Einführung in Künstliche Intelligenz SS 2017, 2.0 VU, 184.735

Exercise Sheet 2 – Learning and Neural Networks

For the presentation part of this exercise, mark your solved exercises in **TUWEL** until Monday, June 05, 23:55 CET. Be sure that you tick only those exercises that you can solve and explain on the blackboard!

Please ask questions in the **TISS** Forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 2.1: Consider a classification problem with three attributes A, B, C with the domains $V(A) = \{a_1, a_2, a_3\}$, $V(B) = \{b_1, b_2, b_3\}$, $V(C) = \{c_1, c_2, c_3\}$ and two classes $K = \{T, F\}$. For this problem, build a decision tree using the rule from the lecture of choosing the attribute maximising information gain in each step.

	A	B	C	Class
1	a_1	b_1	c_2	F
2	a_3	b_2	c_3	F
3	a_2	b_3	c_2	T
4	a_3	b_1	c_1	F
5	a_2	b_1	c_3	F
6	a_3	b_2	c_1	T
7	a_1	b_3	c_3	T

Exercise 2.2: Compute the information gain for an attribute A that takes a different value for each example. What problem do you see?

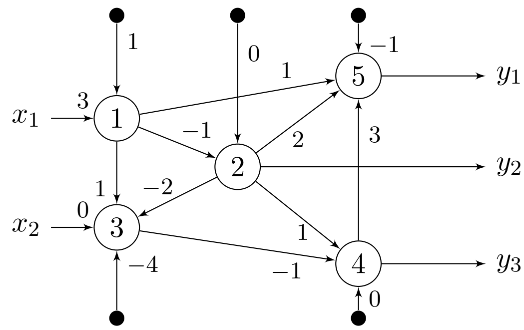
Exercise 2.3: Consider again the problem in Exercise 2.1. Again construct a decision tree, but in every step choose the argument that maximises the *relative information gain*, i.e., the ratio between their gain and their own intrinsic information.

$$GainR(A) = \frac{Gain(A)}{H(A)}$$

Thereby we have $H(A) := \sum_{a \in V(A)} \frac{|E_a|}{V} \log_2 \frac{V}{|E_a|}$, where $V(A)$ denotes the set of possible values for the attribute A and E_a is the set of samples with $A = a$ and $V := \sum_{a \in V(A)} |E_a|$ is the total number of samples.

Exercise 2.4: Suppose that an attribute splits the set of examples E into subsets E_k and that each subset has p_k positive and n_k negative examples. Show that the attribute has zero information gain if the ratio $\frac{p_k}{p_k+n_k}$ is equal for all k.

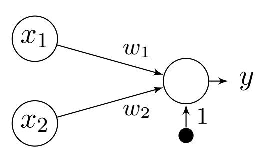
Exercise 2.5: Consider a neural network with five nodes of the following form:



The numbers beneath the arrows denote the respective weights. Nodes 1 and 2 have the *identity function* as their activation function. Nodes 3 and 4 use the hard limiter with threshold 0 and node 5 uses the function ($\frac{2+x}{x^2}$). The arrows from the black dots give the respective bias weights. What is the output produced by the network when the input is $(x_1, x_2) = (1, 1)$?

Exercise 2.6: Construct a neural network with 3 input nodes x_1, x_2, x_3 and one output node y which produces a parity bit of the respective input values, namely $y = x_1$ XOR x_2 XOR x_3 holds. You are allowed to introduce a single hidden layer with additional nodes, but every node should use the hard limiter with threshold 1 as activation function. Please note that the fixed input a_0 is 1, as presented in the lecture.

Exercise 2.7: Consider a 2-layer perceptron with two input neurons and one output neuron of the following form:



Train the perceptron using the *Perceptron learning rule* and the identity transfer function on the following training data: $f(0, 0) = 1$, $f(1, 0) = 0$ and $f(0, 1) = 0$. The weights are initialised to 0 and the learning rate α is 1. The bias weight will not be learned and stays 1.

Exercise 2.8: Suppose you have a neural network with linear activation functions. That is, for each unit the output is some constant c times the weighted sum of the inputs. For simplicity, assume that the activation function is the same linear function at each node: $g(x) = c \cdot x + d$.

Assume that the neural network has one hidden layer. For a given assignment to the weights w, write down equations for the value of the units in the output layer as a function of w and the input layer x, without any explicit mention of the output of the hidden layer. Show that there is a network with no hidden units that computes the same function.