

<b>Examination for “Logic and Computability”</b> <b>January 20, 2020</b> — first Exam for WS20/21		
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**Task 1:**

Formalize the following sentence in classical logic

“If not everybody gets a covid vaccine, there is at least someone who does not get a covid vaccine.”

Exhibit either a sequent calculus or a natural deduction derivation for the resulting formula (in case it is valid) or an interpretation that falsifies it (in case it is not valid).

**Task 2:**

Consider the following formulas of classical logic:

- (a)  $\forall x \forall y (P(x, y) \rightarrow P(y, x))$
- (b)  $\forall x \forall y \forall z (P(x, y) \wedge P(y, z) \rightarrow P(x, z))$
- (c)  $\forall x \neg P(x, x)$

Does  $(a), (b) \models (c)$  hold? Motivate your answer.

**Task 3:**

Let  $A$  be the set

$$\{x \mid \Phi_x(x) \downarrow \text{ and } \Phi_x(x) > 2\}$$

Is  $A$  recursive, r.e. but not recursive, or none of them? (provide a formal proof)

**Task 4:**

Let  $I$  be an infinite set. Prove the following statement

” $I$  is recursively enumerable if and only if  $I = R(f)$  for some  $f$  that is total, computable, and 1-1.”  
(Recall that  $R(f)$  denotes the range of the function  $f$ .)

**Task 5:**

Let  $G$  be the modal formula  $\Diamond \Box A \vee \Diamond \neg A$ . Prove or refute:

- (1)  $G$  is valid in every reflexive frame.
- (2) If  $\mathcal{F} \models G$  for a frame  $\mathcal{F}$ , then  $\mathcal{F}$  is reflexive.

**Task 6:**

Show by Robinson-resolution that the clause set  $\{C, D, E\}$  is unsatisfiable, where

$$\begin{aligned} C &= \neg q(f(x), y) \vee \neg q(y, f(z)) \\ D &= q(x, y) \vee q(y, x) \vee p(x, g(z)), \\ E &= \neg p(x, g(x)). \end{aligned}$$

Specify all used factors, MGUs, and unified literals.

**Task 7:**

Use the proof of the ADRF theorem to prove that the function  $f(g(x), 2) + 3$  is arithmetically definable, if  $f$  and  $g$  are arithmetically definable. *Hint:* Specify the witnessing formula, following slide 23 of the last set of slides (on incompleteness).