.34	–0.11
111	120.2	13.33	–0.11

A graph of s' as a function of s follows.



(*c*) The images are real and inverted for this range of object distances.

(*d*) The asymptotes of the graph of s' versus s correspond to the focal length of the lens. The horizontal asymptote of the graph of m versus s indicates the fact that, as the object moves away from the lens, the image formed by the lens approaches the far focal point and its size approaches zero.

63 ••

Picture the Problem We can plot the first graph by solving the thin-lens equation for the image distance s' and the second graph by using the definition of the magnification of the image.

(*a*) and (*b*) Solve the thin-lens equation for s' to obtain:

$$s' = \frac{fs}{s - f}$$

The magnification of the image is given by:

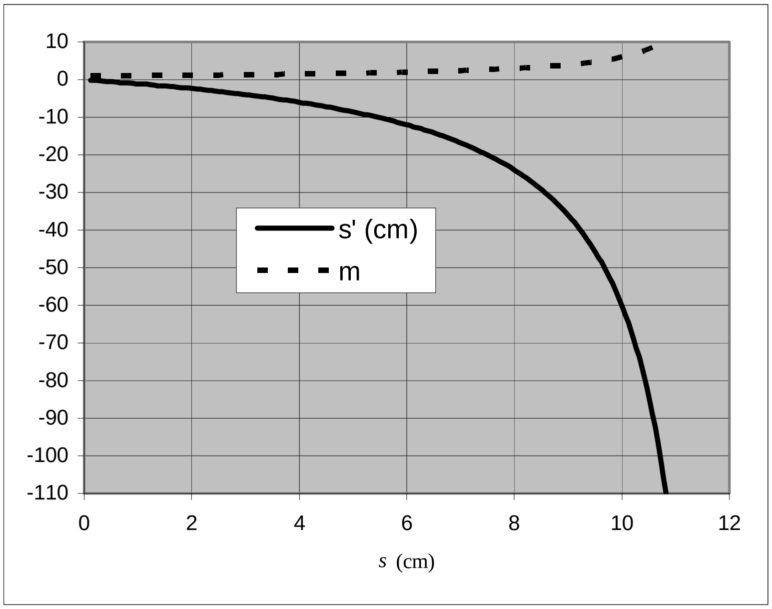
$$m = -\frac{s'}{s}$$

A spreadsheet program to calculate s' as a function of s is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	12	f
A4	0.12	s
A5	A4 + 0.1	$s + \Delta s$
B4	$B$1*A4/(A4 – $B$1)	$\frac{fs}{s - f}$
C5	–B4/A4	$-\frac{s'}{s}$

	A	B	C
1	f=	12	cm
2			
3	s	s'	m
4	0.12	–0.12	1.01
5	0.22	–0.22	1.02
6	0.32	–0.33	1.03
7	0.42	–0.44	1.04
8	0.52	–0.54	1.05
9	0.62	–0.65	1.05
108	10.52	–85.30	8.11
109	10.62	–92.35	8.70
110	10.72	–100.50	9.37
111	10.82	–110.03	10.17

A graph of s' as a function of s follows.



(*c*) The images are virtual and erect for this range of object distances.

(*d*) The asymptote of the graph of s' versus s corresponds to the image approaching infinity as the object distance approaches the focal length of the lens. The horizontal asymptote of the graph of m versus s indicates that, as the object moves toward the lens, the height of the image formed by the lens approaches the height of the object.

***64 ••**

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \quad (1)$$

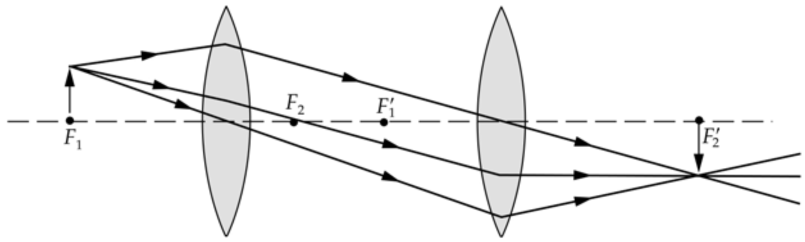
Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(15\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 15\,\text{cm}} = \infty$$

With $s_1' = \infty$, the thin-lens equation applied to the second lens becomes:

$$\frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = f_2 = \boxed{15.0\,\text{cm}}$$

A ray diagram is shown below:



The final image is 50 cm from the object, real, inverted, and the same size as the object.

65 ••

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

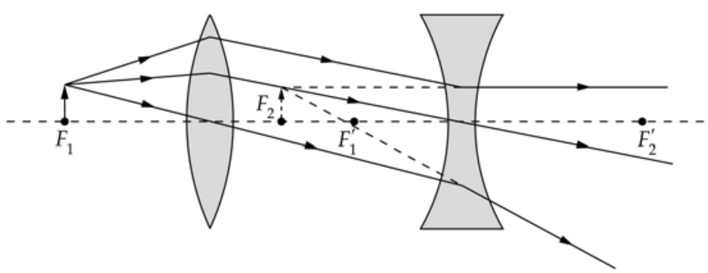
Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(15\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 15\,\text{cm}} = \infty$$

With $s'_1 = \infty$, the thin-lens equation applied to the second lens becomes:

$$\frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = f_2 = \boxed{15.0\,\text{cm}}$$

A ray diagram is shown below:



The final image is 50 cm from the object, real, inverted, and the same size as the object.

66 •••

Picture the Problem We can substitute $x = s - f$ and $x' = s' - f$ in the thin-lens equation and the equation for the lateral magnification of an image to obtain Newton's equations.

Express the thin-lens equation:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

If $x = s - f$ and $x' = s' - f$:

$$\frac{1}{x+f}+\frac{1}{x'+f}=\frac{1}{f}$$

Expand this expression to obtain:

$$f\left(x'+x+2f\right)=\left(x+f\right)\left(x'+f\right)$$
$$=xx'+xf+x'f+f^2$$

or, simplifying, $\boxed{xx'=f^2}$ (1)

The lateral magnification is:

$$m=-\frac{s'}{s}$$

or, because $x = s - f$ and $x' = s' - f$,

$$m=-\frac{x'+f}{x+f}$$

Solve equation (1) for x:

$$x=\frac{f^2}{x'}$$

Substitute for x and simplify to obtain:

$$m=-\frac{x'+f}{\frac{f^2}{x'}+f}=-\frac{x'+f}{\frac{f\left(f+x'\right)}{x'}}$$
$$=\boxed{-\frac{x'}{f}}$$

The lateral magnification is also given by:

$$m=-\frac{x'+f}{x+f}$$

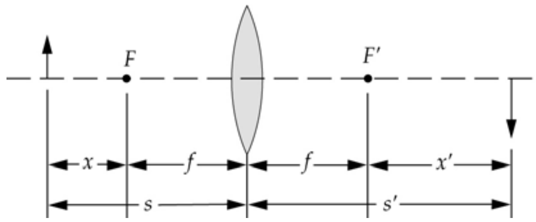
From equation (1) we have:

$$x'=\frac{f^2}{x}$$

Substitute to obtain:

$$m=-\frac{\frac{f^2}{x}+f}{x+f}=-\frac{f\left(\frac{f}{x}+1\right)}{x\left(1+\frac{f}{x}\right)}=\boxed{-\frac{f}{x}}$$

The variables x, f, s,and s' are shown in the sketch below:



67 •••

Picture the Problem The ray diagram shows the two lens positions and the corresponding image and object distances (denoted by the numerals 1 and 2). We can use the thin-lens equation relate the two sets of image and object distances to the focal length of the lens and then use the hint to express the relationships between these distances and the distances D and L to eliminate s_1, s_1', s_2, and s_2' and obtain an expression relating f, D, and L.

Relate the image and object distances for the two lens positions to the focal length of the lens:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f} \text{ and } \frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f}$$

Solve for f to obtain:

$$f=\frac{s_1s_1'}{s_1+s_1'}=\frac{s_2s_2'}{s_2+s_2'} \qquad (1)$$

The distances D and L can be expressed in terms of the image and object distances:

$$D=s_1+s_1'=s_2+s_2'$$

and

$$L=s_2-s_1=s_1'-s_2'$$

Substitute for the sums of the image and object distances in equation (1) to obtain:

$$f=\frac{s_1s_1'}{D}=\frac{s_2s_2'}{D}$$

From the hint:

$$s_1=s_2' \text{ and } s_1'=s_2$$

Hence $D = s_1 + s_2$ and:

$$D-L=2s_1 \text{ and } D+L=2s_2$$

Take the product of $D - L$ and $D + L$ to obtain:

$$(D-L)(D+L)=D^2-L^2$$
$$=4s_1s_2=4s_1s_1'$$

From the thin-lens equation:

$$4s_1s_2=rs_1s_1'=4fD$$

Substitute to obtain:

$$4fD=D^2-L^2$$

Solve for f:

$$\boxed{f = \frac{D^2 - L^2}{4D}}$$

68 ••

Picture the Problem We can use results obtained in Problem 67 to find the focal length of the lens and the two locations of the lens with respect to the object.

(*a*) From Problem 77 we have:

$$f = \frac{D^2 - L^2}{4D}$$

Substitute numerical values and evaluate f:

$$f = \frac{(1.7\,\text{m})^2 - (0.72\,\text{m})^2}{4(1.7\,\text{m})} = \boxed{34.9\,\text{cm}}$$

(*b*) Solve the thin-lens equation for the image distance to obtain:

$$s' = \frac{fs}{f - s} \qquad (1)$$

In Problem 77 it was established that:

$$D - L = 2s_1 \text{ and } D + L = 2s_2$$

Solve for s_1 and s_2:

$$s_1 = \frac{D - L}{2} \text{ and } s_2 = \frac{D + L}{2}$$

Substitute numerical values and evaluate s_1 and s_2:

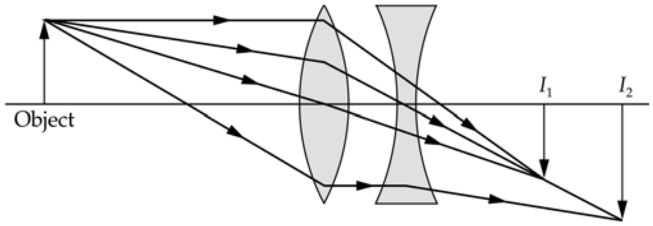
$$s_1 = \frac{170\,\text{cm} - 72\,\text{cm}}{2} = \boxed{49.0\,\text{cm}}$$

and

$$s_2 = \frac{170\,\text{cm} + 72\,\text{cm}}{2} = \boxed{121\,\text{cm}}$$

69 •••

Picture the Problem The ray diagram shows four rays from the head of the object that locate images I_1 and I_2. We can use the thin-lens equation to find the location of the image formed in the positive lens and then, knowing the separation of the two lenses, determine the object distance for the second lens and apply the thin lens a second time to find the location of the final image.



(*a*) Express the object-to-image distance *d*:

$$d = s_1 + 5\,\text{cm} + s_2' \qquad (1)$$

Apply the thin-lens equation to the positive lens:

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

Solve for s_1':

$$s_1' = \frac{f_1 s_1}{s_1 - f_1}$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(8.5\,\text{cm})(17.5\,\text{cm})}{17.5\,\text{cm} - 8.5\,\text{cm}} = 16.53\,\text{cm}$$

Find the object distance for the negative lens:

$$s_2 = 5\,\text{cm} - s_1' = 5\,\text{cm} - 16.53\,\text{cm}$$
$$= -11.53\,\text{cm}$$

The image distance s_2' is given by:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(-30\,\text{cm})(-11.53\,\text{cm})}{-11.53\,\text{cm} - (-30\,\text{cm})} = 18.7\,\text{cm}$$

Substitute numerical values in equation (1) and evaluate *d*:

$$d = 17.5\,\text{cm} + 5\,\text{cm} + 18.7\,\text{cm}$$
$$= \boxed{41.2\,\text{cm}}$$

(*b*) The overall lateral magnification is given by:

$$m = m_1 m_2$$

Express m_1 and m_2:

$$m_1 = -\frac{s_1'}{s_1} \text{ and } m_2 = -\frac{s_2'}{s_2}$$

Substitute to obtain:

$$m = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \frac{s_1' s_2'}{s_1 s_2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{(16.53\,\text{cm})(18.7\,\text{cm})}{(17.5\,\text{cm})(-11.53\,\text{cm})} = \boxed{-1.53}$$

$$\boxed{\text{Because } m < 0 \text{, the image is inverted. Because } s_2' > 0 \text{, the image is real.}}$$

Aberrations

***70** •

Determine the Concept Chromatic aberrations are a consequence of the differential refraction of light of differing wavelengths by lenses. $\boxed{(a)\text{ is correct.}}$

71 •

(*a*) False. Aberrations are a consequence of imperfections in lenses.

(*b*) True.

72 •

Picture the Problem We can use the lens-maker's equation to find the focal length the this lens for the two colors of light.

The lens-maker's equation relates the radii of curvature and the index of refraction to the focal length of the lens:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

(*a*) For red light:

$$\frac{1}{f_{\text{red}}}=(1.47-1)\left(\frac{1}{10\,\text{cm}}-\frac{1}{-10\,\text{cm}}\right)$$

and

$$f_{\text{red}}=\boxed{10.6\,\text{cm}}$$

(*b*) For blue light:

$$\frac{1}{f_{\text{blue}}}=(1.53-1)\left(\frac{1}{10\,\text{cm}}-\frac{1}{-10\,\text{cm}}\right)$$

and

$$f_{\text{blue}}=\boxed{9.43\,\text{cm}}$$

The Eye

***73** ••

Picture the Problem The thin-lens equation relates the image and object distances to the power of a lens.

(*a*) Use the thin-lens equation to relate the image and object distances to the power of the lens:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}=P$$

Because $s' = d$ and, for a distance object, $s = \infty$:

$$P_{\min} = \frac{1}{s'} = \boxed{\frac{1}{d}}$$

(*b*) If x_{np} is the closest distance an object could be and still remain in clear focus on the screen, equation (1) becomes:

$$P_{\max} = \boxed{\frac{1}{x_{\text{np}}} + \frac{1}{d}}$$

(*c*) Use our result in (*a*) to obtain:

$$P_{\min} = \frac{1}{2.5\,\text{cm}} = \boxed{40.0\,\text{D}}$$

Use the results of (*a*) and (*b*) to express the accommodation of the model eye:

$$A = P_{\max} - P_{\min} = \frac{1}{x_{\text{np}}} + \frac{1}{d} - \frac{1}{d} = \frac{1}{x_{\text{np}}}$$

Substitute numerical values and evaluate *A*:

$$A = \frac{1}{25\,\text{cm}} = \boxed{4.00\,\text{D}}$$

74 ••

Picture the Problem The thin-lens equation relates the image and object distances to the power of a lens.

(*a*) Use the thin-lens equation to relate the image and object distances to the power of the lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = P$$

Because $s' = d$ and $s = x_{\text{fp}}$:

$$P_{\min} = \boxed{\frac{1}{x_{\text{fp}}} + \frac{1}{d}}$$

(*b*) To correct for the nearsightedness of this eye, we need a lens that will form an image 25 cm in front of the eye of an object at the eye's far point:

$$P_{\min} = \frac{1}{50\,\text{cm}} + \frac{1}{-25\,\text{cm}} = \boxed{-2.00\,\text{D}}$$

75 ••

Picture the Problem The thin-lens equation relates the image and object distances to the power of a lens.

(*a*) Use the thin-lens equation to relate the image and object distances to the power of the lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = P$$

Because $s' = d$ and $s = x'_{np}$:

$$P'_{max} = \boxed{\frac{1}{x'_{np}} + \frac{1}{d}} \quad (1)$$

(*b*) For a normal eye:

$$P_{max} = \frac{1}{x_{np}} + \frac{1}{d} \quad (2)$$

The amount by which the power of the lens is too small is the difference between equations (2) and (1):

$$P_{max} - P'_{max} = \frac{1}{x_{np}} + \frac{1}{d} - \left(\frac{1}{x'_{np}} + \frac{1}{d}\right) = \boxed{\frac{1}{x_{np}} - \frac{1}{x'_{np}}}$$

(*c*) For x_{np} = 15 cm and x'_{np} = 150 cm:

$$P_{max} - P'_{max} = \frac{1}{15\,\text{cm}} - \frac{1}{150\,\text{cm}} = \boxed{6.00\,\text{D}}$$

76 •

Picture the Problem We can use the thin-lens equation to find the distance from the lens to the image and then take their difference to find the distance the lens would have to be moved.

Express the distance *d* that the lens would have to move:

$$d = s' - f$$

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

Substitute to obtain:

$$d = \frac{fs}{s - f} - f$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(2.5\,\text{cm})(25\,\text{cm})}{25\,\text{cm} - 2.5\,\text{cm}} - 2.5\,\text{cm} = \boxed{0.278\,\text{cm}}$$

That is, the lens would have to move 0.278 cm toward the object.

77 •

Picture the Problem We can apply the thin-lens equation for the two values of *s* to find Δf.

Express the change Δf in the focal length:

$$\Delta f = f_{s=3\,\text{m}} - f_{s=0.3\,\text{m}}$$

Solve the thin-lens equation for *s*:

$$f = \frac{ss'}{s'+s}$$

Substitute to obtain:

$$\Delta f = \frac{s_{3\text{m}}s'_{3\text{m}}}{s'_{3\text{m}} + s_{3\text{m}}} - \frac{s_{0.3\text{m}}s'_{0.3\text{m}}}{s'_{0.3\text{m}} + s_{0.3\text{m}}}$$

or, because $s'_{3\text{ m}} = s'_{0.3\text{ m}}$,

$$\Delta f = s'_{3\text{m}}\left[\frac{s_{3\text{m}}}{s'_{3\text{m}} + s_{3\text{m}}} - \frac{s_{0.3\text{m}}}{s'_{0.3\text{m}} + s_{0.3\text{m}}}\right]$$

Substitute numerical values and evaluate Δf:

$$\Delta f = (2.5\,\text{cm})\left[\frac{300\,\text{cm}}{2.5\,\text{cm} + 300\,\text{cm}} - \frac{30\,\text{cm}}{2.5\,\text{cm} + 30\,\text{cm}}\right] = 0.172\,\text{cm} = \boxed{1.72\,\text{mm}}$$

78 •

Picture the Problem We can use the thin-lens equation and the definition of the power of a lens to express the near point distance as a function of *P* and *s*.

From the thin-lens equation we have:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = P$$

Solve for s':

$$s' = \frac{s}{Ps - 1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{25\,\text{cm}}{(1.75\,\text{m}^{-1})(0.25\,\text{m}) - 1} = -44.4\,\text{cm}$$

$$\boxed{\text{The person's near point with lenses is 44.4 cm.}}$$

***79** •

Picture the Problem We can use the relationship between a distance measured along the arc of a circle and the angle subtended at its center to approximate the smallest angle the two points can subtend and the separation of the two points 20 m from the eye.

(*a*) Relate θ_{min} to the diameter of the eye and the distance between the activated cones:

$$d_{\text{eye}}\theta_{\text{min}} \approx 2\,\mu\text{m}$$

Solve for θ_{min}:

$$\theta_{min} = \frac{2\,\mu m}{d_{eye}}$$

Substitute numerical values and evaluate θ_{min}:

$$\theta_{min} = \frac{2\,\mu m}{2.5\,cm} = \boxed{80.0\,\mu rad}$$

(*b*) Let *D* represent the separation of the points *R* = 20 m from the eye to obtain:

$$D = R\theta_{min} = (20\,m)(80\,\mu rad) = \boxed{1.60\,mm}$$

80 ••

Picture the Problem We can use the thin-lens equation to find *f* and the definition of the power of a lens to find *P*.

(*a*) Solve the thin-lens equation for *f*:

$$f = \frac{ss'}{s' + s}$$

Noting that $s' < 0$, substitute numerical values and evaluate *f*:

$$f = \frac{(45\,cm)(-80\,cm)}{-80\,cm + 45\,cm} = \boxed{103\,cm}$$

(*b*) Use the definition of the power of a lens to obtain:

$$P = \frac{1}{f} = \frac{1}{1.03\,m} = \boxed{0.971\,diopters}$$

81 ••

Picture the Problem We can use the thin-lens equation to find *f* and the definition of the power of a lens to find *P*.

Express the required power of the lens:

$$P = \frac{1}{f}$$

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

For $s = \infty$:

$$\frac{1}{s'} = \frac{1}{f} \Rightarrow f = s'$$

Substitute for *f* to obtain:

$$P = \frac{1}{s'}$$

Substitute for s' and evaluate *P*:

$$P = \frac{1}{2.25\,m} = \boxed{0.444\,diopters}$$

82 ••

Picture the Problem We can use the lens-maker's equation with $s = \infty$ to find the radius of the cornea modeled as a homogeneous sphere with an index of refraction of 1.4.

Use the lens-maker's equation to relate the radius of the cornea to its index of refraction and that of air:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

Because $n_2 = n$, $n_1 = 1$, and $s = \infty$:

$$\frac{n}{s'} = \frac{n-1}{r}$$

Solve for r:

$$r = \frac{s'(n-1)}{n} = \left(1 - \frac{1}{n}\right)s'$$

Substitute numerical values and evaluate r:

$$r = \left(1 - \frac{1}{1.4}\right)(2.5\,\text{cm}) = \boxed{0.714\,\text{cm}}$$

The eye is not a homogeneous sphere. It is filled with a transparent liquid (vitreous humor) which has an index of refraction that is not known. If that index of refraction differs from 1.4, there is refraction at the inner surface of the cornea which will result in the formation of the image nearer the cornea's surface if $n > 1.4$ and farther if $n < 1.4$, where n is the index of refraction of the vitreous humor. If $n < 1.4$, then r as calculated above is too small.

83 ••

Picture the Problem We can use the definition of the power of a lens and the thin-lens equation to find the power of the lens that should be used in the glasses.

Express the power of the lens that should be used in the glasses:

$$P = P_{\text{eye}} + P_{\text{lens}} = \frac{1}{f_{\text{eye}}} + \frac{1}{f_{\text{glasses}}} \qquad (1)$$

Because the glasses are 2 cm from the eye:

$$s' = -80\,\text{cm} + 2\,\text{cm} = -78\,\text{cm}$$

and

$$s = 25\,\text{cm} - 2\,\text{cm} = 23\,\text{cm}$$

Apply the thin-lens equation to the eye with $s' = \infty$:

$$\frac{1}{s} = \frac{1}{f_{\text{eye}}} \Rightarrow f_{\text{eye}} = s$$

Apply the thin-lens equation to the glasses with $s = \infty$:

$$\frac{1}{s'} = \frac{1}{f_{\text{glasses}}} \Rightarrow f_{\text{glasses}} = s'$$

Substitute for f_{eye} and f_{glasses} in equation (1) to obtain:

$$P = \frac{1}{s} + \frac{1}{s'}$$

Substitute numerical values and evaluate P:

$$P = \frac{1}{0.23\,\text{m}} + \frac{1}{-0.78\,\text{m}} = \boxed{3.07\,\text{D}}$$

84 •••

Picture the Problem We can use the thin-lens equation and the distance from her eyes to her glasses to derive an expression for the location of her near point.

(*a*) Express her near point, x_{np}, at age 45 in terms of the location of her glasses:

$$x_{\text{np}} = |s'| + 2.2\,\text{cm} \qquad (1)$$

Because the glasses are 2.2 cm from her eye:

$$s = 25\,\text{cm} - 2.2\,\text{cm} = 22.8\,\text{cm}$$

Apply the thin-lens equation to the glasses:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_{\text{glasses}}} = P$$

Solve for s':

$$s' = \frac{s}{Ps - 1} = \frac{1}{P - \frac{1}{s}}$$

Substitute in equation (1) to obtain:

$$x_{\text{np}} = \left|\frac{1}{P - \frac{1}{s}}\right| + 2.2\,\text{cm} \qquad (2)$$

Substitute numerical values and evaluate x_{np}:

$$x_{\text{np}} = \left|\frac{1}{2.1\,\text{m}^{-1} - \frac{1}{0.228\,\text{m}}}\right| + 2.2\,\text{cm} = \boxed{45.9\,\text{cm}}$$

(*b*) At age 55:

$$s = 40\,\text{cm} - 2.2\,\text{cm} = 37.8\,\text{cm}$$

Substitute numerical values in equation (2) and evaluate *s*′:

$$x_{np} = \left|\frac{1}{2.1\,\text{m}^{-1} - \dfrac{1}{0.378\,\text{m}}}\right| + 2.2\,\text{cm}$$

$$= \boxed{185\,\text{cm}}$$

(*c*) Solve the thin-lens equation for *f*:

$$f = \frac{ss'}{s' + s}$$

From the definition of *P*:

$$P = \frac{1}{f} = \frac{s' + s}{s's}$$

For *s* = 22.8 cm and *s*′ = 183.3 cm:

$$P = \frac{183.3\,\text{cm} + 22.8\,\text{cm}}{(183.3\,\text{cm})(22.8\,\text{cm})} = \boxed{4.93\,\text{D}}$$

The Simple Magnifier

***85** •

Picture the Problem We can use the definitions of the magnifying power of a lens ($M = x_{np}/f$) and of the power of a lens ($P = 1/f$) to find the magnifying power of the given lens.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f} = Px_{np}$$

where *P* is the power of the lens.

Substitute numerical values and evaluate *M*:

$$M = (20\,\text{m}^{-1})(0.3\,\text{m}) = \boxed{6.00}$$

86 •

Picture the Problem We can use the definition of the magnifying power of a lens to find the required focal length so that this person’s lens will have magnification power of 5.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f}$$

Solve for *f*:

$$f = \frac{x_{np}}{M}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{25\,\text{cm}}{5} = \boxed{5.00\,\text{cm}}$$

87 •

Picture the Problem We can use the definition of the magnifying power of a lens to find the magnifying power of this lens.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{35\,\text{cm}}{7\,\text{cm}} = \boxed{5.00}$$

88 ••

Picture the Problem Let the numerals 1 and 2 denote the 1st and 2nd persons, respectively. We can use the definition of magnifying power to find the effective magnifying power of the lens for each person. The relative height of the images on the retinas of the two persons is given by the ratio of the effective magnifying powers.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f}$$

Substitute numerical values and evaluate M_1 and M_2:

$$M_1 = \frac{25\,\text{cm}}{6\,\text{cm}} = \boxed{4.17}$$

and

$$M_2 = \frac{40\,\text{cm}}{6\,\text{cm}} = \boxed{6.67}$$

From the definition of magnifying power we have:

$$\frac{M_1}{M_2} = \frac{\frac{y_1}{f}}{\frac{y_2}{f}} = \frac{y_1}{y_2}$$

Substitute for M_1 and M_2 and evaluate the ratio of y_1 to y_2:

$$\frac{y_1}{y_2} = \frac{4.17}{6.67} = \boxed{0.625}$$

89 ••

Picture the Problem We can use the definition of angular magnification to find the expected angular magnification if the final image is at infinity and the thin-lens equation and the expression for the magnification of a thin lens to find the angular magnification when the final image is at 25 cm.

(*a*) Express the angular magnification when the final image is at infinity:

$$M = \frac{x_{\text{np}}}{f} = x_{\text{np}}P$$

where *P* is the power of the lens.

Substitute numerical values and evaluate *M*:

$$M = (25\,\text{cm})(12\,\text{m}^{-1}) = \boxed{3.00}$$

(*b*) Express the magnification of the lens when the final image is at 25 cm:

$$m = -\frac{s'}{s}$$

Solve the thin-lens equation for *s*:

$$s = \frac{fs'}{s' - f}$$

Substitute to obtain:

$$m = -\frac{s'}{\dfrac{fs'}{s'-f}} = -\frac{s'-f}{f} = -\frac{s'}{f} + 1$$
$$= 1 - s'P$$

Substitute numerical values and evaluate *m*:

$$m = 1 - (-0.25\,\text{m})(12\,\text{m}^{-1}) = \boxed{4}$$

***90 ••**

Picture the Problem We can use the definition of the angular magnification of a lens and the thin-lens equation to show that $M = \frac{x_{\text{np}}}{f} + 1$.

(*a*) Express the angular magnification of the simple magnifier in terms of the angles subtended by the object and the image:

$$M = \frac{\theta}{\theta_0} \qquad (1)$$

Solve the thin-lens equation for *s*:

$$s = \frac{fs'}{s' - f}$$

Because the image is virtual:

$$s' = -x_{\text{np}}$$

Substitute to obtain:

$$s = \frac{f(-x_{\text{np}})}{-x_{\text{np}} - f} = \frac{fx_{\text{np}}}{x_{\text{np}} + f}$$

Express the angle subtended by the object:

$$\theta_0 = \frac{y}{x_{np}}$$

where y is the height of the object.

Express the angle subtended by the image:

$$\theta = \frac{y}{s}$$

Substitute for s to obtain:

$$\theta = \frac{y}{\dfrac{f x_{np}}{x_{np} + f}} = \frac{y(x_{np} + f)}{f x_{np}}$$

Substitute in equation (1) and simplify:

$$M = \frac{\dfrac{y(x_{np} + f)}{f x_{np}}}{\dfrac{y}{x_{np}}} = \frac{x_{np} + f}{f} = \boxed{\frac{x_{np}}{f} + 1}$$

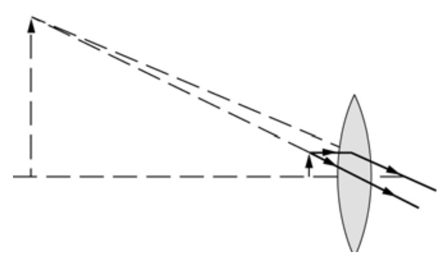
(*b*) In terms of the power of the magnifying lens:

$$M = x_{np} P + 1$$

The magnification of a 20-D lens for a person with a near point of 30 cm and the final image at the near point is:

$$M = (0.3\,\text{m})(20\,\text{m}^{-1}) + 1 = \boxed{7.00}$$

A ray diagram for this situation is shown to the right:



91 ••

Picture the Problem We can use the definitions of lateral and angular magnification and the result given in Problem 82 to show that, when the image of a simple magnifier is viewed at the near point, the lateral and angular magnifications are equal.

Express the lateral magnification of the lens:

$$M = \frac{x_{np}}{f}$$

Because the image is viewed at the near point, $f = s$ and:

$$M = \frac{x_{np}}{s}$$

From Problem 32-82:

$$M = \frac{x_{np}}{f} + 1$$

and

$$\frac{x_{np}}{s} = \frac{x_{np}}{f} + 1 \text{ or } \boxed{M_{lateral} = M_{angular}}$$

The Microscope

92 ••

Picture the Problem We can use the thin-lens equation to find the location of the object and the expression for the magnifying power of a microscope to find the magnifying power of the given microscope for a person whose near point is at 25 cm.

(*a*) Using the thin-lens equation, relate the object distance *s* to the focal length of the objective lens f_0:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_0}$$

Solve for *s* to obtain:

$$s = \frac{f_0 s'}{s' - f_0}$$

From Figure 32-48, the image distance for the image formed by the objective lens is:

$$s' = f_0 + L = 1.7\,\text{cm} + 16\,\text{cm} = 17.7\,\text{cm}$$

Substitute numerical values and evaluate *s*:

$$s = \frac{(1.7\,\text{cm})(17.7\,\text{cm})}{17.7\,\text{cm} - 1.7\,\text{cm}} = \boxed{1.88\,\text{cm}}$$

(*b*) Express the magnifying power of a microscope:

$$M = -\frac{L}{f_0}\frac{x_{np}}{f_e}$$

Substitute numerical values and evaluate *M*:

$$M = -\frac{16\,\text{cm}}{1.7\,\text{cm}}\frac{25\,\text{cm}}{5.1\,\text{cm}} = \boxed{-46.1}$$

***93 ••**

Picture the Problem The lateral magnification of the objective is $m_o = -L/f_o$ and the magnifying power of the microscope is $M = m_o M_e$.

(*a*) The lateral magnification of the objective is given by:

$$m_o = -\frac{L}{f_o}$$

Substitute numerical values and evaluate m_o:

$$m_o = -\frac{16\,\text{cm}}{8.5\,\text{mm}} = \boxed{-1.88}$$

(*b*) The magnifying power of the microscope is given by:

$$M = m_o M_e$$

where M_e is the angular magnification of the lens.

Substitute numerical values and evaluate M:

$$M = (-1.88)(10) = \boxed{-18.8}$$

94 ••

Picture the Problem We can find the tube length from the length of the tube to which the lenses are fastened and the focal lengths of the objective and eyepiece. We can use their definitions to find the lateral magnification of the objective and the magnifying power of the microscope. The distance of the object from the objective can be found using the thin-lens equation.

(*a*) The tube length L is given by:

$$L = D - f_o - f_e$$

$$= 0.30\,\text{m} - \frac{2}{20\,\text{m}^{-1}} = \boxed{20.0\,\text{cm}}$$

(*b*) The lateral magnification of the objective m_o is given by:

$$m_o = -\frac{L}{f_o} = -\frac{20\,\text{cm}}{5\,\text{cm}} = \boxed{-4.00}$$

(*c*) The magnifying power of the microscope is given by:

$$M = m_o M_e = m_o \frac{x_{np}}{f_e}$$

Substitute numerical values and evaluate M:

$$M = (-4)\frac{25\,\text{cm}}{5\,\text{cm}} = \boxed{-20.0}$$

(*d*) From the thin-lens equation we have:

$$\frac{1}{s_o} + \frac{1}{s_o'} = \frac{1}{f_o}$$

where $s_o' = f_o + L$

Substitute to obtain:

$$\frac{1}{s_o} + \frac{1}{f_o + L} = \frac{1}{f_o}$$

Solve for s_o:

$$s_o = \frac{f_o(f_o + L)}{L}$$

Substitute numerical values and evaluate s_o:

$$s_o = \frac{(5\,\text{cm})(5\,\text{cm} + 20\,\text{cm})}{20\,\text{cm}} = \boxed{6.25\,\text{cm}}$$

***95 ••**

Picture the Problem The magnifying power of a compound microscope is the product of the magnifying powers of the objective and the eyepiece.

Express the magnifying power of the microscope in terms of the magnifying powers of the objective and eyepiece:

$$M = m_o m_e \qquad (1)$$

From Problem 82, the magnification of the eyepiece is given by:

$$m_e = \frac{x_{np}}{f_e} + 1 = P_e x_{np} + 1$$

The magnification of the objective is given by:

$$m_o = -\frac{L}{f_o}$$

where $L = D - f_o - f_e$

Substitute to obtain:

$$m_o = -\frac{D - f_o - f_e}{f_o}$$

Substitute for m_e and m_o in equation (1) to obtain:

$$M = (P_e x_{np} + 1)\left(-\frac{D - f_o - f_e}{f_o}\right)$$

Substitute numerical values and evaluate M:

$$M = [(80\,\text{D})(0.25\,\text{m}) + 1]\left(-\frac{28\,\text{cm} - 2.22\,\text{cm} - 1.25\,\text{cm}}{2.22\,\text{cm}}\right) = \boxed{-232}$$

96 •••

Picture the Problem We can find the focal length of the eyepiece from its angular magnification and the near point of a normal eye. The location of the object such that it is in focus for a normal relaxed eye can be found from the lateral magnification of the eyepiece and the magnifying power of the microscope. Finally, we can use the thin-lens equation to find the focal length of the objective lens.

(*a*) Relate the focal length of the eyepiece to its angular magnifying power:

$$M_e = \frac{x_{np}}{f_e} \Rightarrow f_e = \frac{x_{np}}{M_e}$$

Substitute numerical values and evaluate f_e:

$$f_e = \frac{25\,\text{cm}}{15} = \boxed{1.67\,\text{cm}}$$

(*b*) Relate *s* to *s*′ through the lateral magnification of the objective:

$$m_o = -\frac{s'}{s} \Rightarrow s = -\frac{s'}{m_o}$$

Relate the magnifying power of the microscope *M* to the lateral magnification of its objective m_0 and the angular magnification of its eyepiece M_e:

$$M = m_o M_e$$

Solve for m_o:

$$m_o = \frac{M}{M_e}$$

Substitute to obtain:

$$s = -\frac{s' M_e}{M}$$

Evaluate *s*′:

$$s' = 22\,\text{cm} - f_e$$
$$= 22\,\text{cm} - 1.67\,\text{cm} = 20.33\,\text{cm}$$

Substitute numerical values and evaluate *s*:

$$s = -\frac{(20.33\,\text{cm})(15)}{-600} = \boxed{0.508\,\text{cm}}$$

(*c*) Solve the thin-lens equation for f_o:

$$f_o = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate f_o:

$$f_o = \frac{(0.508\,\text{cm})(20.33\,\text{cm})}{20.33\,\text{cm} + 0.508\,\text{cm}}$$
$$= \boxed{0.496\,\text{cm}}$$

The Telescope

97 •

Picture the Problem Because of the great distance to the moon, its image formed by the objective lens is at the focal point of the objective lens and we can use $D = f_o\theta$ to find the diameter D of the image of the moon. Because angle subtended by the final image at infinity is given by $\theta_e = M\theta_o = M\theta$, we can solve (*b*) and (*c*) together by first using $M = -f_o/f_e$ to find the magnifying power of the telescope.

(*a*) Relate the diameter D of the image of the moon to the image distance and the angle subtended by the moon:

$$D = s_o'\theta$$

Because the image of the moon is at the focal point of the objective lens:

$$s_o' = f_o$$

and

$$D = f_o\theta$$

Substitute numerical values and evaluate D:

$$D = (100\,\text{cm})(0.009\,\text{rad}) = \boxed{9.00\,\text{mm}}$$

(*b*) and (*c*) Relate the angle subtended by the final image at infinity to the magnification of the telescope and the angle subtended at the objective:

$$\theta_e = M\theta_o = M\theta$$

Express the magnifying power of the telescope:

$$M = -\frac{f_o}{f_e}$$

Substitute numerical values and evaluate M and θ_e:

$$M = -\frac{100\,\text{cm}}{5\,\text{cm}} = \boxed{-20.0}$$

and

$$\theta_e = (-20)(0.009\,\text{rad}) = \boxed{-0.180\,\text{rad}}$$

98 •

Picture the Problem Because of the great distance to the moon, its image formed by the objective lens is at the focal point of the objective lens and we can use $D = f_o\theta$ to find the diameter D of the image of the moon.

Relate the diameter D of the image of the moon to the image distance and the angle subtended by the moon:

$$D = s_o'\theta$$

Because the image of the moon is at

$$s_o' = f_o$$

the focal point of the objective lens: and

$$D = f_o\theta$$

Substitute numerical values and evaluate D:

$$D = (19.5\,\text{m})(0.009\,\text{rad}) = \boxed{17.6\,\text{cm}}$$

***99 ••**

Picture the Problem Because the light-gathering power of a mirror is proportional to its area, we can compare the light-gathering powers of these mirrors by finding the ratio of their areas. We can use the ratio of the focal lengths of the objective and eyepiece lenses to find the magnifying power of the Palomar telescope.

(*a*) Express the ratio of the light-gathering powers of the Palomar and Yerkes mirrors:

$$\frac{P_{\text{Palomar}}}{P_{\text{Yerkes}}} = \frac{A_{\text{Palomar mirror}}}{A_{\text{Yerkes mirror}}} = \frac{\frac{\pi}{4}d^2_{\text{Palomar mirror}}}{\frac{\pi}{4}d^2_{\text{Yerkes mirror}}} = \frac{d^2_{\text{Palomar mirror}}}{d^2_{\text{Yerkes mirror}}}$$

Substitute numerical values and evaluate $P_{\text{Palomar}}/P_{\text{Yerkes}}$:

$$\frac{P_{\text{Palomar}}}{P_{\text{Yerkes}}} = \frac{(200\,\text{in})^2}{(40\,\text{in})^2} = 25.0$$

or

$$P_{\text{Palomar}} = \boxed{(25.0)P_{\text{Yerkes}}}$$

(*b*) Express the magnifying power of the Palomar telescope:

$$M = -\frac{f_o}{f_e}$$

Substitute numerical values and evaluate M:

$$M = -\frac{1.68\,\text{m}}{1.25\,\text{cm}} = \boxed{-134}$$

100 ••

Picture the Problem We can use the expression for the magnifying power of a telescope and the fact that the length of a telescope is the sum of focal lengths of its objective and eyepiece lenses to obtain simultaneous equations in f_o and f_e.

The magnifying power of the telescope is given by:

$$M = -\frac{f_o}{f_e} = 7$$

The length of the telescope is the sum of the focal lengths of the objective and eyepiece lenses:

$$L = f_o + f_e = 32\,\text{cm}$$

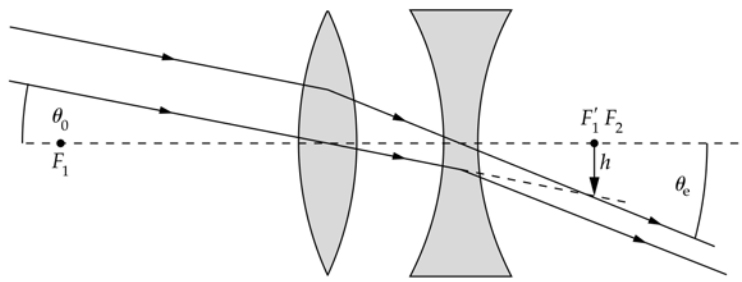
Solve these equations simultaneously to obtain:

$$f_o = \boxed{28.0\,\text{cm}} \text{ and } f_e = \boxed{4.00\,\text{cm}}$$

101 ••

Picture the Problem The magnification of a telescope is the ratio of the angle subtended at the eyepiece lens to the angle subtended at the objective lens. We can use the geometry of the ray diagram to express both θ_e and θ_o.

(*b*) The ray diagram is shown below:



(*a*) Express the magnifying power *M* of the telescope:

$$M = \frac{\theta_e}{\theta_o}$$

Because the image formed by the objective lens is at the focal point, F'_1:

$$\theta_o = \frac{h}{f_o}$$

where we have assumed that $\theta_o << 1$ so that $\tan\theta_o \approx \theta_o$.

Express the angle subtended by the eyepiece:

$$\theta_e = \frac{h}{f_e}$$ where f_e is negative.

Substitute to obtain:

$$M = \frac{\frac{h}{f_e}}{\frac{h}{f_o}} = \frac{f_o}{f_e}$$

and $M = \boxed{-\frac{f_o}{f_e}}$ is positive.

Remarks: Because the object for the eyepiece is at its focal point, the image is at infinity. As is also evident from the ray diagram, the image is virtual and upright.

102 ••

Picture the Problem We can use the thin-lens equation to find the image distance for the objective lens and the object distance for the eyepiece lens. The separation of the lenses is the sum of these distances. We can use the definition of the angular magnification and the angles subtended at the objective and eyepiece lenses to find the height of the final image.

(*a*) Solve the thin-lens equation for s_o':

$$s_o' = \frac{f_o s_o}{s_o - f_o}$$

Substitute numerical values and evaluate s_o':

$$s_o' = \frac{(1\,\text{m})(30\,\text{m})}{30\,\text{m} - 1\,\text{m}} = \boxed{103.45\,\text{cm}}$$

where we have kept more than three significant figures in the answer for use in (*c*) and (*d*).

(*b*) Solve the thin-lens equation for s_e:

$$s_e = \frac{f_e s_e'}{s_e' - f_e}$$

Noting that $s_e' = -25$ cm, substitute numerical values and evaluate s_e:

$$s_e = \frac{(-5\,\text{cm})(-25\,\text{cm})}{-25\,\text{cm} - (-5\,\text{cm})_e} = \boxed{-6.25\,\text{cm}}$$

where the minus sign tells us that the object of the eyepiece is virtual.

(*c*) Express the separation *D* of the lenses:

$$D = s_o' + s_e$$

Substitute numerical values and evaluate *D*:

$$D = 103.45\,\text{cm} - 6.25\,\text{cm} = \boxed{97.2\,\text{cm}}$$

(*d*) Express the height *h*′ of the final image in terms of the magnification *M* of the telescope:

$$h' = Mh$$

The magnification of the telescope is the product of the magnifications of the objective and eyepiece lenses:

$$M = m_o m_e = \frac{s_o'}{s_o}\frac{s_e'}{s_e}$$

Substitute to obtain:

$$h' = \frac{s_o'}{s_o}\frac{s_e'}{s_e}h$$

Substitute numerical values and evaluate h':

$$h' = \left(\frac{103.45\,\text{cm}}{3000\,\text{cm}}\right)\left(\frac{-25\,\text{cm}}{-6.25\,\text{cm}}\right)(1.5\,\text{m}) = \boxed{20.7\,\text{cm}}$$

Express the angular magnification of the telescope:

$$M = \frac{\theta_e}{\theta_o}$$

The angle subtended by the object is:

$$\theta_o = \frac{h}{s_o}$$

The angle subtended by the image is:

$$\theta_e = \tan^{-1}\left(\frac{h'}{s_e}\right)$$

Substitute to obtain:

$$M = \frac{\tan^{-1}\left(\frac{h'}{s_e}\right)}{\frac{h}{s_o}} = \frac{s_o}{h}\tan^{-1}\left(\frac{h'}{s_e}\right)$$

Substitute numerical values and evaluate M:

$$M = \frac{30\,\text{m}}{1.5\,\text{m}}\tan^{-1}\left(\frac{20.7\,\text{cm}}{6.25\,\text{cm}}\right) = \boxed{25.6}$$

103 •••

Picture the Problem The roles of the objective and eyepiece lenses are reversed.

Express the magnifying power of the ″wrong end″ telescope:

$$M = -\frac{f_e}{f_o}$$

Substitute numerical values and evaluate M:

$$M = -\frac{1.5\,\text{cm}}{2.25\,\text{m}} = -6.67\times10^{-3} = \boxed{-1/150}$$

General Problems

104 •

Picture the Problem We can solve the thin-lens equation for s' and then argue that the signs of the numerator and denominator are such that their quotient is always negative.

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

For a diverging lens: $f < 0$ and $s > 0$ for a real object.

$$\boxed{\text{Consequently, the denominator is positive and the numerator is negative, so } s' \text{ must always be negative.}}$$

***105** •

Picture the Problem We can express the distance Δs that the lens must move as the difference between the image distances when the object is at 30 m and when it is at infinity and then express these image distances using the thin-lens equation.

Express the distance Δs that the lens must move to change from focusing on an object at infinity to one at a distance of 30 m:

$$\Delta s = s'_{30} - s'_{\infty}$$

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

Substitute and simplify to obtain:

$$\Delta s = \frac{fs_{30}}{s_{30} - f} - \frac{fs_{\infty}}{s_{\infty} - f} = \frac{fs_{30}}{s_{30} - f} - \frac{f}{1 - f/s_{\infty}} = f\left[\frac{s_{30}}{s_{30} - f} - 1\right]$$

Substitute numerical values and evaluate Δs:

$$\Delta s = (200\,\text{mm})\left[\frac{30\,\text{m}}{30\,\text{m} - 0.2\,\text{m}} - 1\right] = \boxed{1.34\,\text{mm}}$$

106 •

Picture the Problem We can express the distance Δs that the lens must move as the difference between the image distances when the object is at 30 m and when it is at infinity and then express these image distances using the thin-lens equation.

Express the distance Δs that the lens must move to change from focusing on an object at infinity to one at a

$$\Delta s = s'_{5} - s'_{\infty}$$

distance of 5 m:

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

Substitute and simplify to obtain:

$$\Delta s = \frac{fs_5}{s_5 - f} - \frac{fs_\infty}{s_\infty - f} = \frac{fs_5}{s_5 - f} - \frac{f}{1 - f/s_\infty} = f\left[\frac{s_5}{s_5 - f} - 1\right]$$

Substitute numerical values and evaluate Δs:

$$\Delta s = (28\,\text{mm})\left[\frac{5\,\text{m}}{5\,\text{m} - 0.028\,\text{m}} - 1\right] = \boxed{0.158\,\text{mm}}$$

107 •

Picture the Problem We can use the thin-lens and magnification equations to obtain simultaneous equations that we can solve to find the image and object distances for the two situations described in the problem statement.

(*a*) Use the thin-lens equation to relate the image and object distances to the focal length of the lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Because the image is twice as large as the object:

$$m = -\frac{s'}{s} \Rightarrow s' = -2s$$

Substitute to obtain:

$$\frac{1}{s} + \frac{1}{-2s} = \frac{1}{f}$$

Solve for s:

$$s = \tfrac{1}{2} f$$

Substitute numerical values and evaluate s and s':

$$s = \tfrac{1}{2}(10\,\text{cm}) = \boxed{5.00\,\text{cm}}$$

and

$$s' = -2(5\,\text{cm}) = \boxed{-10.0\,\text{cm}}$$

(*b*) If the image is inverted, then: $s' = 2s$ and $\frac{1}{s} + \frac{1}{2s} = \frac{1}{f}$

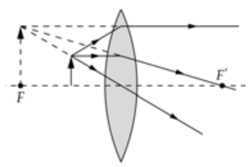
Solve for *s*: $s = \frac{3}{2} f$

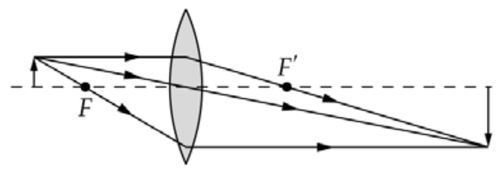
Substitute numerical values and evaluate *s* and *s*′: $s = \frac{1}{2} 3(10\,\text{cm}) = \boxed{15.0\,\text{cm}}$

and

$s' = 2(15\,\text{cm}) = \boxed{30.0\,\text{cm}}$

The ray diagrams for (*a*) (left) and (*b*) (right) are shown below:

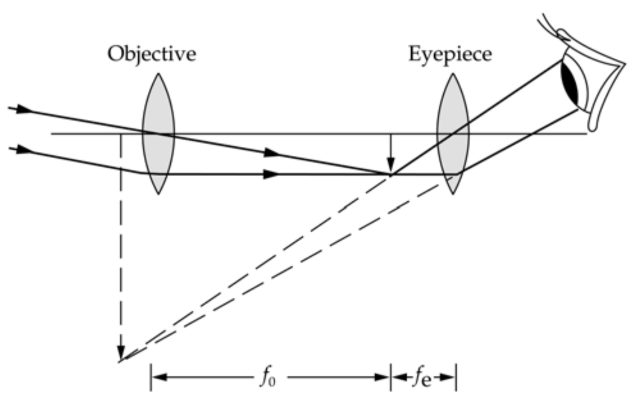




108 ••

(*a*) In an astronomical telescope the eyepiece (short focal length) and objective (long focal length) lenses are separated by the sum of their focal lengths. Given these two lenses, we'll use the 25 mm lens as the eyepiece lens and the 75 mm lens as the objective lens and mount them 100 mm apart. The angular magnification is then $M = \frac{f_0}{f_e} = \frac{75\,\text{mm}}{25\,\text{mm}} = \boxed{3}$.

(*b*) A ray diagram showing how rays from a distant object are magnified by an astronomical telescope follows. A real and inverted image of the distant object is formed by the objective lens near its second focal point. The eyepiece lens forms an enlarged and inverted image of the image formed by the objective lens.



109 ••

Determine the Concept

(*a*) Because the focal lengths appear in the magnification formula as a product, it would appear that it does not matter in which order we use them. The usual arrangement would be to use the shorter focal length lens as the objective but we get the same magnification in the reverse order. What difference does it make then? None in this problem. However, it is generally true that the smaller the focal length of a lens, the smaller its diameter. This condition makes it harder to use the shorter focal length lens, with its smaller diameter, as the eyepiece lens. If we separate the objective and eyepiece lenses by $L + f_e + f_o$ = 16 cm + 7.5 cm + 2.5 cm = $\boxed{26.0\,\text{cm},}$ the overall magnification will be

$$M = m_0 M_e = -\frac{L}{f_0}\frac{x_{np}}{f_e} = -\frac{16\,\text{cm}}{7.5\,\text{cm}}\frac{25\,\text{cm}}{2.5\,\text{cm}} = \boxed{-21.3}\,.$$

In a compound microscope, the lenses are separated by:

$$\delta = L + f_e + f_0$$

Substitute numerical values and evaluate δ:

$$\delta = 16\,\text{cm} + 7.5\,\text{cm} + 2.5\,\text{cm} = \boxed{26.0\,\text{cm}}$$

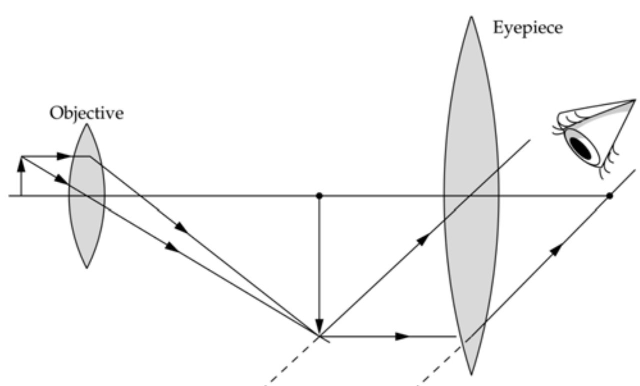
The overall magnification of a compound microscope is given by:

$$M = m_0 M_e = -\frac{L}{f_0}\frac{x_{np}}{f_e}$$

Substitute numerical values and evaluate M:

$$M = -\frac{16\,\text{cm}}{7.5\,\text{cm}}\frac{25\,\text{cm}}{2.5\,\text{cm}} = \boxed{-21.3}$$

(*b*) A ray diagram showing how rays from a near-by object are magnified by a compound microscope follows. A real and inverted image of the near-by object is formed by the objective lens at the first focal point of the eyepiece lens. The eyepiece lens forms an inverted and virtual image of this image at infinity.



***110 ••**

Picture the Problem We can use the equation for refraction at a single surface to locate the image of the fish and the expression for the magnification due to refraction at a spherical surface to find the magnification of the image.

(*a*) Use the equation describing refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r}$$

Solve for s':

$$s'=\frac{n_2 rs}{(n_2-n_1)s-n_1 r}$$

Substitute numerical values and evaluate s':

$$s'=\frac{(1)(0.5\,\text{m})(2.5\,\text{m})}{(1-1.33)(2.5\,\text{m})-(1.33)(0.5\,\text{m})}=\boxed{-0.839\,\text{m}}$$

Note that the fish appears to be much closer to the diver than it actually is.

(*b*) Express the magnification due to refraction at a spherical surface:

$$m=-\frac{n_1 s'}{n_2 s}$$

Substitute numerical values and evaluate m:

$$m=-\frac{(1.33)(-0.839\,\text{m})}{(1)(2.5\,\text{m})}=\boxed{0.446}$$

Note that the fish appears to be smaller than it actually is.

111 ••

Picture the Problem We can use the thin-lens equation and the definition of the magnification of an image to determine where the person should stand.

Use the thin-lens equation to relate s and s':

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

The magnification of the image is given by:

$$m=-\frac{s'}{s}=-\frac{2.4\,\text{cm}}{175\,\text{cm}}=-1.37\times10^{-2}$$

and

$$s'=-ms$$

Substitute to obtain:

$$\frac{1}{s}-\frac{1}{ms}=\frac{1}{f}$$

Solve for *s*:

$$s=\left(1-\frac{1}{m}\right)f$$

Substitute numerical values and evaluate *s*:

$$s=\left(1-\frac{1}{-1.37\times10^{-2}}\right)(50\,\text{mm})$$
$$=\boxed{3.70\,\text{m}}$$

112 ••

Picture the Problem We can use the thin-lens equation and the definition of the magnification of an image to determine the ideal focal length of the lens.

Use the thin-lens equation to relate *s* and *s*′:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

The magnification of the image is given by:

$$m=-\frac{s'}{s}=-\frac{3.6\,\text{cm}}{200\,\text{cm}}=-1.80\times10^{-2}$$

and

$$s'=-ms$$

Substitute to obtain:

$$\frac{1}{s}-\frac{1}{ms}=\frac{1}{f}$$

Solve for *f*:

$$f=\frac{s}{1-\frac{1}{m}}$$

Substitute numerical values and evaluate *f*:

$$f=\frac{30\,\text{m}}{1-\frac{1}{-1.80\times10^{-2}}}=\boxed{0.530\,\text{m}}$$

113 ••

Picture the Problem Let the numeral 1 refer to the first lens and the numeral 2 to the second lens. We apply the thin-lens equation twice; once to locate the image formed by the first lens and a second time to find the image formed by the second lens. The magnification of the image is the product of the magnifications produced by the two lenses.

(*a*) Solve the thin-lens equation for the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1}$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - 10\,\text{cm}} = 60.0\,\text{cm}$$

Because the second lens is 20 cm to the right of the first lens:

$$s_2 = 20\,\text{cm} - 60\,\text{cm} = -40\,\text{cm}$$

Solve the thin-lens equation for the location of the image formed by the second lens:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(12.5\,\text{cm})(-40\,\text{cm})}{-40\,\text{cm} - 12.5\,\text{cm}} = \boxed{9.52\,\text{cm}}$$

i.e., the final image is 9.52 cm to the right of the second lens.

(*b*) Express the magnification of the final image:

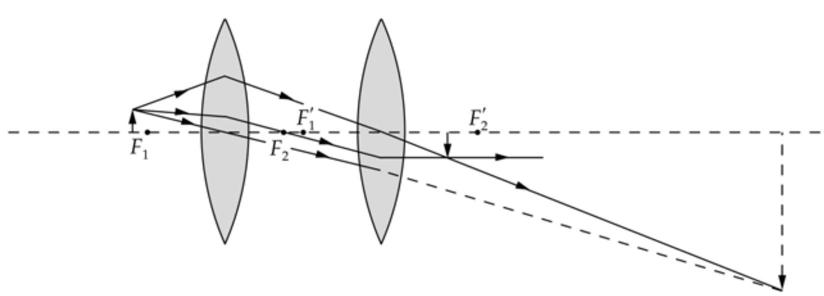
$$m = m_1 m_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \frac{s_1' s_2'}{s_1 s_2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{(60\,\text{cm})(9.52\,\text{cm})}{(12\,\text{cm})(-40\,\text{cm})} = \boxed{-1.19}$$

i.e., the final image is about 20% larger than the object and is inverted.

(*c*) The ray diagram is shown in the figure. The enlarged, inverted image formed by the first lens serves as a virtual object for the second lens. The image formed from this virtual object is the real, inverted image shown in the ray diagram.



114 ••

Picture the Problem We can apply the equation for refraction at a surface to both surfaces of the lens and add the resulting equations to obtain an equation relating the image and object distances to the indices of refraction. We can then use the lens maker's equation to complete the derivation of the given relationship between *f* ′ and *f*.

(*a*) Relate *s* and *s*′ at the water-lens interface:

$$\frac{n_w}{s}+\frac{n}{s_1'}=\frac{n-n_w}{r_1}$$

Relate *s* and *s*′ at the lens-water interface:

$$\frac{n}{-s_1'}+\frac{n}{s'}=\frac{n_w-n}{r_2}$$

Add these equations to obtain:

$$n_w\left(\frac{1}{s}+\frac{1}{s'}\right)=(n-n_w)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Let $\frac{1}{f'}=\frac{1}{s}+\frac{1}{s'}$ to obtain:

$$\frac{n_w}{f'}=(n-n_w)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

and

$$\frac{1}{r_1}-\frac{1}{r_2}=\frac{1}{(n-1)f}$$

Substitute to obtain:

$$\frac{n_w}{f'}=(n-n_w)\left(\frac{1}{(n-1)f}\right)$$

Solve for *f* ′:

$$f'=\boxed{\frac{n_w(n-1)}{n-n_w}f}$$

(*b*) Use the lens-maker's equation to find the focal length of the lens in air:

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{-30\,\text{cm}}-\frac{1}{35\,\text{cm}}\right)$$

and

$$f=\boxed{-32.3\,\text{cm}}$$

Use the result derived in (*a*) to find *f* ′:

$$f'=\frac{(1.33)(1.5-1)}{1.5-1.33}(-32.3\,\text{cm})$$

$$=\boxed{-126\,\text{cm}}$$

***115 ••**

Picture the Problem Here we must consider refraction at each surface separately. To find the focal length we imagine the object at $s = \infty$, and find the image from the first refracting surface at s'_1. That image serves as the object for the second refracting surface. We'll find that this is a virtual image for the second refracting surface, i.e., s_2 is negative. Using the equation for refraction at a single surface a second time, we can locate the image formed by the second refracting surface by the virtual object at s_2. The location of that image is then the focal point of the thick lens. We'll let the numeral 1 denote the first surface and the numeral 2 the second surface. In part (*b*) we can proceed as in part (*a*) (except that now $n_1 = 1.33$ for the first refraction and $n_2 = 1.33$ for the second refraction) to determine the focal length in water, which we denote by f_w.

(*a*) Use the equation for refraction at a single surface to relate s_1 and s_1':

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{r_1}$$

For $s_1 = \infty$:

$$\frac{n_2}{s_1'} = \frac{n_2 - n_1}{r_1}$$

Solve for s_1':

$$s_1' = \frac{n_2 r_1}{n_2 - n_1} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(1.5)(20\,\text{cm})}{1.5 - 1} = 60.0\,\text{cm}$$

The object distance s_2 for the second lens is:

$$s_2 = -(s_1' - 4\,\text{cm}) = -(60\,\text{cm} - 4\,\text{cm}) = -56\,\text{cm}$$

Solve the equation for refraction at a single surface for s_2':

$$s_2' = \frac{n_2 r_2 s_2}{(n_2 - n_1)s_2 - n_1 r_2} \qquad (2)$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(1)(-20\,\text{cm})(-56\,\text{cm})}{(1 - 1.5)(-56\,\text{cm}) - (1.5)(-20\,\text{cm})} = 19.3\,\text{cm}$$

Because f is measured from the center of the lens:

$$f = s_2' + 2\,\text{cm} = 19.3\,\text{cm} + 2\,\text{cm} = \boxed{21.3\,\text{cm}}$$

(*b*) Substitute numerical values in equation (1) and evaluate s_1':

$$s_1' = \frac{(1.5)(20\,\text{cm})}{1.5 - 1.33} = 176\,\text{cm}$$

The object distance s_2 for the second lens is:

$$s_2 = -(s_1' - 4\,\text{cm}) = -(176\,\text{cm} - 4\,\text{cm}) = -172\,\text{cm}$$

Substitute numerical values in equation (2) and evaluate s'_2:

$$s_2' = \frac{(1.33)(-20\,\text{cm})(-172\,\text{cm})}{(1.33 - 1.5)(-172\,\text{cm}) - (1.5)(-20\,\text{cm})} = 77.2\,\text{cm}$$

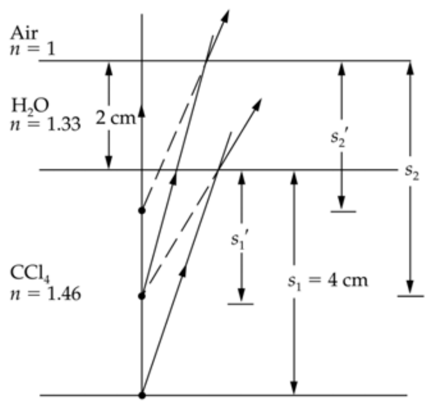
Because f_w is measured from the center of the lens:

$$f_w = s_2' + 2\,\text{cm} = 77.2\,\text{cm} + 2\,\text{cm} = \boxed{79.2\,\text{cm}}$$

Remarks: Note that if we use the expression given in Problem 114 we obtain f_w = 83.3 cm, in only moderate agreement with the exact result given above.

116 ••

Picture the Problem Let the numeral 1 denote the CCl_4-H_2O interface and the numeral 2 the H_2O-air interface. We can locate the final image by applying the equation for refraction at a single surface to both interfaces. The ray diagram shown below shows a spot at the bottom of the tank and the rays of light emanating from it that form the intermediate and final images.



Use the equation for refraction at a single surface to relate s and s' at the CCl_4-H_2O interface:

$$\frac{n_{CCl_4}}{s_1} + \frac{n_{H_2O}}{s_1'} = \frac{n_{H_2O} - n_{CCl_4}}{r}$$

or, because $r = \infty$,

$$\frac{n_{CCl_4}}{s_1} + \frac{n_{H_2O}}{s_1'} = 0$$

Solve for s_1':

$$s_1' = -\frac{n_{H_2O}s_1}{n_{CCl_4}} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = -\frac{(1.33)(4\,\text{cm})}{1.46} = -3.64\,\text{cm}$$

The depth of this image, as viewed from the H_2O-air interface is:

$$s_2 = 2\,\text{cm} - s_1' = 2\,\text{cm} - (-3.64\,\text{cm}) = 5.64\,\text{cm}$$

At the H_2O-air interface equation (1) becomes:

$$s_2' = -\frac{n_{\text{air}} s_2}{n_{H_2O}}$$

Substitute numerical values and evaluate s_2':

$$s_2' = -\frac{(1)(5.64\,\text{cm})}{1.33} = -4.24\,\text{cm}$$

The apparent depth is 4.24 cm.

117 ••

Picture the Problem The speed of the jogger as seen in the mirror is $v' = ds'/dt$. We can use the mirror equation to derive an expression for v' in terms of f and ds/dt.

Solve the mirror equation for s':

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} \quad (1)$$

Differentiate s' with respect to time to obtain:

$$v' = \frac{ds'}{dt} = \frac{d}{dt}\left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = -\left(\frac{1}{f} - \frac{1}{s}\right)^{-2}\left(\frac{1}{s^2}\right)\frac{ds}{dt}$$

Simplify this result to obtain:

$$v' = -\left(\frac{s'}{s}\right)^2 v \quad (2)$$

Rewrite equation (1) in terms of r:

$$s' = \left(\frac{2}{r} - \frac{1}{s}\right)^{-1}$$

Find s' when s = 5 m:

$$s' = \left(\frac{2}{-2\,\text{m}} - \frac{1}{5\,\text{m}}\right)^{-1} = -0.833\,\text{m}$$

Use equation (2) to find $|v'|$ when $|v|$ = 3.5 m/s:

$$|v'| = \left|-\left(\frac{-0.833\,\text{m}}{5\,\text{m}}\right)^2 (3.5\,\text{m/s})\right| = \boxed{0.0971\,\text{m/s}}$$

118 ••

Picture the Problem Let the numerals 1 and 2 denote to the first and second refracting surfaces of the spherical lens, respectively, and follow the steps given in the hint.

Use the equation for refraction at a single surface to relate s_1 and s_1':

$$\frac{n_1}{s_1}+\frac{n_2}{s_1'}=\frac{n_2-n_1}{r_1}$$

When $s_1 = \infty$:

$$\frac{n_2}{s_1'}=\frac{n_2-n_1}{r_1}$$

Solve for s_1':

$$s_1'=\frac{n_2 r_1}{n_2-n_1}$$

Substitute numerical values and evaluate s_1':

$$s_1'=\frac{(1.5)(2\text{ mm})}{1.5-1}=6.00\text{ mm}$$

Because the thickness of the glass sphere is 4 mm:

$$s_2=4\text{ mm}-s_1'=4\text{ mm}-6\text{ mm}=-2\text{ mm}$$

Use the equation for refraction at a single surface to relate s_2 and s_2':

$$\frac{n_2}{s_2}+\frac{n_1}{s_2'}=\frac{n_1-n_2}{r_2}$$

Solve for s_2':

$$s_2'=\frac{n_1 r_2 s_2}{(n_1-n_2)s_2-n_2 r_2}$$

Substitute numerical values and evaluate s_2':

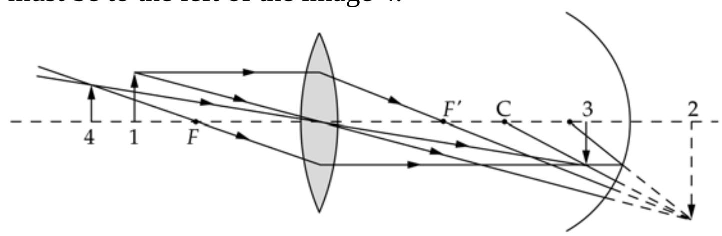
$$s_2'=\frac{(1)(-2\text{ mm})(-2\text{ mm})}{(1-1.5)(-2\text{ mm})-[1.5](-2\text{ mm})}=1.00\text{ mm}$$

Because $s_2'=1.00\text{ mm}=r/2,\ f=1.00\text{ mm}.$

119 •••

Picture the Problem We can use the thin-lens equation to locate the first image formed by the lens, the mirror equation to locate the image formed in the mirror, and the thin-lens equation a second time to locate the final image formed by the lens as the rays pass back through it.

(*b*) and (*c*) The ray diagram is shown below. The numeral 1 represents the object. The parallel and central rays from 1 are shown; one passes through the center of the lens, the other is paraxial and then passes through the focal point *F*′. The two rays intersect behind the mirror, and the image formed there, identified by the numeral 2, serves as a virtual object for the mirror. Two rays are shown emanating from this virtual image, one through the center of the mirror, the other passing through its focal point (halfway between *C* and the mirror surface) and then continuing as a paraxial ray. These two rays intersect in front of the mirror, forming a real image, identified by the numeral 3. Finally, the image 3 serves as a real object for the lens; again we show two rays, a paraxial ray that then passes through the focal point *F* and a ray through the center of the lens. These two rays intersect to form the final real, upright, and diminished image, identified as 4. To see this image the eye must be to the left of the image 4.



(*a*) Solve the thin-lens equation for s_1':

$$s_1' = \frac{fs_1}{s_1 - f}$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 10\,\text{cm}} = 30\,\text{cm}$$

Because the image formed by the lens is behind the mirror:

$$s_2 = 25\,\text{cm} - 30\,\text{cm} = -5\,\text{cm}$$

Solve the mirror equation for s_2':

$$s_2' = \frac{fs_2}{s_2 - f}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(5\,\text{cm})(-5\,\text{cm})}{-5\,\text{cm} - 5\,\text{cm}} = 2.50\,\text{cm}$$ and the image is 22.5 cm from the lens; i.e., s_3 = 22.5 cm.

Solve the thin-lens equation for s_3':

$$s_3' = \frac{fs_3}{s_3 - f}$$

Substitute numerical values and evaluate s_3':

$$s_3' = \frac{(10\,\text{cm})(22.5\,\text{cm})}{22.5\,\text{cm} - 10\,\text{cm}} = \boxed{18.0\,\text{cm}}$$

***120** •••

Picture the Problem The mirror surfaces must be concave to create inverted images on reflection. Therefore, the lens is a diverging lens. Let the numeral 1 denote the lens in its initial orientation and the numeral 2 the lens in its second orientation. We can use the mirror equation to find the magnitudes of the radii of the lens' surfaces, the thin-lens equation to find its focal length, and the lens maker's equation to find its index of refraction.

Solve the mirror equation for $|r_1|$:

$$|r_1| = \frac{2s_1 s_1'}{s_1' + s_1}$$

Substitute numerical values and evaluate $|r_1|$:

$$|r_1| = \frac{2(30\,\text{cm})(6\,\text{cm})}{6\,\text{cm} + 30\,\text{cm}} = 10.0\,\text{cm}$$

Solve the mirror equation for $|r_2|$:

$$|r_2| = \frac{2s_2 s_2'}{s_2' + s_2}$$

Substitute numerical values and evaluate $|r_2|$:

$$|r_2| = \frac{2(30\,\text{cm})(10\,\text{cm})}{10\,\text{cm} + 30\,\text{cm}} = 15.0\,\text{cm}$$

Solve the thin-lens equation for *f*:

$$f = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{(30\,\text{cm})(-7.5\,\text{cm})}{-7.5\,\text{cm} + 30\,\text{cm}} = -10.0\,\text{cm}$$

Solve the lens-maker's equation for *n* to obtain:

$$n = \frac{1}{f\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + 1$$

Because the lens is a diverging lens, $r_1 = -10$ cm and $r_2 = 15$ cm. Substitute numerical values and evaluate *n*:

$$n = \frac{1}{(-10\,\text{cm})\left(\frac{1}{-10\,\text{cm}} - \frac{1}{15\,\text{cm}}\right)} + 1$$

$$= \boxed{1.60}$$

121 •••

Picture the Problem Assume that the object is very small compared to *r* so that all incident and reflected rays traverse 1 cm of water. The problem involves two refractions at the air–water interface and one reflection at the mirror. Let the numeral 1 refer to the first refraction at the air–water interface, the numeral 2 to the reflection in the mirror surface, and the numeral 3 to the second refraction at the water–air interface.

Use the equation for refraction at a single surface to relate s_1 and s_1':

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{r}$$

or, because $r = \infty$,

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = 0$$

Solve for s_1':

$$s_1' = -\frac{n_2 s_1}{n_1}$$

Let $n_2 = n$. Because $n_1 = 1$:

$$s_1' = -ns_1$$

Find the object distance for the mirror:

$$s_2 = 1 - s_1' = 1 + ns_1$$

where 1 has units of cm.

Solve the mirror equation for s_2':

$$s_2' = \left(\frac{2}{r} - \frac{1}{s_2}\right)^{-1}$$

Substitute for s_2:

$$s_2' = \left(\frac{2}{r} - \frac{1}{1+ns_1}\right)^{-1}$$

Find the object distance s_3 for the water–air interface:

$$s_3 = 1 - s_2' = 1 - \left(\frac{2}{r} - \frac{1}{1+ns_1}\right)^{-1}$$

Use the equation for refraction at a single surface to relate s_3 and s_3':

$$\frac{n_1}{s_3} + \frac{n_2}{s_3'} = \frac{n_2 - n_1}{r}$$

or, because $r = \infty$,

$$\frac{n_1}{s_3} + \frac{n_2}{s_3'} = 0$$

Solve for s_3':

$$s_3' = -\frac{n_2 s_3}{n_1}$$

Because $n_2 = 1$ and $n_1 = n$:

$$s_3' = -\frac{s_3}{n} = -\frac{1 - \left(\frac{2}{r} - \frac{1}{1+ns_1}\right)^{-1}}{n}$$

Equate s_3' and s_1:

$$s_1 = -\frac{1 - \left(\frac{2}{r} - \frac{1}{1+ns_1}\right)^{-1}}{n}$$

Simplify to obtain:

$$s_1^2 + \frac{2-r}{n}s_1 + \frac{1-r}{n^2} = 0$$

Substitute numerical values and simplify:

$$s_1^2 + \frac{2\,\text{cm} - 50\,\text{cm}}{1.33} s_1 + \frac{1\,\text{cm} - 50\,\text{cm}}{(1.33)^2} = 0$$

or

$$s_1^2 - 36.09 s_1 - 27.70 = 0$$

where s_1 is in cm.

Solve for the positive value of s_1:

$$s_1 = \boxed{36.8\,\text{cm}}$$

122 •••

Picture the Problem We can use the lens maker's equation, in conjunction with the result given in Problem 114, to find the index of refraction of the liquid.

Solve the lens-maker's equation for n:

$$n = \frac{1}{f\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + 1$$

Substitute numerical values and evaluate n:

$$n = \frac{1}{(27.5\,\text{cm})\left(\frac{1}{-17\,\text{cm}} - \frac{1}{-8\,\text{cm}}\right)} + 1$$

$$= 1.55$$

From Problem 114, the focal length of the lens in the liquid, f_L, is related to the focal length of the lens in air, f, according to:

$$f_L = \frac{n_L(n-1)}{n - n_L} f$$

Solve for n_L:

$$n_L = \frac{n f_L}{(n-1)f + f_L}$$

Substitute numerical values and evaluate n_L:

$$n_L = \frac{(1.55)(109\,\text{cm})}{(1.55-1)(27.5\,\text{cm}) + 109\,\text{cm}}$$

$$= \boxed{1.36}$$

123 •••

Picture the Problem The problem involves two refractions and one reflection. We can use the refraction at spherical surface equation and the mirror equation to find the images formed in the two refractions and one reflection. Let the numeral 1 refer to the first refraction at the air-glass interface, the numeral 2 to the reflection from the silvered surface, and the numeral 3 refer to the refraction at the glass-air interface.

(*a*) The image and object distances for the first refraction are related according to:

$$\frac{n_1}{s_1}+\frac{n_2}{s_1'}=\frac{n_2-n_1}{r}$$

Solve for s_1' to obtain:

$$s_1'=\frac{n_2 r s_1}{(n_2-n_1)s_1-n_1 r}$$

Substitute numerical values and evaluate s_1':

$$s_1'=\frac{(1.5)(10\,\text{cm})(30\,\text{cm})}{(1.5-1)(30\,\text{cm})-(1)(10\,\text{cm})}=90.0\,\text{cm}$$

The object for the mirror surface is behind the mirror and its distance from the surface of the mirror is:

$$s_2=20\,\text{cm}-90\,\text{cm}=-70\,\text{cm}$$

Use the mirror equation to relate s_2 and s_2':

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{2}{r}$$

Solve for s_2':

$$s_2'=\frac{r s_2}{2s_2-r}$$

Substitute numerical values and evaluate s_2':

$$s_2'=\frac{(10\,\text{cm})(-70\,\text{cm})}{2(-70\,\text{cm})-10\,\text{cm}}=4.67\,\text{cm}$$

The object for the second refraction at the glass-air interface is in front of the mirrored surface and its distance from the glass-air interface is:

$$s_3=20\,\text{cm}-4.67\,\text{cm}=15.3\,\text{cm}$$

The image and object distances for the second refraction are related according to:

$$\frac{n_1}{s_3}+\frac{n_2}{s_3'}=\frac{n_2-n_1}{r}$$

Solve for s_3' to obtain:

$$s_3'=\frac{n_2 r s_3}{(n_2-n_1)s_3-n_1 r}$$

Noting that $r = -10$ cm, substitute numerical values and evaluate s_3':

$$s_3'=\frac{(1)(-10\,\text{cm})(15.3\,\text{cm})}{(1-1.5)(15.3\,\text{cm})-(1.5)(-10\,\text{cm})}=-20.8\,\text{cm}$$

The final image is $-20.8\text{ cm} + 20\text{ cm} = 0.8\text{ cm}$ behind the mirror surface.

(*b*) Proceed as in (*a*) with $s_1 = 20\text{ cm}$ to obtain $s_3 = -20\text{ cm}$ and the final image to be at the mirror surface.

124 •••

Picture the Problem We can solve the lens maker's equation for *f* and then differentiate with respect to *n* and simplify to obtain $df/f = -dn/(n-1)$.

(*a*) The lens maker's equation is:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{C}(n-1)$$

where $\frac{1}{C} = \frac{1}{r_1} - \frac{1}{r_2}$

Solve for *f*:

$$f = C(n-1)^{-1}$$

Differentiate *f* with respect to *n* and simplify:

$$\frac{df}{dn} = \frac{d}{dn}\left[C(n-1)^{-1}\right] = -C(n-1)^{-2} = -\frac{f(n-1)^{-2}}{(n-1)^{-1}} = -\frac{f}{n-1}$$

Solve for *df*/*f* :

$$\frac{df}{f} = \boxed{-\frac{dn}{n-1}}$$

(*b*) Express the focal length for blue light in terms of the focal length for red light:

$$f_{\text{blue}} = f_{\text{red}} + \Delta f \qquad (1)$$

Approximate *df*/*f* by Δ*f*/*f* and *dn* by Δ*n* to obtain:

$$\frac{\Delta f}{f} \approx -\frac{\Delta n}{n-1}$$

Solve for Δ*f* :

$$\Delta f = -\frac{f\,\Delta n}{n-1}$$

Substitute for Δ*f* in equation (1) to obtain:

$$f_{\text{blue}} = f_{\text{red}} - \frac{f_{\text{red}}\,\Delta n}{n-1} = f_{\text{red}}\left(1 - \frac{\Delta n}{n_{\text{red}} - 1}\right)$$

Substitute numerical values and evaluate f_{blue}:

$$f_{\text{blue}} = (20\,\text{cm})\left(1 - \frac{1.53 - 1.47}{1.47 - 1}\right) = \boxed{17.4\,\text{cm}}$$

***125 •••**

Picture the Problem We examine the amount by which the image distance *s*′ changes due to a change in *s*.

Solve the thin-lens equation for *s*′:

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1}$$

Differentiate *s*′ with respect to *s*:

$$\frac{ds'}{ds} = \frac{d}{ds}\left[\left(\frac{1}{f} - \frac{1}{s}\right)^{-1}\right] = -\frac{1}{\left(\frac{1}{f} - \frac{1}{s}\right)^{2}}\frac{1}{s^{2}} = -\frac{s'^{2}}{s^{2}} = -m^{2}$$

$$\boxed{\text{The image of an object of length } \Delta s \text{ will have a length } -m^{2}\Delta s.}$$

Chapter 33
Interference and Diffraction

Conceptual Problems

***1 •**

Determine the Concept The energy is distributed nonuniformly in space; in some regions the energy is below average (destructive interference), in others it is higher than average (constructive interference).

2 •

Determine the Concept Coherent sources have a constant phase difference. The pairs of light sources that satisfy this criterion are (*b*), (*c*), and (*e*).

3 •

Determine the Concept The thickness of the air space between the flat glass and the lens is approximately proportional to the square of *d*, the diameter of the ring. Consequently, the separation between adjacent rings is proportional to $1/d$.

4 ••

Determine the Concept The distance between adjacent fringes is so small that the fringes are not resolved by the eye.

5 ••

Determine the Concept If the film is thick, the various colors (i.e., different wavelengths) will give constructive and destructive interference at that thickness. Consequently, what one observes is the reflected intensity of white light.

***6 •**

(*a*) The phase change on reflection from the front surface of the film is 180°; the phase change on reflection from the back surface of the film is 0°. As the film thins toward the top, the phase change associated with the film's thickness becomes negligible and the two reflected waves interfere destructively.

(*b*) The first constructive interference will arise when $t = \lambda/4$. Therefore, the first band will be violet (shortest visible wavelength).

(*c*) When viewed in transmitted light, the top of the film is white, since no light is reflected. The colors of the bands are those complementary to the colors seen in reflected light; i.e., the top band will be red.

7 •

Determine the Concept The first zeroes in the intensity occur at angles given by $\sin\theta = \lambda/a$. Hence, decreasing *a* increases θ and the diffraction pattern becomes wider.

8 •

Determine the Concept Equation 33-2 expresses the condition for an intensity maximum i slit interference. Here *d* is the slit separation, λ the wavelength of the light, *m* an integer, a the angle at which the interference maximum appears.

Equation 33-11 expresses the condition for the first minimum in single-slit diffraction. Here *a* is the width of the slit, λ the wavelength of the light, and θ the angle at which the first minimum appears, assuming $m = 1$.

9 •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for θ with $m = 1$ to express the location of the first-order maximum as a function of the wavelength of the light.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where *d* is the separation of the slits and $m = 0, 1, 2, \ldots$

Solve for the angular location θ_1 of the first-order maximum :

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

Because $\lambda_{\text{green light}} < \lambda_{\text{red light}}$:

$\theta_{\text{green light}} < \theta_{\text{red light}}$ and $\boxed{(a)\text{ is correct.}}$

***10** •

Determine the Concept The distance on the screen to *m*th bright fringe is given by $y_m = m\dfrac{\lambda L}{d}$, where *L* is the distance from the slits to the screen and *d* is the separation of the slits. Because the index of refraction of air is slightly larger than the index of refraction of a vacuum, the introduction of air reduces λ to λ/n and decreases y_m. Because the separation of the fringes is $y_m - y_{m-1}$, the separation of the fringes decreases and $\boxed{(b)\text{ is correct.}}$

11 •

(*a*) False. When destructive interference of light waves occurs, the energy is no longer distributed evenly. For example, light from a two-slit device forms a pattern with very bright and very dark parts. There is practically no energy at the dark fringes and a great deal of energy at the bright fringe. The total energy over the entire pattern equals the energy from one slit plus the energy from the second slit. Interference re-distributes the energy.

(*b*) True

(*c*) True

(*d*) True

(*e*) True

Estimation and Approximation

***12** •

Picture the Problem We'll assume that the diameter of the pupil of the eye is 5 mm and that the wavelength of light is 600 nm. Then we can use the expression for the minimum angular separation of two objects than can be resolved by the eye and the relationship between this angle and the width of an object and the distance from which it is viewed to support the claim.

Relate the width w of an object that can be seen at a height h to the critical angular separation α_c:

$$\tan\alpha_c = \frac{w}{h}$$

Solve for w:

$$w = h\tan\alpha_c$$

The minimum angular separation α_c of two point objects that can just be resolved by an eye depends on the diameter D of the eye and the wavelength λ of light:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Substitute for α_c in the expression for w to obtain:

$$w = h\tan\left(1.22\frac{\lambda}{D}\right)$$

In low-earth orbit:

$$w = (400\,\text{km})\tan\left(1.22\frac{600\,\text{nm}}{5\,\text{mm}}\right) = 58.6\,\text{m}$$

Because the width of the Great Wall is about 5 m, a naked eye would not be able to see it from the moon.

At a distance equal to that of the distance of the moon from earth:

$$w = (3.84\times10^{8}\ \text{m})\tan\left(1.22\frac{600\,\text{nm}}{5\,\text{mm}}\right) = 56.2\,\text{km}$$

Because the width of the Great Wall is about 5 m, a naked eye would not be able to see it from the moon.

13 •

Picture the Problem We can use $\sin\theta = 1.22\frac{\lambda}{D}$ to relate the diameter D of the opaque-disk water droplets to the angular diameter θ of a coronal ring and to the wavelength of light. We'll assume a wavelength of 500 nm.

The angle θ subtended by the first diffraction minimum is related to the wavelength λ of light and the diameter D of the opaque-disk water droplet:

$$\sin\theta = 1.22\frac{\lambda}{D}$$

Because of the great distance to the cloud of water droplets, $\theta << 1$ and:

$$\theta \approx 1.22\frac{\lambda}{D}$$

Solve for D to obtain:

$$D = \frac{1.22\lambda}{\theta}$$

Substitute numerical values and evaluate D:

$$D = \frac{1.22(500\,\text{nm})}{10°\times\frac{\pi\ \text{rad}}{180°}} = \boxed{3.50\ \mu\text{m}}$$

14 •

Picture the Problem We can use $\sin\theta = 1.22\frac{\lambda_n}{D}$ to relate the diameter D of a microsphere to the angular diameter θ of a coronal ring and to the wavelength of light in water.

The angle θ subtended by the first diffraction minimum is related to the wavelength λ_n of light in water and the diameter D of the microspheres:

$$\sin\theta = 1.22\frac{\lambda_n}{D} = 1.22\frac{\lambda}{nD}$$

Because $\theta << 1$:

$$\theta \approx 1.22\frac{\lambda}{nD}$$

Substitute numerical values and evaluate θ:

$$\theta \approx \frac{1.22(632.8\,\text{nm})}{1.33(5\,\mu\text{m})} = 0.116\,\text{rad} = \boxed{6.65°}$$

15 •

Picture the Problem We can use $\sin\theta = 1.22\dfrac{\lambda}{D}$ to relate the diameter D of a pollen grain to the angular diameter θ of a coronal ring and to the wavelength of light. We'll assume a wavelength of 450 nm for blue light and 650 nm for red light.

The angle θ subtended by the first diffraction minimum is related to the wavelength λ of light and to the diameter D of the microspheres:

$$\sin\theta = 1.22\frac{\lambda}{D}$$

Because $\theta << 1$:

$$\theta \approx 1.22\frac{\lambda}{nD}$$

Substitute numerical values and evaluate θ for red light:

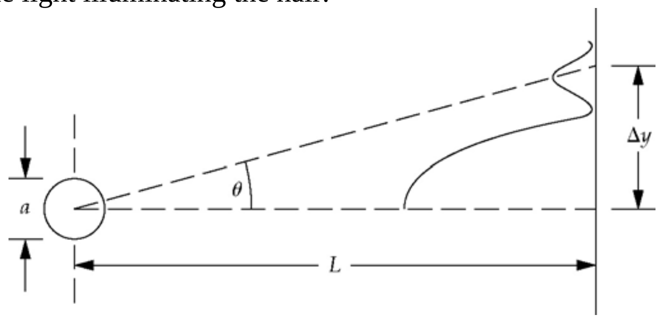
$$\theta_{\text{red}} \approx \frac{1.22(650\,\text{nm})}{25\,\mu\text{m}} = 3.17\times10^{-2}\,\text{rad} = \boxed{1.82°}$$

Substitute numerical values and evaluate θ for blue light:

$$\theta_{\text{blue}} \approx \frac{1.22(450\,\text{nm})}{25\,\mu\text{m}} = 2.20\times10^{-2}\,\text{rad} = \boxed{1.26°}$$

***16** ••

Picture the Problem The diagram shows the hair whose diameter $d = a$, the screen a distance L from the hair, and the separation Δy of the first diffraction peak from the center. We can use the geometry of the experiment to relate Δy to L and a and the condition for diffraction maxima to express θ in terms of the diameter of the hair and the wavelength of the light illuminating the hair.



Relate θ to Δy:

$$\tan\theta = \frac{\Delta y}{L}$$

Solve for Δy: $\Delta y = L\tan\theta$

Diffraction maxima occur where: $a\sin\theta = \left(m+\frac{1}{2}\right)\lambda$ where m = 1, 2, 3, …

Solve for θ to obtain: $\theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{a}\right]$

Substitute for θ in the expression for Δy to obtain: $\Delta y = L\tan\left\{\sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{a}\right]\right\}$

For the first peak, m = 1. Substitute numerical values and evaluate Δy:

$$\Delta y = (10\,\text{m})\tan\left\{\sin^{-1}\left[\frac{\left(1+\frac{1}{2}\right)(632.8\,\text{nm})}{70\,\mu\text{m}}\right]\right\} = \boxed{13.6\,\text{cm}}$$

Phase Difference and Coherence

17 •

Picture the Problem A path difference Δr contributes a difference δ given by $\delta = \frac{\Delta r}{\lambda}360°$.

(*a*) Relate a path difference Δr to a phase shift δ: $\delta = \frac{\Delta r}{\lambda}360°$ (1)

Solve for Δr: $\Delta r = \frac{\delta\lambda}{360°}$

Substitute numerical values and evaluate Δr: $\Delta r = \frac{(180°)(600\,\text{nm})}{360°} = \boxed{300\,\text{nm}}$

(*b*) Substitute numerical values in equation (1) and evaluate δ: $\delta = \frac{300\,\text{nm}}{800\,\text{nm}}360° = \boxed{135°}$

18 •

Picture the Problem The wavelength of light in a medium whose index of refraction is n is the ratio of the wavelength of the light in air divided by n. The number of wavelengths of light contained in a given distance is the ratio of the distance to the wavelength of light in the given medium. The difference in phase between the two waves is the sum of a π phase shift in the reflected wave and a phase shift due to the additional distance traveled by the wave reflected from the bottom of the water–air interface.

(*a*) Express the wavelength of light in water in terms of the wavelength of light in air:

$$\lambda_n = \frac{\lambda}{n} = \frac{500\,\text{nm}}{1.33} = \boxed{376\,\text{nm}}$$

(*b*) Relate the number of wavelengths *N* to the thickness *t* of the film and the wavelength of light in water:

$$N = \frac{2t}{\lambda_n} = \frac{2\times10^{-4}\,\text{cm}}{376\,\text{nm}} = \boxed{5.32}$$

(*c*) Express the phase difference as the sum of the phase shift due to reflection and the phase shift due to the additional distance traveled by the wave reflected from the bottom of the water–air interface:

$$\delta = \delta_{\text{reflection}} + \delta_{\text{additional distance traveled}}$$
$$= \pi + \frac{2t}{\lambda_n}2\pi = \pi + 2\pi N$$

Substitute for *N* and evaluate δ:

$$\delta = \pi\,\text{rad} + 2\pi(5.32\,\text{rad}) = \boxed{11.6\pi\,\text{rad}}$$

or, subtracting 11.6π rad from 12π rad,
$\delta = \boxed{0.4\pi\,\text{rad}}$

***19 ••**

Picture the Problem The difference in phase depends on the path difference according to $\delta = \frac{\Delta r}{\lambda}360°$. The path difference is the difference in the distances of (0, 15 cm) and (3 cm, 14 cm) from the origin.

Relate a path difference Δr to a phase shift δ:

$$\delta = \frac{\Delta r}{\lambda}360°$$

The path difference Δr is:

$$\Delta r = 15\,\text{cm} - \sqrt{(3\,\text{cm})^2 + (14\,\text{cm})^2}$$
$$= 0.682\,\text{cm}$$

Substitute numerical values and evaluate δ:

$$\delta = \frac{0.682\,\text{cm}}{1.5\,\text{cm}}360° = \boxed{164°}$$

Interference in Thin Films

20 •

Picture the Problem Because the mth fringe occurs when the path difference $2t$ equals m wavelengths, we can express the additional distance traveled by the light in air as an $m\lambda$. The thickness of the wedge, in turn, is related to the angle of the wedge and the distance from its vertex to the mth fringe.

(*a*) The first band is dark because the phase difference due to reflection by the back surface of the top plate and the top surface of the bottom plate is 180°

(*b*) The mth fringe occurs when the path difference $2t$ equals m wavelengths:

$$2t = m\lambda$$

Relate the thickness of the air wedge to the angle of the wedge:

$$\theta = \frac{t}{x} \Rightarrow t = x\theta$$

where we've used a small-angle approximation to replace an arc length by the length of a chord.

Substitute to obtain:

$$2x\theta = m\lambda$$

Solve for θ:

$$\theta = \frac{m\lambda}{2x} = \frac{1}{2}\frac{m}{x}\lambda$$

Substitute numerical values and evaluate θ:

$$\theta = \frac{1}{2}\left(\frac{5}{\text{cm}}\right)(700\,\text{nm}) = \boxed{1.75\times10^{-4}\,\text{rad}}$$

***21 ••**

Picture the Problem The condition that one sees m fringes requires that the path difference between light reflected from the bottom surface of the top slide and the top surface of the bottom slide is an integer multiple of a wavelength of the light.

The mth fringe occurs when the path difference $2d$ equals m wavelengths:

$$2d = m\lambda \Rightarrow d = \frac{m\lambda}{2}$$

Because the nineteenth (but not the twentieth) bright fringe can be seen,

$$\left(m - \tfrac{1}{2}\right)\frac{\lambda}{2} < d < \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2}$$

the limits on d must be: where $m = 19$

Substitute numerical values to obtain:

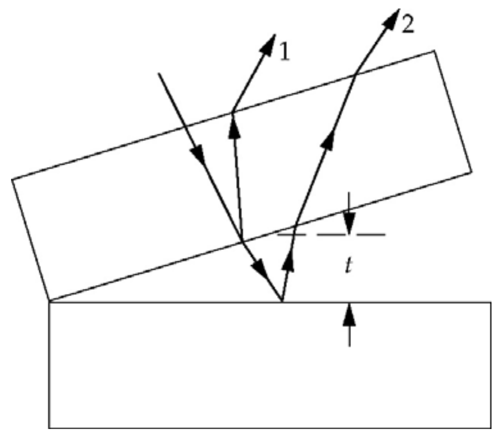
$$\left(19-\tfrac{1}{2}\right)\frac{590\,\text{nm}}{2} < d < \left(19+\tfrac{1}{2}\right)\frac{590\,\text{nm}}{2}$$

or

$$\boxed{5.46\,\mu\text{m} < d < 5.75\,\mu\text{m}}$$

22 ••

Picture the Problem The light reflected from the top surface of the bottom plate (wave 2 in the diagram) is phase shifted relative to the light reflected from the bottom surface of the top plate (wave 1 in the diagram). This phase difference is the sum of a phase shift of π (equivalent to a $\lambda/2$ path difference) resulting from reflection plus a phase shift due to the additional distance traveled.



Relate the extra distance traveled by wave 2 to the distance equivalent to the phase change due to reflection and to the condition for constructive interference:

$$2t + \tfrac{1}{2}\lambda = \lambda, 2\lambda, 3\lambda, \ldots$$

or

$$2t = \tfrac{1}{2}\lambda, \tfrac{3}{2}\lambda, \tfrac{5}{2}\lambda, \ldots$$

and

$2t = \left(m+\tfrac{1}{2}\right)\lambda$ where m = 0, 1, 2, … and λ is the wavelength of light in air.

Solve for m:

$$m = \frac{2t}{\lambda} - \frac{1}{2} = \frac{2(2r)}{\lambda} - \frac{1}{2} = \frac{4r}{\lambda} - \frac{1}{2}$$

where r is the radius of the wire.

Substitute numerical values and evaluate m:

$$m = \frac{4(0.025\,\text{mm})}{600\,\text{nm}} - \frac{1}{2} = \boxed{166}$$

23 ••

Picture the Problem We can use the condition for destructive interference in a thin film to find its thickness. Once we've found the thickness of the film, we can use the condition for constructive interference to find the wavelengths in the visible portion of the spectrum that will be brightest in the reflected interference pattern and the condition for destructive interference to find the wavelengths of light missing from the reflected light when the film is placed on glass with an index of refraction greater than that of the film.

(*a*) Express the condition for destructive interference in the thin film:

$$2t+\tfrac{1}{2}\lambda'=\tfrac{1}{2}\lambda',\tfrac{3}{2}\lambda',\tfrac{5}{2}\lambda',...$$

or

$$2t=\lambda',2\lambda',3\lambda',...$$

or

$$2t=m\lambda'=m\frac{\lambda}{n} \quad (1)$$

where m = 1, 2, 3, … and λ' is the wavelength of the light in the film.

Solve for λ:

$$\lambda=\frac{2nt}{m}$$

Substitute for the missing wavelengths to obtain:

$$450\,\text{nm}=\frac{2nt}{m} \text{ and } 360\,\text{nm}=\frac{2nt}{m+1}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{450\,\text{nm}}{360\,\text{nm}}=\frac{\frac{2nt}{m}}{\frac{2nt}{m+1}}=\frac{m+1}{m}$$

Solve for m:

$$m=4 \text{ for } \lambda=450\,\text{nm}$$

Solve equation (1) for t:

$$t=\frac{m\lambda}{2n}$$

Substitute numerical values and evaluate t:

$$t=\frac{4(450\,\text{nm})}{2(1.5)}=\boxed{600\,\text{nm}}$$

(*b*) Express the condition for constructive interference in the thin film:

$$2t+\tfrac{1}{2}\lambda'=\lambda',2\lambda',3\lambda',...$$

or

$$2t=\tfrac{1}{2}\lambda',\tfrac{3}{2}\lambda',\tfrac{5}{2}\lambda',...=\left(m+\tfrac{1}{2}\right)\lambda' \quad (1)$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' to obtain:

$$2t=\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n}$$

where n is the index of refraction of the film.

Solve for λ:

$$\lambda = \frac{2nt}{m+\frac{1}{2}}$$

Substitute numerical values and simplify to obtain:

$$\lambda = \frac{2(1.5)(600\text{ nm})}{m+\frac{1}{2}} = \frac{1800\text{ nm}}{m+\frac{1}{2}}$$

Substitute for *m* and evaluate λ to obtain the following table:

m	0	1	2	3	4	5
λ (nm)	3600	1200	720	514	400	327

From the table, we see that the only wavelengths in the visible spectrum are 720 nm, 514 nm, and 400 nm.

(*c*) Because the index of refraction of the glass is greater than that of the film, the light reflected from the film-glass interface will be shifted by $\frac{1}{2}\lambda$ (as is the wave reflected from the top surface) and the condition for destructive interference becomes:

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots$$

or

$$2t = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{n}$$

where *n* is the index of refraction of the film and *m* = 0, 1, 2, …

Solve for λ:

$$\lambda = \frac{2nt}{m+\frac{1}{2}}$$

Substitute numerical values and simplify to obtain:

$$\lambda = \frac{2(1.5)(600\text{ nm})}{m+\frac{1}{2}} = \frac{1800\text{ nm}}{m+\frac{1}{2}}$$

Substitute for *m* and evaluate λ to obtain the following table:

m	0	1	2	3	4	5
λ (nm)	3600	1200	720	514	400	327

From the table we see that the missing wavelengths in the visible spectrum are 720 nm, 514 nm, and 400 nm.

24 ••

Picture the Problem Because there is a $\frac{1}{2}\lambda$ phase change due to reflection at both the air-oil and oil-water interfaces, the condition for constructive interference is that twice the thickness of the oil film equal an integer multiple of the wavelength of light in the film.

Express the condition for constructive interference:

$$2t = \lambda', 2\lambda', 3\lambda', ...$$

or

$$2t = m\lambda' \quad (1)$$

where λ' is the wavelength of light in the oil = 1, 2, 3, …

Substitute for λ' to obtain:

$$2t = m\frac{\lambda}{n}$$

Solve for t:

$$t = \frac{m\lambda}{2n}$$

Substitute numerical values and evaluate t:

$$t = \frac{(2)(650\,\text{nm})}{2(1.22)} = \boxed{533\,\text{nm}}$$

25 ••

Picture the Problem Because there is a $\frac{1}{2}\lambda$ phase change due to reflection at both the air-oil and oil-glass interfaces, the condition for constructive interference is that twice the thickness of the oil film equal an integer multiple of the wavelength of light in the film.

Express the condition for constructive interference:

$$2t = \lambda', 2\lambda', 3\lambda', ... = m\lambda' \quad (1)$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' to obtain:

$$2t = m\frac{\lambda}{n}$$

where n is the index of refraction of the oil.

Solve for λ:

$$\lambda = \frac{2nt}{m}$$

Substitute for the predominant wavelengths to obtain:

$$690\,\text{nm} = \frac{2nt}{m} \text{ and } 460\,\text{nm} = \frac{2nt}{m+1}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{690\,\text{nm}}{460\,\text{nm}} = \frac{\frac{2nt}{m}}{\frac{2nt}{m+1}} = \frac{m+1}{m}$$

Solve for *m*:

$m = 2$ for $\lambda = 690\,\text{nm}$

Solve equation (1) for *t*:

$$t = \frac{m\lambda}{2n}$$

Substitute numerical values and evaluate *t*:

$$t = \frac{(2)(690\,\text{nm})}{2(1.45)} = \boxed{476\,\text{nm}}$$

***26 ••**

Picture the Problem Because the index of refraction of air is less than that of the oil, there is a phase shift of π rad ($\frac{1}{2}\lambda$) in the light reflected at the air-oil interface. Because the index of refraction of the oil is greater than that of the glass, there is no phase shift in the light reflected from the oil-glass interface. We can use the condition for constructive interference to determine *m* for λ = 700 nm and then use this value in our equation describing constructive interference to find the thickness *t* of the oil film.

Express the condition for constructive interference between the waves reflected from the air-oil interface and the oil-glass interface:

$$2t + \tfrac{1}{2}\lambda' = \lambda', 2\lambda', 3\lambda', \ldots$$

or

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots = \left(m + \tfrac{1}{2}\right)\lambda' \qquad (1)$$

where λ' is the wavelength of light in the oil and *m* = 0, 1, 2, …

Substitute for λ' and solve for λ to obtain:

$$\lambda = \frac{2nt}{m + \frac{1}{2}}$$

Substitute the predominant wavelengths to obtain:

$$700\,\text{nm} = \frac{2nt}{m + \frac{1}{2}} \text{ and } 500\,\text{nm} = \frac{2nt}{m + \frac{3}{2}}$$

Divide the first of these equations by the second to obtain:

$$\frac{700\,\text{nm}}{500\,\text{nm}} = \frac{\dfrac{2nt}{m + \frac{1}{2}}}{\dfrac{2nt}{m + \frac{3}{2}}} = \frac{m + \frac{3}{2}}{m + \frac{1}{2}}$$

Solve for *m*:

$m = 2$ for $\lambda = 700$ nm

Solve equation (1) for *t*:

$$t = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2n}$$

Substitute numerical values and evaluate *t*:

$$t = \left(2 + \tfrac{1}{2}\right)\frac{700\,\text{nm}}{2(1.45)} = \boxed{603\,\text{nm}}$$

Newton's Rings

***27** ••

Picture the Problem This arrangement is essentially identical to a ″thin film″ configuration, except that the ″film″ is air. A phase change of 180° ($\frac{1}{2}\lambda$) occurs at the top of the flat glass plate. We can use the condition for constructive interference to derive the result given in (*a*) and use the geometry of the lens on the plate to obtain the result given in (*b*). We can then use these results in the remaining parts of the problem.

(*a*) The condition for constructive interference is:

$$2t + \tfrac{1}{2}\lambda = \lambda, 2\lambda, 3\lambda, ...$$

or

$$2t = \tfrac{1}{2}\lambda, \tfrac{3}{2}\lambda, \tfrac{5}{2}\lambda, ... = \left(m + \tfrac{1}{2}\right)\lambda$$

where λ is the wavelength of light in air and m = 0, 1, 2, …

Solve for t:

$$t = \boxed{\left(m + \tfrac{1}{2}\right)\frac{\lambda}{2}, m = 0, 1, 2, ...} \quad (1)$$

(*b*) From Figure 33-39 we have:

$$r^2 + (R - t)^2 = R^2$$

or

$$R^2 = r^2 + R^2 - 2Rt + t^2$$

For $t \ll R$ we can neglect the last term to obtain:

$$R^2 \approx r^2 + R^2 - 2Rt$$

Solve for r:

$$r = \boxed{\sqrt{2Rt}} \quad (2)$$

(*c*) $\boxed{\text{The transmitted pattern is complementary to the reflected pattern.}}$

(*d*) Square equation (2) and substitute for t from equation (1) to obtain:

$$r^2 = \left(m + \tfrac{1}{2}\right)R\lambda$$

Solve for m:

$$m = \frac{r^2}{R\lambda} - \frac{1}{2}$$

Substitute numerical values and evaluate m:

$$m = \frac{(2\,\text{cm})^2}{(10\,\text{m})(590\,\text{nm})} - \frac{1}{2} = 67$$

and so there will be $\boxed{68}$ bright fringes.

(*e*) The diameter of the m^{th} fringe is:

$$D = 2r = 2\sqrt{\left(m+\tfrac{1}{2}\right)R\lambda}$$

Noting that $m = 5$ for the sixth fringe, substitute numerical values and evaluate D:

$$D = 2\sqrt{\left(5+\tfrac{1}{2}\right)(10\,\text{m})(590\,\text{nm})}$$
$$= \boxed{1.14\,\text{cm}}$$

(*f*) The wavelength of the light in the film becomes $\lambda_{\text{air}}/n = 444$ nm. The separation between fringes is reduced and the number of fringes that will be seen is increased by the factor $n = 1.33$.

