16 c) $\quad g(t) = e^t \quad , 0 \le t < T$

$$h(t) = \begin{cases} g(t) & 0 \le t < T \\ g(-t) & -T < t < 0 \end{cases} \qquad h(t) \Rightarrow 2T \text{ periodisch}$$

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_n \cdot \cos(n t) + b_n \cdot \sin(n t)\right)$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cdot \cos(n t)\, dt \qquad b_n = 0$$

$$a_n = \frac{1}{T} \int_0^T e^t \cos(nt)\, dt + \frac{1}{T} \int_{-T}^0 e^{-t} \cdot \cos(nt)$$

$$\int \underbrace{e^t}_{u} \cdot \underbrace{\cos\left(\frac{n\pi}{T} t\right)}_{v'} = \frac{T \sin\left(\frac{n\pi}{T} t\right)}{n\pi} e^t - \int \frac{\sin \frac{n\pi t}{T}}{n\pi} \cdot T \cdot e^t \qquad u = e^t \quad v = \frac{\sin\left(\frac{\pi n t}{T}\right)}{n\pi} \cdot T \qquad u' = e^t \quad v' = \cos\left(\frac{n\pi t}{T}\right)$$

$$\int e^t \cdot \cos\left(\frac{n\pi t}{T}\right) = \frac{T \cdot \sin\left(\frac{n\pi t}{T}\right)}{n\pi} \cdot e^t - \left(\frac{-T^2 \cdot \cos\left(\frac{n\pi t}{T}\right) \cdot e^t}{n^2\pi^2} - \right. \qquad u_1 = e^t \quad v_1 = -\frac{T^2 \cdot \cos\left(\frac{n\pi t}{T}\right)}{n^2\pi^2} \qquad u_1' = e^t \quad v_1' = \frac{\sin\left(\frac{\pi n t}{T}\right)}{n\pi} T$$

$$- \left(\frac{+T^2}{n^2\pi^2}\right) \cdot \int \cos\left(\frac{n\pi t}{T}\right) e^t \quad \Big| + \left(\frac{-T^2}{n^2\pi^2}\right) \cdot \int \cos\left(\frac{n\pi t}{T}\right) e^t$$

$$\left(\int e^t \cos\left(\frac{n\pi t}{T}\right)\right)\left(1 + \frac{T^2}{n^2\pi^2}\right) = \frac{T \cdot \sin\left(\frac{n\pi t}{T}\right) \cdot e^t}{n\pi} + \frac{T^2 \cos\left(\frac{n\pi t}{T}\right) e^t}{n^2\pi^2} \quad \Big| : \left(\frac{n^2\pi^2 + T^2}{n^2\pi^2}\right)$$

$$\int e^t \cos\left(\frac{n\pi t}{T}\right) dt = \frac{\frac{T \cdot \sin\left(\frac{n\pi t}{T}\right) \cdot e^t}{n\pi}}{\frac{n^2\pi^2 + T^2}{n^2\pi^2}} + \frac{\frac{T^2 \cdot \cos\left(\frac{n\pi t}{T}\right) \cdot e^t}{n^2\pi^2}}{\frac{n^2\pi^2 + T^2}{n^2\pi^2}} =$$

$$= \frac{n^{\cancel{2}} \pi^{\cancel{2}} \cdot T \cdot \sin\left(\frac{n\pi t}{T}\right) e^t}{\cancel{n\pi}\,(n^2\pi^2 + T^2)} + \frac{\cancel{n^2\pi^2} \cdot T^2 \cdot \cos\left(\frac{n\pi t}{T}\right) \cdot e^t}{\cancel{n^2\pi^2}\,(n^2\pi^2 + T^2)}$$

$$\int_0^T e^t \cdot \cos\left(\frac{n\pi t}{T}\right) dt = \frac{n\pi \cdot T \cdot \overbrace{\sin(n\pi)}^{0} \cdot e^T}{n^2\pi^2 + T^2} + \frac{T^2 \cdot \overbrace{\cos n\pi}^{(-1)^n} e^T}{n^2\pi^2 + T^2} - 0 - \frac{T^2}{n^2\pi^2 + T^2}$$

$$\int e^{-t} \cdot \cos\left(\frac{n\pi t}{T}\right) dt = \frac{T \cdot e^{-t}\left(\pi n \cdot \sin\left(\frac{n\pi t}{T}\right) - T \cos\left(\frac{\pi n t}{T}\right)\right)}{n^2\pi^2 + T^2}$$

$$\int_{-T}^{0} e^t \cos\left(\frac{n\pi t}{T}\right) dt = \frac{-T \cdot e^T \left(\overbrace{\pi n \cdot \sin(-n\pi)}^{0} - T \cdot (-1)^n\right) - T^2}{n^2\pi^2 + T^2} = \frac{e^T \cdot T^2 \cdot (-1)^n - T^2}{n^2\pi^2 + T^2}$$

$$\int_0^T e^t \cdot \cos\left(\frac{n\pi t}{T}\right) dt + \int_{-T}^{0} e^t \cdot \cos\left(\frac{n\pi t}{T}\right) dt = \frac{e^T \cdot T^2 \cdot (-1)^n - T^2 + e^T \cdot T^2 \cdot (-1)^n - T^2}{n^2\pi^2 + T^2} =$$

$$= \frac{2\left(e^T \cdot T^2 \cdot (-1)^n - T^2\right)}{n^2\pi^2 + T^2} = \frac{2T^2\left(e^T (-1)^n - 1\right)}{\pi^2 n^2 + T^2} = K$$

$$a_n = \frac{1}{T} \cdot K = \frac{2T\left(e^T \cdot (-1)^n - 1\right)}{n^2\pi^2 + T^2}$$

$$a_0 = \frac{2(e^T - 1)}{T}$$

$$S_{f(t)} = \frac{e^T - 1}{T} + \sum_{n=1}^{\infty} \frac{2T\left(e^T \cdot (-1)^n - 1\right)}{n^2\pi^2 + T^2}$$