

Examination for “Logic and Computability”
January 20, 2020 — first Exam for WS20/21

Matrikelnummer	FAMILY NAME	First Name
----------------	-------------	------------

Task 1:

Formalize the following sentence in classical logic

“If not everybody gets a covid vaccine, there is at least someone who does not get a covid vaccine.”

Exhibit either a sequent calculus or a natural deduction derivation for the resulting formula (in case it is valid) or an interpretation that falsifies it (in case it is not valid).

Task 2:

Consider the following formulas of classical logic:

- (a) $\forall x \forall y (P(x, y) \rightarrow P(y, x))$
- (b) $\forall x \forall y \forall z (P(x, y) \wedge P(y, z) \rightarrow P(x, z))$
- (c) $\forall x \neg P(x, x)$

Does (a), (b) \models (c) hold? Motivate your answer.

Task 3:

Let A be the set

$$\{x \mid \Phi_x(x) \downarrow \text{ and } \Phi_x(x) > 2\}$$

Is A recursive, r.e. but not recursive, or none of them? (provide a formal proof)

Task 4:

Let I be an infinite set. Prove the following statement

“ I is recursively enumerable if and only if $I = R(f)$ for some f that is total, computable, and 1-1.”
 (Recall that $R(f)$ denotes the range of the function f .)

Task 5:

Let G be the modal formula $\Diamond \Box A \vee \Diamond \neg A$. Prove or refute:

- (1) G is valid in every reflexive frame.
- (2) If $\mathcal{F} \models G$ for a frame \mathcal{F} , then \mathcal{F} is reflexive.

Task 6:

Show by Robinson-resolution that the clause set $\{C, D, E\}$ is unsatisfiable, where

$$\begin{aligned} C &= \neg q(f(x), y) \vee \neg q(y, f(z)) \\ D &= q(x, y) \vee q(y, x) \vee p(x, g(z)), \\ E &= \neg p(x, g(x)). \end{aligned}$$

Specify all used factors, MGUs, and unified literals.

Task 7:

Use the proof of the ADRF theorem to prove that the function $f(g(x), 2) + 3$ is arithmetically definable, if f and g are arithmetically definable. *Hint:* Specify the witnessing formula, following slide 23 of the last set of slides (on incompleteness).