28 ••

Picture the Problem This arrangement is essentially identical to a ″thin film″ configuration, except that the ″film″ is air. A phase change of 180° ($\tfrac{1}{2}\lambda$) occurs at the top of the flat glass plate. We can use the condition for constructive interference and the results of Problem 27(*b*) to determine the radii of the first and second bright fringes in the reflected light.

The condition for constructive interference is:

$$2t + \tfrac{1}{2}\lambda = \lambda, 2\lambda, 3\lambda, \ldots$$

or

$$2t = \tfrac{1}{2}\lambda, \tfrac{3}{2}\lambda, \tfrac{5}{2}\lambda, \ldots = \left(m+\tfrac{1}{2}\right)\lambda$$

where λ is the wavelength of light in air and $m = 0, 1, 2, \ldots$

Solve for t:

$$t = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{2}, m = 0, 1, 2, \ldots$$

From Problem 27(*b*):

$$r = \sqrt{2tR}$$

Substitute for t to obtain:

$$r = \sqrt{\left(m+\tfrac{1}{2}\right)\lambda R}$$

The first fringe corresponds to $m = 0$:

$$r = \sqrt{\tfrac{1}{2}(520\,\text{nm})(2\,\text{m})} = \boxed{0.721\,\text{mm}}$$

The second fringe corresponds to $m = 1$:

$$r = \sqrt{\tfrac{3}{2}(520\,\text{nm})(2\,\text{m})} = \boxed{1.25\,\text{mm}}$$

29 ••

Picture the Problem This arrangement is essentially identical to a ″thin film″ configuration, except that the ″film″ is oil. A phase change of 180° ($\frac{1}{2}\lambda$) occurs at lens-oil interface. We can use the condition for constructive interference and the results from Problem 27(*b*) to determine the radii of the first and second bright fringes in the reflected light.

The condition for constructive interference is:

$$2t + \tfrac{1}{2}\lambda' = \lambda', 2\lambda', 3\lambda', \ldots$$

or

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots = \left(m + \tfrac{1}{2}\right)\lambda'$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' and solve for t:

$$t = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2n}, m = 0, 1, 2, \ldots$$

where λ is the wavelength of light in air.

From Equation 33-29:

$$r = \sqrt{2tR}$$

Substitute for t to obtain:

$$r = \sqrt{\left(m + \tfrac{1}{2}\right)\frac{\lambda R}{n}}$$

The first fringe corresponds to m = 0:

$$r = \sqrt{\frac{1}{2}\frac{(520\,\text{nm})(2\,\text{m})}{1.82}} = \boxed{0.535\,\text{mm}}$$

The second fringe corresponds to m = 1:

$$r = \sqrt{\frac{3}{2}\frac{(520\,\text{nm})(2\,\text{m})}{1.82}} = \boxed{0.926\,\text{mm}}$$

Two-Slit Interference Pattern

***30** •

Picture the Problem The number of bright fringes per unit distance is the reciprocal of the separation of the fringes. We can use the expression for the distance on the screen to the *m*th fringe to find the separation of the fringes.

Express the number N of bright fringes per centimeter in terms of the separation of the fringes:

$$N = \frac{1}{\Delta y} \qquad (1)$$

Express the distance on the screen to the *m*th and (m + 1)st bright fringe:

$$y_m = m\frac{\lambda L}{d} \text{ and } y_{m+1} = (m+1)\frac{\lambda L}{d}$$

Subtract the second of these equations from the first to obtain:

$$\Delta y = \frac{\lambda L}{d}$$

Substitute in equation (1) to obtain:

$$N = \frac{d}{\lambda L}$$

Substitute numerical values and evaluate N:

$$N = \frac{1\,\text{mm}}{(600\,\text{nm})(2\,\text{m})} = \boxed{8.33\,\text{cm}^{-1}}$$

31 •

Picture the Problem We can use the expression for the distance on the screen to the *m*th and (*m* + 1)st bright fringes to obtain an expression for the separation Δy of the fringes as a function of the separation of the slits *d*. Because the number of bright fringes per unit length *N* is the reciprocal of Δy, we can find *d* from *N*, λ, and *L*.

Express the distance on the screen to the *m*th and (*m* + 1)st bright fringe:

$$y_m = m\frac{\lambda L}{d} \text{ and } y_{m+1} = (m+1)\frac{\lambda L}{d}$$

Subtract the second of these equations from the first to obtain:

$$\Delta y = \frac{\lambda L}{d}$$

Solve for *d*:

$$d = \frac{\lambda L}{\Delta y}$$

Because the number of fringes per unit length *N* is the reciprocal of Δy:

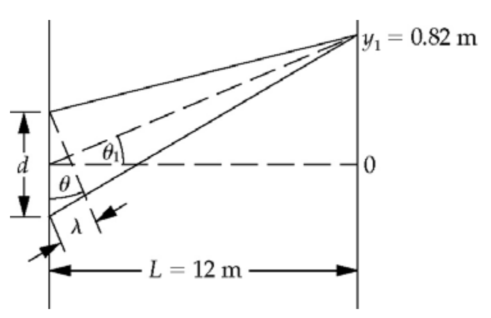
$$d = N\lambda L$$

Substitute numerical values and evaluate *d*:

$$d = (28\,\text{cm}^{-1})(589\,\text{nm})(3\,\text{m}) = \boxed{4.95\,\text{mm}}$$

32 •

Picture the Problem We can use the geometry of the setup, represented to the right, to find the separation of the slits. To find the number of interference maxima that can be observed we can apply the equation describing two-slit interference maxima and require that $\sin\theta \leq 1$.



Because $d << L$, we can approximate $\sin\theta_1$ as:

$$\sin\theta_1 \approx \frac{\lambda}{d}$$

Solve for d to obtain:

$$d \approx \frac{\lambda}{\sin\theta_1} \quad (1)$$

From the right triangle whose sides are L and y_1 we have:

$$\sin\theta_1 = \frac{0.82\,\text{m}}{\sqrt{(12\,\text{m})^2 + (0.82\,\text{m})^2}} = 0.06817$$

Substitute numerical values in equation (1) and evaluate d:

$$d \approx \frac{633\,\text{nm}}{0.06817} = \boxed{9.29\,\mu\text{m}}$$

(b) The equation describing two-slit interference maxima is:

$$d\sin\theta = m\lambda, m = 0, 1, 2, \ldots$$

Because $\sin\theta \le 1$ determines the maximum number of interference fringes that can be seen:

$$d = m_{\text{max}}\lambda$$

Solve for m_{max}:

$$m_{\text{max}} = \frac{d}{\lambda}$$

Substitute numerical values and evaluate m_{max}:

$$m_{\text{max}} = \frac{9.29\,\mu\text{m}}{633\,\text{nm}} = 14 \text{ because } m \text{ must be}$$

an integer.

Because there are 14 fringes on either side of the central maximum:

$$N = 2m_{\text{max}} + 1 = 2(14) + 1 = \boxed{29}$$

33 ••

Picture the Problem We can use the equation for the distance on a screen to the *m*th bright fringe to derive an expression for the spacing of the maxima on the screen. In (c) we can use this same relationship to express the slit separation d.

(a) Express the distance on the screen to the *m*th and (m + 1)st bright fringe:

$$y_m = m\frac{\lambda L}{d} \text{ and } y_{m+1} = (m+1)\frac{\lambda L}{d}$$

Subtract the second of these equations from the first to obtain:

$$\Delta y = \frac{\lambda L}{d} \quad (1)$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{(500\,\text{nm})(1\text{m})}{1\text{cm}} = \boxed{50.0\,\mu\text{m}}$$

(*b*) Not with the unaided eye. The separation is too small to be observed with the naked eye.

(*c*) Solve equation (1) for *d*:

$$d = \frac{\lambda L}{\Delta y}$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(500\,\text{nm})(1\,\text{m})}{1\,\text{mm}} = \boxed{0.500\,\text{mm}}$$

34 ••

Picture the Problem Let the separation of the slits be *d*. We can find the total path difference when the light is incident at an angle ϕ and set this result equal to an integer multiple of the wavelength of the light to obtain the given equation.

Express the total path difference:

$$\Delta\ell = d\sin\phi + d\sin\theta_{\text{m}}$$

The condition for constructive interference is:

$$\Delta\ell = m\lambda$$

where *m* is an integer.

Substitute to obtain:

$$d\sin\phi + d\sin\theta_{\text{m}} = m\lambda$$

Divide both sides of the equation by *d* to obtain:

$$\boxed{\sin\phi + \sin\theta_{\text{m}} = \frac{m\lambda}{d}}$$

***35** ••

Picture the Problem Let the separation of the slits be *d*. We can find the total path difference when the light is incident at an angle ϕ and set this result equal to an integer multiple of the wavelength of the light to relate the angle of incidence on the slits to the direction of the transmitted light and its wavelength.

Express the total path difference:

$$\Delta\ell = d\sin\phi + d\sin\theta$$

The condition for constructive interference is:

$$\Delta\ell = m\lambda$$

where *m* is an integer.

Substitute to obtain:

$$d\sin\phi + d\sin\theta = m\lambda$$

Divide both sides of the equation by *d* to obtain:

$$\sin\phi + \sin\theta = \frac{m\lambda}{d}$$

Set $\theta = 0$ and solve for λ:

$$\lambda = \frac{d \sin\phi}{m}$$

Substitute numerical values and simplify to obtain:

$$\lambda = \frac{(2.5\,\mu\text{m})\sin 30°}{m} = \frac{1.25\,\mu\text{m}}{m}$$

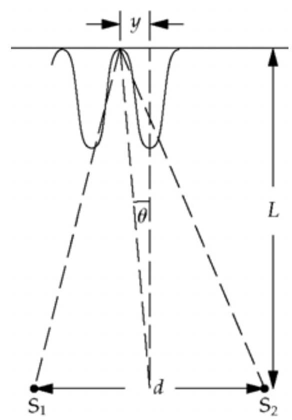
Evaluate λ for positive integral values of m:

m	λ (nm)
1	1250
2	625
3	417
4	313

From the table we can see that 625 nm and 417 nm are in the visible portion of the electromagnetic spectrum.

36 ••

Picture the Problem The diagram shows the two speakers, S_1 and S_2, the central-bright image and the first-order image to the left of the central-bright image. The distance y is measured from the center of the central-bright image. We can apply the conditions for constructive and destructive interference from two sources and use the geometry of the speakers and microphone to find the distance to the first interference minimum and the distance to the first interference maximum.



Relate the distance Δy to the first minimum from the center of the central maximum to θ and the distance L from the speakers to the plane of the microphone:

$$\tan\theta = \frac{y}{L}$$

Solve for y to obtain:

$$y = L\tan\theta \qquad (1)$$

Interference minima occur where:

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

where m = 0, 1, 2, 3, …

Solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{\left(m + \frac{1}{2}\right)\lambda}{d}\right]$$

Relate the wavelength λ of the sound waves to the speed of sound v and the frequency f of the sound:

$$\lambda = \frac{v}{f}$$

Substitute for λ in the expression for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)v}{df}\right]$$

Substitute for θ in equation (1):

$$y = L\tan\left\{\sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)v}{df}\right]\right\} \quad (2)$$

Noting that the first minimum corresponds to m = 0, substitute numerical values and evaluate Δy:

$$y_{\text{1st min}} = (1\,\text{m})\tan\left\{\sin^{-1}\left[\frac{\left(\frac{1}{2}\right)(343\,\text{m/s})}{(5\,\text{cm})(10\,\text{kHz})}\right]\right\} = \boxed{0.365\,\text{m}}$$

The maxima occur where:

$$d\sin\theta = m\lambda$$

where m = 1, 2, 3, …

For diffraction maxima, equation (2) becomes:

$$\Delta y = L\tan\left\{\sin^{-1}\left[\frac{mv}{af}\right]\right\}$$

Noting that the first maximum corresponds to m = 1, substitute numerical values and evaluate Δy:

$$y_{\text{1st max}} = (1\,\text{m})\tan\left\{\sin^{-1}\left[\frac{(1)(343\,\text{m/s})}{(5\,\text{cm})(10\,\text{kHz})}\right]\right\} = \boxed{0.943\,\text{m}}$$

Diffraction Pattern of a Single Slit

37 •

Picture the Problem We can use the expression locating the first zeroes in the intensity to find the angles at which these zeroes occur as a function of the slit width a.

The first zeroes in the intensity occur at angles given by:

$$\sin\theta = \frac{\lambda}{a}$$

Solve for θ:

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

(a) For a = 1 mm:

$$\theta = \sin^{-1}\left(\frac{600\,\text{nm}}{1\,\text{mm}}\right) = \boxed{0.600\,\text{mrad}}$$

(*b*) For *a* = 0.1 mm:

$$\theta = \sin^{-1}\left(\frac{600\,\text{nm}}{0.1\text{mm}}\right) = \boxed{6.00\,\text{mrad}}$$

(*c*) For *a* = 0.01 mm:

$$\theta = \sin^{-1}\left(\frac{600\,\text{nm}}{0.01\text{mm}}\right) = \boxed{60.0\,\text{mrad}}$$

38 •

Picture the Problem We can use the expression locating the first zeroes in the intensity to find the wavelength of the radiation as a function of the angle at which the first diffraction minimum is observed and the width of the plate.

The first zeroes in the intensity occur at angles given by:

$$\sin\theta = \frac{\lambda}{a}$$

Solve for λ:

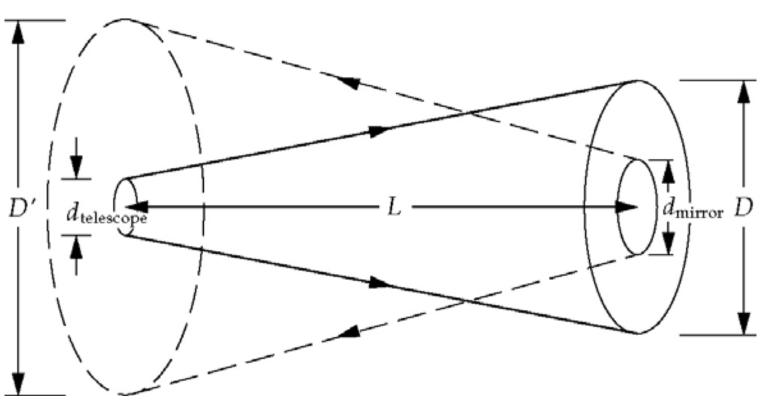
$$\lambda = a\sin\theta$$

Substitute numerical values and evaluate λ:

$$\lambda = (5\,\text{cm})\sin 37° = \boxed{3.01\text{cm}}$$

***39 ••**

Picture the Problem The diagram shows the beam expanding as it travels to the moon and that portion of it that is reflected from the mirror on the moon expanding as it returns to earth. We can express the diameter of the beam at the moon as the product of the beam divergence angle and the distance to the moon and use the equation describing diffraction at a circular aperture to find the beam divergence angle. We can follow this same procedure to find the diameter of the beam when it gets back to the earth. In Parts (*c*) and (*d*) we can use the dependence of the power in a beam on its cross-sectional area to find the fraction of the power of the beam that is reflected back to earth and the fraction of the original beam energy that is recaptured upon return to earth.



(*a*) Relate the diameter *D* of the beam at the moon to the distance to the moon *L* and the beam divergence angle θ:

$$D \approx \theta L$$

The angle θ subtended by the first diffraction minimum is related to the wavelength λ of the light and the diameter of the telescope opening $d_{\text{telescope}}$ by:

$$\sin\theta = 1.22\frac{\lambda}{d_{\text{telescope}}}$$

Because $\theta << 1$, $\sin\theta \approx \theta$ and:

$$\theta \approx 1.22\frac{\lambda}{d_{\text{telescope}}}$$

Substitute for θ in equation (1) to obtain:

$$D = \frac{1.22L\lambda}{d_{\text{telescope}}}$$

Substitute numerical values and evaluate D:

$$D = \left(3.82\times10^{8}\text{ m}\right)\left[\frac{1.22(500\text{ nm})}{6\text{ in}\times\frac{2.54\text{ cm}}{\text{in}}\times\frac{1\text{ m}}{10^{2}\text{ cm}}}\right] = \boxed{1.53\text{ km}}$$

(b) The portion of the beam reflected back to the earth will be that portion incident on the mirror, so the diffraction angle is:

$$\theta \approx 1.22\frac{\lambda}{d_{\text{mirror}}}$$

The beam will expand back to:

$$D' = L\left[1.22\frac{\lambda}{d_{\text{mirror}}}\right]$$

Substitute numerical values and evaluate D':

$$D' = \left(3.82\times10^{8}\text{ m}\right)\left[\frac{1.22(500\text{ nm})}{20\text{ in}\times\frac{2.54\text{ cm}}{\text{in}}\times\frac{1\text{ m}}{10^{2}\text{ cm}}}\right] = \boxed{459\text{ m}}$$

(c) Because the power of the beam is proportional to its cross-sectional area, the fraction of the power that is reflected back to the earth is the ratio of the area of the mirror to the area of the expanded beam at the moon:

$$\frac{P'}{P} = \frac{A_{\text{mirror}}}{A_{\text{beam}}} = \frac{\frac{\pi}{4}d_{\text{mirror}}^{2}}{\frac{\pi}{4}D^{2}} = \left(\frac{d_{\text{mirror}}}{D}\right)^{2}$$

Substitute for D to obtain:

$$\frac{P'}{P}=\left(\frac{\dfrac{d_{\text{mirror}}}{1.22L\lambda}}{d_{\text{telescope}}}\right)^2=\left(\frac{d_{\text{mirror}}d_{\text{telescope}}}{1.22L\lambda}\right)^2 \quad (1)$$

Substitute numerical values and evaluate P'/P:

$$\frac{P'}{P}=\left[\frac{(20\,\text{in})(6\,\text{in})\left(\dfrac{2.54\,\text{cm}}{\text{in}}\right)^2}{1.22(3.82\times10^8\text{ m})(500\text{ nm})}\right]^2$$

$$=\boxed{1.10\times10^{-7}}$$

(d) The angular spread of the beam from reflection from the 20-in mirror is given by:

$$\theta\approx1.22\frac{\lambda}{d_{\text{mirror}}}$$

The diameter D' of the beam on return to earth will be:

$$D'\approx1.22L\frac{\lambda}{d_{\text{mirror}}}$$

Letting P'' represent the power intercepted by the telescope, we have:

$$\frac{P''}{P'}=\frac{A_{\text{telescope}}}{A_{\text{beam}}}=\frac{\dfrac{\pi}{4}d^2_{\text{telescope}}}{\dfrac{\pi}{4}D'^2}$$

$$=\left(\frac{d_{\text{telescope}}}{D'}\right)^2$$

Substitute for D' and simplify:

$$\frac{P''}{P'}=\left(\frac{d_{\text{telescope}}d_{\text{mirror}}}{1.22L\lambda}\right)^2 \quad (2)$$

Multiply equation (2) by equation (1) and simplify to obtain:

$$\frac{P''}{P'}\frac{P'}{P}=\frac{P''}{P}=\left(\frac{d_{\text{telescope}}d_{\text{mirror}}}{1.22L\lambda}\right)^2\left(\frac{d_{\text{mirror}}d_{\text{telescope}}}{1.22L\lambda}\right)^2=\left(\frac{d_{\text{mirror}}d_{\text{telescope}}}{1.22L\lambda}\right)^4$$

Substitute numerical values and evaluate P''/P:

$$\frac{P''}{P}=\left[\frac{(20\,\text{in})(6\,\text{in})\left(\dfrac{2.54\,\text{cm}}{\text{in}}\right)^2}{1.22(3.82\times10^8\text{ m})(500\text{ nm})}\right]^4$$

$$=\boxed{1.21\times10^{-14}}$$

Interference-Diffraction Pattern of Two Slits

40 •

Picture the Problem We need to find the value of m for which the mth interference maximum coincides with the first diffraction minimum. Then there will be $N = 2m - 1$ fringes in the central maximum.

The number of fringes N in the central maximum is:

$$N = 2m - 1 \qquad (1)$$

Relate the angle θ_1 of the first diffraction minimum to the width a of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_m corresponding to the mth interference maxima in terms of the separation d of the slits:

$$\sin\theta_m = \frac{m\lambda}{d}$$

Because we require that $\theta_1 = \theta_m$, we can equate these expressions to obtain:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

Solve for and evaluate m:

$$m = \frac{d}{a} = \frac{5a}{a} = 5$$

Substitute in equation (1) to obtain:

$$N = 2(5) - 1 = \boxed{9}$$

If $d = na$:

$$m = \frac{d}{a} = \frac{na}{a} = n$$

and

$$N = \boxed{2n - 1}$$

41 ••

Picture the Problem We can equate the sine of the angle at which the first diffraction minimum occurs to the sine of the angle at which the fifth interference maximum occurs to find a. We can then find the number of bright interference fringes seen in the central diffraction maximum using $N = 2m - 1$.

(*a*) Relate the angle θ_1 of the first diffraction minimum to the width a of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_5 corresponding to the mth fifth interference maxima maximum in terms of the separation *d* of the slits:

$$\sin\theta_5 = \frac{5\lambda}{d}$$

Because we require that $\theta_1 = \theta_{m5}$, we can equate these expressions to obtain:

$$\frac{5\lambda}{d} = \frac{\lambda}{a}$$

Solve for and evaluate *ma*:

$$a = \frac{d}{5} = \frac{0.1\text{mm}}{5} = \boxed{20.0\,\mu\text{m}}$$

(*b*) Because *m* = 5:

$$N = 2m - 1 = 2(5) - 1 = \boxed{9}$$

42 ••

Picture the Problem We can equate the sine of the angle at which the first diffraction minimum occurs to the sine of the angle at which the *m*th interference maximum occurs to find *m*. We can then find the number of bright interference fringes seen in the central diffraction maximum using $N = 2m - 1$.

The number of fringes *N* in the central maximum is:

$$N = 2m - 1 \qquad (1)$$

Relate the angle θ_1 of the first diffraction minimum to the width *a* of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_m corresponding to the *m*th interference maxima in terms of the separation *d* of the slits:

$$\sin\theta_m = \frac{m\lambda}{d}$$

Because we require that $\theta_1 = \theta_m$, we can equate these expressions to obtain:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

Solve for *m*:

$$m = \frac{d}{a}$$

Substitute in equation (1) to obtain:

$$N = \frac{2d}{a} - 1$$

Substitute numerical values and evaluate N:

$$N = \frac{2(0.2\,\text{mm})}{0.01\,\text{mm}} - 1 = \boxed{39}$$

***43 ••**

Determine the ConceptPicture the Problem There are 8 interference fringes on each side of the central maximum. The secondary diffraction maximum is half as wide as the central one. It follows that it will contain 8 interference maxima.

44 ••

Picture the Problem We can equate the sine of the angle at which the first diffraction minimum occurs to the sine of the angle at which the *m*th interference maximum occurs to find *m*. We can then find the number of bright interference fringes seen in the central diffraction maximum using $N = 2m - 1.$ In (*b*) we can use the expression relating the intensity in a single-slit diffraction pattern to phase constant $\phi = \frac{2\pi}{\lambda} a \sin\theta$ to find the ratio of the intensity of the third interference maximum to the side of the centerline to the intensity of the center interference maximum.

(*a*) The number of fringes N in the central maximum is:

$$N = 2m - 1 \qquad (1)$$

Relate the angle θ_1 of the first diffraction minimum to the width a of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_m corresponding to the *m*th interference maxima in terms of the separation d of the slits:

$$\sin\theta_m = \frac{m\lambda}{d}$$

Because we require that $\theta_1 = \theta_m$, we can equate these expressions to obtain:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

Solve for m:

$$m = \frac{d}{a}$$

Substitute in equation (1) to obtain:

$$N = \frac{2d}{a} - 1$$

Substitute numerical values and evaluate N:

$$N = \frac{2(0.15\,\text{mm})}{0.03\,\text{mm}} - 1 = \boxed{9}$$

(*b*) Express the intensity for a single-slit diffraction pattern as a function of the phase difference ϕ:

$$I = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2 \qquad (2)$$

where $\phi = \frac{2\pi}{\lambda}a\sin\theta$

For $m = 3$:

$$\sin\theta_3 = \frac{3\lambda}{d}$$

and

$$\phi = \frac{2\pi}{\lambda}a\sin\theta_3 = \frac{2\pi}{\lambda}a\left(\frac{3\lambda}{d}\right) = 6\pi\left(\frac{a}{d}\right)$$

Substitute numerical values and evaluate ϕ:

$$\phi = 6\pi\left(\frac{0.03\,\text{mm}}{0.15\,\text{mm}}\right) = \frac{6\pi}{5}$$

Solve equation (2) for the ratio of I_3 to I_0:

$$\frac{I}{I_0} = \left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$$

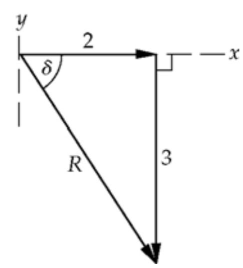
Substitute numerical values and evaluate I_3/I_0:

$$\frac{I_3}{I_0} = \left[\frac{\sin\frac{1}{2}\left(\frac{6\pi}{5}\right)}{\frac{1}{2}\left(\frac{6\pi}{5}\right)}\right]^2 = \boxed{0.255}$$

Using Phasors to Add Harmonic Waves

45 •

Picture the Problem Chose the coordinate system shown in the phasor diagram. We can use the standard methods of vector addition to find the resultant of the two waves.



The resultant of the two waves is of the form:

$$E = R\sin(\omega t + \delta)$$

Express $\vec{R}$ in vector form:

$$\vec{R} = 2\hat{i} - 3\hat{j}$$

Find the magnitude of $\vec{R}$:

$$R = \sqrt{(2)^2 + (-3)^2} = 3.61$$

Find the phase angle δ between $\vec{R}$ and $\vec{E}_1$:

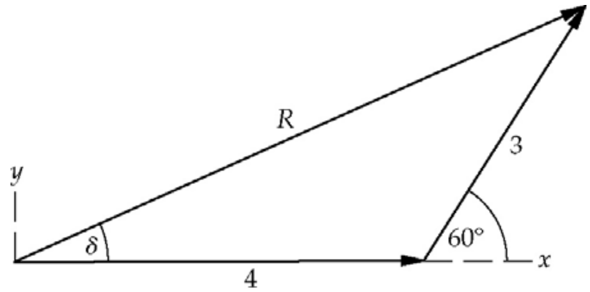
$$\delta = \tan^{-1}\left(\frac{-3}{2}\right) = -56.3°$$

Substitute to obtain:

$$E = \boxed{3.61\sin(\omega t - 56.3°)}$$

***46** •

Picture the Problem Chose the coordinate system shown in the phasor diagram. We can use the standard methods of vector addition to find the resultant of the two waves.



The resultant of the two waves is of the form:

$$E = R\sin(\omega t + \delta)$$

Express the x component of $\vec{R}$:

$$R_x = 4 + 3\cos 60° = 5.50$$

Express the y component of $\vec{R}$:

$$R_y = 0 + 3\sin 60° = 2.60$$

Find the magnitude of $\vec{R}$:

$$R = \sqrt{(5.50)^2 + (2.60)^2} = 6.08$$

Find the phase angle δ between $\vec{R}$ and $\vec{E}_1$:

$$\delta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.60}{5.50}\right) = 25.3°$$

Substitute to obtain:

$$E = \boxed{6.08\sin(\omega t + 25.3°)}$$

Remarks: We could have used the law of cosines to find *R* and the law of sines to find δ.

47 ••

Picture the Problem We can evaluate the expression for the intensity for a single-slit diffraction pattern at the second secondary maximum to express I_2 in terms of I_0.

The intensity at the second secondary maximum is given by:

$$I_2 = I_0\left[\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right]^2$$

where

$$\phi = \frac{2\pi}{\lambda} a \sin\theta$$

At this second secondary maximum:

$$a\sin\theta = \frac{5}{2}\lambda$$

and

$$\phi = \frac{2\pi}{\lambda}\left(\frac{5\lambda}{2}\right) = 5\pi$$

Substitute for ϕ and evaluate I_2:

$$I_2 = I_0\left[\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}}\right]^2 = \boxed{0.0162 I_0}$$

48 ••

Picture the Problem We can use phasor concepts to find the phase angle δ in terms of the number of phasors N (three in this problem) forming a closed polygon of N sides at the minima and then use this information to express the path difference Δr for each of these locations. Applying a small angle approximation, we can obtain an expression for y that we can evaluate for enough of the path differences to establish the pattern given in the problem statement.

Express the phase angle δ in terms of the number of phasors N forming a closed polygon of N sides at the first minimum:

$$\delta = \frac{2\pi}{N}$$

Express the path difference Δr in terms of $\sin\theta$ and the separation d of the slits:

$$\Delta r = d\sin\theta$$

or, provided the small angle approximation is valid,

$$\Delta r = \frac{yd}{L}$$

where L is the distance to the screen.

Solve for y:

$$y = \frac{L}{d}\Delta r \quad y = \frac{L}{2d}\delta$$

For three equally spaced sources, the phase angle corresponding to the first minimum is:

$$\delta = \frac{2\pi}{3} \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{1}{3}\lambda$$

Substitute to obtain:

$$y_1 = \frac{L}{d}\left(\frac{\lambda}{3}\right) = (1)\frac{\lambda L}{3d}$$

The phase angle corresponding to the second minimum is:

$$\delta = \frac{1}{2}\left(\frac{2\pi}{3}\right) \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{2}{3}\lambda$$

Substitute to obtain:

$$y_2 = \frac{L}{d}\left(\frac{2\lambda}{3}\right) = (2)\frac{\lambda L}{3d}$$

When the path difference is λ, we have an interference maximum.

The path difference corresponding to the fourth minimum is:

$$\Delta r = \tfrac{4}{3}\lambda$$

Substitute to obtain:

$$y_2 = \frac{L}{d}\left(\frac{4\lambda}{3}\right) = (4)\frac{\lambda L}{3d}$$

Continue in this manner to obtain:

$$\boxed{y_{\text{min}} = \frac{n\lambda L}{3d}, n = 1, 2, 4, 5, 7, 8, ...}$$

(*b*) For L = 1 m, $\lambda = 5\times10^{-7}$ m, and d = 0.1 mm:

$$2y_{\text{min}} = \frac{2(500\,\text{nm})(1\,\text{m})}{3(0.1\,\text{mm})} = \boxed{3.33\,\text{mm}}$$

49 ••

Picture the Problem We can use phasor concepts to find the phase angle δ in terms of the number of phasors N (four in this problem) forming a closed polygon of N sides at the minima and then use this information to express the path difference Δr for each of these locations. Applying a small angle approximation, we can obtain an expression for y that we can evaluate for enough of the path differences to establish the pattern given in the problem statement.

Express the phase angle δ in terms of the number of phasors N forming a closed polygon of N sides at the first minimum:

$$\delta = \frac{2\pi}{N}$$

Express the path difference Δr in terms of $\sin\theta$ and the separation d of the slits:

$$\Delta r = d\sin\theta$$

or, provided the small angle approximation is valid,

$$\Delta r = \frac{yd}{L}$$

where L is the distance to the screen.

Solve for y:

$$y = \frac{L}{d}\Delta r$$

For four equally spaced sources, the phase angle corresponding to the first minimum is:

$$\delta = \frac{\pi}{2} \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{1}{4}\lambda$$

Substitute to obtain:

$$y_1 = \frac{L}{d}\left(\frac{\lambda}{4}\right) = (1)\frac{\lambda L}{4d}$$

The phase angle corresponding to the second minimum is:

$$\delta = \pi \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{1}{2}\lambda$$

Substitute to obtain:

$$y_2 = \frac{L}{d}\left(\frac{\lambda}{2}\right) = (2)\frac{\lambda L}{4d}$$

The phase angle angle corresponding to the third minimum is:

$$\delta = \frac{3\pi}{2} \text{ and } \Delta r = \frac{\lambda}{2\pi}\left(\frac{3\pi}{2}\right) = \frac{3\lambda}{4}$$

Substitute to obtain:

$$y_3 = \frac{L}{d}\left(\frac{3\lambda}{4}\right) = (3)\frac{\lambda L}{4d}$$

Continue in this manner to obtain:

$$\boxed{y_{\text{min}} = \frac{n\lambda L}{4d}, n = 1, 2, 3, 5, 6, 7, 9, \ldots}$$

(*b*) For L = 2 m, $\lambda = 6\times10^{-7}$ m, d = 0.1 mm, and n = 1:

$$2y_{\text{min}} = \frac{2(600\,\text{nm})(2\,\text{m})}{4(0.1\text{mm})} = \boxed{6.00\,\text{mm}}$$

For two slits:

$$2y_{\text{min}} = \frac{2\left(m + \frac{1}{2}\right)\lambda L}{d}$$

For L = 2 m, $\lambda = 6\times10^{-7}$ m, d = 0.1 mm, and m = 0:

$$2y_{\text{min}} = \frac{(600\,\text{nm})(2\,\text{m})}{0.1\text{mm}} = 12.0\,\text{mm}$$

$$\boxed{\text{The width for four sources is half the width for two sources.}}$$

50 ••

Picture the Problem We can use $\sin\theta = \lambda/a$ to find the first zeros in the intensity pattern. The four-slit interference maxima occur at angles given by $d\sin\theta = m\lambda, m = 0,1,2,\ldots$. In (*c*) we can use the result of Problem 49 to find the angular spread between the central interference maximum and the first interference minimum on either side of it. In (*d*) we'll proceed as in Example 33-6, using a phasor diagram for a four-slit grating, to find the resultant amplitude at a given point in the intensity pattern as a function of the phase constant δ, that, in turn, is a function of the angle θ that determines the location of a point in the interference pattern.

(*a*) The first zeros in the intensity occur at angles given by:

$$\sin\theta = \frac{\lambda}{a}$$

Solve for θ:

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{480\,\text{nm}}{2\,\mu\text{m}}\right) = \boxed{0.242\,\text{rad}}$$

(*b*) The four-slit interference maxima occur at angles given by:

$$d\sin\theta = m\lambda, m = 0,1,2,\ldots$$

Solve for θ_m:

$$\theta_m = \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Substitute numerical values to obtain:

$$\theta_m = \sin^{-1}\left[\frac{m(480\,\text{nm})}{6\,\mu\text{m}}\right] = \sin^{-1}(0.08m)$$

Evaluate θ_m for m = 0, 1, 2, and 3:

$$\theta_0 = \sin^{-1}[0(0.08)] = \boxed{0}$$

$$\theta_1 = \sin^{-1}[1(0.08)] = \boxed{80.1\,\text{mrad}}$$

$$\theta_2 = \sin^{-1}[2(0.08)] = \boxed{0.161\,\text{rad}}$$

$$\theta_3 = \sin^{-1}[3(0.08)] = 0.242\,\text{rad}$$

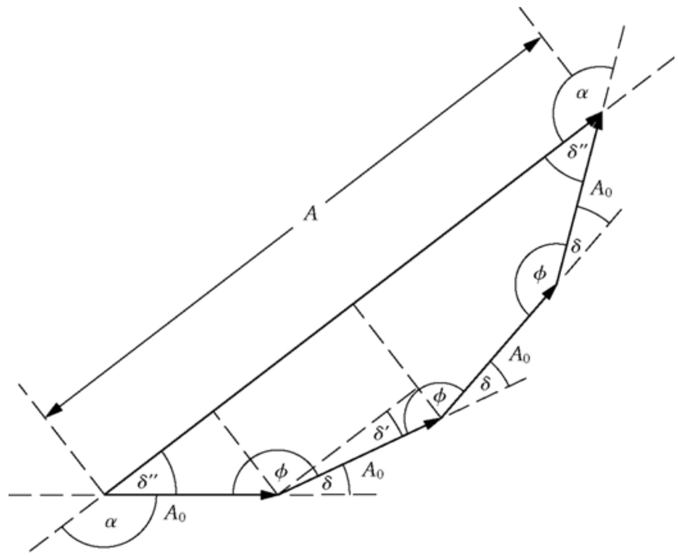
where θ_3 will not be seen as it coincides with the first minimum in the diffraction pattern.

(*c*) From Problem 49 we have:

$$\theta_{\min} = \frac{n\lambda}{4d}$$

For $n = 1$: $$\theta_{\text{min}} = \frac{480\,\text{nm}}{4(6\,\mu\text{m})} = \boxed{0.0200\,\text{rad}}$$

(*d*) Use the phasor method to show the superposition of four waves of the same amplitude A_0 and constant phase difference $\delta = \frac{2\pi}{\lambda} d\sin\theta$.



Express A in terms of δ' and δ'': $$A = 2\left(A_0\cos\delta'' + A_0\cos\delta'\right) \quad (1)$$

Because the sum of the external angles of a polygon equals 2π: $$2\alpha + 3\delta = 2\pi$$

Examining the phasor diagram we see that: $$\alpha + \delta'' = \pi$$

Eliminate α and solve for δ'' to obtain: $$\delta'' = \tfrac{3}{2}\delta$$

Because the sum of the internal angles of a polygon of n sides is $(n - 2)\pi$: $$3\phi + 2\delta'' = 3\pi$$

From the definition of a straight angle we have: $$\phi - \delta' + \delta = \pi$$

Eliminate ϕ between these equations to obtain: $$\delta' = \tfrac{1}{2}\delta$$

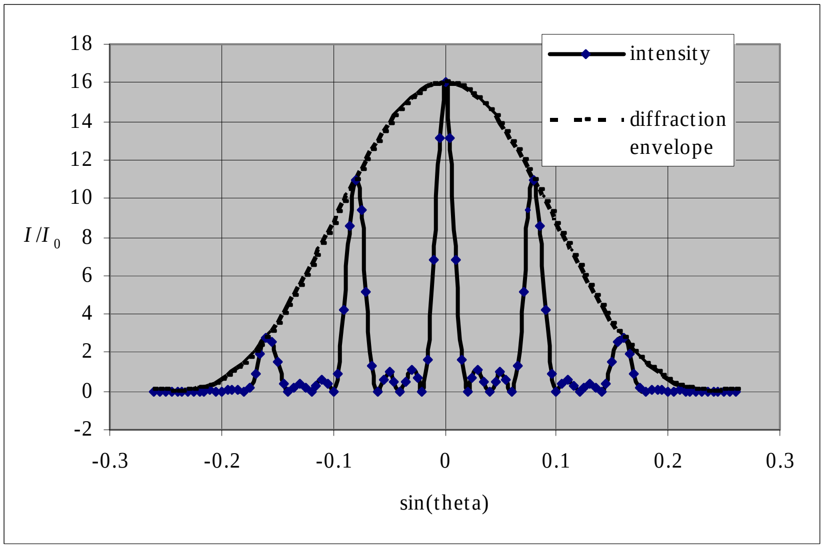
Substitute for δ'' and δ' in equation (1) to obtain:

$$A = 2A_0\left(\cos\tfrac{3}{2}\delta + \cos\tfrac{1}{2}\delta\right)$$

Because the intensity is proportional to the square of the amplitude of the resultant wave:

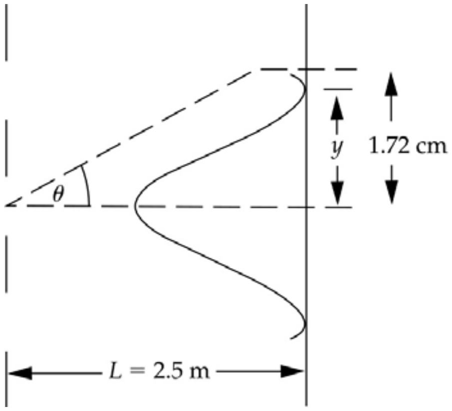
$$I = 4I_0\left(\cos\tfrac{3}{2}\delta + \cos\tfrac{1}{2}\delta\right)^2$$

The following graph of I/I_0 as a function of $\sin\theta$ was plotted using a spreadsheet program. The diffraction envelope was plotted using $\frac{I}{I_0} = 4^2\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$, where $\phi = \frac{2\pi}{\lambda}a\sin\theta$. Note the excellent agreement with the results calculated in (*a*), (*b*), and (*c*).



51 •••

Picture the Problem We can find the phase constant δ from the geometry of the diagram to the right. Using the value of δ found in this fashion we can express the intensity at the point 1.72 cm from the centerline in terms of the intensity on the centerline. On the centerline, the amplitude of the resultant wave is 3 times that of each individual wave and the intensity is 9 times that of each source acting separately.



(*a*) Express δ for the adjacent slits:

$$\delta = \frac{2\pi}{\lambda} d \sin\theta$$

For small angles, $\sin\theta \approx \tan\theta$:

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

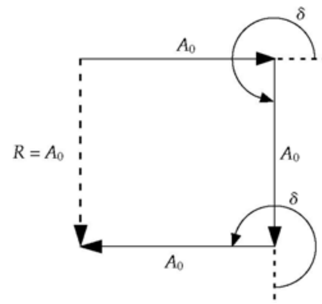
Substitute to obtain:

$$\delta = \frac{2\pi dy}{\lambda L}$$

Substitute numerical values and evaluate δ:

$$\delta = \frac{2\pi(0.06\,\text{mm})(1.72\,\text{cm})}{(550\,\text{nm})(2.5\,\text{m})} = \frac{3\pi}{2}\,\text{rad} = 270°$$

The three phasors, 270° apart, are shown in the diagram to the right. Note that they form three sides of a square. Consequently, their sum, shown as the resultant *R*, equals the magnitude of one of the phasors.



(*b*) Express the intensity at the point 1.72 cm from the centerline:

$$I \propto R^2$$

Because $I_0 \propto 9R^2$:

$$\frac{I}{I_0} = \frac{R^2}{9R^2} \Rightarrow I = \frac{I_0}{9}$$

Substitute for I_0 and evaluate *I*:

$$I = \frac{0.05\,\text{W/m}^2}{9} = \boxed{5.56\,\text{mW/m}^2}$$

***52 •••**

Picture the Problem We can use the phasor diagram shown in Figure 33-26 to determine the first three values of ϕ that produce subsidiary maxima. Setting the derivative of Equation 33-19 equal to zero will yield a transcendental equation whose roots are the values of ϕ corresponding to the maxima in the diffraction pattern.

(*a*) Referring to Figure 33-26 we see that the first subsidiary maximum occurs when:

$$\phi = 3\pi$$

A minimum occurs when:

$$\phi = 4\pi$$

Another maximum occurs when: $\phi = 5\pi$

Thus, subsidiary maxima occur when: $\phi = (2n+1)\pi,\ n = 1, 2, 3, \ldots$
and the first three subsidiary maxima are at $\phi = 3\pi$, 5π, and 7π.

(*b*) The intensity in the single-slit diffraction pattern is given by:

$$I = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$$

Set the derivative of this expression equal to zero for extrema:

$$\frac{dI}{d\phi} = 2I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)\left[\frac{\frac{1}{4}\phi\cos\frac{1}{2}\phi - \frac{1}{2}\sin\frac{1}{2}\phi}{\left(\frac{1}{2}\phi\right)^2}\right] = 0 \text{ for relative maxima and minima}$$

Simplify to obtain the transcendental equation: $\tan\frac{1}{2}\phi = \frac{1}{2}\phi$

Solve this equation numerically (use the "Solver" function of your calculator) to obtain:

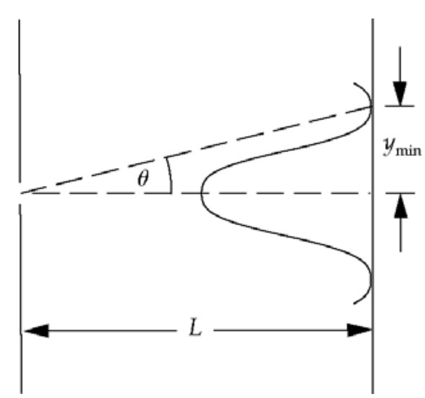
$$\phi = \boxed{2.86\pi, 4.92\pi, \text{and } 6.94\pi}$$

Remarks: Note that our results in (*b*) are smaller than the approximate values found in (*a*) by 4.80%, 1.63%, and 0.865% and that the agreement improves as *n* increases.

Diffraction and Resolution

53 •

Picture the Problem We can use $\theta = 1.22\frac{\lambda}{D}$ to find the angle between the central maximum and the first diffraction minimum for a Fraunhofer diffraction pattern and the diagram to the right to find the distance between the central maximum and the first diffraction minimum on a screen 8 m away from the pinhole.



(*a*) The angle between the central maximum and the first diffraction

$$\theta = 1.22\frac{\lambda}{D}$$

minimum for a Fraunhofer diffraction pattern is given by:

Substitute numerical values and evaluate θ:

$$\theta = 1.22\frac{700\text{ nm}}{0.1\text{mm}} = \boxed{8.54\text{ mrad}}$$

(*b*) Referring to the diagram, we see that:

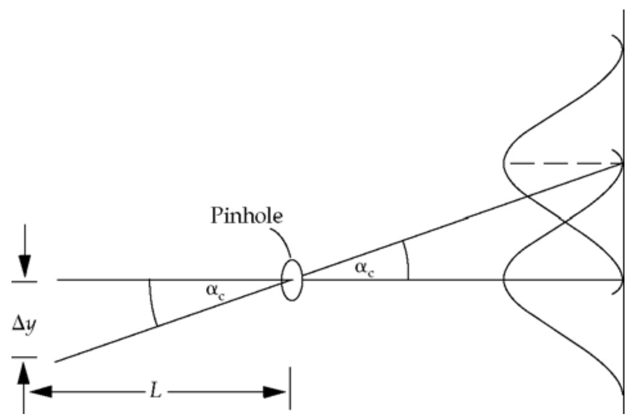
$$y = L\tan\theta$$

Substitute numerical values and evaluate *y*:

$$y = (8\text{ m})\tan(8.54\text{ mrad}) = \boxed{6.83\text{ cm}}$$

54 •

Picture the Problem We can apply Rayleigh's criterion to the overlapping diffraction patterns and to the diameter *D* of the pinhole to obtain an expression that we can solve for Δy.



Rayleigh's criterion is satisfied provided:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Relate α_c to the separation Δy of the light sources:

$$\alpha_c \approx \frac{\Delta y}{L} \text{ provided } \alpha_c << 1.$$

Equate these expressions to obtain:

$$\frac{\Delta y}{L} = 1.22\frac{\lambda}{D}$$

Solve for Δy:

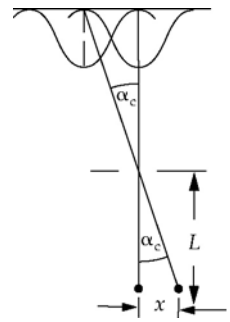
$$\Delta y = 1.22\frac{\lambda L}{D}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = 1.22\frac{(700\text{nm})(10\text{ m})}{0.1\text{mm}} = \boxed{8.54\text{cm}}$$

***55 •**

Picture the Problem We can use Rayleigh's criterion for slits and the geometry of the diagram to the right showing the overlapping diffraction patterns to express x in terms of λ, L, and the width a of the slit.



Referring to the diagram, relate α_c, L, and x: $\alpha_c \approx \frac{x}{L}$

For slits, Rayleigh's criterion is: $\alpha_c = \frac{\lambda}{a}$

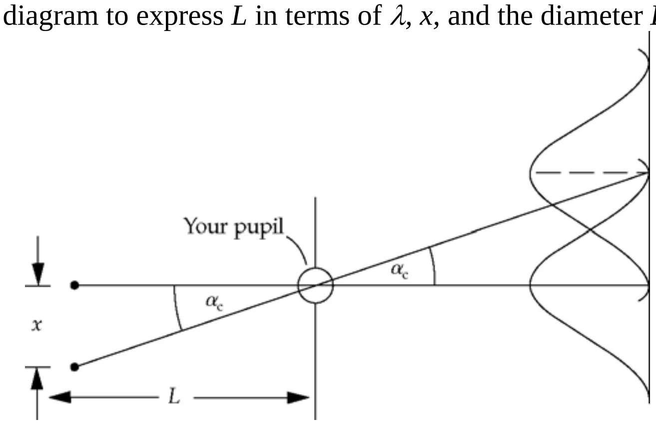
Equate these two expressions to obtain: $\frac{x}{L} = \frac{\lambda}{a}$

Solve for x: $x = \frac{\lambda L}{a}$

Substitute numerical values and evaluate x: $x = \frac{(700\ \text{nm})(5\ \text{m})}{0.5\ \text{mm}} = \boxed{7.00\ \text{mm}}$

56 •

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to express L in terms of λ, x, and the diameter D of your pupil.



Referring to the diagram, relate α_c, L, and x: $\alpha_c \approx \frac{x}{L}$

For circular apertures, Rayleigh's criterion is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = 1.22\frac{\lambda}{D}$$

Solve for L:

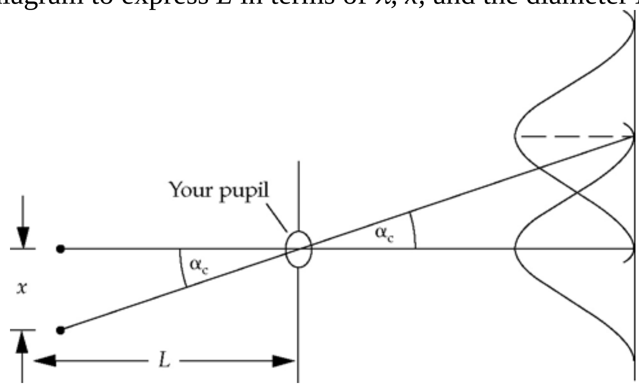
$$L = \frac{xD}{1.22\lambda}$$

Substitute numerical values and evaluate L:

$$L = \frac{(112\,\text{cm})(5\,\text{mm})}{1.22(550\,\text{nm})} = \boxed{8.35\,\text{km}}$$

57 •

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to express L in terms of λ, x, and the diameter D of your pupil.



Referring to the diagram, relate α_c, L, and x:

$$\alpha_c \approx \frac{x}{L}$$

For circular apertures, Rayleigh's criterion is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = 1.22\frac{\lambda}{D}$$

Solve for L:

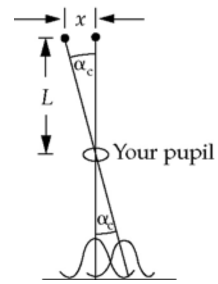
$$L = \frac{xD}{1.22\lambda}$$

Substitute numerical values and evaluate L:

$$L = \frac{(6.5\,\text{cm})(5\,\text{mm})}{1.22(550\,\text{nm})} = \boxed{484\,\text{m}}$$

58 ••

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to the right showing the overlapping diffraction patterns to express L in terms of λ, x, and the diameter D of your pupil.



(*a*) Referring to the diagram, relate α_c, L, and x:

$$\alpha_c \approx \frac{x}{L} \text{ provided } \alpha << 1$$

For circular apertures, Rayleigh's criterion is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = 1.22\frac{\lambda}{D}$$

Solve for L:

$$L = \frac{xD}{1.22\lambda}$$

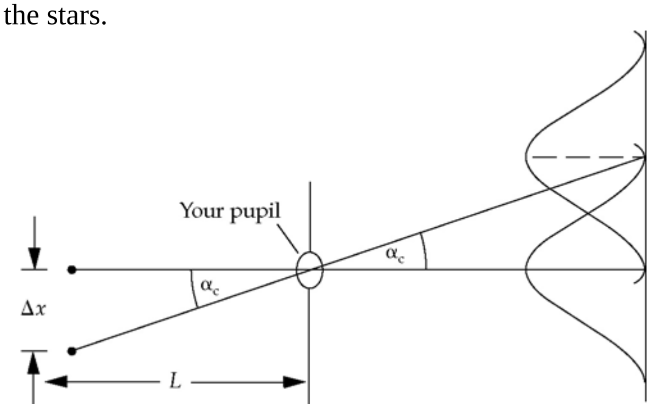
Substitute numerical values and evaluate L:

$$L = \frac{(6\text{ mm})(5\text{ mm})}{1.22(500\text{ nm})} = \boxed{49.2\text{ m}}$$

(*b*) Because L is inversely proportional to λ, the holes can be resolved better with violet light which has a shorter wavelength.

59 ••

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to obtain an expression we can solve for the minimum separation Δx of the stars.



(*a*) Rayleigh's criterion is satisfied provided:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Relate α_c to the separation Δx of the light sources:

$$\alpha_c \approx \frac{\Delta x}{L} \text{ because } \alpha_c << 1$$

Equate these expressions to obtain:

$$\frac{\Delta x}{L} = 1.22\frac{\lambda}{D}$$

Solve for Δx:

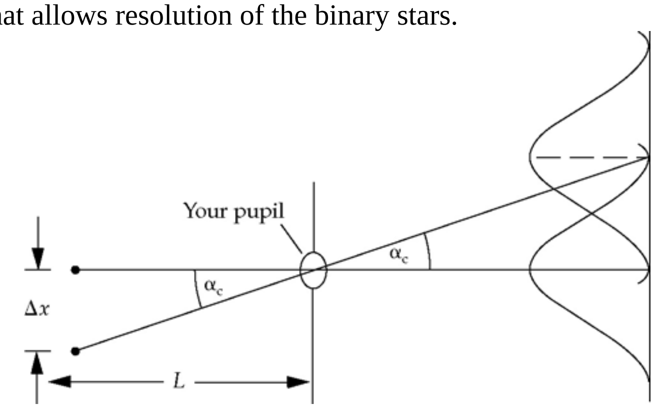
$$\Delta x = 1.22\frac{\lambda L}{D}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = 1.22\frac{(550\,\text{nm})\left(4c\cdot\text{y}\times\frac{9.461\times10^{15}\,\text{m}}{1c\cdot\text{y}}\right)}{200\,\text{in}\times\frac{2.54\,\text{cm}}{1\,\text{in}}} = \boxed{5.00\times10^{9}\,\text{m}}$$

***60** ••

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to obtain an expression we can solve for the minimum diameter *D* of the pupil that allows resolution of the binary stars.



(*a*) Rayleigh's criterion is satisfied provided:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Solve for *D*:

$$D = 1.22\frac{\lambda}{\alpha_c}$$

Substitute numerical values and evaluate D:

$$D = 1.22\frac{550\,\text{nm}}{14''\times\frac{1^\circ}{3600''}\times\frac{\pi\,\text{rad}}{180^\circ}} = \boxed{9.89\,\text{mm}} \approx 1\,\text{cm}$$

Diffraction Gratings

61 •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for θ with $m = 1$ to express the location of the first-order maximum as a function of the wavelength of the light.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where d is the separation of the slits and m = 0, 1, 2, …

Solve for the angular location θ_m of the maxima :

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Relate the number of slits N per centimeter to the separation d of the slits:

$$N = \frac{1}{d}$$

Substitute to obtain:

$$\theta_m = \sin^{-1}(mN\lambda)$$

Evaluate θ_1 for λ = 434 nm:

$$\theta_1 = \sin^{-1}\left[\left(2000\,\text{cm}^{-1}\right)\left(434\,\text{nm}\right)\right] = \boxed{86.9\,\text{mrad}}$$

Evaluate θ_1 for λ = 410 nm:

$$\theta_1 = \sin^{-1}\left[\left(2000\,\text{cm}^{-1}\right)\left(410\,\text{nm}\right)\right] = \boxed{82.1\,\text{mrad}}$$

***62** •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for λ with $m = 1$ to express the location of the first-order maximum as a function of the angles at which the first-order images are found.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where d is the separation of the slits and m = 0, 1, 2, …

Solve for λ:

$$\lambda = \frac{d\sin\theta}{m}$$

Relate the number of slits N per centimeter to the separation d of the slits:

$$N = \frac{1}{d}$$

Let m =1 and substitute for d to obtain:

$$\lambda = \frac{d\sin\theta}{N}$$

Substitute numerical values and evaluate λ_1 for $\theta_1 = 9.72\times10^{-2}$ rad:

$$\lambda_1 = \frac{\sin\left(9.72\times10^{-2}\text{ rad}\right)}{2000\text{ cm}^{-1}} = \boxed{485\text{ nm}}$$

Substitute numerical values and evaluate λ_1 for $\theta_2 = 1.32\times10^{-1}$ rad:

$$\lambda_1 = \frac{\sin\left(1.32\times10^{-1}\text{ rad}\right)}{2000\text{ cm}^{-1}} = \boxed{658\text{ nm}}$$

63 •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for θ with m = 1 to express the location of the first-order maximum as a function of the wavelength of the light.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where d is the separation of the slits and m = 0, 1, 2, …

Solve for the angular location θ_m of the maxima :

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Relate the number of slits N per centimeter to the separation d of the slits:

$$N = \frac{1}{d}$$

Substitute to obtain:

$$\theta_m = \sin^{-1}(mN\lambda)$$

Evaluate θ_1 for λ = 434 nm:

$$\theta_1 = \sin^{-1}\left[\left(15000\text{ cm}^{-1}\right)(434\text{ nm})\right] = 0.7089\text{ rad} = \boxed{40.6°}$$

Evaluate θ_1 for λ = 410 nm:

$$\theta_1 = \sin^{-1}\left[\left(15000\text{ cm}^{-1}\right)(410\text{ nm})\right] = 0.6624\text{ rad} = \boxed{38.0°}$$

64 •

Picture the Problem We can use the grating equation with $\sin\theta = 1$ and $m = 5$ to find the longest wavelength that can be observed in the fifth-order spectrum with the given grating spacing.

The interference maxima are at angles θ given by:

$$d\sin\theta = m\lambda, \quad m = 1, 2, 3, \ldots$$

Solve for λ:

$$\lambda = \frac{d\sin\theta}{m}$$

Evaluate λ for $\sin\theta = 1$ and $m = 5$:

$$\lambda = \frac{d}{5} = \frac{\frac{1}{4000\,\text{cm}^{-1}}}{5} = \boxed{500\,\text{nm}}$$

65 •

Picture the Problem We can use the grating equation to find the angle at which normally incident blue light will be diffracted by the *Morpho's* wings.

The grating equation is:

$$d\sin\theta = m\lambda$$

where $m = 1, 2, 3, \ldots$

Solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Substitute numerical values and evaluate θ_1:

$$\theta = \sin^{-1}\left[\frac{(1)(440\,\text{nm})}{880\,\text{nm}}\right] = \boxed{30.0°}$$

66 ••

Picture the Problem We can use the grating equation to find the angular separation of the first-order spectrum of the two lines. In (b) we can apply the definition of the resolving power of the grating to find the width of the grating that must be illuminated for the lines to be resolved.

(*a*) Express the angular separation in the first-order spectrum of the two lines:

$$\Delta\theta = \theta_{579} - \theta_{577}$$

Solve the grating equation for θ:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Substitute to obtain:

$$\Delta\theta = \sin^{-1}\left[\frac{m(579\,\text{nm})}{\frac{1}{2000\,\text{cm}^{-1}}}\right] - \sin^{-1}\left[\frac{m(577\,\text{nm})}{\frac{1}{2000\,\text{cm}^{-1}}}\right]$$

For $m = 1$:

$$\Delta\theta = \sin^{-1}\left[\frac{(1)(579\,\text{nm})}{\frac{1}{2000\,\text{cm}^{-1}}}\right] - \sin^{-1}\left[\frac{(1)(577\,\text{nm})}{\frac{1}{2000\,\text{cm}^{-1}}}\right] = \boxed{0.0231°}$$

(*b*) Express the width of the beam necessary for these lines to be resolved:

$$w = Nd \qquad (1)$$

Relate the resolving power of the diffraction grating to the number of slits *N* that must be illuminated in order to resolve these wavelengths in the *m*th order:

$$\frac{\lambda}{\Delta\lambda} = mN$$

For $m = 1$:

$$N = \frac{\lambda}{\Delta\lambda}$$

Substitute in equation (1) to obtain:

$$w = \frac{\lambda d}{\Delta\lambda}$$

Letting λ be the average of the two wavelengths, substitute numerical values and evaluate *w*:

$$w = \frac{(578\,\text{nm})\left(\frac{1}{2000\,\text{cm}^{-1}}\right)}{2\,\text{nm}} = \boxed{1.45\,\text{mm}}$$

***67 ••**

Picture the Problem We can use the grating equation $d\sin\theta = m\lambda, m = 1, 2, 3, \ldots$ to express the order number in terms of the slit separation *d*, the wavelength of the light λ, and the angle θ.

The interference maxima in the diffraction pattern are at angles θ

$$d\sin\theta = m\lambda, m = 1, 2, 3, \ldots$$

given by:

Solve for m:

$$m = \frac{d\sin\theta}{\lambda}$$

If one is to see the complete spectrum:

$$\sin\theta \le 1 \text{ and } m \le \frac{d}{\lambda}$$

Evaluate m_{max}:

$$m_{max}\ \frac{\dfrac{1}{4800\,\text{cm}^{-1}}}{\lambda_{max}} = \frac{\dfrac{1}{4800\,\text{cm}^{-1}}}{700\,\text{nm}} = 2.98$$

Because $m_{max} = 2.98$, one can see the complete spectrum only for $m = 1$ and 2.

Express the condition for overlap:

$$m_1\lambda_1 \ge m_2\lambda_2$$

Because 700 nm < 2×400 nm, there is no overlap of the second-order spectrum into the first-order spectrum; however, there is overlap of long wavelengths in the second order with short wavelengths in the third-order spectrum.

68 ••

Picture the Problem We can use the grating equation and the resolving power of the grating to derive an expression for the angle at which you should look to see a wavelength of 510 nm in the fourth order.

The interference maxima in the diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda, m = 1, 2, 3, \ldots \qquad (1)$$

The resolving power R is given by:

$$R = mN$$

where N is the number of slits and m is the order number.

Relate d to the width w of the grating:

$$d = \frac{w}{N}$$

Substitute for N to obtain:

$$d = \frac{mw}{R}$$

Substitute for d in equation (1) to obtain:

$$\frac{mw}{R}\sin\theta = m\lambda$$

Solve for θ:

$$\theta = \sin^{-1}\left(\frac{R\lambda}{w}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(22{,}000)(510\,\text{nm})}{5\,\text{cm}}\right] = \boxed{13.0°}$$

69 ••

Picture the Problem The distance on the screen to the *m*th bright fringe can be found using $y_m = m\lambda L/d$,where d is the slit separation. We can use $\theta_{\text{min}} = \lambda/Nd = \Delta y/2L$ to find the width of the central maximum and the $R = mN$, where N is the number of slits in the grating, to find the resolution in the first order.

(*a*) The distance on the screen to the *m*th bright fringe is given by:

$$y_m = m\frac{\lambda L}{d}$$

or, because $d = n^{-1}$,

$$y_m = mn\lambda L$$

Substitute numerical values to obtain:

$$y_m = m\left(4000\,\text{cm}^{-1}\right)(589\,\text{nm})(1.5\,\text{m}) = (0.353\,\text{m})m$$

Evaluate y_1 and y_2:

$$y_1 = (0.353\,\text{m})(1) = \boxed{0.353\,\text{m}}$$

and

$$y_2 = (0.353\,\text{m})(2) = \boxed{0.706\,\text{m}}$$

(*b*) The angle θ_{min} that locates the first minima in the diffraction pattern is given by:

$$\theta_{\text{min}} = \frac{\lambda}{Nd} = \frac{\Delta y}{2L}$$

where Δy is the width of the central maximum.

Solve for Δy:

$$\Delta y = \frac{2L\lambda}{Nd}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{2(1.5\,\text{m})(589\,\text{nm})}{(8000\,\text{lines})\left(\dfrac{1}{4000\,\text{cm}^{-1}}\right)} = \boxed{88.4\,\mu\text{m}}$$

(*c*) The resolution *R* in the *m*th order is given by:

$$R = mN$$

Substitute numerical values and evaluate *R*:

$$R = (1)(8000) = \boxed{8000}$$

70 ••

Picture the Problem The width of the grating *w* is the product of its number of lines *N* and the separation of its slits *d*. Because the resolution of the grating is a function of the average wavelength, the difference in the wavelengths, and the order number, we can express *w* in terms of these quantities.

Express the width *w* of the grating as a function of the number of lines *N* and the slit separation *d*:

$$w = Nd$$

The resolving power *R* of the grating is given by:

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

Solve for *N* to obtain:

$$N = \frac{\lambda}{m\Delta\lambda}$$

Substitute for *N* in the expression for *w* to obtain:

$$w = \frac{\lambda d}{m\Delta\lambda}$$

Letting λ be the average of the given wavelengths, substitute numerical values and evaluate *w*:

$$w = \frac{\frac{1}{2}(519.313\,\text{nm} + 519.322\,\text{nm})\left(\dfrac{1}{8400\,\text{cm}^{-1}}\right)}{2(519.322\,\text{nm} - 519.313\,\text{nm})} = \boxed{3.43\,\text{cm}}$$

***71** ••

Picture the Problem We can use the expression for the resolving power of a grating to find the resolving power of the grating capable of resolving these two isotopic lines in the third-order spectrum. Because the total number of the slits of the grating *N* is related to width *w* of the illuminated region and the number of lines per centimeter of the grating and the resolving power *R* of the grating, we can use this relationship to find the number of lines per centimeter of the grating

The resolving power of a diffraction grating is given by:

$$R = \frac{\lambda}{|\Delta\lambda|} = mN \qquad (1)$$

Substitute numerical values and evaluate *R*:

$$R = \frac{546.07532}{|546.07532 - 546.07355|} = \boxed{3.09 \times 10^5}$$

Express *n*, be the number of lines per centimeter of the grating, in terms of the total number of slits *N* of the grating and the width *w* of the grating:

$$n = \frac{N}{w}$$

From equation (1) we have:

$$N = \frac{R}{m}$$

Substitute to obtain:

$$n = \frac{R}{mw}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{3.09 \times 10^5}{(3)(2\,\text{cm})} = \boxed{5.15 \times 10^4\ \text{cm}^{-1}}$$

72 ••

Picture the Problem We can differentiate the grating equation implicitly to obtain an expression for the number of lines per centimeter *n* as a function of $\cos\theta$ and $d\theta/d\lambda$. We can use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and the grating equation to write $\cos\theta$ in terms of *n*, *m*, and λ. Making this substitution and approximating $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$ will yield an expression for *n* in terms of *m*, λ, $\Delta\lambda$, and $\Delta\theta$.

(*a*) The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, \ldots \qquad (1)$$

Differentiate both sides of this equation with respect to λ:

$$\frac{d}{d\lambda}(d\sin\theta) = \frac{d}{d\lambda}(m\lambda)$$

or

$$d\cos\theta\frac{d\theta}{d\lambda} = m$$

Because $n = 1/d$:

$$\cos\theta\frac{d\theta}{d\lambda} = nm$$

Solve for n to obtain: $$n=\frac{1}{m}\cos\theta\frac{d\theta}{d\lambda}$$

Approximate $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$: $$n=\frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\cos\theta$$

Substitute for $\cos\theta$: $$n=\frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\sqrt{1-\sin^2\theta}$$

From equation (1): $$\sin\theta=\frac{m\lambda}{d}=nm\lambda$$

Substitute to obtain: $$n=\frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\sqrt{1-n^2m^2\lambda^2}$$

Solve for n: $$n=\frac{1}{m\sqrt{\lambda^2+\left(\frac{\Delta\lambda}{\Delta\theta}\right)^2}}$$

Substitute numerical values and evaluate n:

$$n=\frac{1}{3\sqrt{\left(\frac{480\,\text{nm}+500\,\text{nm}}{2}\right)^2+\left(\frac{500\,\text{nm}-480\,\text{nm}}{12^\circ\times\frac{\pi\,\text{rad}}{180^\circ}}\right)^2}}=6.677\times10^5\ \text{m}^{-1}$$

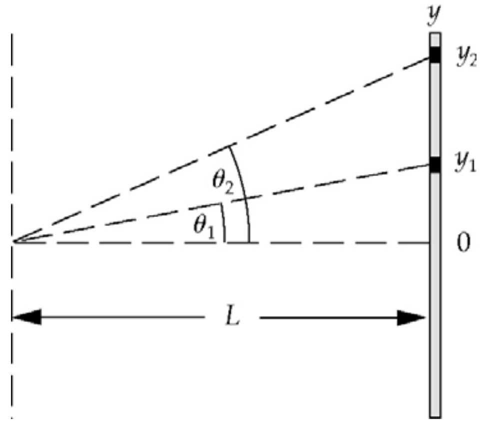
$$=\boxed{6677\,\text{cm}^{-1}}$$

(*b*) Express m_{max} in terms of d and λ_{max}: $$m_{\text{max}}=\frac{d}{\lambda_{\text{max}}}=\frac{1}{n\lambda_{\text{max}}}$$

Substitute numerical values and evaluate m_{max}: $$m_{\text{max}}=\frac{1}{\left(6677\,\text{cm}^{-1}\right)\left(500\,\text{nm}\right)}=\boxed{3}$$

73 ••

Picture the Problem We can use the grating equation and the geometry of the diagram to derive an expression for the separation $\Delta y = y_2 - y_1$ of the spectral lines in terms of the distance L to the screen, the wavelengths of the resolved lines, and the number of grating slits per centimeter n. We will assume that the angle θ_2 is small and then verify that this is a justified assumption.



(*a*) The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, ...$$

Assuming that $\theta_2 << 1$ and $m = 2$:

$$\sin\theta_2 \approx \tan\theta_2 = \frac{y}{L}$$

Substitute to obtain:

$$d\frac{y}{L} = m\lambda$$

Solve for y:

$$y = \frac{mL\lambda}{d}$$

Letting the numerals 1 and 2 refer to the spectral lines, express $y_2 - y_1$:

$$\Delta y = y_2 - y_1 = \frac{mL}{d}(\lambda_2 - \lambda_1)$$

Solve for d to obtain:

$$d = \frac{mL}{y_2 - y_1}(\lambda_2 - \lambda_1)$$

The number of lines per centimeter n is the reciprocal of d:

$$n = \frac{y_2 - y_1}{mL(\lambda_2 - \lambda_1)}$$

Substitute numerical values and evaluate n:

$$n = \frac{8.4\,\text{cm}}{(2)(8\,\text{m})(590\,\text{nm} - 520\,\text{nm})}$$
$$= \boxed{750\,\text{cm}^{-1}}$$

To confirm our assumption that $\theta_2 << 1$, solve the grating equation for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda}{d}\right) = \sin^{-1}(2\lambda n)$$

Substitute numerical values and evaluate θ_2:

$$\theta_2 = \sin^{-1}\left[2(590\,\text{nm})(750\,\text{cm}^{-1})\right] = 8.86\times10^{-2} << 1$$

Because θ_2 << 1:

$\sin\theta_2 \approx \tan\theta_2 \approx \theta_2$, as was assumed above.

(*b*) The separation of the wavelengths is given by:

$$\Delta y = \frac{mL}{d}(\lambda_2 - \lambda_1) = mLn(\lambda_2 - \lambda_1)$$

For $m = 1$:

$$\Delta y = (1)(8\,\text{m})(750\,\text{cm}^{-1})(590\,\text{nm} - 520\,\text{nm}) = \boxed{4.20\,\text{cm}}$$

For $m = 3$:

$$\Delta y = (3)(8\,\text{m})(750\,\text{cm}^{-1})(590\,\text{nm} - 520\,\text{nm}) = \boxed{12.6\,\text{cm}}$$

74 •••

Picture the Problem We can differentiate the grating equation implicitly and approximate $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$ to obtain an expression $\Delta\theta$ as a function of *m*, *n*, $\Delta\lambda$, and $\cos\theta$. We can use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and the grating equation to write $\cos\theta$ in terms of *n*, *m*, and λ. Making these substitutions will yield the given equation.

The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, \ldots \qquad (1)$$

Differentiate both sides of this equation with respect to λ:

$$\frac{d}{d\lambda}(d\sin\theta) = \frac{d}{d\lambda}(m\lambda)$$

or

$$d\cos\theta\frac{d\theta}{d\lambda} = m$$

Because $n = 1/d$:

$$\cos\theta\frac{d\theta}{d\lambda} = nm$$

Solve for *n* to obtain:

$$n = \frac{1}{m}\cos\theta\frac{d\theta}{d\lambda}$$

Approximate $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$:

$$n = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\cos\theta$$

Solve for $\Delta\theta$:

$$\Delta\theta = \frac{nm\Delta\lambda}{\cos\theta}$$

Substitute for $\cos\theta$:

$$\Delta\theta = \frac{nm\Delta\lambda}{\sqrt{1-\sin^2\theta}}$$

From equation (1):

$$\sin\theta = \frac{m\lambda}{d} = nm\lambda$$

Substitute to obtain:

$$\Delta\theta = \frac{nm\Delta\lambda}{\sqrt{1-n^2m^2\lambda^2}}$$

Simplify by dividing the numerator and denominator by *nm*:

$$\Delta\theta = \frac{\Delta\lambda}{\frac{1}{nm}\sqrt{1-n^2m^2\lambda^2}} = \frac{\Delta\lambda}{\sqrt{\frac{1-n^2m^2\lambda^2}{n^2m^2}}} = \boxed{\frac{\Delta\lambda}{\sqrt{\frac{1}{n^2m^2}-\lambda^2}}}$$

75 •••

Picture the Problem We can use the grating equation and the geometry of the grating to derive an expression for ϕ_m in terms of the order number *m*, the wavelength of the light λ, and the groove separation *a*.

(*a*) The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, \ldots \qquad (1)$$

Because ϕ and θ_i have their left and right sides mutually perpendicular:

$$\theta_i = \phi_m$$

Substitute to obtain:

$$d\sin\phi_m = m\lambda$$

Solve for ϕ_m:

$$\phi_m = \boxed{\sin^{-1}\left(\frac{m\lambda}{d}\right)}$$

(*b*) For *m* = 2:

$$\phi_2 = \sin^{-1}\left(\frac{(2)(450\,\text{nm})}{\frac{1}{10{,}000\,\text{cm}^{-1}}}\right) = \boxed{64.2°}$$

76 •••

Picture the Problem We can follow the procedure outlined in the problem statement to obtain $R = \lambda/\Delta\lambda = mN$.

(*a*) Express the relationship between the phase difference ϕ and the path difference Δr:

$$\frac{\phi}{2\pi} = \frac{\Delta r}{\lambda} \Rightarrow \phi = \frac{2\pi\Delta r}{\lambda}$$

Because $\Delta r = d\sin\theta$:

$$\phi = \boxed{\frac{2\pi d}{\lambda}\sin\theta}$$

(*b*) Differentiate this expression with respect to θ to obtain:

$$\frac{d\phi}{d\theta} = \frac{d}{d\theta}\left[\frac{2\pi d}{\lambda}\sin\theta\right] = \frac{2\pi d}{\lambda}\cos\theta$$

Solve for $d\phi$:

$$d\phi = \boxed{\frac{2\pi d}{\lambda}\cos\theta\, d\theta}$$

(*c*) From (*b*):

$$d\theta = \frac{\lambda d\phi}{2\pi d\cos\theta}$$

Substitute $2\pi/N$ for $d\phi$ to obtain:

$$d\theta = \boxed{\frac{\lambda}{Nd\cos\theta}} \qquad \text{33-30}$$

(*d*) Equation 33-27 is:

$$d\sin\theta = m\lambda, m = 0, 1, 2, \ldots$$

Differentiate this expression implicitly with respect to λ to obtain:

$$\frac{d}{d\lambda}[d\sin\theta] = \frac{d}{d\lambda}[m\lambda]$$

or

$$d\cos\theta\frac{d\theta}{d\lambda} = m$$

Solve for $d\theta$ to obtain:

$$d\theta = \boxed{\frac{m d\lambda}{d\cos\theta}} \qquad \text{33-31}$$

(*e*) Equate the two expressions for $d\theta$ obtained in (*c*) and (*d*):

$$\frac{\lambda}{Nd\cos\theta} = \frac{m d\lambda}{d\cos\theta}$$

Solve for $R = \lambda/\Delta\lambda$:

$$R = \boxed{\frac{\lambda}{d\lambda} = mN}$$

General Problems

***77** •

Picture the Problem We can apply the condition for constructive interference to find the angular position of the first maximum on the screen. Note that, due to reflection, the wave from the image is 180° out of phase with that from the source.

(*a*) Because $y_0 << L$, the distance from the mirror to the first maximum is given by:

$$y_0 = L\theta_0 \quad (1)$$

Express the condition for constructive interference:

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda, m = 0, 1, 2, \ldots$$

Solve for θ:

$$\theta = \sin^{-1}\left[\left(m + \tfrac{1}{2}\right)\frac{\lambda}{d}\right]$$

For the first maximum, $m = 0$ and:

$$\theta_0 = \sin^{-1}\left[\left(\tfrac{1}{2}\right)\frac{\lambda}{d}\right]$$

Substitute in equation (1) to obtain:

$$y_0 = L\sin^{-1}\left[\left(\tfrac{1}{2}\right)\frac{\lambda}{d}\right]$$

Because the image of the slit is as far behind the mirror's surface as the slit is in front of it, $d = 2$ mm. Substitute numerical values and evaluate y_0:

$$y_0 = (1\,\text{m})\sin^{-1}\left[\left(\tfrac{1}{2}\right)\frac{600\,\text{nm}}{2\,\text{mm}}\right] = \boxed{0.150\,\text{mm}}$$

(*b*) The separation of the fringes on the screen is given by:

$$\Delta y = \frac{\lambda L}{d}$$

The number of dark bands per centimeter is the reciprocal of the fringe separation:

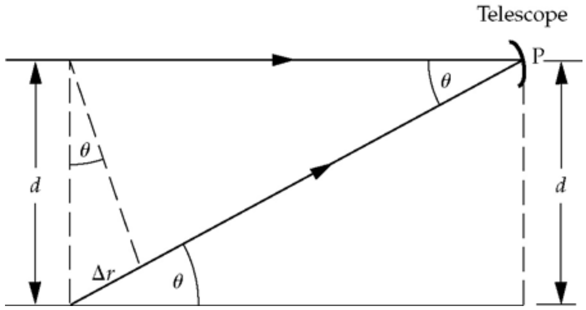
$$n = \frac{1}{\Delta y} = \frac{d}{\lambda L}$$

Substitute numerical values and evaluate n:

$$n = \frac{2\,\text{mm}}{(600\,\text{nm})(1\,\text{m})} = \boxed{3.33\times10^{3}\,\text{m}^{-1}}$$

78 ••

Picture the Problem The light from the radio galaxy reaches the radio telescope by two paths; one coming directly from the galaxy and the other reflected from the surface of the lake. The latter is phase shifted 180°, relative to the former, by reflection from the surface of the lake. We can use the condition for constructive interference of two waves to find the angle above the horizon at which the light from the galaxy will interfere constructively.



Because the reflected light is phase shifted by 180°, the condition for constructive interference at point P is:

$$\Delta r = \left(m + \tfrac{1}{2}\right)\lambda$$
where $m = 0, 1, 2, \ldots$

Referring to the figure, note that:

$$\sin\theta \approx \frac{\Delta r}{d} \Rightarrow \theta = \sin^{-1}\left[\frac{\Delta r}{d}\right]$$

Substitute for Δr to obtain:

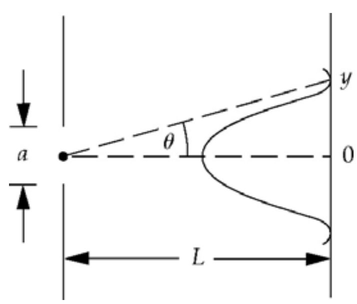
$$\theta = \sin^{-1}\left[\frac{\left(m + \frac{1}{2}\right)\lambda}{d}\right]$$

Noting that $m = 0$ for the first interference maximum, substitute numerical values and evaluate θ_0:

$$\theta_0 = \sin^{-1}\left[\frac{\frac{1}{2}(20\,\text{cm})}{20\,\text{m}}\right] = 5.00\times10^{-3}\,\text{rad}$$
$$= \boxed{0.286°}$$

79 •

Picture the Problem We can use the condition determining the location of points of zero intensity in a diffraction pattern to express the location of the first zero in terms of y and L. The width of the central maximum can then be found from $\Delta y = 2y$.



Express the horizontal length of the principal diffraction maximum on the screen:

$$\Delta y = 2y \qquad (1)$$

Referring to the diagram, relate the angle θ to the distances y and L:

$$\tan\theta = \frac{y}{L}$$
or, because $\theta \ll 1$, $\tan\theta \approx \sin\theta$ and

$$\sin\theta = \frac{y}{L}$$

The points of zero intensity for a single-slit diffraction pattern are determined by the condition:

$$a\sin\theta = m\lambda, m = 1, 2, \ldots$$

Substitute for $\sin\theta$ to obtain:

$$\frac{ay}{L} = m\lambda$$

Solve for y:

$$y = m\frac{\lambda L}{a}$$

Substitute for y in equation (1):

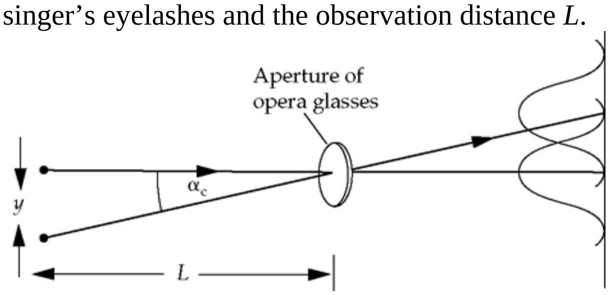
$$\Delta y = 2m\frac{\lambda L}{a}$$

At the first diffraction minimum, m = 1. Substitute numerical values and evaluate Δy:

$$\Delta y = 2(1)\frac{(700\,\text{nm})(6\,\text{m})}{0.5\,\text{mm}} = \boxed{1.68\,\text{cm}}$$

80 •

Picture the Problem We can use the Rayleigh criterion to express α_c in terms of λ and the diameter of the opera glasses lens D and the geometry of the problem to relate α_c to separation y of the singer's eyelashes and the observation distance L.



The critical angular separation, according to Rayleigh's criterion, is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Given that $\alpha_c << 1$, it is also given by:

$$\alpha_c \approx \frac{y}{L}$$

Equating these two expressions yields:

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

Solve for D to obtain:

$$D = 1.22\frac{\lambda L}{y}$$

Substitute numerical values and evaluate D:

$$D = 1.22\frac{(550\,\text{nm})(25\,\text{m})}{0.5\,\text{mm}} = \boxed{33.6\,\text{mm}}$$

81 •

Picture the Problem The resolving power of a telescope is the ability of the instrument to resolve two objects that are close together. Hence we can use Rayleigh's criterion as the resolving power of the Arecibo telescope.

Rayleigh's criterion for resolution is:

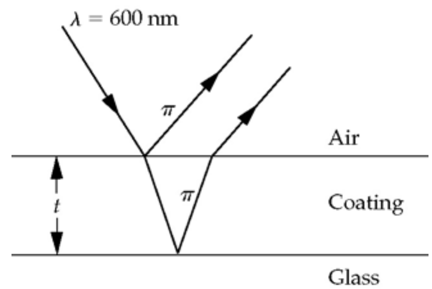
$$\alpha_c = 1.22\frac{\lambda}{D}$$

Substitute numerical values and evaluate α_c:

$$\alpha_c = 1.22\frac{3.2\,\text{cm}}{300\,\text{m}} = \boxed{0.130\,\text{mrad}}$$

***82** ••

Picture the Problem Note that reflection at both surfaces involves a phase shift of π rad. We can apply the condition for destructive interference to find the thickness t of the nonreflective coating.



The condition for destructive interference is:

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_{\text{coating}} = \left(m + \tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{n_{\text{coating}}}$$

Solve for t:

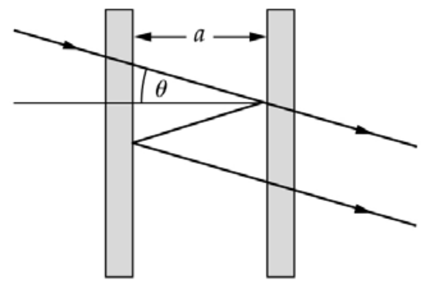
$$t = \left(m + \tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{2n_{\text{coating}}}$$

Evaluate t for $m = 0$:

$$t = \left(\tfrac{1}{2}\right)\frac{600\,\text{nm}}{2(1.30)} = \boxed{115\,\text{nm}}$$

83 ••

Picture the Problem The *Fabry-Perot interferometer* is shown in the figure. For constructive interference in the transmitted light the path difference must be an integral multiple of the wavelength of the light. This path difference can be found using the geometry of the interferometer.



Express the path difference between the two rays that emerge from the interferometer:

$$\Delta r = \frac{2a}{\cos\theta}$$

For constructive interference we Require that:

$$\Delta r = m\lambda, m = 0, 1, 2, \ldots$$

Equate these expressions to obtain:

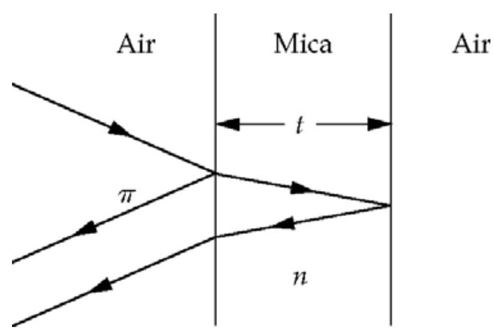
$$m\lambda = \frac{2a}{\cos\theta}$$

Solve for *a* to obtain:

$$a = \boxed{\frac{m\lambda}{2}\cos\theta}$$

84 ••

Picture the Problem The gaps in the spectrum of the visible light are the result of destructive interference between the incident light and the reflected light. Noting that there is a π rad phase shift at the first air-mica interface, we can use the condition for destructive interference to find the index of refraction *n* of the mica sheet.



Because there is a π rad phase shift at the first air-mica interface, the condition for destructive interference is:

$$2t = m\lambda_{\text{mica}} = m\frac{\lambda_{\text{air}}}{n}, m = 1, 2, 3, \ldots$$

Solve for *n*:

$$n = m\frac{\lambda_{\text{air}}}{2t} \qquad (1)$$

For λ = 474 nm:

$$2t = (474\,\text{nm})m$$

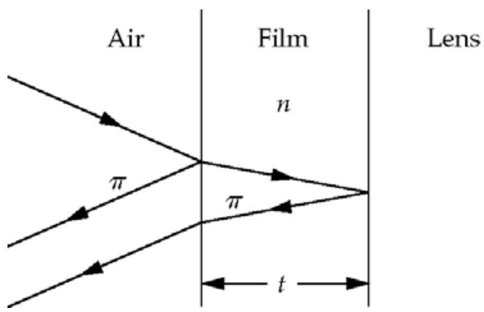
For λ = 421 nm: $2t = (421\,\text{nm})(m+1)$

Equate these two expressions for 2*t* and solve for *m* to obtain: $m = 8$ for λ = 474 nm

Substitute numerical values in equation (1) and evaluate *n*: $n = 8\dfrac{474\,\text{nm}}{2(1.2\,\mu\text{m})} = \boxed{1.58}$

85 ••

Picture the Problem Note that the light reflected at both the air-film and film-lens interfaces undergoes a π rad phase shift. We can use the condition for destructive interference between the light reflected from the air-film interface and the film-lens interface to find the thickness of the film. In (*c*) we can find the factor by which light of the given wavelengths is reduced by this film from $I \propto \cos^2 \frac{1}{2}\delta$.



(*a*) Express the condition for destructive interference between the light reflected from the air-film interface and the film-lens interface:

$$2t = \left(m+\tfrac{1}{2}\right)\lambda_{\text{film}} = \left(m+\tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{n} \qquad (1)$$

where *m* = 0, 1, 2, …

Solve for *t*: $t = \left(m+\tfrac{1}{2}\right)\dfrac{\lambda_{\text{air}}}{2n}$

Evaluate *t* for *m* = 0: $t = \left(\dfrac{1}{2}\right)\dfrac{540\,\text{nm}}{2(1.38)} = \boxed{97.8\,\text{nm}}$

(*b*) Solve equation (1) for λ_{air}: $\lambda_{\text{air}} = \dfrac{2tn}{m+\frac{1}{2}}$

Evaluate λ_{air} for *m* = 1: $\lambda_{\text{air}} = \dfrac{2(97.8\,\text{nm})(1.38)}{1+\frac{1}{2}} = 180\,\text{nm}$

$$\boxed{\text{No; because 180 nm is not in the visible portion of the spectrum.}}$$

(*c*) Express the reduction factor *f* as $f = \cos^2 \frac{1}{2}\delta \qquad (2)$

a function of the phase difference δ between the two reflected waves:

Relate the phase difference to the path difference Δr:

$$\frac{\delta}{2\pi}=\frac{\Delta r}{\lambda_{\text{film}}}\Rightarrow \delta=2\pi\left(\frac{\Delta r}{\lambda_{\text{film}}}\right)$$

Because $\Delta r = 2t$:

$$\delta=2\pi\left(\frac{2t}{\lambda_{\text{film}}}\right)$$

Substitute in equation (2) to obtain:

$$f=\cos^2\left[\tfrac{1}{2}2\pi\left(\frac{2t}{\lambda_{\text{film}}}\right)\right]=\cos^2\left[\frac{2\pi t}{\lambda_{\text{film}}}\right]$$
$$=\cos^2\left[\frac{2\pi nt}{\lambda_{\text{air}}}\right]$$

Evaluate f for $\lambda = 400$ nm:

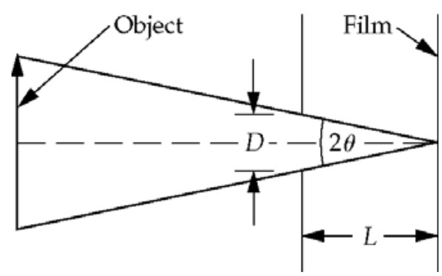
$$f_{400}=\cos^2\left[\frac{2\pi(1.38)(97.8\,\text{nm})}{400\,\text{nm}}\right]$$
$$=\boxed{0.273}$$

Evaluate f for $\lambda = 700$ nm:

$$f_{400}=\cos^2\left[\frac{2\pi(1.38)(97.8\,\text{nm})}{700\,\text{nm}}\right]$$
$$=\boxed{0.124}$$

86 ••

Picture the Problem As indicated in the problem statement, we can find the optimal size of the pinhole by equating the angular width of the object at the film and the angular width of the diffraction pattern.



Express the angular width of the a distant object at the film in terms of the diameter D of the pinhole and the distance L from the pinhole to the object:

$$2\theta=\frac{D}{L}\Rightarrow\theta=\frac{D}{2L}$$

Using Rayleigh's criterion, express the angular width of the diffraction

$$\theta_{\text{diffraction}}=1.22\frac{\lambda}{D}$$

pattern:

Equate these two expressions to obtain:

$$\frac{D}{2L} = 1.22\frac{\lambda}{D}$$

Solving for *D* yields:

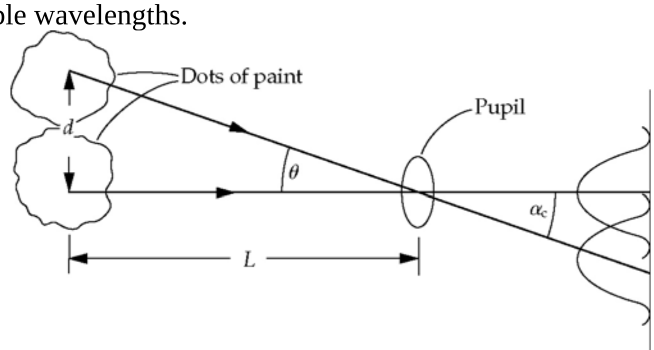
$$D = \sqrt{2.44\lambda L}$$

Substitute numerical values and evaluate *D*:

$$D = \sqrt{2.44(550\,\text{nm})(10\,\text{cm})}$$
$$= \boxed{0.366\,\text{mm}}$$

***87 ••**

Picture the Problem We can use the geometry of the dots and the pupil of the eye and Rayleigh's criterion to find the greatest viewing distance that ensures that the effect will work for all visible wavelengths.



Referring to the diagram, express the angle subtended by the adjacent dots:

$$\theta \approx \frac{d}{L}$$

Letting the diameter of the pupil of the eye be *D*, apply Rayleigh's criterion to obtain:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Set $\theta = \alpha_c$ to obtain:

$$\frac{d}{L} = 1.22\frac{\lambda}{D}$$

Solve for *L*:

$$L = \frac{Dd}{1.22\lambda}$$

Evaluate *L* for the *shortest* wavelength light in the visible portion of the spectrum:

$$L = \frac{(3\,\text{mm})(2\,\text{mm})}{1.22(400\,\text{nm})} = \boxed{12.3\,\text{m}}$$

***88 •••**

Picture the Problem It is given that with one tube evacuated and one full of air at 1-atm pressure, there are 198 more wavelengths of light in the tube full of air than in the evacuated tube of the same length. We can use this condition to obtain an equation that expresses this difference in terms of L, λ_n, and λ_0. We can obtain a second equation relating λ_n, n, and λ_0 ($\lambda_n = \frac{\lambda_0}{n}$) and solve the two equations simultaneously to find n.

(*a*) The wavelengths are related by:

$$\lambda_n = \frac{\lambda_0}{n}$$

The number of wavelengths in length L is the length L divided by the wavelength. Thus:

$$\frac{L}{\lambda_n} - \frac{L}{\lambda_0} = 198$$

Substitute for λ_n:

$$\frac{nL}{\lambda_0} - \frac{L}{\lambda_0} = 198$$

Solve for λ_n to obtain:

$$n = 1 + \frac{198\lambda_0}{L}$$

Substitute numerical values and evaluate n:

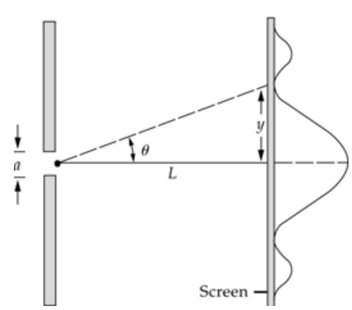
$$n = 1 + 198\left(\frac{589\text{ nm}}{0.4\text{ m}}\right) = \boxed{1.0002916}$$

(*b*) Replace 198 with 198 ± 0.25 and assume that the uncertainties in L and λ_0 are negligible:

$$n = 1 + \frac{\lambda_0}{L}(198 \pm 0.25) = \boxed{1.0002916 \pm 0.0000004}$$

89 •••

Picture the Problem We can use the condition that determines points of zero intensity for a single slit diffraction pattern and the geometry of the slit and screen shown in the diagram to derive the given width of the central maximum on the screen.



(*a*) The points of zero intensity for a single-slit diffraction pattern are given by:

$$a\sin\theta = m\lambda, m = 1, 2, 3, \ldots \qquad (1)$$

Relate the half-width y of the diffraction pattern to θ and L:

$$\tan\theta = \frac{y}{L}$$

Because θ is very small, $\tan\theta \approx \sin\theta$ and:

$$\sin\theta \approx \frac{y}{L}$$

Substitute for $\sin\theta$ in equation (1) to obtain:

$$a\frac{y}{L} \approx m\lambda$$

Solve for y:

$$y \approx m\frac{\lambda L}{a}$$

The width of the central maximum (m = 1) is:

$$2y \approx \boxed{\frac{2\lambda L}{a}}$$

(b) Set $a = \dfrac{2L\lambda}{a}$ and simplify to obtain:

$$2y \approx \frac{2\lambda L}{\dfrac{2L\lambda}{a}} = \boxed{a}$$

Chapter 34
Wave-Particle Duality and Quantum Physics

Conceptual Problems

***1 •**

Determine the Concept The Young double-slit experiment, the diffraction of light by a small aperture, and the J.J. Thomson cathode-ray experiment all demonstrated the wave nature of electromagnetic radiation. Only the photoelectric effect requires an explanation based on the quantization of electromagnetic radiation. $\boxed{(c)\text{ is correct.}}$

2 ••

Determine the Concept Since the power radiated by a source is the energy radiated per unit area and per unit time, it is directly proportional to the energy. The energy radiated varies inversely with the wavelength ($E = hc/\lambda$); i.e., the longer the wavelength, the less energy is associated with the electromagnetic radiation. $\boxed{(b)\text{ is correct.}}$

3 •

(*a*) True

(*b*) False. The work function of a metal is a property of the metal and is independent of the frequency of the incident light.

(*c*) True

(*d*) True

4 •

Determine the Concept In the photoelectric effect, the number of electrons emitted per second is a function of the light intensity, proportional to the light intensity, independent of the work function of the emitting surface and independent of the frequency of the light. $\boxed{(b)\text{ is correct.}}$

***5 •**

Determine the Concept The threshold wavelength for emission of photoelectrons is related to the work function of a metal through $\phi = hc/\lambda_t$. Hence $\lambda_t = hc/\phi$ and $\boxed{(a)\text{ is correct.}}$

6 ••

Determine the Concept In order for electrons to be emitted hc/λ must be greater than ϕ. Evidently, $hc/\lambda_1 < \phi$, but $hc/\lambda_2 > \phi$.

7 •

(*a*) True

(*b*) True

(*c*) True

(*d*) False. Electrons are too small to be resolved by an electron microscope.

8 •

Determine the Concept If the de Broglie wavelengths of an electron and a proton are equal, their momenta must be equal. Since $m_p > m_e$, $v_p < v_e$. Response (*c*) is correct.

9 •

Picture the Problem The kinetic energy of a particle can be expressed, in terms of its momentum, as $K = \frac{p^2}{2m}$. We can use the equality of the kinetic energies and the fact that $m_e < m_p$ to determine the relative sizes of their de Broglie wavelengths.

Express the equality of the kinetic energies of the proton and electron in terms of their momenta and masses:

$$\frac{p_p^2}{2m_p} = \frac{p_e^2}{2m_e}$$

Use the de Broglie relation for the wavelength of matter waves to obtain:

$$\frac{h^2}{2m_p\lambda_p^2} = \frac{h^2}{2m_e\lambda_e^2}$$

or

$$m_p\lambda_p^2 = m_e\lambda_e^2$$

Since $m_e < m_p$:

$$\lambda_p^2 < \lambda_e^2 \text{ and } \lambda_e > \lambda_p$$

and $\boxed{(c)\text{ is correct.}}$

10 •

Determine the Concept Yes. <*x*> can equal a value for which $P(x)$ is zero. An example is the asymmetric well for all even numbered states.

***11 •**

Determine the Concept In the photoelectric effect, an electron absorbs the energy of a single photon. Therefore, $K_{max} = hf - \phi$, independently of the number of photons incident on the surface. However, the number of photons incident on the surface determines the number of electrons that are emitted.

12 ••

Picture the Problem The probability of a particular event occurring is the number of ways that event can occur divided by the number of possible outcomes. The expectation value, on the other hand, is the average value of the experiment.

(*a*) Find the probability of a 1 coming up when the die is thrown:

$$P(1) = \frac{3}{6} = \boxed{\frac{1}{2}}$$

(*b*) Find the average value of a large number of throws of the die:

$$\langle n \rangle = \frac{3\times 1 + 3\times 2}{6} = \boxed{1.5}$$

13 ••

Determine the Concept According to quantum theory, the average value of many measurements of the same quantity will yield the expectation value of that quantity. However, any single measurement may differ from the expectation value.

Estimation and Approximation

14 ••

Picture the Problem From Einstein's photoelectric equation we have $K_{max} = hf - \phi$, which is of the form $y = mx + b$, where the slope is h and the K_{max}-intercept is the work function. Hence we should plot a graph of K_{max} versus f in order to obtain a straight line whose slope will be an experimental value for Planck's constant.

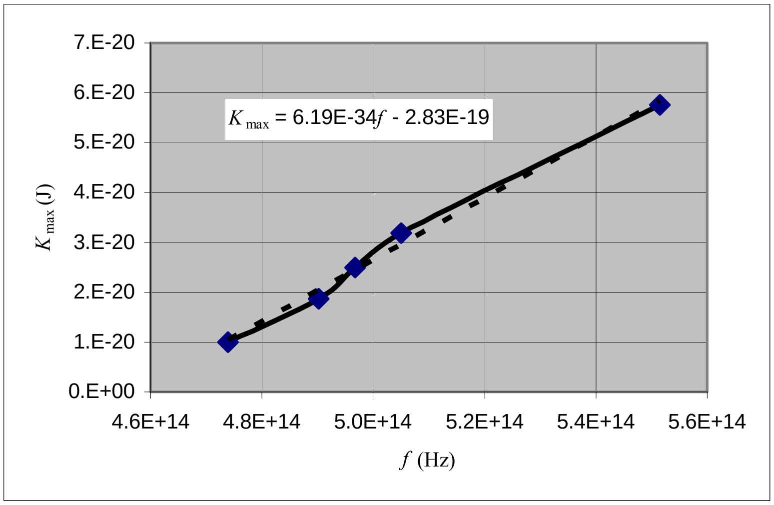
(*a*) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A3	544	λ (nm)
B3	0.36	K_{max} (eV)
C3	A3*10^−19	λ (m)
D3	3*10^8/C3	c/λ
E3	B3*1.6*10^−19	K_{max} (J)

lambda	Kmax	lambda	f=c/lambda	Kmax
(nm)	(eV)	(m)	(Hz)	(J)

544	0.36	5.44E–07	5.51E+14	5.76E–20
594	0.199	5.94E–07	5.05E+14	3.18E–20
604	0.156	6.04E–07	4.97E+14	2.50E–20
612	0.117	6.12E–07	4.90E+14	1.87E–20
633	0.062	6.33E–07	4.74E+14	9.92E–21

The following graph was plotted from the data shown in the above table. Excel's "Add Trendline" was used to fit a linear function to the data.



(*b*) From the regression line we note that the experimental value for Planck's constant is:

$$h_{\text{exp}} = \boxed{6.19\times10^{-34}\ \text{J}\cdot\text{s}}$$

(*c*) Express the percent difference between h_{exp} and h:

$$\%\,\text{diff} = \frac{h - h_{\text{exp}}}{h} = 1 - \frac{h_{\text{exp}}}{h}$$

$$= 1 - \frac{6.19\times10^{-34}\ \text{J}\cdot\text{s}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{6.64\%}$$

15 ••

Picture the Problem From Einstein's photoelectric equation we have $K_{\max} = hf - \phi$, which is of the form $y = mx + b$, where the slope is h and the $K_{\max}$-intercept is the work function. Hence we should plot a graph of $K_{\max}$ versus f in order to obtain a straight line whose intercept will be an experimental value for the work function.

(*a*) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A3	544	λ (nm)

B3	0.36	K_{max} (eV)
C3	A3*10^–19	λ (m)
D3	3*10^8/C3	c/λ
E3	B3*1.6*10^–19	K_{max} (J)

lambda (nm)	Kmax (eV)	lambda (m)	f=c/lambda (Hz)	Kmax (J)
544	0.36	5.44E–07	5.51E+14	5.76E–20
594	0.199	5.94E–07	5.05E+14	3.18E–20
604	0.156	6.04E–07	4.97E+14	2.50E–20
612	0.117	6.12E–07	4.90E+14	1.87E–20
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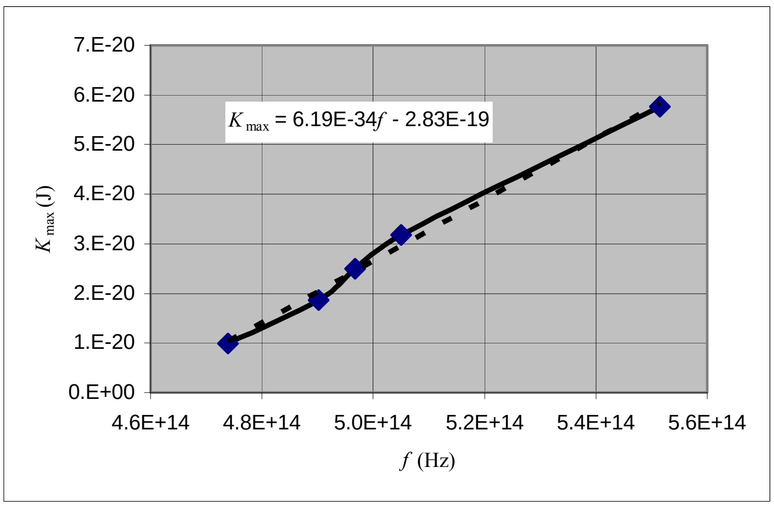
The following graph was plotted from the data shown in the above table. Excel's "Add Trendline" was used to fit a linear function to the data.



(*b*) From the regression line we note that the experimental value for the work function ϕ is:

$$\phi_{exp} = 2.83\times10^{-19}\text{ J}\times\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}$$

$$= \boxed{1.77\text{ eV}}$$

(*c*) $\boxed{\text{The value of } \phi_{exp} = 1.77\text{ eV is closest to the work function for cesium.}}$

***16** ••

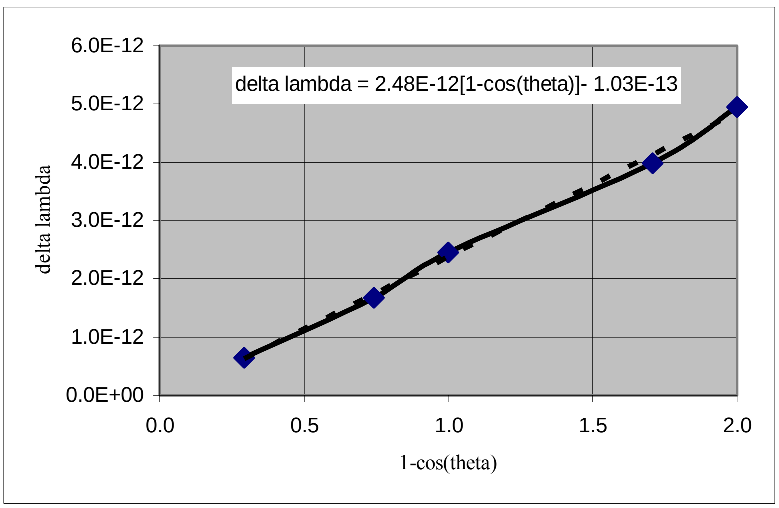
Picture the Problem From the Compton-scattering equation we have $\lambda_2 - \lambda_1 = \lambda_C(1-\cos\theta)$, where $\lambda_C = h/m_e c$ is the Compton wavelength. Note that this equation is of the form $y = mx + b$ provided we let $y = \lambda_2 - \lambda_1$ and $x = 1 - \cos\theta$. Thus, we can linearize the Compton equation by plotting $\Delta\lambda = \lambda_2 - \lambda_1$ as a function of $1-\cos\theta$. The slope of the resulting graph will yield an experimental value for the Compton wavelength.

