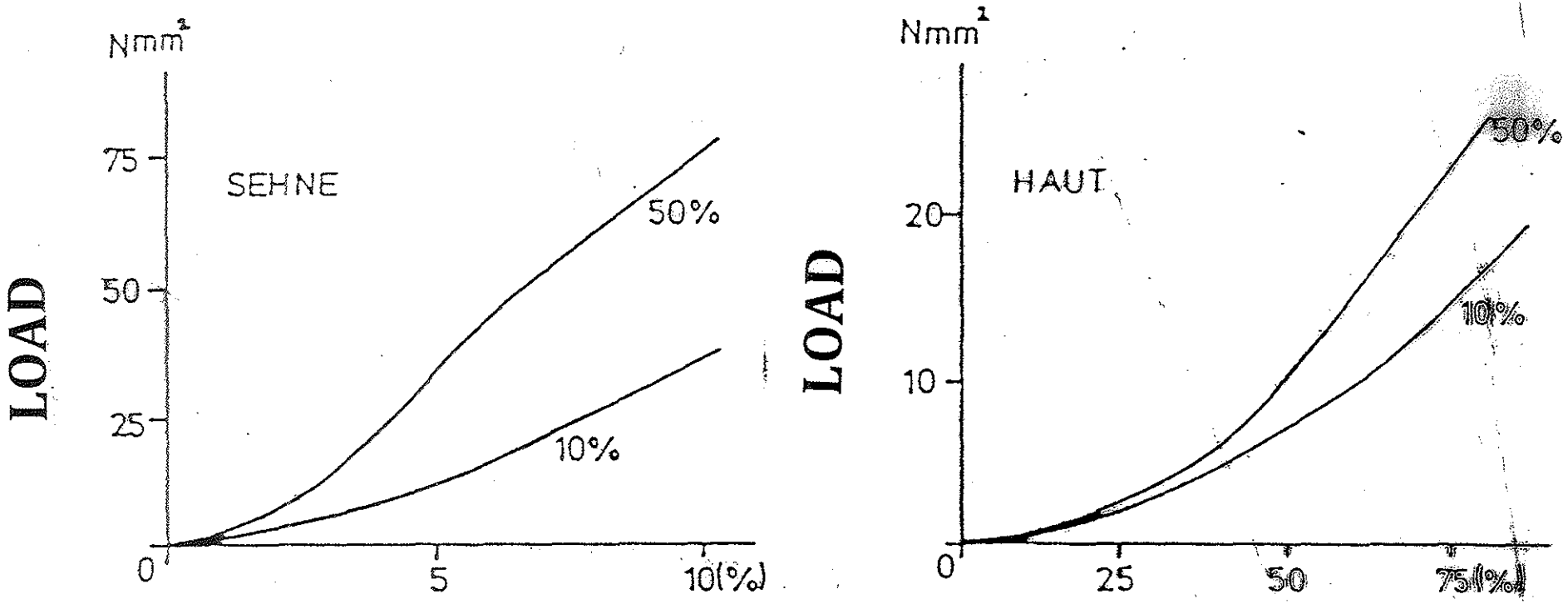
Viscoelastic behavior of soft connective tissues

We observe that

- the stress-strain relationship depends on the *strain rate*
- tissues show the phenomenon of *stress relaxation*; the load or stress is monitored as a function of time while strain is kept constant
- tissues show the phenomenon of *creep*; the strain or elongation is monitored as a function of time while stress is kept constant
- the stress-strain relationship of the loading differs from that of the unloading mode; *hysteresis loop*
- after *preconditioning*, i.e. the repetition of a certain experimental scheme, the stress-strain relationship achieves a stationary state.



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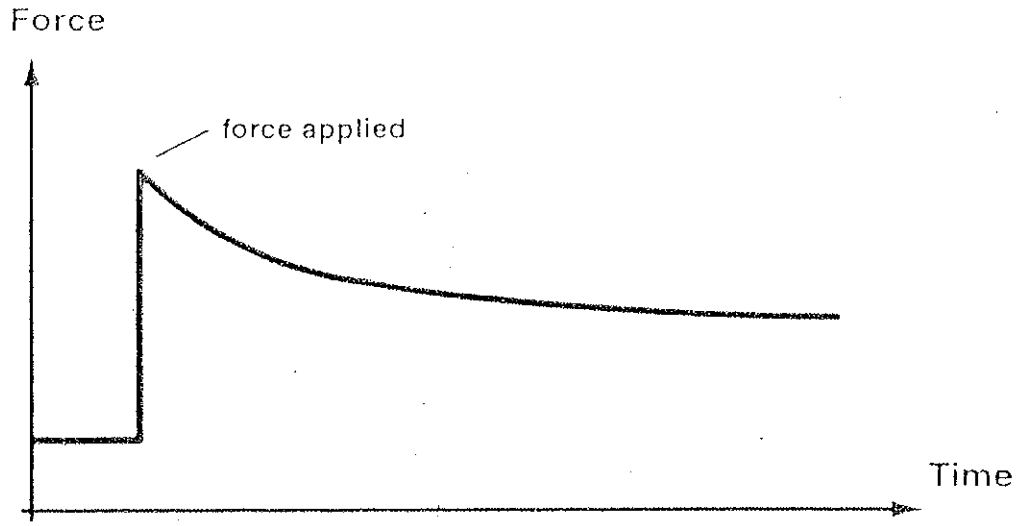


Figure 2.5.7 Schematic force-relaxation curve for ligament.

e.g., temperature and solution in which the test is carried out. The faster a force is applied, the less time there is for the viscous component to dissipate. A ligament will appear stronger and slightly stiffer under rapid force than under slow force. *Creep* is the analogous behaviour of a ligament under a fixed force when the force is either held or reached repetitively in a cyclic fashion. Creep is the increase in length over time under a constant force. With creep, as with force relaxation, manifestation of the viscous component through time-dependent force or strain changes eventually ceases (Fig. 2.5.8).

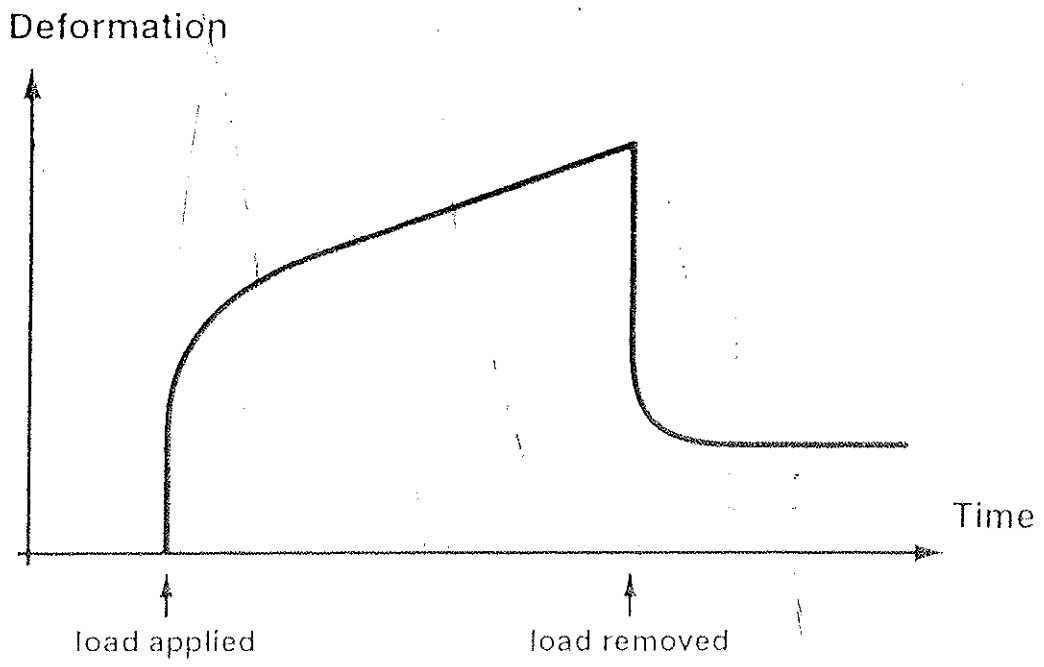


Figure 2.5.8 Schematic creep curve for ligament.

During cyclic force, however, some of the viscous component can be recovered in each cycle (Fig. 2.5.9). When the ligament is unforced, the viscous component, while never recovering completely (at least during *in-vitro* tests), can recover to over 90% of its original state after many hours in a relaxed condition. One can speculate about what is being

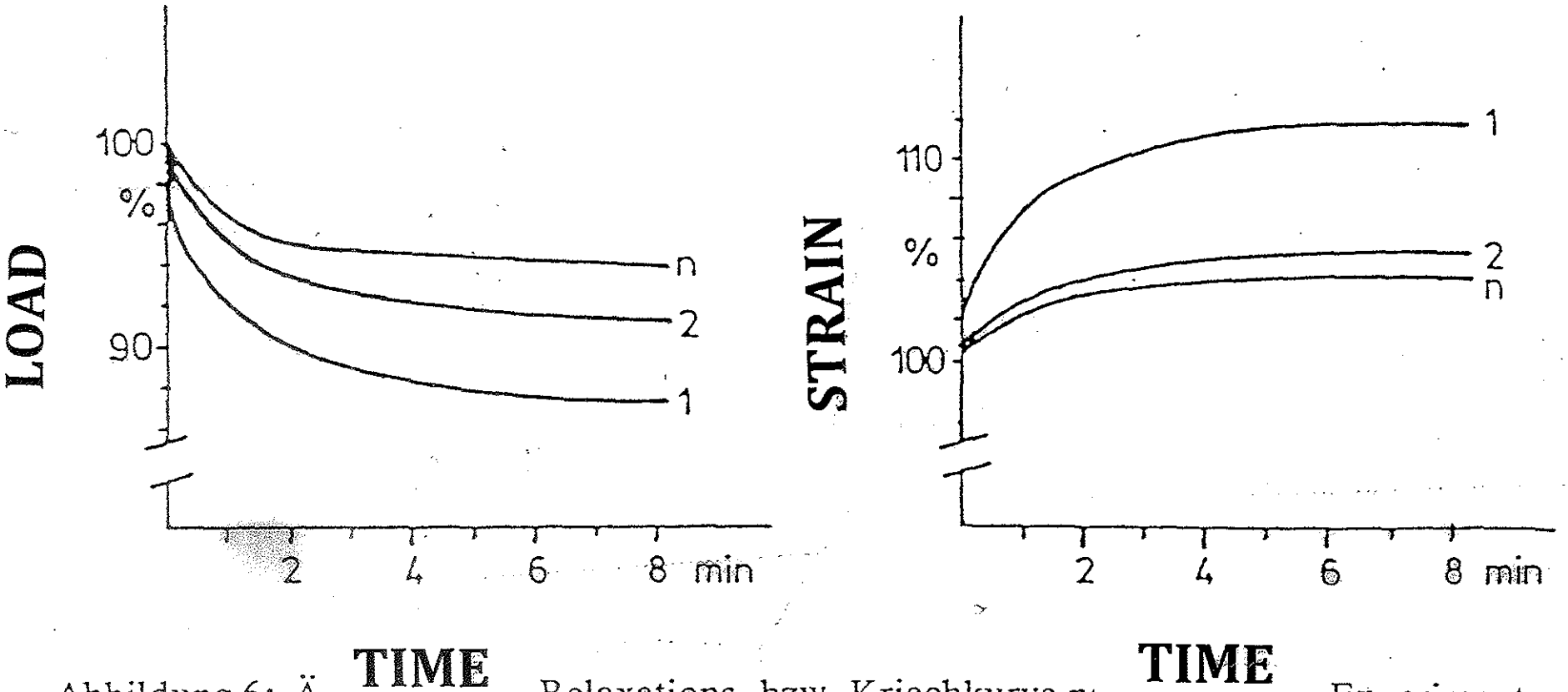


Abbildung 6: Ä Relaxations- bzw. Kriechkurve n Experimenten zu gleichem Dehnungs- bzw. Lastniveau

recovered, but this recovery probably involves some combination of water influx, returning collagen crimp, elastin tensile force, and increasing collagenous disorganization under unforced conditions.

