Problem Set 6

Problem 6.1 Consider a discrete random variable $\mathsf{x} \in \{-1, 1\}$ with $\mathrm{P}\{X = 1\} = 1/2$ and a Gaussian distributed random variable $\mathsf{v} \sim \mathcal{N}(0, \sigma^2)$. x and v are statistically independent.

a) Calculate the joint probability density function $f_{\mathsf{x},\mathsf{z}}(x, z)$ for $\mathsf{z} = \mathsf{x}\mathsf{v}$.

b) Calculate the marginal pdf $f_{\mathsf{z}}(z)$.

c) Find out whether x and z are statistically independent and/or uncorrelated and/or orthogonal. Justify your answers.

Problem 6.2 Three iid random variables x_1, x_2 and x_3 are uniformly distributed on the interval $[0, a]$, where $a > 0$. Find and sketch the probability density function of the random variable $\mathsf{y} = \mathsf{x}_1 + \mathsf{x}_2 + \mathsf{x}_3$.

Problem 6.3 Let $\mathsf{m} \in \mathcal{M} \subset \mathbb{N}$ be a positive, integer-valued random variable, and let $\mathsf{x}_1, \mathsf{x}_2, \ldots$ be iid random variables. Furthermore, assume that m is independent of x_i for every i. A new random variable y is given by the *random* sum

$$\mathsf{y} = \sum_{i=1}^{\mathsf{m}} \mathsf{x}_i.$$

a) Find the mean μ_{y}.

b) Find the variance σ_{y}^2.

Problem 6.4 Let x_1, x_2 and x_3 be three iid random variables, each with probability density function $f_{\mathsf{x}_i}(x_i) = e^{-x_i}u(x_i)$, where $u(\cdot)$ is the unit-step function $(i = 1, 2, 3)$. Let y_1, y_2 and y_3 be random variables given by

$$\begin{aligned} \mathsf{y}_1 &= \frac{\mathsf{x}_1}{\mathsf{x}_1 + \mathsf{x}_2} \\ \mathsf{y}_2 &= \frac{\mathsf{x}_1 + \mathsf{x}_2}{\mathsf{x}_1 + \mathsf{x}_2 + \mathsf{x}_3} \\ \mathsf{y}_3 &= \mathsf{x}_1 + \mathsf{x}_2 + \mathsf{x}_3. \end{aligned}$$

a) Find the joint density $f_{\mathsf{y}_1,\mathsf{y}_2,\mathsf{y}_3}(y_1, y_2, y_3)$.

b) Are y_1, y_2 and y_3 statistically independent? Justify your answer.