Knowledge Based Systems, 4.0 VU, 184.730

Exercise Sheet 1

This exercise sheet serves as a **preparation for the mandatory exercise test**, which covers the exercises and related background knowledge. You do *not* need to submit solutions.

For questions regarding exercises please visit the tutoring sessions (times are announced in **TUWEL**). You can find them in Conference room Hahn (room HG 03 06), Favoritenstr. 9-11, Stiege 3, 3rd floor, Institute of Information Systems. For questions of general interest, please use the **TISS** Forum or contact us on `kbsci-2018s@kr.tuwien.ac.at`.

Exercise 1.1: To cut personnel costs, a bank hires you to write rules in CLIPS-related pseudocode (like the examples on the lecture slides) such that the RBS outputs the correct decision whether a person receives a loan. You can assume that the input facts (i.e., the facts about a person asking for a loan) contain all relevant information. Additionally, draw the RETE-network of your solution.

> Rich people are eligible for a loan. People with more than 250.000 euro employed earnings or more than 200.000 euro self-employed a year are rich.
> A person is also eligible if they own a house that is worth more than 300.000 euro.
> Poor people earn less than 15.000 Euro as an employee and less than 5.000 Euro self-employed per year. Poor people may also receive a loan if they use it to pay for a house, except if they are married and their spouse already took out a loan.
> Eligible persons only receive a loan if they do not currently have another one.

Exercise 1.2: Recap FOL Entailment: Consider the following knowledge base KB and the formula φ where x is a variable and a, b are constants.

$$KB = Q(a) \land P(b) \land (\exists x\, Q(x) \rightarrow \forall x\, (\neg Q(x) \lor \neg P(x)))$$
$$\varphi = \neg P(a)$$

Prove or refute semantically whether $KB \models \varphi$ holds.

Exercise 1.3: Interpretation structures: For the following sentences, where c is a constant and P a binary predicate:

1. Give an interpretation structure under which sentence (1) becomes false and the other two sentences become true.
2. How can these types of problems be solved using TC1? Examine what happens when searching for an interpretation (using TC1) under which sentence (1) is false and sentence (2) is true. (Ignore sentence (3) for this part of the exercise)

$$\forall x \forall y \forall z\, (R(x,y) \land R(y,z) \rightarrow R(x,z)) \tag{1}$$
$$\forall x \exists y\, R(x,y) \tag{2}$$
$$\forall x \forall y\, (R(x,c) \rightarrow R(c,y)) \tag{3}$$

Exercise 1.4: Let T be a knowledge-base and $\varphi, \psi, \chi(x)$ be formulas. Prove or refute:

1. $T \models \psi$ and $T \cup \{\psi\} \models \varphi$ implies $T \models \varphi$;
2. $T \models \forall x\, (\varphi \wedge \psi)$ implies $T \models (\forall x\, \varphi) \wedge (\forall x\, \psi)$;
3. $T \cup \{\neg\varphi\} \models \bot$ iff $T \models \varphi$;
4. if c does not occur in T, then $T \models \chi(c)$ implies $T \models \forall x\, \chi(x)$.

Exercise 1.5: As we have seen in the lecture, *labeling* is a technique used in structure-preserving normal form translation where each subformula is replaced by a label.
For atomic formulas $\varphi = a$, we let $D_\varphi = l_a \leftrightarrow a$ and for composed formulas $\varphi = \varphi_1 \circ \varphi_2$ we let $D_\varphi = l_\varphi \leftrightarrow (l_{\varphi_1} \circ l_{\varphi_2})$, where l_{φ_1} and l_{φ_2} are the labels for subformulas φ_1 and φ_2, respectively.
Prove the following: A formula φ is satisfiable iff $\bigwedge_{\psi \text{ is a subformula of } \varphi} D_\psi \wedge l_\varphi$ is satisfiable.

Exercise 1.6:

(a) Formalize the following sentences of natural language in first-order logic. We devise a knowledge base concerning mathematicians and their doctoral students. Use the following predicate symbols with their intended meanings:

$\mathit{supervises}(x, y)$	x supervises y.
$\mathit{Mathematician}(x)$	x is a mathematician.
$\mathit{Philosopher}(x)$	x is a philosopher.
$\mathit{ComputerScientist}(x)$	x is a computer scientist.
$\mathit{Logician}(x)$	x is a logician.
$\mathit{worksIn}(x, y)$	x works in y.

Furthermore use true equality as a binary relation symbol of our logical language[1] and introduce appropriate constant symbols for the persons and subjects named.

- Everyone is a mathematician, a philosopher, a logician, or a computer scientist.
- Logicians are mathematicians and, additionally, (i) philosophers or (ii) computer scientists.
- Someone is a logician if and only if he/she works in logic.
- Everybody is supervised by somebody. Furthermore, no one is supervised by him- or herself.
- People working in number theory only supervise number theorists.
- Furtwängler is supervised by Klein. Furtwängler works in number theory.
- Gödel is a logician, but no computer scientist.
- Gödel has no students.
- Furtwängler, Klein, Menger, and Gödel are all different people.
- Menger and Gödel have a common supervisor.
- Nobody supervises him- or herself.

[1]Recall that true equality is interpreted in every interpretation as the equality relation on the underlying domain.

(b) Let I be an interpretation interpreting the knowledge base you devised in (a) and let $\mathcal{U}$ be its domain. We say that $a \in \mathcal{U}$ is an *ancestor* of $b \in \mathcal{U}$ if there exists a sequence $x_1, x_2, \ldots, x_n$ ($x_k \in \mathcal{U}$ for $k = 1, \ldots, n$) such that

   (i) for $k = 1, \ldots, n-1$ we have $x_k I(\mathit{supervises}) x_{k+1}$, and

   (ii) $a = x_1$, $b = x_n$.

   Use the compactness theorem[2] to prove that there is no formula $\varphi(x, y)$ in the language of the knowledge base of (a) such that for any interpretation I with domain $\mathcal{U}$ and any $a, b \in \mathcal{U}$ we have

$$I_{\{x \leftarrow a, y \leftarrow b\}} \models \varphi(x, y) \iff a \text{ is an ancestor of } b.$$

   Hint: You may consult the literature on how to prove that *graph reachability* is not first-order expressible and use the same proof idea here!

Exercise 1.7: 1. Let φ be a formula of first-order logic where (true) equality does not occur. Prove the following: For all $n \in \mathbb{N}$, if φ has a model of size n, then it also has a model of size $n + 1$.

2. Give a formula φ without equality such that φ has a model of size n for each $n \geq 3$, and no models of size $n < 3$.

3. Give a (satisfiable) formula ψ such that all models of ψ are of size 3.

4. Argue that adding true equality to the logical language increases expressivity.

[2]Recall that the compactness theorem states that a set of formulas is satisfiable iff every finite subset of it is satisfiable.