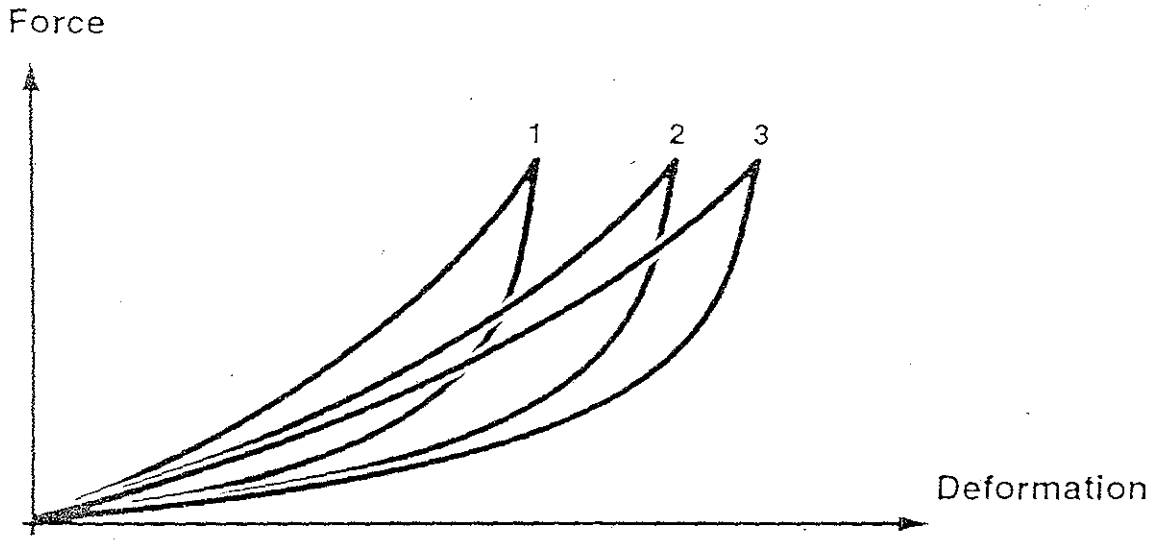


Figure 2.5.9 Schematic force-deformation graph showing three successive cycles of forcing and unforcing, illustrating the viscoelastic creep effect of cycling upon a ligament.

To modell these biomechanical properties a network of viscoelastic elements is formed

The components (elements) of this network are

The spring (Hooke element), characterized by a linear relationship of stress and strain (or force and elongation)

$$\sigma = E^{*}\varepsilon$$

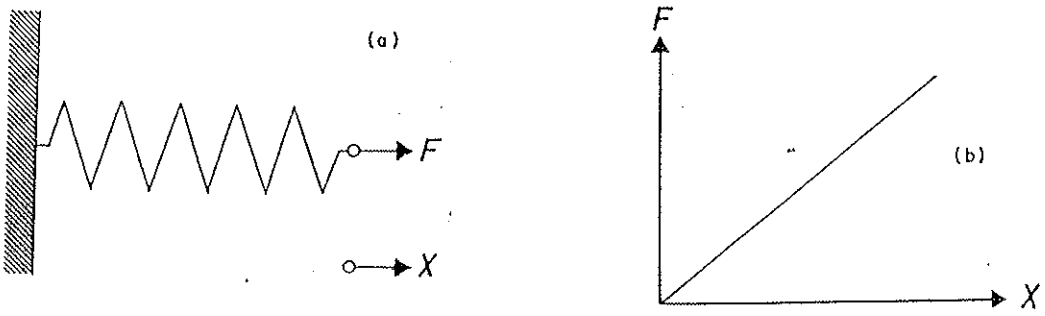
The dashpot (Newton element), characterized by a linear relationship of stress and strain rate (or force and rate of elongation)

$$\sigma = \eta^{*}d\varepsilon/dt$$

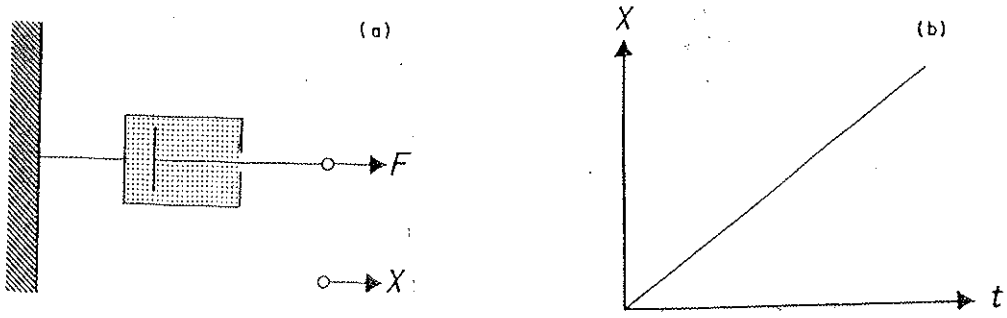
The dry friction element (Coulomb element), characterizing ideal plasticity, i.e. strain remains zero until a certain level of load is achieved, and then strain is indeterminable

The mentioned viscoelastic elements are symbolized as follows

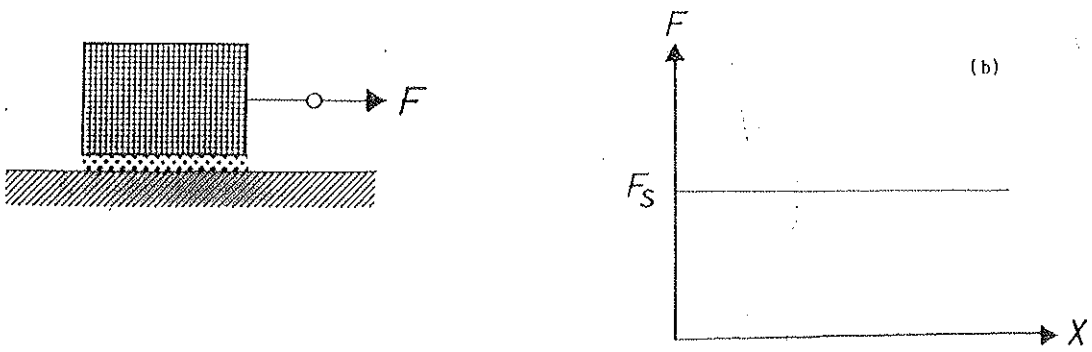
The spring (Hooke element)



The dashpot (Newton element)

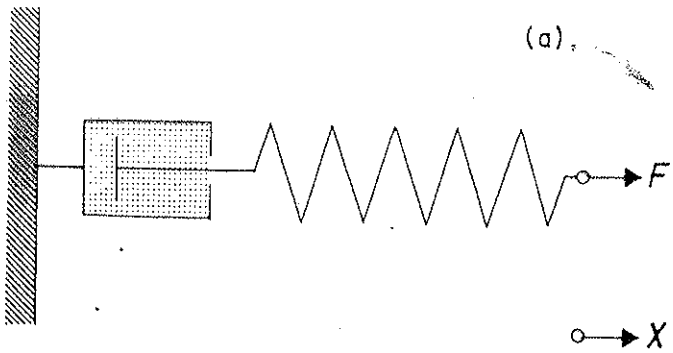


The dry friction element (Coulomb element)



There are two possible ways to combine the Hooke and Newton element to a simple viscoelastic model

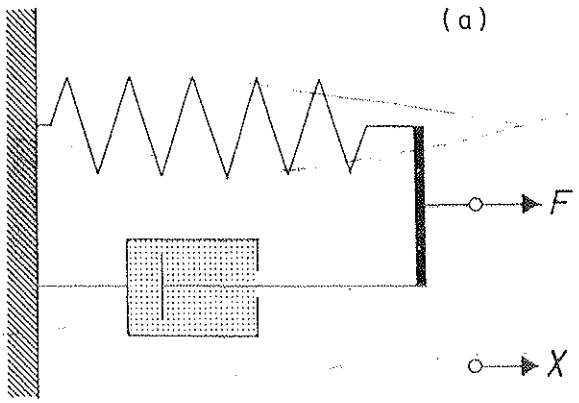
In series strains are added (Maxwell model)



$$\varepsilon_1+\varepsilon_2 = \varepsilon = E^{-1}*\sigma + (\eta*d/dt)^{-1}*\sigma$$

or $$E\eta*d\varepsilon/dt = (E+\eta*d/dt)*\sigma$$

In parallel stresses are added (Kelvin model or Voigt model)

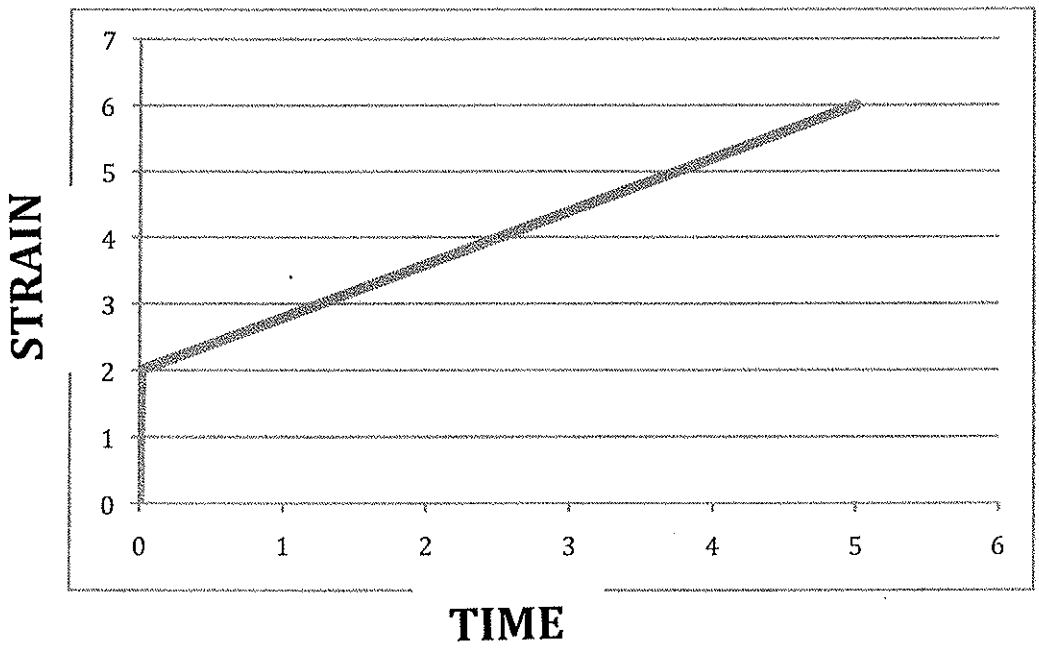


$$\sigma = E*\varepsilon + \eta*d\varepsilon/dt$$

If stress is applied suddenly (Heaviside step)

