Put your student ID card here

B1	B2	B3	B4	B5	B6	$\sum_{\mathrm{B}i}$

Name	
Student ID	

184.730 VU Knowledge Based Systems

Exercise Test – 10.06.2021

- The test covers the **exercise sheets** and **background questions**.
- Please **read the questions carefully** and give **precise answers**.
  Note that there might be **differences compared to the exercise sheets**.
- Write legibly and with a pen or a fountain-pen *(no pencil allowed)!*
- You need 14 of 27 points to pass the exercise part.

Good luck!

Exercise 1 (4 pts.):

(i) Convert the following first-order formula to negation normal form (NNF):

$$\big(\exists x(\neg P(x)) \leftrightarrow \exists y(q(y))\big) \lor \neg\big(\exists u(P(u) \land \neg Q(u))\big)$$

(ii) Given formula $\varphi : \forall x \exists y \big((\neg P(x,x) \land \neg P(x,y)) \lor (P(x,y) \land P(y,y)) \big)$. The NNF of the negation of φ is the following:

$$\exists x \forall y \big((P(x,x) \lor P(x,y)) \land (\neg P(x,y) \lor \neg P(y,y)) \big)$$

Use TC1 to show that φ is valid. Clarify the logical relation between the formulas in each transformation step.

Exercise 2 (5 pts.):

(a) Give a formal definition of an interpretation structure in first-order logic in general. Explain the elements it consists of.

(b) Give one interpretation structure under which all the following sentences become true.

$$\exists x \forall y \exists z \big(A(x) \to (\neg A(y) \lor B(z)) \to C(x,y,z)\big),$$
$$\exists x \forall y (A(x) \to \neg A(y)),$$
$$\exists x A(x) \land \forall x B(x).$$

Exercise 3 (5 pts.):

(a) Use the tableau procedure for $\mathcal{ALC}$ to check whether the concept

$$C_0 := \exists R.\big((C \sqcup D) \sqcap \exists S.(A \sqcup B)\big) \sqcap \forall R.\big(\forall S.(A \sqcup \exists R.C) \sqcap \forall S.(\neg C \sqcap \neg C)\big)$$

is satisfiable.

(b) Consider the knowledge base $\mathcal{K} := \langle \{B \sqsubseteq \exists R.(\exists R.B)\}, \{B(a)\}\rangle$. Apply the $\mathcal{ALC}$ tableau algorithm for KB-satisfiability to $\mathcal{K}$ and give an interpretation induced by the resulting completion graph.

Exercise 4 (4 pts.):

(a) Provide the definition of an admissible model M for a JTMS $\mathcal{T}$. Properly define each criterion.

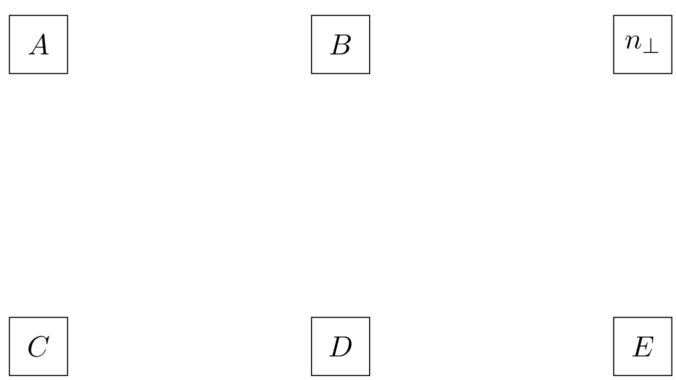
(b) Consider the JTMS $\mathcal{T} := (\{A, B, C, D, E, n_\perp\}, \mathcal{J})$, where $N_\perp = \{n_\perp\}$ and $\mathcal{J}$ contains

$$\begin{aligned}
J_1 &:= \langle B \mid \emptyset \rightarrow A \rangle, \\
J_2 &:= \langle A \mid \emptyset \rightarrow B \rangle, \\
J_3 &:= \langle \emptyset \mid C, E \rightarrow D \rangle, \\
J_4 &:= \langle \emptyset \mid D, E \rightarrow C \rangle, \\
J_5 &:= \langle D \mid C \rightarrow B \rangle. \\
J_6 &:= \langle \emptyset \mid A, B \rightarrow E \rangle. \\
J_7 &:= \langle E \mid D \rightarrow n_\perp \rangle.
\end{aligned}$$

Represent $\mathcal{T}$ graphically (use the layout below).

Check whether the following sets are admissible models of $\mathcal{T}$. Explain for each set why, or why not, this is an admissible model.

(i) $M_1 := \{A, B, C\}$
(ii) $M_2 := \{A, B, D\}$
(iii) $M_3 := \{E, n_\perp\}$



Exercise 5 (4 pts):

(a) How many stable models does the following normal logic program $\mathcal{P}$ have? Justify your answer without evaluating the program! (The signature of this program contains only the variable symbol X, the constant symbols a and b, and the predicates P, Q, and S.)

$$\begin{aligned}\mathcal{P} := \{&P(a) \leftarrow Q(b), \textit{not } Q(a).,\\ &Q(a) \leftarrow P(b).,\\ &S(X) \leftarrow Q(X), \textit{not } P(X).\}.\end{aligned}$$

(b) For the logic program $\mathcal{P}$ above, write down the Herbrand universe and the Herbrand base $\mathcal{H}(\mathcal{P})$.

Exercise 6 (5 pts.):

(a) Consider the following normal logic program:

$$\begin{aligned}\mathcal{P}_1 := \{&T(a).,\ T(b).,\\ &P(X) \leftarrow T(X), not\ Q(X).,\\ &Q(X) \leftarrow P(X), Q(Y).,\\ &S(X) \leftarrow T(X), not\ P(X).,\\ &\leftarrow S(X).\}.\end{aligned}$$

First, compute the grounding $grnd(\mathcal{P}_1)$ of the program $\mathcal{P}_1$.

Second, which of the following sets are stable models of $\mathcal{P}_1$?

(i) $S_1 = \{T(a), P(a), P(b)\}$,

(ii) $S_2 = \{T(a), T(b), P(a), P(b)\}$,

(iii) $S_3 = \{T(a), T(b), S(a), P(a), Q(b)\}$.

Explain your answers. If a set is a stable model, also compute its Gelfond-Lifschitz reduct.

(b) Consider a normal logic program $\mathcal{P}_2$ with the stable models $\{a, c, f\}$, $\{a, e, f\}$ and $\{a, b, c\}$. Which of the following inferences hold?

(i) $\mathcal{P}_2 \models^{stab} a$,

(ii) $\mathcal{P}_2 \models^{stab} c$,

(iii) $\mathcal{P}_2 \not\models^{stab} b$,

(iv) $\mathcal{P}_2 \models^{stab} \neg d$.