(*a*) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A3	45	θ (deg)
B3	1 − cos(A3*PI()/180)	$1-\cos\theta$
C3	6.47E^−13	$\Delta\lambda = \lambda_2 - \lambda_1$

θ (deg)	$1-\cos\theta$	$\lambda_2-\lambda_1$
45	0.293	6.47E−13
75	0.741	1.67E−12
90	1.000	2.45E−12
135	1.707	3.98E−12
180	2.000	4.95E−12

The following graph was plotted from the data shown in the above table. Excel's ″Add Trendline″ was used to fit a linear function to the data. The regression line is $\Delta\lambda = 2.48\times10^{-12}(1-\cos\theta)-1.03\times10^{-13}$



From the regression line we note that the experimental value for the Compton wavelength $\lambda_{C,exp}$ is:

$$\lambda_{C,exp} = \boxed{2.48\times10^{-12}\ \text{m}}$$

The Compton wavelength is given by:

$$\lambda_C = \frac{h}{m_e c} = \frac{hc}{m_e c^2}$$

Substitute numerical values and evaluate λ_C:

$$\lambda_C = \frac{1240\ \text{eV}\cdot\text{nm}}{5.11\times10^5\ \text{eV}} = 2.43\times10^{-12}\ \text{m}$$

Express the percent difference between λ_C and $\lambda_{C,exp}$:

$$\%\,\text{diff} = \frac{\lambda_{C,exp} - \lambda_{exp}}{\lambda_{exp}} = \frac{\lambda_{C,exp}}{\lambda_{exp}} - 1$$

$$= \frac{2.48\times10^{-12}\ \text{m}}{2.43\times10^{-12}\ \text{m}} - 1 = \boxed{2.06\%}$$

***17** ••

Picture the Problem The de Broglie wavelength of an object is given by $\lambda = h/p$, where p is the momentum of the object.

The de Broglie wavelength of an object, in terms of its mass m and speed v, is:

$$\lambda = \frac{h}{mv}$$

The values in the following table were obtained using the internet:

Type of ball	m	v_{max}
	(g)	(m/s)
Baseball	142	44
Tennis	57	54
Golf	57	42
Soccer	250	31

The de Broglie wavelength of a baseball, moving with its maximum speed, is:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(0.142\,\text{kg})(44\,\text{m/s})} = 1.06\times10^{-34}\ \text{m}$$

Proceed as above to obtain the values shown in the table:

Type of ball	m	v_{max}	λ
	(g)	(m/s)	(m)
Baseball	142	44	1.06×10^{-34}
Tennis	57	54	2.15×10^{-34}
Golf	57	42	2.77×10^{-34}
Soccer	250	31	0.855×10^{-34}

Examination of the table indicates that the soccer ball has the shortest de Broglie wavelength.

The Particle Nature of Light: Photons

18 •

Picture the Problem We can find the photon energy for an electromagnetic wave of a given frequency f from $E = hf$ where h is Planck's constant.

(*a*) For *f* = 100 MHz:

$$E = hf = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(100\ \text{MHz}) = \boxed{6.63\times10^{-26}\ \text{J}}$$

$$= 6.63\times10^{-26}\ \text{J}\times\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}} = \boxed{4.14\times10^{-7}\ \text{eV}}$$

(*b*) For *f* = 900 kHz:

$$E = hf = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(900\ \text{kHz}) = \boxed{5.96\times10^{-28}\ \text{J}}$$

$$= 5.96\times10^{-28}\ \text{J}\times\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}} = \boxed{3.73\times10^{-9}\ \text{eV}}$$

19 •

Picture the Problem The energy of a photon, in terms of its frequency, is given by $E=hf$.

(*a*) Express the frequency of a photon in terms of its energy and evaluate f for E = 1 eV:

$$f = \frac{E}{h} = \frac{1\ \text{eV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} = \boxed{2.42\times10^{14}\ \text{Hz}}$$

(*b*) For E = 1 keV:

$$f = \frac{1\ \text{keV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} = \boxed{2.42\times10^{17}\ \text{Hz}}$$

(*c*) For E = 1 MeV:

$$f = \frac{1\ \text{MeV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} = \boxed{2.42\times10^{20}\ \text{Hz}}$$

***20** •

Picture the Problem We can use $E = hc/\lambda$ to find the photon energy when we are given the wavelength of the radiation.

(*a*) Express the photon energy as a function of wavelength and evaluate E for λ = 450 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{450\ \text{nm}} = \boxed{2.76\ \text{eV}}$$

(*b*) For λ = 550 nm:

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{550\,\text{nm}} = \boxed{2.25\,\text{eV}}$$

(*c*) For λ = 650 nm:

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{650\,\text{nm}} = \boxed{1.91\,\text{eV}}$$

21 •

Picture the Problem We can use $E = hc/\lambda$ to find the photon energy when we are given the wavelength of the radiation.

(*a*) Express the photon energy as a function of wavelength and evaluate E for λ = 0.1 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.1\,\text{nm}} = \boxed{12.4\,\text{keV}}$$

(*b*) For λ =1 fm:

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{10^{-6}\,\text{nm}} = \boxed{1.24\,\text{GeV}}$$

22 ••

Picture the Problem We can express the density of photons in the beam as the number of photons per unit volume. The number of photons per unit volume is, in turn, the ratio of the power of the laser to the energy of the photons and the volume occupied by the photons emitted in one second is the product of the cross-sectional area of the beam and the speed at which the photons travel, i.e., the speed of light.

Express the density of photons in the beam as a function of the number of photons emitted per second and the volume occupied by those photons:

$$\rho = \frac{N}{V}$$

Relate the number of photons emitted per second to the power of the laser and the energy of the photons:

$$N = \frac{P}{E} = \frac{P\lambda}{hc}$$

Express the volume containing the photons emitted in one second as a function of the cross sectional area of the beam:

$$V = Ac$$

Substitute to obtain:

$$\rho = \frac{P\lambda}{hc^2 A}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{(3\,\text{mW})(632\,\text{nm})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^{8}\ \text{m/s})^2\left(\frac{\pi}{4}(1\,\text{mm})^2\right)} = \boxed{4.05\times10^{13}\ \text{m}^{-3}}$$

***23 •**

Picture the Problem The number of photons per unit volume is, in turn, the ratio of the power of the laser to the energy of the photons and the volume occupied by the photons emitted in one second is the product of the cross-sectional area of the beam and the speed at which the photons travel; i.e., the speed of light.

Relate the number of photons emitted per second to the power of the laser and the energy of the photons:

$$N = \frac{P}{E} = \frac{P\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{(2.5\,\text{mW})(1.55\,\mu\text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^{8}\ \text{m/s})} = \boxed{1.95\times10^{16}\ \text{s}^{-1}}$$

The Photoelectric Effect

24 •

Picture the Problem The threshold wavelength and frequency for emission of photoelectrons is related to the work function of a metal through $\phi = hf_t = hc/\lambda_t$. We can use Einstein's photoelectric equation $K_{max} = \frac{hc}{\lambda} - \phi$ to find the maximum kinetic energy of the electrons for the given wavelengths of the incident light.

(*a*) Express the threshold frequency in terms of the work function for tungsten and evaluate f_t:

$$f_t = \frac{\phi}{h} = \frac{4.58\,\text{eV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} = \boxed{1.11\times10^{15}\ \text{Hz}}$$

Using $v = f\lambda$, express the threshold wavelength in terms of the threshold

$$\lambda_t = \frac{v}{f_t} = \frac{3\times10^{8}\ \text{m/s}}{1.11\times10^{15}\ \text{Hz}} = \boxed{270\,\text{nm}}$$

frequency and evaluate λ_t:

(b) Using Einstein's photoelectric equation, relate the maximum kinetic energy of the electrons to their wavelengths and evaluate K_{max}:

$$K_{max} = E - \phi = hf - \phi = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{200\,\text{nm}} - 4.58\,\text{eV}$$
$$= \boxed{1.62\,\text{eV}}$$

(c) Evaluate K_{max} for λ = 250 nm:

$$K_{max} = \frac{1240\,\text{eV}\cdot\text{nm}}{250\,\text{nm}} - 4.58\,\text{eV}$$
$$= \boxed{0.380\,\text{eV}}$$

25 •

Picture the Problem We can use the Einstein equation for photon energy to find the energy of an incident photon and his photoelectric equation to relate the work function for potassium to the maximum energy of the photoelectrons. The threshold wavelength can be found from $\lambda_t = hc/\phi$.

(a) Use the Einstein equation for photon energy to relate the energy of the incident photon to its wavelength:

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{300\,\text{nm}} = \boxed{4.13\,\text{eV}}$$

(b) Using Einstein's photoelectric equation, relate the work function for potassium to the maximum kinetic energy of the photoelectrons:

$$K_{max} = E - \phi$$

Solve for and evaluate ϕ:

$$\phi = E - K_{max} = 4.13\,\text{eV} - 2.03\,\text{eV}$$
$$= \boxed{2.10\,\text{eV}}$$

(c) Proceed as in (b) with $E = hc/\lambda$:

$$K_{max} = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{430\,\text{nm}} - 2.10\,\text{eV}$$
$$= \boxed{0.784\,\text{eV}}$$

(*d*) Express the threshold wavelength as a function of potassium's work function and evaluate λ_t:

$$\lambda_t = \frac{hc}{\phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{2.10\,\text{eV}} = \boxed{590\,\text{nm}}$$

26 •

Picture the Problem We can find the work function for silver using $\phi = hc/\lambda_t$ and the maximum kinetic energy of the electrons using Einstein's photoelectric equation.

(*a*) Express the work function for silver as a function of the threshold wavelength:

$$\phi = \frac{hc}{\lambda_t} = \frac{1240\,\text{eV}\cdot\text{nm}}{262\,\text{nm}} = \boxed{4.73\,\text{eV}}$$

(*b*) Using Einstein's photoelectric equation, relate the work function for silver to the maximum kinetic energy of the photoelectrons:

$$K_{\text{max}} = E - \phi = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{175\,\text{nm}} - 4.73\,\text{eV}$$
$$= \boxed{2.36\,\text{eV}}$$

27 •

Picture the Problem We can find the threshold frequency and wavelength for cesium using $\phi = hf_t = hc/\lambda_t$ and the maximum kinetic energy of the electrons using Einstein's photoelectric equation.

(*a*) Use the Einstein equation for photon energy to express and evaluate the threshold wavelength for cesium:

$$\lambda_t = \frac{hc}{\phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.9\,\text{eV}} = \boxed{653\,\text{nm}}$$

Use $v = f\lambda$ to find the threshold frequency:

$$f_t = \frac{v}{\lambda_t} = \frac{3\times10^8\ \text{m/s}}{653\,\text{nm}}$$
$$= \boxed{4.59\times10^{14}\ \text{Hz}}$$

(*b*) Using Einstein's photoelectric equation, relate the maximum kinetic energy of the photoelectrons to the wavelength of the incident light and evaluate K_{max} for $\lambda = 250$ nm:

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{250\,\text{nm}} - 1.90\,\text{eV}$$
$$= \boxed{3.06\,\text{eV}}$$

(*c*) Proceed as above with $\lambda = 350$ nm:

$$K_{\max} = \frac{1240\,\text{eV}\cdot\text{nm}}{350\,\text{nm}} - 1.90\,\text{eV} = \boxed{1.64\,\text{eV}}$$

***28 ••**

Picture the Problem We can use Einstein's photoelectric equation to find the work function of this surface and then apply it a second time to find the maximum kinetic energy of the photoelectrons when the surface is illuminated with light of wavelength 365 nm.

Use Einstein's photoelectric equation to relate the maximum kinetic energy of the emitted electrons to their total energy and the work function of the surface:

$$K_{\max} = \frac{hc}{\lambda} - \phi$$

Using Einstein's photoelectric equation, find the work function of the surface:

$$\phi = E - K_{\max} = \frac{hc}{\lambda} - K_{\max} = \frac{1240\,\text{eV}\cdot\text{nm}}{780\,\text{nm}} - 0.37\,\text{eV} = 1.22\,\text{eV}$$

Substitute for ϕ and λ and evaluate $K_{\max}$:

$$K_{\max} = \frac{1240\,\text{eV}\cdot\text{nm}}{410\,\text{nm}} - 1.22\,\text{eV} = \boxed{1.80\,\text{eV}}$$

Compton Scattering

29 •

Picture the Problem We can calculate the shift in wavelength using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$.

The shift in wavelength is given by:

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Substitute numerical values and evaluate $\Delta\lambda$:

$$\Delta\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3\times10^{8}\ \text{m/s})}(1-\cos60°) = \boxed{1.21\,\text{pm}}$$

30 •

Picture the Problem We can calculate the scattering angle using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$.

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Solve for θ:

$$\theta = \cos^{-1}\left(1-\frac{m_e c}{h}\Delta\lambda\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left(1-\frac{(9.11\times10^{-31}\ \text{kg})(3\times10^{8}\ \text{m/s})}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}(0.33\,\text{pm})\right) = \boxed{30.2°}$$

31 •

Picture the Problem We can calculate the shift in wavelength using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$.

Express the wavelength of the incident photons in terms of the fractional change in wavelength:

$$\frac{\Delta\lambda}{\lambda} = 2.3\% \Rightarrow \lambda = \frac{\Delta\lambda}{0.023}$$

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.023(9.11\times10^{-31}\ \text{kg})(3\times10^{8}\ \text{m/s})}(1-\cos135°) = \boxed{180\,\text{pm}}$$

***32 •**

Picture the Problem We can use the Einstein equation for photon energy to find the energy of both the incident and scattered photon and the Compton scattering equation to find the wavelength of the scattered photon.

(*a*) Use the Einstein equation for photon energy to obtain:

$$E=\frac{hc}{\lambda_1}=\frac{1240\,\text{eV}\cdot\text{nm}}{0.0711\,\text{nm}}=\boxed{17.4\,\text{keV}}$$

(*b*) Express the wavelength of the scattered photon in terms of its pre-scattering wavelength and the shift in its wavelength during scattering:

$$\lambda_2=\lambda_1+\Delta\lambda=\lambda_1+\frac{h}{m_\text{e}c}(1-\cos\theta)$$

Substitute numerical values and evaluate λ_2:

$$\lambda_2=0.0711\,\text{nm}+\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{(9.11\times10^{-31}\,\text{kg})(3\times10^{8}\,\text{m/s})}(1-\cos180°)=\boxed{0.0760\,\text{nm}}$$

(*c*) Use the Einstein equation for photon energy to obtain:

$$E=\frac{hc}{\lambda_2}=\frac{1240\,\text{eV}\cdot\text{nm}}{0.0760\,\text{nm}}=\boxed{16.3\,\text{keV}}$$

33 •

Picture the Problem Compton used X rays of wavelength 71.1 pm. Let the direction the incident photon (and the recoiling electron) is moving be the positive direction. We can use $p = h/\lambda$ to find the momentum of the incident photon and the conservation of momentum to find its momentum after colliding with the electron.

Use the expression for the momentum of a photon to find the momentum of Compton's photons:

$$p_1=\frac{h}{\lambda_1}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{71.1\,\text{pm}}$$
$$=\boxed{9.32\times10^{-24}\,\text{kg}\cdot\text{m/s}}$$

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

$$\lambda_2=\lambda_1+\lambda_\text{C}(1-\cos\theta)$$

Substitute numerical values and evaluate λ_2:

$$\lambda_2=71.1\,\text{pm}+(2.43\times10^{-12}\,\text{m})(1-\cos180°)=76.0\,\text{pm}$$

Apply conservation of momentum to obtain:

$$p_1 = p_e - p_2 \Rightarrow p_e = p_1 - p_2$$

Substitute for p_1 and p_2 and evaluate p_e:

$$p_e = 9.32\times10^{-24}\ \text{kg}\cdot\text{m/s} - \left(-\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{76.0\ \text{pm}}\right) = \boxed{1.80\times10^{-23}\ \text{kg}\cdot\text{m/s}}$$

34 ••

Picture the Problem We can calculate the shift in wavelength using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_C(1-\cos\theta)$ and use conservation of energy to find the kinetic energy of the scattered electron.

(*a*) Use the Compton scattering equation to find the change in wavelength of the photon:

$$\Delta\lambda = \lambda_C(1-\cos\theta) = (2.43\times10^{-12}\ \text{m})(1-\cos 90°) = \boxed{2.43\ \text{pm}}$$

(*b*) Use conservation of energy to relate the change in the kinetic energy of the electron to the energies of the incident and scattered photon:

$$\Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

Find the wavelength of the scattered photon:

$$\lambda_2 = \lambda_1 + \Delta\lambda = 6\ \text{pm} + 2.43\ \text{pm} = 8.43\ \text{pm}$$

Substitute and evaluate the kinetic energy of the electron (equal to the change in its energy since it was stationary prior to the collision with the photon):

$$\Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = 1240\ \text{eV}\cdot\text{nm}\left(\frac{1}{6\ \text{pm}} - \frac{1}{8.43\ \text{pm}}\right) = \boxed{59.6\ \text{keV}}$$

35 ••

Picture the Problem We can find the number of head-on collisions required to double the wavelength of the incident photon by dividing the required change in wavelength by the change in wavelength per collision. The change in wavelength per collision can be found using the Compton scattering equation.

Express the number of collisions required in terms of the change in wavelength per collision:

$$N=\frac{\Delta\lambda}{\Delta\lambda/\text{collision}}$$

Using the Compton scattering equation, express the wavelength shift per collision:

$$\Delta\lambda=\lambda_C(1-\cos\theta)$$

Substitute numerical values and evaluate $\Delta\lambda$:

$$\Delta\lambda=(2.43\times10^{-12}\text{ m})(1-\cos180°)=4.86\text{ pm}$$

Substitute and evaluate N:

$$N=\frac{200\text{ pm}}{4.86\text{ pm}}=\boxed{42}$$

Electrons and Matter Waves

36 •

Picture the Problem From Equation 34-16 we have $\lambda=\frac{1.226}{\sqrt{K}}\text{nm}$ provided K is in electron volts.

(*a*) For K = 2.5 eV:

$$\lambda=\frac{1.226}{\sqrt{2.5}}\text{nm}=\boxed{0.775\text{ nm}}$$

(*b*) For K = 250 eV:

$$\lambda=\frac{1.226}{\sqrt{250}}\text{nm}=\boxed{0.0775\text{ nm}}$$

(*c*) For K = 2.5 keV:

$$\lambda=\frac{1.226}{\sqrt{2500}}\text{nm}=\boxed{0.0245\text{ nm}}$$

(*d*) For K = 25 keV:

$$\lambda=\frac{1.226}{\sqrt{25000}}\text{nm}=\boxed{7.75\text{ pm}}$$

37 •

Picture the Problem We can use its definition to find the de Broglie wavelength of this electron.

Use its definition to express the de Broglie wavelength of the electron in terms of its momentum:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(2.5\times10^{5}\ \text{m/s})} = \boxed{2.91\,\text{nm}}$$

38 •

Picture the Problem We can find the momentum of the electron from the de Broglie equation and its kinetic energy from $\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$, where K is in eV.

(*a*) Use the de Broglie relation to express the momentum of the electron:

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{200\,\text{nm}} = \boxed{3.31\times10^{-27}\ \text{kg}\cdot\text{m/s}}$$

(*b*) Use the electron wavelength equation to relate the electron's wavelength to its kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$$

Solve for and evaluate K:

$$K = \left(\frac{1.226\,\text{eV}^{1/2}\text{nm}}{200\,\text{nm}}\right)^2 = \boxed{3.76\times10^{-5}\ \text{eV}}$$

***39** ••

Picture the Problem The momenta of these particles can be found from their kinetic energies and speeds. Their de Broglie wavelengths are given by $\lambda = h/p$.

(*a*) The momentum of a particle p, in terms of its kinetic energy K, is given by:

$$p = \sqrt{2mK}$$

Substitute numerical values and evaluate p_e:

$$p_e = \sqrt{2m_e K} = \sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(150\text{ keV}\times\frac{1.6\times10^{-19}\text{ C}}{\text{eV}}\right)}$$
$$= \boxed{2.09\times10^{-22}\text{ N}\cdot\text{s}}$$

Substitute numerical values and evaluate p_p:

$$p_p = \sqrt{2m_p K} = \sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)\left(150\text{ keV}\times\frac{1.6\times10^{-19}\text{ C}}{\text{eV}}\right)}$$
$$= \boxed{8.95\times10^{-21}\text{ N}\cdot\text{s}}$$

Substitute numerical values and evaluate p_α:

$$p_\alpha = \sqrt{2m_\alpha K} = \sqrt{2\left(4\text{ u}\times\frac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right)\left(150\text{ keV}\times\frac{1.6\times10^{-19}\text{ C}}{\text{eV}}\right)}$$
$$= \boxed{1.79\times10^{-20}\text{ N}\cdot\text{s}}$$

(*b*) The de Broglie wavelengths of the particles are given by:

$$\lambda = \frac{h}{p}$$

Substitute numerical values and evaluate λ_p:

$$\lambda_p = \frac{h}{p_p} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{8.95\times10^{-21}\text{ N}\cdot\text{s}} = \boxed{7.41\times10^{-14}\text{ m}}$$

Substitute numerical values and evaluate λ_e:

$$\lambda_e = \frac{h}{p_e} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2.09\times10^{-22}\text{ N}\cdot\text{s}} = \boxed{3.17\times10^{-12}\text{ m}}$$

Substitute numerical values and evaluate λ_α:

$$\lambda_\alpha = \frac{h}{p_\alpha} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{1.79\times10^{-20}\text{ N}\cdot\text{s}} = \boxed{3.70\times10^{-14}\text{ m}}$$

40 •

Picture the Problem The wavelength associated with a particle of mass *m* and kinetic energy *K* is given by Equation 34-15 as $\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$.

Substitute numerical data in Equation 34-15 to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(940\,\text{MeV})(0.02\,\text{eV})}} = \boxed{0.202\,\text{nm}}$$

41 •

Picture the Problem The wavelength associated with a particle of mass *m* and kinetic energy *K* is given by Equation 34-15 as $\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$.

Substitute numerical data in Equation 34-15 to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(938\,\text{MeV})(2\,\text{MeV})}} = 2.02\times10^{-5}\text{ nm} = \boxed{20.2\,\text{fm}}$$

***42** •

Picture the Problem We can use its definition to calculate the de Broglie wavelength of this proton.

Use its definition to express the de Broglie wavelength of the proton:

$$\lambda_{\text{p}} = \frac{h}{p_{\text{p}}} = \frac{h}{m_{\text{p}}v_{\text{p}}}$$

Substitute numerical values and evaluate λ_{p}:

$$\lambda = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(1.67\times10^{-27}\text{ kg})[0.003(3\times10^{8}\text{ m/s})]} = \boxed{0.441\,\text{pm}}$$

43 •

Picture the Problem We can solve Equation 34-15 ($\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$) for the kinetic energy of the proton and use the rest energy of a proton mc^2 = 938 MeV to simplify our computation.

Solve Equation 34-15 for the kinetic energy of the proton:

$$K = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{2mc^2\lambda^2}$$

(*a*) Substitute numerical values and evaluate K for $\lambda = 1$ nm:

$$K = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{2(938\,\text{MeV})(1\,\text{nm})^2} = \boxed{0.820\,\text{meV}}$$

(*b*) Evaluate K for $\lambda = 1$ nm:

$$K = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{2(938\,\text{MeV})(10^{-6}\,\text{nm})^2} = \boxed{820\,\text{MeV}}$$

44 •

Picture the Problem We'll need to convert oz and mph into SI units. Then we can use its definition to calculate the de Broglie wavelength of the baseball.

Use its definition to express the de Broglie wavelength of the baseball:

$$\lambda_{\text{baseball}} = \frac{h}{p_{\text{baseball}}} = \frac{h}{m_{\text{baseball}}v_{\text{baseball}}}$$

Substitute numerical values and evaluate $\lambda_{\text{baseball}}$:

$$\lambda_{\text{baseball}} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(5\,\text{oz}\times\frac{1\,\text{lb}}{16\,\text{oz}}\times\frac{1\,\text{kg}}{2.20\,\text{lb}}\right)\left(95\frac{\text{mi}}{\text{h}}\times\frac{0.447\frac{\text{m}}{\text{s}}}{\frac{\text{mi}}{\text{h}}}\right)} = 1.10\times10^{-34}\,\text{m}$$

For the tennis ball:

$$\lambda_{\text{tennis ball}} = \frac{h}{p_{\text{tennis ball}}} = \frac{h}{m_{\text{tennis ball}}v_{\text{tennis balll}}}$$

Substitute numerical values and evaluate $\lambda_{\text{tennis ball}}$:

$$\lambda_{\text{tennis balll}} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(2\,\text{oz}\times\frac{1\,\text{lb}}{16\,\text{oz}}\times\frac{1\,\text{kg}}{2.20\,\text{lb}}\right)\left(130\frac{\text{mi}}{\text{h}}\times\frac{0.447\frac{\text{m}}{\text{s}}}{\frac{\text{mi}}{\text{h}}}\right)} = 2.01\times10^{-34}\,\text{m}$$

$$\boxed{\text{The tennis ball has the longer de Broglie wavelength.}}$$

Remarks: Because $\lambda = h/p$, we could have solved the problem by determining which ball has the smaller momentum.

45 •

Picture the Problem If K is in electron volts, the wavelength of a particle is given by $\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$ provided K is in eV.

Evaluate λ for K = 54 eV:

$$\lambda = \frac{1.226}{\sqrt{K}}\text{nm} = \frac{1.226}{\sqrt{54}}\text{nm} = \boxed{0.167\,\text{nm}}$$

46 •

Picture the Problem We can use $\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$, where K is in eV, to find the energy of electrons whose wavelength is λ.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$$

Solve for K:

$$K = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\lambda}\right)^2$$

Substitute numerical values and evaluate K:

$$K = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{0.257\,\text{nm}}\right)^2 = \boxed{22.8\,\text{eV}}$$

***47** •

Picture the Problem We can use $\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$, where K is in eV, to find the wavelength of 70-keV electrons.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.23}{\sqrt{K}}\text{nm}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1.226}{\sqrt{70\times10^3\,\text{eV}}}\text{nm} = \boxed{4.63\,\text{pm}}$$

48 •

Picture the Problem We can use its definition to calculate the de Broglie wavelength of a neutron with speed 10^6 m/s.

Use its definition to express the de Broglie wavelength of the neutron:

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_n v_n}$$

Substitute numerical values and evaluate λ_n:

$$\lambda_n = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(10^{6}\ \text{m/s})} = \boxed{0.397\ \text{pm}}$$

Wave-Particle Duality

49 •

Picture the Problem In order for diffraction to occur, the diameter of the aperture *d* must be approximately equal to the de Broglie wavelength of the spherical object. We can use the de Broglie relationship to find the size of the aperture necessary for this object to show diffraction.

Express the de Broglie wavelength of the spherical object:

$$\lambda_{\text{object}} = \frac{h}{p_{\text{object}}} = \frac{h}{m_{\text{object}} v_{\text{object}}}$$

Substitute numerical values and evaluate λ_{object}:

$$\lambda_{\text{object}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(4\times10^{-3}\ \text{kg})(100\ \text{m/s})} = \boxed{1.66\times10^{-33}\ \text{m}}$$

This is many orders of magnitude smaller than even the diameter of a proton and so no common objects would be able to squeeze through such an aperture.

50 •

Picture the Problem In order for diffraction to occur, the size of the object must be approximately λ. The wavelength of the neutron is given by $\lambda = \dfrac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$. The rest energy of the neutron is $mc^2 = 940$ MeV.

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2(940\ \text{MeV})(10\ \text{MeV})}} = 9.04\times10^{-6}\ \text{nm} \approx \boxed{10\ \text{fm}}$$

This wavelength is of the same order-of-magnitude as a nuclear diameter and so nuclei would be suitable targets to demonstrate the wave nature of neutrons with this energy.

51 •

Picture the Problem We can use $\lambda = \frac{1.226}{\sqrt{K}}\,\text{nm}$, where K is in eV, to find the wavelength of 200-eV electrons. In order for diffraction to occur, the size of the target must be approximately λ.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}}\,\text{nm}$$

Substitute numerical values and evaluate λ:

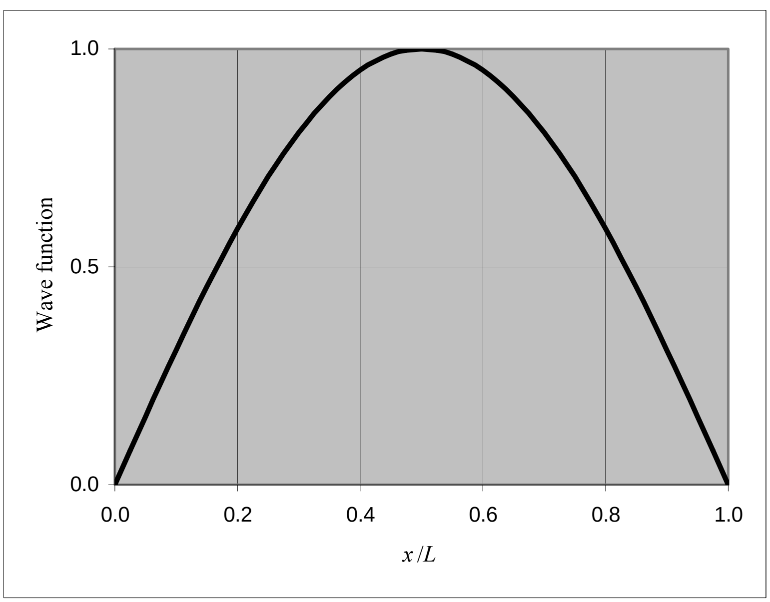
$$\lambda = \frac{1.226}{\sqrt{200\,\text{eV}}}\,\text{nm} = \boxed{0.0867\,\text{nm}}$$

This distance is of the order of the size of an atom and so atoms would be suitable targets to demonstrate the wave nature of electrons with this energy.

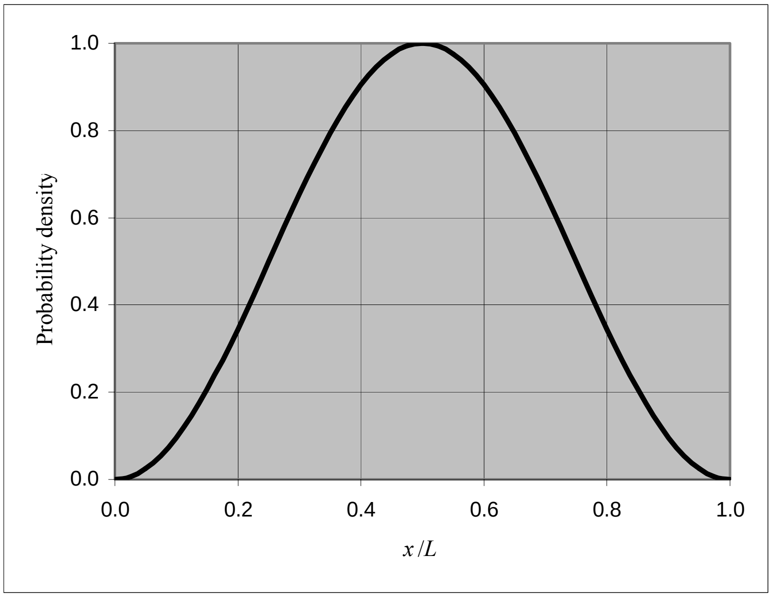
A Particle in a Box

***52** ••

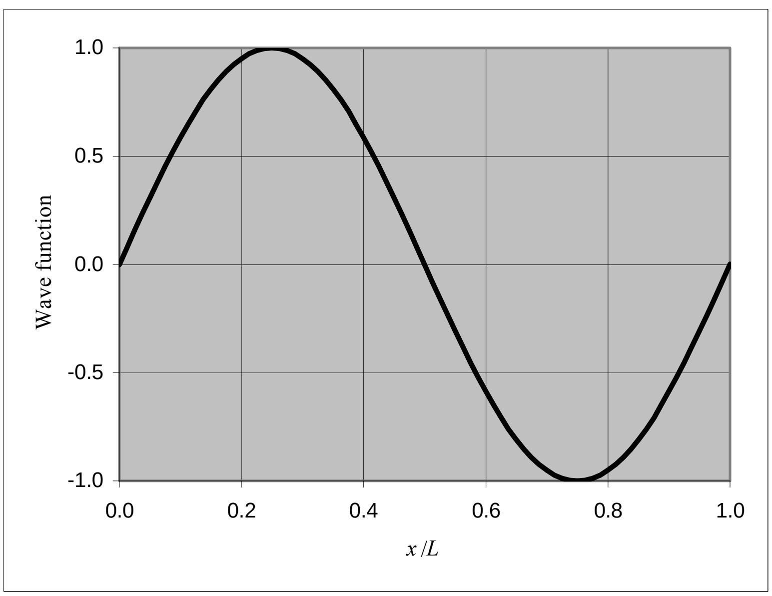
Picture the Problem The wave function for state n is $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$. The following graphs were plotted using a spreadsheet program. The graph of $\psi(x)$ for $n = 1$ is shown below:



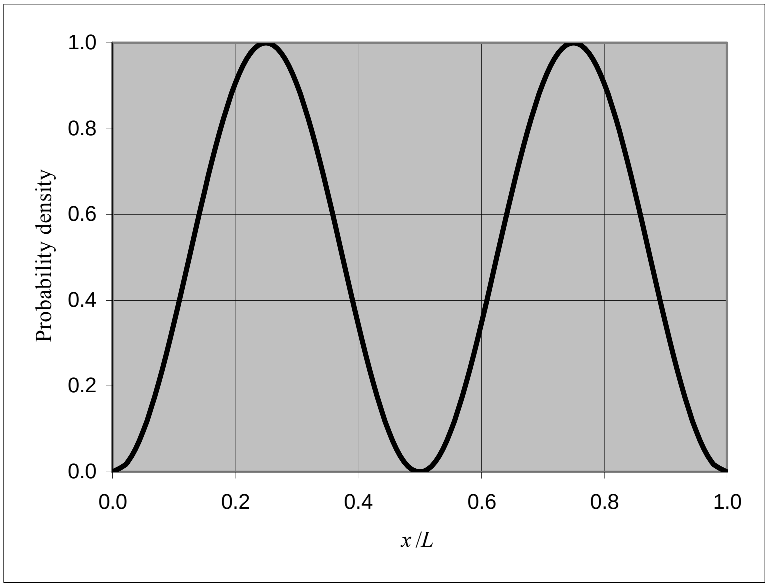
The graph of $\psi^2(x)$ for $n = 1$ is shown below:



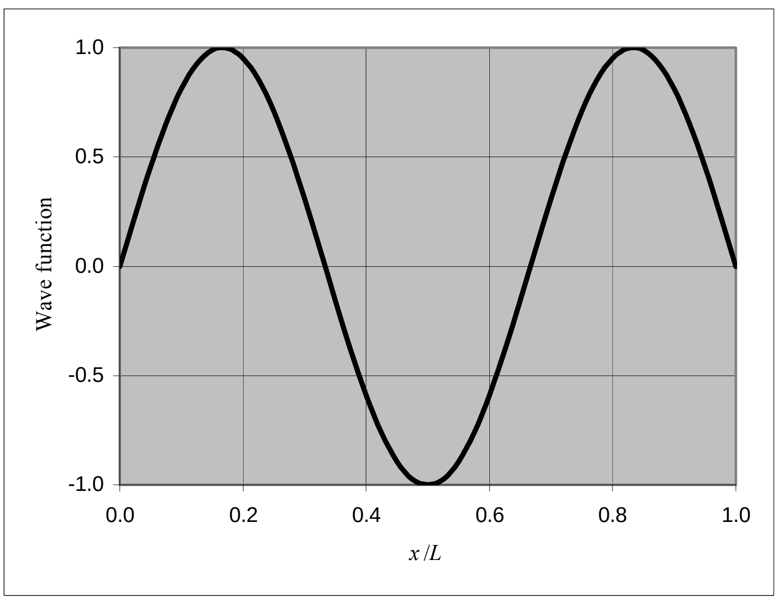
The graph of $\psi(x)$ for $n = 2$ is shown below:



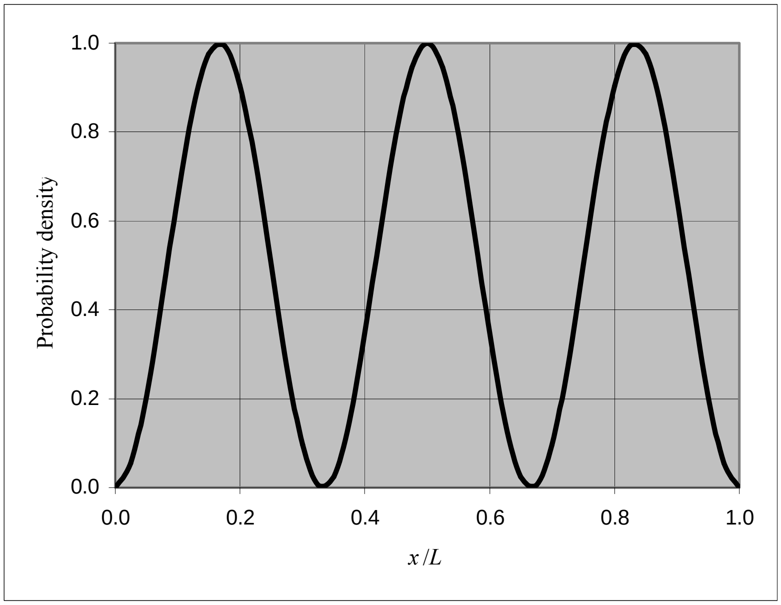
The graph of $\psi^2(x)$ for $n = 2$ is shown below:



The graph of $\psi(x)$ for $n = 3$ is shown below:



The graph of $\psi^2(x)$ for n = 3 is shown below:



53 ••

Picture the Problem We can find the ground-state energy using $E_1 = \frac{h^2}{8mL^2}$ and the energies of the excited states using $E_n = n^2 E_1$. The wavelength of the electromagnetic radiation emitted when the proton transitions from one state to another is given by the Einstein equation for photon energy ($E = \frac{hc}{\lambda}$).

(*a*) Express the ground-state energy:

$$E_1 = \frac{h^2}{8m_{\text{p}}L^2}$$

Substitute numerical values and evaluate E_1:

$$E_1 = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(10^{-15}\ \text{m}\right)^2}\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}} = \boxed{206\,\text{MeV}}$$

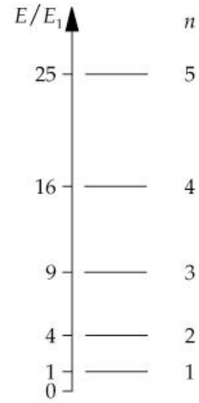
Find the energies of the first two excited states:

$$E_2 = 2^2 E_1 = 4(206\,\text{MeV}) = \boxed{824\,\text{MeV}}$$

and

$$E_3 = 3^2 E_1 = 9(206\,\text{MeV}) = \boxed{1.85\,\text{GeV}}$$

The energy-level diagram for this system is shown to the right:



(*b*) Relate the wavelength of the electromagnetic radiation emitted during a proton transition to the energy released in the transition:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E}$$

For the $n = 2$ to $n = 1$ transition:

$$\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1$$

Substitute numerical values and evaluate $\lambda_{2\to1}$:

$$\lambda_{2\to1} = \frac{1240\,\text{eV}\cdot\text{nm}}{3E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{3(206\,\text{MeV})} = \boxed{2.01\,\text{fm}}$$

(*c*) For the $n = 3$ to $n = 2$ transition:

$$\Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1$$

Substitute numerical values and evaluate $\lambda_{3\to2}$:

$$\lambda_{3\to2} = \frac{1240\,\text{eV}\cdot\text{nm}}{5E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{5(206\,\text{MeV})} = \boxed{1.20\,\text{fm}}$$

(*d*) For the $n = 3$ to $n = 1$ transition:

$$\Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1$$

Substitute numerical values and evaluate $\lambda_{3\to1}$:

$$\lambda_{3\to1} = \frac{1240\,\text{eV}\cdot\text{nm}}{8E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{8(206\,\text{MeV})} = \boxed{0.752\,\text{fm}}$$

54 ••

Picture the Problem We can find the ground-state energy using $E_1 = \frac{h^2}{8mL^2}$ and the energies of the excited states using $E_n = n^2 E_1$. The wavelength of the electromagnetic radiation emitted when the proton transitions from one state to another is given by the

Einstein equation for photon energy ($E = \dfrac{hc}{\lambda}$).

(*a*) Express the ground-state energy:

$$E_1 = \frac{h^2}{8m_\text{p}L^2}$$

Substitute numerical values and evaluate E_1:

$$E_1 = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.2\ \text{nm}\right)^2}\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} = \boxed{5.14\,\text{meV}}$$

Find the energies of the first two excited states:

$$E_2 = 2^2 E_1 = 4(5.14\,\text{meV}) = \boxed{20.6\,\text{meV}}$$

and

$$E_3 = 3^2 E_1 = 9(5.14\,\text{meV}) = \boxed{46.3\,\text{meV}}$$

(*b*) Relate the wavelength of the electromagnetic radiation emitted during a proton transition to the energy released in the transition:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E}$$

For the $n = 2$ to $n = 1$ transition:

$$\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1$$

Substitute numerical values and evaluate $\lambda_{2\to1}$:

$$\lambda_{2\to1} = \frac{1240\,\text{eV}\cdot\text{nm}}{3E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{3(5.14\,\text{meV})} = \boxed{80.4\,\mu\text{m}}$$

(*c*) For the $n = 3$ to $n = 2$ transition:

$$\Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1$$

Substitute numerical values and evaluate $\lambda_{3\to2}$:

$$\lambda_{3\to2} = \frac{1240\,\text{eV}\cdot\text{nm}}{5E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{5(5.14\,\text{meV})} = \boxed{48.2\,\mu\text{m}}$$

(*d*) For the $n = 3$ to $n = 1$ transition:

$$\Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1$$

Substitute numerical values and evaluate $\lambda_{3\to1}$:

$$\lambda_{3\to1} = \frac{1240\,\text{eV}\cdot\text{nm}}{8E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{8(5.14\,\text{meV})} = \boxed{30.2\,\mu\text{m}}$$

Calculating Probabilities and Expectation Values

55 ••

Picture the Problem The probability of finding the particle in some range Δx is $\psi^2 dx$. The interval $\Delta x = 0.002L$ is so small that we can neglect the variation in $\psi(x)$ and just compute $\psi^2\Delta x$.

Express the probability of finding the particle in the interval Δx:

$$P = P(x)\Delta x = \psi^2(x)\Delta x$$

Express the wave function for a particle in the ground state:

$$\psi_1(x) = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$$

Substitute to obtain:

$$P = \frac{2}{L}\sin^2\frac{\pi x}{L}\Delta x = \frac{2}{L}\left(\sin^2\frac{\pi x}{L}\right)(0.002L) = 0.004\sin^2\frac{\pi x}{L}$$

(*a*) Evaluate P at $x = L/2$:

$$P = 0.004\sin^2\frac{\pi L}{2L} = 0.004\sin^2\frac{\pi}{2} = \boxed{0.004}$$

(*b*) Evaluate P at $x = 2L/3$:

$$P = 0.004\sin^2\frac{2\pi L}{3L} = 0.004\sin^2\frac{2\pi}{3} = \boxed{0.003}$$

(*c*) Evaluate P at $x = L$:

$$P = 0.004\sin^2\frac{\pi L}{L} = 0.004\sin^2\pi = \boxed{0}$$

***56** ••

Picture the Problem The probability of finding the particle in some range Δx is $\psi^2 dx$. The interval $\Delta x = 0.002L$ is so small that we can neglect the variation in $\psi(x)$ and just compute $\psi^2\Delta x$.

Express the probability of finding the particle in the interval Δx:

$$P = P(x)\Delta x = \psi^2(x)\Delta x$$

Express the wave function for a particle in its first excited state:

$$\psi_2(x)=\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Substitute to obtain:

$$P=\frac{2}{L}\sin^2\frac{2\pi x}{L}\Delta x=\frac{2}{L}\left(\sin^2\frac{2\pi x}{L}\right)(0.002L)=0.004\sin^2\frac{2\pi x}{L}$$

(*a*) Evaluate *P* at $x = L/2$:

$$P=0.004\sin^2\frac{2\pi L}{2L}=0.004\sin^2\pi=\boxed{0}$$

(*b*) Evaluate *P* at $x = 2L/3$:

$$P=0.004\sin^2\frac{4\pi L}{3L}=0.004\sin^2\frac{4\pi}{3}=\boxed{0.003}$$

(*c*) Evaluate *P* at $x = L$:

$$P=0.004\sin^2\frac{2\pi L}{L}=0.004\sin^2 2\pi=\boxed{0}$$

57 ••

Picture the Problem We'll use $\langle f(x)\rangle=\int f(x)\psi^2(x)dx$ with $\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$.

(*a*) Express $\psi(x)$ for the $n = 2$ state:

$$\psi_2(x)=\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Express $\langle x\rangle$ using the $n = 2$ wave function:

$$\langle x\rangle=\int_0^L\frac{2x}{L}\sin^2\frac{2\pi x}{L}dx$$

Change variables by letting $\theta=\frac{2\pi x}{L}$. Then:

$$x=\frac{L}{2\pi}\theta,$$

$$d\theta=\frac{2\pi}{L}dx,\text{ and}$$

$$dx=\frac{L}{2\pi}d\theta$$

and the limits on θ are 0 and 2π.

Substitute to obtain:

$$\langle x\rangle = \frac{2}{L}\int_0^L\left(\frac{L}{2\pi}\theta\right)\sin^2\theta\left(\frac{L}{2\pi}d\theta\right) = \frac{L}{2\pi^2}\int_0^{2\pi}\theta\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x\rangle = \frac{L}{2\pi^2}\left[\frac{\theta^2}{4}-\frac{\theta\sin 2\theta}{4}-\frac{\cos 2\theta}{8}\right]_0^{2\pi} = \frac{L}{2\pi^2}\left[\pi^2-\frac{1}{8}+\frac{1}{8}\right] = \boxed{\frac{L}{2}}$$

(b) Express $\langle x^2\rangle$ using the n = 2 wave function:

$$\langle x^2\rangle = \int_0^L \frac{2x^2}{L}\sin^2\frac{2\pi x}{L}dx$$

Change variables as in (a) and substitute to obtain:

$$\langle x^2\rangle = \frac{2}{L}\int_0^L\left(\frac{L}{2\pi}\theta\right)^2\sin^2\theta\left(\frac{L}{2\pi}d\theta\right) = \frac{L^2}{4\pi^3}\int_0^{2\pi}\theta^2\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x^2\rangle = \frac{L^2}{4\pi^3}\left[\frac{\theta^3}{6}-\left(\frac{\theta^2}{4}-\frac{1}{8}\right)\sin 2\theta-\frac{\theta\cos 2\theta}{4}\right]_0^{2\pi} = \frac{L^2}{4\pi^3}\left[\frac{4\pi^3}{3}-\frac{\pi}{2}\right]$$

$$= L^2\left(\frac{1}{3}-\frac{1}{8\pi^2}\right) = \boxed{0.321L^2}$$

58 ••

Picture the Problem We'll use $\langle f(x)\rangle = \int f(x)\psi^2(x)dx$ with $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$. In Part ($c$) we'll use $P(x) = \psi_2^2(x)$ to determine the probability of finding the particle in some small region dx centered at $x = \frac{1}{2}L$.

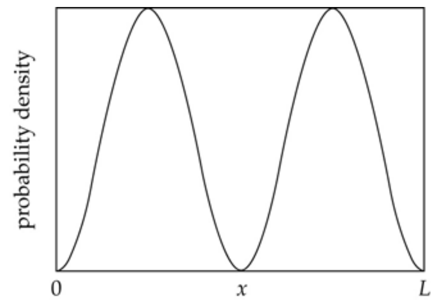
(a) Express the wave function for a particle in its first excited state:

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Square both sides of the equation to obtain:

$$\psi_2^2(x) = \frac{2}{L}\sin^2\frac{2\pi x}{L}$$

The graph of $\psi^2(x)$ as a function of x is shown to the right:



(*b*) Express $\langle x \rangle$ using the $n = 2$ wave function:

$$\langle x \rangle = \int_0^L \frac{2x}{L}\sin^2\frac{2\pi x}{L}\,dx$$

Change variables by letting $\theta = \frac{2\pi x}{L}$. Then:

$$x = \frac{L}{3\pi}\theta,$$

$$d\theta = \frac{3\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{3\pi}d\theta$$

and the limits on θ are 0 and 2π.

Substitute to obtain:

$$\langle x \rangle = \frac{2}{L}\int_0^L \left(\frac{L}{2\pi}\theta\right)\sin^2\theta\left(\frac{L}{2\pi}d\theta\right) = \frac{L}{2\pi^2}\int_0^{2\pi}\theta\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x \rangle = \frac{L}{2\pi^2}\left[\frac{\theta^2}{4} - \frac{\theta\sin 2\theta}{4} - \frac{\cos 2\theta}{8}\right]_0^{2\pi} = \frac{L}{2\pi^2}\left[\pi^2 - \frac{1}{8} + \frac{1}{8}\right] = \boxed{\frac{L}{2}}$$

(*c*) Express $P(x)$:

$$P(x) = \psi_2^2(x) = \frac{2}{L}\sin^2\frac{2\pi x}{L}dx$$

Evaluate $P(L/2)$:

$$P\left(\frac{L}{2}\right) = \frac{2}{L}\sin^2\frac{2\pi}{L}\cdot\frac{L}{2} = \frac{2}{L}\sin^2\pi = 0$$

Because $P(L/2) = 0$:

$$P\left(\frac{L}{2}\right)dx = \boxed{0}$$

(*d*) The answers to Parts (*b*) and (*c*) are not contradictory. (*b*) states that the average value of measurements of the position of the particle will yield $L/2$, even though the probability that any one measurement of position will yield $L/2$ is zero.

59 ••

Picture the Problem We can find the constant A by applying the normalization condition $\int_{-\infty}^{\infty}\psi^2(x)dx = 1$ and finding the value for A that satisfies this condition. As soon as we have found the normalization constant, we can calculate the probability of the finding the particle in the region $-a \le x \le a$ using $P = \int_{-a}^{a}\psi^2(x)dx$.

(*a*) Express the normalization condition:

$$\int_{-\infty}^{\infty}\psi^2(x)dx = 1$$

Substitute $\psi(x) = Ae^{-|x|/a}$:

$$\int_{-\infty}^{\infty}\left(Ae^{-|x|/a}\right)^2 dx = 2A^2\int_0^{\infty} e^{-2x/a}dx$$

From integral tables:

$$\int_0^{\infty} e^{-\alpha x}dx = \frac{1}{\alpha}$$

Therefore:

$$2A^2\int_0^{\infty} e^{-2x/a}dx = 2A^2\left(\frac{a}{2}\right) = aA^2 = 1$$

Solve for A:

$$A = \boxed{\frac{1}{\sqrt{a}}}$$

(*b*) Express the normalized wave function:

$$\psi(x) = \frac{1}{\sqrt{a}}e^{-|x|/a}$$

The probability of finding the particle in the region $-a \le x \le a$ is:

$$P = \int_{-a}^{a}\psi^2(x)dx = 2\int_0^{a}\frac{1}{a}e^{-2x/a}dx$$

$$= \frac{2}{a}\int_0^{a} e^{-2x/a}dx = 1 - e^{-2} = \boxed{0.865}$$

60 ••

Picture the Problem The probability density for the particle in its ground state is given by $P(x)=\frac{2}{L}\sin^2\frac{\pi}{L}x$. We'll evaluate the integral of $P(x)$ between the limits specified in (*a*), (*b*), and (*c*).

Express $P(x)$ for $0 < x < d$:

$$P(x)=\frac{2}{L}\int_0^d \sin^2\frac{\pi}{L}x\,dx$$

Change variables by letting $\theta=\frac{\pi}{L}x$. Then:

$$x=\frac{L}{\pi}\theta,$$

$$d\theta=\frac{\pi}{L}dx, \text{ and}$$

$$dx=\frac{L}{\pi}d\theta$$

Substitute to obtain:

$$P(x)=\frac{2}{L}\int_0^{\theta'}\sin^2\theta\left(\frac{L}{\pi}d\theta\right)=\frac{2}{\pi}\int_0^{\theta'}\sin^2\theta d\theta$$

Using a table of integrals, evaluate $\int_0^{\theta'}\sin^2\theta\,d\theta$:

$$P(x)=\frac{2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\theta'} \qquad (1)$$

(*a*) Noting that the limits on θ are 0 and $\pi/2$, evaluate equation (1) over the interval $0 < x < \frac{1}{2}L$:

$$P(x)=\frac{2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/2}=\frac{2}{\pi}\left[\frac{\pi}{4}\right]=\boxed{0.500}$$

(*b*) Noting that the limits on θ are 0 and $\pi/3$, Evaluate equation (1) over the interval $0 < x < L/3$:

$$P(x)=\frac{2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/3}=\frac{2}{\pi}\left[\frac{\pi}{6}-\frac{\sin 2\pi/3}{4}\right]=\frac{1}{3}-\frac{\sqrt{3}}{4\pi}=\boxed{0.196}$$

(*c*) Noting that the limits on θ are 0 and $3\pi/4$, Evaluate equation (1) over the interval $0 < x < 3L/4$:

$$P(x) = \frac{2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{3\pi/4}$$

$$= \frac{2}{\pi}\left[\frac{3\pi}{8} - \frac{\sin 6\pi/4}{4}\right]$$

$$= \frac{3}{4} + \frac{1}{2\pi} = \boxed{0.909}$$

61 ••

Picture the Problem The probability density for the particle in its first excited state is given by $P(x) = \frac{2}{L}\sin^2\frac{2\pi}{L}x$. We'll evaluate the integral of $P(x)$ between the limits specified in (*a*), (*b*), and (*c*).

Express $P(x)$ for $0 < x < d$:

$$P(x) = \frac{2}{L}\int_0^d \sin^2\frac{2\pi}{L}x\,dx$$

Change variables by letting $\theta = \frac{2\pi}{L}x$. Then:

$$x = \frac{L}{2\pi}\theta,$$

$$d\theta = \frac{2\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{2\pi}d\theta$$

Substitute to obtain:

$$P(x) = \frac{2}{L}\int_0^{\theta'} \sin^2\theta\left(\frac{L}{2\pi}d\theta\right)$$

$$= \frac{1}{\pi}\int_0^{\theta'} \sin^2\theta d\theta$$

Using a table of integrals, evaluate $\int_0^{\theta'} \sin^2\theta\, d\theta$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\theta'} \quad (1)$$

(*a*) Noting that the limits on θ are 0 and π, evaluate equation (1) over the interval $0 < x < ½L$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi} = \frac{1}{\pi}\left[\frac{\pi}{2}\right]$$

$$= \boxed{0.500}$$

(b) Noting that the limits on θ are 0 and $2\pi/3$, evaluate equation (1) over the interval $0 < x < L/3$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi/3}$$
$$= \frac{1}{\pi}\left[\frac{2\pi}{6} - \frac{\sin 4\pi/3}{4}\right]$$
$$= \frac{1}{3} + \frac{\sqrt{3}/2}{4\pi} = \boxed{0.402}$$

(c) Noting that the limits on θ are 0 and $3\pi/2$, evaluate equation (1) over the interval $0 < x < 3L/4$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{3\pi/2} = \frac{1}{\pi}\left[\frac{3\pi}{4}\right]$$
$$= \boxed{0.750}$$

62 ••

Picture the Problem Classically, $\langle x \rangle = \int xP(x)dx$ and $\langle x^2 \rangle = \int x^2 P(x)dx$.

Evaluate $\langle x \rangle$ with $P(x) = 1/L$:

$$\langle x \rangle = \int_0^L \frac{x}{L}dx = \left[\frac{x^2}{2L}\right]_0^L = \boxed{\frac{L}{2}}$$

Evaluate $\langle x^2 \rangle$ with $P(x) = 1/L$:

$$\langle x^2 \rangle = \int_0^L \frac{x^2}{L}dx = \left[\frac{x^3}{3L}\right]_0^L = \boxed{\frac{L^2}{3}}$$

63 ••

Picture the Problem We'll use $\langle f(x) \rangle = \int f(x)\psi^2(x)dx$ with $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ to show that $\langle x \rangle = \frac{L}{2}$ and $\langle x^2 \rangle = \frac{L^2}{3} = \frac{L^2}{2n^2\pi^2}$.

(a) Express $\langle x \rangle$ for a particle in the nth state:

$$\langle x \rangle = \int_0^L \frac{2x}{L}\sin^2\frac{n\pi x}{L}dx$$

Change variables by letting $\theta = \frac{n\pi x}{L}$. Then:

$$x = \frac{L}{n\pi}\theta,$$
$$d\theta = \frac{n\pi}{L}dx, \text{ and}$$
$$dx = \frac{L}{n\pi}d\theta$$

and the limits on θ are 0 and $n\pi$.

Substitute to obtain:

$$\langle x\rangle = \frac{2}{L}\int_0^{n\pi}\left(\frac{L}{n\pi}\theta\right)\sin^2\theta\left(\frac{L}{n\pi}d\theta\right)$$
$$= \frac{2L}{n^2\pi^2}\int_0^{n\pi}\theta\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x\rangle = \frac{2L}{n^2\pi^2}\left[\frac{\theta^2}{4}-\frac{\theta\sin 2\theta}{4}-\frac{\cos 2\theta}{8}\right]_0^{n\pi}$$
$$= \frac{2L}{n^2\pi^2}\left[\frac{n^2\pi^2}{4}-\frac{n\pi\sin 2n\pi}{4}-\frac{\cos 2n\pi}{8}+\frac{1}{8}\right] = \frac{2L}{n^2\pi^2}\left[\frac{n^2\pi^2}{4}-\frac{1}{8}+\frac{1}{8}\right]$$
$$= \boxed{\frac{L}{2}}$$

Express $\langle x^2\rangle$ for a particle in the nth state:

$$\langle x^2\rangle = \int_0^L \frac{2x^2}{L}\sin^2\frac{n\pi x}{L}dx$$

Change variables by letting $\theta = \frac{n\pi x}{L}$. Then:

$$x = \frac{L}{n\pi}\theta,$$
$$d\theta = \frac{n\pi}{L}dx, \text{ and}$$
$$dx = \frac{L}{n\pi}d\theta$$

and the limits on θ are 0 and $n\pi$.

Substitute to obtain:

$$\langle x^2\rangle = \frac{2}{L}\int_0^{n\pi}\left(\frac{L}{n\pi}\theta\right)^2\sin^2\theta\left(\frac{L}{n\pi}d\theta\right)$$
$$= \frac{2L^2}{n^3\pi^3}\int_0^{n\pi}\theta^2\sin^2\theta\,d\theta$$

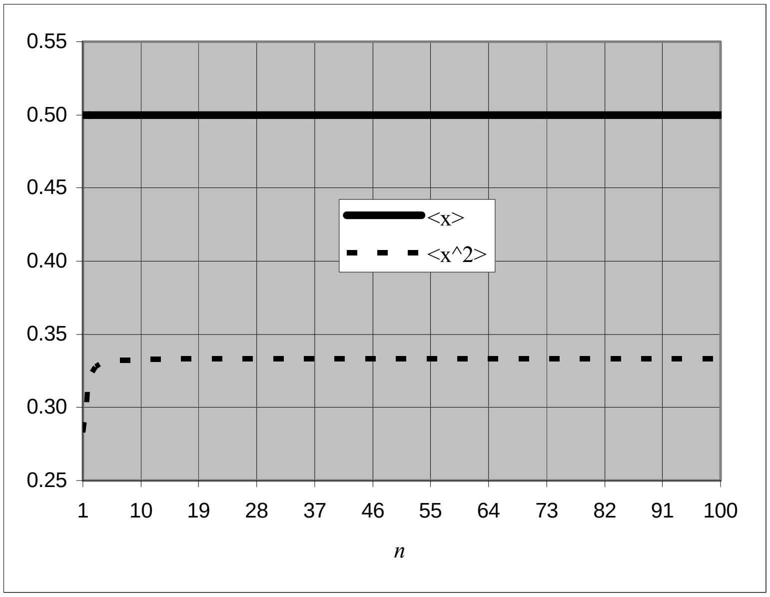
Using a table of integrals, evaluate the integral:

$$\langle x^2\rangle = \frac{2L^2}{n^3\pi^3}\left[\frac{\theta^3}{6}-\left(\frac{\theta^2}{4}-\frac{1}{8}\right)\sin 2\theta-\frac{\theta\cos 2\theta}{4}\right]_0^{n\pi} = \frac{2L^2}{n^3\pi^3}\left[\frac{n^3\pi^3}{6}-\frac{n\pi}{4}\right]$$
$$= \boxed{\frac{L^2}{3}-\frac{L^2}{2n^2\pi^2}}$$

(*b*) For large values of n, the result agrees with the classical value of $L^2/3$ given in Problem 62.

***64 ••**

Picture the Problem From Problem 63 we have $\langle x\rangle = \frac{L}{2}$ and $\langle x^2\rangle = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$. A spreadsheet program was used to plot the following graphs of <*x*> and <*x*^{2}> as a function of *n*.



$$\text{As } n \to \infty,\ \langle x^2\rangle \to \frac{L^2}{3}$$

65 ••

Picture the Problem For the ground state, $n = 1$ and so we'll evaluate $\langle f(x)\rangle = \int f(x)\psi^2(x)dx$ using $\psi_1(x) = \sqrt{\frac{2}{L}}\cos\frac{\pi x}{L}$.

Because $\psi_1^2(x)$ is an even function of x, $x\psi_1^2(x)$ is an odd function of x. It follows that the integral of $x\psi_1^2(x)$ between $-L/2$ and $L/2$ is zero. Thus:

$\langle x\rangle = \boxed{0}$ for all values of n.

Express $\langle x^2 \rangle$:

$$\langle x^2 \rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \cos^2 \frac{\pi}{L} x dx$$

Change variables by letting $\theta = \dfrac{\pi x}{L}$.

$$x = \frac{L}{\pi}\theta,$$

Then:

$$d\theta = \frac{\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{\pi}d\theta$$

and the limits on θ are $-\pi/2$ and $\pi/2$.

Substitute to obtain:

$$\langle x^2 \rangle = \frac{2}{L}\int_{-\pi/2}^{\pi/2}\left(\frac{L}{\pi}\theta\right)^2 \cos^2\theta\left(\frac{L}{\pi}d\theta\right)$$

$$= \frac{2L^2}{\pi^3}\int_{-\pi/2}^{\pi/2}\theta^2\cos^2\theta\, d\theta$$

Use a trigonometric identity to rewrite the integrand:

$$\langle x^2 \rangle = \frac{2L^2}{\pi^3}\int_{-\pi/2}^{\pi/2}\theta^2\left(1-\sin^2\theta\right)d\theta$$

$$= \frac{2L^2}{\pi^3}\left[\int_{-\pi/2}^{\pi/2}\theta^2 d\theta - \int_{-\pi/2}^{\pi/2}\theta^2\sin^2\theta\, d\theta\right]$$

Evaluate the second integral by looking it up in the tables:

$$\langle x^2 \rangle = \frac{2L^2}{\pi^3}\left[\frac{\theta^3}{3} - \left\{\frac{\theta^3}{6} - \left(\frac{\theta^2}{4} - \frac{1}{8}\right)\sin 2\theta - \frac{\theta\cos 2\theta}{4}\right\}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\theta^3}{6} + \left(\frac{\theta^2}{4} - \frac{1}{8}\right)\sin 2\theta + \frac{\theta\cos 2\theta}{4}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\pi^3}{48} + \left(\frac{\pi^2}{16} - \frac{1}{8}\right)\sin\pi + \frac{\pi\cos\pi}{8}\right.$$

$$\left. + \frac{\pi^3}{48} + \left(\frac{\pi^2}{16} - \frac{1}{8}\right)\sin\pi + \frac{\pi\cos\pi}{8}\right]$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \boxed{L^2\left[\frac{1}{12} - \frac{1}{2\pi^2}\right]}$$

Remarks: The result differs from that of Example 34-8. Since we have shifted the origin by $\Delta x = L/2$, we could have arrived at the above result, without performing the integration, by subtracting $(\Delta x)^2 = L^2/4$ from $\langle x^2 \rangle$ as given in Example 34-8.

66 ••

Picture the Problem For the first excited state, $n = 2$, and so we'll evaluate $\langle f(x) \rangle = \int f(x)\psi^2(x)dx$ using $\psi_2(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$.

Since $\psi_2^2(x)$ is an even function of x, $x\psi_2^2(x)$ is an odd function of x. It follows that the integral of $x\psi_2^2(x)$ between $-L/2$ and $L/2$ is zero. Thus:

$$\langle x \rangle = \boxed{0}$$

Express $\langle x^2 \rangle$:

$$\langle x^2 \rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2\frac{2\pi}{L}x dx$$

Change variables by letting $\theta = \frac{2\pi x}{L}$. Then:

$$x = \frac{L}{2\pi}\theta,$$

$$d\theta = \frac{2\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{2\pi}d\theta$$

and the limits on θ are $-\pi$ and π.

Substitute to obtain:

$$\langle x^2 \rangle = \frac{2}{L}\int_{-\pi}^{\pi}\left(\frac{L}{2\pi}\theta\right)^2 \sin^2\theta\left(\frac{L}{2\pi}d\theta\right)$$

$$= \frac{L^2}{4\pi^3}\int_{-\pi}^{\pi}\theta^2 \sin^2\theta \, d\theta$$

Evaluate the integral by looking it up in the tables:

$$\langle x^2 \rangle = \frac{L^2}{4\pi^3}\left[\frac{\theta^3}{6} - \left(\frac{\theta^2}{4} - \frac{1}{8}\right)\sin 2\theta - \frac{\theta\cos 2\theta}{4}\right]_{-\pi}^{\pi}$$

$$= \frac{L^2}{4\pi^3}\left[\frac{\pi^3}{6} - \left(\frac{\pi^2}{4} - \frac{1}{8}\right)\sin 2\pi - \frac{\pi\cos 2\pi}{4}\right.$$

$$\left. + \frac{\pi^3}{6} - \left(\frac{\pi^2}{4} - \frac{1}{8}\right)\sin 2\pi - \frac{\pi\cos 2\pi}{4}\right]$$

$$= \frac{L^2}{4\pi^3}\left[\frac{\pi^3}{6} - \frac{\pi}{4} + \frac{\pi^3}{6} - \frac{\pi}{4}\right]$$

$$= \frac{L^2}{4\pi^3}\left[\frac{\pi^3}{3} - \frac{\pi}{2}\right] = \boxed{L^2\left[\frac{1}{12} - \frac{1}{8\pi^2}\right]}$$

Remarks: The result differs from that of Example 34-8. Since we have shifted the origin by $\Delta x = L/2$, we could have arrived at the above result, without performing the integration, by subtracting $(\Delta x)^2 = L^2/4$ from $\langle x^2 \rangle$ as given in Example 34-8.

General Problems

***67** •

Picture the Problem We can use the Einstein equation for photon energy to find the energy of each photon in the beam. The intensity of the energy incident on the surface is the ratio of the power delivered by the beam to its delivery time. Hence, we can express the energy incident on the surface in terms of the intensity of the beam.

(*a*) Use the Einstein equation for photon energy to express the energy of each photon in the beam:

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate E_{photon}:

$$E_{\text{photon}} = \frac{1240\,\text{eV}\cdot\text{nm}}{400\,\text{nm}} = \boxed{3.10\,\text{eV}}$$

(*b*) Relate the energy incident on a surface of area A to the intensity of the beam:

$$E = IA\Delta t$$

Substitute numerical values and evaluate E:

$$E = (100\ \text{W/m}^2)(10^{-4}\ \text{m}^2)(1\,\text{s})$$
$$= 0.01\,\text{J} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}}$$
$$= \boxed{6.25\times10^{16}\ \text{eV}}$$

(*c*) Express the number of photons striking this area in 1 s as the ratio of the total energy incident on the surface to the energy delivered by each photon:

$$N = \frac{E}{E_{\text{photon}}} = \frac{6.25\times10^{16}\ \text{ev}}{3.10\,\text{eV}}$$
$$= \boxed{2.02\times10^{16}}$$

68 •

Picture the Problem The particle's *n*th-state energy is $E_n = n^2\dfrac{h^2}{8mL^2}$. We can find n by solving this equation for n and substituting the particle's kinetic energy for E_n.

Express the energy of the particle when it is in its *n*th state:

$$E_n = n^2\frac{h^2}{8mL^2}$$

Solve for n:

$$n = \frac{L}{h}\sqrt{8mE_n}$$

Express the energy (kinetic) of the particle:

$$E_n = \tfrac{1}{2}mv^2$$

Substitute to obtain:

$$n = \frac{2mvL}{h}$$

Substitute numerical values and evaluate n:

$$n = \frac{2(10^{-9}\ \text{kg})(10^{-3}\ \text{m/s})(10^{-2}\ \text{m})}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 3.02\times10^{19} \approx \boxed{3\times10^{19}}$$

69 •

Picture the Problem We can use the fact that the uncertainties are given by $\Delta x/L = 0.01$ percent and $\Delta p/p = 0.01$ percent to find Δx and Δp.

(*a*) Assuming that $\Delta x/L = 0.01$ percent, find Δx:

$$\Delta x = 10^{-4}(L) = 10^{-4}(10^{-2}\ \text{m}) = \boxed{1.00\ \mu\text{m}}$$

Assuming that $\Delta p/p = 0.01$ percent, find Δp:

$$\Delta p = 10^{-4}mv = 10^{-4}\left(10^{-9}\text{ kg}\right)\left(10^{-3}\text{ m/s}\right) = \boxed{10^{-16}\text{ kg}\cdot\text{m/s}}$$

(*b*) Evaluate $(\Delta x\Delta p)/\hbar$:

$$\frac{\Delta x\Delta p}{\hbar} = \frac{(1\ \mu\text{m})\left(10^{-16}\text{ kg}\cdot\text{m/s}\right)}{1.054\times10^{-34}} = \boxed{0.949\times10^{12}}$$

70 •

Picture the Problem We can estimate the number of emitted photons from the ratio of the total energy in the flash to the energy of a single photon.

Letting N be the number of emitted photons, express the ratio of the total energy in the flash to the energy of a single photon:

$$N = \frac{E}{E_{\text{photon}}}$$

Relate the energy in the flash to the power produced:

$$E = P\Delta t$$

Express the energy of a single photon as a function of its wavelength:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Substitute to obtain:

$$N = \frac{P\Delta t\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{\left(5\times10^{15}\text{ W}\right)\left(10^{-12}\text{ s}\right)\left(400\times10^{-9}\text{ m}\right)}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)} = \boxed{1.01\times10^{22}}$$

71 •

Picture the Problem We can use the electron wavelength equation $\lambda = \frac{1.23}{\sqrt{K}}\text{nm},$ where K is in eV to find the minimum energy required to see an atom.

Relate the energy of the electron to the size of an atom (the wavelength of the electron):

$$\lambda = \frac{1.23}{\sqrt{K}}\text{nm}$$

provided K is in eV.

Solve for K:

$$K = \frac{\left(1.23\,\text{eV}^{1/2}\cdot\text{nm}\right)^2}{\lambda^2}$$

Substitute numerical values and evaluate K:

$$K = \frac{\left(1.23\,\text{eV}^{1/2}\cdot\text{nm}\right)^2}{\left(0.1\,\text{nm}\right)^2} = \boxed{151\,\text{eV}}$$

72 •

Picture the Problem The flea's de Broglie wavelength is $\lambda = h/p$, where p is the flea's momentum immediately after takeoff. We can use a constant acceleration equation to find the flea's speed and, hence, momentum immediately after takeoff.

Express the de Broglie wavelength of the flea immediately after takeoff:

$$\lambda = \frac{h}{p} = \frac{h}{mv_0}$$

Using a constant acceleration equation, express the height the flea can jump as a function of its takeoff speed:

$$v^2 = v_0^2 + 2a\Delta y$$

or, since $v = 0$ and $a = -g$,

$$v_0 = \sqrt{2g\Delta y}$$

Substitute to obtain:

$$\lambda = \frac{h}{m\sqrt{2g\Delta y}}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(8\times10^{-6}\,\text{kg}\right)\sqrt{2\left(9.81\,\text{m/s}^2\right)\left(0.2\,\text{m}\right)}} = \boxed{4.18\times10^{-29}\,\text{m}}$$

***73 ••**

Picture the Problem We can relate the fraction of the photons entering the eye to ratio of the area of the pupil to the area of a sphere of radius R. We can find the number of photons emitted by the source from the rate at which it emits and the energy of each photon which we can find using the Einstein equation.

Letting r be the radius of the pupil, $N_{\text{entering eye}}$ the number of photons per second entering the eye, and N_{emitted} the number of photons emitted by the source per second, express the fraction of the light energy entering the eye at a distance R from the

$$\frac{N_{\text{entering eye}}}{N_{\text{emitted}}} = \frac{A_{\text{eye}}}{4\pi R^2} = \frac{\pi r^2}{4\pi R^2} = \frac{r^2}{4R^2}$$

source:

Solve for R to obtain:

$$R = \frac{r}{2}\sqrt{\frac{N_{\text{emitted}}}{N_{\text{entering eye}}}} \qquad (1)$$

Find the number of photons emitted by the source per second:

$$N_{\text{emitted}} = \frac{P}{E_{\text{photon}}}$$

Using the Einstein equation, express the energy of the photons:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate E_{photon}:

$$E_{\text{photon}} = \frac{1240\,\text{eV}\cdot\text{nm}}{600\,\text{nm}} = 2.07\,\text{eV}$$

Substitute and evaluate N_{emitted}:

$$N_{\text{emitted}} = \frac{100\,\text{W}}{(2.07\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 3.02\times10^{20}\ \text{s}^{-1}$$

Substitute for N_{emitted} in equation (1) and evaluate R:

$$R = \frac{3.5\,\text{mm}}{2}\sqrt{\frac{3.02\times10^{20}\ \text{s}^{-1}}{20\,\text{s}^{-1}}} = \boxed{6.80\times10^{3}\ \text{km}}$$

74 ••

Picture the Problem The intensity of the light such that one photon per second passes through the pupil is the ratio of the energy of one photon to the product of the area of the pupil and time interval during which the photon passes through the pupil. We'll use the Einstein equation to express the energy of the photon.

Use its definition to relate the intensity of the light to the energy of a 600-nm photon:

$$I_{1\,\text{photon}} = \frac{P}{A} = \frac{E_{1\,\text{photon}}}{A\Delta t}$$

Using the Einstein equation, express the energy of a 600-nm photon:

$$E_{1\,\text{photon}} = \frac{hc}{\lambda}$$

Substitute for $E_{1\ \text{photon}}$ to obtain:

$$I_{1\,\text{photon}} = \frac{hc}{\lambda A\Delta t}$$

Substitute numerical values and evaluate $I_{1\text{ photon}}$:

$$I_{1\text{photon}} = \frac{(1240\,\text{eV}\cdot\text{nm})(1.602\times10^{-19}\ \text{J/eV})}{(600\,\text{nm})\left[\frac{\pi}{4}(5\times10^{-3}\ \text{m})^2\right](1\,\text{s})} = \boxed{1.69\times10^{-14}\ \text{W/m}^2}$$

75 ••

Picture the Problem We can find the intensity at a distance of 1.5 m directly from its definition. The number of photons striking the surface each second can be found from the ratio of the energy incident on the surface to the energy of a 650-nm photon.

(*a*) Use its definition to express the intensity of the light as a function of distance from the light bulb:

$$I = \frac{P}{A} = \frac{P}{4\pi R^2}$$

Substitute numerical data to obtain:

$$I = \frac{90\ \text{W}}{4\pi(1.5\,\text{m})^2} = \boxed{3.18\ \text{W/m}^2}$$

(*b*) Express the number of photons per second that strike the surface as the ratio of the energy incident on the surface to the energy of a 650-nm photon:

$$N = \frac{IA}{E_{\text{photon}}}$$

where A is the area of the surface.

Use the Einstein equation to express the energy of the 650-nm photons:

$$E = \frac{hc}{\lambda}$$

Substitute to obtain:

$$N = \frac{IA\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{(3.18\ \text{W/m}^2)(10^{-4}\ \text{m}^2)(650\,\text{nm})}{(1240\,\text{eV}\cdot\text{nm})(1.60\times10^{-19}\ \text{J/eV})} = \boxed{1.04\times10^{15}}$$

76 ••

Picture the Problem The maximum kinetic energy of the photoelectrons is related to the frequency of the incident photons and the work function of the cathode material through the Einstein equation. We can apply this equation to the two sets of data and solve the resulting equations simultaneously for the work function.

Using the Einstein equation, relate the maximum kinetic energy of the emitted electrons to the frequency of the incident photons and the work function of the cathode material:

$$K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$$

Substitute numerical data for the light of wavelength λ_1:

$$1.8\,\text{eV} = \frac{hc}{\lambda_1} - \phi$$

Substitute numerical data for the light of wavelength $\lambda_1/2$:

$$5.5\,\text{eV} = \frac{hc}{\lambda_1/2} - \phi = \frac{2hc}{\lambda_1} - \phi$$

Solve these equations simultaneously for ϕ to obtain:

$$\phi = \boxed{1.90\,\text{eV}}$$

77 ••

Picture the Problem We can use the Einstein equation to express the energy of the scattered photon in terms of its wavelength and the Compton scattering equation to relate this wavelength to the scattering angle and the pre-scattering wavelength.

Express the energy of the scattered photon E' as a function of their wavelength λ':

$$E' = \frac{hc}{\lambda'}$$

Express the wavelength of the scattered photon as a function of the scattering angle θ:

$$\lambda' = \frac{h}{m_e c}(1 - \cos\theta) + \lambda$$

where λ is the wavelength of the incident photon.

Substitute and simplify to obtain:

$$E' = \frac{hc}{\frac{h}{m_e c}(1 - \cos\theta) + \lambda} = \frac{\frac{hc}{\lambda}}{\frac{hc}{m_e c^2 \lambda}(1 - \cos\theta) + 1} = \boxed{\frac{E}{\frac{E}{m_e c^2}(1 - \cos\theta) + 1}}$$

78 ••

Picture the Problem While we can work with either of the transitions described in the problem statement, we'll use the first transition in which radiation of wavelength 114.8 nm is emitted. We can express the energy released in the transition in terms of the difference between the energies in the two states and solve the resulting equation for *n*.

Express the energy of the emitted radiation as the particle goes from the *n*th to *n* – 1 state:

$$\Delta E = E_n - E_{n-1}$$

Express the energy of the particle in *n*th state:

$$E_n = n^2 E_1$$

Express the energy of the particle in the *n* – 1 state:

$$E_{n-1} = (n-1)^2 E_1$$

Substitute and simplify to obtain:

$$\Delta E = n^2 E_1 - (n-1)^2 E_1 = (2n-1)E_1 = \frac{hc}{\lambda}$$

Solve for *n*:

$$n = \frac{hc}{2\lambda E_1} + \frac{1}{2}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{1240\,\text{eV}\cdot\text{nm}}{2(114.8\,\text{nm})(1.2\,\text{eV})} + \frac{1}{2} = \boxed{5}$$

***79** ••

Picture the Problem We can use the expression for the energy of a particle in a well to find the energy of the most energetic electron in the uranium atom.

Relate the energy of an electron in the uranium atom to its quantum number *n*:

$$E_n = n^2\left(\frac{h^2}{8mL^2}\right)$$

Substitute numerical values and evaluate E_{92}:

$$E_{92} = (92)^2\left[\frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\,\text{kg})(0.05\,\text{nm})^2}\times\frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}}\right] = \boxed{1.28\,\text{MeV}}$$

The rest energy of an electron is:

$$m_ec^2 = (9.11\times10^{-31}\text{ kg})(3\times10^8\text{ m/s})^2\left(\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}\right) = 0.512\text{ MeV}$$

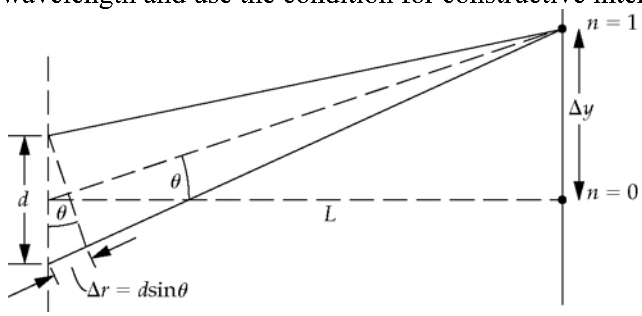
Express the ratio of E_{92} to m_ec^2:

$$\frac{E_{92}}{m_ec^2} = \frac{1.28\text{ MeV}}{0.512\text{ MeV}} = 2.50$$

The energy of the most energetic electron is approximately 2.5 times the rest - mass energy of an electron.

80 ••

Picture the Problem We can express the kinetic energy of an electron in the beam in terms of its momentum. We can use the de Broglie relationship to relate the electron's momentum to its wavelength and use the condition for constructive interference to find λ.



Express the kinetic energy of an electron in terms of its momentum:

$$K = \frac{p^2}{2m} \qquad (1)$$

Using the de Broglie relationship, relate the momentum of an electron to its momentum:

$$p = \frac{h}{\lambda}$$

Substitute for p in equation (1) to obtain:

$$K = \frac{h^2}{2m\lambda^2} \qquad (2)$$

The condition for constructive interference is:

$$d\sin\theta = n\lambda$$

where d is the slit separation and $n = 0, 1, 2, \ldots$

Solve for λ:

$$\lambda = \frac{d\sin\theta}{n}$$

For $\theta << 1$, $\sin\theta$ is also given by:

$$\sin\theta \approx \frac{\Delta y}{L}$$

Substitute for $\sin\theta$ to obtain: $\lambda = \frac{d\Delta y}{nL}$

Substitute for λ in equation (2) to obtain: $K = \frac{n^2L^2h^2}{2md^2(\Delta y)^2}$

Substitute numerical values (n = 1) and evaluate K:

$$K = \frac{(1)^2(1.5\,\text{m})^2(6.63\times10^{-34}\,\text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\,\text{kg})(54\,\text{nm})^2(0.68\,\text{mm})^2}\frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}} = \boxed{2.52\,\text{keV}}$$

81 ••

Picture the Problem The maximum kinetic energy of the photoelectrons is related to the frequency of the incident photons and the work function of the illuminated surface through the Einstein equation. We can apply this equation to either set of data and solve the resulting equations simultaneously for the work function of the surface and the wavelength of the incident photons.

Using the Einstein equation, relate the maximum kinetic energy of the emitted electrons to the frequency of the incident photons and the work function of the cathode material:

$$K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi$$

Substitute numerical data for the light of wavelength λ: $1.2\,\text{eV} = \frac{hc}{\lambda} - \phi$

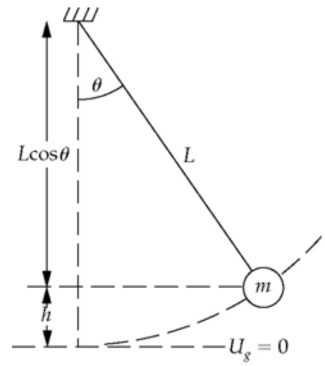
Substitute numerical values for the light of wavelength λ': $1.76\,\text{eV} = \frac{hc}{\lambda'} - \phi = \frac{hc}{0.8\lambda} - \phi$

Solve these equations simultaneously for ϕ to obtain: $\phi = \boxed{1.04\,\text{eV}}$

Substitute in either of the equations and solve for λ: $\lambda = \boxed{554\,\text{nm}}$

82 ••

Picture the Problem The diagram shows the pendulum with an angular displacement θ. The energy of the oscillator is equal to its initial potential energy $mgh = mgL(1 - \cos\theta)$. We can find n by equating this initial energy to $E_n = \left(n + \frac{1}{2}\right)hf_0$ and solving for n. In part (*b*) we'll express the ratio of ΔE_n to E_n and solve for Δn.



(*a*) Express the nth-state energy as a function of the frequency of the pendulum:

$$E_n = \left(n + \tfrac{1}{2}\right)hf_0 = \left(n + \tfrac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{g}{L}}$$

Express the energy of the pendulum:

$$E_n = mgL(1 - \cos\theta)$$

Substitute to obtain:

$$mgL(1 - \cos\theta) = \left(n + \tfrac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{g}{L}}$$

Solve for n:

$$n = \frac{2\pi m\sqrt{g}L^{3/2}(1 - \cos\theta)}{h} - \frac{1}{2}$$

Substitute numerical values and evaluate n:

$$n = \frac{2\pi(0.3\,\text{kg})\sqrt{9.81\,\text{m/s}^2}\,(1\,\text{m})^{3/2}(1 - \cos 10°)}{6.63\times10^{-34}\,\text{J}\cdot\text{s}} - \frac{1}{2} = \boxed{1.35\times10^{32}}$$

(*b*) Express the ratio of ΔE_n to E_n:

$$\frac{\Delta E_n}{E_n} = \frac{\left(n + \Delta n + \frac{1}{2}\right)hf_0 - \left(n + \frac{1}{2}\right)hf_0}{\left(n + \frac{1}{2}\right)hf_0} = \frac{\Delta n}{n + \frac{1}{2}} = 10^{-4}$$

Solve for and evaluate Δn:

$$\Delta n = 10^{-4}\left(n + \tfrac{1}{2}\right) \approx 10^{-4}n = \boxed{1.35\times10^{28}}$$

***83** ••

Picture the Problem We can use the fact that the energy of the nth state is related to the energy of the ground state according to $E_n = n^2E_1$ to express the fractional change in energy in terms of n and then examine this ratio as n grows without bound.

(*a*) Express the ratio $(E_{n+1} - E_n)/E_n$:

$$\frac{E_{n+1} - E_n}{E_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{2n+1}{n^2}$$

$$= \frac{2}{n} + \frac{1}{n^2} \approx \boxed{\frac{2}{n}}$$

for $n >> 1$.

(*b*) Evaluate $\frac{E_{1001} - E_{1000}}{E_{1000}}$:

$$\frac{E_{1001} - E_{1000}}{E_{1000}} \approx \frac{2}{1000} = \boxed{0.2\%}$$

(*c*) Classically, the energy is continuous. For very large values of *n*, the energy difference between adjacent levels is infinitesimal.

84 ••

Picture the Problem We can apply the definition of power in conjunction with the de Broglie equation for the energy of a photon to derive an expression for the average power produced by the laser.

The average power produced by the laser is:

$$P = \frac{\Delta E}{\Delta t}$$

Use the de Broglie equation to express the energy of the emitted photons:

$$\Delta E = Nhf = \frac{Nhc}{\lambda}$$

where N is number of photons in each pulse.

Substitute for ΔE to obtain:

$$P = \frac{Nhc}{\lambda \Delta t}$$

Substitute numerical values and evaluate P:

$$P = \frac{(5\times10^9)(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^8\ \text{m/s})}{(850\,\text{nm})(10^{-8}\,\text{s})} = \boxed{117\,\text{mW}}$$

Remarks: Note that the pulse length has no bearing on the solution.

85 ••

Picture the Problem We can find the rate at which energy is delivered to the atom using the definitions of power and intensity. We can also use the definition of power to determine how much time is required for an amount of energy equal to the work function to fall on one atom.

(*a*) Relate the energy per second (power) falling on an atom to the intensity of the incident radiation:

$$P = \frac{\Delta E}{\Delta t} = IA$$

Substitute numerical values and evaluate P:

$$P = \left(0.01\,\mathrm{W/m^2}\right)\left(0.01\times10^{-18}\,\mathrm{m^2}\right)$$

$$= 10^{-22}\,\frac{\mathrm{J}}{\mathrm{s}}\times\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}$$

$$= \boxed{6.25\times10^{-4}\,\mathrm{eV/s}}$$

(*b*) Classically:

$$\Delta t = \frac{\Delta E}{P} = \frac{\phi}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2\,\mathrm{eV}}{6.25\times10^{-4}\,\mathrm{eV/s}} = 3200\,\mathrm{s}$$

$$= \boxed{53.3\,\mathrm{min}}$$

Chapter 35
Applications of the Schrödinger Equation

Conceptual Problems

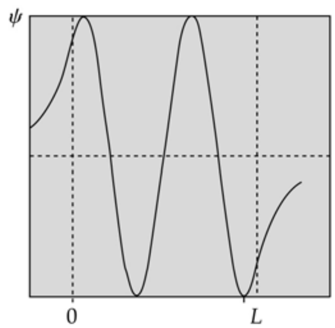
1 •

True

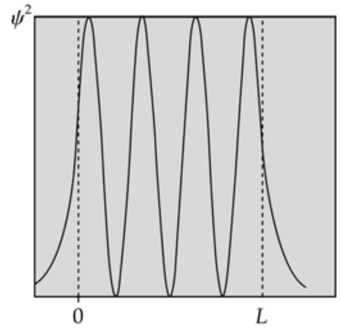
2 •

Determine the Concept Looking at the graphs in the text for the $n = 1$, 2, and 3 states, we note that the $n = 4$ state graph of the wave function must have four extrema in the region $0 < x < L$ and decay in toward zero in the regions $x < 0$ and $x > L$.

(*a*)



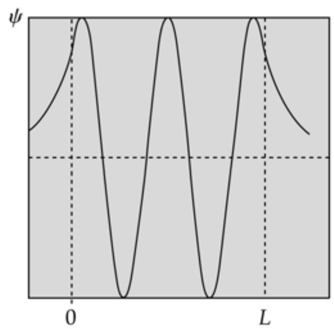
(*b*)



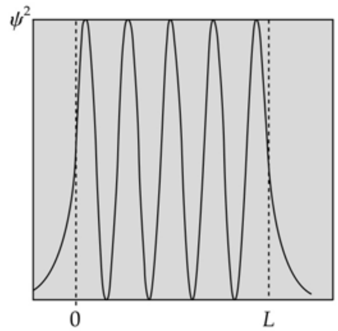
3 •

Determine the Concept Looking at the graphs in the text for the $n = 1$, 2, and 3 states, we note that the $n = 5$ state graph of the wave function must have five extrema in the region $0 < x < L$ and decay in toward zero in the regions $x < 0$ and $x > L$.

(*a*)



(*b*)



Estimation and Approximation

***4 •**

Picture the Problem Assume a mass of 150 g for the baseball, 30 cm for the width of the locker, and 1 cm/s for the speed of the ball, and equate the kinetic energy of the ball and the quantum-mechanical energy and solve for the quantum number *n*.

The allowed energy states of a particle of mass *m* in a 1-dimensional infinite potential well of width *L* are given by:

$$E_n = n^2\left(\frac{h^2}{8mL^2}\right)$$

The kinetic energy of the ball is:

$$K = \frac{1}{2}mv^2$$

For $E_n = K$:

$$n^2\left(\frac{h^2}{8mL^2}\right) = \frac{1}{2}mv^2$$

Solve for the quantum number *n*:

$$n = \frac{2mvL}{h}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{2(0.15\,\text{kg})(0.01\,\text{m/s})(0.3\,\text{m})}{6.63\times10^{-34}\,\text{J}\cdot\text{s}}$$

$$= 1.36\times10^{30} \approx \boxed{10^{30}}$$

The Schrödinger Equation

5 ••

Picture the Problem We can show that $\psi_3(x)$ is a solution to the time-independent Schrödinger equation by differentiating it twice and substituting in Equation 35-4.

Equation 35-4 is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Because $\psi_1(x)$ and $\psi_2(x)$ are solutions of Equation 35-4:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_1(x)}{dx^2} + U(x)\psi_1(x) = E\psi_1(x)$$

and

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_2(x)}{dx^2} + U(x)\psi_2(x) = E\psi_2(x)$$

Add these equations to obtain:

$$-\frac{\hbar^2}{2m}\left[\frac{d^2\psi_1(x)}{dx^2}+\frac{d^2\psi_2(x)}{dx^2}\right]+U(x)[\psi_1(x)+\psi_2(x)]=E[\psi_1(x)+\psi_2(x)] \quad (1)$$

Differentiate $\psi_3(x)=\psi_1(x)+\psi_2(x)$ twice with respect to x to obtain:

$$\frac{d\psi_3(x)}{dx}=\frac{d\psi_1(x)}{dx}+\frac{d\psi_2(x)}{dx}$$

and

$$\frac{d^2\psi_3(x)}{dx^2}=\frac{d^2\psi_1(x)}{dx^2}+\frac{d^2\psi_2(x)}{dx^2}$$

Substitute in equation (1) to obtain:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_3(x)}{dx^2}+U(x)\psi_3(x)=E\psi_3(x)$$

which shows that $\psi_3(x)=\psi_1(x)+\psi_2(x)$ satisfies Equation 35-4.

The Harmonic Oscillator

6 ••

Picture the Problem We can relate the spring constant to the mass of the hydrogen atom and its angular frequency and then use the relationship between the allowed energy levels and the angular frequency ω to derive an expression for the spring constant *k*.

The spring constant *k* is related to the mass *m* of the hydrogen molecule and its angular frequency ω:

$$k=m\omega^2 \quad (1)$$

Relate the energy spacing ΔE to the angular frequency ω:

$$\Delta E=hf=\frac{h\omega}{2\pi}=\hbar\omega$$

Solve for ω:

$$\omega=\frac{\Delta E}{\hbar}$$

Substitute for ω in equation (1) to obtain:

$$k=m\left(\frac{\Delta E}{\hbar}\right)^2$$

Substitute numerical values and evaluate *k*:

$$k = \left(1\,\text{u}\times\frac{1.66\times10^{-27}\ \text{kg}}{\text{u}}\right)\left(\frac{8.7\times10^{-20}\ \text{J}}{1.05\times10^{-34}\ \text{J}\cdot\text{s}}\right)^2 = \boxed{1.14\,\text{kN/m}}$$

Remarks: Our result is very similar to the stiffness constant of typical macroscopic springs. Note that strictly speaking one should use the reduced mass of a hydrogen molecule rather than the simpler model of a single atom attached to a fixed point.

7 ••

Determine the Concept The integral $\langle x\rangle = \int x|\psi|^2 dx = 0$ because the integrand is an odd function of x for the ground state as well as any excited state of the harmonic oscillator.

***8** ••

Picture the Problem We can differentiate $\psi(x)$ twice and substitute in the Schrödinger equation for the harmonic oscillator. Substitution of the given value for a will lead us to an expression for E_1.

The wave function for the first excited state of the harmonic oscillator is:

$$\psi_1(x) = A_1 x e^{-ax^2}$$

Compute $d\psi_1(x)/dx$:

$$\frac{d\psi_1(x)}{dx} = \frac{d}{dx}\left[A_1 x e^{-ax^2}\right] = A_1 e^{-ax^2}$$

Compute $d^2\psi_1(x)/dx^2$:

$$\frac{d^2\psi_1(x)}{dx^2} = \frac{d}{dx}\left[A_1 e^{-ax^2}\right] = -2axA_1e^{-ax^2} - 4axA_1e^{-ax^2} + 4a^2x^3A_1e^{-ax^2}$$
$$= \left(4a^2x^3 - 6ax\right)A_1e^{-ax^2}$$

Substitute in the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\left[\left(4a^2x^3 - 6ax\right)A_1e^{-ax^2}\right] + \tfrac{1}{2}m\omega_0^2x^2A_1xe^{-ax^2} = E_1A_1xe^{-ax^2}$$

Divide out $A_1e^{-ax^2}$ to obtain:

$$-\frac{\hbar^2}{2m}\left[\left(4a^2x^3 - 6ax\right)\right] + \tfrac{1}{2}m\omega_0^2x^3 = E_1x$$

or

$$-\frac{\hbar^2}{2m}\left(4a^2x^3\right)+\frac{\hbar^2}{2m}(6ax)+\tfrac{1}{2}m\omega_0^2x^3=E_1x$$

Substitute for a to obtain:

$$-\frac{\hbar^2}{2m}4\left(\frac{m\omega_0}{2\hbar}\right)^2x^3+\frac{\hbar^2}{2m}6\left(\frac{m\omega_0}{2\hbar}\right)x+\tfrac{1}{2}m\omega_0^2x^3=E_1x$$

Solve for E_1 to obtain:

$$E_1=\boxed{\tfrac{3}{2}\hbar\omega_0}=3E_0$$

9 •••

Picture the Problem We must show that, with $A_0 = (2m\omega_0/h)^{1/4}$, the normalization condition $\int_{-\infty}^{\infty}|\psi_0(x)|^2\,dx=\int_{-\infty}^{\infty}\left|A_0e^{-ax^2}\right|^2dx=1$ is satisfied.

We need to show that:

$$\int_{-\infty}^{\infty}|\psi_0(x)|^2\,dx=\int_{-\infty}^{\infty}\left|A_0e^{-ax^2}\right|^2dx=1$$

With $A_0=\left(\frac{2m\omega_0}{h}\right)^{1/4}=\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4}$, the normalization condition becomes:

$$\int_{-\infty}^{\infty}|\psi_0(x)|^2\,dx=\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty}e^{-2ax^2}dx$$

In Example 35-1 it is shown that:

$$a=\frac{m\omega_0}{2\hbar}$$

Substitute to obtain:

$$\int_{-\infty}^{\infty}|\psi_0(x)|^2\,dx=\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty}e^{-\frac{m\omega_0}{\hbar}x^2}dx$$

Let $s=\left(\frac{m\omega_0}{\hbar}\right)^{1/2}x$. Then:

$$ds=\left(\frac{m\omega_0}{\hbar}\right)^{1/2}dx$$

and

$$dx=\left(\frac{m\omega_0}{\hbar}\right)^{-1/2}ds$$

Substitute for dx and simplify to obtain:

$$\int_{-\infty}^{\infty}|\psi_0(x)|^2\,dx=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}e^{-s^2}ds$$

From integral tables (see Table D-5): $\int_{-\infty}^{\infty} e^{-s^2} ds = \sqrt{\pi}$

Therefore: $\int_{-\infty}^{\infty} |\psi_0(x)|^2 dx = 1$ provided $A_0 = \left(\frac{2m\omega_0}{h}\right)^{1/4}$

10 •••

Picture the Problem We are required to evaluate $\langle x^2 \rangle = \int x^2 |\psi_0(x)|^2$ with $\psi_0(x) = A_0 e^{-ax^2}$, where $a = \frac{m\omega_0}{2\hbar}$. We can then use $U_{\text{av}} = \frac{1}{2} m\omega_0^2 \langle x^2 \rangle$ to find the average potential energy of the harmonic oscillator.

We need to evaluate: $$\langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2 |\psi|^2 dx$$

For the ground state of the harmonic oscillator: $$|\psi|^2 = A_0^2 e^{-2ax^2}$$

Substitute for $|\psi|^2$ to obtain: $$\langle x^2 \rangle = A_0^2 \int_{-\infty}^{+\infty} x^2 e^{-2ax^2} dx = 2A_0^2 \int_0^{+\infty} x^2 e^{-2ax^2} dx$$

Use the appropriate integral from the inside of the back cover of the text to obtain: $$\langle x^2 \rangle = 2A_0^2 \frac{1}{4}\sqrt{\frac{\pi}{(2a)^2}} = \frac{A_0^2}{4a}\sqrt{\frac{\pi}{2a}} \quad (1)$$

The normalization condition is: $$1 = \int_{-\infty}^{+\infty} |\psi|^2 dx = A_0^2 \int_{-\infty}^{+\infty} e^{-2ax^2} dx = 2A_0^2 \int_0^{+\infty} e^{-2ax^2} dx$$

Again, use the appropriate integral from the inside of the back cover of the text to obtain: $$1 = 2A_0^2 \frac{1}{2}\sqrt{\frac{\pi}{2a}} = A_0^2 \sqrt{\frac{\pi}{2a}}$$

Solve for A_0^2 to obtain: $$A_0^2 = \sqrt{\frac{2a}{\pi}}$$

Substitute in equation (1) to obtain:

$$\langle x^2\rangle = \frac{1}{4a}\sqrt{\frac{2a}{\pi}}\sqrt{\frac{\pi}{2a}} = \boxed{\frac{1}{4a}} \quad (2)$$

From Example 36-1:

$$a = \frac{m\omega_0}{2\hbar}$$

Substitute for a in equation (2) to obtain:

$$\langle x^2\rangle = \frac{2\hbar}{4m\omega_0} = \boxed{\frac{\hbar}{2m\omega_0}}$$

The average potential energy of the oscillator is:

$$U_{\text{av}} = \tfrac{1}{2}m\omega_0^2\langle x^2\rangle$$

Substitute for $\langle x^2\rangle$ and simplify:

$$U_{\text{av}} = \tfrac{1}{2}m\omega_0^2\left(\frac{\hbar}{2m\omega_0}\right) = \boxed{\tfrac{1}{4}\hbar\omega_0}$$
$$= \tfrac{1}{2}E_0$$

11 ••

Picture the Problem We can combine the result for $\sqrt{\langle x^2\rangle}$ from Problem 10 and the result for $\langle x\rangle$ from Problem 7 to obtain an expression for $\sqrt{\langle x^2\rangle - \langle x\rangle^2}$. The lowest energy of the electron in an infinite potential well is given by $E_1 = \dfrac{h^2}{8mL^2}$.

(*a*) From Problem 10 we have:

$$\langle x^2\rangle = \frac{\hbar}{2m\omega_0} \quad (1)$$

The ground-state energy is given by:

$$E_0 = \frac{1}{2}\hbar\omega_0$$

Solve for ω_0 to obtain:

$$\omega_0 = \frac{2E_0}{\hbar}$$

Substitute in equation (1) and simplify to obtain:

$$\langle x^2\rangle = \frac{\hbar^2}{4mE_0}$$

From Problem 7 we have:

$$\langle x\rangle = 0 \quad (2)$$

Substitute equations (1) and (2) in the expression $\sqrt{\langle x^2\rangle - \langle x\rangle^2}$ to obtain:

$$\sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{\hbar^2}{4mE_0} - 0} = \frac{\hbar}{2}\sqrt{\frac{1}{mE_0}}$$

Substitute numerical values and evaluate $\sqrt{\langle x^2\rangle - \langle x\rangle^2}$:

$$\sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{1.05\times10^{-34}\ \text{J}\cdot\text{s}}{2}\sqrt{\frac{1}{(9.11\times10^{-31}\ \text{kg})\left(2.1\times10^{-4}\ \text{eV}\times\frac{1.6\times10^{-19}\ \text{J}}{\text{eV}}\right)}}$$
$$= \boxed{9.49\ \text{nm}}$$

(*b*) The lowest energy of an electron trapped in an infinite potential well is:

$$E_1 = \frac{h^2}{8mL^2}$$

Letting $L = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ yields:

$$E_1 = \frac{h^2}{8m\sqrt{\langle x^2\rangle - \langle x\rangle^2}}$$

Substitute numerical values and evaluate E_1:

$$E_1 = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(9.49\times10^{-9}\ \text{m})^2}\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}} = \boxed{4.19\,\text{meV}}$$

12 •••

Picture the Problem We can begin by equating the average kinetic energy of the harmonic oscillator and its average potential energy and solving for $\langle p^2\rangle$ and then evaluating and substituting for $\langle x^2\rangle$.

According to the problem statement:

$$\frac{\langle p^2\rangle}{2m} = \frac{1}{2}k\langle x^2\rangle$$

Solve for $\langle p^2\rangle$:

$$\langle p^2\rangle = mk\langle x^2\rangle$$

or, because $\omega_0^2 = \frac{k}{m}$,

$$\langle p^2\rangle = m^2\omega_0^2\langle x^2\rangle \qquad (1)$$

We need to evaluate:

$$\langle x^2\rangle = \int x^2|\psi|^2 = A_0^2\int_{-\infty}^{\infty} x^2 e^{-2ax^2}\,dx \qquad (2)$$

where $a = \frac{m\omega_0}{2\hbar}$

Let $y^2 = 2ax^2$. Then $x^2 = \dfrac{y^2}{2a}$ and:

$$dx = \frac{ydy}{2ax}$$

With appropriate substitutions, the integral becomes:

$$\int_{-\infty}^{\infty} \frac{y^2}{2a} e^{-y^2} \frac{ydy}{2ax} = \int_{-\infty}^{\infty} \frac{y^3}{2a} e^{-y^2} \frac{\sqrt{2a}dy}{2ay} = \frac{1}{(2a)^{3/2}} \int_{-\infty}^{\infty} y^2 e^{-y^2} dy$$

From integral tables (see Table D-5):

$$\int_{-\infty}^{\infty} y^2 e^{-y^2} dy = \frac{1}{2}\sqrt{\pi}$$

In Problem 35-9 it was given that:

$$A_0 = \left(\frac{2m\omega_0}{h}\right)^{1/4}$$

Substitute in equation (2) to obtain:

$$\langle x^2 \rangle = \frac{1}{(2a)^{3/2}} \left(\frac{2m\omega_0}{h}\right)^{1/2} \left(\frac{1}{2}\sqrt{\pi}\right)$$

Substitute for a and simplify:

$$\langle x^2 \rangle = \frac{1}{\left(\frac{m\omega_0}{\hbar}\right)^{3/2}} \left(\frac{2m\omega_0}{h}\right)^{1/2} \left(\frac{1}{2}\sqrt{\pi}\right) = \frac{\hbar}{2m\omega_0}$$

Substitute for $\langle x^2 \rangle$ in equation (1):

$$\langle p^2 \rangle = m^2\omega_0^2 \left(\frac{\hbar}{2m\omega_0}\right) = \boxed{\tfrac{1}{2}\hbar m\omega_0}$$

13 •••

Picture the Problem We can use the definition of the standard deviation of Δx and Δp and the results of Problems 7, 10, and 12 to determine the uncertainty product $\Delta x\, \Delta p$ for the ground state of the harmonic oscillator.

