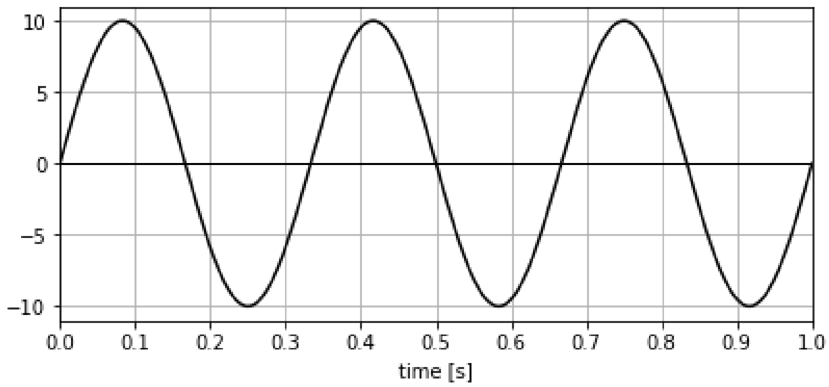
Signal and Image Processing - Practice Exam

Voluntary and not graded

Task 1 (General questions) [19 points]

(a) The plot below shows a signal $x(t)$. Draw the amplitude and phase spectrum of $x(t)$. Carefully label all axes!



Amplitude spectrum

Phase spectrum

[/ 6]

(b) Let $z(t)$ be the analytic signal of $x(t)$ from the previous exercise. Draw the amplitude spectrum of $z(t)$. Carefully label all axes!

[/ 3]

(c) Roughly explain one way how you could find out whether a musical instrument is correctly tuned with the methods learned in this course.

[/ 6]

(d) Name one example where the discrete cosine transform (DCT) is preferable to the discrete fourier transform (DFT) and explain why this is the case.

[/ 4]

Task 2 (Time-frequency analysis) [20 points]

The short time Fourier transform (STFT) and the discrete wavelet transform (DWT) are two of the most common time-frequency analysis methods.

(a) Name one difference between the STFT and the DWT.

[/ 4]

(b) Is it possible to have a fine frequency domain resolution and a narrow time window simultaneously? Justify your answer.

[/ 4]

(c) Given a mother wavelet $h_{m=0,k=0}[n]$ as shown below, sketch the derived wavelets for the following three cases. (Note: m and k represent the scaling factor and the shift respectively.)

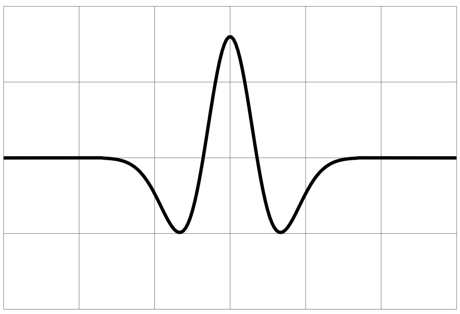


Figure 1: Mother wavelet $h_{m=0,k=0}[n]$

Case 1: $h_{m=1,k=0}[n]$

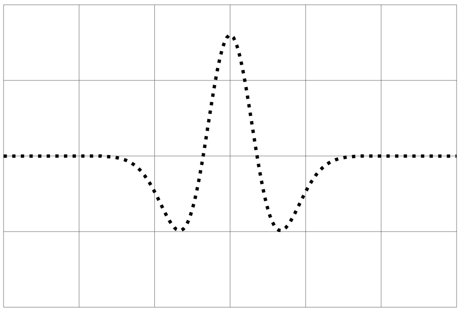


Figure 2: Dotted line: Mother wavelet $h_{m=0,k=0}[n]$

Case 2: $h_{m=0,k=1}[n]$

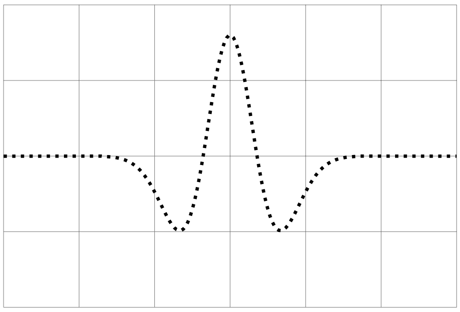


Figure 3: Dotted line: Mother wavelet $h_{m=0,k=0}[n]$

Case 3: $h_{m=1,k=1}[n]$

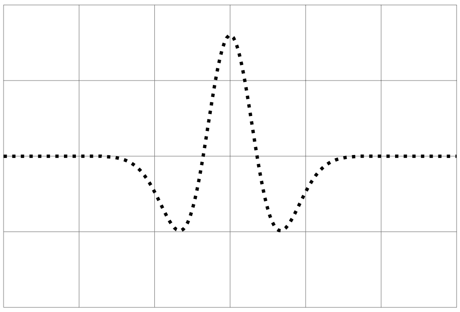


Figure 4: Dotted line: Mother wavelet $h_{m=0,k=0}[n]$

[/ 12]

Task 3 (Filters) [24 points]

(a) Sketch the frequency response of an ideal lowpass filter with cut-off frequency ω_c.