$\sigma(t) = \sigma_0 * \theta(t)$

then strain versus time for the Maxwell model



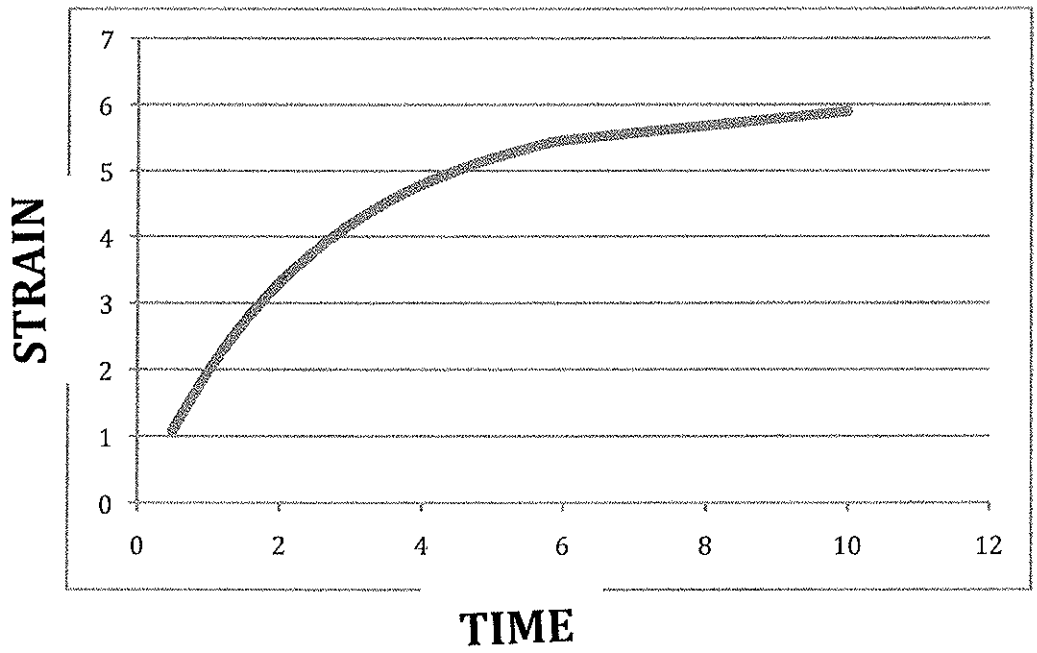
- the first phase is the elastic reaction: instantaneous reaction of strain due to suddenly applied load

- followed by the reaction of the dashpot: strain is a linear function of time as the load is constant after the Heaviside step

If stress is applied suddenly (Heaviside step)

$$\sigma(t) = \sigma_0 * \theta(t)$$

then strain versus time for the Kelvin model



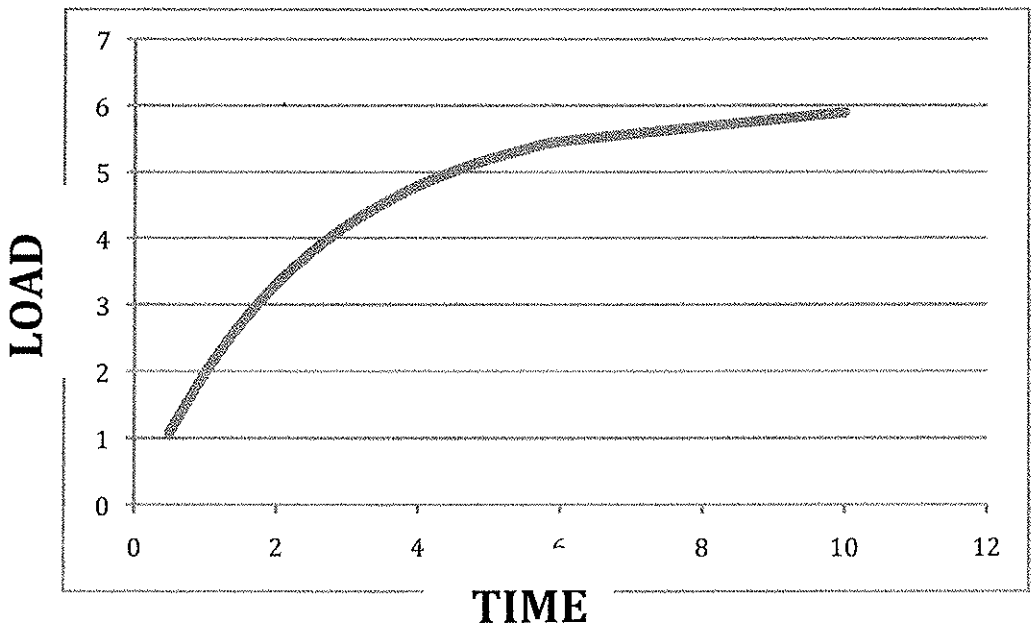
The strain is an exponential function with time converging asymptotically to the final value.

The final value of strain corresponds to that of the spring when the dashpot is thought to be removed. The dashpot slows down the elongation of the Kelvin model.

In case of the application of strain at a constant rate (as used often in experiments with bio tissues) we observe the load (or stress) as a function of time.

$\varepsilon(t)=r*t$ r is for rate

load versus time for the Maxwell model

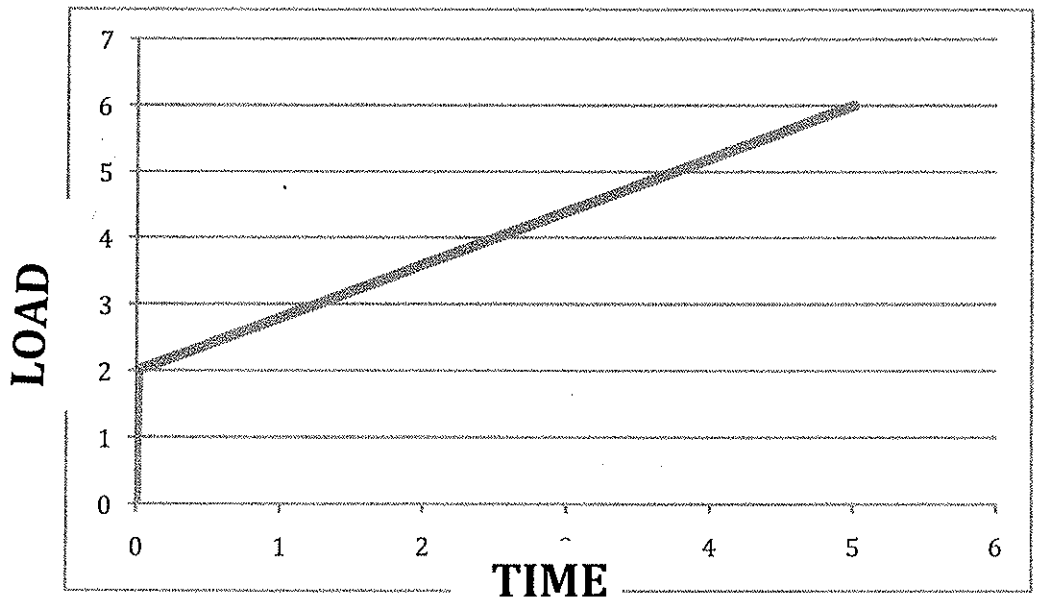


The load achieves asymptotically a final value, which corresponds to that when the spring would be removed.

In case of the application of strain at a constant rate we observe the load (or stress) as a function of time.

$\varepsilon(t)=r*t$ r is for rate

load versus time for the Kelvin model



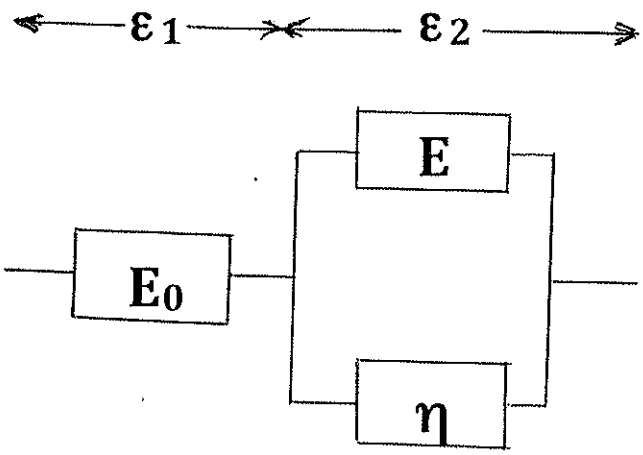
There is an instantaneous reaction of the dashpot which is added to the linearily increasing load of the spring.

Now to models using three elements.

There are the following ways to form a viscoelastic solid with 3 elements.

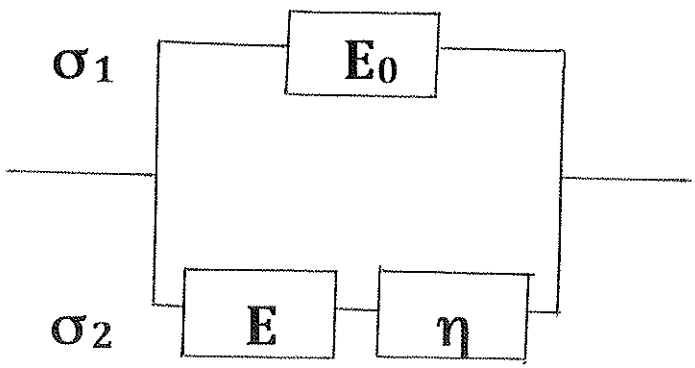
Version 1: spring element with Kelvin model in series

Strains are added



Version 2: spring element with Maxwell model in parallel

Stresses are added



Now to models using three elements.

There are the following ways to form a viscoelastic solid with 3 elements.

Version 1: spring element with Kelvin model in series

Strains are added

$$\varepsilon_1+\varepsilon_2 = E_0^{-1}*\sigma + (E+\eta*d/dt)^{-1}*\sigma = \varepsilon$$

or

$$(E_0+E+\eta*d/dt)*\sigma = E_0*(E+\eta*d/dt)*\varepsilon$$

Version 2: spring element with Maxwell model in parallel

Stresses are added

$$\sigma_1 + \sigma_2 = (E^{-1} + (\eta*d/dt)^{-1})^{-1}*\varepsilon + E_0*\varepsilon = \sigma$$

or

$$(E + \eta*d/dt)\sigma = (E_0*E + (E+E_0)\eta*d/dt)\varepsilon$$

Both **versions of three parameter solids** are symetric with respect to the occurring derivatives of stress and strain. If load is given as a function of time one has to solve a differential equation of first order for strain, and for strain as a function of time solve equation for stress.

The equations may be written in terms of the time constants for stress relaxation and creep.