Express the standard deviation of Δp (see Equation 17-35a):

$$(\Delta p)^2 = \left[(p - p_{\text{av}})^2\right]_{\text{av}} = \left[p^2 - 2pp_{\text{av}} - p_{\text{av}}^2\right]_{\text{av}}$$

Because $p_{\text{av}} = 0$:

$$(\Delta p)^2 = \left(p^2\right)_{\text{av}}$$

Express the standard deviation of Δx (see Equation 17-35*a*):

$$(\Delta x)^2 = \left[(x - x_{\text{av}})^2\right]_{\text{av}} = \left[x^2 - 2xx_{\text{av}} - x_{\text{av}}^2\right]_{\text{av}}$$

Because $x_{\text{av}} = 0$:

$$(\Delta x)^2 = \left(x^2\right)_{\text{av}}$$

We have, from Problems 10 and 12, for the ground state of the harmonic oscillator:

$$\left\langle x^2 \right\rangle = (\Delta x)^2 = \frac{\hbar}{2m\omega_0}$$

and

$$\left\langle p^2 \right\rangle = (\Delta p)^2 = \frac{\hbar m\omega_0}{2}$$

Express the product of $(\Delta x)^2$ and $(\Delta p)^2$:

$$(\Delta x)^2 (\Delta p)^2 = \left(\frac{\hbar}{2m\omega_0}\right)\left(\frac{\hbar m\omega_0}{2}\right) = \frac{\hbar^2}{4}$$

Take the square root of both sides of the equation to obtain:

$$\boxed{\Delta x \Delta p = \frac{\hbar}{2}}$$

Reflection and Transmission of Electron Waves: Barrier Penetration

***14 ••**

Picture the Problem We can use the total energy of the particle in the region $x > 0$ to express k_2 in terms of α and k_1. Knowing k_2 in terms of k_1, we can use $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$ to find R and $T = 1 - R$ to determine the transmission coefficient T.

(*a*) Using conservation of energy, express the energy of the particle in the region $x > 0$:

$$\frac{\hbar^2 k_2^2}{2m} + U_0 = \alpha U_0$$

Solve for k_2:

$$k_2 = \frac{\sqrt{2mU_0(\alpha - 1)}}{\hbar}$$

From the equation for the total energy of the particle:

$$k_1 = \frac{\sqrt{2m\alpha U_0}}{\hbar}$$

Express the ratio of k_2 to k_1:

$$\frac{k_2}{k_1}=\frac{\frac{\sqrt{2mU_0(\alpha-1)}}{\hbar}}{\frac{\sqrt{2m\alpha U_0}}{\hbar}}=\sqrt{\frac{\alpha-1}{\alpha}}$$

and $k_2=\boxed{\sqrt{\frac{\alpha-1}{\alpha}}k_1}$

(*b*) The reflection coefficient R is given by:

$$R=\frac{(k_1-k_2)^2}{(k_1+k_2)^2}$$

Factor k_1 from the numerator and denominator to obtain:

$$R=\frac{\left(1-\frac{k_2}{k_1}\right)^2}{\left(1+\frac{k_2}{k_1}\right)^2}$$

Substitute our result from (*a*) for k_2/k_1:

$$R=\frac{\left(1-\sqrt{\frac{\alpha-1}{\alpha}}\right)^2}{\left(1+\sqrt{\frac{\alpha-1}{\alpha}}\right)^2}=\left(\frac{1-\sqrt{\frac{\alpha-1}{\alpha}}}{1+\sqrt{\frac{\alpha-1}{\alpha}}}\right)^2$$

The transmission coefficient is given by:

$$T=1-R=1-\left(\frac{1-\sqrt{\frac{\alpha-1}{\alpha}}}{1+\sqrt{\frac{\alpha-1}{\alpha}}}\right)^2$$

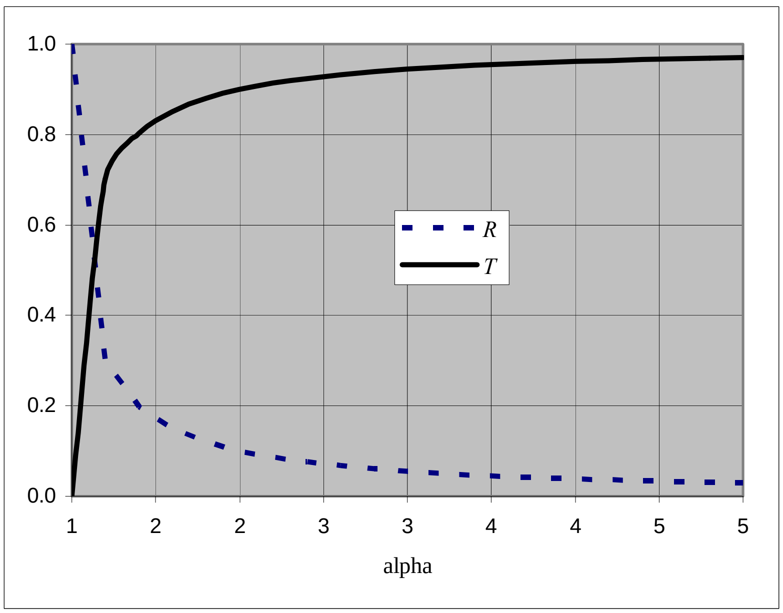
A spreadsheet program to plot R and T as functions of α is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A2	1.0	α
B2	(1–SQRT((A2–1)/A2))/ (1+SQRT((A2–1)/A2))^2	$\left(\frac{1-\sqrt{\frac{\alpha-1}{\alpha}}}{1+\sqrt{\frac{\alpha-1}{\alpha}}}\right)^2$

C2	1–B2	$1-\left(\dfrac{1-\sqrt{\dfrac{\alpha-1}{\alpha}}}{1+\sqrt{\dfrac{\alpha-1}{\alpha}}}\right)^2$

	A	B	C
1	alpha	R	T
2	1.0	1.000	0.000
3	1.2	0.298	0.702
4	1.4	0.198	0.802
5	1.6	0.149	0.851
18	4.2	0.036	0.964
19	4.4	0.034	0.966
20	4.6	0.032	0.968
21	4.8	0.031	0.969
22	5.0	0.029	0.971

The following graph was plotted using the data in the above table:



15 ••

Picture the Problem We can use the total energy of the particle in the region $x > 0$ to find k_2. Knowing k_2, we can use $R = \dfrac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$ to find R and $T = 1 - R$ to determine the transmission coefficient T.

(*a*) Using conservation of energy, express the particle in the region $x > 0$:

$$\frac{\hbar^2 k_2^2}{2m} - U_0 = 2U_0$$

Solve for k_2:

$$k_2 = \frac{\sqrt{6mU_0}}{\hbar}$$

From the equation for the total energy of the particle:

$$k_1 = \frac{\sqrt{4mU_0}}{\hbar}$$

Express the ratio of k_2 to k_1:

$$\frac{k_2}{k_1} = \frac{\dfrac{\sqrt{6mU_0}}{\hbar}}{\dfrac{\sqrt{4mU_0}}{\hbar}} = \sqrt{\frac{3}{2}} \Rightarrow k_2 = \boxed{\sqrt{\frac{3}{2}}k_1}$$

(*b*) The reflection coefficient R is given by:

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{\left(1 - \dfrac{k_2}{k_1}\right)^2}{\left(1 + \dfrac{k_2}{k_1}\right)^2}$$

Substitute for k_2/k_1 and evaluate R:

$$R = \frac{\left(1 - \sqrt{\dfrac{3}{2}}\right)^2}{\left(1 + \sqrt{\dfrac{3}{2}}\right)^2} = \boxed{0.0102}$$

(*c*) Because $R + T = 1$:

$$T = 1 - R = 1 - 0.0102 = \boxed{0.990}$$

(*d*) If we let N_0 represent the number of particles incident upon the potential step, then the number that continue beyond is:

$$N_0 T = 10^6 \times 0.990 = \boxed{9.90 \times 10^5}$$

$$\boxed{\text{Classically, all } 10^6 \text{ would continue to move past the potential step.}}$$

16 ••

Picture the Problem We can use the energies in the regions $U = 0$ and $U = U_0$ to express the ratio of the potential energy to the total energy in terms of the ratio of the wave numbers. We can also express this ratio in terms of the reflection coefficient R to obtain an expression for the ratio of E to U in terms of R.

In the region $U = 0$:

$$E = \frac{\hbar^2 k_1^2}{2m} \Rightarrow k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

In the region $U = U_0$:

$$E - U_0 = \frac{\hbar^2 k_2^2}{2m} \Rightarrow k_2 = \sqrt{\frac{2m(E - U_0)}{\hbar^2}}$$

Let r equal the ratio of k_2 to k_1:

$$r = \frac{k_2}{k_1} = \frac{\sqrt{\frac{2m(E - U_0)}{\hbar^2}}}{\sqrt{\frac{2mE}{\hbar^2}}} = \sqrt{1 - \frac{U_0}{E}}$$

Letting $U_0 = U,$ solve for U/E:

$$\frac{U}{E} = 1 - r^2 \qquad (1)$$

Write the reflection coefficient R as a function of $r = \frac{k_2}{k_1}$:

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{\left(1 - \frac{k_2}{k_1}\right)^2}{\left(1 + \frac{k_2}{k_1}\right)^2} = \frac{(1 - r)^2}{(1 + r)^2}$$

Solve for r to obtain:

$$r = \frac{1 - \sqrt{R}}{1 + \sqrt{R}}$$

Substitute for r in equation (1):

$$\frac{U}{E} = 1 - \left(\frac{1 - \sqrt{R}}{1 + \sqrt{R}}\right)^2$$

and

$$\frac{E}{U} = \left[1 - \left(\frac{1 - \sqrt{R}}{1 + \sqrt{R}}\right)^2\right]^{-1}$$

Substitute a numerical value for R and evaluate E/U:

$$\frac{E}{U} = \left[1 - \left(\frac{1 - \sqrt{0.5}}{1 + \sqrt{0.5}}\right)^2\right]^{-1} = \boxed{1.03}$$

17 ••

Picture the Problem The probability that a proton will tunnel out of a nucleus in one collision with a nuclear barrier if it has a given energy is given by Equation 35-29.

Equation 35-29 is:

$$T = e^{-2\alpha a}$$

where

$$\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

Multiply the numerator and denominator of α by c to obtain:

$$\alpha = \frac{\sqrt{2mc^2(U_0 - E)}}{\hbar c}$$

where

$$\hbar c = 1.974\times 10^{-13}\ \text{MeV}\cdot\text{m}$$

Using $m_\text{p}c^2 = 938\,\text{MeV}$, evaluate T:

$$T = \exp\left\{-2\left(10^{-15}\ \text{m}\right)\frac{\sqrt{2(938\,\text{MeV})(6\,\text{MeV})}}{1.974\times 10^{-13}\ \text{MeV}\cdot\text{m}}\right\} = \boxed{0.341}$$

***18** ••

Picture the Problem The probability that the electron with a given energy will tunnel through the given barrier is given by Equation 35-29.

(*a*) Equation 35-29 is:

$$T = e^{-2\alpha a}$$

where

$$\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

Multiply the numerator and denominator of α by c to obtain:

$$\alpha = \frac{\sqrt{2mc^2(U_0 - E)}}{\hbar c}$$

where

$$\hbar c = 1.974\times 10^{-13}\ \text{MeV}\cdot\text{m}$$

Using $m_\text{e}c^2 = 511\,\text{keV}$, evaluate T:

$$T = \exp\left\{-2\left(10^{-9}\ \text{m}\right)\frac{\sqrt{2(511\,\text{keV})(25\,\text{eV} - 10\,\text{eV})}}{1.974\times 10^{-13}\ \text{MeV}\cdot\text{m}}\right\} = \boxed{5.91\times 10^{-18}}$$

(*b*) Repeat with $a = 10^{-10}$ m:

$$T = \exp\left\{-2\left(10^{-10}\text{ m}\right)\frac{\sqrt{2(511\text{ keV})(25\text{ eV} - 10\text{ eV})}}{1.974\times10^{-13}\text{ MeV}\cdot\text{m}}\right\} = \boxed{1.89\times10^{-2}}$$

19 •••

Picture the Problems We can find the distance of closest approach by equating the kinetic energy of the alpha particle and the Coulomb potential energy. The probability that the electron with a given energy will tunnel through the given barrier is given by $T = e^{-2\alpha a}$, where α is the transmission coefficient and depends on ΔE.

(*a*) The distance of closest approach is related to the kinetic energy E of the alpha particles:

$$E = \frac{k2eZe}{r_1}$$

Solve for r_1:

$$r_1 = \frac{2kZe^2}{E}$$

For E = 4 MeV:

$$r_{1,4\text{ MeV}} = \frac{2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)(92)\left(1.6\times10^{-19}\text{ C}\right)^2}{4\text{ MeV}\times\dfrac{1.6\times10^{-19}\text{ J}}{\text{eV}}} = \boxed{6.62\times10^{-14}\text{ m}}$$

For K = 7 MeV:

$$r_{1,7\text{ MeV}} = \frac{2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)(92)\left(1.6\times10^{-19}\text{ C}\right)^2}{7\text{ MeV}\times\dfrac{1.6\times10^{-19}\text{ J}}{\text{eV}}} = \boxed{3.78\times10^{-14}\text{ m}}$$

(*b*) The transmission coefficient T is given by:

$$T = e^{-2\alpha a} \qquad (1)$$

where

$$\alpha = \sqrt{\frac{2m\Delta E}{\hbar^2}} = \frac{\sqrt{2m\Delta E}}{\hbar}$$

Evaluate $\alpha_{4\text{ MeV}}$ for ΔE = 4 MeV:

$$\alpha_{4\text{ MeV}} = \frac{\sqrt{2\left(4\text{ u}\times\dfrac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right)\left(4\text{ MeV}\times\dfrac{1.6\times10^{-19}\text{ J}}{\text{eV}}\right)}}{1.05\times10^{-34}\text{ J}\cdot\text{s}} = 8.78\times10^{14}\text{ m}^{-1}$$

Evaluate $\alpha_{7\,\text{MeV}}$ for ΔE = 7 MeV:

$$\alpha_{7\,\text{MeV}} = \frac{\sqrt{2\left(4\,\text{u}\times\frac{1.66\times10^{-27}\,\text{kg}}{\text{u}}\right)\left(7\,\text{MeV}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}\right)}}{1.05\times10^{-34}\,\text{J}\cdot\text{s}} = 1.16\times10^{15}\,\text{m}^{-1}$$

Substitute numerical values in equation (1) and evaluate $T_{4\,\text{MeV}}$:

$$T_{4\,\text{MeV}} = e^{-2\left(8.78\times10^{14}\,\text{m}^{-1}\right)\left(6.62\times10^{-14}\,\text{m}\right)} = \boxed{3.27\times10^{-51}}$$

Substitute numerical values in equation (1) and evaluate $T_{7\,\text{MeV}}$:

$$T_{7\,\text{MeV}} = e^{-2\left(1.16\times10^{15}\,\text{m}^{-1}\right)\left(3.78\times10^{-14}\,\text{m}\right)} = \boxed{8.21\times10^{-39}}$$

The Schrödinger Equation in Three Dimensions

20 •

Picture the Problem We can use $E_{n_1,n_2,n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$ with the given sides of the box to find the quantum numbers n_1, n_2, n_3 that correspond to the lowest ten quantum states of this box.

The energies of the quantum states are given by Equation 35-34:

$$E_{n_1,n_2,n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$$

For a box with sides L_1, $L_2 = 2L_1$, and $L_3 = 3L_1$:

$$E_{n_1,n_2,n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{4L_1^2}+\frac{n_3^2}{9L_1^2}\right) = \frac{h^2}{8mL_1^2}\left(n_1^2+\frac{n_2^2}{4}+\frac{n_3^2}{9}\right) = \frac{h^2}{288mL_1^2}\left(36n_1^2+9n_2^2+n_3^2\right)$$

The energies in units of $\frac{h^2}{288mL_1^2}$ are listed in the following table:

n_1	n_2	n_3	E
1	1	1	49
1	1	2	61

1	2	1	76
1	1	3	81
1	2	2	88
1	2	3	108
1	1	4	109
1	3	1	121
1	3	2	133
1	2	4	136

21 •

Picture the Problem The wave functions are of the form

$$\psi = A\sin\left(\frac{n_1\pi}{L_1}x\right)\sin\left(\frac{n_2\pi}{2L_1}y\right)\sin\left(\frac{n_3\pi}{3L_1}z\right)$$

22 •

Picture the Problem We can use $E_{n_1,n_2,n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$ with the given sides of the box to find the quantum numbers n_1, n_2, n_3 that correspond to the lowest ten quantum states of this box.

(*a*) The energies of the quantum states are given by Equation 35-34:

$$E_{n_1,n_2,n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$$

For a box with sides L_1, $L_2 = 2L_1$, and $L_3 = 4L_1$:

$$E_{n_1,n_2,n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{4L_1^2}+\frac{n_3^2}{16L_1^2}\right) = \frac{h^2}{8mL_1^2}\left(n_1^2+\frac{n_2^2}{4}+\frac{n_3^2}{16}\right) = \frac{h^2}{128mL_1^2}\left(16n_1^2+4n_2^2+n_3^2\right)$$

The energies in units of $\frac{h^2}{128mL_1^2}$ are listed in the following table:

n_1	n_2	n_3	E
1	1	1	21
1	1	2	24
1	1	3	29
1	2	1	33

1	1	4	36
1	2	2	36
1	2	3	41
1	1	5	45
1	2	4	48
1	3	1	53
1	1	6	56
1	3	2	56

Referring to the table, we see that there are two degenerate levels:

$$\boxed{(1,1,4)\text{and}(1,2,2)}$$

and

$$\boxed{(1,1,6)\text{and}(1,3,2)}$$

***23** •

Picture the Problem The wave functions are of the form

$$\psi = A\sin\left(\frac{n_1\pi}{L_1}x\right)\sin\left(\frac{n_2\pi}{2L_1}y\right)\sin\left(\frac{n_3\pi}{4L_1}z\right)$$

24 •

Picture the Problem The boundary conditions in the y and z directions are the same those in Figure 35-1. In the x direction, we'll require the $\psi = 0$ at $-L/2$ and $L/2$.

(*a*) The boundary conditions in the x direction are:

$$\psi\left(-\tfrac{1}{2}L\right) = \psi\left(\tfrac{1}{2}L\right) = 0$$

The general solution of the time-independent Schrödinger equation is:

$$\psi(x) = A\sin kx + B\cos kx$$

Apply the boundary conditions to obtain:

$$\psi\left(-\tfrac{1}{2}L\right) = -A\sin\frac{kL}{2} + B\cos\frac{kL}{2} = 0$$

and

$$\psi\left(\tfrac{1}{2}L\right) = A\sin\frac{kL}{2} + B\cos\frac{kL}{2} = 0$$

Eliminate the terms in B by subtracting the equations:

$$A\sin\frac{kL}{2} = 0$$

For $A \neq 0$:

$$\frac{kL}{2} = \sin^{-1}0 = 0, \pi, 2\pi, \ldots$$

or

$$k=\frac{n_1\pi}{L}, n_1=0, 2, 4, \ldots$$

Eliminate the terms in A by adding the equations:

$$B\sin\frac{kL}{2}=0$$

For $B\neq 0$:

$$\frac{kL}{2}=\cos^{-1}0=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}, \ldots$$

or

$$k=\frac{n_2\pi}{L}, n_2=1, 3, 5, \ldots$$

Thus:

$$\psi(x,y,z)=B\cos\left(\frac{n_1\pi}{L}x\right)\sin\left(\frac{n_2\pi}{L}y\right)\sin\left(\frac{n_3\pi}{L}z\right), n_1=2n+1$$

and

$$\psi(x,y,z)=A\sin\left(\frac{n_1\pi}{L}x\right)\sin\left(\frac{n_2\pi}{L}y\right)\sin\left(\frac{n_3\pi}{L}z\right),\ n_1=2n$$

The ground-state wave function is:

$$\boxed{\psi(1,1,1)=A\cos\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{L}y\right)\cos\left(\frac{\pi}{L}z\right)}$$

(*b*) The allowed energies are the same as those for a well with $U=0$ for $0<x<$ L.

25 ••

Picture the Problem We can apply the solution to the time-independent Schrödinger equation in three dimensions to obtain the wave function and the allowed energies for the given two-dimensional region. In (*c*), we must find three different sets of quantum numbers (*m,n*) for which the sum of the squares are the same.

(*a*) The solution to the time-independent Schrödinger equation in two-dimensions is:

$$\psi(x,y)=A\sin k_1x\sin k_2y=\boxed{A\sin\frac{n\pi}{L}x\sin\frac{m\pi}{L}y}$$

where n and m are integers.

(*b*) The energy is quantized to the values:

$$E_{n,m} = \boxed{\frac{h^2}{8mL^2}\left(n^2 + m^2\right)}$$

(*c*) The lowest two states that are degenerate are:

$$\boxed{E_{1,2} = E_{2,1} = \frac{5h^2}{8mL^2}}$$

(*d*) The energies of three lowest states that have the same energies (in units of $\frac{h^2}{8mL^2}$) are listed in the table to the right:

n	m	$E_{n,m}$
1	7	50
7	1	50
5	5	50

The quantum numbers for the three states are:

$\boxed{(1,7),}$ $\boxed{(7,1),}$ and $\boxed{(5,5)}$

and their energies are

$$E = (50)\frac{h^2}{8mL^2} = \boxed{\frac{25h^2}{4mL^2}}$$

The Schrödinger Equation for Two Identical Particles

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Picture the Problem We must differentiate Equation 35-37 twice and substitute these derivatives in this equation to show that it is a solution.

With $U = 0$, Equation 35-35 becomes:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x_1,x_2)}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2\psi(x_1,x_2)}{\partial x_2^2} = E\psi(x_1,x_2) \quad (1)$$

Differentiate Equation 35-7 with respect to x_1:

$$\frac{\partial\psi}{\partial x_1} = \frac{\partial}{\partial x_1}\left[A\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}\right] = \frac{A\pi}{L}\cos\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}$$

Compute the second derivative with respect to x_1:

$$\frac{\partial^2\psi}{\partial x_1^2} = \frac{\partial}{\partial x_1}\left[\frac{A\pi}{L}\cos\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}\right] = -\frac{A\pi^2}{L^2}\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}$$

Differentiate Equation 35-7 with respect to x_2:

$$\frac{\partial\psi}{\partial x_2} = \frac{\partial}{\partial x_2}\left[A\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}\right] = \frac{2A\pi}{L}\sin\frac{\pi x_1}{L}\cos\frac{2\pi x_2}{L}$$

Compute the second derivative with respect to x_2:

$$\frac{\partial^2\psi}{\partial x_2^2} = \frac{\partial}{\partial x_2}\left[\frac{2A\pi}{L}\sin\frac{\pi x_1}{L}\cos\frac{2\pi x_2}{L}\right] = -\frac{4A\pi^2}{L^2}\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}$$

Substitute in equation (1) to obtain:

$$-\frac{\hbar^2}{2m}\left[-\frac{A\pi^2}{L^2}\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}\right]-\frac{\hbar^2}{2m}\left[-\frac{4A\pi^2}{L^2}\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}\right] = EA\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}$$

Solve for E to obtain:

$$E = \frac{5\hbar^2\pi^2}{2mL^2}$$

Thus we've shown that Equation 35-37 satisfies Equation 35-35 provided $E = \dfrac{5\hbar^2\pi^2}{2mL^2}$.

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Picture the Problem Because bosons have symmetric wave functions and do not obey the Pauli exclusion principle, they can occupy the same ground state.

The ground-state energy of a single particle in a one-dimensional box of length L is:

$$E_{0,1\,\text{particle}} = \frac{h^2}{8mL^2}$$

For 10 bosons:

$$E_{0,10\,\text{bosons}} = \frac{10h^2}{8mL^2} = \boxed{\frac{5h^2}{4mL^2}}$$

***28** •

Picture the Problem For fermions, such as neutrons for which the spin quantum number is ½, two particles can occupy the same spatial state.

The lowest total energy for the 10 fermions is:

$$E = 2E_1\left(1^2 + 2^2 + 3^2 + 4^2 + 5^2\right)$$
$$= 2\left(\frac{h^2}{8mL^2}\right)(55)$$
$$= \boxed{\frac{55h^2}{4mL^2}}$$

Orthogonality of Wave Functions

29 ••

Picture the Problem We need to show that $\int_{-\infty}^{\infty} \psi_0(x)\psi_1(x)dx = 0$, where $\psi_0(x)$ and $\psi_1(x)$ are given by Equations 35-23 and 35-25, respectively.

Equations 35-23 and 35-25 are:

$$\psi_0(x) = A_0 e^{-ax^2} \qquad 35\text{-}23$$

and

$$\psi_1(x) = A_1 x e^{-ax^2} \qquad 35\text{-}25$$

Note that $\psi_1(x)$ is antisymmetric, whereas $\psi_0(x)$ is symmetric. Because the product of an antisymmetric function and a symmetric function is antisymmetric:

$\psi_0(x)\psi_1(x)$ is antisymmetric

Because the integral of an antisymmetric function over symmetric limits is zero:

$$\boxed{\int_{-\infty}^{\infty} \psi_0(x)\psi_1(x)dx = 0}$$

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Picture the Problem We need to show that $\int_{-\infty}^{\infty} \psi_1(x)\psi_2(x)dx = 0$, where $\psi_2(x)$ is given in the problem statement and $\psi_1(x) = A_1 x e^{-ax^2}$.

Note that $\psi_1(x)$ is antisymmetric, whereas $\psi_2(x)$ is symmetric. Because the product of a symmetric function and an antisymmetric function is antisymmetric:

$\psi_1(x)\psi_2(x)$ is antisymmetric.

Because the integral of an antisymmetric function over symmetric limits is zero:

$$\int_{-\infty}^{\infty} \psi_1(x)\psi_2(x)dx = 0$$

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Picture the Problem We need to show that $\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = 0.$

Use the trigonometric identity $(\sin a\alpha)(\sin b\alpha) = \frac{1}{2}\{\cos[(a-b)\alpha] - \cos[(a+b)\alpha]\}$ to rewrite the product of the two sine functions as the difference of two cosine functions:

$$\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right) = \frac{1}{2}\left\{\cos\left[(n-m)\frac{\pi x}{L}\right] - \cos\left[(n+m)\frac{\pi x}{L}\right]\right\}$$

Substitute for $\sin\left(\frac{n\pi x}{L}\right)$ and $\sin\left(\frac{m\pi x}{L}\right)$ and evaluate $\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx$:

$$\int_{-\infty}^{\infty} \frac{1}{2}\left\{\cos\left[(n-m)\frac{\pi x}{L}\right] - \cos\left[(n+m)\frac{\pi x}{L}\right]\right\}dx = \frac{L}{\pi}\frac{\sin\left[(n-m)\frac{\pi x}{L}\right]}{n-m} - \frac{L}{\pi}\frac{\sin\left[(n+m)\frac{\pi x}{L}\right]}{n+m}$$

Because n and m are integers and $n \neq m$, the sine functions vanish at the two limits $x = 0$ and $x = L$. Therefore, $\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = 0$ for $n \neq m$.

General Problems

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Picture the Problem We can use the wave functions $\psi_1(x)$ and $\psi_2(x)$ and the definitions of $\langle x\rangle$ and $\langle x^2\rangle$ to evaluate these quantities and the wave functions at $x = 0$.

(*a*) The wave functions $\psi_1(x)$ and $\psi_2(x)$ are:

$$\psi_m = \sqrt{\frac{2}{L}}\sin\left(\frac{m\pi}{L}x\right), m = 2n$$

and

$$\psi_m = \sqrt{\frac{2}{L}}\cos\left(\frac{m\pi}{L}x\right), m = 2n+1$$

where $n = 0, 1, 2, \ldots$

Evaluate these functions at $x = 0$ to obtain:

$$\psi_1(0) = \sqrt{\frac{2}{L}}\sin\left[\frac{\pi}{L}(0)\right] = \boxed{0}$$

and

$$\psi_2(0) = \sqrt{\frac{2}{L}}\cos\left[\frac{\pi}{L}(0)\right] = \boxed{\sqrt{\frac{2}{L}}}$$

(*b*) Because $\left|\psi_m(x)\right|^2$ is an even function of x in all cases, $x\psi_m^2(x)$ is an odd function of x and:

$$\langle x\rangle = \int_{-L/2}^{L/2} x\psi_m^2(x)\,dx = \boxed{0}$$

(*c*) For $n = 1$:

$$\left\langle x^2\right\rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2\frac{\pi}{L}x\,dx$$

From integral tables:

$$\int x^2\sin^2(ax)\,dx = \frac{x^3}{6} - \left(\frac{x^3}{4a} - \frac{1}{8a^3}\right)\sin(2ax) - \frac{x\cos(2ax)}{4a^2}$$

Use this integral with $a = \pi/L$ to obtain:

$$\left\langle x^2\right\rangle = \boxed{\frac{L^2}{12}\left(1 + \frac{6}{\pi^2}\right)}$$

For $n = 2$:

$$\left\langle x^2\right\rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \cos^2\frac{2\pi}{L}x\,dx$$

From integral tables:

$$\int x^2\cos^2(ax)\,dx = \frac{x^3}{6} - \left(\frac{x^3}{4a} - \frac{1}{8a^3}\right)\sin(2ax) + \frac{x\cos(2ax)}{4a^2}$$

Use this integral with $a = 2\pi/L$ to obtain:

$$\left\langle x^2\right\rangle = \boxed{\frac{L^2}{12}\left(1 + \frac{3}{2\pi^2}\right)}$$

Remarks: Note that for any value of *m*, $\left\langle x^2\right\rangle = \frac{L^2}{12}\left(1 + \frac{6}{m^2\pi^2}\right)$.

***33 ••**

Picture the Problem We can determine the energies of the state by identifying the four lowest quantum states that are occupied in the ground state and computing their combined energies. We can then find the energy difference between the ground state and the first excited state and use this information to find the energy of the excited state.

Each *n*, *m* state can accommodate only 2 particles. Therefore, in the ground state of the system of 8 fermions, the four lowest quantum states are occupied. These are:

(1,1), (1,2), (2,1) and (2,2)

Note that the states (1,2) and (2,1) are distinctly different states because the *x* and *y* directions are distinguishable.

The energies are quantized to the values given by:

$$E_{n_1,n_2} = 2\left(\frac{h^2}{8mL^2}\right)\left(n_1^2 + n_2^2\right)$$

The energy of the ground state is the sum of the energies of the four lowest quantum states:

$$\begin{aligned} E_0 &= E_{1,1} + E_{1,2} + E_{2,1} + E_{2,2} \\ &= 2\left(\frac{h^2}{8mL^2}\right)\left(1^2 + 1^2\right) + 2\left(\frac{h^2}{8mL^2}\right)\left(1^2 + 2^2\right) + 2\left(\frac{h^2}{8mL^2}\right)\left(2^2 + 1^2\right) + 2\left(\frac{h^2}{8mL^2}\right)\left(2^2 + 2^2\right) \\ &= 2\left(\frac{h^2}{8mL^2}\right)\left(2 + 5 + 5 + 8\right) \\ &= \frac{5h^2}{mL^2} \end{aligned}$$

The next higher state is achieved by taking one fermion from the (2,2) state and raising it to the next higher unoccupied state. That state is the (1,3) state. The energy difference between the ground state and this state is:

$$\begin{aligned} \Delta E &= E_{1,3} - E_{2,2} \\ &= \frac{h^2}{8mL^2}\left(1^2 + 3^2\right) - \frac{h^2}{8mL^2}\left(2^2 + 2^2\right) \\ &= \frac{h^2}{8mL^2}\left(10 - 8\right) = \frac{h^2}{4mL^2} \end{aligned}$$

Hence, the energies of the degenerate states (1,3) and (3,1) are:

$$\begin{aligned} E_{1,3} &= E_{3,1} = E_0 + \Delta E \\ &= \frac{5h^2}{mL^2} + \frac{h^2}{4mL^2} = \frac{21h^2}{4mL^2} \end{aligned}$$

The three lowest energy levels are therefore:

$$E_0 = \boxed{\frac{5h^2}{mL^2}}$$

and two states of energy

$$E_1 = E_2 = \boxed{\frac{21h^2}{4mL^2}}$$

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Picture the Problem The energy levels are the same as for a two-dimensional box of widths L and $3L$.

(*a*) The energies of the bound states are given by:

$$E_{n,m} = \frac{h^2}{8m}\left(\frac{n^2}{L^2} + \frac{m^2}{9L^2}\right) = \frac{h^2}{72mL^2}\left(9n^2 + m^2\right)$$

The three lowest energy states are:

$$E_{1,1} = \frac{h^2}{72mL^2}(9+1) = \boxed{\frac{5h^2}{36mL^2}}$$

$$E_{1,2} = \frac{h^2}{72mL^2}(9+4) = \boxed{\frac{13h^2}{72mL^2}}$$

and

$$E_{1,3} = \frac{h^2}{72mL^2}(9+9) = \boxed{\frac{h^2}{4mL^2}}$$

$$\boxed{\text{None of these states are degenerate.}}$$

(*b*) Express the condition that must be satisfied for two states to be degenerate:

$$9\left(n_1^2 - n_2^2\right) = m_2^2 - m_1^2$$

This condition is first satisfied for:

$$n_1 = 2, m_1 = 3, \text{and } n_2 = 1 \text{ and } m_2 = 6$$

Find the energy of this doubly degenerate state:

$$E_{2,3} = \frac{h^2}{72mL^2}(36+9) = \boxed{\frac{5h^2}{8mL^2}}$$

35 •••

Picture the Problem We can use the definition of the classical expectation value (average value) to show that the classical expectation value of x^2 for a particle in a one-dimensional box of length L centered at the origin is $L^2/12$. In (*b*) we'll proceed as in (*a*) using the definition of the quantum expectation value of x^2.

(*a*) The classical expectation value is given by:

$$\left(x^2\right)_{\text{av}} = \frac{1}{\frac{1}{2}L-\left(-\frac{1}{2}L\right)}\int_{-L/2}^{L/2} x^2\,dx = \frac{1}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{1}{L}\left(\frac{L^3}{12}\right) = \boxed{\frac{L^2}{12}}$$

(*b*) For a particle in the *n*th state in a one-dimensional box:

$$\left\langle x^2\right\rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2\frac{n\pi}{L}x\,dx$$

From integral tables:

$$\int x^2\sin^2(ax)\,dx = \frac{x^3}{6}-\left(\frac{x^3}{4a}-\frac{1}{8a^3}\right)\sin(2ax)-\frac{x\cos(2ax)}{4a^2}$$

In the limit $n >> 1$:

$$\left\langle x^2\right\rangle = \boxed{\frac{L^2}{12}}$$

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Picture the Problem We can solve Equation 35-28 for T and substitute for R using Equation 35-27. Letting $r = k_2/k_1$ and simplifying will lead to the given result.

Equation 35-28 is:

$$T+R=1 \Rightarrow T=1-R$$

From Equation 35-27:

$$R=\frac{(k_1-k_2)^2}{(k_1+k_2)^2}=\frac{\left(1-\frac{k_2}{k_1}\right)^2}{\left(1+\frac{k_2}{k_1}\right)^2}=\frac{(1-r)^2}{(1+r)^2}$$

where $r = k_2/k_1$

Substitute for R to obtain:

$$T=1-\frac{(1-r)^2}{(1+r)^2}=\frac{(1+r)^2-(1-r)^2}{(1+r)^2} = \boxed{\frac{4r}{(1+r)^2}}$$

Substitute for k_2/k_1 for r and simplify to obtain:

$$T=\boxed{\frac{4k_1k_2}{(k_1+k_2)^2}}$$

37 ••

Picture the Problem We can use the energies in the regions $U = 0$ and $U = U_0$ to express the ratio of the wave numbers k_1 and k_2 in these regions in terms of E and U_0 and the definition of the reflection coefficient R to show that $R = \frac{(1-r)^2}{(1+r)^2}$.

In the region $U_0 = 0$:

$$E = \frac{\hbar^2 k_1^2}{2m} \Rightarrow k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

In the region $U = U_0$:

$$E - U_0 = \frac{\hbar^2 k_2^2}{2m} \Rightarrow k_2 = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

Let r equal the ratio of k_2 to k_1:

$$r = \frac{k_2}{k_1} = \frac{\sqrt{\frac{2m(E-U_0)}{\hbar^2}}}{\sqrt{\frac{2mE}{\hbar^2}}} = \boxed{\sqrt{1 - \frac{U_0}{E}}}$$

The reflection coefficient R is given by:

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$$

Factor k_1 from the numerator and denominator to obtain:

$$R = \frac{\left(1 - \frac{k_2}{k_1}\right)^2}{\left(1 + \frac{k_2}{k_1}\right)^2}$$

Substitute for k_2/k_1 to obtain:

$$R = \boxed{\frac{(1-r)^2}{(1+r)^2}}$$

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Picture the Problem

(*a*) From Problem 37 we have:

$$R = \frac{(1-r)^2}{(1+r)^2}, \text{ where } r = \sqrt{1 - \frac{U_0}{E}}$$

Because $E = \alpha U_0$, R can be written:

$$r = \sqrt{1 - \frac{1}{\alpha}}$$

From Problem 36 we have:

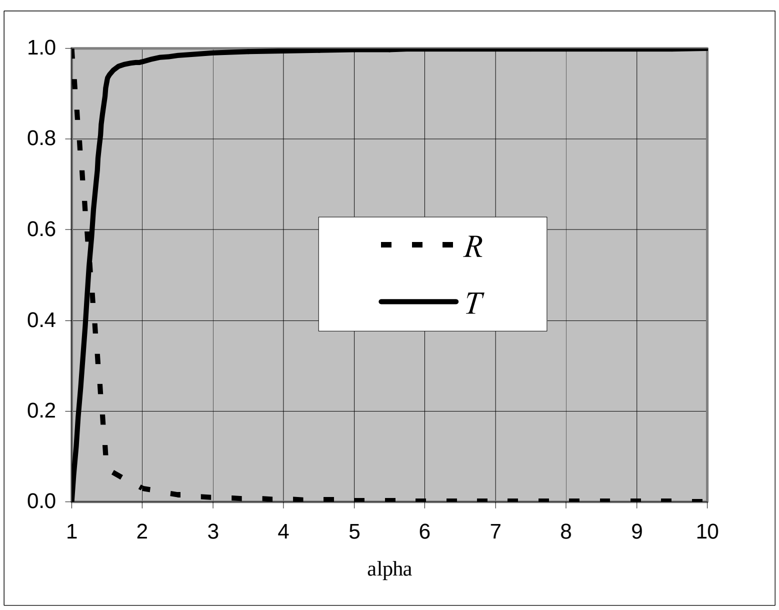
$$T = \frac{4r}{(1+r)^2}$$

A spreadsheet program to plot R and T as functions of α is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A2	1.0	α
B2	SQRT(1–1/A2)	$\sqrt{1-\frac{1}{\alpha}}$
C2	(1–B2)^2/(1+B2)^2	$\frac{(1-r)^2}{(1+r)^2}$
D2	4*B2//(1+B2)^2	$\frac{4r}{(1+r)^2}$

	A	B	C	D
1	alpha	r	R	T
2	1.0	0.000	1.000	0.000
3	1.5	0.577	0.072	0.928
4	2.0	0.707	0.029	0.971
5	2.5	0.775	0.016	0.984
16	8.0	0.935	0.001	0.999
17	8.5	0.939	0.001	0.999
18	9.0	0.943	0.001	0.999
19	9.5	0.946	0.001	0.999
20	10.0	0.949	0.001	0.999

The following graph of R and T as functions of α was plotted using the data in the table:



(*b*) From the graph, we note that, as $\alpha \to \infty, T \to 1$ and $R \to 0$. The graph also shows that, as $\alpha \to 1, T \to 0$ and $R \to 1$.

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Picture the Problem We require that

$$A_2^2 \int_{-\infty}^{\infty} \left(2ax^2 - \tfrac{1}{2}\right)^2 e^{-2ax^2} dx = 2A_2^2 \int_0^{\infty} \left(2ax^2 - \tfrac{1}{2}\right)^2 e^{-2ax^2} dx = 1.$$

Expand the integrand to obtain:

$$\left(2ax^2 - \tfrac{1}{2}\right)^2 e^{-2ax^2} = \left(4a^2x^4 - 2ax^2 + \tfrac{1}{4}\right)e^{-2ax^2} = 4a^2x^4e^{-2ax^2} - 2ax^2e^{-2ax^2} + \tfrac{1}{4}e^{-2ax^2}$$

Substitute in the integral expression:

$$2A_2^2 \int_0^{\infty} \left(4a^2x^4e^{-2ax^2} - 2ax^2e^{-2ax^2} + \tfrac{1}{4}e^{-2ax^2}\right)dx = 1$$

or

$$8a^2A_2^2 \int_0^{\infty} x^4e^{-2ax^2}dx - 4aA_2^2 \int_0^{\infty} x^2e^{-2ax^2}dx + \tfrac{1}{2}A_2^2 \int_0^{\infty} e^{-2ax^2}dx = 1 \qquad (1)$$

Use the definite integrals $\int_0^{\infty} e^{-bx^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{b}}$ and $\int_0^{\infty} x^{2n}e^{-bx^2}dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n+1}b^n}\sqrt{\frac{\pi}{b}}, n \geq 1$ (see Table D-5) to integrate equation (1) term by term:

$$8a^2A_2^2\left[\frac{3}{2^3(2a)^2}\sqrt{\frac{\pi}{2a}}\right] - 4aA_2^2\left[\frac{1}{2^2(2a)}\sqrt{\frac{\pi}{2a}}\right] + \tfrac{1}{2}A_2^2\left[\frac{1}{2}\sqrt{\frac{\pi}{2a}}\right] = 1$$

or

$$A_2^2\left[\frac{3}{4}\sqrt{\frac{\pi}{2a}} - \frac{1}{2}\sqrt{\frac{\pi}{2a}} + \frac{1}{4}\sqrt{\frac{\pi}{2a}}\right] = 1$$

or

$$A_2^2\left[\frac{1}{2}\sqrt{\frac{\pi}{2a}}\right] = 1$$

Solve for A_2:

$$A_2 = \sqrt{2\sqrt{\frac{2a}{\pi}}}$$

Because $a = \dfrac{m\omega_0}{2\hbar} = \dfrac{m\omega_0\pi}{h}$ $\qquad$ $\boxed{A_2 = \sqrt[4]{\dfrac{8m\omega_0}{h}}}$

40 •••

(*a*) Let $x = -x$. The second derivative is an even operator, that is, $d^2\psi(-x)/d(-x)^2 = d^2\psi(x)/dx^2$. Therefore, if $U(-x) = U(x)$, the Schrödinger equation for $\psi(-x) = \psi(x)$ and must give the same values for the energy E. If $\psi(-x)$ differs from $\psi(x)$, the ratio $\psi(-x)/\psi(x)$ cannot be a function of x and must be a constant. Hence, $\psi(x) = C\psi(-x)$.

(*b*) The previous result means that replacing the argument of the wave function by its negative is equivalent to multiplication by C. Thus, if $C\psi(-x)$ is a good wave function and we replace its argument by its negative, that is, by x, we must multiply by C again. Thus, $\psi(x) = C^2\psi(x)$, $C^2 = 1$, and $C = \pm 1$.

***41** •••

Picture the Problem We can follow the step-by-step procedure outlined in the problem statement to show that $(E_{\text{av}})_{\text{min}} = +\frac{1}{2}\hbar\omega$.

1. The total classical energy is:

$$E_{\text{av}} = U_{\text{av}} + K_{\text{av}} = \tfrac{1}{2}m\omega^2\left(x^2\right)_{\text{av}} + \frac{\left(p^2\right)_{\text{av}}}{2m} \qquad (1)$$

2. Express the standard deviation of Δp:

$$(\Delta p)^2 = \left[(p - p_{\text{av}})^2\right]_{\text{av}} = \left[p^2 - 2pp_{\text{av}} - p_{\text{av}}^2\right]_{\text{av}}$$

Because $p_{\text{av}} = 0$:

$$(\Delta p)^2 = \left(p^2\right)_{\text{av}}$$

3. Express the standard deviation of Δx:

$$(\Delta x)^2 = \left[(x - x_{\text{av}})^2\right]_{\text{av}} = \left[x^2 - 2xx_{\text{av}} - x_{\text{av}}^2\right]_{\text{av}}$$

Because $x_{\text{av}} = 0$:

$$(\Delta x)^2 = \left(x^2\right)_{\text{av}}$$

4. Use the uncertainty principle $\Delta p = \hbar/2\Delta x$ to eliminate $(p^2)_{av}$ from the average energy in equation (1):

$$E_{av} = \tfrac{1}{2}m\omega^2\left(x^2\right)_{av} + \frac{\left(\Delta p^2\right)}{2m} = \tfrac{1}{2}m\omega^2\left(x^2\right)_{av} + \frac{1}{2m}\left[\frac{\hbar^2}{4(\Delta x)^2}\right] = \tfrac{1}{2}m\omega^2\left(x^2\right)_{av} + \frac{\hbar^2}{8m\left(x^2\right)_{av}}$$

Let $Z = (x^2)_{av}$ to obtain:

$$E_{av} = \tfrac{1}{2}m\omega^2 Z + \frac{\hbar^2}{8mZ}$$

5. Differentiate E_{av} with respect to Z and set this derivative equal to zero:

$$\frac{dE_{av}}{dZ} = \frac{d}{dZ}\left[\tfrac{1}{2}m\omega^2 Z + \frac{\hbar^2}{8mZ}\right] = \tfrac{1}{2}m\omega^2 - \frac{\hbar^2}{8mZ^2} = 0 \text{ for extrema}$$

Solve for Z to find the value of Z that minimizes E_{av} (see the remark below):

$$Z = \frac{\hbar}{2m\omega}$$

6. Evaluate E_{av} when $Z = \hbar/2m\omega$:

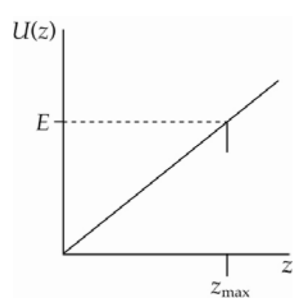
$$\left(E_{av}\right)_{min} = \tfrac{1}{2}m\omega^2\left(\frac{\hbar}{2m\omega}\right) + \frac{\hbar^2}{8m}\left(\frac{2m\omega}{\hbar}\right) = \boxed{\tfrac{1}{2}\hbar\omega}$$

Remarks: All we've shown is that $Z = \hbar/2m\omega$ is an extreme value, i.e., either a *maximum* or a *minimum*. To show that $Z = \hbar/2m\omega$ minimizes E_{av}, we must either 1) show that the second derivative of E_{av} with respect to Z evaluated at $Z = \hbar/2m\omega$ is positive, or 2) confirm that the graph of E_{av} as a function of Z opens upward at $Z = \hbar/2m\omega$.

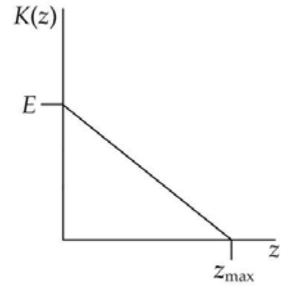
42 •••

Picture the Problem

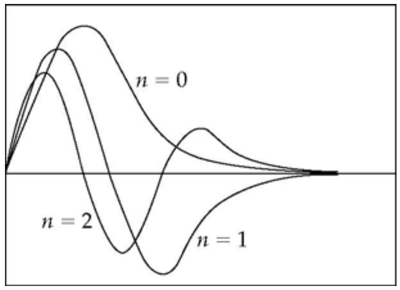
The classically allowed region is for $E \geq U(z)$. In the figure below, this region extends from $z = 0$ to $z = z_{max}$.



The kinetic energy is $E - U(z)$. In this case, $K(z)$ is a straight line extending from E at $z = 0$ to 0 at $z = z_{max}$.



A sketch of the wave functions for the lowest three energy states is shown to the right:



43 ••

Picture the Problem If $f(x) = 0$ everywhere on the interval $1 < x < 2$, then the slope of $f(x)$ is zero everywhere on the interval; and if the slope remains zero everywhere on the interval, then the rate of change of the slope (with respect to x) also remains zero everywhere on the interval; the rate of change of slope remains zero everywhere on the interval, then the rate of change of the rate of change of the slope also remains zero everywhere on the interval; and so on. More concisely, if $f(x) = 0$ everywhere on the interval $1 < x < 2$, then derivatives of $f(x)$ with respect to x of order 1, 2, 3, … are each equal to zero everywhere on the interval.

Calculating the first three derivatives of f we obtain:

$$\frac{df}{dx} = 3Ax^2 + 2Bx + Cx$$

$$\frac{d^2f}{dx^2} = 6Ax + 2B$$

and

$$\frac{d^3f}{dx^3} = 6A$$

Using $d^3f/df^3 = 0$ and solving for A one obtains:

$A = 0$

Substituting 0 for A in the expression for d^2f/dx^2 gives:

$$\frac{d^2f}{dx^2} = 0 + 2B = 2B$$

Using $d^2f/dx^2 = 0$ and solving for B yields:

$B = 0$

Substituting 0 for both A and B in the expression for df/dx yields:

$$\frac{df}{dx} = 0 + 0 + Cx = Cx$$

Using $df/dx = 0$ and solving for C

$C = 0$

one obtains:

Substituting 0 for *A, B,* and *C* in the expression for *f* gives: $f = 0 + 0 + 0 + D = D$

Using $f = 0$ and solving for *D* gives: $D = 0$

Thus, we've shown that if $f(x) = Ax^3 + Bx^2 + Cx + D = 0$ everywhere on the interval $1 < x < 2$, it follows that $A = B = C = D = 0$.

Chapter 36
Atoms

Conceptual Problems

***1 •**

Determine the Concept Examination of Figure 35-4 indicates that as n increases, the spacing of adjacent energy levels decreases**.**

2 •

Picture the Problem The energy of an atom of atomic number Z, with exactly one electron in its nth energy state is given by $E_n = -Z^2 \frac{E_0^2}{n^2}, n = 1, 2, 3, \ldots$.

Express the energy of an atom of atomic number Z, with exactly one electron, in its nth energy state:

$$E_n = -Z^2 \frac{E_0^2}{n^2}, n = 1, 2, 3, \ldots$$

where E_0 is the atom's ground state energy.

For lithium (Z = 3) in its first excited state (n = 1) this expression becomes:

$$E_2 = -(3)^2 \frac{E_0}{1^2} = -9E_0$$

and $\boxed{(a) \text{ is correct.}}$

3 •

Determine the Concept Bohr's postulates are 1) the electron in the hydrogen atom can move only in certain non-radiating, circular orbits called *stationary states*, 2) if E_i and E_f are the initial and final energies of the atom, the frequency f of the emitted radiation during a transition is given by $f = [E_i - E_f]/h$, and 3) the angular momentum of a circular orbit is constrained by $mvr = n\hbar$. $\boxed{(a) \text{ is correct.}}$

4 ••

Picture the Problem We can express the kinetic energy of the orbiting electron as well as its total energy as functions of its radius r.

Express the total energy of an orbiting electron:

$$E = K + U$$

Express the orbital kinetic energy of an electron:

$$K = \frac{kZe^2}{2r} \qquad (1)$$

Express the potential energy of an orbiting electron:

$$U = -\frac{kZe^2}{r}$$

Substitute and simplify to obtain:

$$E = \frac{kZe^2}{2r} - \frac{kZe^2}{r} = \frac{kZe^2}{2r} - \frac{2kZe^2}{2r} = -\frac{kZe^2}{2r}$$

Thus, as r increases, E becomes less negative and therefore *increases*.

Examination of the expression for K makes it clear that if r increases, K decreases.

5 •

Picture the Problem We can relate the kinetic energy of the electron in the n = 2 state to its total energy using $E_2 = K_2 + U_2$.

Express the total energy of the hydrogen atom in its n = 2 state:

$$E_2 = K_2 + U_2 = K_2 - 2K_2 = -K_2$$

or

$$K_2 = -E_2$$

Express the energy of hydrogen in its nth energy state:

$$E_n = -Z^2 \frac{E_0^2}{n^2} = -(1)^2 \frac{E_0^2}{n^2} = -\frac{E_0^2}{n^2}$$

where E_0 is hydrogen's ground state energy and Z = 1.

Substitute to obtain:

$$K_n = \frac{E_0^2}{n^2} \text{ and } K_2 = \frac{E_0}{2^2} = \frac{E_0}{4}$$

(d) is correct.

6 •

Picture the Problem The orbital radius r depends on the n = 1 orbital radius a_0, the atomic number Z, and the orbital quantum number n according to $r = n^2 a_0/Z$.

The radius of the n = 5 orbit is:

$$r_5 = 5^2 \frac{a_0}{1} = 25a_0$$

because Z = 1 for hydrogen.

(b) is correct.

*7 •

Determine the Concept We can find the possible values of ℓ by using the constraints on the quantum numbers n and ℓ.

The allowed values for the orbital quantum number ℓ for n = 1, 2, 3, and 4 are summarized in table shown to the right:

n	ℓ
1	0
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3

From the table it is clear that ℓ can have 4 values.

(a) is correct.

8 •

Picture the Problem We can find the number of different values m_ℓ can have by enumerating the possibilities when the principal quantum number $n = 4$.

The allowed values for the orbital quantum number ℓ and the magnetic quantum number m_ℓ for $n = 4$ are summarized to the right:

$$\ell = 0, 1, 2, 3$$
and
$$m_\ell = -3, -2, -1, 0, 1, 2, 3$$

From this enumeration we can see that m can have 7 values.

$\boxed{(c) \text{ is correct.}}$

9 •

Picture the Problem We can visualize the relationship between the quantum number ℓ and the electronic configuration as shown in the table below.

	s	p	d	f	g	h
ℓ value	0	1	2	3	4	5

Because the p state corresponds to $\ell = 1$, $\boxed{(c) \text{ is correct.}}$

***10** ••

Determine the Concept The s state, with $\ell = 0$, is a ″penetrating″ state in which the probability density near the nucleus is significant. Consequently, the 3s electron in sodium is in a region of low potential energy for a significant portion of the time. In the state $\ell = 1$, the probability density at the nucleus is zero, so the 2p electron of sodium is shielded from the nuclear charge by the 1s electrons. In hydrogen, the 3s and 2p electrons experience the same nuclear potential.

11 ••

Determine the Concept In conformity with the exclusion principle, the total number of electrons that can be accommodated in states of quantum number n is n^2 (see Problem 48). The fact that closed shells correspond to $2n^2$ electrons indicates that there is another quantum number that can have two possible values.

12 ••

Picture the Problem We can group these elements by using Table 35-1 to look for a common outer electronic configuration in the ground states.

The following elements have an outer $4s^2$ configuration in the ground state:

$\boxed{\text{titanium, manganese, and calcium}}$

The following elements have an outer 4s configuration in the ground state: $\boxed{\text{potassium, chromium, and copper.}}$

Remarks: It is to be expected that atoms of the first group will have similar properties, and, likewise, that atoms of the second group will have similar properties.

13 •

Picture the Problem We can use the fact that the sum of the exponents in the electronic configuration representation is the atomic number to identify these two elements.

(*a*) Adding the exponents yields a sum of 15. Because this sum is the atomic number, *Z*, the element must be $\boxed{\text{phosphorus.}}$

(*b*) Adding the exponents yields a sum of 24. Because this sum is the atomic number, *Z*, the element must be $\boxed{\text{chromium.}}$

Remarks: Checking the electronic configurations in Table 35-1 further confirms these conclusions.

***14 •**

Picture the Problem We can apply the constraints on the quantum numbers ℓ and m_ℓ to find the possible values for each when $n = 3$.

Express the constraints on the quantum numbers n, ℓ, and m_ℓ:

$$n = 1, 2, 3, \ldots,$$
$$\ell = 0, 1, 2, \ldots, n-1,$$
and
$$m_\ell = -\ell, -\ell+1, \ldots, \ell$$

So, for $n = 3$, the constraints on ℓ limit it to the values:

$$\ell = \boxed{0, 1, \text{and } 2.}$$

m_ℓ can take on the values:

$$m_\ell = \boxed{-2, -1, 0, 1, 2}$$

15 •

Determine the Concept The correspondence between the letter designations K, L, M, N, O, and P for the shells and the principal quantum number *n* is summarized in the table below.

Shell designation	K	L	M	N	O	P	Q
n	1	2	3	4	5	6	7
ℓ	0	0	0	0	0	0	0
		1	1	1	1	1	1
			2	2	2	2	2
				3	3	3	3
					4	4	4
						5	5
							6

While $n = 2$ for the L shell, ℓ can be either 0 or 1. $\boxed{(d) \text{ is correct.}}$

16 ••

Picture the Problem The strengths and weaknesses of each model are summarized in the following table.

	Bohr Theory	**Schrödinger Theory**
Ease of application	Easy	Difficult
Prediction of stationary state energies	Correct predictions	Correct predictions
Prediction of angular momenta	Predicts incorrect results	Predicts correct results
Spatial distribution of electrons	Predicts incorrect results	Predicts correct probabilistic distribution

17 ••

Determine the Concept The optical spectrum of any atom is due to the configuration of its outer-shell electrons. Ionizing the next atom in the periodic table gives you an ion with the same number of outer-shell electrons, and almost the same nuclear charge. Hence, the spectra should be very similar.

***18** ••

Determine the Concept The Ritz combination principle is due to the quantization of energy levels in the atom. We can use the relationship between the wavelength of the emitted photon and the difference in energy levels within the atom that results in the emission of the photon to express each of the wavelengths and then the sum of the reciprocals of the first and second wavelengths and the sum of the reciprocals of the third and fourth wavelengths.

Express the wavelengths of the spectral lines λ_1, λ_2, λ_3, and λ_4 in terms of the corresponding energy transitions:

$$\lambda_1 = \frac{hc}{E_3 - E_2}$$

$$\lambda_2 = \frac{hc}{E_2 - E_0}$$

$$\lambda_3 = \frac{hc}{E_3 - E_1}$$

and

$$\lambda_4 = \frac{hc}{E_1 - E_0}$$

Add the reciprocals of λ_1 and λ_2 to obtain:

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{E_3 - E_2}{hc} + \frac{E_2 - E_0}{hc} = \frac{E_3 - E_0}{hc} \quad (1)$$

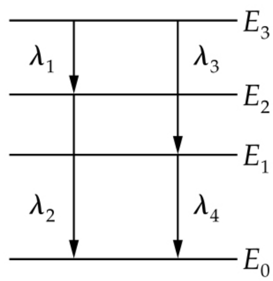
Add the reciprocals of λ_3 and λ_4 to obtain:

$$\frac{1}{\lambda_3} + \frac{1}{\lambda_4} = \frac{E_3 - E_1}{hc} + \frac{E_1 - E_0}{hc} = \frac{E_3 - E_0}{hc} \quad (2)$$

Because the right-hand sides of equations (1) and (2) are equal:

$$\boxed{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3} + \frac{1}{\lambda_4}}$$

One possible set of energy levels is shown to the right:



19 •

Determine the Concept An allowed transition must satisfy the selection rules $\Delta m_\ell = 0$ or ±1 and $\Delta\ell = \pm1$.

(*a*) $\Delta\ell = -1$ and $\Delta m_\ell = 0$: $\boxed{\text{The transition is allowed.}}$

(*b*) (3,0,1) does not exist. $\boxed{\text{The transition is not allowed.}}$

(*c*) $\Delta\ell = -1$ and $\Delta m_\ell = 2$: $\boxed{\text{The transition is not allowed.}}$

(*d*) $\Delta\ell = +1$ and $\Delta m_\ell = 1$: $\boxed{\text{The transition is allowed.}}$

(*e*) $\Delta\ell = -1$ and $\Delta m_\ell = 0$: $\boxed{\text{The transition is allowed.}}$

Estimation and Approximation

***20 ••**

Picture the Problem The number of photons need to stop a ^{85}Rb atom traveling at 300 m/s is the ratio of its momentum to that of a typical photon.

(*a*) The number *N* of photon-atom collisions needed to bring an atom to rest is the ratio of the change in the momentum of the atom as it stops to the momentum brought to the collision by each photon:

$$N = \frac{\Delta p_{\text{atom}}}{p_{\text{photon}}} = \frac{mv}{\frac{E}{c}} = \frac{mvc}{E}$$

where *m* is the mass of the atom.

The kinetic energy of an atom whose temperature is *T* is:

$$\tfrac{1}{2}mv^2 = \tfrac{3}{2}kT \Rightarrow v = \sqrt{\frac{3kT}{m}}$$

Substitute for *v* to obtain:

$$N = \frac{mc}{E}\sqrt{\frac{3kT}{m}} = \frac{c}{E}\sqrt{3mkT}$$

For an atom use mass is 50 u:

$$N = \frac{3\times10^8\text{ m/s}}{1\text{eV}\times\frac{1.6\times10^{-19}\text{ J}}{\text{eV}}}\sqrt{3\left(50\text{ u}\times\frac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right)\left(1.38\times10^{-23}\text{ J/K}\right)\left(500\text{ K}\right)} \approx \boxed{10^5}$$

(*b*) The number *N* of ping-pong ball-bowling ball collisions needed to bring the bowling ball to rest is the ratio of the change in the momentum of the bowling ball as it stops to the momentum brought to the collision by each ping-pong ball:

$$N = \frac{\Delta p_{\text{bowling ball}}}{p_{\text{ping-pong ball}}} = \frac{m_{\text{bb}}v_{\text{bb}}}{m_{\text{ppb}}v_{\text{ppb}}}$$

Provided the speeds of the approaching bowling ball and ping-pong ball are approximately the same:

$$N = \frac{\Delta p_{\text{bowling ball}}}{p_{\text{ping-pong ball}}} \approx \frac{m_{\text{bb}}}{m_{\text{ppb}}} \approx \frac{6\text{ kg}}{4\text{ g}} \approx \boxed{10^3}$$

(*c*) The number of photons *N* needed to stop a ^{85}Rb atom is the ratio of the change in the momentum of the atom to the momentum brought to the collision by each photon:

$$N = \frac{\Delta p_{\text{atom}}}{p_{\text{photon}}} = \frac{mv}{\frac{h}{\lambda}} = \frac{mv\lambda}{h}$$

Substitute numerical values and evaluate *N*:

$$N = \frac{85(1.66\times10^{-27}\text{ kg})(300\text{ m/s})(780.24\text{ nm})}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{4.98\times10^{4}}$$

21 ••

Picture the Problem We can use the relationship between the kinetic energy of an atom and its momentum, together with the de Broglie equation, to derive the expression for the thermal de Broglie wavelength. In Part (*b*), we can use the definition of the number density of atoms and the result from Part (*a*), with the interatomic spacing set equal to the thermal de Broglie wavelength, to estimate the temperature needed to create a Bose condensate.

(*a*) Express the kinetic energy of an atom in terms of its momentum:

$$K = \frac{p^2}{2m}$$

Use the de Broglie relationship to express the atom's momentum in terms of its de Broglie wavelength:

$$p = \frac{h}{\lambda_T}$$

where λ_T is the thermal de Broglie wavelength.

Substitute for p to obtain:

$$K = \frac{h^2}{2m\lambda_T^2}$$

The kinetic energy of an atom is also a function of its temperature T:

$$K = \frac{3}{2}kT$$

Equate these expressions for K to obtain:

$$\frac{3}{2}kT = \frac{h^2}{2m\lambda_T^2}$$

Solve for λ_T:

$$\lambda_T = \boxed{\sqrt{\frac{h^2}{3mkT}}}$$

(*b*) The number density of atoms ρ is given by:

$$\rho = \frac{N}{V}$$

where N is the number of atoms and V is the volume they occupy.

Assume that the atoms are arrayed on a cubic lattice of lattice spacing d to obtain:

$$V = Nd^3 \text{ and } \rho = \frac{N}{Nd^3} = \frac{1}{d^3}$$

Solve for d to obtain:

$$d = \rho^{-1/3}$$

Setting $d = \lambda_T$ yields:

$$\rho^{-1/3} = \sqrt{\frac{h^2}{3mkT}}$$

Solve for T to obtain:

$$T = \frac{h^2 \rho^{2/3}}{3mk}$$

Substitute numerical values and evaluate T:

$$T = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2\left(10^{12}\ \dfrac{\text{atoms}}{\text{cm}^3}\times\dfrac{10^6\ \text{cm}^3}{\text{m}^3}\right)^{2/3}}{3(85\,\text{u})\left(1.66\times10^{-27}\ \dfrac{\text{kg}}{\text{u}}\right)\left(1.38\times10^{-23}\ \dfrac{\text{J}}{\text{K}}\right)} = \boxed{75.2\,\text{nK}}$$

The Bohr Model of the Hydrogen Atom

22 •

Picture the Problem The radius of the first Bohr orbit is given by $a_0 = \dfrac{\hbar^2}{mke^2}$.

Equation 36-12 is:

$$a_0 = \frac{\hbar^2}{mke^2}$$

Substitute numerical values and evaluate a_0:

$$a_0 = \frac{\left(1.05\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{\left(9.11\times10^{-31}\text{kg}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2} = 5.26\times10^{-11}\ \text{m}$$
$$= \boxed{0.0526\,\text{nm}}$$

23 •

Picture the Problem We can use the equation relating the wavelength of the radiation emitted during a transition between two energy states to find the wavelengths for the transitions specified in the problem statement.

Express the wavelength of the radiation emitted during an energy transformation from one energy state to another:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{E_\text{i} - E_\text{f}}$$

provided the energies are expressed in eV. Note that this relationship tells us that the longest wavelength corresponds to the smallest energy difference.

Because $E_\text{f} = -13.6$ eV:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{E_\text{i} + 13.6\,\text{eV}}$$

Express the energy of the nth energy state of the atom:

$$E_n = -\frac{E_0}{n^2} = -\frac{13.6\,\text{eV}}{n^2}$$

Substitute to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{-\frac{13.6\,\text{eV}}{n^2} + 13.6\,\text{eV}} = \frac{1240\,\text{eV}\cdot\text{nm}}{13.6\,\text{eV}\left(1 - \frac{1}{n^2}\right)} \quad (1)$$

(*a*) Evaluate equation (1) for $n = n_1 = 3$:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{13.6\,\text{eV}\left(1 - \frac{1}{3^2}\right)} = \boxed{103\,\text{nm}}$$

(*b*) Evaluate equation (1) for $n = n_1 = 4$:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{13.6\,\text{eV}\left(1 - \frac{1}{4^2}\right)} = \boxed{97.3\,\text{nm}}$$

24 •

Picture the Problem For the Balmer series, $E_f = E_2 = -3.40$ eV. The wavelength associated with each transition is related to the difference in energy between the states by $\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{E_i - E_f}$.

Express the wavelength of the radiation emitted during an energy transformation from one energy state to another:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} \quad (1)$$

provided the energies are expressed in eV. Note that this relationship tells us that the longest wavelength corresponds to the smallest energy difference.

Evaluate ΔE for the transition $n = 3$ to $n = 2$:

$$\Delta E_{n\to 2} = E_n - E_2 = \frac{E_0}{n^2} - E_2$$

Because $E_f = E_2 = -3.40$ eV and $E_0 = -13.6$ eV:

$$\Delta E_{n\to 2} = -\frac{13.6\,\text{eV}}{n^2} + 3.40\,\text{eV} \quad (2)$$

Evaluate equation (2) for $n = 3$:

$$\Delta E_{3\to 2} = -\frac{13.6\,\text{eV}}{3^2} + 3.40\,\text{eV} = \boxed{1.89\,\text{eV}}$$

Substitute in equation (1) to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{1.89\,\text{eV}} = \boxed{656\,\text{nm}}$$

Evaluate equation (2) for $n = 4$:

$$\Delta E_{4\to 2} = -\frac{13.6\,\text{eV}}{4^2} + 3.40\,\text{eV} = \boxed{2.55\,\text{eV}}$$

Substitute in equation (1) to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{2.55\,\text{eV}} = \boxed{486\,\text{nm}}$$

Evaluate equation (2) for $n = 5$:

$$\Delta E_{5\to 2} = -\frac{13.6\,\text{eV}}{5^2} + 3.40\,\text{eV} = \boxed{2.86\,\text{eV}}$$

Substitute in equation (1) to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{2.86\,\text{eV}} = \boxed{434\,\text{nm}}$$

25 ••

Picture the Problem We can use Bohr's second postulate to relate the photon energy to its frequency and use $\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{E_\text{i} - E_\text{f}}$ to find the wavelengths of the three longest wavelengths in the Paschen series.

(*a*) Use Bohr's second postulate to express the energy of the photons in the Paschen series:

$$hf = \Delta E = E_\text{i} - E_\text{f}$$

For the series limit:

$$n = \infty \text{ and } E_\text{i} = 0$$

Substitute to obtain:

$$\Delta E = -E_\text{f} = -\left(-\frac{E_0}{n_2^2}\right) = \frac{E_0}{n_2^2} \qquad (1)$$

Evaluate the photon energy for $n_2 = 3$:

$$hf = \frac{13.6\,\text{eV}}{3^2} = \boxed{1.51\,\text{eV}}$$

Express the wavelength of the radiation resulting from an energy transition $\Delta E = hf$:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} \qquad (2)$$

provided the energies are expressed in eV.

Evaluate λ_{min} for the transition $n = \infty$ to $n_2 = 3$:

$$\lambda_{min} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.51\,\text{eV}} = \boxed{821\,\text{nm}}$$

(*b*) For the three longest wavelengths:

$$n_i = 4, 5, \text{and } 6$$

Equation (1) becomes:

$$\begin{aligned} hf = E_i - E_f &= -\frac{E_0}{n_i^2} - \left(-\frac{E_0}{n_2^2}\right) \\ &= E_0\left(\frac{1}{n_2^2} - \frac{1}{n_i^2}\right) = E_0\left(\frac{1}{9} - \frac{1}{n_i^2}\right) \end{aligned} \quad (3)$$

Evaluate equation (3) for $n = 4$:

$$\Delta E_{4\to3} = (13.6\,\text{eV})\left(\frac{1}{9} - \frac{1}{16}\right) = \boxed{0.661\,\text{eV}}$$

Evaluate equation (2) for $\Delta E = 0.661$ eV:

$$\lambda_{4\to3} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.661\,\text{eV}} = \boxed{1876\,\text{nm}}$$

Evaluate equation (3) for $n = 5$:

$$\Delta E_{5\to3} = (13.6\,\text{eV})\left(\frac{1}{9} - \frac{1}{25}\right) = \boxed{0.967\,\text{eV}}$$

Evaluate equation (2) for $\Delta E = 0.967$ eV:

$$\lambda_{5\to3} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.967\,\text{eV}} = \boxed{1282\,\text{nm}}$$

Evaluate equation (3) for $n = 6$:

$$\Delta E_{6\to3} = (13.6\,\text{eV})\left(\frac{1}{9} - \frac{1}{36}\right) = \boxed{1.13\,\text{eV}}$$

Evaluate equation (2) for $\Delta E = 1.13$ eV:

$$\lambda_{6\to3} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.13\,\text{eV}} = \boxed{1097\,\text{nm}}$$

The positions of these lines on a horizontal linear scale are shown below with the wavelengths and transitions indicated.

```
   6→3       5→3                              4→3
---|-------------|-----------------------------------------|-----
1097 nm    1282 nm                           1876 nm
```

***26 ••**

Picture the Problem We can use Bohr's second postulate to relate the photon energy to its frequency and use $\lambda = \dfrac{1240\,\text{eV}\cdot\text{nm}}{E_\text{i} - E_\text{f}}$ to find the wavelengths of the three longest wavelengths in the Brackett series.

(*a*) Use Bohr's second postulate to express the energy of the photons in the Paschen series:

$$hf = \Delta E = E_\text{i} - E_\text{f}$$

For the series limit:

$$n = \infty \text{ and } E_\text{i} = 0$$

Substitute to obtain:

$$\Delta E = -E_\text{f} = -\left(-\frac{E_0}{n_2^2}\right) = \frac{E_0}{n_2^2} \qquad (1)$$

Evaluate the photon energy for $n_2 = 4$:

$$hf = \frac{13.6\,\text{eV}}{4^2} = \boxed{0.850\,\text{eV}}$$

Express the wavelength of the radiation resulting from an energy transition $\Delta E = hf$:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} \qquad (2)$$

provided the energies are expressed in eV.

Evaluate λ_min for the transition $n = \infty$ to $n_2 = 4$:

$$\lambda_\text{min} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.850\,\text{eV}} = \boxed{1459\,\text{nm}}$$

(*b*) For the three longest wavelengths:

$$n_\text{i} = 5, 6, \text{and } 7$$

Equation (1) becomes:

$$\Delta E = E_\text{i} - E_\text{f} = -\frac{E_0}{n_\text{i}^2} - \left(-\frac{E_0}{n_2^2}\right)$$
$$= E_0\left(\frac{1}{n_2^2} - \frac{1}{n_\text{i}^2}\right) = E_0\left(\frac{1}{16} - \frac{1}{n_\text{i}^2}\right) \qquad (3)$$

Evaluate equation (3) for $n = 5$:

$$\Delta E_{5\to4} = (13.6\,\text{eV})\left(\frac{1}{16} - \frac{1}{25}\right) = \boxed{0.306\,\text{eV}}$$

Evaluate equation (2) for $\Delta E = 0.306$ eV:

$$\lambda_{5\to4} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.306\,\text{eV}} = \boxed{4052\,\text{nm}}$$

Evaluate equation (3) for $n = 6$:

$$\Delta E_{6\to 4} = (13.6\,\text{eV})\left(\frac{1}{16} - \frac{1}{36}\right) = \boxed{0.472\,\text{eV}}$$

Evaluate equation (2) for $\Delta E = 0.472$ eV:

$$\lambda_{6\to 4} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.472\,\text{eV}} = \boxed{2627\,\text{nm}}$$

Evaluate equation (3) for $n = 7$:

$$\Delta E_{7\to 4} = (13.6\,\text{eV})\left(\frac{1}{16} - \frac{1}{49}\right) = \boxed{0.572\,\text{eV}}$$

Evaluate equation (2) for $\Delta E = 0.572$ eV:

$$\lambda_{7\to 4} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.572\,\text{eV}} = \boxed{2168\,\text{nm}}$$

The positions of these lines on a horizontal linear scale are shown below with the wavelengths and transitions indicated.

```
   7→4    6→4                                    5→4
---|---------|---------------------------------------------|-----
2168 nm 2627 nm                                  4052 nm
```

27 ••

Picture the Problem We can use the grating equation to find the wavelength of the given spectral line and the Rydberg-Ritz formula to evaluate R.

(*a*) The grating equation is:

$$m\lambda = d\sin\theta$$

where $m = 1, 2, 3, \ldots$

Solve for λ:

$$\lambda = \frac{d\sin\theta}{m}$$

Substitute numerical values and evaluate λ for $m = 1$:

$$\lambda = \frac{(3.377\,\mu\text{m})\sin 11.233^\circ}{1} = \boxed{657.8\,\text{nm}}$$

(*b*) The Rydberg-Ritz formula is:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

Solve for R to obtain:

$$R = \frac{1}{\lambda}\left(\frac{1}{\frac{1}{n_2^2} - \frac{1}{n_1^2}}\right)$$

Substitute numerical values and evaluate R:

$$R = \frac{1}{0.6578\,\mu\text{m}}\left(\frac{1}{\frac{1}{2^2} - \frac{1}{3^2}}\right)$$

$$= 10.946\,\mu\text{m}^{-1} = \boxed{1.0946\times 10^{7}\,\text{m}^{-1}}$$

Remarks: The data used here came from a real experiment. The value for *R* differs by approximately 0.2% from the commonly accepted value.

28 •••

Picture the Problem This is an extreme value problem in which we need to identify the relationship between E and r, differentiate it with respect to r, and set that derivative equal to zero. Solving the latter expression for r will give us r_m.

Express the total energy of the electron:

$$E = \frac{\hbar^2}{2mr^2} - \frac{ke^2}{r}$$

Differentiate this expression with respect to r to obtain:

$$\frac{dE}{dr} = \frac{d}{dr}\left[\frac{\hbar^2}{2mr^2} - \frac{ke^2}{r}\right]$$

$$= \frac{d}{dr}\left[\frac{\hbar^2}{2mr^2}\right] - \frac{d}{dr}\left[\frac{ke^2}{r}\right]$$

$$= -\frac{\hbar^2}{mr^3} + \frac{ke^2}{r^2}$$

$$= 0 \text{ for extreme values}$$

Solve for r to obtain:

$$r = \frac{\hbar^2}{ke^2m}$$

Differentiate E a second time to obtain:

$$\frac{d^2E}{dr^2} = \frac{3\hbar^2}{mr^4} - \frac{2ke^2}{r^3}$$

Evaluate d^2E/dr^2 at r to obtain:

$$\left.\frac{d^2E}{dr^2}\right|_{r=\frac{\hbar^2}{ke^2m}} = \frac{k^4e^8m^3}{\hbar^6} > 0$$

Therefore, our extreme value is a minimum and the value for r that minimizes the energy is $r = \boxed{\dfrac{\hbar^2}{ke^2 m}}$

Note that this is just the Bohr radius a_0. Consequently, the energy is the ground state energy of the hydrogen atom and:

$$E_{\min} = \boxed{13.6\,\text{eV}}$$

***29** •••

Picture the Problem We can express the total kinetic energy of the electron-nucleus system as the sum of the kinetic energies of the electron and the nucleus. Rewriting these kinetic energies in terms of the momenta of the electron and nucleus will lead to $K = p^2/2m_r$.

(*a*) Express the total kinetic energy of the electron-nucleus system:

$$K = K_e + K_n$$

Express the kinetic energies of the electron and the nucleus in terms of their momenta:

$$K_e = \frac{p^2}{2m_e} \text{ and } K_n = \frac{p^2}{2M}$$

Substitute to obtain:

$$K = \frac{p^2}{2m_e} + \frac{p^2}{2M} = \frac{p^2}{2}\left(\frac{1}{m_e} + \frac{1}{M}\right)$$

$$= \frac{p^2}{2}\left(\frac{M + m_e}{m_e M}\right) = \frac{p^2}{2\left(\dfrac{m_e M}{M + m_e}\right)}$$

$$= \boxed{\frac{p^2}{2m_r}}$$

provided we define $\mu = m_e M/(M + m_e)$.

(*b*) From Equation 36-14 we have:

$$R = \frac{m_r k^2 e^4}{4\pi c\hbar^3} = C\left(\frac{m_e}{1 + \dfrac{m_e}{M}}\right) \qquad (1)$$

where

$$C = \frac{k^2 e^4}{4\pi c\hbar^3}$$

Use the Table of Physical Constants at the end of the text to obtain:

$$C = 1.204663 \times 10^{37}\ \text{m}^{-1}/\text{kg}$$

For H:

$$R_H = C\left(\frac{m_e}{1+\frac{m_e}{m_p}}\right)$$

Substitute numerical values and evaluate R_H:

$$R_H = \left(1.204663\times10^{37}\ \text{m}^{-1}/\text{kg}\right)\left(\frac{9.11\times10^{-31}\ \text{kg}}{1+\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}}\right) = \boxed{1.096850\times10^{7}\ \text{m}^{-1}}$$

Let $M\to\infty$ in equation (1) to obtain $R_{H,approx}$:

$$R_{H,approx} = Cm_e$$

Substitute numerical values and evaluate $R_{H,approx}$:

$$R_{H,approx} = \left(1.204663\times10^{37}\ \text{m}^{-1}/\text{kg}\right)\left(9.11\times10^{-31}\ \text{kg}\right) = \boxed{1.097448\times10^{7}\ \text{m}^{-1}}$$

$$\boxed{R_H \text{ and } R_{H,approx} \text{ agree to three significant figures.}}$$

(*c*) Express the ratio of the kinetic energy K of the electron in its orbit about a stationary nucleus to the kinetic energy of the reduced-mass system K':

$$\frac{K}{K'} = \frac{\frac{p^2}{2m_e}}{\frac{p^2}{2m_r}} = \frac{\mu}{m_e} = \frac{1}{m_e}\left(\frac{m_p m_e}{m_p + m_e}\right)$$

$$= \frac{m_p}{m_p + m_e} = \frac{1}{1+\frac{m_e}{m_p}}$$

Substitute numerical values and evaluate the ratio of the kinetic energies:

$$\frac{K}{K'} = \frac{1}{1+\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}}$$

$$= 0.999455$$

or

$$K = 0.999455K'$$

and the correction factor is the ratio of the masses or $\boxed{0.0545\%}$

Remarks: **The correct energy is slightly less than that calculated neglecting the motion of the nucleus.**

***30 ••**

Picture the Problem We can use Equation 36-15 with $Z = 2$ to explain how it is that every other line of the Pickering series is very close to a line in the Balmer series. We can use the relationship between the energy difference between two quantum states and the wavelength of the photon emitted during a transition from the higher state to the lower state to find the wavelength of the photon corresponding to a transition from the $n = 6$ to the $n = 4$ level of He^+.

(*a*) From Equation 36-15, the energy levels of an atom are given by:

$$E_n = -Z^2 \frac{E_0}{n^2}$$

where E_0 is the Rydberg constant (13.6 eV).

For He^+, $Z = 2$ and:

$$E_n = -4\frac{E_0}{n^2}$$

Because of this, an energy level with even principal quantum number n in He^+ will have the same energy as a level with quantum number $n/2$ in H. Therefore, a transition between levels with principal quantum numbers $2m$ and $2p$ in He^+ will have almost the same energy as a transition between level m and p in H. In particular, transitions from $2m$ to $2p = 4$ in He^+ will have the same energy as transitions from m to $n = 2$ in H (the Balmer series).

(*b*) Transitions between these energy levels result in the emission or absorption of a photon whose wavelength is given by:

$$\lambda = \frac{hc}{E_6 - E_4} \qquad (1)$$

Evaluate E_6 and E_4:

$$E_6 = -4\left(\frac{13.6\,\text{eV}}{6^2}\right) = -1.51\,\text{eV}$$

and

$$E_4 = -4\left(\frac{13.6\,\text{eV}}{4^2}\right) = -3.40\,\text{eV}$$

Substitute for E_6 and E_4 in equation (1) and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{-1.51\,\text{eV} - (-3.40\,\text{eV})} = \boxed{656\,\text{nm}}$$

which is the same as the $n = 3$ to $n = 2$ transition in H.

Quantum Numbers in Spherical Coordinates

31 •

Picture the Problem We can use the expression relating L to ℓ to find the magnitude of the angular momentum and the constraints on the quantum numbers to determine the allowed values for m.

(a) Express the angular momentum as a function of ℓ:

$$L = \sqrt{\ell(\ell+1)}\hbar$$

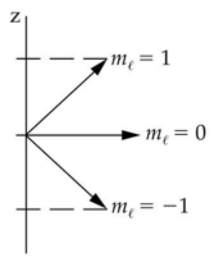
Substitute numerical values and evaluate L:

$$L = \sqrt{1(1+1)}\hbar = \sqrt{2}\hbar$$
$$= \sqrt{2}\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)$$
$$= \boxed{1.49\times10^{-34}\ \text{J}\cdot\text{s}}$$

(b) Because $m_\ell = -\ell, \ldots, 0, \ldots, \ell$ the allowed values for $\ell = 1$ are:

$$m_\ell = \boxed{-1, 0, +1}$$

(c) The vector diagram is shown on the right. Note that because $L_z = m_\ell\hbar$ and $L = \sqrt{2}\hbar$, the vectors for $m_\ell = -1$ and $m_\ell = 1$ must make angles of 45° with the z axis.



32 •

Picture the Problem We can use the expression relating L to ℓ to find the magnitude of the angular momentum and the constraints on the quantum numbers to determine the allowed values for m.

(a) Express the angular momentum as a function of ℓ:

$$L = \sqrt{\ell(\ell+1)}\hbar$$

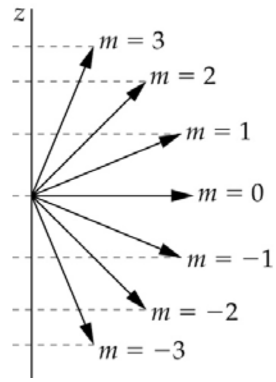
Substitute numerical values and evaluate L:

$$L = \sqrt{3(3+1)}\hbar = 2\sqrt{3}\hbar$$
$$= 2\sqrt{3}\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)$$
$$= \boxed{3.65\times10^{-34}\ \text{J}\cdot\text{s}}$$

(b) Because $m_\ell = -\ell, \ldots, 0, \ldots, \ell$ the allowed values for $\ell = 1$ are:

$$m_\ell = \boxed{-3, -2, -1, 0, +1, +2, +3}$$

(c) The vector diagram is shown on the right. Note that because $L_z = m_\ell \hbar$ and $L = 2\sqrt{3}\hbar$, the angles between the vectors and the z axis are determined by $\cos\theta_m = m_\ell / 2\sqrt{3}$. Thus $\theta_3 = 30°$, $\theta_2 = 54.7°$, and $\theta_1 = 73.2°$. The spacing between the allowed values of L_z is constant and equal to $\hbar$.



33 •

Picture the Problem We can find the possible values of ℓ by using the constraints on the quantum numbers. A more analytical solution is to first derive the number of electron states for an arbitrary value of n and then substitute the specific value of n.

(a) When $n = 3$: $\ell = \boxed{0, 1, 2}$

(b) For $\ell = 0$: $m_\ell = \boxed{0}$

For $\ell = 1$: $m_\ell = \boxed{-1, 0, +1}$

For $\ell = 2$: $m_\ell = \boxed{-2, -1, 0, +1, +2}$

(c) We can find the total number of electron states by enumerating the possibilities as shown in the table.

n	ℓ	m_ℓ
3	0	0
3	1	–1
3	1	0
3	1	1
3	2	–2
3	2	–1
3	2	0
3	2	1
3	2	2

$$\boxed{\text{Note that there are 9 } m \text{ states. Because } N\text{, the number of electron states is twice the number of } m_\ell \text{ states, the number of electron states is 18.}}$$

Alternatively, we can derive an expression for the number of electron states for an arbitrary value of n and then substitute specific values of n:

The number of m_ℓ states for a given n is given by:

$$N_m = \sum_{\ell=0}^{n-1}(2\ell+1) = 2\sum_{\ell=0}^{n-1}\ell + \sum_{\ell=0}^{n-1}(1)$$

Express the sum of all integers from 0 to p:

$$\sum_{\ell=0}^{p} \ell = \tfrac{1}{2} p(p+1)$$

Use this result to evaluate $2\sum_{\ell=0}^{n-1} \ell$:

$$2\sum_{\ell=0}^{n-1} \ell = 2\left[\tfrac{1}{2}(n-1)n\right] = n^2 - n$$

Evaluate the second term to obtain:

$$\sum_{\ell=0}^{n-1} (1) = n$$

Substitute to obtain:

$$N_m = n^2 - n + n = n^2$$

and, because N, the number of electron states is twice the number of m states, the number of electron states is $N = 2n^2$.

Hence, for $n = 3$, the number of electron states is:

$$N = 2n^2 = 2(3)^2 = \boxed{18}$$

34 •

Picture the Problem While we could find the number of electron states by finding the possible values of ℓ from the constraints on the quantum numbers and then enumerating the states, we'll take a more analytical approach by deriving an expression for the number of electron states for an arbitrary value of n and then substitute specific values of n.

The number of m states for a given n is given by:

$$N_m = \sum_{\ell=0}^{n-1} (2\ell + 1) = 2\sum_{\ell=0}^{n-1} \ell + \sum_{\ell=0}^{n-1} (1)$$

Express the sum of all integers from 0 to p:

$$\sum_{\ell=0}^{p} \ell = \tfrac{1}{2} p(p+1)$$

Use this result to evaluate $2\sum_{\ell=0}^{n-1} \ell$:

$$2\sum_{\ell=0}^{n-1} \ell = 2\left[\tfrac{1}{2}(n-1)n\right] = n^2 - n$$

Evaluate the second term to obtain:

$$\sum_{\ell=0}^{n-1} (1) = n$$

Substitute to obtain:

$$N_m = n^2 - n + n = n^2$$

and, because N, the number of electron states is twice the number of m states, the number of electron states is $N = 2n^2$.

(*a*) For $n = 4$, the number of electron states is:

$$N = 2n^2 = 2(4)^2 = \boxed{32}$$

(*b*) For $n = 2$, the number of electron states is:

$$N = 2n^2 = 2(2)^2 = \boxed{8}$$

***35** ••

Picture the Problem The minimum angle between the z axis and $\vec{L}$ is the angle between the $\vec{L}$ vector for $m = \ell$ and the z axis.

Express the angle θ as a function of L_z and L:

$$\theta = \cos^{-1}\left(\frac{L_z}{L}\right)$$

Relate the z component of $\vec{L}$ to m_ℓ and ℓ:

$$L_z = m_\ell \hbar = \ell\hbar$$

Express the angular momentum L:

$$L = \sqrt{\ell(\ell+1)}\hbar$$

Substitute to obtain:

$$\theta = \cos^{-1}\left(\frac{\ell\hbar}{\sqrt{\ell(\ell+1)}\hbar}\right) = \cos^{-1}\left(\sqrt{\frac{\ell}{\ell+1}}\right)$$

(*a*) For $\ell = 1$:

$$\theta = \cos^{-1}\left(\sqrt{\frac{1}{1+1}}\right) = \boxed{45.0°}$$

(*b*) For $\ell = 4$:

$$\theta = \cos^{-1}\left(\sqrt{\frac{4}{4+1}}\right) = \boxed{26.6°}$$

(*c*) For $\ell = 50$:

$$\theta = \cos^{-1}\left(\sqrt{\frac{50}{50+1}}\right) = \boxed{8.05°}$$

36 ••

Picture the Problem We can use constraints on the quantum numbers in spherical coordinates to find the possible values of n and m_ℓ for each of the values of ℓ.

The constraints on n, m_ℓ, and ℓ are:

$n = 1, 2, 3, \ldots$
$\ell = 0, 1, 2, \ldots, n - 1$
$m_\ell = -\ell, (-\ell + 1), \ldots 0, 1, 2, \ldots \ell$

(*a*) For $\ell = 3$: $\boxed{n \geq 4 \text{ and } m_\ell = -3, -2, -1, 0, 1, 2, 3}$

(*b*) For $\ell = 4$: $\boxed{n \geq 5 \text{ and } m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4}$

(*c*) For $\ell = 0$: $\boxed{n \geq 1 \text{ and } m_\ell = 0}$

37 ••

Picture the Problem The magnitude of the orbital angular momentum L of an electron is related to orbital quantum number ℓ by $L = \sqrt{\ell(\ell+1)}\hbar$ and the z component of the angular momentum of the electron is given by $L_z = m\hbar$.

(*a*) For the $\ell = 2$ state, the square magnitude of the angular momentum is:

$$L^2 = 2(2+1)\hbar^2 = \boxed{6\hbar^2}$$

(*b*) For the $\ell = 2$ state, the maximum value of L_z^2 is:

$$L_z^2 = 2^2\hbar^2 = \boxed{4\hbar^2}$$

(*c*) The smallest value of $L_x^2 + L_y^2$ is given by:

$$L_x^2 + L_y^2 = L^2 - L_z^2 = 6\hbar^2 - 4\hbar^2 = \boxed{2\hbar^2}$$

Quantum Theory of the Hydrogen Atom

38 •

Picture the Problem We can use $\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$ to evaluate the normalized ground-state wave function and its square at $r = a_0$ and $P(r) = 4\pi r^2|\psi|^2$ to find the radial probability density at the same location.

(*a*) Noting that $Z = 1$ for hydrogen, evaluate $\psi(a_0)$ to obtain:

$$\psi(a_0) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-a_0/a_0}$$

$$= \boxed{\frac{1}{ea_0\sqrt{\pi a_0}}}$$

(*b*) Square $\psi(a_0)$ to obtain:

$$\psi^2(a_0) = \left(\frac{1}{ea_0\sqrt{\pi a_0}}\right)^2 = \boxed{\frac{1}{e^2 a_0^3 \pi}}$$

(*c*) Use the result from part (*b*) to evaluate $P(a_0)$:

$$P(a_0) = 4\pi a_0^2 |\psi|^2 = 4\pi a_0^2 \left(\frac{1}{e^2 a_0^3 \pi}\right)$$

$$= \boxed{\frac{4}{e^2 a_0}}$$

***39 •**

Picture the Problem We can use the constraints on *n*, ℓ, and *m* to determine the number of different wave functions, excluding spin, corresponding to the first excited energy state of hydrogen.

For $n = 2$: $\ell = 0 \text{ or } 1$

(*a*) For $\ell = 0$, $m_\ell = 0$ and we have: 1 state

For $\ell = 1$, $m_\ell = -1, 0, +1$ and we have: 3 states

Hence, for $n = 2$ we have: $\boxed{4\text{ states}}$

(*b*) The four wave functions are summarized to the right.

n	ℓ	m_ℓ	(n, ℓ, m_ℓ)
2	0	0	(2,0,0)
2	1	–1	(2,1,–1)
2	1	0	(2,1,0)
2	1	1	(2,1,1)

40 ••

Picture the Problem We can use $\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$ to evaluate the normalized ground-state wave function and its square and $P(r) = 4\pi r^2 |\psi|^2$ to find the radial probability density at the same location. Because the range Δr is so small, the variation in the radial probability density $P(r)$ can be neglected. The probability of finding the electron in some small range Δr is then $P(r)\,\Delta r$.

Express the probability of finding the electron in the range Δr:

$$\text{Probability} = \int P(r)dr \qquad (1)$$

where $P(r)$ is the radial probability density function.

The radial probability density function is:

$$P(r) = 4\pi r^2 |\psi|^2 \qquad (2)$$

Express the normalized ground-state wave function:

$$\psi(r)=\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}e^{-Zr/a_0}$$

Evaluate the normalized ground-state wave function evaluated at $r = a_0$ to obtain:

$$\psi(a_0)=\frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}e^{-a_0/a_0}=\frac{1}{ea_0\sqrt{\pi a_0}}$$

Square $\psi(a_0)$ to obtain:

$$\psi^2(a_0)=\left(\frac{1}{ea_0\sqrt{\pi a_0}}\right)^2=\frac{1}{e^2a_0^3\pi}$$

Substitute in equation (2) to obtain:

$$P(a_0)=4\pi a_0^2|\psi|^2=4\pi a_0^2\left(\frac{1}{e^2a_0^3\pi}\right)=\frac{4}{e^2a_0}$$

(*a*) Substitute in equation (1) to find the probability of finding the electron in the small range $\Delta r = 0.03a_0$:

$$\text{Probability}=\int P(a_0)dr\approx P(a_0)\Delta r$$

$$=\frac{4}{e^2a_0}(0.03a_0)=\boxed{0.0162}$$

(*b*) Evaluate the normalized ground-state wave function at $r = 2a_0$ to obtain:

$$\psi(2a_0)=\frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}e^{-2a_0/a_0}=\frac{1}{e^2a_0\sqrt{\pi a_0}}$$

Square $\psi(2a_0)$ to obtain:

$$\psi^2(2a_0)=\left(\frac{1}{e^2a_0\sqrt{\pi a_0}}\right)^2=\frac{1}{e^4a_0^3\pi}$$

Substitute in equation (2) to obtain:

$$P(2a_0)=4\pi(2a_0)^2|\psi|^2=16\pi a_0^2\left(\frac{1}{e^4a_0^3\pi}\right)$$

$$=\frac{16}{e^4a_0}$$

Substitute in equation (1) to find the probability of finding the electron in some small range $\Delta r = 0.03a_0$:

$$\text{Probability}=\int P(2a_0)dr\approx P(2a_0)\Delta r$$

$$=\frac{16}{e^4a_0}(0.03a_0)=\boxed{0.00879}$$

Remarks: There is about a 2% chance of finding the electron in this range at $r = a_0$, but at $r = 2a_0$, the chance is only about 0.9%.

41 ••

Picture the Problem We can use Equation 36-36 and the given expression for $C_{2,0,0}$ to evaluate the spherically symmetric wave function ψ for $n = 2$, $\ell = 0$, $m_\ell = 0$, and $Z = 1$ and then use this result to evaluate ψ^2 and $P(r)$ for $r = a_0$.

(*a*) Express the spherically symmetric wave function for $n = 2$, $\ell = 0$, $m_\ell = 0$, and $Z = 1$ (Equation 35-36):

$$\psi_{2,0,0} = C_{2,0,0}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

Evaluate this expression for $r = a_0$:

$$\psi_{2,0,0}(a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{a_0}{a_0}\right)e^{-a_0/2a_0} = \frac{1}{4\sqrt{2e\pi}\,a_0^{3/2}} = \boxed{\frac{0.0605}{a_0^{3/2}}}$$

(*b*) Square $\psi_{2,0,0}(a_0)$ to obtain:

$$\left[\psi_{2,0,0}(a_0)\right]^2 = \left(\frac{0.0605}{a_0^{3/2}}\right)^2 = \boxed{\frac{0.00366}{a_0^3}}$$

(*c*) Express the radial probability density:

$$P(r) = 4\pi r^2\psi^2(r)$$

Substitute to obtain:

$$P(a_0) = 4\pi a_0^2\frac{0.00366}{a_0^3} = \boxed{\frac{0.0460}{a_0}}$$

42 •••

Picture the Problem We can use the definition of the radial probability density and the wave function (Equation 35-37) for the state (2, 1, 0) to obtain the result given in the problem statement.

Using Equation 35-37, express the wave function for the state (2, 1, 0):

$$\psi_{2,1,0}(r) = C_{2,1,0}\frac{Zr}{a_0}e^{-Zr/2a_0}\cos\theta$$

Square $\psi_{2,1,0}(r)$ to obtain:

$$\left[\psi_{2,1,0}(r)\right]^2 = \left(C_{2,1,0}\frac{Zr}{a_0}e^{-Zr/2a_0}\cos\theta\right)^2 = C_{2,1,0}^2\frac{Z^2r^2}{a_0^2}e^{-Zr/a_0}\cos^2\theta$$

Express the radial probability density:

$$P(r) = 4\pi r^2 \psi^2(r)$$

Substitute and simplify to obtain:

$$P(r) = 4\pi r^2 C_{2,1,0}^2 \frac{Z^2 r^2}{a_0^2} e^{-Zr/a_0} \cos^2\theta$$

$$= \boxed{Ar^4 \cos^2\theta \, e^{-Zr/a_0}}$$

where

$$A == \boxed{\frac{4\pi C_{2,1,0}^2 Z^2}{a_0^2}}$$

43 •••

Picture the Problem In this instance, $\int P(r)dr$ extends over a sufficiently narrow interval $\Delta r << 0.02a_0$ that we can neglect the dependence of $P(r)$ on r. Hence, we can set $\int P(r)dr = P(r)\Delta r$ and use the wave function (Equation 36-36) for the state (2, 0, 0) and the expression for $C_{2,0,0}$ from Problem 41 to find $\psi_{2,0,0}$, $\psi_{2,0,0}^2$, $P(r)$, and the probability of finding the electron in the range specified at $r = a_0$ and $r = 2a_0$.

(*a*) Express the probability of finding the electron in the range Δr:

$$\text{Probability} = \int P(r)dr \qquad (1)$$

where $P(r)$ is the radial probability density function.

The radial probability density function is:

$$P(r) = 4\pi r^2 |\psi|^2 \qquad (2)$$

The normalized wave function for the (2, 0, 0) state of hydrogen is given by Equation 36-36:

$$\psi_{2,0,0}(r) = C_{2,0,0}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

From Problem 41 we have, for hydrogen:

$$C_{2,0,0} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}$$

Substitute to obtain:

$$\psi_{2,0,0}(r) = C_{2,0,0}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

$$= \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

Evaluate the normalized ground-state wave function at $r = a_0$ to obtain:

$$\psi_{2,0,0}(a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-a_0/2a_0} = \frac{1}{4\sqrt{2\pi e}a_0^{3/2}} = \frac{0.0605}{a_0^{3/2}}$$

Square $\psi_{2,0,0}(a_0)$ to obtain:

$$\left[\psi_{2,0,0}(a_0)\right]^2 = \left(\frac{0.0605}{a_0^{3/2}}\right)^2 = \frac{0.00366}{a_0^3}$$

Substitute in equation (2) to obtain:

$$P(a_0) = 4\pi a_0^2\psi^2(a_0) = 4\pi a_0^2\left(\frac{0.00366}{a_0^3}\right) = \frac{4\pi(0.00366)}{a_0} = \frac{0.0460}{a_0}$$

Substitute in equation (1) to find the probability of finding the electron in some small range $\Delta r = 0.02\ a_0$:

$$\text{Probability} = \int P(a_0)dr \approx P(a_0)\Delta r = \frac{0.0460}{a_0}(0.02a_0) = \boxed{9.20\times10^{-4}}$$

(*b*) Evaluate the normalized ground-state wave function at $r = 2a_0$ to obtain:

$$\psi_{2,0,0}(2a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{2a_0}{a_0}\right)e^{-2a_0/2a_0} = 0$$

Square $\psi_{2,0,0}(2a_0)$ to obtain:

$$\left[\psi_{2,0,0}(a_0)\right]^2 = 0$$

Substitute in equation (2) to obtain:

$$P(2a_0) = 0$$

Substitute in equation (1) to find the probability of finding the electron in some small range $\Delta r = 0.02\ a_0$:

$$\text{Probability} = \int P(2a_0)dr \approx P(2a_0)\Delta r = \boxed{0}$$

***44 ••**

Picture the Problem We wish to show that $\psi_{1,0,0} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0} = Ce^{-Zr/a_0}$ is a solution to $-\frac{\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + U(r)\psi = E\psi$, where $U(r) = -\frac{kZe^2}{r}$. Because the ground state is spherically symmetric, we do not need to consider the angular partial derivatives in Equation 36-21.

The normalized ground-state wave function is:

$$\psi_{1,0,0} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0} = Ce^{-Zr/a_0}$$

Differentiate this expression with respect to r to obtain:

$$\frac{\partial\psi_{1,0,0}}{\partial r} = C\frac{\partial}{\partial r}\left[e^{-Zr/a_0}\right] = -C\frac{Z}{a_0}e^{-Zr/a_0}$$

Multiply both sides of this equation by r^2:

$$r^2\frac{\partial\psi_{1,0,0}}{\partial r} = -C\frac{Z}{a_0}r^2e^{-Zr/a_0}$$

Differentiate this expression with respect to r to obtain:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_{1,0,0}}{\partial r}\right) = -C\frac{Z}{a_0}\frac{\partial}{\partial r}\left(r^2e^{-Zr/a_0}\right) = \left[-\frac{2Zr}{a_0} + r^2\left(\frac{Z}{a_0}\right)^2\right]Ce^{-Zr/a_0}$$

Substitute in Schrödinger's equation to obtain:

$$-\frac{\hbar^2}{2mr^2}\left[-\frac{2Zr}{a_0} + r^2\left(\frac{Z}{a_0}\right)^2\right]Ce^{-Zr/a_0} - \frac{kZe^2}{r}Ce^{-Zr/a_0} = ECe^{-Zr/a_0}$$

Solve for E:

$$E = -\frac{\hbar^2}{2mr^2}\left[-\frac{2Zr}{a_0} + r^2\left(\frac{Z}{a_0}\right)^2\right] - \frac{kZe^2}{r}$$

Because $a_0 = \dfrac{\hbar^2}{mke^2}$:

$$E = -\frac{\hbar^2}{2mr^2}\left[-\frac{2mke^2Zr}{\hbar^2} + r^2\left(\frac{Zmke^2}{\hbar^2}\right)^2\right] - \frac{kZe^2}{r} = \frac{kZe^2}{r} - \frac{Z^2k^2e^4m}{2\hbar^2} - \frac{kZe^2}{r}$$

$$= \boxed{-\frac{Z^2k^2e^4m}{2\hbar^2}}$$

Because this is the correct ground state energy, we have shown that Equation 36-33, is a solution to Schrödinger's Equation 36-21 with the potential energy function Equation 36-26.

45 ••

Picture the Problem We can substitute the dimensions of the physical quantities for the physical quantities in Equation 36-28 and simplify the resulting expression to show that it has dimensions of energy.

Equation 36-28 is:

$$E_0 = \frac{mk^2e^4}{2\hbar^2}$$

The units of this equation are:

$$\frac{[\mathrm{kg}]\left[\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right]^2[\mathrm{C}]^4}{[\mathrm{J\cdot s}]^2} = \frac{[\mathrm{kg}][\mathrm{N\cdot m^2}]^2}{[\mathrm{N\cdot m\cdot s}]^2} = \frac{[\mathrm{kg}][\mathrm{m}]^2}{[\mathrm{s}]^2} = \boxed{[\mathrm{J}]}$$

46 ••

Picture the Problem The Bohr radius is $a_0 = \frac{\hbar^2}{mke^2}$. We can substitute the dimensions of the physical quantities for the physical quantities in this equation and simplify the resulting expression to show that it has dimensions of length.

Because the SI units of $\hbar$ are J·s, its dimensions are:

$$\frac{\mathrm{M\cdot L^2}}{\mathrm{T^2}}\cdot \mathrm{T} = \frac{\mathrm{M\cdot L^2}}{\mathrm{T}}$$

Because the SI units of k are $\mathrm{N\cdot m^2/C^2}$, its dimensions are:

$$\frac{\mathrm{M\cdot L}}{\mathrm{T^2}}\cdot \mathrm{L^2}\cdot\frac{1}{\mathrm{Q^2}} = \frac{\mathrm{M\cdot L^3}}{\mathrm{T^2\cdot Q^2}}$$

where Q is the dimension of charge.

Substitute the dimensions in $a_0 = \frac{\hbar^2}{mke^2}$ to obtain:

$$\frac{\left[\frac{\mathrm{M\cdot L^2}}{\mathrm{T}}\right]^2}{\mathrm{M}\cdot\left[\frac{\mathrm{M\cdot L^3}}{\mathrm{T^2\cdot Q^2}}\right]\cdot \mathrm{Q^2}} = \frac{\frac{\mathrm{M^2\cdot L^4}}{\mathrm{T^2}}}{\frac{\mathrm{M^2\cdot L^3}}{\mathrm{T^2}}} = \boxed{\mathrm{L}}$$

47 ••

Picture the Problem This is an extreme value problem. We'll begin its solution with the radial probability distribution function, differentiate it with respect to its independent variable r, set this derivative equal to zero, and solve for the value for an extreme value for r. We can show that this value corresponds to a maximum by evaluating the second derivative of $P(r)$ at the location found from the first derivative.

