

This is the third homework assignment. The problems are to be presented on exercise session on **November 8, 2022**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions. The problems should be ticked by **20:00 on November 7, 2022**.

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(1) **Basketball free throws**

Two professional basketball players, Tom and John, each throw ten free throws with a basketball. Tom makes 80% of the free throws he tries, while John makes 85% of the free throws he tries. Player who achieves the highest score wins the game. It is assumed that the two players do not influence each other when throwing. What is the probability that Tom will win the game?

*Hint:* R-function `dbinom()` can be used in computations.

(2) **Hurricane insurance**

An insurance company needs to assess the risk associated with providing hurricane insurance. During 22 years from 1990 through 2011, Florida was hit by 27 major hurricanes (level 3 and above). The insurance company assumed Poisson distribution for modeling number of hurricanes.

- (a) If hurricanes are independent and the mean has not changed, what is the probability of having a year in Florida with each of the following?
  - (1) No hits.
  - (2) Exactly one hit.
  - (3) More than two hits.
- (b) Use R to estimate the number of hurricane hits that will occur with the probability 97.5%.

(3) **Tire company**

A tire manufacturer believes that the tread life of its snow tires can be distributed by a Normal model with a mean of 32 000 miles and a standard deviation of 2 500 miles.

Use the values of the cumulative distribution function  $\Phi(z) = P(Z \leq z)$  of the standard normal distribution  $Z$  given in Table 1 to answer the following questions.

- (a) Approximately what fraction of these tires can be expected to last between 30 000 and 35 000 miles?
- (b) Calculate the interquartile range of this distribution.  
Recall, the interquartile range is the difference between the upper  $x_{0.75}$  and the lower quartile  $x_{0.25}$ , i.e.,  $\text{IQR} = x_{0.75} - x_{0.25}$ .
- (c) In a marketing strategy, a local tire dealer wants to offer a refund to any customer whose tires fail to last a certain number of miles. However, the dealer does not want to take too big risk. If the dealer is willing to give refunds to no more than one of every 15 customers, for what mileage can he guarantee these tires to last?

(4) **Drug company**

Manufacturing and selling drugs that claim to reduce an individual's cholesterol level is big business. A company would like to market their drug to women if their cholesterol is in the top 15%. Assume the cholesterol levels of adult American women can be described by a Normal model with a mean of 188 mg/dL and a standard deviation of 24.

Use R to answer the following questions.

- (a) Draw and label the cumulative distribution function of this model.
- (b) Approximate the probability that cholesterol levels of adult American women are over 200 mg/dL?
- (c) Approximate the probability that cholesterol levels of adult American women are between 155 and 170 mg/dL?
- (d) Compute the interquartile range of the cholesterol levels.
- (e) Above what value are the highest 10% of the cholesterol levels of adult American women?

*Hint:* R commands `pnorm()`, `qnorm()`, and `dnorm()` are useful.

(5) **Uniform-exponential relationship**

Let  $X \sim \mathcal{U}(0, 1)$ .

- (a) Find the cumulative distribution function of  $X$ .
- (b) Show that the random variable  $Y = -\ln X$  has exponential distribution  $\exp(1)$ , where  $\ln$  denotes the natural logarithm.