Version 1 of three parameter solid

$$\tau_{relaxation} = \eta/(E_0+E) = \tau_r$$

$$\tau_{creep} = \eta/E = \tau_c$$

then to the equation

$$(E_0+E)*(1 + \tau_r *d/dt)\sigma = E_0*E*(1 + \tau_c *d/dt)\varepsilon$$

Version 2 of three parameter solid

$$\tau_{relaxation} = \eta/E = \tau_r$$

$$\tau_{creep} = (E_0+E)\eta/(E_0*E) = \tau_c$$

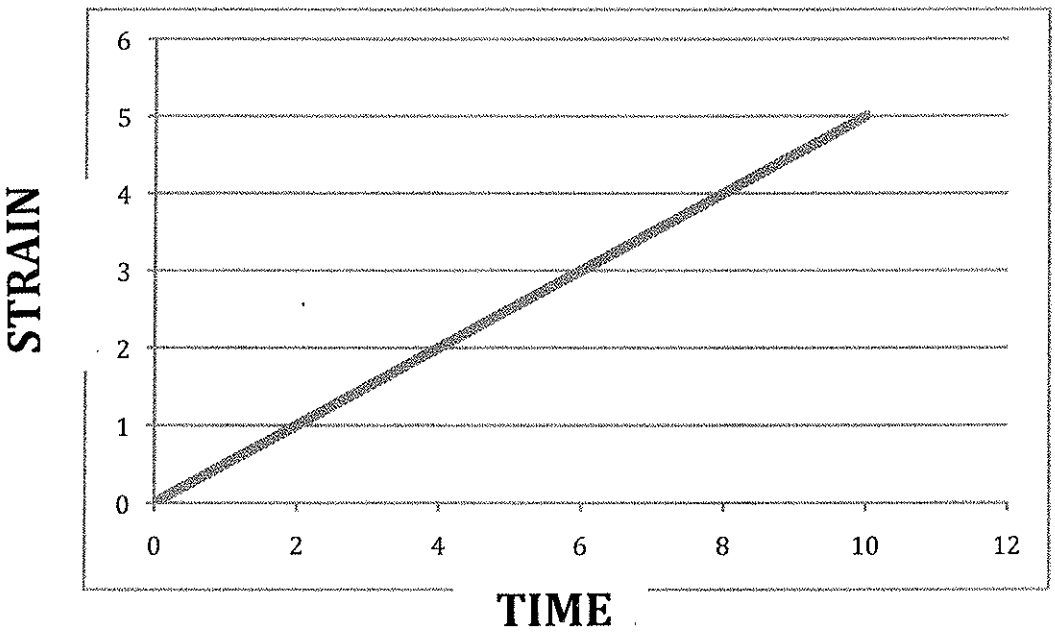
then to the equation

$$(1 + \tau_r *d/dt)\sigma = E_0*(1 + \tau_c *d/dt)\varepsilon$$

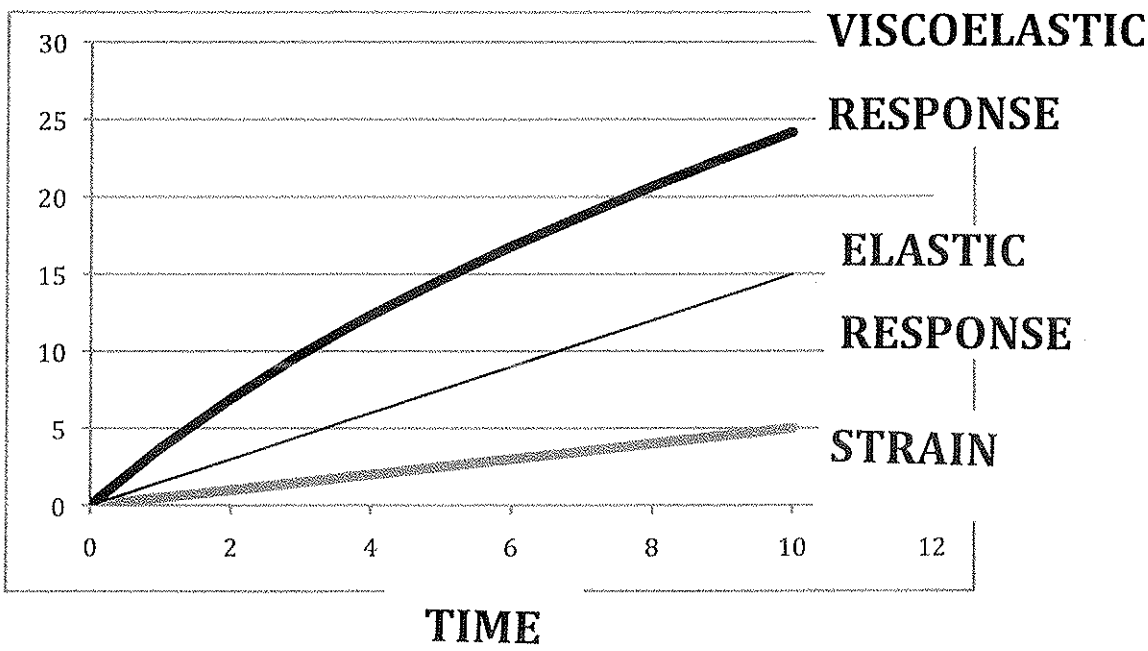
In case of an experiment carried out at constant strain rate we have

A Ramp with constant strain rate r, t time

$\mathbf{d\varepsilon/dt = r}$ and $\boldsymbol{\varepsilon = r*t}$

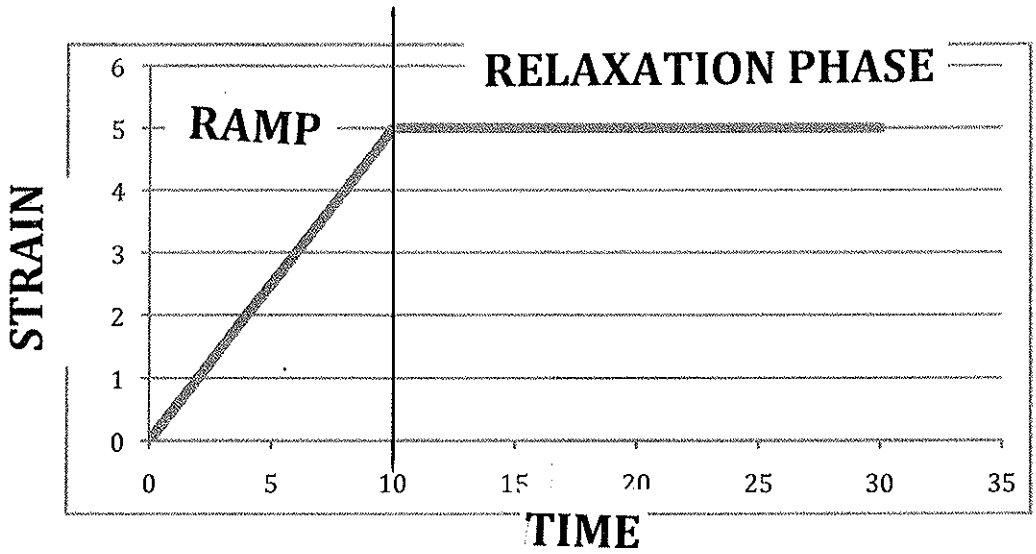


The stress response is composed of a linear and an exponential function of time

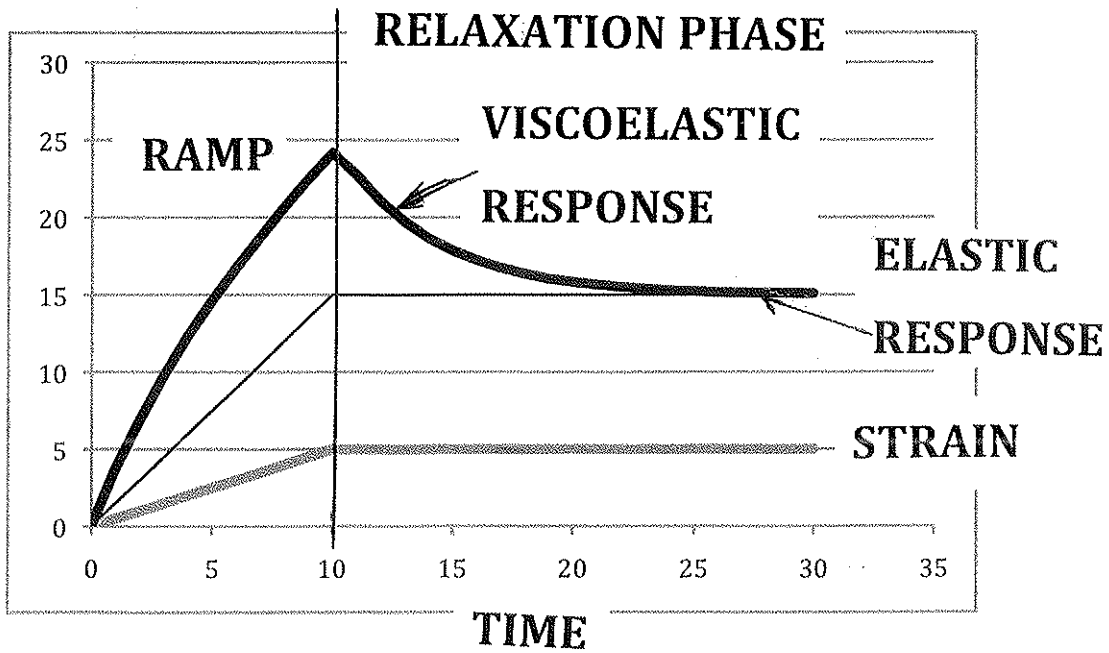


For **stress relaxation tests** we have a ramp phase (strain rate constant) followed by the relaxation tests (strain constant)

The ramp phase as before



During the relaxation phase the contribution of the dashpot diminish (the viscous contribution to stress decays to result in the final elastic contribution)



Viscoelastic model for the nonlinear stress-strain behavior of bio-tissues

- use nonlinear Hooke and Newton elements
- use successively engaged spring elments -> figure

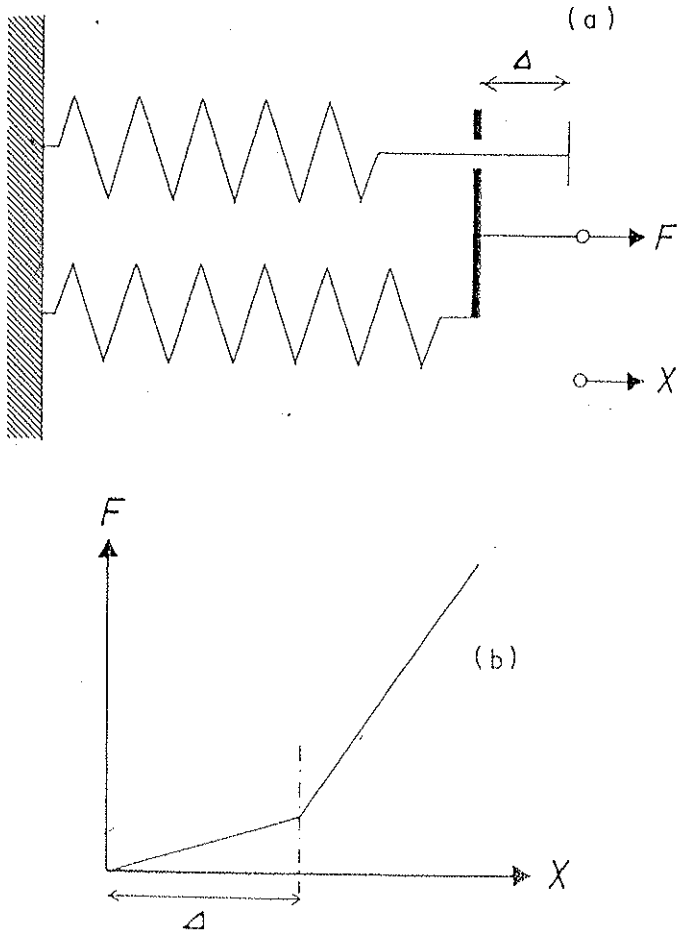


Fig. 6. A system for nonlinear spring (Hooke-element) action. (a) Two Hooke elements arranged in parallel; the upper one comes into action after a deformation of Δ; (b) Force-deformation diagram of the system.

A dashpot (Newton element) is added

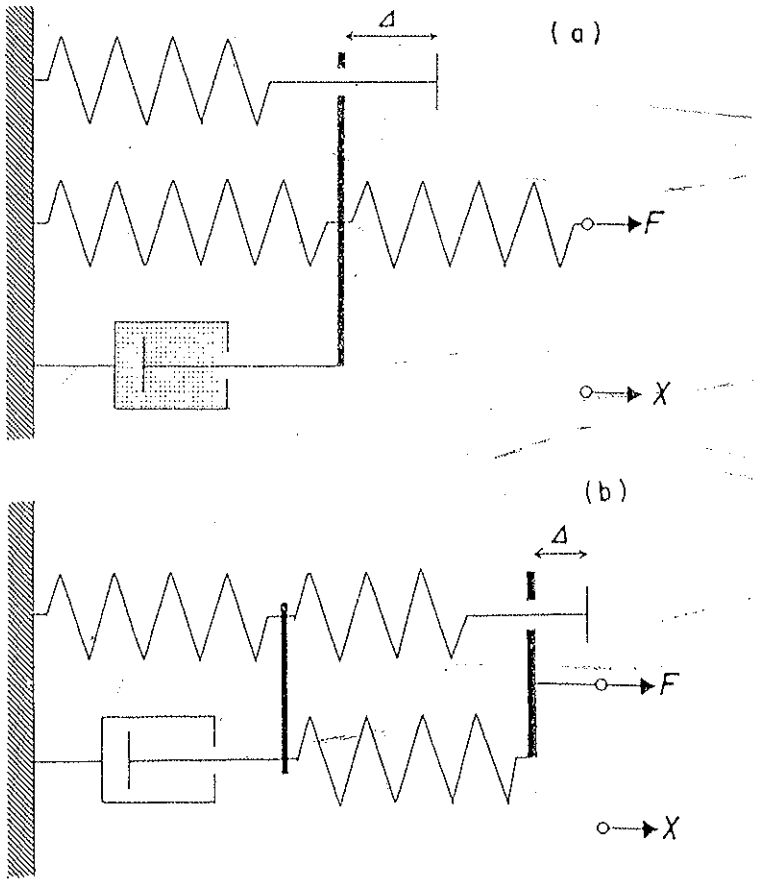
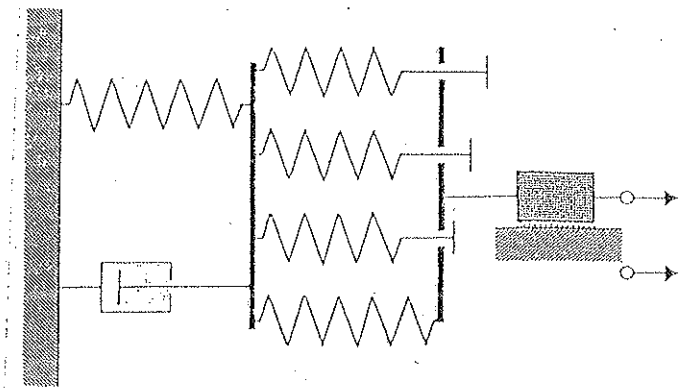


Fig. 10.(a) and (b). Illustrate the two possibilities for combining the nonlinear Hooke-element action (cf. Fig. 6) with the Newton element in the model.