Differentiate the radial probability distribution function with respect to r to obtain:

$$\frac{dP(r)}{dr} = C\frac{d}{dr}\left[r^2 e^{-2Zr/a_0}\right] = C\left[2re^{-2Zr/a_0} - \frac{2Zr^2}{a_0}e^{-2Zr/a_0}\right] = \frac{2CZr}{a_0}e^{-2Zr/a_0}\left(\frac{a_0}{Z} - r\right) = 0 \text{ for extrema}$$

Solve for r to obtain:

$$r = \frac{a_0}{Z}$$

To show that this value for r corresponds to a maximum, differentiate $dP(r)/dr$ to obtain:

$$\frac{d^2P(r)}{dr^2} = -\frac{2CZr}{a_0}e^{-2Zr/a_0} + \left(\frac{a_0}{Z} - r\right) \times \left(-\frac{4CZ^2}{a_0^2} + \frac{2CZ}{a_0}\right)e^{-2Zr/a_0}$$

Evaluate this derivative at $r = a_0/Z$:

$$\left.\frac{d^2P(r)}{dr^2}\right|_{r=\frac{a_0}{Z}} = -2Ce^{-2} < 0$$

because C is a positive constant. Hence, $P(r)$ has its maximum value at $\boxed{r = \frac{a_0}{Z}}$

48 •••

Picture the Problem We can double the sum of the number of m states for a given n to show that the number of states in the hydrogen atom for a given n is $2n^2$.

The number of m_ℓ states for a given n is:

$$N_{m_\ell} = \sum_{\ell=0}^{n-1}(2\ell + 1) = 2\sum_{\ell=0}^{n-1}\ell + \sum_{\ell=0}^{n-1}(1)$$

The sum of all integers from 0 to p is:

$$\sum_{\ell=0}^{p}\ell = \tfrac{1}{2}p(p+1)$$

and

$$2\sum_{\ell=0}^{n-1}\ell = 2\left(\tfrac{1}{2}\right)(n-1)(n) = n^2 - n$$

The second term is:

$$\sum_{\ell=0}^{n-1}(1) = n$$

Substitute to obtain:

$$N_{m_\ell} = n^2 - n + n = n^2$$

Because N, the number of electron states, is twice the number of m_ℓ states, the number of electron states is:

$$N = 2N_{m_\ell} = \boxed{2n^2}$$

49 •••

Picture the Problem The ground state of a hydrogen atom is the state described by $n = 1$, $\ell = 0$, $m_\ell = 0$. We can calculate the probability that the electron in the ground state of the hydrogen atom is in the region $0 < r < a_0$ by evaluating the integral $\int_0^{a_0} 4\pi r^2 \psi_{1,0,0}^2(r)dr$.

Express the probability that the electron in the ground state of a hydrogen atom is in the region $0 < r < a_0$:

$$\text{Probability} = \int_0^{a_0} 4\pi r^2 \psi_{1,0,0}^2(r)dr$$

Express the ground-state wave function for hydrogen:

$$\psi_{1,0,0}(r) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$$

Square the wave function to obtain:

$$\psi_{1,0,0}^2(r) = \frac{1}{\pi a_0^3} e^{-2r/a_0}$$

Substitute to obtain:

$$\text{Probability} = \int_0^{a_0} 4\pi r^2 \left(\frac{1}{\pi a_0^3} e^{-2r/a_0}\right)dr$$

$$= \frac{4}{a_0^3}\int_0^{a_0} r^2 e^{-2r/a_0} dr$$

Use a table of integrals to find:

$$\int x^2 e^{bx} dx = \frac{e^{bx}}{b^3}\left(b^2x^2 - 2bx + 2\right)$$

Use this integral to show that:

$$\text{Probability} = -e^{-2r/a_0}\left(\frac{2r^2}{a_0^2} + \frac{2r}{a_0} + 1\right)\Bigg|_0^{a_0}$$

$$= 1 - 5e^{-2} = \boxed{0.323}$$

The Spin-Orbit Effect and Fine Structure

***50** •

Picture the Problem The energy difference between the two possible orientations of an electron in a magnetic field is $2\mu B$ and the wavelength of the photons required to induce a spin-flip transition can be found from $hc/\Delta E$. The magnetic moment μ_B associated with the spin of an electron is 5.79×10^{-5} eV/T.

(*a*) Relate the difference in energy between the two spin orientations in terms of the difference in the potential energies of the two states:

$$\Delta E = 2\mu B = 2\left(5.79\times10^{-5}\text{ eV/T}\right)\left(0.6\text{ T}\right) = \boxed{6.95\times10^{-5}\text{ eV}}$$

(*b*) Relate the wavelength of the photon needed to induce such a transition to the energy required:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\text{ eV}\cdot\text{nm}}{6.95\times10^{-5}\text{ eV}} = 1.78\times10^{7}\text{ nm} = \boxed{1.78\text{ cm}}$$

51 •

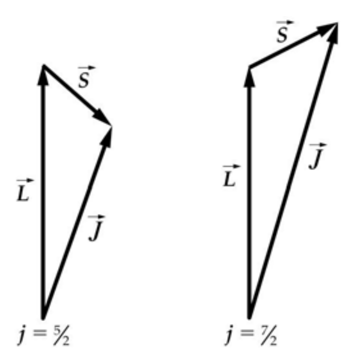
Determine the Concept j and ℓ are constrained according to $j = \ell \pm \frac{1}{2}$. For $j = ½$, $\ell = \frac{1}{2} \pm \frac{1}{2}$ or $\ell = \boxed{0 \text{ or } 1}$.

52 •

Determine the Concept j and ℓ are constrained according to $j = \ell \pm \frac{1}{2}$. For $\ell = 2$, $j = 2 \pm \frac{1}{2}$, or $j = \boxed{\frac{3}{2} \text{ or } \frac{5}{2}}$

53 •

Picture the Problem The total angular momentum vector $\vec{J}$ is the sum of the orbital momentum vector $\vec{L}$ and the spin orbital angular momentum vector $\vec{S}$. The quantum number j can be either $\ell + \frac{1}{2}$ or $\ell - \frac{1}{2}$, where $\ell \neq 0$. Hence, j can take on the values 3 + 1/2 = 7/2 and 3 – 1/2 = 5/2. The scaled vector diagrams are shown to the right.



The Periodic Table

54 •

Determine the Concept The total number of quantum states of hydrogen with quantum number n is $2n^2$. For n =4, we have $2(4)^2 = 32$. $\boxed{(c)\text{ is correct.}}$

55 •

Determine the Concept From Table 36-1, oxygen's electronic configuration is $1s^22s^22p^4$. Because there are 4 electrons in the p state, $\boxed{(c)\text{ is correct.}}$

***56** •

Determine the Concept We can use the atomic numbers of carbon and oxygen to determine the sum of the exponents in their electronic configurations and then use the rules for the filling of the shells to find their electronic configurations.

(*a*) The atomic number Z of carbon is 6. So we must fill the subshells of the electronic configuration until we have placed its 6 electrons. This is accomplished by writing:

$$\boxed{1s^2\,2s^2\,2p^2}$$

(*b*) The atomic number Z of oxygen is 8. So we must fill the subshells of the electronic configuration until we have placed its 8 electrons. This is accomplished by writing:

$$\boxed{1s^2\,2s^2\,2p^4}$$

57 •

Determine the Concept We can find the z component of the orbital angular momentum using $L_z = m\hbar$ and the relationship between the quantum numbers ℓ (which we know from the state of the electrons) and m_ℓ (which is related to ℓ through $m_\ell = -\ell, (-\ell + 1), \ldots, 0, 1, 2, \ldots \ell$).

(*a*) For a p electron $\ell = 1$. For $\ell = 1$, $m_\ell = -1, 0,$ or 1. Because $L_z = -m\hbar, \ldots, m\hbar$:

$$L_z = \boxed{-2\hbar, -\hbar, 0, \hbar, 2\hbar}$$

(*b*) For an f electron, $\ell = 4$. For $\ell = 4$, $m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4$. Because $L_z = -m\hbar, \ldots, m\hbar$:

$$L_z = \boxed{-4\hbar, -3\hbar, -2\hbar, -\hbar, 0, \hbar, 2\hbar, 3\hbar, 4\hbar}$$

Optical Spectra and X-Ray Spectra

58 •

Determine the Concept Lithium, sodium, potassium, chromium, and cesium have one outer s electron and hence belong in the same group. Beryllium, magnesium, calcium, nickel, and barium have two outer s electrons and, hence, belong in the same group.

59 •

Determine the Concept We can use Table 35-1 to find the electronic configurations for the first excited states of these elements.

(*a*) For H, *E* depends only on *n* and the lowest excited state is: $\boxed{\text{2s or 2p}}$

(*b*) For Na, the 3p state is higher energy than the 3s state and the lowest excited state is: $\boxed{1s^2 2s^2 2p^6 3p}$

(*c*) For He, the lowest excited state has one electron in the 2s state and the lowest excited state is: $\boxed{1s2s}$

60 •

Determine the Concept Atoms with one outer electron have spectra similar to H: Li, Rb, Ag, Fr. Atoms with two outer electrons have spectra similar to He: Ca, Ti, Hg, Cd, Ba, Ra. Therefore, the table should be completed as shown below:

Optical Spectra Similar to Hydrogen	Optical Spectra Similar to Helium
Li, Rb, Ag, Fr	Ca, Ti, Hg, Cd, Ba, Ra

***61** •

Picture the Problem When an electron from state *n* drops into a vacated state in the *n* = 1 shell, a photon of energy $\Delta E = E_n - E_1$ is emitted. We can find the wavelength of this photon using $\lambda = hc/\Delta E$. The second and third longest wavelengths in the *K* series correspond to transitions from *n* = 3 to *n* = 1 and *n* = 4 to *n* = 1 and the shortest wavelength to the transition from $n = \infty$ to *n* = 1.

Express the wavelength of the emitted photon in terms of the energy transition within the atom:

$$\lambda = \frac{hc}{E_n - E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{E_n - E_1}$$

Express the energy of the *n*th energy state:

$$E_n = -(Z-1)^2 \frac{E_0}{n^2}$$

where n = 1, 2, …

Substitute to obtain:

$$\lambda = \frac{hc}{E_n - E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{-(Z-1)^2 \frac{E_0}{n^2} - \left(-(Z-1)^2 \frac{E_0}{1^2}\right)} = \frac{1240\,\text{eV}\cdot\text{nm}}{(Z-1)^2 E_0\left(1-\frac{1}{n^2}\right)}$$

(*a*) Evaluate this expression with n = 3 and Z = 42 to obtain:

$$\lambda_3 = \frac{1240\,\text{eV}\cdot\text{nm}}{(42-1)^2(13.6\,\text{eV})\left(1-\frac{1}{3^2}\right)} = \boxed{0.0610\,\text{nm}}$$

Use n = 4 and Z = 42 to obtain:

$$\lambda_4 = \frac{1240\,\text{eV}\cdot\text{nm}}{(42-1)^2(13.6\,\text{eV})\left(1-\frac{1}{4^2}\right)} = \boxed{0.0578\,\text{nm}}$$

(*b*) The shortest wavelength in the series corresponds to the largest energy difference between the initial and final states. Repeat the calculation in part (*a*) with $n = \infty$ to obtain:

$$\lambda_\infty = \frac{1240\,\text{eV}\cdot\text{nm}}{(42-1)^2(13.6\,\text{eV})(1-0)} = \boxed{0.0542\,\text{nm}}$$

62 •

Picture the Problem When an electron from state n drops into the vacated state in the n = 1 shell, a photon of energy $E_n - E_1$ is emitted. The wavelength of this photon is $\lambda = \frac{hc}{(Z-1)^2(13.6\,\text{eV})\left(1-\frac{1}{n^2}\right)}$. Hence, if we know the wavelength of the K_α line we can solve for the atomic number of the element and use its value to identify the element.

Express the wavelength of the K_α line as a function of the atomic number of the element:

$$\lambda = \frac{hc}{(Z-1)^2(13.6\,\text{eV})\left(1-\frac{1}{n^2}\right)}$$

Solve for Z:

$$Z = 1 + \sqrt{\frac{hc}{\lambda(13.6\,\text{eV})\left(1-\frac{1}{n^2}\right)}}$$

Substitute numerical values and evaluate Z:

$$Z = 1 + \sqrt{\frac{1240\,\text{eV}\cdot\text{nm}}{(0.3368\,\text{nm})(13.6\,\text{eV})\left(1-\frac{1}{2^2}\right)}} = 20$$

$$\boxed{\text{The element whose atomic number is 20 is calcium.}}$$

63 •

Picture the Problem The K_α corresponds to a transition from n = 2 to n = 1. Equation 36-16 relates the atomic number Z to the wavelength of the emitted photon When an electron from state n drops into a vacated state in the n = 1 shell, a photon of energy $E_n - E_1$ is emitted. We can find the wavelength of this photon using $\lambda = hc/(E_n - E_1)$ and E_n from $E_n = -Z^2(E_0/n^2)$.

Express the wavelength of the K_α line:

$$\lambda = \frac{hc}{E_n - E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{E_n - E_1}$$

Express the energy of the atom's nth energy state:

$$E_n = -Z^2\frac{E_0}{n^2}$$

Substitute and simplify to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{-Z^2\frac{E_0}{n^2} + Z^2E_0} = \frac{1240\,\text{eV}\cdot\text{nm}}{Z^2E_0\left(1-\frac{1}{n^2}\right)}$$

(*a*) Substitute n = 2, Z = 12, and E_0 = 13.6 eV to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{11^2(13.6\,\text{eV})\left(1-\frac{1}{2^2}\right)} = \boxed{1.00\,\text{nm}}$$

(*b*) Substitute n = 2, Z = 29, and E_0 = 13.6 eV to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{28^2(13.6\,\text{eV})\left(1-\frac{1}{2^2}\right)} = \boxed{0.155\,\text{nm}}$$

General Problems

64 •

Picture the Problem The energy associated with a transition from an initial state to some final state is given by $\Delta E = E_i - E_f$ and the wavelength λ of a photon emitted in such a transition is given by $\lambda = hc/\Delta E$. Hence, the shortest wavelength corresponds to the largest energy difference.

Express the wavelength of the emitted photon in terms of the energy difference ΔE between the atom's initial and final states:

$$\lambda = \frac{hc}{\Delta E} \text{ or } \Delta E = \frac{hc}{\lambda}$$

For λ_{min}, ΔE will be the energy required to ionize a hydrogen atom:

$$\Delta E_{max} = \frac{hc}{\lambda_{min}} = \boxed{13.6\,\text{eV}}$$

65 •

Picture the Problem This spectral line is due to a transition from some initial state n_i to a final state n_f (we're given that the final state is the ground state). The wavelength of the spectral line is related to the difference in energy ΔE between these states according to $\lambda = 1240\,\text{eV}\cdot\text{nm}/\Delta E$ and the energy of the nth state is given (for hydrogen, Z = 1) by $E_n = (1^2)(-13.6\text{ eV})/n^2$.

Relate the wavelength of a spectral line to the energy transition within the atom:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} \quad (1)$$

Express the energy difference ΔE in a transition:

$$\Delta E = E_i - E_f = -\frac{Z^2E_0}{n_i^2} + \frac{Z^2E_0}{n_f^2} = Z^2E_0\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For Z = 1 and E_0 = 13.6 eV:

$$\Delta E = (13.6\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Substitute in equation (1) to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{(13.6\,\text{eV})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)} = \frac{91.2\,\text{nm}}{\frac{1}{n_f^2}-\frac{1}{n_i^2}}$$

or

$$\frac{1}{n_f^2}-\frac{1}{n_i^2} = \frac{91.2\,\text{nm}}{\lambda}$$

For λ = 97.254 nm and n_f = 1 this expression simplifies to:

$$1-\frac{1}{n_i^2} = \frac{91.2\,\text{nm}}{97.254\,\text{nm}} = 0.938$$

Solve for n_i to obtain:

$$n_i = 4$$

The transition that produced the given wavelength was from $n_i = 4$ to $n_f = 1$.

66 •

Picture the Problem This spectral line is due to a transition from some initial state n_i to a final state n_f (we're given that the final state is the ground state). The wavelength of the spectral line is related to the difference in energy ΔE between these states according to $\lambda = 1240\,\text{eV}\cdot\text{nm}/\Delta E$ and the energy of the nth state is given (for hydrogen, Z = 1) by $E_n = (1^2)(-13.6\text{ eV})/n^2$.

Relate the wavelength of a spectral line to the energy transition within the atom:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} \qquad (1)$$

Express the energy difference ΔE in a transition:

$$\Delta E = E_i - E_f = -\frac{Z^2E_0}{n_i^2}+\frac{Z^2E_0}{n_f^2} = Z^2E_0\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

For Z = 1 and E_0 = 13.6 eV:

$$\Delta E = (13.6\,\text{eV})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

Substitute in equation (1) to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{(13.6\,\text{eV})\left(\frac{1}{n_\text{f}^2}-\frac{1}{n_\text{i}^2}\right)} = \frac{91.2\,\text{nm}}{\frac{1}{n_\text{f}^2}-\frac{1}{n_\text{i}^2}}$$

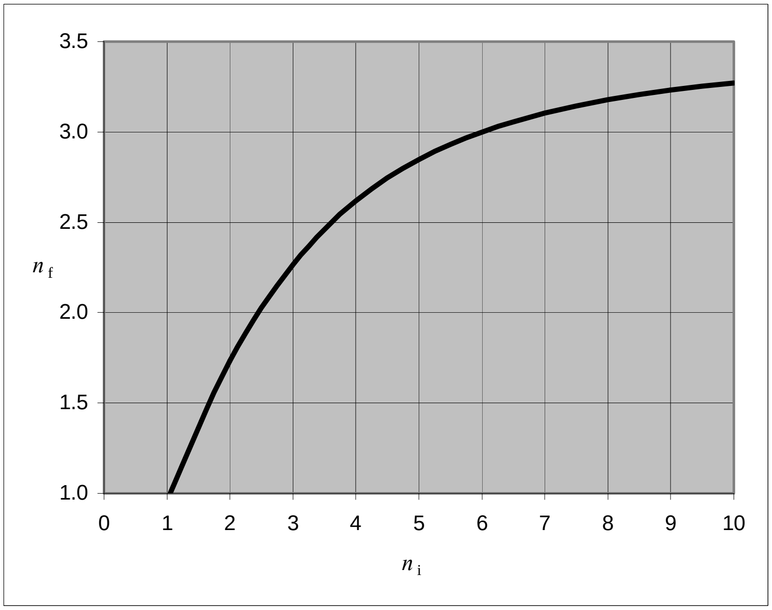
or

$$\frac{1}{n_\text{f}^2}-\frac{1}{n_\text{i}^2} = \frac{91.2\,\text{nm}}{\lambda}$$

For λ = 1093.8 nm this expression simplifies to:

$$\frac{1}{n_\text{f}^2}-\frac{1}{n_\text{i}^2} = \frac{91.2\,\text{nm}}{1093.8\,\text{nm}} = 0.0834$$

Because the only constraints on n_f and n_i are that they be integers, we can solve this equation by trial and error. One way to do this is to plot a graph of n_i as a function of n_f and look for integer solutions visually or with a trace of the trajectory of the curve. The following graph was plotted using a spreadsheet program. Note that a solution to our equation is n_i = 6 and n_f = 3.



Thus, the transition that produces the given wavelength is from $n_\text{i} = 6$ to $n_\text{f} = 3$.

67 •

Picture the Problem These spectral lines are due to transitions in singly ionized helium from some initial state n_i to a final state n_f. The wavelengths of the spectral lines are related to the difference in energy ΔE between these states according to $\lambda = 1240\,\text{eV}\cdot\text{nm}/\Delta E$ and the energy of the nth state is given (for helium, $Z = 2$) by $E_n = (2^2)(-13.6\text{ eV})/n^2$.

Relate the wavelength of a spectral line to the energy transition within the atom:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E} \quad (1)$$

Express the energy difference ΔE in a transition:

$$\Delta E = E_i - E_f = -\frac{Z^2E_0}{n_i^2} + \frac{Z^2E_0}{n_f^2} = Z^2E_0\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For $Z = 2$ and $E_0 = 13.6$ eV:

$$\Delta E = 2^2(13.6\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (54.4\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Substitute in equation (1) to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{(54.4\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)} = \frac{22.8\,\text{nm}}{\frac{1}{n_f^2} - \frac{1}{n_i^2}}$$

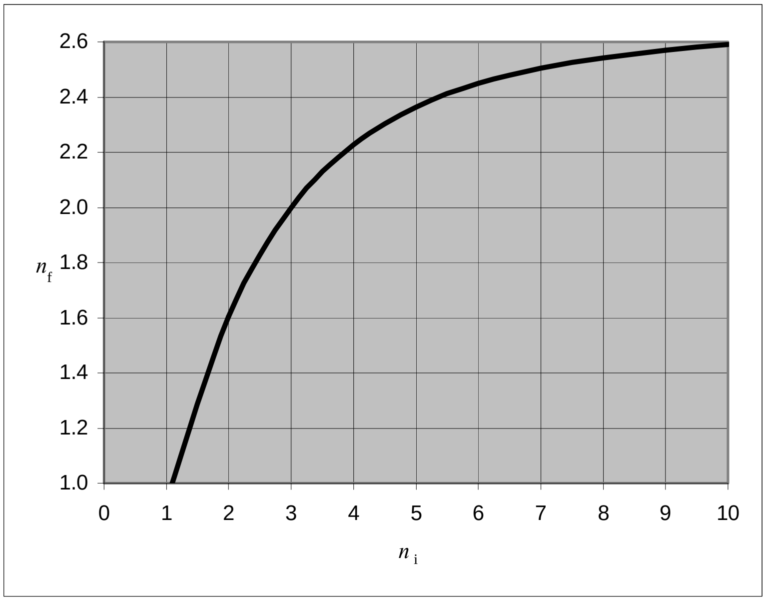
or

$$\frac{1}{n_f^2} - \frac{1}{n_i^2} = \frac{22.8\,\text{nm}}{\lambda}$$

For $\lambda = 164$ nm this expression becomes:

$$\frac{1}{n_f^2} - \frac{1}{n_i^2} = \frac{22.8\,\text{nm}}{164\,\text{nm}} = 0.139$$

Because the only constraints on n_f and n_i are that they be integers, we can solve this equation by trial and error. One way to do this is to plot a graph of n_i as a function of n_f and look for integer solutions visually or with a trace of the trajectory of the curve. The following graph was plotted using a spreadsheet program. Note that a solution to our equation is $n_i = 3$ and $n_f = 2$.



Thus, the transition that produces the given wavelength is from $n_i = 3$ to $n_f = 2$.

Similarly, for λ = 230.6 nm:

The transition that produces the given wavelength is from $n_i = 9$ to $n_f = 3$.

For λ = 541 nm:

The transition that produces the given wavelength is from $n_i = 7$ to $n_f = 4$.

***68 ••**

Picture the Problem We can show that ke^2 = 1.44 eV·nm by solving the equation for the ground state energy of an atom for ke^2.

Express the ground state energy of an atom as a function of k, e, and a_0:

$$E_0 = \frac{ke^2}{2a_0}$$

Solve for ke^2:

$$ke^2 = 2E_0 a_0$$

Substitute for E_0 and a_0 to obtain:

$$ke^2 = 2(13.6\,\text{eV})(0.0529\,\text{nm}) = \boxed{1.44\,\text{eV}\cdot\text{nm}}$$

69 ••

Picture the Problem Because the energies of the photons emitted by potassium during these transitions are related to their wavelengths through $hf = (1240\text{ eV}\cdot\text{nm}/\lambda)$ eV where λ is in nm, we can use this relationship to find the energies of the given photons. The difference in energy between these states can be found using its definition and is related to the magnetic field through $\Delta E = 2\mu_B B$.

(*a*) For λ = 766.41 nm:

$$hf = \frac{1240\,\text{eV}\cdot\text{nm}}{766.41\,\text{nm}} = \boxed{1.6179\,\text{eV}}$$

For λ = 769.90 nm:

$$hf = \frac{1240\,\text{eV}\cdot\text{nm}}{769.90\,\text{nm}} = \boxed{1.6106\,\text{eV}}$$

(*b*) Using its definition, express the difference in energy between these two states:

$$\Delta E = 1.6179\,\text{eV} - 1.6106\,\text{eV} = \boxed{0.00730\,\text{eV}}$$

(*c*) Relate the energy difference between these states ΔE to the magnetic field B and the quantum unit of magnetic moment (a Bohr magneton) μ_B:

$$\Delta E = 2\mu_B B$$

Solve for B:

$$B = \frac{\Delta E}{2\mu_B}$$

Substitute numerical values and evaluate B:

$$B = \frac{0.0073\,\text{eV}}{2\left(5.79\times10^{-5}\,\text{eV/T}\right)} = \boxed{63.0\,\text{T}}$$

Remarks: This magnetic field is about 42 times that of commercial magnetic resonance imagers.

70 ••

Picture the Problem One 1s electron must be released from the atom. It is shielded from the nuclear charge Z by one other 1s electron. Thus, the effective charge is $Z - 1$, and the ionization energy for that 1s electron is $E_{min} = (Z - 1)^2 E_0$.

(*a*) For tungsten, Z = 74, and:

$$E_{min} = 73^2(13.6\,\text{eV}) = \boxed{72.5\,\text{keV}}$$

(*b*) For molybdenum, Z = 42, and:

$$E_{min} = 41^2(13.6\,\text{eV}) = \boxed{22.9\,\text{keV}}$$

(*c*) For copper, $Z = 29$, and:

$$E_{\min} = 28^2(13.6\,\text{eV}) = \boxed{10.7\,\text{keV}}$$

***71 ••**

Picture the Problem We can show that α is dimensionless by showing that it has no units. In part (*b*) we can use Bohr's 3rd postulate and the expression for the radii of the Bohr orbits, together with the definition of α, to show that the speed of the electron in a stationary state of quantum number *n* is related to α according to $v_n = c\alpha/n$.

(*a*) Express the units of α:

$$\frac{C^2\left(\dfrac{N\cdot m^2}{C^2}\right)}{(J\cdot s)\dfrac{m}{s}} = \frac{N\cdot m^2}{J\cdot m} = 1$$

$$\boxed{\text{Because } \alpha \text{ is unitless, it is also dimensionless.}}$$

(*b*) Apply the quantization of angular momentum postulate to obtain:

$$v_n = \frac{n\hbar}{mr_n}$$

The radii of the Bohr orbits are given by:

$$r_n = n^2\frac{\hbar^2}{mkZe^2}$$

or, because $Z = 1$ for hydrogen,

$$r_n = n^2\frac{\hbar^2}{mke^2}$$

Substitute and simplify to obtain:

$$v_n = \frac{n\hbar}{mn^2\dfrac{\hbar^2}{mke^2}} = \frac{ke^2}{n\hbar}$$

Divide this expression by the definition of α to obtain:

$$\frac{v_n}{\alpha} = \frac{\dfrac{ke^2}{n\hbar}}{\dfrac{e^2k}{\hbar c}} = \frac{c}{n}$$

Solve for v_n:

$$v_n = \boxed{\frac{\alpha c}{n}}$$

72 ••

Picture the Problem We can use Problem 29 to express the energy levels of positronium in terms of the reduced mass of the electron-positron system. In part (*b*) we can find the energies corresponding to 400 nm and 700 nm to decide whether the transitions between any of the levels found in (*a*) fall in the visible range of wavelengths

Express the energy of positronium as a function of the quantum number *n*:

$$E_n = -\frac{m_r k^2 e^4}{2\hbar^2}\frac{Z^2}{n^2}$$

From Problem 29 we have:

$$m_r = \frac{m_e m_{pos}}{m_e + m_{pos}}$$

Because $m_e = m_{pos}$:

$$m_r = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$$

Substitute and simplify to obtain:

$$E_n = -\frac{m_e k^2 e^4}{4\hbar^2}\frac{1}{n^2} = -\frac{E_0}{2n^2}$$

(*a*) Evaluate E_n for n = 1, 2, 3, 4, and 5 to obtain:

n	E_n
	(eV)
1	–6.80
2	–1.70
3	–0.756
4	–0.425
5	–0.272

Relate the wavelength of the emitted photons to the energy-level differences:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\ \text{eV}\cdot\text{nm}}{\Delta E}$$

Solve for ΔE:

$$\Delta E = \frac{1240\ \text{eV}\cdot\text{nm}}{\lambda}$$

Evaluate ΔE for λ = 400 nm and λ = 700 nm:

$$\Delta E_{400\ \text{nm}} = \frac{1240\ \text{eV}\cdot\text{nm}}{400\ \text{nm}} = 3.10\ \text{eV}$$

and

$$\Delta E_{700\ \text{nm}} = \frac{1240\ \text{eV}\cdot\text{nm}}{700\ \text{nm}} = 1.77\ \text{eV}$$

Because none of the energies in the table shown above are in the interval 1.77 eV to 3.10 eV, no transitions are in the visible range of wavelengths.

73 •

Picture the Problem We can use $E = hf$ to find the frequency of the photon and $\lambda = hc/E$ to find its wavelength.

(*a*) The energy of the photon whose energy is equal to the Lamb shift energy is given by:

$$E = hf$$

Solve for f to obtain:

$$f = \frac{E}{h}$$

Substitute numerical values and evaluate f:

$$f = \frac{4.372\times10^{-6}\ \text{eV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} = \boxed{1.06\,\text{GHz}}$$

(*b*) The wavelength of this photon is given by:

$$\lambda = \frac{hc}{E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{4.372\times10^{-6}\ \text{eV}} = \boxed{28.4\,\text{cm}}$$

This wavelength is in the microwave portion of the electromagnetic spectrum.

***74** •

Picture the Problem The ionization energy of the electron is the magnitude of the energy of the atom in the given state. We can use $E = -E_0/n^2$, where E_0 is the ground-state energy, to find the energy levels in the 44th and 45th states and, hence, the energy level separation between the states. The wavelength of a photon resonant with this transition can be found from $\lambda = hc/\Delta E$. We'll approximate the size of the atom in the $n = 45$ state by finding the radius of the outer-shell electron.

(*a*) The energy of the atom in its nth state is:

$$E_n = -\frac{E_0}{n^2}$$

The energy of the atom in the $n = 45$ state is:

$$E_{45} = -\frac{13.6\,\text{eV}}{(45)^2} = -6.72\,\text{meV}$$

The ionization energy is the negative of the energy in the state $n = 45$:

$$E_{\text{ionizing}} = -E_{45} = \boxed{6.72\,\text{meV}}$$

(*b*) The energy level separation between the $n = 45$ and $n = 44$ state is:

$$E_{45\to44} = -\left(\frac{13.6\,\text{eV}}{(45)^2} - \frac{13.6\,\text{eV}}{(44)^2}\right)$$

$$= \boxed{3.09\times10^{-4}\ \text{eV}}$$

(*c*) The photon wavelength is:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{3.09\times10^{-4}\ \text{eV}} = \boxed{4.01\times10^{6}\ \text{nm}}$$

(*d*) The radii of the Bohr orbits are given by:

$$r = n^2\frac{a_0}{Z}$$

Substitute numerical values and evaluate the radius of the 45th Bohr orbit:

$$r = (45)^2\frac{0.0529\,\text{nm}}{1} = \boxed{107\,\text{nm}}$$

75 ••

Picture the Problem We can use the definition of the Rydberg constant and the equation for the reduced mass from Problem 29 to calculate the Rydberg constant for hydrogen and for deuterium. We can find the wavelength difference between the longest wavelength Balmer lines of hydrogen and deuterium by finding the longest wavelengths from the Rydberg-Ritz equation, using the appropriate value for *R*, and taking their difference.

(*a*) From Equation 36-14 we have:

$$R = \frac{m_\text{r}k^2e^4}{4\pi c\hbar^3} = C\left(\frac{m_\text{e}}{1+\dfrac{m_\text{e}}{M}}\right)$$

where

$$C = \frac{k^2e^4}{4\pi c\hbar^3} = 1.204662\times10^{37}\ \text{m}^{-1}/\text{kg}$$

For H:

$$R_\text{H} = C\left(\frac{m_\text{e}}{1+\dfrac{m_\text{e}}{m_\text{p}}}\right)$$

Substitute numerical values and evaluate R_H:

$$R_\text{H} = \left(1.204662\times10^{37}\ \text{m}^{-1}/\text{kg}\right)\times\left(\frac{9.109390\times10^{-31}\ \text{kg}}{1+\dfrac{9.109390\times10^{-31}\ \text{kg}}{1.672623\times10^{-27}\ \text{kg}}}\right) = \boxed{1.096776\times10^{7}\ \text{m}^{-1}}$$

For deuterium:

$$R_D = C\left(\frac{m_e}{1+\frac{m_e}{2m_p}}\right)$$

Substitute numerical values and evaluate R_D:

$$R_D = \left(1.204662\times10^{37}\ \text{m}^{-1}/\text{kg}\right)\times\left(\frac{9.109390\times10^{-31}\ \text{kg}}{1+\frac{9.109390\times10^{-31}\ \text{kg}}{2\left(1.672623\times10^{-27}\ \text{kg}\right)}}\right)$$

$$= \boxed{1.097075\times10^{7}\ \text{m}^{-1}}$$

(*b*) Express the wavelength difference between the longest wavelength Balmer lines of hydrogen and deuterium:

$$\Delta\lambda = \lambda_{\text{longest, H}} - \lambda_{\text{longest, D}}$$

Use the Rydberg-Ritz formula to express the reciprocal wavelength:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where n_1 and n_2 are integers and $n_1 > n_2$.

Solve for λ to obtain:

$$\lambda = \frac{n_1^2 n_2^2}{R\left(n_1^2 - n_2^2\right)}$$

The longest wavelength in the Balmer series corresponds to a transition from $n_1 = 3$ to $n_2 = 2$. Use $R = R_H$ to evaluate $\lambda_{\text{longest, H}}$:

$$\lambda_{\text{longest, H}} = \frac{3^2\left(2^2\right)}{\left(1.096776\times10^{7}\ \text{m}^{-1}\right)\left(3^2 - 2^2\right)} = 656.470\ \text{nm}$$

Find $\lambda_{\text{longest,D}}$ using $R = R_D$:

$$\lambda_{\text{longest, D}} = \frac{3^2\left(2^2\right)}{\left(1.097075\times10^{7}\ \text{m}^{-1}\right)\left(3^2 - 2^2\right)} = 656.291\ \text{nm}$$

Substitute to obtain:

$$\Delta\lambda = 656.470\ \text{nm} - 656.291\ \text{nm} = \boxed{0.179\ \text{nm}}$$

76 ••

Picture the Problem We can use Problem 29 to express the energy levels of muonium in terms of the reduced mass of the muonium-proton system. In Part (*b*) we can find the energies corresponding to 400 nm and 700 nm to decide whether the transitions between any of the levels found in (*a*) fall in the visible range of wavelengths

Express the energy of muonium as a function of the quantum number *n*:

$$E_n = -\frac{m_r k^2 e^4}{2\hbar^2}\frac{Z^2}{n^2} \qquad (1)$$

From Equation 35-47 in Problem 17 we have:

$$m_r = \frac{m_\mathrm{p} m_{\mu^{-1}}}{m_\mathrm{p} + m_{\mu^{-1}}}$$

Because $m_{\mu^{-1}} = 207 m_\mathrm{e}$:

$$m_r = \frac{207 m_\mathrm{p} m_\mathrm{e}}{m_\mathrm{p} + 207 m_\mathrm{e}} = \frac{207 m_\mathrm{e}}{1 + 207\dfrac{m_\mathrm{e}}{m_\mathrm{p}}}$$

Because $m_\mathrm{p} = 1836 m_\mathrm{e}$:

$$m_r = \frac{207 m_\mathrm{e}}{1 + \dfrac{207}{1836}} = 186 m_\mathrm{e}$$

Substitute in equation (1) and simplify to obtain:

$$E_n = -\frac{186 m_\mathrm{e} k^2 e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{186 E_0}{n^2}$$

(*a*) Evaluate E_n for n = 1, 2, 3, 4, and 5 to obtain:

n	E_n
	(keV)
1	–2.53
2	–0.633
3	–0.281
4	–0.158
5	–0.101

Relate the wavelength of the emitted photons to the energy-level differences:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\mathrm{eV\cdot nm}}{\Delta E}$$

Solve for ΔE:

$$\Delta E = \frac{1240\,\mathrm{eV\cdot nm}}{\lambda}$$

Evaluate ΔE for λ = 400 nm and λ = 700 nm:

$$\Delta E_{400\,\mathrm{nm}} = \frac{1240\,\mathrm{eV\cdot nm}}{400\,\mathrm{nm}} = 3.10\,\mathrm{eV}$$

and

$$\Delta E_{700\,\text{nm}} = \frac{1240\,\text{eV}\cdot\text{nm}}{700\,\text{nm}} = 1.77\,\text{eV}$$

Because none of the energies in the table shown above are in the interval 1.77 eV to 3.10 eV, no transitions are in the visible range of wavelengths.

77 ••

Picture the Problem We can use the definition of the Rydberg constant and the equation for the reduced mass from Problem 29 to calculate the Rydberg constant for hydrogen, tritium, and deuterium. We can find the wavelength difference between the longest wavelength Balmer lines of tritium and deuterium and tritium and hydrogen by finding the longest wavelengths from the Rydberg-Ritz equation, using the appropriate value for *R*, and taking their difference.

(*a*) From Problem 29 we have:

$$R = \frac{m_r k^2 e^4}{4\pi c\hbar^3} = C\left(\frac{m_e}{1+\frac{m_e}{M}}\right)$$

where

$$C = \frac{k^2 e^4}{4\pi c\hbar^3} = 1.204662\times10^{37}\ \text{m}^{-1}/\text{kg}$$

For tritium:

$$R_T = C\left(\frac{m_e}{1+\frac{m_e}{m_p + 2m_n}}\right)$$

Evaluate the expression in parentheses to obtain:

$$m_r = 9.107738\times10^{-31}\ \text{kg}$$

Substitute numerical values and evaluate R_T:

$$R_T = \left(1.204662\times10^{37}\ \text{m}^{-1}/\text{kg}\right)\times\left(9.107738\times10^{-31}\ \text{kg}\right) = \boxed{1.097175\times10^{7}\ \text{m}^{-1}}$$

For deuterium:

$$R_D = C\left(\frac{m_e}{1+\frac{m_e}{2m_p}}\right)$$

Substitute numerical values and evaluate R_D:

$$R_D = \left(1.204662\times10^{37}\ \text{m}^{-1}/\text{kg}\right) \times \left(\frac{9.109390\times10^{-31}\ \text{kg}}{1+\dfrac{9.109390\times10^{-31}\ \text{kg}}{2\left(1.672623\times10^{-27}\ \text{kg}\right)}}\right) = 1.097075\times10^{7}\ \text{m}^{-1}$$

(*b*) Express the wavelength difference between the longest wavelength Balmer lines of hydrogen and deuterium:

$$\Delta\lambda = \lambda_{\text{longest, D}} - \lambda_{\text{longest, T}}$$

Use the Rydberg-Ritz formula to express the reciprocal wavelength:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where n_1 and n_2 are integers and $n_1 > n_2$.

Solve for λ to obtain:

$$\lambda = \frac{n_1^2 n_2^2}{R\left(n_1^2 - n_2^2\right)}$$

The longest wavelength in the Balmer series corresponds to a transition from $n_1 = 3$ to $n_2 = 2$. Use $R = R_T$ to evaluate $\lambda_{\text{longest, T}}$:

$$\lambda_{\text{longest, T}} = \frac{3^2\left(2^2\right)}{\left(1.097175\times10^{7}\ \text{m}^{-1}\right)\left(3^2 - 2^2\right)} = 656.231\,\text{nm}$$

Find $\lambda_{\text{longest,D}}$ using $R = R_D$:

$$\lambda_{\text{longest, D}} = \frac{3^2\left(2^2\right)}{\left(1.097075\times10^{7}\ \text{m}^{-1}\right)\left(3^2 - 2^2\right)} = 656.291\,\text{nm}$$

Substitute to obtain:

$$\Delta\lambda = 656.291\,\text{nm} - 656.231\,\text{nm} = \boxed{0.0600\,\text{nm}}$$

Proceed similarly to show that for hydrogen and hydrogen:

$$\Delta\lambda = 656.4695\,\text{nm} - 656.2314\,\text{nm} = \boxed{0.238\,\text{nm}}$$

Chapter 37
Molecules

Conceptual Problems

***1 •**

Determine the Concept Yes. Because the center of charge of the positive Na ion does not coincide with the center of charge for the negative Cl ion, the NaCl molecule has a permanent dipole moment. Hence, it is a polar molecule.

2 •

Determine the Concept Because a N_2 molecule has no permanent dipole moment, it is a non-polar molecule.

3 •

Determine the Concept No. Neon occurs naturally as Ne, not Ne_2. Neon is a rare gas atom with a closed shell electron configuration.

4 •

Determine the Concept

(*a*) Because an electron is transferred from the H atom to the F atom, the bonding mechanism is ionic.

(*b*) Because an electron is transferred from the K atom to the Br atom, the bonding mechanism is ionic.

(*c*) Because the atoms share two electrons, the bonding mechanism is covalent.

(*d*) Because each valence electron is shared by many atoms, the bonding mechanism is metallic bonding.

***5 ••**

Determine the Concept The diagram would consist of a non-bonding ground state with no vibrational or rotational states for ArF (similar to the upper curve in Figure 37-4) but for ArF* there should be a bonding excited state with a definite minimum with respect to inter-nuclear separation and several vibrational states as in the excited state curve of Figure 37-13.

6 •

Determine the Concept Elements similar to carbon in outer shell configurations are silicon, germanium, tin, and lead. We would expect the same hybridization for these as for carbon, and this is indeed the case for silicon and germanium whose crystal structure is the diamond structure. Tin and lead, however, are metallic and here the metallic bond is dominant.

7 •

Determine the Concept The effective force constant from Example 37-4 is 1.85×10^{3} N/m. This value is about 25% larger than the given value of the force constant of the suspension springs on a typical automobile.

8 •

Determine the Concept As the angular momentum increases, the separation between the nuclei also increases (the effective force between the nuclei is similar to that of a stiff spring). Consequently, the moment of inertia also increases.

9 •

Determine the Concept For H_2, the concentration of negative charge between the two protons holds the protons together. In the H_2^+ ion, there is only one electron that is shared by the two positive charges such that most of the electronic charge is again between the two protons. However, the negative charge in the H_2^+ ion is not as effective as the larger charge in the H_2 molecule, and the protons should be farther apart. The experimental values support this argument. For H_2, r_0 = 0.074 nm, while for H_2^+, r_0 = 0.106 nm.

10 •

Determine the Concept The energy of the first excited state of an atom is orders of magnitude greater than *kT* at ordinary temperatures. Consequently, practically all atoms are in the ground state. By contrast, the energy separation between the ground rotational state and nearby higher rotational states is less than or roughly equal to *kT* at ordinary temperatures, and so these higher states are thermally excited and occupied.

11 ••

Determine the Concept With more than two atoms in the molecule there will be more than just one frequency of vibration because there are more possible relative motions. In advanced mechanics these are known as normal modes of vibration.

Estimation and Approximation

12 ••

Picture the Problem We can estimate the value of the quantum number ν for which the improved formula corrects the original formula by 10 percent by setting the ratio of the correction term to the first term equal to 10 percent and solving for ν.

Express the ratio of the correction term to the first term of the expression for E_ν and simplify to obtain:

$$\frac{\left(\nu+\frac{1}{2}\right)^2 hf\alpha}{\left(\nu+\frac{1}{2}\right)hf}=\left(\nu+\tfrac{1}{2}\right)\alpha$$

For a correction of 10 percent:

$$\left(\nu+\tfrac{1}{2}\right)\alpha=0.1$$

Solve for ν to obtain:

$$\nu=\frac{1}{10\alpha}-\frac{1}{2}$$

Substitute numerical values and evaluate ν:

$$\nu=\frac{1}{10\left(7.6\times10^{-3}\right)}-\frac{1}{2}=12.7\approx\boxed{13}$$

13 ••

Picture the Problem We can solve Equation 37-12 for ℓ and substitute for the moment of inertia and rotational kinetic energy of the baseball to estimate the quantum number ℓ and spacing between adjacent energy levels for a baseball spinning about its own axis.

The rotational energy levels are given by Equation 37-12:

$$E=\frac{\ell(\ell+1)\hbar^2}{2I}$$

where ℓ = 0, 1, 2, … is the rotational quantum number and I is the moment of inertia of the ball.

Solve for $\ell(\ell+1)$:

$$\ell(\ell+1)=\frac{2IE}{\hbar^2}$$

Factor ℓ from the parentheses to obtain:

$$\ell^2\left(1+\frac{1}{\ell}\right)=\frac{2IE}{\hbar^2}$$

The result of our calculation of ℓ will show that ℓ >> 1. Assuming for the moment that this is the case:

$$\ell^2\approx\frac{2IE}{\hbar^2}\ \text{and}\ \ell\approx\frac{\sqrt{2IE}}{\hbar}$$

Because the energy of the ball is rotational kinetic energy:

$$E=K_{\text{rot}}=\tfrac{1}{2}I\omega^2$$

Substitute for E in the expression for ℓ to obtain:

$$\ell\approx\frac{\sqrt{2I\left(\frac{1}{2}I\omega^2\right)}}{\hbar}=\frac{\sqrt{\left(I^2\omega^2\right)}}{\hbar}=\frac{I\omega}{\hbar}$$

The moment of inertia of a ball about an axis through its diameter is (see Table 9-1):

$$I=\tfrac{2}{5}mr^2$$

Substitute for I to obtain:

$$\ell\approx\frac{2mr^2\omega}{5\hbar}$$

Substitute numerical values and evaluate ℓ:

$$\ell \approx \frac{2(0.3\,\text{kg})(0.03\,\text{m})^2\left(\frac{20\,\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)}{5(1.05\times10^{-34}\,\text{J}\cdot\text{s})} = \boxed{2.15\times10^{30}}$$

Set $\ell = 0$ to express the spacing between adjacent energy levels:

$$E_{0\text{r}} = \frac{\hbar^2}{2I} = \frac{5\hbar^2}{4mr^2}$$

Substitute numerical values and evaluate $E_{0\text{r}}$:

$$E_{0\text{r}} = \frac{5(1.05\times10^{-34}\,\text{J}\cdot\text{s})^2}{4(0.3\,\text{kg})(0.03\,\text{m})^2} = \boxed{5.10\times10^{-65}\,\text{J}}$$

Remarks: Note that our value for ℓ justifies our assumption that ℓ >> 1.

***14 ••**

Picture the Problem We can solve Equation 37-18 for ν and substitute for the frequency of the mass-and-spring oscillator to estimate the quantum number ν and spacing between adjacent energy levels for this system.

The vibrational energy levels are given by Equation 37-18:

$$E_\nu = \left(\nu + \tfrac{1}{2}\right)hf$$

where ν = 0, 1, 2, …

Solve for ν:

$$\nu = \frac{E_\nu}{hf} - \frac{1}{2}$$

or, because ν >> 1,

$$\nu \approx \frac{E_\nu}{hf}$$

The vibrational energy of the object attached to the spring is:

$$E_\nu = \tfrac{1}{2}kA^2$$

where A is the amplitude of its motion.

Substitute for E_ν in the expression for ν to obtain:

$$\nu = \frac{kA^2}{2hf}$$

The frequency of oscillation f of the mass-and-spring oscillator is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\nu = \frac{\pi kA^2}{h}\sqrt{\frac{m}{k}} = \frac{\pi A^2}{h}\sqrt{mk}$$

Substitute numerical values and evaluate ν:

$$\nu = \frac{\pi(0.02\,\text{m})^2}{6.63\times10^{-34}\,\text{J}\cdot\text{s}}\sqrt{(5\,\text{kg})(1500\,\text{N/m})} = \boxed{1.64\times10^{32}}$$

Set $\nu = 0$ in Equation 37-18 to express the spacing between adjacent energy levels:

$$E_{0\nu} = \frac{1}{2}hf = \frac{h}{4\pi}\sqrt{\frac{k}{m}}$$

Substitute numerical values and evaluate $E_{0\nu}$:

$$E_{0\nu} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{4\pi}\sqrt{\frac{1500\,\text{N/m}}{5\,\text{kg}}} = \boxed{9.14\times10^{-34}\,\text{J}}$$

Remarks: Note that our value for ν justifies our assumption that ν >> 1.

Molecular Bonding

15 •

Picture the Problem The electrostatic potential energy with U at infinity is given by $U = -ke^2/r$.

Relate the electrostatic potential energy of the ions to their separation:

$$U_e = -\frac{ke^2}{r}$$

Solve for r:

$$r = -\frac{ke^2}{U_e}$$

Substitute numerical values and evaluate r:

$$r = -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\,\text{C})^2}{(-1.52\,\text{eV})(1.6\times10^{-19}\,\text{J/eV})} = \boxed{0.946\,\text{nm}}$$

16 •

Picture the Problem We can find the energy absorbed or released per molecule by computing the difference between dissociation energy of Cl and the binding energy of NaCl.

Noting that the dissociation energy per Cl atom is 1.24 eV, express the net energy change per molecule ΔE:

$$\Delta E = 1.24\,\text{eV} - E_{\text{binding, NaCl}}$$

The binding energy of NaCl is (see page 1210):

4.27 eV

Substitute and evaluate ΔE:

$$\Delta E = 1.24\,\text{eV} - 4.27\,\text{eV} = \boxed{-3.03\,\text{eV}}$$

$$\boxed{\text{Because } \Delta E < 0 \text{, energy is released. The reaction is } \textit{exothermic}.}$$

17 •

Picture the Problem We can use conversion factors to convert eV/molecule into kcal/mol.

(*a*)
$$1\frac{\text{eV}}{\text{molecule}} = 1\frac{\text{eV}}{\text{molecule}} \times \frac{1\,\text{kcal}}{4184\,\text{J}} \times \frac{6.02\times10^{23}\text{ molecules}}{\text{mole}} \times \frac{1.60\times10^{-19}\text{ J}}{\text{eV}}$$
$$= \boxed{23.0\,\text{kcal/mol}}$$

(*b*) The dissociation energy of NaCl, in eV/molecule, is (see page 1205):

4.27 eV/molecule

Using the conversion factor in (*a*), express this energy in kcal/mol:

$$\frac{4.27\,\text{eV}}{\text{molecule}} \times \frac{23.0\,\text{kcal}}{\text{mol}} = \boxed{98.2\,\text{kcal/mol}}$$

***18** •

Picture the Problem The percentage of the bonding that is ionic is given by $100\left(\frac{p_{\text{meas}}}{p_{100}}\right)$.

Express the percentage of the bonding that is ionic:

$$\text{Percent ionic bonding} = 100\left(\frac{p_{\text{meas}}}{p_{100}}\right)$$

Express the dipole moment for 100% ionic bonding:

$$p_{100} = er$$

Substitute to obtain:

$$\text{Percent ionic bonding} = 100\left(\frac{p_{\text{meas}}}{er}\right)$$

Substitute numerical values and evaluate the percent ionic bonding:

$$\text{Percent ionic bonding} = 100\left[\frac{6.40\times10^{-30}\ \text{C}\cdot\text{m}}{(1.60\times10^{-19}\ \text{C})(0.0917\ \text{nm})}\right] = \boxed{43.6\%}$$

19 ••

Picture the Problem If we choose the potential energy at infinity to be ΔE, the total potential energy is $U_{\text{tot}} = U_{\text{e}} + \Delta E + U_{\text{rep}}$, where U_{rep} is the energy of repulsion, which is found by setting the dissociation energy equal to $-U_{\text{tot}}$.

Express the total potential energy of the molecule:

$$U_{\text{tot}} = U_{\text{e}} + \Delta E + U_{\text{rep}}$$

The core-repulsive energy is :

$$U_{\text{rep}} = -(\Delta E + U_{\text{e}} + E_{\text{d}})$$

Calculate the energy ΔE needed to form Rb^+ and F^- ions from neutral rubidium and fluorine atoms:

$$\Delta E = 4.18\ \text{eV} - 3.40\ \text{eV} = 0.78\ \text{eV}$$

Express the electrostatic potential energy is:

$$U_{\text{e}} = -\frac{ke^2}{r}$$

Substitute numerical values and evaluate U_{e}:

$$U_{\text{e}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(0.227\ \text{nm})(1.60\times10^{-19}\ \text{J/eV})} = -6.34\ \text{eV}$$

Substitute numerical values and evaluate U_{rep}:

$$U_{\text{rep}} = -(0.78\ \text{eV} - 6.34\ \text{eV} + 5.12\ \text{eV}) = \boxed{0.44\ \text{eV}}$$

20 ••

Picture the Problem The potential energy of attraction of the ions is $U_{\text{e}} = -ke^2/r$. We can find the dissociation energy from the negative of the sum of the potential energy of attraction and the difference between the ionization energy of potassium and the electron affinity of chlorine.

(*a*) The potential energy of attraction of the ions is given by:

$$U_e = -\frac{ke^2}{r}$$

where $ke^2 = 1.44$ eV·nm

Substitute numerical values and evaluate U_e:

$$U_e = -\frac{1.44\,\text{eV}\cdot\text{nm}}{0.267\,\text{nm}} = \boxed{-5.39\,\text{eV}}$$

(*b*) Express the total potential energy of the molecule:

$$U_{tot} = U_e + \Delta E + U_{rep}$$

or, neglecting any energy of repulsion,

$$U_{tot} = U_e + \Delta E$$

The dissociation energy is the negative of the total potential energy:

$$E_{d,calc} = -U_{tot} = -(U_e + \Delta E)$$

ΔE is the difference between the ionization energy of potassium and the electron affinity of Cl:

$$\Delta E = 4.34\,\text{eV} - 3.62\,\text{eV} = 0.72\,\text{eV}$$

Substitute numerical values and evaluate $E_{d,calc}$:

$$E_{d,calc} = -(-5.39\,\text{eV} + 0.72\,\text{eV})$$
$$= \boxed{4.67\,\text{eV}}$$

The energy due to repulsion of the ions at equilibrium separation is given by:

$$U_{rep} = E_{d,calc} - E_{d,meas}$$

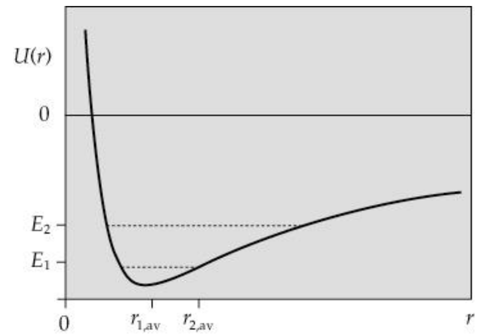
Substitute numerical values and evaluate U_{rep}:

$$U_{rep} = 4.67\,\text{eV} - 4.49\,\text{eV} = \boxed{0.18\,\text{eV}}$$

21 ••

Picture the Problem Assume that $U(r)$ is of the form given in Problem 24 with $n = 6$.

The potential energy curve is shown in the figure. The turning points for vibrations of energy E_1 and E_2 are at the values of r, where the energies equal $U(r)$. It is apparent that the average value of r depends on the energy and that $r_{2,\text{av}}$ is greater than $r_{1,\text{av}}$.



22 ••

Picture the Problem We can use $U_e = -ke^2/r_0$ to calculate the potential energy of attraction between the Na^+ and Cl^- ions at the equilibrium separation r_0 = 0.236 nm. We can find the energy due to repulsion of the ions at the equilibrium separation from $U_{\text{rep}} = -(U_e + E_d + \Delta E)$.

The potential energy of attraction between the Na^+ and Cl^- ions at the equilibrium separation r_0 is given by:

$$U_e = -\frac{ke^2}{r_0}$$

where ke^2 = 1.44 eV·nm.

Substitute numerical values and evaluate U_e:

$$U_e = -\frac{1.44\,\text{eV}\cdot\text{nm}}{0.236\,\text{nm}} = \boxed{-6.10\,\text{eV}}$$

From Figure 37-1:

$$E_d = 4.27\,\text{eV}$$

The ratio of the magnitude of the potential energy of attraction to the dissociation energy is:

$$\frac{|U_e|}{E_d} = \frac{6.10\,\text{eV}}{4.27\,\text{eV}} = \boxed{1.43}$$

U_{rep} is related to U_e, E_d, and ΔE according to:

$$U_{\text{rep}} = -(U_e + E_d + \Delta E)$$

From Figure 37-1:

$$\Delta E = 1.52\,\text{eV}$$

Substitute numerical values and evaluate U_{rep}:

$$U_{\text{rep}} = -(-6.10\,\text{eV} + 4.27\,\text{eV} + 1.52\,\text{eV}) = \boxed{0.310\,\text{eV}}$$

23 ••

Picture the Problem The potential energy of attraction of the ions is $U_e = -ke^2/r$. We can find the dissociation energy from the negative of the sum of the potential energy of attraction and the difference between the ionization energy of potassium and the electron affinity of fluorine.

(*a*) The potential energy of attraction between the K^+ and F^- ions at the equilibrium separation r_0 is given by:

$$U_e = -\frac{ke^2}{r_0}$$

where $ke^2 = 1.44$ eV·nm.

Substitute numerical values and evaluate U_e:

$$U_e = -\frac{1.44\,\text{eV}\cdot\text{nm}}{0.217\,\text{nm}} = \boxed{-6.64\,\text{eV}}$$

(*b*) Express the total potential energy of the molecule:

$$U_{tot} = U_e + \Delta E + U_{rep}$$

or, neglecting any energy of repulsion,

$$U_{tot} = U_e + \Delta E$$

The dissociation energy is the negative of the total potential energy:

$$E_{d,calc} = -U_{tot} = -(U_e + \Delta E)$$

ΔE is the difference between the ionization energy of potassium and the electron affinity of fluorine:

$$\Delta E = 4.34\,\text{eV} - 3.40\,\text{eV} = 0.94\,\text{eV}$$

Substitute numerical values and evaluate $E_{d,calc}$:

$$E_{d,calc} = -(-6.64\,\text{eV} + 0.94\,\text{eV}) = \boxed{5.70\,\text{eV}}$$

The energy due to repulsion of the ions at equilibrium separation is given by:

$$U_{rep} = E_{d,calc} - E_{d,meas}$$

Substitute numerical values and evaluate U_{rep}:

$$U_{rep} = 5.70\,\text{eV} - 5.07\,\text{eV} = \boxed{0.63\,\text{eV}}$$

***24** •••

Picture the Problem $U(r)$ is the potential energy of the two ions as a function of separation distance r. $U(r)$ is chosen so $U(\infty) = -\Delta E$, where ΔE is the negative of the energy required to form two ions at infinite separation from two neutral atoms also at infinite separation. $U_{rep}(r)$ is the potential energy of the two ions due to the repulsion of the two closed-shell cores. E_d is the disassociation energy, the energy required to separate the two ions plus the energy ΔE required to form two neutral atoms from the two ions at infinite separation. The net force acting on the ions is the sum of F_{rep} and F_e. We can find F_{rep} from U_{rep} and F_e from Coulomb's law and then use $dU/dr = F_{net} = 0$ at $r = r_0$ to solve for n.

Express the net force acting on the ions:

$$F_{net} = F_{rep} + F_e \qquad (1)$$

Find F_{rep} from U_{rep}:

$$F_{rep} = \frac{dU_{rep}}{dr} = \frac{d}{dr}\left[Cr^{-n}\right] = -nCr^{-n-1} = -\frac{nC}{r^{n+1}}$$

The electrostatic potential energy of the two ions as a function of separation distance is given by:

$$U_e = -\frac{ke^2}{r}$$

Find the electrostatic force of attraction F_e from U_e:

$$F_e = \frac{dU_e}{dr} = \frac{d}{dr}\left[-\frac{ke^2}{r}\right] = \frac{ke^2}{r^2}$$

Substitute for F_{rep} and F_e in equation (1) to obtain:

$$F_{net} = -\frac{nC}{r^{n+1}} + \frac{ke^2}{r^2}$$

Because $dU/dr = F_{net} = 0$ at $r = r_0$:

$$0 = -\frac{nC}{r_0^{n+1}} + \frac{ke^2}{r_0^2}$$

Multiply both sides of this equation by r_0 to obtain:

$$0 = -\frac{nC}{r_0^n} + \frac{ke^2}{r_0} = -nU_{rep}(r_0) + \left|U_e(r_0)\right|$$

Solve for n to obtain:

$$n = \boxed{\frac{\left|U_e(r_0)\right|}{U_{rep}(r_0)}}$$

25 •••

Picture the Problem U_{rep} at $r = r_0$ is related to U_e, E_d, and ΔE through $U_{rep} = -(U_e + E_d + \Delta E)$. The net force is the sum of F_{rep} and F_e. We can find F_{rep} from U_{rep} and F_e from Coulomb's law. Because $F_{net} = 0$ at $r = r_0$, we can obtain simultaneous equations in C and n that we can solve for each of these quantities.

(*a*) U_{rep} is related to U_e, E_d, and ΔE according to:

$$U_{rep} = -(U_e + E_d + \Delta E)$$

where $\Delta E_{NaCl} = 1.52$ eV

$U_e(r_0)$ is given by:

$$U_e = -\frac{ke^2}{r_0}$$

where $ke^2 = 1.44$ eV·nm

Substitute numerical values and evaluate U_e:

$$U_e = -\frac{1.44\,\text{eV}\cdot\text{nm}}{0.236\,\text{nm}} = -6.10\,\text{eV}$$

Substitute numerical values and evaluate U_{rep}:

$$U_{rep} = -(-6.10\,\text{eV} + 4.27\,\text{eV} + 1.52\,\text{eV})$$
$$= \boxed{0.31\,\text{eV}}$$

(*b*) Express the net force acting on the Na^+ and Cl^- ions:

$$F_{net} = F_{rep} + F_e \qquad (1)$$

Find F_{rep} from U_{rep}:

$$F_{rep} = \frac{dU_{rep}}{dr} = \frac{d}{dr}\left[Cr^{-n}\right] = -nCr^{-n-1}$$
$$= -\frac{nC}{r^{n+1}}$$

The electrostatic force of attraction is:

$$F_e = \frac{ke^2}{r^2}$$

Substitute for F_{rep} and F_e in equation (1) to obtain:

$$F_{net} = -\frac{nC}{r^{n+1}} + \frac{ke^2}{r^2}$$

Because $F_{net} = 0$ at $r = r_0$:

$$0 = -\frac{nC}{r_0^{n+1}} + \frac{ke^2}{r_0^2}$$

or

$$|U_e(r_0)| = n\frac{C}{r_0^n} = nU_{rep}(r_0)$$

Solve for n and C to obtain:

$$n = \frac{|U_e(r_0)|}{U_{rep}(r_0)}$$

and

$$C = U_{rep}(r_0)r_0^n$$

From (*a*):

$$U_e(r_0) = -6.10\,\text{eV}$$
$$U_{rep}(r_0) = 0.31\,\text{eV}$$

and, from Figure 37-1,

$$r_0 = 0.236\,\text{nm}$$

Substitute for $U_e(r_0)$ and $U_{rep}(r_0)$ and evaluate n and C:

$$n = \frac{6.10\,\text{eV}}{0.31\,\text{eV}} = \boxed{19.7}$$

and

$$C = (0.31\,\text{eV})(0.236\,\text{nm})^{19.7}$$
$$= \boxed{1.37\times10^{-13}\,\text{eV}\cdot\text{nm}^{19.7}}$$

Energy Levels of Spectra of Diatomic Molecules

26 •

Picture the Problem We can relate the characteristic rotational energy E_{0r} to the moment of inertia of the molecule and model the moment of inertia of the N_2 molecule as two point objects separated by a distance r.

The characteristic rotational energy of a molecule is given by:

$$E_{0r} = \frac{\hbar^2}{2I}$$

Express the moment of inertia of the molecule:

$$I = 2M_N\left(\frac{r}{2}\right)^2 = \tfrac{1}{2}M_N r^2$$

Substitute for I to obtain:

$$E_{0r} = \frac{\hbar^2}{2\left(\frac{1}{2}M_N r^2\right)} = \frac{\hbar^2}{M_N r^2} = \frac{\hbar^2}{14m_p r^2}$$

Solve for r:

$$r = \hbar\sqrt{\frac{1}{14E_{0r}m_p}}$$

Substitute numerical values and evaluate r:

$$r = (1.055\times10^{-34}\,\text{J}\cdot\text{s})\sqrt{\frac{1}{14(2.48\times10^{-4}\,\text{eV})(1.602\times10^{-19}\,\text{J/eV})(1.673\times10^{-27}\,\text{kg})}}$$
$$= \boxed{0.109\,\text{nm}}$$

***27** •

Picture the Problem We can relate the characteristic rotational energy E_{0r} to the moment of inertia of the molecule and model the moment of inertia of the O_2 molecule as two point objects separated by a distance r.

The characteristic rotational energy of a molecule is given by:

$$E_{0r} = \frac{\hbar^2}{2I}$$

Express the moment of inertia of the molecule:

$$I = 2M_{\text{O}}\left(\frac{r}{2}\right)^2 = \tfrac{1}{2}M_{\text{O}}r^2$$

Substitute for I to obtain:

$$E_{0r} = \frac{\hbar^2}{2\left(\frac{1}{2}M_{\text{O}}r^2\right)} = \frac{\hbar^2}{M_{\text{O}}r^2} = \frac{\hbar^2}{16m_{\text{p}}r^2}$$

Solve for r:

$$r = \frac{\hbar}{4}\sqrt{\frac{1}{E_{0r}m_{\text{p}}}}$$

Substitute numerical values and evaluate r:

$$r = \left(\frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{4}\right)\sqrt{\frac{1}{\left(1.78\times10^{-4}\text{ eV}\right)\left(1.6\times10^{-19}\text{ J/eV}\right)\left(1.67\times10^{-27}\text{ kg}\right)}}$$
$$= \boxed{0.121\text{nm}}$$

28 ••

Picture the Problem We can use the definition of the reduced mass to show that the reduced mass is smaller than either mass in a diatomic molecule.

Express the reduced mass of a two-body system:

$$\mu = \frac{m_1m_2}{m_1+m_2}$$

Divide the numerator and denominator of this expression by m_2 to obtain:

$$\mu = \frac{m_1}{1+\frac{m_1}{m_2}} \qquad (1)$$

Divide the numerator and denominator of this expression by m_1 to obtain:

$$\mu = \frac{m_2}{1+\frac{m_2}{m_1}} \qquad (2)$$

Because the denominator is greater than 1 in equations (1) and (2):

$$\boxed{\mu < m_1} \text{ and } \boxed{\mu < m_2}$$

(*a*) For H_2, $m_1 = m_2 = 1$ u:

$$\mu_{\text{H}_2} = \frac{(1\text{u})(1\text{u})}{1\text{u}+1\text{u}} = \boxed{0.500\text{u}}$$

(*b*) For N_2, $m_1 = m_2 = 14$ u:

$$\mu_{N_2} = \frac{(14\,\text{u})(14\,\text{u})}{14\,\text{u} + 14\,\text{u}} = \boxed{7.00\,\text{u}}$$

(*c*) For CO, $m_1 = 12$ u and $m_2 = 16$ u:

$$\mu_{CO} = \frac{(12\,\text{u})(16\,\text{u})}{12\,\text{u} + 16\,\text{u}} = \boxed{6.86\,\text{u}}$$

(*d*) For HCl, $m_1 = 1$ u and $m_2 = 35.5$ u:

$$\mu_{HCl} = \frac{(1\,\text{u})(35.5\,\text{u})}{1\,\text{u} + 35.5\,\text{u}} = \boxed{0.973\,\text{u}}$$

29 ••

Picture the Problem We can solve Equation 37-18 for ν and substitute for the frequency of the CO molecule (see Example 37-4) and its binding energy to estimate the quantum number ν.

The vibrational energy levels are given by Equation 37-18:

$$E_\nu = \left(\nu + \tfrac{1}{2}\right)hf$$

where $\nu = 0, 1, 2, \ldots$

Solve for ν:

$$\nu = \frac{E_\nu}{hf} - \frac{1}{2}$$

Substitute numerical values and evaluate ν:

$$\nu = \frac{11\,\text{eV} \times \dfrac{1.60 \times 10^{-19}\,\text{J}}{\text{eV}}}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(6.42 \times 10^{13}\,\text{Hz})} - \frac{1}{2}$$

$$= 40.8 \approx \boxed{41}$$

***30** ••

Picture the Problem We can use the expression for the rotational energy levels of the diatomic molecule to express the energy separation ΔE between the $\ell = 3$ and $\ell = 2$ rotational levels and model the moment of inertia of the LiH molecule as two point objects separated by a distance r_0.

The energy separation between the $\ell = 3$ and $\ell = 2$ rotational levels of this diatomic molecule is given by:

$$\Delta E = E_{\ell=3} - E_{\ell=2}$$

Express the rotational energy levels $E_{\ell=3}$ and $E_{\ell=2}$ in terms of E_{0r}:

$$E_{\ell=3} = 3(3+1)E_{0r} = 12E_{0r}$$

and

$$E_{\ell=2} = 2(2+1)E_{0r} = 6E_{0r}$$

Substitute for $E_{\ell=3}$ and $E_{\ell=2}$ to

$$\Delta E = 12E_{0r} - 6E_{0r} = 6E_{0r}$$

obtain: or

$$E_{0r} = \tfrac{1}{6}\Delta E$$

The characteristic rotational energy of a molecule is given by:

$$E_{0r} = \frac{\hbar^2}{2I} = \tfrac{1}{6}\Delta E \Rightarrow \Delta E = \frac{3\hbar^2}{I}$$

Express the moment of inertia of the molecule:

$$I = \mu r_0^2$$

where μ is the reduced mass of the molecule.

Substitute for I to obtain:

$$\Delta E = \frac{3\hbar^2}{\mu r_0^2} = \frac{3\hbar^2}{\frac{m_{\text{Li}} m_{\text{H}}}{m_{\text{Li}} + m_{\text{H}}} r_0^2} = \frac{3\hbar^2 (m_{\text{Li}} + m_{\text{H}})}{m_{\text{Li}} m_{\text{H}}\, r_0^2}$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \frac{3(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2(6.94\,\text{u} + 1\,\text{u})}{(6.94\,\text{u})(1\,\text{u})(0.16\,\text{nm})^2(1.602\times10^{-19}\ \text{J/eV})(1.660\times10^{-27}\ \text{kg/u})} = \boxed{5.61\,\text{meV}}$$

***31 ••**

Picture the Problem Let the origin of coordinates be at the point mass m_1 and point mass m_2 be at a distance r_0 from the origin. We can express the moment of inertia of a diatomic molecule with respect to its center of mass using the definitions of the center of mass and the moment of inertia of point particles.

Express the moment of inertia of a diatomic molecule:

$$I = m_1 r_1^2 + m_2 r_2^2 \qquad (1)$$

The r coordinate of the center of mass is:

$$r_{\text{CM}} = \frac{m_2}{m_1 + m_2} r_0$$

The distances of m_1 and m_2 from the center of mass are:

$$r_1 = r_{\text{CM}}$$

and

$$r_2 = r_0 - r_{\text{CM}} = r_0 - \frac{m_2}{m_1 + m_2} r_0 = \frac{m_1}{m_1 + m_2} r_0$$

Substitute for r_1 and r_2 in equation (1) to obtain:

$$I = m_1\left(\frac{m_2}{m_1+m_2}r_0\right)^2 + m_2\left(\frac{m_1}{m_1+m_2}r_0\right)^2$$

Simplifying this expression leads to:

$$I = \frac{m_1 m_2}{m_1+m_2}r_0^2$$

or

$$\boxed{I = \mu r_0^2} \qquad 36\text{-}14$$

where

$$\boxed{\mu = \frac{m_1 m_2}{m_1+m_2}} \qquad 36\text{-}15$$

32 ••

Picture the Problem We can relate the characteristic rotational energy E_{0r} to the moment of inertia of the molecule and model the moment of inertia of the KCl molecule as two point objects of reduced mass μ separated by a distance r.

The characteristic rotational energy of a molecule is given by:

$$E_{0r} = \frac{\hbar^2}{2I}$$

Express the moment of inertia of the molecule:

$$I = \mu r_0^2$$

where $\mu = \dfrac{m_K m_{Cl}}{m_K + m_{Cl}}$

Substitute for I to obtain:

$$E_{0r} = \frac{\hbar^2}{2\mu r_0^2} = \frac{\hbar^2(m_K + m_{Cl})}{2m_k m_{Cl} r_0^2}$$

Substitute numerical values and evaluate E_{0r}:

$$E_{0r} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2(39.1\,\text{u}+35.5\,\text{u})}{2(39.1\,\text{u})(35.5\,\text{u})(0.267\,\text{nm})^2(1.660\times10^{-27}\ \text{kg/u})}$$
$$= 2.53\times10^{-24}\ \text{J}\times\frac{1}{1.602\times10^{-19}\ \text{J/eV}} = \boxed{0.0158\,\text{meV}}$$

33 ••

Picture the Problem We can use the expression for the vibrational energies of a molecule to find the lowest vibrational energy. Because the difference in the vibrational energy levels depends on both Δf and the moment of inertia I of the molecule, we can relate these quantities and solve for I. Finally, we can use $I = \mu r^2$, with μ representing the reduced mass of the molecule, to find the equilibrium separation of the atoms.

(*a*) The vibrational energy levels are given by:

$$E_\nu = \left(\nu + \tfrac{1}{2}\right)hf, \quad \nu = 0,1,2,\ldots$$

The lowest vibrational energy corresponds to $\nu = 0$:

$$E_0 = \tfrac{1}{2}hf$$

Substitute numerical values and evaluate E_0:

$$E_0 = \tfrac{1}{2}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.66\times10^{13}\ \text{Hz}\right) = 2.87\times10^{-20}\ \text{J}\times\frac{1}{1.6\times10^{-19}\ \text{J/eV}}$$
$$= \boxed{0.179\,\text{eV}}$$

(*b*) For $\Delta\ell = \pm1$:

$$\Delta E_\ell = \frac{\ell\hbar^2}{I} = \ell h\Delta f$$

Solve for I:

$$I = \frac{\hbar^2}{h\Delta f} = \frac{h^2}{4\pi^2 h\Delta f} = \frac{h}{4\pi^2\Delta f}$$

Substitute numerical values and evaluate I:

$$I = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi^2\left(6\times10^{11}\ \text{Hz}\right)} = \boxed{2.80\times10^{-47}\ \text{kg}\cdot\text{m}^2}$$

(*c*) The moment of inertia of a HCL molecule is given by:

$$I = \mu r^2$$

Replace μ by the reduced mass of a HCl molecule and r by r_0 to obtain:

$$I = \frac{m_\text{H}m_\text{Cl}}{m_\text{H} + m_\text{Cl}}r_0^2$$

Solve for r_0:

$$r_0 = \sqrt{\frac{m_\text{H} + m_\text{Cl}}{m_\text{H}m_\text{Cl}}I}$$

Substitute numerical values and evaluate r_0:

$$r_0 = \sqrt{\left[\frac{1\,\text{u} + 34.453\,\text{u}}{(1\,\text{u})(34.453\,\text{u})}\right]\frac{\left(2.80\times10^{-47}\ \text{kg}\cdot\text{m}^2\right)}{1.66\times10^{-27}\ \text{kg/u}}} = \boxed{0.132\,\text{nm}}$$

34 ••

Picture the Problem Let the numeral 1 refer to the H^+ and the numeral 2 to the Cl^- ion. For a two-mass and spring system on which no external forces are acting, the center of mass must remain fixed. We can use this condition to express the net force acting on either the H^+ or Cl^- ion. Because this force is a linear restoring force, we can conclude that the motion of the object whose mass is m_1 will be simple harmonic with an angular frequency given by $\omega = \sqrt{K_{\text{eff}}/m_1}$. Substitution for K_{eff} will lead us to the result given in (*b*).

If the particle whose mass is m_1 moves a distance r_1 from (or toward) the center of mass, then the particle whose mass is m_2 must move a distance:

$\Delta r_2 = \frac{m_1}{m_2}\Delta r_1$ from (or toward) the center of mass.

Express the force exerted by the spring:

$$F = -K\Delta r = -K(\Delta r_1 + \Delta r_2)$$

Substitute for Δr_2 to obtain:

$$F = -K\left(\Delta r_1 + \frac{m_1}{m_2}\Delta r_1\right) = -K\left(\frac{m_1+m_2}{m_2}\right)\Delta r_1$$

A displacement Δr_1 of m_1 results in a restoring force:

$$F = -K\left(\frac{m_1+m_2}{m_2}\right)\Delta r_1 = -K_{\text{eff}}\Delta r_1$$

where $K_{\text{eff}} = K\left(\frac{m_1+m_2}{m_2}\right)$

Because this is a linear restoring force, we know that the motion will be simple harmonic with:

$$\omega = \sqrt{\frac{K_{\text{eff}}}{m_1}}$$

or

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K_{\text{eff}}}{m_1}}$$

Substitute for K_{eff} and simplify to obtain:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{K\left(\frac{m_1+m_2}{m_1 m_2}\right)}$$

or, because $\mu = \frac{m_1 m_2}{m_1+m_2}$ is the reduced mass of the two-particle system,

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$$

Solve for K:

$$K = 4\pi^2 f^2 \mu = 4\pi^2 f^2 \frac{m_{\mathrm{H}} m_{\mathrm{Cl}}}{m_{\mathrm{H}} + m_{\mathrm{Cl}}}$$

Substitute numerical values and evaluate K:

$$K = \frac{4\pi^2 (8.66\times10^{13}\ \mathrm{Hz})^2 (1\,\mathrm{u})(35.453\,\mathrm{u})(1.66\times10^{-27}\ \mathrm{kg/u})}{(1\,\mathrm{u} + 35.453\,\mathrm{u})} = \boxed{478\,\mathrm{N/m}}$$

35 ••

Picture the Problem

We're given the population of rotational states function:

$$f(\ell) = (2\ell + 1)e^{-E_l/kT}$$

where

$$E_\ell = \ell(\ell+1)E_{0r} \text{ and } E_{0\mathrm{r}} = \frac{\hbar^2}{2I}$$

The moment of inertia I of an oxygen molecule is given by:

$$I = \tfrac{1}{2} m r_0^2$$

where m is the reduced mass and r_0 is the separation of the atoms in a molecule.

We'll assume, as in Example 37-3, that:

$$r_0 = 0.1\,\mathrm{nm}$$

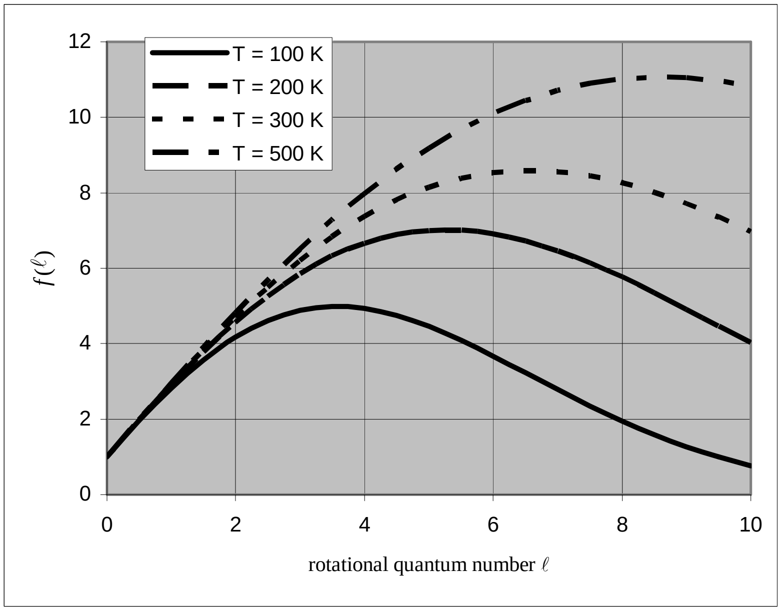
A spreadsheet program to plot $f(\ell)$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.00E–10	r_0
B2	16	m (u)
B3	2.66E–26	m (kg)
B4	1.05E–34	$\hbar$
B5	1.38E–23	k
B6	4.15E–23	$E_{0\mathrm{r}}$
B7	100	T (K)
B8	200	T (K)
B9	300	T (K)
B10	500	T (K)
A13	0	ℓ

B13	A13*(A13+1)*$B$6	$\ell(\ell+1)E_{0r}$
C13	(2*A13 + 1)*EXP(–B13/($B$5*$B$7))	$f(\ell, T = 100\text{ K})$
D13	(2*A13 + 1)*EXP(–B13/($B$5*$B$8))	$f(\ell, T = 200\text{ K})$
E13	(2*A13 + 1)*EXP(–B13/($B$5*$B$9))	$f(\ell, T = 300\text{ K})$
F13	(2*A13 + 1)*EXP(–B13/($B$5*$B$10))	$f(\ell, T = 500\text{ K})$

	A	B	C	D	E	F
1	r_0=	1.00E–10	m			
2	m=	16	u			
3	m=	2.656E–26	kg			
4	h_bar=	1.05E–34	J.s			
5	k=	1.38E–23	J/K			
6	E_0r=	4.15E–23	eV			
7	T=	100	K			
8	T=	200	K			
9	T=	300	K			
10	T=	500	K			
11						
12	l	E_l	E_100 K	E_200 K	E_300 K	E_500 K
13	0.0	0.00E+00	1.00	1.00	1.00	1.00
14	0.5	3.11E–23	1.96	1.98	1.99	1.99
15	1.0	8.30E–23	2.82	2.91	2.94	2.96
16	1.5	1.56E–22	3.57	3.78	3.85	3.91
17	2.0	2.49E–22	4.17	4.57	4.71	4.82
29	8.0	2.99E–21	1.95	5.76	8.26	11.02
30	8.5	3.35E–21	1.59	5.34	8.01	11.07
31	9.0	3.74E–21	1.27	4.91	7.71	11.06
32	9.5	4.14E–21	1.00	4.46	7.36	10.98
33	10.0	4.57E–21	0.77	4.02	6.97	10.83

The following graph shows $f(\ell)$ as a function of temperature.



***36** ••

Picture the Problem For a two-mass and spring system on which no external forces are acting, the center of mass must remain fixed. We can use this condition to express the net force acting on either object. Because this force is a linear restoring force, we can conclude that the motion of the object whose mass is m_1 will be simple harmonic with an angular frequency given by $\omega = \sqrt{\frac{k_{\text{eff}}}{m_1}}$. Substitution for k_{eff} will lead us to the result given in (*b*).

(*a*) If the particle whose mass is m_1 moves a distance Δr_1 from (or toward) the center of mass, then the particle whose mass is m_2 must move a distance:

$\Delta r_2 = \frac{m_1}{m_2}\Delta r_1$ from (or toward) the center of mass.

Express the force exerted by the spring:

$$F = -k\Delta r = -k(\Delta r_1 + \Delta r_2)$$

Substitute for Δr_2 to obtain:

$$F = -k\left(\Delta r_1 + \frac{m_1}{m_2}\Delta r_1\right) = \boxed{-k\left(\frac{m_1 + m_2}{m_2}\right)\Delta r_1}$$

(*b*) A displacement Δr_1 of m_1 results in a restoring force:

$$F = -k\left(\frac{m_1 + m_2}{m_2}\right)\Delta r_1 = -k_{\text{eff}}\Delta r_1$$

where $k_{\text{eff}} = k\left(\frac{m_1 + m_2}{m_2}\right)$

Because this is a linear restoring force, we know that the motion will be simple harmonic with:

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m_1}}$$

or

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m_1}}$$

Substitute for k_{eff} and simplify to obtain:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k\left(\frac{m_1 + m_2}{m_1 m_2}\right)}$$

or, because $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the two-particle system,

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}}.$$

37 •••

Picture the Problem We can use the definition of the reduced mass to find the reduced mass for the $H^{35}Cl$ and $H^{37}Cl$ molecules and the fractional difference $\Delta\mu/\mu$. Because the rotational frequency is proportional to $1/I$, where I is the moment of inertia of the system, and I is proportional to μ, we can obtain an expression for f as a function of μ that we differentiate implicitly to show that $\Delta f/f = -\Delta\mu/\mu$.

For $H^{35}Cl$:

$$\mu = \frac{(35\,\text{u})(1\,\text{u})}{35\,\text{u} + 1\,\text{u}} = \frac{35}{36}\,\text{u} = \boxed{0.9722\,\text{u}}$$

For $H^{37}Cl$:

$$\mu = \frac{(37\,\text{u})(1\,\text{u})}{37\,\text{u} + 1\,\text{u}} = \frac{37}{38}\,\text{u} = \boxed{0.9737\,\text{u}}$$

The fractional difference is:

$$\frac{\Delta\mu}{\mu} = \frac{\frac{37}{38}\,\text{u} - \frac{35}{36}\,\text{u}}{\frac{1}{2}\left(\frac{35}{36}\,\text{u} + \frac{37}{38}\,\text{u}\right)} = \frac{\frac{36\times37 - 35\times38}{36\times38}\,\text{u}}{\frac{35\times38 + 36\times37}{2(36)(38)}\,\text{u}} = \boxed{0.00150}$$

The rotational frequency is proportional to $1/I$, where I is the moment of inertia of the system. Because I is proportional to μ:

$$f = \frac{C}{\mu}$$

and

$$df = -C\mu^{-2}d\mu$$

Divide df by f to obtain:

$$\frac{df}{f} = -\frac{d\mu}{\mu} \text{ and } \frac{\Delta f}{f} \approx -\frac{\Delta\mu}{\mu}$$

From Figure 36-17:

$$\Delta f \approx 0.01\times10^{13}\text{ Hz} = 10^{11}\text{ Hz}$$

For $f = 8.40\times10^{13}$ Hz:

$$\frac{\Delta f}{f} \approx \frac{10^{11}\text{ Hz}}{8.40\times10^{13}\text{ Hz}} = \boxed{0.00119}$$

This result is in fair agreement (about 21% difference) with the calculated result. Note that Δf is difficult to determine precisely from Figure 36-17.

General Problems

38 •

Picture the Problem We can use the definition of the reduced mass to show that when one atom in a diatomic molecule is much more massive than the other the reduced mass is approximately equal to the mass of the lighter atom.

Express the reduced mass of a two-body system:

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Divide the numerator and denominator of this expression by m_2 to obtain:

$$\mu = \frac{m_1}{1 + \frac{m_1}{m_2}}$$

If $m_2 >> m_1$, then:

$$\frac{m_1}{m_2} << 1 \text{ and } \boxed{\mu \approx m_1}$$

39 ••

Picture the Problem The rotational energy levels are given by

$$E = \frac{\ell(\ell+1)\hbar^2}{2I}, \ell = 0, 1, 2, \ldots.$$

Express the energy difference between these rotational energy

$$\Delta E_{1,2} = E_2 - E_1$$

levels:

Express E_2 and E_1:

$$E_2 = \frac{2(2+1)\hbar^2}{2I} = \frac{3\hbar^2}{I}$$

and

$$E_1 = \frac{1(1+1)\hbar^2}{2I} = \frac{\hbar^2}{I}$$

Substitute to obtain:

$$\Delta E_{1,2} = \frac{3\hbar^2}{I} - \frac{\hbar^2}{I} = \frac{2\hbar^2}{I}$$

The moment of inertia of the molecule is:

$$I = \mu r_0^2$$

where μ is the reduced mass of the molecule.

Substitute for I to obtain:

$$\Delta E_{1,2} = \frac{2\hbar^2}{\mu r_0^2} = \frac{2\hbar^2}{\dfrac{m_C m_O}{m_C + m_O} r_0^2} = \frac{2\hbar^2 (m_C + m_O)}{m_C m_O r_0^2}$$

Substitute numerical values and evaluate $\Delta E_{1,2}$:

$$\Delta E_{1,2} = \frac{2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2(12\text{ u} + 16\text{ u})}{(16\text{ u})(12\text{ u})(0.113\text{ nm})^2(1.66\times10^{-27}\text{ kg/u})} = \frac{1.53\times10^{-22}\text{ J}}{1.6\times10^{-19}\text{ J/eV}} = \boxed{0.955\text{ meV}}$$

***40 ••**

Picture the Problem We can use the result of Problem 36 to find the frequency of vibration of the HF molecule.

In Problem 36 it was established that:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

The reduced mass is:

$$\mu = \frac{m_H m_F}{m_H + m_F}$$

Substitute for μ to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{m_H m_F}{m_H + m_F}}} = \frac{1}{2\pi}\sqrt{\frac{k(m_H + m_F)}{m_H m_F}}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{1}{2\pi}\sqrt{\frac{(970\,\text{N/m})(1\,\text{u} + 19\,\text{u})}{(1\,\text{u})(19\,\text{u})(1.66\times10^{-27}\,\text{kg/u})}} = \boxed{1.25\times10^{14}\,\text{Hz}}$$

41 ••

Picture the Problem We can use the result of Problem 36 to find the effective force constant for NO.

In Problem 36 it was established that:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

Solve for *k*:

$$k = 4\pi^2 f^2 \mu$$

The reduced mass is:

$$\mu = \frac{m_N m_O}{m_N + m_O}$$

Substitute for μ to obtain:

$$k = \frac{4\pi^2 f^2 m_N m_O}{m_N + m_O}$$

Substitute numerical values and evaluate *k*:

$$k = \frac{4\pi^2 (5.63\times10^{13}\,\text{s}^{-1})^2 (14\,\text{u})(16\,\text{u})(1.66\times10^{-27}\,\text{kg/u})}{14\,\text{u} + 16\,\text{u}} = \boxed{1.55\,\text{kN/m}}$$

42 ••

Picture the Problem We can use the expression for the vibrational energy levels of a molecule and the expression for the frequency of oscillation from Problem 36 to find the four lowest vibrational levels of the given molecules.

The vibrational energy levels are given by:

$$E_\nu = (\nu + \tfrac{1}{2})hf, \nu = 0, 1, 2, \ldots$$

In Problem 36 we showed that the frequency of oscillation is:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

where

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Substitute for f and μ to obtain:

$$E_\nu = \frac{\left(\nu + \frac{1}{2}\right)h}{2\pi}\sqrt{\frac{(m_1 + m_2)k}{m_1 m_2}} = \frac{\left(\nu + \frac{1}{2}\right)h}{2\pi}\sqrt{k}\sqrt{\frac{(m_1 + m_2)}{m_1 m_2}}$$

Substitute numerical values to obtain:

$$E_\nu = \frac{\left(\nu + \frac{1}{2}\right)\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)}{2\pi}\sqrt{\frac{580\ \text{N/m}}{1.661\times10^{-27}\ \text{kg/u}}}\sqrt{\frac{(m_1 + m_2)}{m_1 m_2}} = \left(\nu + \tfrac{1}{2}\right)(0.389\,\text{eV}\cdot\text{u})\sqrt{\frac{(m_1 + m_2)}{m_1 m_2}}$$

Substitute for m_1 and m_2 and evaluate E_0 for H_2:

$$E_0 = \tfrac{1}{2}(0.389\,\text{eV}\cdot\text{u})\sqrt{\frac{(1\text{u} + 1\text{u})}{(1\text{u})(1\text{u})}} = 0.275\,\text{eV}$$

Proceed similarly to complete the table to the right:

	H_2	HD	D_2
	(eV)	(eV)	(eV)
0	0.275	0.238	0.195
1	0.825	0.715	0.584
2	1.375	1.191	0.973
3	1.925	1.667	1.362

The energies of the photons resulting from transitions between adjacent vibrational levels of these molecules are given by:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Evaluate $\lambda(H_2)$:

$$\lambda(H_2) = \frac{1240\,\text{eV}\cdot\text{nm}}{0.550\,\text{eV}} = \boxed{2.25\,\mu\text{m}}$$

Evaluate $\lambda(HD)$:

$$\lambda(HD) = \frac{1240\,\text{eV}\cdot\text{nm}}{0.477\,\text{eV}} = \boxed{2.60\,\mu\text{m}}$$

Evaluate $\lambda(D_2)$:

$$\lambda(D_2) = \frac{1240\,\text{eV}\cdot\text{nm}}{0.389\,\text{eV}} = \boxed{3.19\,\mu\text{m}}$$

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Picture the Problem We can set the derivative of the potential energy function equal to zero to find the value of r for which it is either a maximum or a minimum. Examination of the second derivative of this function at the value for r obtained from setting the first derivative equal to zero will establish whether the function is a relative maximum or relative minimum at this point.

Differentiate the potential energy function with respect to r:

$$\frac{dU}{dr} = \frac{d}{dr}\left\{U_0\left[\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6}\right]\right\} = -\frac{U_0}{r}\left[12\left(\frac{a}{r}\right)^{11} - 12\left(\frac{a}{r}\right)^{5}\right]$$

Set the derivative equal to zero:

$$\frac{dU}{dr} = -\frac{U_0}{r_0}\left[12\left(\frac{a}{r_0}\right)^{11} - 12\left(\frac{a}{r_0}\right)^{5}\right] = 0 \text{ for extrema}$$

Solve for r_0 to obtain, as our candidate for r that minimizes the Lenard-Jones potential:

$$r_0 = \boxed{a}$$

To show that $r_0 = a$ corresponds to a minimum, differentiate U a second time to obtain:

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left\{-\frac{U_0}{r}\left[12\left(\frac{a}{r}\right)^{11} - 12\left(\frac{a}{r}\right)^{5}\right]\right\} = \frac{U_0}{r^2}\left[132\left(\frac{a}{r}\right)^{10} - 60\left(\frac{a}{r}\right)^{4}\right]$$

Evaluate this second derivative of the potential at $r_0 = a$:

$$\left.\frac{d^2U}{dr^2}\right|_{r=a} = \frac{U_0}{a^2}[132 - 60] = \frac{72U_0}{a^2} > 0$$

Therefore, we can conclude that $r_0 = a$

minimizes the potential function.

Evaluate U_{min}:

$$U_{min} = U(a) = U_0\left[\left(\frac{a}{a}\right)^{12} - 2\left(\frac{a}{a}\right)^{6}\right] = \boxed{-U_0}$$

From Figure 37-4:

$$r_0 = \boxed{0.074\,\text{nm}}$$

and

$$U_0 = \boxed{4.52\,\text{eV}}$$

44 ••

Picture the Problem We can use Equation 21-10 to establish the dependence of E on x and the dependence of an induced dipole on the field that induces it to establish the dependence of p and U on x.

(*a*) In terms of the dipole moment, the electric field on the axis of the dipole at a point a great distance $|x|$ away has the magnitude (see Equation 21-10):

$$E = \frac{2kp}{|x|^3}$$

or

$$E \propto \boxed{\frac{1}{|x|^3}}$$

(*b*) Because the induced dipole moment is proportional to the field that induces it:

$$p \propto \boxed{\frac{1}{x^3}}$$

and

$$U = -\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{E}} \propto \boxed{\frac{1}{x^6}}$$

(*c*) Differentiate U with respect to x to obtain:

$$F_x = -\frac{dU}{dx} \propto \boxed{\frac{1}{x^7}}$$

45 ••

Picture the Problem the case of two polar molecules, p does not depend on the field E.

Because p does not depend on the electric field in which the polar molecules find themselves:

$$U \propto \frac{1}{x^3}$$

Differentiate U with respect to x to obtain:

$$F_x = -\frac{dU}{dx} \propto \boxed{\frac{1}{x^4}}$$

46 ••

Picture the Problem We can use the expression for the vibrational and rotational energies of a molecule, in conjunction with Figure 37-17 to find E_{0r}, f, and hf.

(a) Except for a gap of $4E_{0r}/h$ at the vibrational frequency f, the absorption spectrum contains frequencies equally spaced at:

$$f = \frac{2E_{0r}}{h}$$

Solve for E_{0r}:

$$E_{0r} = \tfrac{1}{2}hf$$

From Figure 37-17:

$$f = 8.66\times10^{13}\ \text{Hz}$$

Substitute numerical values and evaluate E_{0r}:

$$E_{0r} = \tfrac{1}{2}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.66\times10^{13}\ \text{Hz}\right) = 2.87\times10^{-20}\,\text{J}\times\frac{1}{1.6\times10^{-19}\ \text{J/eV}}$$
$$= \boxed{0.179\,\text{eV}}$$

(b) The vibrational energy levels are given by:

$$E_\nu = \left(\nu + \tfrac{1}{2}\right)hf, \quad \nu = 0, 1, 2, \ldots$$

The lowest vibrational energy corresponds to $\nu = 0$:

$$E_0 = \tfrac{1}{2}hf$$
and
$$hf = 2E_0 \qquad (1)$$

Determine f from Figure 37-17:

$$f = \boxed{8.66\times10^{13}\ \text{Hz}}$$

Substitute for f and h and evaluate E_0:

$$E_0 = \tfrac{1}{2}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.66\times10^{13}\ \text{Hz}\right) = 2.87\times10^{-20}\,\text{J}\times\frac{1}{1.6\times10^{-19}\ \text{J/eV}}$$
$$= 0.179\ \text{eV}$$

Substitute in equation (1) and evaluate hf:

$$hf = 2\left(0.179\,\text{eV}\right) = \boxed{0.358\,\text{eV}}$$

***47 ••**

Picture the Problem We can find the reduced mass of CO and the moment of inertia of a CO molecule from their definitions. The energy level diagram for the rotational levels for $\ell = 0$ to $\ell = 5$ can be found using $\Delta E_{\ell,\ell-1} = 2\ell E_{0r}$. Finally, we can find the wavelength of the photons emitted for each transition using $\lambda_{\ell,\ell-1} = \dfrac{hc}{\Delta E_{\ell,\ell-1}} = \dfrac{hc}{2\ell \Delta E_{0r}}$.

(*a*) Express the moment of inertia of CO:

$$I = \mu r_0^2$$

where μ is the reduced mass of the CO molecule.

Find μ:

$$\mu = \frac{m_C m_O}{m_C + m_O} = \frac{(12\,\text{u})(16\,\text{u})}{12\,\text{u} + 16\,\text{u}} = 6.86\,\text{u}$$

In Problem 39 it was established that r_0 = 0.113 nm. Use this result to evaluate I:

$$I = (6.86\,\text{u})(1.66\times10^{-27}\ \text{kg/u})(0.113\,\text{nm})^2 = \boxed{1.45\times10^{-46}\ \text{kg}\cdot\text{m}^2}$$

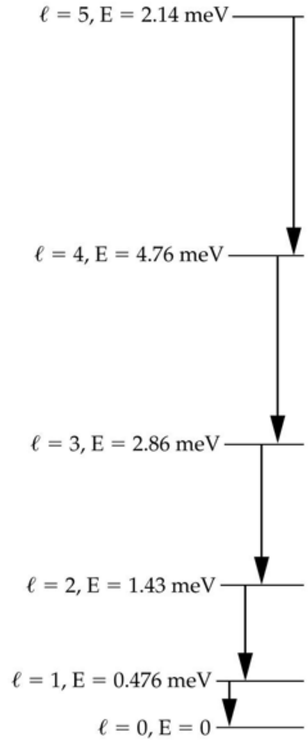
The characteristic rotational energy E_{0r} is given by:

$$E_{0r} = \frac{\hbar^2}{2I}$$

Substitute numerical values and evaluate E_{0r}:

$$E_{0r} = \frac{(6.58\times10^{-16}\ \text{eV}\cdot\text{s})^2(1.6\times10^{-19}\ \text{J/eV})}{2(1.45\times10^{-46}\ \text{kg}\cdot\text{m}^2)} = \boxed{0.239\,\text{meV}}$$

(*b*) The energy level diagram is shown to the right. Note that $\Delta E_{\ell,\ell-1}$, the energy difference between adjacent levels for $\Delta\ell = -1$, is $\Delta E_{\ell,\ell-1} = 2\ell E_{0r}$.



(*c*) Express the energy difference $\Delta E_{\ell.\ell-1}$ between energy levels in terms of the frequency of the emitted radiation:

$$\Delta E_{\ell,\ell-1} = hf_{\ell,\ell-1}$$

Because $c = f_{\ell,\ell-1}\lambda_{\ell,\ell-1}$:

$$\lambda_{\ell,\ell-1} = \frac{hc}{\Delta E_{\ell,\ell-1}} = \frac{hc}{2\ell\Delta E_{0r}}$$

Substitute numerical values to obtain:

$$\lambda_{\ell,\ell-1} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3\times10^{8}\ \text{m/s})}{2\ell(0.239\,\text{meV})} = \frac{2596\,\mu\text{m}}{\ell}$$

For $\ell = 1$:

$$\lambda_{1,0} = \frac{2596\,\mu\text{m}}{1} = \boxed{2596\,\mu\text{m}}$$

For $\ell = 2$:

$$\lambda_{2,1} = \frac{2596\,\mu\text{m}}{2} = \boxed{1298\,\mu\text{m}}$$

For $\ell = 3$:

$$\lambda_{3,2} = \frac{2596\,\mu\text{m}}{3} = \boxed{865\,\mu\text{m}}$$

For $\ell = 4$: $$\lambda_{4,3} = \frac{2596\,\mu\text{m}}{4} = \boxed{649\,\mu\text{m}}$$

For $\ell = 5$: $$\lambda_{5,4} = \frac{2596\,\mu\text{m}}{5} = \boxed{519\,\mu\text{m}}$$

These wavelengths fall in the microwave region of the spectrum.

***48 •••**

Picture the Problem The wavelength resulting from transitions between adjacent harmonic oscillator levels of a LiCl molecule is given by $\lambda = \frac{2\pi c}{\omega}$. We can find an expression for ω by following the procedure outlined in the problem statement.

The wavelength resulting from transitions between adjacent harmonic oscillator levels of this molecule is given by:

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{\hbar\omega} = \frac{2\pi c}{\omega} \qquad (1)$$

From Problem 24 we have:

$$U(r) = -\frac{ke^2}{r} + \frac{C}{r^n}\text{, where } \Delta E \text{ is constant.}$$

The Taylor expansion of $U(r)$ about $r = r_0$ is:

$$U(r) = U(r_0) + \left(\frac{dU}{dr}\right)_{r_0}(r - r_0) + \frac{1}{2}\left(\frac{d^2U}{dr^2}\right)_{r_0}(r - r_0)^2 + \ldots$$

Because $U(r_0)$ is a constant, it can be dropped without affecting the physical results and because $\left(\frac{dU}{dr}\right)_{r_0} = 0$:

$$U(r) \approx \frac{1}{2}\left(\frac{d^2U}{dr^2}\right)_{r_0}(r - r_0)^2 \qquad (2)$$

Differentiate $U(r)$ twice to obtain:

$$\frac{d^2U}{dr^2} = -2\frac{ke^2}{r^3} + n(n-1)\frac{C}{r^{n+2}}$$

Because $dU/dr = F_{\text{net}} = 0$ at $r = r_0$:

$$0 = -\frac{nC}{r_0^{n+1}} + \frac{ke^2}{r_0^2}$$

Solving for C yields:

$$C = \frac{ke^2 r_0^{n+1}}{n r_0^2} = \frac{ke^2 r_0^{n-1}}{n}$$

Substitute for C and evaluate $\left(\frac{d^2U}{dr^2}\right)_{r_0}$ to obtain:

$$\left(\frac{d^2U}{dr^2}\right)_{r_0} = -2\frac{ke^2}{r_0^3}(n-1) + \frac{n(n-1)}{r_0^{n+2}}\frac{ke^2 r_0^{n-1}}{n}$$
$$= \frac{ke^2}{r_0^3}(n-1)$$

Substitute for $\left(\frac{d^2U}{dr^2}\right)_{r_0}$ in equation (2):

$$U(r) \approx \frac{1}{2}\left[\frac{ke^2}{r_0^3}(n-1)\right](r-r_0)^2$$

Because the potential energy of a simple harmonic oscillator is given by $U_{\text{SHO}} = \frac{1}{2}m\omega^2(r-r_0)^2$:

$$\frac{1}{2}m\omega^2(r-r_0)^2 \approx \frac{1}{2}\left[\frac{ke^2}{r_0^3}(n-1)\right](r-r_0)^2$$

Solve for ω to obtain:

$$\omega \approx \sqrt{\frac{(n-1)ke^2}{mr_0^3}}$$

Substitute μ_{LiCl} for m to obtain:

$$\omega \approx \sqrt{\frac{(n-1)ke^2}{\frac{m_{\text{Li}}m_{\text{Cl}}}{m_{\text{Li}}+m_{\text{Cl}}}r_0^3}} = \sqrt{\frac{(n-1)(m_{\text{Li}}+m_{\text{Cl}})ke^2}{m_{\text{Li}}m_{\text{Cl}}r_0^3}} \quad (3)$$

From Problem 24:

$$n = \frac{|U_{\text{e}}(r_0)|}{U_{\text{rep}}(r_0)} \quad (4)$$

U_{rep} is related to U_{e}, E_{d}, and ΔE according to:

$$U_{\text{rep}} = -(U_{\text{e}} + E_{\text{d}} + \Delta E) \quad (5)$$

The energy needed to form Li^+ and Cl^- from neutral lithium and chlorine atoms is:

$$\Delta E = E_{\text{ionization}} - E_{\text{electron affinity}}$$
$$= 5.39\,\text{eV} - 3.62\,\text{eV} = 1.77\,\text{eV}$$

$U_{\text{e}}(r_0)$ is given by:

$$U_{\text{e}} = -\frac{ke^2}{r_0} = -\frac{1.44\,\text{eV}\cdot\text{nm}}{r_0}$$

Substitute r_0 and evaluate U_e:

$$U_e = -\frac{1.44\,\text{eV}\cdot\text{nm}}{0.202\,\text{nm}} = -7.13\,\text{eV}$$

Substitute numerical values in equation (5) and evaluate U_{rep}:

$$U_{rep} = -\left(-7.13\,\text{eV} + 4.86\,\text{eV} + 1.77\,\text{eV}\right) = 0.500\,\text{eV}$$

Substitute for $U_{rep}(r_0)$ and $U_e(r_0)$ in equation (4) and evaluate n:

$$n = \frac{\left|-7.13\,\text{eV}\right|}{0.500\,\text{eV}} = 14.3$$

Substitute numerical values in equation (3) and evaluate ω:

$$\omega \approx \sqrt{\frac{(14.3-1)(6.941\,\text{u} + 35.453\,\text{u})(1.44\,\text{eV}\cdot\text{nm})\left(1.60\times10^{-19}\ \text{J/eV}\right)}{(6.941\,\text{u})(35.453\,\text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)(0.202\,\text{nm})^3}} = \boxed{1.96\times10^{14}\ \text{s}^{-1}}$$

Substitute numerical values in equation (1) and evaluate λ:

$$\lambda = \frac{2\pi\left(3\times10^{8}\ \text{m/s}\right)}{1.96\times10^{14}\ \text{s}^{-1}} = \boxed{9.62\ \mu\text{m}}$$

Chapter 38
Solids and the Theory of Conduction

Conceptual Problems

1 •

Determine the Concept The energy lost by the electrons in collision with the ions of the crystal lattice appears as Joule heat (I^2R).

