VU Discrete Mathematics

Exercises for 16th December 2025

55) Which of the following polynomials are primitive over $\mathbb{Z}_3$?

$$x^3 + x^2 + x + 1, \quad x^3 + x^2 + x + 2, \quad x^3 + 2x + 1.$$

56) Show that the repetition code of order r (*i.e.* each bit of the original word is sent r times) is a linear code. Determine a generator matrix and a check matrix of this code.

57) Let $g(x) = x^3 + 2$ be the generator polynomial of a cyclic $(9, 6)$ linear code over $\mathbb{Z}_3$. Determine the check polynomial $h(x)$ of the code and find out whether every code word with Hamming weight 1 can be chosen as coset leader.

58) Let $\mathcal{L}^{[d,k]}$ denote the set of words over the alphabet $\{a, b\}$ that contain exactly k occurrences of b such that between any two consecutive b's there are no more than d many a's. Find a specification of $\mathcal{L}^{[d,k]}$ as combinatorial construction and use generating functions to compute the number of words in $\mathcal{L}^{[d,k]}$ having exactly n letters.

Remark: The result is an alternating sum which cannot be simplified further.

59) Prove: Given a set $\{a_1, a_2, \ldots, a_{n+1}\} \subseteq \{1, \ldots, 2n\}$ with $n + 1$ elements, then there exist i, j with $i \neq j$ such that a_i divides a_j.

60) Given a set A with n elements and $B = \{A_1, A_2, \ldots, A_n\} \subseteq 2^A$. Prove that there exists an injective mapping $f : B \to A$ such that $f(A_i) \in A_i$ for all $i \in \{1, 2, \ldots, n\}$ if and only if for all $I \subseteq \{1, 2, \ldots, n\}$ the cardinality of $\bigcup_{i \in I} A_i$ is at least equal to the cardinality of I.