A dry friction element (Coulomb) is added



To the complete model which can describe the nonlinear stress-strain relationship and the preconditioning behavior

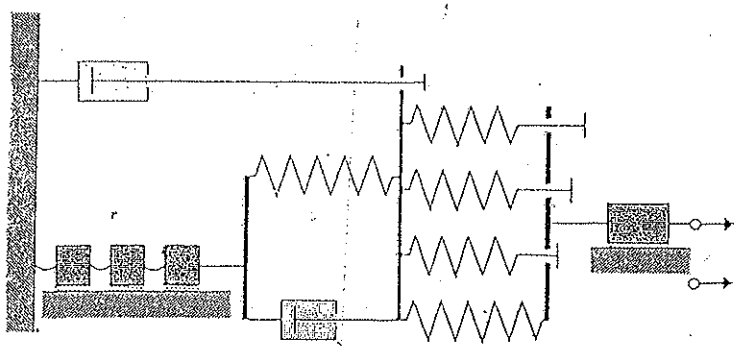


Fig. 14. The complete model for the mechanical behavior of ligamentous tissue. The part on the left accounts for the building-up process.

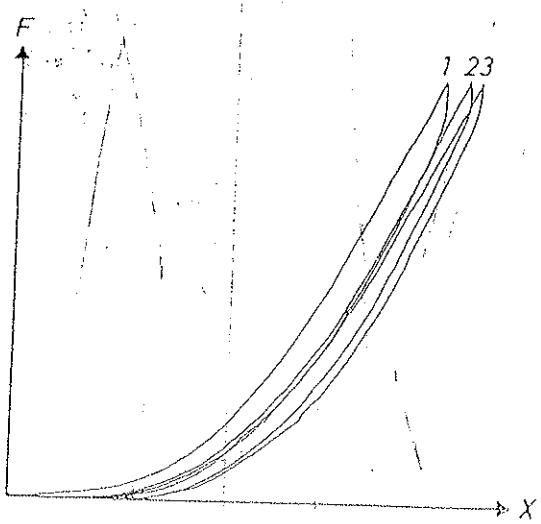


Fig. 13. Load-deformation diagrams (cf. Fig. 14) for three consecutive cycles of loading and unloading of the same specimen.

Improved description of the relaxation behavior of bio-tissues

Instead of a single relaxation time constant employ a spectrum of amplitudes and time constants τ

That means instead of a viscous stress decay of the form

$$\mathbf{exp(-t/\tau)}$$

the relaxation phase is described with a discrete spectrum, i.e.

$$\mathbf{\sum a_i * exp(-t/\tau_i)}$$

or the relaxation phase is described with a continuous spectrum of time constants, i.e.

$$\mathbf{\int d\tau\ a(\tau)*exp(-t/\tau)}$$

It is known from literature that the **continuous spectrum** should be preferred. The following terms are found

$$\mathbf{a(\tau) = a_0 / \tau}$$ for τ within the interval (τ_1, τ_2)

that means that one has to determine 3 parameters a_0, τ_1, τ_2 with a least mean square fit

or the **lognorm distribution**, i.e. the logarithms of the relaxation time constant are normal distributed and the amplitudes are also following a $1/\tau$ law.

For a comparison with dynamic tests it is shown that the imaginary part of the Young`s modulus (describing the **phase shift** between excitation and answer of the material) is nearly **constant within 2-3 orders of magnitude** of frequency

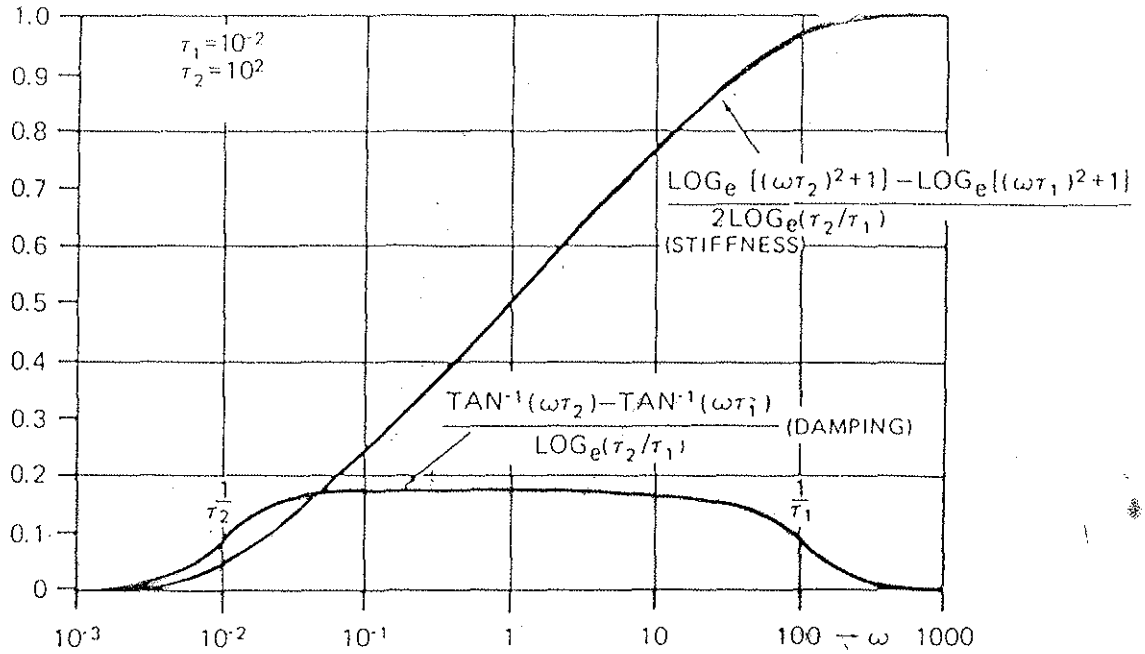


Figure 7.6:2 The stiffness (real part of the complex modulus $\mathscr{M}$) and the damping plotted as functions of the logarithm of the frequency ω; corresponding to a continuous relaxation spectrum $S(\tau) = c/\tau$ for $\tau_1 \leq \tau \leq \tau_2$ and zero elsewhere. $\tau_1 = 10^{-2}$, $\tau_2 = 10^{2}$. From Neubert (1963).

$$\mathscr{M}(\omega) = \left\{1 + \int_{\tau_1}^{\tau_2} \left[c\frac{\omega\tau}{1+(\omega\tau)^2} + \frac{ic}{1+(\omega\tau)^2}\right] d(\omega\tau)\right\}\left\{1 + \int_{\tau_1}^{\tau_2} \frac{c}{\tau}\, d\tau\right\}^{-1}$$

$$= \left\{1 + \frac{c}{2}\left[\ln(1+\omega^2\tau_2^2) - \ln(1+\omega^2\tau_1^2)\right] + ic\left[\tan^{-1}(\omega\tau_2) - \tan^{-1}(\omega\tau_1)\right]\right\}\left\{1 + c\ln\frac{\tau_2}{\tau_1}\right\}^{-1}.$$

Stiffness is derived from the real part of the complex Young`s modulus

Damping results from phase shift as mentioned before

If a three parameter solid is used the phase shift would show a pronounced peak at a certain frequency in the dynamic test

The equation for the three parameter solid may be expressed by the time constants for stress relaxation and creep as followes

$$\sigma + \tau_{Relaxation} * d\sigma/dt = E_0*(\varepsilon + \tau_{Creep} *d\varepsilon /dt)$$

The following figure shows a summary of different viscoelastic models: hysteresis loop versus frequency (strain rate)

Comparison of Maxwell, Kelvin model and three parameter solid

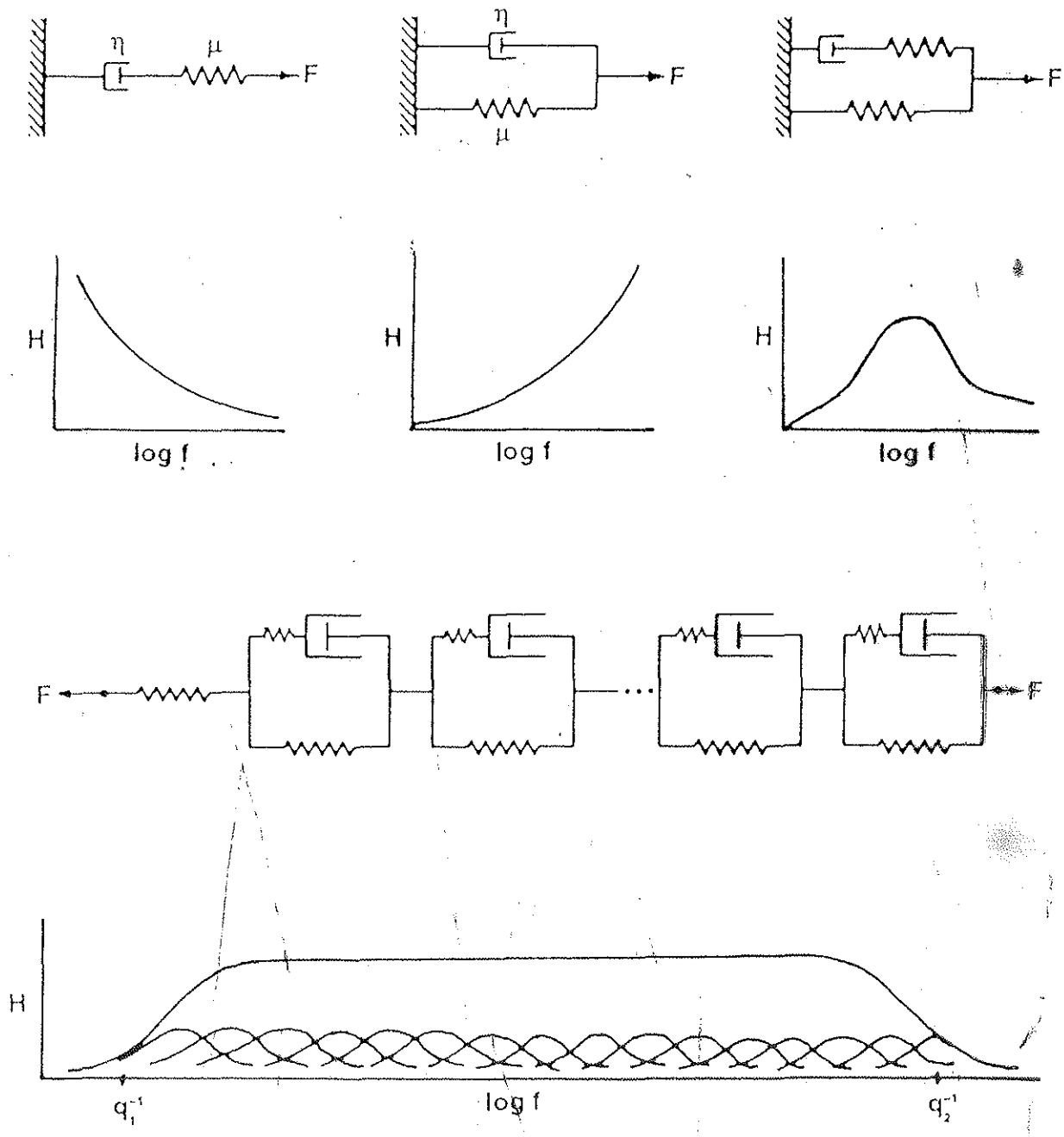


Figure 7.6:5 A summary of the principal features of viscoelastic models. Three standard viscoelastic models, namely, the Maxwell, Volgt and Kelvin models are shown in the top row, and a mathematical model of the viscoelasticity of biological soft tissues is shown in the third row. Figures in the second row show the relationships between the hysteresis (H) and the logarithm of frequency (ln f) of the three models immediately above. The figure in the bottom row shows the general hystersis-log frequency relationship of most living soft tissues, corresponding to the model shown in the third row. For the soft tissue model the springs are nonlinear, and each Kelvin unit contributes a small bell-shaped curve, the sum of which is flat over a wide range of frequencies.