***2** •

Determine the Concept The resistivity of brass at 4 K is almost entirely due to the ″residual resistance,″ the resistance due to impurities and other imperfections of the crystal lattice. In brass, the zinc ions act as impurities in copper. In pure copper, the resistivity at 4 K is due to its residual resistance, which is very low if the copper is very pure.

3 •

Picture the Problem The contact potential is given by $V_{\text{contact}} = \dfrac{\phi_1 - \phi_2}{e}$, where ϕ_1 and ϕ_2 are the work functions of the two different metals in contact with each other.

(*a*) Express the contact potential in terms of the work functions of the metals:

$$V_{\text{contact}} = \frac{\phi_1 - \phi_2}{e}$$

Examining Table 38-2, we see that the greatest difference between the work functions will occur when potassium and nickel are joined.

(*b*) Substitute numerical values and evaluate V_{contact}:

$$V_{\text{contact}} = \frac{(5.2\,\text{eV} - 2.1\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{3.10\,\text{V}}$$

4 •

Picture the Problem The contact potential is given by $V_{\text{contact}} = \dfrac{\phi_1 - \phi_2}{e}$, where ϕ_1 and ϕ_2 are the work functions of the two different metals in contact with each other.

(*a*) Express the contact potential in terms of the work functions of the metals:

$$V_{\text{contact}} = \frac{\phi_1 - \phi_2}{e}$$

Examining Table 38 - 2, we see that the least difference between the work functions will occur when silver and gold are joined.

(*b*) Substitute numerical values and evaluate V_{contact}:

$$V_{\text{contact}} = \frac{(4.8\,\text{eV} - 4.7\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{0.100\,\text{V}}$$

5 •

Determine the Concept If the valence band is only partially full, there are many available empty energy states in the band, and the electrons in the band can easily be raised to a higher energy state by an electric field. $\boxed{(c)\ \text{is correct.}}$

6 •

Determine the Concept Insulators are poor conductors of electricity because there is a large energy gap between the full valence band and the next higher band where electrons can exist. $\boxed{(b)\ \text{is correct.}}$

7 •

(*a*) True

(*b*) False. The classical free-electron theory predicts heat capacities for metals that are not observed experimentally.

(*c*) True

(*d*) False. The Fermi energy is the energy of the last filled (or half-filled) level at $T = 0$.

(*e*) True

(*f*) True

(*g*) False. Because semiconductors conduct current by electrons and holes, their conduction is in both directions.

***8** •

Determine the Concept The resistivity of copper increases with increasing temperature; the resistivity of (pure) silicon decreases with increasing temperature because the number of charge carriers increases.

9 •

Determine the Concept Because a gallium atom can accept electrons from the valence band of germanium to complete its four covalent bonds, $\boxed{(b)\ \text{is correct.}}$

10 •

Determine the Concept Because phosphorus has 3 electrons that it can donate to the conduction band of germanium without leaving holes in the valence band, $\boxed{(d)\,\text{is correct.}}$

11 •

Determine the Concept The excited electron is in the conduction band and can conduct electricity. A hole is left in the valence band allowing the positive hole to move through the band also contributing to the current.

12 •

Determine the Concept

(a) Phosphorus and antimony will make n-type semiconductors since each has one more valence electron than silicon.

(b) Boron and thallium will make p-type semiconductors since each has one less valence electron than silicon.

13 •

Determine the Concept Because a *pn* junction solar cell has donor impurities on one side and acceptor impurities on the other, both electrons and holes are created. $\boxed{(c)\,\text{is correct.}}$

Estimation and Approximation

14 •

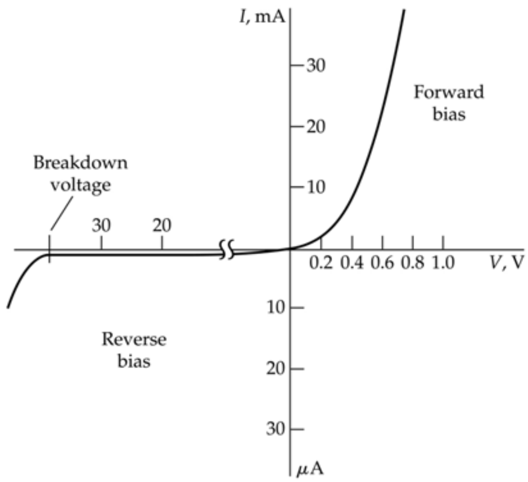
Picture the Problem We can use the list of tables on the inside back covers of volumes 1 and 2 to find tables of material properties. A representative sample is included in the following table in which all the units are SI:

Table	Material property	Largest value	Smallest value	Ratio (order of magnitude)
13-1	Mass density	22.5×10^{3} (Osmium)	0.08994 (Hydrogen)	10^{5}
20-3	Thermal conductivity	429 (Ag)	0.026 (air)	10^{4}
20-1	Thermal expansion	51×10^{-6} (ice)	10^{-6} (invar)	10^{2}
12-8	Tensile strength	520 (steel)	2 (concrete)	10^{2}

12-8	Young's modulus	200 (steel)	9 (bone)	10
18-1	Heat capacity	4.18 (water)	0.900 (Al)	1

15 •

Picture the Problem Figure 38-21 is reproduced below. We can draw tangent lines at each of the voltages and estimate the slope. The differential resistance is the reciprocal of the slope.



V (V)	1/slope (Ω)
–20	$\boxed{\infty}$
+0.2	$\boxed{40}$
+0.4	$\boxed{20}$
+0.6	$\boxed{10}$
+0.8	$\boxed{5}$

Remarks: Note that, because of the difficulty in determining the slopes, these results are only approximations.

The Structure of Solids

16 •

Picture the Problem We can use the geometry of the ion to relate the volume per mole to the length of its side r_0 and the definition of density to express the volume per mole in terms of its molar mass and density.

Because the cube length/ion is r_0, the volume/mole is given by:

$$V_{mol} = 2N_A r_0^3$$

Solve for r_0:

$$r_0 = \sqrt[3]{\frac{V_{mol}}{2N_A}}$$

The volume/mole is given by:

$$V_{mol} = \frac{M}{\rho}$$

where M is the molar mass of KCl.

Substitute for V_{mol} in the expression for r_0:

$$r_0 = \sqrt[3]{\frac{M}{2\rho N_A}}$$

Substitute numerical values and evaluate r_0:

$$r_0 = \sqrt[3]{\frac{74.55\,\text{g/mol}}{2(1.984\,\text{g/cm}^3)(6.02\times10^{23}\,\text{particles/mol})}} = \boxed{0.315\,\text{nm}}$$

17 •

Picture the Problem We can use the definition of density and the geometry of the ions to compute the density of LiCl.

The density of LiCl is given by:

$$\rho = \frac{M_{\text{unit cell}}}{V_{\text{unit cell}}}$$

Express the volume of the unit cell:

$$V_{\text{unit cell}} = (2r_0)^3$$

Because the unit cell has four molecules, its mass is given by:

$$M_{\text{unit cell}} = \frac{4M}{N_A}$$

Substitute for $V_{\text{unit cell}}$ and $M_{\text{unit cell}}$ to obtain:

$$\rho = \frac{\frac{4M}{N_A}}{(2r_0)^3} = \frac{4M}{N_A(2r_0)^3}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{4(42.4\,\text{g/mol})}{(6.02\times10^{23}\,\text{particles/mol})[2(0.257\times10^{-9}\,\text{m})]^3} = 2.07\times10^{6}\,\text{g/m}^3\times\left(\frac{1\,\text{m}}{10^2\,\text{cm}}\right)^3$$
$$= \boxed{2.07\,\text{g/cm}^3}$$

***18** •

Picture the Problem We can solve Equation 38-6 for *n*.

Equation 38-6 is:

$$U(r_0) = -\alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

and

$$|U(r_0)| = \alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

Solve for *n* to obtain:

$$n = \frac{1}{1-\dfrac{|U(r_0)|r_0}{\alpha ke^2}}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{1}{1-\dfrac{(741\,\text{kJ/mol})\left(\dfrac{1\,\text{eV/ion pair}}{96.47\,\text{kJ/mol}}\right)(0.257\,\text{nm})}{(1.7476)(1.44\,\text{eV}\cdot\text{nm})}} = \boxed{4.64}$$

19 ••

Picture the Problem We can substitute numerical values in Equation 38-6 to evaluate $U(r_o)$ for $n = 8$ and $n = 10$.

(*a*) Equation 38-6 is:

$$U(r_0) = -\alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

Substitute numerical values and evaluate $U(r_0)$:

$$U(r_o) = -\frac{(1.7476)(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{J})^2}{0.208\times10^{-9}\,\text{m}}\left(1-\frac{1}{8}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)$$
$$= \boxed{-10.6\,\text{eV}}$$

(*b*) The fractional change is given by:

$$\frac{\Delta U(r_0)}{U(r_0)} = \frac{U_{n=10}-U_{n=8}}{U_{n=8}} = \frac{U_{n=10}}{U_{n=8}} - 1$$

Substitute numerical values and evaluate $U(r_0)$ for $n = 10$:

$$U_{n=10}(r_o) = -\frac{(1.7476)(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{J})^2}{0.208\times10^{-9}\,\text{m}}\left(1-\frac{1}{10}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)$$
$$= -10.9\,\text{eV}$$

Substitute numerical values and evaluate the fractional change in $U(r_0)$:

$$\frac{\Delta U(r_0)}{U(r_0)} = \frac{-10.9\,\text{eV}}{-10.6\,\text{eV}} - 1 = \boxed{-2.83\%}$$

A Microscopic Picture of Conduction

20 •

Picture the Problem We can use the expression for the volume occupied by one electron to show that $r_s = \left(\frac{3}{4\pi n}\right)^{1/3}$.

(*a*) The volume occupied by one electron is:

$$\frac{1}{n} = \frac{4}{3}\pi r_s^3$$

Solve for r_s:

$$r_s = \boxed{\left(\frac{3}{4\pi n}\right)^{1/3}}$$

(*b*) From Table 38-1:

$$n_{\text{Cu}} = 8.47\times10^{28}\,\text{m}^{-3}$$

Substitute numerical values and evaluate r_s for copper:

$$r_s = \sqrt[3]{\frac{3}{4\pi(8.47\times10^{28}\,\text{m}^{-3})}} = \boxed{0.141\,\text{nm}}$$

21 •

Picture the Problem We can use the expression for the resistivity of the copper in terms of v_{av} and λ to find the classical value for the resistivity ρ of copper. In (*b*) we can use $v_{av} = \sqrt{\frac{3kT}{m_e}}$ to relate the average speed to the temperature.

(*a*) In terms of the mean free path and the mean speed, the resistivity is:

$$\rho = \frac{m_e v_{av}}{n_e e^2 \lambda}$$

Substitute numerical values and evaluate ρ (see Table 38-1 for the free-electron number density of copper):

$$\rho = \frac{(9.11\times10^{-31}\,\text{kg})(1.17\times10^{5}\,\text{m/s})}{(8.47\times10^{28}\,\text{m}^{-3})(1.60\times10^{-19}\,\text{C})^2(0.4\,\text{nm})} = \boxed{0.123\,\mu\Omega\cdot\text{m}}$$

(*b*) Relate the average speed of the electrons to the temperature:

$$v_{\text{av}} = \sqrt{\frac{3kT}{m_{\text{e}}}}$$

Substitute for v_{av} in the expression for ρ to obtain:

$$\rho = \frac{m_{\text{e}}}{n_{\text{e}}e^2\lambda}\sqrt{\frac{3kT}{m_{\text{e}}}} = \frac{1}{n_{\text{e}}e^2\lambda}\sqrt{3m_{\text{e}}kT}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{\sqrt{3(9.11\times10^{-31}\,\text{kg})(1.38\times10^{-23}\,\text{J/K})(100\,\text{K})}}{(8.47\times10^{28}\,\text{m}^{-3})(1.60\times10^{-19}\,\text{C})^2(0.4\,\text{nm})} = \boxed{0.0708\,\mu\Omega\cdot\text{m}}$$

***22** ••

Picture the Problem We can use Equation 38-14 to estimate the resistivity of silicon.

(*a*) From Equation 38-14:

$$\rho = \frac{m_{\text{e}}v_{\text{av}}}{n_{\text{e}}e^2\lambda} \qquad (1)$$

The speed of the electrons is given by:

$$v_{\text{av}} = v_{\text{F}} = \sqrt{\frac{2E_{\text{F}}}{m_{\text{e}}}}$$

Substitute numerical values and evaluate v_{av}:

$$v_{\text{av}} = \sqrt{\frac{2(4.88\,\text{eV})}{(9.11\times10^{-31}\,\text{kg})}\frac{1.60\times10^{-19}\,\text{J}}{1\,\text{eV}}} = 1.31\times10^{6}\,\text{m/s}$$

The electron density of Si is given by:

$$n_{\text{e}} = MN_{\text{A}}N_{\text{atom}}$$

where N_{atom} is the number of electrons per atom.

Substitute numerical values and evaluate n_{e}:

$$n_{\text{e}} = \left(2.41\times10^{3}\,\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{0.02809\,\text{kg}}\right)\left(\frac{2\,\text{e}}{\text{atom}}\right) = 1.03\times10^{29}\,\text{e/m}^3$$

Substitute numerical values in equation (1) and evaluate ρ:

$$\rho = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.31\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)^{2}\left(1.03\times10^{29}\,\text{e/m}^{3}\right)\left(27.0\times10^{-9}\,\text{m}\right)} = \boxed{1.66\times10^{-8}\,\Omega\cdot\text{m}}$$

(*b*) The accepted resistivity of 640 Ω·m is much greater than the calculated value. We assume that valence electrons will produce conduction in the material. Silicon is a semiconductor and a gap between the valence band and conduction band exists. Only electrons with sufficient energies will be found in the conduction band.

The Fermi Electron Gas

23 •

Picture the Problem The number density of free electrons is given by $n = \rho N_\text{A}/M$, where N_A is Avogadro's number, ρ is the density of the element, and M is its molar mass.

Relate the number density of free electrons to the density ρ and molar mass M of the element:

$$\frac{n}{N_\text{A}} = \frac{\rho}{M} \Rightarrow n = \frac{\rho N_\text{A}}{M}$$

(*a*) For Ag:

$$n_\text{Ag} = \frac{\left(10.5\,\text{g/cm}^3\right)\left(6.02\times10^{23}\,\text{electrons/mol}\right)}{107.87\,\text{g/mol}} = \boxed{5.86\times10^{22}\,\text{electrons/cm}^3}$$

(*b*) For Au:

$$n_\text{Ag} = \frac{\left(19.3\,\text{g/cm}^3\right)\left(6.02\times10^{23}\,\text{electrons/mol}\right)}{196.97\,\text{g/mol}} = \boxed{5.90\times10^{22}\,\text{electrons/cm}^3}$$

$$\boxed{\text{Both these results agree with the values in Table 38-1.}}$$

24 •

Picture the Problem The number of free electrons per atom n_e is given by $n_\text{e} = nM/\rho N_\text{A}$, where N_A is Avogadro's number, ρ is the density of the element, M is its molar mass, and n is the free electron number density for the element.

The number of free electrons per atom is given by:

$$n_\text{e} = \frac{nM}{\rho N_\text{A}}$$

Substitute numerical values and evaluate n_e:

$$n_e = \frac{(18.1\times10^{22}\ \text{electrons/cm}^3)(26.98\,\text{g/mol})}{(2.7\,\text{g/cm}^3)(6.02\times10^{23}\ \text{electrons/mol})} = \boxed{3.00}$$

25 •

Picture the Problem The number of free electrons per atom n_e is given by $n_e = nM/\rho N_A$,where N_A is Avogadro's number, ρ is the density of the element, M is its molar mass, and n is the free electron number density for the element.

The number of free electrons per atom is given by:

$$n_e = \frac{nM}{\rho N_A}$$

Substitute numerical values and evaluate n_e:

$$n_e = \frac{(14.8\times10^{22}\ \text{electrons/cm}^3)(118.69\,\text{g/mol})}{(7.3\,\text{g/cm}^3)(6.022\times10^{23}\ \text{electrons/mol})} = \boxed{4.00}$$

***26 •**

Picture the Problem The Fermi temperature T_F is defined by $kT_F = E_F$, where E_F is the Fermi energy.

The Fermi temperature is given by:

$$T_F = \frac{E_F}{k}$$

(*a*) For Al:

$$T_F = \frac{11.7\,\text{eV}}{8.62\times10^{-5}\ \text{eV/K}} = \boxed{1.36\times10^5\ \text{K}}$$

(*b*) For K:

$$T_F = \frac{2.11\,\text{eV}}{8.62\times10^{-5}\ \text{eV/K}} = \boxed{2.45\times10^4\ \text{K}}$$

(*c*) For Sn:

$$T_F = \frac{10.2\,\text{eV}}{8.62\times10^{-5}\ \text{eV/K}} = \boxed{1.18\times10^5\ \text{K}}$$

27 •

Picture the Problem We can solve the expression for the Fermi energy for the speed of a conduction electron.

Express the Fermi energy in terms of the Fermi speed a conduction electron:

$$E_F = \tfrac{1}{2}m_e u_F^2$$

Solve for u_F:

$$u_F = \sqrt{\frac{2E_F}{m_e}}$$

(*a*) Substitute numerical values (see Table 38-1 for E_F) and evaluate u_F for Na:

$$u_F = \sqrt{\frac{2(3.24\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.07\times10^{6}\ \text{m/s}}$$

(*b*) Substitute numerical values and evaluate u_F for Au:

$$u_F = \sqrt{\frac{2(5.53\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.39\times10^{6}\ \text{m/s}}$$

(*c*) Substitute numerical values and evaluate u_F for Sn:

$$u_F = \sqrt{\frac{2(10.2\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.89\times10^{6}\ \text{m/s}}$$

28 •

Picture the Problem The Fermi energy at $T = 0$ depends on the number of electrons per unit volume (the number density) according to $E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(N/V)^{2/3}$.

The Fermi energy at $T = 0$ is given by:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)\left(\frac{N}{V}\right)^{2/3}$$

(*a*) For Al, $N/V = 18.1\times10^{22}$ electrons/cm^3 (see Table 38-1) and:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(18.1\times10^{22}\ \text{electrons/cm}^3)^{2/3} = \boxed{11.7\,\text{eV}}$$

(*b*) For K, $N/V = 1.4\times10^{22}$ electrons/cm^3 and:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(1.4\times10^{22}\ \text{electrons/cm}^3)^{2/3} = \boxed{2.12\,\text{eV}}$$

(*c*) For Sn, $N/V = 14.8\times10^{22}$ electrons/cm^3 and:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(14.8\times10^{22}\ \text{electrons/cm}^3)^{2/3} = \boxed{10.2\,\text{eV}}$$

29 •

Picture the Problem The average energy of electrons in a Fermi gas at $T = 0$ is three-fifths of the Fermi energy.

The average energy at $T = 0$ is given by:

$$E_{av} = \tfrac{3}{5}E_F$$

(*a*) For copper, $E_F = 7.04$ eV (see Table 38-1) and:

$$E_{av} = (0.6)(7.04\,\text{eV}) = \boxed{4.22\,\text{eV}}$$

(*b*) For lithium, $E_F = 4.75$ eV and:

$$E_{av} = (0.6)(4.75\,\text{eV}) = \boxed{2.85\,\text{eV}}$$

30 •

Picture the Problem The Fermi energy at $T = 0$ is given by $E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(N/V)^{3/2}$, where N/V is the free-electron number density and the Fermi temperature is related to the Fermi energy according to $kT_F = E_F$.

(*a*) The Fermi temperature for iron is given by:

$$T_F = \frac{E_F}{k}$$

Substitute numerical values (see Table 38-1) and evaluate T_F:

$$T_F = \frac{11.2\,\text{eV}}{8.62\times10^{-5}\,\text{eV/K}} = \boxed{1.30\times10^5\,\text{K}}$$

(*b*) The Fermi energy at $T = 0$ is given by:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)\left(\frac{N}{V}\right)^{3/2}$$

Substitute numerical values (see Table 38-1) and evaluate E_F:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(17.0\times10^{22}\,\text{electrons/cm}^3)^{2/3} = \boxed{11.2\,\text{eV}}$$

***31** ••

Picture the Problem We can use $n_e = \rho V = \dfrac{\rho N_A N_{atom}}{m}$, where N_{atom} is the number of electrons per atom, to calculate the electron density of gold. The Fermi energy is given by $E_F = \frac{1}{2}m_e v_F^2$.

(*a*) The electron density of gold is given by:

$$n_e = \rho V = \frac{\rho N_A N_{atom}}{m}$$

Substitute numerical values and evaluate n_e:

$$n_e = \frac{\left(19.3\times10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(6.02\times10^{23}\,\text{atoms}\right)\left(\frac{1\text{e}}{1\,\text{atom}}\right)}{0.197\ \text{kg}} = \boxed{5.90\times10^{28}\ \text{e/m}^3}$$

(*b*) The Fermi energy is given by: $E_F = \frac{1}{2}m_e v_F^2$

Substitute numerical values and evaluate E_F:

$$E_F = \frac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.39\times10^6\,\text{m/s}\right)^2\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{5.50\,\text{eV}}$$

(*c*) The factor by which the Fermi energy is higher than the *kT* energy at room temperature is: $f = \frac{E_F}{kT}$

At room temperature kT = 0.026 eV. Substitute numerical values and evaluate *f*: $f = \frac{5.50\,\text{eV}}{0.026\,\text{eV}} = \boxed{212}$

(*d*) E_F is 212 times *kT* at room temperature. There are so many free electrons present that most of them are crowded, as described by the Pauli exclusion principle, up to energies far higher than they would be according to the classical model.

***32 ••**

Picture the Problem We can solve $PV = \frac{2}{3}NE_{av}$ for *P* and substitute for E_{av} in order to express *P* in terms of *N*/*V* and E_F.

Solve $PV = \frac{2}{3}NE_{av}$ for *P*: $P = \frac{2}{3}\left(\frac{N}{V}\right)E_{av}$

Because $E_{av} = \frac{3}{5}E_F$: $P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{3}{5}E_F\right) = \frac{2}{5}\left(\frac{N}{V}\right)E_F$

Substitute numerical values (see Table 38-1) and evaluate *P*:

$$P = \frac{2}{5}\left(8.47\times10^{22}\ \text{electrons/cm}^3\right)\left(7.04\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)$$

$$= \boxed{3.82\times10^{10}\ \text{N/m}^2} = 3.82\times10^{10}\ \text{N/m}^2 \times \frac{1\text{atm}}{101.325\times10^3\ \text{N/m}^2}$$

$$= \boxed{3.77\times10^5\ \text{atm}}$$

33 ••

Picture the Problem We can follow the procedure given in the problem statement to show that $P = \frac{2NE_F}{5V} = CV^{-5/3}$ and $B = \frac{5}{3}P = \frac{2NE_F}{3V}$.

(*a*) From Problem 32 we have:

$$PV = \tfrac{2}{3}NE_{av}$$

Because $E_{av} = \frac{3}{5}E_F$:

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{3}{5}E_F\right) = \frac{2}{5}\left(\frac{N}{V}\right)E_F \qquad (1)$$

The Fermi energy is given by:

$$E_F = \frac{h^2}{8m_e}\left(\frac{3N}{\pi V}\right)^{2/3} = \frac{h^2}{8m_e}\left(\frac{3N}{\pi}\right)^{2/3} V^{-2/3}$$

Substitute to obtain:

$$P = \frac{2}{5}\left(\frac{N}{V}\right)\frac{h^2}{8m_e}\left(\frac{3N}{\pi}\right)^{2/3} V^{-2/3} = \frac{N^{5/3}h^2}{20m_e}\left(\frac{3}{\pi}\right)^{2/3} V^{-5/3} = \boxed{CV^{-5/3}}$$

where $C = \frac{N^{5/3}h^2}{20m_e}\left(\frac{3}{\pi}\right)^{2/3}$ is a constant.

(*b*) The bulk modulus is given by:

$$B = -V\frac{dP}{dV} = -V\frac{d}{dV}\left[CV^{-5/3}\right] = -CV\left(-\frac{5}{3}V^{-8/3}\right) = \frac{5}{3}CV^{-5/3} = \frac{5}{3}P$$

Substitute for P from equation (1) to obtain:

$$B = \frac{5}{3}\left[\frac{2}{5}\left(\frac{N}{V}\right)E_F\right] = \boxed{\frac{2}{3}\left(\frac{N}{V}\right)E_F}$$

(*c*) Substitute numerical values and evaluate B for copper:

$$B = \frac{2}{3}\left(8.47\times10^{22}\ \text{electrons/cm}^3\right)\left(7.04\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right) = \boxed{63.6\times10^9\ \text{N/m}^2}$$

From Table 13-2:

$$B_{Cu} = 140\ \text{GN/m}^2$$

Divide the calculated value for B by the value from Table 13-2 to obtain:

$$\frac{B}{B_{\text{Cu}}} = \frac{63.6\,\text{GN/m}^2}{140\,\text{GN/m}^2} = 0.455$$

or

$$B = \boxed{0.454 B_{\text{Cu}}}$$

34 •

Picture the Problem The contact potential is given by $V_{\text{contact}} = \frac{\phi_1 - \phi_2}{e}$, where ϕ_1 and ϕ_2 are the work functions of the two different metals in contact with each other.

The contact potential is given by:

$$V_{\text{contact}} = \frac{\phi_1 - \phi_2}{e}$$

(a) For Ag and Cu (see Table 38-1):

$$V_{\text{contact}} = \frac{(4.7\,\text{eV} - 4.1\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{0.6\,\text{V}}$$

(b) For Ag and Ni:

$$V_{\text{contact}} = \frac{(5.2\,\text{eV} - 4.7\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{0.5\,\text{V}}$$

(c) For Ca and Cu (see Table 38-1):

$$V_{\text{contact}} = \frac{(4.1\,\text{eV} - 3.2\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{0.9\,\text{V}}$$

Heat Capacity Due to Electrons in a Metal

***35** ••

Picture the Problem We can use Equation 38-29 to find the molar specific heat of gold at constant volume and room temperature.

The molar specific heat is given by Equation 38-29:

$$c'_{\text{V}} = \frac{\pi^2 RT}{2T_{\text{F}}}$$

The Fermi energy is given by:

$$E_{\text{F}} = kT_{\text{F}} \Rightarrow T_{\text{F}} = \frac{E_{\text{F}}}{k}$$

Substitute for T_F to obtain:

$$c'_V = \frac{\pi^2 RkT}{2E_F}$$

Substitute numerical values and evaluate c'_V :

$$c'_V = \frac{\pi^2(8.31\,\text{J/mol K})(1.38\times10^{-23}\,\text{J/mol})\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)(300\,\text{K})}{2(5.53\,\text{eV})}$$
$$= \boxed{0.192\,\text{J/mol}\cdot\text{K}}$$

Remarks: The value 0.192 J/mol K is for a mole of gold atoms. Since each gold atom contributes one electron to the metal, a mole of gold corresponds to a mole of electrons.

Quantum Theory of Electrical Conduction

36 •

Picture the Problem We can solve the equation giving the resistivity of a conductor in terms of the mean free path and the mean speed for the mean free path and use the Fermi speeds from Problem 27 as the mean speeds.

In terms of the mean free path and the mean speed, the resistivity is:

$$\rho = \frac{m_e v_{av}}{ne^2\lambda}$$

Solve for λ to obtain:

$$\lambda = \frac{m_e v_{av}}{ne^2\rho}$$

From Problem 27 we have:

$$\mu_{F,Na} = 1.07\times10^6\ \text{m/s}$$
$$\mu_{F,Au} = 1.39\times10^6\ \text{m/s}$$
and
$$\mu_{F,Sn} = 1.89\times10^6\ \text{m/s}$$

Using the Fermi speeds as the average speeds, substitute numerical values and evaluate the mean free path of Na:

$$\lambda_{Na} = \frac{(9.11\times10^{-31}\,\text{kg})(1.07\times10^6\ \text{m/s})}{(2.65\times10^{22}\ \text{electrons/cm}^3)(1.60\times10^{-19}\,\text{C})^2(4.2\,\mu\Omega\cdot\text{cm})} = \boxed{34.2\,\text{nm}}$$

Proceed as above for Au:

$$\lambda_{\text{Au}} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.39\times10^{6}\ \text{m/s}\right)}{\left(5.90\times10^{22}\ \text{electrons/cm}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)^2\left(2.04\ \mu\Omega\cdot\text{cm}\right)} = \boxed{41.1\,\text{nm}}$$

Proceed as above for Sn:

$$\lambda_{\text{Sn}} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.89\times10^{6}\ \text{m/s}\right)}{\left(14.8\times10^{22}\ \text{electrons/cm}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)^2\left(10.6\ \mu\Omega\cdot\text{cm}\right)} = \boxed{4.29\,\text{nm}}$$

***37** ••

Picture the Problem We can solve the resistivity equation for the mean free path and then substitute the Fermi speed for the average speed to express the mean free path as a function of the Fermi energy.

(*a*) In terms of the mean free path and the mean speed, the resistivity is:

$$\rho_{\text{i}} = \frac{m_{\text{e}}v_{\text{av}}}{ne^2\lambda_{\text{i}}} = \frac{m_{\text{e}}u_{\text{F}}}{ne^2\lambda_{\text{i}}}$$

Solve for λ to obtain:

$$\lambda_{\text{i}} = \frac{m_{\text{e}}u_{\text{F}}}{ne^2\rho_{\text{i}}}$$

Express the Fermi speed u_{F} in terms of the Fermi energy E_{F}:

$$u_{\text{F}} = \sqrt{\frac{2E_{\text{F}}}{m_{\text{e}}}}$$

Substitute to obtain:

$$\lambda_{\text{i}} = \frac{\sqrt{2m_{\text{e}}E_{\text{F}}}}{ne^2\rho_{\text{i}}}$$

Substitute numerical values (see Table 38-1) and evaluate λ_{i}:

$$\lambda_{\text{i}} = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(7.04\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}}{\left(8.47\times10^{28}\ \text{electrons/m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)^2\left(10^{-8}\ \Omega\cdot\text{m}\right)} = \boxed{66.1\,\text{nm}}$$

(*b*) From Equation 38-16 we have:

$$\lambda = \frac{1}{n\pi r^2}$$

Solve for πr^2:

$$\pi r^2 = \frac{1}{n\lambda}$$

Substitute numerical values and evaluate πr^2:

$$\pi r^2 = \frac{1}{\left(8.47\times10^{28}\text{ m}^{-3}\right)\left(66.1\text{nm}\right)}$$
$$= 1.79\times10^{-22}\text{ m}^2 = \boxed{1.79\times10^{-4}\text{ nm}^2}$$

Band Theory of Solids

38 •

Picture the Problem We can use $E_g = hc/\lambda$ to calculate the energy gap for this semiconductor.

The lowest photon energy to increase conductivity is given by:

$$E_g = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate E_g:

$$E_g = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)}{\left(380\times10^{-9}\text{ m}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}$$
$$= \boxed{3.27\text{ eV}}$$

***39 •**

Picture the Problem We can relate the maximum photon wavelength to the energy gag using $\Delta E = hf = hc/\lambda$.

Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\text{ eV}\cdot\text{nm}}{1.14\text{ eV}} = \boxed{1.09\ \mu\text{m}}$$

40 •

Picture the Problem We can relate the maximum photon wavelength to the energy gag using $\Delta E = hf = hc/\lambda$.

Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{0.74\,\text{eV}} = \boxed{1.68\,\mu\text{m}}$$

41 •

Picture the Problem We can relate the maximum photon wavelength to the energy gag using $\Delta E = hf = hc/\lambda$.

Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{7.0\,\text{eV}} = \boxed{177\,\text{nm}}$$

42 ••

Picture the Problem We can use $\Delta E = hf = hc/\lambda$ to find the energy gap between these bands and $T = E_g/k$ to find the temperature for which kT equals this energy gap.

(*a*) Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \frac{1240\,\text{eV}\cdot\text{nm}}{3.35\,\mu\text{m}} = \boxed{0.370\,\text{eV}}$$

(*b*) The temperature is related to the energy gap E_g according to:

$$T = \frac{E_g}{k}$$

Substitute numerical values and evaluate T:

$$T = \frac{0.370\,\text{eV}}{8.617\times10^{-5}\,\text{eV/K}} = \boxed{4.29\times10^{3}\,\text{K}}$$

Semiconductors

43 •

Picture the Problem We can use $\Delta E = kT$ to find the temperature for which $kT = 0.01$ eV

Express the temperature T in terms of the energy gap ΔE:

$$T = \frac{\Delta E}{k}$$

Substitute numerical values and evaluate T:

$$T = \frac{0.01\,\text{eV}}{1.38\times10^{-23}\,\text{J/K}\times\dfrac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}} = \boxed{116\,\text{K}}$$

***44 ••**

Picture the Problem We can use $E = hf$ to find the energy gap of this semiconductor.

The energy gap of the semiconductor is given by:

$$E_g = hf = \frac{hc}{\lambda}$$

where

hc = 1240 eV·nm

Substitute numerical values and evaluate E_g:

$$E_g = \frac{1240\,\text{eV}\cdot\text{nm}}{1.85\,\mu\text{m}} = \boxed{0.670\,\text{eV}}$$

45 ••

Picture the Problem We can make the indicated substitutions in the expression for a_0 (= 0.0529 nm) to obtain an expression the Bohr radii for the outer electron as it orbits the impurity arsenic atom in silicon and germanium.

Make the indicated substitutions in the expression for a_0 to obtain:

$$a_B = \frac{\kappa \in_0 h^2}{\pi m_{eff} e^2} = \frac{\kappa \in_0 m_e h^2}{\pi m_e m_{eff} e^2} = \frac{\kappa m_e}{m_{eff}}\frac{\in_0 h^2}{\pi m_e e^2} = \frac{\kappa m_e}{m_{eff}} a_0$$

For silicon:

$$a_B = \frac{12 m_e}{0.2 m_e}(0.0529\,\text{nm}) = \boxed{3.17\,\text{nm}}$$

For germanium:

$$a_B = \frac{16 m_e}{0.1 m_e}(0.0529\,\text{nm}) = \boxed{8.46\,\text{nm}}$$

***46 ••**

Picture the Problem We can make the same substitutions we made in Problem 45 in the expression for E_1 (= 13.6 eV) to obtain an expression that we can use to estimate the binding energy of the extra electron of an impurity arsenic atom in silicon and germanium.

Make the indicated substitutions in the expression for E_1 to obtain:

$$E_1 = -\frac{e^2 m_{\text{eff}}}{8(\kappa \in_0)^2 h^2} = -\frac{e^2 m_e m_{\text{eff}}}{8 m_e \kappa^2 \in_0^2 h^2}$$
$$= -\frac{m_{\text{eff}}}{m_e \kappa^2 \in_0^2}\frac{e^2 m_e}{8 \in_0^2 h^2}$$
$$= -\frac{m_{\text{eff}}}{m_e \kappa^2 \in_0^2} E_1$$

(*a*) For silicon:

$$E_1 = -\frac{0.2 m_e}{m_e (12)^2}(13.6\,\text{eV}) = \boxed{18.9\,\text{meV}}$$

(*b*) For germanium:

$$E_1 = -\frac{0.1 m_e}{m_e (16)^2}(13.6\,\text{eV}) = \boxed{5.31\,\text{meV}}$$

47 ••

Picture the Problem We can use the expression for the resistivity ρ of the sample as a function of the mean free path λ of the conduction electrons in conjunction with the expression for the average speed v_{av} of the electrons to derive an expression that we can use to calculate the mean free path of the electrons.

Express the resistivity ρ of the sample as a function of the mean free path λ of the conduction electrons:

$$\rho = \frac{m_e v_{av}}{n_e e^2 \lambda}$$

Solve for λ to obtain:

$$\lambda = \frac{m_e v_{av}}{n_e e^2 \rho} \qquad (1)$$

The average speed v_{av} of the electrons is given by:

$$v_{av} \approx v_{rms} = \sqrt{\frac{3kT}{m_e}}$$

Substitute for v_{av} in the expression for λ to obtain:

$$\lambda = \frac{m_e}{n_e e^2 \rho}\sqrt{\frac{3kT}{m_e}} = \frac{\sqrt{3km_e T}}{n_e e^2 \rho}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{\sqrt{3(1.38\times10^{-23}\,\text{J/K})(0.2)(9.11\times10^{-31}\,\text{kg})(300\,\text{K})}}{(10^{16}\,\text{cm}^{-3})(1.6\times10^{-19}\,\text{C})^2(5\times10^{-3}\,\Omega\cdot\text{m})} = \boxed{37.2\,\text{nm}}$$

The number density of electrons n_e is related to the mass density ρ_m, Avogadro's number N_A, and the molar mass M.:

$$n_e = \frac{\rho_m N_A}{M}$$

Substitute numerical values (For copper, ρ = 8.93 g/cm^3 and M = 63.5 g/mol.) and evaluate n_e:

$$n_e = \frac{(8.93\,\text{g/cm}^3)(6.02\times10^{23}\,\text{electrons/mol})}{63.5\,\text{g/mol}} = 8.47\times10^{28}\,\text{electrons/m}^3$$

Using equation (1), evaluate λ_{Cu} (see Table 25-1 for the resistivity of copper and Example 38-4 for u_F):

$$\lambda_{Cu} = \frac{(9.11\times10^{-31}\,\text{kg})(1.57\times10^{6}\,\text{m/s})}{(8.47\times10^{28}\,\text{electrons/m}^3)(1.6\times10^{-19}\,\text{C})^2(1.7\times10^{-8}\,\Omega\cdot\text{m})} = \boxed{38.8\,\text{nm}}$$

$$\boxed{\text{The mean free paths agree to within 4.02\%.}}$$

48 ••

Picture the Problem We can use the expression for the Hall coefficient to determine the type of impurity and the concentration of these impurities.

(*a*) and (*b*) The Hall coefficient is given by:

$$R = \frac{1}{nq} \qquad (1)$$

$$\boxed{\text{Because } R > 0,\ q > 0 \text{ and conduction is by holes and the sample contains acceptor impurities.}}$$

Solve equation (1) for n:

$$n = \frac{1}{Rq}$$

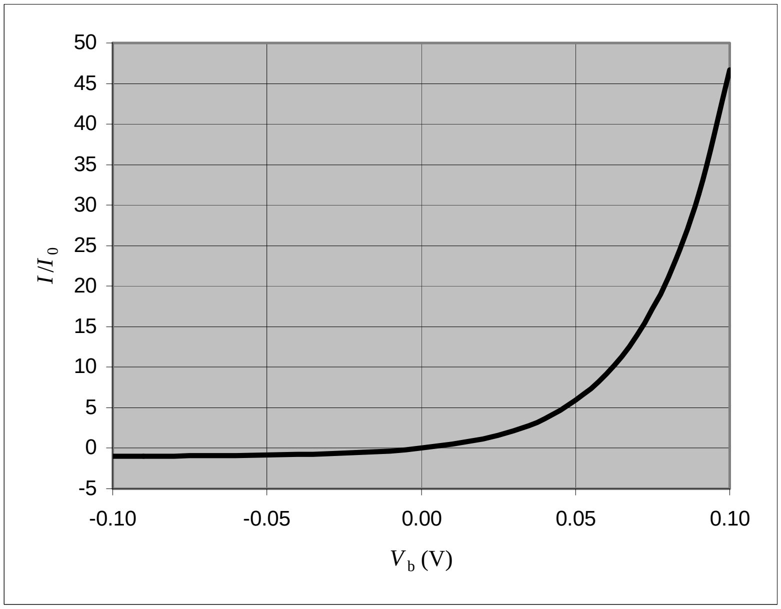
Substitute numerical values and evaluate n:

$$n = \frac{1}{(0.04\,\text{V}\cdot\text{m/A}\cdot\text{T})(1.6\times10^{-19}\,\text{C})} = \boxed{1.56\times10^{20}\,\text{m}^{-3}}$$

Semiconductor Junctions and Devices

49 ••

Picture the Problem The following graph of I/I_0 versus V_b was plotted using a spreadsheet program.



50 •

Picture the Problem The base current is the difference between the emitter current and the plate current.

The base current I_B is given by:

$$I_B = I - I_C$$

We're given that:

$$I_C = 0.88I = 25.0\text{ mA}$$

Solve for I to obtain:

$$I = \frac{25.0\text{ mA}}{0.88} = 28.4\text{ mA}$$

Substitute numerical values for I and I_C and evaluate I_B:

$$I_B = 28.4\text{ mA} - 25.0\text{ mA} = \boxed{3.4\text{ mA}}$$

***51** ••

Picture the Problem We can use its definition to compute the voltage gain of the amplifier.

The voltage gain of the amplifier is given by:

$$\text{Voltage gain} = \frac{I_c R_L}{I_b R_b}$$

Substitute numerical values and evaluate the voltage gain:

$$\text{Voltage gain} = \frac{(0.5\text{ mA})(10\text{ k}\Omega)}{(10\ \mu\text{A})(2\text{ k}\Omega)} = \boxed{250}$$

52 ••

Picture the Problem The number of electron-hole pairs N is related to the energy E of the incident beam and the energy gap E_g.

(*a*) The number of electron-hole pairs N is given by:

$$N = \frac{E}{E_g}$$

Substitute numerical values and evaluate N:

$$N = \frac{660\,\text{keV}}{0.72\,\text{eV}} = \boxed{9.17\times10^5}$$

(*b*) The energy resolution of the detector is given by:

$$\frac{\Delta E}{E} = \frac{\Delta N}{N}$$

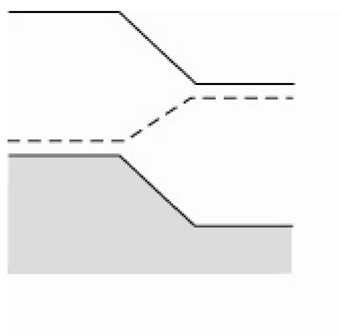
For $\Delta N = 1$ and $N = \sqrt{9.17\times10^5}$:

$$\frac{\Delta E}{E} = \frac{1}{\sqrt{9.17\times10^5}} = 1.04\times10^{-3} = \boxed{0.104\%}$$

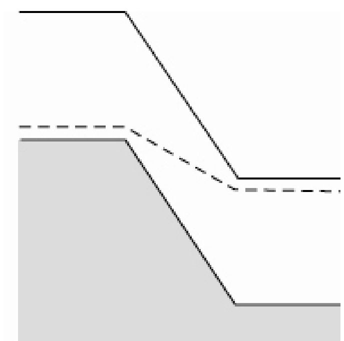
53 ••

Picture the Problem The nearly full valence band is shown shaded. The Fermi level is shown by the dashed line.

(*a*)



(*b*)



***54** ••

Picture the Problem We can use Ohm's law and the expression for the current from Problem 49 to find the resistance for small reverse-and-forward bias voltages.

(*a*) Use Ohm's law to express the resistance:

$$R = \frac{V_b}{I} \qquad (1)$$

From Problem 47, the current across a *pn* junction is given by:

$$I = I_0\left(e^{eV_b/kT} - 1\right) \qquad (2)$$

For $eV_b << kT$:

$$e^{eV_b/kT} - 1 \approx 1 + \frac{eV_b}{kT} - 1 = \frac{eV_b}{kT}$$

Substitute to obtain:

$$I = I_0\frac{eV_b}{kT}$$

Substitute for I in equation (1) and simplify:

$$R=\frac{V_b}{I_0\frac{eV_b}{kT}}=\frac{kT}{eI_0}$$

Substitute numerical values and evaluate R:

$$R=\frac{(0.025\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.60\times10^{-19}\,\text{C})(10^{-9}\,\text{A})}$$
$$=\boxed{25.0\,\text{M}\Omega}$$

(b) Substitute equation (2) in equation (1) to obtain:

$$R=\frac{V_b}{I_0\left(e^{eV_b/kT}-1\right)} \qquad (3)$$

Evaluate $\frac{eV_b}{kT}$ for $V_b = -0.5$ V:

$$\frac{eV_b}{kT}=\frac{(1.60\times10^{-19}\,\text{C})(-0.5\,\text{V})}{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}=-19.8$$

Evaluate equation (3) for $V_b = -0.5$ V:

$$R=\frac{-0.5\,\text{V}}{(10^{-9}\,\text{A})(e^{-19.8}-1)}=\boxed{500\,\text{M}\Omega}$$

(c) Evaluate $\frac{eV_b}{kT}$ for $V_b = +0.5$ V:

$$\frac{eV_b}{kT}=\frac{(1.60\times10^{-19}\,\text{C})(0.5\,\text{V})}{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}=19.8$$

Evaluate equation (3) for $V_b = +0.5$ V:

$$R=\frac{0.5\,\text{V}}{(10^{-9}\,\text{A})(e^{19.8}-1)}=\boxed{1.26\,\Omega}$$

(d) Evaluate $R_{ac} = dV/dI$ to obtain:

$$R_{ac}=\frac{dV}{dI}=\left(\frac{dI}{dV}\right)^{-1}$$
$$=\left\{\frac{d}{dV}\left[I_0\left(e^{eV_b/kT}-1\right)\right]\right\}^{-1}$$
$$=\left\{\frac{eI_0}{kT}e^{eV_b/kT}\right\}^{-1}=\frac{kT}{eI_0}e^{-eV_b/kT}$$

Substitute numerical values and evaluate R_{ac}:

$$R_{ac}=(25\,\text{M}\Omega)e^{-19.8}=\boxed{0.0629\,\Omega}$$

55 ••

Picture the Problem We can use the Hall-effect equation to find the concentration of charge carriers in the slab of silicon. We can determine the semiconductor type by determining the directions of the magnetic and electric fields.

Use the expression for the Hall-effect voltage to relate the concentration of charge carriers n to

$$V_H=\frac{IB}{nte}$$

the Hall voltage V_H:

Solve for n to obtain:

$$n = \frac{IB}{teV_H}$$

Substitute numerical values and evaluate n:

$$n = \frac{(0.2\,\text{A})(0.4\,\text{T})}{(1.0\,\text{mm})(1.60\times10^{-19}\,\text{C})(5\,\text{mV})} = \boxed{1.00\times10^{23}\,\text{m}^{-3}}$$

> Referring to Figure 26-28, note that $\vec{B}$ points out of the page. $\vec{E}$ is in the y direction. Therefore, the charge carriers are holes and the semiconductor is p-type.

The BCS Theory

56 •

Picture the Problem We can calculate E_g using $E_g = 3.5kT_c$ and find the wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ using $\lambda = hc/E_g$.

(*a*) From Equation 38-24 we have:

$$E_g = 3.5kT_c$$

Substitute numerical values and evaluate E_g:

$$E_g = 3.5(8.617\times10^{-5}\,\text{eV/K})(3.72\,\text{K}) = \boxed{1.12\,\text{meV}}$$

Express the ratio of E_g to $E_{g,\text{measured}}$:

$$\frac{E_g}{E_{g,\text{measured}}} = \frac{1.12\,\text{meV}}{6\times10^{-4}\,\text{eV}} = 1.87$$

or

$$E_g \approx \boxed{2E_{g,\text{measured}}}$$

(*b*) The wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ is given by:

$$\lambda = \frac{hc}{E_g}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{6\times10^{-4}\,\text{eV}} = 2.07\times10^{6}\,\text{nm} = \boxed{2.07\,\text{mm}}$$

***57** •

Picture the Problem We can calculate E_g using $E_g = 3.5kT_c$ and find the wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ using $\lambda = hc/E_g$.

(*a*) From Equation 38-24 we have:

$$E_g = 3.5kT_c$$

Substitute numerical values and evaluate E_g:

$$E_g = 3.5\left(8.62\times10^{-5}\text{ eV/K}\right)\left(7.19\text{ K}\right) = \boxed{2.17\text{ meV}}$$

Express the ratio of E_g to $E_{g,measured}$:

$$\frac{E_g}{E_{g,measured}} = \frac{2.17\text{ meV}}{2.73\times10^{-3}\text{ eV}} = 0.795$$

or

$$E_g \approx \boxed{0.8E_{g,measured}}$$

(*b*) The wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ is given by:

$$\lambda = \frac{hc}{E_g}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\text{ eV}\cdot\text{nm}}{2.73\times10^{-3}\text{ eV}} = 4.54\times10^{5}\text{ nm} = \boxed{0.454\text{ mm}}$$

The Fermi-Dirac Distribution

58 ••

Picture the Problem We can evaluate the Fermi factor at the bottom of the conduction band for T near room temperature to show that this factor is given by $\exp(-E_g/2kT)$.

(*a*) At the bottom of the conduction band:

$e^{(E-E_F)/kT} = e^{E_g/2kT} >>> 1$ for T near room temperature.

We can neglect the 1 in the denominator of the Fermi function to obtain:

$$f\left(\tfrac{1}{2}E_g\right) = \frac{1}{e^{E_g/2kT}} = \boxed{e^{-E_g/2kT}}$$

Substitute numerical values and evaluate $f\left(\frac{1}{2}E_g\right)$ for T = 300 K:

$$f\left(\tfrac{1}{2}E_g\right)=\exp\left[\frac{-1\,\text{eV}}{2\left(8.62\times10^{-5}\ \text{eV/K}\right)\left(300\,\text{K}\right)}\right]=\boxed{4.01\times10^{-9}}$$

Given that low a probability of finding an electron in a state near the bottom of the conduction band, the exclusion principle has no significant impact on the distribution function. With 10^{22} valence electrons per cubic centimeter, the number of electrons in the conduction band will be about 4×10^{13} per cm^3.

(b) Evaluate $f\left(\frac{1}{2}E_g\right)$ for T = 300 K and E_g = 6 eV:

$$f\left(\tfrac{1}{2}E_g\right)=\exp\left[\frac{-6\,\text{eV}}{2\left(8.62\times10^{-5}\ \text{eV/K}\right)\left(300\,\text{K}\right)}\right]=\boxed{4.15\times10^{-51}}$$

The probability of finding even one electron in the conduction band is negligibly small (approximately 4×10^{-51}).

59 ••

Picture the Problem The number of energy states per unit volume per unit energy interval N is given by $N\approx g(E)\Delta E$, where N is only approximate, because ΔE is not infinitesimal and $g(E)=\frac{8\sqrt{2}\pi m_e^{3/2}V}{h^3}E^{1/2}$ is the density of states.

The number of states N is the product of the density of states and the energy interval:

$$N\approx g(E)\Delta E \qquad (1)$$

The density of states is given by:

$$g(E)=\frac{8\sqrt{2}\pi m_e^{3/2}V}{h^3}E^{1/2}$$

Substitute numerical values and evaluate $g(E)$:

$$g(E)=\frac{8\sqrt{2}\pi\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}\left(1\times10^{-3}\ \text{m}\right)^3}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^3}\left(2.1\,\text{eV}\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}\right)^{1/2}$$
$$=6.15\times10^{37}\ \text{J}^{-1}$$

Substitute numerical values in equation (1) and evaluate N:

$$N \approx \left(6.15\times10^{37}\ \text{J}^{-1}\right)\left(2.20\,\text{eV}-2.00\,\text{eV}\right)\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}=\boxed{1.97\times10^{18}}$$

***60 ••**

Picture the Problem Equation 38-22*a* expresses the dependence of the Fermi energy E_F on the number density of free electrons. Once we've determined the Fermi energy for silver, we can find the average electron energy from the Fermi energy for silver and then use the average electron energy to find the Fermi speed for silver.

(*a*) From Equation 38-22*a* we have:

$$E_F=\frac{h^2}{8m_e}\left(\frac{3N}{\pi V}\right)^{3/2}$$

Use Table 27-1 to find the free-electron number density N/V for silver:

$$\frac{N}{V}=5.86\text{x}10^{22}\ \frac{\text{electrons}}{\text{cm}^3}=5.86\text{x}10^{28}\ \frac{\text{electrons}}{\text{m}^3}$$

Substitute numerical values and evaluate E_F:

$$E_F=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\left[\frac{3\left(5.86\times10^{28}\ \text{electrons/m}^3\right)}{\pi}\right]^{2/3}\left(\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=\boxed{5.51\text{eV}}$$

(*b*) The average electron energy is given by:

$$E_{av}=\frac{3}{5}E_F$$

Substitute numerical values and evaluate *E*av:

$$E_{av}=\frac{3}{5}\left(5.51\,\text{eV}\right)=\boxed{3.31\text{eV}}$$

(*c*) Express the Fermi energy in terms of the Fermi speed of the electrons:

$$E_F=\tfrac{1}{2}m_e v_F^2$$

Solve for v_F:

$$v_F=\sqrt{\frac{2E_F}{m_e}}$$

Substitute numerical values and evaluate v_F:

$$v_F=\sqrt{\frac{2\left(3.31\text{eV}\right)}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\text{eV}}\right)}=\boxed{1.08\times10^{6}\ \text{m/s}}$$

61 ••

Picture the Problem We can evaluate the $f(E_F)$ at $E = E_F$ to show that $F = 0.5$.

The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$$

Evaluate $f(E_F)$:

$$f(E_F)=\frac{1}{e^{(E_F-E_F)/kT}+1}=\frac{1}{1+1}=\boxed{0.5}$$

62 ••

Picture the Problem We can find the difference between the energies at which the Fermi factor has the given values by solving the expression for Fermi factor for E and then deriving an expression for ΔE.

(*a*) The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$$

Solve for E:

$$E = E_F + kT\ln\left(\frac{1}{f(E)}-1\right)$$

The difference between the energies is given by:

$$\begin{aligned}\Delta E &= E(0.1)-E(0.9)\\ &= E_F+\frac{(1.38\times10^{-23}\text{ J/K})(300\text{ K})}{1.60\times10^{-19}\text{ J/eV}}\ln\left(\frac{1}{0.1}-1\right)\\ &\quad-\left\{E_F+\frac{(1.38\times10^{-23}\text{ J/K})(300\text{ K})}{1.60\times10^{-19}\text{ J/eV}}\ln\left(\frac{1}{0.9}-1\right)\right\}\\ &=\frac{(1.38\times10^{-23}\text{ J/K})(300\text{ K})}{1.60\times10^{-19}\text{ J/eV}}\left[\ln\left(\frac{1}{0.1}-1\right)-\ln\left(\frac{1}{0.9}-1\right)\right]\\ &=\boxed{0.114\text{ eV}}\end{aligned}$$

(*b*) and (*c*) $\boxed{\text{Because } \Delta E \text{ is independent of } E_F,\ \Delta E \text{ is the same as in } (a).}$

***63 ••**

Picture the Problem The probability that a conduction electron will have a given kinetic energy is given by the Fermi factor.

The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_{\mathrm{F}})/kT}+1}$$

Because $E_{\mathrm{F}} - 4.9$ eV >> 300k:

$$f(4.9\,\mathrm{eV})=\frac{1}{0+1}=\boxed{1}$$

64 ••

Picture the Problem We can solve Equation 38-22a for V and substitute in Equation 38-41 to show that $g(E)=(3N/2)E_{\mathrm{F}}^{-3/2}E^{1/2}$.

From Equation 38-22a we have:

$$E_{\mathrm{F}}=\frac{h^2}{8m_{\mathrm{e}}}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Solve for V to obtain:

$$V=\frac{3N}{\pi}\left(\frac{h^2}{8m_{\mathrm{e}}E_{\mathrm{F}}}\right)^{3/2}$$

The density $g(E)$ is given by Equation 38-41:

$$g(E)=\frac{8\pi\sqrt{2}m_{\mathrm{e}}^{3/2}V}{h^3}E^{1/2}$$

Substitute for V and simplify to obtain:

$$g(E)=\frac{8\pi\sqrt{2}m_{\mathrm{e}}^{3/2}}{h^3}\left[\frac{3N}{\pi}\left(\frac{h^2}{8m_{\mathrm{e}}E_{\mathrm{F}}}\right)^{3/2}\right]E^{1/2}$$

$$=\boxed{\frac{3N}{2}E_{\mathrm{F}}^{-3/2}E^{1/2}}$$

65 ••

Picture the Problem We can use the expression for $g(E)$ from Problem 64 to show that the average energy at $T = 0$ is $\frac{3}{5}E_{\mathrm{F}}$.

From Problem 64 we have:

$$g(E)=\frac{3N}{2}E_{\mathrm{F}}^{-3/2}E^{1/2}$$

Substitute in the expression for E_{av} and simplify to obtain:

$$E_{av} = \frac{1}{N}\int_0^{E_F} Eg(E)dE = \frac{1}{N}\int_0^{E_F} E\left(\frac{3N}{2}E_F^{-3/2}E^{1/2}\right)dE = \frac{3}{2}E_F^{-3/2}\int_0^{E_F} E^{3/2}dE$$

Integrate the expression for E_{av}:

$$E_{av} = \frac{3}{2}E_F^{-3/2}\int_0^{E_F} E^{3/2}dE = \frac{3}{2}E_F^{-3/2}\,\frac{2}{5}E_F^{5/2} = \boxed{\frac{3}{5}E_F}$$

66 ••

Picture the Problem We can integrate $g(E)$ from 0 to E_F to show that the total number of states is $\frac{2}{3}AE_F^{3/2}$.

(*a*) Integrate $g(E)$ from 0 to E_F:

$$N = \int_0^{E_F} AE^{1/2}dE = \boxed{\tfrac{2}{3}AE_F^{3/2}}$$

(*b*) Express the fraction of N within kT of E_F:

$$\frac{kTg(E_F)}{N} = \frac{kTAE_F^{1/2}}{\frac{2}{3}AE_F^{3/2}} = \boxed{\frac{3kT}{2E_F}}$$

(*c*) Substitute numerical values and evaluate the expression obtained in (*b*) for copper:

$$\frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\text{ eV/K})(300\text{ K})}{2(7.04\text{ eV})} = \boxed{5.51\times10^{-3}}$$

67 ••

Picture the Problem The probability that a conduction electron in metal is the Fermi factor.

Express the Fermi factor:

$$f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

Calculate the dimensionless exponent in the Fermi factor:

$$\frac{E-E_F}{kT} = \frac{5.49\text{ eV} - 5.50\text{ eV}}{(8.62\times10^{-5}\text{ eV/K})(300\text{ K})} = -0.387$$

Use this result to calculate the Fermi factor:

$$f(5.49\,\text{eV}) = \frac{1}{e^{-0.387}+1} = \boxed{0.596}$$

68 ••

Picture the Problem We can integrate the density-of-states function, Equation 38-41, to find the number of occupied states *N*. The fraction of these states that are within *kT* of E_F can then be found from the ratio of $kTg(E_F)$ to *N*.

The density of states function is:

$$g(E) = AE^{1/2}$$

where

$$A = \frac{8\pi\sqrt{2}m_e^{3/2}V}{h^3}$$

Integrate $g(E)$ from 0 to E_F to find the total number of occupied states:

$$N = \int_0^{E_F} AE^{1/2}dE = \tfrac{2}{3}AE_{E_F}^{3/2}$$

Express the fraction of *N* within *kT* of E_F:

$$\frac{kTg(E_F)}{N} = \frac{kTAE_F^{1/2}}{\frac{2}{3}AE_F^{3/2}} = \boxed{\frac{3kT}{2E_F}}$$

(*a*) Substitute numerical values and evaluate the expression obtained above for copper at $T = 77$ K:

$$\frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\,\text{eV/K})(77\,\text{K})}{2(7.04\,\text{eV})} = \boxed{1.41\times10^{-3}}$$

(*b*) At $T = 300$ K:

$$\frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}{2(7.04\,\text{eV})} = \boxed{5.51\times10^{-3}}$$

69 ••

Picture the Problem The distribution function of electrons in the conduction band is given by $n(E) = g(E)f(E)$ where $f(E)$ is the Fermi factor and $g(E)$ is the density of states in terms of E_F.

Express the number of electrons *n* with energy *E*:

$$n(E) = g(E)f(E) \qquad (1)$$

where

$$g(E) = \frac{3N}{2}E_{E_F}^{-3/2}E^{1/2}$$

and

$$f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$$

The dimensionless exponent in the Fermi factor is:

$$\frac{E-E_F}{kT}=\frac{E-\frac{1}{2}E_g}{kT}>>1$$

and

$$\exp\left(\frac{E-\frac{1}{2}E_g}{kT}\right)>>1$$

Hence:

$$f(E)=\frac{1}{e^{\left(E-\frac{1}{2}E_g\right)/kT}}\approx e^{E_g/2kT}e^{-E/kT}$$

Substitute in equation (1) and simplify to obtain:

$$n(E)=\boxed{\left(\tfrac{3}{2}NE_F^{-3/2}e^{E_g/2kT}\right)E^{1/2}e^{-E/kT}}$$

There is an additional temperature dependence that arises from the fact that E_F depends on T. At room temperature, $\exp[(E-E_g/2)/kT]\geq\exp(0.35\text{ eV}/0.0259\text{ eV})=7.4\times10^5$, so the approximation leading to the Boltzmann distribution is justified.

***70 •••**

Picture the Problem We can follow the step-by-step procedure outlined in the problem statement to obtain the indicated results.

(*a*) The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_F)/kT}+1}=\frac{1}{e^{-E_F/kT}e^{E/kT}+1}$$

$$=\boxed{\frac{1}{Ce^{E/kT}+1}}$$

provided $C=e^{-E_F/kT}$

(*b*) If $C>>e^{-E/kT}$:

$$f(E)=\frac{1}{Ce^{E/kT}+1}\approx\frac{1}{Ce^{E/kT}}=\boxed{Ae^{-E/kT}}$$

where $A=1/C$

(*c*) The energy distribution function is:

$$n(E)dE=g(E)dE\,f(E)$$

where

$$g(E)=\frac{8\pi\sqrt{2}m_e^{3/2}V}{h^3}E^{1/2}$$

Substitute for $g(E)dE$ *and* $f(E)$ in the expression for N to obtain:

$$N=A\frac{8\pi\sqrt{2}m_e^{3/2}V}{h^3}\int_0^\infty E^{1/2}e^{-E/kT}dE$$

The definite integral has the value: $\int_0^\infty E^{1/2}e^{-E/kT}dE = \frac{(kT)^{3/2}}{2}\sqrt{\pi}$

Substitute to obtain: $N = A\frac{8\pi\sqrt{2}m_e^{3/2}V}{h^3}\frac{(kT)^{3/2}}{2}\sqrt{\pi}$

Solve for A: $\boxed{A = \frac{\sqrt{2}h^3}{8\pi^{3/2}m_e^{3/2}}\left(\frac{N}{V}\right)\frac{1}{(kT)^{3/2}}}$

(*d*) Evaluate A at T = 300 K:

$$A = \frac{\sqrt{2}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^3 n}{8\pi^{3/2}\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}\left[\left(1.38\times10^{-23}\ \text{J/K}\right)(300\ \text{K})\right]^{3/2}} \approx 4\times10^{-26}n$$

where the units are SI.

The valence electron concentration is typically about $10^{39}\ \text{m}^{-3}$. To satisfy the condition that $A << 1$ at room temperature, n should be less than $10^{23}\ \text{m}^{-3}$, or about one millionth of the valence electron concentration. Because A depends on $T^{-3/2}$, the electron concentration may be greater the higher the temperature.

(*e*) $10^{17}\ \text{cm}^{-3} = 10^{23}\ \text{m}^{-3}$. So, according to the criterion in (*d*), the classical approximation is applicable.

71 •••

Picture the Problem We can approximate the separation of electrons in the gas by $(V/N)^{1/3}$ and use the for A from Problem 70 and de Broglie's equation to express the separation d of electrons in terms of the de Broglie wavelength λ and the constant A.

The separation d of electrons is approximately: $d = \left(\frac{V}{N}\right)^{1/3}$

From Problem 70: $\left(\frac{V}{N}\right)^{1/3} = \frac{2^{1/6}h}{8^{1/3}\pi^{1/2}m_e^{1/2}}\frac{1}{(kT)^{1/2}A^{1/3}}$

Substitute to obtain:

$$d = \frac{2^{1/6}h}{8^{1/3}\pi^{1/2}m_e^{1/2}}\frac{1}{(kT)^{1/2}A^{1/3}} = \frac{2^{1/6}h}{\pi^{1/2}}\frac{1}{\sqrt{2m_e kT}\,A^{1/3}}$$

Express the momentum of an electron in the gas in terms of its de Broglie wavelength λ:

$$p = \frac{h}{\lambda} = \sqrt{2mK} = \sqrt{2m_e kT}$$

Substitute for $\sqrt{2m_e kT}$ in the expression for d to obtain:

$$d = \frac{2^{1/6}h}{\pi^{1/2}}\frac{\lambda}{hA^{1/3}} = \frac{2^{1/6}}{\pi^{1/2}}\frac{\lambda}{A^{1/3}}$$
$$= 0.633\frac{\lambda}{A^{1/3}}$$

$$\boxed{\text{Thus, if } A << 1, d >> \lambda}$$

72 •••

Picture the Problem We can follow the procedure outlined in the problem statement to determine the rms energy of a Fermi distribution.

Express the E_{rms} in terms of $g(E)$:

$$E_{rms} = \left(\frac{1}{N}\int_0^{E_F} g(E)E^2 dE\right)^{1/2}$$

The density of states $g(E)$ is given by:

$$g(E) = \frac{3N}{2}E_F^{-3/2}E^{1/2}$$

Substitute to obtain:

$$E_{rms} = \left(\frac{1}{2NE_F^{3/2}}\int_0^{E_F} E^{3/2}dE\right)^{1/2}$$

Evaluate the integral and simplify:

$$E_{rms} = \sqrt{\frac{3}{7}}E_F = \boxed{0.655E_F}$$

$\boxed{E_{rms} > E_{av} \text{ because the process of averaging the square of the energy weighs larger energies more heavily.}}$

General Problems

73 •

Picture the Problem The number of free electrons per atom n_e is given by $n_e = nM/\rho N_A$ where N_A is Avogadro's number, ρ is the density of the element, M is its molar mass, and n is the free electron number density for the element.

The number of electrons per atom is given by:

$$n_e = \frac{nM}{\rho N_A}$$

Substitute numerical values and evaluate n_e:

$$n_e = \frac{\left(1.4\times10^{22}\text{ electrons/cm}^3\right)\left(39.098\text{ g/mol}\right)}{\left(0.851\text{g/cm}^3\right)\left(6.02\times10^{23}\text{ electrons/mol}\right)} = \boxed{1.07}$$

74 •

Picture the Problem The number of free electrons per atom n_e is related to the number density of free electrons n by $n_e = nM/\rho N_A$, where N_A is Avogadro's number, ρ is the density of the element, and M is its molar mass.

The number of electrons per atom is given by:

$$n_e = \frac{nM}{\rho N_A}$$

Solve for n to obtain:

$$n = \frac{n_e \rho N_A}{M}$$

(a) Substitute numerical values and evaluate n for Mg:

$$n = \frac{(2)\left(1.74\text{ g/cm}^3\right)\left(6.02\times10^{23}\text{ electrons/mol}\right)}{24.31\text{ g/mol}} = \boxed{8.62\times10^{22}\text{ electrons/cm}^3}$$

(b) Substitute numerical values and evaluate n for Zn:

$$n = \frac{(2)\left(7.1\text{g/cm}^3\right)\left(6.02\times10^{23}\text{ electrons/mol}\right)}{65.38\text{ g/mol}} = \boxed{13.1\times10^{22}\text{ electrons/cm}^3}$$

$$\boxed{\text{Both results agree with the values in Table 38 - 1 to within 1\%.}}$$

75 ••

Picture the Problem We can integrate $g(E)$ from 0 to E_F to show that the total number of states is $\frac{2}{3}AE_F^{3/2}$ and then use this result to find the fraction of the free electrons that are above the Fermi energy at the given temperatures.

Integrate $g(E)$ from 0 to E_F:

$$N = \int_0^{E_F} AE^{1/2}dE = \tfrac{2}{3}AE_{E_F}^{3/2}$$

Express the fraction of N within kT of E_F:

$$\frac{kTg(E_F)}{N} = \frac{kTAE_F^{1/2}}{\frac{2}{3}AE_F^{3/2}} = \frac{3kT}{2E_F}$$

(*a*) Substitute numerical values and evaluate this fraction for copper at 300 K:

$$\frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\text{ eV/K})(300\text{ K})}{2(7.04\text{ eV})} = \boxed{5.51\times10^{-3}}$$

(*b*) Evaluate the same fraction at 1000 K:

$$\frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\text{ eV/K})(1000\text{ K})}{2(7.04\text{ eV})} = \boxed{1.84\times10^{-2}}$$

***76 ••**

Picture the Problem The Fermi factor gives the probability of an energy state being occupied as a function of the energy of the state *E*, the Fermi energy E_F for the particular material, and the temperature *T*.

The Fermi factor is:

$$f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

For 10 percent probability:

$$0.1 = \frac{1}{e^{(E-E_F)/kT}+1}$$

or

$$e^{(E-E_F)/kT} = 9$$

Take the natural logarithm of both sides of the equation to obtain:

$$\frac{E-E_F}{kT} = \ln 9$$

Solve for *E* to obtain:

$$E = E_F + kT\ln 9$$

From Table 37-1, E_F(Mn) = 11.0 eV. Substitute numerical values and evaluate *E*:

$$E = 11.0\text{ eV} + (1.38\times10^{-23}\text{ J/K})(1300\text{ K})\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)\ln 9 = \boxed{11.2\text{ eV}}$$

77 ••

Picture the Problem The energy gap for the semiconductor is related to the wavelength of the emitted light according to $E_g = hc/\lambda$.

Express the energy gap *E*g in terms of the wavelength λ of the emitted light:

$$E_g = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{E_g}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{1.8\,\text{eV}} = \boxed{689\,\text{nm}}$$

Remarks: This wavelength is in the red portion of the visible spectrum.

***78** •••

Picture the Problem The rate of production of electron-hole pairs is the ratio of the incident energy to the energy required to produce an electron-hole pair.

(*a*)The number of electron-hole pairs N produced in one second is:

$$N = \frac{IA}{\frac{hc}{\lambda}} = \frac{IA\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{(4.0\,\text{W/m}^2)(2\times10^{-4}\,\text{m}^2)(775\,\text{nm})}{(1240\,\text{eV}\cdot\text{nm})(1.60\times10^{-19}\,\text{J/eV})} = \boxed{3.12\times10^{15}\,\text{s}^{-1}}$$

(*b*) In the steady state, the rate of recombination equals the rate of generation. Therefore:

$$N = \boxed{3.12\times10^{15}\,\text{s}^{-1}}$$

(*c*) The power radiated equals the power absorbed:

$$P_{\text{rad}} = IA$$

Substitute numerical values and evaluate P_{rad}:

$$P_{\text{rad}} = (4.0\,\text{W/m}^2)(2\times10^{-4}\,\text{m}^2) = \boxed{0.800\,\text{mJ/s}}$$

Chapter 39
Relativity

Conceptual Problems

***1 •**

Picture the Problem The total relativistic energy E of a particle is defined to be the sum of its kinetic and rest energies.

The total relativistic energy of a particle is given by:

$$E = K + mc^2 = \tfrac{1}{2}mu^2 + mc^2$$

and $\boxed{(a)\text{ is correct.}}$

***2 •**

Determine the Concept The gravitational field of the earth is slightly greater in the basement of the office building than it is at the top floor. Because clocks run more slowly in regions of low gravitational potential, clocks in the basement will run more slowly than clocks on the top floor. Hence, the twin who works on the top floor will age more quickly. $\boxed{(b)\text{ is correct.}}$

3 •

(*a*) True

(*b*) True

(*c*) False. The shortening of the length of an object in the direction in which it is moving is independent of the velocity of the frame of reference from which it is observed.

(*d*) True

(*e*) False. Consider two explosions equidistant, but in opposite directions, from an observer in the observer's frame of reference.

(*f*) False. Whether events appear to be simultaneous depends on the motion of the observer.

(*g*) True

4 •

Determine the Concept Because the clock is moving with respect to the first observer, a time interval will be longer for this observer than for the observer moving with the spring-and-mass oscillator. Hence, the observer moving with the system will measure a period that is less than T. $\boxed{(b)\text{ is correct.}}$

5 •

Determine the Concept Although $\Delta y = \Delta y'$, $\Delta t \neq \Delta t'$. Consequently, $u_y = \Delta y/\Delta t' \neq \Delta y'/\Delta t' = u_y'$.

Estimation and Approximation

6 ••

Picture the Problem We can calculate the sun's loss of mass per day from the number of reactions per second and the loss of mass per reaction.

Express the rate at which the sun loses mass:

$$\frac{\Delta M}{\Delta t} = N\Delta m$$

where N is the number of reactions per second and Δm is the loss of mass per reaction.

Solve for ΔM:

$$\Delta M = N\Delta m\Delta t \qquad (1)$$

Find the number of reactions per second, N:

$$N = \frac{P}{E\,/\,\text{reaction}} = \frac{4\times10^{26}\ \text{J/s}}{25\dfrac{\text{MeV}}{\text{reaction}}\times1.60\times10^{-19}\dfrac{\text{J}}{\text{eV}}} = 10^{38}\ \text{s}^{-1}$$

The loss of mass per reaction Δm is:

$$\Delta m = \frac{E\,/\,\text{reaction}}{c^2} = \frac{25\dfrac{\text{MeV}}{\text{reaction}}\times1.60\times10^{-19}\dfrac{\text{J}}{\text{eV}}}{\left(3\times10^{8}\ \text{m/s}\right)^2} = 4.44\times10^{-29}\ \text{kg}$$

Substitute numerical values in equation (1) and evaluate ΔM:

$$\Delta M = \left(10^{38}\ \text{s}^{-1}\right)\left(4.44\times10^{-29}\ \text{kg}\right)\left(1\,\text{d}\right)\left(86.4\,\text{ks/d}\right) = \boxed{3.84\times10^{14}\ \text{kg}}$$

***7** ••

Picture the Problem We can use the result from Problem 30, for light that is Doppler-shifted with respect to an observer, $v = c\left(\frac{u^2-1}{u^2+1}\right)$, where $u = z + 1$ and z is the red-shift parameter, to find the ratio of v to c. In (*b*) we can solve Hubble's law for x and substitute our result from (*a*) to estimate the distance to the galaxy.

(*a*) Use the result of Problem 30 to express v/c as a function of z:

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

Substitute for z and evaluate v/c:

$$\frac{v}{c} = \frac{(5+1)^2 - 1}{(5+1)^2 + 1} = \boxed{0.946}$$

(*b*) Solve Hubble's law for x:

$$x = \frac{v}{H}$$

Substitute numerical values and evaluate x:

$$x = \frac{0.946c}{H} = \frac{0.946(3\times10^5\ \text{km/s})}{75\,\frac{\text{km/s}}{\text{Mpc}}}$$

$$= 3.78\times10^3\ \text{Mpc}\times\frac{3.26\times10^6\ c\cdot\text{y}}{\text{Mpc}}$$

$$= \boxed{12.3\,\text{G}c\cdot\text{y}}$$

Time Dilation and Length Contraction

8 •

Picture the Problem We can find the mean lifetime of a muon as measured in the laboratory using $t' = \gamma t$ where $\gamma = 1\big/\sqrt{1-(v/c)^2}$ and t is the proper mean lifetime of the muon. The distance L that the muon travels is the product of its speed and its mean lifetime in the laboratory.

(*a*) The mean lifetime of the muon, as measured in the laboratory, is given by:

$$t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Substitute numerical values and evaluate t':

$$t' = \frac{2\,\mu\text{s}}{\sqrt{1-\left(\frac{0.95c}{c}\right)^2}} = \boxed{6.41\,\mu\text{s}}$$

(*b*) The distance L that the muon travels is related to its mean lifetime in the laboratory:

$$L = vt'$$

Substitute numerical values and evaluate L:

$$L = 0.95ct' = 0.95\left(3\times10^{8}\text{ m/s}\right)\left(6.41\,\mu\text{s}\right) = \boxed{1.83\,\text{km}}$$

9 ••

Picture the Problem The proper length L_p of the beam is its length as measured in a reference frame in which it is not moving. The proper length is related to its length in the frame in which it is measured by $L_p = \gamma L$.

(*a*) Relate the proper length L_p of the beam to its length L in the laboratory frame of reference:

$$L_p = \gamma L$$

The energy of the beam also depends on γ:

$$E = \gamma mc^2$$

Solve for and evaluate γ:

$$\gamma = \frac{E}{mc^2} = \frac{50\,\text{GeV}}{0.511\,\text{MeV}} = 9.785\times10^{4}$$

Substitute numerical values and evaluate L_p:

$$L_p = \left(9.785\times10^{4}\right)\left(1\,\text{cm}\right) = \boxed{978.5\,\text{m}}$$

and

$\boxed{\text{The width } w \text{ of the beam is unchanged.}}$

(*b*) Express the length of the accelerator in the electron beam's frame of reference:

$$L_{acc} = \frac{L_{acc,p}}{\gamma}$$

Set $L_{acc} = L_p$:

$$L_p = \frac{L_{acc,p}}{\gamma}$$

Solve for $L_{acc,p}$:

$$L_{acc,p} = \gamma L_p$$

Substitute numerical values and evaluate L_p:

$$L_{acc,p} = (9.785\times10^4)(978.5\,\text{m}) = \boxed{9.57\times10^7\ \text{m}}$$

(*c*) The length of the positron bundle in the electron's frame of reference is:

$$L_{pos} = \frac{L}{\gamma}$$

Substitute numerical values and evaluate L_{pos}:

$$L_{pos} = \frac{1\,\text{cm}}{9.785\times10^4} = \boxed{0.102\,\mu\text{m}}$$

***10** ••

Picture the Problem The time required for the particles to reach the detector, as measured in the laboratory frame of reference is the ratio of the distance they must travel to their speed. The half life of the particles is the trip time as measured in a frame traveling with the particles. We can find the speed at which the particles must move if they are to reach the more distant detector by equating their half life to the ratio of the distance to the detector in the particle's frame of reference to their speed.

(*a*) The time required to reach the detector is the ratio of the distance to the detector and the speed with which the particles are traveling:

$$\Delta t = \frac{\Delta x}{v} = \frac{\Delta x}{0.866c}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1000\,\text{m}}{0.866(3\times10^8\ \text{m/s})} = \boxed{3.85\,\mu\text{s}}$$

(*b*) The half life is the trip time as measured in a frame traveling with the particles:

$$\Delta t' = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1-\left(\frac{v}{c}\right)^2}$$

Substitute numerical values and evaluate $\Delta t'$:

$$\Delta t' = 3.85\,\mu\text{s}\sqrt{1-\left(\frac{0.866c}{c}\right)^2} = \boxed{1.93\,\mu\text{s}}$$

(*c*) In order for half the particles to reach the detector:

$$\Delta t' = \frac{\Delta x'}{\gamma v} = \frac{\Delta x'\sqrt{1-\left(\frac{v}{c}\right)^2}}{v}$$

where $\Delta x'$ is the distance to the new detector.

Rewrite this expression to obtain:

$$\frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=\frac{\Delta x'}{\Delta t'}$$

Squaring both sides of the equation yields:

$$\frac{v^2}{1-\left(\frac{v}{c}\right)^2}=\left(\frac{\Delta x'}{\Delta t'}\right)^2$$

Substitute numerical values for $\Delta x'$ and $\Delta t'$ and simplify to obtain:

$$\frac{v^2}{1-\left(\frac{v}{c}\right)^2}=\left(\frac{10^4\text{ m}}{1.93\,\mu\text{s}}\right)^2=(17.3c)^2$$

Divide both sides of the equation by c^2 to obtain:

$$\frac{\frac{v^2}{c^2}}{1-\left(\frac{v}{c}\right)^2}=(17.3)^2$$

Solve this equation for v^2/c^2:

$$\frac{v^2}{c^2}=\frac{(17.3)^2}{1+(17.3)^2}=0.9967$$

Finally, solving for v yields:

$$v=\boxed{0.998c}$$

11 ••

Picture the Problem We can use the time-dilation relationship to find the speed of the spacecraft. The distance to the second star is the product of the new gamma factor, the speed of the spacecraft, and the elapsed time. Finally, the time that has elapsed on earth (your age) is the sum of the elapsed times for the three legs of the journey.

(*a*) From the point of view of an observer on earth, the time for the trip will be:

$$\Delta t=\frac{L}{v}$$

From the point of view of an observer on the spaceship, the time for the trip will be:

$$\Delta t'=\frac{\Delta t}{\gamma}=\frac{L}{\gamma v}$$

Substitute for γ to obtain:

$$\Delta t'=\frac{L}{v}\sqrt{1-\frac{v^2}{c^2}}$$

Solve for v:

$$v = \frac{Lc}{\sqrt{L^2 + c^2(\Delta t')^2}}$$

Substitute numerical values and evaluate v:

$$v = \frac{(27\,c\cdot\text{y})c}{\sqrt{(27\,c\cdot\text{y})^2 + c^2(12\,\text{y})^2}} = \boxed{0.914c}$$

Note that from the point of view of an earth observer, this part of the trip has taken 27 $c\cdot$ y/0.914c = 29.5 y.

(b) The distance the ship travels, from the point of view of an earth observer, in 5 y is:

$$\Delta L' = 2\gamma\Delta L = 2\gamma v\Delta t$$

where γ is the gamma factor for the first part of the trip.

The gamma factor in Part (a) is:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{0.914c}{c}\right)^2}} = 2.46$$

Substitute numerical values and evaluate $\Delta L'$:

$$\Delta L' = 2(2.46)(0.914c)(5\,\text{y}) = \boxed{22.5\,c\cdot\text{y}}$$

(c) The elapsed time Δt on earth (your age) is the sum of the times for the spacecraft to travel to the star 27 $c\cdot$ y away, l to the second star, and to return home from the second star:

$$\Delta t = 29.5\,\text{y} + 22.5\,\text{y} + \Delta t_{\text{returning home}}$$

The elapsed time on earth while the spacecraft is returning to earth is:

$$\Delta t_{\text{returning home}} = 2\gamma\Delta t_{\text{ship's time}} = 2(2.46)(10\,\text{y}) = 49.2\,\text{y}$$

Substitute for $\Delta t_{\text{returning}}$ home and evaluate Δt:

$$\Delta t = 29.5\,\text{y} + 22.5\,\text{y} + 49.2\,\text{y} = \boxed{101\,\text{y}}$$

12 •

Picture the Problem We can use $\Delta t = L/v$, where L is the distance to the star and v is the speed of the spaceship to find the time Δt for the trip as measured on earth. The travel time as measured by a passenger on the spaceship can be found using $\Delta t' = \Delta t/\gamma$.

(a) The travel time as measured on earth is the ratio of the distance

$$\Delta t = \frac{L}{v}$$

traveled L to speed of the spaceship:

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{35c\cdot\text{y}}{2.7\times10^8\ \text{m/s}} = \frac{35\,\text{y}}{\dfrac{2.7\times10^8\ \text{m/s}}{c}}$$

$$= \frac{35\,\text{y}}{0.9} = \boxed{38.9\,\text{y}}$$

(a) The travel time as measured by a passenger on the spaceship is given by:

$$\Delta t' = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1-\left(\frac{v}{c}\right)^2}$$

Substitute numerical values and evaluate $\Delta t'$:

$$\Delta t' = (38.9\,\text{y})\sqrt{1-(0.9)^2} = \boxed{17.0\,\text{y}}$$

13 •

Picture the Problem We can use the definition of γ and the binomial expansion of $(1 + x)^n$ to show that each of these relationships holds provided $v << c$.

(a) Express the gamma factor:

$$\gamma = \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

Expand the radical factor binomially to obtain:

$$\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$= 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \text{higher order terms}$$

For $v << c$:

$$\gamma \approx \boxed{1+\frac{1}{2}\frac{v^2}{c^2}}$$

(b) Express the reciprocal of γ:

$$\frac{1}{\gamma} = \sqrt{1-\frac{v^2}{c^2}}$$

Expand the radical binomially to obtain:

$$\frac{1}{\gamma} = \left(1-\frac{v^2}{c^2}\right)^{1/2}$$

$$= 1 + \left(\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \text{higher order terms}$$

For $v << c$:

$$\frac{1}{\gamma} \approx \boxed{1 - \frac{1}{2}\frac{v^2}{c^2}}$$

(*c*) Express the gamma factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Subtract one from both sides of the equation to obtain:

$$\gamma - 1 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1$$

Expand the radical binomially to obtain:

$$\gamma - 1 = 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) - 1 + \text{higher order terms}$$

For $v << c$:

$$\gamma - 1 \approx \boxed{\frac{1}{2}\frac{v^2}{c^2}}$$

14 ••

Picture the Problem We can express the fractional difference in your time-interval measurements as a function of γ and solve the resulting equation for the relative speed of the two spaceships.

Express the fractional difference in the time-interval measurements of the two observers:

$$\frac{\Delta t - \Delta t'}{\Delta t} = 1 - \frac{\Delta t'}{\Delta t} = 0.01$$

Since $\Delta t'/\Delta t = 1/\gamma$:

$$\frac{\Delta t - \Delta t'}{\Delta t} = 1 - \frac{1}{\gamma} = 0.01$$

From Problem 13(*b*) we have:

$$\frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

Substitute to obtain:

$$1 - \frac{1}{\gamma} \approx 1 - \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right) = 0.01$$

or

$$\frac{1}{2}\frac{v^2}{c^2} = 0.01$$

Solve for v to obtain:

$$v = \sqrt{0.02}c = 0.141c = \boxed{4.23\times10^{7}\text{ m/s}}$$

15 ••

Picture the Problem We can use the time dilation equation to relate the time lost by the clock to the speed of the plane and the time it must fly.

Express the time δt lost by the clock:

$$\delta t = \Delta t - \Delta t_{\text{p}} = \Delta t - \frac{\Delta t}{\gamma} = \Delta t\left(1 - \frac{1}{\gamma}\right)$$

Because $V << c$, we can use part (*b*) of Problem 13:

$$\frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{V^2}{c^2}$$

Substitute to obtain:

$$\delta t = \Delta t\left[1 - \left(1 - \frac{1}{2}\frac{V^2}{c^2}\right)\right] = \frac{1}{2}\frac{V^2}{c^2}\Delta t$$

Solve for Δt:

$$\Delta t = \frac{2\delta t\, c^2}{V^2}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(1\text{ s})(3\times10^{8}\text{ m/s})^2}{(2000\text{ km/h}\times1\text{h/3600 s})^2}$$
$$= 5.83\times10^{11}\text{ s}\times\frac{1\text{ y}}{31.56\text{ Ms}}$$
$$= \boxed{1.85\times10^{4}\text{ y}}$$

The Lorentz Transformation, Clock Synchronization, and Simultaneity

16 ••

Picture the Problem We can use the inverse Lorentz transformations and the result of Problem 13(*c*) to show that when $u << c$ the transformation equations for x, t, and u reduce to the Galilean equations.

The inverse transformation for x is:

$$x' = \gamma(x - vt)$$

From Problem 13(*c*):

$$\gamma = 1 + \frac{1}{2}\frac{v^2}{c^2}$$

Substitute for γ and expand to obtain:

$$x' = \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)(x - vt)$$

$$= x - vt + \frac{1}{2}\frac{v^2}{c^2}x - \frac{1}{2}\frac{v^3}{c^2}$$

When $v << c$:

$$\boxed{x' \approx x - vt}$$

The inverse transformation for t is:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Substitute for γ and expand to obtain:

$$t' = \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)\left(t - \frac{vx}{c^2}\right)$$

$$= t - \frac{vx}{c^2} + \frac{1}{2}\frac{v^2}{c^2}t - \frac{1}{2}\frac{v^3}{c^4}x$$

When $v << c$:

$$\boxed{t' \approx t}$$

The inverse velocity transformation for motion in the x direction is:

$$u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

When $v << c$:

$$\boxed{u_x' \approx u_x - v}$$

The inverse velocity transformation for motion in the y direction is:

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

Substitute for γ and expand to obtain:

$$u_y' = \frac{u_y}{\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)\left(1 - \frac{vu_x}{c^2}\right)}$$

$$= \frac{u_y}{1 - \frac{vu_x}{c^2} + \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{2}\frac{v^3u_x}{c^4}}$$

When $v << c$:

$$\boxed{u_y' \approx u_y}$$

Proceed similarly to show that:

$$\boxed{u_z' \approx u_z}$$

***17** ••

Picture the Problem Let S be the reference frame of the spaceship and S' be that of the earth (transmitter station). Let event A be the emission of the light pulse and event B the reception of the light pulse at the nose of the spaceship. In (*a*) and (*c*) we can use the classical distance, rate, and time relationship and in (*b*) and (*d*) we can apply the inverse Lorentz transformations.

(*a*) In both *S* and *S*′ the pulse travels at the speed *c*. Thus:

$$t_A = \frac{L_p}{v} = \frac{400\,\text{m}}{0.76\text{c}} = \boxed{1.76\,\mu\text{s}}$$

(*c*) The elapsed time, according to the clock on the ship is:

$$t_B = t_{\text{pulse to travel length of ship}} + t_\text{A}$$

Find the time of travel of the pulse to the nose of the ship:

$$t_{\text{pulse to travel length of ship}} = \frac{400\,\text{m}}{2.998\times10^8\ \text{m/s}} = 1.33\,\mu\text{s}$$

Substitute numerical values and evaluate t_B:

$$t_B = 1.33\,\mu\text{s} + 1.76\,\mu\text{s} = \boxed{3.09\,\mu\text{s}}$$

(*b*) The inverse time transformation is:

$$t_B' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.76c)^2}{c^2}}} = 1.54$$

Substitute numerical values and evaluate t'_B:

$$t_B' = (1.54)\left(3.09\,\mu\text{s} - \frac{(-0.76c)(400\,\text{m})}{c^2}\right)$$

$$= (1.54)\left(3.09\,\mu\text{s} - \frac{(-0.76)(400\,\text{m})}{3\times10^8\ \text{m/s}}\right)$$

$$= \boxed{6.32\,\mu\text{s}}$$

(*d*) The inverse transformation for *x* is:

$$x' = \gamma(x - vt)$$

Substitute numerical values and evaluate x':

$$x' = (1.54)\left[400\,\text{m} - (-0.76)(3\times10^8\ \text{m/s})(3.09\times10^{-6}\ \text{s})\right] = \boxed{1.70\,\text{km}}$$

18 ••

Picture the Problem We can use Equation 39-12, the inverse time transformation equation, to find the required speed of the observer.

Use Equation 39-12 to obtain:

$$t_B' - t_A' = \gamma\left[(t_B - t_A) - \frac{v}{c^2}(x_B - x_A)\right]$$
$$= \boxed{\gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)}$$

where $\Delta t = t_B - t_A$ and $\Delta x = x_B - x_A$.

Events A and B are simultaneous if:

$$\Delta t - \frac{v\Delta x}{c^2} = 0$$

Solve for v:

$$v = \frac{c^2\Delta t}{\Delta x}$$

Substitute numerical values and evaluate v:

$$v = \frac{(3\times10^8\text{ m/s})^2(2\,\mu\text{s})}{1.5\,\text{km}}$$
$$= 1.20\times10^8\text{ m/s} = \boxed{0.4c}$$

$$\boxed{\text{Yes, } t_B' \text{ will be less than } t_A' \text{ if } V > 0.4c.}$$

19 ••

Picture the Problem We can use Equation 39-12, the inverse time transformation equation, to express the separation in time between the two explosions as measured in S' as a function of the speed of the observer and Equation 39-11, the inverse position transformation equation, to find the speed of the observer.

Use Equation 39-12 to obtain:

$$\Delta t' = \gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]$$
$$= \frac{\Delta t - \frac{v}{c^2}\Delta x}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad (1)$$

From Equation 39-11:

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

Because the explosions occur at the same point in space, $\Delta x' = 0$:

$$0 = \gamma(\Delta x - v\Delta t)$$

Solve for v:

$$v = \frac{\Delta x}{\Delta t}$$

Substitute numerical values and evaluate v:

$$v=\frac{1200\,\text{m}-480\,\text{m}}{5\,\mu\text{s}}=1.44\times10^{8}\ \text{m/s}$$

Substitute numerical values in equation (1) and evaluate $\Delta t'$:

$$\Delta t'=\frac{5\,\mu\text{s}-\dfrac{1.44\times10^{8}\ \text{m/s}}{\left(3\times10^{8}\ \text{m/s}\right)^{2}}\left(1200\,\text{m}-480\,\text{m}\right)}{\sqrt{1-\left(\dfrac{1.44\times10^{8}\ \text{m/s}}{3\times10^{8}\ \text{m/s}}\right)^{2}}}=\boxed{4.39\,\mu\text{s}}$$

20 •••

Picture the Problem We can use Equation 39-12, the inverse time transformation equation, to establish the results called for in this problem.

(*a*) Use Equation 39-12 to obtain:

$$t_2'-t_1'=\gamma\left[\left(t_2-t_1\right)-\frac{v}{c^2}\left(x_2-x_1\right)\right]$$

$$=\boxed{\gamma\left(T-\frac{vD}{c^2}\right)}$$

where $T = t_2 - t_1$ and $D = x_2 - x_1$.

(*b*) Events 1 and 2 are simultaneous in S' if:

$$t_2'=t_1'$$

or

$$T-\frac{vD}{c^2}=0\Rightarrow D=\frac{c^2T}{v}$$

Because $v \leq c$:

$$D\geq\boxed{cT}$$

(*c*) $\boxed{\text{If } D<cT\text{, then } t_2'>t_1' \text{ and the events are not simultaneous in } S'.}$

(*d*) If $D = c'T > cT,$ then:

$$T-\frac{vD}{c^2}=T\left[1-\frac{v}{c}\frac{c'}{c}\right]=t_2'-t_1'$$

$\boxed{\text{In this case, } t_2'-t_1' \text{ could be negative; i.e., } t_2' \text{ could be less than } t_1'\text{, or the effect could precede the cause.}}$

21 •••

Picture the Problem Let S be the ground reference frame, S' the reference frame of the rocket, and $v = 0.9c$ be the speed of the rocket relative to S. Denote the tail and nose of the rocket by T and N, respectively. The initial conditions in S' are $t_N' = 0$, $x_N' = 0$, $x_T' = 0$, and $x_T' = -L' = -700\,\text{m}$.

(*a*) The reading of the tail clock is given by:

$$t_T' = \gamma\left(t_T - \frac{vx_T}{c^2}\right) = -\frac{\gamma v x_T}{c^2}$$

because $t_T = 0$

We can find x_T using the length contraction equation:

$$x_T = -\frac{L'}{\gamma}$$

Substitute to obtain:

$$t_T' = \frac{vL'}{c^2}$$

Substitute numerical values and evaluate t_T':

$$t_T' = \frac{(0.9)(700\,\text{m})}{3\times10^8\,\text{m/s}} = \boxed{2.10\,\mu\text{s}}$$

(*b*) The time for the rocket to move a distance L' is given by:

$$t_T' = \frac{L'}{v} = \frac{L'}{0.9c}$$

Substitute numerical values and evaluate t_T':

$$t_T' = \frac{700\,\text{m}}{0.9(3\times10^8\,\text{m/s})} = \boxed{2.59\,\mu\text{s}}$$

(*c*) As seen by an observer on the ground:

$$t_N = \Delta t' = 2.59\,\mu\text{s} - 2.10\,\mu\text{s} = \boxed{0.49\,\mu\text{s}}$$

(*d*) Because the clocks are synchronized in S':

$$t_N' = t_T' = \boxed{2.59\,\mu\text{s}}$$

(*e*) The time the signal is received on the ground is the sum of the time when the signal is sent and the time for it to travel to the ground:

$$t_{\text{rec}} = \Delta t + \Delta t_{\text{travel}}$$

Find Δt, the time the signal is sent:

$$\Delta t = \gamma \Delta t_{\text{p}} = \frac{1\,\text{h}}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = 2.294\,\text{h}$$

Find Δt_{travel}, the time for the signal to travel to the ground:

$$\Delta t_{\text{travel}} = \frac{\Delta x}{c} = \frac{(2.294\,\text{h})(0.9c)}{c} = 2.065\,\text{h}$$

Substitute for Δt and Δt_{travel} and evaluate t_{rec}:

$$t_{\text{rec}} = 2.294\,\text{h} + 2.065\,\text{h} = \boxed{4.36\,\text{h}}$$

(*f*) Find Δx when the signal is sent:

$$\Delta x = (4.36\,\text{h})(0.9c) = 3.924\,c\cdot\text{h}$$

In *S*, the signal arrives at 0.1*c* relative to the rocket. The time required for the signal to travel to the rocket is:

$$\Delta t = \frac{\Delta x}{0.1c} = \frac{3.924\,c\cdot\text{h}}{0.1c} = 39.24\,\text{h}$$

Find the time when the signal reaches the rocket:

$$t = 39.24\,\text{h} + 3.924\,\text{h} = 43.16\,\text{h}$$

Finally, use the time dilation equation to find t_N':

$$t_N' = \frac{t}{\gamma} = (43.16\,\text{h})\sqrt{1-\frac{(0.9c)^2}{c^2}} = \boxed{18.8\,\text{h}}$$

***22 •••**

Picture the Problem We can use the inverse time dilation equation to derive an expression for the elapsed time between the flashes in *S*′ in terms of the elapsed time between the flashes in *S*, their separation in space, and the speed *v* with which *S*′ is moving.

From the inverse time transformation we have:

$$\Delta t' = \gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]$$

where $\Delta t'$ is the time between the flashes in *S*′ and Δt and Δx are the elapsed time between the flashes and their separation in *S*.

Set $\Delta t' = -\Delta t$ to obtain:

$$\frac{-\Delta t}{\gamma} = \Delta t - \frac{v}{c^2}\Delta x$$

or

$$-\Delta t\sqrt{1-\frac{v^2}{c^2}} = \Delta t - \frac{v}{c^2}\Delta x$$

Square both sides of the equation:

$$(\Delta t)^2 - \frac{v^2}{c^2}(\Delta t)^2 = (\Delta t)^2 - 2\frac{v}{c^2}\Delta x\Delta t + \frac{v^2}{c^4}(\Delta x)^2$$

Simplify to obtain:

$$-v(\Delta t)^2 = -2\Delta x\Delta t + \frac{v}{c^2}(\Delta x)^2$$

Solve for v:

$$v = \frac{2\frac{\Delta x}{\Delta t}}{1+\left(\frac{1}{c}\frac{\Delta x}{\Delta t}\right)^2}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\left(\frac{2400\,\text{m}}{5\,\mu\text{s}}\right)}{1+\left[\frac{1}{3\times10^8\ \text{m/s}}\left(\frac{2400\,\text{m}}{5\,\mu\text{s}}\right)\right]^2} = 2.697\times10^8\ \text{m/s} = \boxed{0.899c}$$

Because v is positive, S' is moving in the positive x direction.

The Velocity Transformation

23 ••

Picture the Problem We can make the substitutions given in the hint in Equation 39-18*a* and simplify the resulting expression to show that $u_x < c$.

Equation 39-18*a* gives the x direction relativistic velocity transformation:

$$u_x = \frac{u_x' + v}{1+\frac{vu_x'}{c^2}} \quad \text{or} \quad \frac{u_x}{c} = \frac{u_x' + v}{c+\frac{vu_x'}{c}}$$

Make the substitutions given in the hint to obtain:

$$\frac{u_x}{c} = \frac{(1-\varepsilon_1)c + (1-\varepsilon_2)c}{c+\frac{(1-\varepsilon_2)c(1-\varepsilon_1)c}{c}} = \frac{2-(\varepsilon_1+\varepsilon_2)}{1+(1-\varepsilon_2)(1-\varepsilon_1)} = \frac{2-(\varepsilon_1+\varepsilon_2)}{2-(\varepsilon_1+\varepsilon_2)+\varepsilon_1\varepsilon_2}$$

Because ε_1 and ε_2 are small positive numbers that are less than 1:

$$\frac{u_x}{c} < 1 \Rightarrow \boxed{u_x < c}$$

***24** ••

Picture the Problem We'll let the velocity (in *S*) of the spaceship after the ith boost be v_i and derive an expression for the ratio of v to c after the spaceship's (i + 1)th boost as a function of *N*. We can use the definition of γ, in terms of v/c to plot γ as a function of *N*.

(*a*) and (*b*) The velocity of the spaceship after the (i + 1)th boost is given by relativistic velocity addition equation:

$$v_{i+1} = \frac{v_i + 0.5c}{1 + \frac{(0.5c)v_i}{c^2}}$$

Factor c from both the numerator and denominator to obtain:

$$v_{i+1} = \frac{\frac{v_i}{c} + 0.5}{1 + 0.5\frac{v_i}{c}}$$

γ_i is given by:

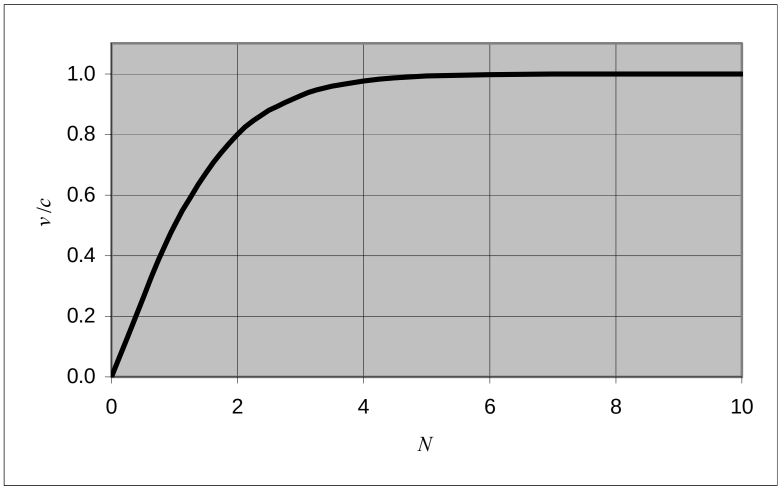
$$\gamma_i = \frac{1}{\sqrt{1 - \left(\frac{v_i}{c}\right)}}$$

A spreadsheet program to calculate v/c and γ as functions of the number of boosts *N* is shown below. The formulas used to calculate the quantities in the columns are as follows:

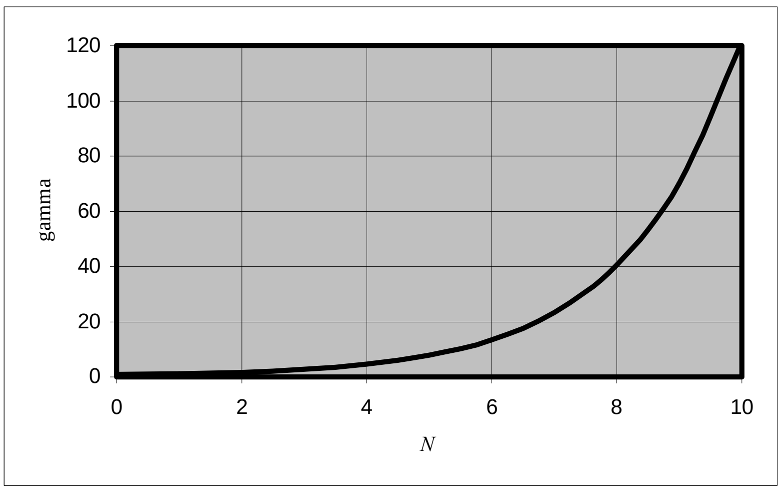
Cell	Content/Formula	Algebraic Form
A3	0	N
B2	0	v_0
B3	(B2+0.5)/(1+0.5*B2)	v_{i+1}
C1	1/(1–B2^2)^0.5	γ

	A	B	C
1	boost	v/c	gamma
2	0	0.000	1.00
3	1	0.500	1.15
4	2	0.800	1.67
5	3	0.929	2.69
6	4	0.976	4.56
7	5	0.992	7.83
8	6	0.997	13.52
9	7	0.999	23.39
10	8	1.000	40.51
11	9	1.000	70.15
12	10	1.000	121.50

A graph of v/c as a function of N is shown below:



A graph of γ as a function of N is shown below:



(*c*) Examination of the spreadsheet or of the graph of v/c as a function of N indicates that, after 8 boosts, the velocity of the spaceship is greater than $0.999c$.

(*d*) After 5 boosts, the spaceship has traveled a distance Δx, measured in the earth frame of reference (S), given by:

$$\begin{aligned}\Delta x &= \Delta x_{1\to 2} + \Delta x_{2\to 3} + \Delta x_{3\to 4} + \Delta x_{4\to 5} \\ &= (0.5c)(10\,\text{s})\gamma_{1\to 2} + (0.8c)(10\,\text{s})\gamma_{2\to 3} + (0.929c)(10\,\text{s})\gamma_{3\to 4} + (0.976c)(10\,\text{s})\gamma_{4\to 5} \\ &\quad + (0.992c)(10\,\text{s})\gamma_{5\to 6} \\ &= (0.5c)(10\,\text{s})(1.15) + (0.8c)(10\,\text{s})(1.67) + (0.929c)(10\,\text{s})(2.69) \\ &\quad + (0.976c)(10\,\text{s})(4.56) + (0.992c)(10\,\text{s})(7.83) \\ &= \boxed{166\,c\cdot\text{s}}\end{aligned}$$

The average speed of the spaceship, between boost 1 and boost 5, as measured in S is given by:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t}$$

where Δt is the travel time as measured in the earth frame of reference.

