

Exercise 1.7

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Setting up R

```
testscores <- data.frame(hours = c(5,7.5,3,10.5,6,8,4,9), score = c(72,70,55,80,68,75,60,78))
```

(a)

Estimate the relationship between the test score as a dependent variable and the hours studied as an explanatory variable by computing the corresponding least-squares estimators β_1 and β_2 .

β_1 is a measure of how much the dependent variable changes when you change the independent variable by one unit.

β_2 is the “starting point” for the relationship between the independent and dependent variables, for example if β_2 is 5 that means when the independent variable is zero the dependent variable is 5.

```
model <- lm(score ~ hours, data = testscores)
summary(model)
```

```
##
## Call:
## lm(formula = score ~ hours, data = testscores)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.5526 -1.9030 -0.7305  0.9360  7.2695
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  49.2857     3.7561  13.121 1.21e-05 ***
## hours        3.0889     0.5329   5.797 0.00115 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.629 on 6 degrees of freedom
## Multiple R-squared:  0.8485, Adjusted R-squared:  0.8232
## F-statistic: 33.6 on 1 and 6 DF, p-value: 0.001155
```

The depended variable is score and the explanatory variable is hours.

I created a data frame (testscores) containing these variables.

I used the `lm()` function to fit a linear regression model.

The `summary()` function provides the summary of the linear regression model, which includes the estimated coefficients β_1 and β_2 .

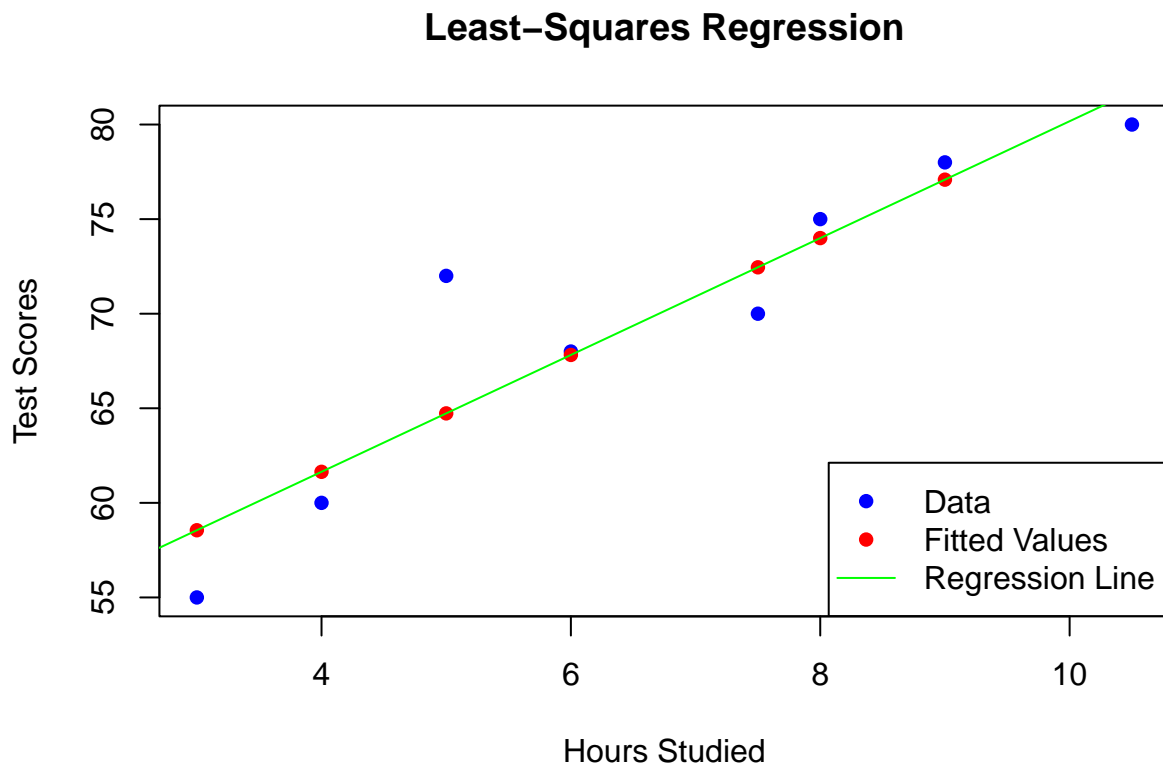
The coefficient for hours represents the estimate for β_1 and the intercept term represents the estimator of β_2 .

$\beta_1 = 3.0889$
 $\beta_2 = 49.2857$

(b)

Produce a plot of your results: plot the least-squares regression line together with the points (x_i, y_i) and (x_i, \hat{y}_i) . Comment on the “direction” of the relationship, i.e., the sign of the estimated slope parameter β_2

```
fitted_values <- predict(model)
plot(testscores, xlab = "Hours Studied", ylab = "Test Scores", main = "Least-Squares Regression",
col = "blue", pch = 16)
points(testscores$hours, fitted_values, pch = 16, col = "red")
abline(model, col = "green")
legend("bottomright", legend = c("Data", "Fitted Values", "Regression Line"),
col = c("blue", "red", "green"), pch = c(16,16, NA), lty = c(NA, NA, 1))
```



Since β_2 is positive it indicates a positive relationship.

(c)

What is the expected increase in the test score if the student studied an extra hours?

See β_1 : If a student studied an extra hour he/she would gain 3.0889 Testpoints.