Further models to describe the nonlinear stress strain behavior of bio tissues

A normal distribution of reference lengths for the collagen fibers is assumed. The collagen fibers are for simplicity seen as elastic elements. This would result in

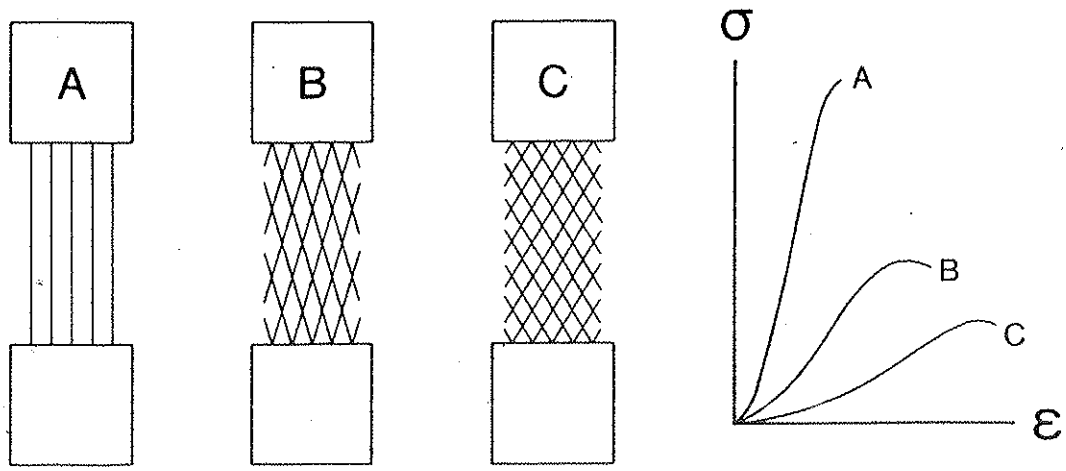
- an increasing number of fibers engaged when strain increases
- a nonlinearly increasing stress as the tissue is strained

$$\mathrm{d}F(\tau) = G(0) \int_{l_i = l_0}^{l_i = l(\tau)} \mathrm{d}\frac{l(\tau) - l_i}{l_i} \times \frac{N}{\sqrt{2\pi}\, s} \mathrm{e}^{-(\mu - l_i)^2/2s^2}\, \mathrm{d}l_i.$$

An improvement of the model is to assume the collagen fibers themselves are viscoelastic.

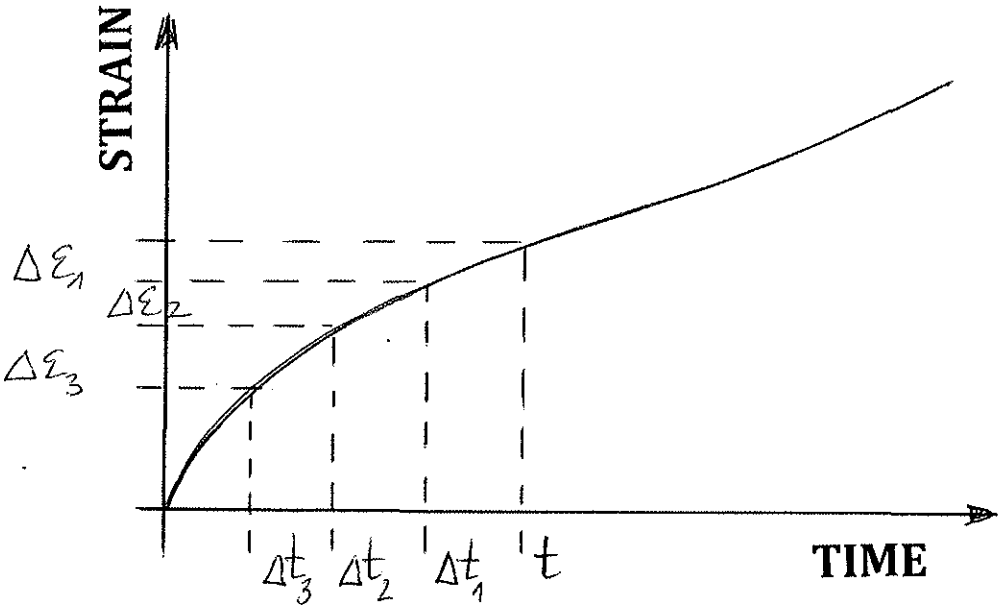
As summary for the description of the biomechanical behavior of soft connective tissues we have

- elastic behavior modelled with successively engaged spring elements (distribution of reference lengths of collagen fibers)
- viscous contribution given by a set of 3 parameter solids with different time constants for relaxation and creep
- for flat tissues there will be added a distribution of angles of the collagen fibers to the axis of load application



Desccription of the stress relaxation with hereditary integrals

A strain controlled loading history is assumed



The function strain versus time is devided into several steps. The stress taken at time instant t will depend on all the time instants $t' < t$ via the relaxation function G(t):

$$\sigma(t) = G(t) + \Delta\varepsilon_1 G(t-\Delta t_1) + \Delta\varepsilon_2 G(t-\Delta t_1-\Delta t_2) + \Delta\varepsilon_3 G(t-\Delta t_1-\Delta t_2-\Delta t_3) + ..$$

In the limit $\Delta t_i \to 0$ the hereditary integral results

$$\sigma(t) = \int dt' G(t-t') * d\varepsilon/dt\,(t') + \sigma(t=0)$$

The initial value of stress $\sigma(t=0)$ should be a state of equilibrium, i.e. a pure elastic value after the relaxation of previous steps of loading is completed.

Some simple cases for the relaxation function

$$G(t) = E + \eta \exp(-t/\tau)$$

Without viscous component->Hooke element

$$\sigma(t) = \int dt'\, E\, d\varepsilon/dt(t') = E*\varepsilon(t)$$

and without elastic component at constant strain rate

$$d\varepsilon/dt = r = const$$

$$\sigma(t) = \int dt'\, \eta \exp[(t'-t)/\tau]\, d\varepsilon/dt(t') =$$

$$\eta r\tau*[1-\exp(-t/\tau]$$

In the limit of large t the reponse of a Newton element results, i.e. $s=\eta r\tau=const$ at const strain rate.

Some examples of stress relaxation tests with different bio materials

Stress relaxation tests carried out at successively increased strain levels

Specimens of aorta, skin, and tendon were tested

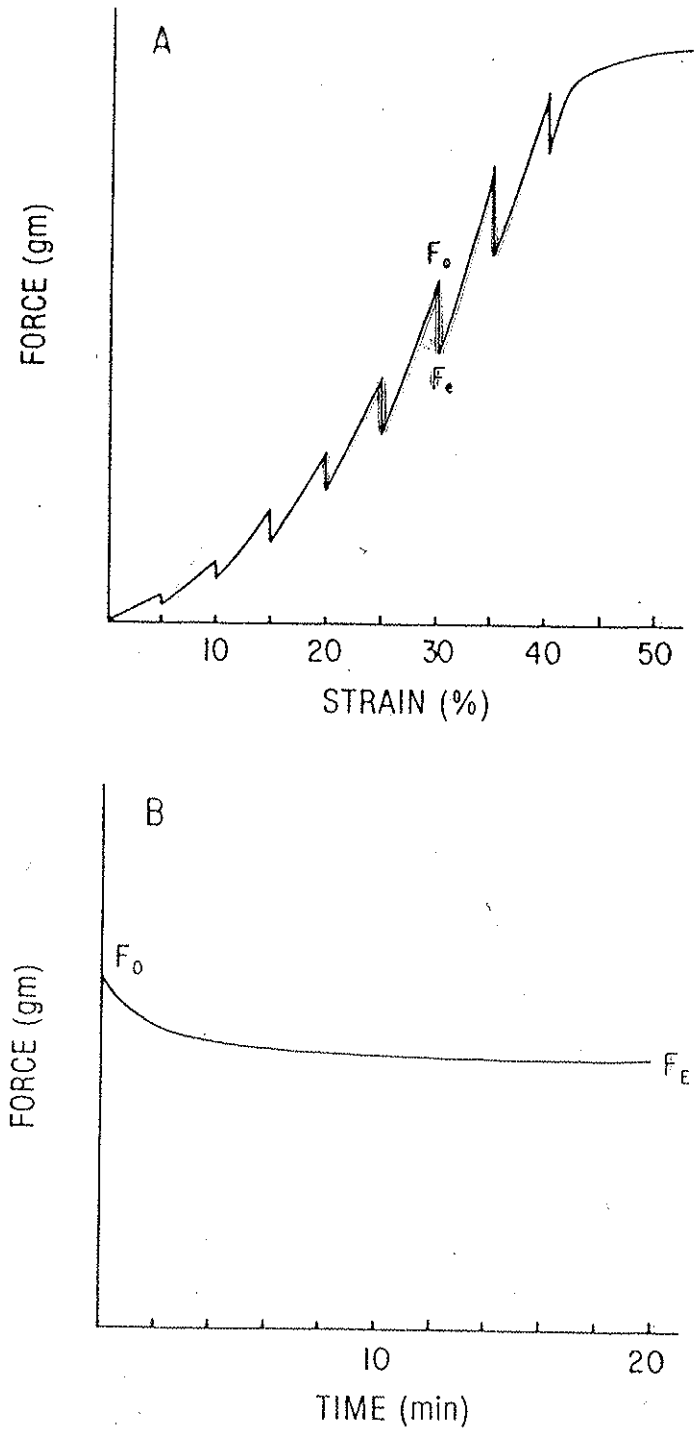
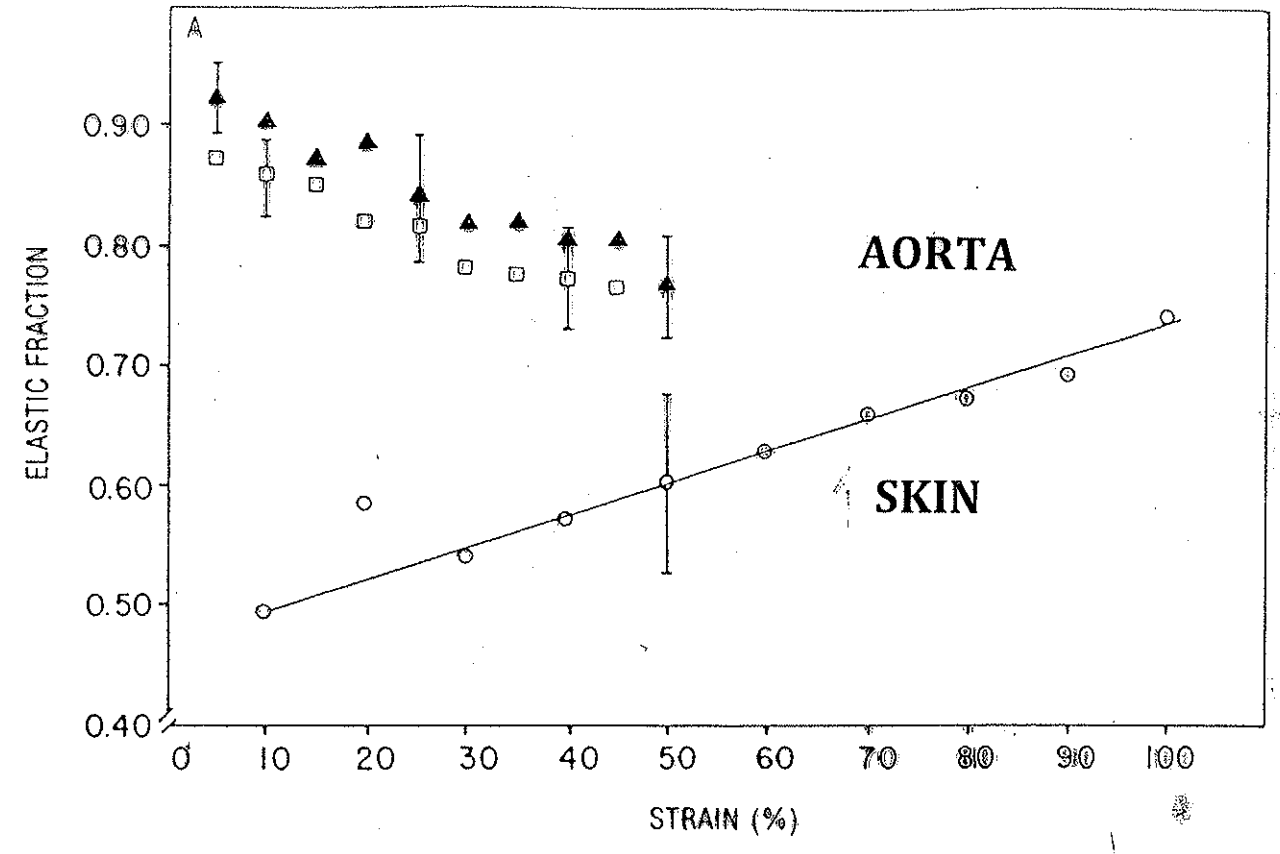