Express Δt as the sum of the times the spaceship travels during each 10-s interval following a boost in its speed:

$$\begin{aligned}\Delta t &= \Delta t_{1\to2} + \Delta t_{2\to3} + \Delta t_{3\to4} + \Delta t_{4\to5} \\ &= (10\,\text{s})\gamma_{1\to2} + (10\,\text{s})\gamma_{2\to3} + (10\,\text{s})\gamma_{3\to4} + (10\,\text{s})\gamma_{4\to5} + (10\,\text{s})\gamma_{5\to6} \\ &= (10\,\text{s})(\gamma_{1\to2} + \gamma_{2\to3} + \gamma_{3\to4} + \gamma_{4\to5} + \gamma_{5\to6})\end{aligned}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = (10\,\text{s})(1.15 + 1.67 + 2.69 + 4.56 + 7.83) = 179\,\text{s}$$

Substitute for Δx and Δt and evaluate v_{av}:

$$v_{\text{av}} = \frac{166\,c\cdot\text{s}}{179\,\text{s}} = \boxed{0.927c}$$

Remarks: This result seems to be reasonable. Relativistic time dilation implies that the spacecraft will be spending larger amounts of time at high speed (as seen in reference frame *S*).

The Relativistic Doppler Shift

25 •

Picture the Problem We can substitute, using $v = f\lambda$, in the relativistic Doppler shift equation and solve for the speed of the source.

Using the expression for the relativistic Doppler shift, express f' as a function of v:

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}}$$

Substitute using $v = f\lambda$ and simplify to obtain:

$$\frac{v}{\lambda'} = \frac{v}{\lambda}\sqrt{\frac{1+v/c}{1-v/c}}$$

$$\frac{\lambda}{\lambda'} = \sqrt{\frac{1+v/c}{1-v/c}}$$

or

$$\left(\frac{\lambda}{\lambda'}\right)^2 = \frac{1+v/c}{1-v/c}$$

Solve for v to obtain:

$$v=\left[\frac{\left(\frac{\lambda}{\lambda'}\right)^2-1}{\left(\frac{\lambda}{\lambda'}\right)^2+1}\right]c$$

Substitute numerical values and evaluate v:

$$v=\left[\frac{\left(\frac{589\,\text{nm}}{547\,\text{nm}}\right)^2-1}{\left(\frac{589\,\text{nm}}{547\,\text{nm}}\right)^2+1}\right]\left(3\times10^8\ \text{m/s}\right)$$
$$=\boxed{2.22\times10^7\ \text{m/s}}$$

26 •

Picture the Problem We can use the relativistic Doppler shift, when the source and the receiver are receding, to relate the frequencies of the two wavelengths and $c = f\lambda$ to express the ratio of the wavelengths as a function of the speed of the galaxy.

When the source and receiver are moving away from each other, the relativistic Doppler shift is given by:

$$f'=\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}f_0$$

Use the relationship between the wavelength and frequency to obtain:

$$\frac{c}{\lambda'}=\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\frac{c}{\lambda_0}\Rightarrow\frac{\lambda_0}{\lambda'}=\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Solve for λ'/λ_0:

$$\frac{\lambda'}{\lambda_0}=\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Express the fractional redshift:

$$\frac{\lambda'-\lambda_0}{\lambda_0}=\frac{\lambda'}{\lambda_0}-1=\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}-1$$

Substitute numerical values and evaluate $(\lambda' - \lambda_0)/\lambda_0$:

$$\frac{\lambda' - \lambda_0}{\lambda_0} = \sqrt{\frac{1 + \dfrac{1.85\times10^7\ \text{m/s}}{2.998\times10^8\ \text{m/s}}}{1 - \dfrac{1.85\times10^7\ \text{m/s}}{2.998\times10^8\ \text{m/s}}}} - 1$$

$$= \boxed{0.0637}$$

27 ••

Picture the Problem We can begin the derivation by expressing the number of waves encountered by the observer, in the rest frame of the source, in a time interval Δt. We can then relate this time interval to the time interval in the rest frame of the observer to complete the derivation of Equation 39-16a.

Express the number of waves n encountered by the observer, in the rest frame of the source, in a time interval Δt_s:

$$n = \frac{(c+v)\Delta t_s}{\lambda} = \frac{(c+v) f_o \Delta t_s}{c}$$

$$= f_o\left(1 + \frac{v}{c}\right)\Delta t_s$$

This time interval in the rest frame of the observer is given by:

$$\Delta t_o = \frac{\Delta t_s}{\gamma}$$

Express the frequency heard by the observer and simplify to obtain:

$$f_o = \frac{n}{\Delta t_o} = \gamma\left(1 + \frac{v}{c}\right) f_o = \frac{1 + \dfrac{v}{c}}{\sqrt{1 - \dfrac{v^2}{c^2}}} f_o = \sqrt{\frac{1 + \dfrac{v}{c}}{1 - \dfrac{v}{c}}} f_o = \boxed{\frac{\sqrt{1 - \dfrac{v^2}{c^2}}}{1 - \dfrac{v}{c}} f_o}$$

28 •

Picture the Problem We can use the expression for the relativistic Doppler shift to show that, to a good approximation, $\Delta f/f \approx \pm v/c$.

Express the fractional Doppler shift in terms of f and f_0:

$$\frac{\Delta f}{f_0} = \frac{f - f_0}{f_0} = \frac{f}{f_0} - 1$$

When the source and receiver are approaching each other, the relativistic Doppler shift is given by:

$$f = \sqrt{\frac{1 + \dfrac{v}{c}}{1 - \dfrac{v}{c}}} f_0 \Rightarrow \frac{f}{f_0} = \sqrt{\frac{1 + \dfrac{v}{c}}{1 - \dfrac{v}{c}}}$$

Substitute in the expression for $\Delta f/f_0$ to obtain:

$$\frac{\Delta f}{f_0}=\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}-1$$

$$=\left(1+\frac{v}{c}\right)^{1/2}\left(1-\frac{v}{c}\right)^{-1/2}-1$$

Keeping just the lowest order terms in v/c, expand binomially to obtain:

$$\frac{\Delta f}{f_0}=\left(1+\frac{1}{2}\frac{v}{c}\right)\left(1+\frac{1}{2}\frac{v}{c}\right)-1$$

$$\approx 1+\frac{v}{c}-1=\boxed{\frac{v}{c}}$$

The sign depends on whether the source and receiver are approaching or receding. Here we have assumed that they are approaching.

***29 ••**

Picture the Problem Due to its motion, the orbiting clock will run more slowly than the earth-bound clock. We can use Kepler's third law to find the radius of the satellite's orbit in terms of its period, the definition of speed to find the orbital speed of the satellite from the radius of its orbit, and the time dilation equation to find the difference δ in the readings of the two clocks.

Express the time δ lost by the clock:

$$\delta=\Delta t-\Delta t_{\rm p}=\Delta t-\frac{\Delta t}{\gamma}=\Delta t\left(1-\frac{1}{\gamma}\right)$$

Because $v \ll c$, we can use part (*b*) of Problem 13:

$$\frac{1}{\gamma}\approx 1-\frac{1}{2}\frac{v^2}{c^2}$$

Substitute to obtain:

$$\delta=\Delta t\left[1-\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)\right]=\frac{1}{2}\frac{v^2}{c^2}\Delta t \qquad (1)$$

Express the square of the speed of the satellite in its orbit:

$$v^2=\left(\frac{2\pi r}{T}\right)^2=\frac{4\pi^2 r^2}{T^2} \qquad (2)$$

where T is its period and r is the radius of its (assumed) circular orbit.

Use Kepler's third law to relate the period of the satellite to the radius of its orbit about the earth:

$$T^2=\frac{4\pi^2}{GM_{\rm e}}r^3=\frac{4\pi^2}{gR_{\rm e}^2}r^3$$

Solve for r:

$$r = \sqrt[3]{\frac{gR_e^2T^2}{4\pi^2}}$$

Substitute numerical values and evaluate r:

$$r = \sqrt[3]{\frac{(9.81\,\text{m/s}^2)(6370\,\text{km})^2(90\,\text{min}\times 60\,\text{s/min})^2}{4\pi^2}} = 6.65\times 10^6\ \text{m}$$

Substitute numerical values in equation (2) and evaluate v^2:

$$v^2 = \frac{4\pi^2(6.65\times 10^6\ \text{m})^2}{(90\,\text{min}\times 60\,\text{s/min})^2} = 5.99\times 10^7\ \text{m}^2/\text{s}^2$$

Finally, substitute for v^2 in equation (1) and evaluate δ:

$$\delta = \frac{1}{2}\frac{(5.99\times 10^7\ \text{m}^2/\text{s}^2)(1\,\text{y}\times 31.56\,\text{Ms/y})}{(3\times 10^8\ \text{m/s})^2} = \boxed{10.5\,\text{ms}}$$

30 ••

Picture the Problem We can use the definition of the redshift parameter and the relativistic Doppler shift equation to show that $v = c\left(\frac{u^2 - 1}{u^2 + 1}\right)$, where $u = z + 1$.

The red-shift parameter is defined to be:

$$z = \frac{f - f'}{f'}$$

The relativistic Doppler shift is given by:

$$f' = f\sqrt{\frac{1 + v/c}{1 - v/c}}$$

Substitute to obtain:

$$z = \frac{f - f\sqrt{\frac{1+v/c}{1-v/c}}}{f\sqrt{\frac{1+v/c}{1-v/c}}} = \frac{1 - \sqrt{\frac{1+v/c}{1-v/c}}}{\sqrt{\frac{1+v/c}{1-v/c}}} = \sqrt{\frac{1+v/c}{1-v/c}} - 1$$

Letting $u = z + 1$:

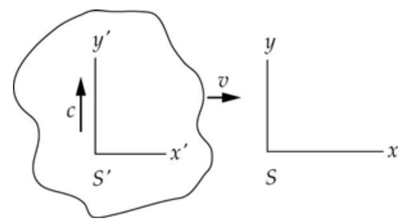
$$u = z + 1 = \sqrt{\frac{1+v/c}{1-v/c}}$$

Solve for v to obtain:

$$v = \boxed{c\left(\frac{u^2 - 1}{u^2 + 1}\right)}$$

31 •

Picture the Problem We can use the velocity transformation equations for the x and y directions to express the x and y components of the velocity of the light beam in frame S.



(*a*) The x and y components of the velocity of the light beam in frame S are:

$$u_x = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}}$$

and

$$u_y = \frac{u_y'}{\gamma\left(1 + \frac{vu_x'}{c^2}\right)}$$

Because $u_x' = 0$:

$$u_x = \boxed{v} \text{ and } u_y = \boxed{\frac{c}{\gamma}}$$

(*b*) The magnitude of the velocity of the light beam in S is given by:

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{v^2 + \frac{c^2}{\gamma^2}}$$

$$= \sqrt{v^2 + \left(1 - \frac{v^2}{c^2}\right)c^2} = \boxed{c}$$

32 •

Picture the Problem Let S be the earth reference frame and S' be that of the ship traveling east (positive x direction). Then in the reference frame S', the velocity of S is directed west, i.e., $v = -u_x$. We can apply the inverse velocity transformation equation to express u'_x in terms of u_x.

Apply the inverse velocity transformation equation to obtain:

$$u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

Substitute for v:

$$u_x' = \frac{u_x + u_x}{1 + \frac{u_x^2}{c^2}} = \frac{2u_x}{1 + \frac{u_x^2}{c^2}}$$

Because $u_x = 0.90c$:

$$u_x' = \frac{2(0.90c)}{1+\frac{(0.90c)^2}{c^2}} = \boxed{0.994c}$$

Picture the Problem We can apply the inverse velocity transformation equation to express the speed of the particle relative to both frames of reference.

(*a*) Express u_x' in terms of u_x'':

$$u_x' = \frac{u_x'' + v}{1+\frac{vu_x''}{c^2}}$$

where V of S', relative to S'', is 0.8*c*.

Substitute numerical values and evaluate u_x':

$$u_x' = \frac{0.8c + 0.8c}{1+\frac{(0.8c)^2}{c^2}} = \frac{1.6c}{1.64} = \boxed{0.976c}$$

(*b*) Express u_x in terms of u_x':

$$u_x = \frac{u_x' + v}{1+\frac{vu_x'}{c^2}}$$

where v of S, relative to S', is 0.8*c*.

Substitute numerical values and evaluate u_x:

$$u_x = \frac{0.976c + 0.8c}{1+\frac{(0.8c)(0.976c)}{c^2}} = \frac{1.776c}{1.781}$$

$$= \boxed{0.997c}$$

Relativistic Momentum and Relativistic Energy

***34** •

Picture the Problem We can use the relation for the total energy, momentum, and rest energy to find the momentum of the proton and Equation 39-26 to relate the speed of the proton to its energy and momentum.

Relate the energy of the proton to its momentum:

$$E^2 = p^2c^2 + (mc^2)^2$$

(*b*) Solve for p to obtain:

$$p = \sqrt{\frac{E^2 - (mc^2)^2}{c^2}}$$

Substitute numerical values and evaluate p:

$$p = \frac{\sqrt{(2200\,\text{MeV})^2 - (938\,\text{MeV})^2}}{c} = \boxed{1.99\,\frac{\text{GeV}}{c}}$$

(a) From Equation 39-26 we have:

$$\frac{v}{c} = \frac{pc}{E}$$

Solve for v to obtain:

$$v = \frac{pc}{E}c$$

Substitute numerical values and evaluate v:

$$v = \frac{1.99\,\text{GeV}}{2200\,\text{MeV}}c = \boxed{0.905c}$$

35 •

Picture the Problem We can use $E^2 = p^2c^2 + (mc^2)^2$ (from Problem R-37) to find the relativistic momentum of the particle in terms of γ and the fact that the kinetic energy of the particle equals twice its rest energy to find the error made in using mv for the momentum of the particle.

Express the error e in using $p' = mv$ for the momentum of the particle:

$$e = \frac{p - p'}{p} = 1 - \frac{p'}{p} \qquad (1)$$

From Problem R-37, the relationship between the total energy E, momentum p, and rest energy mc^2 of the particle is:

$$E^2 = p^2c^2 + (mc^2)^2$$

Solve for p to obtain:

$$p = \sqrt{\frac{E^2}{c^2} - m^2c^2} = \sqrt{m^2c^2\left(\frac{E^2}{m^2c^4} - 1\right)}$$

Because $E = \gamma mc^2$:

$$p = mc\sqrt{\gamma^2 - 1}$$

Substitute for p and p' in equation (1) to obtain:

$$e = 1 - \frac{mu}{mc\sqrt{\gamma^2 - 1}} = 1 - \frac{u}{c\sqrt{\gamma^2 - 1}} \qquad (2)$$

From the definition of γ:

$$\frac{u}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

Eliminate v/c in equation (2) to obtain:

$$e = 1 - \frac{1}{\sqrt{\gamma^2 - 1}}\sqrt{1 - \frac{1}{\gamma^2}} = 1 - \frac{1}{\gamma} \qquad (3)$$

The kinetic energy of the particle is related to its rest energy:

$$K = (\gamma - 1)mc^2$$

Solve for γ to obtain:

$$\gamma = 1 + \frac{K}{mc^2}$$

Because the kinetic energy of the particle is twice its rest energy:

$$\gamma = 1 + \frac{2mc^2}{mc^2} = 3$$

Substitute for γ in equation (3) and evaluate e:

$$e = 1 - \frac{1}{3} = 0.667 = \boxed{66.7\%}$$

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Picture the Problem We can use the result of Problem R-37 to find the energy of the particle and its energy in a reference frame in which its momentum is 4 MeV/c. We can apply the inverse velocity transformation equation to find the relative velocities of the two reference frames.

(a) From Problem R-37 we have:

$$E^2 = p^2c^2 + m_0^2c^4 = p^2c^2 + E_0^2$$

Solve for E_0:

$$E_0 = \sqrt{E^2 - p^2c^2}$$

Substitute numerical values and evaluate E_0:

$$E_0 = \sqrt{(8\,\text{MeV})^2 - (6\,\text{MeV}/c)^2c^2} = \sqrt{(8\,\text{MeV})^2 - (6\,\text{MeV})^2} = \boxed{5.29\,\text{MeV}}$$

(b) Because E_0 is independent of the reference frame:

$$E = \sqrt{p^2c^2 + E_0^2}$$

Substitute numerical values and evaluate E:

$$E = \sqrt{(4\,\text{MeV}/c)^2c^2 + (5.29\,\text{MeV})^2} = \boxed{6.63\,\text{MeV}}$$

(c) The inverse velocity transformation is:

$$u_b = \frac{u_a - v}{1 - \dfrac{vu_a}{c^2}}$$

where the subscripts refer to the velocities in parts (*a*) and (*b*) of the problem.

Solve for *v* to obtain:

$$v = \frac{u_a - u_b}{1 - \frac{u_a u_b}{c^2}} \qquad (1)$$

Relate the relativistic energy of the particle in (*a*) to its velocity:

$$E = \frac{E_0}{\sqrt{1 - \frac{u_a^2}{c^2}}} \Rightarrow u_a = c\sqrt{1 - \left(\frac{E_0}{E}\right)^2}$$

Substitute numerical values and evaluate u_a:

$$u_a = c\sqrt{1 - \left(\frac{5.29\,\text{MeV}}{8\,\text{MeV}}\right)^2} = 0.750c$$

Relate the relativistic momentum of the particle in (*b*) to its velocity:

$$p = \frac{m_0 u_b}{\sqrt{1 - \frac{u_b^2}{c^2}}} \Rightarrow u_b = \frac{pc}{\sqrt{p^2 + m_0 c^2}}$$

Substitute numerical values and evaluate u_b:

$$u_b = \frac{(4\,\text{MeV}/c)c}{\sqrt{(4\,\text{MeV}/c)^2 + (5.29\,\text{MeV}/c)^2}} = 0.603c$$

Substitute in equation (1) and evaluate *V*:

$$V = \frac{0.750c - 0.603c}{1 - \frac{(0.750c)(0.603c)}{c^2}} = \boxed{0.268c}$$

37 ••

Picture the Problem We can use the rule for the derivative of a quotient to establish the result given in the problem statement.

Use the expression for the derivative of a quotient to obtain:

$$\frac{d}{du}\left(\frac{mu}{\sqrt{1-u^2/c^2}}\right) = \frac{\sqrt{1-\frac{u^2}{c^2}}m + \frac{mu^2}{c^2}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}}{1-\frac{u^2}{c^2}}$$

Multiply the numerator and denominator of the right-hand side of this expression by

$\sqrt{1-\frac{u^2}{c^2}}$ and simplify to obtain:

$$\frac{d}{du}\left(\frac{mu}{\sqrt{1-u^2/c^2}}\right)=\frac{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}m+\frac{mu^2}{c^2}\frac{\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{u^2}{c^2}}}}{\left(1-\frac{u^2}{c^2}\right)\sqrt{1-\frac{u^2}{c^2}}}=\frac{\left(1-\frac{u^2}{c^2}\right)m+\frac{mu^2}{c^2}}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

$$=m\left(1-\frac{u^2}{c^2}\right)^{-3/2}$$

and

$$d\left(\frac{mu}{\sqrt{1-u^2/c^2}}\right)=\boxed{m\left(1-\frac{u^2}{c^2}\right)^{-3/2}du}$$

38 ••

Picture the Problem We will first consider the decay process in the center of mass reference frame and then transform to the laboratory reference frame in which one of the pions is at rest.

Apply energy conservation in the center of mass frame of reference to obtain:

$$m_{K_0}c^2=2m_{\pi_0}\gamma c^2$$

Solve for γ:

$$\gamma=\frac{m_{K_0}}{2m_{\pi_0}}$$

Substitute numerical values and evaluate γ:

$$\gamma=\frac{497.7\,\text{MeV}/c^2}{2(139.6\,\text{MeV}/c^2)}=1.78$$

Because one of the pions is at rest in the laboratory frame, γ = 1.78 for the transformation to the laboratory frame. The kinetic energy of the K^0 particle is:

$$K_{K^0}=(\gamma-1)E$$
$$=(1.78-1)(497.7\,\text{MeV})$$
$$=\boxed{388.2\,\text{MeV}}$$

The total initial energy in the laboratory frame is:

$$E=497.7\,\text{MeV}+388.2\,\text{MeV}$$
$$=885.9\,\text{MeV}$$

Express the energy of the other pion:

$$E_\pi = E - 2m_{0\pi}c^2$$

Substitute numerical values and evaluate E_π:

$$E_\pi = 885.9\,\text{MeV} - 2(139.6\,\text{MeV}) = \boxed{607\,\text{MeV}}$$

***39 ••**

Picture the Problem The total kinetic energy of the two protons in part (*a*) is the sum of their kinetic energies and is given by $K = 2(\gamma - 1)E_0$. Part (*b*) differs from part (*a*) in that we need to find the speed of the moving proton relative to frame *S*.

(*a*) The total kinetic energy of the protons in frame *S*′ is given by:

$$K = 2(\gamma - 1)E_0$$

Substitute for γ and E_0 and evaluate *K*:

$$K = 2\left(\frac{1}{\sqrt{1-\frac{(0.5c)^2}{c^2}}} - 1\right)(938.28\,\text{MeV}) = \boxed{290\,\text{MeV}}$$

(*b*) The kinetic energy of the moving proton in frame *S* is given by:

$$K = (\gamma - 1)E_0 \qquad (1)$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{uv}{c^2}}}$$

Express the speed *u* of the proton in frame *S*:

$$u = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}}$$

Substitute numerical values and evaluate *u*:

$$u = \frac{0.5c + 0.5c}{1 + \frac{(0.5c)(0.5c)}{c^2}} = 0.800c$$

Evaluate γ:

$$\gamma = \frac{1}{\sqrt{1-\frac{(0.8c)(0.8c)}{c^2}}} = 1.67$$

Substitute numerical values in equation (1) and evaluate *K*:

$$K = (1.67 - 1)(938.28\,\text{MeV}) = \boxed{629\,\text{MeV}}$$

40 ••

Picture the Problem We can find the speed of each proton by equating their total relativistic kinetic energy to $2mc^2$. In (*b*) we can use the inverse velocity transformation with $V = u$ and $u_x = -u$ to find u_x'. In part (*c*) we'll need to evaluate γ' for the kinetic energy transformation $K_\text{L} = (\gamma' - 1)E_0$.

(*a*) Set the relativistic kinetic energy of the protons equal to $2mc^2$ to obtain:

$$2(\gamma - 1)E_0 = 2mc^2 \Rightarrow \gamma = 2$$

Substitute for γ:

$$\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = 2$$

Solve for *u* to obtain:

$$u = \frac{\sqrt{3}}{2}c = \boxed{0.866c}$$

(*b*) Use the inverse velocity transformation with $v = u$ and $u_x = -u$ to find u_x':

$$u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} = \frac{-\frac{\sqrt{3}}{2}c - \frac{\sqrt{3}}{2}c}{1 - \frac{\left(-\frac{\sqrt{3}}{2}c\right)\left(\frac{\sqrt{3}}{2}c\right)}{c^2}}$$

$$= \frac{-4\sqrt{3}c}{7} = \boxed{-0.990c}$$

(*c*) The kinetic energy of the moving proton in the laboratory's frame is given by:

$$K_\text{L} = (\gamma' - 1)E_0$$

where

$$\gamma' = \frac{1}{\sqrt{1 - \frac{(u_x')^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{\left(-\frac{4\sqrt{3}}{7}c\right)^2}{c^2}}} = 7$$

Substitute for γ' and E_0 and evaluate K_L:

$$K_\text{L} = (7 - 1)mc^2 = \boxed{6mc^2}$$

41 •••

Picture the Problem (*a*) and (*b*) The initial speed of the particle can be found from its total energy and its total energy found using $E = K + E_0 = \gamma E_0$. (*c*) We can solve $E^2 = p^2c^2 + (mc^2)^2$ for the initial momentum of the system. In (*d*) and (*e*) we can use conservation of energy and conservation of momentum to find the total kinetic energy after the collision and the mass of the system after the collision.

(*a*) Express the total energy of the particle:

$$E = K + E_0 = \gamma E_0$$

Because the kinetic energy of the particle is twice its energy:

$$2E_0 + E_0 = \gamma E_0 \text{ and } \gamma = 3$$

Solve the factor γ for u:

$$u = c\sqrt{1 - \frac{1}{\gamma^2}}$$

Substitute for γ and evaluate u:

$$u = c\sqrt{1 - \frac{1}{3^2}} = c\sqrt{\frac{8}{9}} = \boxed{0.943c}$$

(*b*) The total energy of the particle is:

$$E = K + E_0 = \gamma E_0 = 3E_0$$

Substitute for E_0 and evaluate E:

$$E = 3(1\,\text{MeV}) = \boxed{3\,\text{MeV}}$$

(*c*) The initial momentum of the incoming particle is related to its energy and mass according to:

$$E^2 = p^2c^2 + (mc^2)^2$$

Solve for p:

$$p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2}$$

Substitute for E and mc^2 and simplify to obtain:

$$p = \frac{1}{c}\sqrt{(3E_0)^2 - (E_0)^2} = \frac{\sqrt{8}E_0}{c}$$

Substitute for E_0 and evaluate p:

$$p = \frac{\sqrt{8}(1\,\text{MeV})}{c} = \boxed{2.83\,\text{MeV}/c}$$

(*d*) and (*e*) From conservation of energy we have:

$$E_\text{f} = E_\text{i} = 5\,\text{MeV}$$

From conservation of momentum we have:

$$p_f = p_i$$

The final momentum of the system is related to its energy and mass according to:

$$E_f^2 = p_f^2 c^2 + E_{f0}^2$$

Solve for E_{f0}:

$$E_{f0} = \sqrt{E_f^2 - p_f^2 c^2}$$

Substitute numerical values and evaluate E_{f0}:

$$E_{f0} = \sqrt{(5\,\text{MeV})^2 - (2.83\,\text{MeV}/c)^2 c^2}$$
$$= 4.122\,\text{MeV}$$

Because $E_{f0} = m_{f0}c^2$:

$$m_{f0} = \frac{E_{f0}}{c^2} = \boxed{4.12\,\text{MeV}/c^2}$$

The total kinetic energy after the collision is given by:

$$K_f = E_f - E_{f0} = 5\,\text{MeV} - 4.122\,\text{MeV}$$
$$= \boxed{0.878\,\text{MeV}}$$

General Relativity

***42 ••**

Picture the Problem Let m represent the mass equivalent of a photon. We can equate the change in the gravitational potential energy of a photon as it rises a distance L in the gravitational field to $h\Delta f$ and then express the wavelength shift in terms of the frequency shift.

The speed of the photons in the light beam are related to their frequency and wavelength:

$$c = f\lambda \Rightarrow f = \frac{c}{\lambda}$$

Differentiate this expression with respect to λ to obtain:

$$\frac{df}{d\lambda} = -c\lambda^{-2} = -\frac{c}{\lambda^2}$$

Approximate $df/d\lambda$ by $\Delta f/\Delta\lambda$ and solve for Δf:

$$\Delta f = -\frac{c}{\lambda^2}\Delta\lambda$$

Divide both sides of this equation by f to obtain:

$$\frac{\Delta f}{f} = \frac{-\frac{c}{\lambda^2}\Delta\lambda}{\frac{c}{\lambda}} = -\frac{\Delta\lambda}{\lambda}$$

Solve for $\Delta\lambda$:

$$\Delta\lambda = -\lambda\frac{\Delta f}{f} \qquad (1)$$

The change in the energy of the photon as it rises a distance L in a gravitational field is given by:

$$\Delta E = \Delta U = mgL$$

Because $\Delta E = h\Delta f$:

$$h\Delta f = mgL \qquad (2)$$

Letting m represent the mass equivalent of the photon:

$$E = mc^2 = hf \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$\frac{h\Delta f}{hf} = \frac{mgL}{mc^2} \Rightarrow \frac{\Delta f}{f} = \frac{gL}{c^2}$$

Substitute for $\Delta f/f$ in equation (1):

$$\Delta\lambda = -\frac{gL\lambda}{c^2}$$

Substitute numerical values and evaluate $\Delta\lambda$:

$$\Delta\lambda = -\frac{(9.81\,\text{m/s}^2)(100\,\text{m})(632.8\,\text{nm})}{(3\times10^8\,\text{m/s})^2} = \boxed{-6.90\times10^{-12}\,\text{nm}}$$

43 ••

Picture the Problem In a freely falling reference frame, both cannonballs travel along straight lines, so they must hit each other, as they were pointed at each other when they were fired.

44 •••

Picture the Problem Consider the turntable to be a giant hollow cylinder in space that is spinning about its axis. Someone on the inside surface of the cylinder would experience a centripetal acceleration caused by the normal force of the surface pushing them toward the rotation axis. Alternatively, they can consider that they are not accelerating but a gravitational field $\vec{g} = \omega^2 r\hat{\mathbf{r}}$ is pushing them away from the axis ($\hat{\mathbf{r}}$ is away from the axis). This is the principle of equivalence. From this perspective, up is toward the axis and the points closer to the axis are at the higher gravitational potential. (Just like the electric field points in the direction of decreasing electric potential, the gravitational field points in the direction of decreasing gravitational potential.)

(*a*) From the time dilation equation we have:

$$\Delta t_\text{r} = \frac{\Delta t_0}{\gamma}$$

and

$$\frac{\Delta t_r - \Delta t_0}{\Delta t_0} = \frac{1}{\gamma} - 1$$

Because $r\omega/c << 1$ (see Problem 14):

$$\frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{u^2}{c^2} = 1 - \frac{r^2\omega^2}{2c^2}$$

Substitute to obtain:

$$\frac{\Delta t_r - \Delta t_0}{\Delta t_0} = 1 - \frac{r^2\omega^2}{2c^2} - 1 = \boxed{-\frac{r^2\omega^2}{2c^2}}$$

(*b*) The pseudoforce is given by:

$$F_P = -ma$$

where a is the acceleration of the non-inertial reference frame.

In this case a is the centripetal acceleration:

$$a = -r\omega^2 \Rightarrow F_P = F_r = \boxed{mr\omega^2}$$

To relate this problem to Equation 39-31, point 2 is a distance r from the axis and point 1 is on the axis. The term in parentheses on the right hand side of Equation 2 is $\phi_2 - \phi_1$, which translates to $\phi_r - \phi_0$. Because ϕ_r is at a lower potential than ϕ_0, this term is negative. Hence:

$$\phi_r - \phi_0 = -\int_0^r \vec{g} \cdot d\vec{\ell} = -\int_0^r \omega^2 r\,\hat{r} \cdot d\vec{\ell} = -\int_0^r \omega^2 r\,dr = \boxed{-\tfrac{1}{2}r^2\omega^2}$$

From Equation 39-31:

$$\frac{\Delta t_r - \Delta t_0}{\Delta t_0} = \frac{1}{c^2}\left(\phi_r - \phi_0\right)$$

$$= \frac{1}{c^2}\left(-\tfrac{1}{2}r^2\omega^2\right)$$

$$= \boxed{-\frac{r^2\omega^2}{2c^2}}$$

General Problems

45 •

Picture the Problem We can use the definition of γ and the time dilation equation to find the speed of the muon.

(*a*) From the definition of γ we have:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

Solve for *u*/*c*:

$$\frac{u}{c} = \sqrt{1-\frac{1}{\gamma^2}}$$

Relate the mean lifetime of the muon to its proper lifetime:

$$\Delta t = \gamma\Delta t_{\mathrm{p}} \Rightarrow \gamma = \frac{\Delta t}{\Delta t_{\mathrm{p}}}$$

Substitute in the expression for *u*/*c* to obtain:

$$\frac{u}{c} = \sqrt{1-\left(\frac{\Delta t_{\mathrm{p}}}{\Delta t}\right)^2}$$

Substitute numerical values and evaluate *u*/*c*:

$$\frac{u}{c} = \sqrt{1-\left(\frac{2\ \mu\mathrm{s}}{46\ \mu\mathrm{s}}\right)^2} = 0.999$$

or

$$u = \boxed{0.999c}$$

46 •

Picture the Problem We can use the relativistic Doppler shift, when the source and the receiver are receding, to relate the frequencies of the two wavelengths and $c = f\lambda$ to express the ratio of the wavelengths as a function of the speed of the galaxy.

When the source and receiver are moving away from each other, the relativistic Doppler shift is given by:

$$f' = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}f_0$$

Use the relationship between the wavelength and frequency to obtain:

$$\frac{c}{\lambda'} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\frac{c}{\lambda_0} \Rightarrow \frac{\lambda_0}{\lambda'} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Because $\lambda' = 2\lambda_0$:

$$\frac{\lambda_0}{2\lambda_0} = \frac{1}{2} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Solve for v/c:

$$\frac{v}{c}=\frac{3}{5}\Rightarrow v=\boxed{0.600c}$$

***47 ••**

Picture the Problem We can use Equation 39-12, the inverse time transformation equation, to relate the elapsed times and separations of the events in the two systems to the velocity of *S′* relative to *S*. We can use this same relationship in (*b*) to find the time at which these events occur as measured in *S′*.

(*a*) Use Equation 39-12 to obtain:

$$\Delta t'=t_2'-t_1'=\gamma\left[(t_2-t_1)-\frac{v}{c^2}(x_2-x_1)\right]$$
$$=\gamma\left[\Delta t-\frac{v}{c^2}\Delta x\right]$$

Because the events occur simultaneously in frame *S′*, $\Delta t'=0$ and:

$$0=\Delta t-\frac{v}{c^2}\Delta x$$

Solve for v to obtain:

$$v=\frac{c^2\Delta t}{\Delta x}$$

Substitute for Δt and Δx and evaluate V:

$$v=\frac{c^2(0.5\,\text{y}-1\,\text{y})}{2.0\,c\cdot\text{y}-1.0\,c\cdot\text{y}}=\boxed{-0.5c}$$

Because $\Delta t=t_2-t_1=-0.5\,\text{y}$:

$$\boxed{S'\text{ moves in the negative } x \text{ direction.}}$$

(*b*) Use the inverse time transformation to obtain:

$$t_2'=\gamma\left(t_2-\frac{vx_2}{c^2}\right)=\frac{t_2-\frac{vx_2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Substitute numerical values and evaluate t_2' and t_1':

$$t_2'=t_1'=\frac{0.5\,\text{y}-\frac{(-0.5c)(2.0\,c\cdot\text{y})}{c^2}}{\sqrt{1-\frac{(-0.5c)^2}{c^2}}}$$
$$=\boxed{1.73\,\text{y}}$$

48 ••

Picture the Problem We can use the relationship between distance, speed, and time and the length contraction relationship to find the speed of the ship relative to the earth. The elapsed time between the departure of the spaceship and the receipt of the signal at earth is the sum of the travel time to the distant star system and the time it takes the signal to return to earth.

(*a*) Express the travel time as measured on the spaceship:

$$\Delta t' = \frac{L'}{u} = \frac{L}{\gamma u}$$

Solve for γu:

$$\gamma u = \frac{L}{\Delta t'}$$

Substitute numerical values and evaluate γu:

$$\gamma u = \frac{12\,c\cdot \text{y}}{15\,\text{y}} = 0.8c$$

or

$$\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{0.8c}{u}$$

Solve for u to obtain:

$$u = \boxed{0.625c}$$

(*b*) The elapsed time T before earth receives the signal is the sum of the travel time to the distant star system and the time it takes the signal to return:

$$T = \frac{L}{u} + \frac{L}{c}$$

Substitute numerical values and evaluate T:

$$T = \frac{12\,c\cdot \text{y}}{0.625c} + \frac{12\,c\cdot \text{y}}{c} = \boxed{31.2\,\text{y}}$$

49 ••

Picture the Problem We can use conservation of energy to find γ in the CM frame of reference and then use the definition of γ to find the speed u of the projectile proton. We can then use the velocity transformation equation to find the speed and kinetic energy of this proton in the laboratory frame of reference.

Use conservation of energy to find γ in the CM frame of reference:

$$\gamma E_\text{i} = E_\text{f} \Rightarrow \gamma = \frac{E_\text{f}}{E_\text{i}}$$

E_i and E_f are:

$$E_\text{i} = 938\,\text{Mev} + 938\,\text{MeV} = 1876\,\text{MeV}$$

and

$$E_f = 938\,\text{Mev} + 938\,\text{MeV} + 135\,\text{MeV} = 2011\,\text{MeV}$$

Substitute E_i and E_f and evaluate γ:

$$\gamma = \frac{2011\,\text{MeV}}{1876\,\text{MeV}} = 1.072$$

Express γ as a function of the speed u of the projectile proton:

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Solve for u to obtain:

$$u = c\sqrt{1 - \frac{1}{\gamma^2}}$$

Substitute for γ and evaluate u:

$$u = c\sqrt{1 - \frac{1}{(1.072)^2}} = 0.360c$$

Transform to the laboratory frame and find u':

$$u' = \frac{u - v}{1 - \frac{vu}{c^2}} = \frac{0.360c - (-0.360c)}{1 - \frac{(0.360c)(-0.360\text{c})}{c^2}} = 0.637c$$

The kinetic energy of the moving proton in the laboratory's frame is given by:

$$K_L = (\gamma_L - 1)E_0$$

where

$$\gamma_L = \frac{1}{\sqrt{1 - \frac{(u')^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.637c)^2}{c^2}}} = 1.30$$

Substitute for γ_L and E_0 and evaluate K_L:

$$K_L = (1.30 - 1)(938\,\text{MeV}) = \boxed{281\,\text{MeV}}$$

Remarks: In Problem 55 we show that the threshold kinetic energy of the projectile is given by $\boldsymbol{K_{\text{th}} = \frac{\left(\sum m_{\text{in}} + \sum m_{\text{fin}}\right)\left(\sum m_{\text{fin}} - \sum m_{\text{in}}\right)c^2}{2m_{\text{target}}}}$.

50 ••

Picture the Problem We can use $\Delta t_{\text{p}} = L_{\text{p}}/u$, where u is the speed of the bullet relative to the rocket, to find the elapsed time in the frame of the rocket. In (*b*) and (*c*) we can proceed similarly, finding the speed of the bullet relative to the rocket as seen from the ground frame in (*b*) and, in (*c*), using the speed of the bullet relative to the rocket.

(*a*) In the rocket frame:

$$\Delta t = \Delta t_{\text{p}} = \frac{L_{\text{p}}}{u} = \frac{L_{\text{p}}}{0.8c}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1000\,\text{m}}{0.8\left(2.998\times10^{8}\,\text{m/s}\right)} = \boxed{4.17\,\mu\text{s}}$$

(*b*) In the ground frame of reference, the elapsed time is given by:

$$\Delta t_{\text{ground}} = \frac{L}{u'} \qquad (1)$$

where u' is the speed of the bullet relative to the rocket as seen from the ground.

The speed of the ground is given by:

$$u_{\text{ground}} = \frac{u_{\text{rocket}} + V}{1 + \dfrac{Vu_{\text{ground}}}{c^{2}}}$$

Substitute for u_{rocket} and V and evaluate u_{ground}:

$$u_{\text{ground}} = \frac{0.6c + 0.8c}{1 + \dfrac{(0.8c)(0.6c)}{c^{2}}} = 0.946c$$

The speed of the bullet relative to the rocket as seen from the ground is:

$$u' = 0.946c - 0.6c = 0.346c$$

Relate L_{ground} to L_{p}:

$$L_{\text{ground}} = \frac{L_{\text{p}}}{\gamma} = L_{\text{p}}\sqrt{1 - \frac{u_{\text{ground}}^{2}}{\text{c}^{2}}}$$

Substitute for L_{p} and u_{ground} and evaluate L_{ground}:

$$L_{\text{ground}} = (1000\,\text{m})\sqrt{1 - \frac{(0.6c)^{2}}{\text{c}^{2}}} = 800\,\text{m}$$

Substitute numerical values in equation (1) and evaluate Δt_{ground}:

$$\Delta t_{\text{ground}} = \frac{800\,\text{m}}{0.346\left(2.998\times10^{8}\,\text{m/s}\right)} = \boxed{7.71\,\mu\text{s}}$$

(*c*) In the bullet's frame of reference, the elapsed time is given by:

$$\Delta t_{\text{bullet}} = \frac{L'}{u_{\text{bullet}}} \quad (2)$$

The length L' of the rocket in the bullet's frame is given by:

$$L' = \frac{L_{\text{p}}}{\gamma} = L_{\text{p}}\sqrt{1-\frac{u_{\text{bullet}}^2}{c^2}}$$

Substitute in equation (2) to obtain:

$$\Delta t_{\text{bullet}} = \frac{L_{\text{p}}}{u_{\text{bullet}}}\sqrt{1-\frac{u_{\text{bullet}}^2}{c^2}}$$

Substitute numerical values and evaluate Δt_{bullet}:

$$\Delta t_{\text{bullet}} = \frac{1000\,\text{m}}{0.8\left(2.998\times10^{8}\ \text{m/s}\right)}\sqrt{1-\frac{(0.8c)^2}{c^2}} = \boxed{2.50\,\mu\text{s}}$$

***51 •••**

Picture the Problem We can use conservation of energy to express the recoil velocity of the box and the relationship between distance, speed, and time to find the distance traveled by the box in time $\Delta t = L/c$. Equating the initial and final locations of the center of mass will allow us to show that the radiation must carry mass $m = E/c^2$.

(*a*) Apply conservation of momentum to obtain:

$$\frac{E}{c} + Mv = p_{\text{i}} = 0$$

Solve for v:

$$v = \boxed{-\frac{E}{Mc}}$$

(*b*) The distance traveled by the box in time $\Delta t = L/c$ is:

$$d = v\Delta t = \frac{vL}{c}$$

Substitute for v from (*a*):

$$d = \frac{L}{c}\left(-\frac{E}{Mc}\right) = \boxed{-\frac{LE}{Mc^2}}$$

(*c*) Let $x = 0$ be at the center of the box and let the mass of the photon be m. Then initially the center of mass is at:

$$x_{\text{CM}} = \frac{-\frac{1}{2}mL}{M+m}$$

When the photon is absorbed at the other end of the box, the center of mass is at:

$$x_{\rm CM} = \frac{\left[\dfrac{-MEL}{Mc^2} + m\left(\frac{1}{2}L - \dfrac{EL}{Mc^2}\right)\right]}{M+m}$$

Because no external forces act on the system, these expressions for $x_{\rm CM}$ must be equal:

$$\frac{-\frac{1}{2}mL}{M+m} = \frac{\left[\dfrac{-MEL}{Mc^2} + m\left(\frac{1}{2}L - \dfrac{EL}{Mc^2}\right)\right]}{M+m}$$

Solve for m to obtain:

$$m = \frac{E}{c^2\left(1 - \dfrac{E}{Mc^2}\right)}$$

Because Mc^2 is of the order of 10^{16} J and $E = hf$ is of the order of 1 J for reasonable values of f, $E/Mc^2 << 1$ and:

$$m = \boxed{\frac{E}{c^2}}$$

52 •••

Picture the Problem We can apply a velocity transformation equation to find the speed of the particle and use the distance and time transformation equations to find the distance and direction the particle traveled from t'_1 to t'_2 and the time the particle traveled as observed in frame S.

(*a*) The velocity transformation equation for motion at speed v along the x axis is:

$$u_x = \frac{u_x' + v}{1 + \dfrac{vu_x'}{c^2}}$$

Evaluate u_x for $u_x' = -c/3$ and $v = 0.6$c:

$$u_x = \frac{-\dfrac{1}{3}c + \dfrac{3}{5}c}{1 + \dfrac{\left(\dfrac{3}{5}c\right)\left(-\dfrac{1}{3}c\right)}{c^2}} = \boxed{\tfrac{1}{3}c}$$

(*b*) The distance traveled by the particle from t'_1 to t'_2 is given by:

$$\Delta x = x_2 - x_1 \qquad (1)$$

To find x_2, we must first find x_2' and $\Delta t'$:

$$x_2' = 10\,\text{m} - (60\,\text{m}/c)\left(\frac{c}{3}\right) = -10\,\text{m}$$

and

$$\Delta t' = t_2' - t_1' = 60\,\text{m}/c = 200\,\text{ns}$$

x_2 is related to x_2' through the relativistic transformation:

$$x_2 = \gamma(x_2' + v\Delta t') = \frac{x_2' + v\Delta t'}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Substitute numerical values and evaluate x_2:

$$x_2 = \frac{-10\,\text{m} + (0.6c)(200\,\text{ns})}{\sqrt{1-\dfrac{(0.6c)^2}{c^2}}} = 32.5\,\text{m}$$

x_1 is given by:

$$x_1 = \gamma x_1' = \frac{x_1'}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Substitute numerical values and evaluate x_1:

$$x_1 = \frac{10\,\text{m}}{\sqrt{1-\dfrac{(0.6c)^2}{c^2}}} = 12.5\,\text{m}$$

Substitute numerical values in equation (1) and evaluate Δx:

$$\Delta x = 32.5\,\text{m} - 12.5\,\text{m} = \boxed{20.0\,\text{m}}$$

(c) The time the particle traveled is given by:

$$\Delta t = t_2 - t_1 \qquad (2)$$

Express and evaluate t_1:

$$t_1 = \gamma t_1' = \frac{t_1'}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

$$= \frac{6\,\text{m}/c}{\sqrt{1-\dfrac{(0.6c)^2}{c^2}}} = 7.50\,\text{m}/c = 25.0\,\text{ns}$$

Express and evaluate t_2:

$$t_2 = \gamma\left(t' + \frac{vt'}{c^2}\right) = \frac{t' + \dfrac{vt'}{c^2}}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

$$= \frac{60\,\text{m}/c + (-6\,\text{m}/c)}{\sqrt{1-\dfrac{(0.6c)^2}{c^2}}} = 225\,\text{ns}$$

Substitute in equation (2) and evaluate Δt:

$$\Delta t = 225\,\text{ns} - 25\,\text{ns} = \boxed{200\,\text{ns}}$$

53 •••

Picture the Problem We can evaluate the differentials of Equations 39-19*a, b,* and *c* and 39-10 and express their ratio to obtain expressions for a_x', a_y', and a_z'.

From Equation 39-19*a* we have:

$$du_x' = d\left(\frac{u_x - v}{1-\frac{vu_x}{c^2}}\right) = \frac{\left(1-\frac{vu_x}{c^2}\right)du_x + \left(u_x - v\right)\left(\frac{v}{c^2}\right)du_x}{\left(1-\frac{vu_x}{c^2}\right)^2} = \frac{\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vu_x}{c^2}\right)^2}du_x$$

From Equation 39-10:

$$dt' = \gamma d\left(t - \frac{vx}{c^2}\right) = \gamma dt - \frac{\gamma v}{c^2}dx = \frac{\gamma v}{c^2}\frac{dx}{dt}dt = \gamma\left(1-\frac{vu_x}{c^2}\right)dt$$

Divide du_x' by dt' to obtain:

$$a_x' = \frac{du_x'}{dt'} = \frac{\frac{\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vu_x}{c^2}\right)^2}du_x}{\gamma\left(1-\frac{vu_x}{c^2}\right)dt} = \frac{\left(1-\frac{v^2}{c^2}\right)}{\gamma\left(1-\frac{vu_x}{c^2}\right)^3}\frac{du_x}{dt} = \boxed{\frac{1}{\gamma^3\delta^3}a_x}$$

where $\delta = 1 - \frac{vu_x}{c^2}$

Proceeding in exactly the same manner, one obtains:

$$a_y' = \boxed{\frac{1}{\gamma^2\delta^2}a_y + \frac{vu_y}{\gamma^3\delta^3c^2}a_x}$$

and an identical expression for a_z' with *z* replacing *y.*

54 •••

Picture the Problem Without loss of generality, we'll consider the absorption case. We'll assume that the electron is initially at rest and that it travels with a speed *v* after it absorbs the photon. Applying the conservation of energy and the conservation of momentum will lead us to an absurd conclusion that, in turn, will force us to abandon our initial assumption that an electron can absorb a photon. Such an argument is known as a *reductio ad absurdum* argument.

When the electron absorbs a photon, the conservation of relativistic momentum requires that its momentum become:

$$p = \gamma mv$$

From the conservation of energy:

$$mc^2 + pc = \gamma mc^2 \Rightarrow p = (\gamma - 1)mc$$

Equate these expression for p to obtain:

$$\gamma mv = (\gamma - 1)mc$$

Solving for v yields:

$$v = \left(\frac{\gamma - 1}{\gamma}\right)c \qquad (1)$$

Square both sides of the equation to obtain:

$$v^2 = \left(\frac{\gamma - 1}{\gamma}\right)^2 c^2 = \frac{\gamma^2 - 2\gamma + 1}{\gamma^2}c^2 \qquad (2)$$

From the definition of γ:

$$\gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{c^2}{c^2 - v^2}$$

Solve for v^2 to obtain:

$$v^2 = \frac{\gamma^2 - 1}{\gamma^2}c^2$$

Substitute for v^2 in equation (1) and simplify to obtain:

$$\frac{\gamma^2 - 1}{\gamma^2}c^2 = \frac{\gamma^2 - 2\gamma + 1}{\gamma^2}c^2$$

or

$$-1 = -2\gamma + 1 \Rightarrow \gamma = 1$$

Substitute for γ in equation (1) and evaluate v:

$$v = \left(\frac{1 - 1}{1}\right)c = 0$$

Our assumption that an electron can absorb a photon has led to the contradictory conclusion that its speed after the absorption is zero. Hence, we must conclude that the electron cannot absorb a photon.

***55** •••

Picture the Problem Let m_i denote the mass of the incident (projectile) particle. Then $\sum m_{in} = m_i + m_{target}$ and we can use this expression to determine the threshold kinetic energy of protons incident on a stationary proton target for the production of a proton–antiproton pair.

Consider the situation in the center of mass reference frame. At threshold we have:

$$E^2 - p^2c^2 = \sum m_{\text{fin}}c^2$$

Note that this is a relativistically invariant expression.

In the laboratory frame, the target is at rest so:

$$E_{\text{target}} = E_{\text{t}} = E_{\text{t,0}}$$

We can, therefore, write:

$$\left(E_{\text{i}} + E_{\text{t,0}}\right)^2 - p_{\text{i}}^2c^2 = \left(\sum m_{\text{fin}}c^2\right)^2$$

For the incident particle:

$$E_{\text{i}}^2 - p_{\text{i}}^2c^2 = E_{\text{i,0}}^2$$

and

$$E_{\text{i}} = E_{\text{i,0}} + K_{\text{th}}$$

where K_{th} is the threshold kinetic energy of tl incident particle in the laboratory frame.

Express K_{th} in terms of the rest energies:

$$\left(E_{\text{t,0}} + E_{\text{i.0}}\right)^2 + 2K_{\text{th}}E_{\text{t,0}} = \left(\sum m_{\text{fin}}c^2\right)^2$$

where

$$E_{\text{t,0}} + E_{\text{i.0}} = \sum m_{\text{fin}}c^2$$

and

$$E_{\text{t,0}} = m_{\text{target}}c^2$$

Substitute to obtain:

$$\left(\sum m_{\text{fin}}c^2\right)^2 + 2K_{\text{th}}m_{\text{target}}c^2 = \left(\sum m_{\text{fin}}c^2\right)^2$$

Solve for K_{th} to obtain:

$$K_{\text{th}} = \boxed{\frac{\left(\sum m_{\text{in}} + \sum m_{\text{fin}}\right)\left(\sum m_{\text{fin}} - \sum m_{\text{in}}\right)c^2}{2m_{\text{target}}}}$$

For the creation of a proton - antiproton pair in a proton - proton collision:

$$\sum m_{\text{in}} = 2m_{\text{p}}$$

$$\sum m_{\text{fin}} = 4m_{\text{p}}$$

and

$$m_{\text{target}} = m_{\text{p}}$$

Substitute to obtain:

$$K_{\text{th}} = \frac{\left(2m_{\text{p}} + 4m_{\text{p}}\right)\left(4m_{\text{p}} - 2m_{\text{p}}\right)c^2}{2m_{\text{p}}}$$

$$= \frac{\left(6m_{\text{p}}\right)\left(2m_{\text{p}}\right)c^2}{2m_{\text{p}}} = \boxed{6m_{\text{p}}c^2}$$

in agreement with Problem 40.

56 •••

Picture the Problem We'll solve the problem for the general case of a particle of rest mass *M* decaying into two identical particles each of rest mass *m*.

In the center of mass reference frame:

$$Mc^2 = 2mc^2 = 2\gamma mc^2$$

Solve for *u*/*c* to obtain:

$$\frac{u}{c} = \sqrt{1-\left(\frac{2m}{M}\right)^2}$$

where *u* is the speed of each of the decay particles in the CM frame.

Next we determine the speed *v* of the laboratory frame relative to the CM frame. The energy of the particle of rest mass *M* is:

$$\gamma_{CM}Mc^2$$

where

$$\gamma_{CM} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

and

$$\frac{v}{c} = \beta_{CM} = \sqrt{1-\frac{1}{\gamma_{CM}^2}}$$

Use Equation 39-18*a* to express u_{lab}, the speeds of the decay products in the laboratory reference frame:

$$u_{lab} = \frac{\beta_{CM} \pm \frac{u}{c}}{1 \pm \beta_{CM}\frac{u}{c}}c$$

where ± refers to the fact that one of the decay particles will travel in the direction of *M*, and the other in the direction opposite to that of *M*.

In this problem we have:

$$\gamma_{CM} = 4\,,\ \beta_{CM} = 0.968\,,\ \frac{2m_0}{M_0} = 0.6\,,$$

and $\frac{u}{c} = 0.8$

Substitute to obtain:

$$u_{lab} = \frac{0.968+0.8}{1+(0.968)(0.8)}c = \boxed{0.996c}$$

and

$$u_{\text{lab}} = \frac{0.968 - 0.8}{1 - (0.968)(0.8)}c = \boxed{0.745c}$$

57 •••

Picture the Problem We can write the components of the stick in its reference frame and then apply the Lorentz length contraction equation to obtain the given result.

In its reference frame, the stick has x and y components:

$$L_{\text{p}x} = L_{\text{p}} \cos\theta$$

and

$$L_{\text{p}y} = L_{\text{p}} \sin\theta$$

Only $L_{\text{p}x}$ is Lorentz contracted to:

$$L_x' = \frac{L_{\text{p}x}}{\gamma}$$

Hence, the length in the reference frame S' is:

$$L' = \left[(L_x')^2 + (L_y')^2\right]^{1/2} = \boxed{L_{\text{p}}\left(\frac{\cos^2\theta}{\gamma^2} + \sin^2\theta\right)^{1/2}}$$

The angle that L' makes with the x' axis is given by:

$$\tan\theta' = \frac{L_y'}{L_x'} = \frac{\sin\theta}{\frac{\cos\theta}{\gamma}} = \boxed{\gamma\tan\theta}$$

58 •••

Picture the Problem We can express the tangent of the angle u' makes with the x' axis and then use the velocity transformation equations to obtain the given result.

Express the tangent of the angle u' makes with the x' axis:

$$\tan\theta' = \frac{u_y'}{u_x'}$$

Substitute for u_y' and u_x':

$$\tan\theta' = \frac{\dfrac{u_y}{\gamma\left(1 - \dfrac{vu_x}{c^2}\right)}}{\dfrac{u_x - v}{1 - \dfrac{vu_x}{c^2}}} = \frac{u_y}{\gamma(u_x - v)}$$

Substitute for u_y and u_x and simplify to obtain:

$$\tan\theta' = \frac{u\sin\theta}{\gamma(u\cos\theta - v)} = \boxed{\frac{\sin\theta}{\gamma(\cos\theta - v/u)}}$$

***59** •••

Picture the Problem We can use the expressions for $\vec{p}$ and E in S together with the relations we wish to verify and the inverse velocity transformation equations to establish the condition $u'^2 = (u_x')^2 + (u_y')^2 + (u_z')^2 = v^2 + \frac{u^2}{\gamma^2}$ and then use this result to verify the given expressions for p_x', p_y', p_z' and E'/c.

In any inertial frame the momentum and energy are given by:

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}} \text{ and } E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

where $\vec{u}$ is the velocity of the particle and u is its speed.

The components of $\vec{p}$ in S are:

$$p_x = \frac{mu_x}{\sqrt{1-\frac{u^2}{c^2}}},\ p_y = \frac{mu_y}{\sqrt{1-\frac{u^2}{c^2}}}, \text{ and}$$

$$p_z = \frac{mu_z}{\sqrt{1-\frac{u^2}{c^2}}}$$

Because $u_x = u_z = 0$ and $u_y = u$:

$$p_x = p_z = 0$$

and

$$p_y = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}}$$

Substituting zeros for p_x and p_z in the relations we are trying to show yields:

$$p_x' = \gamma\left(0 - \frac{vE}{c^2}\right) = -\gamma\frac{vE}{c^2},\ p_y' = p_y,$$

$p_z' = 0$, and

$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - 0\right) = \gamma\frac{E}{c}$$

In S' the momentum components are:

$$p_x' = \frac{mu_x'}{\sqrt{1-\frac{u'^2}{c^2}}},\ p_y' = \frac{mu_y'}{\sqrt{1-\frac{u'^2}{c^2}}}, \text{ and}$$

$$p_z' = \frac{mu_z'}{\sqrt{1-\frac{u'^2}{c^2}}}$$

The inverse velocity transformations are:

$$u_x' = \frac{u_x - v}{\sqrt{1 - \frac{vu_x}{c^2}}},\ u_y' = \frac{u_y}{\sqrt{1 - \frac{vu_y}{c^2}}},\ \text{and}$$

$$u_z' = \frac{u_z}{\sqrt{1 - \frac{vu_z}{c^2}}}$$

Substitute $u_x = u_z = 0$ and $u_y = u$ to obtain:

$$u_x' = -v,\ u_y' = \gamma u,\ \text{and}\ u_z' = 0$$

Thus:

$$u'^2 = (u_x')^2 + (u_y')^2 + (u_z')^2 = v^2 + \frac{u^2}{\gamma^2}$$

First we verify that $p_z' = p_z = 0$:

$$p_z' = \frac{m(0)}{\sqrt{1 - \frac{u'^2}{c^2}}} = p_z = \boxed{0}$$

Next we verify that $p_y' = p_y$:

$$p_y' = \frac{mu_y'}{\sqrt{1 - \frac{u'^2}{c^2}}} = \frac{mu}{\gamma\sqrt{1 - \frac{v^2}{c^2} - \frac{u^2}{\gamma^2 c^2}}} = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}}\,\frac{\sqrt{1 - \frac{u^2}{c^2}}}{\gamma\sqrt{1 - \frac{v^2}{c^2} - \frac{u^2}{\gamma^2 c^2}}}$$

$$= \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}}\sqrt{\frac{\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2} - \frac{u^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)}} = p_y\sqrt{\frac{1 - \frac{v^2}{c^2} - \frac{u^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2} - \frac{u^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \boxed{p_y}$$

Next we verify that $p_x' = \gamma\left(p_x - \frac{vE}{c^2}\right)$:

$$p_x{}' = \frac{mu_x{}'}{\sqrt{1-\frac{u'^2}{c^2}}} = \frac{-mv}{\gamma\sqrt{1-\frac{v^2}{c^2}-\frac{u^2}{\gamma^2c^2}}} = -\frac{\gamma v}{c^2}\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}\frac{\gamma^{-1}\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}-\frac{u^2}{\gamma^2c^2}}}$$

$$= -\frac{\gamma v}{c^2}E\sqrt{\frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}} = -\frac{\gamma v}{c^2}E\sqrt{\frac{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}}$$

$$= \boxed{-\frac{\gamma v}{c^2}E}$$

Finally, we verify that $\frac{E'}{c} = \gamma\left(\frac{E}{c}-\frac{vp_x}{c}\right) = \gamma\frac{E}{c}$, or $E' = \gamma E$:

$$E' = \frac{mc^2}{\sqrt{1-\frac{u'^2}{c^2}}} = \frac{\gamma mc^2}{\sqrt{1-\frac{u^2}{c^2}}}\frac{\gamma^{-1}\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{u'^2}{c^2}}} = \gamma E\frac{\gamma^{-1}\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}-\frac{u^2}{\gamma^2c^2}}}$$

$$= \gamma E\sqrt{\frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}} = \gamma E\sqrt{\frac{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}}$$

$$= \boxed{\gamma E}$$

The *x, y, z,* and *t* transformation equations are:

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
and
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

The *x, y, z,* and *ct* transformation equations are:

$$x' = \gamma\left(x - \frac{v}{c}ct\right)$$
$$y' = y$$
$$z' = z$$
and

$$ct' = \gamma\left(ct - \frac{v}{c}x\right)$$

The p_x, p_y, p_z, and E/c transformation equations are:

$$p_x' = \gamma\left(p_x - \frac{v}{c}\frac{E}{c}\right)$$

$$p_y' = p_y$$

$$p_z' = p_z$$

and

$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \frac{v}{c}p_x\right)$$

Note that the transformation equations for *x, y, z,* and *ct* and the transformation equations for p_x, p_y, p_z, and E/c are identical.

60 •••

Picture the Problem The Lorentz transformation was derived on the basis of the postulate that the speed of light is *c* in any inertial reference frame. Thus, if the clocks in *S* and *S′* are synchronized at $t = t' = 0$, then it follows from the Einstein postulate that $r^2 = c^2t^2$ and $r'^2 = c^2t'^2$ or $r^2 - c^2t^2 = 0 = r'^2 - c^2t'^2$. In other words, the quantity $s^2 = r^2 - c^2t^2 = 0$ is a relativistic invariant, which can also be written as $x^2 + y^2 + z^2 - c^2t^2 = 0$.

Using the Lorentz transformation equations for *x*, *y*, *z*, and *t* we have:

$$x'^2 + y'^2 + z'^2 - (ct')^2 = \gamma^2(x^2 - 2vxt + v^2t^2) + y^2 + z^2 - \gamma^2(c^2t^2 - 2vxt + v^2x^2/c^2)$$

The terms linear in *x* cancel and the terms in x^2 combine to give:

$$\gamma^2x^2(1 - v^2/c^2) = x^2$$

The coefficients of the terms in $(ct)^2$ give:

$$\gamma^2(v^2/c^2 - 1) = -1$$

Thus, $r^2 - c^2t^2 = r'^2 - c^2t'^2$ as required by the Einstein postulate.

61 •••

Picture the Problem We'll use Equation 39-27 to show that this quantity has the value $-mc^2$ in both the *S* and *S′* reference frames.

From Equation 39-27, the relationship between total energy *E,* momentum *p,* and rest energy mc^2 is:

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

or

$$p^2c^2 - E^2 = -\left(mc^2\right)^2$$

Divide both sides of this equation by c^2 to obtain:

$$p^2 - \left(\frac{E}{c}\right)^2 = -(mc)^2 \qquad (1)$$

We can relate p to p_x, p_y, and p_z:

$$p^2 = p_x^2 + p_y^2 + p_z^2$$

Substitute for p^2 in equation (1) to obtain:

$$p_x^2 + p_y^2 + p_z^2 - \left(\frac{E}{c}\right)^2 = -m^2c^2$$

Because m is the mass of the particle in its rest frame, it is constant. Hence:

$p^2 - \left(\frac{E}{c}\right)^2$ must be a relativistic invariant.

Also, in Problem 59 we saw that the components of p and the quantity E/c transform like the components of r and the quantity ct. In Problem 60 we demonstrated that $r^2 - (ct)^2$ is a relativistic invariant. Consequently, $p^2 - (E/c)^2$ must also be relativistically invariant.

***62 •••**

Picture the Problem We can use the inverse Lorentz transformation for time to show that the observer will conclude that the rod is bent into a parabolic shape.

In frame S where the rod is not moving along the x axis, the height of the rod at time t is:

$$y(t) = -\tfrac{1}{2}gt^2$$

The inverse Lorentz time transformation is:

$$t = \gamma\left(t' + \frac{vx}{c^2}\right)$$

Express $y'(t)$ in the moving frame of reference:

$$y'(t) = -\tfrac{1}{2}g\gamma\left(t' + \frac{vx}{c^2}\right)^2$$

Evaluate $y'(t)$ at $t' = 0$ to obtain:

$$y'(t) = -\frac{g\gamma v^2}{2c^2}x^2 \qquad (1)$$

Because equation (1) is the equation of a parabola, we've shown that the moving observer will conclude that the rod is bent into a parabolic shape. Because the coefficient of x^2 is negative, the parabola is concave downward.

Chapter 40
Nuclear Physics

Conceptual Problems

1 •

Determine the Concept Two or more nuclides with the same atomic number Z but different N and A numbers are called isotopes.

(a) Two other isotopes of ^{14}N are: ^{15}N, ^{16}N

(b) Two other isotopes of ^{56}Fe are: ^{54}Fe, ^{55}Fe

(c) Two other isotopes of ^{118}Sn are: ^{54}Fe, ^{55}Fe

2 •

Determine the Concept The parent of that series, ^{237}Np, has a half-life of 2×10^{6} y that is much shorter than the age of the earth. There is no naturally occurring Np remaining on earth.

3 •

Determine the Concept Generally, β-decay leaves the daughter nucleus neutron rich, i.e., above the line of stability. The daughter nucleus therefore tends to decay via β^- emission which converts a nuclear neutron to a proton.

***4** •

Determine the Concept ^{14}C is found on earth because it is constantly being formed by cosmic rays in the upper atmosphere in the reaction $^{14}\text{N} + \text{n} \rightarrow {}^{14}\text{C} + {}^{1}\text{H}$.

5 •

Determine the Concept It would make the dating unreliable because the current concentration of ^{14}C is not equal to that at some earlier time.

6 •

Determine the Concept An element with such a high Z value would either fission spontaneously or decay almost immediately by α emission (see Figure 40-3).

7 •

Determine the Concept The probability for neutron capture by the fissionable nucleus is large only for slow (thermal) neutrons. The neutrons emitted in the fission process are fast (high energy) neutrons and must be slowed to thermal neutrons before they are likely to be captured by another fissionable nucleus.

8 •

Determine the Concept The process of "slowing down" involves the sharing of energy of a fast neutron and another nucleus in an elastic collision. The fast particle will lose maximum energy in such a collision if the target particle is of the same mass as the incident particle. Hence, neutron-proton collisions are most effective in slowing down neutrons. However, ordinary water cannot be used as a moderator because protons will capture the slow neutrons and form deuterons.

9 •

Determine the Concept Beta decay occurs in nuclei that have too many or too few neutrons for stability. In β decay, A remains the same while Z either increases by 1 (β^- decay) or decreases by 1 (β^+ decay).

(*a*) The reaction is: $${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{+1}\beta \Rightarrow \boxed{\beta^+} \text{ decay}$$

(*b*) The reaction is: $${}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + {}^{0}_{-1}\beta \Rightarrow \boxed{\beta^-} \text{ decay}$$

10 •

Advantages	Disadvantages
The reactor uses ^{238}U, which, by neutron capture and subsequent decays, produces ^{239}Pu. Thus plutonium isotope fissions by fast neutron capture. Thus, the breeder reactor uses the plentiful uranium isotope and does not need a moderator to slow the neutrons needed for fission.	The fraction of delayed neutrons emitted in the fission of ^{239}Pu is very small. Consequently, control of the fission reaction is very difficult, and the safety hazards are more severe than for the ordinary reactor that uses ^{235}U as fuel.

11 •

(*a*) False. The nucleus does not contain electrons.

(*b*) True.

(*c*) False. After two half-lives, three-fourths of the radioactive nuclei in a given sample have decayed.

(*d*) True (given an unlimited supply of ^{238}U).

12 •

Determine the Concept Pressure and temperature changes have no effect on the internal structure of the nucleus. They do have an effect on the electronic configuration; consequently, they can influence K-capture processes.

***13** •

Determine the Concept Knowing the parent nucleus and one of the decay products, we can use the conservation of charge, the conservation of energy, and the conservation of the number of nucleons to identify the participants in the decay.

(*a*) beta decay of ^{16}N $\boxed{{}^{16}_{7}\text{N}\to{}^{16}_{8}\text{O}+{}^{0}_{-1}\beta+{}^{0}_{0}\overline{\nu}+Q}$

(*b*) alpha decay of ^{248}Fm $\boxed{{}^{248}_{100}\text{Fm}\to{}^{244}_{98}\text{Cf}+{}^{4}_{2}\text{He}+Q}$

(*c*) positron decay of ^{12}N $\boxed{{}^{12}_{7}\text{N}\to{}^{12}_{6}\text{C}+{}^{0}_{+1}\beta+{}^{0}_{0}\nu+Q}$

(*d*) beta decay of ^{81}Se $\boxed{{}^{81}_{34}\text{Se}\to{}^{81}_{35}\text{Br}+{}^{0}_{-1}\beta+{}^{0}_{0}\overline{\nu}+Q}$

(*e*) positron decay of ^{61}Cu $\boxed{{}^{61}_{29}\text{Cu}\to{}^{61}_{28}\text{Ni}+{}^{0}_{+1}\beta+{}^{0}_{0}\nu+Q}$

(*f*) alpha decay of $^{228\text{Th}}$ $\boxed{{}^{228}_{90}\text{Th}\to{}^{224}_{88}\text{Ra}+{}^{4}_{2}\text{He}+Q}$

***14** •

Determine the Concept We can use the information regarding the daughter nuclei to write and balance equations for each of the reactions.

(*a*) $\boxed{{}^{240}_{94}\text{Pu}\to 3{}^{1}_{0}\text{n}+{}^{90}_{38}\text{Sr}+{}^{147}_{56}\text{Ba}}$

(*b*) $\boxed{{}^{72}_{32}\text{Ge}+{}^{4}_{2}\text{He}\to{}^{1}_{0}\text{n}+{}^{75}_{34}\text{Se}}$

(*c*) $\boxed{{}^{127}_{53}\text{I}+{}^{2}_{1}\text{H}\to{}^{1}_{0}\text{n}+{}^{128}_{54}\text{Xe}}$

(*d*) $\boxed{{}^{235}_{92}\text{U}+{}^{1}_{0}\text{n}\to 2{}^{1}_{0}\text{n}+{}^{113}_{47}\text{Ag}+{}^{121}_{45}\text{Rh}}$

(*e*) $\boxed{{}^{55}_{25}\text{Mn}+{}^{7}_{3}\text{Li}\to{}^{3}_{1}\text{H}+{}^{59}_{27}\text{Co}}$

(*f*) ${}^{238}_{92}\text{U}+{}^{1}_{0}\text{n}\to{}^{239}_{92}\text{U};\ {}^{239}_{92}\text{U}\to{}^{0}_{-1}\beta+{}^{0}_{0}\nu+{}^{239}_{93}\text{Np};\ {}^{239}_{93}\text{Np}\to{}^{0}_{-1}\beta+{}^{0}_{0}\nu+\boxed{{}^{239}_{94}\text{Pu}}$

Estimation and Approximation

15 •

Picture the Problem There is no table of half lives in the text although the information is mentioned in the alpha particle discussion for alpha decay (about 15 orders of magnitude). Mass density in an atom ranges roughly as the cube of the radius of an atom to that of the nucleus, also about 15 orders of magnitude. Nuclear masses only range 2 orders of magnitude.

Material property	Ratio (order of magnitude)
Mass density	10^{15}
Half life	10^{15}
Nuclear masses	2

16 ••

Picture the Problem The mass of ^{235}U required is given by $m_{235} = \frac{N}{N_A} M_{235}$, where M_{235} is the molecular mass of ^{235}U and N is the number of fissions required to produce 10^{20} J. The mass of deuterium and tritium required can be found similarly.

(*a*) Relate the mass of ^{235}U required to the number of fissions N required:

$$m_{235} = \frac{N}{N_A} M_{235} \qquad (1)$$

where M_{235} is the molecular mass of ^{235}U.

Determine N:

$$N = \frac{E_{\text{annual}}}{E_{\text{per fission}}}$$

Substitute numerical values and evaluate N:

$$N = \frac{10^{20}\text{ J}}{200\,\text{MeV} \times \dfrac{1.60\times10^{-19}\text{ J}}{\text{eV}}} = 3.13\times10^{30}$$

Substitute numerical values in equation (1) and evaluate m_{235}:

$$m_{235} = \frac{3.13\times10^{30}}{6.02\times10^{23}\text{ nuclei/mol}}(235\,\text{g/mol}) = \boxed{5.20\times10^{6}\text{ kg}}$$

(*b*) Relate the mass of ^{2}H and ^{3}H required to the number of fusions N required:

$$m_{^2\text{H}+^3\text{H}} = \frac{N}{N_\text{A}} M_{^2\text{H}+^3\text{H}} \qquad (2)$$

where $M_{^2\text{H}+^3\text{H}}$ is the molecular mass of ^{2}H + ^{3}H.

Determine N:

$$N = \frac{E_\text{annual}}{E_\text{per fission}}$$

Substitute numerical values and evaluate N:

$$N = \frac{10^{20}\ \text{J}}{18\,\text{MeV} \times \dfrac{1.60\times10^{-19}\ \text{J}}{\text{eV}}}$$

$$= 3.47\times10^{31}$$

Substitute numerical values in equation (2) and evaluate $m_{^2\text{H}+^3\text{H}}$:

$$m_{235} = \frac{3.47\times10^{31}}{6.02\times10^{23}\ \text{nuclei/mol}}(5\,\text{g/mol}) = \boxed{2.88\times10^{6}\ \text{kg}}$$

Properties of Nuclei

***17 •**

Picture the Problem To find the binding energy of a nucleus we add the mass of its neutrons to the mass of its protons and then subtract the mass of the nucleus and multiply by c^2. To convert to MeV we multiply this result by 931.5 MeV/u. The binding energy per nucleon is the ratio of the binding energy to the mass number of the nucleus.

(*a*) For ^{12}C, Z = 6 and N = 6. Add the mass of the neutrons to that of the protons:

$$6m_\text{p} + 6m_\text{n} = 6\times1.007825\,\text{u} + 6\times1.008665\,\text{u} = 12.098940\,\text{u}$$

Subtract the mass of ^{12}C from this result:

$$(6m_\text{p} + 6m_\text{n}) - m_{^{12}\text{C}} = 12.098940\,\text{u} - 12\,\text{u} = 0.098940\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (0.098940\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\text{u}} = \boxed{92.2\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\text{b}}{A} = \dfrac{92.2\,\text{MeV}}{12} = \boxed{7.68\,\text{MeV}}$

(b) For ^{56}Fe, Z = 26 and N = 30. Add the mass of the neutrons to that of the protons:

$$26m_\text{p} + 30m_\text{n} = 26\times1.007825\,\text{u} + 30\times1.008665\,\text{u} = 56.463400\,\text{u}$$

Subtract the mass of ^{56}Fe from this result:

$$\left(26m_\text{p} + 30m_\text{n}\right) - m_{^{12}\text{C}} = 56.463400\,\text{u} - 55.934942\,\text{u} = 0.528458\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (0.528458\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}} = \boxed{492\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\text{b}}{A} = \dfrac{492\,\text{MeV}}{56} = \boxed{8.79\,\text{MeV}}$

(c) For ^{238}U, Z = 92 and N = 146. Add the mass of the neutrons to that of the protons:

$$92m_\text{p} + 146m_\text{n} = 92\times1.007825\,\text{u} + 146\times1.008665\,\text{u} = 239.984990\,\text{u}$$

Subtract the mass of ^{238}U from this result:

$$\left(92m_\text{p} + 146m_\text{n}\right) - m_{^{238}\text{U}} = 239.984990\,\text{u} - 238.050783\,\text{u} = 1.934207\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (1.934207\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}} = \boxed{1802\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\text{b}}{A} = \dfrac{1802\,\text{MeV}}{238} = \boxed{7.57\,\text{MeV}}$

18 •

Picture the Problem To find the binding energy of a nucleus we add the mass of its neutrons to the mass of its protons and then subtract the mass of the nucleus and multiply by c^2. To convert to MeV we multiply this result by 931.5 MeV/u. The binding energy per nucleon is the ratio of the binding energy to the mass number of the nucleus.

(*a*) For ^{6}Li, Z = 3 and *N* = 3. Add the mass of the neutrons to that of the protons:

$$3m_\text{p} + 3m_\text{n} = 3\times 1.007825\,\text{u} + 3\times 1.008665\,\text{u} = 6.049470\,\text{u}$$

Subtract the mass of ^{6}Li from this result:

$$(3m_\text{p} + 3m_\text{n}) - m_{^6\text{L}i} = 6.049470\,\text{u} - 6.015122\,\text{u} = 0.034348\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (0.034348\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\text{u}} = \boxed{32.0\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\text{b}}{A} = \dfrac{32.0\,\text{MeV}}{6} = \boxed{5.33\,\text{MeV}}$

(*b*) For ^{39}K, Z = 19 and *N* = 20. Add the mass of the neutrons to that of the protons:

$$19m_\text{p} + 20m_\text{n} = 19\times 1.007825\,\text{u} + 20\times 1.008665\,\text{u} = 39.321975\,\text{u}$$

Subtract the mass of ^{39}K from this result:

$$(19m_\text{p} + 20m_\text{n}) - m_{^{39}\text{K}} = 39.321975\,\text{u} - 38.963707\,\text{u} = 0.358268\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (0.358268\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\text{u}} = \boxed{334\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\text{b}}{A} = \dfrac{334\,\text{MeV}}{39} = \boxed{8.56\,\text{MeV}}$

(*c*) For ^{208}Pb, Z = 82 and *N* = 126. Add the mass of the neutrons to that of the protons:

$$82m_p + 126m_n = 82\times 1.007825\,\text{u} + 126\times 1.008665\,\text{u} = 209.733440\,\text{u}$$

Subtract the mass of ^{208}Pb from this result:

$$(82m_p + 126m_n) - m_{^{208}\text{Pb}} = 209.733440\,\text{u} - 207.976636\,\text{u} = 1.756804\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_b = (\Delta m)c^2 = (1.756804\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}} = \boxed{1636\,\text{MeV}}$$

and the binding energy per nucleon is $\frac{E_b}{A} = \frac{1636\,\text{MeV}}{208} = \boxed{7.87\,\text{MeV}}$

19 •

Picture the Problem The nuclear radius is given by $R = R_0 A^{1/3}$ where R_0 = 1.2 fm.

(*a*) The radius of ^{16}O is:

$$R_{^{16}\text{O}} = (1.2\,\text{fm})(16)^{1/3} = \boxed{3.02\,\text{fm}}$$

(*b*) The radius of ^{56}Fe is:

$$R_{^{56}\text{Fe}} = (1.2\,\text{fm})(56)^{1/3} = \boxed{4.59\,\text{fm}}$$

(*c*) The radius of ^{197}Au is:

$$R_{^{197}\text{Au}} = (1.2\,\text{fm})(197)^{1/3} = \boxed{6.98\,\text{fm}}$$

20 •

Picture the Problem The nuclear radius is given by $R = R_0 A^{1/3}$ where R_0 = 1.2 fm.

The radii of the daughter nuclei are given by:

$$R = R_0 A^{1/3} \qquad (1)$$

Because the ratio of the mass numbers of the daughter nuclei is 3 to 1:

$$A_1 = \frac{3}{4}(239) \text{ and } A_2 = \frac{1}{4}(239)$$

Substitute in equation (1) to obtain:

$$R_1 = (1.2\,\text{fm})\left(\frac{3\times 239}{4}\right)^{1/3} = \boxed{6.77\,\text{fm}}$$

and

$$R_2 = (1.2\,\text{fm})\left(\frac{239}{4}\right)^{1/3} = \boxed{4.69\,\text{fm}}$$

***21** ••

Picture the Problem The speed of the neutrons can be found from their thermal energy. The time taken to reduce the intensity of the beam by one-half, from *I* to *I*/2, is the half-life of the neutron. Because the beam is monoenergetic, the neutrons all travel at the same speed.

(*a*) The thermal energy of the neutron is:

$$\begin{aligned} E_{\text{thermal}} &= kT \\ &= \left(1.38\times10^{-23}\ \text{J/K}\right)\left(25+273\right)\text{K} \\ &= \boxed{4.11\times10^{-21}\ \text{J}} \\ &= 4.11\times10^{-21}\ \text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{25.7\,\text{meV}} \end{aligned}$$

(*b*) Equate E_{thermal} and the kinetic energy of the neutron to obtain:

$$E_{\text{thermal}} = \tfrac{1}{2}m_{\text{n}}v^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2E_{\text{thermal}}}{m_{\text{n}}}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2\left(4.11\times10^{-21}\,\text{J}\right)}{1.67\times10^{-27}\,\text{kg}}} = \boxed{2.22\,\text{km/s}}$$

(*c*) Relate the half-life, $t_{1/2}$, to the speed of the neutrons in the beam:

$$t_{1/2} = \frac{x}{v}$$

Substitute numerical values and evaluate $t_{1/2}$:

$$t_{1/2} = \frac{1350\text{ km}}{2.22\,\text{km/s}} = 608\,\text{s}\times\frac{1\text{min}}{60\,\text{s}} = \boxed{10.1\text{min}}$$

22 •

Picture the Problem We can use the definition of density, the equation for the volume of a sphere, and the given approximation to calculate the density of nuclear matter in grams per cubic centimeter.

Express the density of a spherical nucleus:

$$\rho = \frac{m}{V}$$

The approximate mass is:

$$m = \left(1.66\times10^{-27}\text{ kg}\right)A$$

Express the volume of the nucleus:

$$V = \tfrac{4}{3}\pi\left(R_0A^{1/3}\right)^3 = \tfrac{4}{3}\pi R_0^3A$$

Substitute for *m* and *V* to obtain:

$$\rho = \frac{(1.66\times10^{-27}\text{ kg})A}{\frac{4}{3}\pi R_0^3 A} = \frac{3(1.66\times10^{-27}\text{ kg})}{4\pi R_0^3}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{3(1.66\times10^{-27}\text{ kg})}{4\pi(1.2\text{ fm})^3} = 2.29\times10^{17}\text{ kg/m}^3 = \boxed{2.29\times10^{14}\text{ g/cm}^3}$$

23 ••

Picture the Problem The separation of the nuclei when they are just touching is the sum of their radii, which is given by $R = R_0 A^{1/3}$.

The electrostatic potential energy of the system is given by:

$$U = k\frac{q_1q_2}{R} = k\frac{(Z_{\text{Mo}}e)(Z_{\text{La}}e)}{R_{\text{Mo}} + R_{\text{La}}}$$

where *R* is the distance from the center of the ^{95}Mo nucleus to the center of the ^{139}La nucleus.

The radii of the nuclei are:

$$R_{\text{Mo}} = R_o A_{\text{Mo}}^{1/3} \text{ and } R_{\text{La}} = R_o A_{\text{La}}^{1/3}$$

Substitute for R_{Mo} and R_{La} and simplify to obtain:

$$U = ke^2\frac{(Z_{\text{Mo}})(Z_{\text{La}})}{R_o A_{\text{Mo}}^{1/3} + R_o A_{\text{La}}^{1/3}} = \frac{ke^2}{R_o}\frac{(Z_{\text{Mo}})(Z_{\text{La}})}{\left(A_{\text{Mo}}^{1/3} + A_{\text{La}}^{1/3}\right)}$$

Substitute numerical values and evaluate *U*:

$$U = \frac{(8.99\times10^9\text{ Nm}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(1.2\times10^{-15}\text{ m})}\frac{(42)(57)}{\left(95^{1/3}+139^{1/3}\right)}$$
$$= 4.71\times10^{-11}\text{ J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{ J}} = \boxed{295\text{ MeV}}$$

***24 ••**

Picture the Problem The Heisenberg uncertainty principle relates the uncertainty in position, Δx, to the uncertainty in momentum, Δp, by $\Delta x\Delta p \geq \frac{1}{2}\hbar$.

Solve the Heisenberg equation for Δp:

$$\Delta p \approx \frac{\hbar}{2\Delta x}$$

Substitute numerical values and evaluate Δp:

$$\Delta p \approx \frac{1.05\times10^{-34}\,\text{J}\cdot\text{s}}{2\left(10\times10^{-15}\,\text{m}\right)} = 5.25\times10^{-21}\,\text{kg}\cdot\text{m/s}$$

The kinetic energy of the electron is given by:

$$K = pc$$

Substitute numerical values and evaluate K:

$$K = \left(5.25\times10^{-21}\,\text{kg}\cdot\text{m/s}\right)\left(3\times10^{8}\,\text{m/s}\right) = 1.58\times10^{-12}\,\text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = 9.88\,\text{MeV}$$

This result contradicts experimental observations that show that the energy of electrons in unstable atoms is of the order of 1 to 1000 eV.

Radioactivity

25 •

Picture the Problem The counting rate, as a function of the number of half-lives *n*, is given by $R = \left(\frac{1}{2}\right)^n R_0$.

(*a*) The counting rate after *n* half-lives is:

$$R = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for *n* to obtain:

$$n = \frac{\ln\left(R/R_0\right)}{\ln\left(\frac{1}{2}\right)}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{\ln\left(\dfrac{1000\,\text{counts/s}}{4000\,\text{counts/s}}\right)}{\ln\left(\frac{1}{2}\right)} = 2$$

Because there are two half-lives in 10 min:

$$t_{1/2} = \boxed{5\,\text{min}}$$

(*b*) At the end of 4 half-lives:

$$R = \left(\tfrac{1}{2}\right)^4\left(4000\,\text{counts/s}\right) = \boxed{250\,\text{Bq}}$$

26 •

Picture the Problem The counting rate, as a function of the number of half-lives *n*, is given by $R = \left(\frac{1}{2}\right)^n R_0$.

(*a*) When t = 4 min, two half-lives will have passed and n = 2:

$$R = \left(\tfrac{1}{2}\right)^2\left(2000\,\text{counts/s}\right) = \boxed{500\,\text{Bq}}$$

(*b*) When $t = 6$ min, three half-lives will have passed and $n = 3$:

$$R = \left(\tfrac{1}{2}\right)^3 (2000\,\text{counts/s}) = \boxed{250\,\text{Bq}}$$

(*c*) When $t = 8$ min, four half-lives will have passed and $n = 4$:

$$R = \left(\tfrac{1}{2}\right)^4 (2000\,\text{counts/s}) = \boxed{125\,\text{Bq}}$$

27 •

Picture the Problem The counting rate, as a function of the number of half-lives *n*, is given by $R = \left(\tfrac{1}{2}\right)^n R_0$ and the decay constant λ is related to the half-life by $t_{1/2} = \ln 2/\lambda$.

(*a*) Relate the counting rate at time $t = 10$ min to the counting rate at $t = 0$:

$$R_{10\,\text{min}} = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for *n*:

$$n = \frac{\ln\left(R_{10\,\text{min}}/R_0\right)}{\ln\left(\tfrac{1}{2}\right)}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{\ln\left(\dfrac{1000\,\text{counts/s}}{8000\,\text{counts/s}}\right)}{\ln\left(\tfrac{1}{2}\right)} = 3$$

Therefore, 3 half-lives have passed in 10 min:

$$3t_{1/2} = 10\,\text{min} \Rightarrow t_{1/2} = \boxed{200\,\text{s}}$$

(*b*) The decay constant λ is related to the half-life by:

$$t_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for $t_{1/2}$ and evaluate λ:

$$\lambda = \frac{\ln 2}{200\,\text{s}} = \boxed{3.47\times10^{-3}\,\text{s}^{-1}}$$

(*c*) Six half-lives will have passed in 20 min:

$$R_{20\,\text{min}} = \left(\tfrac{1}{2}\right)^6 (8000\,\text{Bq}) = \boxed{125\,\text{Bq}}$$

28 •

Picture the Problem We can use $R = \lambda N_0 e^{-\lambda t}$ to show that the disintegration rate is approximately 1 Ci.

The decay rate is given by:

$$R = \lambda N_0 e^{-\lambda t}$$

where N_0 is the number of nuclei at $t = 0$.

The decay constant λ is related to the half-life by:

$$t_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for $t_{1/2}$ and evaluate λ:

$$\lambda = \frac{\ln 2}{1620\,\text{y}} = 4.28\times10^{-4}\,\text{y}^{-1}\times\frac{\text{y}}{31.56\,\text{Ms}}$$
$$= 1.356\times10^{-11}\,\text{s}^{-1}$$

The number of nuclei at $t = 0$ is given by:

$N_0 = N_A/M$
where M is the atomic mass of radium and N_A is Avogadro's number.

Substitute numerical values and evaluate N_0:

$$N_0 = \frac{6.02\times10^{23}}{226} = 2.664\times10^{21}$$

Substitute numerical values for λ and N_0 and evaluate R:

$$R = \left(1.356\times10^{-11}\,\text{s}^{-1}\right)\left(2.664\times10^{21}\right)e^{-\left(1.356\times10^{-11}\,\text{s}^{-1}\right)(1200\,\text{s})} = 3.61\times10^{10}\,\text{s}^{-1}$$
$$\approx 3.7\times10^{10}\,\text{s}^{-1} = \boxed{1\,\text{Ci}}$$

29 •

Picture the Problem We can use $R = \left(\frac{1}{2}\right)^n R_0$ to relate the counting rate R to the number of half-lives n that have passed since $t = 0$. The detection efficiency depends on the probability that a radioactive decay particle will enter the detector and the probability that upon entering the detector it will produce a count. If the efficiency is 20 percent, the decay rate must be 5 times the counting rate.

(*a*) When $t = 2.4$ min, $n = 1$ and:

$$R_{2.4\,\text{min}} = \left(\tfrac{1}{2}\right)^1\left(1000\,\text{counts/s}\right) = \boxed{500\,\text{Bq}}$$

When $t = 4.8$ min, $n = 2$ and:

$$R_{4.8\,\text{min}} = \left(\tfrac{1}{2}\right)^2\left(1000\,\text{counts/s}\right) = \boxed{250\,\text{Bq}}$$

(*b*) The number of radioactive nuclei is related to the decay rate R, and the decay constant λ:

$$R = \lambda N \Rightarrow N = \frac{R}{\lambda} \qquad (1)$$

The decay constant is related to the half-life:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.4\,\text{min}} = \frac{0.693}{144\,\text{s}}$$
$$= 4.813\times10^{-3}\,\text{s}^{-1}$$

Calculate the decay rate at $t = 0$

$$R_0 = 5\times1000\,\text{counts/s} = 5000\,\text{s}^{-1}$$

from the counting rate:

Substitute in equation (1) and evaluate N_0 at $t = 0$:

$$N_0 = \frac{R_0}{\lambda} = \frac{5000\,\text{s}^{-1}}{4.813\times10^{-3}\,\text{s}^{-1}} = \boxed{1.04\times10^{6}}$$

Calculate the decay rate at $t = 2.4$ min from the counting rate:

$$R_{2.4\,\text{min}} = 5\times500\,\text{counts/s} = 2500\,\text{s}^{-1}$$

Substitute in equation (1) and evaluate $N_{2.4\,\text{min}}$ at $t = 0$:

$$N_{2.4\,\text{min}} = \frac{R_{2.4\,\text{min}}}{\lambda} = \frac{2500\,\text{s}^{-1}}{4.813\times10^{-3}\,\text{s}^{-1}} = \boxed{5.19\times10^{5}}$$

(*c*) The time at which the counting rate will be about 30 counts/s is the product of the number of half-lives that will have passed and the half-life:

$$t = nt_{1/2} \qquad (2)$$

The counting rate R after n half-lives is related to the counting rate at $t = 0$ by:

$$R = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for n:

$$n = \frac{\ln(R/R_0)}{\ln\left(\frac{1}{2}\right)}$$

Substitute numerical values and evaluate n:

$$n = \frac{\ln(30\,\text{counts/s}/1000\,\text{counts/s})}{\ln\left(\frac{1}{2}\right)} = 5.059$$

Substitute numerical values for n and $t_{1/2}$ in equation (2) and evaluate t:

$$t = (5.059)(2.4\,\text{min}) = \boxed{12.1\,\text{min}}$$

30 •

Picture the Problem Knowing each of these reactions, we can use Table 40-1 to find the differences in the masses of the nuclei and then convert this difference into the energy released in each reaction.

(*a*) Write the reaction:

$$^{226}\text{Ra}\rightarrow{}^{222}\text{Rn}+{}^{4}\text{He}$$

Use Table 40-1 to find ΔE:

$$\Delta E = \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}}\left(226.025403\,\text{u} - 222.017571\,\text{u} - 4.002603\,\text{u}\right)c^2 = \boxed{4.87\,\text{MeV}}$$

(b) Write the reaction:

$${}^{242}\text{Pu} \rightarrow {}^{238}\text{U} + {}^{4}\text{He}$$

Use Table 40-1 to find ΔE:

$$\Delta E = \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}}\left(242.058737\,\text{u} - 238.050783\,\text{u} - 4.002603\,\text{u}\right)c^2 = \boxed{4.98\,\text{MeV}}$$

***31** ••

Picture the Problem Each ^{239}Pu nucleus emits an alpha particle whose activity, A, depends on the decay constant of ^{239}Pu and on the number N of nuclei present in the ingested ^{239}Pu. We can find the decay constant from the half-life and the number of nuclei present from the mass ingested and the atomic mass of ^{239}Pu. Finally, we can use the dependence of the activity on time to find the time at which the activity be 1000 alpha particles per second.

(a) The activity of the nuclei present in the ingested ^{239}Pu is given by:

$$A = \lambda N \qquad (1)$$

Find the constant for the decay of ^{239}Pu:

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{(24360\,\text{y})(31.56\,\text{Ms/y})}$$
$$= 9.02\times10^{-13}\,\text{s}^{-1}$$

Express the number of nuclei present in the quantity of ^{239}Pu ingested:

$$N = m_{\text{Pu}}\frac{N_{\text{A}}}{M_{\text{Pu}}}$$

where M_{Pu} is the atomic mass of ^{239}Pu.

Substitute numerical values and evaluate N:

$$N = (2.0\,\mu\text{g})\left(\frac{6.02\times10^{23}\,\text{nuclei/mol}}{239\,\text{g/mol}}\right)$$
$$= 5.04\times10^{15}\,\text{nuclei}$$

Substitute numerical values in equation (1) and evaluate A:

$$A = \left(9.02\times10^{-13}\,\text{s}^{-1}\right)\left(5.04\times10^{15}\,\alpha\right)$$
$$= \boxed{4.55\times10^{3}\,\alpha/\text{s}}$$

(b) The activity varies with time according to:

$$A = A_o e^{-\lambda t}$$

Solve for t to obtain:

$$t = \frac{\ln\left(\frac{A}{A_o}\right)}{-\lambda}$$

Substitute numerical values and evaluate t:

$$t = \frac{\ln\left(\frac{1\times10^3\,\alpha/\text{s}}{4.55\times10^3\,\alpha/\text{s}}\right)}{-\left(9.02\times10^{-13}\,\text{s}^{-1}\right)\left(\frac{31.56\,\text{Ms}}{1\,\text{y}}\right)} = \boxed{5.32\times10^4\ \text{y}}$$

32 ••

Picture the Problem We can use conservation of energy and conservation of linear momentum to relate the momenta and kinetic energies of the nuclei to the decay's Q value

(a) Express the kinetic energies of the alpha particle and daughter nucleus:

$$K_\alpha = \tfrac{1}{2}m_\alpha v_\alpha^2 = \frac{p_\alpha^2}{2m_\alpha} \qquad (1)$$

and

$$K_\text{Y} = \tfrac{1}{2}m_\text{Y} v_\text{Y}^2 = \frac{p_\text{Y}^2}{2m_\text{Y}} \qquad (2)$$

Solve equations (1) and (2) for p_α^2 and p_D^2:

$$p_\alpha^2 = 2m_\alpha K_\alpha$$

and

$$p_\text{Y}^2 = 2m_\text{Y} K_\text{Y}$$

From the conservation of linear momentum we have:

$$\vec{p}_\text{i} = \vec{p}_\text{f}$$

or, because the parent is initially at rest, $0 = p_\alpha - p_\text{Y}$ and $p_\alpha = p_\text{Y}$

Because the momenta are equal:

$$2m_\text{Y}K_\text{Y} = 2m_\alpha K_\alpha$$

Solve for K_Y:

$$K_\text{Y} = \frac{m_\alpha}{m_\text{Y}}K_\alpha = \frac{4}{A-4}K_\alpha$$

Because the daughter nucleus and the alpha particle share the Q-value:

$$Q = K_\text{Y} + K_\alpha = \frac{4}{A-4}K_\alpha + K_\alpha = \left(\frac{4}{A-4}+1\right)K_\alpha = \left(\frac{A}{A-4}\right)K_\alpha$$

Solve for K_α:

$$K_\alpha = \boxed{\left(\frac{A-4}{A}\right)Q}$$

(*b*) Substitute for K_α in the expression for Q to obtain:

$$Q = K_Y + \left(\frac{A-4}{A}\right)Q$$

Solve for K_Y:

$$K_Y = Q - \left(\frac{A-4}{A}\right)Q = \boxed{\frac{4Q}{A}}$$

***33 ••**

Picture the Problem We can write the equation of the decay process by using the fact that the post-decay sum of the *Z* and *A* numbers must equal the pre-decay values of the parent nucleus. The Q value in the equations from Problem 32 is given by $Q = -(\Delta m)c^2$.

^{239}Pu undergoes alpha decay according to:

$$\boxed{{}^{239}_{94}\text{Pu} \rightarrow {}^{235}_{92}\text{U} + {}^{4}_{2}\alpha + Q}$$

The Q value for the decay is given by:

$$Q = \left[(m_{Pu}) - (m_U + m_\alpha)\right]\left(\frac{931.5\,\text{MeV}}{1\text{u}}\right)$$

Substitute numerical values and evaluate Q:

$$Q = \left[(239.052156\,\text{u}) - (235.043923\,\text{u} + 4.002603\,\text{u})\right]\left(\frac{931.5\text{MeV}}{1\text{u}}\right) = \boxed{5.24\,\text{MeV}}$$

From Problem 32, the kinetic energy of the alpha particle is given by:

$$K_\alpha = \left(\frac{A-4}{A}\right)Q$$

Substitute numerical values and evaluate K_α:

$$K_\alpha = \left(\frac{239-4}{239}\right)(5.24\,\text{MeV}) = \boxed{5.15\,\text{MeV}}$$

From Problem 32, the kinetic energy of the ^{239}U is given by:

$$K_U = \frac{4Q}{A}$$

Substitute numerical values and evaluate K_U:

$$K_U = \frac{4(5.24\,\text{MeV})}{239} = \boxed{87.7\,\text{keV}}$$

34 •

Picture the Problem We can find the age of the sample using $R_n = \left(\tfrac{1}{2}\right)^n R_0$ to find n and then applying $t = nt_{1/2}$.

Express the age of the bone in terms of the half-life of ^{14}C and the

$$t = nt_{1/2} \qquad (1)$$

number n of half-lives that have elapsed:

The decay rate R_n after n half-lives is related to the counting rate R_0 at $t = 0$ by:

$$R_n = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for n:

$$n = \frac{\ln(R/R_0)}{\ln\left(\frac{1}{2}\right)}$$

Because there are 15.0 decays per minute per gram of carbon in a living organism:

$$R_0 = 15.0\,\frac{\text{decays}}{\text{min}\cdot\text{g}} \times \frac{1\,\text{min}}{60\,\text{s}} \times 175\,\text{g} = 43.75\,\text{Bq}$$

Substitute numerical values for R and R_0 and evaluate n:

$$n = \frac{\ln\left(\dfrac{8.1\,\text{Bq}}{43.75\,\text{Bq}}\right)}{\ln\left(\frac{1}{2}\right)} = 2.433$$

Substitute numerical values in equation (1) and evaluate t:

$$t = (2.433)(5730\,\text{y}) = \boxed{13{,}940\,\text{y}}$$

35 •

Picture the Problem We can solve $R = R_0 e^{-\lambda t}$ for λ to find the decay constant of the sample and use $t_{1/2} = \dfrac{\ln 2}{\lambda}$ to find its half-life. The number of radioactive nuclei in the sample initially can be found from $R_0 = \lambda N_0$.

(*a*) The decay rate is given by:

$$R = R_0 e^{-\lambda t}$$

Solve for λ to obtain:

$$\lambda = \frac{\ln\left(\dfrac{R}{R_0}\right)}{-t}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{\ln\left(\dfrac{85.2\,\text{Bq}}{115\,\text{Bq}}\right)}{-2.25\,\text{h}} = \boxed{0.133\,\text{h}^{-1}}$$

The half-life is related to the decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.133\,\text{h}^{-1}} = \boxed{5.20\,\text{h}}$$

(*b*) The number N_0 of radioactive nuclei in the sample initially is related to the decay constant λ and the initial decay rate R_0:

$$R_0 = \lambda N_0 \Rightarrow N_0 = \frac{R_0}{\lambda}$$

Substitute numerical values and evaluate N_0:

$$N_0 = \frac{115\,\text{Bq}}{0.133\,\text{h}^{-1} \times \frac{1\,\text{h}}{3600\,\text{s}}} = \boxed{3.11 \times 10^6}$$

***36 ••**

Picture the Problem We can use $R_0 = \lambda N$ to find the initial activity of the sample and $R = R_0 e^{-\lambda t}$ to find the activity of the sample after 1.75 y.

(*a*) The initial activity of the sample is the product of the decay constant λ for ^{60}Co and the number of atoms N of ^{60}Co initially present in the sample:

$$R_0 = \lambda N \qquad (1)$$

Express N in terms of the mass m of the sample, the molar mass M of ^{60}Co, and Avogadro's number N_A:

$$N = \frac{m}{M} N_A$$

Substitute numerical values and evaluate N:

$$N = \left(\frac{1.00 \times 10^{-6}\,\text{g}}{60\,\text{g/mol}}\right)\left(6.02 \times 10^{23}\,\text{nuclei/mol}\right) = 1.00 \times 10^{16}\ \text{nuclei}$$

The decay constant is given by:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{0.693}{(5.27\ \text{y})(31.56\,\text{Ms/y})} = 4.17 \times 10^{-9}\,\text{s}^{-1}$$

Substitute numerical values in equation (1) and evaluate A_0:

$$R_0 = \left(4.17 \times 10^{-9}\,\text{s}^{-1}\right)\left(1.00 \times 10^{16}\,\text{nuclei}\right) = 4.17 \times 10^{7}\ \text{s}^{-1} \times \frac{1\,\text{Ci}}{3.7 \times 10^{10}\ \text{s}^{-1}} = \boxed{1.13\,\text{mCi}}$$

(*b*) The activity varies with time according to:

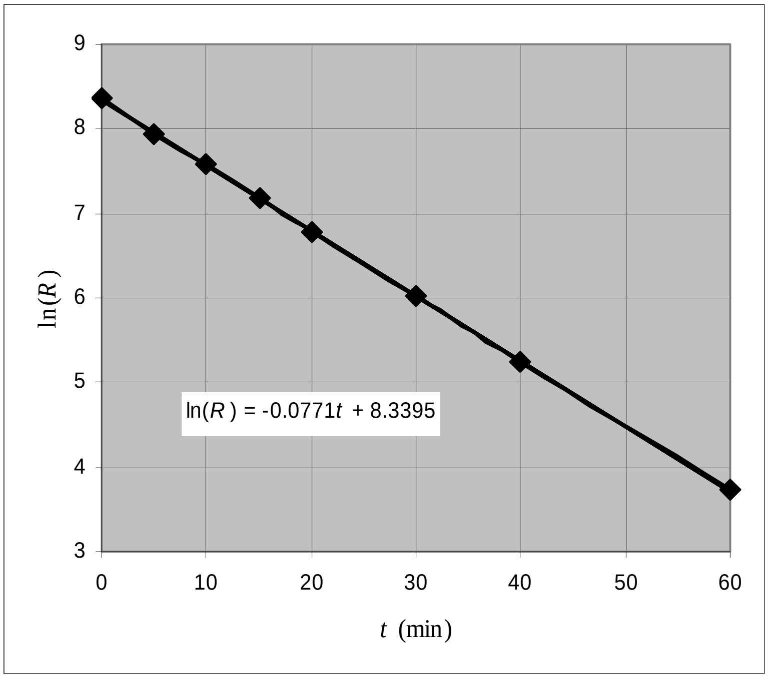
$$R = R_o e^{-\lambda t} = R_o e^{-\left(\frac{0.693t}{5.27\,\text{y}}\right)}$$

Evaluate R at t = 1.75 y:

$$R = (1.13\,\text{mCi})e^{-\left(\frac{0.693\times 1.75\,\text{y}}{5.27\,\text{y}}\right)} = \boxed{0.898\,\text{mCi}}$$

37 ••

Picture the Problem The following graph was plotted using a spreadsheet program. Excel's "Add Trendline" feature was used to determine the equation of the line.



The linearity and negative slope of this graph tell us that it represents an exponential decay.

The decay rate equation is:

$$R = R_0 e^{-\lambda t}$$

Take the natural logarithm of both sides of the equation to obtain:

$$\ln R = \ln e^{-\lambda t} + \ln R_0 = -\lambda t + \ln R_0$$

This equation is of the form:

$$y = mx + b$$

where $y = \ln R$, $x = t$, $m = -\lambda$, and $b = \ln R_0$.

The decay constant is the negative of the slope of the graph:

$$\lambda = \boxed{0.0771\,\text{min}^{-1}}$$

The half-life of the radioisotope is:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.0771\,\text{min}^{-1}} = \boxed{8.99\,\text{min}}$$

38 ••

Picture the Problem We can solve Equation 40-7 for λ to show that $\lambda = t_1^{-1}\ln(R_0/R_1)$.

(*a*) Express the half-life as a function of the decay constant λ:

$$t_{1/2} = \frac{\ln 2}{\lambda} \qquad (1)$$

From Equation 40-7 it follows that:

$$\frac{R_0}{R_1} = e^{\lambda t}$$

Solve for λ:

$$\lambda = \frac{\ln\left(\frac{R_0}{R_1}\right)}{t} = \boxed{t^{-1}\ln\left(\frac{R_0}{R_1}\right)}$$

(*b*) Substitute numerical values for t, R_1, and R_0 and evaluate λ:

$$\lambda = \frac{1}{60\,\text{s}}\ln\left(\frac{1200\,\text{Bq}}{800\,\text{Bq}}\right) = \boxed{0.00676\,\text{s}^{-1}}$$

Use the decay constant to find the half-life:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.00676\,\text{s}^{-1}} = \boxed{103\,\text{s}}$$

39 ••

Picture the Problem The required mass is given by M = (5 counts/min)/R, where R is the current counting rate per gram of carbon. We can use the assumed age of the casket to find the number of half-lives that have elapsed and $R = \left(\frac{1}{2}\right)^n R_0$ to find the current counting rate per gram of ^{14}C.

