Technische Universität Wien
Fakultät für Informatik
Assist. Prof. Florian Zuleger
Assist. Prof. Georg Weissenbacher
Yulia Demyanova, Mag.
Moritz Sinn, M.Sc.; Ivan Radiček, M.Sc.

SS 2015
Exercise 3
Tuesday, 23 April 2015

Exercises on Semantics of Programming Languages

Solutions are to be handed in at the lecture on May 11th. Later submissions will not be accepted.

<u>Exercise 1</u> Weakest Precondition **(5 Points)**

Use the rules of First Order Logic calculus and the definition of *wlp* to prove the following claim:

$$wlp(C,P) \wedge wlp(C,Q) \quad \equiv \quad wlp(C, P \wedge Q)$$

<u>Exercise 2</u> Completeness of Hoare Logic **(10 Points)**

(a) The proof of relative completeness (in the sense of Cook) of Hoare Logic presented in the lecture does not address the conditional rule:

$$\frac{\{B \wedge P\}\, C_1\, \{Q\} \qquad \{\neg B \wedge P\}\, C_2\, \{Q\}}{\{P\}\ \texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2\, \{Q\}}$$

Complete the proof by showing that

$$\vDash_{\mathcal{T}} \{P\}\ \texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2\, \{Q\} \quad \Rightarrow \quad \vdash_{\mathcal{T},\mathcal{H}} \{P\}\ \texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2\, \{Q\}\,.$$

(b) Let

$$\texttt{do}\ C\ \texttt{while}\ B \quad \stackrel{\text{def}}{=} \quad C;\ \texttt{while}\ B\ \texttt{do}\ C\,.$$

The corresponding Hoare rule is

$$\frac{\{P\}\, C\, \{P\}}{\{P\}\ \texttt{do}\ C\ \texttt{while}\ B\, \{P \wedge \neg B\}}\ \textsc{Do-While}$$

Show that

$$\vDash_{\mathcal{T}} \{P\}\ \texttt{do}\ C\ \texttt{while}\ B\, \{P \wedge \neg B\} \quad \Rightarrow \quad \vdash_{\mathcal{T},\mathcal{H}} \{P\}\ \texttt{do}\ C\ \texttt{while}\ B\, \{P \wedge \neg B\}\,.$$

Exercise 3 Weakest liberal pre-condition **(5 Points)**

Recall from the lecture that

$$s \vDash wlp(C, Q) \Leftrightarrow s \in \{s \,|\, \langle C, s\rangle \Downarrow s' \Rightarrow s' \vDash Q\}\,.$$

Prove the following equivalence (which shows that the *transition relation* $T_C \subseteq S \times S$ of a statement C is determined by the weakest liberal pre-condition):

$$\neg wlp(C, \bigvee_{x \in \mathsf{Var}} x \neq x') \qquad \equiv \qquad (\langle C, s\rangle \Downarrow s') \wedge \bigwedge_{x \in \mathsf{Var}} x = s(x) \wedge x' = s'(x)$$