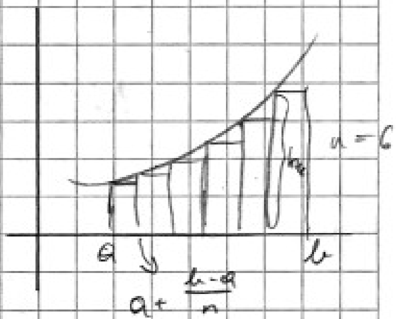
$$\left[\ \cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}\ \right]$$

$$= \arcsin(x)\,\frac{x^2}{2} + \frac{x\sqrt{1-x^2} - \arcsin(x)}{4} + C$$

25)
$$\lim_{n\to\infty} \frac{1}{n^2}\sum_{k=1}^{n}\sqrt{k(n-k)} \qquad R.\,25$$



$$A = \sum_{k=1}^{n} b_k \cdot h_k$$

$$b_k = \frac{b-a}{n}$$

$$A = \frac{b-a}{n}\sum_{k=1}^{n} h_k$$

$$h_k = f\left(a + k\cdot\frac{b-a}{n}\right)$$

$$A = \frac{b-a}{n}\sum_{k=1}^{n} f\left(a + k\,\frac{b-a}{n}\right)$$

$$b-a=1 \rightarrow b=1 \quad a=0$$

$$A = \frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$$

$$f\left(\frac{k}{n}\right) = \frac{\sqrt{k(n-k)}}{n} = \sqrt{\frac{k\cancel{n}\left(1-\frac{k}{n}\right)}{n^{\cancel{2}}}} = \sqrt{\frac{k}{n}\left(1-\frac{k}{n}\right)}$$

$$x = \frac{k}{n}$$

$$f\left(\frac{k}{n}\right) = f(x) = \sqrt{x(1-x)}$$

$$A = \int_0^1 \sqrt{x(1-x)}\,dx$$

$$x = \frac{1+t}{2} \qquad dx = \frac{1}{2}\,dt$$

$$A = \frac{1}{2}\int_a^b \sqrt{\frac{t+1}{2}\left(1-\frac{t+1}{2}\right)}\,dt = \frac{1}{4}\int_a^b \sqrt{1-t^2}\,dt$$

$t = \sin u \qquad dt = \cos u \, du$

$$A = \frac{1}{4}\int_c^d \sqrt{1-\sin^2(u)}\,\cos u \, du = \frac{1}{4}\int_c^d \cos^2(u)\,du$$

<u>NR:</u> $\int \cos^2(u)\,du =$

$x = \cos u \qquad dy = \cos u \, du$

$dx = -\sin u \, du \qquad y = \sin u \, du$

$$\int \cos^2(u)\,du = \cos(u)\sin(u) + \int \sin^2(u)\,du$$

$$\sin^2(u) = 1-\cos^2 u$$

$$\int \cos^2(u)\,du = \cos(u)\sin(u) + \int (1-\cos^2 u)\,du$$

$$\int \cos^2(u)\,du = \cos(u)\sin(u) + u - \int \cos^2(u)\,du$$

$$2\int \cos^2(u)\,du = \cos(u)\sin(u) + u$$

$$\underline{\int \cos^2(u)\,du = \frac{1}{2}\left(\cos(u)\sin u + u\right)}$$

$$A = \frac{1}{8}\left(\cos(u)\sin(u) + u\right)_c^d = \frac{1}{8}\left(\sqrt{1-\sin^2(u)}\,\sin(u) + u\right)_c^d$$

$$A = \frac{1}{8}\left(\sqrt{1-t^2}\cdot t + \arcsin(t)\right)_a^b$$

$$A = \frac{1}{8}\left(\sqrt{1-(2x-1)^2}\,(2x-1) + \arcsin(2x-1)\right)_0^1$$

A_{NR}: $x = \frac{1+t}{2}$

$t = 2x-1$

$\underline{\underline{A = 0{,}3926}}$