

Einführung in Wissensbasierte Systeme WS 2012/13, 3.0 VU, 184.737

Exercise Sheet 2 - Logic, Part 2

For the presentation part of this exercise, mark your solved exercises in **TUWEL** until Friday, November 16, 13:00 CET. Be sure that you tick only those exercises which you can solve and explain on the blackboard!

Exercise 1 (2 pts.): Translate the following arguments into propositional logic and show by TC0 that the argument is either correct or else extract an interpretation from the tableau showing that the argument is not correct.

- (a) The Joker is incarcerated in Arkham Asylum (I) if he was caught by the police (P) or by Batman (B). He is not incarcerated in Arkham Asylum. Therefore, he was neither caught by the police nor by Batman. (1pt.)
- (b) If you work as a physician (P), you must have studied medicine (M) or forged a diploma (F). You do not work as a physician. Therefore, you neither studied medicine nor forged a diploma. (1pt.)

Exercise 2 (1 pt.): Let x be a variable, $\phi(x)$ be a formula containing x free, and ψ be a formula not containing x free. Show:

If $\models \psi \rightarrow \phi(x)$, then $\models \psi \rightarrow \forall x \phi(x)$.

Exercise 3 (1 pt.): Translate the following argument into predicate logic and establish the validity or invalidity of the result:

One person being the father of another person is an asymmetric relation (i.e., for all persons x and y , if x is father of y then y is not father of x). Therefore, no person is his own father.

Exercise 4 (2 pts.): Consider the following statements:

- (i) Every politician is bribed by someone.
- (ii) There is someone who bribes all politicians.

Represent (i) and (ii) by first-order formulas ϕ and ψ , respectively, using the unary predicate $p(x)$ expressing that an object is a politician and the binary predicate $bribes(x, y)$ expressing that x bribes y .

- (a) Show using TC1 that $\psi \models \phi$ holds. (1 pt.)
- (b) Show that $\phi \models \psi$ does not hold. (1 pt.)

Exercise 5 (2 pts.): An *enthymeme* is an argument where not all premisses are explicitly stated or where the conclusion is not mentioned. That is, premisses can be left out because they can be assumed to be generally known or for rhetoric reasons where problematic premisses, which the listener may doubt, are suppressed. The suppression of a conclusion is often done because of not wanting to give bold comments explicitly, rather to let the listener make this conclusion himself (this is also referred to as engaging in *innuendo*).

Translate the following enthymemes into logical symbolism and supply missing premisses or conclusions in order to obtain valid arguments. Justify your results.

(a) We have no proof that he committed the crime. Therefore he must be acquitted. (1 pt.)

(b) General G.I. Brassbottom: "Can Roger Ramjet sorten out the mess?"
Lance Crossfire: "Well, he is too soft-hearted." (1 pt.)

Exercise 6 (2 pts.): Let L be a language of propositional logic where formulas are build only from Boolean variables using the primitive connectives $\neg, \wedge, \vee, \rightarrow$, and \leftrightarrow (thus \top and \perp are not part of the language). Furthermore, let A be a formula of L containing no occurrence of \neg and let B be any formula of L .

Show the following propositions:

(a) Let I be an interpretation assigning to all atomic formulas of A the truth value 1. Then, A is true under I . (1pt.)

(b) If $\models B \leftrightarrow \neg A$, then B contains at least one occurrence of \neg . (1pt.)

Hint: Show Item (a) by induction on the logical complexity of A (i.e., on the number of occurrences of logical connectives in A). Show Item (b) using Item (a).