FIGURE 1 (A) Typical loading program used to determine elastic and viscous components. Specimen was strained at 5% increments and the force was allowed to relax at constant strain. (B) Typical time course of relaxation at a set strain. Elastic Fraction was defined as Fe/Fo.



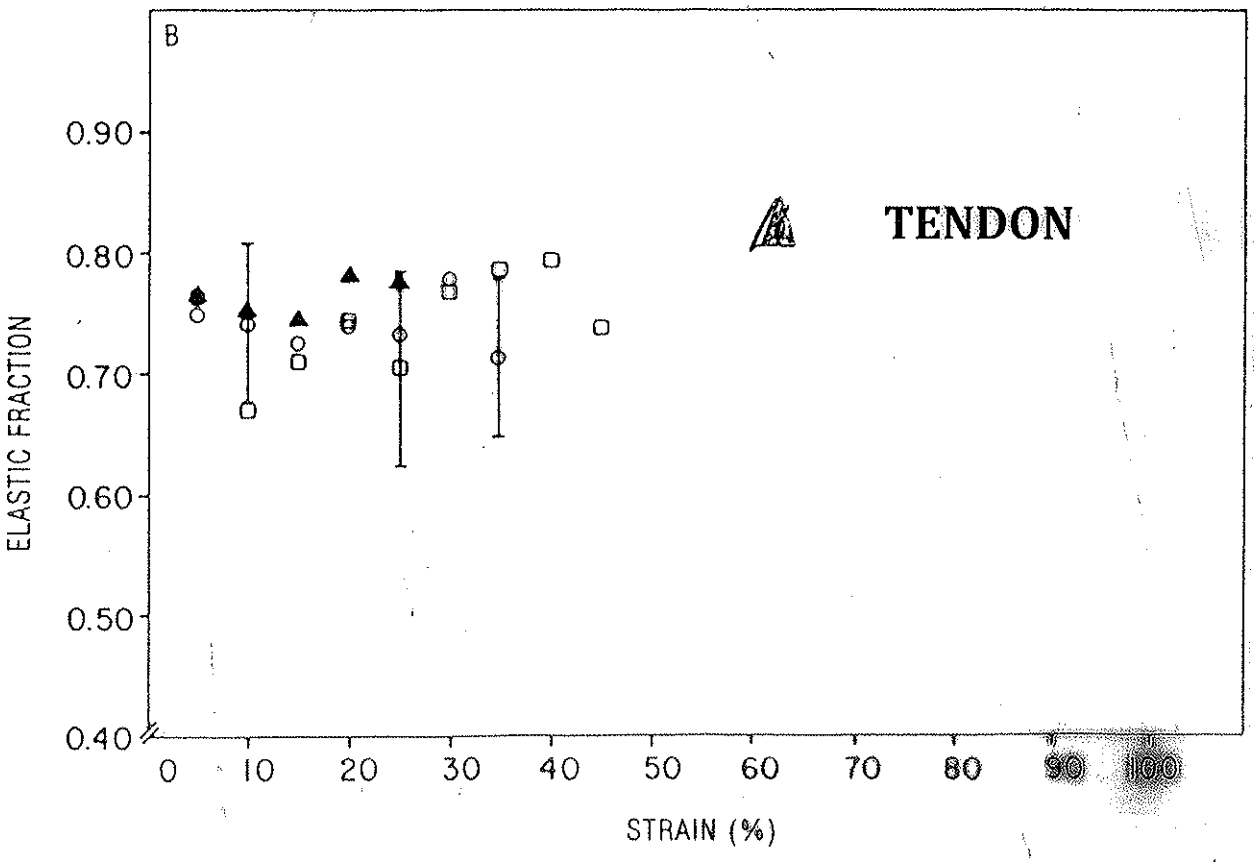
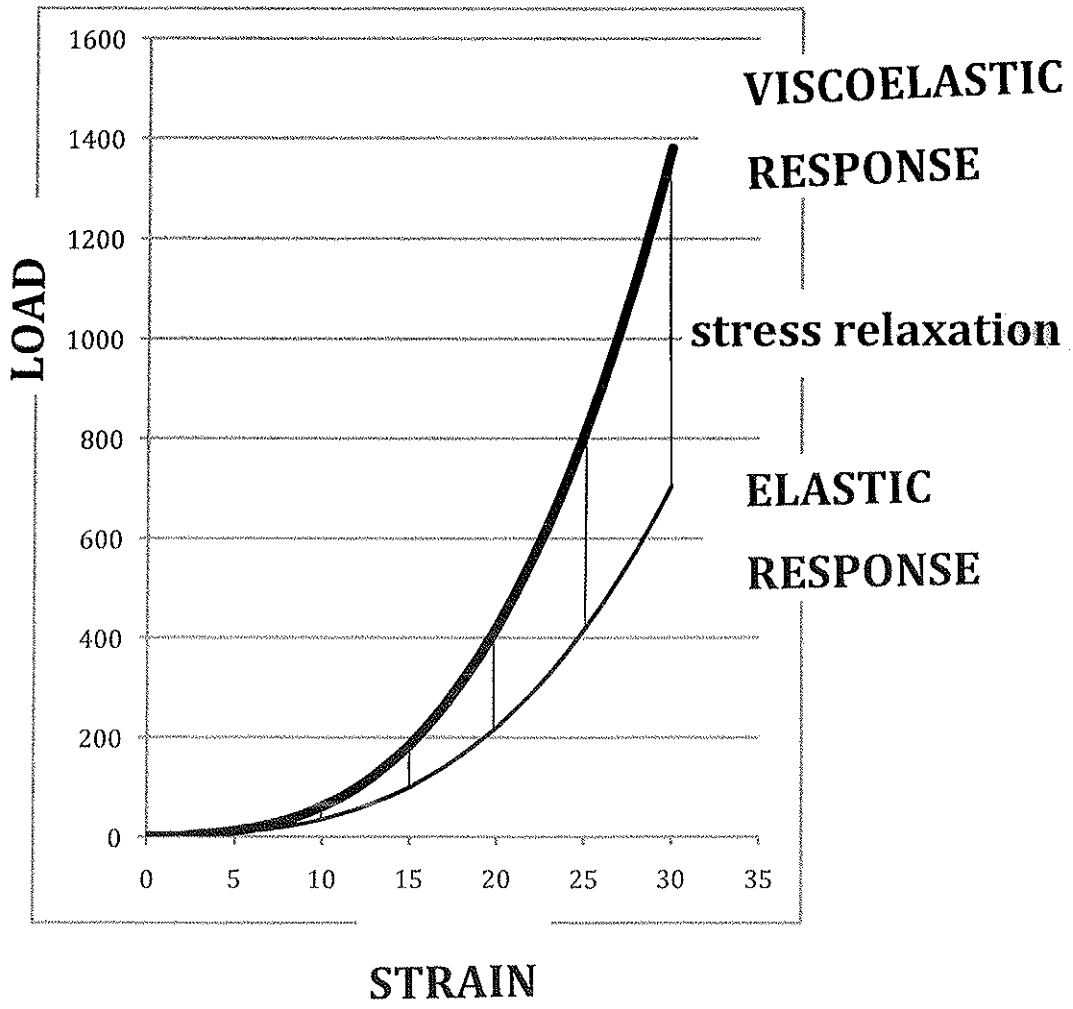


FIGURE 3 Plot of ratio of equilibrium force to initial force Fe/Fo, termed elastic fraction, vs strain in % for (A) transverse aorta (▲), longitudinal aorta (□), and skin (○). (B) pericardium (□), psoas major tendon (▲), and dura mater (○). Mean values are plotted with typical standard deviations indicated. The number of observations was ten for most points plotted.

The value of stress at the begining of the relaxation phase represents a sum of the elastic and viscous component of stress; this component will diminish and the elastic component will remain after relaxation is completed. The final values of stress plotted versus strain will give nearly the elastic stress strain graph. This corresponds to an experiment with the limit strain rate -> zero.



The determination of stress relaxation time constants and the viscous (or vice versa the elastic) fraction may help to characterize the biomechanical status of tissue samples. One dont need to determine the cross sectional area of samples for computing stresses. Wether time constants need to be normalized nor a quotient of forces. This quotient will, of course, be the same for the corresponding quotient of stresses.

Elastic fraction =

$$F_{elastic}/F_{viscoelastic} = \sigma_{elastic}/\sigma_{viscoelastic}$$

The viscous fraction is then 100% minus the elastic fraction.