[/ 4]

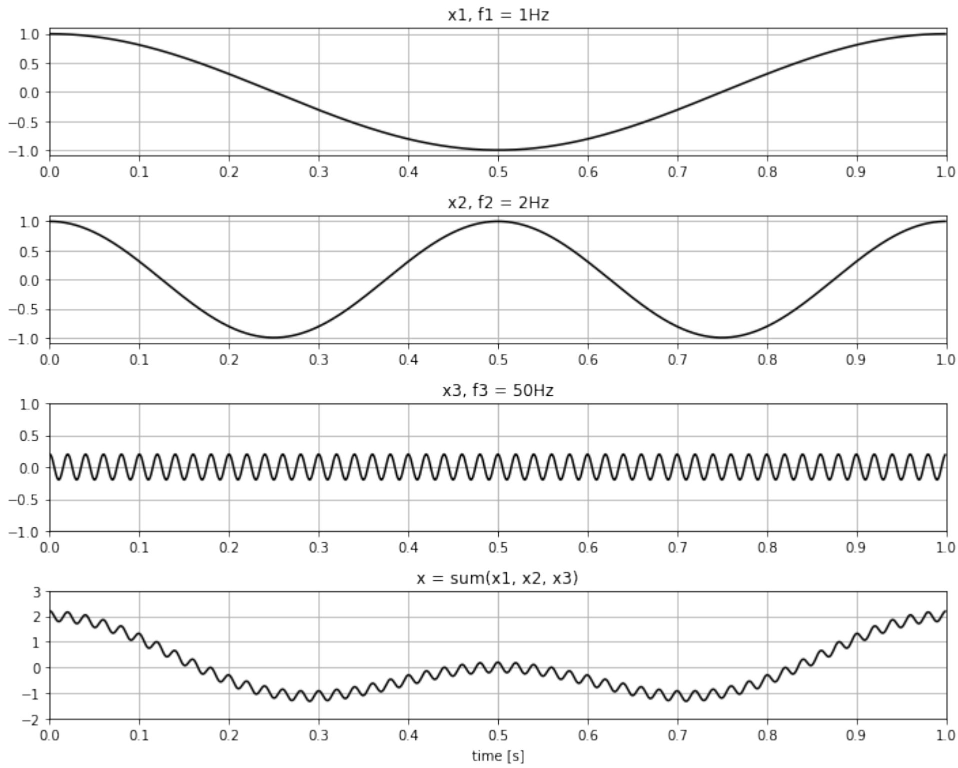
(b) Describe how to obtain a $100 - 200$Hz band-pass filter using two low-pass filters.

[/ 4]

(c) Name three differences between finite impulse response filters (FIR) and infinite impulse response filters (IIR).

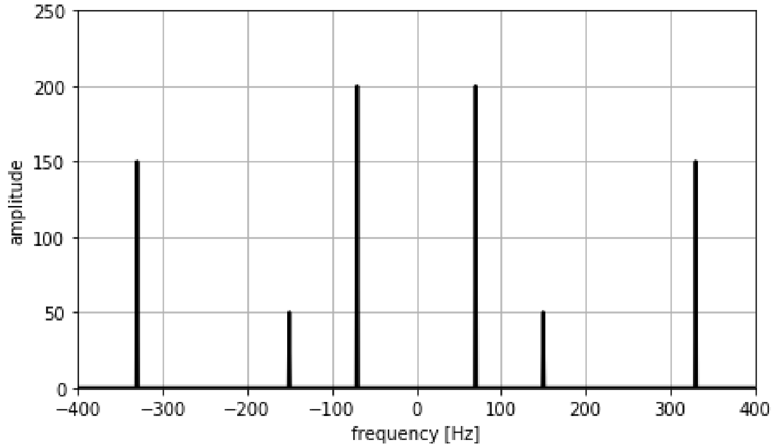
[/ 6]

(d) The plot below shows 1s long sections of three infinite signals $x_1(t), x_2(t)$ and $x_3(t)$, as well as their sum $x(t) = \sum_{i=1}^{3} x_i(t)$. Sketch the signal which would result from applying an ideal 30Hz low-pass filter with zero-phase to $x(t)$. Carefully label all axes!



[/ 6]

(e) The plot below shows the amplitude spectrum of a signal. Sketch the amplitude spectrum which would result from applying an ideal 100Hz high-pass filter to it. Carefully label all axes!



[/ 4]

Task 4 (Information theory) [22 points]

To transmit messages between computers, we usually need to encode them into binary or hexadecimal code. To increase the efficiency of data transmission, many data compression techniques have been proposed, and one of the most famous algorithms is arithmetic coding. Assume that we are going to transmit messages containing only the letters a, b, c, d, e and f with arithmetic coding, where each of the characters occurs with the following probability:

a	b	c	d	e	f
0.2	0.1	0.1	0.2	0.3	0.1

(a) Compute the corresponding probability range of each character.

[/ 6]

(b) Using the previously calculated character probability ranges, calculate the probability range of the word `hello` and find its most efficient binary code representation.

[/ 12]

(c) Name one disadvantage of arithmetic coding, as well as one coding method which does not have this disadvantage.

[/ 4]

Task 5 (Application: AM radio) [15 points]

The term *amplitude modulation* (AM) refers to the process of scaling a signal with a high, constant frequency (carrier signal) by the amplitude of a lower frequency signal (modulator signal).

(a) You are tasked to broadcast an AM radio signal and are given a speech signal with a range from 80Hz to 320Hz. Using a carrier frequency of 800kHz, describe how you can transfer the speech signal to the carrier frequency.

[/ 6]

(b) Which frequency components does the resulting modulated signal contain?

[/ 3]

(c) Describe how the receiver can reconstruct the original signal from the broadcasted signal.

[/ 6]