The mass of carbon required is:

$$M = \frac{5\,\text{counts/min}}{R} \qquad (1)$$

Because there were about 15.0 decays per minute per gram of the living wood, the counting rate per gram is:

$$R = \left(\tfrac{1}{2}\right)^n R_0 = \left(\tfrac{1}{2}\right)^n (15\,\text{counts/min}\cdot\text{g})$$

We can find n from the assumed age of the casket and the half-life of ^{14}C:

$$n = \frac{18{,}000\,\text{y}}{5730\,\text{y}} = 3.141$$

Substitute for *n* and evaluate *R*:

$$R = \left(\tfrac{1}{2}\right)^{3.141}(15\,\text{counts/min}\cdot\text{g}) = 1.70\,\text{counts/min}\cdot\text{g}$$

Substitute for *R* in equation (1) and evaluate *M*:

$$M = \frac{5\,\text{counts/min}}{1.70\,\text{counts/min}\cdot\text{g}} = \boxed{2.94\,\text{g}}$$

40 ••

Picture the Problem The decay constant λ can be found from the decay rate *R* and the number of radioactive nuclei *N* at the moment of interest and the half-life, in turn, can be found from the decay constant.

The decay rate *R* is related to the decay constant λ and the number of radioactive nuclei *N* at the moment of interest:

$$R = \lambda N \Rightarrow \lambda = \frac{R}{N} \qquad (1)$$

The number of radioactive nuclei *N* at the moment of interest can be found from Avogadro's number, the mass *m* of the sample, and the molar mass *M* of the sample:

$$N = N_A \frac{m}{M}$$

Substitute numerical values and evaluate *N*:

$$N = (6.02\times10^{23}\ \text{nuclei/mol})\frac{10^{-3}\ \text{g}}{59.934\,\text{g/mol}} = 1.004\times10^{19}$$

Substitute numerical values in equation (1) and evaluate λ:

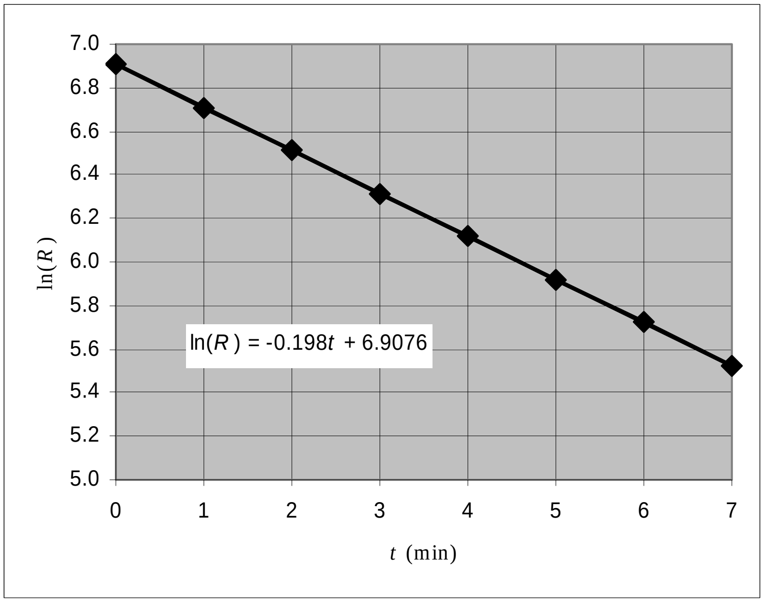
$$\lambda = \frac{1.131\,\text{Ci}\times\dfrac{3.7\times10^{10}\ \text{Bq}}{\text{Ci}}}{1.004\times10^{19}} = \boxed{4.17\times10^{-9}\ \text{s}^{-1}}$$

Find the half-life from the decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17\times10^{-9}\ \text{s}^{-1}} = 1.67\times10^{8}\ \text{s}\times\frac{1\,\text{y}}{31.56\,\text{Ms}} = \boxed{5.27\,\text{y}}$$

***41** ••

Picture the Problem The following graph was plotted using a spreadsheet program. Excel's "Add Trendline" feature was used to determine the equation of the line.



The linearity and negative slope of this graph tells us that it represents an exponential decay.

The decay rate equation is: $R = R_0 e^{-\lambda t}$

Take the natural logarithm of both sides of the equation to obtain:

$$\ln R = \ln e^{-\lambda t} + \ln R_0$$
$$= -\lambda t + \ln R_0$$

This equation is of the form: $y = mx + b$

where $y = \ln R$, $x = t$, $m = -\lambda$, and $b = \ln R_0$.

The half-life of the radioisotope is:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.198\,\text{min}^{-1}} = \boxed{3.50\,\text{min}}$$

42 ••

Picture the Problem We can use the decay rate equation $R = R_0 e^{-\lambda t}$ and the expression relating the half-life of a source to its decay constant to find the half-life of the sample. Solving the decay-rate equation for t will yield the time at which the activity level drops to any given value.

(*a*) The half-life of the material is given by:

$$t_{1/2}=\frac{\ln 2}{\lambda} \qquad (1)$$

The decay rate is given by:

$$R=R_0e^{-\lambda t} \qquad (2)$$

Solve for λ:

$$\lambda=\frac{\ln\left(\frac{R_0}{R}\right)}{t}$$

Substitute for λ in equation (1) to obtain:

$$t_{1/2}=\frac{\ln 2}{\frac{\ln\left(\frac{R_0}{R}\right)}{t}}=\frac{\ln 2}{\ln\left(\frac{R_0}{R}\right)}t$$

Substitute numerical values and evaluate $t_{1/2}$:

$$t_{1/2}=\frac{\ln 2}{\ln\left(\frac{115\,\text{decays/min}}{73.5\,\text{decays/min}}\right)}(101\,\text{h})=\boxed{156\,\text{h}}$$

(*b*) Solve equation (2) for *t*:

$$t=\frac{\ln\left(\frac{R}{R_0}\right)}{-\lambda}$$

Express λ in terms of $t_{1/2}$:

$$\lambda=\frac{\ln 2}{t_{1/2}}$$

Substitute for λ in the expression for *t* to obtain:

$$t=\frac{\ln\left(\frac{R}{R_0}\right)}{-\ln 2}t_{1/2}$$

Substitute numerical values and evaluate *t*:

$$t=\frac{\ln\left(\frac{10\,\text{decays/min}}{115\,\text{decays/min}}\right)}{-\ln 2}(156\,\text{h})=550\,\text{h}\times\frac{1\,\text{d}}{24\,\text{h}}=\boxed{22.9\,\text{d}}$$

43 ••

Picture the Problem We can use the decay rate equation $R=R_0e^{-\lambda t}$ and the expression relating the half-life of a source to its decay constant to find the age of the fossils.

The decay rate is given by:

$$R = R_0 e^{-\lambda t}$$

Solve for t to obtain:

$$t = \frac{\ln\left(\frac{R}{R_0}\right)}{-\lambda}$$

Express λ in terms of $t_{1/2}$:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for λ in the expression for t to obtain:

$$t = \frac{\ln\left(\frac{R}{R_0}\right)}{-\ln 2} t_{1/2}$$

or, because the activity at any time is proportional to the number of radioactive nuclei present,

$$t = \frac{\ln\left(\frac{N_{\text{Rb}}}{N_{0,\text{Rb}}}\right)}{-\ln 2} t_{1/2} \quad (1)$$

The number of ^{87}Sr nuclei present in the rocks is given by:

$$N_{\text{Sr}} = N_{0,\text{Rb}} - N_{\text{Rb}} \Rightarrow N_{0,\text{Rb}} = N_{\text{Sr}} + N_{\text{Rb}}$$

We're given that:

$$N_{\text{Sr}} = 0.01 N_{\text{Rb}} \Rightarrow \frac{N_{\text{Sr}}}{N_{\text{Rb}}} = 0.01$$

Express the ratio of $N_{0,\text{Rb}}$ to N_{Rb}:

$$\frac{N_{0,\text{Rb}}}{N_{\text{Rb}}} = \frac{N_{\text{Sr}} + N_{\text{Rb}}}{N_{\text{Rb}}} = \frac{N_{\text{Sr}}}{N_{\text{Rb}}} + 1$$
$$= 0.01 + 1 = 1.01$$

Substitute numerical values in equation (1) and evaluate the age of the fossils:

$$t = \frac{\ln\left(\frac{1}{1.01}\right)}{-\ln 2}\left(4.9\times10^{10}\text{ y}\right)$$
$$= \boxed{7.03\times10^{8}\text{ y}}$$

44 •••

Picture the Problem We can evaluate this integral by changing variables to obtain a form that we can find in a table of integrals.

Change variables by letting:

$$x = \lambda t$$

Then: $dx = \lambda dt$, $dt = \dfrac{dx}{\lambda}$, and $t = \dfrac{x}{\lambda}$

Substitute to obtain:

$$\tau = \int_0^\infty t\lambda e^{-\lambda t} dt = \int_0^\infty \frac{x}{\lambda}\lambda e^{-x}\frac{dx}{\lambda} = \frac{1}{\lambda}\int_0^\infty xe^{-x} dx$$

From integral tables:

$$\int_0^\infty xe^{-x} dx = 1$$

Substitute in the expression for τ to obtain:

$$\tau = \boxed{\frac{1}{\lambda}}$$

Nuclear Reactions

45 •

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for these reactions.

(*a*) Find the mass of each atom from Table 40-1:

$$m_{^1\text{H}} = 1.007825 \text{ u}$$
$$m_{^3\text{H}} = 3.016049 \text{ u}$$
$$m_{^3\text{He}} = 3.016029 \text{ u}$$
$$m_\text{n} = 1.008665 \text{ u}$$

Calculate the initial mass m_i of the incoming particles:

$$m_\text{i} = 1.007825 \text{ u} + 3.016049 \text{ u} = 4.023874 \text{ u}$$

Calculate the final mass m_f:

$$m_\text{f} = 3.016029 \text{ u} + 1.008665 \text{ u} = 4.024694 \text{ u}$$

Calculate the increase in mass:

$$\Delta m = m_\text{f} - m_\text{i} = 4.024694 \text{ u} - 4.023874 \text{ u} = 0.000820 \text{ u}$$

Calculate the Q value:

$$Q = -(\Delta m)c^2 = -(0.000820\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{u}\right) = \boxed{-0.764\,\text{MeV}}$$

(*b*) Proceed as in (*a*) to obtain:

$$Q = (0.003510\,\text{u})\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{3.27\,\text{MeV}}$$

Remarks: Because *Q* < 0 for the first reaction, it is endothermic. Because *Q* > 0 for the second reaction, it is exothermic.

46 •

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for these reactions.

(*a*) Find the mass of each atom from Table 40-1:

$$m_{^2\text{H}} = 2.014102\ \text{u}$$
$$m_{^3\text{H}} = 3.016049\ \text{u}$$
$$m_{^1\text{H}} = 1.007825\ \text{u}$$

Calculate the initial mass m_i of the incoming particles:

$$m_\text{i} = 2(2.014102\ \text{u}) = 4.028204\ \text{u}$$

Calculate the final mass m_f:

$$m_\text{f} = 3.016049\ \text{u} + 1.007825\ \text{u} = 4.023874\ \text{u}$$

Calculate the increase in mass:

$$\Delta m = m_\text{f} - m_\text{i} = 4.023874\ \text{u} - 4.028204\ \text{u} = -0.004330\ \text{u}$$

Calculate the Q value:

$$Q = -(\Delta m)c^2 = -(-0.004330\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{u}\right) = \boxed{4.03\,\text{MeV}}$$

(*b*) Proceed as in (*a*) to obtain:

$$Q = -(\Delta m)c^2 = -(-0.019703\,\text{u})\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{18.4\,\text{MeV}}$$

(*c*) Proceed as in (*a*) to obtain:

$$Q = -(\Delta m)c^2 = -(-0.005135\,\text{u})\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{4.78\,\text{MeV}}$$

***47 ••**

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for this reaction.

(*a*) The masses of the atoms are:

$$m_{^{14}\text{C}} = 14.003242\,\text{u}$$
$$m_{^{14}\text{N}} = 14.003074\,\text{u}$$

Calculate the increase in mass:

$$\Delta m = m_\text{f} - m_\text{i} = 14.003074\,\text{u} - 14.003242\,\text{u} = -0.000168\,\text{u}$$

Calculate the Q value:

$$Q = -(\Delta m)c^2 = -(-0.000168\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right) = \boxed{0.156\,\text{MeV}}$$

(*b*)

The masses given are for atoms, not nuclei, so for nuclear masses the masses are too large by the atomic number times the mass of an electron. For the given nuclear reaction, the mass of the carbon atom is too large by $6m_\text{e}$ and the mass of the nitrogen atom is too large by $7m_\text{e}$. Subtracting $6m_\text{e}$ from both sides of the reaction equation leaves an extra electron mass on the right. Not including the mass of the beta particle (electron) is mathematically equivalent to explicitly subtracting $1m_\text{e}$ from the right side of the equation.

48 ••

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for this reaction.

(*a*) The masses of the atoms are:

$$m_{^{13}\text{N}} = 13.005738\,\text{u}$$
$$m_{^{13}\text{C}} = 13.003354\,\text{u}$$

For β^+ decay:

$$Q = (m_i - m_f - 2m_e)c^2$$
$$= (m_i - m_f)c^2 - 2m_e c^2$$

Calculate $m_i - m_f$:

$$m_i - m_f = 13.005738\,\text{u} - 13.003354\,\text{u}$$
$$= 0.002384\,\text{u}$$

Calculate the Q value:

$$Q = (0.002384\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{u}\right) - 2(0.511\,\text{MeV}) = \boxed{1.20\,\text{MeV}}$$

(*b*) The atomic masses include the masses of the electrons of the neutral atoms. In this reaction the initial atom has 7 electrons and the final atom only has 6 electrons. Moreover, in addition to one electron not included in the atomic masses, a positron of mass equal to that of an electron is created. Consequently, one must add the rest energies of two electrons to the rest energy of the daughter atomic mass when calculating Q.

Fission and Fusion

***49** •

Picture the Problem The power output of the reactor is the product of the number of fissions per second and energy liberated per fission.

Express the required number N of fissions per second in terms of the power output P and the energy released per fission $E_{\text{per fission}}$:

$$N = \frac{P}{E_{\text{per fission}}}$$

Substitute numerical values and evaluate N:

$$N = \frac{500\,\text{MW}}{200\,\text{MeV}}$$
$$= \frac{5\times10^8\,\frac{\text{J}}{\text{s}} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}}{200\,\text{MeV}}$$
$$= \boxed{1.56\times10^{19}\,\text{s}^{-1}}$$

50 •

Picture the Problem If k = 1.1, the reaction rate after N generations is 1.1^N. We can find the number of generations by setting 1.1^N equal, in turn, to 2, 10, and 100 and solving for N. The time to increase by a given factor is the number of generations N needed to increase by that factor times the generation time.

(*a*) Set 1.1^N equal to 2 and solve for N:

$$(1.1)^N = 2$$
$$N \ln 1.1 = \ln 2$$
$$N = \frac{\ln 2}{\ln 1.1} = \boxed{7.27}$$

(*b*) Set 1.1^N equal to 10 and solve for N:

$$(1.1)^N = 10$$
$$N \ln 1.1 = \ln 10$$
$$N = \frac{\ln 10}{\ln 1.1} = \boxed{24.2}$$

(*c*) Set 1.1^N equal to 100 and solve for N:

$$(1.1)^N = 100$$
$$N \ln 1.1 = \ln 100$$
$$N = \frac{\ln 100}{\ln 1.1} = \boxed{48.3}$$

(*d*) Multiply the number of generations by the generation time:

$$t_2 = Nt_1 = (7.27)(1\,\text{ms}) = \boxed{7.27\,\text{ms}}$$
$$t_{10} = Nt_1 = (24.2)(1\,\text{ms}) = \boxed{24.2\,\text{ms}}$$
$$t_{100} = Nt_1 = (48.3)(1\,\text{ms}) = \boxed{48.3\,\text{ms}}$$

(*e*) Multiply the number of generations by the generation time:

$$t_2 = Nt_1 = (7.27)(100\,\text{ms}) = \boxed{0.727\,\text{s}}$$
$$t_{10} = Nt_1 = (24.2)(100\,\text{ms}) = \boxed{2.42\,\text{s}}$$
$$t_{100} = Nt_1 = (48.3)(100\,\text{ms}) = \boxed{4.83\,\text{s}}$$

***51** ••

Picture the Problem We can use $Q = -(\Delta m)c^2$, where $\Delta m = m_f - m_i$, to calculate the Q value.

The Q value is given by:

$$Q = -(\Delta m)c^2 \times \frac{931.5\,\text{MeV/c}^2}{1\text{u}}$$

Calculate the change in mass Δm:

$$\Delta m = m_f - m_i$$
$$= 94.905842\ \text{u} + 138.906348\ \text{u} + 2(1.008665\ \text{u}) - (235.043923\ \text{u} + 1.008665\ \text{u})$$
$$= -0.223068\ \text{u}$$

Substitute for Δm and evaluate Q:

$$Q = -(-0.223068\ \text{u}) \times \frac{931.5\,\text{MeV}}{1\text{u}} = \boxed{208\ \text{MeV}}$$

The ratio of Q to U found in Problem 23 is:

$$\frac{Q}{U} = \frac{208\ \text{MeV}}{236\,\text{MeV}} = \boxed{88.1\%}$$

52 ••

Picture the Problem We can find the number of neutrons per second in the generation of 4 W of power from the number of reactions per second.

The number of neutrons emitted per second is:

$$N_n = \tfrac{1}{2} N$$

where N is the number of reactions per second.

The number of reactions per second is:

$$N = 2\left(\frac{4\frac{\text{J}}{\text{s}} \times \frac{1\text{eV}}{1.60 \times 10^{-19}\ \text{J}}}{3.27\,\text{MeV} + 4.03\,\text{MeV}}\right) = 6.85 \times 10^{12}\ \text{s}^{-1}$$

Substitute for N and evaluate N_n:

$$N_n = \tfrac{1}{2}\left(6.85 \times 10^{12}\ \text{s}^{-1}\right) = \boxed{3.43 \times 10^{12}\ \text{neutrons/s}}$$

53 ••

Picture the Problem We can use the energy released in the reactions of Problem 50, together with the 17.6 MeV released in the reaction described in this problem, to find the energy released using 5 ^{2}H nuclei. Finding the number of D atoms in 4 L of H_2O, we can then find the energy produced if all of the ^{2}H nuclei undergo fusion.

Find the energy released using 5 ^{2}H nuclei:

$$Q = 3.27\,\text{MeV} + 4.03\,\text{MeV} + 17.6\,\text{MeV} = 24.9\,\text{MeV}$$

The number of H atoms in 4 L of H_2O is:

$$N_H = 2\left(\frac{m_{H_2O}}{18\,g/mol}\right)N_A$$

Substitute numerical values and evaluate N_H:

$$N_H = 2\left(\frac{4\,kg}{18\,g/mol}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right) = 2.676\times10^{26}$$

The number of D atoms in 4 L of H_2O is:

$$\begin{aligned} N_D &= \left(1.5\times10^{-4}\right)N_H \\ &= \left(1.5\times10^{-4}\right)\left(2.676\times10^{26}\right) \\ &= 4.01\times10^{22} \end{aligned}$$

The energy produced is given by:

$$E = \frac{N_D}{5}Q$$

Substitute numerical values and evaluate E:

$$\begin{aligned} E &= \frac{4.01\times10^{22}}{5}(24.9\,MeV) \\ &= 1.997\times10^{23}\,MeV\times\frac{1.60\times10^{-19}\,J}{eV} \\ &= \boxed{3.20\times10^{10}\,J} \end{aligned}$$

***54** •••

Picture the Problem We can use the conservation of momentum and the given Q value to find the final energies of both the ^{4}He nucleus and the neutron, assuming that the initial momentum of the system is zero.

Apply conservation of energy to obtain:

$$\begin{aligned} 18.6\,MeV &= \tfrac{1}{2}m_{He}v_{He}^2 + \tfrac{1}{2}m_n v_n^2 \\ &= K_{He} + K_n \end{aligned} \qquad (1)$$

Apply conservation of momentum to obtain:

$$m_{He}v_{He} + m_n v_n = 0 \qquad (2)$$

Solve equation (2) for v_{He}:

$$v_{He} = -\frac{m_n v_n}{m_{He}} \Rightarrow v_{He}^2 = \left(\frac{m_n}{m_{He}}\right)^2 v_n^2$$

Substitute for v_{He}^2 in equation (1):

$$18.6\,MeV = \tfrac{1}{2}m_{He}\left(\frac{m_n}{m_{He}}\right)^2 v_n^2 + \tfrac{1}{2}m_n v_n^2$$

or

$$18.6\,\text{MeV} = \tfrac{1}{2}m_\text{n}v_\text{n}^2\left(1+\frac{m_\text{n}}{m_\text{He}}\right)$$
$$= K_\text{n}\left(1+\frac{m_\text{n}}{m_\text{He}}\right)$$

Solve for K_n:

$$K_\text{n} = \frac{18.6\,\text{MeV}}{1+\dfrac{m_\text{n}}{m_\text{He}}}$$

Substitute numerical values for m_n and m_He and evaluate K_n:

$$K_\text{n} = \frac{18.6\,\text{MeV}}{1+\dfrac{1.008665\,\text{u}}{4.002603\,\text{u}}} = \boxed{14.86\,\text{MeV}}$$

Use equation (1) to find K_He:

$$K_\text{He} = 18.6\,\text{MeV} - K_\text{n}$$
$$= 18.6\,\text{MeV} - 14.86\,\text{MeV}$$
$$= \boxed{3.74\,\text{MeV}}$$

55 •••

Picture the Problem Adding the three reactions will yield their net effect. We can use $(\Delta m)c^2$ to find the rest energy released in the cycle and find the rate of proton consumption from the ratio of the sun's power output to the released per proton in fusion.

(*a*) Add the three reactions to obtain:

$${}^1\text{H} + {}^1\text{H} + {}^1\text{H} + {}^2\text{H} + {}^1\text{H} + {}^3\text{He} \rightarrow {}^2\text{H} + \beta^+ + \nu_\text{e} + {}^3\text{He} + \gamma + {}^4\text{He} + \beta^+ + \nu_\text{e}$$

Simplify to obtain:

$$\boxed{4\,{}^1\text{H} \rightarrow {}^4\text{He} + 2\beta^+ + 2\nu_\text{e} + \gamma}$$

(*b*) Express the rest energy released in this cycle:

$$(\Delta m)c^2 = (4m_\text{p} - m_\alpha - 4m_\text{e})c^2$$

Use Table 40-1 to find the masses of the participants in the reaction and evaluate $(\Delta m)c^2$:

$$(\Delta m)c^2 = [4(1.007825\text{ u}) - 4.002603\text{ u}]c^2 \times \frac{931.5\,\text{MeV}/c^2}{\text{u}} - 4(0.511\,\text{MeV})$$
$$= \boxed{24.7\,\text{MeV}}$$

(*c*) Express the rate *R* of proton consumption:

$$R = \frac{P}{E} \qquad (1)$$

where *E* is the energy released per proton in fusion.

Find *N*, the number of protons in the sun:

$$N = \frac{\frac{1}{2}m_{\text{sun}}}{m_{\text{p}}} = \frac{\frac{1}{2}\left(1.99\times10^{30}\ \text{kg}\right)}{1.67\times10^{-27}\ \text{kg}}$$
$$= 5.96\times10^{56}$$

where we have assumed that protons constitute about half of the total mass of the sun.

The energy released per proton in fusion is:

$$E = \tfrac{1}{4}(26.7\,\text{MeV}) = 6.675\,\text{MeV}$$
$$= 6.675\,\text{MeV}\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}$$
$$= 1.07\times10^{-12}\ \text{J}$$

Substitute numerical values in equation (1) and evaluate *R*:

$$R = \frac{4\times10^{26}\ \text{W}}{1.07\times10^{-12}\ \text{J}} = \boxed{3.74\times10^{38}\ \text{s}^{-1}}$$

The time *T* for the consumption of all protons is:

$$T = \frac{N}{R} = \frac{5.96\times10^{56}}{3.74\times10^{38}\ \text{s}^{-1}}$$
$$= 1.59\times10^{18}\ \text{s}\times\frac{1\,\text{y}}{31.56\,\text{Ms}}$$
$$= \boxed{5.04\times10^{10}\ \text{y}}$$

General Problems

56 •

Picture the Problem We can use the values of *k*, *e*, *h*, and *c* and the appropriate conversion factors to show that ke^2 = 1.44 MeV·fm and *hc* = 1240 MeV·fm

(*a*) Evaluate ke^2 to obtain:

$$ke^2 = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2 = 2.307\times10^{-28}\ \text{N}\cdot\text{m}^2$$
$$= 2.307\times10^{-28}\ \text{J}\cdot\text{m}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} = 1.44\times10^{-9}\ \text{eV}\cdot\text{m}$$
$$= 1.44\times10^{-9}\ \text{eV}\cdot\text{m}\times\frac{1\,\text{fm}}{10^{-15}\ \text{m}}\times\frac{1\,\text{MeV}}{10^{6}\ \text{eV}} = \boxed{1.44\,\text{MeV}\cdot\text{fm}}$$

(*b*) Evaluate *hc* to obtain:

$$hc = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right) = 1.99\times10^{-25}\ \text{J}\cdot\text{m}$$
$$= 1.99\times10^{-25}\ \text{J}\cdot\text{m}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}} = 1240\times10^{-9}\ \text{eV}\cdot\text{m}$$
$$= 1240\times10^{-9}\ \text{eV}\cdot\text{m}\times\frac{1\text{fm}}{10^{-15}\ \text{m}}\times\frac{1\text{MeV}}{10^{6}\ \text{eV}} = \boxed{1240\,\text{MeV}\cdot\text{fm}}$$

***57 •**

Picture the Problem We can use the given information regarding the half-life of the source to find its decay constant. We can then plot a graph of the counting rate as a function of time.

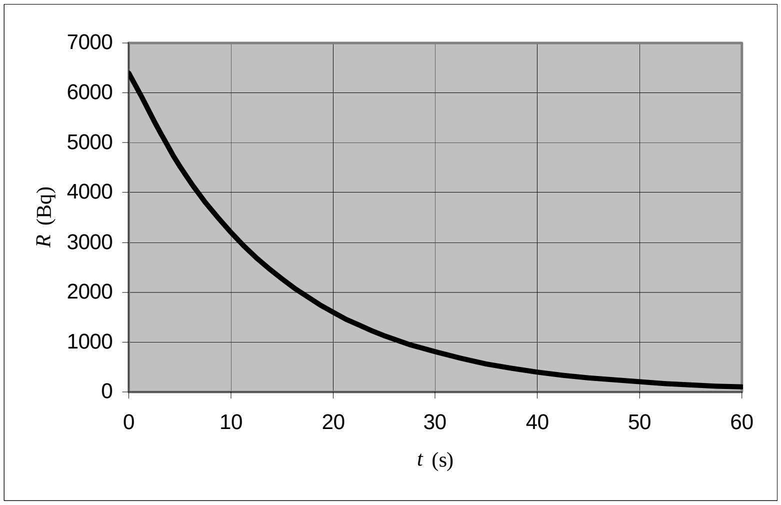
The decay constant is related to the half-life of the source:

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{10\,\text{s}} = \boxed{0.0693\text{s}^{-1}}$$

The activity of the source is given by:

$$R = R_0 e^{-\lambda t} = \left(6400\,\text{Bq}\right)e^{-\left(0.0693\,\text{s}^{-1}\right)t}$$

The following graph of $R = \left(6400\,\text{Bq}\right)e^{-\left(0.0693\,\text{s}^{-1}\right)t}$ was plotted using a spreadsheet program.



58 •

Picture the Problem The energy needed to remove a neutron is given by $Q = \left(\Delta m\right)c^2$ where Δm is the difference between the sum of the masses of the reaction products and the mass of the target nucleus.

(*a*) The reaction is: $^{4}\text{He}+\text{Q}={}^{3}\text{He}+\text{n}$

The masses are (see Table 40-1):

$$m_{^4\text{He}}=4.002603\text{ u}$$
$$m_{^3\text{He}}=3.016029\text{ u}$$
$$m_{\text{n}}=1.008665\,\text{u}$$

Calculate the final mass:

$$m_{\text{f}}=3.016029\text{ u}+1.008665\text{ u}=4.024694\text{ u}$$

Calculate the increase in mass:

$$\Delta m=m_{\text{f}}-m_{\text{i}}=4.024694\text{ u}-4.002603\text{ u}=0.022091\text{ u}$$

Calculate the energy *Q* needed to remove a neutron from ^{4}He:

$$Q=(\Delta m)c^2=(0.022091\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)=\boxed{20.6\,\text{MeV}}$$

(*b*) The reaction is: $^{7}\text{Li}+\text{Q}={}^{6}\text{Li}+\text{n}$

The masses are (see Table 40-1):

$$m_{^7\text{Li}}=7.016004\text{ u}$$
$$m_{^6\text{Li}}=6.015122\text{ u}$$
$$m_{\text{n}}=1.008665\,\text{u}$$

Calculate the final mass:

$$m_{\text{f}}=6.015122\text{ u}+1.008665\text{ u}=7.023787\text{ u}$$

Calculate the increase in mass:

$$\Delta m=m_{\text{f}}-m_{\text{i}}=7.023787\text{ u}-7.016004\text{ u}=0.007783\text{ u}$$

Calculate the energy *Q* needed to remove a neutron from ^{4}He:

$$Q=(\Delta m)c^2=(0.007783\text{u})c^2\left(\frac{931.5\,\text{MeV}/c^2}{1\text{u}}\right)=\boxed{7.25\,\text{MeV}}$$

59 •

Picture the Problem The maximum kinetic energy of the electron is given by $K_{\text{max}} = Q = \left(m_{^{14}\text{C}} - m_{^{14}\text{N}}\right)c^2$.

The maximum kinetic energy of the electron is the Q value for the reaction:

$$K_{\text{max}} = Q = \left(m_{^{14}\text{C}} - m_{^{14}\text{N}}\right)c^2$$

Find the mass of each atom from Table 40-1:

$$m_{^{14}\text{C}} = 14.003242\ \text{u}$$
$$m_{^{14}\text{N}} = 14.003074\ \text{u}$$

Calculate $\Delta m = m_{^{14}\text{C}} - m_{^{14}\text{N}}$:

$$\Delta m = 14.003242\ \text{u} - 14.003074\ \text{u} = 0.000168\,\text{u}$$

Calculate the maximum kinetic energy of the electron:

$$Q = (\Delta m)c^2 = (0.000168\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right) = \boxed{156\ \text{keV}}$$

60 •

Picture the Problem We can use the definition density to find the radius of the neutron star.

Relate the mass of the neutron star to the mass of the sun M, the volume V of the star and the nuclear density ρ:

$$M = \rho V = \tfrac{4}{3}\pi\rho R^3$$

where R is the radius of the star.

Solve for R:

$$R = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

In Problem 20 it was established that:

$$\rho = 1.174\times10^{17}\ \text{kg/m}^3$$

Substitute numerical values and evaluate R:

$$R = \sqrt[3]{\frac{3\left(1.99\times10^{30}\ \text{kg}\right)}{4\pi\left(1.174\times10^{17}\ \text{kg/m}^3\right)}} = \boxed{15.9\,\text{km}}$$

***61 ••**

Picture the Problem We can show that ^{109}Ag is stable against alpha decay by demonstrating that its Q value is negative.

The Q value for this reaction is:

$$Q = -\left[\left(m_{\text{Rh}} + m_\alpha\right) - m_{\text{Ag}}\right]c^2\left(931.5\,\frac{\text{MeV}/c^2}{\text{u}}\right)$$

Substitute numerical values and evaluate Q:

$$\begin{aligned}Q &= -\left[\left(4.002603\text{ u} + 104.905250\text{ u}\right) - 108.904756\text{ u}\right]\left(931.5\text{MeV/u}\right)\\ &= \boxed{-2.88\text{ MeV}}\end{aligned}$$

Remarks: Alpha decay occurs spontaneously and the Q value will equal the sum of the kinetic energies of the alpha particle and the recoiling daughter nucleus, $Q = K_\alpha + K_{\text{D}}$. Kinetic energy cannot be negative; hence, alpha decay cannot occur unless the mass of the parent nucleus is greater than the sum of the masses of the alpha particle and daughter nucleus, $m_{\text{P}} > m_\alpha + m_{\text{D}}$. Alpha decay cannot take place unless the total rest mass decreases.

62 ••

Picture the Problem We can use $E_{\text{threshold}} = hf_{\text{threshold}} = hc/\lambda_{\text{threshold}}$, where $E_{\text{threshold}}$ is the binding energy of the deuteron, to find the threshold wavelength for the given nuclear reaction.

Express the threshold energy of the photon:

$$E_{\text{threshold}} = hf_{\text{threshold}} = \frac{hc}{\lambda_{\text{threshold}}}$$

Solve for the threshold wavelength:

$$\lambda_{\text{threshold}} = \frac{hc}{E_{\text{threshold}}} \qquad (1)$$

The threshold energy equals the binding energy of the deuteron:

$$E_{\text{threshold}} = E_{\text{B}} = \left[m_{\text{D}} - \left(m_{\text{p}} + m_{\text{n}}\right)\right]c^2 \times \frac{931.5\,\text{MeV}/c^2}{\text{u}}$$

Substitute numerical values and evaluate E_{th}, the energy that must be added to the deuteron that will cause it to fission:

$$\begin{aligned}E_{\text{threshold}} &= \left[2.014102\text{ u} - \left(1.007825\text{ u} + 1.008665\text{ u}\right)\right]\left(931.5\,\text{MeV/u}\right)\\ &= -2.22\text{ MeV} \times \frac{1.60\times10^{-19}\text{ J}}{\text{eV}} = -3.55\times10^{-13}\text{ J}\end{aligned}$$

Substitute numerical values in equation (1) and evaluate $\lambda_{\text{threshold}}$:

$$\lambda_{\text{threshold}} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{3.55\times10^{-13}\,\text{J}} = 5.60\times10^{-13}\,\text{m} = \boxed{0.560\,\text{pm}}$$

63 •

Picture the Problem The activity of a radioactive source is the product of the number of radioactive nuclei present and their decay constant.

The activity of the isotope ^{40}K in the student is:

$$R = N_{40}\lambda = \frac{N_{40}\ln 2}{t_{1/2}} \quad (1)$$

Find N, the number of K nuclei in the student:

$$N = 0.0036\frac{mN_A}{M}$$

where m is the mass of the student and M is the atomic mass of K.

Substitute numerical values and evaluate N:

$$N = 0.0036\frac{(60\,\text{kg})\left(6.02\times10^{23}\,\text{nuclei/mol}\right)}{39.098\ \text{g/mol}} = 3.326\times10^{24}$$

The number N_{40} of ^{40}K nuclei in the student is the product of the relative abundance and the number of K nuclei in the student:

$$\begin{aligned}N_{40} &= \text{Relative abundance}\times N\\ &= \left(1.2\times10^{-4}\right)\left(3.326\times10^{24}\right)\\ &= 3.991\times10^{20}\end{aligned}$$

Substitute numerical values in equation (1) and evaluate R:

$$R = \frac{\left(3.991\times10^{20}\right)\ln 2}{1.3\times10^{9}\ \text{y}\times\frac{31.56\,\text{Ms}}{\text{y}}} = \boxed{6.74\times10^{3}\,\text{Bq}}$$

64 ••

Picture the Problem We can find the energy released in the reaction $\beta^{+} + \beta^{-} \rightarrow Q$ by recognizing that a total of 2 electron masses are converted into energy in this annihilation.

The energy released when a positron-electron pair annihilate is given by:

$$Q = E = 2m_e c^2$$

Substitute numerical values and evaluate Q:

$$Q = 2\left(9.11\times10^{-31}\,\text{kg}\right)\left(3\times10^{8}\,\text{m/s}\right)^2 = 1.64\times10^{-13}\,\text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{1.02\,\text{MeV}}$$

65 ••

Picture the Problem We can use the fact that, after *n* half-lives, the decay rate of the ^{24}Na isotope is $R=\left(\frac{1}{2}\right)^n R_0$, where R_0 is its decay rate at t = 0.

The counting rate after *n* half-lives is related to the initial counting rate:

$$R=\left(\tfrac{1}{2}\right)^n R_0$$

Divide both sides of the equation by the volume *V* of blood in the patient:

$$\frac{R}{V}=\left(\tfrac{1}{2}\right)^n \frac{R_0}{V}$$

We're given that n = 2/3, R_0 = 600 kBq, and, after *n* half-lives, the decay rate per unit volume is 60 Bq/mL:

$$60\,\text{Bq/mL}=\left(\tfrac{1}{2}\right)^{2/3}\frac{600\,\text{kBq}}{V}$$

Solve for and evaluate *V*:

$$V=\left(\tfrac{1}{2}\right)^{2/3}\frac{600\,\text{kBq}}{60\,\text{Bq/mL}}=6.30\times10^{3}\,\text{mL}$$
$$=\boxed{6.30\,\text{L}}$$

***66** ••

Picture the Problem We can solve this problem in the center of mass reference frame for the general case of an α particle in a head-on collision with a nucleus of atomic mass *M* u and then substitute data for a nucleus of ^{197}Au and a nucleus of ^{10}B.

In the CM frame, the kinetic energy is:

$$K_{\text{CM}}=\frac{K_{\text{lab}}}{1+\dfrac{m_\alpha}{M}}=\frac{K_{\text{lab}}}{1+\dfrac{4\,\text{u}}{M}}$$

At the point of closest approach:

$$K_{\text{CM}}=\frac{kq_1q_2}{R_{\min}}=\frac{k(2e)(Ze)}{R_{\min}}=\frac{ke^2\,2Z}{R_{\min}}$$

or, because ke^2 = 1.44 MeV·fm,

$$K_{\text{CM}}=\frac{(1.44\,\text{MeV}\cdot\text{fm})(2Z)}{R_{\min}}$$

Solve for $R_{\min}$ to obtain:

$$R_{\min}=\frac{(1.44\,\text{MeV}\cdot\text{fm})(2Z)}{K_{\text{CM}}}\qquad(1)$$

(*a*) Neglecting the recoil of the target nucleus is equivalent to replacing K_{CM} by K_{lab}. Evaluate equation (1) for ^{197}Au:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 79)}{8\,\text{MeV}} = \boxed{28.4\,\text{fm}}$$

Evaluate equation (1) for ^{10}B:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 5)}{8\,\text{MeV}} = \boxed{1.80\,\text{fm}}$$

(*b*) Find K_{CM} for the ^{197}Au nucleus:

$$K_{CM} = \frac{8\,\text{MeV}}{1+\frac{4\,\text{u}}{197\,\text{u}}} = 7.841\,\text{MeV}$$

Substitute numerical values in equation (1) and evaluate R_{min}:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 79)}{7.841\,\text{MeV}} = \boxed{29.0\,\text{fm}}$$

Note that this result is about 2% greater that R_{min} calculated ignoring recoil.

Find K_{CM} for the ^{10}B nucleus:

$$K_{CM} = \frac{8\,\text{MeV}}{1+\frac{4\,\text{u}}{10\,\text{u}}} = 5.714\,\text{MeV}$$

Substitute numerical values in equation (1) and evaluate R_{min}:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 5)}{5.714\,\text{MeV}} = \boxed{2.52\,\text{fm}}$$

Note that this result is about 40% greater that R_{min} calculated ignoring recoil.

67 ••

Picture the Problem The allowed energy levels in a one-dimensional infinite square well are given by Equation 35-13: $E_n = n^2\left(\frac{h^2}{8mL^2}\right)$.

(*a*) The lowest energy of a nucleon of mass 1 u in the well corresponds to $n = 1$:

$$E_1 = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8(1\,\text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)(3\,\text{fm})^2}$$
$$= 3.678\times10^{-12}\ \text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}}$$
$$= \boxed{23.0\,\text{MeV}}$$

(*b*) Because neutrons are fermions, there can be only two per state:

$$E = 2\left(E_1 + E_2 + E_3 + E_4 + E_5 + E_6\right) = 2\left(E_1 + 2^2E_1 + 3^2E_1 + 4^2E_1 + 5^2E_1 + 6^2E_1\right)$$
$$= 182E_1 = 182(23.0\,\text{MeV}) = \boxed{4.19\,\text{GeV}}$$

(c) Find E for 4 protons and 4 neutrons:

$$E = 4\left(E_1 + E_2 + E_3\right) = 4\left(E_1 + 2^2E_1 + 3^2E_1\right)$$
$$= 56E_1 = 56(23.0\,\text{MeV}) = \boxed{1.29\,\text{GeV}}$$

68 ••

Picture the Problem We can apply BE = $(\Delta m)c^2$ to the model to find the binding energies and the binding energies/bond.

(*a*) Find the binding energy BE for this model:

$$\text{BE} = \left(4m_\alpha - m_{^{16}\text{O}}\right)c^2$$
$$= \left[4(4.002603\ \text{u}) - 15.994915\ \text{u}\right]c^2$$
$$= (0.015497\ \text{u})c^2$$

There are 6 bonds for the regular tetrahedron:

$$\frac{\text{BE}}{\text{bond}} = \tfrac{1}{6}\text{BE} = \tfrac{1}{6}(0.015497\,\text{u})c^2$$
$$= \tfrac{1}{6}(0.015497\,\text{u})c^2 \times \frac{931.5\,\text{MeV/u}}{c^2}$$
$$= \boxed{2.406\,\text{MeV}}$$

(*b*) ^{12}C has 3 pairwise α particle bonds. Find the total BE for ^{12}C with this model:

$$\text{BE}\left(^{12}\text{C}\right) = 3\times\text{BE}\left(^{4}\text{He}\right) + 3(2.406\,\text{MeV})$$

Calculate BE(^{4}He):

$$\text{BE}\left({}^{4}\text{He}\right)=\left[2\left(m_{\text{p}}+m_{\text{n}}\right)-m_{{}^{4}\text{He}}\right]c^{2}$$

$$=\left[2\left(1.007825\,\text{u}+1.008665\,\text{u}\right)-4.002603\,\text{u}\right]c^{2}\times\frac{931.5\,\text{MeV}/c^{2}}{\text{u}}$$

$$=28.30\,\text{MeV}$$

Substitute numerical values and evaluate $\text{BE}\left({}^{12}\text{C}\right)$:

$$\text{BE}\left({}^{12}\text{C}\right)=3\left(28.30\,\text{MeV}\right)+3\left(2.406\,\text{MeV}\right)=\boxed{92.1\,\text{MeV}}$$

Use Table 40-1 to find $\text{BE}\left({}^{12}\text{C}\right)$:

$$\text{BE}\left({}^{12}\text{C}\right)=\left[6\left(m_{\text{p}}+m_{\text{n}}\right)-m_{{}^{12}\text{C}}\right]c^{2}$$

$$=\left[6\left(1.007825\,\text{u}+1.008665\,\text{u}\right)-12.000000\,\text{u}\right]c^{2}\times\frac{931.5\,\text{MeV}/c^{2}}{\text{u}}$$

$$=92.2\,\text{MeV}$$

Note that this result is good agreement with the model.

69 ••

Picture the Problem We can separate the variables in the differential equation $dN/dt = R_{\text{p}} - \lambda N$ and integrate to express N as a function of t. When $dN/dt \approx 0$, $R_{\text{p}} - \lambda N_{\infty} = 0$, a condition we can use to find N_{∞}.

(*a*) Separate the variables in the differential equation to obtain:

$$\frac{dN}{R_{\text{p}}-\lambda N}=dt$$

Integrate the left side of the equation from 0 to N and the right side from 0 to t to obtain:

$$\int_{0}^{N}\frac{dN'}{R_{\text{p}}-\lambda N'}=\int_{0}^{t}dt'$$

Let $u = R_{\text{p}} - \lambda N'$. Then:

$$du=-\lambda dN'$$

and

$$\int_{0}^{N}\frac{dN'}{R_{\text{p}}-\lambda N'}=-\frac{1}{\lambda}\int_{\ell_1}^{\ell_2}\frac{du}{u}=-\frac{1}{\lambda}\ln u\Big|_{\ell_1}^{\ell_2}$$

$$=-\frac{1}{\lambda}\ln\left(R_{\text{p}}-\lambda N'\right)\Big|_{0}^{N}$$

$$=-\frac{1}{\lambda}\ln\left(R_{\text{p}}-\lambda N\right)+\frac{1}{\lambda}\ln\left(R_{\text{p}}\right)$$

$$=\frac{1}{\lambda}\ln\left(\frac{R_{\text{p}}}{R_{\text{p}}-\lambda N}\right)$$

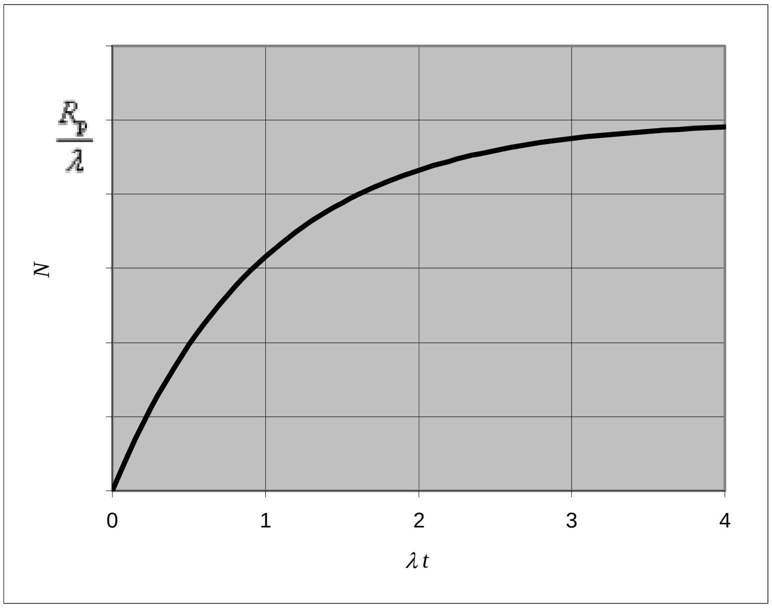
Because $\int_0^t dt' = t$:

$$\frac{1}{\lambda}\ln\left(\frac{R_p}{R_p - \lambda N}\right) = t$$

Solve for N to obtain:

$$N = \frac{R_p}{\lambda}\left(1 - e^{-\lambda t}\right)$$

The following graph of $N(t) = (R_p/\lambda)(1 - e^{-\lambda t})$ was plotted using a spreadsheet program. Note that $N(t)$ approaches R_p/λ in the same manner that the charge on a capacitor approaches the value CV.



(*b*) When $dN/dt = 0$:

$$R_p - \lambda N_\infty = 0 \Rightarrow N_\infty = \frac{R_p}{\lambda}$$

The decay constant is:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for λ to obtain:

$$N_\infty = \frac{R_p}{\ln 2} t_{1/2}$$

Substitute numerical values and evaluate N_∞:

$$N_\infty = \frac{100\,\text{s}^{-1}}{\ln 2}\left(10\,\text{min} \times \frac{60\,\text{s}}{\text{min}}\right) = \boxed{8.66 \times 10^4}$$

***70 ••**

Picture the Problem The mass of ^{235}U required is given by $m_{235} = \frac{N}{N_A} M_{235}$, where M_{235} is the molecular mass of ^{235}U and N is the number of fissions required to produce 7.0×10^{19} J.

Relate the mass of ^{235}U required to the number of fissions N required:

$$m_{235} = \frac{N}{N_A} M_{235} \quad (1)$$

where M_{235} is the molecular mass of ^{235}U.

Determine N:

$$N = \frac{E_{\text{annual}}}{E_{\text{per fission}}}$$

Substitute numerical values and evaluate N:

$$N = \frac{7.0\times10^{19}\ \text{J}}{200\,\text{MeV}\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}} = 2.18\times10^{30}$$

Substitute numerical values in equation (1) and evaluate m_{235}:

$$m_{235} = \frac{2.18\times10^{30}}{6.02\times10^{23}\ \text{nuclei/mol}}(235\,\text{g/mol}) = \boxed{8.51\times10^{5}\ \text{kg}}$$

71 ••

Picture the Problem In the ground state of a one-dimensional infinite square well of length L the wavelength of a particle is $2L$. We can use de Broglie's equation to find p for the particle and the relationship $E^2 = E_0^2 + p^2c^2$ with $E_0 << pc$ to show that $E \approx pc$.

(*a*) In the ground state of a one-dimensional infinite square well of length L:

$$\lambda = 2L = 2(2\,\text{fm}) = \boxed{4.00\,\text{fm}}$$

(*b*) Use de Broglie's relation to obtain:

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c}$$

Substitute numerical values and evaluate p:

$$p = \frac{1240\,\text{eV}\cdot\text{nm}}{(4\,\text{fm})c} = \boxed{310\,\text{MeV}/c}$$

(*c*) Relate the total energy of the electron to its rest energy and

$$E^2 = E_0^2 + p^2c^2 = p^2c^2\left(1+\frac{E_0^2}{p^2c^2}\right)$$

momentum:

Because $E_0 << pc$: $E^2 \approx p^2c^2 \Rightarrow E \approx \boxed{pc}$

(*d*) The kinetic energy of an electron in the ground state of this well is given by:

$$K = E - E_0 \approx E = pc$$
$$= (310\,\text{MeV}/c)c = \boxed{310\,\text{MeV}}$$

72 ••

Picture the Problem When a single proton is removed from a ^{12}C nucleus, a ^{11}B nucleus remains and we can use $Q = \Delta mc^2$ to determine the minimum energy required to remove a proton.

The nuclear reaction is: ${}^{12}_{6}\text{C} + Q \rightarrow {}^{11}_{5}\text{B} + {}^{1}_{1}\text{H}$

The minimum energy Q required is: $Q = \left(m_{^{11}\text{B}} + m_{^{1}\text{H}} - m_{^{12}\text{C}}\right)c^2$

Substitute numerical values and evaluate Q:

$$Q = [(11.009306\,\text{u} + 1.007825\,\text{u}) - 12.000000\,\text{u}]\left(\frac{931.5\,\text{MeV}}{\text{u}}\right) = \boxed{16.0\,\text{MeV}}$$

***73** •••

Picture the Problem The momentum of the electron is related to its total energy through $E^2 = p^2c^2 + E_0^2$ and its total relativistic energy E is the sum of its kinetic and rest energies.

(*a*) Relate the total energy of the electron to its momentum and rest energy:

$$E^2 = p^2c^2 + E_0^2 \qquad (1)$$

The total relativistic energy E of the electron is the sum of its kinetic energy and its rest energy:

$$E = K + E_0$$

Substitute for E in equation (1) to obtain:

$$(K + E_0)^2 = p^2c^2 + E_0^2$$

Solve for p:

$$p = \frac{\sqrt{K(K + 2E_0)}}{c}$$

Substitute numerical values and evaluate p:

$$p = \frac{\sqrt{(0.782\,\text{MeV})(0.782\,\text{MeV} + 2\times 0.511\,\text{MeV})}}{c} = \boxed{1.188\,\text{MeV}/c}$$

(*b*) Because $p_p = -p_e$:

$$K_p = \frac{p_p^2}{2m_p}$$

Substitute numerical values (see Table 7-1 for the rest energy of a proton) and evaluate K_p:

$$K_p = \frac{(1.188\,\text{MeV}/c)^2}{2(938.28\,\text{MeV}/c^2)} = \boxed{752\,\text{eV}}$$

(*c*) The percent correction is:

$$\frac{K_p}{K} = \frac{752\,\text{eV}}{0.782\,\text{MeV}} = \boxed{0.0962\%}$$

74 •••

Picture the Problem Conservation of momentum and conservation of energy allow us to find the final velocities. Because the initial kinetic energy of the nucleus is zero, its final kinetic energy equals the energy lost by the neutron.

(*a*) Apply conservation of momentum to the collision to obtain:

$$(m + M)V = mv_L$$

Solve for *V*:

$$V = \boxed{\frac{mv_L}{m+M}}$$

(*b*) In the CM frame, $V_{Mi} = V$ and so:

$$V_{Mi} = \boxed{V}$$

In the CM frame, $V_f = -V_i$ and so:

$$V_{Mf} = \boxed{-V}$$

(*c*) Use conservation of momentum to obtain one relation for the final velocities:

$$mv_L = mv_f + MV_{Mf} \qquad (1)$$

The equality of the initial and final kinetic energies provides a second equation relating the two final velocities. This is implemented by equating the speeds of recession and approach:

$$V_{Mf} - v_f = -(V_{Mi} - v_L) = 0 + v_L$$

and so

$$v_f = V_{Mf} - v_L$$

To eliminate v_f, substitute in equation (1) :

$$mv_L = m(V_{Mf} - v_L) + MV_{Mf}$$

Solve for V_{Mf}:

$$V_{Mf} = \boxed{\frac{2m}{M+m}v_L}$$

(*d*) The kinetic energy of the nucleus after the collision in the laboratory frame is:

$$K_M = \tfrac{1}{2}MV_{Mf}^2$$

Substitute for V_{Mf} and simplify to obtain:

$$K_{Mf} = \tfrac{1}{2}M\left(\frac{2m}{M+m}v_L\right)^2 = \boxed{\frac{4mM}{(M+m)^2}\left(\tfrac{1}{2}mv_L^2\right)}$$

(*e*) The fraction of the energy lost by the neutron in the elastic collision is given by:

$$\frac{\Delta E}{E} = \frac{-K_{Mf}}{\tfrac{1}{2}mv_L^2} = \frac{\frac{4mM}{(M+m)^2}\left(\tfrac{1}{2}mv_L^2\right)}{\tfrac{1}{2}mv_L^2} = \frac{4mM}{(M+m)^2} = \frac{4mM}{M^2\left(1+\frac{m}{M}\right)^2} = \boxed{\frac{\frac{4m}{M}}{\left(1+\frac{m}{M}\right)^2}}$$

75 •••

Picture the Problem We can use the result of Problem 74, part (*e*), to find the fraction $f = E_f/E_0$ of its initial energy lost per collision and then use this result to show that, after N collisions, $E = (0.714)^N E_0$.

(*a*) Determine $f = E_f/E_0$ per collision:

$$f = \frac{E_0 - \Delta E}{E_0} = 1 - \frac{\Delta E}{E_0}$$

From Problem 74, part (*e*):

$$\frac{\Delta E}{E_0} = \frac{4m}{M\left(1+\frac{m}{M}\right)^2}$$

Substitute for $\Delta E/E_0$ in the expression for f to obtain:

$$f = 1 - \frac{4m}{M\left(1+\frac{m}{M}\right)^2}$$

Substitute numerical values and evaluate *f*:

$$f = 1 - \frac{4(1.008665\,\text{u})}{12.000000\,\text{u}\left(1 + \frac{1.008665\,\text{u}}{12.000000\,\text{u}}\right)^2} = 0.714$$

After *N* collisions:

$$E_{fN} = f^N E_0 = \boxed{(0.714)^N E_0} \qquad (1)$$

(*b*) Solve equation (1) for *N*:

$$N = \frac{\ln\left(\frac{E_{fN}}{E_0}\right)}{\ln(0.714)}$$

Substitute numerical values and evaluate *N*:

$$N = \frac{\ln\left(\frac{0.02\,\text{eV}}{2\,\text{MeV}}\right)}{\ln(0.714)} = 54.7$$

55 head-on collisions are required to reduce the energy of the neutron from 2 MeV to 0.02 eV.

76 •••

Picture the Problem We can use the result of Problem 74, part (*e*), to find the fraction $f = E_f/E_0$ of its initial energy lost per collision. Note the difference between the energy loss per collision specified here and that of the preceding problem. In the preceding problem it was assumed that all collisions are head-on collisions.

(*a*) Determine $f = E_f/E_0$ per collision:

$$f = \frac{E_0 - \Delta E}{E_0} = 1 - \frac{\Delta E}{E_0}$$

In a collision with a hydrogen atom:

$$\frac{\Delta E}{E_0} = 0.63$$

and so

$$f = 0.37$$

After *N* collisions:

$$E_{fN} = f^N E_0 = (0.37)^N E_0 \qquad (1)$$

Solve equation (1) for *N*:

$$N = \frac{\ln\left(\frac{E_{fN}}{E_0}\right)}{\ln(0.37)} \qquad (2)$$

Substitute numerical values and evaluate N:

$$N = \frac{\ln\left(\frac{0.02\,\text{eV}}{2\,\text{MeV}}\right)}{\ln(0.37)} = 18.5$$

$$\boxed{\begin{array}{l}\text{19 head-on collisions with an atom of hydrogen are required to reduce the}\\ \text{energy of the neutron from 2 MeV to 0.02 eV.}\end{array}}$$

(*b*) In a collision with a carbon atom:

$$\frac{\Delta E}{E_0} = 0.11$$

and so

$$f = 0.89$$

Equation (2) becomes:

$$N = \frac{\ln\left(\frac{E_{fN}}{E_0}\right)}{\ln(0.89)}$$

Substitute numerical values and evaluate N:

$$N = \frac{\ln\left(\frac{0.02\,\text{eV}}{2\,\text{MeV}}\right)}{\ln(0.89)} = 158$$

$$\boxed{\begin{array}{l}\text{158 head-on collisions with an atom of carbon are required to reduce the}\\ \text{energy of the neutron from 2 MeV to 0.02 eV.}\end{array}}$$

***77 •••**

Picture the Problem We can differentiate $N_B(t) = \frac{N_{A0}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$ with respect to t to show that it is the solution to the differential equation $dN_B/dt = \lambda_A N_A - \lambda_B N_B$.

(*a*) The rate of change of N_B is the rate of generation of B nuclei minus the rate of decay of B nuclei. The generation rate is equal to the decay rate of A nuclei, which equals $\lambda_A N_A$. The decay rate of B nuclei is $\lambda_B N_B$.

(*b*) We're given that:

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad (1)$$

$$N_B(t) = \frac{N_{A0}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) \quad (2)$$

$$N_A = N_{A0}e^{-\lambda_A t} \quad (3)$$

Differentiate equation (2) with respect to t to obtain:

$$\frac{d}{dt}\left[N_{\mathrm{B}}(t)\right]=\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\frac{d}{dt}\left[\left(e^{-\lambda_{\mathrm{A}}t}-e^{-\lambda_{\mathrm{B}}t}\right)\right]=\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]$$

Substitute this derivative in equation (1) to get:

$$\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]=\lambda_{\mathrm{A}}N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}-\lambda_{\mathrm{B}}\left[\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\left(e^{-\lambda_{\mathrm{A}}t}-e^{-\lambda_{\mathrm{B}}t}\right)\right]$$

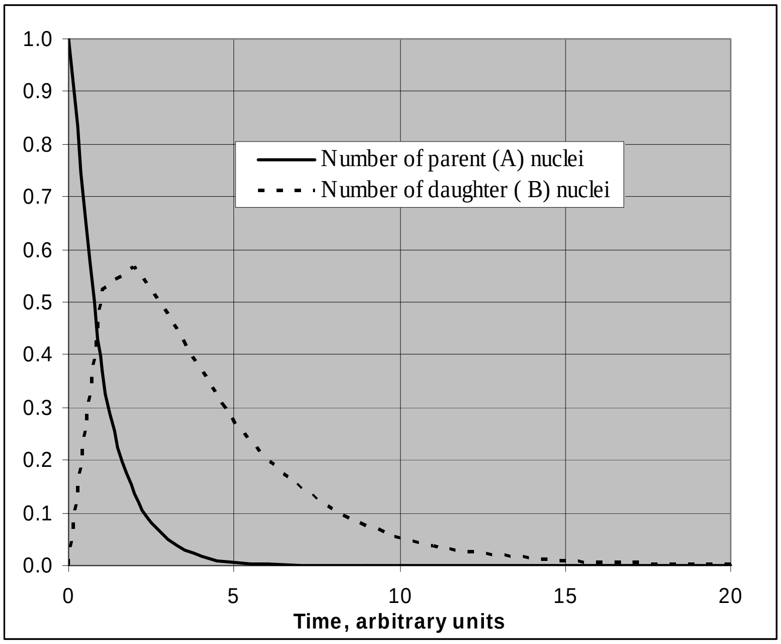
Multiply both sides by $\frac{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}\lambda_{\mathrm{A}}}$ and simplify to obtain:

$$\begin{aligned}\frac{N_{\mathrm{A0}}}{\lambda_{\mathrm{B}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]&=\frac{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}}N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}-N_{\mathrm{A0}}\left(e^{-\lambda_{\mathrm{A}}t}-e^{-\lambda_{\mathrm{B}}t}\right)\\&=N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}-\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}}e^{-\lambda_{\mathrm{A}}t}-N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}+N_{\mathrm{A0}}e^{-\lambda_{\mathrm{B}}t}\\&=-\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}}e^{-\lambda_{\mathrm{A}}t}+N_{\mathrm{A0}}e^{-\lambda_{\mathrm{B}}t}\\&=\frac{N_{\mathrm{A0}}}{\lambda_{\mathrm{B}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]\end{aligned}$$

which is an identity and confirms that equation (2) is the solution to equation (1).

(*c*) If $\lambda_{\mathrm{A}}>\lambda_{\mathrm{B}}$ the denominator and the expression in the parentheses are both negative for $t>0$. If $\lambda_{\mathrm{A}}<\lambda_{\mathrm{B}}$ the denominator and the expression in the parentheses are both positive for $t>0$.

(*d*) The following graph was plotted using a spreadsheet program.



78 •••

Picture the Problem We can express the time at which the number of isotope B nuclei will be a maximum by setting dN_B/dt equal to zero and solving for t.

From Problem 77 we have:

$$\frac{dN_B}{dt}=\lambda_A N_A-\lambda_B N_B=0 \text{ for extrema}$$

Replace $\lambda_A N_A$ by $\lambda_A N_{A0}e^{-\lambda_A t}$ and N_B by $\frac{N_{A0}\lambda_A}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)$:

$$\lambda_A N_{A0}e^{-\lambda_A t}-\lambda_B\frac{N_{A0}\lambda_A}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)=0$$

Simplify to obtain:

$$e^{-\lambda_A t}-\frac{\lambda_B}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)=0$$

$$\left(\lambda_B-\lambda_A\right)e^{-\lambda_A t}-\lambda_B\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)=0$$

Remove the parentheses and combine like terms to obtain:

$$\lambda_A e^{-\lambda_A t}=\lambda_B e^{-\lambda_B t}$$

Solve for t:

$$t=\boxed{\frac{\ln\left(\lambda_B/\lambda_A\right)}{\lambda_B-\lambda_A}}$$

Remarks: Note that all we've shown is that an *extreme value* exists at $t=\frac{\ln(\lambda_B/\lambda_A)}{\lambda_B-\lambda_A}$. To show that this value for *t* maximizes N_B, we need to either 1) examine the second derivative at this value for *t*, or 2) plot a graph of N_B as a function of time (see Problem 77) .

79 •••

Picture the Problem We can show that, provided $\tau_A >> \tau_B$, $e^{-\lambda_A t}-e^{-\lambda_B t}\approx 1$ and $\frac{\lambda_A}{\lambda_B-\lambda_A}\approx\frac{\lambda_A}{\lambda_B}$ and, hence, that $N_B = (\lambda_A/\lambda_B)N_A$.

We have, from Problem 77 (*b*):

$$N_B(t)=\frac{N_A\lambda_A}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right) \quad (1)$$

Because $\tau_A >> \tau_B$:

$$\lambda_A << \lambda_B$$

When several years have passed, because $\lambda_A t << 1$:

$$e^{-\lambda_A t}-e^{-\lambda_B t}\approx 1 \quad (2)$$

Also, when $\lambda_A << \lambda_B$:

$$\frac{\lambda_A}{\lambda_B-\lambda_A}\approx\frac{\lambda_A}{\lambda_B} \quad (3)$$

Substitute (2) and (3) in (1) to obtain:

$$N_B(t)=\boxed{\frac{\lambda_A}{\lambda_B}N_A}$$

Chapter 41
Elementary Particles and the Beginning of the Universe

Conceptual Problems

1 •

Similarities	Differences
Baryons and mesons are hadrons, i.e., they participate in the strong interaction. Both are composed of quarks.	Baryons consist of three quarks and are fermions. Mesons consist of two quarks and are bosons. Baryons have baryon number +1 or −1. Mesons have baryon number 0.

2 •

Determine the Concept The muon is a lepton. It is a spin-½ particle and is a fermion. It does not participate in strong interactions. It appears to be an elementary particle like the electron. The pion is a meson. Its spin is 0 and it is a boson. It does participate in strong interactions and is composed of quarks.

***3** •

Determine the Concept A decay process involving the strong interaction has a very short lifetime (~10^{-23} s), whereas decay processes that proceed via the weak interaction have lifetimes of order 10^{-10} s.

4 •

(*a*) True

(*b*) False. There are two kinds of hadrons-baryons, which have spin $\frac{1}{2}$ (or $\frac{3}{2}, \frac{5}{2}$, and so on), and mesons, which have zero or integral spin.

5 •

False. Mesons have zero or integral spins.

6 •

Determine the Concept A meson has 2 quarks, a baryon has 3 quarks.

7 •

Determine the Concept No; from Table 41-2 it is evident that any quark-antiquark combination always results in an integral or zero charge.

8 •

(*a*) False. Leptons are not made up of quarks.

(*b*) True

(*c*) False. Electrons are leptons and leptons interact via the weak interaction.

(*d*) True

(*e*) True

***9** •

Determine the Concept No. Such a reaction is impossible. A proton requires three quarks. Three quarks are not available because a pion is made of a quark and an antiquark and the antiproton consists of three antiquarks.

Estimation and Approximation

10 ••

Picture the Problem Assuming that the lifetime of a proton is 10^{32} y, one proton out of every 10^{32} protons should decay every year on average. Hence, we can estimate the expected time between proton-decays that occur in the water of a filled Olympic-size swimming pool by determining the number of protons *N* in the pool and dividing 10^{32} y by this number.

The mean time between disintegrations is the ratio of the lifetime of the protons to the number of protons *N* in the pool:

$$\Delta t_{\text{mean}} = \frac{10^{32}\ \text{y}}{N} \qquad (1)$$

The number of protons *N* in the pool is related to the mass of water in the pool M_{water}, the molar mass of water $m_{\text{molar, water}}$, and the number of protons per molecule *n*:

$$\frac{N}{M_{\text{water}}} = \frac{nN_{\text{A}}}{m_{\text{molar, water}}}$$

Solve for *N* to obtain:

$$N = \frac{nN_{\text{A}}M_{\text{water}}}{m_{\text{molar, water}}}$$

Because the mass of the water is the product of its density and the volume of the pool:

$$N = \frac{nN_{\text{A}}\rho_{\text{water}}V_{\text{pool}}}{m_{\text{molar, water}}}$$

Substituting for N in equation (1) yields:

$$\Delta t_{\text{mean}} = \frac{10^{32}\text{ y}}{\dfrac{nN_{\text{A}}\rho_{\text{water}}V_{\text{pool}}}{m_{\text{molar, water}}}} = \frac{\left(10^{32}\text{ y}\right)m_{\text{molar, water}}}{nN_{\text{A}}\rho_{\text{water}}V_{\text{pool}}}$$

Because each molecule of water has 10 protons:

$$n = 10\,\frac{\text{protons}}{\text{molecule}}$$

Substitute numerical values and evaluate Δt_{mean}:

$$\Delta t_{\text{mean}} = \frac{\left(10^{32}\text{ y}\right)\left(18\,\dfrac{\text{g}}{\text{mol}}\times\dfrac{1\,\text{kg}}{10^{3}\,\text{g}}\right)}{\left(10\,\dfrac{\text{protons}}{\text{molecule}}\right)\left(6.02\times10^{23}\,\dfrac{\text{molecules}}{\text{mol}}\right)\left(10^{3}\,\dfrac{\text{kg}}{\text{m}^{3}}\right)(100\,\text{m})(25\,\text{m})(2\,\text{m})}$$

$$= 0.0598\,\text{y} = 0.0598\,\text{y}\times\frac{365.24\,\text{d}}{\text{y}} = \boxed{21.8\,\text{d}}$$

11 •

Picture the Problem We can use $F_{\text{em}} = kq_{\text{proton}}^2/r_{\text{nucleus}}^2$ and $F_{\text{grav}} = Gm_{\text{proton}}^2/r_{\text{nucleus}}^2$ to estimate the ratio of the electromagnetic and gravitational forces between two protons located in a nucleus.

The electromagnetic force between two protons located in a nucleus is given by:

$$F_{\text{em}} = \frac{kq_{\text{proton}}^2}{r_{\text{nucleus}}^2}$$

The gravitational force between these same protons is given by:

$$F_{\text{grav}} = \frac{Gm_{\text{proton}}^2}{r_{\text{nucleus}}^2}$$

Divide F_{em} by F_{grav} to obtain:

$$\frac{F_{\text{em}}}{F_{\text{grav}}} = \frac{\dfrac{kq_{\text{proton}}^2}{r_{\text{nucleus}}^2}}{\dfrac{Gm_{\text{proton}}^2}{r_{\text{nucleus}}^2}} = \frac{kq_{\text{proton}}^2}{Gm_{\text{proton}}^2}$$

Substitute numerical values and evaluate $F_{\text{em}}/F_{\text{grav}}$:

$$\frac{F_{\text{em}}}{F_{\text{grav}}}=\frac{\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.67\times10^{-27}\,\text{kg}\right)^2}=\boxed{1.24\times10^{36}}$$

Spin and Antiparticles

***12** •

Picture the Problem We can use both conservation of energy and momentum to explain why the energies of the two γ-rays must be equal. We can find the energy of each γ-ray in Table 41-1 and find their wavelengths using $\lambda = hc/E$.

(*a*) The initial momentum is zero; therefore, the final momentum must be zero. The momentum of a photon is E/c. To conserve both momentum and energy the two photons must have the same momentum magnitude. Hence, they have the same energy.

(*b*) From Table 41-1:

$$E_\gamma=\boxed{139.6\,\text{MeV}}$$

(*c*) The wavelength of each γ ray is given by:

$$\lambda=\frac{hc}{E}=\frac{1240\,\text{MeV}\cdot\text{fm}}{E}$$

Substitute numerical values and evaluate λ:

$$\lambda=\frac{1240\,\text{MeV}\cdot\text{fm}}{139.6\,\text{MeV}}=\boxed{8.88\,\text{fm}}$$

13 •

Picture the Problem In each case, the required energy is given by $E=2mc^2$ where *m* is mass of each particle produced in the pair-production reaction. These masses can be found in Tables 41-1 and 41-3.

(*a*) For $\gamma \rightarrow \pi^+ + \pi^-$:

$$E=2m_\pi c^2=2\left(139.6\,\text{MeV}/c^2\right)c^2=\boxed{279.2\,\text{MeV}}$$

(*b*) For $\gamma \rightarrow \text{p}+\text{p}^-$:

$$E=2m_\text{p}c^2=2\left(938.3\,\text{MeV}/c^2\right)c^2=\boxed{1877\,\text{MeV}}$$

(*c*) For $\gamma \rightarrow \mu^- + \mu^+$:

$$E=2m_\mu c^2=2\left(105.659\,\text{MeV}/c^2\right)c^2=\boxed{211.3\,\text{MeV}}$$

The Conservation Laws

14 •

Picture the Problem We need to check for conservation of energy, charge, baryon number, and lepton number.

(*a*) Energy conservation:

Because $m_p < m_n$, energy conservation is violated.

Charge conservation:
$+e \to 0 + e + 0 = +e$

Because the net charge is $+e$ before and after the decay, charge is conserved.

Baryon number:
$+1 \to +1 + 0 + 0 = +1$

Because B is $+1$ before and after the decay, baryon number is conserved.

Lepton number; electrons:
$0 \to 0 + 0 + 0 = 0$

Because $L_e = 0$ before and after the decay, the lepton number for electrons is conserved.

The process is not allowed because it violates conservation of energy.

(*b*) Energy conservation:

Because $m_n < m_p + m_{\pi^-}$, energy conservation is violated.

Charge conservation:
$0 \to +e + (-e) = 0$

Because the net charge is 0 before and after the decay, charge is conserved.

Baryon number:
$1 \to 1 + 0 = 1$

Because $B = 0$ before and after the decay, baryon number is conserved.

Lepton number; electrons:
$0 \to 0 + 0 = 0$

Because $L = 0$ before and after the decay, lepton number is conserved.

Because energy is not conserved, this decay is not allowed.

(*c*) Momentum conservation is violated; two (or more) γ rays must be emitted to conserve momentum.

(*d*) Energy conservation: Energy is conserved.

Charge conservation:
$+1 + (-1) \to 0 + 0 = 0$

Because the net charge is *zero* before and after the decay, charge is conserved.

Baryon number:
$+1 + (-1) \to 0 + 0 = 0$

Because $B = 0$ before and after the decay, baryon number is conserved.

Lepton number; electrons:
$0 \to 0 + 0 + 0 = 0$

Because $L_e = 0$ before and after the decay, the lepton number for electrons is conserved.

Because none of the conservation laws are violated, this is an allowed process.

(*e*) Energy conservation: Because $m_p > m_n + m_{e^+}$, energy is conserved.

Charge conservation:
$0 + 1 \to 0 + 1 = 1$

Because the net charge is one before and after the decay, charge is conserved.

Baryon number:
$0 + 1 \to 1 + 0 = 0$

Because $B = 1$ before and after the decay, baryon number is conserved.

Lepton number; electrons:
$-1 + 0 \to 0 + (-1) = -1$

Because $L_e = -1$ before and after the decay, the lepton number for electrons is conserved.

Because none of the conservation laws are violated, this is an allowed process.

15 •

Picture the Problem The decay will occur via the strong interaction if strangeness is conserved. If $\Delta S = \pm 1$, it will occur via the weak interaction. If S changes by more than 1, the decay will not occur.

(*a*) List the strangeness of Ω^-, Ξ^0, and π^-:

$\Omega^-: S = -3$
$\Xi^0: S = -2$
$\pi^-: S = 0$

Determine ΔS:

$$\Delta S = -2 - (-3) = \boxed{+1}$$

$$\boxed{\text{Because } \Delta S = +1, \text{ the reaction can proceed via the weak interaction.}}$$

(*b*) List the strangeness of Ξ^0, p, π^-, and π^0:

$\Xi^0: S = -2$
$\text{p}: S = 0$
$\pi^-: S = 0$
$\pi^0: S = 0$

Determine ΔS:

$$\Delta S = 0 - (-2) = \boxed{+2}$$

$$\boxed{\text{Because } \Delta S = +2, \text{ the reaction is not allowed.}}$$

(*c*) List the strangeness of Λ^0, p^+, and π^-:

$\Lambda^0: S = -1$
$\text{p}^+: S = 0$
$\pi^-: S = 0$

Determine ΔS:

$$\Delta S = 0 - (-1) = \boxed{+1}$$

$$\boxed{\text{Because } \Delta S = +1, \text{ the reaction can proceed via the weak interaction.}}$$

16 •

Picture the Problem The decay will occur via the strong interaction if strangeness is conserved. If $\Delta S = \pm 1$, it will occur via the weak interaction. If S changes by more than 1, the decay will not occur.

(*a*) List the strangeness of Ω^-, Λ^0, and K^-:

$\Omega^-: S = -3$
$\Lambda^0: S = -1$
$\text{K}^-: S = -1$

Determine ΔS:

$$\Delta S = -1 - 1 - (-3) = \boxed{+1}$$

Because $\Delta S = +1$, the reaction can proceed via the weak interaction.

(*b*) List the strangeness of Ξ^0, p, and π^-:

Ξ^0: $S = -2$
p: $S = 0$
π^-: $S = 0$

Determine ΔS:

$$\Delta S = 0-(-2) = \boxed{+2}$$

Because $\Delta S = +2$, the reaction is not allowed.

17 •

Picture the Problem The decay will occur via the strong interaction if strangeness is conserved. If $\Delta S = \pm 1$, it will occur via the weak interaction. If S changes by more than 1, the decay will not occur.

(*a*) List the strangeness of Ω^-, Λ^0, $\bar{\nu}_e$, and e^-:

Ω^-: $S = -3$
Λ^0: $S = -1$
$\bar{\nu}_e$: $S = 0$
e^-: $S = 0$

Determine ΔS:

$$\Delta S = -1-(-3) = \boxed{+2}$$

Because $\Delta S = +2$, the reaction is not allowed.

(*b*) List the strangeness of Σ^+, p, and π^0:

Σ^+: $S = -1$
p: $S = 0$
π^0: $S = 0$

Determine ΔS:

$$\Delta S = 0-(-1) = \boxed{+1}$$

Because $\Delta S = +1$, the reaction can proceed via the weak interaction.

18 •

Picture the Problem We can decide whether the given decays of the τ particle are possible by determining whether energy conservation is satisfied and whether conservation of both the τ and μ lepton numbers is satisfied.

(*a*) The first decay is allowed. It satisfies energy conservation and conservation of both the τ and μ lepton numbers.

(*b*) The second decay scheme is not allowed because it does not conserve τ and μ lepton numbers.

(*c*) The total kinetic for the decay in (*a*) is:

$$K_{\text{tot}} = m_\tau c^2 - m_\mu c^2$$

From Table 41-3 we have:

$$m_\tau = 1784\,\text{MeV}/c^2$$

and

$$m_\mu = 105.659\,\text{MeV}/c^2$$

Substitute numerical values and evaluate K_{tot}:

$$K_{\text{tot}} = \left(1784\,\text{MeV}/c^2\right)c^2 - \left(106\,\text{MeV}/c^2\right)c^2 = \boxed{1678\,\text{MeV}}$$

Remarks: Note that the kinetic energy of the individual decay products cannot be determined from the decay scheme alone.

19 ••

Picture the Problem Examination of the decay products will reveal whether all the final products are stable. A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) No; the neutron is not stable:

$$\boxed{n \to p^+ + e^- + \bar{\nu}_e}$$

(*b*) Add the reactions to obtain:

$$\boxed{\Omega^- \to p^+ + e^+ + 3e^- + \nu_e + 3\bar{\nu}_e + 2\bar{\nu}_\mu + 2\nu_\mu}$$

(*c*) Charge conservation:
$-1 \to 1 + 1 - 3 + 0 + 0 + 0 + 0 = -1$

Because $Q = -1$ before and after the decay, charge is conserved.

Baryon number:
$1 \to 1 + 0 + 0 + 0 + 0 + 0 + 0 = 1$

Because $B = 1$ before and after the decay, baryon number is conserved.

Lepton number:
$0 \to 0 - 1 + 3 + 1 - 3 - 2 + 2 = 0$

Because $L_e = 0$ before and after the decay, the lepton number for electrons is conserved.

Strangeness:
$-3 \rightarrow 0$

Strangeness is not conserved. However, in each baryon decay $\Delta S = +1$, and each decay is allowed via the weak interaction.

***20** ••

Picture the Problem A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) Energy conservation:

Because $m_n > 2m_\pi + 2m_\mu$, energy conservation is not violated.

Charge conservation:
$0 \rightarrow +1 + (-1) + 0 + 0 = 0$

Because the total charge is 0 before and after the decay, charge is conserved.

Baryon number:
$1 \rightarrow 0 + 0 + 0 + 0 = 0$

Because baryon number changes from $+1$ to 0, conservation of baryon number is violated.

Lepton number:
$0 \rightarrow 0 + 0 + 1 + (-1) = 0$

Because $L_\mu = 0$ before and after the decay, the lepton number for muons is conserved.

The process is not allowed because it violates conservation of baryon number.

(*b*) Energy conservation:

Because $m_\pi > 2m_e$, energy conservation is not violated.

Charge conservation:
$0 \rightarrow +1 + (-1) + 0 + 0 = 0$

Because the total charge is 0 before and after the decay, charge is conserved.

Baryon number:
$0 \rightarrow 0 + 0 + 0 + 0 = 0$

Because $B = 0$ before and after the decay, the baryon number is conserved.

Lepton number:
$0 \rightarrow 0+0+0+0=0$

Because $L_e = 0$ before and after the decay, the lepton number is conserved.

The decay satisfies all conservation laws and is allowed.

Quarks

21 •

Picture the Problem For each quark combination we can determine the baryon number *B*, the charge *Q*, and the strangeness *S* and then use Table 41-1 to find a match and complete the following table.

	Combination	B	Q	S	hadron
(*a*)	*uud*	1	+1	0	p^+
(*b*)	*udd*	1	0	0	n
(*c*)	*uus*	1	+1	−1	Σ^+
(*d*)	*dds*	1	−1	−1	Σ^-
(*e*)	*uss*	1	0	−2	Ξ^0
(*f*)	*dss*	1	−1	−2	Ξ^-

22 •

Picture the Problem For each quark combination we can determine the baryon number *B*, the charge *Q*, and the strangeness *S* and then use Table 41-1 to find a match and complete the following table.

	Combination	B	Q	S	hadron
(*a*)	$u\bar{d}$	0	+1	0	π^+
(*b*)	$\bar{u}d$	0	−1	0	π^-
(*c*)	$u\bar{s}$	0	+1	+1	K^+
(*d*)	$\bar{u}s$	0	−1	−1	K^-

23 •

Determine the Concept From Table 41-2 we see that to satisfy the conditions of charge = +2 and zero strangeness, charm, topness, and bottomness, the quark combination must be *uuu*.

24 •

Picture the Problem Because K^+ and K^0 are mesons, they consist of a quark and an antiquark. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(*a*) For K^+ we need:

$Q = +1$
$B = 0$
$S = +1$

A combination of quarks with these properties is $\boxed{u\bar{s}.}$

(*b*) For K^0 we need:

$Q = 0$
$B = 0$
$S = +1$

A combination of quarks with these properties is $\boxed{d\bar{s}.}$

25 •

Determine the Concept Because D^+ and D^- are mesons, they consist of a quark and an antiquark.

(*a*) $B = 0$, so we must look for a combination of quark and antiquark. Because it has charm of +1, one of the quarks must be *c*. Because the charge is +*e*, the antiquark must be $\bar{d}$. The possible combination for D^+ is $\boxed{c\bar{d}.}$.

(*b*) Because D^- is the antiparticle of D^+, the quark combination is $\boxed{\bar{c}d.}$

26 •

Picture the Problem We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles. Because K^- and $\overline{K}^0$ are mesons, they consist of a quark and an antiquark.

(*a*) For K^- we need:

$Q = -1$
$B = 0$
$S = -1$

A combination of quarks with these properties is $\boxed{\bar{u}s.}$

(*b*) For $\overline{K}^0$ we need:

$Q = 0$
$B = 0$
$S = -1$

A combination of quarks with these properties is $\boxed{\bar{d}s.}$

Remarks: An alternative solution could take advantage of our results in Problem 20 for the antiparticles K^+ and K^0.

***27 ••**

Picture the Problem Because Λ^0, p^-, and Σ^- are baryons, they are made up of three quarks. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(*a*) For Λ^0 we need:

$Q = 0$
$B = +1$
$S = -1$

The quark combination that satisfies these conditions is $\boxed{uds.}$

(*b*) For p^- we need:

$Q = -1$
$B = -1$
$S = +1$

The quark combination that satisfies these conditions is $\boxed{\bar{u}\bar{u}\bar{d}.}$

(*c*) For Σ^- we need:

$Q = -1$
$B = +1$
$S = -1$

The quark combination that satisfies these conditions is $\boxed{dds.}$

28 ••

Picture the Problem Because $\bar{n}$, Ξ^0, and Σ^+ are baryons, they are made up of three quarks. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(*a*) For $\bar{n}$ we need:

$Q = 0$
$B = -1$
$S = 0$

The quark combination that satisfies these conditions is $\boxed{\bar{u}\bar{d}\bar{d}.}$

(*b*) For Ξ^0 we need:

$Q = 0$
$B = +1$
$S = -2$

The quark combination that satisfies these conditions is $\boxed{uss.}$

(c) For Σ^+ we need:

$$Q = +1$$
$$B = +1$$
$$S = -1$$

The quark combination that satisfies these conditions is $\boxed{uus.}$

29 ••

Picture the Problem Because Ω^- and Ξ^- are baryons, they are made up of three quarks. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(a) For Ω^- we need:

$$Q = -1$$
$$B = +1$$
$$S = -3$$

The quark combination that satisfies these conditions is $\boxed{sss.}$

(b) For Ξ^- we need:

$$Q = -1$$
$$B = +1$$
$$S = -2$$

The quark combination that satisfies these conditions is $\boxed{ssd.}$

30 ••

Picture the Problem We can use Table 41-2 to identify the properties of the particles made up of the given quarks.

(a) For ddd:

$$Q = \boxed{-1}$$
$$B = \boxed{+1}$$
$$S = \boxed{0}$$

(b) For $u\bar{c}$:

$$Q = \boxed{0}$$
$$B = \boxed{0}$$
$$S = \boxed{0}$$
$$\text{charm} = \boxed{-1}$$

(*c*) For $u\bar{b}$:

$Q = \boxed{+1}$

$B = \boxed{0}$

$S = \boxed{0}$

$\text{bottomness} = \boxed{-1}$

(*d*) For $\bar{s}\bar{s}\bar{s}$:

$Q = \boxed{+1}$

$B = \boxed{-1}$

$S = \boxed{+3}$

The Evolution of the Universe

***31 •**

Picture the Problem We can use Hubble's law to find the distance from the earth to this galaxy.

The recessional velocity of galaxy is related to its distance by Hubble's law:

$$v = Hr$$

Solve for *r*:

$$r = \frac{v}{H}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{(0.025)c}{\frac{23\,\text{km/s}}{10^6\,c\cdot\text{y}}} = \frac{(0.025)(3\times10^5\ \text{km/s})}{\frac{23\,\text{km/s}}{10^6\,c\cdot\text{y}}}$$

$$= \boxed{3.26\times10^8\,c\cdot\text{y}}$$

32 •

Picture the Problem We can use Hubble's law to find the speed of the galaxy.

The recessional velocity of galaxy is related to its distance by Hubble's law:

$$v = Hr$$

Substitute numerical values and evaluate *v*:

$$v = \left(\frac{23\,\text{km/s}}{10^6\,c\cdot\text{y}}\right)\left(12\times10^9\,c\cdot\text{y}\right)\left(\frac{c}{3.00\times10^5\ \text{km/s}}\right) = \boxed{0.920c}$$

33 ••

Picture the Problem We can substitute for f' and f_0, using $v = f\lambda$, in Equation 39-16*b* to show that the relativistic wavelength shift is $\lambda' = \lambda_0\sqrt{\dfrac{1+v/c}{1-v/c}}$.

From Equation 39-16*b*:

$$f' = f_0\sqrt{\frac{1-v/c}{1+v/c}}$$

Express f' and f_0 in terms of λ' and λ_0:

$$f' = \frac{c}{\lambda'} \text{ and } f_0 = \frac{c}{\lambda_0}$$

Substitute for f' and f_0 to obtain:

$$\frac{c}{\lambda'} = \frac{c}{\lambda_0}\sqrt{\frac{1-v/c}{1+v/c}}$$

Solve for λ':

$$\lambda' = \boxed{\lambda_0\sqrt{\frac{1+v/c}{1-v/c}}}$$

***34** ••

Picture the Problem Using Hubble's law, we can rewrite the equation from Problem 31 as $\lambda' = \lambda_0\sqrt{\dfrac{1+Hr/c}{1-Hr/c}}$.

From Problem 33 we have:

$$\lambda' = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}}$$

Use Hubble's law to relate v to r:

$$v = Hr$$

Substitute for v to obtain:

$$\lambda' = \lambda_0\sqrt{\frac{1+Hr/c}{1-Hr/c}}$$

(*a*) For $r = 5\times10^6\ c{\cdot}\text{y}$:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{(5\times10^6\,\text{c}\cdot\text{y})}{(3\times10^5\,\text{km/s})}}{1-\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{(5\times10^6\,\text{c}\cdot\text{y})}{(3\times10^5\,\text{km/s})}}} = \boxed{656.6\,\text{nm}}$$

(*b*) For $r = 50\times10^6\ c{\cdot}\text{y}$:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\frac{23\text{ km/s}}{10^6\text{ c}\cdot\text{y}}\right)\frac{\left(50\times10^6\text{ c}\cdot\text{y}\right)}{\left(3\times10^5\text{ km/s}\right)}}{1-\left(\frac{23\text{ km/s}}{10^6\text{ c}\cdot\text{y}}\right)\frac{\left(50\times10^6\text{ c}\cdot\text{y}\right)}{\left(3\times10^5\text{ km/s}\right)}}} = \boxed{658.8\,\text{nm}}$$

(*c*) For $r = 500\times10^6\ c\cdot$y:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\frac{23\text{ km/s}}{10^6\text{ c}\cdot\text{y}}\right)\frac{\left(500\times10^6\text{ c}\cdot\text{y}\right)}{\left(3\times10^5\text{ km/s}\right)}}{1-\left(\frac{23\text{ km/s}}{10^6\text{ c}\cdot\text{y}}\right)\frac{\left(500\times10^6\text{ c}\cdot\text{y}\right)}{\left(3\times10^5\text{ km/s}\right)}}} = \boxed{682.0\,\text{nm}}$$

(*d*) For $r = 5\times10^9\ c\cdot$y:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\frac{23\text{ km/s}}{10^6\text{ c}\cdot\text{y}}\right)\frac{\left(5\times10^9\text{ c}\cdot\text{y}\right)}{\left(3\times10^5\text{ km/s}\right)}}{1-\left(\frac{23\text{ km/s}}{10^6\text{ c}\cdot\text{y}}\right)\frac{\left(5\times10^9\text{ c}\cdot\text{y}\right)}{\left(3\times10^5\text{ km/s}\right)}}} = \boxed{983.0\,\text{nm}}$$

General Problems

35 •

Determine the Concept

(*a*) It must be a meson, and it must consist of a quark and its antiquark.

(*b*) The π^0 is its own antiparticle.

(*c*) The Ξ^0 is a baryon; it cannot be its own antiparticle; the antiparticle is the $\Xi^0 = \overline{u}\overline{s}\overline{s}$.

36 ••

Picture the Problem Examination of the decay products will reveal whether all the final products are stable. A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) $\boxed{\text{Yes, the final products are stable.}}$

(*b*) Add the reactions to obtain:

$$\Xi^0 \to p^+ + e^- + \overline{\nu}_e + \nu_\mu + \overline{\nu}_\mu + 2\gamma$$

(*c*) Charge conservation:
$0 \to e^+ + e^- = 0$

Charge is conserved.

Baryon number:
$1 \to 1 + 0 = 1$

Baryon number is conserved.

Lepton number:
$0 \to 0 + 1 - 1 + 1 - 1 = 0$

Lepton number is conserved.

Strangeness:
$-2 \to 0$

Strangeness is not conserved. However, the reaction is allowed via the weak interaction because in the first two decays $\Delta S = +1$.

(*d*) No; the rest masses of the decay products would be greater than the rest mass of the Ξ^0, violating energy conservation.

***37** ••

Picture the Problem The π^o particle is composed of two quarks, $u\overline{u}$. Hence, the reaction $\pi^o \to \gamma + \gamma$ is equivalent to $u\overline{u} \to \gamma + \gamma$.

(*a*) The u and $\overline{u}$ annihilate resulting in the photons.

(*b*) Two or more photons are required to conserve linear momentum.

38 ••

Picture the Problem A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) Energy conservation:

Because $m_\Lambda > m_p + m_\pi$, energy conservation is not violated.

Charge conservation:
$0 \to 1 - 1 = 0$

Because the total charge is 0 before and after the decay, charge conservation is not violated.

Baryon number:
$1 \to 1 + 0 = 1$

Because there is no change in baryon number, baryon number is conserved.

Lepton number:
$0 \to 0 + 0 = 0$

Because lepton number is 0 on both sides, lepton number is conserved.

The decay satisfied all conservation laws and is allowed.

(*b*) Energy conservation:

Because $m_\Sigma < m_n + m_p$, energy is not conserved.

Charge conservation:
$-1 \to 0 - 1 = -1$

Because the total charge does not change, charge is conserved.

Baryon number:
$+1 \to 1 - 1 = 0$

Because B changes from $+1$ to 0, baryon number is not conserved.

Lepton number:
$0 \to 0 + 0 = 0$

Because L is 0 on both sides, lepton number is conserved.

Because the decay violates both energy conservation and baryon number, it is not allowed.

(*c*) Energy conservation:

Energy is conserved.

Charge conservation:
$-1 \to -1 + 0 + 0 = -1$

Because the total charge does not change, charge is conserved.

Baryon number:
$0 \to 0 + 0 + 0 = 0$

Because B does not change, baryon number is conserved.

Lepton number:
$1 \to 1 - 1 + 1 = 1$

Because L does not change, lepton number is conserved.

The decay satisfied all conservation laws and is allowed.

Remarks: The decay in Part (*c*) is the decay process for the muon μ^- (see Example 41-2).

***39 ••**

Picture the Problem We can systematically determine *Q*, *B*, *S*, and s for each reaction and then use these values to identify the unknown particles.

(*a*) For the strong reaction: $\mathrm{p} + \pi^- \to \Sigma^o + ?$

Charge number: $+1 - 1 = 0 + Q \Rightarrow Q = 0$

Baryon number: $+1 + 0 = +1 + B \Rightarrow B = 0$

Strangeness: $0 + 0 = -1 + S \Rightarrow S = +1$

Spin: $+\frac{1}{2} + 0 = +\frac{1}{2} + s \Rightarrow s = 0$

$$\boxed{\text{These properties indicate that the particle is the kaon, } \mathrm{K}^0.}$$

(*b*) For the strong reaction: $\mathrm{p} + \mathrm{p} \to \pi^+ + \mathrm{n} + \mathrm{K}^+ + ?$

Charge number: $+1 + 1 = +1 + 0 + 1 + Q \Rightarrow Q = 0$

Baryon number: $+1 + 1 = 0 + 1 + 0 + B \Rightarrow B = +1$

Strangeness: $0 + 0 = 0 + 0 + 1 + S \Rightarrow S = -1$

Spin: $+\frac{1}{2} + \frac{1}{2} = 0 + \frac{1}{2} + 0 + s \Rightarrow s = +\frac{1}{2}$

$$\boxed{\text{These properties indicate that the particle is either the } \Sigma^0 \text{ or the } \Lambda^0 \text{ baryon.}}$$

(*c*) For the strong reaction: $\mathrm{p} + \overline{\mathrm{K}}^- \to \Xi^- + ?$

Charge number: $+1 - 1 = -1 + Q \Rightarrow Q = +1$

Baryon number: $+1 + 0 = +1 + B \Rightarrow B = 0$

Strangeness: $0 - 1 = -2 + S \Rightarrow S = -1$

Spin: $+\frac{1}{2} + 0 = +\frac{1}{2} + s \Rightarrow s = 0$

$$\boxed{\text{These properties indicate that the particle is the kaon, } \mathrm{K}^+.}$$

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Picture the Problem We can systematically determine *Q*, *B*, *S*, and *s* for the reaction and then use these values to identify the unknown particle. The *Q* value for the reaction is given by $Q = -(\Delta m)c^2$ and the expression for the threshold energy for the reaction is given in the problem statement.

(a) For the strong reaction: $\mathrm{p}+\mathrm{p}\to\Lambda^{o}+\mathrm{K}^{o}+\mathrm{p}+?$

Charge number: $+1+1=0+0+1+Q\Rightarrow Q=+1$

baryon number: $+1+1=+1+0+1+B\Rightarrow B=0$

strangeness: $0+0=-1+1+0+S\Rightarrow S=0$

spin: $+\frac{1}{2}+\frac{1}{2}=+\frac{1}{2}+0+\frac{1}{2}+s\Rightarrow s=0$

These properties indicate that the unknown particle is a pion, π^{+}.

(b) The reaction is: $\mathrm{p}+\mathrm{p}\to\Lambda^{o}+\mathrm{K}^{o}+\mathrm{p}+\pi^{+}$

The Q-value for the reaction is:

$$Q=\left[\left(m_{\mathrm{p}}+m_{\mathrm{p}}\right)-\left(M_{\Lambda^{o}}+M_{\mathrm{K}^{o}}+M_{\mathrm{p}}+M_{\pi^{+}}\right)\right]c^{2}$$

Use Table 41-1 to find the mass-energy values:

$$Q=\left[(938.3+938.3)-(1116+497.7+938.3+139.6)\right]\mathrm{MeV}=\boxed{-815\,\mathrm{MeV}}$$

Because $Q<0$, the reaction is endothermic.

(c) The threshold energy for this reaction is:

$$K_{\mathrm{th}}=-\frac{Q}{2m_{\mathrm{p}}}\left(m_{\mathrm{p}}+m_{\mathrm{p}}+M_{\Lambda^{o}}+M_{\mathrm{K}^{o}}+M_{\mathrm{p}}+M_{\pi^{+}}\right)$$

Using Table 41-1 to find the mass-energy values, substitute numerical values and evaluate K_{th}:

$$K_{\mathrm{th}}=-\frac{-815\,\mathrm{MeV}}{2(938.3\,\mathrm{MeV})}(938.3+938.3+1116+497.7+938.3+139.6)\mathrm{MeV}$$
$$=1984\,\mathrm{MeV}=\boxed{1.984\,\mathrm{GeV}}$$

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Picture the Problem We can solve the equation derived in Problem 31 for the recessional velocity of the galaxy and then use Hubble’s equation to estimate the distance to the galaxy.

(a) From Problem 31 we have: $\lambda'=\lambda_{0}\sqrt{\dfrac{1+v/c}{1-v/c}}$

Solve for the radicand:

$$\frac{1+v/c}{1-v/c}=\left(\frac{\lambda'}{\lambda_0}\right)^2$$

Substitute numerical values for λ' and λ_0:

$$\frac{1+v/c}{1-v/c}=\left(\frac{1458\text{ nm}}{656.3\text{ nm}}\right)^2=4.935$$

Simplify to obtain:

$$4.953\left(1-\frac{v}{c}\right)=1+\frac{v}{c}$$

and

$$5.953\frac{v}{c}=3.953$$

Solve for v:

$$v=0.664c=0.664\left(3\times10^8\text{ m/s}\right)$$
$$=1.99\times10^8\text{ m/s}=\boxed{1.99\times10^5\text{ km/s}}$$

(*b*) From the Hubble equation we have:

$$r=\frac{v}{H}$$

Substitute numerical values and evaluate r:

$$r=\frac{1.99\times10^5\text{ km/s}}{\dfrac{23\text{ km/s}}{10^6\,c\cdot\text{y}}}=\boxed{8.65\times10^9\,c\cdot\text{y}}$$

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Picture the Problem We can use the masses of the parent and daughters to find the total kinetic energy of the decay products under the assumption that the Λ^0 is initially at rest. Application of conservation of energy and the definition of kinetic energy will yield the ratio of the kinetic energy of the pion to the kinetic energy of the proton. Finally, we can use our results in (*a*) and (*b*) to find the kinetic energies of the proton and the pion for this decay.

(*a*) The total kinetic energy of the decay products is given by:

$$K_{\text{tot}}=\left(m_\Lambda-m_{\text{p}}-m_\pi\right)c^2$$

Substitute numerical values (see Table 41-1) and evaluate K_{tot}:

$$K_{\text{tot}}=\left(1116\frac{\text{MeV}}{c^2}-938.3\frac{\text{MeV}}{c^2}-139.6\frac{\text{MeV}}{c^2}\right)c^2=\boxed{38.1\text{MeV}}$$

(*b*) The ratio of the kinetic energies is given by:

$$\frac{K_\pi}{K_{\text{p}}}=\frac{\frac{1}{2}m_\pi v_\pi^2}{\frac{1}{2}m_{\text{p}}v_{\text{p}}^2}=\frac{m_\pi v_\pi^2}{m_{\text{p}}v_{\text{p}}^2}$$

Use conservation of momentum (nonrelativistic) to obtain:

$$m_\pi v_\pi = m_p v_p \Rightarrow \frac{v_\pi}{v_p} = \frac{m_p}{m_\pi}$$

Substitute for the ratio of the speeds to obtain:

$$\frac{K_\pi}{K_p} = \frac{m_\pi}{m_p}\left(\frac{m_p}{m_\pi}\right)^2 = \frac{m_p}{m_\pi}$$

Substitute numerical values and evaluate the ratio of the kinetic energies:

$$\frac{K_\pi}{K_p} = \frac{938.3\,\frac{\text{MeV}}{c^2}}{139.6\,\frac{\text{MeV}}{c^2}} = \boxed{6.72}$$

(*c*) Express the total kinetic energy in terms of K_π and K_p:

$$K_p + K_\pi = K_{tot} \qquad (1)$$

Use our results in (*a*) and (*b*) to obtain:

$$K_p + 6.72K_p = 38.1\,\text{MeV}$$

Solve for K_p:

$$K_p = \boxed{4.94\,\text{MeV}}$$

Substitute in equation (1) to obtain:

$$K_\pi = K_{tot} - K_p = \boxed{33.2\,\text{MeV}}$$

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Picture the Problem The total kinetic energy of the decay products is the rest energy of the Σ^0 particle. We can find the momentum of the photon from its energy and use the conservation of momentum to calculate the kinetic energy of the Λ^0.

(*a*) The total kinetic energy of the decay products is given by:

$$K_{tot} = (m_\Sigma)c^2$$

Substitute numerical values (see Table 41-1) and evaluate K_{tot}:

$$K_{tot} = \left(1193\,\frac{\text{MeV}}{c^2}\right)c^2 = \boxed{1193\,\text{MeV}}$$

(*b*) The momentum of the photon is given by:

$$p_\gamma = \frac{E_\gamma}{c} = \frac{E - m_\Lambda c^2}{c}$$

Substitute numerical values and evaluate p_γ:

$$p_\gamma = \frac{1193\,\text{MeV} - \left(1116\,\frac{\text{MeV}}{c^2}\right)c^2}{c} = \boxed{77.0\,\frac{\text{MeV}}{c}}$$

(*c*) The kinetic energy of the Λ^0 is given by:

$$K_\Lambda = \frac{p_\Lambda^2}{2m_\Lambda}$$

or, because $p_\Lambda = p_\gamma$,

$$K_\Lambda = \frac{p_\gamma^2}{2m_\Lambda}$$

Substitute numerical values and evaluate K_Λ:

$$K_\Lambda = \frac{\left(77.0\,\frac{\text{MeV}}{c}\right)^2}{2\left(1116\,\frac{\text{MeV}}{c^2}\right)} = \boxed{2.66\,\text{MeV}}$$

(*d*) A better estimate of the energy of the photon is:

$$E_\gamma = E - m_\Lambda c^2 - K_\Lambda$$

Substitute numerical values and evaluate E_γ:

$$E_\gamma = 1193\,\text{MeV} - \left(1116\,\frac{\text{MeV}}{c^2}\right)c^2 - 2.66\,\text{MeV} = \boxed{74.3\,\text{MeV}}$$

The improved estimate of the momentum of the photon is:

$$p_\gamma = \frac{E_\lambda}{c} = \frac{74.3\,\text{MeV}}{c} = \boxed{74.3\,\frac{\text{MeV}}{c}}$$

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Picture the Problem The solution strategy is outlined in the problem statement.

(*a*) Express $\Delta t = t_2 - t_1$ in terms of u_2 and u_1:

$$\Delta t = t_2 - t_1 = \frac{x}{u_2} - \frac{x}{u_1} = \frac{x(u_1 - u_2)}{u_1 u_2}$$

Noting that $u_1 u_2 \approx c^2$, we have:

$$\Delta t \approx \boxed{\frac{x\Delta u}{c^2}} \qquad (1)$$

where $\Delta u = u_1 - u_2$

(*b*) From Equation 39-25 we have:

$$\frac{u}{c} = \sqrt{1 - \left(\frac{mc^2}{E}\right)^2} = \left(1 - \left(\frac{mc^2}{E}\right)^2\right)^{1/2}$$

Expand binomially to obtain:

$$\frac{u}{c} = \boxed{1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2}$$

(c) Express $u_1 - u_2$ in terms of E_1, E_2, and mc^2:

$$u_1 - u_2 = \tfrac{1}{2}\left(mc^2\right)^2\left(\frac{1}{E_2^2} - \frac{1}{E_1^2}\right) = \frac{c\left(mc^2\right)^2\left(E_1^2 - E_2^2\right)}{2E_1^2E_2^2}$$

Substitute numerical values and evaluate Δu:

$$\Delta u = \frac{c\left(20\,\frac{\text{eV}}{c^2}c^2\right)^2\left[(20\,\text{MeV})^2 - (5\,\text{MeV})^2\right]}{2(20\,\text{MeV})^2(5\,\text{MeV})^2} = \boxed{7.50\times10^{-12}c}$$

Use equation (1) to evaluate Δt:

$$\Delta t \approx \frac{\left(1.7\times10^5\,c\cdot\text{y}\right)\left(7.5\times10^{-12}\,c\right)}{c^2} = 1.275\times10^{-6}\,\text{y}\times\frac{31.56\,\text{Ms}}{\text{y}} = \boxed{40.2\,\text{s}}$$

(d) Using mc^2 = 40 eV for the rest energy of a neutrino:

$$\Delta u = \frac{c\left(40\,\frac{\text{eV}}{c^2}c^2\right)^2\left[(20\,\text{MeV})^2 - (5\,\text{MeV})^2\right]}{2(20\,\text{MeV})^2(5\,\text{MeV})^2} = \boxed{3.00\times10^{-11}c}$$

Use equation (1) to evaluate Δt:

$$\Delta t \approx \frac{\left(1.7\times10^5\,c\cdot\text{y}\right)\left(3\times10^{-11}\,c\right)}{c^2} = 5.1\times10^{-6}\,\text{y}\times\frac{31.56\,\text{Ms}}{\text{y}} = \boxed{161\,\text{s}}$$

Remarks: Note that the spread in the arrival time for neutrinos from a supernova can be used to estimate the mass of a neutrino.