Results of a study of Dupuytren´s disease (apparently normal tissue and contracture bands) compared to normal segments of palmar aponeurosis (from patients with carpal tunnel syndrome)

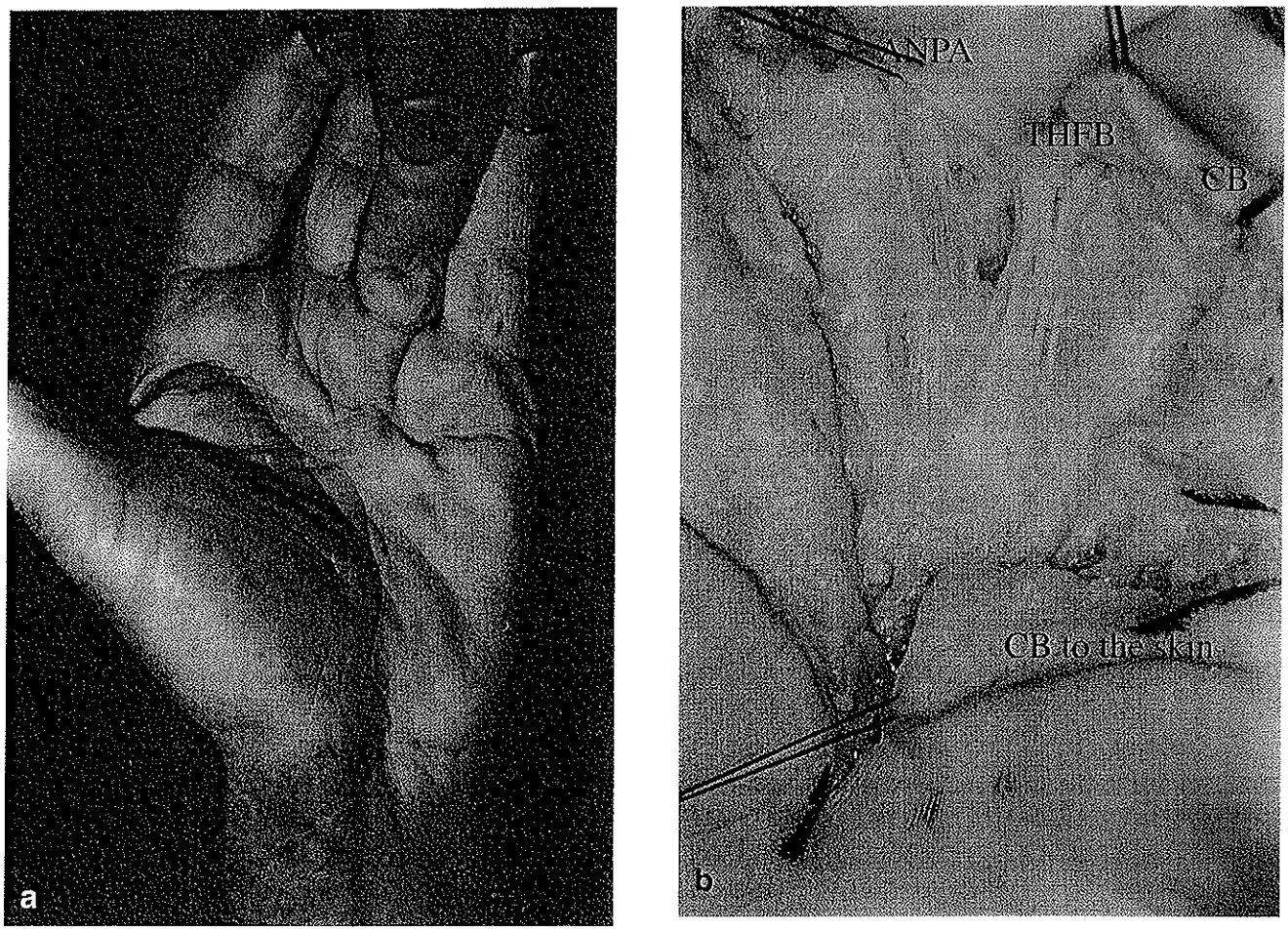


Fig 1 (a) A patient with marked Dupuytren's contracture of the ring finger. Another contracture band merges into the skin causing a 'funnel-like' retraction. The index and middle fingers are not contracted. (b) The palmar aponeurosis of this hand is exposed by a Y-shaped incision. One can easily distinguish the normal looking, transparent fibre bundles to the index finger (ANPA), the thickened, opaque fibre bundles to the middle finger (THFB), the contracture bands (CB) to the ring finger and the contracture band to the skin over the little finger ray (CB to the skin).

Normal tendons from the palmaris longus muscle (n = 14) and normal palmar aponeuroses (n = 23) were obtained during surgery of carpal tunnel syndrome (18 patients, 2 male and 16 female; age range: 41–78 years). Apparently normal palmar aponeuroses (n = 9) and contracture bands (n = 20) were obtained at surgery of 16 male patients (age range: 42–70 years) with Dupuytren's disease. Generally, specimens of the apparently normal areas were excised from the 2nd or

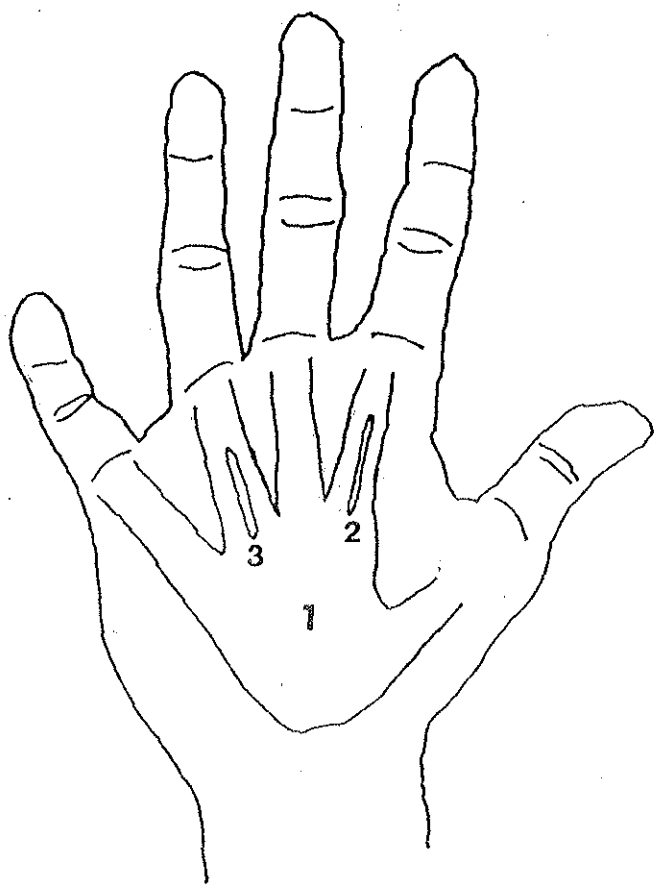
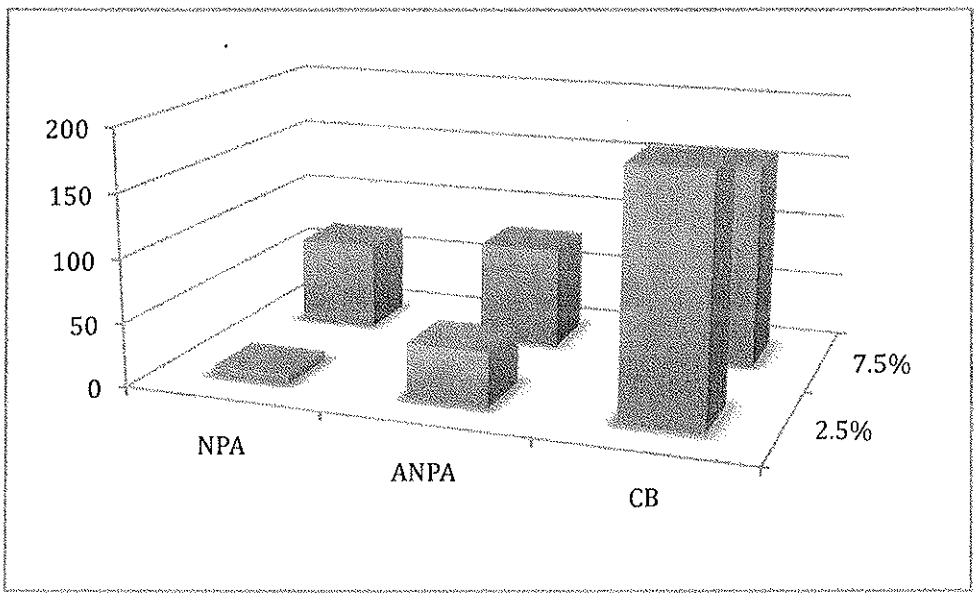
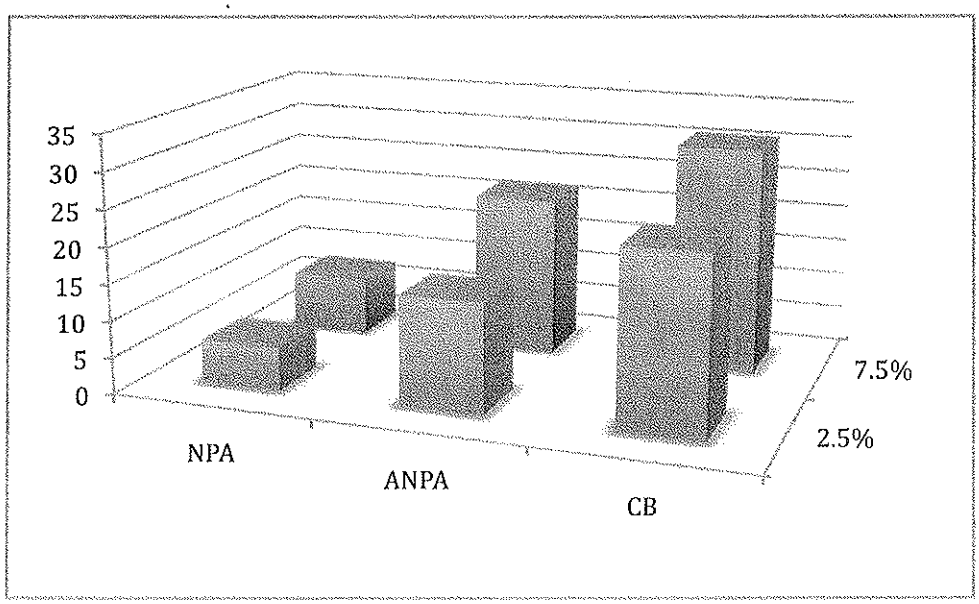


Figure 1. Palmar aponeurosis **1**, including **2**, a specimen of an apparently normal tissue from the 2nd finger, and **3**, a sample of a contracture band from the 4th finger.

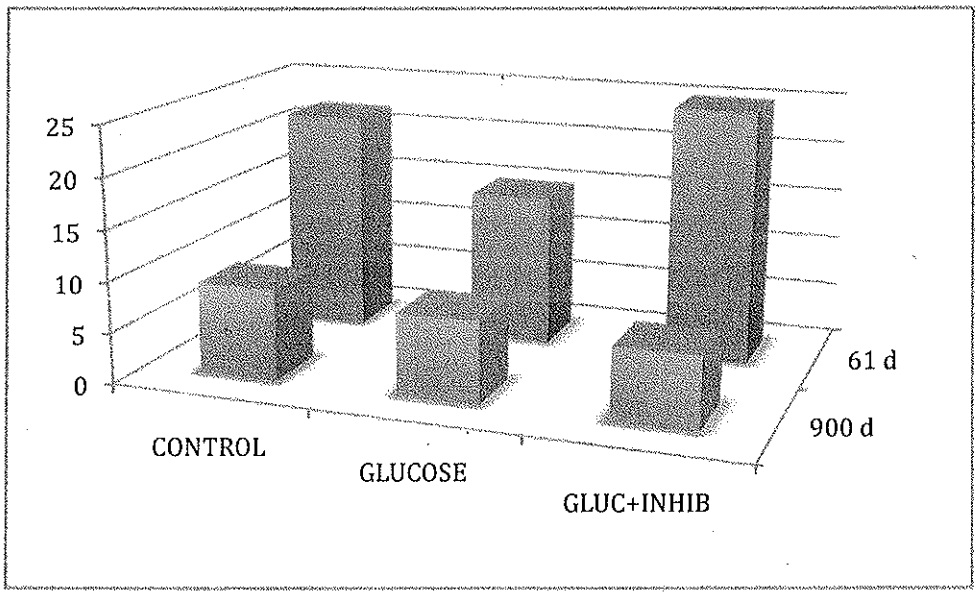
The **average time constant for stress relaxation** with specimens of palmar aponeurosis was significantly higher for the contracture bands



This was also observed for the **viscous fraction**, i.e. the viscous stress component related to both the elastic and viscous component of stress (initial value at the begining of the relaxation phase)



As to a study of non enzymatic glycation with rat tail tendons (incubated in glucose in presence or absence of an inhibitor we found for the **viscous fraction**



The viscous fraction of samples from old animals is not affected by glucose and also there is nothing to inhibit. But there is a marked decrease of the viscous fraction by glucose for the young animals, this decrease is inhibited.