248) $\int_1^\infty \ln\left(1+\frac{1}{x}\right) \mathrm{d}x$

$v = x$

Partielle Integr. $u = \ln(1+\frac{1}{x})$ $v' = 1$

$u' = \frac{1}{\frac{1}{x}+1} \cdot (\frac{1}{x}+1)' = \frac{-x^{-2}}{\frac{1}{x}+1}$

$\Rightarrow x \cdot \ln(1+\frac{1}{x}) - \int - \frac{1}{(\frac{1}{x}+1)x^2} \cdot x$

$= x\ln(1+\frac{1}{x}) + \ln(x+1) \Big|_1^\infty$

$= \lim_{n\to\infty} \underbrace{n \cdot \ln(1+\frac{1}{n})}_{\xrightarrow{1} \text{ De L'Hospital}} + \underbrace{\ln(n+1)}_{\to\infty} \overbrace{- \ln(2) - \ln(2)}^{\text{const}}$

$= \int_1^\infty \ln(1+\frac{1}{x}) dx = \underline{\underline{\infty}}$

$\frac{0}{0}$

$\lim = \frac{\ln(1+\frac{1}{n})}{n^{-1}} = \lim \frac{-\frac{1}{n(n+1)}}{\frac{-1}{n^2}}$

$= \lim \frac{x}{x+1} \overset{\frac{\infty}{\infty}}{=} \lim \frac{1}{1} = 1$