244) $\int_{-1}^{1} x^2\sqrt{1-x^2}\,\mathrm{d}x$

Subst $x = \sin(u)$

$dx = \cos(u)\,du$

I $1-\sin^2(u) = \cos^2(u)$

II

$\sin^2(u) = \frac{1}{2}(1-\cos(2u))$

$\cos^2(u) = \frac{1}{2}(\cos(2u)+1)$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(u)\cdot\sqrt{1-\sin^2(u)}\cdot\cos(u)\,du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(u)\cdot\cos^2(u)\,du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\cdot(1-\cos(2u))\cdot\frac{1}{2}(\cos(2u)+1)\,du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(2u)+1-\cos^2(2u)-\cos(2u)}{4}\,du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1-\left[\frac{1}{2}(\cos(4u)+1)\right]}{4}\,du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1-\cos(4u)}{8}\,du$$

$$= \frac{1}{8}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\,dx - \frac{1}{8}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(4u)$$

$$= \left[\frac{u}{8} - \frac{\sin(4u)}{32}\right]\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{16} + \frac{\pi}{16} = \frac{\pi}{8}$$

$\int \cos(4u)$ $v = 4u$

$du = \frac{1}{4}\,dv$

$= \frac{1}{4}\int \cos(v)\,dv$

$= \frac{\sin(v)}{4} = \frac{\sin(4u)}{4}$